ANALYZING THE POTENTIAL FOR UNSTABLE MINE FAILURES
WITH THE CALCULATION OF RELEASED ENERGY
IN NUMERICAL MODELS

by

Eric C. Poeck
Unstable failure in underground mining occurs when a volume of material is loaded beyond its strength and displaces suddenly. It is recognized on various scales, from small rock bursts to the collapse of pillars or entire sections of a mine. The energy that is released during smaller scale events is manifested through the ejection of material, which can pose a hazard to the safety of miners. Larger scale events generate seismic waves as mine workings are damaged and may entrap miners or terminate production.

This dissertation focuses on the analysis of unstable failure in an underground room and pillar mining environment. The potential for violent pillar failure is assessed using numerical modeling techniques and a parametric approach to loading conditions and material strength properties. The magnitude of instability is quantified by calculating the release of kinetic energy that occurs as failure progresses in each simulation.

Fundamental mechanisms associated with the release of kinetic energy are analyzed in a series of finite difference models, and the results are compared with analytical solutions to illustrate the applicability of the energy calculations to increasingly complex modes of failure. Back analyses are performed on two room and pillar mine collapse events from the western United States by constructing large-scale models and reproducing widespread failure. The values of energy released in two-dimensional models are extrapolated by assuming a depth of failure in the third direction, and the total energy values are compared to the documented seismic magnitudes from each collapse through empirical equations. With further development of this numerical modeling approach, energy consideration may be used to study the potential for instability in a wide variety of mining excavations and identify the associated range of hazards.
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Newtons ....................................................................................................................................................  26
Kilograms ................................................................................................................................................... kg
Meters ........................................................................................................................................................ m
Pascals ........................................................................................................................................................ Pa
Joules ........................................................................................................................................................... J
Bulk modulus .............................................................................................................................................. K
Shear modulus ............................................................................................................................................. G
Young’s modulus ........................................................................................................................................ E
Poisson’s ratio .............................................................................................................................................. v
Stress ........................................................................................................................................................... σ
Normal stress ............................................................................................................................................. σn
Major principle stress ............................................................................................................................... σ1
Minor principle stress ............................................................................................................................... σ3
Shear stress .............................................................................................................................................. τ
Friction angle ............................................................................................................................................. φ
Cohesion ...................................................................................................................................................... c
Released seismic energy .......................................................................................................................... $E_S$
Seismic magnitude ................................................................................................................................... M
Local seismic magnitude ............................................................................................................................ $M_L$
Released energy ....................................................................................................................................... $W_R$
Total boundary work ................................................................................................................................. $W$
Total stored strain energy .......................................................................................................................... $U_C$
Total change in potential energy ............................................................................................................... $U_B$
Total energy dissipated in joint shear ....................................................................................................... $W_J$
Total work dissipated in plastic deformation of intact material \( W_p \)

Current value of kinetic energy \( U_K \)

Total work dissipated by damping \( W_K \)

Work done by viscous boundaries \( W_V \)

Total strain energy in excavated material \( U_M \)
LIST OF ABBREVIATIONS

Megapascals ........................................................................................................................................... MPa
Gigapascals ............................................................................................................................................. GPa
Megajoules ............................................................................................................................................... MJ
Fixed contact conditions ........................................................................................................................... fix
Continuously-Yielding joint condition ................................................................................................... CY
Coulomb Slip joint condition ................................................................................................................... CS
Mohr Coulomb material constitutive model ............................................................................................... Mohr
Mohr Coulomb strain softening material constitutive model ................................................................. MCss
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CHAPTER 1
INTRODUCTION

The challenges associated with the design and operation of a safe and profitable mine increase as near-surface deposits are depleted and the pursuit of mineral resources is pushed deeper underground. Safe development of an underground mine requires an anticipation of the loads generated by deep cover and a detailed analysis of the load-bearing capacity of the materials in and around the mine workings. An improper or incomplete evaluation of loads and load-bearing capacities can lead to the sudden failure of mine workings, which poses a threat to the safety of the miners and the productivity of the mine.

1.1 Problem Statement

Room and pillar mining involves the extraction of a series of rooms, or entry ways, intersected by a series of perpendicular or slightly angled entries that leaves a pattern of unmined pillars. Room and pillar mining has been conducted in a wide range of deposits, under a wide range of depths, for centuries with the basic understanding that some quantity of material must be left behind to support the roof. The extent of extraction is optimized in modern operations to ensure the highest productivity under safe operating conditions.

In many cases, pillars are designed within a set of empirical guidelines to meet a minimum safety factor, which means that the estimated load bearing capacity is sufficiently larger than the anticipated load of the overlying material, and a small amount of risk is accepted for the time period in which mining will be conducted nearby. More challenging operating conditions are met with a more technical approach, employing numerical models to analyze every detail allowable by the quality and availability of geologic data. The fundamental laws that govern the strength of materials are applied in a number of commercially-available software packages and are capable of identifying discrepancies in predicted loads and the support capacities of pillars in a proposed layout.

The failure of pillars may occur gradually, or after an area has been abandoned, indicating an adequate understanding of the loads and a successful pillar design. In some cases, however, the lack of sufficient data or the inherent uncertainty associated with geologic materials may result in an incomplete understanding of pillar support capacity or an unanticipated concentration of load. Despite the level of detail afforded by modern computing techniques, pillars sometimes fail in an unstable manner and the mining community must determine how to further reduce the associated injuries, deaths, and loss of production.
Numerical modeling procedures are typically focused on predicting the strengths of pillars and may take geologic variability and uncertainty into consideration for an idea of the pillar behaviors that may be observed during operation of the mine. The criteria for unstable pillar failure is well understood, but little research has been conducted to better understand the mechanisms of unstable pillar failure in numerical models and assess the potential for instability in the design of room and pillar workings. Predictive analyses can be improved by applying the known criteria for instability in numerical models with the use of energy calculations to identify hazardous conditions and quantify the magnitude of potentially destructive events.

1.2 Definition of Unstable Failure

Unstable failure refers to the outburst or collapse of material that occurs when a volume of rock has been suddenly loaded beyond its strength. It is important to note that not all failures are unstable or unexpected. Pillars often fail gradually as excavation is carried out nearby and loads are progressively increased.

Unstable failure requires two fundamental material characteristics: brittle material behavior and a soft loading system. Brittle material behavior, which is exhibited in most rocks, refers to the loss of structural integrity and strength that occurs when a volume of material is compressed beyond its strength. The material tends to fracture and may no longer be capable of supporting its own weight.

A soft loading system is one that is capable of sudden displacements when opposing force is taken away. A stack of free weights represents a perfectly soft loading system on a brittle material. Once failure is initiated within the material, the stack of weights will fall rapidly. In contrast, a stiff loading system is one that may be capable of supplying tremendous pressure, but after small increments of displacement within the failing medium, the load is reduced and failure is arrested. A screw clamp represents a stiff loading system on a brittle material. When the clamping force exceeds the strength of the material and it fails, the excess load is relieved and will not increase until the screw is turned again. The loading conditions in an underground mine vary with local geology and generally exhibit some degree of softness or stiffness between the two extremes.

Unstable failures and their associated magnitudes of energy are recognized on various scales in an underground environment. Small scale events can be manifested as audible cracks and pops, while failures on a larger scale can be mistaken for earthquakes. In either case, the sudden ejection or collapse of material poses a hazard to the safety of miners.
1.3 Research Objective

Calculation of the transfer and release of energy has been fundamental to the study of unstable failure and discussed at length by several pioneering researchers (Griffith, 1921), (Cook, 1963), (Duvall and Stephenson, 1965), and (Salamon 1970). What is seen as severity or violence in the failure of materials, whether in laboratory compression tests or the failure of underground workings, is the release of stored energy through sudden displacement. On both large and small scales, the sudden movement of material represents the release of kinetic energy.

The objective of the research contained herein is to develop a method of geomechanical analysis that incorporates energy calculation to the study of instability in room and pillar mine workings. By considering the release of kinetic energy, along with traditional measures of material integrity including stress and displacement, the occurrence of unstable failures can be identified in a properly calibrated model. The magnitude of kinetic energy released during the simulation of failure determines the severity of an event and the potential for damage to structures or harm to individuals.

This research serves to demonstrate the fundamentals of an energy-based approach to rock mechanics, from the calibration of appropriate material strength parameters to the analysis of numerical modeling results. The back analysis of two documented mine collapse events illustrates the effectiveness of the approach to quantify the severity of instability in terms of energy release and associated seismic magnitude.

1.4 Methodology

Unstable failure requires brittle material behavior, and as such, the simulation of instability requires the calibration of brittle or softening material properties. As with any method of analysis, the quality of results depends heavily on the quality of the input data. The strain softening material properties utilized in this research are calibrated based on published data for the materials present in each analysis. For materials in which the rock mass or pillar-scale behavior is assessed by previous researchers, pillar-scale models are calibrated to match the peak strengths observed in a range of pillar geometries. For materials in which laboratory compression test data is available, assumptions are made regarding the characteristics of the rock mass to formulate pillar-scale strength properties. In either case, vertical loads are slowly increased upon pillar-scale models to induce failure and match observations from the mine site which is being studied.

Energy calculations are performed within a commercially available distinct element software package, and a series of simple models are constructed to investigate the release of energy associated with
various modes of unstable failure. The results of the simple models are compared to analytic solutions to demonstrate the accuracy of the calculations under controlled loading conditions.

Back analyses are performed on two room and pillar mine collapse events from the western United States by constructing large scale models and simulating the excavation sequence or loading conditions that precluded the documented events. Results of the two-dimensional analyses are extrapolated in the third direction by a distance that reflects the estimated three-dimensional extent of the documented collapse area. Total values of released energy are then converted to seismic magnitudes through empirical equations to quantify the severity of the simulated failure in relative terms. The simulated and documented seismic magnitudes are compared in order to demonstrate the effectiveness of the energy-based approach in the study of instability.
Although numerical modeling is an important tool for rock mechanics engineers and has accommodated improvements in the quality and complexity of analyses, a significant aspect of this research involves the understanding of unstable failure. Major contributions to this field of study began early in the 20th century.

2.1 Energy and Unstable Failure

The understanding of brittle failure was explored by Griffith in the 1920’s in studies concerning the propagation of a crack in tension (Griffith, 1920). Griffith’s criterion stated that the growth of a crack will occur when growth reduces the potential energy of the crack and the material around it. The formulae derived in this work were based on principle stresses and stored strain energy terms. McClintock and Walsch (1962) expanded upon Griffith’s work by examining the closing of a crack during compression and the development of frictional forces.

Unstable failure in the form of rockburst has long been recognized as a hazard in various types of underground mining. Over time, the progression of mining into deeper ore bodies increased interest in the concepts behind rockburst, and several researchers developed theories and formulae to define the mechanics associated with brittle failure (Cook, 1964), (Cook, 1965), and (Salamon, 1970).

Cook (1965) conducted a series of laboratory experiments and proved that excess energy during failure of rock samples was supplied by the resilience, or stiffness, of the testing machine. The process of loading a specimen results in compression of the platens as well, which stores strain energy in the material, usually steel, over small displacements. When the stiffness of the testing machine is lower than that of the specimen, the energy stored in the platens is transferred to the specimen at the moment of failure.

Salamon elaborated on the concepts introduced by Cook and proved that a pillar will remain stable, regardless of the level of convergence predicted, if the stiffness of the loading system is greater than the post-peak stiffness of the pillar. The concepts of system stiffness, stability, and instability are illustrated in Figure 2.1. Stability is achieved when the strength of the pillar or sample is greater than that of the load and no failure occurs. Unstable failure is introduced when the load exceeds the strength of the pillar and a continuation of energy, or rather displacement, is supplied by the loading system. This is evident when the slope of the load line is lower than that of the post-peak stress/strain curve.
Analytical solutions were derived Duvall and Stephenson for energy that is radiated outward when an underground cavity in a uniform stress field is created or enlarged (Duvall and Stephenson, 1965). The study showed that radiated energy can be calculated based on the elastic properties of the rock and that the magnitude was directly proportional to the volume of rock excavated. The study provided valuable insight toward rockburst and energy release as the concepts were first being explored, but calculation was limited to an elastic rock mass, a uniform stress field, and excavations of cylindrical or spherical shape.

Cook (1966) developed a method of quantifying rockburst severity by calculating the energy release rate (ERR) for excavations in assumed homogeneous, elastic ground. The calculation of ERR did not account for geologic features that are known to enhance or trigger rockburst activity, such as faults, but results correlated well with events observed in deep mining fronts. The ERR concept proved to be a valuable platform for rockburst studies over the course of many years.

Research was also conducted to better understand unstable failure in the form of fault slip and earthquake motion (Rice, 1983). Constitutive relations were developed to incorporate the weakening of shear surfaces as slip was induced, and terms were derived for the release of elastic strain energy and friction work along sliding surfaces.

Energy balance equations were developed for work done during a change in the state of equilibrium for an elastic rock mass (Salamon, 1984). Formulae were derived to calculate various components of energy change without restriction on the size, shape, or number of mine workings. Work performed by support in an excavation was taken into account, and equations were modified to accommodate tabular excavations. Salamon’s analytical solution was limited to specific loading conditions, and numerical methods were not yet used to evaluate realistic mining conditions.
Further research into the mechanisms of unstable failure built upon the concepts introduced by Cook and Salamon (Ryder, 1988). Ryder recognized the role of excess shear stress (ESS) in deep mining fronts and discussed the increase in magnitude of crushing-type rockburst events when a shear slip mechanism was involved. Additionally, excess shear stress was characterized and grouped into five possible categories, and a methodology was presented for the calculation of ESS with the assistance of numerical stress analysis programs.

2.2 Numerical Modeling of Instability

Numerical modeling software has advanced the complexity of rock mechanics analyses. Researchers have studied unstable failure by incorporating non-linear material behavior, which is required for unstable failure to occur, in a number of different ways. Tang and Kaiser used a finite element software package called Rock Failure Process Analysis (RFPA) to model the brittle failure of UCS samples and simple excavations and pillars in 2D (Tang, 1997), (Tang et al, 1998), and (Kaiser et al, 1998). The elements were given an elastic-brittle constitutive model with a heterogeneous distribution of rock parameters to simulate non-linear progression of failure. The heterogeneous strength parameters are illustrated in Figure 2.2 with elastic-brittle stress/strain curves for high and low strength elements respectively.

Figure 2.2: Heterogeneous distribution of parameters for simulation of non-linear behavior using elastic-brittle elements. Example stress/strain curves shown at bottom. (Tang, 1997)

Tang and Kaiser recorded the frequency of element failure during loading on UCS test specimens and correlated the results with acoustic emission signatures of laboratory tests. Additionally, the energy associated with failure was estimated by calculation of the strain energy surrendered by failed elements. This calculation was simplified by the elastic-brittle constitutive law in which the elements were programmed to fail completely and their post-peak strength was zero. Results of the energy calculations were not presented in terms of empirical or SI units, but rather, on a timeline of a given simulation as
percentages of the total strain energy released. Tang and Kaiser’s approach has not been applied to a mine-scale failure analysis.

Board and Damjanac (2007) conducted an investigation of the February 3, 1995 collapse at the Solvay trona mine by running a series of numerical models. The stress/strain behavior of individual pillars was characterized in a series of FLAC3D models by using a Mohr-Coulomb strain-softening constitutive model and loading the pillars to failure. The stiffness of the loading system was evaluated separately in UDEC models by generating a ground reaction curve for the assumed span of overburden that failed. The 2D mine-scale UDEC model exhibited signs of weakness in the overburden when support pressure was reduced below a threshold value, as shown in Figure 2.3.

![Figure 2.3: Velocity vectors indicating failure in model of Solvay Mine (Board, 2007)](image)

Board and Damjanac compared the stress strain curves of pillars from various extraction ratios with the ground reaction curve of the overburden. They observed the conditions that likely accounted for instability in the mine but did not consider the calculation of released energy in the study.

The partial collapse of the Crandall Canyon Mine in August of 2007 was followed by several numerical modeling studies to help determine the cause of the accident. As part of the Mine Safety and Health Administration (MSHA) accident investigation, Pariseau constructed a 2D finite element model through a vertical cross section of the mine (Gates, 2008). The goal of the study was to analyze the
distribution of stresses and estimate factors of safety for pillars. A close-up view of the model is shown in Figure 2.4 with the geometry of the Main West section of the mine.

![Figure 2.4: Cross section of the Main West in a 2D finite element study (Gates, 2008)](image)

Pariseau considered all excavations in the chosen cross section, including the longwall panels to the north and south, and a detailed stratigraphy. The analysis was performed within an elastic rock mass and conclusions were based on observed stresses and assumed pillar strengths alone. The study did not consider brittle failure or shear slip along discontinuities in any portion of the model and the release of energy was not discussed.

Heasley (2008) conducted an analysis of the Crandall Canyon collapse using a boundary-element software called LaModel. The numerical model, oriented in plan-view, accounted for the sequential development of entries and retreat of appropriate pillars in the Main West section. Abutment loading from the longwall panels to the north and south was accounted for numerically. The stated goal of the LaModel analysis was to analyze stress distributions and calculate pillar safety factors. Although significant changes in stress distribution between stages of excavations indicated potentially unsafe conditions, the software was not optimized for modeling unstable failures.

A strain softening constitutive law was applied in the model for consideration of post-peak pillar strength. Safety factors were calculated for individual elements based on peak and applied strain and averaged for a given pillar. Shear slip along discontinuities, and the release of energy were not considered in the study. A color contoured diagram of pillar safety factor in the boundary-element model is shown in Figure 2.5.

The study of rockburst also extends into the field of tunneling as ambitious civil projects have pushed the development of roadways and water transport systems into deeper rock worldwide. The review of literature for the purpose of this research proposal included a study from the Chinese Academy of Sciences concerning rockburst in the pilot tunnels of the Jinping II hydro-station (Jiang, 2010). In this work, 3D tunnel excavation was simulated in elastic-brittle-plastic materials and analyzed for the
potential of rockburst under loading conditions with high horizontal stress. The intensity of potential rockburst was quantified by the calculation of Local Energy Release Rate (LERR), a term coined by the authors.

![Figure 2.5: Pillar safety factors calculated in a boundary-element model of the Crandall Canyon Main West section (Heasley, 2008).](image)

LERR accounts for the change in strain energy before and after failure of an element, similar to the approach used by Tang, but in this study the authors provide color contoured plots of expected energy release in units of Joules and correlate their results with observations made at the dam site. Comparison is made between predicted energy release values and severity of rock bursts witnessed during excavation. While the study builds upon concepts introduced by Cook and correlates energy release values with field observations, it does not address failures on the mine scale. Additionally, the numerical models only account for changes in strain energy and do not consider the effects of faults or other shear mechanisms which may contribute to the release of kinetic energy.

Recent studies at the Colorado School of Mines have served as a foundation of the research demonstrated in this thesis. Garvey (2013) constructed a series of discrete element and finite difference models, using PFC3D and FLAC3D respectively, to explore the numerical identifiers of unstable failure. Garvey also implemented functions to calculate and record the release of excess energy in finite difference models and presented results for laboratory and pillar-scale simulations of unstable failure. Energy balance equations were derived for both static and dynamic terms.

Gu (2013) examined the role of unstable shear slip in underground coal mine failures by conducting a thorough investigation into the stability of discontinuities. A continuously-yielding (displacement-softening) joint constitutive model was used in UDEC simulations of small-scale direct shear tests and room-scale excavations. Stability of discontinuities was studied by parametric evaluation of model parameters including rock mass stiffness, joint stiffness, friction angle, and joint roughness. Stability was
also assessed by monitoring and recording the mass-damped work within the models, which relates to the magnitudes of velocity at grid points and kinetic energy being released. The application of kinetic energy concepts is carried through to the research presented here.
CHAPTER 3
NUMERICAL METHODS

The numerical models built for the purpose of this study are constructed using a software called the Universal Distinct Element Code (UDEC) (Itasca, 2014). From the Introduction section of the software user’s guide:

“The Universal Distinct Element Code (UDEC) is a two-dimensional numerical program based on the distinct element method for discontinuum modeling. UDEC simulates the response of discontinuous media (such as a jointed rock mass) subjected to either static or dynamic loading. The discontinuous medium is represented as an assemblage of discrete blocks. The discontinuities are treated as boundary conditions between blocks; large displacements along discontinuities and rotations of blocks are allowed. Individual blocks behave as either rigid or deformable material. Deformable blocks are subdivided into a mesh of finite-difference elements, and each element responds according to a prescribed linear or nonlinear stress-strain law. The relative motion of the discontinuities is also governed by linear or nonlinear force-displacement relations for movement in both the normal and shear directions.

UDEC has several built-in material behavior models, for both the intact blocks and the discontinuities, which permit the simulation of response representative of discontinuous geologic (or similar) materials. UDEC is based on a Lagrangian calculation scheme that is well-suited to model the large movements and deformations of a blocky system. UDEC also contains the powerful built-in programming language FISH. With FISH, you can write your own functions to extend UDEC’s usefulness. FISH offers a unique capability to UDEC users who wish to tailor analyses to suit specific needs.”

UDEC was selected for its ability to model softening behavior in blocks and discontinuities, which is essential for the study of unstable failure. In addition, UDEC contains an energy calculation routine that accounts for the transfer and release of energy as changes in loading conditions or excavation geometry are simulated. Details regarding the calculation of energy terms are provided in CHAPTER 4.

3.1 Model Construction

UDEC is a command-driven application, meaning that the construction and execution of a model requires a list of commands to be entered into the command prompt or be read from an external data file by the software. Although there is a Graphical Interface for Itasca Codes (GIIC) to assist users in the construction of models in a menu-driven mode, the principles behind the operation of the program remain the same and the commands used in the creation of a model in menu-driven mode will be saved internally. Simulations of complex mining processes are not typically suited to the use of the menu-driven interface, as they will potentially require hundreds of sequenced commands, custom FISH
functions for the management of data, and several hours of computation time between stages that require input.

A model is created in UDEC by defining the 2D space required for an analysis and generating one block that encompasses the entire area. From that point, the block can be split into smaller blocks or sculpted to nearly any shape desired through the use of joint commands and the deletion of unwanted blocks. Joints can serve as deformable geologic discontinuities if given the necessary constitutive law and properties, or they can be used to establish geometric limits and then be removed.

### 3.2 Boundary Conditions

Boundary conditions must be used to fix a model in space or limit displacement in a particular direction. For instance, when gravity is applied to a model, vertical motion at one or more boundaries must be restricted to avoid infinite displacement when the model is cycled. Boundaries that are restricted in the direction normal to their surface are referred to as roller boundaries, while those that are restricted in both orthogonal directions are referred to as fixed. Open boundaries, such as the ground surface or the sides of an unconfined compression test specimen, are allowed.

Boundary conditions can also be used to prescribe a loading condition and will consist of either a stress or velocity designation. A stress boundary condition will apply constant forces to all of the grid points within a specified range. The value of force applied to a grid point will depend upon the desired stress value and the spacing between adjacent grid points. A velocity boundary condition will apply a constant displacement to the grid points within a specified range. The amount of displacement incremented at each calculation step, or cycle, is determined by multiplying the designated velocity by the mechanical time step. In other words, if a mechanical time step of 0.001 seconds is being used by UDEC to cycle the model and a velocity boundary condition of 0.5 meters per second is specified at the boundary, the displacement of grid points will be incremented 0.0005 meters at the beginning of each cycle. The displacement of the boundary will reach 1.0 meters after 2000 calculation steps.

A stress boundary represents an infinitely soft loading condition, as displacements in the direction of the applied load will occur freely and rapidly if opposing forces are lost. Such would be the case if a stress condition of appropriate magnitude were placed on an unconfined compression test specimen. The velocity boundary condition, if applied using appropriately small values, is a very stiff loading condition. The failure of a compression test specimen can be very well controlled through small incremental displacements.
3.3 Mesh Generation

To be made deformable, blocks must be divided into finite difference zones. *Quad* zoning in UDEC will generate square or rectangular zones with four internal triangular elements and is said to provide the most accurate solution for plasticity problems (Itasca, 2014). *Edge* zoning will generate individual triangular zones of irregular edge lengths in order to fill irregularly-shaped blocks and is recommended for any application in which *quad* zones cannot be used. Examples of *quad* and *edge* zoning are illustrated in Figure 2.1.

![Figure 3.1: Examples of Quad and Edge zoning in UDEC.](image)

In the models constructed for the purpose of this study, zones are generated in such a way that they form equilateral right triangles wherever possible, as zones with a large aspect ratio, approaching 10:1, may reduce solution accuracy.

3.4 Block Constitutive models

Blocks that are made deformable must be assigned a constitutive law and be given material properties. Three different constitutive laws are utilized throughout the course of the study, including an elastic block model, a Mohr-Coulomb model, and a Mohr-Coulomb strain softening model. An elastic block exhibits a linear increase in strain as load is applied and has no strength limit. Mohr Coulomb elements exhibit a linear increase in strain as load is applied, but when the peak strength is reached, further deformation occurs at a constant stress. This relation is also referred to as nonlinear or elastic-plastic behavior. The strain softening variant of the Mohr-Coulomb constitutive model, also referred to as elastic-brittle-plastic, exhibits a loss in load-bearing capacity after the peak strength has been reached and reduces to a residual strength value as deformation continues. The idealized stress versus strain characteristics of each block constitutive model are shown in Figure 3.2.
The shapes of the Mohr-Coulomb and strain softening curves illustrated in the figure represent the idealized behavior of a single finite difference zone and will vary with the size and shape of a larger failing medium. For instance, wide pillars containing hundreds of Mohr-Coulomb zones may not exhibit a perfectly elastic-plastic response because the failure of the pillar occurs gradually as individual zones reach their peak strength throughout the pillar.

### 3.4.1 Elastic Blocks

Elastic blocks are useful for analyzing stress distributions around excavations that are assumed to be in equilibrium and estimating deformations that result purely from elastic strain. Elastic elements have infinite strength and therefore are not used in materials expected to fail. The most common applications for elastic blocks throughout the course of this study are to extend the far field region of the model to prevent symmetry effects at the boundary or to serve as dead weight in the overburden.

Elastic blocks require only three material properties to be used in a model. Density is applied using the keyword `dens` and is measured in units of mass per unit of volume. When using International Standard (SI) units, the mass is measured in kilograms, while in empirical units, it is measured in slugs. The stiffness of a block is defined in UDEC by assigning a bulk modulus, using keywords `bulk` or `K`, and shear modulus, keywords `shear` or `G`, which are both measured in units of force per unit of area. Bulk and shear modulus can be calculated using Young’s modulus (E) and Poisson’s ratio (ν) through the equations:

\[
K = \frac{E}{\frac{3(1-\nu)}{2}}
\]  

\[
G = \frac{E}{\frac{2(1+\nu)}{2}}
\]  

\[\text{(3.1)}\] 

\[\text{(3.2)}\]
UDEC does not accept Young’s modulus or Poisson’s ratio as input and will not automatically calculate the equivalent bulk and shear moduli. The programming language FISH, which is built into UDEC, can be used to calculate the bulk and shear moduli if desired.

### 3.4.2 Mohr-Coulomb Blocks

The Mohr-Coulomb strength criterion describes the formation of a failure plane under normal and shear loading based upon internal strength characteristics that may be calculated through compressive laboratory testing. In combination with numerical modeling codes, the criterion is widely used in geotechnical, structural, and mechanical engineering to determine the strength of complex 2D and 3D objects under various loading conditions.

The biaxial, compressive loading of an idealized specimen of material, as shown in Figure 3.3a, results in the formation of shear plane $ab$ at failure. The shear stress along the failure plane is denoted by $\tau$, and the angle at which the plane develops with respect to the horizontal axis in the specimen is denoted by $\beta$. Figure 3.3b contains a diagram of Mohr’s circle for biaxial loading conditions and illustrates that for a given value of the maximum principle stress, $\sigma_1$, a linear envelope exists for which a value of the minor principle stress, $\sigma_3$, will result in shear failure. Failure is characterized by the intersection of Mohr’s circle with the linear failure envelope. The value of the shear stress that will be acting upon the fracture plane is noted by the Y coordinate of the intersection point.

![Figure 3.3: Biaxial, compressive loading of an idealized specimen (A) and a diagram of Mohr’s circle for biaxial loading conditions (B)](image)

The linear envelope in Figure 3.3b is defined by the internal friction angle, $\phi$, and the cohesion, $c$, of the block material. Equation 3.3 summarizes the Mohr-Coulomb criterion by defining the shear stress
that will be developed along a failure plane as a function of the resultant normal stress on the plane, $\sigma_n$, the friction angle, $\phi$, and the cohesion, $c$. Additional derivations can be made to relate the principle stresses to the shear stress or normal stress at failure.

$$\tau = c + \sigma_n (\tan \phi)$$

(3.3)

In UDEC, the density and elastic properties of Mohr-Coulomb blocks are defined using the same parameters as elastic blocks. The strength characteristics of the blocks are assigned using the keywords *fric* for a friction angle in units of degrees and *coh* for a cohesion value in units of force per unit of area. Tensile strength and a dilation angle can be specified if desired.

### 3.4.3 Mohr-Coulomb Strain-Softening Blocks

In the Mohr-Coulomb strain softening constitutive model, the values of cohesion, friction angle, dilation angle, and tensile strength can be increased or decreased after the onset of plastic yield. In a traditional Mohr-Coulomb model, these properties remain constant. The difference in stress-strain behavior of these two constitutive laws is conceptualized in Figure 3.2. In order to mimic the behaviors exhibited by most rocks under compressive loading, the strength parameters are adjusted such that the load bearing capacity of the modeled material is reduced after failure.

The simplest way of accomplishing a strength reduction is by lowering the cohesion after an appropriate level of plastic strain. The desired values of cohesion are stored in a table in UDEC along with the value of plastic strain at which the values of cohesion are to take effect. The software calculates the value of strain on each zone in the model at each calculation step and interpolates an appropriate value of cohesion from the values stored in the softening table. Figure 3.4 illustrates a set of simple cohesion softening parameters, with the values of cohesion plotted against the corresponding strain values.

![Cohesion vs Plastic Strain](image)

**Figure 3.4: Simplified cohesion softening parameters.**
In more advanced calibration procedures, the values of friction angle and dilation angle can be increased or decreased to promote behavior observed in laboratory tests or in site-specific conditions (Edelbro, 2009), (Walton, 2012). Regardless of the parameters used, the selection of an appropriate zone size is important when modeling strain softening materials. The smallest dimension of a failing medium should be transected by as many strain softening zones as possible to portray a realistic mode of failure without resulting in unreasonably long computation times. Once a set of softening properties has been calibrated for a particular zone size, it should be implemented consistently throughout the modeled material.

The reduction in strength exhibited by strain softening material is essential to the study and understanding of unstable failure. If the loading conditions are suitable, the failure of a softening material will accommodate sudden displacements, which is the manifestation of the release of kinetic energy. The process of energy release is further described in CHAPTER 4 of this thesis.

3.5 Joint Constitutive Models

Three different joint constitutive models are utilized throughout the course of this study. Shear slip along a discontinuity is governed by either a fixed condition, Coulomb Slip (CS) criteria, or a Continuously Yielding (CY) displacement-softening law. A fixed condition is one in which shear slip is entirely restricted and any deformation across the joint is the result of elastic strain. A Coulomb slip model exhibits elastic-plastic behavior under shear loading, deforming at a constant shear stress after failure. The Continuously-Yielding joint model, which was developed for the UDEC software (Cundall and Hart, 1984), exhibits a post-peak reduction in shear strength with continued deformation. Idealized shear stress versus shear strain behavior for each of the three joint models is illustrated in Figure 3.5.

![Figure 3.5: Generalized shear stress-displacement behavior of joint constitutive models](image-url)
Each of the joint constitutive models require input parameters for normal and shear stiffness. The shear stiffness controls the amount of elastic deformation that will take place before the contact reaches its peak strength. The normal stiffness parameter controls the amount of overlap that occurs between grid points on each side of the contact. Low normal stiffness can lead to numerical error, while excessively high stiffness can increase computation time. Further explanation of the joint constitutive models and the input parameters required for their application are provided in the following sections.

3.5.1 **Fixed Joints**

Fixed joint conditions can be accomplished in UDEC in two ways. One is to issue the `join_contact` command for a joint or set of joints within the model. The other way is implement a Coulomb slip constitutive model at the desired joint locations and assign artificially high strength properties.

Joined contacts are automatically prevented from failing in shear or tension and are given a normal stiffness, $j_{kn}$, equal to 100 times the average block stiffness in the model. Shear stiffness, $j_{ks}$, is set at a value equal to half of the calculated normal stiffness. Alternatively, by using a Coulomb slip constitutive model with an extremely high failure envelope, shear and tensile failure are prevented and the effect of varying joint stiffness values on the modeling results can be evaluated if necessary.

3.5.2 **Coulomb Slip Joints**

A Coulomb slip joint is identical to a failure plane generated in a Mohr-Coulomb block, but it is analyzed as a pre-existing contact between two independent blocks. The stiffness of a CS joint is governed by the same normal and shear parameters described for fixed joints. The strength in shear is controlled by the friction angle, $j_{fric}$, measured in degrees and a cohesion value, $j_{coh}$, in units of force per unit area. Tensile strength, $j_{tens}$, in the normal direction can be assigned in units of force per unit of area or left with a default value of zero. Joint dilation in units of degrees can be assigned if desired.

3.5.3 **Continuously-Yielding Joint**

The CY model is intended to reproduce the types of phenomena observed in shear experiments with rock joints, such as post-peak softening and dilation (Cundall and Lemos, 1990). From the *Theory and Background* section of the UDEC manual (Itasca, 2014):

> “The continuously yielding model is considered more “realistic” than the standard Mohr-Coulomb joint model in that the continuously yielding model attempts to account for some nonlinear behavior observed in physical tests (such as joint shearing damage, normal stiffness dependence on normal stress, and decrease in dilation angle with plastic shear displacement). The essential features of the continuously yielding model include the following.
1. The curve of shear stress/shear displacement is always tending toward a “target” shear strength for the joint (i.e., the instantaneous gradient of the curve depends directly on the difference between strength and stress).

2. The target shear strength decreases continuously as a function of accumulated plastic displacement (a measure of damage).

3. Dilation angle is taken as the difference between the apparent friction angle (determined by the current shear stress and normal stress) and the residual friction angle.

As a consequence of these assumptions, the model exhibits, automatically, the commonly observed peak/residual behavior of rock joints. Also, hysteresis is displayed for unloading and reloading cycles of all strain levels, no matter how small.

The initial stiffness of a CY joint is governed by the same normal and shear parameters as the other joint models. Figure 3.5 illustrates that the effective stiffness of the CY joint reduces gradually as shear strain occurs during loading. The peak shear strength is determined by three parameters including an initial friction angle, $j_{fric}$, measured in units of degrees, an intrinsic friction angle, $j_{fric}$, measured in degrees, and a joint roughness value, $j_{rough}$, with units in meters. As with other constitutive laws governing shear failure, the peak shear strength of a CY joint also depends upon the magnitude of normal force present. Tensile strength of a CY joint is always assumed to be zero in UDEC.

The CY joint exhibits the behavior of a discontinuity with asperities that must be overcome or damaged in order for shear displacement to occur. Sliding on a CY joint is initially controlled by the joint roughness and the frictional properties between asperities, as shown in Figure 3.6a. As loading increases in the shear direction, the asperities are damaged and the residual strength of the joint is controlled by the intrinsic friction angle value. The residual state of the joint is conceptualized in Figure 3.6b.

Figure 3.6: Conceptual diagram of a rock discontinuity with asperities. Initial strength is controlled by asperities (a) and residual strength is reduced as asperities are damaged (b).

The CY joint provides the ability to model unstable slip if the surrounding rock or loading system provide the necessary conditions for instability to occur. In such instances, the post-peak reduction in
shear strength exhibited by the CY joint allows the loading system to undergo sudden displacements which result in the release of kinetic energy.

3.6 Localization and Path Dependence

It is important to note that the finite difference method employed in UDEC uses explicit time-marching in order to solve the equations of motion for a system of grid points under specified loading and boundary conditions. The final solution for a system modeled in UDEC may take different paths depending upon the initial conditions used in the simulation. The phenomenon is referred to as bifurcation and is commonly seen in the results of simple compressive tests on Mohr-Coulomb materials (Itasca, 2014). Rather than exhibiting uniform distributions of stress and deformation, the specimen may develop shear strains through a localized band of zones, which is similar to the physical behavior seen in geologic materials under comparable loading conditions. However, the exact path of failure observed in a simulation can be affected by small changes in modeling parameters. From the section of the software User’s Guide titled Problem Solving with UDEC (Itasca, 2014):

“In most nonlinear, inelastic systems, there are an infinite number of solutions that satisfy equilibrium, compatibility and the constitutive relations. There is no “correct” solution to the physical problem unless the path is specified. If the path is not specified, all possible solutions are correct. This situation can cause endless debate among modelers and users, particularly if a seemingly irrelevant parameter in the solution process (e.g., damping) is seen to affect the final result. All of the solutions are valid numerically. For example, a simulation done of a mining excavation with low damping may show a large overshoot and, hence, large final displacements, while high damping will eliminate the overshoot and give lower final displacements."

Strain softening materials are especially prone to exhibit the effects of localization. Their load-displacement behavior will be affected by the zone size used in the model, which governs the thickness of the shear band that forms. The path of failure in a strain softening material with sufficiently small zone size may appear randomly oriented depending on the model geometry and the complexity of loading conditions.
In the numerical models constructed for this study, the energy associated with unstable failure is calculated and recorded by UDEC. Several energy terms can be kept as histories during a simulation in UDEC by simply engaging the energy calculation option and initializing the histories. The energy calculation procedures are explained in the following sections, and further details can be found in Section 3 of the *Theory and Background* manual provided with the software (Itasca, 2014).

### 4.1 Energy Balance

The energy balance in a UDEC simulation can be calculated either through static or kinetic terms to account for all of the energy that is added, stored, consumed, or released during a sequence of modeled events. The result of the balance calculation, the released energy, should be the same whether it is calculated through static or kinetic terms in a given simulation. The energy balance based on static terms is governed by the following equation in UDEC (Itasca, 2014):

\[
W_R = W - (U_C + U_B + W_J + W_P)
\]

Where:
- \(W_R\) = released energy
- \(W\) = total boundary loading work supplied to the system
- \(U_C\) = total stored strain energy in material
- \(U_B\) = total change in potential energy of the system
- \(W_J\) = total dissipated energy in joint shear
- \(W_P\) = total dissipated work in plastic deformation of intact rock

In this expression, the boundary work, \(W\), represents the energy made available, or added, to the system and each of the terms in parentheses represent mechanisms that can store, consume, or allow the discharge of energy. The condition for instability is such that the total boundary loading work, \(W\), applied to the model is greater than the amount of energy that can be stored or consumed by the materials. In such a case, the difference equates to a value of energy, \(W_R\), that must be released.

