APPLICATION OF THE SEISMIC QUALITY FACTOR VERSUS
OFFSET AND AZIMUTH (QVOA) FOR FRACTURED
RESERVOIR CHARACTERIZATION

by

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science (Engineering Geophysics).

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Fracture characterization of a reservoir is very important because the presence of fractures determines the flow of hydrocarbons during production. Accurate modeling of the fracture network can help in optimizing the production of the reservoir. Fractures affect the amplitudes of the seismic waves, therefore, seismic attenuation is used to determine their characteristics. Here, I use a new technique called QVOA which involves the evaluation of the seismic attenuation and its variation with offset (O), and azimuth (A). Variation of seismic attenuation from QVOA methodologies can help in determining fracture characteristics where conventional methods fail.

The QVOA method is a two step process where seismic attenuation is computed first and then its variation is determined with respect to offset and azimuth. I compute seismic attenuation using four different techniques based on the spectral ratio and frequency-shift methods. The variation with respect to offset and azimuth is determined using approximate method of sectors (ASM) and approximate truncate method (ATM). Orientation and the B-gradient of the fracture characteristics are obtained using this QVOA technique.

I apply this QVOA technique to 3D seismic data acquired over the Gulf of Mexico region where the target is a naturally fractured carbonate reservoir. Fracture orientation in the reservoir region obtained using the QVOA technique are verified with well log data. Finally, a comparative analysis of different techniques of seismic attenuation computation is provided, where the frequency-shift methods perform better than the spectral ratio method, and are more stable in the presence of noise. Variation of the B-gradient versus the azimuth suggests the presence of attenuation anisotropy in this reservoir.
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Karla Vizuett
# TABLE OF CONTENTS

ABSTRACT ................................................................. iii

ACKNOWLEDGMENTS ....................................................... iv

LIST OF FIGURES ............................................................. viii

LIST OF TABLES .............................................................. xiii

LIST OF SYMBOLS ........................................................... xiv

LIST OF ABBREVIATIONS .................................................... xvi

CHAPTER 1 INTRODUCTION .................................................. 1

CHAPTER 2 NATURALLY FRACTURED RESERVOIR (NFRs) .................. 3

2.1 Classification of Naturally Fractured Reservoirs (NFRs) .............. 5

2.2 Geophysical Methods for NFRs Characterization ...................... 7

CHAPTER 3 INTERPRETATION AND VELOCITY MODEL ..................... 11

3.1 Geology of the Area of Interest ...................................... 11

3.2 Database .................................................................... 14

3.3 Interpretation and Velocity Cube ..................................... 15

CHAPTER 4 QUALITY FACTOR AND ATTENUATION IN SEISMIC TRACES .. 19

4.1 Seismic Attenuation in Isotropic Media .............................. 21

4.1.1 Phase and Group Attenuation Coefficients .................... 21

4.1.2 Isotropic Media ..................................................... 22

4.1.2.1 Angle $\xi$: Small and Moderate Variation .............. 23

4.1.2.2 Angle $\xi$: Large values .................................... 24
4.2 Methods Used to Calculate Attenuation in Surface Seismic Data . . . . . . . 25
  4.2.1 Spectral-Ratio Method (SR) . . . . . . . . . . . . . . . . . . . . . . . . 26
  4.2.2 Centroid Frequency Shift Method (CFS) . . . . . . . . . . . . . . . . . 29
  4.2.3 Peak Frequency Shift Method (PFS) . . . . . . . . . . . . . . . . . . . 30
  4.2.4 Dominant and Central Frequency Shift Method (DCFS) . . . . . . . . 30

4.3 Synthetic Model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 32

4.4 Real Data . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 33

CHAPTER 5 SEISMIC QUALITY FACTOR VERSUS OFFSET AND AZIMUTH . . 41
  5.1 Theory . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 42
  5.2 Numerical Methods to Solve QVOA . . . . . . . . . . . . . . . . . . . . . . . . 48
    5.2.1 Approximate Method of Sectors (ASM) . . . . . . . . . . . . . . . . . 49
    5.2.2 Approximate Truncate Method (ATM) . . . . . . . . . . . . . . . . . . 51

5.3 Gradient B . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 52

CHAPTER 6 QVOA PROGRAM . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 57
  6.1 QVOA Approximate Truncate Method (ATM) . . . . . . . . . . . . . . . . . 57
  6.2 QVOA Approximate Method of Sectors (AMS) . . . . . . . . . . . . . . . . 66

CHAPTER 7 QVOA-APPLICATION TO REAL DATA . . . . . . . . . . . . . . . . . 67
  7.1 Well-3 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 72
    7.1.1 Fracture direction: Gather 1256-1369 . . . . . . . . . . . . . . . . . . 75
    7.1.2 Fracture direction: Gather 1256-1372 . . . . . . . . . . . . . . . . . . 77
    7.1.3 Fracture direction: Gather 1253-1369 . . . . . . . . . . . . . . . . . . 80
    7.1.4 Fracture direction: Gather 1253-1369 . . . . . . . . . . . . . . . . . . 82
    7.1.5 B-gradient: Gather 1256-1369 . . . . . . . . . . . . . . . . . . . . . . 86
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 2.1</td>
<td>Relative positions of fractures reservoir types based on petrophysical properties of rock fractures. Modified from</td>
<td>6</td>
</tr>
<tr>
<td>Figure 2.2</td>
<td>Sketch of an HTI model. With two vertical symmetry planes, called symmetry axis plane and the isotropy plane</td>
<td>9</td>
</tr>
<tr>
<td>Figure 2.3</td>
<td>Sketch of an HTI model. With two vertical symmetry planes, called symmetry axis plane and the isotropy plane</td>
<td>10</td>
</tr>
<tr>
<td>Figure 3.1</td>
<td>An Inline and a Xline from the 3D seismic cube</td>
<td>15</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>Five surfaces where created for this project. The second surface from bottom to top, corresponds to the top of the reservoir. Wells with their velocity log from synthetic seismograms are also displayed. The color bar correspond to the depth of the surfaces.</td>
<td>16</td>
</tr>
<tr>
<td>Figure 3.3</td>
<td>Eight synthetic seismograms were generated for this project. Velocity logs obtained from the synthetic seismograms are displayed.</td>
<td>17</td>
</tr>
<tr>
<td>Figure 3.4</td>
<td>An Inline and a Xline showing the velocities in m/s from the velocity cube created for this project. This velocity model was created using the synthetic seismograms from well logs (density and sonic logs), tops from the wells and horizons. Velocity curves from the wells are also displayed. The color bar represents the velocity values of the velocity cube.</td>
<td>18</td>
</tr>
<tr>
<td>Figure 4.1</td>
<td>Spectral -Ratio: The incident wave is affected by instrument and medium response</td>
<td>27</td>
</tr>
<tr>
<td>Figure 4.2</td>
<td>Select window over the amplitude spectra where the ratio of the amplitudes is constant</td>
<td>29</td>
</tr>
<tr>
<td>Figure 4.3</td>
<td>Synthetic model composed of three layers with different values of quality factors (Q). The second layer corresponds to the interval of interest</td>
<td>33</td>
</tr>
<tr>
<td>Figure 4.4</td>
<td>Synthetic trace extracted from the synthetic CMP gather that represents the model in the Figure 4.3</td>
<td>34</td>
</tr>
<tr>
<td>Figure 4.5</td>
<td>Values obtained after calculating the attenuation with each method used in this project</td>
<td>35</td>
</tr>
</tbody>
</table>
Figure 4.6 The way to apply the different methods in real data is: selecting two windows in time at the top and at the bottom of the interval of interest.

Figure 4.7 Amplitude spectrum from the top (blue line) and bottom (red line) of the interval of interest. The yellow line corresponds to the ratio between the two amplitude spectrum.

Figure 4.8 A window in frequency must be selected from the amplitude spectrum.

Figure 4.9 Values of attenuation from spectral ratio method, selecting different sizes of windows in frequency over the amplitude spectrum.

Figure 4.10 Values of attenuation from centroid frequency shift method, selecting different sizes of windows in frequency over the amplitude spectrum.

Figure 4.11 Values of attenuation from dominant and centroid frequency shift method, selecting different sizes of windows in frequency over the amplitude spectrum.

Figure 4.12 Two layer model for travel times calculation.

Figure 5.1 $B$ gradient vs azimuth.

Figure 5.2 Curves representing the variation of the $B$ gradient with azimuth and Vs/Vp ratio (figure in the top), crack filling fluids (figure in the middle) and crack densities (figure in the lower part).

Figure 6.1 Common Mid Point gather (CMP).

Figure 6.2 Superbin. The files that are used for this project use 3X3 bins, in total 9 bins compose a superbin in this case.

Figure 6.3 Five surfaces where created for this project. The second surface from bottom to top, corresponds to the top of the reservoir. Wells with velocity logs are also displayed. The bar color represents the depths from the surfaces.

Figure 6.4 An Inline and Xline showing the velocities from the velocity cube created for this project. This velocity model was created using the synthetic seismograms from well logs (density and sonic logs), tops from the wells and the horizons showed in Figure 6.3.

Figure 6.5 To upload a file into the program, it must be by two columns: number of the trace, and the time values from the top and the bottom of the interval of interest.
Figure 6.6  Example of the graph where top and bottom values must be picked. . . . 62

Figure 6.7  Zoom of the anterior figure. In this second figure a cross that is activated when pressing the bottom “enter” is appreciated. . . . . . . . . 63

Figure 7.1  Common mid point gathers (CMP) located over the top of the reservoir. The gathers were located using the central coordinate of the superbin (white dots over the surface). . . . . . . . . . . . . . . . . . . . . . . . . 69

Figure 7.2  Wells displayed over the top of the reservoir . . . . . . . . . . . . . . . . 70

Figure 7.3  Well logs plots of fractures are displayed over the top of the reservoir. The seismic attribute “edge evidence” was calculated over the surface. The color bar represents the values of the “edge evidence” attribute. . . 71

Figure 7.4  Results from QVOA equation. The parameter b must be carefully interpreted. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 72

Figure 7.5  Base-map that correspond to Well-3. Location of gathers are represented with black fill-in. The rose graph corresponding to the well tadpoles is displayed in the location of the well. . . . . . . . . . . . . . 75

Figure 7.6  General rose graph of fractures within the reservoir from the tadpoles in Well-3. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 76

Figure 7.7  The first gather processed is colored with red. . . . . . . . . . . . . . . . . 77

Figure 7.8  ATM (left) and AMS (right) methods, using spectral ratio for attenuation determination . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 78

Figure 7.9  ATM (left) and AMS (right) methods, using centroid frequency shift for attenuation determination . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 79

Figure 7.10 ATM (left) and AMS (right) methods, using peak frequency shift for attenuation determination . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 80

Figure 7.11 ATM (left) and AMS (right) methods, using dominant and centroid frequency shift for attenuation calculation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 81

Figure 7.12 The second gather processed is colored with red. . . . . . . . . . . . . . . . . 82

Figure 7.13 ATM (left) and AMS (right) methods, using spectral ratio for attenuation determination. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 83
Figure 7.14 ATM (left) and AMS (right) methods, using centroid frequency shift for attenuation determination
Figure 7.15 AMS method, using peak frequency shift for attenuation determination
Figure 7.16 ATM (left) and AMS (right) methods, using dominant and centroid frequency shift for attenuation calculation
Figure 7.17 The third gather processed is colored with red.
Figure 7.18 ATM (left) and AMS (right) methods, using spectral ratio for attenuation determination.
Figure 7.19 ATM (left) and AMS (right) methods, using centroid frequency shift for attenuation determination
Figure 7.20 AMS method, using peak frequency shift for attenuation determination
Figure 7.21 ATM (left) and AMS (right) methods, using dominant and centroid frequency shift for attenuation calculation
Figure 7.22 The fourth gather processed is colored with red.
Figure 7.23 ATM (left) and AMS (right) methods, using spectral ratio for attenuation determination.
Figure 7.24 ATM (left) and AMS (right) methods, using centroid frequency shift for attenuation determination
Figure 7.25 ATM (left) and AMS (right) methods, using peak frequency shift for attenuation determination
Figure 7.26 ATM (left) and AMS (right) methods, using dominant and centroid frequency shift for attenuation calculation
Figure 7.27 Gradient from the gather 1256-1369 obtained from the four different methods to determine attenuation.
Figure 7.28 $\epsilon_Q$ and $V_s/V_p$ obtained from the B gradient for each method. "B perp is the value for $B^\perp$.
Figure 7.29 Gradient from the gather 1256-1372 obtained from the four different methods to determine attenuation.
Figure 7.30  $\epsilon_Q$ and $V_s/V_p$ obtained from the B gradient for each method. $B_{\perp}$ is the value for $B_{\perp}$. 98

Figure 7.31  Gradient from the gather 1253-1369 obtained from the four different methods to determine attenuation. 99

Figure 7.32  $\epsilon_Q$ and $V_s/V_p$ obtained from the B gradient for each method. $B_{\perp}$ is the value for $B_{\perp}$. 99

Figure 7.33  Gradient from the gather 1256-1372 obtained from the four different methods to determine attenuation. 100

Figure 7.34  $\epsilon_Q$ and $V_s/V_p$ obtained from the B gradient for each method. $B_{\perp}$ is the value for $B_{\perp}$. 100

Figure 7.35  Data with (right column) and without (left column) filtering for ATM (up) and AMS (down) methods. Spectral ratio method. 101

Figure 7.36  Data with (right) and without (left) filtering for dominant and centroid frequency shift for ATM method. 102

Figure 7.37  Gathers represented by points, over the surface of the top of the reservoir of interest. 103

Figure 7.38  Top of the reservoirs surface with the rose diagrams from the QVOA analysis. 103

Figure 7.39  Well-3 final fracture orientations from QVOA. 104
Table 2.1 Classification of Naturally Fractured Reservoirs .................. 5
Table 3.1 Production in the area of interest ................................. 14
Table 7.1 Information available for the application of the QVOA ........ 68
LIST OF SYMBOLS

Attenuation ......................................................... $\alpha$
Gradient ................................................................. $B$
Intercept ................................................................. $A$
Quality Factor .......................................................... $Q$
Stiffness Matrix .......................................................... $C$
Stiffness Tensors ......................................................... $c_{ij}$
Peak strain energy ....................................................... $E$
Energy loss per cycle ................................................... $-\delta E$
Velocity ................................................................. $c$
Angular Frequency ...................................................... $w$
Frequency ............................................................... $f$
Distance ................................................................. $x$
Time ................................................................. $t$
Inhomogeneous Angle .................................................. $\xi$
Normalized Phase Attenuation Coefficient ................................ $A$
Propagation Vector Real Part of the Seismic Wave ................. $k^R$
Attenuation Vector Imaginary Part of the Seismic Wave .......... $k^R$
Reflection ................................................................. $G$
Average Group Attenuation ........................................... $K_g$
Distance between two receivers ......................................... $l$
Velocity Group \( V_g \)

Real Part of the Medium Velocity \( V \)

Density- Normalized Stiffness Tensor \( a_{ij} \)

Dominant Frequency \( f_m \)

Central Frequency \( f_c \)

Amplitude Spectra from the top of the interval \( f_t \)

Amplitude Spectra from the bottom of the interval \( f_b \)

Variance \( \sigma_i^2 \)
LIST OF ABBREVIATIONS

Quality Factor Versus Offset and Azimuth ........................................ QVOA
Amplitude Versus Offset ........................................................................ AVO
Amplitude Versus Offset and Azimuth .................................................... AVOA
Transversely isotropic media ................................................................. TI
Horizontal transverse isotropy ............................................................... HTI
Vertical transverse isotropy ................................................................. HTI
Normal Moveout .................................................................................... NMO
Root Mean Square .................................................................................... rms
Dip Moveout ............................................................................................ DMO
Common Midpoint .................................................................................. CMP
Signal to Noise Ratio ............................................................................... SNR
Spectral Ratio .......................................................................................... SR
Centroid Frequency Shift ......................................................................... CFS
Peak Frequency Shift ............................................................................... PFS
Dominant and Central Frequency Shift .................................................. DCFS
Fractures impact the permeability of a reservoir and their characterization is important for characterizing the reservoir. Variation of velocity with azimuth, and amplitude variation with offset and azimuth (AVOA) are used to determine fracture parameters (fracture orientation, density, etc.) (Rueger and Tsvankin, 1997). Similarly, attenuation variation with offset and azimuth (QVOA) can provide better approximations of fracture parameters. Seismic attenuation is an inelastic phenomena and its response is sensitive to different physical properties in the subsurface. These physical properties are the changes in the water saturation, the clay content, porosity, pore geometry, permeability, microfracturing, and pressure (Behura, 2009). This is used to determine properties such as lithology, pore structure, fractures, fluid saturation (oil, brine, water or gas), and mobility of the fluids (Bratton et al., 2006). Attenuation can also be used as a direct hydrocarbon detector (Behura, 2009).

The preferential flow of fluids (Bratton et al., 2006) and orientation of fractures to pores in the host rock, lead to azimuthally varying attenuation. The QVOA method relates the variation of attenuation with offset and azimuth. And, it is based on a model by Chichinina et al. (2006) where the azimuthal anisotropic attenuation in a homogeneous transversely isotropic (TI) is described by Schoenberg’s linear slip model (Schoenberg and Sayers, 1995) with complex-valued normal and tangential fracture stiffnesses. The background medium is taken to be isotropic and the fluid flow between microcracks is the only source of attenuation according to Hudson’s model (Carcione et al., 2012).

The QVOA method is similar to the AVOA method for PP-reflection (Rüger, 1997) since it includes intercept, gradient and fracture orientation. It also incorporates Q-anisotropy by using complex-valued velocity which defines attenuation and quality factors uniquely (Carcione et al., 2012). This method operates on CMP gathers and provides the attenuation
of target interval, which subsequently gives an approximation of the fracture orientation (Chichinina et al., 2006).

