STRUCTURAL DAMAGE DETECTION USING
ADVANCED DATA PROCESSING
AND ANALYSIS

by
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CHAPTER I
INTRODUCTION

Damage detection and diagnosis of civil infrastructure is a topic that is constantly undergoing further research to improve reliability and cost effectiveness. Regular inspections are essential to ensure reliable performance and durability of aging civil infrastructure, (e.g., Mori and Ellingwood, 1994a,b; and Olson and Wright, 1990). In modern analysis, a technician has to visually examine each available major member and certify its safety, especially after hazards such as earthquakes. This approach is neither objective nor reliable, containing great uncertainty in identifying the existence, location, and degree of damage, (e.g., Doebling et al., 1996, 1997, and 1998). This costly and ineffective approach stimulated recent research and development in new technologies for the nondestructive evaluation of civil infrastructures (e.g., Chang, 1997; Chase and washer, 1997) in the areas of infrared thermography, ground-penetrating radar, acoustic emission monitoring, and eddy current detection. These technologies are feasible technically, but the locations of damage need to first be identified before they are efficient. In a complicated structure, locating damage can be difficult since some members are unreachable and sometimes invisible. Many sensors are required with instrumentation that is costly, needs frequent recalibration, and requires extensive data analysis.

Relating this information to bridge structures, many state departments of transportation have initiated and accomplished significant improvements in inspection, rating, and management operations. According to the Federal Highway Administration (FHWA), a fundamental weakness in current bridge management systems is that the data input to the system is based on visual inspection and subjective condition assessment (Chase, 1997). As a result of this, inspections cannot evaluate damage in the absence of visible symptoms such as concrete deterioration, corrosion, and scour. Since most condition assessment tests have been related to bridge superstructures, little testing of the
substructure of the bridge has been performed. The most common cause of bridge failure is from floods, in which scour causes failure of bridge piers and abutments (Richardson et al, 1993). A national study of 383 bridge failures caused by catastrophic floods showed that 25% involved pier damage and 72% involved abutment damage (Chang, 1973). Over 100,000 bridges over water in the United States have “unknown foundations” (Lagasse, et al, 1995), which means that their vulnerability to scour cannot be calculated by normal hydraulic and geotechnical analysis procedures. It is therefore necessary to perform testing on bridge substructures in order to analyze their signature characteristics. Dynamic testing of bridge substructures is essential to reliably analyze and diagnose damage within a structure.

Using only vibration measurements is a simpler and less costly technique (Zhang, 2001). Using vibration data can also require costly instrumentation, which, without appropriate data analysis methods, will not identify damage precisely. With the use of the Hilbert-Huang Transform (HHT), a cost-efficient, reliable structural damage detection procedure can be performed to extract the signature response from civil infrastructures. In this study, the HHT analysis method is used to perform damage detection analysis and condition assessment on model-based FEM tests on the Trinity River Relief (TRR) Bridge in Texas. Field tests were previously performed on the actual bridge and the results helped provide a preliminary procedure for damage detection (Zhang, King, Olson, 2001). To validate the theories on vibration analysis and answer any questions to the preliminary field study using HHT, the FEM tests and analyses are used. A variety of tests are performed in order to fully examine the usefulness and limitations of the HHT method. These tests will include model structure states of intact, minor-, and severe-damage conditions.

In chapter II, a review of conventional vibration data analysis methods are presented as well as their deficiencies in analyzing nonstationary data series. The HHT method is then introduced, which features two sections: Empirical Mode Decomposition (EMD) and Hilbert Spectral Analysis (HSA). Simple examples are presented to validate
HHT’s efficiency and feasibility in analyzing nonstationary time series compared to that of conventional methods.

Chapter III presents an introduction into damage detection and analysis with the use of HHT. Then a comprehensive analysis of the actual field-testing of the TRR Bridge is given based on a previous study (Zhang, King, Olson, 2001). Vibration recordings produced from the sensors at locations on the bridge piles are analyzed using the HHT method and are compared to conventional analysis methods. These analyses will begin to validate HHT’s use in damage detection and will show that the conventional methods are inadequate in properly analyzing nonstationary vibration data sets. These tests involve the TRR bridge in its intact, minor-, and severe-damage states. The results from chapter III serve as the introduction into the main topic of research for this paper which includes model-based analyses of the TRR Bridge.

Chapter IV examines the model-based analysis method using HHT. A finite element modeling (FEM) simulation program called ANSYS is used to create intact, minor-, and severe damaged models of the TRR bridge and simulates the response of each given an excitation. A modal analysis is first performed to reveal global frequency characteristics of each model. An actual forcing function used in the TRR bridge field test is then applied to the model and simulated in ANSYS to produce acceleration data for analysis. The HHT method is used to analyze these simulated responses and is compared to conventional Fourier-based techniques. A thorough analysis involving signature response characteristics, damage detection, and damping effects is performed to show HHT’s full capabilities in damage detection. The analyses will verify the theories and hypotheses previously created concerning HHT’s use in field testing. The results will show HHT’s effectiveness in reliable damage diagnosis and offer some possible limitations. Chapter IV represents the majority of the research offered in this thesis.

Chapter V presents conclusions to this analysis and offers future investigations into damage detection using HHT. These investigations will involve the attempt to estimate parameter values of a structure (i.e. stiffness properties) from the HHT analysis...
of vibration data produced from a loading cycle. Future directions in research will be offered in this area.
CHAPTER II
VIBRATION DATA PROCESSING BY CONVENTIONAL
AND HHT METHODS

2.1 Introduction

In this section the newly developed Hilbert-Huang Transform (HHT) will be
introduced to analyze examples of vibration data. The key part of the method is the
Empirical Mode Decomposition (EMD) with which any complicated time series can be
decomposed into a finite and often small number of Intrinsic Mode Functions (IMF) that
admit well-behaved Hilbert transforms. Along with the Hilbert transform, the IMF
components yield instantaneous frequencies as a function of time. Since the EMD is
based on the local characteristic time scale of the data, the HHT method is particularly
useful in analyzing nonstationary processes. Before showing this method, a review of
conventional methods is given below.

2.2 Conventional Techniques for Vibration Data Analysis

In obtaining frequency related characteristics from motion data, engineers primarily
use the major methods listed below. Also listed are shortcomings of each method, which
will be contrasted with the HHT approach.

2.2.1 Fourier Spectral Analysis

The most widely used method for analyzing vibration recordings is the Fourier
spectral analysis. This approach can reveal the frequency content and amplitude, or
energy distribution, of the motion via Fourier transformation. However, the problem is that Fourier analysis is meaningful only for stationary data. Stationary data is defined as data sets, which may contain multiple frequencies, but the frequencies do not vary over time. Nonstationary data frequencies vary over time. An example of nonstationary data would be the output of an accelerometer mounted on a vehicle driving over rough terrain. The Fourier spectrum defines harmonic components globally. Therefore, it yields average characteristics over the duration of the data window analyzed, even if those detailed characteristics, e.g., dominant frequencies, may change significantly over different portions of the window. Also, because Fourier spectral analysis explains data in terms of a superposition of trigonometric functions, it needs many harmonic components to simulate deformed or non-sinusoidal wavetrains. A typical example is the Delta(δ) function for modeling impulse forces; its Fourier spectrum spreads out the energy uniformly over all frequency ranges.

Because vibration data obtained from testing (e.g. damage detection on a structure) are nonstationary, Fourier spectral analysis distorts the energy distribution from the recordings. Specifically, the Fourier transform spreads the energy over a wider frequency range and loses the locality of the energy distribution.

### 2.2.2 Windowed Fourier Spectral Analysis

To improve the use of Fourier spectral analysis over a whole time history of vibration data, windowed Fourier spectral analysis is used. In this approach, Fourier spectral analysis is performed over restricted windows centered on specific features of interest in the data. In this way, the analysis provides information on frequency content locally in the data (e.g., evolutionary power spectra), thus reducing the problem of nonstationarity. However, the frequency resolution of Fourier spectral analysis is reduced when the length of the window is restricted. Thus, one is faced with a trade-off.
shorter window means better locality of the Fourier spectrum, but poorer frequency resolution. In addition, selection of window length and placement is highly subjective.

2.2.3 Wavelet Analysis

Based on recognition of the disadvantages of Fourier analysis, especially the non-localization characteristic in time, wavelet analysis has been used in the past decade or so to recover more accurate local information from nonstationary strong motion data (e.g., Onsay and Haddow, 1994; Chakraborty and Okaya, 1995). The wavelet approach is essentially an adjustable window Fourier-type spectral analysis (e.g., Huang et al, 1998). It therefore suffers from similar deficiencies as the windowed Fourier spectral analyses. Accordingly, wavelet analysis can provide a sound tool in situations where better time-resolution at high frequencies than at low frequencies is desirable (Daubechies, 1990). Since the low-frequency features of vibration data (e.g., a structure’s natural frequency) are more important in engineering applications than are those at high frequencies, the use of wavelet analysis is not likely to be better than those of the conventional or windowed Fourier spectral analysis.

Furthermore, wavelet decomposition is strongly conditional on the selected mother wavelet. Consequently, the physical interpretation of each wavelet decomposition and spectrum is meaningful only relative to the selected mother wavelet, but not directly to the original data. This point will be elaborated in the numerical examples later.
2.2.4 Summary

In short, conventional frequency domain approaches provide distorted, indirect, or incomplete information on recorded acceleration data. The problem lies in the lack of an appropriate method for analysis of nonstationary vibration data.

2.3 Hilbert-Huang Transform (HHT)

The Hilbert-Huang Transform (HHT), developed by Huang et al. (1998 and 1999), can represent nonstationary data such as vibration recordings from exciting a structure. The HHT method consists of two parts: Empirical Mode Decomposition (EMD) and Hilbert Spectral Analysis (HSA), which are summarized from Huang et al. (1998 and 1999) below, and focused on applications to bridge column vibration data.

2.3.1 Empirical Mode Decomposition (EMD)

The EMD builds on the assumption that any data set consists of different, simple, intrinsic modes of oscillation that need not be sinusoidal, and the non-sinusoidal character of each mode of oscillation is derived in an objective manner from the data. At any given time, the recorded data may have many different co-existing “modes of oscillation” which may relate to different vibrational phases. Each of these oscillatory modes, called an Intrinsic Mode Function (IMF), is defined by the following conditions:

(1) over the entire data set, the number of extrema and the number of zero-crossings must be equal or differ at most by one, and
(2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.
A typical IMF is shown in figure 2.1. An IMF represents a simple oscillatory mode similar to a component in the Fourier-based simple harmonic function, but more general. One can decompose any waveform as follows.

Figure 2.1 A typical intrinsic mode function with the same numbers of zero crossings and extrema, and symmetry of the upper and lower envelopes with respect to zero (from Huang et al., 1998).

First, identify all the local extrema. Connect all the local maxima by a cubic spline to produce the upper envelope, and repeat the procedure for the local minima to produce the lower envelope. The upper and lower envelopes should encompass all the data between them. The mean of these two envelopes is designated as $m_1$, and the difference between the data $X$ and $m_1$ is the first component $h_1$; i.e.,

$$h_1(t) = X(t) - m_1(t).$$ \hspace{1cm} (2.1)

Ideally, $h_1$ should be an IMF, since the construction of $h_1$ described above should have made it satisfy all the conditions set in the definition of an IMF. Yet, in practice, all the conditions of an IMF cannot be achieved until the previous process, called sifting process, is repeated. In the subsequent sifting process, $h_1$ is treated as the data, then
\[ h_{11}(t) = h_1(t) - m_{11}(t) \]  

(2.2)

where \( m_{11} \) is the mean of the upper and lower envelopes of \( h_1 \). After repeated sifting, up to \( k \) times until \( h_{1k} \) is an IMF, given by

\[ h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t) \]  

(2.3)

It is designated as the first IMF component \( c_1 \) from the data, or \( c_1(t) = h_{1k}(t) \).

(2.4)

The procedure is illustrated in figures 2.2, 2.3. Typically, \( c_1 \) will contain the finest-scale or the shortest-period component of the signal. One then removes \( c_1 \) from the rest of the data to obtain the residue

\[ r_1(t) = X(t) - c_1(t). \]  

(2.5)

The residue \( r_1 \), which contains longer-period components, is treated as the new data and subjected to the same sifting process as described above. This procedure can be repeated to obtain all the subsequent \( r_j \)'s as follows:

\[ r_{j-1}(t) - c_j(t) = r_j(t) \quad ; \quad j = 2,3,\ldots,n \]  

(2.6)

The sifting process can be terminated by any of the following predetermined criteria: (1) either the component \( c_n \) or the residue \( r_n \) becomes so small that it is less than a predetermined value of consequence, and (2) the residue \( r_n \) becomes a monotonic function, from which no more IMF can be extracted. If the data have a trend, the final residue will be that trend. The original data is thus the sum of the IMF components plus the final residue:

\[ X(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \]  

(2.7)

Thus, the data are decomposed into \( n \) IMF components, and a residue \( r_n \) that can be either the mean trend or a constant.
Figure 2.2 (left) Illustration of the sifting processes: (a) the original data; (b) the data in thin solid line, with the upper and lower envelopes in dot-dashed lines and the mean, $m_1$, in thick solid line; (c) $h_1$, the difference between the data (X) and $m_1$. This is still not an IMF, for there are negative local maxima and positive minima suggesting riding waves (from Huang et al., 1998)

Figure 2.3 (right) Illustration of the effects of repeated sifting process: (a) after one more sifting of the result in figure 2.2(c), the result is still asymmetric and still not an IMF; (b) after three siftings, the result is much improved, but more sifting needed to eliminate the asymmetry. The final IMF is shown in figure 2.1 after nine siftings (from Huang et al., 1998)
2.3.2 Hilbert Spectral Analysis (HSA)

To provide a method for analyzing the IMF components in the EMD, the Hilbert Spectrum is used to contour the frequency-energy-time information.