The kinetic energy balance terms essentially account for the rate at which displacement and stress transfer occur in a model, rather than the sum of stored and consumed energy values before and after failure. The energy balance based on kinetic terms is governed by the following equation in UDEC (Itasca, 2014).
\[ W_R = U_K + W_K + W_V + U_M \] (4.2)

Where:
- \( W_R \) = released energy
- \( U_K \) = current value of kinetic energy in the system
- \( W_K \) = total work dissipated by damping
- \( W_V \) = work done by viscous boundaries
- \( U_M \) = total strain energy in excavated material

The current value of kinetic energy, \( U_K \), is determined by the mass and velocity of grid points according to the equation:

\[ U_K = \frac{1}{2} m v^2 \] (4.3)

Where:
- \( m \) = grid point mass
- \( v \) = grid point velocity

Unstable failure of material results in the sudden displacement of grid points, and thus kinetic energy, as the equation of motion is solved within the model. The value of kinetic energy is not cumulative, meaning it will be large during material failure and near zero when the model is in a state of equilibrium.

The value of work dissipated by damping, \( W_K \), is cumulative and accounts for energy that is extracted by UDEC during computation in order to reach a solution of static equilibrium. The default damping coefficient of 0.8 reduces the velocity of a grid point by 80% at each calculation step, and because each grid point has an assigned mass, the damping routine effectively extracts 80% of its kinetic energy. Without a damping factor, unbalanced forces would be allowed to oscillate between the boundaries of a model.

The work done by viscous boundaries, \( W_V \), is zero, as no viscous boundaries are used, and the strain energy in excavated material, \( U_M \), is disregarded because excavations are carried out very slowly, such that the strain energy is released gradually and stresses in the surrounding material are allowed to redistribute accordingly.

### 4.1.1 Mechanical Damping

For the simulations of unstable failure presented in this study, the discussion of released energy relies entirely on the kinetic energy balance. Once a simulation has arrived at a final state of equilibrium, the value of released energy is essentially accounted for by the damped energy term, and thus it is
important to understand the application of damping and how it may affect the results of a simulation. For static analysis, the equations of motion are damped to reach a force equilibrium state as quickly as possible under the applied initial and boundary conditions (Itasca, 2014). The use of traditional, velocity-proportional damping introduces problems when one area of a model is stable and another is failing, as the selection of an appropriate damping constant at any given point in the simulation can require intensive computational effort.

In order to overcome these difficulties, a method of damping, referred to as *local damping*, was developed in which the damping force is proportional to the unbalanced force present at a given node (Cundall, 1987). In this way, damping can be applied separately to areas of stability and instability within a given model, which makes local damping ideal for problems involving sudden load changes or failure.

While UDEC allows a user to modify the damping coefficient, it is important to realize that lower values may result in unrealistic stress wave transmission and oscillations within a modeled system. In a simple elastic model with no discontinuities, the instantaneous application of a boundary stress condition may cause oscillations as the model seeks a new state of equilibrium. If the final stress distribution is the only objective of the elastic analysis, then the oscillations will be of no concern. However, when blocks are made deformable or discontinuities with low tensile strength are present, the transmission of stress waves through the model with a low damping coefficient may cause unrealistic tensile failure in the material or the contacts. In contrast, the use of a higher damping coefficient may prevent the development of yield to its fullest extent and require additional calculation steps to find a new state of static equilibrium.

For the study of instability in quasi-static simulations, no advantage has been found to alter the local damping coefficient in UDEC. In addition, the released energy calculations utilized in this study have been validated for various modes of failure, under various loading conditions, with the default local damping parameters in use.

### 4.1.2 Energy Balance during Unstable Failure

The value of released energy in a simulation involving unstable failure should be the same whether calculated through static or kinetic terms. This concept can be demonstrated in a simple model of compressive stress on an unconfined specimen of strain-softening material bound by elastic platens. The data file used to run the simulation can be found in Appendix A. The low stiffness of the platens, relative to the specimen, allows the release of stored strain energy during failure. Figure 4.1 illustrates the failed specimen and stress-strain data from the simulation. The color of the block represents the value of
cohesion within the zones, and the diagonal band indicates a reduction of cohesion along the plane of failure. Vectors indicate the direction of grid point displacement.

![Figure 4.1: Compressive test on a strain softening sample with resultant stress-strain data.](image)

The specimen is 75 meters wide and 150 meters tall and discretized with 7.5 meter zones. The platens at top and bottom are each 60 meters tall and modeled as elastic material with a Young’s modulus that is 60% lower than that of the specimen. The interfaces between the platens and the specimens are modeled as coulomb slip joints with normal stiffness 100 times higher than the Young’s modulus of the specimen in order to limit the storage and release of strain energy. The strength parameters of the interface are set sufficiently high to prevent shear slip.

The bottom of the model is fixed and a slow velocity boundary condition is applied at the top of the upper platen as the model is cycled for 2.1 million calculation steps. Average vertical stress is measured through a band of zones near the center of the specimen, and the simulation is initialized with 1.5 MPa vertical stress to save computation time. Strain is measured over the length of the specimen and not the platens.

The energy terms that contribute to the static energy balance during the simulation of unstable failure are shown graphically in Figure 4.2, where the boundary loading work applied to the model accounts for the largest energy term.
Figure 4.2: Static energy terms during the compressive test on a strain softening sample

The blue line in the graph illustrates the increase in strain energy as the specimen is loaded and the release of strain energy as the specimen fails. The block strain energy term accounts for the material in the specimen as well as the platens, which gets released and drives shear failure of the specimen once its peak strength has been exceeded. Most of the released strain energy gets consumed by plastic deformation of the specimen along the plane of failure, as indicated by the simultaneous rise of the green line, but a small portion is released through rapid displacement of material as indicated by the small step in the red line.

For the simulation considered here, the final values of the energy terms within the static energy balance are shown in Table 4.1. By subtracting the stored strain energy and plastic deformation work from the total boundary loading work according to Equation 4.1, the difference of 1.1 MJ matches the final value of the released energy curve shown in the graph. The actual magnitude of energy released during failure of the specimen is accounted for by the near-vertical increase of the curve. The remainder of the energy is accumulated as a result of the velocity boundary loading condition.

Table 4.1: Final static energy term values after a compressive test on a strain softening sample

<table>
<thead>
<tr>
<th>Energy Term</th>
<th>Value (MJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Loading Work, $W$</td>
<td>23.6</td>
</tr>
<tr>
<td>Work Dissipated in Plastic Deformation, $W_p$</td>
<td>20.9</td>
</tr>
<tr>
<td>Stored Strain Energy, $U_c$</td>
<td>1.6</td>
</tr>
<tr>
<td>Change in Potential Energy, $U_b$</td>
<td>-</td>
</tr>
<tr>
<td>Energy Dissipated in Joint Shear, $W_j$</td>
<td>-</td>
</tr>
<tr>
<td>Released Energy, $W_r$</td>
<td>1.1</td>
</tr>
</tbody>
</table>
The components of the kinetic energy balance are shown graphically in Figure 4.3, with the current kinetic energy plotted on a separate axis to account for a large difference in scale. The curve representing released energy essentially coincides with that of the damped work, though they actually differ by a very small percentage depending on the value of current kinetic energy according to Equation 4.2. Note that the changes in energy terms associated with failure of the specimen occur within the same range of numerical time between Figure 4.2 and Figure 4.3.

Figure 4.3: Kinetic energy terms during the compressive test on a strain softening sample

The graph of current kinetic energy exhibits a peak value of 2655 Joules during failure of the block and coincides with the rapid increase in damped work. As expected, the total value of released energy in the kinetic energy balance matches that of the static energy balance at the end of the simulation. The final values of the terms within the kinetic energy balance are shown in Table 4.2. The relatively small value of current kinetic energy at the end of the simulation illustrates the very small percentage by which the damped work and the released energy differ.

Table 4.2: Final kinetic energy term values after a compressive test on a strain softening sample

<table>
<thead>
<tr>
<th>Energy Term</th>
<th>Value (MJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Extracted by Damping, $W_k$</td>
<td>1.1</td>
</tr>
<tr>
<td>Current Kinetic Energy in System, $U_k$</td>
<td>$1.4 \times 10^5$</td>
</tr>
<tr>
<td>Work Dissipated by Viscous Boundaries, $W_v$</td>
<td>-</td>
</tr>
<tr>
<td>Strain Energy in Excavated Material, $U_m$</td>
<td>-</td>
</tr>
<tr>
<td>Released Energy, $W_r$</td>
<td>1.1</td>
</tr>
</tbody>
</table>
The velocity boundary condition, which implements a small displacement at every time step, is responsible for the generation of kinetic energy indicated by the gradual slope of the damped work curve before and after block failure. For the loading procedure exhibited in this example, the release of energy that is of most interest is accounted for by the near-vertical portion of the released energy curve. The actual magnitude of energy released during failure of the strain-softening block is approximately 0.8 MJ, which is 3.4% of the total boundary loading work applied to the model. Classifying the released energy as a percentage of boundary loading work can be useful when comparing the effects of various inputs such as strength criteria, stiffness, and loading conditions in parametric models, as the value of boundary loading work required to induce failure in different models can vary.

4.1.3 Energy Balance during Stable Failure

When material fails in a stable manner, the amount of energy made available by the loading system is fully stored or consumed, and the value of released energy is either zero or insignificant. This concept can be illustrated by performing a simple unconfined compressive test on a Mohr-Coulomb material, which continues to deform at a constant stress once its peak strength has been reached and does not allow the release of stored strain energy. The data file used to construct the model is identical to that of the strain softening specimen, with a different constitutive law applied to the failing medium, and can be found in Appendix A. The failed specimen and resultant stress-strain graph are shown in Figure 4.4, where the color of the block indicates the cohesion value, which does not change during failure, and the vectors indicate the direction of displacement.

Figure 4.4: Compressive test on a Mohr-Coulomb sample with resultant stress-strain data.
The geometry and loading conditions in the Mohr-Coulomb test are identical to those described previously for the demonstration of unstable failure on the strain softening sample. The terms of the static energy balance recorded during the simulation of stable failure are shown graphically in Figure 4.5.

Figure 4.5: Static energy terms during the compressive test on a Mohr-Coulomb sample

The boundary loading work is higher at the end of this simulation because the Mohr-Coulomb material has a higher post-peak strength and consumes more energy as further displacement is applied by the loading condition at the boundary. The final values of the static energy balance components are shown in Table 4.3.

Table 4.3: Final static energy term values after a compressive test on a Mohr-Coulomb sample

<table>
<thead>
<tr>
<th>Energy Term</th>
<th>Value (MJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Loading Work, ( W )</td>
<td>40.2</td>
</tr>
<tr>
<td>Work Dissipated in Plastic Deformation, ( W_p )</td>
<td>28.9</td>
</tr>
<tr>
<td>Stored Strain Energy, ( U_c )</td>
<td>11.0</td>
</tr>
<tr>
<td>Change in Potential Energy, ( U_b )</td>
<td>-</td>
</tr>
<tr>
<td>Energy Dissipated in Joint Shear, ( W_j )</td>
<td>-</td>
</tr>
<tr>
<td>Released Energy, ( W_r )</td>
<td>0.3</td>
</tr>
</tbody>
</table>

It can be seen that the stored strain energy and plastic deformation work account for nearly all of the boundary loading work applied to the model. The value of released energy in the simulation is reported as approximately 0.28 MJ, which results from the application of the velocity boundary loading condition and does not reflect any degree of unstable failure. This concept can be more clearly illustrated by examining the kinetic energy balance terms, which are shown graphically in Figure 4.6. The current
kinetic energy is plotted on a separate axis to emphasize the large difference in scale, and the vertical axes are plotted with the same values as those in Figure 4.3 for comparison.

![Diagram of Dynamic Terms - Stable Failure]

**Figure 4.6: Kinetic energy terms during the compressive test on a Mohr-Coulomb sample**

As the graph illustrates, the failure of a Mohr-Coulomb material does not generate a sudden increase in kinetic energy during failure like the strain-softening material does. The value of released energy shown as 0.3 MJ matches that of the static energy balance, as expected, but the steady slope of the damped work curve and absence of perturbations in the kinetic energy graph indicate that no instability has occurred.

### 4.2 Energy Release Mechanisms

The values of released energy discussed later in this dissertation with regard to mine-scale simulations rely on the damped work term in the kinetic energy balance. It is worth noting, however, that the mechanisms of energy storage and discharge quantified in the static energy balance are the mechanisms responsible for the generation of grid point velocities quantified in the kinetic energy balance. The following sections will further describe the role of various mechanisms in the release of kinetic energy.

#### 4.2.1 Gravitational Potential Energy

In mine-scale simulations involving gravity, a majority of the energy available for unstable failure may come from gravitational potential. A simple model can be constructed to ensure that the kinetic energy associated with material displaced rapidly under gravitational loading is calculated accurately by UDEC. A block with a known mass is assigned a value of gravity, and the model is cycled for one second of mechanical time without fixing the boundaries.
With zero mechanical damping, the total energy released at the end of a one-second simulation will be dependent entirely upon the mass and velocity of the block, as governed by the equation for kinetic energy:

\[ U_k = \frac{1}{2} \times (mass) \times (velocity)^2 \]  

(4.4)

With the gravitational acceleration set at 10.0 m/s\(^2\), the velocity of a block is 10.0 m/s at the end of a one second simulation. For a block with a mass of 1000 kg, the kinetic energy calculated according to Equation 4.4 will match the current kinetic energy reported in UDEC at 50,000 Joules. The total released energy will also be 50,000 Joules, as damped work is zero.

When mechanical damping is engaged, the default coefficient of 0.8 reduces the velocity of each grid point by 80% at each calculation step, thus the velocity at the end of a one second simulation is only 2.0 m/s. The value of kinetic energy at the end of the simulation, according to Equation 4.4, will be 2000 Joules. The total released energy in UDEC, as governed by Equation 4.2, will be 10,000 Joules, with 8000 Joules extracted as damped work. With damping applied, the model must be cycled longer in order for the block to reach the same displacement, and thus release the same magnitude of potential energy.

The distance that an object falls under gravitational loading in a certain period of time is determined by the equation:

\[ Distance = \frac{1}{2} \times (gravity) \times (time)^2 \]

(4.5)

In the simulation with no damping, the distance traveled by the block after one second is 5.0 meters. Since the acceleration of gravity is essentially 2.0 m/s\(^2\) in the simulation with damping applied, the time required for the block to travel 5.0 meters is 2.236 seconds. If the damped model is cycled 2.236 seconds of numerical time, the total released energy of the block will be 50,000 Joules, matching the results of the undamped model. The released energy term will account for 40,000 Joules extracted through damping and 10,000 Joules of current kinetic energy, as the velocity of the block will have reached 4.472 m/s. These results illustrate that the energy released by falling ground is calculated accurately for mine-scale simulations of unstable failure.

4.2.2 Release of Stored Strain Energy

Stored strain energy accounts for elastic deformation, often in compression, that occur as loading is applied at the boundary of a model or increased through other mechanisms such as excavation. Under complex loading conditions, stresses and strains in each principal direction are taken into account at each
calculation step. The formula governing elastic strain energy in a given zone, over one time step, is shown in the equation below (Itasca, 2014).

\[
U_c = \frac{A}{2} \left[ (\sigma_{11} + \sigma'_{11})e_{11} + 2(\sigma_{12} + \sigma'_{12})e_{12} + (\sigma_{22} + \sigma'_{22})e_{22} \right] 
\] (4.6)

Where:
- \(\sigma\) = current zone stress
- \(\sigma'\) = stress from previous calculation step
- \(e\) = incremental strains over the current calculation step
- \(A\) = area of zone

The history of stored strain energy in UDEC is recorded as a change of strain energy from the initial conditions in which energy calculation is engaged. The storage of additional strain energy in compression is reported as a positive energy value, and the release of compressed strain energy results in a negative energy value. The value is cumulative for all zones in the model.

The relationship between stored strain energy and the release of kinetic energy is derived by Duvall and Stephenson (1965) for both cylindrical and spherical cavities in an infinite elastic medium. For excavations in a uniform stress field, with no pre-existing opening, the magnitude of radiated seismic energy, \(W_K\), is equal to the total work done by the far field stresses, \(W\), minus the change in stored strain energy, \(U_C\), around the perimeter of the excavation. In addition, the derivations show that the value of released energy is equal to half of the total work done. The relationships are summarized in the following equations:

\[
W_K = W - U_C \quad (4.7)
\]

\[
W_K = \frac{1}{2} W \quad (4.8)
\]

In short, the formulae state that when an excavation is created, the total work done by the stressed rock in the far field is equally consumed by the storage of strain energy around the perimeter of the excavation and the release of kinetic energy during the physical movement of the excavation walls.

Salamon (1984) improved upon the analytical relationship by deriving formulae for the release of energy that occurs when an excavation, of any size or shape, is created or enlarged in an infinite elastic medium. The result of Salamon’s derivation maintains that the total work done by the far field is the sum of additional stored strain energy and the release of kinetic energy at the perimeter of the excavation, though they are not necessarily equal. The result of the Salamon’s derivations take the same form as Equation 4.7.
Agreement between these analytical solutions and the numerical simulation of circular openings in elastic ground are illustrated in the UDEC software Theory and Background manual (Itasca, 2014). The model is shown in Figure 4.7 after excavation of a circular volume at center. The vectors indicate direction and relative magnitude of displacement.

![Figure 4.7: Analysis of energy released by excavation in an elastic medium (Itasca, 2014)](image)

The simulation involves the instantaneous removal of a circular volume at the center of the model, as shown in the figure, followed by a secondary excavation of material around the perimeter of the first opening. The excavations are referred to as Stage I and Stage II respectively. For both stages of excavation, the values of released kinetic energy, stored strain energy, and far field work are compared to analytical solutions by Salamon (1984). The results of the example problem are shown in Figure 4.8 from the Theory and Background manual, with the relevant energy values encircled and the names of the energy terms noted at left.

Although the analytical values are provided through Salamon’s formulae, the results of Stage I illustrate Duvall and Stephenson’s derivations in that the value of stored strain energy is equal to the amount of kinetic energy released during creation of the initial opening. In addition, the sum of the two terms is equal to the total work done by the far field material.
The results of Stage II illustrate Salamon’s derivations for the enlargement of a cavity in that the total work done by the far field is equal to the sum of the stored strain energy and the release of kinetic energy, though they are not equal. The overall agreement between the analytical solutions and the calculations performed by UDEC show that the software is capable of accurately recording the transfer and release of stored strain energy.

### 4.2.3 Compressive Failure of a Softening Material

The failure of a strain-softening material under simple loading conditions is demonstrated on the unconfined specimen in Section 4.1.2. The storage and release of strain energy by the platens in the simulation helps illustrate the concepts put forth in Cook’s (1965) laboratory studies, which revealed that the stiffness of a loading machine supplies the energy that is released when rock samples fail unstably. The magnitude of energy released during the failure of a rock specimen can be quantified by comparing the stress-versus-displacement behavior of the rock specimen with the load-versus-displacement curve of the machine.

An analogy can be made to pillars failing under vertical load, where the post-peak behavior of the pillar represents that of a rock specimen and the load-displacement characteristic of the elastic overburden represents that of a loading machine (Salamon, 1970). In order to validate the calculation of energy released in such a situation, a model is constructed with a strain-softening pillar bound by two rectangular excavations in elastic ground. The data files used to construct the pillar model can be found in Appendix A. The pillar is 5 m tall and 5 m wide, and the excavations are 10 m wide. The bottom of the model is fixed in the vertical direction, and the sides of the model are fixed in the horizontal direction. Loading is provided by a constant boundary stress of 2.0 MPa at the top of the model, and no gravity is present. The model is shown in Figure 4.9 with a majority of the surrounding rock excluded from view.
Elastic properties for the overburden rock are shown in Table 4.4 along with strength parameters for the pillar. The Young’s modulus of the surrounding rock mass is set much lower than that of the pillar in order to promote instability.

Table 4.4: Material properties used in the simulation of pillar failure.

<table>
<thead>
<tr>
<th></th>
<th>Dens (kg/m³)</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson ratio</th>
<th>Friction angle (deg)</th>
<th>Cohesion (MPa)</th>
<th>Residual Cohesion (MPa)</th>
<th>Tensile Str. (MPa)</th>
<th>Dilation angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Rock</td>
<td>2500</td>
<td>5.0</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pillar</td>
<td>1500</td>
<td>25.0</td>
<td>0.2</td>
<td>25.0</td>
<td>1.5</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The stress-strain behavior of the pillar is recorded during a very slow excavation procedure. The zones in the two entries are deleted simultaneously, replaced with equivalent support pressures along the inner boundaries, and the pressures are reduced to zero over 1000 increments. The model is solved to equilibrium at each interval. Results of the simulation reveal that the pillar reaches an average vertical stress just above 5.0 MPa and fails when support forces in the entries are reduced by 94.6%.

In order to obtain a graphical representation of the overburden load vs deformation, or Ground Reaction Curve (GRC), an identical model is constructed with the loading conditions that existed during failure of the pillar model. Excavation support loads of 5.4% of their original value are applied at each grid point in the entries of the GRC model and held constant for the duration of the GRC simulation. Additionally, the vertical forces present within the pillar just before failure are extracted and applied to their respective grid points in the GRC model. Figure 4.10 shows the GRC model in pre-pillar-failure equilibrium with support loads marked by arrows at each grid point along the roof and floor. The arrow size is adjusted in the figure to illustrate that the virtual pillar support forces are larger than the supports within the entries.
The color of the zones in the figure represents vertical stress, which is concentrated above the virtual pillar in the center and reduced to nearly zero above the entries. The ground reaction curve is obtained by reducing the pillar supports uniformly over 1000 increments. The resultant load vs vertical strain data from the GRC model is compared to pillar stress-strain data in Figure 4.11.

The magnitude of energy released during pillar failure is obtained by calculating the area between the two curves in the figure. For this purpose, 11 points are selected from the graph and plotted as a polygon in a CAD package capable of calculating areas. With units of Pascals (N/m²) on the y-axis and strain on the x-axis, the product of the area calculation will be in units of N/m², which is an energy density. In order to obtain units of energy in N-m (Joules), the result must be multiplied by the volume of the pillar, which is 25 m³ assuming a depth of 1 m in the third direction.
The energy density calculated between the curves is approximately 1,630 N/m², and when multiplied by the volume of the pillar, the total energy is 40,750 J. The increase in damped work that occurs during failure of the pillar is calculated by UDEC as approximately 41,300 J. For verification, Figure 4.12 shows the history of damped work during the simulation.

![Damped Work Graph]

Figure 4.12: Damped work recorded during simulation of pillar failure.

The difference of approximately 1.3% is likely introduced by calculating the graphical area using only 11 data points. Regardless, the relative agreement between the energy calculation performed by UDEC and the energy magnitude calculated through stress and displacement data provides confidence that the calculation of released kinetic energy in UDEC remains accurate when complex mechanisms of material failure are simulated.

### 4.2.4 Shear Slip on a Softening Joint

The examples in Sections 4.1.2 and 4.2.3 both illustrate the failure of a strain-softening material in compression, which allows the release of stored strain energy and sudden displacement of material. Strain energy can also be released during failure of a discontinuity, which results in sudden shear displacement and the generation of kinetic energy. Just as the simulation of unstable failure of a material in compression requires a strain-softening constitutive model, unstable failure of a discontinuity in shear requires a displacement-softening constitutive model, which is provided in UDEC as the Continuously-Yielding joint.

To illustrate the effect of unstable shear slip, a model is constructed of two elastic blocks with one shearing discontinuity between them. The data file associated with the model can be found in Appendix A. The large lower block is fixed and the upper block has a vertical stress of 2.0 MPa applied to the top when the model is brought to an initial state of equilibrium. A gradually-increasing horizontal force...
vector is then applied at the top left corner of the upper block to induce shear. The model is shown in Figure 4.13 with a graph of the resultant shear stress versus shear displacement at the discontinuity.

![Figure 4.13: Simulation of unstable shear slip and resultant shear stress-displacement data.]

The faint dashed lines in the model illustrate the coarse zoning used in each of the blocks. The model is discretized in such a way that the discontinuity between the blocks consists of only two contacts. The stress and displacement data shown in the graph represents an average of the values recorded for each contact. The sliding block is one meter square and is separated from the fixed block by a lateral distance of 30 millimeters before loading is applied. The purpose of the 30 millimeter gap is to limit displacement once the joint fails and allow the sliding block to reach a state of final equilibrium.

The sliding block is given a Poisson’s ratio of 0.2 and a shear modulus of 4.16 GPa. The fixed block is given a stiffness that is 100 larger in order to reduce its influence on the accumulation and release of strain energy. In this way, the loading system can be characterized by evaluation of the average shear stress within the zones of the sliding block. The properties applied to the continuously-yielding joint are listed in Table 4.5.

**Table 4.5: Properties of continuously yielding joint used in simulation.**

<table>
<thead>
<tr>
<th>Joint</th>
<th>Normal stiffness (GPa)</th>
<th>Shear stiffness (GPa)</th>
<th>Initial Friction Angle (deg)</th>
<th>Final Friction Angle (deg)</th>
<th>Joint Roughness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CY Joint</td>
<td>20.0</td>
<td>10.0</td>
<td>40.0</td>
<td>15.0</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

The release of kinetic energy during failure of the discontinuity can be quantified by comparing the shear stress-displacement curve of the joint with the load-displacement characteristic of the loading system. The shear load on the discontinuity is provided by the elastic deformation of the upper block with a force vector applied at one corner. In this example, the application of a constant force vector at the top left corner of the sliding block provides a “perfectly soft” loading system, meaning that the load versus displacement characteristic is a horizontal line. No drop in shear stress will occur as the block...
slides because the applied load is always present and capable of infinite displacement. The complete shear stress versus shear displacement of the contact is shown in Figure 4.14 with the horizontal load versus displacement line exhibited by the block after shear slip has occurred. 

![Contact Shear Stress/Disp vs Load Line](image)

Figure 4.14: Shear stress-displacement data with load-displacement behavior of the block.

As the value of shear displacement on the horizontal axis illustrates, the upper block slides until the 30 millimeter gap is closed. The magnitude of released energy is obtained by calculating the area between the two curves. For this purpose, 9 points are selected from the graph and plotted as a polygon in a CAD package capable of calculating areas. With units of Pascals (N/m$^2$) on the y-axis and displacement on the x-axis, the product of the area calculation will be in units of N/m. In order to obtain units of energy in N-m (Joules), the result must be multiplied by the length of the contact and the depth of the model in the third direction, which are each 1.0 meters.

The area calculated between the curves in Figure 4.14 is approximately 12,690 N/m, which results in a total energy of 12,690 Joules when multiplied by one meter in two dimensions. The increase in damped work that occurs during failure of the joint is calculated by UDEC as approximately 12,625 Joules. For verification of the results obtained by UDEC, Figure 4.15 shows the history of damped work recorded during the simulation.
The difference of approximately 0.5% between the damped work and the graphical energy value is likely introduced during calculation of the area in the graph. The agreement in the results provides further assurance that the calculation of released kinetic energy in UDEC remains accurate during the simulation of unstable failure on discontinuities in shear.

4.3 Comparing Released Energy and Seismic Magnitude

The energy released in a large-scale numerical simulation of unstable failure can be compared to seismic magnitude through the use of an empirical formula. Several empirical equations have been proposed by previous researchers in the field of seismology, which define the relationship between seismic magnitude and energy based on parameters such as amplitude, wavelength, velocity, and duration of seismic waves detected by various instruments. Empirical relations also account for the distance at which signals are detected from their source and the assumed rates of signal attenuation exhibited by the earth materials present. For the purpose of this study, two different empirical formulae are considered.

It is worth noting that the equations will be used for the comparison of seismic magnitudes associated with mine collapse events, rather than earthquakes. Since the formulae were developed through analysis of raw seismic wave data, it is assumed that the estimate of energy release associated with ground motion, which is inherently a very broad estimate, is applicable regardless of the source.

4.3.1 Gutenberg and Richter, 1956

Gutenberg and Richter (1956) present a nonlinear formula for the comparison of seismic magnitude with the base-10 logarithm of total released energy. The work published in 1956 supersedes previous research on the same topic, with the inclusion of additional data points and revised methods of estimating
energy and magnitude separately. For seismic events detected by multiple monitoring stations, correction
factors are assigned to account for signal attenuation rates and travel distances. The revised equation is:

\[
\log E_s = 9.14 + 2.14M - 0.054M^2
\]  

Where:
\( E_s \) = released energy (ergs)
\( M \) = seismic magnitude

Note that the equation is constructed with energy in units of ergs. Energy values in units of Joules
must be multiplied by 1.0e7 in order to obtain units of ergs before solving for a value of magnitude.

Gutenberg and Richter state that previously-derived, linear formulae, including those derived by
themselves and others, appear applicable to groups of earthquakes at either high or low magnitudes but
lack continuity over both groups. The nonlinear relation was chosen as an alternative to a bilinear
equation in an attempt to fit a wide range of data. Figure 4.16 shows a small set of data points used by
Bath (1955) to formulate a linear relation between energy and seismic magnitude, with the Gutenberg and
Richter nonlinear curve overlain for comparison.

![Figure 4.16: Comparison of empirical formulas by Bath and Gutenberg-Richter.](image)

Although the Gutenberg and Richter equation is based on a much larger database of earthquakes, the
figure illustrates that the nonlinear formula is able to fit the data points for which a specific linear
equation was derived. The linear curve proposed by Bath intersects the horizontal axis at a seismic
magnitude that does not correlate well with the value of energy at the base of the vertical axis and thus, a lack of applicability to lower magnitude events.

Since the empirical relation will be used to back-calculate seismic magnitude from a given value of released energy in this study, the magnitudes obtained through the Gutenberg and Richter equation are assumed to represent local magnitudes.

4.3.2 Kanamori, 1993

Kanamori (1993) presents a linear formula based on a large database of small scale earthquakes with multiple methods of data collection and improvements in data processing specifically for the purpose of estimating released energy. The empirical relation is as follows:

$$\log E_S = 1.96M_L + 9.05$$  \hspace{1cm} (4.10)

Where:
- $E_S$ = released energy (ergs)
- $M_L$ = local seismic magnitude

Kanamori demonstrates the linearity of the relationship between released energy and magnitude by independently calculating the two values through a specific set of procedures for 66 small earthquakes in southern California. The formula is said to improve upon previous relations, including those by Gutenberg and Richter, through the rigorous calibration of various correction factors. Figure 4.17 shows the results of the study.

Figure 4.17: Correlation of Kanamori's empirical formula with a wide range of data points.
The calculations of energy are based on raw seismic wave data and account for attenuation rates and correction factors specific to the sites at which the signals were measured. Magnitudes are estimated through several traditional methods and consider data collected from different instruments. For local magnitudes between 1.5 and 6.0, the linear equation closely fits the data considered in the study. The linear equation derived by Kanamori is considered appropriate for the comparison of mining induced seismicity because all known mine collapse events have exhibited magnitudes of 6.0 or less.
This chapter presents a back analysis of the Crandall Canyon Coal Mine collapse, which occurred in August of 2007. The first section provides information pertaining to the layout of the mine and the extent of damage incurred during the collapse for readers unfamiliar with the incident and as a basis for the approach used in the study. The remainder of the chapter focuses on the calibration of material properties, setup of models, and the evaluation of released energy during the simulation of collapse.

5.1 Background Information

The following section contains a brief history of the events that led to the collapse of the Crandall Canyon Mine and summarizes the geomechanical setting in which the collapse occurred. The information and figures presented here are taken primarily from the Mine Safety and Health Administration (MSHA) investigation report (Gates, 2008).

5.1.1 Mine History

The Crandall Canyon Coal Mine was located in the Wasatch Plateau coal field of central Utah. Mining began in 1983 with the development of rooms and pillars, which were recovered upon retreat, under depths of overburden ranging from 120 to 480 meters. Longwall mining was implemented in 1995 and conducted extensively in the western portion of the property beneath overburden depths up to 670 meters. As reserves became scarce in 2005, the final years of production were limited to pillar recovery operations in the mains that ran to the southern and western boundaries of the property. Figure 5.1 shows the state of extraction in the Main West section at the far western extent of the property around the year 2000, when longwall Panel 13 to the south was completed. At that point in time, the mains were separated from the longwall panels to the north and south by 135-meter barriers of intact coal. Contours of overburden depth, in units of feet, are shown in gold.

As the contours in the figure suggest, the steeply-sloped mountains at the surface increased the depth of cover toward the center of the longwall panels. Although the west mains were sealed in 2004 due to deteriorating roof and pillar conditions, the mine operator developed a plan to recover additional resources from the adjacent north and south barriers of intact coal. In October of 2006, a series of four entries were developed in the north barrier and advanced west to cross cut number 159. Crosscut numbers are labeled above the mains in Figure 5.1. Pillar retreat operations were then advanced back to the east, toward deeper cover, until February of 2007 when operations were suspended due to a coal outburst event that damaged utilities near cross cut number 134.
Mining proceeded in the south barrier with the development of a series of four entries similar to that in the north. The cross cuts were spaced farther apart to increase the size and load-bearing capacity of the pillars. Retreat operations were then advanced back to the east, toward deeper cover. On August 6, 2007, the widespread collapse of the Main West area trapped six miners and generated a 3.9 local magnitude seismic event. Figure 5.2 shows the state of extraction at the time of the collapse overlain by subsidence contours, in units of centimeters, measured after the event.
A brown dotted line in the south barrier section of Figure 5.2 shows where travel was completely blocked from the east by rubble. The 15cm subsidence contour was estimated to be a good indication of the other areas in which pillar damage was severe enough to completely block travel through the entries (Gates, 2008). Figure 5.3 shows an entry in the south barrier that was found to be completely blocked during the subsequent rescue effort, where the coal comes within a meter of the roof in some areas and is touching in others.

Figure 5.3: An entry in the south barrier completely blocked by rubble.

Search and rescue teams attempted to reach the last known location of the mining crew by excavating through the rubble in the entry farthest to the south. Excavation subsequently destabilized the surrounding coal, and on August 16, 2007, a secondary collapse took the lives of two rescue workers and one mine safety inspector. Efforts to locate the six entrapped miners through boreholes were suspended on August 30th, and within a matter of weeks, the mine was permanently sealed.

A maximum subsidence of 30 centimeters was measured directly above the south barrier pillar, near the peak of the overlying mountain and the greatest depth of cover. Subsidence was measured using Interferometric Synthetic Aperture Radar (InSAR) data obtained from the United States Geological Survey (USGS), which relies on measurements taken by satellite before and after an event. The difference in ground elevation is determined by the wavelengths of radio waves reflected off the earth’s surface. Average closure between the roof and floor of the coal seam was estimated to be 0.3 meters over an area of approximately 0.2 square kilometers (Pechmann, 2008).
5.1.2 Geology

The Hiawatha coal seam was typically between 1.5 and 4.0 meters thick and mined only in areas where its height exceeded 2.3 meters. The Blackhawk formation overlying the Hiawatha coal seam consisted of 200 meters of interbedded sandstones and siltstones with various thin coal seams that were not mined. The Castlegate Sandstone was located above the Blackhawk formation and consisted of approximately 75 meters of competent sandstone. The Price River and North Horn formations, consisting of alternating sandstone, siltstone, and shale, made up the remainder of the material above, with the overall thickness dependent upon location. Figure 5.4 shows a generalized stratigraphic column from the central portion of the mine.

Figure 5.4: Geologic cross section above the Crandall Canyon Mine.

As the stratigraphic column suggests, the geology of the Wasatch Plateau contained a significant amount of sandstone, which lends itself to the formation of steep slopes and cliffs in areas where it is
exposed at the surface. The quantity and competence of sandstone in the overburden was known to affect the process of caving above the longwall panels. Subsidence measurements from Panels 13 through 15, just south of the west mains, indicated that very little subsidence had occurred over the middle of Panel 13 when longwall mining in the west was complete. The data suggests that the stresses within the south barrier section may have been elevated by the additional load of the cantilevered sandstones. The overburden of the Main West section of the mine acts as a stiff beam above mining excavations and is capable of transferring abutment loads over distances of several hundred meters.

5.1.3 Excavation Geometry

The entries in the Main West were mined at a height of 2.5 meters, and in areas where the thickness of the coal seam was greater than the mining height, coal was left in the floor or the roof. Pillars in the mains were typically 21 meters square when mined, though gradual damage due to abutment loading may have reduced their effective size. The north and south barriers of intact coal, which separated the mains from the longwall panels, were approximately 135 meters in width when longwall mining was complete.

The entries in the north barrier section were approximately 6.0 meters wide. Excavation of the four-entry system, with pillars 18 meters wide, resulted in a remnant barrier width of approximately 41 meters adjacent to longwall Panel 12 and a strip pillar 16 meters wide separating the new development from the sealed mains. Figure 5.5 illustrates the final excavation geometry in the Main West section with the dimensions of pillars and entries in the north barrier noted below.

![Diagram of excavation geometry](image)

Figure 5.5: Excavation geometry of the Main West section of the mine.

Similar to the development in the north, the excavation of the four-entry system in the south barrier resulted in a remnant barrier width of approximately 37 meters adjacent to longwall Panel 13 and a strip pillar 19 meters in width adjacent to the sealed mains.
5.2 Approach

The goal of the back analysis is to reproduce the widespread, unstable failure of the Main West section of the Crandall Canyon Mine in a 2D model and calculate the associated release of kinetic energy. Mine-scale models are constructed using the UDEC (Itasca, 2014) distinct element software, and the mine geometry is based on the dimensions described in Section 5.1.3.

Material properties are calibrated for Mohr-Coulomb and strain-softening constitutive laws in the coal. Properties are also calibrated for the coal-rock interface above and below the coal seam to evaluate the effect of shear slip on the failure of the pillar, which is compared between fixed conditions, Coulomb slip, and Continuously-Yielding constitutive laws. Parametric simulations are run using a single pillar geometry from the Main West to evaluate the effect of each of the material and joint constitutive models on the strength and deformation characteristics of a large width-to-height pillar.

The same combinations of material and joint constitutive laws are applied in parametric mine-scale simulations to evaluate the potential for unstable failure as excavation is progressed. Excavations are carried out in a sequence that reflects the history of mining in the area, which can be categorized into four main events for the purpose of back analysis. They are:

1. Development of original five entries in west mains
2. Abutment loading from long wall mining north and south
3. Development of north barrier entries and pillar recovery
4. Development of south barrier entries and pillar recovery

The release of kinetic energy is calculated throughout the process of simulated excavation. Instability is quantified by the amount of energy released during the excavation of any single entry or pillar, and the largest of these values from the parametric series of models will be compared to the magnitude of the seismic event at the mine through the use of empirical formulas. The results of the 2D model are extrapolated in the third direction in order to make this comparison.