The application of the QVOA method requires two steps: the computation of attenuation coefficient with respect to offset, and the computation of the variation of attenuation with respect to azimuth. I calculate the attenuation coefficient from seismic traces individually by selecting an interval of interest which potentially represents the reservoir. Four techniques are used to calculate the attenuation coefficients: spectral ratio (Johnston and Toksöz, 1981), centroid frequency shift (Quan and Harris, 1993), peak frequency shift (Zhang and Ulrych, 2002), and dominant and central frequency shift (Li et al., 2015b). The second step in the QVOA method application is the determination of the variation of the attenuation versus azimuth. Two methods are used to accomplish this task: approximate method of sectors (ASM), and approximate truncate method (ATM) (Sabinin, 2013).

In this thesis, naturally fractured reservoirs (NFRs) and their classification, as well as the characteristics of the field, and reservoir of interest are described. An important step in the attenuation calculation is a velocity model that must be obtained, and is presented and explained in detail. The velocities obtained are used to calculated travel times, which are necessary for determining the attenuation. The four methods used for calculating the attenuation are described thoroughly as well. The theory of the QVOA method, and the techniques used to implement it are also explained. The QVOA method was applied on 3D seismic data acquired over the Gulf of Mexico region and the results are verified with well information.
Naturally fractured reservoirs (NFRs) are very important for the oil industry. A naturally fractured reservoir is a hydrocarbon-bearing formation that contains fractures (planar discontinuities) created by nature, due to diastrophism (folding and faulting) and volume shrink (Aguilera, 1980). They are distributed as a consistent network of various degrees of fractures throughout the reservoir (Mazzullo and Chilingarian, 1996). Almost all hydrocarbon reservoirs are affected by natural fractures. Some of the important fractured reservoirs may be found in cherts, shales, limestones, siltstones, sandstones, igneous and metamorphic rocks (Aguilera, 1980).

The complexity of these reservoirs creates the need for geoscientists and petroleum engineers to find new ways to understand and handle them by integrating all geoscience and engineering drilling completion reservoir concepts. In real life this integration barely happens, creating a gap in the conception of the fracture network of the field. The fractures in the NFRs have the main role in the flow of fluids during the extraction of hydrocarbons. The poor understanding of the reservoir’s fracture network can cause problems when a well is being drilled, translated into losses (wasting resources, for example: infill drilling) which will affect the amount of produced oil.

It is clear now that the integrated understanding of the reservoir as a whole is very important. This understanding of the fracture network of the reservoir must start from the beginning of exploration to have an early assessment of the reservoir’s potential. During the first stages of the exploration, the study of outcrops by geologists, and the study of gravity and magnetic studies by geophysicists are a first big important step to characterize the reservoir in a regional sense. This will help to determine the state of stress in the zone of exploration. The state of stress largely dictates whether fractures are open enough to
conduct reservoirs fluids (Bratton et al., 2006).

After the regional characterization, when the first seismic survey is available, geophysicists and seismic interpreters are in charge of giving the first general approximation of the fracture network in a local sense. This local characterization of the field will let the first exploration well to be drilled. Having exploration wells, cores and well logging (formation logs, borehole seismic surveys, drillstem tests, initial flow test, etc) will help to test and calibrate previous approximations about the direction and type of the fractures in the oilfield. At this point, most of the models generated still have a big uncertainty. Other exploration wells must be drilled to reduce that uncertainty and delimit the reservoir.

In the next stages, during production, the optimization of well locations and paths are highly dependent on the knowledge of the fracture systems in the oilfield. During this stage, more sophisticated techniques are required to get important parameters, such as: field rates and recoveries, and the prediction of economic depletion of the field, that will also depend on the models generated during the reservoir characterization. These sophisticated techniques could be: well-test data, production data, and passive and time-lapse seismic data. The extrapolation and interpolation of these parameters (including the fracture systems) are an issue in any stage creating uncertainty, but every time a new well is drilled, or new information is acquired, uncertainty would be updated.

Not accounting for fractures during the reservoir modeling is ill-advised for all the reasons mentioned before. Typically in the oil industry, the practice of denying the presence of fractures occurs. This is because having fractures creates a lot of technical problems and takes a lot of time to study. "Industry people have all the time there is".

The NFRs are classified to have a better idea of what type of reservoir is being modeled during the reservoir characterization. The type of reservoir will largely dictates what type of techniques are needed to be used during the characterization. In the next section this classification is discussed.
2.1 Classification of Naturally Fractured Reservoirs (NFRs)

Classification of NFRs is based on the influence of fractures on the fluid flow through the reservoir and the interaction between the relative porosity and permeability contributions (Donnez, 2012). Table 2.1 shows this classification giving 6 different types of NFRs.

Table 2.1: Classification of Naturally Fractured Reservoirs (Donnez, 2012)

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>Fractures provide both the porosity and permeability elements. Drainage areas per well are large.</td>
</tr>
<tr>
<td>Type 2</td>
<td>With low porosity and low permeability in the matrix. Fractures provide the essential pathway for productivity.</td>
</tr>
<tr>
<td>Type 3</td>
<td>Have high porosity and may produce without fractures. In these reservoirs fractures provide added permeability and define anisotropy.</td>
</tr>
<tr>
<td>Type M</td>
<td>Have high matrix porosity and permeability, so open fractures can enhance permeability.</td>
</tr>
<tr>
<td>Type 4</td>
<td>Fractures add no significant additional porosity and permeability, but instead are usually barriers to flow creating baffles, barriers and compartments (flow and saturation)</td>
</tr>
<tr>
<td>Type G</td>
<td>Created for unconventional fractured gas reservoirs, such as CBM, and fractures gas condensate reservoirs (Most of this type fall within or near the type 2 reservoir).</td>
</tr>
</tbody>
</table>

In the type one reservoir, fractures are responsible for both the porosity and permeability. These reservoirs need few wells and a very good static description because their production is highly variable in 4D. Production of water is a huge problem in this type of reservoir. Type 2 has very low porosity so the presence of natural fractures is very important for the hydrocarbon to flow. Because of this, water influx must be monitored and fracture closure must be controlled. Type G is very similar to type 2 but it is focused on unconventional fractured gas reservoirs.

Type 3 has a high porosity which, in this case will give a good permeability. If this type of reservoir has natural fractures, they are going to behave in two ways: 1. Giving extra permeability to the reservoir (which is Type 3) and improved continuity or 2. Complicating fluid flow by forming barriers (which is Type M). Types 3 and M share the same character-
istics with the exception of the behavior of the fractures. In type 4, reservoir porosity and permeability will stay constant even with the presence of fractures.

![Figure 2.1](image)

Figure 2.1: Relative positions of fractures reservoirs types based on petrophysical properties of rock fractures. Modified from (Nelson, 2001)

Figure 2.1 displays the types of reservoirs positioned according to the percentage of total porosity (increasing from left to right) versus percentage of total permeability (increasing from bottom to top). Type 1 is considered as the one with more fracture porosity and fracture permeability. The field that will be analyzed in this thesis corresponds to a reservoir of type 1. Its a massive carbonate structure where natural fractures are responsible for all production as well as for a rapid decline and water breakthrough.

Two definitions of natural fractures are given by (Donnez, 2012) and (Bansal, 2007):

- “A natural fracture is a macroscopic planar discontinuity that results from stresses that exceed the rupture strength of the rock” (Donnez, 2012).

- “An imperfectly welded interface where only traction is continuous, and the displacement field is not continuous across the interface” (Bansal, 2007).
Natural fractures have different properties that can be divided into geometric properties and physical properties, all of them are of a big interest for geoscientist and engineers. The geometric properties or parameters of a fracture in a rock mass are:

- size or length,
- aperture (fracture permeability is proportional to the cube of fracture aperture (Donnez, 2012)),
- fractional area of the fracture walls in contact (fractures could terminate into another or could be infinitely extensive),
- orientation (this parameter helps to control the direction of fluid flow in a reservoir), and
- fracture porosity and morphology (open, deformed, mineral-filled, or vuggy).

The physical properties of the fractures are:

- fracture density, and
- fracture-length density.

The properties of the natural fractures are difficult to measure. Geophysical methods based on seismic data at large and medium scale, help the geoscientists to approximate the values of the properties for a better characterization.

2.2 Geophysical Methods for NFRs Characterization

In order to characterize the fractures, geoscientists apply techniques from small to large scales. The large scale includes magnetic, gravity or seismic methods. The medium scale includes well logs, like pressure testing (providing information on fractures and fracture-related flow), injected tracers, water-composition analysis, and vertical seismic profiles (VSPs)
(from multioffset to multiazimuth). The small scale refers to logs that are based on fracture evaluation techniques using ultrasonic and resistivity borehole imaging and cores.

From the geophysics point of view, fractures affect the velocity, amplitude, and attenuation of the seismic wavelet. That is why seismic techniques are of major interest in the characterization of reservoirs. It is important to understand that these techniques don’t detect individual faults or fractures but the average response. The most important techniques used in seismic for fracture characterization are velocity-anisotropy determination, amplitude variation with offset (AVOA) and azimuth, and normal moveout (NMO) variation with azimuth.

The AVO technique is more robust than velocity based techniques and it has higher vertical resolution. AVO analysis has proven useful in characterizing changes in material properties along a reflector (Minsley et al., 2004). The AVO studies the reflection amplitude, or reflectivity that will depend on the effective elastic properties of the rock (seismic scale). With the AVO method, it is possible to determine the direction of fractures and fracture density. Using the AVO method combined with the NMO technique through weak anisotropy approximation, it is possible to estimate the intercept and gradient, which will lead to the interpretation of the fluid properties in the reservoir.

The NMO technique allows the interpreter to study the direction of the fractures when they are not vertical and when the beds of the rock are dipping. NMO uses an ellipse in the horizontal plane using a minimum of three azimuthal measurements. This ellipse helps the interpreter to analyze the velocities in all azimuthal directions. Because this method is based on the study of velocities, it makes the NMO suffer from velocity related degradation of vertical resolution (Bratton et al., 2006).

The velocity based techniques, such as S-wave splitting, requires two component stacked S-wave surface seismic or VSP data. This technique will analyze the difference between $S^{\parallel}$ and $S^{\perp}$, and will do an estimation of the orientation of $S^{\parallel}$ letting the estimation of parameters such as density and fracture orientation.
When applying any of the geophysical techniques mentioned before, some assumptions must be done to simplify the understanding of the fracture network in the reservoir. Those assumptions are:

1. The medium is considered to be homogeneous.

2. The scale of observation, or seismic wavelength, is much larger than the scale length of the heterogeneities present in the subsurface (Bansal, 2007).

3. The medium is elastic (short time scale, moderate pressure, temperature and very small deformation).

A major assumption is that the parallel vertical cracks are embedded in a homogeneous isotropic matrix, where the symmetry-axis plane is a vertical plane that intersects the crack set perpendicularly, which makes it parallel to the fracture orientation (HTI media) (see Figure 2.2). The isotropy plane is normal to the fracture strike (Rueger and Tsvankin, 1997).

Figure 2.2: Sketch of an HTI model. With two vertical symmetry planes, called symmetry axis plane and the isotropy plane (Rueger and Tsvankin, 1997)
“The majority of existing studies of seismic anisotropy are performed for transversely isotropic medium” (Tsvankin, 2012) and it is broadly used in industry. More realistic models are being considered for every day use in the oil industry. One of those realistic models is the orthorhombic medium, where the presence of two or three mutually orthogonal fracture systems can be analyzed. These models are fully described using three mutually orthogonal planes of mirror symmetry (Figure 2.3). Orthorhombic media is actually considered as the simplest realistic symmetry that can be used for solving geophysical problems. Models with less symmetry must be taken into account to have a better approximation of the parameters of the fractures in the reservoir.

Figure 2.3: Sketch of an HTI model. With two vertical symmetry planes, called symmetry axis plane and the isotropy plane (Rueger and Tsvankin, 1997)

The analysis of attenuation in the seismic data is another way to understand the fractures in the reservoir. Just like AVOA, QVOA uses mostly the same approximations, and assumptions focusing on the seismic attenuation. The QVOA technique is supposed to help the interpreter to find the dominant direction of the fractures in the reservoir.
CHAPTER 3
INTERPRETATION AND VELOCITY MODEL

In this chapter, the stratigraphy and the structural style of the area of interest for this project are explained. Also a representative velocity model is generated.

3.1 Geology of the Area of Interest

The area of the project located in the Gulf of Mexico, is a geologically complex structure that resulted from three main phases of deformation:

1. rifting during the Triassic- Early Cretaceous,

2. compression during the Cretaceous- Miocene,

3. subsidence during the Pliocene, and

4. progradation and aggradation during the Pleistocene.

The center portion of the structure, which is where this project is focused on, shows a large fault-bend fold. The upper and lower blocks of the fault-bend fold structure in the central part has a NW-SE regional orientation converging to NE. Its crest is cut by several normal faults with a dominant orientation between N40W and N20E. Both blocks are delimited in the their west flank by a right-lateral fault with an orientation N and N-NW, with a steep inclination (Garcia Avendano, 2010).

The basement in the area of study consist of the boundary between transitional to thick crust towards the east and thin transitional crust basinwards, possibly composed of palaeozoic metavolcanics rocks, quartzite and schists (Meneses, 1980). The area of study is underlain by thick succession of salt distributed throughout in an irregular manner, reaching 160 meters of thickness in the northeast near the area. The stratigraphy of the Oxfordian is uniform and is composed by (Ricoy-Paramo, 2005):
• anhydrite layer 6m thick with sporadic alternations of arenite,

• sandstone body 53m thick,

• anhydrite layer 82m thick with alternations of shale and calcareous shale,

• carbonate layer consisting of 27m of limestone, argillaceous limestone, with limestone and arenite, and

• terrigenous layer 68m with sporadic alternations of argillaceous limestone.

The Kimmeridgian has sediments with 500 to 600 meters of thickness in the area of study and it consist of (Meneses, 1980):

• carbonates and terrigenous rocks characterized by oolitic and partially dolomitized limestones,

• shales, and

• betonitic mudstones.

The thickness of the Tithonian is 130 meters average. During the Tithonian periods of sea-level highstands happened. Its deposits consist consist mainly of (Ricoy-Paramo, 2005):

• shaly limestones with inclusions of black shale,

• organic rich-mudstone, and

• betonitic mudstones.

During the Cretaceous, deep water carbonates were deposited. These carbonates have been strongly dolomitized through diagenetic processes. The Carbonate rocks are characterized in terms of seismic data by greater velocities and densities compared with siliciclast rocks that are at the top of the carbonates. In the presence of faster velocities the vertical and spatial resolution of the seismic data is giving a low resolution in the area of study
Another characteristic of this area is that the carbonates were exposed subaerially very often, having problems of diagenesis and karstification creating networks of macroscopic pores. The filling of these pores creates back-scattering seismic noise, influencing the final seismic image. Upper Cretaceous is the reservoir of interest and the main interest of this project (Ricoy-Paramo, 2005).

The Tertiary in the area is composed of:

- Cretaceous/Palaeocene boundary is composed of: dolomitized breccia,
- Palaeocene to Oligocene section consisting of shales,
- Eocene-Oligocene sequence consisting of conglomerates deposited as turbidity currents in submarine fans and bentonitic greenish-grey shales with pyrite grains,
- Miocene is composed of alternating sandstones and bluish grey, fossiliferous shales, probably deposited as submarine fans in the bathyal zone,
- Cascadian composed of grey fossiliferous shales with abundant intercalations of micaeous quartz sandstones deposited in a platform environment, and
- Pleistocene composed of prograding and aggradational sequences of alternating shales and sandstones with occasional conglomerates.

The history of production of the field is summarized in Table 3.1. In general the production in the reservoir that consists of heavy crude oil of 19-22 American Petroleum Institute (API), have been dropping in the last nine years. The injection of Nitrogen to force the reservoir to fluid faster is suggested as one of the main reasons of the declination of the reservoir. The production of gas from the reservoir nowadays is complicated because is contaminated with the presence of nitrogen. The oil zone is approximately 1,000 meters thick. As it was discussed before, in this type of reservoir fractures gives the porosity and the permeability which in this case is ranging from 1 millidarcy (md) to more than 1 Darcy. The porosity is considered to be 8 percent on average and water saturation is 21 percent. The presence of
water in the Southeastern part of the reservoir is already affecting the production (Romo, 2015).

Table 3.1: Production in the area of interest (Donnez, 2012)

<table>
<thead>
<tr>
<th>Year</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>From the Upper Cretaceous calcareous breccia interval, 1.16 million barrels per day were produced (180,000 m³/d).</td>
</tr>
<tr>
<td>1995</td>
<td>From the same interval the production rate lowered one million (160,000 m³/d).</td>
</tr>
<tr>
<td>1998</td>
<td>First well that discover a repeated section of Oligocene to Mid Cretaceous strata underlying Oxfordian sediments were drilled. This suggested the occurrence of a major thrust structure. The production from the underlying strata consist on heavy oil 30°API and 22°API. 8 more wells were drilled to confirm the presence of a significant thrust structure.</td>
</tr>
<tr>
<td>2000</td>
<td>Nitrogen was injected to increase the production rate. It reached 1.6 million barrels per day (250,000 m³/d).</td>
</tr>
<tr>
<td>2002</td>
<td>The production increased to 1.9 million barrels per day (300,000 m³/d).</td>
</tr>
<tr>
<td>2003</td>
<td>The production increased again, reaching 2.1 million barrels per day (330,000 m³/d).</td>
</tr>
<tr>
<td>2007</td>
<td>The production decreased by this year to 1.526 million barrels per day (243,000 m³/d).</td>
</tr>
<tr>
<td>2008</td>
<td>Production of 973,668 barrels per day (155,000 m³/d).</td>
</tr>
<tr>
<td>2009</td>
<td>Production of 772,000 barrels per day (123,000 m³/d).</td>
</tr>
<tr>
<td>2012</td>
<td>Production dropped to 408,000 (60,000 m³/d).</td>
</tr>
<tr>
<td>2014</td>
<td>Production of 340,000 barrels per day.</td>
</tr>
<tr>
<td>2015</td>
<td>Production of 256,000 barrels per day.</td>
</tr>
</tbody>
</table>

3.2 Database

The database consists of a 3D ocean bottom cable seismic cube. The information is given in two way time and it is migrated. Its amplitude is registered at 8 bit. It has been processed with filters and gain algorithms to illuminate the structure of interest. No other information about the processing sequence is given. Figure 6.7 displays an Inline and a Xline of the 3D cube.
The database also includes 8 wells with density and sonic logs. No VSP or Checkshots were available for this project. These 8 wells also came with a whole set of stratigraphy well tops.