For given data, $X(t)$, the Hilbert Transform, $Y(t)$, is defined as

$$
Y(t) = \frac{1}{\pi} P \int_{t-t'}^{t+t'} \frac{X(t')}{t-t'} dt'.
$$

(2.8)

where $P$ denotes the Cauchy principal value. With this definition, $X(t)$ and $Y(t)$ can be combined to form the analytical signal $Z(t)$, given by

$$
Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)},
$$

(2.9)

where time-dependent amplitude $a(t)$ and phase $\theta(t)$ are found as

$$
a(t) = \sqrt{X^2(t) + Y^2(t)}
$$

(2.10)

$$
\theta(t) = \arctan \frac{Y(t)}{X(t)}.
$$

(2.11)

From the polar coordinate expression of Eq. (2.9), the instantaneous frequency can be defined as

$$
\omega(t) = \frac{d\theta(t)}{dt}.
$$

(2.12)

Applying the Hilbert transform to the $n$ IMF components of $X(t)$ in Eq. (2.7), the data $X(t)$ can be written as

$$
X(t) = \Re \sum_{j=1}^{n} a_j(t)e^{i\omega_j(t)\frac{t}{t}} dt,
$$

(2.13)

where $\Re$ denotes the real part of the value to be calculated and $a_j$ is the analytic signal associated with the $j$th IMF. The residue $r_n$ is not included because of its monotonic property (Huang et al., 1998). Eq. (2.13) is written in terms of amplitude and instantaneous frequency associated with each component as functions of time, which
differ from the time-independent amplitude and phase in the Fourier series representation of

\[ X(t) = \Re \sum_{j=1}^{n} A_j e^{i\Omega_j t} \quad (2.14) \]

where \( A_j \) is the Fourier transform of \( X(t) \), a function of frequency \( \Omega_j \). Comparison of the two representations in Eqs. (2.13) and (2.14) suggests that the IMF represents a generalized Fourier expansion. The time-dependent amplitude and instantaneous frequency in Eq. (2.13) might not only improve the flexibility of the expansion, but also enable the expansion to accommodate nonstationary data.

With this equation, we are also able to represent the amplitude and instantaneous frequency as functions of time in a 3D plot, in which the amplitude can be contoured on the frequency-time plane. This frequency-time distribution of the amplitude \( a_j \) is designated as the Hilbert Amplitude Spectrum, \( H(\omega, t) \), or simply Hilbert Spectrum.

The marginal spectrum, \( h(\omega) \), defined as

\[ h(\omega) = \int_{0}^{T} H(\omega, t) dt \quad (2.15) \]

provides a measure of total amplitude or energy contribution from each frequency value, in which \( T \) denotes the time duration of data. The Hilbert spectrum or marginal spectrum has a totally different meaning from the Fourier spectral analysis. In the Fourier representation, the existence of energy at a frequency refers to a component of a sine or a cosine function that has persisted through the time span of the data. The Hilbert spectrum is a weighted non-normalized joint frequency-time distribution of amplitude. Consequently, the frequency in the marginal spectrum indicates only the probability that an oscillation with such a frequency exists.

It should also be noted that the Hilbert transform described in Eqs. (2.8)-(2.12) is not new. However, the incorporation of Hilbert transform into the IMF components and thus the HHT representation of data in Eq. (2.13) are entirely novel. The reason of doing
EMD of original data is to allow correct calculations of instantaneous frequency. Without the EMD, the instantaneous frequency out of the Hilbert spectrum is not meaningful. It requires that, before the invention of the EMD, $X(t)$ (original data) in the equation (2.8) is symmetric locally with respect to zero mean level in order to apply the Hilbert spectrum, which most data series are not. The Hilbert spectrum, therefore, has not been used broadly. But consequently, it became the start point for Huang to invent the EMD and HHT method.

2.3.3 Summary

The EMD of HHT is a local-scale-based decomposition of the original data, rather than global-based decomposition as in the conventional or windowed Fourier transformation approach, or mother-wavelet-based decomposition as in the wavelet transformation approach. The HSA of the HHT defines the instantaneous or time-dependent frequency of the data, a general version of Fourier-based fixed or time-independent frequency. These two unique properties enable HHT analysis to reveal the physical meaning of each IMF component and Hilbert spectrum, as opposed to each Fourier or wavelet component and spectrum. Further details of this method, including some applications in geophysics and engineering, can be seen in Huang et al. (1998, 1999).

2.4 Physical Interpretation of the EMD of HHT

To fully understand the concept of the Hilbert-Huang transform, the physics of the EMD of HHT will be examined here. Figure 2.4a shows a hypothetical water wave recording $y(t) = y_1(t) + y_2(t)$, consisting of decaying water waves $y_1(t) = \cos[2\pi + 0.5\sin(2\pi)t]e^{-0.2t}$ with frequency being $[1 + 0.5\cos(2\pi)]$ in Hz and
noises \( y_2(t) = 0.05 \sin(30\pi t) \) with a frequency of 15 Hz. It is noted that the nature of the water wave is a typical non-sinusoidal waveform with sharp crests and rounded-off troughs. Although the dominant frequencies of water waves and noise are 1 and 15 Hz, respectively, its Fourier spectrum in figure 2.4b indicates that the frequency content of the water waves is spread out with the primary energy at 1, 2, and 3 Hz. The Fourier components at five selected frequencies are shown in figure 2.5. While mathematically correct, these Fourier components do not provide information that correctly represents the nature of the waves nor the time-dependent features (i.e., nonstationarity).

**Figure 2.4** Plots of (a) A hypothetical water wave recording \( y(t) = y_1(t) + y_2(t) \), consisting of water waves \( y_1(t) = \cos(2\pi t + 0.5\sin(2\pi t))e^{-0.2t} \) and high-frequency noise \( y_2(t) = 0.05 \sin(30\pi t) \) with unitless amplitude; (b) Fourier spectrum of the hypothetical water wave recording (frequency vs. energy).
Figure 2.5 Fourier components \( f_j, j=1,2,3,4,5 \) at selected frequencies (i.e., 10, 5, 2, 1 and 0.5Hz).

Figures 2.6a-d show the wavelet components corresponding to the mother wavelet of db1, db5, db10 in the Daubechies family and sym2 in the Symmlet family, respectively. The decompositions are much better than the Fourier decompositions. However, again the wavelet components (e.g., \( w_5 \)) are strongly dependent on the selected mother wavelet. Furthermore, none of components (e.g., \( w_5 \)) in each set gives the right amplitude in the beginning of the record.

In contrast, HHT can reveal the nature of the waves with just a few IMF components. Figure 2.7 shows the five IMF components decomposed from the original data by EMD. Carefully examining figure 2.7 indicates that IMF components contain observable physical meanings. In particular, the first and second components (\( c_1 \) and \( c_2 \)) represent the noise and water waves, while the rest of them with negligible amplitudes represent decomposition numerical error.
Quite often, Fourier-based low- and high-pass filters are used in such a case to separate the high-frequency noise from the true waveform. Figure 2.9a shows the high- and low-frequency components of the data by using a 10\textsuperscript{th} order butterworth filter with cutoff (or corner) frequency 2 Hz, indicating clearly that the separated low-frequency water waves (i.e., component f2) are significantly distorted in terms of peak, mean trend, and waveform. Such distortion might disappear (fully or partially) as an appropriate cutoff frequency is chosen, as shown in figures. 2.9b and 2.9c with cutoff frequencies 3 and 10 Hz. Nevertheless, these figures indicate that selection of cutoff frequency in the filters is subjective. By the same token, HHT can also achieve the same purpose by grouping the second to fifth IMF components (referred to as EMD-based low-frequency component), as shown in figure 2.8.
Figure 2.6 Wavelet Components with the use of wavelet of (a) db1, (b) db2, (c) db10, and (d) sym2
Figure 2.7 (left) Five IMF components of the hypothetical water wave recording
Figure 2.8 (right) EMD-based components

Figure 2.9 Fourier components with a cutoff frequency (a) 2Hz, (b) 3Hz, and (c) 10Hz
2.5 Hilbert Spectrum

Before examining the temporal-frequency energy distribution of the data by the Hilbert spectra, four simple examples are presented to show the characteristics of HSA, in comparison with wavelet spectral analysis. The first example as shown in figure 2.10a, involves frequency-shifted sinusoidal waves, a simple representation of frequency-evolutionary characteristic of wave motions. The corresponding Morlet wavelet and Hilbert spectra are respectively plotted in figures 2.10b-e. The reason of using the Morlet wavelet (Figure 2.10b) is that it is a Gaussian modulated wavelet with the definition of

\[
morl(x) = e^{\frac{-x^2}{2}} \cos(5x)
\]

It is able to resolve a sinusoidal wave better. The 3D plot of the wavelet spectrum in figure 2.10c is presented in order to facilitate understanding of the contour plot in figure 2.10d. From figures 2.10d and 2.10e, one sees that both spectra can capture the main feature of the shifting frequency. However, the Morlet wavelet spectrum is less precise than the Hilbert spectrum.

**Figure 2.10a** (left) Frequency-shifted sinusoidal waves, a simple representation of frequency-evolutionary characteristics of earthquake wave motions due to various body- and surface-wave arrivals; **b** (right) Morlet wavelet
Figure 2.10 (c) (left) Morlet wavelet spectrum of frequency-shift sinusoidal waves depicted in Figure 2.10a; (d) (right) a contour plot for the Morlet wavelet spectrum of frequency-shift sinusoidal waves depicted in Figure 2.10a. The lighter the color, the larger the energy in the spectra, which applies to all the subsequent contour plots.

Figure 2.10e A contour plot of the Hilbert spectrum of frequency-shift sinusoidal waves depicted in Fig. 2.10a.
A second example is shown in figure 2.11a, a double sinusoidal pulse as a simple example of a low frequency vibration recording. The corresponding Morlet wavelet and Hilbert spectra are respectively plotted in figures 2.11b and 2.11c. These results clearly show that the Hilbert spectrum is much better in identifying temporal-frequency content than is the Morlet wavelet spectrum.

While the previous two examples use sinusoidal waves, figure 2.12a, the third example, is a non-sinusoidal, nonstationary time history. While the wavelet spectrum almost loses the trace of frequency evolution and intensity with time, due to the deficiency of wavelet theory to handle low-frequency nonstationary data, the Hilbert spectrum shows that HSA clearly and precisely captures the frequency evolution and its intensity over time. The lower frequencies are shown to exist in longer time intervals because of the duration of the frequency with respect to time. In addition, computational time for Hilbert spectra (less than 30 sec) is significantly less than that for wavelet spectra (15 minutes on average, depending on the selected time- and frequency-resolutions).

We use the hypothetical water wave as another example to show the efficiency of HSA. Figure 2.13 shows the db10 wavelet spectrum of the hypothetical water wave (Figure. 2.4) as a function of time and frequency. Note that the frequency in figure 2.13 is in the wavelet sense and obtainable as the inverse of scale factor. So are the previous wavelet spectrum plots. Each mother wavelet has its own frequency band in the conventional frequency sense (i.e., Fourier-based and thus time-independent), as shown in figures 2.15a to 2.15h. Therefore, the scale factor in the wavelet spectrum is no longer simply (i.e., inversely) related to the conventional time-independent frequency. Converting the scale in wavelet spectrum to the conventional Fourier-based frequency content requires a much greater amount of customizability. Therefore, the wavelet spectrum essentially provides indirect information of temporal-frequency energy distribution of the data. In contrast, Hilbert spectrum in HSA shows a much clearer picture of temporal-frequency energy distribution. As depicted in figure 2.14, the true
water waves have dominant energy with intrawave frequency modulation around 1 Hz and a noise frequency at 15 Hz.

**Figure 2.11a** A double sinusoidal pulse as a simple example of a LFPL wave in earthquake recordings.

**Figure 2.11** contour plot of (b, left) Morlet wavelet spectrum, (c, right) Hilbert spectrum of sinusoidal-type pulse waves depicted in Figure 2.11a.
Figure 2.12 (a) A time history of nonlinear and nonstationary waves \( y = e^{-0.5t} \sin(t^2) \).

Figure 2.12 A contour plot for (b, left) the Morlet wavelet spectrum, (c, right) Hilbert spectrum of nonstationary waves depicted in Figure 2.12a.
Figure 2.13 The db10 Wavelet spectrum of the hypothetical water wave recording in Figure 2.4.

Figure 2.14 The Hilbert spectrum of the hypothetical water wave recording in Figure 2.4.
Figure 2.15 Plots of (left to right starting at top) (a) wavelet db1, and (b) its Fourier spectrum; (c) wavelet db5, and (d) its Fourier spectrum; (e) wavelet db10, and (f) its Fourier spectrum; (g) wavelet sym2, and (h) its Fourier spectrum
3.1 Current Vibration-Based Signature-Recognition Techniques

Identification of certain vibration patterns (or signature recognition) from recordings can reveal structural dynamic characteristics (e.g., natural frequencies and modes), which are of paramount importance to broad-based applications in structural engineering, such as structural model validation and updating, condition assessment and monitoring, and damage diagnosis and detection. With a few exceptions, two sets of large-array vibration data recorded at multiple points (pre- and post-damage) are usually required. Analysis and comparison of these two data sets can reveal different dynamic characteristics in the structure, and thus lead to the identification of damage location and severity. This kind of approach builds on the incorporation of the experimental data process with simplified structural modal models (e.g., Aktan et al.; Caicedo et al.; Chang; Hou, et al; Kim and Stubbs; Park et al.; Shi et al; Shah et al.; Smyth et al.; Vanik et al., Sohn et al.; Vestroni and Capecchi; and Yun et al.; all in 2000).

The above currently used signature-recognition methods can, however, be improved in the following three aspects.

3.1.1 Experimental Data Process

Traditional data processing (e.g., Fourier-based) provides distorted and/or indirect information of nonstationary data of measured vibration that is also nonlinear for a
damaged structure. This will mislead the consequent use of the data for signature recognition. Since this issue has been detailed in Chapter 2, it will not be repeated here again.

3.1.2 Signature of Damaged Structural Members

Modal analysis is often used together with data processing of in-situ vibration responses for structural identification or signature recognition. Such an approach is neither efficient nor sensitive in identifying the change of local dynamic properties of a structure from recordings.