5.2.1 Coal Material Properties

The material properties used in the coal seam are derived from a thorough calibration process. In this process, a series of individual pillar models with varying width-to-height ratios are failed under compression to ensure that their peak or nominal strength is consistent with the predicted strength of similarly-sized coal pillars (Salamon-Munro, 1967) and (Mark, 2000). These tests are performed on three-dimensional pillar models using the FLAC3D software package (Itasca, 2012).
FLAC3D is three-dimensional explicit finite difference program designed for the analysis of continuum materials. The principles that govern 3D analyses in FLAC3D, such as the specification of boundary conditions, loading, and material properties, are similar to those for 2D analyses in UDEC, but the generation of a mesh requires the use of pre-defined primitive blocks. Discontinuities can be modeled in FLAC3D by specifying an interface between adjacent continuum blocks, although the simulation of block separation and rotation is better handled by the distinct element method employed in UDEC. For the purpose of material property calibration, the contact between the coal and surrounding rock is fixed in the FLAC3D pillar models. An example of the FLAC3D pillar geometry is shown in Figure 5.6.

![Figure 5.6: Example of geometry used to calibrate pillar strength in FLAC3D (Ozbay, 2015).](image)

The pillars are 2.4 meters in height with no additional coal modeled above or below the entry and a fixed interface between the coal and rock. The pillars are assigned strain softening material properties, and the surrounding rock is modeled as elastic material. The FLAC3D models are discretized using a zone size of 0.4 meters in the coal so that the height of the pillars contain six finite element zones. The complete list of material properties calibrated for the coal seam are listed in Table 5.1 along with the elastic properties used for the surrounding rock. The hardening and softening parameters of the cohesion, friction angle, and dilation angle are listed in Table 5.2 along with their associated levels of strain.

**Table 5.1: Material properties applied to coal and surrounding rock in FLAC3D pillar models.**

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg / m$^3$)</th>
<th>Young’s Mod. (Pa)</th>
<th>Poisson Ratio</th>
<th>Friction Angle (deg)</th>
<th>Cohesion (Pa)</th>
<th>Dilation Angle (deg)</th>
<th>Tensile Strength (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden</td>
<td>2350</td>
<td>23.4e9</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Coal</td>
<td>1313</td>
<td>3.0e9</td>
<td>0.2</td>
<td>23.0</td>
<td>1.48e6</td>
<td>2.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Table 5.2: Strain hardening and softening parameters applied to coal in FLAC3D pillar models.

<table>
<thead>
<tr>
<th>Strain</th>
<th>Cohesion (Pa)</th>
<th>Strain</th>
<th>Friction Angle (deg)</th>
<th>Strain</th>
<th>Dilation Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>1.48E+06</td>
<td>0.0000</td>
<td>23</td>
<td>0.0000</td>
<td>2</td>
</tr>
<tr>
<td>0.0005</td>
<td>1.48E+06</td>
<td>0.0005</td>
<td>30</td>
<td>0.0005</td>
<td>10</td>
</tr>
<tr>
<td>0.0136</td>
<td>2.00E+05</td>
<td>1.0000</td>
<td>30</td>
<td>0.0136</td>
<td>10</td>
</tr>
<tr>
<td>1.0000</td>
<td>2.00E+05</td>
<td></td>
<td></td>
<td>0.0141</td>
<td>2</td>
</tr>
</tbody>
</table>

The results of the calibration procedure in FLAC3D are shown in Figure 5.7 for pillars with width-to-height (W:H) ratios between 0.5 and 6.0 and compared graphically to empirically-estimated pillar strengths. The strengths of the pillars modeled in FLAC3D show excellent agreement with the Mark-Bieniawski formula at lower width-to-height ratios and gradually trend toward the values predicted by Salamon-Munro as the pillars get larger.

![Calibrated Pillar vs. Pillar Strength Formulas](image)

Figure 5.7: Modeled pillar strengths versus empirically estimated strengths (Ozbay, 2015).

The properties calibrated in the FLAC3D models are applied to 2D models in UDEC with slight adjustments in the softening parameters to compensate for the inherent differences between finite element and finite difference mesh generation. A uniform zone size of 0.4 meters in FLAC3D generates a cubic finite element zone that is 0.4 meters in length, while in UDEC, a square element with a length of 0.4 meters is generated and automatically split into four triangular sub-zones in the Quad zoning routine. The resultant difference in zone size and shape affects the calculation of strain and requires slightly different
softening parameters. The strength parameters calibrated for the coal and overburden material in UDEC are shown in Table 5.3. The Mohr Coulomb strain softening parameters applied to the coal are shown in Table 5.4 with their associated strain values.

Table 5.3: Material properties applied to coal and surrounding rock in UDEC models.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg / m³)</th>
<th>Young’s Mod. (Pa)</th>
<th>Poisson Ratio</th>
<th>Friction Angle (deg)</th>
<th>Cohesion (Pa)</th>
<th>Dilation Angle (deg)</th>
<th>Tensile Strength (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden</td>
<td>2350</td>
<td>23.4e9</td>
<td>0.26</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Coal</td>
<td>1313</td>
<td>3.0e9</td>
<td>0.20</td>
<td>23.0</td>
<td>1.69e6</td>
<td>2.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 5.4: Strain hardening and softening parameters applied to coal in UDEC models.

<table>
<thead>
<tr>
<th>Strain</th>
<th>Cohesion (Pa)</th>
<th>Strain</th>
<th>Friction Angle (deg)</th>
<th>Strain</th>
<th>Dilation Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000</td>
<td>1.69E+06</td>
<td>0.00000</td>
<td>23</td>
<td>0.00000</td>
<td>2</td>
</tr>
<tr>
<td>0.00006</td>
<td>1.54E+06</td>
<td>0.00007</td>
<td>27.5</td>
<td>0.00007</td>
<td>10</td>
</tr>
<tr>
<td>0.00008</td>
<td>1.47E+06</td>
<td>0.00010</td>
<td>30</td>
<td>0.01360</td>
<td>10</td>
</tr>
<tr>
<td>0.03500</td>
<td>2.00E+05</td>
<td>1.00000</td>
<td>30</td>
<td>0.01413</td>
<td>2</td>
</tr>
<tr>
<td>1.00000</td>
<td>2.00E+05</td>
<td>1.00000</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For simulations in which strain softening is not considered, the strength of the coal is defined simply by constant values of the friction angle, cohesion, dilation angle, and tensile strength listed above.

5.2.2 Joint Material Properties

The strength properties used in the coal-rock interface are derived from literature regarding previous research on the topic and calibrated in a series of simple numerical models. A friction angle of 20 degrees and cohesion of zero are used for the strength of the Coulomb slip interface. This friction angle lies within the range of 10 to 25 degrees suggested for fault gouge and smooth rock surfaces, respectively (Iannacchione, 1990), and is more conservative than the value of 25 degrees that has been suggested for pillar strength studies with a coal-rock interface (Esterhuizen, 2010). A cohesion value of zero is used as a conservative measure.

A series of direct shear tests are performed in UDEC in order to calibrate a Continuously-Yielding joint with peak and residual strengths that compare with the strength of the Coulomb slip joint. In the direct shear tests, two blocks with a single contact surface between them are loaded with 25 MPa of normal stress. Data files associated with the direct shear tests can be found in Appendix A. Figure 5.8 shows a diagram of the direct shear test model.
Figure 5.8: Direct shear test model geometry.

A very small, constant horizontal velocity is applied to the upper block until failure occurs and the strength of the joint has reached a residual value. The shear stress / shear displacement graphs for each of the joints used in the mine-scale analyses are shown in Figure 5.9. The data file used to obtain the values in the graph can be found in Appendix A.

Figure 5.9: Comparison of Coulomb Slip (CS) and Continuously Yielding (CY) joints in shear.

The two joint constitutive models are given the same initial values of shear and normal stiffness, but their strength criteria require fundamentally different input parameters. The strength parameters of the CY joint are adjusted through trial and error until the results of the test exhibit a peak shear strength that is slightly higher than that of the Coulomb Slip joint and a residual strength that is slightly lower under the same loading conditions. The properties used in each of the joints are shown in Table 5.5.
Table 5.5: Properties applied to respective joint constitutive models.

<table>
<thead>
<tr>
<th></th>
<th>Fixed</th>
<th>Coulomb slip</th>
<th>Continuously Yielding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Stiffness (Pa)</td>
<td>50.0e9</td>
<td>50.0e9</td>
<td>50.0e9</td>
</tr>
<tr>
<td>Normal Stiffness (Pa)</td>
<td>50.0e9</td>
<td>50.0e9</td>
<td>50.0e9</td>
</tr>
<tr>
<td>Initial Friction Angle (deg)</td>
<td>60.0</td>
<td>20.0</td>
<td>40.0</td>
</tr>
<tr>
<td>Final Friction Angle (deg)</td>
<td>-</td>
<td>-</td>
<td>15.0</td>
</tr>
<tr>
<td>Joint Roughness (m)</td>
<td>-</td>
<td>-</td>
<td>0.00015</td>
</tr>
<tr>
<td>Cohesion (Pa)</td>
<td>10.0e9</td>
<td>0.0</td>
<td>-</td>
</tr>
<tr>
<td>Dilation Angle (deg)</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
</tr>
<tr>
<td>Tensile Strength (Pa)</td>
<td>10.0e9</td>
<td>0.0</td>
<td>-</td>
</tr>
</tbody>
</table>

For simulations in which the coal-rock interface conditions are fixed, a Coulomb slip constitutive model is used with extremely high strength criteria to prevent slip or separation from occurring.

5.2.3 Evaluation of Material Properties in a Single Pillar

Combinations of each material and joint constitutive model are applied to a single pillar geometry to evaluate their effect on the peak strength and deformation. The half-symmetry model represents typical excavation geometry in the north and south barrier sections of the Main West, where pillars were approximately 18 meters wide and 2.4 meters tall. The modeled pillar is actually 9.2 meters wide in order to accommodate a 0.4 meter zone size, which makes the effective width-to-height ratio 7.7. The entry is 3.2 meters wide in the model, and 1.6 meters of floor coal is left below the excavation. The geometry of the pillar model is represented in Figure 5.10 with the coal seam colored in pink and a majority of the surrounding rock excluded from view. The single pillar data file can be found in Appendix A.

![Figure 5.10: Single Pillar geometry used to evaluate effects of material and joint properties.](image-url)
The bottom and sides of the model are fixed in the perpendicular direction and the top surface is slowly loaded with a velocity boundary condition. Average stress is measured through a horizontal band of zones just below the centerline of the pillar, and closure is measured over the entire coal seam.

The use of two material models in the coal and three joint models at the coal-rock interface results in a parametric series of six simulations. Each simulation is given a name that consists of abbreviations representing the constitutive models used in the coal and the interface respectively. For example, Mohr-fix represents a model with Mohr-Coulomb properties assigned in the coal and fixed conditions at the interface. The use of a strain softening material is abbreviated as MCss. Coulomb slip conditions at the interface are abbreviated with the initials CS, and a Continuously Yielding interface is labeled as CY.

Figure 5.11 shows the stress versus closure measured in each of the pillar loading simulations with different combination of material and joint constitutive laws. Although the models were loaded until closure reached approximately 0.3 meters, the horizontal axis is limited to 0.2 meters closure and the vertical axis is limited to 60 MPa to enhance visualization.

![Stress vs Closure - 18 meter Wide Pillar](image)

Figure 5.11: Stress versus closure measured with different material and joint properties.

The results illustrate a significant difference in strength depending on the properties of both the coal and the interface. Each of the pillars with strain softening parameters applied in the coal exhibits signs of strength reduction between 0.02 and 0.05 meters of closure, and their strengths are collectively lower than those of the Mohr-Coulomb pillars. In addition, as closure increases, models with Continuously Yielding properties at the interface exhibit a significantly lower strength than pillars with Coulomb slip properties. The two pillars with fixed conditions at the interface exhibit the highest strength, suggesting that the failure of the pillar is accommodated by some degree of shear slip.
The combinations of material and joint properties shown here are applied in parametric mine-scale simulations to gauge their effect on the stability of failure as excavation is carried out. Further description of the mine-scale modeling approach is provided in the following section.

5.2.4 Mine-Scale Model Geometry

The mine-scale models represent a 2D, north-south cross section through the entire Main West section of the mine, which can be drawn from left to right through the plan view of mine workings in Figure 5.5. The left and right extents of the model are formed by a vertical plane of symmetry through the center of the bleeder entries that were developed for the adjacent longwall panels. The coal seam is 4.0 meters thick, and the overburden is modeled with a depth of 609 meters (2000 feet). The selected depth of cover represents a conservative average of the values shown in Figure 5.1 over the entire Main West area. An example of the mine-scale model data files can be found in Appendix A.

The top of the model is a free surface, the bottom is fixed in the vertical direction, and the left and right boundaries of the model are fixed in the horizontal direction. The material above and below the coal seam is modeled as an elastic material without any vertical jointing, though horizontal joints exist for zone discretization and are fixed. The continuous elastic blocks reduce simulation time and focus the analysis on the failure of the pillars alone. The use of a stiff overburden is also analogous to the presence of stiff sandstone units at the mine, which distribute loads across large distances. A schematic of the mine-scale model geometry is shown in Figure 5.12.

![Figure 5.12: Schematic of the Main West mine-scale 2D model geometry.](image-url)
The models are brought to an initial state of equilibrium under gravitational loading with elastic material properties. The model is then brought to equilibrium with either Mohr-Coulomb or strain softening properties in the coal seam and deformable joint properties at the coal-rock interface. The excavations and abutment loading procedures are carried out in the following order:

1. Excavation of Main West entries 1 – 5
2. Application of abutment load on south boundary of model
3. Application of abutment load on north boundary of model
4. Excavation of entries 1 – 4 in north barrier
5. Retreat mining of two pillars in north barrier
6. Excavation of entries 1 – 4 in south barrier
7. Retreat mining of two pillars and floor coal in south barrier
8. Slabbing of south barrier remnant pillar

Each mine entry is divided into vertical slices 0.4 meters wide and mined one slice at a time from left to right. Furthermore, each slice is deleted and replaced with equivalent forces, which are reduced incrementally in 100 steps for as little dynamic effect as possible on the model. Figure 5.13 illustrates the division of a single entry into slices for progressive excavation.

![Figure 5.13: Mine entry divided into slices for progressive excavation.](image)

Excavation is controlled by a FISH function, which deletes a single slice of the entry and gradually reduces support pressure with the SOLVE RELAX command. Execution of the SOLVE RELAX command automatically calculates and applies the appropriate forces required for a given excavation and desired number of relaxation increments.

### 5.2.5 Abutment Loading

Abutment loading from longwall panels to the north and south of the Main West area is simulated in the models through the use of grid point forces along the right and left boundaries. The use of grid point forces allows the abutment loads to be applied to the model after excavation of the west mains in virgin ground conditions.
An abutment angle of 21 degrees was assumed for normal caving conditions, as suggested in the Analysis of Retreat Mining Pillar Stability (ARMPS) program (Mark, 1997). Although surface subsidence patterns at the mine suggested the strata overlying the longwall panel to the south were cantilevered, which may have further increased the load on the south barrier pillar before the collapse, the abutment loads are modeled uniformly at 21 degrees for a lack of additional data. Figure 5.14 shows the elastic mine-scale model used for calibration of abutment loads, where a 21-degree wedge is physically attached at the left and right sides.

This full-scale abutment wedge model is brought to equilibrium under gravitational loading with elastic material properties. The wedges are then removed instantaneously and the appropriate grid point forces are extracted through the use of a FISH function. The data file used to analyze abutment loads can be found in Appendix A.

Figure 5.14: Illustration of physical abutment wedges and equivalent abutment loads.

The distribution of vertical stress throughout the coal seam is compared between the model with physical wedges and the model with representative grid point forces to ensure that the effect within the coal seam is the same. The distribution of vertical stress in the coal seam, measured in a state of equilibrium with either physical wedges or grid point loads applied, is shown in Figure 5.15. The units of meters on the horizontal axis represent x-coordinates from left to right within the 2D model, where the south barrier is positioned at left.
Figure 5.15: Comparison of stress distribution in coal seam with physical and simulated wedges.

The graph illustrates the elevated levels of vertical stress toward the north and south boundaries of the model when the abutment loads are present. It also shows average vertical stresses of approximately 21 MPa in the mains at the center of the model, which represents a 50% increase with the abutment loads present. Virgin stress conditions, under 609 meters of overburden with an average density of 2350 kg/m$^3$ are approximately 14 MPa. The magnitude of stress shown in the abutment loading simulation offers insight to the conditions that may have existed in the original five west mains entries, which were sealed due to deteriorating ground conditions after completion of the longwall panels.

5.3 Analysis of Mine-Scale Modeling Results

The most direct way of determining the relative significance of each of the mine-scale models is to examine the total values of energy released after the full sequence of excavation steps described in Section 5.2.4. In this way, the occurrence of a potential instability can be identified in models with larger energy values. Figure 5.16 shows the cumulative energy released in each of the mine-scale simulations. The names of the individual models are noted by the constitutive laws used in the coal and the coal-rock interface respectively.
The graph identifies the \textit{MCss-CY} model, which has strain softening properties applied in the coal and a Continuously-Yielding joint at the coal-rock interface, as hosting the highest degree of instability. The next highest release of energy, which occurs in the \textit{MCss-CS} model, is approximately 85% lower in value but may still exhibit unstable failure to some extent. Each of the remaining models exhibit relatively similar magnitudes of released energy and are excluded from further investigative analysis. The \textit{MCss-CS} and \textit{MCss-CY} models are colored blue and orange, respectively, for the purpose of visualization in the comparison of other results.

Further analysis of the energy results in Figure 5.16 shows that for each series of simulations in which the properties of the coal are held constant, there is an increased level of instability when the coal-rock interface is modeled with a Coulomb slip joint instead of a fixed joint. Likewise, an additional increase is observed when a Continuously-Yielding joint is used at the interface. The increasing trend in released energy with deformable and softening joint properties suggests that shearing mechanisms at the coal-rock interface affect the overall stability of the coal pillars as excavation progresses. The graph also illustrates, as expected, that the use of strain softening properties in the coal results in a higher degree of instability and energy release.

It is worth noting that the Mohr-Coulomb constitutive law is not capable of modeling unstable compressive failure, and in that regard, the values of energy released in the simulations with Mohr-Coulomb coal properties should be near zero, especially with fixed or Coulomb slip interface parameters. Although the excavations in the mine scale models are progressed gradually and in small increments, the
initiation of each excavation slice results in an instantaneous deconfinement of nearby zones because the support forces implemented automatically by the SOLVE RELAX command are not perfectly matched to the loads at each grid point. A release of energy occurs because the strength of the nearby zones, which is dependent upon confinement, temporarily drops below the existing vertical load, resulting in a small but sudden displacement until equilibrium is regained by the applied forces. Thus, the quantities of energy released in the Mohr Coulomb simulations result primarily from a relatively sudden change in support that cannot be avoided. This can be illustrated by plotting the increase of damped work that occurred during excavation of the NB2 entry in the Mohr-CS model, shown in Figure 5.17.

![Damped work during NB2 mining — MOHR-CS](image)

Figure 5.17: Increase in damped work that occurs during initiation of each excavation slice.

The NB2 entry consists of 16 excavation slices, and the initiation of each slice can be correlated with a small increase in damped work.

5.3.1 Energy Released at Each Stage of Excavation

The results of the mine-scale analyses can be better visualized by comparing the amounts of energy that are released in each stage of excavation from the MCss-CS and MCss-CY models. Figure 5.18 shows the individual values of energy released in all stages of the excavation sequence. The location of each of the entries can be found in Figure 5.12 for reference.
The original five entries through the west mains are developed in virgin ground conditions, and thus the corresponding release of energy is relatively small in both models. With the subsequent addition of abutment loads to simulate the presence of longwall mining nearby, increasing magnitudes of energy are released as entries are developed in the highly stressed ground of the north barrier. Energy values in the range of megajoules may be evident of potential instability, and the magnitudes shown in the north barrier stages of excavation may be related to the bump event that occurred in the north barrier in March of 2007, prior to the collapse in August (Gates, 2008). A smaller-scale model of the area, with more detailed geologic data and deformable properties in the immediate roof, would be required in order to correlate pillar damage and energy release in the models to the documented coal bump event.

Pillar recovery operations in the north barrier are simulated in the mine-scale models at the NB Pillar 1 and NB Pillar 2 stages, but the complete removal of two pillars does not generate significant levels of kinetic energy relative to other excavation stages. This is attributed to several factors. The primary factor is the use of an elastic constitutive law in the overburden, which easily spans large voids and distributes load to other areas of the coal seam as the pillars are being mined. Further analysis of the $MC_{ss-CY}$ model results reveals that the pillars in the north barrier are fully failed when recovery operations are initiated, meaning their support capacity is already reduced to residual levels, associated roof displacements have already occurred, and the stiff overburden is already being supported by more
competent areas of the coal seam. In effect, the immediate roof is no longer capable of a significant unstable response.

Unlike the MCss-CY model, the magnitudes of energy released in the MCss-CS model during pillar recovery are similar in magnitude to those shown for the development of the entries. In addition, the magnitudes of energy in the MCss-CS model are lower than those in the other model at nearly every stage of excavation. Both of these trends are attributed to the fact that the pillars in the MCss-CS model exhibit a much higher support capacity throughout the excavation sequence. Stress-strain data from the mine scale simulations is presented in Section 5.3.2 for further comparison of pillar strengths from each of the models.

The most prominent release of energy in Figure 5.18 occurs during excavation of the south barrier entries in the MCss-CY model. The energy values ranges from 2.1 megajoules during development of the first entry to 54 megajoules during excavation of the fourth entry, signifying a magnitude of failure greater than all previous stages of excavation combined. The extent of excavation in the model, which includes pillar recovery in the north barrier and the development of four entries in the south barrier, correlates well with the extent of excavation in the area of the Main West section of the mine that lay beneath 600 meters of overburden at the time of the collapse. For this reason, the modeling results will be further analyzed for evidence of a collapse event at the SB4 stage of excavation. If the extent of collapse at the SB4 stage of mining correlates with data and observations from the mine, then the release of energy that occurs during further stages of excavation, such as slabbing of the south barrier pillar, may be disregarded considering that it is generated by an unrealistic continuation of mining.

5.3.2 Pillar Stress and Strain Data

This section makes further comparison between the results of the MCss-CS and MCss-CY mine-scale models by illustrating stress and displacement data from different areas of the models. Data is presented with emphasis on the changes that took place in the MCss-CY model.

In order to account for the difference in the values of energy released in the two models, data regarding average pillar stress and strain is compared between the same pillars in each simulation. In each model, the average pillar stress is measured by a FISH function through a band of zones near the center height of the pillar. Average strain is calculated by measuring the closure, which is the difference in vertical displacement between the upper and lower extents of the coal seam, and dividing by the coal seam height of 4.0 meters.
Stress-strain histories are presented for three different pillars from the stages of excavation in which their failure is relevant to the release of energy. Figure 5.19 shows the layout of the mine-scale model with the locations of the three pillars noted by arrows and labels.

![Figure 5.19: Locations of three pillars for which stress-strain data is further analyzed.](image)

The fundamental difference in pillar behavior between the two models can be illustrated by plotting stress-strain data from pillar P1 in the mains after partial excavation of the north barrier. Figure 5.20 shows the data collected at pillar P1 from each of the models. The solid portion of the lines indicate the data that is recorded from the beginning of the simulations through excavation of entry NB2, while the dotted portions indicate the data that was recorded during the subsequent excavation of entry NB3 alone.

![Figure 5.20: Stress-strain data from pillar P1 in two mine scale models.](image)

The pillars exhibit similar behavior in the early stages of the simulation, with a peak stress of 19 MPa followed by a slight reduction in strength as the adjacent entries are mined in virgin ground conditions. The subsequent application of abutment loads and the excavation of entries NB1 and NB2 increase the level of stress in each of the pillars, with gradual failure of the MCss-CY pillar noted by incremental drops in stress and a higher overall level of strain. Development of entry NB3 causes
significant failure of pillar P1 in the MCss-CY model, while very little change in stress or strain occur in the pillar bound by the CS interface.

The failure of the MCss-CY pillar illustrates a potential failure mode of large width-to-height coal pillars that is dependent upon shear slip, or shear weakening, at the coal-rock interface. As the pillar is compressed, slip at the interface results in a loss of confining stress in the coal near the ribs, which in turn results in additional shear slip along the interface. The release of energy in the MCss-CY model suggests that this process occurs relatively rapidly and is accommodated specifically by the softening parameters applied at the interface.

As mentioned previously, the greater support capacity of the pillars in the MCss-CS model contributes to the lower values of released energy at each stage of excavation compared to the other model. The softening behavior exhibited by pillar P1 in the MCss-CY model contributes to the release of energy that occurs at the NB3 stage, which is shown previously in Figure 5.18 as approximately 5.0 megajoules. The failure of the pillars in the north barrier, much closer to the excavation of entry NB3, also contribute to the release of energy. Figure 5.21 shows the stress-strain behavior of pillar P2 throughout the same time periods of excavation. Again, the dotted portions of the curve indicate the data was recorded during excavation of the NB3 entry alone.

Figure 5.21: Stress-strain data from pillar P2 in two mine scale simulations.

The difference of approximately 24 MPa between the peak and residual strengths of the MCss-CY pillar helps to justify the release of energy at the NB3 stage. As the strength of the pillar reduces, the state of stress in the immediate floor and overburden reduces, allowing the relaxation of the surrounding rock and a release of stored strain energy. Additionally, the vertical strain of 5.5% equates to more than
66 centimeters of closure on the coal seam, which implies significant roof displacements and a likely release of potential energy of the overburden.

With the data from P2, it is evident that the pillar in the north barrier of the *MCss-CS* model exhibits softening behavior during failure, while pillar P1 exhibits hardening behavior and has a slightly higher load-bearing capacity at the end of the same time period. The difference in behavior and load-bearing capacity between pillar P2 in the north and P1 in the mains can be explained by the difference in the widths of the pillars, which are modeled at 18 and 21 meters respectively.

Since the greatest release of energy occurs in the *MCss-CY* model during excavation of the final entry in the south barrier, the behavior of pillars in the south barrier is of special interest. Figure 5.22 shows stress-strain data from Pillar P3, which is formed by the excavation of the final two entries in the model, SB3 and SB4. The dotted portions of the curves show the changes that take place as entry SB4 is excavated.

![Stress-Strain of P3 Through Entire Simulation](image)

Figure 5.22: Stress-strain data from pillar P3 in two mine scale simulations.

The graph illustrates a reduction in support capacity of nearly 50 MPa in the *MCss-CY* pillar, which accommodates a significant relaxation of the surrounding rock and a large release of stored strain energy. The increase in vertical strain from 4% to 19% accounts for an increase in closure on the coal seam from 16 to 76 centimeters, signifying a significant change in the potential energy of the overburden. In contrast, the pillar in the *MCss-CS* model exhibits a 7 MPa reduction in strength and a maximum overall closure of roughly 18 centimeters at the end of the same excavation sequence.

To emphasize the changes that take place during excavation of entry SB4, Figure 5.23 shows the stress-strain behavior of all three pillars for the duration of the excavation sequence in the *MCss-CY*
model. The dotted portions of the curves indicate the data that is recorded during excavation of the SB4 entry.

![Stress-Strain of Each Pillar Through MCss-CY Simulation](image)

Figure 5.23: Stress-strain data from pillars P1, P2, and P3 in the \(MCss-CY\) model.

It is evident by the simultaneous increase of vertical strain on each pillar that significant roof displacements occurred in different areas of the model at the SB4 stage of excavation, signifying the likely occurrence of a collapse event. The distance of nearly 200 meters that separates the pillars in Figure 5.23 indicates that the collapse affects a wide portion of the model from left to right. Examination of the roof displacements that occur over the lateral extents of the model provide a more thorough demonstration of the scope of the collapse.

### 5.3.3 Roof Displacements

Seam closure can be measured after any stage of the excavation sequence by restoring the saved state of the model and running a FISH function that queries vertical displacements at the grid points along the top and bottom of the coal seam. The values of closure are saved to a text file, along with the x-coordinates of the grid points to which they belong, so that the data can be plotted and positioned above or below the 2D mine layout for visualization. Figure 5.24 shows the change in seam closure that occurred in the \(MCss-CS\) and \(MCss-CY\) models during excavation of the first three entries in the south barrier. The mine layout diagrams positioned above the curves illustrate the extent of mining at the two stages.
Figure 5.24: Seam closure during first three stages of south barrier excavation.

The axes in the figure are oriented such that the values of seam closure appear as downward roof displacement below their respective locations in the mine layout. The dotted lines indicate the values of closure that are measured before any excavation in the south barrier, at which point the $MC_{ss-CY}$ model shows approximately 10 centimeters more closure over the mains and the north barrier. Neither of the models show significant closure in the south barrier before mining is commenced there. At the SB3 stage of excavation, the difference in closure between the models increases to approximately 20 centimeters in most areas of the seam, as indicated by the distance between the solid curves.

The roof displacements that occur in each model during excavation of the SB4 entry are illustrated in Figure 5.25. In this graph, the SB3 stage of each model is represented by the dotted lines and the final, SB4 stage is shown in solid colors.

As suggested by the pillar stress-strain data in Section 5.3.2, the lower value of energy released in the $MC_{ss-CS}$ model correlates with less extreme changes in roof displacements, and the amount of vertical closure that takes place in the $MC_{ss-CY}$ model during the SB4 stage of mining is significant all across the seam. The graph illustrates an increase in seam closure of roughly 30-40 centimeters over the mains, 20 centimeters over the north barrier, and even 10 centimeters closure over the 40-meter wide barrier pillars at the far left and right sides of the model.
Figure 5.25: Seam closure during the final stage of excavation in the south barrier.

The average of closure values across the seam before and after the excavation of SB4 in the $MC_{ss-CY}$ model are 0.32 and 0.62 meters respectively. The change of 0.3 meters closure during the simulated collapse correlates well with the estimate of average seam closure made by seismologists during investigation of the mine collapse (Pechmann, 2008). For this reason the excavation of SB4 is considered the end of the simulation, and the additional stages of excavation are disregarded.

5.4 Extrapolation of energy results

Pechmann (2008) estimated that the collapsed workings comprised an area of approximately 0.2 square kilometers, consistent with the area that exhibited 15 centimeters of more of subsidence. The estimated area was also based on direct observations of blocked entries at the eastern end of the Main West and bore-hole measurements of seam level voids toward the western end. Since the width of the model is approximately 400 meters, the value of released energy from the 2D models is multiplied by 500 meters in the third direction to match the horizontal extents of collapse. The release of 54 million Joules during excavation of the SB4 entry, multiplied by 500 meters in the third direction, results in a total energy value of 27 billion Joules.

Gutenberg and Richter’s (1956) and Kanamori’s (1993) equations, explained in Section 4.3, are used to estimate an associated seismic magnitude based on a value of released energy. After converting the total released energy from the model to units of ergs, Gutenberg’s equation yields a 4.2 seismic magnitude. Kanamori’s equation associates the release of 27 billion Joules with a 4.3 magnitude event.
The collapse of the mine was estimated by Pechmann (2008) to be a 3.9 local magnitude event, illustrating that the release of energy in the 2D model is exaggerated to some degree. The values of energy associated with a 3.9 magnitude event can be calculated from Gutenberg and Richter’s and Kanamori’s equations as 8.4 billion and 4.9 billion Joules respectively. Although this highlights the broad range of energy values estimated by different researchers to be released during the same magnitude event, the quantity of energy released in the model is still three to five times higher than expected.

The exaggerated release of energy in the model may be attributed to the simplified approach used in determining the model extents and defining the boundary conditions. The left and right boundaries of the model are constructed near the outer limits of the barrier pillars to avoid the complexities associated with simulating gob compaction and stress distribution in the adjacent longwall panels. The boundaries are restricted from horizontal displacement but allowed to move freely in the vertical direction, and as such, the displacement of the overburden that occurs during widespread pillar failure is nearly uniform from left to right and slightly exaggerated. The mechanisms of bending and separation in the overburden units are entirely disregarded, and the subsidence profile observed at the surface of the mine cannot be reproduced in the model.

Regardless of the limitations exposed in the setup of the model and the exaggeration of the seismic magnitude associated with the failure, the sequential excavation sequence performed under carefully selected loading conditions clearly results in a collapse event that closely correlates with observations at the mine. An exaggeration of energy release may be a favorable byproduct of simplified model geometry if the study of instability is to be conducted in a predictive context.

5.5 Conclusions

Coal pillar strength is calibrated for a range of pillar geometries based on previous research regarding the support capacities observed in a database of mines. A mine scale model is constructed of the Crandall Canyon Main West area with careful consideration of the loading conditions that existed and the excavation sequence that commenced prior to collapse in August of 2007. Parametric simulations are run to evaluate the effect of different material and joint constitutive behaviors on the strength of the pillars and the degree of instability associated with failure.

The results of this study illustrate that for high stresses conditions on squat coal pillars, the use of softening material properties in both the coal and the coal-rock interface reveals a failure mode and a degree of instability that were not demonstrated by other modeling methods. Stress and displacement data from the MCSS-CY model depict the occurrence of a collapse under loading conditions and an extent
of excavation that correlate with those of the Main West area of the mine at the time of the documented seismic event.

The release of 54 million Joules during the final stage of excavation in the MCss-CY model signifies a magnitude of unstable failure greater than all of the previous excavation stages combined and is an order of magnitude greater than the release of energy that occurs in the most comparable model. By extrapolating the results of the model in the third direction to satisfy the estimated horizontal extents of the mine collapse, the associated seismic magnitude is calculated to be 4.2 or 4.3 depending on the empirical formula used to relate energy and magnitude. Further agreement with the documented local magnitude of 3.9 could be achieved by including detailed overburden geology and caved longwall panels adjacent to the Main West section, but computation time and model complexity would increase drastically. Regardless, the collapse is well illustrated by the calculation of energy during a staged excavation sequence and validated by stress and displacement data from the model.
CHAPTER 6
SOLVAY ANALYSIS

This chapter presents a back analysis of the collapse of the Solvay Trona Mine, which occurred on February 3, 1995. Background information regarding observations and measurements made after collapse are provided as a basis for the approach used in the study, and detailed procedures are explained for the calibration of material properties and numerical investigation of pillar failure. An energy-based approach is used to investigate the magnitude of instability in a mine scale simulation of widespread pillar failure and the seismic magnitude of the documented collapse is compared to modeling results.

6.1 Background Information

The Solvay Trona Mine is located in southwestern Wyoming, approximately 15 miles west of the town of Green River. Trona is a common reference to the raw mineral known as sodium sesquicarbonate, which is processed into soda ash for use in the production of glass, detergents, paper, water softeners, and baking soda. Production began in 1981 with a room and pillar layout excavated by continuous-mining equipment, and the arrangement of underground workings has continued to expand.

The sudden collapse of the southwest panel occurred on February 3, 1995 with no apparent warning. The collapse damaged mine workings over an area of nearly two square kilometers and generated a local magnitude 5.3 seismic event. The air blast that resulted from the collapse blew out ventilation stoppings all around the southwest panel and reversed the direction of air flow in the intake shaft for approximately 17 minutes (Ferriter, 1996). A total of 54 miners were underground at the time of the collapse, and all but two were evacuated safely. One of the entrapped miners perished during the subsequent rescue effort.

Mining operations were taking place at a depth of approximately 480 meters. The trona in the southwest panel was on the order of 3.5 meters thick, and ovaloid-shaped production rooms 2.5 meters in height were excavated using twin-rotor, bore-type miners. The material directly above the trona consisted of shales and mudstones, with more competent sandstones higher in the overburden. The material directly below the trona consisted of a layer of oil shale approximately two meters thick underlain by mudstone. Mining induced stresses typically resulted in minor rib spalling and floor heave as the trona pillars punched into the weak floor materials. Floor heave varied between 0.1 and 0.4 meters before the collapse occurred, as shown in Figure 6.1.
The closure that resulted from the collapse was extensive, ranging from approximately one to two meters of floor heave in some areas to complete closure due to roof fall in other areas. The lower half of the trona pillars were damaged as they were pushed farther into the floor, but many of the ceilings remained relatively intact. Figure 6.2 illustrates a heavily damaged floor and an intact ceiling after collapse.

Subsidence was measured by two rows of pins at the surface, which ran north-south and were spaced 400 meters east-west, roughly dividing the southwest panel into thirds. The pins in the northern half of the panel were spaced at 60 meters, while those in the southern half of the panel were spaced somewhat irregularly around 120 meter or larger intervals. Due to the orientation of the pins, the subsidence
contours are relatively well defined in the north-south direction, but the shape of the contours in the east-west direction is based upon interpretation. The maximum subsidence measured was 1.0 meters, and average was estimated to be 0.6 meters (Pechmann, 1995). Figure 6.3 shows the subsidence contours in increments of 0.1 feet (0.03 meters), with measurement points at the surface marked by red dots.

![Figure 6.3: Interpreted subsidence curves, with measurement points in red. (Ferriter, 1996)](image)

The National Earthquake Information Center (NEIC) estimated the local magnitude ($M_L$) to be 5.3 based on the compressional wave amplitudes measured by various seismographs (Swanson, 1995). The seismic signatures matched well with a collapse or implosion-type event and did not match well with a typical fault-slip event due to the relative lack of shear waves.

### 6.2 Approach

The goal of this analysis is to determine whether the release of energy calculated in a 2D numerical simulation of the collapse can be extrapolated in the third direction and quantitatively compared with the magnitude of the seismic event at the mine. To accomplish this, the source of available energy and the mechanisms of failure related to the release of energy are evaluated.

The energy associated with the collapse of the Southwest Panel can be approximated by considering the volume of material in the overburden that subsided and how far it moved under gravitational loading. By assuming a volume of 2000 x 1000 x 485 meters, an average material density of 2300 kg/m$^3$, and an average falling distance of 0.6 meters under gravitational loading, the potential energy of the block is $1.3 \times 10^{13}$ Joules (Ferriter, 1996). This value represents an upper limit for the calculation of released energy.
to be extrapolated from numerical models, as a majority of the available energy in a seismic event is consumed by plastic deformation and friction.

The size of the southwest panel presents a challenge in the development of an appropriate numerical model. The source of energy during the collapse involves thousands of meters of overburden, but the mechanisms of failure occur in a relatively thin seam of pillars. Although the problem geometry lends itself to a 2D analysis, computation time can still be unreasonably long if the model is large enough to account for the span of overburden but detailed enough at seam level to simulate the failure of individual pillars. Consistent with other studies regarding the collapse of room and pillar workings in Wyoming trona mines (Zipf, 1999), (Board, 2007), and (Damjanac, 2014), it is logical to first assess loading conditions and failure mechanisms at the pillar scale. Detailed numerical models of the pillar-floor system, calibrated to match deformations observed at site, can provide useful information about the change in stresses that occurred at seam level during collapse. If these changes in stress can be accurately represented through grid point forces in a larger mine-scale model, the release of energy associated with collapse can then be analyzed more accurately and efficiently.

The approach to the back analysis of the Solvay Mine collapse includes the following tasks:
1. Develop strain-softening parameters for the materials present at seam level.
2. Construct detailed pillar-scale models and load them to failure.
3. Evaluate the change in average roof support being provided by the pillars during failure.
4. Construct a 2D mine-scale numerical model, representative of an east-west cross section through the southwest panel.
5. Apply the average roof support characteristics from pillar-scale models to mine-scale models through a system of grid point forces at seam level.
6. Calculate the energy released during the simulation of widespread pillar failure.