### 3.3 Interpretation and Velocity Cube

Three Tertiary horizons, which represent changes in the velocities, were correlated. They are not attached to any stratigraphic top from the wells. In Figure 3.2, two of these horizons are displayed. The first horizon, which is not displayed in the figure, corresponds with the sea level of the area.

Other two deeper horizons were correlated and correspond to the top of the Cretaceous of the hanging wall and to the top of the Cretaceous of the footwall of the fault-bend fold structure (See Figure 3.2). Both the hanging wall and the footwall are important reservoirs. This project is focused on the north-east part of the hanging wall. These horizons were used to build the structural framework of the field that were used to calibrate the velocity cube of the area.
Figure 3.2: Five surfaces were created for this project. The second surface from bottom to top, corresponds to the top of the reservoir. Wells with their velocity log from synthetic seismograms are also displayed. The color bar corresponds to the depth of the surfaces.

During the first part of the project, synthetic seismograms of the 8 wells were generated, where density and sonic logs were available in the area of interest. In the wells shown in Figure 3.3, the location of the wells and the velocity logs extracted from the synthetic seismograms are shown. Velocities go from slower (in pink) to faster (in red). The higher velocities are located in the lowest part of the wells. These velocities correspond to the reservoir.

Using the horizons, synthetic seismograms, and well tops, an isotropic velocity model representative of the area was generated. Usually velocity models are also calibrated with Root Mean Square (RMS) velocities that come from the processing of the seismic data. That was not possible in this project because those velocities were not available.

Figure 3.4 shows an Inline and a Xline from the velocity cube. Four major velocity intervals are recognized. The first interval is in the upper part of the section which is colored in light green. Knowing the lithology of the area, the first interval is composed of sands which are increasing their velocity from 1900 m/s to 2200 m/s.
Figure 3.3: Eight synthetic seismograms were generated for this project. Velocity logs obtained from the synthetic seismograms are displayed.

The second interval is just below the first interval and is comprised of low velocities. These velocities correspond to the shales present in the Oligocene age. The third interval is just below the lower velocity zone. In this interval, the velocities are higher than the previous ones. They are in between 2200 m/s to 2600 m/s.

The last interval starts with the top of carbonates that correspond to the reservoir with the highest velocities in the field (from 2700 m/s to 3000m/s). The higher velocities in contrast with the slower velocities distinguish the top of the carbonates.

The velocity model matches well with the velocity curves displayed over the wells located in the area of interest in Figure 3.4. The velocity model was generated using Petrel.

The velocity model is a very important input for the QVOA programs. The velocities from the velocity cube are used to calculate the travel times of the interval of interests and these travel times are used to calculate the interval attenuation in each one of the traces processed. The velocity model must be carefully built, in order to get a good calibration and logical velocities that match with the lithology present in the field.
Figure 3.4: An Inline and a Xline showing the velocities in m/s from the velocity cube created for this project. This velocity model was created using the synthetic seismograms from well logs (density and sonic logs), tops from the wells and horizons. Velocity curves from the wells are also displayed. The color bar represents the velocity values of the velocity cube.
When a seismic wave is generated, energy will propagate indefinitely if the medium is perfectly elastic. In the case of the Earth, it is really not perfectly elastic so the propagation of the waves will be attenuated with time as they travel. Attenuation can be defined as a gradual loss of the wave energy when it travels through a medium. This loss can be considered as a partial conversion of the seismic energy into some other type of energy and work (e.g., shear heating at the grain boundaries). This causes the decrease of amplitude of the seismic wave and the alteration of its frequencies (Behura, 2009).

The processes by which the attenuation affects the seismic wave are listed below (Bugeja, 2011):

- **Geometrical spreading:** This process is related to the spread of the released energy over an ever-increasing area, making the intensity of the wave decrease with distance. In body waves, for example, the amplitude decays proportional to:

  \[ u = \frac{1}{r} \]  

  (4.1)

  and for surface waves, it decays proportional to:

  \[ u = \frac{1}{\sqrt{r}} \]  

  (4.2)

  Which makes the body waves attenuate with distance faster than the surfaces waves.

- **Intrinsic attenuation:** This process is related to the loss of energy caused by inelastic processes or by internal friction, which means that there is a permanent exchange between potential (displacement) and kinetic (velocity) energy. This process is not
completely reversible because of the presence of shear heating at grain boundaries, mineral dislocations, water, gas etc.

\[
\frac{1}{Q(w)} = \frac{-\delta E}{2\pi E}
\]  

(4.3)

Equation (4.3) is the strength of intrinsic attenuation, where Q is a dimensionless quantity known as quality factor and defined as the inverse of the strength of the attenuation. E is related to peak strain energy and \(\delta E\) is the energy loss per cycle. For seismic application, Q is considered to be much greater than 1 (Q \(\gg\) 1). Considering the amplitude (A) proportional to \(\sqrt{E}\) and giving an initial amplitude or wavelength of \(\frac{2\pi c}{\omega}\) the exponential amplitude decay is giving by:

\[
A(x) = A_0 e^{-\frac{w x}{c Q}}
\]  

(4.4)

where \(c\) is the velocity which could be from P or S-waves, and \(x\) is the distance. This equation specifies the dependence of frequencies with Q constant; the higher the frequency the higher the attenuation. The effect of Q written in terms of time is:

\[
A(t) = A_0 e^{-\frac{\omega t}{2Q}}
\]  

(4.5)

where \(\omega\) is the angular frequency and \(t\) is the time.

S waves have larger values of Q than P waves. This is related to the motion of S-wave (shear motion) which involves frictional heating.

- Scattering: This process is related to heterogeneities of the Earth that will scatter the wavefield in different phases (causing amplitude decay and dispersive effects) depending on material properties.

These processes are included in the formulation of the attenuation, affecting exponentially the decay of the amplitude of the wave. Several models trying to approximate the attenuation don’t take into account all of the processes mentioned before. Most of the different methods
used for attenuation calculation consider the geometrical spreading and the reflectivities to be independent of f, converting them into a constant that facilitates the calculations. To understand more about the behavior of the attenuation in the presence of different mediums, a brief explanation is given in the next section.

4.1 Seismic Attenuation in Isotropic Media

This subsection gives a brief description of the behavior of the attenuation in presence of homogeneity, heterogeneity and isotropic media. The detailed theory on attenuation in anisotropic media can be found in (Behura, 2009). The majority of models used by geoscientists to study the propagation of seismic waves and wave attenuation are based on homogeneous media, in order to avoid the complexity of the angle of inhomogeneity. Attenuation models must be based on inhomogeneous media and include the influence of the inhomogeneity angle ($\xi$).

According to (Behura, 2009), the angle between the propagation vector and the attenuation vector is called the inhomogeneity angle ($\xi$). It will affect the phase and group velocity and the phase and group attenuation, depending on its value and the media of propagation (isotropic, anisotropic or highly attenuative).

4.1.1 Phase and Group Attenuation Coefficients

Phase attenuation per wavelength is described as the ratio between the real part ($k^R$) of the plane-wave propagation and its imaginary part ($k^I$). It is termed as the normalized phase attenuation coefficient $A$ (Behura, 2009).

$$A = \frac{k^I}{k^R} \quad (4.6)$$

When the $\xi>0$, equation 4.6 represents the measure of attenuation along the vector $k^I$ more than $k^R$.

The group attenuation coefficient is described using the spectral ratio method. "If two receivers record the same event at two different locations along a raypath, the attenuation
coefficient can be described from the ratio $S$ of the measured amplitude spectra:

$$InS = InG - k^I_g$$  \hspace{1cm} (4.7)$$

Where $G$ includes: the reflection/transmission coefficients, source/receiver radiation patterns and geometrical spreading along the raypath. $k^I_g$ is the average group attenuation coefficient, and $l$ is the distance between two receivers” (Behura, 2009).

Assuming that the medium is homogeneous and putting equation 4.7 in terms of group velocity $V_g$ and travel time, the equation becomes:

$$InS = InG - \omega A_g t$$  \hspace{1cm} (4.8)$$

Where $A_g$ is the normalized group attenuation coefficient defined as:

$$A_g = \frac{k^I_g}{k^R_g} = \frac{k^I_g}{\frac{\omega}{V_g}}$$  \hspace{1cm} (4.9)$$

The group attenuation along the raypath if the travel time $t$ is known, can be estimated using the slope of $InS$ and $\omega$.

Phase and group attenuation coefficients are going to be affected by the type of media used to describe them. It could be isotropic or anisotropic media, homogeneous or inhomogeneous media, or highly attenuative.

4.1.2 Isotropic Media

To understand the influence of $\xi$ over the group and phase attenuation, Behura (2009) obtains from the wave equation the real ($k^R$) and imaginary ($k^I$) part of vector $k$ assuming that $k^R \cdot k^I > 0$. This assumption forces $\xi$ to be smaller than 90 degrees. Because the discussion is based on a isotropic media, $\xi$ will always be considered larger than zero ($\xi > 0$).

Having all these assumptions in mind, the squared magnitudes of the vectors $k^R$ and $k^I$ are given by:

$$\left(K^R\right)^2 = \frac{\omega^2}{2V^2} \left[ 1 + \frac{1}{(Qcos\xi)^2} \right]$$  \hspace{1cm} (4.10)$$
\[(K^I)^2 = \frac{\omega^2}{2V^2} \left( \sqrt{1 + \frac{1}{(Q\cos\xi)^2}} - 1 \right) \] (4.11)

Where \(V\) is the velocity of the real part of the medium and it is defined as: \(V = \sqrt{a_{33}^R}\), where \(a_{33}^R\) is the density-normalized stiffness tensor (Behura, 2009).

It can be seen that in equations 4.10 and 4.11, the propagation vector \((k^R)\) and the attenuation vector \((k^I)\) are influenced by \(Q\cos\xi\), so the variation from small, moderate to large values in \(\xi\) will produce different interesting results that help understand the behavior of the attenuation (group and phase) in isotropic media.

4.1.2.1 Angle \(\xi\): Small and Moderate Variation

To simplify equations 4.10 and 4.11, and understand the influence of \(Q\cos\xi\) in the propagation and attenuation vector, the following assumptions must be made:

- \(\xi\) is not close to 90 degrees,
- the medium does not have uncommonly strong attenuation, and
- \((Q\cos\xi) >> 1\).

After applying these assumptions Behura (2009) obtained:

\[K^R = \frac{\omega}{V} \] (4.12)

\[K^I = \frac{\omega}{2VQ\cos\xi} \] (4.13)

Looking at equation 4.12, it is clear that the propagation vector \((k^R)\), when values of \(\xi\) are small or moderate, will depend only on the velocity, there is no influence from the angle \(\xi\). It is important to remember that in the case of isotropic media, the phase and group velocity coincide.

On the other hand, with the attenuation vector \((k^I)\) (4.13), not only the angle of inhomogeneity has an influence over this vector, but also the velocity \((V)\) and the attenuation.

\[A = \frac{k^I}{k^R} = \frac{1}{2Q\cos\xi} \] (4.14)
The equation 4.14 represents the normalized phase attenuation coefficient, solved using equations 4.12 and 4.13. The velocity (V) is not present in this equation anymore. This means that the phase attenuation is totally dependent on the inhomogeneous angle and attenuation.

The group velocity is given by:

\[
\frac{w}{V_g} = k^R \cos \psi \tag{4.15}
\]

Using the value of \(k^R\) from the equation 4.12 and the angle: \(\tan \psi = \tan \xi / (1 + 2Q^2) \ll 1\) then:

\[
1 = \frac{V_g}{V} \tag{4.16}
\]

There is an influence of the inhomogeneity angle over the group velocity and the group angle, but this influence is so small that \(V_g\) is considered to be equal to \(V\).

The group attenuation coefficient \(A_g\) is defined as:

\[
A_g = \frac{k^I \cos \xi}{k^R} \tag{4.17}
\]

Using equations 4.12 and 4.13:

\[
A_g = \frac{1}{2Q} = A|_{\xi=0^o} \tag{4.18}
\]

Equation 4.18 is showing that the group attenuation is not influenced by the heterogeneity angle. The influence in the phase attenuation is so small that the group and phase attenuation can be considered as equal. "It also shows that seismic attenuation measurements for isotropic media provide a direct estimate of the quality factor \(Q\)" (Behura, 2009).

4.1.2.2 Angle \(\xi\): Large values

When the angle \(\xi\) approaches 90° but it is still less than 90°, it is no longer correct to assume \((Q\cos \xi) \gg 1\). When the angle \(\xi\) approximates to 90°, \((Q\cos \xi) \ll 1\). Using this assumption, equations 4.10 and 4.11 become:

\[
K^R = \frac{\omega}{V \sqrt{2Q\cos \xi}} (1 + \frac{Q\cos \xi}{2}) \tag{4.19}
\]
\[ K^I = \frac{\omega}{V\sqrt{2Q\cos\xi}}(1 - \frac{Q\cos\xi}{2}) \]  \hspace{1cm} (4.20)

The real and imaginary vectors in equations 4.19 and 4.20 are shown to be heavily influenced by the attenuation and the inhomogeneity angle. Moreover, while angle \( \xi \) approximates to \( 90^\circ \), the wave propagation \( k^R \) goes to zero. Using these equations, phase attenuation, group attenuation, and group angle are given by:

\[ A = \frac{k^I}{k^R} = 1 - Q\cos\xi \]  \hspace{1cm} (4.21)

\[ \tan\psi = \frac{1}{Q} - \cos\xi \]  \hspace{1cm} (4.22)

\[ A_g = \frac{1}{Q} - \cos\xi \]  \hspace{1cm} (4.23)

The group attenuation reduces to angle group (4.23 and 4.22) when \( \xi \) approximates to \( 90^\circ \) and also the group attenuation approaches \( 1/Q \) and is about twice as large as \( A|_{\xi=0^\circ} \) (Behura, 2009).

The behavior of the attenuation in an anisotropic homogeneous and heterogeneous medium is much more complicated. More information related with this topic can be found in (Behura, 2009).

4.2 Methods Used to Calculate Attenuation in Surface Seismic Data

Attenuation is an attribute of the seismic wave that can be correlated with lithology, pore structure, fractures, and fluid content. This is because attenuation varies in response to changes in water saturation, clay content, porosity, pore geometry, permeability, microfracturing, and pressure (Li et al., 2016). Attenuation can also be used as a direct hydrocarbon detector.

There are several methods to calculate attenuation, must of them developed for being used in the well. Some of them have been modified to make it possible to calculate the attenuation from surface seismic data (e.g., wavelet modeling, spectral modeling, spectral ratio method, matching technique, amplitude decay method, risetime method, and analytical...
signal). In this way four of those methods were used in this project.

- Spectral-Ratio Method (SR).
- Peak Frequency Shift Method (PFS).
- Centroid Frequency Shift Method (CFS).
- Dominant and Central Frequency Shift Method (DCFS).

The main reason in using these four methods is to compare the fracture direction obtained from each method and its approximation with well data. Another reason is to compare and understand their stability. The QVOA method is suggested as a tool for fracture characterization, but uses only two of the four methods. This thesis implements an additional idea of adding two other methods to his theory. The four methods implemented in this thesis are related with the spectral amplitude of the seismic trace. In general the values of attenuation $\alpha$ go from 0 to 1 (Sabinin, 2013). In the next sections a review of these methods and its application are provided.

### 4.2.1 Spectral-Ratio Method (SR)

The spectral ratio method (Johnston and Toksöz, 1981) is based on the Q constant theory, where the relation between source and the receiver amplitude spectrum is given by:

$$ R(f) = G(f)H(f)S(f) $$  \hspace{1cm} (4.24)

Equation 4.24 represents the pass of the seismic wave from the source ($S(f)$) through the Earth until the final energy is received ($R(f)$). This final energy is affected by the intrinsic or absorption property ($H(f)$). It is also affected by the geometrical spreading along the ray path, the instrument response, source/receiver, coupling/ radiation patterns, and reflection/transmission coefficients, which are all represented by $G(f)$. In (See Figure 4.1), $G(f)$ and $H(f)$ are shown to be related to instrument and medium response.
Taking equation 4.3 and considering the amplitude (A) proportional to $\sqrt{E}$, the quality factor is related to the changes in amplitude in this way:

$$\frac{1}{Q} = -\frac{\delta A}{\pi A}$$  \hspace{1cm} (4.25)

where $\delta A$ represents the changes in amplitude due to attenuation. If an initial amplitude or wave length is considered to be $\frac{2\pi c}{\omega}$ then:

$$\frac{dA}{dz} = -\frac{\omega}{2cQ} A$$  \hspace{1cm} (4.26)

Where $c$ is the phase velocity and $\omega$ is the frequency. Equation 4.27 represents the exponential decay of the amplitude.

$$A(\omega, z) = A_0(\omega) \exp\left(-\frac{\omega}{2cQ}\right)$$  \hspace{1cm} (4.27)

The following assumptions are made in using this method:

1. the medium is considered to be viscoelastic,
2. the model considers a constant linear frequency, and
3. the model considers a wave whose spectral amplitude exponentially decays with travel time.