The modal analysis builds on multiple-degree-of-freedom modeling of the whole structure. Therefore, the signature of local dynamic properties can be identified only through higher modes. Determining the shape of the higher modes requires a large number of sensors, which makes data collection not only high-cost but also complicated (e.g., issues on optimal locations of sensors). Typically the number of response measurement locations is significantly less than the number of degrees of freedom in a structural model. This mismatch often makes it difficult to precisely identify the local dynamic properties. Even with the detailed data from large sensory systems, the sensitivity is still low for identifying the local dynamic properties from the recordings due to the ubiquitous noise problem and the intrinsic drawback of traditional data analysis methods for nonlinear, non-stationary data processing, as mentioned in Chapter II.

From the perspective of damage detection, most existing signature-recognition techniques require a priori data, (i.e., vibration measured from the same structure in its non-damaged stage). Both a priori (i.e., healthy) and damaged structure data are analyzed by using one of traditional data processes. Then the two are compared to determine the change of natural frequencies, amplitudes and vibration modes, and finally
to locate the damage and estimate its severity. However, obtaining a priori data is not economically practical for most existing structures.

Even if a priori data are obtainable for very important structures (e.g., hospitals and communication buildings), the change of natural frequencies, amplitudes and modes of the structure in its considerably different health- or damage-condition stages is often inconsequential, making the identification ineffective (to be elaborated in an example later). Moreover, since the modal models are inherently linear and time invariant, the structure to be identified conforms to those assumptions as well. Consequently, the modal-based techniques cannot efficiently reveal the nonlinear structural features caused by damage.

### 3.1.3 In-situ Vibration Data

Most importantly, there is lack of destructive tests of a real structure with controlled excitation and damage. The in-situ vibration is typically limited to the low-level response regime of the structure because of difficulties exciting the structure in a controlled manner at higher a level. This makes it difficult, if not impossible, to validate the accuracy, sensitivity, and effectiveness of signature-recognition techniques that are developed primarily on the basis of laboratory experiments and/or computer models.

For example, ambient vibration data are often used in signature recognition. The ambient excitation is, however, not measured. Therefore, it is not known if this excitation source provides input at the frequencies of interest or if the input is uniform over a particular frequency range. Similarly, seismic data are often used to identify the dynamic properties of damaged structures to locate damage and estimate the severity. Since the degree of seismic-induced structural damage is primarily vision-based and difficult to be precisely quantified, some assumptions have to be made in the approach. These and other factors will greatly increase the uncertainties for the use of traditional
signature-recognition techniques for broad-based applications in general and for identifying dynamic properties in particular. The problem is lack of destructive tests of a real structure with controlled excitation and damage. These destructive tests cannot be replaced with laboratory experiments since there are always inherent differences between the models and the as-built structure. These differences arise from simplifications of the boundary and support conditions, connectivity between various structural elements, unknown material properties and constitutive relationships (particularly those associated with soil and concrete), and energy dissipation (damping) mechanisms.

3.2 The HHT Method of Damage Detection

Alternative to the signature recognition with the use of Fourier-based or wavelet data processing, a new view of the Hilbert-Huang Transform (HHT) in structural damage signature from vibration recordings is introduced. In particular, the HHT is used to analyze recordings of controlled field vibration tests of two substructures in Trinity River Relief (TRR) Bridge in Texas and simulated ANSYS responses of the TRR Bridge-based model in its intact, minor- and severe-damage stages. The interpretation of the HHT analysis for the recordings is based on the following observations, assumptions, and theories of vibration and HHT. (1) Structural vibration to a given excitation should reveal its proper frequency content, which includes driving frequencies, natural frequencies of the whole structure and/or local members, and noise frequencies. (2) When a member of the structure is damaged, its stiffness is reduced in comparison with that without damage. Therefore, vibration recorded at the damaged member could show larger vibration amplitude at lower frequency than the vibration at the same member without damage. (3) While the aforementioned damage, or the stiffness reduction, is large enough with respect to that local structural member, it might be still little in comparison with the whole structural stiffness. Consequently, vibration recorded on the
other members might not be sensitive to such a localized damage. In other words, for damaged or undamaged structure, vibrations at the member other than the to-be-damaged or damaged member will show almost the same vibration amplitude at almost the same natural frequencies. (4) Since the HHT method can identify the instantaneous frequency of measured vibration via Hilbert spectrum, the frequency of vibration on damaged member could be observably less than the frequency on other members. This leads to the identification of the damaged structural member.

3.3 Traditional Views of Vibration Testing

In this section, we demonstrate the traditional techniques for signature recognition in recordings and their characteristics by using the in-situ data collected by Olson Engineering, Inc. from controlled field vibration tests on TRR Bridge.

3.4 Field Tests of TRR Bridge

The TRR Bridge was located on old US Hwy 90 on the west side of Liberty, Texas. It was built in the 1920’s and consisted of four bridge structures. During the process of demolishing and replacing the bridge in 1996-1997, the westernmost Relief Structure 4 as shown in figure 3.1a was selected for dynamic testing. The Relief Structure 4 had a 20-foot wide roadway and consisted of 151 concrete panels directly supported by 66 concrete pile bents with a span of 18 ft. The nominal thickness of these concrete panels was 16 inches. These concrete slabs were reinforced with No. 7 steel bars at 5 3/4 inches spacing in the longitudinal direction, and with No. 4 steel bars at less than 2 ft spacing in the transverse direction. These concrete panels had one fixed end and another expansion end, resulting in both fixed and rolled connections sitting on each intermediate bridge bent. The asphalt topping had been removed to the concrete slab
level at the time of field tests. Bent 2 and Bent 12 were tested because they had the same superstructures with different foundation types.

In testing, a portable field computer with Data Physics (DP420) multi-channel FFT analyzer serviced as the data acquisition system. Vibration data recordings were collected by PCB seismic accelerometers Model 393C in 15 locations at columns and beams in each of the two Bents in its intact, minor- and severe-damage stages under recorded different-intensity excitations. The objective was to better understand the frequency ranges involved and consequently to aid in the selection of the appropriate sources and receivers for developing an efficient modal-based structural damage diagnosis.

Figure 3.1b shows the structural configuration of Bent 12 in its severe-damage stage under an excitation in the middle of the deck. As shown in the figure, Bent 12 comprised a concrete beam cap supported by four driven concrete piles. The beam cap was 23.5 ft \times 2 \text{ ft} \times 2 \text{ ft} (\text{length} \times \text{width} \times \text{height}) while the piles had a 14 in. \times 14 in. square cross section. The beam cap was reinforced longitudinally with seven No. 7 steel bars while there were No. 4 steel stirrups at 1’-6” spacing in the transverse direction. The concrete beam spanned 6’-4” symmetrically between the piles. Each pile had four No. 8 steel bars at four corners of its cross section with No. 2 steel hoops at variable spacing from 2 in. to 6 in. along the pile. The pile length between the top of the concrete strip footing and the ground surface was 7’. The three stages of the Bent is in order:

1. **Intact stage**: the column with sensor 15 (the right most pile) was not broken and soil around the pile had the same height as the soil around the pile with sensor 13 (second to the left most pile).

2. **Minor-damage stage**: the pile with sensor 15 was not broken, but soil around the pile was excavated.

3. **Severe-damage stage**: the pile with sensor 15 was broken with the steel bars left only and soil around the pile was excavated.
Figure 3.1 Relief Structure 4 of The Trinity River Relief Bridge, Texas
Figure 3.1b: Vibration tests of Bent 12 in the Trinity Relief River Bridge under excitations at the middle of the deck. The three stages of the bridge are: (1) **Intact stage**: the column with sensor 15 (the most right column) was not broken and soil around the column had the same height as the soil around the column with sensor 13 (second to the most left column), (2) **Minor-damage stage**: the column with sensor 15 was not broken, but soil around the column was excavated, and (3) **Severe-damage stage**: the column with sensor 15 was broken with the steel bars left only and soil around the column was excavated.
3.5 Structural Dynamic Properties

Identifying dynamic properties of a structure involves performing vibration analysis on a global and local level. From vibration recordings, fundamental natural frequencies and modes can be revealed through analysis.

3.5.1 Global Properties

The most widely-used method to identify global structural dynamic properties such as fundamental natural frequency and mode of a structure is Fourier spectral analysis of vibration data measured at limited locations.

Figures 3.2a and 3.2b show the time histories of vertical excitation (or forcing function) and its corresponding vertical (or along-column) vibration response at sensor 15 with the structure in its intact stage respectively. Both time histories are highly nonstationary, with the frequency increasing almost linearly from 5 Hz at 0.5 sec to 75 Hz at 5.5 sec. Therefore, their Fourier spectra, shown in figures 3.3a and 3.3b, are unable to provide faithful information in general and time-dependent frequency content of the data in particular.

Even with such distortion, the two Fourier spectra are still practically used to identify natural frequencies and vibration modes of the structure, which was also the original purpose of this and other field tests. For example, the fundamental natural frequency of the structure is obtained by identifying the first peak location in the Fourier spectrum of vibration normalized by the Fourier spectrum of forcing function, which is 14.8 Hz shown in figure 3.3c (e.g., Olson and Liu, 2001). Note that the peaks associated with frequencies lower than 5 Hz are regarded as vibration features of the bent in non-vertical directions or the like. It is also of interest to note that the identified fundamental natural frequency (14.8 Hz) is very close to the driving frequency content of the
excitation time history during 1-2 sec in figure 3.2a. Since the vibration time history between one and two seconds in figure 3.2b should contain a mixed content of both the driving and natural frequencies, among others, such a phenomenon exists only locally in 1-2 sec, not globally in 0-6 sec in the vibration time history. Therefore, the natural frequencies identified from the global-based (or time-independent) Fourier spectra may not well distinguish the mixed frequency content. This makes the Fourier-based information unclear, sometimes incorrect.

While windowed Fourier spectral analysis can indeed aid in solving partially the above issue such as finding the window-length-dependent frequency, it might not essentially solve the dilemma of time-frequency resolution, i.e., getting accurate low-frequency information at the cost of sacrificing the local time information and vice versa (see section 2.1). Primarily because of this, we do not present the windowed Fourier spectral analysis here.

Alternatively, wavelet analysis could be used to analyze these data and thus recover time-dependent information from the data, especially through the temporal-frequency energy distribution of vibration or simply the wavelet spectrum. figures 3.4b and 3.4c present respectively the wavelet spectra of the forcing function in figure 3.2a (re-plotted in figure 3.4a) with the use of Morlet and Daubechies 5 (Db5) mother wavelets. These results clearly indicate that each of the wavelet spectra is dependent upon the selected mother wavelet. For example, there exists only one dominant scale-decreasing trend of high-energy (white color in figure 3.4b) as time increases from 0 to 1000 in the Morlet wavelet spectrum. This implies that the forcing function has a dominant frequency increasing from that time period since the frequency is inversely proportional to the scale. However, with the same period, there are three such trends in the Db5 wavelet spectra in figure 3.4c. Similar phenomenon is also observed for large scale at time period between 1400 to 1800. More importantly, Morlet and Db5 mother wavelets have their own frequency band in the conventional frequency sense (i.e., Fourier-based). Therefore, the scale factor in the wavelet spectrum will no longer be
simply (i.e., inversely) related to the conventional Fourier-based frequency. Converting the scale in wavelet spectrum to the conventional Fourier-based frequency content requires a great amount of customizability and is thus inefficient.

Figure 3.2 (left): Bridge in intact stage. (a) Forcing function and (b) Recorded vibration at sensor 15.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure32.png}
\caption{Bridge in intact stage. (a) Forcing function and (b) Recorded vibration at sensor 15.}
\end{figure}
Figure 3.3 Bridge in intact stage. (a) Fourier spectrum of forcing function, (b) Fourier spectrum of vibration, and (c) Ratio of Fourier spectra of vibration and forcing function.
**Figure 3.4**: (a, top) forcing function applied at the bridge in intact stage, also shown in figure 3.2a, (b, middle) Morlet wavelet spectrum of the forcing function, and (c, bottom) Db5 wavelet spectrum of the forcing function. Note the lighter the color in (b) and (c), the larger the energy in the wavelet spectra.
3.5.2 Local Properties

Structural vibration at any point to a given excitation should reveal its proper frequencies, i.e., driving frequency content, natural frequency content of the whole structure and its members, and other (e.g., noise) frequency content. If a structural member has been damaged (e.g., one of four piles of the bridge in figure 3.1b), the stiffness and thus the natural frequencies of the member are, proportionally to the degree of the damage, reduced in comparison to those without damage. Therefore, the vibration recorded on the damaged member could show larger vibration amplitude at lower frequency content than the vibration at the same member without damage. That is the signature of damaged structures in recordings collected near the damaged element. Damage may reduce the stiffness of a single member in a large structure; it might not affect the stiffness of the whole structure significantly. Consequently, vibration on those members other than the damaged one might not be sensitive to a small amount of localized damage in another member. Therefore, Bent 12 in its intact, minor- and severe-damage stages should reveal explicitly different signatures of local dynamic properties from vibration data measured at sensor 15, but not at sensor 13.

Based on the normalized Fourier spectra of vibration at sensors 15 shown in figures 3.5a-c, the fundamental natural frequencies identified from Bent 12 in its minor- and severe-damage stages are only 6% and 9% lower than that in the intact stage. This is easily understood since the damage in the bridge is localized on one pile not globally on the whole structure, and consequently the global properties such as fundamental natural frequency will not be greatly affected. However, similar results were also found for higher natural frequencies and modes (e.g., Sanayei and Santini, 1998; Olson and Liu, 2001). With such little change in the natural frequencies and modes, one can hardly tell in practice if it is caused by structural damage, by numerical error, or by other reasons such as temperature change.
Figure 3.5: (a) Normalized Fourier spectrum at sensor 15 in intact bridge with Bent 12, i.e., the ratio of Fourier spectra of vibration and forcing function, (b) Normalized Fourier spectrum at sensor 15 in minor-damage bridge, and (c) Normalized Fourier spectrum at sensor 15 in severe-damage bridge.

In short, traditional techniques, such as combined Fourier-based data process and modal analysis, cannot effectively and sensitively reveal the local dynamic properties that are directly related to the health or damage status of a structure.
3.6 HHT View of Vibration Testing Data

Recently, because of its uniqueness in faithfully representing nonlinear, nonstationary data, the HHT method has found many applications in blood-pressure variation (Huang, et al., 1998b), earthquake engineering (Huang, 2000; Zhang and Ma, 2000; Zhang 2001a, and Zhang et al., 2001b), seismology (Zhang, 2001c, and Ma and Zhang, 2001), earthquake data analysis (Huang, 2000; Zhang and Ma, 2000; and Zhang et al., 2001), building structural identification (Yang et al., 2000; and Yang and Lei, 2000), building damage detection (Vincent et al., 2000; and Loh et al., 2000), bridge system identification (Loh, 2000), among others.