Uncertainty regarding problem geometry and loading conditions is addressed at both the pillar-scale and mine-scale by running parametric simulations.

6.3 Pillar Scale Material Properties

The calibration of the pillar and floor material properties is based on published information and observations documented by investigators at the mine site. The MSHA Technical Investigation Report (Ferriter, 1996) contains material properties, established from laboratory compression tests of intact samples, for the trona, oil shale floor, and the mudstone sub-floor. The average uniaxial compressive strength (UCS) and secant modulus values from Table IV-6 in the MSHA report are summarized here in Table 6.1. The secant modulus is assumed to represent the Young’s modulus of the material, as no further description of the term secant is provided in the report.
Table 6.1: Average strength and stiffness values obtained in laboratory (Ferriter, 1996).

<table>
<thead>
<tr>
<th></th>
<th>Mean UCS (MPa)</th>
<th>Std. Deviation (MPa)</th>
<th>Mean Secant Modulus (GPa)</th>
<th>Std Deviation (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trona</td>
<td>60.1</td>
<td>9.5</td>
<td>18.1</td>
<td>3.6</td>
</tr>
<tr>
<td>Oil shale</td>
<td>42.0</td>
<td>20.3</td>
<td>4.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Mudstone</td>
<td>37.2</td>
<td>3.7</td>
<td>3.3</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Rock mass strength parameters, for the purpose of numerical modeling, are estimated using RocLab software (RocScience, 2013). To estimate these strength parameters, the UCS values from the laboratory tests are provided as input and assumptions are made regarding the Geologic Strength Index (GSI) and material constant (m) values of each material. RocLab provides literature to assist in the selection of these parameters. The Disturbance Factor, D, is assumed zero for all materials. The summary of values entered into RocLab and the suggested Mohr-Coulomb rock mass properties are shown in Table 6.2.

Table 6.2: Rock mass strength parameters determined for pillar and floor materials.

<table>
<thead>
<tr>
<th></th>
<th>Values input to RocLab</th>
<th>Output rock mass strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UCS (MPa)</td>
<td>GSI</td>
</tr>
<tr>
<td>Trona</td>
<td>60.0</td>
<td>70</td>
</tr>
<tr>
<td>Oil shale</td>
<td>42.0</td>
<td>50</td>
</tr>
<tr>
<td>Mudstone</td>
<td>37.0</td>
<td>35</td>
</tr>
</tbody>
</table>

To accommodate a brittle response in the pillar-floor system, Mohr-Coulomb strain-softening parameters are applied in all three materials at seam level. Softening is accomplished by calibrating a cohesion drop of 80 - 100% for each material to exhibit a quasi-brittle reduction in post-peak strength. The complete list of material properties used in the pillar-scale models are shown in Table 6.3 for the pillar and floor materials.

Table 6.3: Complete list of material properties used to model pillar and floor materials.

<table>
<thead>
<tr>
<th></th>
<th>Density (kg/m³)</th>
<th>Young’s Mod. (GPa)</th>
<th>Poisson Ratio</th>
<th>Friction Ang. (deg)</th>
<th>Cohesion (MPa)</th>
<th>Tensile (MPa)</th>
<th>Dilation Ang. (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trona</td>
<td>2150</td>
<td>18.0</td>
<td>0.20</td>
<td>36.3</td>
<td>4.25</td>
<td>0.65</td>
<td>9.0</td>
</tr>
<tr>
<td>Oil Shale</td>
<td>2500</td>
<td>4.1</td>
<td>0.20</td>
<td>26.3</td>
<td>1.80</td>
<td>0.09</td>
<td>3.0</td>
</tr>
<tr>
<td>Mudstone</td>
<td>2500</td>
<td>3.3</td>
<td>0.25</td>
<td>22.1</td>
<td>1.20</td>
<td>0.06</td>
<td>0.0</td>
</tr>
</tbody>
</table>
The tensile strength is set at 15% of the cohesion value for trona and 5% of cohesion values for the oil shale and mudstone. Dilation is set to 9.0 degrees for the trona, as suggested in rock mass classification literature, to be approximately one fourth of the friction angle for a good quality rock mass (Hoek, 1998). The dilation angle of the oil shale is set at 3.0 degrees, which is suggested as approximately one eight of the friction angle for an average quality rock mass, and a dilation angle of zero is used for mudstone, which is assumed to be poor quality rock.

Critical strain values associated with the softening of cohesion are shown in Table 6.4. Each of the materials is given 10%, 20%, and 50% reductions in cohesion at specific strain values to achieve a rounded peak on a triaxial test stress-strain curve. The final reduction in cohesion ranges from 80-100% for each of the materials.

Table 6.4: Softening parameters used in the pillar and floor materials.

<table>
<thead>
<tr>
<th>strain</th>
<th>Trona</th>
<th>Oil Shale</th>
<th>Mudstone</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>4.25e6</td>
<td>1.80e6</td>
<td>1.20e6</td>
</tr>
<tr>
<td>1.50e-4</td>
<td>4.25e6</td>
<td>1.80e6</td>
<td>1.20e6</td>
</tr>
<tr>
<td>7.50e-4</td>
<td>3.80e6</td>
<td>1.60e6</td>
<td>1.10e6</td>
</tr>
<tr>
<td>1.50e-3</td>
<td>3.30e6</td>
<td>1.40e6</td>
<td>1.00e6</td>
</tr>
<tr>
<td>3.00e-3</td>
<td>2.13e6</td>
<td>0.90e6</td>
<td>0.60e6</td>
</tr>
<tr>
<td>2.25e-2</td>
<td>0.85e6</td>
<td>0.18e6</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>0.85e6</td>
<td>0.18e6</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 6.4 shows the stress-strain curves derived from the numerical triaxial tests on specimens of each of the materials. One of the data files used to run the triaxial tests can be found in Appendix A.
Each of the triaxial test simulations are performed with 0.5 MPa confinement and loaded with the same velocity boundary condition. The specimens are 2.5 meters wide, 5.0 meters tall, and discretized with 0.25 meter zones. The results of the triaxial tests help illustrate the relative difference in strength and stiffness of the trona compared to the oil shale and mudstone.

The shale and sandstone units within 100 meters of the roof of the trona seam are represented in the pillar scale models with Mohr-Coulomb material properties. The strength parameters of the materials in the roof are not a significant factor in the failure of the pillars, as they essentially serve to transmit the loading that is applied to the upper boundary of the model and are restricted from horizontal displacement at the left and right boundaries. The stiffness of the roof materials will have a more pronounced effect on the progression of failure in the pillars due to their ability to store and release strain energy. The shales far below the floor of the trona seam are modeled as elastic. The list of material properties used in the roof and sub-floor materials is shown in Table 6.5.

Table 6.5: Material properties used in the overburden and sub-floor of pillar scale models.

<table>
<thead>
<tr>
<th></th>
<th>Density (kg/m³)</th>
<th>Young’s Mod. (GPa)</th>
<th>Poisson Ratio</th>
<th>Friction Ang. (deg)</th>
<th>Cohesion (MPa)</th>
<th>Tensile (MPa)</th>
<th>Dilation Ang. (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstone</td>
<td>2500</td>
<td>10.0</td>
<td>0.3</td>
<td>32.0</td>
<td>3.6</td>
<td>0</td>
<td>4.0</td>
</tr>
<tr>
<td>Roof shale</td>
<td>2500</td>
<td>5.5</td>
<td>0.2</td>
<td>27.0</td>
<td>1.8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sub-floor</td>
<td>2500</td>
<td>5.5</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The contacts between different geologic materials in the pillar scale models are represented by deformable Coulomb-slip joints. The strength parameters are adjusted to be neither significantly stronger nor weaker than the oil shale and mudstone and are applied uniformly to all contacts in the model. In this way, the properties of the geologic contacts do not inhibit or promote shear failure in the floor materials and do not affect the behavior of the overburden, where their strength is irrelevant. The stiffness of the contacts prevents numerical error associated with the overlap of grid points. The list of properties used in the contacts is shown in Table 6.6.

Table 6.6: Properties assigned to geologic contacts in pillar scale models.

<table>
<thead>
<tr>
<th>Geologic Contacts</th>
<th>Normal Stiffness (Gpa)</th>
<th>Shear Stiffness (GPa)</th>
<th>Friction Angle (Deg)</th>
<th>Cohesion (MPa)</th>
<th>Tensile (MPa)</th>
<th>Dilation (Deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geologic Contacts</td>
<td>500</td>
<td>500</td>
<td>20.0</td>
<td>1.6</td>
<td>0.4</td>
<td>6.0</td>
</tr>
</tbody>
</table>
Variations in the constitutive laws and strength parameters of the joints are not considered because observations made at the time of the mine collapse did not suggest a significant component of shear slip along geologic contacts in the failure mode of the pillar-floor system.

6.4 Pillar-Scale Model Geometry

The pillar-scale models are developed with the goal of simulating brittle failure, reproducing the relative amounts of deformation observed at site, and quantifying the average roof support provided by the pillars during failure. For this purpose, the load bearing characteristics of the relatively long, thin panel pillars are analyzed, which account for a majority of the collapsed workings.

Ovaloid-shaped entries are replicated in the pillar-scale models, which resulted from the use of a twin-rotor boring type excavator for production at the mine. The entries are 2.5 meters tall and 4.5 meters wide at the widest point, and the pillars are 3.75 meters wide at their narrowest point. The weight of the remaining overburden is represented by a loading condition at the upper boundary. Overburden material consisting of shale and a thin sandstone unit are modeled up to a distance of 105 meters from the top of the trona. The overburden materials have a density of 2500 kg/m$^3$ and an average Young’s modulus of 5.3 GPa. Geometry of the ovaloid pillar-scale model is shown in Figure 6.5 along with the assumed cross section orientation from a typical group of long, thin panel pillars.

![Figure 6.5: Geometry of pillar scale model with majority of overburden excluded from view.](image)

The trona seam is modeled with a thickness of 3.5 meters. The distance between the floor of the ovaloid entry and bottom of the trona seam is evaluated parametrically at 0.0, 0.25, and 0.5 meters. Because the trona is significantly stronger than the oil shale or mudstone, the bridge formed by material left in the floor of the entry could have an effect on the overall strength of the system or brittleness of failure. The average thickness of trona left in the floor is not mentioned in literature, but use of the term *oil shale floor* may imply that no significant layer of trona was frequently left below excavation (Ferriter, 1996). However, the presence and thickness of trona in the floor may have varied as a result of normal...
mining practices and seam undulation. The pillar-scale model geometry with and without floor trona is illustrated in Figure 6.6. An example of the data files used to execute the pillar-scale simulations can be found in Appendix A.

![Figure 6.6: Pillar scale geometry with variations in floor trona and oil shale thickness.](image)

The thickness of the oil shale is also varied in parametric runs between 2.0 and 1.0 meters, as shown in Figure 6.6. Considering that floor heave reached heights of two meters in various parts of the mine, it is reasonable to assume that shearing of the materials beneath the entries occurred at similar depths, if not greater, depending on the adjacent pillar geometries. The thickness of the oil shale is described in literature as being approximately six feet (1.8 meters) but is varied in this study to account for the possibility of a weaker overall floor structure, since the mudstone is slightly weaker than the shale. The penetration of shearing planes into the mudstone could affect the overall pillar strength and brittleness of failure. The modeled thickness of the mudstone is either 5.0 or 6.0 meters, depending on the thickness of the oil shale.

The combination of three parameters for trona floor thickness and two parameters for oil shale thickness results in a total of six pillar-scale models. The models are numbered 1 to 6 for all future reference and described in Table 6.7.

<table>
<thead>
<tr>
<th>Model</th>
<th>Floor trona thickness (meters)</th>
<th>Oil shale thickness (meters)</th>
<th>Predicted relative strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0</td>
<td>2.0</td>
<td>med-low</td>
</tr>
<tr>
<td>#2</td>
<td>0.25</td>
<td>2.0</td>
<td>med-high</td>
</tr>
<tr>
<td>#3</td>
<td>0.50</td>
<td>2.0</td>
<td>high</td>
</tr>
<tr>
<td>#4</td>
<td>0</td>
<td>1.0</td>
<td>low</td>
</tr>
<tr>
<td>#5</td>
<td>0.25</td>
<td>1.0</td>
<td>med-low</td>
</tr>
<tr>
<td>#6</td>
<td>0.50</td>
<td>1.0</td>
<td>med-high</td>
</tr>
</tbody>
</table>
The mesh is generated with a zone size of 0.25 meters in the seam and floor materials, except in the perimeter of the ovaloid entries where *quad* zoning cannot be used. In these areas, *edge* zoning with a maximum length of 0.5 meters is used. The remainder of the model is given 0.5 meter or larger zones to save computation time.

The pillar-scale models are executed with the following sequence of actions:

1. Define model extents and pillar geometry.
2. Delete blocks in ovaloid entries.
3. Create mesh and apply material properties.
4. Fix bottom and side boundaries with roller conditions.
5. Initialize 90% of in-situ stress and solve to equilibrium.
6. Increase loading by applying a very slow velocity boundary condition to upper surface.
7. Continue loading until displacement at the upper boundary of the model reaches 0.6 meters.

The pillar models are loaded through 0.6 meters of overburden displacement in order to gain insight to the post-peak behavior of the system over a likely range of roof deformations. Considering that subsidence ranged from 0.3 to 1.0 meters above the collapsed workings, with an estimated average of 0.6 meters, target overburden displacements of 0.6 meters at the upper boundary of the model provide the most reasonable criteria for the duration of loading.

### 6.5 Analysis of Pillar-Scale Results

One critical goal of the pillar-scale simulations is to characterize the average support being provided to the roof of the trona seam as failure of the pillar-floor system occurs. It is important to note that the average vertical pressure in the material just above the trona remains constant, regardless of pillar geometry or extraction ratio, as long as the pillars are stable. The volume and density of the overburden provide a constant static load, and the pillars and unmined portions of the seam provide an equal and opposite support as long as the entire system is in equilibrium.

When sudden collapse of the pillars occurs, average support of the roof is significantly reduced for a brief moment of time, until the failing materials have shifted to a position that again provides confinement to the base of the pillars or provides support directly to the roof. The reduction in support that occurs during pillar failure is difficult to estimate, but the simulation of brittle failure in the pillar-scale models provides a likely range of values.

In order to quantify the change in average roof support during collapse, the vertical stress is measured in every zone along a 1.0 meter wide band above the trona seam at every time step during loading of the pillar-scale models. In this way, the results of average roof support can be applied to the roof of the trona seam in a mine-scale model without modeling any of the trona in the panel. The results
of average roof support versus overburden displacement from the parametric pillar-scale study are shown in Figure 6.7.

![Average Vertical Roof Stress vs Overburden Displacement](image)

**Figure 6.7:** Results of pillar scale analyses showing average roof support during pillar failure.

The models are initially brought to equilibrium with 90% of the full overburden load, and further loading is provided by a slow velocity condition at the upper boundary. Full overburden load at the roof of the trona seam is 10.9 MPa, thus the curves in the graph start at 9.8 MPa.

Each pillar scale model exhibits brittle failure, but variations in the thickness of floor trona and oil shale produce a significant difference in the peak support capabilities of the pillars. Model #4 fails at 10.0 MPa average roof stress, meaning it is not strong enough to support full overburden loads. This pillar could only exist in equilibrium if part of the overburden load was being carried by larger pillars or intact trona nearby. While such a scenario may have been possible, no data is available to support that approach to the analysis of the collapse. Model #3 exhibits brittle failure after extensive loading but never drops below 10.9 MPa average support capacity, meaning a system of pillars with such characteristics would not likely facilitate a widespread, violent collapse.

All of the pillars exhibit a drop of 3.5 – 7.0 MPa average roof support during failure. The range of peak support capacities illustrated in the graph, from 10.0 – 16.5 MPa, suggests that an appropriate range of input parameters have been evaluated, but the drop in support falls within a narrower range. It is also worth noting that support pressure does not have to drop to zero to accommodate the deformations associated with the collapse.
The most appropriate pillar-scale model is selected by considering both the peak support capacity and the deformations that occur after failure. For instance, Model #2 reaches a peak strength of 14.5 MPa at 0.17 meters roof displacement, drops to 7.5 MPa of support, and re-achieves full overburden load of 10.9 MPa at 0.37 meters displacement. If the velocity loading condition at the upper boundary of the model had been terminated, the model could have equilibrated at full overburden pressure after just 0.2 meters of overburden displacement, which is not enough to account for the subsidence observed at the mine.

On the other hand, Model #1 reaches a peak strength of 12.0 MPa, drops to 6.9 MPa, and exhibits 0.52 meters of additional displacement before loading is terminated. Considering the average subsidence estimated at the surface of the mine, Model #1 exhibits strength and deformation characteristics that could facilitate collapse and potentially account for the subsidence over the Southwest Panel.

Each of the pillar-scale models exhibits some degree of floor heave, which varies proportionally with the overall strength of the pillars. Figure 6.8 illustrates floor heave in Models #4 and #3, which represent the weakest and the strongest pillar-floor systems respectively. The color scale corresponds to the cohesion value of each zone, and changes in color indicate reduction of cohesion and yield in the strain-softening materials. For instance, intact trona is blue, fully yielded trona is orange-pink, and partially yielded trona may be another color in between.

![Figure 6.8: Floor heave exhibited in the weakest and strongest pillar scale models.](image)

It is worth noting the similarity between the floor heave in Model #4 and that in Figure 6.2, which illustrates conditions in a typical production room after collapse. Additionally, the relatively undisturbed condition of the trona roof in Model #4 correlates with the roof conditions witnessed at the mine. Model
#3, which required unrealistic loads to achieve failure, shows convergence between the ribs of the entry and extensive damage to the trona in the roof that does not correlate with observations.

Each of the diagrams in Figure 6.8 is plotted using the same vertical and horizontal window extents within the UDEC interface. The plots were generated at the end of each simulation, meaning that 0.6 meters of displacement had occurred at the upper boundary of the model. However, the contact between the blue trona and the purple shale above is visibly different in each model, illustrating that seam level vertical displacements are not the same. In fact, the contact is approximately 0.25 meters higher in Model #3, which means that a significant portion of the 0.6 meters displacement at the upper boundary of the model did not get transmitted through the roof of the trona seam. Because the pillar in Model #3 is so much stronger, less roof displacement and floor heave occurred, and more strain energy is being stored in the overburden at the end of the simulation. The graph in Figure 6.7 indicates that the roof material in Model #3 is holding more than twice the vertical stress of Model #4 when loading is terminated.

Considering the average support capacity of the various pillar models, the range of overburden displacements accommodated, and the comparison of floor heave with observations at the mine, Model #1 is most likely to represent an average pillar behavior in the Southwest Panel. It is important to keep in mind that the pillar behavior is assumed average and that both weaker and stronger pillars likely existed in the panel. The average support characteristics of Model #1 are utilized in the mine-scale simulations to match the average subsidence observed at the mine and determine the associated release of energy.

### 6.6 Discussion of Pillar-Scale Results

The support load versus displacement of pillar Model #1 highlights several important concepts related to the potential for collapse. As loading is increased, the average roof stress in Model #1 increases beyond the in-situ overburden pressure of 10.9 MPa, and at 12.0 MPa pressure the pillar fails. If it is assumed that the average group of pillars in the Southwest Panel was carrying full overburden loads before the collapse, then Model #1 shows that they may have been within 10% of their peak support capacity.

An increase in average vertical load beyond full overburden pressure is possible when a group of pillars fails and requires nearby pillars to take additional loads. If the transferred load exceeds the support capacity of the pillars, they too will fail and redistribute excess load to other pillars. Such an event can occur very rapidly and is referred to as a cascading pillar failure. The possibility of such a failure mechanism has been mentioned by several previous researchers in regards to the collapse of the Solvay Mine (Ferriter, 1996), (Swanson, 1995), and (Zipf, 1999).
Additionally, Model #1 exhibits a drop in support capacity from 12 MPa to 6.9 MPa as failure occurs, which would require nearby pillars to compensate and carry the excess 5.1 MPa load. The large reduction in support capacity of a failing pillar, in combination with the narrow margin of excess capacity in a nearby pillar, further illustrate the potential for a cascading failure mechanism.

6.7 Application to Mine-Scale Simulation

In order to apply the pillar support characteristics of Model #1 into a mine-scale simulation and estimate the energy release, an assumption is made regarding the transfer of loads during pillar failure. Although there is a total drop of 5.1 MPa between the peak and the post-peak portions of the pillar support curve in Model #1, the portion of the curve that drops below 10.9 MPa is the only portion of the curve that is relevant to the large-scale simulation of collapse.

The peak of 12 MPa in the pillar response represents a peak capacity and not the actual load that may have existed when failure of the pillars was initiated. The actual loads that may exist at the front of a cascading pillar failure are temporary, and the weight of the overburden is the only load available to drive the pillars into the floor once the cascading failure front has passed. In other words, there is no sustainable source of vertical load beyond that of full overburden pressure. Therefore, it is assumed in this study that the reduction in average support capacity below full overburden pressure exhibited by the pillars will determine the violence of the collapse.

6.8 Mine-Scale Model Development

The 2D mine-scale model represents an east-west cross section through the Southwest panel. A relative location of the cross section on a map of the mine workings is not presented because pillar support will be simulated by an average vertical stress condition at seam level and there are no other features in the model that would imply one location versus another within the mine.

The excavation at seam level spans 855 meters from left to right, similar to the east-west extents of the Southwest panel, and the overall width of the model is 2700 meters, allowing more than 900 meters of far field material on either side of the mined panel. The side and bottom boundaries are fixed in the perpendicular direction, and the top surface is free. Zone sizes range from 15.0 meters in the material near the surface to 5.0 meters near the trona seam. The overall model geometry is shown in Figure 6.9 with an enlarged view of the zoning near the trona seam and the thickness of each geologic layer shown in parentheses.

Each of the geologic units is modeled as a continuous block in the mine-scale simulations, meaning that the horizontal bedding planes are the only discontinuities present in the model. The properties are
assumed homogeneous and isotropic. The data file used to execute a mine-scale simulation can be found in Appendix A.

The overburden geology is based on a generalized cross section through the Wyoming trona mining district (Board, 2007). Each of the materials is given a strain-softening constitutive law in order to accommodate a reduction in strength and promote shearing in areas of high stress. The unmined portion of the trona seam and all floor materials are modeled as elastic. The trona seam is located directly below the Roof Shale layer. The mine-scale simulations are executed with the following sequence of actions:

1. Create block geometry and mesh
2. Define boundary conditions and initialize zone stresses
3. Assign elastic material properties to all materials
4. Bring model to equilibrium under gravitational loading
5. Assign strain softening material properties and bring model to equilibrium
6. Reset displacements
7. Delete 855m span of trona and replace with equivalent internal stresses
8. Bring model to equilibrium
9. Initiate collapse by dropping average roof support to 6.8 MPa
10. Call function to measure average subsidence and continually adjust average support at seam level to reflect pillar behavior during failure
11. Cycle 100 time steps between subsidence measurements

Uncertainty regarding the horizontal stresses native to the Wyoming trona mining district is addressed by running parametric simulations. In order to better understand the effect that horizontal stresses may have on the development of subsidence and release of energy, separate models are initialized with ratios of 0.3, 0.5, and 0.7 horizontal to vertical stress, which is referred to as the K ratio. All other parameters in the mine-scale simulations are held constant.
6.9 Overburden Material Properties

The strength parameters used in the overburden of the mine-scale models are based on information published in a previous study regarding room and pillar stability in Wyoming trona mines (Board, 2007). In particular, Board provides estimates of in-situ rock mass deformation moduli, cohesion values, and friction angles for each of the shales and sandstones. Very little information is available from other sources regarding the physical characteristics or competence of the geologic units above the mine, and no other published studies are known to estimate the strength and stiffness values.

The rock mass stiffness values given by Board are estimated using a technique which relates Rock Mass Rating (RMR) values to a database of rock mass deformation moduli measured through various in-situ testing methods (Serafim and Pereira, 1983). Due to the lack of additional information regarding the overburden at the Solvay Mine, which would allow independent estimates of RMR, deformation moduli, and strength parameters in the overburden, the strength and stiffness values used in the mine-scale models are those estimated by Board.

Tensile strengths of all materials are set to 5% of the cohesion value, and dilation is one fourth of the friction angle in the sandstone alone. All other materials are given a dilation angle of zero. The complete list of rock mass material properties used in the mine-scale analyses is shown in Table 6.8.

Table 6.8: Material properties used in the overburden of the mine scale models.

<table>
<thead>
<tr>
<th></th>
<th>Density (kg/m³)</th>
<th>Young’s Mod. (GPa)</th>
<th>Poisson Ratio</th>
<th>Friction Ang. (deg)</th>
<th>Cohesion (MPa)</th>
<th>Tensile (MPa)</th>
<th>Dilation (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridger Formation</td>
<td>2100</td>
<td>1.4</td>
<td>0.2</td>
<td>22.0</td>
<td>0.9</td>
<td>0.045</td>
<td>0</td>
</tr>
<tr>
<td>Laney Shale</td>
<td>2500</td>
<td>5.5</td>
<td>0.2</td>
<td>26.0</td>
<td>2.0</td>
<td>0.10</td>
<td>0</td>
</tr>
<tr>
<td>Tower Sandstone</td>
<td>2500</td>
<td>3.7</td>
<td>0.2</td>
<td>40.0</td>
<td>5.0</td>
<td>0.25</td>
<td>10.0</td>
</tr>
<tr>
<td>Wilkins Shale</td>
<td>2500</td>
<td>5.5</td>
<td>0.2</td>
<td>26.0</td>
<td>1.2</td>
<td>0.06</td>
<td>0</td>
</tr>
<tr>
<td>D Sandstone</td>
<td>2500</td>
<td>3.7</td>
<td>0.2</td>
<td>40.0</td>
<td>5.0</td>
<td>0.25</td>
<td>10.0</td>
</tr>
<tr>
<td>Roof Shale</td>
<td>2500</td>
<td>5.5</td>
<td>0.2</td>
<td>26.0</td>
<td>0.6</td>
<td>0.03</td>
<td>0</td>
</tr>
</tbody>
</table>

Strain-softening is implemented in the overburden materials in order to promote strength reduction and shearing in areas of high stress. The units are given incremental cohesion reductions of 10%, 20%, and 50% at strain levels specific to each geologic unit. A final cohesion reduction of 95% is used in the shale units and 100% in the sandstone and Bridger units. The critical strain values associated with cohesion drop in each material are shown in Table 6.9 and Table 6.10.
Table 6.9: Cohesion softening parameters applied to Bridger Formation, Laney Shale, and Sandstones.

<table>
<thead>
<tr>
<th>Strain</th>
<th>Cohesion (MPa)</th>
<th>Strain</th>
<th>Cohesion (MPa)</th>
<th>Strain</th>
<th>Cohesion (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.90</td>
<td>0.0</td>
<td>2.0</td>
<td>0.0</td>
<td>5.0</td>
</tr>
<tr>
<td>4.0e-4</td>
<td>0.90</td>
<td>3.0e-4</td>
<td>2.0</td>
<td>7.0e-4</td>
<td>5.0</td>
</tr>
<tr>
<td>2.0e-3</td>
<td>0.81</td>
<td>1.5e-3</td>
<td>1.8</td>
<td>3.5e-3</td>
<td>4.5</td>
</tr>
<tr>
<td>4.0e-3</td>
<td>0.72</td>
<td>3.0e-3</td>
<td>1.6</td>
<td>7.0e-3</td>
<td>4.0</td>
</tr>
<tr>
<td>8.0e-3</td>
<td>0.45</td>
<td>6.0e-3</td>
<td>1.0</td>
<td>1.4e-2</td>
<td>2.5</td>
</tr>
<tr>
<td>6.0e-2</td>
<td>0</td>
<td>4.5e-2</td>
<td>0.2</td>
<td>1.4e-1</td>
<td>0</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
<td>0.2</td>
<td>1.0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.10: Cohesion softening parameters applied to Wilkins Shale and Roof Shale.

<table>
<thead>
<tr>
<th>Strain</th>
<th>Cohesion (MPa)</th>
<th>Strain</th>
<th>Cohesion (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.20</td>
<td>0.0</td>
<td>0.60</td>
</tr>
<tr>
<td>3.0e-4</td>
<td>1.20</td>
<td>3.0e-4</td>
<td>0.60</td>
</tr>
<tr>
<td>1.5e-3</td>
<td>1.08</td>
<td>1.5e-3</td>
<td>0.54</td>
</tr>
<tr>
<td>3.0e-3</td>
<td>0.96</td>
<td>3.0e-3</td>
<td>0.48</td>
</tr>
<tr>
<td>6.0e-3</td>
<td>0.60</td>
<td>6.0e-3</td>
<td>0.30</td>
</tr>
<tr>
<td>4.5e-2</td>
<td>0.12</td>
<td>4.5e-2</td>
<td>0.06</td>
</tr>
<tr>
<td>1.0</td>
<td>0.12</td>
<td>1.0</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Figure 6.10 illustrates the results of triaxial test simulations on each of the overburden materials. The tests are conducted with 0.5 MPa confinement and loaded slowly with a constant velocity condition at the upper boundary. Each of the specimens has a height-to-width ratio of 2.0 and contains 10 zones across the width. They are discretized with the same zone size used in the mine-scale model in order to eliminate any discrepancy in the strength characteristics between triaxial and mine-scale simulations.

The data file used to simulate the triaxial test on the sandstone specimen can be found in Appendix A. The remaining triaxial simulations contain different zone sizes, material properties, and overall model dimensions, but the loading procedures and average stress and strain measurements are identical.
The graph in Figure 6.10 demonstrates the relative difference in strength between the sandstone and shale units. The Tower sandstone, with a thickness of 85 meters, behaves much like a beam in the 2D simulations, transferring vertical stresses to the intact portions of the trona seam as support capacity is reduced in the panel. While high tensile stresses may form in the sandstone above the abutments, the beam is highly resistant to shear because of its estimated strength.

6.10 Support Pressure Versus Average Subsidence

The support pressures implemented at seam level of the mine-scale models are based on the behavior exhibited by Model #1 in the pillar-scale analysis, except that the overburden displacement measured in the pillar-scale model is re-defined as average subsidence above the panel. In this way, seam level support can be adjusted according to average subsidence, which defines when the collapse is complete.

The average subsidence from the collapse is estimated by Pechmann (1995) to be approximately 0.6 meters, but no details are provided regarding the derivation of that value. For the purpose of comparing mine-scale modeling results with the subsidence data, determination of an appropriate average value is based on a cross section taken through the contours in the MSHA report. The location of the cross section, shown in Figure 6.11, is assumed to be an appropriate, or typical, profile through the contours.
Figure 6.11: Cross section A-B taken through subsidence data for further analysis.

The curves in the figure are digitized using the Vulcan 3D mine design software (Maptek, 2016), and the coordinates of each intersection point along the cross section are extracted. The data in the figure does not include a contour for zero subsidence but is extended to zero by assuming a linear continuation of the ends of the curve with the slope of the previous two data points. The extracted coordinates are plotted in Excel and shown in Figure 6.12.

Subsidence Points Through Cross Section

Figure 6.12: Coordinates of subsidence data plotted along cross section A-B.
The area encompassed by the curve is calculated in a spreadsheet using the method of Riemann sums and found to be approximately 730 square meters. The area of the curve is divided by the width of the profile, 1315 meters, for an average subsidence of 0.56 meters.

The support curve from pillar Model #1 is approximated by 14 data points and lengthened proportionally to achieve an overall subsidence value of 0.56 meters. The resultant support curve is shown in Figure 6.13 along with the support data extracted from pillar Model #1. Note that the implemented support values start at a point of zero displacement, which allows the same curve to be used in each parametric mine-scale simulation as long as grid point displacements are reset. The initial support pressure is 6.9 MPa, and simulation of the collapse begins as soon as the support curve is implemented.

![Avg Roof Support vs Overburden Displacement](image)

Figure 6.13: Average support from Pillar #1 and the curve implemented in mine scale models.

The 14 data points that comprise the average support curve are stored as a table in UDEC for implementation in the mine-scale model. At intervals of 100 time steps, a FISH function measures the average subsidence above the panel and interpolates an appropriate increase or reduction of support stress based on the nearest data points of displacement in the table.

6.11 Mine-Scale Model Results

The aim of the mine-scale simulations is to achieve an average subsidence of 0.56 meters through the dynamic implementation of seam level support that reflects the results seen in the pillar scale analysis and estimate the amount of energy released during the simulated collapse. The results indicate that a relatively well-matched subsidence trough and a magnitude of energy consistent with a local magnitude 5.3 seismic event can be generated by simulating widespread pillar failure at seam level.
6.11.1 Subsidence

Variation of the K ratio within the overburden has no significant effect on the shape or extent of the subsidence trough that develops at the surface of the model. Figure 6.14 illustrates a scanline of vertical surface displacements taken from each of the mine-scale models compared to the cross sectional profile used to calculate the target average subsidence.

![Subsidence Data vs Model results](image)

**Figure 6.14**: Modeled subsidence trough with cross section through subsidence data.

Each of the mine-scale models reaches an average subsidence of 0.53 to 0.54 meters, just shy of the 0.56 meter target. The shortcoming occurs because the overburden does not subside any farther once average support at seam level reaches approximately 9.25 MPa. At that stage, approximately 15% of the overburden load above the panel is being transferred to the intact trona to the left and right. This level of abutment loading may correlate with the damage to the panel access entries and parts of the submains observed by MSHA investigators (Ferriter, 1996) and convergence of approximately 16mm over the North-South submains after the collapse (Swanson, 1995). Further comparison would require detailed models of the submain entries under the suggested loading conditions.

The minor differences between the shape of the modeled subsidence curve and the cross section taken from the subsidence data are caused by site-specific factors that affect the slope and extent of the actual subsidence profile. With the geologic layers modeled as continuous blocks in the mine-scale simulation, no further agreement can be achieved, and no data exists regarding the location or orientation of faults or explicit discontinuities in the overburden. The area encompassed by the modeled subsidence profile is calculated in a spreadsheet using the method of Riemann sums and found to be 693 square...
meters, meaning that the volume of subsided ground in the 2D model is within 5% of that in the 2D subsidence profile, assuming a depth of one meter in the third direction.

A plot of displacement vectors from the mine-scale model helps visualize the magnitude and extent of deformations that occurred during the simulation of collapse. Figure 6.15 illustrates a color scale of vertical displacement vectors from the model with a 0.3 K ratio.

![Figure 6.15: Plot of displacement vectors obtained during simulation of widespread pillar failure.](image)

The maximum vertical displacement at the surface is approximately 0.84 meters, which matches the low point of the modeled subsidence profile shown previously in Figure 6.14. Roof displacements at seam level exceed one meter near the middle of the panel.

### 6.11.2 Yield in the Overburden

Plots of plasticity indicators help visualize the extent of damage incurred to the overburden during the simulation of collapse. In this case, the ratio of horizontal to vertical stress has a more pronounced effect because horizontal stresses serve to confine the material as the vertical stresses fluctuate. Figure 6.16 illustrates the damage to the left half of two models at the completion of the simulations. The models with K ratios of 0.3 and 0.7 are shown, which exhibit the highest and lowest extents of yield, respectively.

Most of the yield in the relatively weak Bridger Formation occurs as the model is brought to an initial state of equilibrium in a non-uniform stress field. For that reason, it likely exhibits a lower, residual shear strength as the collapse is simulated.
Figure 6.16: Extent of yield during simulation of widespread pillar failure in two models.

Regardless, the subsidence profile and plots of displacement vectors do not indicate any significant shearing in the Bridger or the units beneath it. Neither of the simulations exhibit any degree of yielding in the Tower Sandstone.

The similarity between subsidence profiles from each model, combined with the relative lack of damage in the model with a 0.7 K ratio, indicates that extensive shearing of the overburden materials is not necessary to produce a subsidence trough at the surface of the model that compares with observations. Geologic variability and discrete fracture patterns may have affected the shape of the subsidence profile above the mine, but the results of the models suggest that the bending of the geologic units accommodates the extent of observed subsidence.

6.11.3 Energy

Variation of the K ratio within the overburden has no significant effect on the magnitude of energy released as the collapse is simulated. Figure 6.17 shows the cumulative damped work from each of the mine-scale models, which accounts for energy that cannot be stored or consumed as average pillar support is varied at seam level. The value of time on the horizontal axis is a product of the numerical time step and the number of calculation cycles required to complete the simulation and has no correlation with the duration of the actual mine collapse.
Figure 6.17: Released energy, in terms of damped work, from each of the mine scale models.

Each of the simulations results in a release of energy between 1.56 and 1.58 billion Joules. Considering the similarity in subsidence profiles and released energy values from each model, with very little dependence upon shearing in the overburden, it can be deduced that the released energy associated with the collapse is attributed almost entirely to the deformation of strata under gravitational loading and the expansion of compressed overburden materials.

The simplification of the energy release mechanisms associated with the collapse can be further illustrated by comparing the average roof support versus roof displacement of the pillars with the average pressure versus displacement of the overburden. In accordance with Salamon’s (1970) theories regarding mine stiffness and the stability of pillar failure, the release of energy can be accounted for by the difference between the slope of the ground reaction curve and the post-peak behavior of the pillar supports. Figure 6.18 compares the pillar and overburden behavior in terms of average roof support versus average roof displacement. The data is taken from the model with a 0.5 K ratio.

The area between the two curves is calculated graphically by entering 17 representative data points into a CAD software package and found to be approximately 1.82e6 N/m. Because the stresses and displacements are averaged in each simulation, effectively dividing the sum of grid point forces and total displacements by 855 meters, this value represents the energy that will be released by a one meter width of pillar and overburden materials.
In order to quantify the energy associated with the entire panel, the value of $1.82 \times 10^6$ N/m is multiplied by 855 meters, and one meter in the third direction, for a total of approximately $1.56 \times 10^9$ Nm or Joules. The cumulative damped work from the mine-scale simulation with a $K$ ratio of 0.5 was $1.56 \times 10^9$ Joules. Although the mine collapse involved numerous pillar geometries and naturally varying ground conditions, and has been broadly simplified for the purpose of this study, the results illustrate that the fundamental mechanisms of brittle pillar failure and elastic overburden displacement can account for the energy associated with the collapse.

6.12 Extrapolation of Energy Results

For the purpose of extrapolating the released energy results into the third direction and estimating the seismic magnitude associated with an equivalent 3D event, the volume of the subsidence trough above the Southwest Panel is estimated and compared to that of the 2D model. In this way, the assumed extent of collapse in the third direction is based on an overall volume of subsidence, rather than the north-south length of the panel or any arbitrary distance.