Equation 4.28 considers the most basic model of a signal response, where a wavelet $\exp(-\frac{\pi ft}{Q})$ have been convoluted with the reflection coefficients $R(f)$ to generate the final signal $A(f, t)$. This model does not contemplate any noise.

$$A(f, t) = S(f)R(f) \exp\left(-\frac{\pi ft}{Q}\right)$$  \hspace{1cm} (4.28)
Considering $S(f)$ to be independent of frequencies and having the final signal from two local subset of reflections:

$$A_1(f, t_1) = S(f) R(f)_1 \exp \left( -\frac{\pi t_1 f}{Q} \right)$$  \hspace{1cm} (4.29)

$$A_2(f, t_2) = S(f) R(f)_2 \exp \left( -\frac{\pi t_2 f}{Q} \right)$$  \hspace{1cm} (4.30)

Taking the logarithm of their ratio, and considering the phase velocity to be independent of the frequencies:

$$\ln \left( \frac{A_2(f, z_2)}{A_1(f, z_1)} \right) = \ln \left( \frac{R(f)_2 R(f)_1^{-1}}{f_2 - f_1} \right) - \frac{\pi f}{Q} (t_2 - t_1)$$  \hspace{1cm} (4.31)

If there is no reflectivity anomaly, the reflectivity coefficient is considered to be independent of the frequencies becoming the constant $c$ (Sabinin, 2013):

$$\ln \left( \frac{A_2(f, z_2)}{A_1(f, z_1)} \right) = -\frac{f d\tau}{Q} + c$$  \hspace{1cm} (4.32)

At this point the values of the attenuation can be measured using the equation 4.32. Where the slope from a regression analysis between the logarithm of the source and receiver amplitude spectra versus the frequency will result in the value of the attenuation.

To use this method (Sabinin, 2013) makes a modification in the equation 4.32, reflected in the value $d\tau$ which represents the travel times in the interval of interest from the surface seismic data. This change is necessary because the original equation assumes that the ratio of the seismic amplitude spectra at two different depths have coincident raypaths which is not true in the case of the seismic surface data.

To apply this method, a correct frequency window must be carefully selected because the spectral ratio method, when applying it to real data, is strongly affected by noise and the shape of the spectrum. It is suggested to select this window over the amplitude spectra where the ratio of the amplitudes is constant (See Figure 4.2).

It is suggested to do this selection manually in order to avoid values that are not desirable. In this project, values are not selected manually, but uses the window between 0.8 of the peak frequency from both the bottom and the top of the interval of interest (Sabinin, 2013).
4.2.2 Centroid Frequency Shift Method (CFS)

This method is proposed by Quan and Harris (1993). Centroid frequency uses the centroid of the amplitude spectrum of the source (S(f)) (in this case the top of the interval of interest $A_t$) and from the receiver R(f) (in this case the bottom of the interval of interest $A_b$). Using these two parameters it calculates the variance. Again, this method depends on the shape of the spectrum. The seismic wave spectrum is represented by the Gauss wavelet.

In a modification of this method to be implemented for this project, the equations are (Sabinin, 2013):

\[ a_i = \sum_f A_i, i = t, b \]  \hspace{1cm} (4.33)

\[ f_i = \frac{1}{a_i} \sum_f f A_i, i = t, b \]  \hspace{1cm} (4.34)

\[ \sigma_t^2 = \frac{1}{a_t} \sum_f (f - f_t)^2 A_t \]  \hspace{1cm} (4.35)
η = \frac{f_t - f_b}{\sigma_t^2} \tag{4.36}

Where \( t \) corresponds to the top and \( b \) to the base of the reservoir, \( A \) represents the amplitudes in the amplitude spectrum, and \( f \) are the frequencies. The parameter \( a_i \) in equation 4.33 refers to the sum of the amplitudes in the amplitude spectrum (See Figure 4.2). The parameter \( f_i \) represents the sum of the amplitudes multiplied by the frequencies in amplitude spectrum divided by the parameter \( a_i \) (equation 4.34).

Finally, the parameter \( \eta \) will be given by the subtraction of the parameters \( f \) from the top and the bottom divided by the variance given by equation 4.35. Using \( \eta = \tau \pi / Q \), values for \( Q \) and attenuation \( \alpha \) can be obtained.

4.2.3 Peak Frequency Shift Method (PFS)

This method was suggested by Zhang and Ulrych (2002). In this method, the seismic wave spectrum is represented by the Ricker wavelet where the maximum amplitude in the spectrum from top and bottom of the interval of interest are defined as peak frequencies \((f_t \text{ and } f_b)\) (equation 4.37) (Hermana et al., 2012).

\[
Q = \frac{\pi t f_b f_t^2}{2(f_t^2 - f_b^2)} \tag{4.37}
\]

Where \( t \) represents the travel time of the wavelet.

The spectrum of Ricker wavelet is (Hu et al., 2013):

\[
H(f) = \frac{2}{\sqrt{\pi} f_m^2} \frac{f^2}{f_m^2} \exp\left(-\frac{f^2}{f_m^2}\right) \tag{4.38}
\]

Where \( f_m \) is the dominant frequency of the wavelet.

4.2.4 Dominant and Central Frequency Shift Method (DCFS)

This method is suggested by Li (2015). In his paper, he discusses the differences between the spectral ratio and the two frequency shift methods discussed above. He concludes that the frequency shift methods are less affected by the signal to noise ratio in the seismic data.
Using these conclusions, he proposed a new method where he takes advantage of the centroid and peak frequency shift methods.

When the signal to noise (SNR) ratio is high (30dB), the spectral ratio method (SR) is reliable, but when the SNR is low, then SR method has several problems as it becomes unstable, sometimes giving negative values.

The CFS method has noticeable systematic errors growing with attenuation (Hu et al., 2013). On the other hand this method is much more robust than PFS. The PFS method yields results closer to the real Q values, but the standard deviations are much more significant compared with CFS method, making this method less robust.

The author combines the robustness of the CFS and the certainty of the PFS. He calculates the error between CFS and PFS methods to know what causes CFS method to be more robust than PFS method. After this, he approaches the peak frequency using central frequencies instead of using the values of the peak frequencies directly.

Combining the Ricker wavelet from equation 4.38 with the equation 4.34 and doing some simplifications and substitutions, the author defines the central frequency as:

\[ f_c = \frac{4}{3} \sqrt{\frac{2}{\pi}} (f_{2m}^2 - f_{2m}^2) \]  

(4.39)

Where the \( f_m \) is the dominant frequency (peak frequency) of the source wavelet.

The Q estimation using this method is given by:

\[ Q \approx \frac{f_m^2 \pi \tau}{4f_m \sqrt{\frac{2}{\pi} - 3f_c \sqrt{\pi}}} \]  

(4.40)

The assumptions used for this method are:

1. A Q constant model

2. Travel time should not be too large to fulfill the first-order Taylor approximation, if the target is too large, it must be deviated and treated as a multi-layer model and the Q estimation must be performed layer-by-layer (Li et al., 2015b)
In the application to real data, this method resulted in more stable results than the previous methods.

### 4.3 Synthetic Model

Four different methods to calculate attenuation were implemented, and a synthetic CMP gather was created to prove they were coded correctly. The synthetic model is based on nearly constant-Q using, the standard linear solid (SLS) model for viscoacoustic isotropic media. An explosive source with a Ricker wavelet of peak frequency 15 Hz was used. The model dimensions are:

- \( nx = nz = 3\text{km}, \)
- \( dx = dz = 20\text{m}, \)
- 151 receivers on the surface at each grid point, and
- 31 shots spacing every 100m starting at \( x=0 \) on the surface.

Figure 4.3 displays the three layer model used for the generation of the synthetic CMP gathers, where the values of the quality factor \( Q \) are varied, giving the minimum value to the middle layer which is the interval of interest. Velocities are also varying through the profile.

The synthetic CMP gather was generated using the program ”sfmpipfwi”, where a proposed technique was used to solve for the quality factor \( Q \) in the time-domain. It uses the fractional-Laplacian approach by low-rank approximation scheme to create more accurate and less expensive (large computational and memory requirements) images (Sun et al., 2015). Figure 4.4 shows the synthetic common mid point gather resulted from the model proposed in the Figure 4.3 using this program.

Using the synthetic common mid point gather generated, the values of quality factor were calculated for the second layer, obtaining results listed in Figure 4.5. The values of
Figure 4.3: Synthetic model composed of three layers with different values of quality factors (Q). The second layer corresponds to the interval of interest.

The quality factor obtained using this code are very close to the one proposed in the 3-layer model.

The methods used to calculate the quality factor values (SR, CFS, PFS and DCFS) in the synthetic gather are always stable when using synthetics without any noise.

But once noise is added to the data, these methods start to behave in very different ways, giving enormous values of attenuation and also negative. (Li et al., 2015b). Some considerations are assumed in order to take into account only those values that are closer to reality.

4.4 Real Data

The following are the steps to calculate the attenuation coefficient using spectral ratio, centroid frequency shift, peak frequency shift, and dominant and central frequency shift methods in real data.

1. Select two windows in time of same size at the top and at the bottom of your reservoir (or interval of interest). In Figure 4.6 a model of three layers is displayed, where the
second layer is the interval of interest. The two windows are selected in black braces.

2. Calculate the amplitude spectrum of the top and the bottom of the interval of interest. Figure 4.7 displays in blue the amplitude spectrum of the top of an interval of interest from one trace. The red line corresponds to the amplitude spectrum of the bottom. The X axis corresponds to the frequencies and Y axis to the magnitude of the amplitude of the trace.

3. Select a frequency window over the amplitude spectrum between 0.8 of the peak frequency for the bottom spectrum and the peak frequency for the top spectrum. In Figure 4.8 there is a black line representing the window selected for the amplitude spectrum displayed in the same figure.

The selection of this window is very important, because it highly influences the results from methods like spectral ratio. Final values can be negative or very large. Figure 4.9 shows the values of attenuation from 10 traces using the spectral ratio method and
Figure 4.5: Values obtained after calculating the attenuation with each method used in this project selecting two different windows. The first window selected includes the whole amplitude spectrum and the second window is selected between the 0.8 of the peak frequency of both amplitude spectrum. The difference between the values is easy to see; negative and higher values are noted.

The centroid frequency method is also affected by the selection of the window. See Figure 4.10 where there are also higher and negative values. The Peak frequency and the dominant and centroid frequency methods are not really affected by the selection of the window, as an example, see Figure 4.10.

4. Extract the dominant amplitude frequency from the values chosen in the previous step.

5. Calculate the travel times $\tau_0$. Using the model in Figure 4.12, travel times can be calculated for a two layer model where the second layer is the target where:

$$ p = 0.5\delta tV_1 + \sqrt{X_0^2 + Z_1^2} $$

(4.41)
where \( X_0 \) is the half of the offset, \( Z_1 \) and \( z_2 \) are the thicknesses of the layers, and \( \theta_1 \) and \( \theta_2 \) are travel (incidence) angles.

In Figure 4.12 the \( X \) represents the offset value of refraction point for the bottom ray given by:

\[
(X_0 - X)(p\sqrt{X^2 + Z_1^2} - X X_0 - Z_1^2) = X Z_2^2
\]  

(4.42)

The non-linear equation 4.43 must be solved numerically to obtain \( X \) and used to calculate the travel time of the ray inside the target layer.

\[
\tau_0 = d\delta t + 2(\sqrt{X_0^2 + Z_1^2} - \sqrt{X^2 + Z_1^2})/V_1
\]  

(4.43)

Where \( V_1 \) is P-phase velocity of the upper layer. The value \( \delta t \) is calculated by using the correlation function between the impulses of the trace. There is a difference between the travel time \( \tau \) and the time \( \delta t \), where the values of the first increases with incident angle \( \theta \), while the value of the second decreases with the incidence angle \( \theta \) (Sabinin,
Figure 4.7: Amplitude spectrum from the top (blue line) and bottom (red line) of the interval of interest. The yellow line corresponds to the ration between the two amplitude spectrum.

6. Calculate attenuation coefficient. This must be done using real data whose amplitudes (and spectrum) are not deformed by the normal moveout (NMO) correction (Sabinin, 2013).

In this chapter the concept of attenuation was reviewed. Four different methods to measure seismic attenuation were analyze. To understand to way of how to apply them in real data, some steps were listened. The implementation of these methods, which give the attenuation, are going to be used as an input for the QVOA equation. The different methods learned in this chapter were implemented using Matlab™.
Figure 4.8: A window in frequency must be selected from the amplitude spectrum.

Figure 4.9: Values of attenuation from spectral ratio method, selecting different sizes of windows in frequency over the amplitude spectrum.
Figure 4.10: Values of attenuation from centroid frequency shift method, selecting different sizes of windows in frequency over the amplitude spectrum.

Figure 4.11: Values of attenuation from dominant and centroid frequency shift method, selecting different sizes of windows in frequency over the amplitude spectrum.
Figure 4.12: Two layer model for travel times calculation (Sabinin, 2012).
CHAPTER 5
SEISMIC QUALITY FACTOR VERSUS OFFSET AND AZIMUTH

The QVOA equation, which studies the variation of the attenuation versus offset and azimuth (Chichinina et al., 2005), is used in this study on real data to confirm the application of this technique in the characterization of fractures in a reservoir. The QVOA equations are a series of linear homogeneous viscoelastic relationships. They are designed for analyzing the attenuation anisotropy caused by the presence of parallel fractures, creating an attenuative media embedded in an isotropic background. The QVOA equations for P-waves, SH-waves and SV-waves were for HTI and VTI media (Chichinina et al., 2009). This project is focused on the P-wave, HTI version of the QVOA.

The QVOA equation is similar to Rüger’s (Rueger and Tsvankin, 1997) approximation for P-wave reflection coefficients which can be found in (Rueger and Tsvankin, 1997). Similarly, QVOA introduces two attributes. The first is intercept A, which corresponds to the direct attenuation, and the second is gradient B, which represents the curvature of the attenuation function.

In section 2.2, AVOA and NMO analysis were shown to be good techniques to characterize fractures using velocities and amplitudes that are influenced by the anisotropy of the media. Why is attenuation important to study if techniques like AVOA are able to solve the problem? What are the additional insights from QVOA analysis? Answers to these questions are related to the presence of strong attenuation normal to the cracks where the increase of the reflectivity can be canceled so that symmetry orientations cannot be determined by azimuthal AVOA methods. This conclusion comes from the analysis of the magnitude of the attenuation anisotropy compared with the magnitude of the velocity anisotropy. The magnitude of attenuation anisotropy is given by:

\[
\frac{Q^\parallel}{Q^\perp} = \frac{1}{[1 - 2(V_p^\parallel)^2]^2} \left(\frac{V_p^\parallel}{V_p^\perp}\right)^2
\]  

(5.1)
Where $V_s$ and $V_p$ are the P and S phase velocities, and $V^\parallel/V^\perp$ is the magnitude of the attenuation velocity. Equation 5.1 shows that the magnitude of the attenuation anisotropy is bigger than the magnitude of the velocity anisotropy. During some laboratory experiments made by (Chichinina et al., 2006) these relations are verified and two other conclusions are obtained. Models saturated with different liquids were used by (Chichinina et al., 2006) to compare the answer of the magnitude of the anisotropy from the p-velocities and the p-attenuation. The results suggest that the anisotropy for p-velocities and p-attenuation are greater in dry models (gas) than in saturated models (oil and brine). On the other hand the magnitude variations measured in the experiment show that the P-wave anisotropy is 10-38% while attenuation anisotropy is 114-153% suggesting that when the velocity anisotropy is small or negligible, the attenuation anisotropy may still be strong enough to measure a response.

5.1 Theory

The attenuation and quality factor magnitudes are given by equations 5.2 and 5.3, (Car-cione, 2000).

$$\alpha = -\omega Im(V^{-1})$$  \hfill (5.2)

$$Q = \frac{Re(\hat{V}^2)}{Im(\hat{V}^2)}$$  \hfill (5.3)

where $Re$ and $Im$ denotes real and imaginary parts. These relations are suggested to de-
scribe attenuation anisotropy for a homogeneous anisotropic viscoelastic medium (Carcione, 2000).

Attenuation and Q factor, assuming that $Q>>1$, are related by:

$$\alpha = \frac{\pi f}{QV_{ph}}$$  \hfill (5.4)

where $V_{ph}$ is the phase velocity given by $V_{ph} = [Re(V^{-1})]^{-1}$ and $f$ is the frequency. Equation 5.3 can be rewritten in terms of components of the complex stiffness matrix $\tilde{C}$.
Where $\tilde{C}_{ij} = C^R_{ij} + iC^I_{ij}$.

\[ Q_{ij} = \frac{C^R_{ij}}{C^I_{ij}} \] (5.5)

The QVOA method considers an HTI medium where the symmetry axis direction lays on the horizontal axis perpendicular to the fractures and the isotropy plane lays in the vertical direction parallel to the fractures. The stiffness matrix $C$ for effective fractures in a HTI medium without attenuation is defined by Shoenberg (1995).

\[
\begin{bmatrix}
M(1 - \Delta_N) & \lambda(1 - \Delta_N) & \lambda(1 - \Delta_N) & 0 & 0 & 0 \\
\lambda(1 - \Delta_N) & M(1 - \xi^2\Delta_N) & \lambda(1 - \xi\Delta_N) & 0 & 0 & 0 \\
\lambda(1 - \Delta_N) & \lambda(1 - \xi\Delta_N) & M(1 - \xi^2\Delta_N) & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu(1 - \delta_T) & 0 \\
0 & 0 & 0 & 0 & 0 & \mu(1 - \Delta_T)
\end{bmatrix}
\]

The QVOA analysis includes the attenuation. To include the attenuation the values of $\Delta_N$ (normal weaknesses) and $\Delta_T$ (tangential weaknesses) must be modified by their complex definitions (Chichinina et al., 2009):

\[ \Delta_N \to \tilde{\Delta}_N = \Delta_N - i\Delta^I_N \] (5.6)

\[ \Delta_T \to \tilde{\Delta}_T = \Delta_T - i\Delta^I_T \] (5.7)

So that the attenuative stiffens matrix $\tilde{C}$ acquires the following form:

\[
\begin{bmatrix}
M(1 - \tilde{\Delta}_N) & \lambda(1 - \tilde{\Delta}_N) & \lambda(1 - \tilde{\Delta}_N) & 0 & 0 & 0 \\
\lambda(1 - \tilde{\Delta}_N) & M(1 - \xi^2\tilde{\Delta}_N) & \lambda(1 - \xi\tilde{\Delta}_N) & 0 & 0 & 0 \\
\lambda(1 - \tilde{\Delta}_N) & \lambda(1 - \xi\tilde{\Delta}_N) & M(1 - \xi^2\tilde{\Delta}_N) & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu(1 - \tilde{\Delta}_T) & 0 \\
0 & 0 & 0 & 0 & 0 & \mu(1 - \tilde{\Delta}_T)
\end{bmatrix}
\]

Where $M = \lambda + 2\mu$ and $\xi = \lambda/M \equiv 1 - 2g$ and $g$ is defined as the squared root of the ratio of the velocities $P$ and $S$:

\[ g = \frac{\mu}{\lambda + 2\mu} = \left(\frac{V_s}{V_p}\right)^2 \] (5.8)
Using equation 5.5, the relationship between the \( Q_p \) in the isotropic (parallel) and anisotropic plane (perpendicular) can be given by (Chichinina et al., 2006):

\[
\frac{1}{Q^\parallel} \equiv \frac{1}{Q_{33}} = \frac{\Delta N(1 - 2g)^2}{1 - \Delta N(1 - 2g)^2} \tag{5.9}
\]

\[
\frac{1}{Q^\perp} \equiv \frac{1}{Q_{11}} = \frac{\Delta N}{1 - \Delta N} \tag{5.10}
\]

In the case of attenuation, the magnitude that is given by equation 5.1 does not depend on the values of the normal weaknesses \( \Delta N \), which represents the attenuation mechanism. On the other hand, it has a strong dependence on the velocity ratio \( V_s/V_p \) (Ronquillo et al., 2013).