3.6.1 Analysis of Data from Bent 12

We now use the HHT to analyze the data from Bent 12 of TRR bridge. Figures 3.6a and 3.6b depict contour plots for the Hilbert spectra of the excitation and vibration time histories shown in figures 3.2a and 3.2b, i.e., the temporal-frequency energy distribution of the excitation and vibration signals. The darker the dot in the figures, the more energy the dot represents at the corresponding frequency and time instant. Figure 3.6a shows that the excitation contains a dominant energy with the frequency increasing with time, i.e., from 5 Hz at 0.5 sec to 75 Hz at 5.5 sec. For convenience in later use, we refer the dominant frequency content to as the dominant increasing driving frequency or DIDF. The excitation also has energy at other frequencies such as high frequencies ranging from 20 to 45 Hz between 0.5 and 1 sec shown in figure 3.6a, which can be verified from the excitation time history shown in figure 3.2a.

According to vibration theory, the frequency content of the vibration response should contain primarily both the driving frequencies depicted in figure 3.6a and the natural frequencies of the structure. This is verified by figure 3.6b which shows the
energy with a primary frequency content linearly increasing with time, which is the signature of excitation corresponding to the DIDF. In addition to the energy at DIDF inherent from excitation, figure 3.6b also illustrates an energy concentration in the frequency range of 10 to 20 Hz (others from 20-75 Hz) between one and two seconds. This phenomenon can be regarded as the energy contributed from the structural vibration modes at a couple of lower (higher) natural frequencies, or the signature of the structure. These vibration modes are likely excited by the excitation at 0.5-1 sec with a low-frequency content of 5-15 Hz that is close to the fundamental natural frequency of the structure. As time goes on (say 2-3 sec), the vibration at those frequencies dies down quickly due to damping, or is too small to be shown in the figure in comparison with strong DIDF energy.

To further demonstrate this point, we examine the IMF components extracted from the vibration response and their corresponding Fourier spectra, shown respectively in figures 3.7a and 3.7b respectively. As seen from these figures, the first two or three IMF components are dominant since their amplitudes are much larger than other IMF components. The first IMF component contains the vibration with a frequency content linearly increasing with time, which could be explained as the vibration component primarily inherited from the excitation at the DIDF. The temporal-frequency energy distribution (i.e., Hilbert spectrum) of the 1st IMF component in figure 8a further supports this clarification. The second and third IMF components contain relatively low frequency content, implying that they are likely the results of a couple of low-frequency vibration modes. Their Fourier-based dominant frequencies in figure 3.7b (around 16 and 10 Hz) also partially support such a hypothesis. Therefore, the fundamental natural frequency can be found from the Hilbert spectrum of the 2nd or 3rd IMF component, particularly around 1-2 sec. Figure 3.8b shows the Hilbert spectrum of the 2nd IMF component and reveals that the lowest frequency of high-energy in 1-2 sec is around 10 Hz.
It should be noted here that HHT theory and software has the issue of mode mixture in IMF components. This is also observed in the 1st and 2nd IMF components in figures 3.8a,b, in which part of motion in 2nd IMF component should belong to part of 1st component and vice versa. This issue can be partially, if not completely due to the complexity nature, solved by using the intermittency criterion or check in the HHT software. With the proper use of intermittency check, all the modes can be separated with more IMF components generated. Such a sifting process with intermittency check will, however, not significantly affect the whole temporal-frequency Hilbert spectra. As a matter of fact, great progress has recently been made of the refinement of HHT itself in terms of theoretical mature and numerical accuracy in general and of unique decomposition of the data into IMF components in particular (Huang 2001). This is able to overcome some drawbacks exposed in the early version of HHT software. Nevertheless, as the identification of frequency content from Hilbert spectra is of sole interest for the problem at hand, we don’t necessarily use the intermittency check at the stage of this study. The sifting process with intermittency check is, however, a subject of continuing study for examining the relationship of each IMF component to traditional vibration mode or the like, as used by others (e.g., Yang and Lei, 2000).

Since we are only interested in identifying the fundamental natural frequencies of the structure from the vibration during 1-2 sec, all the following figures in Hilbert spectra are presented with time less than 3 sec and frequency less than 40 Hz.

Figure 3.9a enlarges the Hilbert spectrum of vibration response at sensor 15 with the structure in its intact stage. Except the DIDF, we believe that all the other energy concentration shown in figure 3.9a is attributed primarily by the structure itself. We now focus on the energy at frequency up to 15 Hz during 1-2 sec. The energy below roughly 10 Hz (light lines or dots) is much less than that above 10 Hz (dark lines or dots). This response energy is believed to be generated by the excitation at the DIDF during 0.5-1 sec and the driving frequency of 6-15 Hz at around 1+ sec in figure 3.6a. If the fundamental natural frequency falls in the driving frequency range of 5-15, the response
energy at the fundamental natural frequency should be much stronger than that at frequency lower, but not necessarily stronger than that at frequency higher since there exist second and higher natural frequencies. Figure 3.9a suggests that the fundamental natural frequency is likely around 10 Hz.

Figure 3.9b shows the Hilbert spectrum of vibration response at sensor 15 with the structure in its minor-damage stage. Besides the inherited DIDF, the vibration energy concentration during 1-2 sec has lowered its frequency to 7 Hz at about 1.4 sec. The observed 7 Hz in the minor-damage stage could be related to the mixed natural frequency of the pile and pier, since the vibration at sensor 15 should reflect the dynamic characteristics of both the whole structure and the local member. The excavated soil, i.e., the structure in the minor-damage stage, reduces the stiffness of the pile and thus the fundamental natural frequency of the bridge pier. This is the signature of local damage in the recordings, which cannot be picked up by the traditional data analysis.

Such an explanation can be further strengthened from figure 3.9c, which shows the Hilbert spectrum of vibration response at sensor 15 with the structure in its severe-damage stage. As seen in the figure, the dominant frequency has decreased to 3 Hz at 1.7 sec.

It should be pointed out that the above explanation is simply based on the observation with the aid of vibration theory without proof. While further model-based validation is the subject of a continuing study, we now provide a very simple, rough estimation of whether the above observation is conceivable.

Since the sensor 15 is amounted on the column with one end connected to the beam and the other end buried in the soil, the dominant vibration recorded should more likely contain the dynamic features of the column itself. To examine the change of fundamental natural frequency of the column at three damage stages, we can roughly model the column with different boundary conditions and lengths. In particular, the boundary conditions are approximated as fixed-fixed with length 7’ for column in the intact stage, fixed-fixed with length 16’ in the minor-damage stage, and fixed-free with
length 7’ in the severe damage. We choose the above simple boundaries not only because they very rudely model the real vibration of the column but also because they have simple exact solutions for the fundamental natural frequency. This simple theoretical model shows that the relative change of the fundamental natural frequency in the minor- and severe-damage stages with respect to that in the intact stage is 56% and 68%, respectively. In contrast, the pertinent relative change with the use of the observed fundamental natural frequency is 50% and 70%. The detailed comparison can be seen in Table 1. The observed results are very close to the theoretical ones. While this comparison suggests that the previous explanation might be permissible, it nevertheless indicates that the explanation is completely correct. Further solid validation is still needed.

The HHT analysis of vibration at sensor 15 with the Bent in three stages shows that the HHT method might clearly recognize the signature difference of sequential damage in a structural member in terms of natural frequency from recordings. As a comparison, we now present the HHT analysis of corresponding vibration at sensor 13, which is removed from the damage location.

Figures 10a-c present the Hilbert spectra of vibration data at sensor 13 for the structure in its intact, minor- and severe-damage stages respectively. All the figures show that the lowest dominant frequency of structural vibration is around 10 Hz in 1-2 sec. This not only confirms that the fundamental natural frequency of the whole structure and its members can likely be well distinguished from the driving frequency by the HHT analysis, it also suggests that the HHT analysis is sensitive to the change in vibration characteristics caused by local damage.

As mentioned before, the second and/or third IMF component of vibration is mostly related to the structural vibration. Therefore, the Hilbert spectra of 2nd IMF components of vibrations at sensors 15 and 13 with structure in its intact, minor- and severe-damage stages are shown in figures 11-13. While the mode mixture of IMF components is observed which results in the mixture of driving and natural frequencies,
these figures still clearly show the signature difference of the structure in three stages in terms of lowest frequency of vibration recordings.

Figure 3.6a  Hilbert spectra of forcing function for vibration at sensor 15 in intact bridge. The contour plot is read in such a way that the darker the color, the larger the energy in the spectra, which is applicable to all the subsequent Hilbert spectra.
Figure 3.6b Hilbert spectra of vibration at sensor 15 in intact bridge.

Figure 3.7a Eight IMF components of vibration at sensor 15 in intact bridge.
Figure 3.7b  Fourier spectra of eight IMF components in Fig. 6a extracted from the vibration at sensor 15 in intact bridge.

Figure 3.8a  Hilbert spectrum of the 1\textsuperscript{st} IMF component extracted from the vibration at sensor 15 in intact bridge.
Figure 3.8b  Hilbert spectrum of the 2nd IMF component extracted from the vibration at sensor 15 in intact bridge.

Figure 3.9a  Hilbert spectra of vibration at sensor 15 in intact bridge.
Figure 3.9b  Hilbert spectra of vibration at sensor 15 in minor-damage bridge.

Figure 3.9c  Hilbert spectra of vibration at sensor 15 in severe-damage bridge.
Figure 3.10a  Hilbert spectra of vibration at sensor 13 in intact bridge.

Figure 3.10b  Hilbert spectra of vibration at sensor 13 in minor-damage bridge.
Figure 3.10c Hilbert spectra of vibration at sensor 13 in severe-damage bridge.

Figure 3.11a Hilbert spectra of the 2nd IMF component of vibration at sensor 15 in intact bridge.
Figure 3.11b  Hilbert spectra of the 2nd IMF component of vibration at sensor 13 in intact bridge.

Figure 3.12a  Hilbert spectra of the 2nd IMF component of vibration at sensor 15 in minor-damage bridge.
Figure 3.12b Hilbert spectra of the 2\textsuperscript{nd} IMF component of vibration at sensor 13 in minor damage bridge.

Figure 3.13a Hilbert spectra of the 2\textsuperscript{nd} IMF component of vibration at sensor 15 in severe-damage bridge.
Figure 3.13b  Hilbert spectrum of the 2\textsuperscript{nd} IMF component of vibration at sensor 13 in severe-damage bridge.
Table 3.1  Fundamental natural frequencies of a column in three stages by simple theoretical models and their comparison with observation from HHT analysis of destructive testing data.

<table>
<thead>
<tr>
<th>Boundary condition with length</th>
<th>Theoretical frequency (F)</th>
<th>Observed Frequencies (f)</th>
<th>Relative change of frequency with respect to frequency at intact stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact Fixed-fixed with l=7'</td>
<td>( F_i = \frac{1}{2l} \sqrt{\frac{E}{\rho}} )</td>
<td>( f_i = 10 ) Hz</td>
<td>N/A</td>
</tr>
<tr>
<td>Minor-damage Fixed-fixed with l=16'</td>
<td>( F_m = \frac{1}{2l} \sqrt{\frac{E}{\rho}} )</td>
<td>( f_m = 5 ) Hz</td>
<td>( \frac{(F_i-F_m)}{F_i} = 56% ) ( \frac{(f_i-f_m)}{f_i} = 50% )</td>
</tr>
<tr>
<td>Severe-damage Fixed-free with l=11'</td>
<td>( F_s = \frac{1}{4l} \sqrt{\frac{E}{\rho}} )</td>
<td>( f_s = 3 ) Hz</td>
<td>( \frac{(F_s-F_i)}{F_i} = 68% ) ( \frac{(f_s-f_i)}{f_i} = 70% )</td>
</tr>
</tbody>
</table>

3.6.2 Analysis of Data from Bent 2

We now show the HHT view of the data from Bent 2 for signature recognition of structural damage. Figure 3.14 shows the structural configuration of Bent 2 in its severe-damage stage under an excitation in the middle of the deck. Bent 2 is different from Bent 12 in that a shallow footing at the ground surface further surrounds the structure’s four piles. All the other pier conditions, including the field test setup, data collection and stages of damage for the two Bents, remain almost the same. We analyzed the data sets from sensors 11 and 9 in two of the four columns (see figure 3.14), both of which are located higher than sensors 15 and 13 on the same columns respectively.

Figure 3.15a-c show the Hilbert spectra of vibration response at sensor 11 with the structure in its intact, minor- and severe-damage stages, while figures 3.16a-16c
present those at sensor 9. Figures 3.16a-c indicate that the fundamental natural frequency of Bent 2 is around 11 Hz, a little higher than 10 Hz of Bent 12. This is understood since Bent 2 has an extra shallow footing and thus a high stiffness in comparison with Bent 12. The signature difference of the Bent 2 in three stages can be seen clearly in Figure 3.15a-c, similar to those from Bent 12. All these results suggest again that the HHT method is able to efficiently diagnose the damage of a local structural member through the signature recognition of a limited number of recordings.
Figure 3.14: Vibration tests of Bent 2 in the Trinity Bridge under excitations at the middle of the deck. The three stages of the bridge are: (1) **Intact stage**: the column with sensor 11 (the most right column) was not broken and soil around the column had the same height as the soil around the column with sensor 9 (second to the most left column), (2) **Minor-damage stage**: the column with sensor 15 was not broken, but soil around the column was excavated, and (3) **Severe-damage stage**: the column with sensor 15 was broken and soil around the column was excavated.
**Figure 3.15a** Hilbert spectra of vibration at sensor 11 in intact bridge (Bent 2).

**Figure 3.15b** Hilbert spectra of vibration at sensor 11 in minor-damage bridge.
Figure 3.15c Hilbert spectra of vibration at sensor 11 in severe-damage bridge.