To estimate the volume of subsidence above the Southwest Panel, the subsidence contours from the MSHA report are digitized and scaled using the Vulcan mine design software package (Maptek, 2016). The scaling factor is based on the linear distance through 25 pillar centers in the East-West Mains with a spacing of 125 feet (Ferriter, 1996). The digitized model is scaled so that the length of the reference line is 3125 feet. Figure 6.19 shows the digitized contours and the reference line used for scaling at the north end.

Figure 6.18: Comparison of ground reaction curve with average pillar support versus roof displacement.
Figure 6.19: Digitized subsidence contours used to determine volume of subsided ground.

The 0.1-foot contour, which is the outermost contour in Figure 6.19, is incomplete in the northeast area of the MSHA diagram and does not form a closed loop. In order to perform calculations of area and volume associated with the subsidence trough, the 0.1-foot contour is given a simple, curved profile between the closest available data points and moved up to an elevation of zero to serve as the upper boundary of the solid. This approach disregards the volume of material at the perimeter of the trough that subsided between 0.0 and 0.1 feet and slightly underestimates the total surface area affected by the collapse.

The volume of the entire subsidence trough is converted from cubic feet and found to be approximately 1.22 million cubic meters, and the surface area of the outermost contour is calculated to be approximately 2.67 million square meters. For the sake of discussion, dividing the total subsided volume by the total surface area of the contours results in an overall average subsidence value of 0.45 meters. This average value is much lower than the previously-mentioned value of 0.6 meters (Pechmann, 1995) and the value of 0.56 meters calculated for the 2D cross section in Figure 6.12. The discrepancy suggests that the cross section selected as “typical” exhibits more subsidence than average. By assuming an average subsidence of 0.56 or 0.6 meters and extrapolating the results of the model by the full north-south distance of the panel in the third direction, the total release of energy associated with the collapse could be over-estimated.

The volume of the subsidence trough in the 2D mine-scale model is actually a 2D area of 693 square meters, multiplied by an assumed one meter in the third direction. Dividing the volume of subsidence at the mine by the volume of subsidence in the 2D model results in a value of approximately 1765. It is therefore assumed that by multiplying the 1.56 billion Joules of released energy from the 2D models by
1765, the energy associated with a 3D collapse, similar in size to that of the Solvay collapse, can be estimated at $2.75 \times 10^{12}$ Joules. The total released energy, based on three-component broadband velocity data, is estimated by Pechmann (1995) to be $1.4 \times 10^{12}$ Joules.

The relative magnitude of an associated seismic event, based on the modeled value of released energy, can be back-calculated using Gutenberg and Richter’s (1956) and Kanamori’s (1993) equations. Details regarding the two equations can be found in Section 4.3. After converting the total released energy to units of ergs, the Gutenberg and Richter calculation produces a 5.4 magnitude. With the total energy in units of Joules, Kanamori’s formula results in a 5.3 magnitude.

While the calculated local magnitudes show good agreement with the estimate of a 5.3 event made by seismologists after the collapse, it is worth noting that total released energy values from $2.3 \times 10^{12}$ to $3.3 \times 10^{12}$ Joules would result in back-calculated local magnitudes of 5.26 to 5.34, respectively, using Kanamori’s equation. Thus, the quantity of energy involved in an estimated 5.3 event can fall within a margin of $+20/-16\%$, and extrapolation of the energy results in the 2D model by distances from 1500 to 2100 meters in the third direction would still produce an estimate of the appropriate magnitude.

Regardless of the margin of error allowed in the logarithmic magnitude scale, the results of the study illustrate that carefully constructed and calibrated 2D numerical models are capable of predicting relevant seismic magnitudes, even if the extent of the collapse in the third direction is poorly understood.

6.13 Conclusions

The energy-based back analysis of the Solvay Mine collapse is approached by quantifying stresses and displacements in pillar scale models before attempting to simulate a widespread collapse. An excavation geometry assumed to represent average conditions within a vast area of collapsed workings is selected for analysis in pillar scale models. Mohr Coulomb rock mass properties are calibrated for the pillar and floor materials based on published laboratory data, and simplified cohesion softening parameters are implemented to accommodate brittle failure. The thickness of the trona and oil shale in the floor of the excavation are evaluated in a parametric series of models to quantify their effect on the peak strength of the pillars and brittleness of failure. The model with geometry most similar to that described in the mine reveals seam level deformations and an extent of damage that are consistent with observations made at the time of the collapse.

Average support forces are extracted from pillar scale models and applied at seam level of a mine scale model to simulate widespread failure. The overburden materials in the mine scale model are given cohesions softening parameters to encourage shearing in areas of high differential stress. Parametric
evaluation of the horizontal stress state in the overburden reveals no significant effect on the subsidence profile or value of energy released during simulation of widespread pillar failure.

The magnitude of released energy and the observed subsidence profile can be accounted for by the change in potential energy of the overburden bending under gravitational loading and the release of stored strain energy that occurs as roof support is reduced in a brittle manner. Results of the simulated collapse demonstrate that the energy released in a 2D mine-scale numerical model, extrapolated in the third direction to achieve a realistic total extent of failure, correlates with the estimated 5.3 seismic magnitude of the collapse.
CHAPTER 7
CONCLUSIONS

The transfer and release of energy has been fundamental to the study of instability since researchers first began exploring the nature of sudden material failure. The goal of the research presented in this dissertation is to develop a method of rock mechanics analysis in which the calculation of released energy is used to assess the potential for instability in underground mine workings.

The calculation of released energy, performed by a commercially available distinct element software package, is validated in a series of simple models by comparing results to analytic solutions under prescribed loading conditions. Back analysis is performed on two documented collapse events from room and pillar mines located within the western United States by constructing large scale models and inducing widespread failure through the appropriate loading conditions or sequence of excavation. The values of energy released during simulations of mine-scale unstable failure are compared to seismic magnitudes through empirical formulas.

7.1 Key findings

The following sections describe the findings of the research and the contributions made to the study of instability in numerical models.

7.1.1 Energy and Unstable Failure

Although the energy release associated with a circular excavation in elastic ground is validated in the software user’s manual, alternative modes of unstable failure are evaluated to verify the accuracy of the energy calculations under increasingly complex conditions. The validation of energy results from each case provides confidence that the energy released during simulations of large scale failure, which involve multiple failure modes at once, is calculated with accuracy.

A simple model of pillar failure illustrates the criteria for instability proposed by Cook (1965) and Salamon (1970). In this modeling approach, a pillar composed of strain softening material is surrounded by elastic ground under simplified vertical loading conditions. After simulating pillar failure through a careful relaxation sequence in the adjacent openings, independently calculated values of average pillar stress and displacement are plotted on the same axes as the average pressure and displacement measured in the overburden. The energy density calculated between the areas under each curve, multiplied by the volume of failed material, results in a total magnitude of energy that correlates well with the values of
damped work reported by the software. The results help visualize the criteria for instability and demonstrate that the software performs as expected.

7.1.2 Crandall Canyon Analysis

The back analysis of the Crandall Canyon Mine collapse of August 2007 involves a parametric assessment of various constitutive laws in the coal seam and the contacts between the coal and the surrounding rock. By comparing the release of energy in each simulation, it is evident that the use of softening constitutive laws in either the coal or the coal-rock interface contributes to greater levels of instability as the excavation sequence is carried out. Although some degree of unstable failure is anticipated in the simulations which include strain softening parameters in the coal seam, the values of released energy in most of the models do not signify a collapse and the extent of failure in the pillars does not match observations at the mine.

The model with softening parameters in both the coal and the coal-rock interface results in a much greater release of energy than any of the other models and demonstrates a single, widespread failure event that correlates with observations at the mine after the collapse. The collapse of the model highlights a potential failure mechanism of large width-to-height coal pillars in which shear slip at the coal-rock interface contributes to deconfinement of the coal material, and the failure of the coal then contributes to additional shear slip. This progression of failure toward the core of the pillar occurs suddenly, due to the softening behavior of both the coal and the shear interface, and drastically reduces its residual support capacity.

The magnitude of energy released in the 2D simulated collapse, multiplied by 500 meters in the third direction to match the estimated extent of the actual collapse, is several times higher than the magnitude of energy believed to accompany a 3.9 local magnitude seismic event. Empirical formulas relating the value of released energy to seismic magnitude show the simulated collapse to be more representative of a 4.2 or 4.3 magnitude event. The discrepancy is attributed to the simplified model geometry and boundary conditions, which disregard the processes of bending and separation in the overlying strata as pillar failure occurs. These mechanisms would serve to limit subsidence and consume energy if additional material were modeled in the far field. However, including additional material in the model would require consideration of gob compaction in the adjacent longwall panels and increase simulation time drastically.

7.1.3 Solvay Analysis

The collapse of the southwest panel of the Solvay Trona Mine in February of 1995 is analyzed by first constructing a detailed model of the interaction between the strong trona pillars and the weak oil shale floor. Strain softening material properties are based on published laboratory test data, and several
pillar-floor geometries are evaluated parametrically under identical loading conditions in order to match resultant deformations with observations at the mine. The failed pillar that most closely resembles conditions after the collapse exhibits a peak load bearing capacity that is only 10% higher than full overburden loads. In addition, the pillar loses more than 40% of its strength during failure, illustrating the likelihood that the analyzed pillar geometry could accommodate a cascading pillar failure.

The average support capacity of the pillar-floor system is recorded throughout the process of loading. This behavior is then applied at seam level of a mine scale model through a system of grid point forces to simulate widespread, brittle pillar failure. The sudden reduction of pillar support results in a large release of energy, and as pillar support is returned to stable levels, the depth and extent of subsidence at the surface of the model shows good agreement with that measured above the collapsed workings. Parametric evaluation of the horizontal stresses within the overburden show that it has very little effect on the magnitude of energy released or extent of subsidence generated.

The magnitude of energy released during the simulation of pillar failure correlates well with the documented local magnitude of 5.3 associated with the mine collapse. The empirical formulas used to relate the two values suggest that the magnitude of the 2D simulated collapse, multiplied by 1765 meters in the third direction to match the total volume of subsided ground at the surface, is representative of a 5.3 or 5.4 magnitude event. Although the overburden units are given strain softening properties to encourage shearing in areas of high stress, the lack of significant yielding in the models suggests that the release of energy is fueled primarily by the change in potential energy of the strata bending under gravitational loading.

7.2 Limitations

The study of unstable failure requires the use of softening parameters, and inherently, assumptions must be made regarding the rate at which the material softens with respect to increments of strain and the degree to which the strength parameters change. Although these parameters can be adjusted in simple models to mimic laboratory stress and strain data, laboratory data is not always available. In addition, the strength and stiffness parameters derived in laboratory tests are often reduced when applied to a rock mass, which requires assumptions regarding the quality of the rock mass and the calibration of entirely new critical strain values. Without careful consideration of the strain increments at which the strength parameters are changed, the zones within a modeled strain softening rock mass may exhibit extreme post-peak brittleness or perfect plasticity. The calibration of a reasonable post-peak stiffness and strength reduction for each relevant material in the model are therefore essential to the study of instability.
The scope of this research and the development of the energy-based approach to studying mine scale instability are thus far limited to 2D analyses. While 2D models are sufficient for the analysis of pillar strength or the response of the overburden through a particular cross section of the mine, the extension of energy results by an assumed distance in the third dimension introduces considerable uncertainty. Three-dimensional models could improve understanding of the unstable failure mechanisms associated with various pillar and excavation geometries, joint orientations, and loading conditions. Unfortunately, the use of strain softening material properties is best suited to applications in which the shortest dimension of a failing medium is transected by multiple zones and the zone size is consistent all throughout the material. Therefore, 3D simulations that involve vast areas of potentially unstable pillars will require an extremely large number of zones and long computation times.

Back analysis of the Crandall Canyon Mine collapse involved various assumptions and simplifications to reduce the number of variables that may distract from the analysis of pillar failure. While the chosen approach was successful in demonstrating a collapse event and an associated value of released energy, the associated seismic magnitude of the simulated collapse was higher than expected. This highlights the difficulty in choosing an ideal 2D modeling approach to analyze complex 3D situations. However, the exaggeration of energy release may attract due attention to potentially unstable conditions if the combination of simplified model geometry and conservative loading conditions are considered in a predictive analysis.

Considerable effort was directed at the calibration of material properties and construction of the pillar scale model for back analysis of the Solvay Mine collapse. The results of the pillar scale investigation offer valuable insight to the support capacity of the stiff trona pillars overriding the weak shale floor. To simulate widespread collapse, the support characteristics of one pillar geometry were applied to the entire 900 meters span of the trona seam in the mine scale model. While no justification could be made for a slightly stronger or weaker average pillar behavior, the application of one support curve disregards the variation in pillar geometry that existed across the southwest panel.

### 7.3 Future Work

Further development of an energy-based approach to the study of instability will benefit by extending the back analysis of unstable failure events into three-dimensions. With more detailed excavation geometry and an improved representation of geologic composition, back analyses could be performed on smaller coal bump events to better understand the associated failure mechanisms and sources of released energy. If the extent of damage and deformation in a 3D model can be matched with site specific observations, the energy values associated with the simulation of failure may be correlated without making assumptions regarding the extension of 2D results.
The study of instability may also be applied to underground excavations beyond room and pillar mines. Analysis of deep excavations in hard rock may improve understanding of smaller rock burst events, especially with the inclusion of softening parameters in discontinuities that are capable of releasing excess shear stress. In any case, the numerical modeling techniques presented in this dissertation can be used to assess the potential for unstable failure in future excavations and ultimately improve the safety of underground mining methods.
REFERENCES CITED


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A.1 UCS test on strain softening specimen

New
title 'UCS test, coh softening'
config
round 0.01
edge 0.02
set ovtol 0.02

; hist nyc 500
;
def _extents
  _rite = 75.0
  _rl = _rite - 0.1
  _rh = _rite + 0.1
  _left = 0.0
  _ll = _left - 0.1
  _lh = _left + 0.1
  ;
  _top = _rite * 2.0
  _tl = _top - 0.1
  _th = _top + 0.1
  _topplat = _top + 60.0
  _tphi = _topplat + 0.1
  _tplo = _topplat - 0.1
  ;
  _bot = 0.0
  _bl = _bot - 0.1
  _bh = _bot + 0.1
  _botplat = _bot - 60.0
  _bphi = _botplat + 0.1
  _bplo = _botplat - 0.1
  ;
end _extents
;
block _left,_botplat _left,_topplat _rite,_topplat _rite,_botplat
crack _left,_top _rite,_top
  crack _left,_bot _rite,_bot
  ;
gen quad 7.501 ; Discretize
;
def mat_props
::
  _d5 = 2500.0
  _E5 = 5.5e9
  _v5 = 0.2
  _K5 = _E5 / (3.0 * (1.0 - 2.0 * _v5))
\_G5 = \_E5 / (2.0 \times (1.0 + \_v5))
\_f5 = 26.0
\_c5 = 0.6e6
\_c5a = 0.54e6
\_c5b = 0.48e6
\_c5c = 0.3e6
\_c5d = 0.0e6
\_t5 = 0.03e6
\_di5 = 0.0

\_d2 = 2500.0
\_E2 = 2.2e9
\_v2 = 0.2
\_K2 = \_E2 / (3.0 \times (1.0 - 2.0 \times \_v2))
\_G2 = \_E2 / (2.0 \times (1.0 + \_v2))
end
mat_props
;
;; softening parameters
table 5 0.0,\_c5 0.0003,\_c5 0.0015,\_c5a 0.003,\_c5b 0.006,\_c5c 0.045,\_c5d 1.0,\_c5d
zone model elas dens _d2 bulk _K2 shear _G2
zone model ss dens _d5 bulk _K5 shear _G5 fric _f5 coh _c5 tens _t5 dil _di5 ctable 5 &
range _ll _rh _bot _top
;
;; All joints assigned parameters
joint model area jkn 500.0e9 jks 500.0e9 jfric 30.0 jcoh 5.0e6 jtens 0.4e6 jdil 0.0
set jcondf = 2
set jmatdf = 1
prop jmat 1 jkn 500.0e9 jks 500.0e9 jfric 25.0
;
;
def _zonecount
\_numz = 0
\_iab = block_head
loop while \_iab # 0
\_iz = b_zone(\_iab)
loop while \_iz # 0
if z_y(\_iz) > 75.0
if z_y(\_iz) < 82.5
\_numz = \_numz + 1
endif
endif
\_iz = z_next(\_iz)
endloop
\_iab = b_next(\_iab)
endloop
end
_zonecount
;
;; Function to generate an array with 1 column and a number of rows equal to
; the number of zones counted
def _build_array
    array all_zones(1, _numz)
end

; ; Function to store the zone ID's of zones at center of pillar for calculation
; ; of average stress
def _zone_id_collector
    _tick = 0
    _iab1 = block_head
    loop while _iab1 # 0
        _iz1 = b_zone(_iab1)
        loop while _iz1 # 0
            if z_y(_iz1) > 75.0
                if z_y(_iz1) < 82.5
                    _tick = _tick + 1
                    all_zones(1, _tick) = _iz1
                endif
            endif
            _iz1 = z_next(_iz1)
        endloop
        _iab1 = b_next(_iab1)
    endloop
end

; ; Function to build array to store grip points. Only 3 grid points
; ; at top and bottom of entry are available to monitor convergence.
def _gp_arrays
    array top_gps(1, 11)
    array bot_gps(1, 11)
end

; ; Function to populate arrays with grid point ID's
def _get_points
    loop m (1, 11)
        _xx = m * 7.5 - 7.5
        top_gps(1, m) = gp_near(_xx, _top)
        bot_gps(1, m) = gp_near(_xx, 0.0)
    endloop
end

; ; Average stress measurement in the pillar
def Calc_stress
    _sumstress = 0.0
    loop i (1, _numz)
        _sumstress = _sumstress + z_syy(all_zones(1, i))
    endloop
Av_stress = _sumstress * (-1.0) / _numz
end

; def Calc_closure
  _sumtop = 0.0
  _sumbot = 0.0
  loop j (1,11)
    _sumtop = _sumtop + gp_ydis(top_gps(1,j))
    _sumbot = _sumbot + gp_ydis(bot_gps(1,j))
  endloop
  _strain = ( _sumbot - _sumtop) / 11.0 / _top
end

; set fishcall 0 Calc_stress
set fishcall 0 Calc_closure
;
; damp local
mscale off
;
set energy on
hist energy
hist Av_stress ;history No.21
hist _strain ;history No.22
;
;=================================================================
; initialize 0.5 MPa hydrostatic stress in all zones
insitu stress 0.0 0.0 -1.5e6 szz -0.5e6
;; top stress
boundary stress 0.0 0.0 -1.5e6 range _ll _rh _tplo _tphi ;; at top
;=================================================================
;
; Fix bottom platen in y direction
boundary yvel 0.0 range _ll _rh _bplo _bphi
; Fix both platens in X-direction
boundary xvel 0.0 range _ll _rh _tplo _tphi
boundary xvel 0.0 range _ll _rh _bplo _bphi
;
solve
;
BOUNDARY yvel -0.0005 range _ll _rh _tplo _tphi
step 4200000
;
save UCS_SS.sav
pl hist 21 vs 22
set pl png size 960 720
copy UCS_SS.png
;
;;;; END OF FILE
A.2 Strain softening pillar excavated in elastic ground

New title 'Single pillar in elastic ground, slowly mined'

; config round 0.005
edge 0.01
set ovf 0.02
set small on
hist ncyc 50
;
def _extent ;; coordinates of model extents and excavation boundaries
    _left = 0
    _ll  = _left - 0.1
    _lh  = _left + 0.1
    _rite = 105.0
    _rl  = _rite - 0.1
    _rh  = _rite + 0.1
    _bot = 0.0
    _bl  = _bot - 0.1
    _bh  = _bot + 0.1
    _top = 105.0
    _tl  = _top - 0.2
    _th  = _top + 0.1
    _roof = 55.0 ;; excavation roof
    _flor = 50.0 ;; excavation floor
    _zonroof1 = _roof + 5.0 ;; for 4x 1.6m zones
    _zonroof2 = _zonroof1 + 10.0
    _zonfloor1 = _flor - 5.0 ;; for zoning below seam
    _zonfloor2 = _zonfloor1 - 10.0
    _rib01L = 40.0 ;; X-coordinates of each pillar rib
    _zoneribL = _rib01L - 10.0
    _zoneribL2 = _zoneribL - 10.0
    _rib01R = 50.0
    _rib02L = 55.0 ;; X-coordinates of each pillar rib
    _rib02R = 65.0
    _zoneribR = _rib02R + 10.0
    _zoneribR2 = _zoneribR + 10.0
    ;
end _extent
;
;
block _left, _bot _left, _top _rite, _top _rite, _bot
;
crack _left, _roof _rite, _roof ;; contact above seam
crack _left, _flor _rite, _flor ;; contact below seam

; crack 48.75 _flor 48.75 _roof
crack 56.25 _flor 56.25 _roof

; crack _rib01L _flor _rib01R _roof ;; vertical cracks at left and right edge of each entry
crack _rib01L _flor _rib01R _roof

; crack _rib02L _flor _rib02R _roof

; crack _zoneribL _zonfloor2 _zoneribR _zonroof2

; crack _zoneribL2 _zonfloor2 _zoneribL2 _zonroof2

; crack _zoneribR2 _zonfloor2 _zoneribR2 _zonroof2

; crack _left, _zonroof1 _rite, _zonroof1

crack _left, _zonfloor1 _rite, _zonfloor1

; crack _left, _zonroof2 _rite, _zonroof2

crack _left, _zonfloor2 _rite, _zonfloor2

; gen quad 1.251 range _zoneribL _zoneribR _zonfloor1 _zonroof1

; gen quad 2.501 range _zoneribL2 _zoneribR2 _zonfloor2 _zonroof2

; gen quad 5.001

; join_contact

; ;; list of groups starting at bottom of model

group zone Rock

group zone Coal range _rib01L _rib02L _flor _roof

; def _parameters

_DensityCoal = 1500.0
_DensityRock = 2500.0
_ECoal = 25.0E9
_ERock = 5.0E9
_vCoal = 0.2
_vRock = 0.2
_CohCoal = 1.5E6
_FricCoal = 25.0
_DiCoal = 0.0
_TenCoal = 0.0

_KCoal = _ECoal/(3*(1-2*_vCoal))
_GCoal = _ECoal/(2*(1+_vCoal))
_KRock = _ERock/(3*(1-2*_vRock))
_GRock = _ERock/(2*(1+_vRock))

;_jkn = 50.0E9 ;; Joint normal stiffness in Pa/m
_jks = 50.0E9 ;; Joint shear stiffness in Pa/m
jiFric CY = 40.0 ; Initial friction angle of CY interface
jiFric CY = 15.0 ; Intrinsic friction angle of CY interface
jr = 1.5E-4 ; Joint roughness of CY interface

end

parameters

table 1 0.0,1.5e6 0.0001,1.5e6 0.001,0.5E6 1.0,0.5e6 ; cohesion

table 1 range group Rock
zone model elas density _DensityRock bulk _KRock shear _GRock range group Rock
zone model ss dens _DensityCoal bulk _KCoal shear _GCoal friction _FricCoal cohesion _CohCoal
table 1 range group Coal

;; Boundary Conditions
boundary yvel 0.0 range _ll _rh _bl _bh ; Fix the bottom of the model in y direction
boundary xvel 0.0 range _ll _lh _bl _th ; Fix the left boundary of the model in x direction
boundary xvel 0.0 range _rl _rh _bl _th ; Fix the right boundary of the model in x direction

Find initial equilibrium
boundary stress 0.0 0.0 -2.0e6 range _ll _rh _tl _th
insitu stress -0.4e6 0.0 -2.0e6

solve
save 01_equil.sav

reset disp

delete range _rib01L _rib01R _flor _roof
delete range _rib02L _rib02R _flor _roof

step 1

; damp local
mscale off
set energy on
hist energy ; histories 1 - 20

; Function to count the number of zones in the pillar,
in order to calculate total average stress.
def _zonecount
_numz = 0
_iab = block_head
loop while _iab # 0

iz = b_zone(_iab)
loop while _iz # 0
  if z_y(_iz) > 50.0
    if z_y(_iz) < 55.0
      if z_x(_iz) > 50.0
        if z_x(_iz) < 55.0
          _numz = _numz + 1
        endif
      endif
    endif
  endif
endif
end

iz = z_next(_iz)
endloop

_iab = b_next(_iab)
endloop
end

_zonecount
;
;
;; Function to generate an array with 1 column and a number of rows equal to
;; the number of zones counted
def _build_array
  array pillar_zones(1,_numz)
end

_build_array
;
;
;; Function to store the zone ID's of zones
def _zone_collector
  _tick = 0
  _iab1 = block_head
  loop while _iab1 # 0
    _iz1 = b_zone(_iab1)
    loop while _iz1 # 0
      if z_y(_iz1) > 50.0
        if z_y(_iz1) < 55.0
          if z_x(_iz1) > 50.0
            if z_x(_iz1) < 55.0
              _tick = _tick + 1
              pillar_zones(1,_tick) = _iz1
            endif
          endif
        endif
      endif
    endif
  endloop
  _iab1 = b_next(_iab1)
endloop
end

_zone_collector
;
;
;; Function to build array to store grip points.
def _gp_arrays
    array top_gps(1,5)
    array bot_gps(1,5)
end

;; Function to populate arrays with grid point ID's
def _get_points
    loop m (1,5)
        _xx = 50.0 + (m * 1.25) - 1.25
        top_gps(1,m) = gp_near(_xx,55.0)
        bot_gps(1,m) = gp_near(_xx,50.0)
    endloop
end

;; Average stress measurement in the pillar
def Calc_stress
    _sumstress = 0.0
    loop i (1,_numz)
        _sumstress = _sumstress + z_syy(pillar_zones(1,i))
    endloop
    Av_stress = _sumstress * (-1.0) / _numz
end

;; Average closure of grid points above and below pillar
def Calc_closure
    _sumtop = 0.0
    _sumbot = 0.0
    loop j (1,5)
        _sumtop = _sumtop + gp_ydis(top_gps(1,j))
        _sumbot = _sumbot + gp_ydis(bot_gps(1,j))
    endloop
    _top_disp = _sumtop / (-5.0)
    _bot_disp = _sumbot / 5.0
    _closure = (_sumbot - _sumtop) / 5.0
    Av_strain = _closure / 5.0
end

set fishcall 0 Calc_stress
set fishcall 0 Calc_closure

hist Av_stress ; hist 21
hist Av_strain ; hist 22
;
hist _closure ; hist 23
hist _top_disp ; hist 24
;
;
.................................................................
;
def mining_arrays
  array top_left_gps(1,7)
  array bot_left_gps(1,7)
  array top_rite_gps(1,7)
  array bot_rite_gps(1,7)
    
  array rib1_left_gps(1,3)
  array rib1_rite_gps(1,3)
  array rib2_left_gps(1,3)
  array rib2_rite_gps(1,3)
    
  array top_left_force(1,7)
  array bot_left_force(1,7)
  array top_rite_force(1,7)
  array bot_rite_force(1,7)
    
  array rib1_left_force(1,3)
  array rib1_rite_force(1,3)
  array rib2_left_force(1,3)
  array rib2_rite_force(1,3)
end
mining_arrays
;
def _get_mining_grid_points
  m = 0
  loop m (1,7)
    _xx_left = 40.0 + m * 1.25
    top_left_gps(1,m) = gp_near(_xx_left,55.0)
    bot_left_gps(1,m) = gp_near(_xx_left,50.0)
    
    _xx_rite = 55.0 + m * 1.25
    top_rite_gps(1,m) = gp_near(_xx_rite,55.0)
    bot_rite_gps(1,m) = gp_near(_xx_rite,50.0)
  endloop
  
  n = 0
  loop n (1,3)
    _yy2 = 50.0 + n * 1.25
    rib1_left_gps(1,n) = gp_near(_yy2,40.0)
    rib1_rite_gps(1,n) = gp_near(_yy2,50.0)
    rib2_left_gps(1,n) = gp_near(_yy2,55.0)
  endloop
rib2_rite_gps(1,n) = gp_near(_yy2,65.0)
endloop
der
_get_mining_grid_points
;
;
;
def what_the_force_top_left
  loop i (1,7)
    top_left_force(1,i) = gp_yforce(top_left_gps(1,i)) * (-1.0)
  endloop
end
what_the_force_top_left
def what_the_force_bottom_left
  loop i (1,7)
    bot_left_force(1,i) = gp_yforce(bot_left_gps(1,i)) * (-1.0)
  endloop
end
what_the_force_bottom_left
def what_the_force_rib1_left
  loop i (1,3)
    rib1_left_force(1,i) = gp_xforce(rib1_left_gps(1,i)) * (-1.0)
  endloop
end
what_the_force_rib1_left
def what_the_force_rib1_rite
  loop i (1,3)
    rib1_rite_force(1,i) = gp_xforce(rib1_rite_gps(1,i)) * (-1.0)
  endloop
end
what_the_force_rib1_rite
def what_the_force_top_rite
  loop i (1,7)
    top_rite_force(1,i) = gp_yforce(top_rite_gps(1,i)) * (-1.0)
  endloop
end
what_the_force_top_rite
def what_the_force_bottom_rite
  loop i (1,7)
    bot_rite_force(1,i) = gp_yforce(bot_rite_gps(1,i)) * (-1.0)
  endloop
end
what_the_force_bottom_rite
def what_the_force_rib2_left
    loop i (1,3)
        rib2_left_force(1,i) = gp_xforce(rib2_left_gps(1,i)) * (-1.0)
    endloop
end
what_the_force_rib2_left
;

def what_the_force_rib2_rite
    loop i (1,3)
        rib2_rite_force(1,i) = gp_xforce(rib2_rite_gps(1,i)) * (-1.0)
    endloop
end
what_the_force_rib2_rite
;

; Function to gradually mine the two excavations
def Let_me_down
    loop n (1,1000)
        _fact = 1.0 - ( n / 1000.0 )
        TLf1 = top_left_force(1,1) * _fact
        TLf2 = top_left_force(1,2) * _fact
        TLf3 = top_left_force(1,3) * _fact
        TLf4 = top_left_force(1,4) * _fact
        TLf5 = top_left_force(1,5) * _fact
        TLf6 = top_left_force(1,6) * _fact
        TLf7 = top_left_force(1,7) * _fact
        ;
        BLf1 = bot_left_force(1,1) * _fact
        BLf2 = bot_left_force(1,2) * _fact
        BLf3 = bot_left_force(1,3) * _fact
        BLf4 = bot_left_force(1,4) * _fact
        BLf5 = bot_left_force(1,5) * _fact
        BLf6 = bot_left_force(1,6) * _fact
        BLf7 = bot_left_force(1,7) * _fact
        ;
        R1L_f1 = rib1_left_force(1,1) * _fact
        R1L_f2 = rib1_left_force(1,2) * _fact
        R1L_f3 = rib1_left_force(1,3) * _fact
        ;
        R1R_f1 = rib1_rite_force(1,1) * _fact
        R1R_f2 = rib1_rite_force(1,2) * _fact
        R1R_f3 = rib1_rite_force(1,3) * _fact
        ;
        TRf1 = top_rite_force(1,1) * _fact
        TRf2 = top_rite_force(1,2) * _fact
        TRf3 = top_rite_force(1,3) * _fact
        TRf4 = top_rite_force(1,4) * _fact
        TRf5 = top_rite_force(1,5) * _fact
TRf6 = top_rite_force(1,6) * _fact
TRf7 = top_rite_force(1,7) * _fact
;
BRf1 = bot_rite_force(1,1) * _fact
BRf2 = bot_rite_force(1,2) * _fact
BRf3 = bot_rite_force(1,3) * _fact
BRf4 = bot_rite_force(1,4) * _fact
BRf5 = bot_rite_force(1,5) * _fact
BRf6 = bot_rite_force(1,6) * _fact
BRf7 = bot_rite_force(1,7) * _fact
;
R2L_f1 = rib2_left_force(1,1) * _fact
R2L_f2 = rib2_left_force(1,2) * _fact
R2L_f3 = rib2_left_force(1,3) * _fact
;
R2R_f1 = rib2_rite_force(1,1) * _fact
R2R_f2 = rib2_rite_force(1,2) * _fact
R2R_f3 = rib2_rite_force(1,3) * _fact
;
calendar
boundary interior yfree range 39.9 50.1 49.9 55.1
boundary interior xfree range 39.9 50.1 49.9 55.1
boundary interior yload TLf1 range 41.15 41.35 54.9 55.1
boundary interior yload TLf2 range 42.4 42.6 54.9 55.1
boundary interior yload TLf3 range 43.65 43.85 54.9 55.1
boundary interior yload TLf4 range 44.9 45.1 54.9 55.1
boundary interior yload TLf5 range 46.15 46.35 54.9 55.1
boundary interior yload TLf6 range 47.4 47.6 54.9 55.1
boundary interior yload TLf7 range 48.65 48.85 54.9 55.1
;
boundary interior yload BLf1 range 41.15 41.35 49.9 50.1
boundary interior yload BLf2 range 42.4 42.6 49.9 50.1
boundary interior yload BLf3 range 43.65 43.85 49.9 50.1
boundary interior yload BLf4 range 44.9 45.1 49.9 50.1
boundary interior yload BLf5 range 46.15 46.35 49.9 50.1
boundary interior yload BLf6 range 47.4 47.6 49.9 50.1
boundary interior yload BLf7 range 48.65 48.85 49.9 50.1
;
boundary interior xload R1L_f1 range 39.9 40.1 51.15 51.35
boundary interior xload R1L_f2 range 39.9 40.1 52.4 52.6
boundary interior xload R1L_f3 range 39.9 40.1 53.65 53.85
;
boundary interior xload R1R_f1 range 49.9 50.1 51.15 51.35
boundary interior xload R1R_f2 range 49.9 50.1 52.4 52.6
boundary interior xload R1R_f3 range 49.9 50.1 53.65 53.85
;
;;;;;
;
boundary interior yfree range 54.9 65.1 49.9 55.1
boundary interior xfree range 54.9 65.1 49.9 55.1
boundary interior yload TRF1 range 56.15 56.35 54.9 55.1

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boundary interior yload TRf2 range 57.4 57.6 54.9 55.1
boundary interior yload TRf3 range 58.65 58.85 54.9 55.1
boundary interior yload TRf4 range 59.9 60.1 54.9 55.1
boundary interior yload TRf5 range 61.15 61.35 54.9 55.1
boundary interior yload TRf6 range 62.4 62.6 54.9 55.1
boundary interior yload TRf7 range 63.65 63.85 54.9 55.1
boundary interior yload BRf1 range 56.15 56.35 49.9 50.1
boundary interior yload BRf2 range 57.4 57.6 49.9 50.1
boundary interior yload BRf3 range 58.65 58.85 49.9 50.1
boundary interior yload BRf4 range 59.9 60.1 49.9 50.1
boundary interior yload BRf5 range 61.15 61.35 49.9 50.1
boundary interior yload BRf6 range 62.4 62.6 49.9 50.1
boundary interior yload BRf7 range 63.65 63.85 49.9 50.1
boundary interior xload R2L_f1 range 54.9 55.1 51.15 51.35
boundary interior xload R2L_f2 range 54.9 55.1 52.4 52.6
boundary interior xload R2L_f3 range 54.9 55.1 53.65 53.85
boundary interior xload R2R_f1 range 64.9 65.1 51.15 51.35
boundary interior xload R2R_f2 range 64.9 65.1 52.4 52.6
boundary interior xload R2R_f3 range 64.9 65.1 53.65 53.85
print n
solve
endcommand
dendloop
end

; hist _fact ; hist 24
;
hist TLf1 ; hist 25
hist TLf2 ; hist 26
hist TLf3 ; hist 27
hist TLf4 ; hist 28
hist TLf5 ; hist 29
hist TLf6 ; hist 30
hist TLf7 ; hist 31
;
hist BLf1 ; hist 32
hist BLf2 ; hist 33
hist BLf3 ; hist 34
hist BLf4 ; hist 35
hist BLf5 ; hist 36
hist BLf6 ; hist 37
hist BLf7 ; hist 38
;
hist R1L_f1 ; hist 39
hist R1L_f2 ; hist 40
hist R1L_f3 ; hist 41
;
hist R1R_f1 ; hist 42
hist R1R_f2 ; hist 43
hist R1R_f3 ; hist 44

; ;
Let_me_down ;
;
;

save 02_failed_pillar.sav
;

pl hist 21 vs 22

set pl png size 960 720

copy Stress_strain_regular_pillar.png
;

;;;;;; END OF FILE
A.3 Direct shear test on a continuously yielding joint with pressure boundary load

;; Data file to model simple direct shear test with only two blocks, where
;; upper block is moving
;;
;; Continuously-Yielding joint model implemented.
;
new

title 'CY Joint test - pressure boundary loading'
config
round 0.001
edge 0.002
set ovtol 0.002
block 0.0 0.2 4.2 4.0
crack 0.0 1.0 4.0 1.0
crack 0.5 1.0 0.5 2.0
crack 1.5 1.0 1.5 2.0
crack 1.53 1.0 1.53 2.0
;
delete 0.0 0.5 1.0 2.0
delete 1.5 1.53 1.0 2.0
;
gen quad 4.1
join_contact range 1.51 4.01 0.9 1.1
;
def mat_props
;;
_d1 = 2000.0
_E1 = 10.0e9
_v1 = 0.2
_K1 = _E1 / (3.0 * (1.0 - 2.0 * _v1))
_G1 = _E1 / (2.0 * (1.0 + _v1))
;;
_d2 = 1000.0
_E2 = 1000.0e9
_v2 = 0.2
_K2 = _E2 / (3.0 * (1.0 - 2.0 * _v2))
_G2 = _E2 / (2.0 * (1.0 + _v2))
end
mat_props
;
zone model elas dens _d2 bulk _K2 shear _G2
zone model elas dens _d1 bulk _K1 shear _G1 range 0.5 1.5 1.0 2.0
;
joint model cy _jkn 20.0e9 _jks 10.0e9 _jfric 40.0 _jifric 15.0 _jr 0.0001
set jcondf = 2
set jmatdf = 1
prop jmat=1 _jkn 40.0e9 _jks 20.0e9 _jfric 15.0
;;
;; Boundary loading conditions
bound stress (0.0,-2.0E6) range 0.4 1.51 1.9 2.1 ; normal stress
bound xload 5.0e5 range 0.4 0.6 1.9 2.1 ; X-load on block
;
;; Fix-type Boundary conditions
bound yvel=0 range -0.1 4.1 -0.1 0.1 ;bottom
bound xvel=0 range -0.1 4.1 -0.1 0.1 ;bottom
bound xvel=0 range -0.1 0.1 -0.1 2.1 ;left
bound xvel=0 range 3.9 4.1 -0.1 2.1 ;right
;
;
damp local
mscale off
;
;; Bring to equilibrium with normal stress applied to top of block
;; and 5.0e5 horizontal load on sliding block to reduce run time later
solve
;
;; lock in vertical stress on sliding block, allow no tipping or rotation
bound yvel=0 range 0.4 1.51 1.9 2.1 ;fix top
;
reset disp jdisp
hist n 200
set energy on
hist energy ;; hist 1 - 20
;
;; shear stress histories along bottom contacts
history sstress 0.5,1.0 ;; hist 21
history sstress 1.5,1.0 ;; hist 22
;
history sdis 0.5,1.0 ;; hist 23
history sdis 1.5,1.0 ;; hist 24
;
;
def _extract
c1 = c_near(0.5,1.0)
c2 = c_near(1.5,1.0)
sf1 = c_sforce(c1)
sf2 = c_sforce(c2)
nf1 = c_nforce(c1)
nf2 = c_nforce(c2)
sd1 = c_sdis(c1)
sd2 = c_sdis(c2)
cL1 = c_length(c1)
cL2 = c_length(c2)
calc_ss1 = sf1 * (-1.0) / cL1
calc_ss2 = sf2 * (-1.0) / cL2
end
;
set fishcall 0 _extract
;
125
; hist calc_ss1          ;; hist 25
hist calc_ss2         ;; hist 26

hist sf1              ;; hist 27
hist sf2              ;; hist 28

hist sd1             ;; hist 29
hist sd2             ;; hist 30

hist cL1             ;; hist 31
hist cL2             ;; hist 32

hist nf1             ;; hist 33
hist nf2             ;; hist 34

;; normal stress history at contacts
history nstress 0.5,1.0  ;; hist 35
history nstress 1.5,1.0  ;; hist 36