The values of \( \Delta N \) and \( \Delta T \) can be related to the fractured medium parameters, as follows (Chichinina et al., 2009):

\[
\Delta N = \frac{16e}{[3(3 - 2g)]} \tag{5.11}
\]

\[
\Delta T = \frac{4e}{3g(1 - g)(1 + K)} \tag{5.12}
\]

\[
K = \frac{K_f}{(\pi \mu a(1 - g))} \tag{5.13}
\]

where \( e \) is the crack density, \( \mu \) is the host rock’s shear modulus, \( \alpha \) is the crack aspect ratio, and \( K_f \) is the fluid bulk modulus that depends on P-wave velocities and densities. (Chichinina et al., 2006). The imaginary parts of the complex deltas can be related to fluid viscosity, rock permeability, porosity, etc (Chichinina et al., 2009).

The behavior of \( \Delta N \) is proportional to the Q-anisotropy parameter \( \epsilon_Q \) (Sabinin, 2013). Both are affected by the infill fluid of the fracture, where in the presence of gas, \( \Delta N \) is greatest and in the presence of liquid (oil or brine), \( \Delta N \) is very small. \( \Delta N \) is also affected by the crack aspect ratio, \( \alpha \). The thinner the crack the smaller the value of \( \Delta N \) (Sabinin, 2013).
The Q-anisotropy parameter $\epsilon_Q$ is introduced by Carcione (2000), where it is defined as analogous with the Thomsen’s anisotropic parameter $\epsilon$.

$$\epsilon_Q = \frac{Q_{11} - Q_{33}}{2Q_{33}}$$

Equation 5.14 represents the fractional difference of the symmetry-axis quality factor $Q_{11}$ and the isotropy-plane quality factor $Q_{33}$.

Using equation 5.5 the relationship between $\epsilon_Q$ and $\Delta_N$ is:

$$\epsilon_Q = \frac{-2g(1-g)}{1 - \Delta_N(1-2g)^2}$$

(5.15)

where $g$ is defined in equation 5.8.

Using the inequality $\Delta_N(1-2g)^2 << 1$ will simplify equation 5.15 to the following form (Sabinin, 2013):

$$\epsilon_Q = -2g(1-g)$$

(5.16)

Using this, the anisotropic parameter $\epsilon_Q$ will depend on the S and P velocity ratio and not on the type of attenuation mechanism.

The QVOA method determines the behavior of the attenuation with the azimuth using Hudson’s fluid-flow mechanisms as a cause of P-wave attenuation. In Hudson’s models, the weaknesses $\Delta_N$ and $\Delta_T$ are redefined with complex frequency-dependent functions: $\hat{M}(\omega)$ and $\hat{K}(\omega)$. Where the function $\hat{M}(\omega)$, which is responsible for predicting viscous energy dissipation, goes to zero for QVOA.

Taking Hudson’s model, where the fluid flow from cracks are in a background porous matrix, the QVOA equation is defined by setting some assumptions. These assumptions are tangential weaknesses $\hat{\Delta}_T$ is real (see equation 5.12), normal weaknesses $\hat{\Delta}_N$ remains complex(see equation 5.17), and its imaginary part dictates the magnitude of P-wave attenuation.

$$\hat{\Delta}_N = \frac{4\epsilon}{3g(1-g)(1 + \hat{K}(\omega))}$$

(5.17)
where \( \hat{K}(\omega) = \frac{K}{1 + \hat{y}(\omega)} \). \( K \) is defined in equation A.17 and \( \hat{y}(\omega) \) is a frequency dependent function that depends on the choice of the fluid-flow model.

Looking at equation 5.3, the attenuation is given by the ratio of the imaginary part of the square complex phase velocity and the real part.

The phase velocity is defined as (Carcione, 2000):

\[
V(\varphi)^2 = V_p^2(1 - \Delta_N[1 - 2g\sin^2\varphi]^2 - \Delta_Tg\sin^22\varphi) \tag{5.18}
\]

The complex velocity is a fundamental quantity that determines uniquely both the attenuation and the quality factors. To get the phase velocity for the dissipative medium, \( \Delta_N \) in equation 5.18 must be replaced by its complex version (see equation 5.6):

\[
\hat{V}(\varphi)^2 = V_p^2(1 - (\Delta_N - i\Delta_N^I)[1 - 2g\sin^2\varphi]^2 - \Delta_Tg\sin^22\varphi) \tag{5.19}
\]

Where \( \Delta_T \) remains real as explained before. Reducing the equation 5.19 making \( \varphi \) to be zero and 90 degrees the anisotropic in-plane phase velocity (perpendicular) and isotropic in-plane phase velocity (parallel) can also be written in complex form as:

\[
\hat{V}_\parallel = V_p\sqrt{1 - \Delta_N} \tag{5.20}
\]

\[
\hat{V}_\perp = V_p\sqrt{1 - \Delta_N(1 - 2g)^2} \tag{5.21}
\]

Equations 5.20 and 5.21 show that the velocity in the isotropic and anisotropic planes are dependent on the values of the normal weaknesses \( \Delta_N \) which involves fractured-medium parameters (crack density, crack aspect ratio, fluid bulk modulus, host rock shear modulus, and \( V_s/V_p \)). The P-velocity in the anisotropic plane will also depend on the ratio of the S and P-velocities (Ronquillo et al., 2013).

The equations 5.18 and 5.6 define the P-wave phase velocity as a function of wave normal angle \( \varphi \) with respect to the symmetry axis. Using equation 5.6, the imaginary and real parts are extracted.

\[
Im(\hat{V})^2 = \Delta_N^IV_p^2[1 - 2g\sin^2\varphi]^2 \tag{5.22}
\]
\[ Re(\tilde{V})^2 = V_p^2(1 - \Delta_N[1 - 2gsin^2\varphi]^2 - \Delta_Tgsin^22\varphi) \equiv V(\varphi)^2 \]  

(5.23)

Again, R and I are the real and imaginary parts and the tilde represents complex numbers (Chichinina et al., 2006). Dividing equations 5.22 and 5.23, making some substitutions (replacing \( \sin^2\varphi \) with \( 1 - \sin^2\theta\cos^2\phi \)), and denoting \( \theta \) as the incidence angle measured with respect to the vertical z-axis, the azimuthal variation of the attenuation is obtained (Chichinina et al., 2006).

\[ Q^{-1}(\phi, \theta) = \frac{\Delta^I_N[1 - 2g(1 - \sin^2\theta\cos^2\phi)]^2}{V^2(\phi, \theta) V_p^2} \]  

(5.24)

The equation 5.24 shows that for all \( \theta \) the attenuation is maximum when \( \phi = 0^\circ(180^\circ) \) and minimum when \( \phi = 90^\circ(270^\circ) \). This equation is applicable for \( \theta = 0^\circ - 40^\circ \).

Setting:

\[ A_0 = \sqrt{\Delta^I_N(1 - 2g)} \]  

(5.25)

and

\[ B^\perp = \sqrt{\Delta^I_N2g} \]  

(5.26)

and taking the next assumptions (Chichinina et al., 2006):

1. Variation of \( V_p/V \) versus \( \theta \) and \( \phi \) in equation 5.24 does not affect the variation of \( Q^{(1/2)}(\theta, \phi) \) significantly. Then, this term can be considered constant. In the definition of \( V_p/V^\perp \) and \( V_p/V^\parallel \) the value \( \Delta_N \) will effect the value of \( V_p/V \).

\[ 1 + 0.5(1 - 2g)^2\Delta_N \leq c \leq 1 + 0.5\Delta_N \]  

(5.27)

2. For thin, liquid-filled cracks with \( \Delta_N \rightarrow 0 \), \( V_p/V(\theta, \phi) \) is approximately equal to one.

A simpler form of the equation 5.24 can be written as:

\[ Q^{(-1/2)} \approx A_0 + B^\perp\cos^2[\phi - \phi_0]\sin^2\theta \]  

(5.28)

When the pores are gas-filled, the equation 5.28 must be rewritten with \( cA_0 \) and \( cB_0 \) to consider the variation of \( V_p/V \). In this new form of the equation, \( A_0 \) is related with the
intercept (normal incidence) and \( B \) is the slope of the line which is called gradient (Ronquillo et al., 2013).

### 5.2 Numerical Methods to Solve QVOA

The QVOA equation, as defined in the last section of this chapter, describes the dependence of the attenuation factor using the information of the incidence angle \( \theta \) and azimuth (source-receiver azimuth \( \phi \)). Here, the incidence angle is defined as the angle between axis \( z \) and the wave ray in the anisotropic medium with attenuation. The equation is valid for values of \( \theta < 37^\circ \) and the interval of interest must be sufficiently thick (2-3 wavelengths). If this is not the case the effect of attenuation is less pronounced and competes with other effects (e.g., interference from short-path multiplies or thin bed influences (Sabinin, 2013)).

\[
\alpha = A + B(\phi)\sin^2 \theta + C(\phi)\sin^4 \theta
\]  

Equation 5.29, as defined before, is a linear approximation of the exact attenuation equation. It includes the intercept \( A \), the gradient \( B \), the far incidence \( C \), and the angle of incidence \( \theta \). Where the exact definitions of \( B \) and \( C \) are given by:

\[
B = B_0 \cos^2(\phi - \phi_0)
\]  

\[
C = C_0 \cos^4(\phi - \phi_0)
\]

Where \( A \), \( B_0 \), and \( C_0 \) are constants, \( \phi \) is the azimuth of the traces in a CMP gather, and \( \phi_0 \) is the symmetry axis angle.

Two parameters must be estimated in order to solve for \( \phi_0 \).

The numerical methods to solve the QVOA equation to obtain the symmetry axis, uses the seismic traces within a bin. This is unless the CMP has too few traces, then a superbin is formed. The information used in this project is all given in superbins with 9 X 9 bins in each one.

There are five numerical methods (Sabinin, 2013):

1. Approximate Method of Sectors (ASM).
2. Method of Sectors (SM).


5. General Method (GM).

Two of the four methods were used in this project: ASM, which is the simplest computationally, and ATM.

5.2.1 Approximate Method of Sectors (ASM)

Mallik et al. (1998) proposed a method to solve for the symmetry axis using the AVO method (Sabinin, 2014). QVOA can also be solved in the same way. To use this method, it is necessary to divide the gather into sectors and rewrite the equation 5.29 as follows:

$$
\alpha_{jk} = A_j + B_j \sin^2 \theta_k + C_j \sin^4 \theta_k
$$

(5.32)

When the gather is divided in sectors, all the traces that are inside each interval are going to adopt the azimuth designated to that corresponding sector. The difference with equation 5.29 is that equation 5.32 will need the value of attenuation $\alpha$, calculated from the trace $k$ in the sector $j$. The minimum traces needed to apply this method is 9 traces from 3 sectors (3 traces from each sector) to have a system of equations and solve them by the least-squares method. The first step is to set the system of equations for one sector and three traces in this way:

$$
\begin{align*}
\alpha_{11} &= A_1 + B_1 \sin^2 \theta_1 + C_1 \sin^4 \theta_1 \\
\alpha_{12} &= A_1 + B_1 \sin^2 \theta_2 + C_1 \sin^4 \theta_2 \\
\alpha_{13} &= A_1 + B_1 \sin^2 \theta_3 + C_1 \sin^4 \theta_3
\end{align*}
$$

The equations in this system can be extended to an infinite number of traces. In the same way, the number of $j$ sectors can vary to any amount but not less than 3. When solving the system of equations using the least-squares method, a value of $A$, $B$ and $C$, which represent
the average values, will be obtained for each sector. The next step is to define the value of gradient $B$ as:

$$B_j = B_0 \cos^2(\phi_j - \phi_0), j = 1, ..., m$$ (5.33)

Equation 5.33 can be used only for precise, theoretical values of the gradient. When using real data, it is necessary to add an error. The error can be written as $B_j = B_j^p + \delta_j$ then, $B_j$ will be given by:

$$B_j = a + b \cos[2(\phi_j - \phi_0)], j = 1, ..., m$$ (5.34)

Using equation 5.34, generates a system of equations that will include all the B gradients obtained in the previous step.

The system for the three sectors is:

$$\begin{cases}
B_1 = a + b \cos[2(\phi_1 - \phi_0)] \\
B_2 = a + b \cos[2(\phi_2 - \phi_0)] \\
B_3 = a + b \cos[2(\phi_3 - \phi_0)]
\end{cases}$$

This system can be extended for all the gradients calculated and must be solved using least-squares method for non-linear equations to get the constants $a$, $b$ and $\phi_0$.

The new constants $a$ and $b$, are important when interpreting final results. First, if the error is not too large, $a$ and $b$ must be close to each other for an HTI medium ($a \approx b$ but $a \neq b$). Second, the condition $b > 0$ must be used to distinguish the symmetry-axis and fracture-strike direction, because in the final results, two solutions of $\phi_0$ values are obtained by differing by $\pi/2$ with the same $b$ but of opposite sign. The best thing to do is, having real data from image logs, calibrate the solutions obtained by the QVOA equation and determine how this second condition is working for the specific reservoir analyzed.

When dividing the gather in sectors, all the traces in between are designated to one sector with one single value in degrees. To have better results while using this technique, it is important to divide the gather in as many sectors as possible taking care of the quantity of traces related with each one of them. No less than 3 traces per sector is required (Ronquillo et al., 2013).
5.2.2 Approximate Truncate Method (ATM)

Jenner (2002) proposed a method to solve for the symmetry axis using the AVOA equation. QVOA can also be solved in the same way. To use this method, it is not necessary to divide the gather in sectors. But still some truncations and substitutions to the equation 5.29 must be done.

Truncating the $C$ term in equation 5.29 and combining it with 5.34, the new relationship is given by:

$$\alpha_i = a + bs_i^2 + dS_i g_i + dS_i h_i$$

(5.35)

where index $i$ refers to the number of traces in the superbin than can not be less than 3.

Defining:

$$s_i = sin^2 \theta_i$$

(5.36)

$$S = sin(2\phi_0)$$

(5.37)

$$C = cos(2\phi_0)$$

(5.38)

$$g_i = cos(2\phi_i)$$

(5.39)

$$h_i = sin(2\phi_i)$$

(5.40)

The equation takes this final form:

$$\alpha_i = a + bs_i + dC s_i g_i + dS s_i h_i$$

(5.41)

Where $s_i$ is easily calculated from the trace information, and $g_i$ and $h_i$ can be calculated using the azimuth $\phi$ that comes in the header of the seismic traces. In this project, because the seismic traces headers did not have this value, it was necessary to use an external program that could calculate this azimuth and post it in the right place in the trace header.

Considering a functional error (Sabinin, 2013):

$$F = \frac{1}{n} \sum_{i=1}^{n} (a + bs_i + dC s_i g_i + dS s_i h_i - \alpha_i)^2$$

(5.42)
This functional $F$ must be minimized over parameters $a$, $b$, $d$ and $\phi_0$ solving a system of four equations (Sabinin, 2013) using least-squares method:

\[
\frac{\partial F}{\partial a} = 0 \quad (5.43)
\]
\[
\frac{\partial F}{\partial b} = 0 \quad (5.44)
\]
\[
\frac{\partial F}{\partial d} = 0 \quad (5.45)
\]
\[
\frac{\partial F}{\partial \phi_0} = 0 \quad (5.46)
\]

The final solutions were derived by the author and are posted in appendix A.

The values for $a$, $b$, $d$ and $\phi_0$ can be obtained using:

\[
tan(2\phi_0) \equiv \frac{S}{C} = \frac{A_2H_1 - A_1H_2}{A_2H_2 - B_1H_1} \quad (5.47)
\]
\[
d = \frac{A_2H_1 - A_1H_2}{S(A_2^2 - A_1B_1)} \quad (5.48)
\]
\[
b = \frac{(F_1 - dCa_2 - dSa_3)}{a_1} \quad (5.49)
\]
\[
a = f_0 - bA - dCD - dSE \quad (5.50)
\]

5.3 Gradient B

The gradient obtained, using the methods mentioned above, can be plotted versus the azimuth. Figure 5.1 is an example applied to real data, according to the QVOA equation, the $\cos2\phi$ should fit this graph.