Figure 3.16a Hilbert spectra of vibration at sensor 9 in intact bridge.
Figure 3.16b  Hilbert spectra of vibration at sensor 9 in minor-damage bridge.

Figure 3.16c  Hilbert spectra of vibration at sensor 9 in severe-damage bridge.
3.7 Proposed Non-Destructive Damage Diagnosis

This above numerical study compares the HHT view of recordings of bridge structures in their intact, minor- and severe-damage stages with traditional ones. It reveals that the HHT-based signature recognition is likely able to identify the local dynamic properties. In addition, comparison of the HHT analysis of two recordings from damaged and health structural elements in one condition stage of the structure can tell the signature difference of local structural members. This eliminates the need of a priori data required in traditional damage diagnosis, and thus significantly improves the efficiency in data collection. Based on the study, a novel signature-recognition technique for structural health monitoring and damage detection is proposed as follows.

1. Two or more similar structural members (e.g., two of four columns in a bridge with the same size, cross-section, and materials) are selected for a non-destructive vibration test, each of which has one sensor mounted (e.g., accelerometer). The test is subjected to a dynamic excitation acting at a location close to, but not in, the members. A computer data acquisition system will digitize and store the sensor output to the excitation. This will result in two sets of data.

2. In the HHT method, the two data sets are first decomposed into a number of IMF components in the EMD process, each of which is then input to the HSA to compute the Hilbert spectrum. Consequently, the driving frequency, natural frequency, and other frequency contents can be identified partially, if not completely, by analyzing the Hilbert spectra and IMF components.

3. If the natural frequencies of the two members are different, the member with the lower frequency has damage while the other is health, or former has more severe damage than the latter. On the other hand, if the frequencies are the same, both members are either undamaged or damaged at the same degree. In that case, the testing will be repeated with a third member involved, or a priori data from testing the same structure is needed as a reference.
(4) After identifying the damaged member, one can exert the excitation at different locations in the damaged member and record a series of vibration data sets at one sensor location.

(5) Analyze the above data sets using step 2.

(6) If one of the above data set shows the lowest natural frequency of the member, the location of the pertinent excitation is exactly or close enough to the damaged spot.

While the first three steps were explained in the previous two sections by analyzing the in-situ data of TRR Bridge tests, the reasons for the last three steps are provided as follows. When the excitation is acting directly on the damaged location, the impact to the structure at that location will be the strongest. Even if the load is not large enough and within the design range, the damaged member at this situation could have nonlinear responses, i.e., the response with lower natural frequencies at certain time period. On the other hand, the vibration responses of the structure could remain elastic and linear if the excitation acts neither directly nor closely enough to the damage spot. Since the HHT is able to identify the instantaneous frequency of data, the damage spot can be found by identifying the lowest natural frequency of vibration from the series of data sets.

3.8 Summary of Field Testing

This study compares the HHT view of recordings of bridge structures in their intact, minor- and severe-damage stages with traditional ones. It reveals the following.

(1) The conventional methods in structural damage diagnosis are found to be less effective and less sensitive for signature recognition of certain type of structural damage (e.g., the one used in the study) than is the method of HHT. While such an observation relies upon further validation, the HHT view on the destructive vibration testing data presented in the study might be useful for pertinent studies.
According to the HHT analysis of the destructive vibration testing data, an HHT-based structural damage diagnosis is proposed. If the effectiveness of the proposed diagnosis is verified, it has at least two unique features different from traditional ones:

(a) It needs only a few sensors (two in general) and no a priori data from undamaged structure, which makes data collection simple and cost-effective.

(b) It uses the HHT method to analyze the above measured data to reveal the change of temporal-frequency energy of various intrinsic oscillation modes that are extracted from the data. This enables the diagnosis to be sensible to a local damage that is associated with certain intrinsic modes.
CHAPTER IV
AN HHT ANALYSIS OF ANSYS SIMULATION MODELS

4.1 Introduction

To get a better understanding of damage detection using the HHT method and to continue and verify the research previously performed on the TRR Bridge field tests, several tests were made using an FEM model based simulation program called ANSYS. Previous tests that have been run using ANSYS involve only modal analysis to extract natural frequencies from a model (Olson Engineering). These tests provide only frequency results for an entire model. To simulate a loading case used in the actual TRR Bridge test, a single time series forcing function used in the TRR Bridge test was used in the simulation to test the frequency characteristics of motion from selected piles. The selected piles and sensor locations (model nodes) match that of the actual TRR Bridge test in the ANSYS analysis. In this case, bent 12 of the TRR Bridge is used which contains pile’s 13 and 15 with the corresponding sensor (node) locations. There are no restrictions to node locations, but for simulation accuracy and comparison equivalency, the nodes were placed in the corresponding locations to the field testing. The results provided in this chapter will verify the use of the HHT method in vibration data analysis and particularly field testing analysis.

Starting with a model previously created by Olson Engineering as a base, three model cases were used in this analysis as were used in the actual TRR Bridge tests. These models created include intact, excavated, and broken states. The models include property values obtained from the design of the TRR Bridge. In the model’s broken state, the boundary conditions have been released on pile 15 allowing it to move freely at its base. This will simulate a severe damage state in the program. In the excavated case, pile 15 is extended to simulate minor damage while retaining its boundary conditions at
its base. Figures 4.1a-c depict these different models used in this analysis. The cap beams and columns of the bent are modeled with beam elements and were rigidly connected to each other. The bridge deck is not included in this 2-D FEM model so its mass and stiffness effects have been lumped onto the cap beam in a way that the resulting dynamic behavior is equivalent to what it would be in the 3-D FEM model. Piles 13 and 15 as shown along with the corresponding nodes are used in these simulations. These two piles are structurally identical in terms of property values for means of comparison. Using the tests results from the ANSYS simulations, it will be shown that the HHT method of analysis is superior to that of the conventional Fourier-based analysis.

**Figure 4.1a** TRR Bridge Model: Intact
Figure 4.1b TRR Bridge Model: Broken

Figure 4.1c TRR Bridge Model: Excavated
4.2 Modal Analysis Results For ANSYS Models

Each model created has been run through a modal analysis using the ANSYS simulation program. These results provide the vertical natural frequency that will be looked for in the simulations that will use the actual loading function used in the TRR Bridge field test. While these results are not necessary for HHT analysis, they are used here to provide a general frequency range for further understanding. The intact, broken, and excavated model cases each have their own distinct vertical natural frequencies that will be able to be identified in later examples using the HHT method. From table 4.1, the vertical natural frequencies are shown for each model. These were determined from both the Fourier and HHT analyses for a zero damping case, which will be fully examined later. For damage comparisons, only the 1st vertical natural frequencies will be used. It can be seen from the intact model, that the 1st vertical natural frequency is 11.98 Hz. In the broken model where the boundary conditions on pile 15 have been released, it can be seen that the 1st vertical natural frequency is 6.95 Hz. When creating the excavated model, a frequency that could be distinguished from the intact and broken modal frequency was needed. While the excavated model used in the ANSYS simulations is not entirely realistic due to the depth of excavation and pile length, a 1st vertical natural frequency of 9.61 Hz was shown to exist in table 4.1. This frequency will allow us to see the distinction for comparison in later examples. It should be noted that from the modal analysis, there is a pseudo-natural frequency present in the first mode. This pseudo-frequency is present because of the original 3-D nature of the ANSYS model. For these 2-D models, the 3-D properties were preserved for simulation accuracy.

In figures 4.2a-i, the corresponding mode deformation shapes can be seen for the vertical natural frequencies in the intact, broken, and excavated cases. Since the loading function will be applied to the model at the center of each model in the vertical direction, these vertical natural frequencies will be extracted and analyzed using the HHT method. The ANSYS program can also input damping cases that will be used in this analysis.
Damping the models in ANSYS will aid in creating a response that is similar to that of the actual field tests of the TRR Bridge.

**Table 4.1** ANSYS modal analysis results for model natural frequencies

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Intact (Undamaged)</th>
<th>Excavated (Minor Damage)</th>
<th>Broken (Severe Damage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo Natural Frequency</td>
<td>3.99</td>
<td>3.51</td>
<td>3.29</td>
</tr>
<tr>
<td>1st Model Natural Frequency (Vertical)</td>
<td>11.98</td>
<td>9.61</td>
<td>6.95</td>
</tr>
<tr>
<td></td>
<td>14.84</td>
<td>12.48</td>
<td>12.47</td>
</tr>
<tr>
<td>2nd Model Natural Frequency (Vertical)</td>
<td>19.68</td>
<td>17.44</td>
<td>18.08</td>
</tr>
<tr>
<td></td>
<td>44.01</td>
<td>26.46</td>
<td>39.19</td>
</tr>
<tr>
<td>3rd Model Natural Frequency (Vertical)</td>
<td>69.22</td>
<td>30.62</td>
<td>52.50</td>
</tr>
<tr>
<td></td>
<td>93.88</td>
<td>44.1</td>
<td>72.50</td>
</tr>
<tr>
<td></td>
<td>104.45</td>
<td>67.08</td>
<td>95.00</td>
</tr>
<tr>
<td></td>
<td>106.37</td>
<td>75.88</td>
<td>105.82</td>
</tr>
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<td></td>
<td>108.49</td>
<td>94.98</td>
<td>108.17</td>
</tr>
<tr>
<td></td>
<td>113.37</td>
<td>105.80</td>
<td>109.89</td>
</tr>
<tr>
<td></td>
<td>128.69</td>
<td>108.14</td>
<td>127.22</td>
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<td>153.39</td>
<td>109.61</td>
<td>152.89</td>
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<tr>
<td></td>
<td>164.27</td>
<td>125.64</td>
<td>163.87</td>
</tr>
<tr>
<td></td>
<td>252.50</td>
<td>144.22</td>
<td>251.76</td>
</tr>
<tr>
<td></td>
<td>278.37</td>
<td>154.16</td>
<td>276.62</td>
</tr>
<tr>
<td></td>
<td>288.65</td>
<td>158.51</td>
<td>284.84</td>
</tr>
<tr>
<td></td>
<td>316.98</td>
<td>183.26</td>
<td>301.45</td>
</tr>
<tr>
<td></td>
<td>355.79</td>
<td>246.26</td>
<td>317.19</td>
</tr>
<tr>
<td></td>
<td>364.91</td>
<td>256.08</td>
<td>359.33</td>
</tr>
</tbody>
</table>
**Figure 4.2a** Mode shape for 1st vertical natural frequency: intact

**Figure 4.2b** Mode shape for 2nd vertical natural frequency: intact
Figure 4.2c  Mode shape for 3rd vertical natural frequency: intact

Figure 4.2d  Mode shape for 1st vertical natural frequency: broken
**Figure 4.2e** Mode shape for 2nd vertical natural frequency: broken

**Figure 4.2f** Mode shape for 3rd vertical natural frequency: broken
Figure 4.2g Mode shape for 1st vertical natural frequency: excavated

Figure 4.2h Mode shape for 2nd vertical natural frequency: excavated
To calculate damping cases for the simulations, the Rayleigh damping method was used. Using the natural frequencies obtained from the intact modal analysis, damping ratios were input and coefficients were calculated and applied to each model in order to simulate responses from the actual field test. The basis for inputting damping values is defined by the equation for a spring-mass system, which is,

\[ [M] \ddot{x} + [C] \dot{x} + [K] x = f(t); \]  

where \( M, C \) and \( K \) are the mass, damping, and stiffness matrix values of the model.

Looking at the equations used for mass and stiffness proportional constants and using the natural frequencies from the modal analysis, these constants can be calculated starting with the equations,

\[ [C] = a_c [M] + a_t [K] \]  

Figure 4.2i Mode shape for 3rd vertical natural frequency: excavated
where \( a_0 \) and \( a_1 \) are the mass and stiffness constants. In order to calculate the mass and stiffness coefficients using a damping ratio, the equation

\[
\xi_n = \frac{a_0}{2\omega_n} + \frac{a_1\omega_n}{2},
\]

is used where \( \xi_n \) is the damping ratio. For simplicity, both coefficients in the ANSYS simulations will be the same (i.e \( a_0 = a_1 = a \)). So to calculate the necessary damping coefficient for a simulation, a user-defined ratio is needed (Clough and Penzien, 1993). The values for damping input into the ANSYS simulations will be referred to in the figures and text as “d”.

### 4.3 ANSYS Simulation Model and Natural Frequency Extraction

To provide an example of how the HHT method can be used to analyze a simulation result from ANSYS and extract natural frequency information from a model, tests were run on the intact TRR Bridge model previously shown in fig 4.1a. To examine this analysis thoroughly, three damping cases were used in this simulation to show that the HHT method can extract natural frequency information in most any situation. These three damping cases include two extreme and one medium damping states. Figures 4.3a-c depict the forcing function used in all the ANSYS simulations as well as the corresponding Fourier and Hilbert spectra of the forcing function. It can be seen from the time history of the forcing function that a broad range of frequencies are being applied over a time range of 0 to 6.4 seconds. Using the time history of the forcing function, the Fourier and Hilbert spectra show that this frequency range is about 0 to 70 hertz. This frequency range will allow us to extract the natural frequency signature of the model by comparing it to the original forcing function. The response should be similar to the forcing function, but the Hilbert Spectrum of the response will allow us to determine the
frequency characteristic of the structure. The sensor location or (node location) that was used in this simulation set can also been seen on the intact model which is node 37 on pile 13. In order to fully investigate the response analysis given by HHT, all IMF components are contoured in the Hilbert Spectrum as shown in these figures and all subsequent figures.