; hist xdisp 0.5 2.0    ;; hist 37
hist xdisp 0.5 1.0    ;; hist 38

; def _some_zones
  _z1 = z_near(0.75,1.5)
  _z2 = z_near(1.0,1.75)
  _z3 = z_near(1.25,1.5)
  _z4 = z_near(1.0,1.25)
end

_def some_zones

; def _get_shears
  _z1sh = z_sxy(_z1)
  _z2sh = z_sxy(_z2)
  _z3sh = z_sxy(_z3)
  _z4sh = z_sxy(_z4)
  _avg_shr = (_z1sh + _z2sh + _z3sh + _z4sh) / 4.0
  _avg_sdis = (sd1 + sd2) / 2.0
end

set fishcall 0 _get_shears

; hist _z1sh               ;; hist 39
hist _z2sh               ;; hist 40
hist _z3sh               ;; hist 41
hist _z4sh               ;; hist 42
hist _avg_shr            ;; hist 43
hist _avg_sdis           ;; hist 44

; def Push_Me
  loop n (251,800)        ;; 5.0e5 N horizontal load already exists at grid point
;
_xforce = n * 2000.0  ;; apply additional 2000 N per iteration
command
bound xfree range 0.4 0.6  1.9 2.1
bound xload _xforce range 0.4 0.6  1.9 2.1
;
print n
solve
endcommand
if sd2 > 0.025    ;; stop loading if displacement already exceeds 2.5 cm
exit
endif
endloop
end
;
hist _xforce   ;; hist 45
;;
Push_Me
;
solve         ;; to make sure system is in equilibrium after sliding has occurred
;
hist write 25 vs 23 Cont1_shear_stress_vs_Disp.txt
hist write 26 vs 24 Cont2_shear_stress_vs_Disp.txt
;
set plot png size 960 720
pl bl iw zon iw disp
copy Block.png
;
pl hist 1
copy Hist_01.png
pl hist 11
copy Hist_11.png
;
save CY_pressure_bound.sav
;
;;;; END OF FILE
A.4  Direct shear test on a continuously yielding joint with velocity boundary load

;; Data file to model simple direct shear test with only two blocks, where
;; upper block is moving
;;
;; This model created in attempt to reduce number of contacts to exactly two
;; so that joint friction work and other terms could be easily calculated
;; in Excel.
;;
;; Continuously-Yielding joint model implemented.
;;
new
title 'CY Joint test - Velocity load - 25Mpa Normal'
config
round 0.001
dx 0.002
set ovtol 0.02
block 0,0 0.3 0.3,0.3 0.3,0
crack 0.0,0.2 0.3,0.2
set 0.0,0.1 0.3,0.1
set 0.05,0.2 0.05,0.1
set 0.25,0.2 0.25,0.1
delete 0.0 0.05 0.1 0.2
delete 0.25 0.3 0.1 0.2
delete 0.0 0.3 0.2 0.3
set -0.0 0.15 -0.3 0.15
set 0.0 0.3 0.15 0.2
;
crack gen quad 0.4
;
change cons 1
prop mat=1 d=2500 k=1111.1E9 g=833.3E9 :E=2000 GPa nu=0.2
change mat=1
;
change jcons 3
; set jcondf=cy
prop jmat=1 jks 50.0e9 jkn 50.0e9 jfric 15.0 jif 40.0 jr 0.00015
change jmat=1
set jcondf 3
;
; apply boundary conditions
bound stress (0,0,-25.0E6) range 0.0 0.3 0.145 0.155 ;top
insitu stress 0.0 0.0 -25.0e6
;
bound yvel=0 range 0.0 0.3 -0.0001 0.0001 ;bottom
bound xvel=0 range 0.0 0.3 -0.0001 0.0001 ;bottom
bound xvel=0 range -0.0001 0.0001 0.0 0.1 ;left
bound xvel=0 range 0.2999 0.3001 0.0 0.1 ;right
;
damp local
mscale off
set small on
;
solve
;
reset disp jdisp
hist n 800
set energy on
hist energy ;; hist 1 - 20
;
;; shear stress histories along bottom contacts
history ststress 0.05,0.10 ;; hist 21
history ststress 0.25,0.10 ;; hist 22
;; shear displacement histories along bottom contacts
history sdis 0.05,0.10 ;; hist 23
history sdis 0.25,0.10 ;; hist 24
;
;; apply shear load by imposing x-velocity on top block
bou xvel=0.002 range 0.049,0.051 0.09,0.21
;
;
def _extract
  c1 = c_near(0.05,0.1)
c2 = c_near(0.25,0.1)
sf1 = c_sforce(c1)
sf2 = c_sforce(c2)
nf1 = c_nforce(c1)
nf2 = c_nforce(c2)
sd1 = c_sdis(c1)
sd2 = c_sdis(c2)
cL1 = c_length(c1)
cL2 = c_length(c2)
calc_ss1 = sf1 / cL1
calc_ss2 = sf2 / cL2
end
_extract
;
set fishcall 0 _extract
;
hist calc_ss1 ;; hist 25
hist calc_ss2 ;; hist 26
;
hist sf1 ;; hist 27
hist sf2 ;; hist 28
;
hist sd1 ;; hist 29
hist sd2 ;; hist 30
;
hist cL1 ;; hist 31
hist cL2 ;; hist 32
hist nf1
hist nf2

;; normal stress history at bottom left contacts
history nstress 0.05,0.10 ;; hist 35
history nstress 0.25,0.10 ;; hist 36

step 850000
save CY_one.sav

set plot png size 960 720

;; block shape, zoning, resultant displacements
plot block zone disp
copy 01_block.png

;; shear stress vs shear displacement
plot hist 21 yrev vs 23
copy 02_Shear_stress_vs_strain_contact1.png

;;;; END OF FILE
A.5  Parameters used in Crandall Canyon simulations

def _parameters ;define the input parameters

_DensityCoal = 1313.0 ; Density of coal
_DensityRock = 2350.0 ; Density of roof and floor
_ECoal = 3.0E9 ; Elastic modulus of coal
_ERock = 23.4E9 ; Elastic modulus of roof and floor
_vCoal = 0.2 ; Poisson’s ratio of coal
_vRock = 0.26 ; Poisson’s ratio of rock
_CohCoal = 1.69E6 ; Cohesion of coal material for MC/MCSS model
_FricCoal = 23.0 ; Friction angle of coal material for MC/MCSS model
_DiCoal = 2.0 ; Dilation angle of coal material for MC/MCSS model
_TenCoal = 0.0 ; Tensile strength of coal material for MC/MCSS model

_KCoal = _ECoal/(3*(1-2*_vCoal)) ; Bulk modulus of coal
_GCoal = _ECoal/(2*(1+_vCoal)) ; Shear modulus of coal
_KRock = _ERock/(3*(1-2*_vRock)) ; Bulk modulus of rock
_GRock = _ERock/(2*(1+_vRock)) ; Shear modulus of rock

_jkn = 50.0E9 ; Joint normal stiffness in Pa/m
_jks = 50.0E9 ; Joint shear stiffness in Pa/m

; Properties for fixed interfaces
_jFricFixed = 90.0 ; Friction angle of fixed interface
_jCohFixed = 1.0E20 ; Cohesion of fixed interface
_jDiFixed = 90.0 ; Dilation angle of fixed interface
_jTenFixed = 1.0E20 ; Tensile strength of fixed interface

; Properties for elastic-perfectly plastic interfaces (Coulomb-slip joint)
_jFricMC = 20.0 ; Friction angle of MC interface
_jCohMC = 0.0 ; Cohesion of MC interface
_jDiMC = 0.0 ; Dilation angle of MC interface
_jTenMC = 0.0 ; Tensile strength of MC interface

; Properties for continuously yielding joint model
_jiFricCY = 40.0 ; Initial friction angle of CY interface
_jFricCY = 15.0 ; Intrinsic friction angle of CY interface
_jr = 1.5E-4 ; Joint roughness of CY interface

; Number of loading steps for pressure boundary loading
_DesiredStress = -30.0E6 ; Desired stress level
_IncreStress = -10.0 ; Increment of stress/step, negative for downward loading
_StepPB = _DesiredStress / _IncreStress ; Number of loading steps

end

_parameters

;;;; END OF FILE
**A.6 Single pillar model, strain softening coal and a continuously yielding interface**

New
call Parameters.dat
config
round 0.005
dean 0.01
set ovto 0.02
set small on
hist ncyc 200
;
title 'Single Pillar, MCss coal - CY interface, small strain'
;
block 0.0,0.0 0.0,24.0 12.4,24.0 12.4,0.0
crack 0.0,14.0 12.4,14.0 ;coal pillar/roof contact
.crack 0.0,11.6 12.4,11.6 ;pillar bottom
.crack 0.0,10.0 12.4,10.0 ;bottom coal/floor rock contact
.crack 9.2,11.6 9.2,14.0 ;pillar edge
delete 9.2,12.4 11.6,14.0
;;;;;;;;;;;;;;
gem quad 0.401 range 0.0,12.4 10.0,14.0 ; Discretize the coal seam
gem quad 1.001 range 0.0,12.4 0.0,10.0 ; Discretize the floor
gem quad 1.001 range 0.0,12.4 14.0,24.0 ; Discretize the roof
;
join_contact range -0.1 12.5 11.5 11.7
;
group zone coal range 0.0,12.4 10.0,14.0
;
table 1 0,1.69E6 0.62E-4,1.54E6 0.8E-4,1.47E6 0.035,2.0E5 1.0,2.0E5 ;cohesion
table 2 0,23.0 0.7E-4,27.5 1.1E-4,30.0 1.0,30.0 ;friction angle
table 3 0,2.0 0.72E-4,10 1.36E-2,10.0 1.41282E-2,2.0 1,2.0 ;dilation
;
zone model elas density _DensityRock bulk _KRock shear _GRock
zone model ss density _DensityCoal bulk _KCoal shear _GCoal friction _FricCoal cohesion _CohCoal
dilation _DiCoal ftable 2 ctable 1 dtable 3 range group coal
;
joint model cy jks _jks jkn _jkn jif _jiFricCY jfric _jFricCY jr _jr
set jcondf = 2
set jmatdf = 1
prop mat 1 jkn 50.0e9 jks 50.0e9
;===============================================
boundary yvel 0.0 range 0.0,12.4 -0.1,0.1 ; Fix the bottom of the model in y direction
boundary xvel 0.0 range -0.1,0.1 0.0,25.0 ; Fix the left boundary of the model in x direction
boundary xvel 0.0 range 12.3,12.5 0.0,25.0 ; Fix the right boundary of the model in x direction
;===============================================
;
;===============================================
;
def _zonecount
    _numz = 0
    _iab = block_head
loop while _iab # 0
    _iz = b_zone(_iab)
loop while _iz # 0
    if z_y(_iz) > 12.4
        if z_y(_iz) < 12.8
            _numz = _numz + 1
            endif
        endif
    _iz = z_next(_iz)
endloop
    _iab = b_next(_iab)
endloop
end
_zonecount

_iz = b_zone(_iab)
loop while _iz # 0
    if z_y(_iz) > 12.4
        if z_y(_iz) < 12.8
            _numz = _numz + 1
            endif
        endif
    _iz = z_next(_iz)
endloop
_end

;; Function to generate an array with 1 column and a number of rows equal to
;; the number of zones counted
def _build_array
    array all_zones(1,_numz)
end
_build_array

;; Function to store the zone ID's of zones at center of pillar for calculation
;; of average stress
def _zone_id_collector
    _tick = 0
    _iab1 = block_head
loop while _iab1 # 0
    _iz1 = b_zone(_iab1)
    loop while _iz1 # 0
        if z_y(_iz1) > 12.4
            if z_y(_iz1) < 12.8
                _tick = _tick + 1
                all_zones(1,_tick) = _iz1
            endif
        endif
        _iz1 = z_next(_iz1)
    endloop
    _iab1 = b_next(_iab1)
endloop
_end

;; Function to build array to store grip points. Only 3 grid points
;; at top and bottom of entry are available to monitor convergence.
def _gp_arrays
    array top_gps(1,24)
    array bot_gps(1,24)
    array surf_gps(1,14)
end

133
gp_arrays
;
;; Function to populate arrays with grid point ID's
def _get_points
    loop m (1,24)
      _xx = m * 0.4 - 0.4
      top_gps(1,m) = gp_near(_xx,14.0)
      bot_gps(1,m) = gp_near(_xx,10.0)
    endloop
    ;
    loop m (1,14)
      _xxs = m * 0.954 - 0.954
      surf_gps(1,m) = gp_near(_xxs,24.0)
    endloop
end
_get_points
;
;; Average stress measurement in the pillar
def Calc_stress
    _sumstress = 0.0
    loop i (1,numz)
      _sumstress = _sumstress + z_syy(all_zones(1,i))
    endloop
    Av_stress = _sumstress * (-1.0) / numz
end
;
def Calc_closure
    _sumtop = 0.0
    _sumbot = 0.0
    loop j (1,24)
      _sumtop = _sumtop + gp_ydis(top_gps(1,j))
      _sumbot = _sumbot + gp_ydis(bot_gps(1,j))
    endloop
    _Closure = ( _sumbot - _sumtop) / 24.0
end
;
def Calc_subsidence
    _sumsurf = 0.0
    loop j (1,14)
      _sumsurf = _sumsurf + gp_ydis(surf_gps(1,j))
    endloop
    _subsidence = _sumsurf / (-14.0)
end
;
set fishcall 0 Calc_stress
set fishcall 0 Calc_closure
set fishcall 0 Calc_subsidence
;
;
insitu stress -4.0e6 0.0 -8.0e6 szz -6.0e6
boundary stress 0.0 0.0 -8.0e6 range -0.1 12.5 23.9 24.1
boundary xvel 0.0 range -0.1,0.1 0.0,25.0 ; Fix the left and right boundary of the model again
boundary xvel 0.0 range 12.3,12.5 0.0,25.0
;
solve
;
;=================================================================================================
;
damp local
mscale off
;
set energy on
hist energy ;; hist 1-20
;
hist Av_stress ;; hist 21
hist _Closure ;; hist 22
;
; Slowly load model to failure... function will stop after 0.5 meters subsidence
;
def _crusher
    desiredDisp = 0.3
    desired_cycles = 100000.0
;
    loop while _subsidence < desiredDisp
        my_timestep = tdel
            my_Yvel = desiredDisp * (-1.0) / ( my_timestep * desired_cycles )
        command
            boundary yvel my_Yvel range -0.1 12.5 23.49 24.1
                print my_timestep
                print my_Yvel
                print Av_stress
                prin _subsidence
        step 1000
    endcommand
endloop
end
;
_crusher
;
save MCSS_CY_small.sav
;
set plot png size 1920 1440
;
plot hist 21 vs 22
copy Stress_strain_graph.png
;
;;;; END OF FILE
A.7  Crandall Canyon abutment loading simulation

;; File for running full-size model with physical abutment wedges attached
;; at south and north boundaries. Elastic material, fixed interfaces.
;;
;; Left wedge is deleted and grid point forces are extracted. Same forces
;; can be used on left and right side of future mine-scale models.
;;
New
title 'Testing physical abutment load'
Set log on
config
round 0.01
edge 0.02
set ovtol 0.02

block -233.77,0.0 -233.77,713.0 626.17,713.0 626.17,0.0

-crack 0.0 0.713
-crack 392.4,0 392.4,713
-crack 0.0,104.0 -233.77,713.0
-crack 392.4,104.0 626.17,713.0

-del -233.77 0.0 0.0 350.0
-del 392.4 626.17 0.0 350.0

-crack 0.0,100.0 393.0,100.0 ;coal seam floor
-crack 0.0,104.0 393.0,104.0 ;coal seam roof

-crack 0.0,64.0 392.4,64.0 ;floor extra
-crack 0.0,84.0 392.4,84.0

-zoning
-crack -233.77,120.0 626.17,120.0 ;roof extra
-zoning
-crack -233.77,144.0 626.17,144.0
-crack -233.77,200.0 626.17,200.0
-crack -233.77,312.0 626.17,312.0
-crack -233.77,408.0 626.17,408.0

-gen quad 2.01 range 0.0,392.4 84.0,100.0 ;floor
-gen quad 4.01 range 0.0,392.4 50.0,84.0 ;floor
-gen quad 8.01 range 0.0,392.4 0.0,64.0 ;floor

-gen quad 2.01 range 0.0 392.4 100.0,104.0 ;coal seam
-gen quad 2.01 range yrange 104.0,120.0 ;roof
-gen quad 4.01 range yrange 120.0,144.0
-gen quad 8.1 range yrange 144.0,200.0 ;roof
-gen quad 16.1 range yrange 200.0,312.0 ;roof
-gen quad 32.1 range yrange 312.0,408.0 ;roof
-gen quad 61.1 range yrange 408.0,713.0 ;roof
gen edge 2.01

; New material properties
call parameters.dat

zone model elas density _DensityCoal bulk _KCoal shear _GCoal range group 'coal'
zone model elas density _DensityRock bulk _KRock shear _GRock range group 'roofandfloor'
joint model area jks 50.0E9 jkn 50.0E9 jfric 60.0 jcoh 10.0e9 jtens 10.0e9

boundary yvel 0.0 range 0.0,392.4 -0.1,0.1     ;bottom of the model
boundary xvel 0.0 range -0.1,0.1 0.0,104.0     ;left boundary of the model
boundary xvel 0.0 range left 0.0,104.1 -233.77,713.1
boundary xvel 0.0 range 392.3,392.5 0.0,104.0  ;right boundary of the model
boundary xvel 0.0 range right 392.4,104.1 626.17,713.1

set gravity 0.0 -9.81
damp auto
;mscale off
set small on
solve
save wedge_geo.sav

; Delete wedge and gather grid point forces
del -240.0 0.0  0.0 715
; fix grid points in horizontal direction
boundary xvel 0.0 range -0.1 0.1 0.0 713.0

; def _scabutl
loop i (0,8)
    gi = 104.0 + i * 2
    g1 = gp_near(0.0,gi)
yfl = gp_yforce(g1)
    command
table 11 gi,yfl
endcommand
endloop

loop i (0,5)
    gi = 124.0 + i * 4
    g1 = gp_near(0.0,gi)
yfl = gp_yforce(g1)
    command
table 11 gi,yfl
endcommand
endloop
;
loop i (0,6)
gi = 152.0 + i * 8
g1 = gp_near(0.0,gi)
yfl = gp_yforce(g1)
command
table 11 gi,yfl
endcommand
endloop
;
loop i (0,6)
gi = 216.0 + i * 16
g1 = gp_near(0.0,gi)
yfl = gp_yforce(g1)
command
table 11 gi,yfl
endcommand
endloop
;
loop i (0,2)
gi = 344.0 + i * 32
g1 = gp_near(0.0,gi)
yfl = gp_yforce(g1)
command
table 11 gi,yfl
endcommand
endloop
;
loop i (0,4)
gi = 469.0 + i * 61
g1 = gp_near(0.0,gi)
yfl = gp_yforce(g1)
command
table 11 gi,yfl
endcommand
endloop
end
;
step 1 ; step one time step to initialize reaction of grid points
set fishcall 0 _scabutl
step 1 ; step one more time step to actually collect forces
;
pl table 11
set pl png size 960 720
copy left_abutment_forces.png
table 11 write 0 Abutment_forces_left.tab
A.8 Crandall Canyon model with strain softening coal and a continuously yielding interface

; File for running full-size Crandall Canyon model with strain-softening coal
; and continuously-yielding coal/rock interface.
;
; Other material and joint constitutive models can be applied
; at Line 188 of this data file.
;
New
title 'Full size Crandall Canyon model - MCSS coal / CY interface'
Set log on
config
round 0.01
edge 0.02
set ovtol 0.02
block 0.0,0.0 0.0,713.0 392.4,713.0 392.4,0.0
; crack 0.0,100.0 393.0,100.0 ;coal seam floor
crack 0.0,104.0 393.0,104.0 ;coal seam roof
;;;; build pillars and entries =====
crack 3.2,100.0 3.2,104.0

; crack 28.0,100.0 28.0,104.0 ; crack for robbing 40ft of south barrier remnant

; crack 40.0,100.0 40.0,104.0

; crack 45.6,100.0 45.6,104.0

; crack 64.4,100.0 64.4,104.0

; crack 70.0,100.0 70.0,104.0

; crack 88.8,100.0 88.8,104.0

; crack 94.4,100.0 94.4,104.0

; crack 113.2,100.0 113.2,104.0

; crack 118.8,100.0 118.8,104.0

; crack 135.6,100.0 135.6,104.0

; crack 142.0,100.0 142.0,104.0

; crack 163.2,100.0 163.2,104.0

; crack 169.6,100.0 169.6,104.0

; crack 190.8,100.0 190.8,104.0

; crack 197.2,100.0 197.2,104.0

; crack 218.4,100.0 218.4,104.0

; crack 224.8,100.0 224.8,104.0

; crack 246.0,100.0 246.0,104.0

; crack 252.4,100.0 252.4,104.0

; crack 268.4,100.0 268.4,104.0
;

; crack 274.8,101.6 274.8,104.0 ; these cracks need not extend to lower interface

; crack 292.8,101.6 292.8,104.0 ;

; crack 299.2,101.6 299.2,104.0 ;

; crack 317.2,101.6 317.2,104.0 ;

; crack 323.6,100.0 323.6,104.0

; crack 341.6,100.0 341.6,104.0

; crack 348.0,100.0 348.0,104.0

; crack 389.2,100.0 389.2,104.0

139
;; bottom coal division
crack 0.0,101.6 3.2,101.6
crack 28.0,101.6 40.0,101.6 ;; crack on floor of SB1 remnant that will be robbed
crack 40.0,101.6 45.6,101.6
crack 45.6,101.6 64.4,101.6 ;; crack for retreating SB1 pillar
crack 64.4,101.6 70.0,101.6
...}

crack 274.8,101.6 292.8,101.6 ;; crack for retreating NB1 pillar

; crack 292.8,101.6 299.2,101.6
...}

crack 389.2,101.6 392.4,101.6 ;; floor extra zoning

crack -233.77,120.0 626.17,120.0 ; roof extra zoning
...}

gen quad 2.01 range 0.0,392.4 84.0,100.0 ; floor
...}

gen quad 0.401 range 0.0 392.4 100.0 104.0 ; coal seam
; gen quad 2.01 range yrange 104.0,120.0 ;roof
gen quad 4.01 range yrange 120.0,144.0
gen quad 8.1 range yrange 144.0,200.0 ;roof
gen quad 16.1 range yrange 200.0,312.0 ;roof
gen quad 32.1 range yrange 312.0,408.0 ;roof
gen quad 61.1 range yrange 408.0,713.0 ;roof
;
group zone 'coal' range 0.0,392.4 100.0,104.0
group zone 'roofandfloor' range 0.0,392.4 0.0,100.0
group zone 'roofandfloor' range 0.0,392.4 104.0,713.0
;
; Material properties
def _parameters ;define the input parameters
    _DensityCoal = 1313.0 ; Density of coal
    _DensityRock = 2350.0 ; Density of roof and floor
    _ECoal = 3.0E9 ; Elastic modulus of coal
    _ERock = 23.4E9 ; Elastic modulus of roof and floor
    _vCoal = 0.2 ; Poisson’s ratio of coal
    _vRock = 0.26 ; Poisson’s ratio of rock
    _CohCoal = 1.69E6 ; Cohesion of coal material for MC/MCSS model
    _FricCoal = 23.0 ; Friction angle of coal material for MC/MCSS model
    _DiCoal = 2.0 ; Dilation angle of coal material for MC/MCSS model
    _TenCoal = 0.0 ; Tensile strength of coal material for MC/MCSS model
    _KCoal = _ECoal/(3*(1-2*_vCoal)) ; Bulk modulus of coal
    _GCoal = _ECoal/(2*(1+_vCoal)) ; Shear modulus of coal
    _KRock = _ERock/(3*(1-2*_vRock)) ; Bulk modulus of rock
    _GRock = _ERock/(2*(1+_vRock)) ; Shear modulus of rock
    _jkn = 50.0E9 ; Joint normal stiffness in Pa/m
    _jks = 50.0E9 ; Joint shear stiffness in Pa/m
;
; Properties for fixed interfaces
    _jFricFixed = 90.0 ; Friction angle of fixed interface
    _jCohFixed = 1.0E20 ; Cohesion of fixed interface
    _jDiFixed = 90.0 ; Dilation angle of fixed interface
    _jTenFixed = 1.0E20 ; Tensile strength of fixed interface
;
; Properties for elastic-perfectly plastic interfaces (Coulomb-slip joint)
    _jFricMC = 20.0 ; Friction angle of MC interface
    _jCohMC = 0.0 ; Cohesion of MC interface
    _jDiMC = 0.0 ; Dilation angle of MC interface
    _jTenMC = 0.0 ; Tensile strength of MC interface
;
; Properties for continuously yielding joint model
    _jiFricCY = 40.0 ; Initial friction angle of CY interface
    _jFricCY = 15.0 ; Intrinsic friction angle of CY interface
    _jr = 1.5E-4 ; Joint roughness of CY interface
;
; Number of loading steps for pressure boundary loading
_DesiredStress = -30.0E6 ; Desired stress level
_IncreStress = -10.0 ; Increment of stress/step, negative for downward loading
_StepPB = _DesiredStress / _IncreStress ; Number of loading steps
;
; Number for skipping histories
_skip = 50.0
;
End
_parameters
;
; elasticity properties for initial equilibrium run
zone model elas density _DensityCoal bulk _KCoal shear _GCoal range group 'coal'
zone model elas density _DensityRock bulk _KRock shear _GRock range group 'roofandfloor'
joint model area jks 50.0E9 jkn 50.0E9 jfric 60.0 jcoh 10.0e9 jtens 10.0e9
;
; Boundary Conditions
boundary yvel 0.0 range 0.0,392.4 -0.1,0.1 ;bottom of the model
boundary xvel 0.0 range -0.1,0.1 0.0,713.0 ;left boundary of the model
boundary xvel 0.0 range 392.3,392.5 0.0,713.0 ;right boundary of the model
;
; damp local
mscale off
set small on
;
set grav 0.0 -9.81
solve
;
save 01_equil.sav
;
zone model ss density _DensityCoal bulk _KCoal shear _GCoal friction _FricCoal &
  cohesion _CohCoal dilation _DiCoal ftable 2.0 ctable 1.0 dtable 3.0 range group 'coal'
joint model cy jks 50.0E9 jkn 50.0E9 jif 40.0 jfric 10.0 jcoh 0.00015 jr 0.00015 range 0.0 392.4 99.9 100.1
joint model cy jks 50.0E9 jkn 50.0E9 jif 40.0 jfric 15.0 jr 0.00015 range 0.0 392.4 103.9 104.1
solve
save 02_equil.sav
;
set energy on
hist energy
;
;
; FISH functions to calculate average stresses and strains on pillars
set echo off
;
def _build_arrays
    array syy_SBs(1,92)
    array syy_SB1(1,47)
    array syy_SB2(1,47)
    array syy_SB3(1,47)
    ;
    array syy_MWs(1,42)
    array syy_MW1(1,53)
    array syy_MW2(1,53)
    array syy_MW3(1,53)
    array syy_MW4(1,53)
    array syy_MWn(1,40)
    ;
    array syy_NB1(1,45)
    array syy_NB2(1,45)
    array syy_NB3(1,45)
    array syy_NBn(1,103)
    ;
    array top_SBs(1,93)
    array top_SB1(1,48)
    array top_SB2(1,48)
    array top_SB3(1,48)
    ;
    array top_MWs(1,43)
    array top_MW1(1,54)
    array top_MW2(1,54)
    array top_MW3(1,54)
    array top_MW4(1,54)
    array top_MWn(1,41)
    ;
    array top_NB1(1,46)
    array top_NB2(1,46)
    array top_NB3(1,46)
    array top_NBn(1,104)
    ;
    ;
    array bot_SBs(1,93)
    array bot_SB1(1,48)
    array bot_SB2(1,48)
    array bot_SB3(1,48)
    ;
    array bot_MWs(1,43)
    array bot_MW1(1,54)
    array bot_MW2(1,54)
    array bot_MW3(1,54)
    array bot_MW4(1,54)
    array bot_MWn(1,41)
; array bot_NB1(1,46)
array bot_NB2(1,46)
array bot_NB3(1,46)
array bot_NBn(1,104)
end
_build_arrays
;
;
def _populate_arrays
loop i (1,92)
    xsbs = 3.0 + (i * 0.4)
    syy_SBs(1,i) = z_near(xsbs,103.8)
endloop
;
loop i (1,47)
    xsb1 = 45.4 + (i * 0.4)
    syy_SB1(1,i) = z_near(xsb1,103.8)
endloop
;
loop i (1,47)
    xsb2 = 69.8 + (i * 0.4)
    syy_SB2(1,i) = z_near(xsb2,103.8)
endloop
;
loop i (1,47)
    xsb3 = 94.2 + (i * 0.4)
    syy_SB3(1,i) = z_near(xsb3,103.8)
endloop
;
loop i (1,42)
    xmws = 118.6 + (i * 0.4)
    syy_mws(1,i) = z_near(xmws,103.8)
endloop
;
loop i (1,53)
    xmw1 = 141.8 + (i * 0.4)
    syy_mw1(1,i) = z_near(xmw1,103.8)
endloop
;
loop i (1,53)
    xmw2 = 169.4 + (i * 0.4)
    syy_mw2(1,i) = z_near(xmw2,103.8)
endloop
;
loop i (1,53)
    xmw3 = 197.0 + (i * 0.4)
    syy_mw3(1,i) = z_near(xmw3,103.8)
endloop
;
loop i (1,53)
xmw4 = 224.6 + (i * 0.4)
    syy_mw4(1,i) = z_near(xmw4,103.8)
endloop
;
loop i (1,40)
    xmwn = 252.2 + (i * 0.4)
    syy_mwn(1,i) = z_near(xmwn,103.8)
endloop
;
loop i (1,45)
    xnb1 = 274.6 + (i * 0.4)
    syy_nb1(1,i) = z_near(xnb1,103.8)
endloop
;
loop i (1,45)
    xnb2 = 299.0 + (i * 0.4)
    syy_nb2(1,i) = z_near(xnb2,103.8)
endloop
;
loop i (1,45)
    xnb3 = 323.4 + (i * 0.4)
    syy_nb3(1,i) = z_near(xnb3,103.8)
endloop
;
loop i (1,103)
    xnbn = 347.8 + (i * 0.4)
    syy_nbn(1,i) = z_near(xnbn,103.8)
endloop
;
loop i (1,93)
    xtsbs = 2.8 + (i * 0.4)
    top_SBs(1,i) = gp_near(xtsbs,104.0)
endloop
;
loop i (1,48)
    xtsb1 = 45.2 + (i * 0.4)
    top_SB1(1,i) = gp_near(xtsb1,104.0)
endloop
;
loop i (1,48)
    xtsb2 = 69.6 + (i * 0.4)
    top_SB2(1,i) = gp_near(xtsb2,104.0)
endloop
;
loop i (1,48)
    xtsb3 = 94.0 + (i * 0.4)
    top_SB3(1,i) = gp_near(xtsb3,104.0)
endloop
;
loop i (1,43)
  xtmws = 118.4 + (i * 0.4)
  top_MWs(1,i) = gp_near(xtmws,104.0)
endloop;

loop i (1,54)
  xtmw1 = 141.6 + (i * 0.4)
  top_MW1(1,i) = gp_near(xtmw1,104.0)
endloop;

loop i (1,54)
  xtmw2 = 169.2 + (i * 0.4)
  top_MW2(1,i) = gp_near(xtmw2,104.0)
endloop;

loop i (1,54)
  xtmw3 = 196.8 + (i * 0.4)
  top_MW3(1,i) = gp_near(xtmw3,104.0)
endloop;

loop i (1,54)
  xtmw4 = 224.4 + (i * 0.4)
  top_MW4(1,i) = gp_near(xtmw4,104.0)
endloop;

loop i (1,41)
  xtmwn = 252.0 + (i * 0.4)
  top_MWn(1,i) = gp_near(xtmwn,104.0)
endloop;

loop i (1,46)
  xtnb1 = 274.4 + (i * 0.4)
  top_NB1(1,i) = gp_near(xtnb1,104.0)
endloop;

loop i (1,46)
  xtnb2 = 298.8 + (i * 0.4)
  top_NB2(1,i) = gp_near(xtnb2,104.0)
endloop;

loop i (1,46)
  xtnb3 = 323.2 + (i * 0.4)
  top_NB3(1,i) = gp_near(xtnb3,104.0)
endloop;

loop i (1,104)
  xtnbn = 347.6 + (i * 0.4)
  top_NBn(1,i) = gp_near(xtnbn,104.0)
endloop;

-----------------------------------------------
; loop i (1,93)
  xbsbs = 2.8 + (i * 0.4)
  bot_SBs(1,i) = gp_near(xbsbs,101.6)
endloop
;
loop i (1,48)
  xbsb1 = 45.2 + (i * 0.4)
  bot_SB1(1,i) = gp_near(xbsb1,101.6)
endloop
;
loop i (1,48)
  xbsb2 = 69.6 + (i * 0.4)
  bot_SB2(1,i) = gp_near(xbsb2,101.6)
endloop
;
loop i (1,48)
  xbsb3 = 94.0 + (i * 0.4)
  bot_SB3(1,i) = gp_near(xbsb3,101.6)
endloop
;
loop i (1,43)
  xbmws = 118.4 + (i * 0.4)
  bot_MWs(1,i) = gp_near(xbmws,101.6)
endloop
;
loop i (1,54)
  xbmw1 = 141.6 + (i * 0.4)
  bot_MW1(1,i) = gp_near(xbmw1,101.6)
endloop
;
loop i (1,54)
  xbmw2 = 169.2 + (i * 0.4)
  bot_MW2(1,i) = gp_near(xbmw2,101.6)
endloop
;
loop i (1,54)
  xbmw3 = 196.8 + (i * 0.4)
  bot_MW3(1,i) = gp_near(xbmw3,101.6)
endloop
;
loop i (1,54)
  xbmw4 = 224.4 + (i * 0.4)
  bot_MW4(1,i) = gp_near(xbmw4,101.6)
endloop
;
loop i (1,41)
  xbmwn = 252.0 + (i * 0.4)
  bot_MWn(1,i) = gp_near(xbmwn,101.6)
endloop
;
loop i (1,46)
    xbnb1 = 274.4 + (i * 0.4)
    bot_NB1(1,i) = gp_near(xbnb1,101.6)
endloop

;.

loop i (1,46)
    xbnb2 = 298.8 + (i * 0.4)
    bot_NB2(1,i) = gp_near(xbnb2,101.6)
endloop

;.

loop i (1,46)
    xbnb3 = 323.2 + (i * 0.4)
    bot_NB3(1,i) = gp_near(xbnb3,101.6)
endloop

;.

loop i (1,104)
    xbnbn = 347.6 + (i * 0.4)
    bot_NBn(1,i) = gp_near(xbnbn,101.6)
endloop
end

_populate_arrays

;.

;=== barrier pillar - south ===
;=== 36.8 m width (3.2 to 40.0 m) ===

_def _record_SBs

    _sumstress = 0.0
    _sumydis_top = 0.0
    _sumydis_bot = 0.0
loop m (1,92)
    _sumstress = _sumstress + z_syy(syy_SBs(1,m))
endloop

loop n (1,93)
    _sumydis_top = _sumydis_top + gp_ydis(top_SBs(1,n))
    _sumydis_bot = _sumydis_bot + gp_ydis(bot_SBs(1,n))
endloop
end

def _calc_SBs
    SBs_SYY = -1.0*_sumstress/92.0
    _ydis = (_sumydis_top - _sumydis_bot)/93.0
    SBs_Strn = -1.0*_ydis/2.4
end

;.