The maximum $B$ gradient value in Figure 5.1 is given at the azimuth $\phi_0 = 50^o$ which corresponds to the symmetry-axis (Hall et al., 2002). The orthogonal direction, $140^o$ gives the fracture-strike orientation or the maximum horizontal permeability (preferred fluid-flow direction). QVOA considers the medium to be HTI, which can be seen in Figure 5.1, when a mirror is used to compare B-gradient from 0 to 180 degrees with the one from 180 to 360 degrees and they are very similar.
The gradient $B$ is an attribute that results from the solution of the QVOA equation that can be interpreted as the Q-anisotropy indicator defined as (Chichinina et al., 2006):

$$B^\perp = \frac{(Q^\perp)^{-1/2} - (Q^\parallel)^{-1/2}}{(Q^\parallel)^{-1/2}}$$  \hspace{1cm} (5.51)

where the gradient is clearly related with the fractional difference in Q along the symmetry direction and the isotropic plane.

The exact expressions for $(Q^\parallel)^{-1/2}$ and $(Q^\perp)^{-1/2}$ are (Chichinina et al., 2006):

$$(Q^\parallel)^{-1/2} = A_0[1 - (1 - 2g)^2 \Delta_N]^{-1/2}$$  \hspace{1cm} (5.52)

$$(Q^\perp)^{-1/2} = (A_0 + B^\perp)[1 - \Delta_N]^{-1/2}$$  \hspace{1cm} (5.53)

Assuming that $\Delta_N \ll 1$ and that $(1-2g)^2\Delta_N \ll 1$, then the exact equations are linearized in this way:

$$(Q^\parallel)^{-1/2} \approx A_0$$  \hspace{1cm} (5.54)

$$(Q^\perp)^{-1/2} \approx A_0 + B^\perp$$  \hspace{1cm} (5.55)
where \((Q^\parallel)^{-1/2}\) is the zero-offset \(Q^{-1/2}\) value and \((Q^\perp)^{-1/2}\) is the symmetry-axis \(Q^{-1/2}\) value for \(\theta = 90^\circ\) and \(\phi - \phi_0 = 0^\circ\).

The magnitude of Q-anisotropy can be expressed in terms of \(B^\perp\) and can be related to the parameter \(\epsilon_Q\) as follows (Chichinina et al., 2006):

\[
\epsilon_Q = 0.5 \left( \frac{1 - \Delta_N}{[B^\perp + 1]^2 (1 - (1 - 2g)^2 \Delta_N)^2} - 1 \right) \tag{5.56}
\]

If \(\Delta_N\) goes to zero, which is the case of thin, liquid-filled cracks, the exact expression in equation 5.56 approximates as follows:

\[
\epsilon_Q = 0.5 \left( \frac{1}{[B^\perp + 1]^2} - 1 \right) \tag{5.57}
\]

Equation 5.57 can be used to determine \(\epsilon\) simply. When inverting, using this equation, the results are going to have a smooth error compared with the initial value. This is due to the restriction of the incidence angle \(\theta = [0^\circ, 40^\circ]\) which was mentioned previously.

To understand the behavior of the gradient when using different values of \(V_s/V_p\) ratios, Chichinina (2006) proposed an experiment where \(V_s/V_p\) ratio, the crack-filling fluids, and the crack densities vary, concluding that the gradient variations are more sensitive to \(V_s/V_p\) ratio changes than to any other parameter (see the graph at the top in Figure 5.2).

Also, in the middle graph of Figure 5.2, the magnitude value of the gradient seems to be higher for the gas than for the oil or brine at the symmetry axis direction which corresponds in this case to \(\phi = 75^\circ\). In the graph at the bottom of Figure 5.2, the gradient values also vary depending on the values of the crack densities, where the bigger the density, the bigger the value of gradient in the symmetry axis direction.

Taking the equations 5.51, 5.52, and 5.53, the exact equation for \(B^\perp\) can be defined as (Sabinin, 2013):

\[
B^\perp = \frac{1}{1 - 2g} \left[ \frac{1 - (1 - 2g)^2 \Delta_N}{1 - \Delta_N} \right]^{1/2} \tag{5.58}
\]
Figure 5.2: Curves representing the variation of the $B$ gradient with azimuth and Vs/Vp ratio (figure in the top), crack filling fluids (figure in the middle) and crack densities (figure in the lower part)
if we consider $\Delta_N$ for thin, liquid-filled cracks, then it goes to zero and the exact equation 5.58 becomes:

$$B^\perp \equiv \frac{2(V_s/V_p)^2}{1 - 2(V_s/V_p)^2} \quad (5.59)$$

Equation 5.59 is the approximation of the $B^\perp$ value that can be solved for the $V_s/V_p$ ratio as follows (Chichinina et al., 2006):

$$\frac{V_s}{V_p} \approx \frac{1}{\sqrt{2(1 + \frac{1}{B^\perp})}} \quad (5.60)$$

From the real data used in this project, it is possible to obtain the B gradient, $V_s/V_p$ ratio, $\epsilon_Q$, and the direction of fracture which comes from $\phi_0$. To compare and calibrate these final results, well data are needed. Modeling theoretical curves would then be possible, as in the Figure 5.2, with real parameters from the well. Then the QVOA results are plotted from the seismic data to find the best fit with these curves, and to have a better idea of the infill fluid. In the chapter 7, the application and results (fracture orientation and B-gradient) from the QVOA method are discussed.
A QVOA program was coded in Matlab\textsuperscript{TM} (Xenophon, 1999).

There are two main programs, one of them is designed to solve QVOA equation using the approximated method of sectors (ASM) called "amplitud\_espectra\_nosectors\_no\_manual", and the other one is designed to solve the equation using Approximate Truncate Method (ATM) called "amplitud\_espectra\_sectors\_no\_manual".

### 6.1 QVOA Approximate Truncate Method (ATM)

To use this program some inputs with specific characteristics must be given:

1. Common Mid Point (CMP) gather files without Normal Move Out (NMO) correction (See Figure 6.1 ). Gathers must be sorted in Superbins with .sgy or .segy format (See Figure 6.2 ) in case only one bin has not enough traces. In the case of this project, the superbins are composed of 9 bins.

2. CMP gather files with NMO correction. Superbins. These gathers must correspond with the same gathers used without NMO correction. In case the same CMP gather is not available, then the closest one can be used. These files are used only for correlation effects (.sgy or .segy) taking the top and bottom time of the reservoir at an specific trace.

3. Surfaces (horizons) of the top and base of your reservoir in time. The inputs must be organized in three different files that must contain (See Figure 6.3 ):
   
   (a) X-coordinates of top and base of the reservoir in all along the horizon,

   (b) Y-coordinates of top and base of the reservoir in all along the horizon, and
Figure 6.1: Common Mid Point gather (CMP)

Figure 6.2: Superbin. The files that are used for this project use 3X3 bins, in total 9 bins compose a superbin in this case
(c) time from horizons.

All of them in .ascii format.

The program looks into the coordinates of the trace that it is working with. Using these coordinates, it looks into the files from the horizon to get the time of the top and bottom of the interval of interest.

Figure 6.3: Five surfaces where created for this project. The second surface from bottom to top, corresponds to the top of the reservoir. Wells with velocity logs are also displayed. The bar color represents the depths from the surfaces.

4. A velocity cube (.sgy or .segy). Figure 6.4 displays a section of the final velocity cube built for the field of interest. In the figure a bar color indicates that from pink (in the lower part of the bar) to red (in the upper part of the bar) there is a variation of velocity from slow (pink) to fast (red). In the figure, there is a zone of lower velocities (in blue) in between two layers of higher velocities, representative of the lithology in the area.

Two other elements are displayed in the (See Figure 6.4), a surface, and wells. Those wells are displaying the velocities that correspond to the synthetic seismogram created for each one of the wells. There is a good match between the velocities in the wells and
Figure 6.4: An Inline and Xline showing the velocities from the velocity cube created for this project. This velocity model was created using the synthetic seismograms from well logs (density and sonic logs), tops from the wells and the horizons showed in Figure 6.3 the velocity. The interval of interest in this study is the one with the higher velocity (red interval), which corresponds to a shallow naturally fractured carbonate. The limits between the different velocity layers, come from the fact that no rms cube velocity was used to calibrate this model. Only layers, wells logs and tops were available.

5. There is a section in the program where one of three different options must be selected.

In case of selecting “to upload a file” with the values of the top and bottom of the interval of interest the file must be created in .ascii format (See Figure 6.5).

This file must contain two columns. The first column corresponds to the traces which are sorted from the minimum trace to the higher trace, twice. The second column corresponds to the value in time of the top and bottom of the interval of interest, arranged from shallower to deeper value per trace.

6. Three different options to calculate azimuths in the seismic traces. When selecting “reading azimuths from previous sectored files”, .sgy or .segy files are needed. This
Figure 6.5: To upload a file into the program, it must be by two columns: number of the trace, and the time values from the top and the bottom of the interval of interest.

files most correspond to the same gather listed as the first item previously sectored. When selecting “Reading azimuths from trace header” a .sgy or .segy file is needed. In this files azimuths of each trace are located in the header from where the program is going to read them. The third option will calculate the azimuths by itself, no need of any file.

The body of the main program consist of 5 main steps:

1. Reading: during the reading process, program will ask for all the inputs listed above.

2. Selecting: during the Selecting process, program will ask to select from various options. The first set of selections is oriented to select the top and bottom of the interval of interest. The three options for this are:

   (a) select top and bottom manually. If this option is selected, the user will have to write directly in the screen the values of the top and bottom of the interval of
interest,

(b) select top and bottom “uploading a file” that contains the values required. If this option is selected, the user must have a file prepared with the values of the top and bottom of the interval of interest (See Figure 6.5), and

(c) select top and bottom picking them directly from a graph. If this option is selected, the user will have to pick the values of the top and bottom of the interval of interest directly from the seismic traces displayed in a graph.

Figure 6.6 is an example of how the graph looks like when you are picking values. In the first trace we can see two blue crosses that are indicating the top and the bottom of the interval of interest. Selecting a value is very simple, pressing the enter bottom and then clicking over the place of interest. The program allows zooming/ unzooming of the traces for better picking (See Figure 6.7).

Figure 6.6 displays the same three traces with a zoom on them. The black cross that cover all the image appears when pressing enter bottom.
Another set of options is oriented to read or calculate the azimuths of each trace. In this way three selections are available: 1.-Reading from previous sectored files: where the user has to give the name of these files. 2.-Let the program to calculate the azimuths using the function “[latitude1,longitude1 ] = coordenadasUTM(lat1,lon1)” (this program is displayed in the appendix) to convert the UTM coordinates to latitude and longitude coordinates. 3.-Reading the azimuth from trace headers: CWP has a program called: “SUAZIMUTH”, that helped me to calculated the azimuths of each traces putting this value in their headers. The program will ask for the name of this file to extract the value of the azimuth from the header.

The last set of options that are used by the program is oriented to let the program to know if it has to filter the data or not. The type of filter used in this program is called “predictive”. 
3. Calling: the main program will call at some point the function named: “atenuation-sects_fornosectors”. The input data for this function are:

(a) seismic traces data (Data),

(b) trace headers (SegyTraceHeaders),

(c) header of the seismic file (SegyHeader),

(d) number of traces (numberoftrace),

(e) the option selected to get the values from top and bottom of the interval of interest (tf1, tf2, tf3),

(f) interval velocities (IntVel),

(g) trace headers of the velocities file (SegyTraceHeadersVel),

(h) the top and bottom time values (timetophor and timebasehor),

(i) seismic traces data from the gather with Vnmo correction (DataVnmo),

(j) from what number to what number of traces the program has to use to make calculations (fromnumber,tonumber),

(k) the total number of traces (xvalueprev),

(l) X and Y coordinate vectors from the velocity file (xvel,yvel),

(m) pair of coordinates from velocity file (pairs), and

(n) the time vector (timeVel).

The data that this function gives back to the main program are: Values of the attenuation from each trace selected from four different methods. alpha11 comes from spectral ratio, alpha22 comes from central shift frequency method, alpha33 comes from peak shift frequency method, and alpha44 comes from central and peak shift frequency method. This function also gives back the angle of incidence from each trace, half of
the length of traces in this case called Xgraphex (Xgra), length of traces (Xgraphex), X and Y middle point of the Receiver and Source in each trace (Xcoord2,Ycoord2).

This function calculates the amplitude spectra, depths and travel times needed for attenuation calculation. This function will call other small functions that are going to give back the attenuation value from each traces. The name of the small functions are:

(a) “Specratio” that corresponds to the spectral ratio method.

Inputs: Spectral amplitude from the top and bottom of the interval of interest (Ampespcvec1,Ampespcvec2), frequencies from the top of the interval of interest (Freqespcvec1) and travel times (traveltime).

Outputs: Quality factor (Q) and attenuation values (alpha1).

(b) “Centralfreq” which corresponds to the central shift frequency method.

Inputs: Spectral amplitude from the top and bottom of the interval of interest (Ampespcvec1,Ampespcvec2), frequencies from the top and bottom of the interval of interest (Freqespcvec1,Freqespcvec2) and travel times (traveltime).

Outputs: Quality factor (Q2), attenuation (alpha2) and eta values.

(c) “Shiftfreq” that corresponds to the peak frequency method.

Inputs: Travel times (traveltime), maximum or peak frequency from top and bottom of the interval of interest (freqmax2,freqmax1).

Outputs: Quality factor (QPF1), attenuation (alpha3).

(d) “Centralpeakfreq” that corresponds to the central and peak shift frequency method.

Inputs: Travel times (traveltime), maximum or peak frequency from the top of the interval of interest (freqmax1).

Outputs: Quality factor (QPF2), attenuation (alpha4).

4. Calculating: in the main program the QVOA equation is solved.
5. Plotting: plots that come from the main program will consist on: Rose plots where the direction of symmetry axis and fractures are plotted, graphs that plot the “B gradient” vs azimuths from the traces.

6. Saving: the main program will save all the following data into .ascii files:

(a) attenuation values from the four methods used,
(b) azimuths of the symmetry axis and fracture direction from the four methods,
(c) top and bottom time values selected from the interval of interest in each trace,
(d) the number of traces that were used by the program as well as their corresponding offset, and
(e) the values resulted from QVOA equation as: a, b and B gradient.

6.2 QVOA Approximate Method of Sectors (AMS)

Everything listed in the previous section applies for this program, with the following minor differences:

Inputs: The CMP gather does not correspond to the whole file, this time the program will ask for the name of the sectored files. The calculation of azimuth is not needed anymore because it is taken from the same files.

The body program consist in the same main steps as in the previous section with the difference that it won’t calculate any azimuth. We will receive the exact same plots and files saved.

All the functions created for this project are listed in the appendix. Other functions that are not of my authorship were used and they are also listed in the appendix. Numerical experiments with real data must not use $\phi_o$, but other parameters for a comparison of the techniques, because of the correct value $\phi_o$ is not known in field data.
Naturally fractured reservoirs are the focus of this project. These reservoirs must be massive or thick enough (two to three wavelengths) to secure that the attenuation effects are measured. The measure of the attenuation in thinner reservoirs can be diminished by the presence of multiples or other thin layer effects. In this project, data from a massive carbonate reservoir with enough thickness is available.

The information available for this project is displayed in Table 7.1. The seismic data available for the application of the QVOA method consist of CMP gathers with and without Vnmo correction. CMP gathers are not available for the whole area, but only around the location of the wells. There are 11 wells total with either seismic information or tadpole logs. In Table 7.1 these wells are displayed. Four wells have image log information, but not seismic data. Five wells only have CMP gather information, but not tadpole logs. Well-3 has information from image logs (tadpoles) and seismic information in the interval of interest. QVOA analysis was performed wherever seismic information was available.

As a result of the interpretation of the image logs, tadpole logs are created. For this project, only the tadpole logs were available. The tadpole logs mainly gives the layer and fracture directions in the interval of interest. These fracture directions help to calibrate the fracture direction obtained from the QVOA method. Other parameters (e.g., anisotropy attenuation, and \( \frac{V_s}{V_p} \) ratio) resulted from the QVOA method can also be calibrated. However, to calibrate the anisotropic parameters obtained by QVOA more information is needed such as: image logs, production reports, etc.

The CMP gathers (with and without \( V_{nmo} \)) for this project, were processed trying to preserve the amplitude and the frequencies as much as possible. The specific process applied to these gathers were not provide but a general idea of it was given. The processing includes:
<table>
<thead>
<tr>
<th>Well</th>
<th>Seismic information</th>
<th>Tadpoles from image logs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CMP gathers (superbin)</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>CMP gathers (superbin)</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>CMP gathers (superbin)</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>CMP gathers (superbin)</td>
<td>No</td>
</tr>
<tr>
<td>6</td>
<td>CMP gathers (superbin)</td>
<td>No</td>
</tr>
<tr>
<td>7</td>
<td>CMP gathers (superbin)</td>
<td>No</td>
</tr>
<tr>
<td>8</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>9</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>10</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>11</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

- spherical divergence,
- geometrical deconvolution gain, and
- filters (e.g., radon).

In Figure 6.1, a real common mid point gather is displayed. To have azimuthal coverage, 9 CMP gathers in square configuration were attached to form a superbin (see Figure 6.2). The central bin coordinates are now the central coordinates of the superbin. These central coordinates are used to localize the superbin over the top horizon of the reservoir. In Figure 7.1 the CMP gathers are displayed, represented by a white dots. Once these gathers are processed with the QVOA program, each white dot will be replace by a rose graph representing the fracture direction.