**Figure 4.3a** Time History of forcing function

**Figure 4.3b** Fourier Spectrum of forcing function
Figure 4.3c Hilbert Spectrum of Forcing Function

Figure 4.3d Hilbert Spectrum of forcing function enlarged
4.3.1 Medium Damping Simulation Response

To understand the process of obtaining natural frequencies via the HHT method, a typical response example with $d=0.00198$ is used and compared to a Fourier analysis. In figure 4.4, the time history acceleration response of node 37 on pile 13 in its intact state with $d=0.00198$ is shown. This time history response shows many similarities to that of the forcing function. The response shows relatively the same shape with a corresponding frequency range. By applying a Fourier analysis, there is a peak energy distribution around 12 Hz as can be seen from the Fourier Spectrum of the response and more clearly from the normalized Fourier Spectrum in figures 4.5a-c. Using the HHT method to extract this frequency information, figures 4.6a-c show the Hilbert spectrum of the response along with enlarged views to see the results more clearly. From the Hilbert Spectrum of the response, we can see similarities to that of the Hilbert Spectrum of the forcing function in figure 4.3c. From the full Hilbert Spectrum view of the response, we can see a clear difference in contour from around 0.5 seconds to around 1.5 seconds. To see this difference clearly, the Hilbert Spectrum is enlarged in figure 4.6b to show that there is an energy distribution at 12 Hz. Figure 4.6c shows an even larger view to clearly illustrate this example. By viewing the Hilbert Spectrum of the response, we are seeing instantaneous time-frequency-energy distributions while the Fourier spectrum gives average frequency characteristics over the entire range of response data. This is a typical analysis of a response using the HHT method. In the next examples, two extreme cases are used to verify the validity of the HHT method in signature response. It will be shown that the HHT method can be used in most any case whereas the Fourier approach has limitations.
Figure 4.4 Time History of response at pile 13 node 37: d=0.00198

Figure 4.5 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 13 node 37: d=0.00198
Figure 4.6a Hilbert Spectrum of response at pile 13 node 37: $d=0.00198$

Figure 4.6b Hilbert Spectrum of response at pile 13 node 37: $d=0.00198$
4.3.2 Zero Damping Simulation Response

In this example, a simulation with zero damping was applied to the intact TRR Bridge model. As before, the same forcing function was used in the simulation. From the time history response in figure 4.7, we can again see similarities with the forcing function in terms of shape and frequency sweep. As expected using zero damping, the Fourier analysis clearly identifies energy peaks in the frequency response. These peaks are shown to exist at frequencies of 12, 19.7 and 69 Hz in figures 4.8a-c. Figure 4.9a shows the corresponding Hilbert Spectrum view of the response. The energy distributions can be seen clearly in this example at 12, 19.7, and 69 Hz as in the Fourier spectrum. Due to zero damping, the natural frequencies of the system are fully excited and the sweeping trend of the forcing function is only slightly recognized in the Hilbert
Spectrum. Because of zero damping, the HHT analysis basically provides the modal results of the structure and reveals the vertical frequency modes. These modal frequencies are listed in table 4.1 as before. This example adds to the HHT method’s validity by proving its effectiveness at one extreme case. To further illustrate the energy distribution, the marginal spectrum of the Hilbert response is shown in figure 4.9b. The frequencies can be identified easily by the corresponding energy peaks. It should be noted that while the marginal spectrum looks similar to that of the Fourier spectrum, it is in fact a different representation because the marginal spectrum is based on EMD and HSA. However, in this example they look alike due to the damping effects.

![Time History Response](image)

Figure 4.7 Time History of response at pile 13 node 37: d = 0.00
Figure 4.8 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 13 node 37: d= 0.00

Figure 4.9a Hilbert Spectrum of response at pile 13 node 37: d= 0.00
4.3.3 High Damping Simulation Response

To illustrate the ability of the HHT method to extract natural frequency information from a response more adequately than the Fourier approach, an extreme high-damping case is used in the ANSYS simulation. In this case, a system damping of $d=0.05305$ is used. While this is a relatively high damping ratio, it is used to prove HHT’s effectiveness in frequency analysis at extreme levels where the Fourier approach might prove to be not as useful. Also, this damping creates a time history response that is similar to that of the actual field test (see figure 3.2). We can see that the time history response shown in figure 4.10 similarly follows the forcing function in shape and frequency sweep again. In this case, the Fourier spectrum of the response in figure 4.11b looks very similar to that of the Forcing Function in figure 4.11a, but there are no distinguishing peaks as before. Figure 4.11c shows the normalized Fourier spectrum in

**Figure 4.9b** Marginal Spectrum of response at pile 13 node 37: $d=0.00$
which no clear peak can be distinguished from the rest. We expect to see a peak at around 12 Hz, but the normalized Fourier Spectrum is unclear and gives no real indication as to what the natural frequency of the intact model is. After applying the HHT analysis method to the response function and displaying the Hilbert spectrum in figure 4.12a, we can see clear differences in comparison to the Hilbert Spectrum of the forcing function. By enlarging the Hilbert Spectrum of the response as shown in figure 4.12b, an energy distribution at 12 Hz can be seen from 0.6 to 1.0 seconds. The determination of 12 Hz in the Hilbert Spectrum can be further clarified in section 4.7. There is clearly an oscillation occurring at 12 Hz in that time range so we can faithfully assert that this is a natural frequency of the modeled system. At times, the Hilbert Spectrum will show energy that is a result of mixed frequency information such as the energy at 1.4 seconds and 10 Hz. This can occur within the HHT calculation when the frequency sweep occurs within a short time range. We generally disregard this information in our analysis conclusions. This example presents further evidence that the HHT method of analysis is indeed better than Fourier analysis in examining frequency characteristics within a structure.

![Time History Response](image)

**Figure 4.10** Time History of response at pile 13 node 37: d=0.05305
Figure 4.11 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 13 node 37: $d=0.05305$

Figure 4.12a Hilbert Spectrum of response at pile 13 node 37: $d=0.05305$
Figure 4.12b  Hilbert Spectrum of response at pile 13 node 37: d=0.05305

4.3.4 Intact Model Simulation Summary

As shown in the previous examples of natural frequency identification, the HHT method of analysis can provide us with accurate information in terms of time, frequency, and energy while the Fourier-based method, at times, cannot. The Fourier approach can reveal accurate frequency peaks at times, but still gives average amplitude characteristics over the range of data. The HHT method provides us with accurate instantaneous time-frequency-energy information even at the most extreme cases.
4.4 Damage Detection Analysis On ANSYS Simulations Using HHT

To further investigate the current study on damage detection using the HHT method, the broken and excavated ANSYS models were used with the single loading simulation. Following the steps outlined previously in section 3.2 on detecting damage in a structure, comparisons are made between undamaged and damaged piles (columns) in this analysis. Using the ANSYS software, two nodes on two different piles were used for comparison. Pile 13 in figures 4.1b-c is used as the undamaged structure and Pile 15 in figures 4.1b-c is used as the broken (severe damage) and excavated (minor damage) structure. These two structures will be compared in terms of signature frequency characteristics during a loading cycle. Nodes 37 and 53 are used in these models as sensor locations on the model to retrieve the response data. Following the outlined method of determining if a structure is damaged in section 3.2 by means of comparison, the following simulation models are analyzed in this exact manner. By comparing the two piles with the use of HHT analysis, we will be able to see contrasts in the frequency characteristics between the damaged and undamaged piles. If a pile is damaged, we should be able to observe a frequency downshift due to the lowering of the stiffness property value of the pile. Since the Hilbert Spectrum reveals instantaneous frequencies at corresponding times, these downshifts in frequency should be easy to identify. In the following cases, pile 15 will always act as the damaged structure and pile 13 will always act as the undamaged structure in the comparisons. As an addition to the Hilbert Spectrum analysis, the Fourier Spectrum analysis is also used in the comparisons to show advantages in using HHT in damage detection.
4.4.1 A Simple Detection Example Using the Broken ANSYS Model

To first get a general idea of how damage detection is performed using the HHT method, a simple example is used where both the Hilbert Spectrum and Fourier Spectrum can identify the frequency downshift. In this simulation case, a damping value of $d=0.003979$ is used during the loading cycle. The forcing function is still the same as described previously in figure 4.3. In figure 4.13 the time history response of pile13 node 37 can be seen and compared closely to the forcing function input. In figures 4.14a-c, the Fourier Spectral analyses of pile 13 node 37 are shown. In this figure, we see an amplitude peak at around 12 Hz which is what we expect based on previous analyses of pile 13 in its undamaged state. A clear energy concentration at 12 Hz can also be seen in the Hilbert Spectrum of the same node at 0.6 to 1 seconds in figures 4.15a-b. This information is expected, but it is needed for the following comparison with pile 15 node 53 in its severely damaged (broken) state. Figure 4.16 shows the time history response at pile 15 node 37. In this figure, a similar shape to the forcing function can be seen, but it is not as comparable as the response from pile 13. This is due to the distance away from the forcing function, pile 15’s broken state, and the damping value. Figures 4.17a-c depict the Fourier analyses of the broken pile. As stated before, the normalized Fourier Spectrum in figure 4.17c can easily identify this frequency downshift at 7 Hz in this specific example as means to show HHT’s validity. Figures 4.18a-b show the full and enlarged view of the Hilbert Spectrum of the response. A frequency downshift can be easily identified in the time range from 0.5 to 1.2 seconds at around 7 Hz. As a supplemental visual, figure 19 shows the marginal spectrum taken from the Hilbert Spectrum and the frequency downshift can be seen easily from the large spike at 7 Hz. When comparing the responses from node 37 and 53 in piles 13 and 15, a difference is highly observable and it is therefore reasonable to assert that pile 15 is the damaged
structure. In the next examples, it will be shown that the Fourier-based analysis methods are incapable of identifying a damaged structure effectively while the HHT method can. In the next examples, another broken model case is used with higher damping and an excavated model case is used to show detection of minor damage within a structure.

Figure 4.13 Time History of response at pile 13 node 37: broken: d= 0.003979

Figure 4.14 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 13 node 37: broken: d= 0.003979
Figure 4.15a  Hilbert Spectrum of response at pile 13 node 37: broken: d= 0.003979

Figure 4.15b  Hilbert Spectrum of response at pile 13 node 37: broken: d= 0.003979
Figure 4.16  Time History of response at pile 15 node 53: broken; $d = 0.003979$

Figure 4.17  Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 15 node 53: broken; $d = 0.003979$
Figure 4.18a Hilbert Spectrum of response at pile 15 node 53: broken: d= 0.003979

Figure 4.18b Hilbert Spectrum of response at pile 15 node 53: broken: d= 0.003979
4.4.2 Second Broken Detection Case

In this case of damage detection, a model damping of $d=0.01326$ is used in the simulation. In figure 4.20 the time history response at pile 13 node 37 is shown. Again, the response is comparable to the forcing function due to the distance from node 37 to the forcing function and the undamaged state of pile 13. In figures 4.21a-c, the Fourier Spectral analyses are shown for pile 13 node 37 in its undamaged state. At this damping, it is becoming unclear as to what the natural frequency of the undamaged column is. There is a crest between 11 and 12 Hz, which is due to the damped natural frequency, but by observation alone, it is not clear whether this crest or the next sharp peak is the natural frequency. In figures 4.22a-b, the full and enlarged views of the Hilbert Spectrum of the response at node 37 are shown. Again, we see energy at the 12 Hz level where there is oscillation at 0.6 to 1 seconds as expected from its signature response. This is not
completely clear to someone with relative inexperience using HHT, but for comparisons in damage detection, there is no confusion. Now we compare the results obtained from pile 15 node 53 to that of pile 13 to identify any observable frequency downshifts. Figure 4.23 shows the time history response at pile 15 node 53. The shape is again similar to the forcing function, but due to the higher damping value, the shape is more similar than the previous example. Figures 4.24a-c show the Fourier Spectral analyses of pile 15 in its broken state. In both the regular and normalized Fourier Spectra of the response, there is no clear indication as to where the frequency downshift occurs. We only see one large smooth crest ranging from 6 to 13 Hz, which could be interpreted as a downshift, but it is not as clear as we would like for reliable detection. However, the Hilbert Spectrum of the response at node 53 in figures 4.25a-b show a clear and identifiable energy concentration at around 7 Hz in the 0.4 to 0.8 second time range. This large energy concentration at around 7 Hz does not appear in the Hilbert Spectrum response of pile 13 node 37. Figure 4.26 shows the marginal spectrum of the response at pile 15. In this figure there is a large peak around 7 Hz. It is not as defined as in the previous example, but it is definitely easy to distinguish. We can therefore assert that pile 15 is in fact the damaged structure.

![Time History Response](image)

**Figure 4.20** Time History of response at pile 13 node 37: broken: d=0.01326
Figure 4.21 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 13 node 37: broken: d=.01326

Figure 4.22a Hilbert Spectrum of response at pile 13 node 37: broken: d=.01326
Figure 4.22b Hilbert Spectrum of response at pile 13 node 37: broken d=.01326

Figure 4.23 Time History of response at pile 15 node 53: broken d=.01326
Figure 4.24  Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 15 node 53: broken: $d=0.01326$

Figure 4.25a  Hilbert Spectrum of response at pile 15 node 53: broken: $d=0.01326$
Figure 4.25b Hilbert Spectrum of response at pile 15 node 53: broken: d=.01326

Figure 4.26 Marginal Spectrum of response at pile 15 node 53: broken: d=.01326
4.4.3 Excavated Detection Case

In order to simulate a minor damage state, the intact model was modified by extending pile 15 and retaining the intact boundary conditions as shown in figure 4.1c. In this case, a model damping of \( d=0.00198 \) was used in the simulation. In the broken cases, the severity of the damage is immense, so Fourier-based analyses can sometimes identify frequency downshifts. But when using Fourier-based analysis on minor damage responses, it becomes difficult to recognize frequency downshifts due to the global characteristics of a Fourier approach. Since the HHT method allows us to view instantaneous frequencies relative to time, we are able to recognize local frequency downshifts due to minor damage. The time history response at pile 13 node 37 is shown in figure 4.27. This time history follows along with the previous time history responses at pile 13 in terms of comparable shape. In figures 4.28a-c the Fourier Spectral analyses on the response from pile 13 node 37 shows an expected peak around 12 Hz, which has been consistent throughout the entire simulation process. The full and enlarged Hilbert Spectra in figures 4.29a-b also show energy concentrations at 12 Hz from 0.8 to 1.4 seconds. Now we compare this result to that obtained from node 53 on pile 15. Figure 4.30 shows the time history response at pile 15 node 53. In contrast to the time history responses from the broken pile cases, the amplitude of this response is higher because pile 15 is in its minor damage state. We still see the distance and damping effects in the time history of pile 15. From figures 4.31a-c we see that the Fourier Spectral analyses on node 53 is very comparable to that of node 37 in the excavated case. The normalized Fourier Spectrum shows no viable evidence that there is any damage in pile 15. Since it is known that there is in fact “minor” damage in pile 15, the Fourier-based approach is shown to be inadequate in damage detection. In figures 4.32a-b, the full and enlarged view of the Hilbert Spectra of the Response from pile 15 node 53, a clear frequency downshift can be observed. In the time range from 0.8 to 1.2 seconds where there was observable energy at 12 Hz in the response from node 37 on pile 13, there is an
observable downshift in frequency from around 7 to 10 Hz in the response from node 53 on pile 15. The marginal spectrum of the response at pile 15 in figure 4.33 shows a peak between 9 and 10 Hz, which helps clarify the visible energy concentration in this region. It is not to the clarity of a severely damaged structure, but we can conclude that there is a downshift occurring and that pile 15 is damaged to some degree.