;=== pillar 1 - south ===
;=== 18.8 m width (45.6 to 64.4 m) ===

_def _record_SB1

    _sumstress1 = 0.0
    _sumydis_top1 = 0.0
    _sumydis_bot1 = 0.0
loop m (1,47)
```python
_sumstress1 = _sumstress1 + z_syy(syy_SB1(1,m))
endloop
loop n (1,48)
    _sumydis_top1 = _sumydis_top1 + gp_ydis(top_SB1(1,n))
    _sumydis_bot1 = _sumydis_bot1 + gp_ydis(bot_SB1(1,n))
endloop
def _calc_SB1
    SB1_SYY = -1.0* _sumstress1/47.0
    _ydis1 = (_sumydis_top1 - _sumydis_bot1)/48.0
    SB1_Strn = -1.0* _ydis1/2.4
end

=== pillar 2 - south ===
=== 18.8 m width (70.0 to 88.8 m) ===
def _record_SB2
    _sumstress2 = 0.0
    _sumydis_top2 = 0.0
    _sumydis_bot2 = 0.0
    loop m (1,47)
        _sumstress2 = _sumstress2 + z_syy(syy_SB2(1,m))
    endloop
    loop n (1,48)
        _sumydis_top2 = _sumydis_top2 + gp_ydis(top_SB2(1,n))
        _sumydis_bot2 = _sumydis_bot2 + gp_ydis(bot_SB2(1,n))
    endloop
    end
def _calc_SB2
    SB2_SYY = -1.0* _sumstress2/47.0
    _ydis2 = (_sumydis_top2 - _sumydis_bot2)/48.0
    SB2_Strn = -1.0* _ydis2/2.4
end

=== pillar 3 - south ===
=== 18.8 m width (94.4 to 113.2 m) ===
def _record_SB3
    _sumstress3 = 0.0
    _sumydis_top3 = 0.0
    _sumydis_bot3 = 0.0
    loop m (1,47)
        _sumstress3 = _sumstress3 + z_syy(syy_SB3(1,m))
    endloop
    loop n (1,48)
        _sumydis_top3 = _sumydis_top3 + gp_ydis(top_SB3(1,n))
        _sumydis_bot3 = _sumydis_bot3 + gp_ydis(bot_SB3(1,n))
    endloop
    end
def _calc_SB3
```

SB3_SYY = -1.0*_sumstress3/47.0
_ydis3 = ( _sumydis_top3 - _sumydis_bot3)/48.0
SB3_Strn = -1.0*_ydis3/2.4
end
;
;
=== barrier pillar between south and main west ===
=== 16.8 m width (118.8 to 135.6 m) ===
def _record_MWs
_sumstress4 = 0.0
_sumydis_top4 = 0.0
_sumydis_bot4 = 0.0
loop m (1,42)
_sumstress4 = _sumstress4 + z_syy(syy_MWs(1,m))
endloop
loop n (1,43)
_sumydis_top4 = _sumydis_top4 + gp_ydis(top_MWs(1,n))
_sumydis_bot4 = _sumydis_bot4 + gp_ydis(bot_MWs(1,n))
endloop
end
def _calc_MWs
MWs_SYY = -1.0*_sumstress4/42.0
_ydis4 = ( _sumydis_top4 - _sumydis_bot4)/43.0
MWs_Strn = -1.0*_ydis4/2.4
end
;
;
=== pillar 1 main west ===
=== 21.2 m width (142.0 to 163.2 m) ===
def _record_MW1
_sumstress5 = 0.0
_sumydis_top5 = 0.0
_sumydis_bot5 = 0.0
loop m (1,53)
_sumstress5 = _sumstress5 + z_syy(syy_MW1(1,m))
endloop
loop n (1,54)
_sumydis_top5 = _sumydis_top5 + gp_ydis(top_MW1(1,n))
_sumydis_bot5 = _sumydis_bot5 + gp_ydis(bot_MW1(1,n))
endloop
end
def _calc_MW1
MW1_SYY = -1.0*_sumstress5/53.0
_ydis5 = ( _sumydis_top5 - _sumydis_bot5)/54.0
MW1_Strn = -1.0*_ydis5/2.4
end
;
;
=== pillar 2 main west ===
=== 21.2 m width (169.6 to 190.8 m) ===
def _record_MW2
_sumstress6 = 0.0
_sumydis_top6 = 0.0
_sumydis_bot6 = 0.0
loop m (1,53)
    _sumstress6 = _sumstress6 + z_syy(syy_MW2(1,m))
endloop
loop n (1,54)
    _sumydis_top6 = _sumydis_top6 + gp_ydis(top_MW2(1,n))
    _sumydis_bot6 = _sumydis_bot6 + gp_ydis(bot_MW2(1,n))
endloop
end
def _calc_MW2
    MW2_SYY = -1.0*_sumstress6/53.0
    _ydis6 = (_sumydis_top6 - _sumydis_bot6)/54.0
    MW2_Strn = -1.0*_ydis6/2.4
end

== pillar 3 main west ==
== 21.2 m width (197.2 to 218.4 m) ==
def _record_MW3
    _sumstress7 = 0.0
    _sumydis_top7 = 0.0
    _sumydis_bot7 = 0.0
loop m (1,53)
    _sumstress7 = _sumstress7 + z_syy(syy_MW3(1,m))
endloop
loop n (1,54)
    _sumydis_top7 = _sumydis_top7 + gp_ydis(top_MW3(1,n))
    _sumydis_bot7 = _sumydis_bot7 + gp_ydis(bot_MW3(1,n))
endloop
end
def _calc_MW3
    MW3_SYY = -1.0*_sumstress7/53.0
    _ydis7 =(_sumydis_top7 - _sumydis_bot7)/54.0
    MW3_Strn = -1.0*_ydis7/2.4
end

== pillar 4 main west ==
== 21.2 m width (224.8 to 246.0 m) ==
def _record_MW4
    _sumstress8 = 0.0
    _sumydis_top8 = 0.0
    _sumydis_bot8 = 0.0
loop m (1,53)
    _sumstress8 = _sumstress8 + z_syy(syy_MW4(1,m))
endloop
loop n (1,54)
    _sumydis_top8 = _sumydis_top8 + gp_ydis(top_MW4(1,n))
    _sumydis_bot8 = _sumydis_bot8 + gp_ydis(bot_MW4(1,n))
endloop
end
endloop
end

def _calc_MW4
    MW4_SYY = -1.0*sumstress8/53.0
    _ydis8 =(_sumydis_top8 - _sumydis_bot8)/54.0
    MW4_Strn = -1.0*_ydis8/2.4
end
;
;
;=== barrier pillar between main west and north ===
;=== 16.0 m width (252.4 to 268.4 m) ===
def _record_MWn
    _sumstress9 = 0.0
    _sumydis_top9 = 0.0
    _sumydis_bot9 = 0.0
    loop m (1,40)
        _sumstress9 = _sumstress9 + z_syy(syy_MWn(1,m))
    endloop
end

def _calc_MWn
    MWn_SYY = -1.0*sumstress9/40.0
    _ydis9 =(_sumydis_top9 - _sumydis_bot9)/41.0
    MWn_Strn = -1.0*_ydis9/2.4
end
;
;=== pillar 1 north ===
;=== 18.0 m width (274.8 to 292.8 m) ===
def _record_NB1
    _sumstress10 = 0.0
    _sumydis_top10 = 0.0
    _sumydis_bot10 = 0.0
    loop m (1,45)
        _sumstress10 = _sumstress10 + z_syy(syy_NB1(1,m))
    endloop
end

def _calc_NB1
    NB1_SYY = -1.0*sumstress10/45.0
    _ydis10 =(_sumydis_top10 - _sumydis_bot10)/46.0
    NB1_Strn = -1.0*_ydis10/2.4
end
;
;=== pillar 2 north ===
;=== 18.0 m width (299.2 to 317.2 m) ===
def _record_NB2
    _sumstress11 = 0.0
    _sumydis_top11 = 0.0
    _sumydis_bot11 = 0.0
    loop m (1,45)
        _sumstress11 = _sumstress11 + z_syy(syy_NB2(1,m))
    endloop
    loop n (1,46)
        _sumydis_top11 = _sumydis_top11 + gp_ydis(top_NB2(1,n))
        _sumydis_bot11 = _sumydis_bot11 + gp_ydis(bot_NB2(1,n))
    endloop
end
def _calc_NB2
    NB2_SYY = -1.0* _sumstress11/45.0
    _ydis11 =(_sumydis_top11 - _sumydis_bot11)/46.0
    NB2_Strn = -1.0* _ydis11/2.4
end ;

;== pillar 3 north ===
;== 18.0 m width (323.6 to 341.6 m) ===
def _record_NB3
    _sumstress12 = 0.0
    _sumydis_top12 = 0.0
    _sumydis_bot12 = 0.0
    loop m (1,45)
        _sumstress12 = _sumstress12 + z_syy(syy_NB3(1,m))
    endloop
    loop n (1,46)
        _sumydis_top12 = _sumydis_top12 + gp_ydis(top_NB3(1,n))
        _sumydis_bot12 = _sumydis_bot12 + gp_ydis(bot_NB3(1,n))
    endloop
end

def _calc_NB3
    NB3_SYY = -1.0* _sumstress12/45.0
    _ydis12 =(_sumydis_top12 - _sumydis_bot12)/46.0
    NB3_Strn = -1.0* _ydis12/2.4
end ;

;== barrier pillar in north ===
;== 41.2 m width (348.0 to 389.2 m) ===
def _record_NBn
    _sumstress13 = 0.0
    _sumydis_top13 = 0.0
    _sumydis_bot13 = 0.0
    loop m (1,103)
        _sumstress13 = _sumstress13 + z_syy(syy_NBn(1,m))
    endloop
    loop n (1,104)
        _sumydis_top13 = _sumydis_top13 + gp_ydis(top_NBn(1,n))
        _sumydis_bot13 = _sumydis_bot13 + gp_ydis(bot_NBn(1,n))
    endloop
end
def _calc_NBn
    NBn_SYY = -1.0*_sumstress13/103.0
    _ydis13 =(_sumydis_top13 - _sumydis_bot13)/104.0
    NBn_Strn = -1.0*_ydis13/2.4
end
set echo on
;
;
;---------------------------------------------------------------------------------------------------------------------------------------;
;
set fishcall 0 _record_SBs
set fishcall 0 _record_SB1
set fishcall 0 _record_SB2
set fishcall 0 _record_SB3
set fishcall 0 _record_MWs
set fishcall 0 _record_MW1
set fishcall 0 _record_MW2
set fishcall 0 _record_MW3
set fishcall 0 _record_MW4
set fishcall 0 _record_MWn
set fishcall 0 _record_NB1
set fishcall 0 _record_NB2
set fishcall 0 _record_NB3
set fishcall 0 _record_NBn
;
set fishcall 0 _calc_SBs
set fishcall 0 _calc_SB1
set fishcall 0 _calc_SB2
set fishcall 0 _calc_SB3
set fishcall 0 _calc_MWs
set fishcall 0 _calc_MW1
set fishcall 0 _calc_MW2
set fishcall 0 _calc_MW3
set fishcall 0 _calc_MW4
set fishcall 0 _calc_MWn
set fishcall 0 _calc_NB1
set fishcall 0 _calc_NB2
set fishcall 0 _calc_NB3
set fishcall 0 _calc_NBn
;
hist n=200
hist SBs_SYY ;history 21
hist SBs_Strn ;history 22
hist SB1_SYY ;history 23
hist SB1_Strn ;history 24
hist SB2_SYY ;history 25
hist SB2_Strn ;history 26
hist SB3_SYY ;history 27
hist SB3_Strn ;history 28
def _MineMW1 ;; Main West #1 Entry
  xS1 = 135.6
  xF1 = 142.0
  xn1 = round((xF1 - xS1) / 0.4)
  loop i (1,xn1)
    xH1 = xS1 + (i * 0.4)
    SRx = xH1 - 0.2
    command
    print xH1
    del xS1 xH1 101.6 104.0
    solve relax SRx 103.0 nsteps 100
  endcommand
end

_save 03_MW1_mined.sav

def _MineMW2 ;; Main West #2 Entry
  xS1 = 163.2
  xF1 = 169.6
  xn1 = round((xF1 - xS1) / 0.4)
  loop i (1,xn1)
    xH1 = xS1 + (i * 0.4)
    SRx = xH1 - 0.2
    command
print xH1
def _MineMW3 ;; Main West #3 Entry
xS1 = 190.8
xF1 = 197.2
xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
  xH1 = xS1 + (i * 0.4)
  SRx = xH1 - 0.2
  command
  print xH1
  del xS1 xH1 101.6 104.0
  solve relax SRx 103.0 nsteps 100
  endcommand
endloop
end
_def _MineMW3
save 03_MW3_mined.sav

;  
def _MineMW4 ;; Main West #4 Entry
xS1 = 218.4
xF1 = 224.8
xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
  xH1 = xS1 + (i * 0.4)
  SRx = xH1 - 0.2
  command
  print xH1
  del xS1 xH1 101.6 104.0
  solve relax SRx 103.0 nsteps 100
  endcommand
endloop
end
_def _MineMW4
save 03_MW4_mined.sav

;  
def _MineMW5 ;; Main West #5 Entry
xS1 = 246.0
xF1 = 252.4
xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
  xH1 = xS1 + (i * 0.4)
  SRx = xH1 - 0.2
  command
print xH1
del xS1 xH1 101.6 104.0
solve relax SRx 103.0 nsteps 100
endcommand
endloop
end
_MineMW5
save 03_MW5_mined.sav
;                        ;; Turn energy OFF during abutment loading
;                        ;; Apply abutment loads to south boundary of model
;                        ; bound yload -5.9410906E+06 range -0.1 0.1 103.9 104.1
;                        ; bound yload -1.6319471E+07 range -0.1 0.1 105.9 106.1
;                        ; bound yload -1.5384637E+07 range -0.1 0.1 107.9 108.1
;                        ; bound yload -1.4701436E+07 range -0.1 0.1 109.9 110.1
;                        ; bound yload -1.4172775E+07 range -0.1 0.1 111.9 112.1
;                        ; bound yload -1.3733595E+07 range -0.1 0.1 113.9 114.1
;                        ; bound yload -1.3292268E+07 range -0.1 0.1 115.9 116.1
;                        ; bound yload -1.2837304E+07 range -0.1 0.1 117.9 118.1
;                        ; bound yload -1.0891607E+07 range -0.1 0.1 119.9 120.1
;                        ; bound yload -2.4876334E+07 range -0.1 0.1 123.9 124.1
;                        ; bound yload -2.4230923E+07 range -0.1 0.1 127.9 128.1
;                        ; bound yload -2.3389783E+07 range -0.1 0.1 131.9 132.1
;                        ; bound yload -2.2591167E+07 range -0.1 0.1 135.9 136.1
;                        ; bound yload -2.1690570E+07 range -0.1 0.1 139.9 140.1
;                        ; bound yload -1.7833068E+07 range -0.1 0.1 143.9 144.1
;                        ; bound yload -4.1914306E+07 range -0.1 0.1 151.9 152.1
;                        ; bound yload -4.0312771E+07 range -0.1 0.1 159.9 160.1
;                        ; bound yload -3.8613239E+07 range -0.1 0.1 167.9 168.1
;                        ; bound yload -3.7073080E+07 range -0.1 0.1 175.9 176.1
;                        ; bound yload -3.5617966E+07 range -0.1 0.1 183.9 184.1
;                        ; bound yload -3.3951129E+07 range -0.1 0.1 191.9 192.1

_xS1 =   0.0
_xF1 =   3.2
_xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
  _xH1 = xS1 + (i * 0.4)
  command
    del xS1 xH1 101.6 104.0
    solve
  endcommand
endloop
end
_MineSBB
save 04_South_bleeder_mined.sav
;                        ; set energy off                        ;
;                        ; def _MineSBB
;                        ; xS1 =   0.0
;                        ; xF1 =   3.2
;                        ;; Turn energy OFF during abutment loading
;                        ;; Apply abutment loads to south boundary of model
;                        ; bound yload -5.9410906E+06 range -0.1 0.1 103.9 104.1
;                        ; bound yload -1.6319471E+07 range -0.1 0.1 105.9 106.1
;                        ; bound yload -1.5384637E+07 range -0.1 0.1 107.9 108.1
;                        ; bound yload -1.4701436E+07 range -0.1 0.1 109.9 110.1
;                        ; bound yload -1.4172775E+07 range -0.1 0.1 111.9 112.1
;                        ; bound yload -1.3733595E+07 range -0.1 0.1 113.9 114.1
;                        ; bound yload -1.3292268E+07 range -0.1 0.1 115.9 116.1
;                        ; bound yload -1.2837304E+07 range -0.1 0.1 117.9 118.1
;                        ; bound yload -1.0891607E+07 range -0.1 0.1 119.9 120.1
;                        ; bound yload -2.4876334E+07 range -0.1 0.1 123.9 124.1
;                        ; bound yload -2.4230923E+07 range -0.1 0.1 127.9 128.1
;                        ; bound yload -2.3389783E+07 range -0.1 0.1 131.9 132.1
;                        ; bound yload -2.2591167E+07 range -0.1 0.1 135.9 136.1
;                        ; bound yload -2.1690570E+07 range -0.1 0.1 139.9 140.1
;                        ; bound yload -1.7833068E+07 range -0.1 0.1 143.9 144.1
;                        ; bound yload -4.1914306E+07 range -0.1 0.1 151.9 152.1
;                        ; bound yload -4.0312771E+07 range -0.1 0.1 159.9 160.1
;                        ; bound yload -3.8613239E+07 range -0.1 0.1 167.9 168.1
;                        ; bound yload -3.7073080E+07 range -0.1 0.1 175.9 176.1
;                        ; bound yload -3.5617966E+07 range -0.1 0.1 183.9 184.1
;                        ; bound yload -3.3951129E+07 range -0.1 0.1 191.9 192.1


bound yload -2.6513226E+07 range -0.1 0.1 199.9 200.1
bound yload -6.4636556E+07 range -0.1 0.1 215.9 216.1
bound yload -6.0878456E+07 range -0.1 0.1 231.9 232.1
bound yload -5.7395239E+07 range -0.1 0.1 247.9 248.1
bound yload -5.4276604E+07 range -0.1 0.1 263.9 264.1
bound yload -5.1392974E+07 range -0.1 0.1 279.9 280.1
bound yload -4.8517353E+07 range -0.1 0.1 295.9 296.1
bound yload -3.9774513E+07 range -0.1 0.1 311.9 312.1
bound yload -9.0678594E+07 range -0.1 0.1 343.9 344.1
bound yload -8.3455741E+07 range -0.1 0.1 369.9 370.1
bound yload -6.7685008E+07 range -0.1 0.1 392.9 393.1
bound yload -1.3903858E+08 range -0.1 0.1 407.9 408.1
bound yload -1.2141245E+08 range -0.1 0.1 468.9 469.1
bound yload -1.0123421E+08 range -0.1 0.1 529.9 530.1
bound yload -7.2478207E+07 range -0.1 0.1 651.9 652.1
bound yload -2.2490606E+07 range -0.1 0.1 712.9 713.1
;
; solve
save 04_South_loaded_abut.sav
;
;
def _MineNBB
  xS1 = 389.2
  xF1 = 392.4
  xn1 = round((xF1 - xS1) / 0.4)
  loop i (1,xn1)
    xH1 = xS1 + (i * 0.4)
    command
      del xS1 xH1 101.6 104.0
    solve
  endcommand
end
_MineNBB
save 05_North_bleeder_mined.sav

;; Apply abutment loads to NORTH boundary of model
bound yload -5.9410906E+06 range 392.3 392.5 103.9 104.1
bound yload -1.6319471E+07 range 392.3 392.5 105.9 106.1
bound yload -1.5384637E+07 range 392.3 392.5 107.9 108.1
bound yload -1.4701436E+07 range 392.3 392.5 109.9 110.1
bound yload -1.4172775E+07 range 392.3 392.5 111.9 112.1
bound yload -1.373595E+07 range 392.3 392.5 113.9 114.1
bound yload -1.3292268E+07 range 392.3 392.5 115.9 116.1
bound yload -1.2837304E+07 range 392.3 392.5 117.9 118.1
bound yload -1.0891607E+07 range 392.3 392.5 119.9 120.1
bound yload -2.4876334E+07 range 392.3 392.5 123.9 124.1
bound yload -2.4230923E+07 range 392.3 392.5 127.9 128.1
bound yload -2.3389783E+07 range 392.3 392.5 131.9 132.1
bound yload -2.2591167E+07 range 392.3 392.5 135.9 136.1
bound yload -2.1690570E+07 range 392.3 392.5 139.9 140.1
bound yload -1.7833068E+07 range 392.3 392.5 143.9 144.1
bound yload -4.1914306E+07 range 392.3 392.5 151.9 152.1
bound yload -4.0312771E+07 range 392.3 392.5 143.9 144.1
bound yload -3.8613239E+07 range 392.3 392.5 167.9 168.1
bound yload -3.7073080E+07 range 392.3 392.5 175.9 176.1
bound yload -3.5617966E+07 range 392.3 392.5 183.9 184.1
bound yload -3.3951129E+07 range 392.3 392.5 191.9 192.1
bound yload -2.6513226E+07 range 392.3 392.5 199.9 200.1
bound yload -6.463656E+07 range 392.3 392.5 215.9 216.1
bound yload -6.0878456E+07 range 392.3 392.5 231.9 232.1
bound yload -5.7395239E+07 range 392.3 392.5 247.9 248.1
bound yload -5.4276604E+07 range 392.3 392.5 263.9 264.1
bound yload -5.1392974E+07 range 392.3 392.5 279.9 280.1
bound yload -4.8517353E+07 range 392.3 392.5 295.9 296.1
bound yload -3.9774513E+07 range 392.3 392.5 311.9 312.1
bound yload -9.0678594E+07 range 392.3 392.5 343.9 344.1
bound yload -8.3455741E+07 range 392.3 392.5 375.9 376.1
bound yload -6.7685008E+07 range 392.3 392.5 407.9 408.1
bound yload -1.3903858E+08 range 392.3 392.5 468.9 469.1
bound yload -1.2141245E+08 range 392.3 392.5 529.9 530.1
bound yload -1.0123421E+08 range 392.3 392.5 590.9 591.1
bound yload -7.2478207E+07 range 392.3 392.5 651.9 652.1
bound yload -2.2490606E+07 range 392.3 392.5 712.9 713.1

solve

save 05_North_loaded_abut.sav

set energy on        ;; Turn energy back ON !!

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

def _MineNB1      ;; North Barrier #1 Entry (farthest south)
xs1 = 268.4
xf1 = 274.8
xn1 = round((xf1 - xs1) / 0.4)
loop i (1,xn1)
    xh1 = xs1 + (i * 0.4)
    srx = xh1 - 0.2
    command
        print xh1
        del xs1 xh1 101.6 104.0
    solve relax SRx 103.0 nsteps 100
endcommand
endloop
_MineNB1
save 06_NB1_mined.sav

def _MineNB2      ;; North Barrier #2 Entry
xs1 = 292.8
xF1 = 299.2
xn1 = round((xF1 - xS1) / 0.4)
loop i (1, xn1)
  xH1 = xS1 + (i * 0.4)
  SRx = xH1 - 0.2
  command
    print xH1
    del xS1 xH1 101.6 104.0
    solve relax SRx 103.0 nsteps 100
  endcommand
endloop
end
_MineNB2
save 06_NB2_mined.sav
;
def _MineNB3 ;; North Barrier #3 Entry
  xS1 = 317.2
  xF1 = 323.6
  xn1 = round((xF1 - xS1) / 0.4)
  loop i (1, xn1)
    xH1 = xS1 + (i * 0.4)
    SRx = xH1 - 0.2
    command
      print xH1
      del xS1 xH1 101.6 104.0
      solve relax SRx 103.0 nsteps 100
    endcommand
  endloop
end
_MineNB3
save 06_NB3_mined.sav
;
def _MineNB4 ;; North Barrier #4 Entry
  xS1 = 341.6
  xF1 = 348.0
  xn1 = round((xF1 - xS1) / 0.4)
  loop i (1, xn1)
    xH1 = xS1 + (i * 0.4)
    SRx = xH1 - 0.2
    command
      print xH1
      del xS1 xH1 101.6 104.0
      solve relax SRx 103.0 nsteps 100
    endcommand
  endloop
end
_MineNB4
save 06_NB4_mined.sav
;
;-------------------------------------
set fishcall 0 remove _record_NB1
set fishcall 0 remove _calc_NB1
def _MineNB1p ;; RETREAT NB1 Pillar!
  xS1 = 274.8
  xF1 = 292.8
  xn1 = round((xF1 - xS1) / 0.4)
  loop i (1,xn1)
    xH1 = xS1 + (i * 0.4)
    SRx = xH1 - 0.2
    command
      print xH1
      del xS1 xH1 101.6 104.0
    solve relax SRx 103.0 nsteps 100
  endcommand
endloop
end _MineNB1p
save 07_NB1_Robbed.sav
;
set fishcall 0 remove _record_NB2
set fishcall 0 remove _calc_NB2
def _MineNB2p ;; RETREAT NB2 Pillar!
  xS1 = 299.2
  xF1 = 317.2
  xn1 = round((xF1 - xS1) / 0.4)
  loop i (1,xn1)
    xH1 = xS1 + (i * 0.4)
    SRx = xH1 - 0.2
    command
      print xH1
      del xS1 xH1 101.6 104.0
    solve relax SRx 103.0 nsteps 100
  endcommand
endloop
end _MineNB2p
save 07_NB2_Robbed.sav
;
;-------------------------------------
;
def _MineSB1 ;; South Barrier #1 Entry (farthest south)
  xS1 = 40.0
  xF1 = 45.6
  xn1 = round((xF1 - xS1) / 0.4)
  loop i (1,xn1)
    xH1 = xS1 + (i * 0.4)
    SRx = xH1 - 0.2
    command
      print xH1
      del xS1 xH1 101.6 104.0
    solve relax SRx 103.0 nsteps 100
endloop
end _MineSB1

endcommand
endloop
end
_MineSB1
save 08_SB1_mined.sav
;
def _MineSB2          ;; South Barrier #2 Entry
xS1 = 64.4
xF1 = 70.0
xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
  xH1 = xS1 + (i * 0.4)
  SRx = xH1 - 0.2
  command
  print xH1
  del xS1 xH1 101.6 104.0
  solve relax SRx 103.0 nsteps 100
endcommand
endloop
end
_MineSB2
save 08_SB2_mined.sav
;
def _MineSB3          ;; South Barrier #3 Entry
xS1 = 88.8
xF1 = 94.4
xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
  xH1 = xS1 + (i * 0.4)
  SRx = xH1 - 0.2
  command
  print xH1
  del xS1 xH1 101.6 104.0
  solve relax SRx 103.0 nsteps 100
endcommand
endloop
end
_MineSB3
save 08_SB3_mined.sav
;
def _MineSB4          ;; South Barrier #4 Entry
xS1 = 113.2
xF1 = 118.8
xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
  xH1 = xS1 + (i * 0.4)
  SRx = xH1 - 0.2
  command
  print xH1
  del xS1 xH1 101.6 104.0
  solve relax SRx 103.0 nsteps 100
endcommand
endloop
end
_MineSB4
save 08_SB4_mined.sav
;
;-------------------------------------
;
set fishcall 0 remove _record_SB1
set fishcall 0 remove _calc_SB1
def _MineSB1p          ;; RETREAT SB1 Pillar !
xS1 =  45.6
xF1 =  64.4
xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
xH1 = xS1 + (i * 0.4)
SRx = xH1 - 0.2
command
print xH1
del xS1 xH1 101.6 104.0
solve relax SRx 103.0 nsteps 100
endcommand
endloop
end
_MineSB1p
save 09_SB1_Robbed.sav
;
delete 40.0,45.6 100.0,101.6    ;; SB #1 Entry Floor mined
solve relax 43.0 101.0 nsteps 200
save 09_SB1_Robbed_entry_floor.sav
;
delete 45.6,64.4 100.0,101.6    ;; SB1 Pillar Floor mined
solve relax 51.0 101.0 nsteps 200
save 09_SB1_Robbed_pillar_floor.sav
;
set fishcall 0 remove _record_SB2
set fishcall 0 remove _calc_SB2
def _MineSB2p          ;; RETREAT SB2 Pillar !
xS1 =  70.0
xF1 =  88.8
xn1 = round((xF1 - xS1) / 0.4)
loop i (1,xn1)
xH1 = xS1 + (i * 0.4)
SRx = xH1 - 0.2
command
print xH1
del xS1 xH1 101.6 104.0
solve relax SRx 103.0 nsteps 100
endcommand
endloop
end
end
end
_MineSB2p
save 09_SB2_Robbed.sav
;
delete 64.4,70.0 100.0,101.6 ;; SB #2 Entry Floor mined
solve relax 67.0 101.0 nsteps 200
save 09_SB2_Robbed_entry_floor.sav
;
delete 70.0,88.8 100.0,101.6 ;; SB2 Pillar Floor mined
solve relax 76.0 101.0 nsteps 200
save 09_SB2_Robbed_pillar_floor.sav
;
set fishcall 0 remove _record_SBs
set fishcall 0 remove _calc_SBs
def _MineSBsp ;; take chunk of SB Remnant Barrier pillar
    xS1 = 28.0
    xF1 = 40.0
    xn1 = round((xF1 - xS1) / 0.4)
    loop i (1,xn1)
        xL1 = xF1 - (i * 0.4)
        SRx = xL1 + 0.2
        command
            print xL1
del xL1 xF1 100.0 104.0
        solve relax SRx 103.0 nsteps 100
        endcommand
    endloop
end
_MineSBsp
save 10_SB_Remnant_Robbed.sav
;
Set plot png size 960 720
plot hist 1
   copy Hist_01_Uk.png
;
plot hist 11
   copy Hist_11_Wk.png
;
plot hist 17
   copy Hist_17_Wr_kinetic.png
;
plot hist 1 8 11 12 17
   copy Energy_Balance_terms_kinetic.png
;
;;;; END OF FILE
A.9 Triaxial test of trona specimen

New title 'Triax test of Trona, 0.5 MPa confinement'
config round 0.005
draw edge 0.01
set ovttol 0.02
set small on
hist ncy 100
;
def _extents
  _rite = 2.5
  _rl = _rite - 0.1
  _rh = _rite + 0.1
  _left = 0.0
  _ll = _left - 0.1
  _lh = _left + 0.1
  _top = _rite * 2.0
  _tl = _top - 0.1
  _th = _top + 0.1
  _bot = 0.0
  _bl = _bot - 0.1
  _bh = _bot + 0.1
end _extents
;
block 0.0,0.0 0.0,_top _rite,_top _rite,0.0
;
---
gena quad 0.251 ; Discretize
;
def mat_props
  ;; Trona
  _d6 = 2150.0
  _E6 = 18.0e9
  _v6 = 0.2
  _K6 = _E6 / (3.0 * (1.0 - 2.0 * _v6))
  _G6 = _E6 / (2.0 * (1.0 + _v6))
  _f6 = 36.3
  _c6 = 4.25e6
  _c6a = 3.8e6
  _c6b = 3.3e6
  _c6c = 2.125e6
  _c6d = 0.85e6
  _t6 = 0.65e6
  _di6 = 9.0
end mat_props
;
;; Trona softening parameters
table 23 0.0,_c6 0.00015_c6 0.00075_c6a 0.0015,_c6b 0.003,_c6c 0.0225,_c6d 1.0,_c6d

165
; zone model ss dens _d6 bulk _K6 shear _G6 fric _f6 coh _c6 tens _t6 dil _di6 ctable 23
; ; Fix the bottom of the model in y direction
boundary yvel 0.0 range -0.1 _rh -0.1 0.1
;
; def _zonecount
  _numz = 0
  _iab = block_head
  loop while _iab # 0
    _iz = b_zone(_iab)
    loop while _iz # 0
      if z_y(_iz) > 2.0
        if z_y(_iz) < 2.5
          _numz = _numz + 1
        endif
      endif
      _iz = z_next(_iz)
    endloop
    _iab = b_next(_iab)
  endloop
end
_zonecount
;
; ; Function to generate an array with 1 column and a number of rows equal to
; ; the number of zones counted
def _build_array
  array all_zones(1,_numz)
end
_build_array
;
; ; Function to store the zone ID's of zones at center of pillar for calculation
; ; of average stress
def _zone_id_collector
  _tick = 0
  _iab1 = block_head
  loop while _iab1 # 0
    _iz1 = b_zone(_iab1)
    loop while _iz1 # 0
      if z_y(_iz1) > 2.0
        if z_y(_iz1) < 2.5
          _tick = _tick + 1
          all_zones(1,_tick) = _iz1
        endif
      endif
      _iz1 = z_next(_iz1)
    endloop
    _iab1 = b_next(_iab1)
  endloop
end
_zone_id_collector
; Function to build array to store grip points. Only 3 grid points
; at top and bottom of entry are available to monitor convergence.
def _gp_arrays
    array top_gps(1,6)
    array bot_gps(1,6)
end

; Function to populate arrays with grid point ID's
def _get_points
    loop m (1,6)
        _xx = m * 0.5 - 0.5
        top_gps(1,m) = gp_near(_xx,_top)
        bot_gps(1,m) = gp_near(_xx,0.0)
    endloop
end

; Average stress measurement in the pillar
def Calc_stress
    _sumstress = 0.0
    loop i (1, numz)
        _sumstress = _sumstress + z_syy(all_zones(1,i))
    endloop
    Av_stress = _sumstress * (-1.0) / numz
end

; def Calc_closure
; _sumtop = 0.0
; _sumbot = 0.0
; loop j (1, 6)
;     _sumtop = _sumtop + gp_ydis(top_gps(1,j))
;     _sumbot = _sumbot + gp_ydis(bot_gps(1,j))
; endloop
; _strain = (_sumbot - _sumtop) / 6.0 / 5.0
end

; set fishcall 0 Calc_scstress
; set fishcall 0 Calc_closure
;
;=====================================================================
; achieve 0.5 MPa vertical stress
; BOUNDARY yvel -0.005 range _ll _rh _tl _th
; step 2000
; boundary yvel  0.0 range _ll _rh _tl _th
;
;; add confinement
boundary stress -0.5e6 0.0 0.0 range _rl _rh _bl _th
boundary stress -0.5e6 0.0 0.0 range _ll _lh _bl _th
solve
;
; fix bottom boundary again
boundary yvel 0.0 range -0.1 _rh -0.1 0.1 ;; y direction
boundary xvel 0.0 range -0.1 _rh -0.1 0.1 ;; x direction
;
; damp local
mscale off
;
set energy on
hist energy
hist Av_stress ;history No.21
hist _strain ;history No.22
;
;
BOUNDARY yvel -0.005 range _ll _rh _tl _th
step 120000
;
save Tri_Trona_0p5_conf.sav
pl hist 21 vs 22
set pl png size 960 720
copy Tri_Trona_0p5_conf.png
;
;;;; END OF FILE
A.10  Single pillar model of Solvay Mine

;; Single pillar model of ovaloid shape in Solvay Mine.
;; Material properties calibrated from MSHA lab data at Solvay

;; Velocity loading maintained through 0.6m surface displacement to find peak strength,
;; stress drop, and re-strengthening behavior of pillars

;; Model is set up with 0.0 meters of trona in the floor of the excavation.
;; Change values on lines 44 and 45 of this data file to increase floor trona.