Figure 7.2 shows the 11 available wells over the top of the reservoir. The majority of the wells available are located in the north-east of the field. Two of them are located in the north-west. Figure 7.3 shows the localization of the four wells that have tadpole logs information displayed over the top of the reservoir. In this case, the surface displayed in Figure 7.3 is showing a geometrical seismic attribute called “edge evidence”. This seismic attribute was obtained using the PETREL software using a time window of 400ms. Other
geometrical attributes were calculated without obtaining good results. Geometrical seismic attributes can help to calibrate the fracture directions obtained from the QVOA analysis with the main fracture directions guided by the surface of interest. The tadpoles located in the most noisy part of the surface in Figure 7.3 are showing different fracture directions, suggesting that there is not a main direction defined in this area. On the other hand, the closer the tadpoles to the center of the structure, the best well defined orientations are being shown.

According to section 5.2, two different methods are used to solve the QVOA equation: Approximate method of sectors (AMS) and approximate truncate method (ATM). In section 4.2, four different methods to calculate attenuation were introduced: Spectral ratio (SR), centroid frequency shift (CFS), peak frequency shift (PFS), and dominant and central frequency shift method (DCFS). The Well-3 is the only well that has both information, CMP gathers and tadpole log. The two methods for solving the QVOA equation and the four methods to calculate attenuation are applied to each one of the gathers in Well-3. The different methods are compared between them and calibrated with the tadpole log from the Well-3.
Two, are the attributes obtained from the QVOA program: fracture direction displayed using rose graphs, and B-gradient. To interpret the fracture direction one must be careful in knowing the rules. Figure 7.4 shows an example of a rose graph representing the fracture direction. Other parameters such as: a, b, and error, are also displayed. The error parameter comes from the relation of the a and b parameters to interpret the fracture direction as follows:

- The parameter b determines which direction (in blue or red) corresponds to symmetry axis or fracture orientation.
  - If b is positive, the blue direction represents the symmetry axis and the red direction represents the fracture orientation.
  - If b is negative, the blue direction represents the fracture orientation and the red direction represents the symmetry axis.
Figure 7.3: Well logs plots of fractures are displayed over the top of the reservoir. The seismic attribute “edge evidence” was calculated over the surface. The color bar represents the values of the “edge evidence” attribute.
• The parameters a and b together, lets the user know if the error of the final results are big or small.
  
  – If b and a have the same sign, the error is small.
  
  – If b and a have different sign, the error might not be small.

In the next section, the seismic information from Well-3 is used and results are shown.

![Figure 7.4: Results from QVOA equation. The parameter b must be carefully interpreted.](image)

7.1 Well-3

In Well-3 the tadpole information from image logs does not coincide in depth with the top of the reservoir, but the interval is located inside the reservoir. Tadpoles are a very important source of information because they measure the direction of fractures and layers in the reservoir. One of the final outputs from the QVOA equation is the direction of faults. In Well-3, a correlation between well information (from tadpoles) and seismic data (output
from QVOA) is possible. In Figure 7.6, the rose graph of Well-3 is displayed. It shows the main direction in Well-3 which is between 0-15 degrees NE-SW. Minor directions are also displayed in the same graph with orientations NE-SW and NW-SE.

Well-3 has a total of 20 gathers around the well (see Figure 7.39). In these section, results and analysis from four gathers processed around the well location are shown. The location of these gathers are represented with black fill-in color in the base map of Figure 7.5. Inlines and xlines are increasing from left to right and from top to bottom. The well is located in between the gathers. Its rose graph from tadpoles is displayed also in the base map. The results from all the gathers are shown in this order:

- Fracture direction.
  - Gather 1256-1369. Results from the ATM and AMS methods, using:
    1. spectral ratio,
    2. centroid frequency shift,
    3. peak frequency shift, and
    4. dominant and centroid frequency shift.
  - Gather 1256-1372. Results from the ATM and AMS methods, using:
    1. spectral ratio,
    2. centroid frequency shift,
    3. peak frequency shift, and
    4. dominant and centroid frequency shift.
  - Gather 1253-1369. Results from the ATM and AMS methods, using:
    1. spectral ratio,
    2. centroid frequency shift,
    3. peak frequency shift, and
    4. dominant and centroid frequency shift.
Gather 1253-1372. Results from the ATM and AMS methods, using:

1. spectral ratio,
2. centroid frequency shift,
3. peak frequency shift, and
4. dominant and centroid frequency shift.

- B- gradient.

Gather 1256-1369. Results from the ATM and AMS methods, using:

1. spectral ratio,
2. centroid frequency shift,
3. peak frequency shift, and
4. dominant and centroid frequency shift.

Gather 1256-1372. Results from the ATM and AMS methods, using:

1. spectral ratio,
2. centroid frequency shift,
3. peak frequency shift, and
4. dominant and centroid frequency shift.

Gather 1253-1369. Results from the ATM and AMS methods, using:

1. spectral ratio,
2. centroid frequency shift,
3. peak frequency shift, and
4. dominant and centroid frequency shift.

Gather 1253-1372. Results from the ATM and AMS methods, using:

1. spectral ratio,
2. centroid frequency shift,
3. peak frequency shift, and
4. dominant and centroid frequency shift.

Figure 7.5: Base-map that correspond to Well-3. Location of gathers are represented with black fill-in. The rose graph corresponding to the well tad poles is displayed in the location of the well.

### 7.1.1 Fracture direction: Gather 1256-1369

The first gather that was processed is colored in red (see Figure 7.7). It corresponds to the gather with inline 1256 and crossline 1369.

1. ATM and AMS using spectral ratio: In Figure 7.8, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing the $a$ and $b$ parameters to be the same sign, but still, the error is 28, which is high. Parameter $b$ is positive, this means that the red direction corresponds to the fracture orientation. The AMS method, on the other hand, is showing a small error of $49 \times 10^{-3}$ in the results. This error is very small compared to the one obtained from the ATM method. Parameter $b$ is negative which means the blue direction represents the fracture direction.
2. ATM and AMS using centroid frequency shift: In Figure 7.9, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 1.4, which is considerably lower compared with the ATM from spectral ratio. Parameter b is negative, this means that the blue direction corresponds to the fracture orientation. The AMS method is showing a lower error in the results (3X10\(^{-3}\)). Parameter b is positive, that which means the red direction represents the fracture direction.

3. ATM and AMS using peak frequency shift: In Figure 7.10, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 0.9, which is considerably lower compared with the ATM from spectral ratio. Parameter b is
negative, this means that the blue direction corresponds to the fracture orientation. The AMS method is showing a smaller error in the results \(45 \times 10^{-4}\). Parameter \(b\) is negative, that means the blue direction represents the fracture direction.

4. ATM and AMS using dominant and centroid frequency shift: In Figure 7.11, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 8.05, which is considerably lower compared with the ATM from spectral ratio. Parameter \(b\) is positive, this means that the red direction corresponds to the fracture orientation. The AMS method is showing a smaller error in the results \(37 \times 10^{-4}\). Parameter \(b\) is positive, that means the red direction represents the fracture direction.

### 7.1.2 Fracture direction: Gather 1256-1372

The second gather that was processed is colored with red (See Figure 7.12). It corresponds to the gather with inline 1256 and crossline 1372.
Figure 7.8: ATM (left) and AMS (right) methods, using spectral ratio for attenuation determination.

1. ATM and AMS using spectral ratio: In Figure 7.13, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing the a and b parameters to be the same sign, but still, the error is 18.67, which is high. Parameter b is positive, this means that the red direction corresponds to the fracture orientation. The AMS method, on the other hand, is showing an error of $39 \times 10^{-3}$ in the results. This error is very small compared with the one obtained from the ATM method. Parameter b is positive which means the red direction to represent the fracture direction.

2. ATM and AMS using centroid frequency shift: In Figure 7.14, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 1.37, which is considerably lower compared with the ATM from spectral ratio. Parameter b is negative, this means that the blue direction corresponds to the fracture orientation. The AMS method is showing a lower error in the results ($27 \times 10^{-4}$). Parameter b is positive, which means the red direction to represent the fracture direction.
3. ATM and AMS using peak frequency shift: In Figure 7.15, one rose graph is displayed. It corresponds to the AMS method which is showing a small error in the results ($26 \times 10^{-3}$). Parameter $b$ is positive, this means the red direction represents the fracture direction. This is a particular case where the peak frequency method from the ATM case fails, because the variation of the attenuation is minimal.

4. ATM and AMS using dominant and centroid frequency shift: In Figure 7.16, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 5.8884, which is considerably lower compared with the ATM from spectral ratio. Parameter $b$ is negative, this means that the blue direction corresponds to the fracture orientation. The AMS method is showing a lower error in the results ($33 \times 10^{-4}$). Parameter $b$ is negative which means the blue direction to represent the fracture direction.
7.1.3 Fracture direction: Gather 1253-1369

The third gather that was processed is colored in red (See Figure 7.17). It corresponds to the gather with inline 1253 and crossline 1369.

1. ATM and AMS using spectral ratio: In Figure 7.18, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing the a and b parameters to be same sing, but still, the error is 18.67, which is high. Parameter b is positive, this means that the red direction corresponds to the fracture orientation. The AMS method, on the other hand, is showing an error of $82 \times 10^{-3}$ in the results. This error is very small compared with the one obtained from the ATM method. Parameter b is negative which means the blue direction to represent the fracture direction.

2. ATM and AMS using centroid frequency shift: In Figure 7.19, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 1.37, which is considerably lower compared with the ATM from spectral ratio. Parameter b
is negative, this means that the blue direction corresponds to the fracture orientation. The AMS method is showing a lower error in the results ($16 \times 10^{-3}$). Parameter $b$ is negative, which means the blue direction to represent the fracture direction.

3. ATM and AMS using peak frequency shift: In Figure 7.20, one rose graph is displayed. It corresponds to the method AMS which is showing an small error in the results ($99 \times 10^{-3}$). Parameter $b$ is negative, that makes the blue direction to represent the fracture direction. This is a particular case or example where the peak frequency method in the ATM case fails, because the variation of the attenuation is minimal.

4. ATM and AMS using dominant and centroid frequency shift: In Figure 7.21, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 5.9, which is considerable lower compared with the ATM from spectral ratio. Parameter $b$ is negative, this means that the blue direction corresponds to the fracture orientation. The AMS method is showing lower error in the results ($19 \times 10^{-4}$). Parameter $b$ is positive which means the red direction to represent the fracture direction.
7.1.4 Fracture direction: Gather 1253-1369

The fourth gather that was processed is colored in red in (See Figure 7.22). It corresponds to the gather with inline 1253 and crossline 1372.

1. ATM and AMS using spectral ratio: In Figure 7.23, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. Here the ATM method is showing different behavior compared with the previous gathers. Here, the error is the highest (30.34), and is also giving different sign in a and b. Parameter b is negative, this means that the blue direction corresponds to the fracture orientation. The AMS method, on the other hand, is showing an error of $17 \times 10^{-3}$ in the results. This error is very small compared with the one obtained from the ATM method. Parameter b is positive which means the red direction to represent the fracture direction.

2. ATM and AMS using centroid frequency shift: In Figure 7.24, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 1.72,
Figure 7.13: ATM (left) and AMS (right) methods, using spectral ratio for attenuation determination.

which is considerably lower compared with the ATM from spectral ratio. Parameter b is negative, this means that the blue direction corresponds to the fracture orientation. The AMS method is showing a lower error in the results ($12 \times 10^{-3}$). Parameter b is negative, which means the blue direction to represent the fracture direction.

3. ATM and AMS using peak frequency shift: In Figure 7.25, two rose graphs are displayed. The one on the left corresponds to the ATM methods, which is showing a large error in the results (3.48). Parameter b is positive, this makes the red direction to represent the fracture direction. The one on the right is from AMS method, it is showing an small error of $14 \times 10^{-3}$. Parameter b is positive, then red corresponds to the fracture orientation.

4. ATM and AMS using dominant and centroid frequency shift: In Figure 7.26, two rose graphs are displayed. The one on the left corresponds to the AMS method and the one on the right corresponds to the ATM method. The ATM method is showing an error of 8.77, which is considerable lower compared with the ATM from spectral ratio. Parameter b is negative, this means that the blue direction corresponds to
the fracture orientation. The AMS method is showing a lower error in the results ($3 \times 10^{-3}$). Parameter $b$ is negative, this makes the blue direction to represent the fracture direction.

Comparing the final results with Figure 7.6 the following conclusions can be made:

1. ATM method is yielding higher errors comparing with the AMS method.

2. Spectral ration gives the highest errors overall.

3. Comparing the four gathers, and focusing only on the spectral ratio, the best approximation to real fracture direction from the well data (See Figure 7.6) is being by the AMS method.

4. Comparing the four gathers and focusing only on the centroid frequency shift method, the best approximation to the fracture direction from the well (See Figure 7.6) is being given by both methods (ATM and AMS), but AMS seems to give less error in its results.
5. Comparing the four gathers, and focusing only on the peak frequency shift method, the best approximation to the fracture direction from the well (See Figure 7.6) is being given by the AMS method. In the case of ATM when applying the peak frequency the method breaks down when the values of the attenuation are small. From these results, it can be concluded that peak frequency is not a robust method (see section 4.2.4).

6. Comparing the four gathers, and focusing only on the dominant and centroid frequency shift method, the best approximation to the fracture direction from the well (See Figure 7.6) is obtained by both methods (ATM and AMS). The approximation with well data is good and the errors in general are low.

7. In general AMS gives the better approximations to real data. The ATM method gives good approximation using the dominant and centroid frequency shift method.

8. The dominant and centroid frequency shift method is more precise and stable compared with the rest of the methods.
Another output from the QVOA equation is the $B$ gradient. It is a very important seismic attribute that can help get values of the anisotropic parameter $\epsilon_Q$. Another value that can be obtained using the $B$ gradient is the $Vs/Vp$ ratio.

This parameter was also calculated for each one of the gathers shown before. The B-gradient is then plotted vs azimuth. The form of the B-gradient curve is fitted by a $\cos^2\phi$ function. The azimuths corresponding with the maximum values in the B gradient represent the direction of the symmetry axis, while the azimuths with the minimum values of B-gradient represent the direction of the fracture in the isotropic plane.

### 7.1.5 B-gradient: Gather 1256-1369

Figure 7.27 shows the B-gradient values obtained for the gather 1256-1369 using the four methods. As concluded before, spectral ratio results show high errors, so it’s logical that the B gradient will also have high errors. The upper-left graph in Figure 7.27, corresponds to the spectral ratio method. It is showing values of gradient from 28 to 32. The minimum gradient value is given at approximately 90 degrees, which corresponds to the attenuation value in the isotropic plane. The maximum value occurs at 10 degrees, which corresponds
to the symmetry axis direction.

The upper-right graph is showing the values of the B-gradient from the centroid frequency shift method. This method has a lower error compared with the spectral ratio method. The maximum value (1.6) occurs at approximately 80 degrees with its mirror occurring at approximately 270 degrees which represent the symmetry axis direction. The minimum value (1.26) occurs at approximately zero degrees and its mirror at 170 degrees, which represents the average direction of the fractures.

The lower-left graph is showing the values of the B-gradient from the peak frequency shift method. The maximum value (1.0) occurs at approximately 70 degrees with its mirror occurring at approximately 250 degrees, which represents the symmetry axis direction. The minimum value (-4.5) occurs at approximately 160 degrees and its mirror at 350 degrees, which represents the average direction of the fractures.

The lower-right graph is showing the values of the B-gradient from the dominant and centroid frequency shift method. This method showed more precision and less error in its final results. The maximum value (9.2) occurs at approximately 40 degrees with its mirror occurring at approximately 220 degrees, which represents the symmetry axis direction. The
minimum value (8.0) occurs at approximately 140 degrees and its mirror at 310 degrees, which represents the average direction of the fractures.

Using the equations 7.1, 7.2 and 7.3, $\epsilon_Q$ and $V_s/V_p$ can be obtained from the B gradient plotted for the different methods.

\[
B^\perp = \frac{(Q^\perp)^{-1/2} - (Q^\parallel)^{-1/2}}{(Q^\parallel)^{-1/2}} \tag{7.1}
\]

\[
\epsilon_Q = 0.5\left(\frac{1}{[B^\perp + 1]^2} - 1\right) \tag{7.2}
\]

\[
\frac{V_s}{V_p} \approx \frac{1}{\sqrt{2(1 + \frac{1}{B^\perp})}} \tag{7.3}
\]

Figure 7.28 shows the values obtained from the B-gradient of the previous graphs. The values representing the $\epsilon_Q$ in all the four methods varies from -0.5 min to -0.375 max. These values must be negative because the attenuation in the isotropic plane is higher then the attenuation in the symmetry axis direction. The values of the $V_s/V_p$ ratio varies from 0.5 min to 0.696 max. The method with the higher error is the SP method. The method with the less error is the DCFS method. SP and DCFS are showing very similar anisotropy values.
The anisotropy values from the PFS method are the smallest.

7.1.6 B-gradient: Gather 1256-1372

Figure 7.29 displays the b-gradient values obtained for the gather: 1256-1372 from all the four methods. The upper-left graph in Figure 7.29, corresponds to the spectral ratio method. It shows values of gradient from 18.5 to 22. The minimum gradient value is given at approximately 100 degrees, which corresponds to the attenuation value in the isotropic plane. The maximum value occurs at 20 degrees, which corresponds to the symmetry axis direction.

The upper-right graph is showing the values of B-gradient from the centroid frequency shift method. This method has lower error compared with the spectral ratio. The maximum value (1.4) occurs at approximately 70 degrees with its mirror occurring at approximately 250 degrees, which represents the symmetry axis direction. The minimum value (1.1) occurs at approximately 150 degrees and its mirror at 340 degrees, which represents the average direction of the fractures.
Figure 7.20: AMS method, using peak frequency shift for attenuation determination

The lower graph is showing the values of B-gradient from the dominant and centroid frequency shift method. This method shows more precision and less error in its final results. The maximum value (5.9) occurs at approximately 100 degrees with its mirror occurring at approximately 280 degrees, which represents the symmetry axis direction. The minimum value (5.2) occurs at approximately 10 degrees and its mirror at 190 degrees, which represents the average direction of the fractures. Using the equations 7.1, 7.2 and 7.3, $\epsilon_Q$ and $V_s/V_p$ can be obtained from the B gradient plotted for the different methods.