Figure 4.27 Time History of response at pile 13 node 37: excavated: d=.00198

Figure 4.28 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 13 node 37: excavated: d=.00198
**Figure 4.29a** Hilbert Spectrum of response at pile 13 node 37: excavated; $d=0.00198$

**Figure 4.29b** Hilbert Spectrum of response at pile 13 node 37: excavated; $d=0.00198$
Figure 4.30  Time History of response at pile 15 node 53: excavated: d = .00198

Figure 4.31  Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response at pile 15 node 53: excavated: d = .00198
Figure 4.32a Hilbert Spectrum of response at pile 15 node 53: excavated: d=0.00198

Figure 4.32b Hilbert Spectrum of response at pile 15 node 53: excavated: d=0.00198
Figure 4.33 Marginal Spectrum of response at pile 15 node 53: excavated: d=.00198

4.4.4 Summary of Damage Detection Using ANSYS Simulations

From these examples using HHT to detect damage in a structure, it has been shown that the conventional methods of frequency analysis (i.e. Fourier analysis) are inadequate in recognizing signature frequency responses in damaged and undamaged structures. By following the steps in determining and identifying the damaged pile and location, the HHT method provides a strong case for its use in modern damage detection.
4.5 Detecting Location of Damage Using ANSYS Simulations

In addition to detecting damage by comparing a damaged structure to an undamaged structure, it is also possible to find the location of damage on a structure. In the previous examples, it was shown that pile 15 had damage relative to pile 13. Since it was known that pile 15 was used as the damaged column in the ANSYS simulations, the location was also known. In a case where the damage location is not known, tests can be run to pinpoint the location of the damage. For example, if a single column was known to have damage, such as pile 15 in the simulations, sensors can be applied to the column in several locations. The location that reveals the highest amplitude at the longest interval of frequency downshift can be assumed to be the location nearest to the damaged portion of the column.

4.5.1 Example of Damage Location Using a Broken ANSYS Model

Within the ANSYS broken model, there are several other node locations as shown in figure 4.1b. For the purpose of this study, only nodes 37 and 53 were used for damage comparison. Since it was known that pile 15 was modeled as the damaged column in the broken model, the responses at nodes closer to node 53 continually showed more detection of damage. As an example, the Hilbert Spectra of the nodes were compared on the bridge beam as well as the bridge columns. To first look at the beam node responses, figures 4.34a-c show the Hilbert Spectrum of the response for nodes 7, 13, and 19. The Hilbert Spectrum at node 7 does not show any clear evidence of damage due to the lack of energy concentration at the 7 Hz level. This is expected because of the distance away from the known damage location. At node 13, the same observation is made as at node 7. At the center of the beam, there is no clear evidence of damage. Again this is expected,
as there is still a considerable distance from node 13 to pile 15, which is the broken location. The response at node 19 shows a clear energy concentration at the 7 Hz level ranging from 0.4 to 1 seconds. Comparing the response at node 19 with the response at nodes 7 and 13 in figures 4.34a-b, it would originally be assumed that node 19 was within a damaged structure, but in fact it is just nearer to the damaged column. Now a comparison between column responses is made. Figure 4.34d shows the Hilbert Spectrum of the response at node 29. Again because of the distance away from the damaged column, there is no clear energy concentration at the 7 Hz level. This is shown to exist at nodes 37 and 45 in figures 4.15b and 4.34e, respectively. Comparing these responses with the Hilbert Spectrum of the response at node 53 shown in figure 4.18b, node 53 is clearly within the damaged column by comparison. An observable energy concentration exists from 0.4 to 1.4 seconds at the 7 Hz level. From these observations, it can be asserted that node 53 is located on the damaged structure. Since node 19 also showed clear energy at the downshift of 7 Hz, it is compared to the response at node 53. The Hilbert Spectrum of the response at node 53 showed a larger range of frequency downshift at higher amplitude. From this comparison, node 53 is clearly closer to the damage, which in this case is known to exist at the base of pile 15. In cases where damage location is uncertain, this type of testing would be needed in order to find the exact location of damage within a structure.
Figure 4.34a  Hilbert Spectrum of response at node 7: broken model: d=.003979

Figure 4.34b  Hilbert Spectrum of response at node 13: broken model: d=.003979
Figure 4.34c Hilbert Spectrum of response at node 19: broken model: d=.003979

Figure 4.34d Hilbert Spectrum of response at node 29: broken model: d=.003979
As described earlier in chapter 1, another approach to using the Fourier-based spectral analysis is using the windowed Fourier spectral analysis. In the case of analyzing vibration data for signature frequency response of a structure, a windowed approach can be used. Once again, the problems facing this approach are the loss of frequency resolution and the subjectivity of the restricted window lengths and placements. When there is a decrease in frequency resolution, both frequency and amplitude can often be distorted in the Fourier Spectrum. For example, if a structure’s signature frequency response occurs at 12 Hz, the windowed Fourier Spectrum could show that peak occurring at 10 Hz. In the case of damage detection, this distortion could cause erroneous conclusions to be made about the structure. A frequency downshift could be detected when there really is no downshift occurring in the vibration data. In
the following examples, the windowed Fourier Spectral analysis will be applied to a single data set involving a broken or “severely damaged” model case. In some cases, the windowed Fourier approach works, but it will be shown that the window length and placement is key in determining the signature response. Due to this subjectivity in length and placement, it will be shown that the HHT method is much quicker and more effective in determining the signature frequency response in a structure as well as detecting damage. In the forcing function used for all of these experiments, it is known that the low driving frequency occurs within the first three seconds of the time series. From this knowledge, the following window lengths and placements were selected to analyze a broken ANSYS model simulation: .5 - 1.5 sec; .5 - 2 sec; .5 - 2.5 sec; .5 - 3 sec; 0.75 - 1.25 sec; 0.25 - 1.75 sec; 0 - 2 sec. These window lengths and placements are set in this order for each set of figures. The simulation used is the same broken case as used in the damage detection with d=0.01326. The following responses show the windowed Fourier Spectrum of the forcing function, response, and normalized response.

4.6.1 Windowed Fourier Spectral Analysis
Pile 13 Node 37-Broken Model

Using the windowed Fourier approach when analyzing pile 13 node 37 on the broken model, we expect to see peaks at 12 Hz as before. In the Fourier Spectrum of the response at node 37, occasionally we see small crests at around 12 Hz. In figures 4.35b-37b, we see these small crests, but it is not highly distinguishable from the full windowed spectrum. In figures 4.38b-41b, we cannot see even a crest at 12 Hz. This is due large in part to the decrease in frequency resolution and placement of the window. The normalized Fourier Spectrum of the response of pile 13 node 37 in figures 4.35c-41c, no useful information in determining the signature response of the structure.
Figure 4.35 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.5 – 1.5 seconds) pile 13 node 37

Figure 4.36 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.5 – 2 seconds) pile 13 node 37
Figure 4.37 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.5 – 2.5 seconds) pile 13 node 37

Figure 4.38 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.5 – 3 seconds) pile 13 node 37
Figure 4.39 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.75 – 1.25 seconds) pile 13 node 37

Figure 4.40 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.25 – 1.75 seconds) pile 13 node 37
4.6.2 Windowed Fourier Spectral Analysis

**Pile 15 Node 53-Broken Model**

Using the windowed Fourier approach when analyzing pile 15 node 53 on the broken model, we expect to see peaks at around 7 Hz as before indicating a frequency downshift. In this set of analyses, the windowed Fourier approach can somewhat identify a downshift using the normalized Fourier Spectrum. However, it is still based on the window length and placement within the response function. In figures 4.42b-48b, we see no distinguishing peaks in the Fourier spectrum of the response at 7 Hz, which is what we would expect to see. Comparing the Fourier Spectrum of the response to the Fourier Spectrum of the forcing function in figures 4.42a-48a, it can be seen that there is more frequency content in the lower than 12 Hz range in the response spectrum. However, there are no distinguishing peaks that can be plainly seen. In this case, the Fourier
spectrum of the response is useless in this analysis. Contrary to the normalized spectrum in the windowed Fourier analysis of pile 13, we can see some distinguishing peaks around 7 Hz in this case using pile 15. In figures 4.42c and 4.45c, peaks can be seen at 7 Hz, but they are not standouts in relative amplitude. It is unclear as to what the signature response is using this information. Figure 3.43c shows a distinguishable peak at 6 Hz and serves as the only windowed example that can be clearly analyzed. Due to the decrease in frequency resolution, the peak shows up at 6 Hz instead of 7 Hz. While 6 Hz is close to 7 Hz, it is a distortion nonetheless. In figures 4.44c and 4.46c, we see no distinguishable peaks within the normalized Fourier response. In figures 4.47c and 4.48c, we see peaks that can be distinguished from others, but the information is not completely clear. It is difficult to make any assertions about the frequency response of the pile in these figures. Figure 4.47c shows a peak at 9 Hz, which in this case is not an expected frequency. If the expected frequency were not known, the information given in this figure would be very misleading. Figure 4.48c shows a peak near 7 Hz, but a similar peak is shown between 4 and 5 Hz so a solid conclusion about the signature frequency cannot be made.

Figure 4.42 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.5 – 1.5 seconds) pile 15 node 53
Figure 4.43 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.5 – 2 seconds) pile 15 node 53

Figure 4.44 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.5 – 2.5 seconds) pile 15 node 53
Figure 4.45 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.5 – 3 seconds) pile 15 node 53

Figure 4.46 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.75 – 1.25 seconds) pile 15 node 53
Figure 4.47 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0.25 – 1.75 seconds) pile 15 node 53

Figure 4.48 Fourier Spectral Response of (a) Forcing Function (b) Response (c) Normalized Response (0– 2 seconds) pile 15 node 53
4.6.3 Summary of Windowed Fourier Analysis of ANSYS Simulations

As illustrated in the previous windowed Fourier Spectral analyses, the information obtained from this approach is neither reliable nor conclusive. It has been shown that while this approach works in some cases, it can also be misleading in revealing accurate frequency content. Due to the deficiencies and the subjectivity of the widowed Fourier approach to this type of analysis, it is not a useful tool in analyzing nonstationary vibration data. The HHT method has been shown to extract the signature frequency response of the pile structures quickly and easily. Referring to figure 4.25b, it took one simple analysis with no subjectivity to produce reliable results that could be interpreted and used for comparison.

4.7 Effects Of Damping on Damage Detection Using ANSYS Simulations

It has already been shown that the HHT method can extract the signature frequency response of a structure at even the most extreme damping values. In Section 4.3, the HHT method was used in conjunction with the Hilbert Spectrum to show response cases of the intact models at extreme damping levels. To further investigate the effects of damping on these ANSYS simulation models, the broken “severely damaged” model (figure 4.1c) was run through a series of damping cases. For these simulations, only node 53 was tested on pile 15 due to the easily viewable results. These damping cases range from d=0 to 0.05305. By looking at selected HHT analyses of responses at selected damping values, it will be shown that the HHT method is capable of providing damage detection analysis and comparison up to about d=0.0265. At this level of damping it was found through investigation that the HHT method is not capable of
providing comparable results for reliable damage detection. These extreme values of damping are not realistic in modern design. From the following examples, it will become evident that the HHT method can indeed faithfully detect local damage within a structure in most situations where there is means for comparison.

4.7.1 Damping Responses to ANSYS Simulations Using HHT

To completely understand this analysis, note that in the section 4.4 involving damage detection that the damaged structure in the damaged model always showed a downshift in frequency to around 7 Hz. We will look at the concentrations of energy at 7Hz for this analysis. Starting at a damping value of d=0, figure 4.49 shows the Hilbert Spectrum of response. As stated before, since the damping is zero, the response does not follow the driving frequency well, but a frequency downshift at 7 Hz can be seen ranging from 0.5 to 6 seconds. A simulation was then performed using d=0.00198. In figure 4.50, we can see the frequency downshift clearly at 7 Hz. In this response, the downshift in frequency occurs from about 0.5 to 2 seconds. This case led to a faithful assertion that this structure was damaged. At d=0.003979, the Hilbert Spectrum looks much like that at d=0.00198, but the frequency downshift occurs up to only 1.4 seconds as shown in figure 4.51. This trend shows that as damping increases, the ability to see the downshift in frequency decreases. A large increase in damping to d=0.0079577 was then used due to the consistently reliable results obtained in the lower damping levels. In figure 4.52 we still see a large amplitude in the Hilbert Spectrum response at 7 Hz. A change was now starting to occur because the concentration of energy was occurring in smaller periods of time. In this case, the downshift in frequency occurred between 0.5 and 0.8 seconds. As the damping was raised again to d=0.01326 a similar response was found. In figure 4.53, a concentration of energy at 7 Hz is seen again and this downshift occurs again between 0.5 and 0.8 seconds. Even at these damping values of d=0.0079577 and 0.01326, reliable
damage detection results were being produced. In order to discover if there was a limit for HHT analysis of damage detection, a higher damping value of $d=0.0265$ was then used. In this case, the limitations were starting to be seen. In figure 4.54, the energy concentration at 7 Hz is not easily detectable. One could assert that there is a frequency downshift occurring at 0.6 and 0.8 seconds, but it is not completely clear and therefore a conclusion cannot be made. Going one-step further to demonstrate the apparent limitation, a damping example of $d=0.05305$ is shown in figure 4.55. In this Hilbert Spectrum analysis, there is no reliable evidence that there is a frequency downshift occurring. An oscillation occurs from 0.5 to 0.6 seconds in the 7 Hz range, but it is not a strong energy concentration. This is not conclusive evidence that there is damage in the structure being analyzed.