;; Model is set up with a 2.0 meter thick oil shale unit below the trona.
;; Change the value on line 59 of this data file to increase of decrease thickness

New

title 'Ovaloid pillars, 2m shale thickness, 00cm floor trona, velocity load'

config
round 0.01
edge 0.02
set ovvtol 0.02

hist ncycle 500

def _extent ;; coordinates of model extents and excavation boundaries

_left = 4.125
_ll  = _left - 0.1
_lh  = _left + 0.1
_rite = 20.625
_rtl = _rite - 0.1
_rh  = _rite + 0.1

_bot = -20.0
_bl  = _bot - 0.1
_bh  = _bot + 0.1
_top = 108.5
_tl  = _top - 0.8
_th  = _top + 0.1

_seamlo = 0.0    ; contact BELOW Trona
_seamlo_plus = _seamlo + 0.1  ; for joining contacts
_seamhi = 3.5    ;; contact ABOVE Trona
_seamplus = _seamhi + 1.0
_seamhi_less = _seamhi - 0.1

_roof = 2.5    ;; excavation roof  **** change to 3.0 to model 0.5m floor trona
_flor  = 0.0    ;; excavation floor  **** change to 0.5 to model 0.5m floor trona

_mid = (_roof + _flor) / 2.0
_midlo = _mid - 0.25
_midhi = _mid + 0.25
_height = _roof - _flor
_radius = _height / 2.0
_toerange = _radius - 0.3

_flr_lim = _flor + 0.01
_roof_lim = _roof - 0.01

_conwilk = 33.5 ;; contact BELOW Wilkins shale (_top - 75m thickness)
_consand = 23.5 ;; contact BELOW D-Sandstone
_zonroof2 = _roof + 5.0 ;; contact ABOVE roof shale (above trona)
_conoil = -2.0 ;; contact BELOW Oil Shale *** change to -1.0 to model thin shale
_conmud = -7.0 ;; contact BELOW Mudstone

_rib01R = 6.0
_rib02L = 10.5
_rib02R = 14.25
_rib03L = 18.75

_cenr01R = _rib01R + _radius
_cenr02L = _rib02L - _radius
_cenr02R = _rib02R + _radius
_cenr03L = _rib03L - _radius

_toe01R = _rib01R + _toerange
_toe02L = _rib02L - _toerange
_toe02R = _rib02R + _toerange
_toe03L = _rib03L - _toerange

end

extent

block _left,_bot _left,_top _rite,_top _rite,_bot

crack _left,_conwilk _rite,_conwilk ;; contact below Wilkins shale

crack _left,_consand _rite,_consand ;; contact below D Sandstone

crack _left,_seamhi _rite,_seamhi ;; contact below roof shale (above trona)

crack _left,_seamlo _rite,_seamlo ;; contact below trona

crack _left,_conoil _rite,_conoil ;; contact below Oil shale

crack _left,_conmud _rite,_conmud ;; contact below mudstone


crack _left,_zonroof2 _rite,_zonroof2 ;; contact above trona for zoning


crack _left _flor _rite _flor ;; crack below entry

crack _left _roof _rite _roof ;; crack above entry


crack _rib01R _flor _rib01R _roof

crack _rib02L _flor _rib02L _roof

crack _rib02R _flor _rib02R _roof

crack _rib03L _flor _rib03L _roof
crack _cenr01R _flor _cenr01R _roof
crack _cenr02L _flor _cenr02L _roof
crack _cenr02R _flor _cenr02R _roof
crack _cenr03L _flor _cenr03L _roof

arc _cenr01R _mid _cenr01R _roof 180.0 4
arc _cenr02L _mid _cenr02L _flor 180.0 4
arc _cenr02R _mid _cenr02R _roof 180.0 4
arc _cenr03L _mid _cenr03L _flor 180.0 4

delete range _rib01R _cenr01R _flor _roof annulus _cenr01R _mid 0.0 _radius
delete range _cenr01R _cenr02L _flor _roof
delete range _cenr02L _rib02L _flor _roof annulus _cenr02L _mid 0.0 _radius

delete range _rib02R _cenr02R _flor _roof annulus _cenr02R _mid 0.0 _radius
delete range _cenr02R _cenr03L _flor _roof
delete range _cenr03L _rib03L _flor _roof annulus _cenr03L _mid 0.0 _radius

; gen quad 7.501 range _left _rite _conwilk _top ;; zoning for Wilkins
gen quad 2.501 range _left _rite _consand _conwilk ;; zoning for D sandstone
gen quad 2.501 range _left _rite _zonroof2 _consand ;; zoning for upper portion of roof shale
gen quad 0.501 range _left _rite _seamhi _zonroof2 ;; zoning for lower portion of roof shale

gen quad 0.251 range _left _rite _conmud _seamhi ;; zoning for trona, oil shale, and mudstone

gen quad 1.001 range _left _rite _bot _conmud ;; zoning for shale bottom

gen edge 0.501 ;; zoning for all pieces left over

join_contact range _ll _rh _flor _roof angle 2.0 178.0 ;; Join all contacts that are not geologic
join_contact range _ll _rh 2.4 2.6

;======================================================================

; list of groups starting at bottom of model

group zone Wilkins range yrange _conwilk _top

group zone Dsands range yrange _consand _conwilk

group zone Roofshale range yrange _seamhi _consand

group zone Trona range yrange _seamlo _seamhi

group zone Oilshale range yrange _conoil _seamlo

group zone Mudstone range yrange _conmud _conoil

group zone Floorshale range yrange _bot _conmud

def mat_props

;; RoofShale
_d3 = 2500.0
_E3 = 5.5e9
_v3 = 0.2
_K3 = _E3 / (3.0 * (1.0 - 2.0 * _v1))
\_G3 = \_E3 / (2.0 * (1.0 + \_v1))
\_f3 = 27.0
\_c3 = 1.8e6
\_t3 = 0.0
\_di3 = 0.0

;; D Sandstone
\_d4 = 2500.0
\_E4 = 10.0e9
\_v4 = 0.3
\_K4 = \_E4 / (3.0 * (1.0 - 2.0 * \_v4))
\_G4 = \_E4 / (2.0 * (1.0 + \_v4))
\_f4 = 32.0
\_c4 = 3.6e6
\_t4 = 0.0
\_di4 = 4.0

;; Trona
\_d6 = 2150.0
\_E6 = 18.0e9
\_v6 = 0.2
\_K6 = \_E6 / (3.0 * (1.0 - 2.0 * \_v6))
\_G6 = \_E6 / (2.0 * (1.0 + \_v6))
\_f6 = 36.3
\_c6 = 4.25e6
\_c6a = 3.8e6
\_c6b = 3.3e6
\_c6c = 2.125e6
\_c6d = 0.85e6
\_t6 = 0.65e6
\_di6 = 9.0

;; Oilshale - GSI assumed 50
\_d9 = 2500.0
\_E9 = 4.1e9
\_v9 = 0.2
\_K9 = \_E9 / (3.0 * (1.0 - 2.0 * \_v9))
\_G9 = \_E9 / (2.0 * (1.0 + \_v9))
\_f9 = 26.3
\_c9 = 1.8e6
\_c9a = 1.6e6
\_c9b = 1.4e6
\_c9c = 0.9e6
\_c9d = 0.18e6
\_t9 = 0.09e6
\_di9 = 3.0

;; Mudstone
\_d7 = 2500.0
\_E7 = 3.3e9
\_v7 = 0.25
\[ _K7 = \frac{E7}{(3.0 \times (1.0 - 2.0 \times _v7))} \]
\[ _G7 = \frac{E7}{(2.0 \times (1.0 + _v7))} \]
\[ _f7 = 22.1 \]
\[ _c7 = 1.2e6 \]
\[ _c7a = 1.1e6 \]
\[ _c7b = 1.0e6 \]
\[ _c7c = 0.6e6 \]
\[ _c7d = 0.0 \]
\[ _t7 = 0.06e6 \]
\[ _di7 = 0.0 \]

end
mat_props

;; Trona softening parameters
table 23 0.0,.c6 0.00015,.c6 0.00075,.c6a 0.0015,.c6b 0.003,.c6c 0.0225,.c6d 1.0,.c6d

;; Mudstone softening parameters
table 24 0.0,.c7 0.0004,.c7 0.002,.c7a 0.004,.c7b 0.008,.c7c 0.06,.c7d 1.0,.c7d

;; Oil shale softening parameters
table 25 0.0,.c9 0.0003,.c9 0.0015,.c9a 0.003,.c9b 0.006,.c9c 0.045,.c9d 1.0,.c9d

zone model mohr dens _d3 bulk _K3 shear _G3 fric _f3 coh _c3 dil _di3 tens _t3 range group Wilkins
zone model mohr dens _d4 bulk _K4 shear _G4 fric _f4 coh _c4 dil _di4 tens _t4 range group Dsands
zone model mohr dens _d3 bulk _K3 shear _G3 fric _f3 coh _c3 dil _di3 tens _t3 range group Roofshale
zone model ss dens _d6 bulk _K6 shear _G6 fric _f6 coh _c6 tens _t6 dil _di6 ctable 23 range group Trona
zone model ss dens _d7 bulk _K7 shear _G7 fric _f7 coh _c7 tens _t7 dil _di7 ctable 24 range group Mudstone
zone model ss dens _d9 bulk _K9 shear _G9 fric _f9 coh _c9 tens _t9 dil _di9 ctable 25 range group Oilshale
zone model elas dens _d3 bulk _K3 shear _G3 range group Floorshale

;; All geologic contacts assigned the same parameters
joint model area jkn 500.0e9 jks 500.0e9 jfric 20.0 jcoh 1.6e6 jtens 0.4e6 jdil 6.0
set jcondf = 2
set jmatdf = 1
prop jmat 1 jkn 500.0e9 jks 250.0e9 jfric 15.0

;;===================================================================

boundary yvel 0.0 range _ll _rh _bl _bh ; Fix the bottom of the model in y direction
boundary xvel 0.0 range _ll _lh _bl _th ; Fix the left boundary of the model in x direction
boundary xvel 0.0 range _rl _rh _bl _th ; Fix the right boundary of the model in x direction

;;===================================================================

;; Function to count the number of normally-shaped zones through center of pillar,
;; ignoring the very thin, oddly-shaped zone at tunnel mid-height. Purpose is to calculate
def _zonecount
    _numz = 0
    _iab = block_head
    loop while _iab # 0
        _iz = b_zone(_iab)
        loop while _iz # 0
            if _y(_iz) > _midlo
                if _y(_iz) < _midhi
                    if _x(_iz) > _rib02L
                        _numz = _numz + 1
                    endif
                endif
            endif
        endloop
        _iz = z_next(_iz)
    endloop
    _iab = b_next(_iab)
end
_zonecount

def _zonecount2 ;; count zones in roof
    _numz2 = 0
    _iab3 = block_head
    loop while _iab3 # 0
        _iz3 = b_zone(_iab3)
        loop while _iz3 # 0
            if _y(_iz3) > _seamhi
                if _y(_iz3) < _seamplus
                    _numz2 = _numz2 + 1
                endif
            endif
        endloop
        _iz3 = z_next(_iz3)
    endloop
    _iab3 = b_next(_iab3)
end
_zonecount2

; ; Function to generate an array with 1 column and a number of rows equal to
; ; the number of zones counted
def _build_array
    array pillar03_zones(1, _numz)
    array all_zones(1, _numz2)
end
_build_array

; ; Function to store the zone ID's of zones at center of pillar for calculation
; ; of average stress
def _zone_collector
    _tick = 0
    _iab1 = block_head
    loop while _iab1 ≠ 0
        _iz1 = b_zone(_iab1)
        loop while _iz1 ≠ 0
            if z_y(_iz1) > _midlo
                if z_y(_iz1) < _midhi
                    if z_x(_iz1) > _rib02L
                        if z_x(_iz1) < _rib02R
                            _tick = _tick + 1
                            pillar03_zones(1, _tick) = _iz1
                        endif
                    endif
                endif
            endif
            _iz1 = z_next(_iz1)
        endloop
        _iab1 = b_next(_iab1)
    endloop
end _zone_collector
;

def _zone_collector2 ;; ID's of zones in roof
    _tick2 = 0
    _iab2 = block_head
    loop while _iab2 ≠ 0
        _iz2 = b_zone(_iab2)
        loop while _iz2 ≠ 0
            if z_y(_iz2) > _seamhi
                if z_y(_iz2) < _seamplus
                    _tick2 = _tick2 + 1
                    all_zones(1, _tick2) = _iz2
                endif
            endif
            _iz2 = z_next(_iz2)
        endloop
        _iab2 = b_next(_iab2)
    endloop
end _zone_collector2
;

;; Function to build array to store grip points. Only 3 grid points
;; at top and bottom of entry are available to monitor convergence.
def _gp_arrays
    array top_gps(1, 8)
    array bot_gps(1, 8)
    array surf_gps(1, 4)
end _gp_arrays
;


;; Function to populate arrays with grid point ID's
def _get_points
  loop m (1,8)
    _xx = 15.375 + m * 0.25
    top_gps(1,m) = gp_near(_xx,3.0)
    bot_gps(1,m) = gp_near(_xx,0.5)
  endloop
  loop m (1,4)
    _xxs = m * 5.5 - 5.5
    surf_gps(1,m) = gp_near(_xxs,108.5)
  endloop
end

;; Average stress measurement in the pillar
def Calc_stress
  _sumstress = 0.0
  loop i (1,_numz)
    _sumstress = _sumstress + z_syy(pillar03_zones(1,i))
  endloop
  Av_stress_03 = _sumstress * (-1.0) / _numz
end

def Calc_stress2
  _sumstress2 = 0.0
  loop j (1,_numz2)
    _sumstress2 = _sumstress2 + z_syy(all_zones(1,j))
  endloop
  Roof_stress = _sumstress2 * (-1.0) / _numz2
end


def Calc_closure
  _sumtop = 0.0
  _sumbot = 0.0
  loop j (1,8)
    _sumtop = _sumtop + gp_ydis(top_gps(1,j))
    _sumbot = _sumbot + gp_ydis(bot_gps(1,j))
  endloop
  _closure = ( _sumbot - _sumtop) / 8.0
end


def Calc_subsidence
  _sumsurf = 0.0
  loop j (1,4)
    _sumsurf = _sumsurf + gp_ydis(surf_gps(1,j))
  endloop
  _subsidence = _sumsurf / (-4.0)
end

;
set fishcall 0 Calc_stress
set fishcall 0 Calc_stress2
set fishcall 0 Calc_closure
set fishcall 0 Calc_subsidence
;
;
;  
;  
;  
; 
;  
;
insitu stress 0.0 0.0 -9.85e6 szz -6.0e6 ygrad 0.0 0.0 24525.0
boundary stress 0.0 0.0 -7.2e6 range _ll _rh _tl _th
boundary xvel 0.0 range _ll _lh _tl _th
boundary xvel 0.0 range _rl _rh _tl _th
set grav 0.0 -9.81
solve
;
save 01_loaded_90_percent.sav
;
reset disp jdisp vel
fraction 0.2
;
;
damp local
mscale off
set energy on
hist energy ; histories 1 - 20
hist Av_stress_03 ; hist 21
hist _closure ; hist 22
hist Roof_stress ; hist 23
hist _subsidence ; hist 24
;
;
;  
;  
;  
;  
;  
;  
;
; Slowly load model to failure... function will stop after 0.6 meters subsidence
;
def _crusher
desired_disp = 0.6
desired_cycles = 5000000.0

;
loop while _subsidence < desired_disp
my_timestep = tdel
my_Yvel = desired_disp * (-1.0) / ( my_timestep * desired_cycles )
command
    boundary yvel my_Yvel range _ll _rh _tl _th
    print my_timestep
    print my_Yvel
    print roof_stress
    print _subsidence
step 1000
endcommand
endloop
end

; _crusher
;
save 03_crushed.sav
;
; ..............................................................
;
set plot png size 960 720
pl hist 1
copy 01_Hist_1.png
pl hist 11
copy 02_Hist_11.png
pl hist 21 vs 22
copy 03_Pillar_stress_vs_closure.png
plot hist 23 vs 24
copy 04_Roof_avg_stress_vs_subsidence.png
window 2.0 22.75 -5.0 15.75
plot coh block
copy 05_Cohesion_plot.png
plot disp block
copy 06_Disp_vectors.png
;
; ;;;;; END OF FILE
A.11 Triaxial test of sandstone specimen

New title 'Triaxial test of Tower sandstone, coh softening, 0.5 MPa conf'
config
round 0.01
diff 0.02
set ovto 0.02
:set small on
hist ncy 100
;
def _extents
  _rite = 50.0
  _rl = _rite - 0.1
  _rh = _rite + 0.1
  _left = 0.0
  _ll = _left - 0.1
  _lh = _left + 0.1
  :
  _top = _reme * 2.0
  _tl = _top - 0.1
  _th = _top + 0.1
  _bot = 0.0
  _bl = _bot - 0.1
  _bh = _bot + 0.1
end
_extents
;
block 0.0,0.0 0.0,_top _rite,_top _rite,0.0
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
gen quad 5.001 ; Discretize
;
def mat_props
  ;;
  _d2 = 2500.0
  _E2 = 3.7e9
  _v2 = 0.2
  _K2 = _E2 / (3.0 * (1.0 - 2.0 * _v2))
  _G2 = _E2 / (2.0 * (1.0 + _v2))
  _f2 = 32.0
  _c2 = 2.1e6
  _c2a = 1.89e6
  _c2b = 1.68e6
  _c2c = 1.05e6
  _c2d = 0.0
  _t2 = 0.21e6
  _di2 = 8.0
end
mat_props
;
;; softening parameters
table 2 0.0_c2 0.0003_c2 0.0015_c2a 0.003_c2b 0.006_c2c 0.045_c2d 1.0_c2d
table 2 0.0_c2 0.000075_c2 0.000375_c2a 0.00075_c2b 0.0015_c2c 0.01125_c2d 1.0_c2d
;
zone model ss dens _d2 bulk _K2 shear _G2 fric _f2 coh _c2 tens _t2 dil _di2 ctable 2
;
;==============================================================================
boundary yvel 0.0 range _ll _rh _bl _bh ; Fix the bottom of the model in x and y direction
;==============================================================================
;
; def _zonecount
  _numz = 0
  _iab = block_head
  loop while _iab # 0
    _iz = b_zone(_iab)
    loop while _iz # 0
      if z_y(_iz) > 50.0
        if z_y(_iz) < 55.0
          _numz = _numz + 1
        endif
      endif
    _iz = z_next(_iz)
  endloop
  _iab = b_next(_iab)
endloop
end _zonecount
;
; Function to generate an array with 1 column and a number of rows equal to
; the number of zones counted
def _build_array
  array all_zones(1,_numz)
end
_build_array
;
; Function to store the zone ID's of zones at center of pillar for calculation
; of average stress
def _zone_id_collector
  _tick = 0
  _iab1 = block_head
  loop while _iab1 # 0
    _iz1 = b_zone(_iab1)
    loop while _iz1 # 0
      if z_y(_iz1) > 50.0
        if z_y(_iz1) < 55.0
          _tick = _tick + 1
        endif
      endif
      all_zones(1,_tick) = _iz1
    endloop
    _iab1 = b_next(_iab1)
  endloop
end _zone_id_collector
180
_iab1 = b_next(_iab1)
endloop
end
_zone_id_collector
;
;
;; Function to build array to store grip points. Only 3 grid points
;; at top and bottom of entry are available to monitor convergence.
def _gp_arrays
    array top_gps(1,11)
    array bot_gps(1,11)
end
_gp_arrays
;
;; Function to populate arrays with grid point ID's
def _get_points
    loop m (1,11)
      _xx = m * 5.0 - 5.0
      top_gps(1,m) = gp_near(_xx,_top)
      bot_gps(1,m) = gp_near(_xx,0.0)
    endloop
end
_get_points
;
;
;; Average stress measurement in the pillar
def Calc_stress
    _sumstress = 0.0
    loop i (1,_numz)
      _sumstress = _sumstress + z_syy(all_zones(1,i))
    endloop
    Av_stress = _sumstress * (-1.0) / _numz
end
;
;
def Calc_closure
    _sumtop = 0.0
    _sumbot = 0.0
    loop j (1,11)
      _sumtop = _sumtop + gp_ydis(top_gps(1,j))
      _sumbot = _sumbot + gp_ydis(bot_gps(1,j))
    endloop
    _strain = ( _sumbot - _sumtop) / 11.0 / _top
end
;
;
set fishcall 0 Calc_stress
set fishcall 0 Calc_closure
;
damp local
mscale off
;
set energy on
hist energy
hist Av_stress ;history No.21
hist _strain ;history No.22
;
;;==========================================================================
;; achieve 0.5 MPa vertical stress
BOUNDARY yvel -0.005 range _ll _rh _tl _th
step 1300
;
boundary yvel 0.0 range _ll _rh _tl _th
;
;; add confinement
boundary stress -0.5e6 0.0 0.0 range _rl _rh _bl _th
boundary stress -0.5e6 0.0 0.0 range _ll _lh _bl _th
;
solve
;
; Fix top and bottom again (corner conditions were erased by confinement commands)
boundary yvel 0.0 range _ll _rh _tl _th
boundary yvel 0.0 range _ll _rh _bl _bh
;
; Fix bottom corners in X-direction to avoid premature slip and failure
boundary xvel 0.0 range _ll _lh _bl _bh
boundary xvel 0.0 range _rl _rh _bl _bh
;
BOUNDARY yvel -0.005 range _ll _rh _tl _th
step 80000
;
save Triax_Dsands_0p5_conf.sav
pl hist 21 vs 22
set pl png size 960 720
copy Triax_Dsands_0p5_conf.png
;
;;;; END OF FILE
A.12 Mine scale model of Solvay Mine Southwest Panel

;; Full 855m span model with brittle properties in overburden.
;; 900m far-field material at left and right.
;; Mining represented by reducing virtual pillar forces at seam level
;; and then increasing support as subsidence increases.
;;
;; Horizontal stress ratio can be changed by using different sets of
;; commands that are commented-out in lines 265-285 of this data file.
;;
; New
title 'Solvay 855m span, Ko=0.3, 4MPa drop in pillar support'

config
round 0.1
edge 0.2
set ovtol 0.05

hist ncyc 100

def _extent
    _left = -900.0  ;; Left boundary X-coordinate of model
    _ll = _left - 0.1
    _lh = _left + 0.1
    _rite = 1800.0
    _rl = _rite - 0.1
    _rh = _rite + 0.1
    _bot = -200.0
    _bl = _bot - 0.1
    _bh = _bot + 0.1
    _top = 488.5  ;; thickness of unconsolidated soils = 260m
    _tl = _top - 0.1
    _th = _top + 0.1

    _startmine = 0.0
    _stl = _startmine - 0.1
    _sth = _startmine + 0.1
    _endmine = 855.0
    _endl = _endmine - 0.1
    _endh = _endmine + 0.1

    _seamlo = 0.0  ;; floor of seam
    _seamhi = 3.5  ;; contact above seam
    _roofshale = 23.5  ;; contact above roof shale
    _Dsands = 33.5  ;; contact above D sandstone
    _Wilkins = 108.5  ;; contact above Wilkins shale
    _Tower = 193.5
    _Laney = 228.5

    _Dhi = _Dsands + 0.3
_Dlo = _Dsands - 0.3
end

extent

block _left_bot _left_top _rite_top _rite_bot

; crack _left_-40.0 _rite_-40.0 ;; contact for zoning
crack _left_-10.0 _rite_-10.0 ;; contact for zoning
crack _left_seamlo _rite_seamlo ;; contact below trona
crack _left_seamhi _rite_seamhi ;; contact above trona
crack _left_roofshale _rite_roofshale ;; contact above roof shale
crack _left_Dsands _rite_Dsands ;; contact above D Sandstone

; crack _startmine_seamlo _startmine_seamhi ;; crack the Trona for excavation

; crack -450.0_bot -450.0_top ;; cracks for zoning in far field
crack 1350.0_bot 1350.0_top

; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ;

; gen quad 40.001 range -900.0 -450.0_bot -40.0 ;; zoning for sub floor (roofshale props)
gen quad 20.001 range -900.0 -450.0 -40.0 -10.0 _seamlo ;; zoning for mudstone 5m @ 2.5 or 1.25
gen quad 7.001 range -900.0 -450.0_seamlo _seamhi 3.75m @ 1.25 ;; zoning for unmined trona

gen quad 10.001 range -900.0 -450.0_seamhi _roofshale _Dsands 10m @ 2.5 ;; zoning for D

gen quad 15.001 range -900.0 -450.0_Dsands _Wilkins 75m @ 5

gen quad 10.001 range -900.0 -450.0_Wilkins _Tower 85m @ 5

gen quad 15.001 range -900.0 -450.0_Tower _Laney 35m @ 5

gen quad 30.001 range -900.0 -450.0_Laney _top

; gen quad 20.001 range -450.0 1350.0_bot -40.0 ;; zoning for sub floor (roofshale props)
gen quad 10.001 range -450.0 1350.0 -40.0 -10.0 _seamlo ;; zoning for mudstone 5m @ 2.5 or 1.25

; gen quad 3.501 range -450.0 1350.0_seamlo _seamhi 3.75m @ 1.25 ;; zoning for unmined trona

gen quad 5.001 range -450.0 1350.0_seamhi _roofshale _Dsands 10m @ 2.5 ;; zoning for roof shale

gen quad 5.001 range -450.0 1350.0_roofshale_Dsands 10m @ 2.5 ;; zoning for D
gen quad 7.501 range -450.0 1350.0 _Dsands _Wilkins ;; zoning for Wilkins Shale 75m @ 5
gen quad 5.001 range -450.0 1350.0 _Wilkins _Tower ;; zoning for Tower Sandstone 85m @ 5
gen quad 7.501 range -450.0 1350.0 _Tower _Laney ;; zoning for Laney Shale 35m @ 5
gen quad 15.001 range -450.0 1350.0 _Laney _top
; gen quad 40.001 range 1350.0 1800.0 _bot -40.0 ;; zoning for sub floor (roofshale props)
gen quad 20.001 range 1350.0 1800.0 -40.0 -10.0 _seamlo ;; zoning for mudstone 5m @ 2.5 or 1.25
gen quad 7.001 range 1350.0 1800.0 _seamlo _seamhi 3.75m @ 1.25
gen quad 10.001 range 1350.0 1800.0 _seamhi _roofshale ;; zoning for roof shale
gen quad 10.001 range 1350.0 1800.0 _roofshale _Dsands ;; zoning for D 10m @ 2.5
gen quad 15.001 range 1350.0 1800.0 _Dsands _Wilkins ;; zoning for Wilkins Shale 75m @ 5
gen quad 10.001 range 1350.0 1800.0 _Wilkins _Tower ;; zoning for Tower Sandstone 85m @ 5
gen quad 15.001 range 1350.0 1800.0 _Tower _Laney ;; zoning for Laney Shale 35m @ 5
gen quad 30.001 range 1350.0 1800.0 _Laney _top
;
join_contact range -451.0 -449.0 _bot _top
join_contact range 1349.0 1351.0 _bot _top
;;=================================

; group zone Soils range _left _rite _Laney _top
group zone LaneyShale range _left _rite _Tower _Laney
group zone TSandstone range _left _rite _Wilkins _Tower
group zone WilkinsShale range _left _rite _Dsands _Wilkins
group zone DSandstone range _left _rite _roofshale _Dsands
group zone RoofShale range _left _rite _seamhi _roofshale
group zone Trona range _left _rite _seamlo _seamhi
group zone Mudstone range _left _rite -10.0 _seamlo
group zone FloorShale range _left _rite _bot -10.0
;
;
def mat_props
;; Bridger
_d0 = 2100.0
_E0 = 1.35e9
_v0 = 0.2
_K0 = _E0 / (3.0 * (1.0 - 2.0 * _v0))
_G0 = _E0 / (2.0 * (1.0 + _v0))
_f0 = 22.0
_c0 = 0.9e6
_c0a = 0.81e6
_c0b = 0.72e6
_c0c = 0.45e6
_c0d = 0.0
_t0 = 0.045e6
_d0 = 0.0

;; LaneyShale
_d1 = 2500.0
_E1 = 5.5e9
_v1 = 0.2
_K1 = _E1 / (3.0 * (1.0 - 2.0 * _v1))
_G1 = _E1 / (2.0 * (1.0 + _v1))
_f1 = 26.0
_c1 = 2.0e6
_c1a = 1.8e6
_c1b = 1.6e6
_c1c = 1.0e6
_c1d = 0.2e6
_t1 = 0.1e6
_d1 = 0.0

;; Tower
_d2 = 2500.0
_E2 = 3.7e9
_v2 = 0.2
_K2 = _E2 / (3.0 * (1.0 - 2.0 * _v2))
_G2 = _E2 / (2.0 * (1.0 + _v2))
_f2 = 40.0
_c2 = 5.0e6
_c2a = 4.5e6
_c2b = 4.0e6
_c2c = 2.5e6
_c2d = 0.0
_t2 = 0.25e6
_d2 = 10.0

;; WilkinsShale
_d3 = 2500.0
_E3 = 5.5e9
_v3 = 0.2
_K3 = _E3 / (3.0 * (1.0 - 2.0 * _v3))
_G3 = _E3 / (2.0 * (1.0 + _v3))
_f3 = 26.0
_c3 = 1.2e6
_c3a = 1.08e6
_c3b = 0.96e6
_c3c = 0.6e6
_c3d = 0.12e6
_t3 = 0.06e6
_d3 = 0.0

;; D sandstone
_d4 = 2500.0
\_E4 = 3.7e9
\_v4 = 0.2
\_K4 = \_E4 / (3.0 * (1.0 - 2.0 * \_v4))
\_G4 = \_E4 / (2.0 * (1.0 + \_v4))
\_f4 = 32.0
\_c4 = 2.1e6
\_c4a = 1.89e6
\_c4b = 1.68e6
\_c4c = 1.05e6
\_c4d = 0.0
\_t4 = 0.21e6
\_di4 = 8.0

;; RoofShale
\_d5 = 2500.0
\_E5 = 5.5e9
\_v5 = 0.2
\_K5 = \_E5 / (3.0 * (1.0 - 2.0 * \_v5))
\_G5 = \_E5 / (2.0 * (1.0 + \_v5))
\_f5 = 26.0
\_c5 = 0.6e6
\_c5a = 0.54e6
\_c5b = 0.48e6
\_c5c = 0.3e6
\_c5d = 0.06e6
\_t5 = 0.03e6
\_di5 = 0.0

;; Trona
\_d6 = 2150.0
\_E6 = 18.0e9
\_v6 = 0.2
\_K6 = \_E6 / (3.0 * (1.0 - 2.0 * \_v6))
\_G6 = \_E6 / (2.0 * (1.0 + \_v6))

;; Mudstone
\_d7 = 2500.0
\_E7 = 3.7e9
\_v7 = 0.25
\_K7 = \_E7 / (3.0 * (1.0 - 2.0 * \_v7))
\_G7 = \_E7 / (2.0 * (1.0 + \_v7))

end
mat_props

;; Soil softening
table 9 0.0,\_c0 0.0004,\_c0 0.002,\_c0a 0.004,\_c0b 0.008,\_c0c 0.06,\_c0d 1.0,\_c0d

;; Laney softening parameters
table 1 0.0,\_c1 0.0003,\_c1 0.0015,\_c1a 0.003,\_c1b 0.006,\_c1c 0.045,\_c1d 1.0,\_c1d
; Tower softening parameters

table 2  0.0, c2  0.0007, c2  0.0035, c2a  0.007, c2b  0.014, c2c  0.14, c2d  1.0, c2d
;

; Wilkins softening parameters

table 3  0.0, c3  0.0003, c3  0.0015, c3a  0.003, c3b  0.006, c3c  0.045, c3d  1.0, c3d
;

; D Sandstone softening parameters

table 4  0.0, c4  0.0003, c4  0.0015, c4a  0.003, c4b  0.006, c4c  0.045, c4d  1.0, c4d
;

; Roof Shale softening parameters

table 5  0.0, c5  0.0003, c5  0.0015, c5a  0.003, c5b  0.006, c5c  0.045, c5d  1.0, c5d
;

zone model elas dens, d0  bulk, K0  shear, G0  range group Soils
zone model elas dens, d1  bulk, K1  shear, G1  range group LaneyShale
zone model elas dens, d2  bulk, K2  shear, G2  range group TSandstone
zone model elas dens, d3  bulk, K3  shear, G3  range group WilkinsShale
zone model elas dens, d4  bulk, K4  shear, G4  range group DSandstone
zone model elas dens, d5  bulk, K5  shear, G5  range group RoofShale
zone model elas dens, d6  bulk, K6  shear, G6  range group Trona
zone model elas dens, d7  bulk, K7  shear, G7  range group Mudstone
zone model elas dens, d8  bulk, K8  shear, G8  range group FloorShale
;

; All bedding joints assigned Coulomb Slip parameters

joint model area, jkn  500.0e9  jks  500.0e9  jfric  20.0  jcoh  1.6e6  jten  0.4e6  jdil  6.0
set jcondf = 2
set jmatdf = 1
prop jmat  1  jkn  500.0e9  jks  250.0e9  jfric  15.0
;

; For Ko = 0.3, use following commands

insitu stress, -3.288e6  0.0  -10.960222e6  szz, -7.0e6  ygrad, 7357.0  0.0  24525.0  ;; initialize shale props for all of model
insitu stress, -3.019e6  0.0  -10.063588e6  szz, -7.0e6  ygrad, 6180.0  0.0  20600.0  range, _left _rite _Laney _top  ;; change for soil
insitu stress, -3.284e6  0.0  -10.948185e6  szz, -7.0e6  ygrad, 6327.5  0.0  21091.5  range, _left _rite _seamlo _seamhi  ;; change for Trona
;

; For Ko = 0.5, use following commands

; ; insitu stress, -5.48e6  0.0  -10.960222e6  szz, -8.0e6  ygrad, 12262.5  0.0  24525.0  ;; initialize shale props for all of model
; ; insitu stress, -5.03e6  0.0  -10.063588e6  szz, -8.0e6  ygrad, 10300.0  0.0  20600.0  range, _left _rite _Laney _top  ;; change for soil
; ; insitu stress, -5.474e6  0.0  -10.948185e6  szz, -8.0e6  ygrad, 10545.7  0.0  21091.5  range, _left _rite _seamlo _seamhi  ;; change for Trona
;

; For Ko = 0.7, use following commands

; ; insitu stress, -7.67e6  0.0  -10.960222e6  szz, -9.0e6  ygrad, 17167.5  0.0  24525.0  ;; initialize shale props for all of model

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;; For Ko = 0.9, use following commands
;; insitu stress -9.86e6 0.0 -10.960222e6 szz -10.4e6 ygrad 22072.5 0.0 24525.0  ;; initialize shale props for all of model
;; insitu stress -9.06e6 0.0 -10.063588e6 szz -10.4e6 ygrad 18982.4 0.0 21091.5 range _left _rite _seamlo _seamhi  ; change for Trona

boundary yvel 0.0 range _left _rite _bl _bh  ; Fix the bottom of the model
boundary xvel 0.0 range _left _rite _bl _bh  ; Fix the bottom of the model

boundary xvel 0.0 range _ll _lh _bl _th  ; Fix the left boundary
boundary xvel 0.0 range _rl _rh _bl _th  ; Fix the right boundary

set grav 0.0 -9.81

solve
save 01_Equil_elas.sav

reset disp jdisp vel

zone model ss dens _d0 bulk _K0 shear _G0 fric _f0 coh _c0 tens _t0 dil _di0 ctable 9 range group Soils
zone model ss dens _d1 bulk _K1 shear _G1 fric _f1 coh _c1 tens _t1 dil _di1 ctable 1 range group LaneyShale
zone model ss dens _d2 bulk _K2 shear _G2 fric _f2 coh _c2 tens _t2 dil _di2 ctable 2 range group TSandstone
zone model ss dens _d3 bulk _K3 shear _G3 fric _f3 coh _c3 tens _t3 dil _di3 ctable 3 range group WilkinsShale
zone model ss dens _d4 bulk _K4 shear _G4 fric _f4 coh _c4 tens _t4 dil _di4 ctable 4 range group DSandstone
zone model ss dens _d5 bulk _K5 shear _G5 fric _f5 coh _c5 tens _t5 dil _di5 ctable 5 range group RoofShale

solve
solve
solve
save 01_Equil_plas.sav

reset vel disp jdisp

del range _startmine _endmine _seamlo _seamhi
boundary interior stress 0.0 0.0 -10.87e6 range _stl _endh 3.3 3.6  ; top of seam
boundary interior stress 0.0 0.0 -10.94e6  range _stl _endh -0.1 0.1 ; bottom of seam
;
solve
save 02_Seam_deleted_equil.sav
;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;
damp local
mscale off
set energy on
hist energy ; histories 1 - 20
;
def _zonecount
    _numz = 0
    _ib = block_head
    loop while _ib # 0
        _iz = b_zone(_ib)
        loop while _iz # 0
            if z_y(_iz) > 3.5
                if z_y(_iz) < 8.5
                    if z_x(_iz) > _startmine
                        if z_x(_iz) < _endmine
                            _numz = _numz + 1
                        endif
                    endif
                endif
            endif
        endloop
    endloop
    _zonecount
end

end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;
def _build_array
    array syy_zones(1,_numz)
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;
def _zone_collector
    _tick = 0
    _iab1 = block_head
    loop while _iab1 # 0
        _iz1 = b_zone(_iab1)
        loop while _iz1 # 0
            if z_y(_iz1) > 3.5
                if z_y(_iz1) < 8.5
                    if z_x(_iz1) > _startmine
                        if z_x(_iz1) < _endmine
                            _tick = _tick + 1
                        endif
                    endif
                endif
            endif
        endloop
    endloop
    _zone_collector
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
syy_zones(1, _tick) = _iz1
    endif
    endif
    endif
    _iz1 = z_next(_iz1)
endloop
    _iab1 = b_next(_iab1)
endloop
end
_zone_collector
;
;
;;
;;; Average vertical stress above seam
def Calc_stress
    _sumstress = 0.0
    loop m (1, _numz)
        _sumstress = _sumstress + z_syy(syy_zones(1, m))
    endloop
    Avg_support = _sumstress * (-1.0) / _numz
end
set fishcall 0 Calc_stress
;
;
;; Average roof displacement and subsidence calculations
def _gp_arrays
    array Subsidence_gps(1, 82)
    array Seam_gps(1, 170)
end
_gp_arrays
;
def _get_points
    loop m (1, 82)
        _xx = m * 15.0 - 195.0
        Subsidence_gps(1, m) = gp_near(_xx, 488.4)
    endloop
    loop m (1, 170)
        _xxs = m * 5.0
        Seam_gps(1, m) = gp_near(_xxs, _seamhi)
    endloop
end
_get_points
;
def Seam_Roof_points
    _rd1 = gp_near(50.0, 3.5)
    _rd2 = gp_near(150.0, 3.5)
    _rd3 = gp_near(250.0, 3.5)
    _rd4 = gp_near(350.0, 3.5)
    _rd5 = gp_near(450.0, 3.5)
end
Seam_Roof_points
;
def Calc_roof_disp
    disp1 = gp_ydis(_rd1)
disp2 = gp_ydis(_rd2)
disp3 = gp_ydis(_rd3)
disp4 = gp_ydis(_rd4)
disp5 = gp_ydis(_rd5)
    _sumsurface = 0.0
    _sumroof = 0.0
    loop j (1,82)
        _sumsurface = _sumsurface + gp_ydis(Subsidence_gps(1,j))
    endloop
    _Subsidence = _sumsurface / (-82.0)
    loop j (1,170)
        _sumroof = _sumroof + gp_ydis(Seam_gps(1,j))
    endloop
    _Roof_Disp = _sumroof / (-170.0)
end
set fishcall 0 Calc_roof_disp
;
; hist Avg_support          ;; Hist 21
hist _Subsidence          ;; Hist 22
hist _Roof_Disp           ;; Hist 23
;
hist disp1
24
hist disp2
25
hist disp3
26
hist disp4
27
hist disp5
28
;
; Table of average surface displacements and average support pressure from
; the pillar-scale model model,
;
; 4MPa-drop support curve
table 101  0.000-6.83E+06
table 101  0.154-7.70E+06
table 101  0.214-7.95E+06
table 101  0.280-7.70E+06
table 101  0.324-7.80E+06
table 101  0.325-7.30E+06
table 101  0.329-7.60E+06
table 101  0.351-7.80E+06
table 101 0.384-8.00E+06
table 101 0.406-8.60E+06
table 101 0.480-9.00E+06
table 101 0.482-8.20E+06
table 101 0.505-8.30E+06
table 101 0.560-9.90E+06
;
;..............................................................................................................
;
def _adjuster
  loop while _Subsidence < 0.1
    _support = table(101,_Subsidence) ;; look-up appropriate support pressure from Table 101,
    depending on value of _Subsidence
    _support_hist = _support * (-1.0)
    command
      print _Subsidence
      print _support
      boundary interior yfree range _sth _endl  1.6 3.6
      boundary interior stress 0.0 0.0 _support range _sth _endl  1.6 3.6
      step 100
  endcommand
endloop
end
hist _support_hist ;; hist 29
_adjuster
save 03_Disp_10cm.sav
;
def _adjuster2
  loop while _Subsidence < 0.2
    _support = table(101,_Subsidence) ;; look-up appropriate support pressure from Table 101,
    depending on value of _Subsidence
    _support_hist = _support * (-1.0)
    _unbalcheck = unbal
    if _unbalcheck > 8.0e4
      command
        print _Subsidence
        print _support
        boundary interior yfree range _sth _endl  1.6 3.6
        boundary interior stress 0.0 0.0 _support range _sth _endl  1.6 3.6
        step 100
      endcommand
    else
      command
        save 04_Equil.sav
        quit
      endcommand
    endif
  endloop
end
_adjuster2
save 03_Disp_20cm.sav

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; def _adjuster3
    loop while _Subsidence < 0.3
        _support = table(101, _Subsidence) ; look-up appropriate support pressure from Table 101,
        _support_hist = _support * (-1.0)
        _unbalcheck = unbal
        if _unbalcheck > 8.0e4
            command
                print _Subsidence
                print _support
                boundary interior yfree range _sth _endl 1.6 3.6
                boundary interior stress 0.0 0.0 _support range _sth _endl 1.6 3.6
                step 100
            endcommand
        else
            command
                save 04_Equil.sav
                quit
            endcommand
        endif
    endloop
end _adjuster3
save 03_Disp_30cm.sav
;
; def _auto_adjust_4
    loop while _Subsidence < 0.4
        _support = table(101, _Subsidence) ; look-up appropriate support pressure from Table 101,
        _support_hist = _support * (-1.0)
        _unbalcheck = unbal
        if _unbalcheck > 8.0e4
            command
                print _Subsidence
                print _support
                boundary interior yfree range _sth _endl 1.6 3.6
                boundary interior stress 0.0 0.0 _support range _sth _endl 1.6 3.6
                step 100
            endcommand
        else
            command
                save 04_Equil.sav
                quit
            endcommand
        endif
    endloop
end _auto_adjust_4
save 03_Disp_40cm.sav
;
def _auto_adjust_5
  loop while _Subsidence < 0.5
    _support = table(101, _Subsidence) ;; look-up appropriate support pressure from Table 101, depending on value of _Subsidence
    _support_hist = _support * (-1.0)
    _unbalcheck = unbal
    if _unbalcheck > 8.0e4
      command
        print _Subsidence
        print _support
        boundary interior yfree range _sth _endl 1.6 3.6
        boundary interior stress 0.0 0.0 _support range _sth _endl 1.6 3.6
        step 100
      endcommand
    else
      command
        save 04_Equil.sav
        quit
      endcommand
    endif
  endloop
end

_auto_adjust_5
save 03_Disp_50cm.sav

; def _auto_adjust_6
  loop while _Subsidence < 0.6
    _support = table(101, _Subsidence) ;; look-up appropriate support pressure from Table 101, depending on value of _Subsidence
    _support_hist = _support * (-1.0)
    _unbalcheck = unbal
    if _unbalcheck > 8.0e4
      command
        print _Subsidence
        print _support
        boundary interior yfree range _sth _endl 1.6 3.6
        boundary interior stress 0.0 0.0 _support range _sth _endl 1.6 3.6
        step 100
      endcommand
    else
      command
        save 04_Equil.sav
        quit
      endcommand
    endif
  endloop
end

_auto_adjust_6
save 03_Disp_60cm.sav

;;;; END OF FILE