Figure 7.30 shows the values obtained from the B-gradient of the previous graphs. The values representing the $\epsilon_Q$ from the three methods varies from -0.499 min to -0.413 max. These values must be negative because the attenuation in the isotropic plane is higher then the attenuation in the symmetry axis direction. The values of the $V_s/V_p$ ratio varies from 0.54 min to 0.692 max. In this case all values are similar with an average of: -0.467 for $\epsilon_Q$ and 0.628 for $V_s/V_p$ ratio.
Figure 7.21: ATM (left) and AMS (right) methods, using dominant and centroid frequency shift for attenuation calculation

7.1.7 B-gradient: Gather 1253-1369

Figure 7.31 shows the b-gradient values obtained for the gather: 1253-1369 from all the four methods. The upper-left graph in the Figure 7.31 corresponds to the spectral ratio method. It is showing values of gradient from 18.5 to 22. The minimum gradient value is given at approximately 100 degrees which corresponds to the attenuation value in the isotropic plane. The maximum value occurs at 30 degrees which corresponds to the symmetry axis direction.

The upper-right graph is showing the values of B-gradient from the centroid frequency shift method. This method has a lower error compared with the spectral ratio. The maximum value (1.4) occurs at approximately 70 degrees with its mirror occurring at approximately 250 degrees, which represents the symmetry axis direction. The minimum value (1.1) occurs at approximately 150 degrees and its mirror at 340 degrees, which represents the average direction of the fractures.

The lower graph is showing the values of B-gradient from the dominant and centroid frequency shift method. This method shows more precision and less error in its final results.
The maximum value (5.9) occurs at approximately 100 degrees, with its mirror occurring at approximately 280 degrees, which represents the symmetry axis direction. The minimum value (5.2) occurs at approximately 10 degrees and its mirror at 190 degrees, which represents the average direction of the fractures.

Using the equations 7.1, 7.2 and 7.3, $\epsilon_Q$ and $V_s/V_p$ can be obtained from the B gradient plotted for the different methods. shows the values obtained from the B-gradient of the previous graphs. The values representing the $\epsilon_Q$ from the three methods varies from -0.499 min to -0.414 max. These values must be negative because the attenuation in the isotropic plane is higher then the attenuation in the symmetry axis direction. The values of the $V_s/V_p$ ratio varies from 0.541 min to 0.692 max. In this case all values are similar with an average of: -0.468 for $\epsilon_Q$ and 0.629 for $V_s/V_p$ ratio.

### 7.1.8 B-gradient: Gather 1253-1372

Figure 7.33 shows the b-gradient values obtained for the gather: 1253-1372 from all the four methods. As concluded before, spectral ratio results show high errors, so it’s logical that the B gradient will also have high errors. The upper-left graph in Figure 7.33, corresponds
Figure 7.23: ATM (left) and AMS (right) methods, using spectral ratio for attenuation determination.

to the spectral ratio method. It shows values of gradient from 22 to 31. The minimum gradient value is given at approximately 0 degrees, which corresponds to the attenuation value in the isotropic plane. The maximum value occurs at 100 degrees, which corresponds to the symmetry axis direction.

The upper-right graph is showing the values of B-gradient from the centroid frequency shift method. This method has a lower error compared with the spectral ratio. The maximum value (1.7) occurs at approximately 120 degrees with its mirror occurring at approximately 300 degrees, which represent the symmetry axis direction. The minimum value (0.3) occurs at approximately 30 degrees and its mirror at 210 degrees, which represents the average direction of the fractures.

The lower-left graph is showing the values of B-gradient from the peak frequency shift method. The maximum value (-1.0) occurs at approximately 160 degrees with its mirror occurring at approximately 320 degrees, which represents the symmetry axis direction. The minimum value (-3.5) occurs at approximately 50 degrees and its mirror at 240 degrees, which represents the average direction of the fractures.
Figure 7.24: ATM (left) and AMS (right) methods, using centroid frequency shift for attenuation determination

The lower-right graph is showing the values of B-gradient from the dominant and centroid frequency shift method. This method shows more precision and less error in its final results. The maximum value (9) occurs at approximately 80 degrees with its mirror occurring at approximately 260 degrees, which represents the symmetry axis direction. The minimum value (3.5) occurs at approximately 170 degrees and its mirror at 350 degrees, which represents the average direction of the fractures.

Using the equations 7.1, 7.2 and 7.3, $\epsilon_Q$ and $V_s/V_p$ can be obtained from the B gradient plotted for the different methods. Figure 7.34 shows the values obtained from the B-gradient of the previous graphs. The values representing the $\epsilon_Q$ from the three methods varies from -0.5 min to -0.431 max. These values must be negative because the attenuation in the isotropic plane is higher then the attenuation in the symmetry axis direction. The values of the $V_s/V_p$ ratio varies from 0.561 min to 0.696 max.

Looking at the results from all the four gathers, the following conclusions can be made:

1. Spectral ratio method yields the highest error values. Apparently these errors does not affect the calculation of the anisotropy term $\epsilon_Q$ and the ratio $V_s/V_p$ value.
2. In each one of the gathers in the four methods, the values of $\epsilon_Q$ and $V_s/V_p$ are close to each other.

3. PFS method showed very different values of $\epsilon_Q$ and $V_s/V_p$ compared with the other methods. Also it was the most unstable.

Results from all the 20 gathers surrounding the Well-3 are displayed in Figure 7.39. Notice that each CMP gather located over the surface is giving a fracture orientation. The area covered by all these gathers is small. It’s important then, to represent the total orientation from all the gathers in only one graph that covers this area, when showing the results over the complete surface.

### 7.1.9 Filtering

To identify how much the results are affected by any type of processing or filtering applied to the data. A simple low pass filter is applied to the seismic data. This filtered information was also analyze using the QVOA program. Figure 7.35 shows the spectral ratio method with and without filtering from both the ATM and AMS methods. The two graphs in the
top are related to the ATM method, to the left without a filter, to the right with a filter. The error in the right graph is smaller than the one without a filter, and the dominant fracture direction is affected slightly. The two graphs in the bottom are behaving in the same way. Still, the approximations obtained without filtering the data are better with respect to the fracture directions from well data.

This exercise was applied to all the other methods giving the same results. Only one method (dominant and centroid frequency shift) was not affected by filtering. In Figure 7.36, the final results with and without filtering for ATM method is displayed. No parameter (error and fracture direction) seemed to be affected by the application of filtering the data.

7.1.10 Wells without image logs

As mentioned before, there are 5 wells which only have seismic information but not image logs. QVOA analysis was applied to these wells as well. The results of this analysis are displayed in Figure 7.38. The surface in Figure 7.38 is showing the geometrical attribute “edge evidence” which reveal the main fracture direction in the area. This surface is used to calibrate the results obtained from the QVOA analysis over the wells without tadpole
Figure 7.27: Gradient from the gather 1256-1369 obtained from the four different methods to determine attenuation.

Figure 7.28: $\epsilon_Q$ and $V_s/V_p$ obtained from the B gradient for each method. "$B_{\perp}$ is the value for $B_{\perp}$. 

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>$\epsilon_Q$</th>
<th>$V_s/V_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>32.00</td>
<td>-0.500</td>
<td>0.696</td>
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<tr>
<td>CFS</td>
<td>1.60</td>
<td>-0.426</td>
<td>0.555</td>
</tr>
<tr>
<td>PFS</td>
<td>1.00</td>
<td>-0.375</td>
<td>0.500</td>
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<tr>
<td>DCFS</td>
<td>9.20</td>
<td>-0.495</td>
<td>0.672</td>
</tr>
</tbody>
</table>
Figure 7.29: Gradient from the gather 1256-1372 obtained from the four different methods to determine attenuation.

Figure 7.30: $\epsilon_Q$ and $V_s/V_p$ obtained from the B gradient for each method. "B perp is the value for $B^\perp$. 

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>$\epsilon_Q$</th>
<th>$V_s/V_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>22.00</td>
<td>-0.499</td>
<td>0.692</td>
</tr>
<tr>
<td>CFS</td>
<td>1.40</td>
<td>-0.413</td>
<td>0.540</td>
</tr>
<tr>
<td>PFS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCFS</td>
<td>5.90</td>
<td>-0.489</td>
<td>0.654</td>
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</table>
Figure 7.31: Gradient from the gather 1253-1369 obtained from the four different methods to determine attenuation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>$\epsilon_Q$</th>
<th>$V_s/V_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>22.10</td>
<td>-0.499</td>
<td>0.692</td>
</tr>
<tr>
<td>CFS</td>
<td>1.41</td>
<td>-0.414</td>
<td>0.541</td>
</tr>
<tr>
<td>PFS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCFS</td>
<td>5.91</td>
<td>-0.490</td>
<td>0.654</td>
</tr>
</tbody>
</table>

Figure 7.32: $\epsilon_Q$ and $V_s/V_p$ obtained from the B gradient for each method. "B perp is the value for $B^\perp$.\"
Figure 7.33: Gradient from the gather 1256-1372 obtained from the four different methods to determine attenuation.

![Gradient plots for different methods](image)

Figure 7.34: $\epsilon_Q$ and $V_s/V_p$ obtained from the B gradient for each method. "B perp is the value for $B^\perp$."

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>$\epsilon_Q$</th>
<th>$V_s/V_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>31.00</td>
<td>-0.500</td>
<td>0.696</td>
</tr>
<tr>
<td>CFS</td>
<td>1.70</td>
<td>-0.431</td>
<td>0.561</td>
</tr>
<tr>
<td>PFS</td>
<td>-1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCFS</td>
<td>9.00</td>
<td>-0.495</td>
<td>0.671</td>
</tr>
</tbody>
</table>
Figure 7.35: Data with (right column) and without (left column) filtering for ATM (up) and AMS (down) methods. Spectral ratio method.
logs information. Five rose graphs are displayed in Figure 7.38. Notice that the rose graphs represent in this case the total fracture directions from all the gathers processed around each well. The orientation obtained by the QVOA method looks very similar to the general fracture orientation of the top of the reservoir.
Figure 7.37: Gathers represented by points, over the surface of the top of the reservoir of interest.

Figure 7.38: Top of the reservoirs surface with the rose diagrams from the QVOA analysis.
Figure 7.39: Well-3 final fracture orientations from QVOA
CHAPTER 8
CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

In this thesis, the behavior of P-wave attenuation as a function of offset and azimuth has been documented as it applies to naturally fractured reservoir characterization. Validating the orientation of fractures obtained from the QVOA method with well information helps to identify the minimum attenuation values as the isotropic plane and the maximum attenuation values as the symmetry axis direction values.

The differences in P-wave attenuation in the symmetry axis direction and the isotropic plane are related to changes in the fracture and in the host rock’s parameters. The P-wave attenuation is more affected by the Vs/Vp ratio, so that it is possible to invert for this parameter, specially for thin, liquid-filled cracks where the influence of $\Delta_N$ is very small and tends to zero when the cracks are thin enough.

Two methods were selected for solving the QVOA equation: approximate method of sectors (AMS) and approximate truncate method (ATM). These two methods proved to be accurate in the determination of the symmetry axis direction and the fracture orientation. However, the ATM method yields higher errors compared with the AMS method. ATM also has some problems in determining fracture orientation when the attenuation values are very small.

Four methods to calculate attenuation were used: spectral ratio, centroid frequency shift, peak frequency shift, and dominant and central frequency shift. Among the four methods, spectral ratio yielded higher errors in the final results compared with the other methods. CFS and DCFS were shown to be stable methods and gave low errors in the calculations and good approximations to fracture orientation validated with well data. PFS method failed in the application to real data, because it takes into account the peak frequencies from
the top and bottom of the interval of interest. When these two peak frequencies are close in value, or equal, a difference can not be seen in the calculation of the attenuation. This makes the PFS method break down and the QVOA equation is not able to solve for fracture orientation.

The anisotropic parameter $\epsilon_Q$ and $V_s/V_p$ are calculated using the B-gradient obtained from the QVOA evaluation. Surprisingly, even with the high errors values accumulated in the spectral ratio method, the values obtained for the two anisotropic parameters are not influenced by it. All of the methods show very similar values. Values obtained for $\epsilon_Q$ are around -0.5 to -0.3, while the values for $V_s/V_p$ ratio are in between 0.5 and 0.69. PFS is the only method that fails in predicting these parameters when the maximum gradient is negative or equal to -1. PFS also gives the lowest values of $\epsilon_Q$ and $V_s/V_p$ while the SR and DCFS methods values of the anisotropic parameters are very similar.

Some geometrical seismic attributes were calculated over the top of the reservoir to enhance the discontinuities related with the presence of fractures. The edge evidence attribute was the best approximation obtained. Fracture orientations obtained from the QVOA method, were compared with those from the edge evidence attribute showing very similar tendencies indicating the value of using QVOA for fracture characterization.
8.2 Recommendations

Only tadpoles oriented with the true north from one well were available. They were used to validate the fracture orientation obtained from the QVOA method. Other parameters such as $\epsilon_Q$ and $V_s/V_p$ also need to be validated. To do so, it is recommended to apply the QVOA method in a set of data that includes more information about the wells (VSP, productivity logs, etc.). These parameters can also be used to model theoretical curves that in theory can help the interpreter to obtain more information about the reservoir, such as, the crack density.

Negative values of attenuation coefficients were obtained in some of the traces used for the QVOA analysis. These negative values were filtered and not taken into account to study the variation of the attenuation with respect to the azimuth. There are many reasons why the attenuation values can result in negative values, and one of them are the assumptions made when calculating them. The methods used in this project for example, do not consider the attenuation to be affected by the presence of reflectivity anomalies. Methods as the one suggested by Li et al. (2015a) where the effect of the reflectivities are taken into account can be implemented to improve the results.

The implementation and application of the QVOA method using the s-wave attenuation coefficients can help the interpreter to characterize the fluids contained in the reservoir, analyzing the values of the $Q_p$ and the $Q_s$ in the presence of oil and dry models versus the incidence angles. Some experiments conducted by (Chichinina et al., 2009) show that the values of the $Q_p$ in dry models are sufficiently large compared with $Q_s$ values, while in the oil model these values are more similar (Toksöz et al., 1979). It would be interesting to test these results using surface seismic data and information from wells.

The attenuation of seismic waves are affected by the intensity of the pressure in the reservoir. This characteristic can be used in studies of 4-D seismic data where the pressure changes due to production. These changes can help to identify the changes in the preferential flow of fluids affecting the permeability of the reservoir. Identifying the fractures where
permeability is higher during the development of the field, new wells can be located with more certainty.
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APPENDIX - QVOA SOLUTIONS FOR THE APPROXIMATE TRUNCATE METHOD

(ATM)

In the approximate truncate method the functional $F$ needed to be minimized over parameters $a$, $b$, $d$, and $\phi_0$, solving a system of four equations, listed below (Sabinin, 2013):

\[
\frac{\partial F}{\partial a} = 0 \quad (A.1)
\]
\[
\frac{\partial F}{\partial b} = 0 \quad (A.2)
\]
\[
\frac{\partial F}{\partial d} = 0 \quad (A.3)
\]
\[
\frac{\partial F}{\partial \phi_0} = 0 \quad (A.4)
\]

The equations were derived by Sabinin (2011) giving the next final results:

\[
f_0 = \frac{1}{n} \sum_{i=1}^{n} \alpha_i \quad (A.5)
\]
\[
f_1 = \frac{1}{n} \sum_{i=1}^{n} s_i \alpha_i \quad (A.6)
\]
\[
f_2 = \frac{1}{n} \sum_{i=1}^{n} g_i s_i \alpha_i \quad (A.7)
\]
\[
f_3 = \frac{1}{n} \sum_{i=1}^{n} h_i s_i \alpha_i \quad (A.8)
\]
\[
A = \frac{1}{n} \sum_{i=1}^{n} s_i \quad (A.9)
\]
\[
B = \frac{1}{n} \sum_{i=1}^{n} s_i^2 \quad (A.10)
\]
\[
D = \frac{1}{n} \sum_{i=1}^{n} g_i s_i \quad (A.11)
\]
\[ E = \frac{1}{n} \sum_{i=1}^{n} h_is_i \]  
\[ G = \frac{1}{n} \sum_{i=1}^{n} g_i^2s_i \]  
\[ H = \frac{1}{n} \sum_{i=1}^{n} h_is_i^2 \]  
\[ I = \frac{1}{n} \sum_{i=1}^{n} g_i^2s_i^2 \]  
\[ J = \frac{1}{n} \sum_{i=1}^{n} h_is_i^2 \]  
\[ K = \frac{1}{n} \sum_{i=1}^{n} g_ih_is_i^2 \]  
\[ F_1 = f_1 - Af_0 \]  
\[ F_2 = f_2 - Df_0 \]  
\[ F_3 = f_3 - Ef_0 \]  
\[ a_1 = B - A^2 \]  
\[ b_1 = I - D^2 \]  
\[ c_1 = J - E^2 \]  
\[ a_2 = G - AD \]  
\[ b_2 = K - ED \]  
\[ a_3 = H - AE \]  
\[ H_1 = F_2a_1 - F_1a_2 \]  
\[ H_2 = F_3a_1 - F_1a_3 \]
\[ A_1 = a_1b_1 - a_2^2 \]  
\[ B_1 = a_1c_1 - a_3^2 \]  
\[ A_2 = a_1b_2 - a_2a_3 \]  

Using all this relations, (Sabinin, 2013) gives the solutions for \( a, a, b, d, \) and \( \phi_0 \) as follow:

\[ \tan(2\phi_0) \equiv \frac{S}{C} = \frac{A_2H_1 - A_1H_2}{A_2H_2 - B_1H_1} \]  
\[ d = \frac{A_2H_1 - A_1H_2}{S(A_2^2 - A_1B_1)} \]  
\[ b = (F_1 - dCa_2 - dSa_3)/a_1 \]  
\[ a = f_0 - bA - dCD - dSE \]