![Hilbert Spectrum](image)

**Figure 4.49** Hilbert Spectrum of response at $d=0.00$
Figure 4.50 Hilbert Spectrum of response at $d = 0.00198$

Figure 4.51 Hilbert Spectrum of response at $d = 0.003979$
Figure 4.52  Hilbert Spectrum of response at $d=0.0079577$

Figure 4.53  Hilbert Spectrum of response at $d=0.01326$
Figure 4.54 Hilbert Spectrum of response at d=.02653

Figure 4.55 Hilbert Spectrum of response at d=.05305
4.7.2 Summary of Damping Effects on ANSYS Simulations

While it has been shown that the HHT method does possess limitations in terms of damage detection due to damping effects, it has also been shown that HHT method produces reliable results at reasonable damping levels. Due to these findings, we can conclude that the HHT method is a useful tool in reliable damage detection.
CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

5.1 Parameter Estimation With The Use Of HHT

In addition to analyzing the signature response of dynamic structures, future research could involve taking the HHT analysis further to calculate the stiffness properties of structures. This would involve structures that undergo kinematic or isotropic hardening during a particular loading cycle. By using the HHT method along with further calculation, it could be determined if a dynamic structure responds with linear stiffness, bilinear stiffness, or nonlinear stiffness and that stiffness value could be found. Using a loading cycle that excites the natural frequencies of a structure, stiffness values can be determined from the HHT analysis on the response of that structure. As a simple illustration of this type of analysis, a single degree of freedom (SDOF) frame with bilinear kinematic hardening is loaded with sine sweep forcing functions and an impulse forcing function that excite the natural frequencies of the frame. Newmarks method of response calculation is used for this SDOF system.

In the following examples, the HHT analysis will be compared to a Fourier analysis of the SDOF system. The SDOF system is based on a frame with two stiffness values. In the sine sweep examples, the stiffness values are 100 and 10 N/m with a frame beam mass of 1kg and a damping of 2%. The corresponding natural frequencies are 1.59 and 0.50 Hz respectively. Both a forward and reverse sine sweep with the same amplitude will be used for these values of stiffness over a range of 40 seconds. A range of 40 seconds was chosen in an attempt to thoroughly analyze the response of the frame system. In the impulse example, the stiffness values are 1000 and 100 N/m with a mass
of 1kg and a damping of 2%. The corresponding natural frequencies are 5 and 1.59 Hz respectively. The sine sweep will allow for the natural frequencies of the SDOF frame to be excited and thus they will be extracted with the use of HHT. The impulse is set from 0 to 0.1 seconds and allowed to oscillate the frame for 5 seconds. The purpose is an attempt to extract these natural frequencies and observe the shift from one to the other with the use of HHT. The shift of stiffness values in the system occurs when a user-defined force limit is exceeded. In figure 5.1, a typical bilinear response is shown from a force vs. displacement plot. The slopes of the force vs. displacement plots are K1 and K2 as shown along with an example of a maximum spring force being exceeded. It will be shown that the HHT method provides a better tool for extracting stiffness parameters than does the Fourier method in a SDOF system.

![Spring Force Vs. Displacement Sweep](image.png)

**Figure 5.1** Force vs.distance bilinear stiffness response example
5.1.1 Forward Sine Sweep Example

The first input into the SDOF frame is the sine sweep in figure 5.2 that ranges from 0 to 4 Hz over a range of 40 seconds at 10N. In figure 5.3, the time history response of the acceleration can be seen. The first stiffness value is exceeded in the first second of the response and continues to be exceeded up about 15 seconds. In this region, the frame is responding in a non-linear fashion by interchanging between both stiffness values of the frame. An observable shift in response can be seen at the 15-second mark where the frame responds linearly with the first stiffness value of 100 to the end of the sweep. Figure 5.4 shows the force vs. displacement plot where the bilinear reaction can easily be seen. The cycling between stiffness values can also be observed from this figure. In figure 5.5, the Fourier Spectrum of the acceleration response is shown. A peak exists at about 0.7 Hz, which is not what we would expect. Many peaks follow which are hard to distinguish from each other. It is not clear what the Fourier Spectrum is providing. In figure 5.6, the Hilbert Spectrum of the second IMF component is shown and we can easily distinguish frequency shifts in this figure. From 0 to 10 seconds, a frequency of around 0.5 Hz is observable. A frequency of 0.5 Hz is expected, and there is some oscillation of frequency under 0.5 Hz, but is possibly due to the damping effects of the frame system. At 15 seconds, a shift can be clearly seen as in the time history response. This energy is located at about 1.6 Hz, which is exactly what is expected due to the natural frequencies of the frame. During the range of time when the frame is cycling between stiffness values, only the lower stiffness value can be seen because its corresponding natural frequency is excited in this region. The reason for using the second IMF component is that it is separated from the driving frequency and interprets the low frequency for the SDOF system better. Figure 5.7 shows the marginal spectrum of the HHT response for the forward sine sweep. The peaks are clearly seen to exist near the expected natural frequency values. The peak at about 0.4 Hz is lower than expected.
This again is possibly due to the damping effects, but this is an effect that will be further researched in order to find meaning.

**Figure 5.2** Sine sweep input to SDOF system.

**Figure 5.3** Time History acceleration response of SDOF bilinear stiffness model: sine sweep
Figure 5.4 Spring force vs. displacement for SDOF bilinear stiffness model: sine sweep

Figure 5.5 Fourier Spectral response to SDOF bilinear stiffness model: sine sweep
Figure 5.6 Hilbert Spectrum of 2\textsuperscript{nd} component response of SDOF bilinear stiffness model: sine sweep

Figure 5.7 Marginal Spectrum of 2\textsuperscript{nd} component response of SDOF bilinear stiffness model: sine sweep
5.1.2 Reverse Sine Sweep Example

The next input into the SDOF frame is the reverse sine sweep, which can be seen in figure 5.8, that ranges from 0 to 4 Hz in 40 seconds at 10N. In figure 5.9, the time history of the acceleration is shown and it can be seen that it follows the reverse sweep trend as the input. In this case, the shift to the second stiffness value can be seen to occur at about 27 seconds. The frame responds in a linear fashion up to this point where it begins to cycle between stiffness values and responds non-linearly. Figure 5.10 shows the force vs. displacement plot where again the bilinear response and cycling can be observed. The Fourier Spectrum of the response is shown in figure 5.11. Again, it is not clear as to what is being shown in the spectrum. The peaks cannot be distinguished from each other. Therefore, no interpretable data is being offered as to the behavior of the bilinear frame. The Hilbert Spectrum in figure 5.12 shows a clear energy concentration from about 0 to 15 seconds centered around 1.6 Hz. Again, this is expected as the frame was responding linearly in this time range. In this figure, the shift does not occur at 27 seconds where it is expected based on the time history response. There is a small amount of energy occurring at 27 seconds, which could be interpreted as the shift, but there is a large amount of energy shown to exist at 0.5 Hz from 35 to 40 seconds, which is a clear observation of the second stiffness value. In figure 5.13, the marginal spectrum of the HHT response for the reverse sine weep is shown. In this figure, the peaks exist a little lower than what is expected again, but as mentioned above, this effect will be further investigated.
Figure 5.8 Reverse sine sweep input to SDOF system

Figure 5.9 Time History acceleration response of SDOF bilinear stiffness model: reverse sine sweep
Figure 5.10 Spring force vs. displacement for SDOF bilinear stiffness model: reverse sine sweep

Figure 5.11 Fourier Spectral response to SDOF bilinear stiffness model: reverse sine sweep
Figure 5.12  Hilbert Spectrum of 2\textsuperscript{nd} component response of SDOF bilinear stiffness model: reverse sine sweep

Figure 5.13  Marginal Spectrum of 2\textsuperscript{nd} component response of SDOF bilinear stiffness model: reverse sine sweep
5.1.3 Impulse Input Example

The next input into the SDOF frame is the pulse, which can be seen in figure 5.14, that is set at a constant $-500 \text{ N}$ for 0.1 seconds and then set back to 0 N in a time range of 5 seconds. Figure 5.15 shows the time history acceleration response to the impulse input. The impulse can be seen in the response occurring from 0 to 0.1 seconds and then the frame is allowed to oscillate for 5 seconds with the corresponding damping. The maximum spring force for the first stiffness value is being exceeded only in the first 0.1 seconds. In figure 5.16, the force vs. displacement plot shows that only 1 cycle of bilinear hardening occurred before the frame responded only linearly. The Fourier response in figure 5.17 shows a peak at 5 Hz, which is expected because the frame was allowed to oscillate at its linear natural frequency for about 4.9 seconds. There is no evidence that there was a bilinear response from the frame in the Fourier Spectrum. The Hilbert Spectrum in figure 5.18 shows a clear energy concentration at 5Hz as expected, with some energy occurring at lower frequencies early in the response. In this example, there were only 2 IMF components so all were shown in this figure. It is not completely clear what the energy at the lower frequencies mean. It is expected that the energy should occur at 1.59 Hz, but the energy at 1.59 Hz cannot be distinguished from the other energy occurring. Therefore, a faithful assertion cannot be made about the stiffness parameters. In figure 5.19, the marginal spectrum of the HHT response for the pulse input also shows no clear evidence of a bilinear response. The sampling rate used in this test was varied for complete analysis, but this example showed the best and most clear results. By increasing the sampling rate, there are more data points to analyze within the impulse portion of the forcing function. But the overall conclusions remained the same for this test over the entire range of sampling rates. Since an impulse input is a typical testing input into systems for response analysis, this type of example will need further research to test the HHT method’s effectiveness.
**Figure 5.14** Pulse input to SDOF system

**Figure 5.15** Time History acceleration response of SDOF bilinear stiffness model: pulse
Figure 5.16 Spring force vs. displacement for SDOF bilinear stiffness model: pulse  

Figure 5.17 Fourier Spectral response to SDOF bilinear stiffness model: pulse
Figure 5.18 Hilbert Spectrum response of SDOF bilinear stiffness model: pulse

Figure 5.19 Marginal Spectrum response of SDOF bilinear stiffness model: pulse
5.1.4 Summary of Future Research

Based on the previous examples, future research in this area will involve more complicated structures in which the HHT method can be used for parameter estimation in a structure. From this process, it can be determined how a structure is responding to given excitations (i.e. linearly, bilinearly, etc.). The type of input that the HHT method can analyze will also have to be examined as it was shown that there might be limitations. In the future, it is possible that structures could be designed more structurally sound with the use of HHT analysis on existing structures and loading conditions.

5.2 Conclusions and Remarks

A comprehensive analysis of HHT and its applications to damage detection and signature response is given in this thesis. For analyzing nonstationary vibration data, the HHT method shows an advantage over the modern conventional techniques such as the Fourier and Wavelet transforms. Vibration data obtained from FEM model based simulations were analyzed using the HHT method. The following observations and conclusions were made:

(1) The conventional analysis techniques are less suited for analyzing nonstationary vibration data than the HHT method based on extensive FEM simulations.

(2) The HHT method provides a sound tool for signature response recognition. In the model-based simulations, there were no observable limitations to signature recognition.

(3) The HHT method is capable of reliable damage detection within a civil infrastructure. Fourier-based methods showed no evidence for reliable damage detection in the FEM simulations. The Fourier method of analysis showed
limitations in damage detection well before the HHT method in cases of increased damping.

(4) In contrast to Fourier analysis, the HHT method can find the location of damage within a structure once damage has been detected based on the model analyses.

(5) Based on the overall results from the ANSYS FEM tests and analyses, the HHT method is a valid tool for use in actual field-testing of dynamic structures. The preliminary procedures for damage detection based on observations made in chapter III have been verified through the model-based simulations.

(6) With the use of EMD, the Hilbert Spectrum of certain IMF components can reveal stiffness values for single degree of freedom structures. It may be possible to extract these parameters in more complicated structures with further research and testing using HHT.


Clough and Pinzien (1993) *Dynamics of Structures*.


(Eds. M. Shinozuka, G.I. Schueller and R. Corotis), June 17-22, 2001, Newport Beach, California, USA.
ABSTRACT

With the use of a newly developed technique called the Hilbert-Huang Transform (HHT) to analyze data from vibration patterns (signature recognition), structural dynamic characteristics such as natural frequencies and modes are revealed. Important broad-based applications in structural engineering such as damage diagnosis and detection, condition assessment and monitoring, and structural model validation are features that the HHT can assess. This technique uses Empirical Mode Decomposition (EMD), which can decompose any complicated time series into a finite number of Intrinsic Mode Functions (IMF). The HHT is applicable to nonstationary data since the decomposition is based on the local characteristics of time scale data. Using the HHT on a time series, the IMF yield instantaneous frequencies as a function of time. The Hilbert Spectrum is the result of the contour of the IMF, which is a frequency-energy-time distribution.

In this study, the Hilbert-Huang Transform is used to analyze vibration data from an ANSYS simulation model of the Trinity River Relief Bridge in Texas. The data is produced from model nodes using intact, minor-, and severe-damaged states of the bridge piles. Comparisons will be made between the Fourier-based method and the HHT method in analyzing vibration data. By using ANSYS simulations, several tests are made in order to verify the validity of the HHT method in modern field-testing. These tests include: modal analysis, signature recognition, damage detection, damping effects and limitations involving HHT, and parameter estimation. Damage detection analysis is also considered on the field testing data from the TRR Bridge which provides preliminary procedures for damage detection that are validated in the model-based research. The
primary focus of this research is to use the model-based simulation results in order to validate the theories produced from the field-testing results.

In addition, this study shows that HHT provides a more precise and accurate analysis of nonstationary vibration data than does the Fourier method. Accordingly, the Hilbert Spectrum reveals temporal-frequency energy distributions for vibration data more clearly and precisely than the wavelet spectra. In the analysis of damage detection on both the actual TRR Bridge and the ANSYS simulation model, the HHT method of detection is closely examined in comparison to that of the Fourier based analysis. The observations, assumptions, and theories of the HHT and vibration are shown to exist in both cases using the HHT analysis on TRR Bridge vibration data.
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