VALUATION OF DEVELOPED GOLD DEPOSITS

by
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ABSTRACT

The Hotelling Valuation Principle, hereinafter known as (HVP), a corollary of the Hotelling “r %” rule, is empirically tested as a valuation method for developed gold deposits, by regressing adjusted market values on their estimated HVP values. The HVP generally fails as a valuation tool for developed gold deposits; it is suggested that a capacity constraint of gold mining is one of the major reasons. The HVP does provide a reliable estimate when the net price (i.e. the spot price of gold minus the unit extraction cost) of the in-situ reserves is greater than $45; below this point the HVP fails completely. It is shown that as the net price approaches zero, the value of the operating option to suspend production increases to a value of $33 per ounce of reserves, thus disturbing the HVP’s prediction of a value of zero. A model of this operating option is presented: the option value falls below $5 per ounce at a net price (P - c) of $50 per ounce, and goes to zero at a net price of $150 per ounce. A combination of the Hotelling Model and an Option Pricing Model is suggested for valuation purposes.
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Chapter 1

INTRODUCTION

An asset is an economic resource that is expected to produce benefits in the future.

The economic value of an asset is the present value of the discounted stream of expected future cash flows produced by the asset. In-situ gold bearing mineral deposits are assets and their valuation is an important topic to many diverse groups of people:

1. Private individual owners of the land containing the deposit would like to know its value so that they can decide whether to lease, sell, or develop the deposit, either immediately or in the future.
2. Owners of a mineral interest via a lease have the same decision; in addition, they may be facing the decision of a temporary shutdown or permanent abandonment of a developed and presently operating deposit; or conversely, they may be contemplating the restart of a developed deposit which is currently shut down due to low gold prices.
3. Gold mining companies need to know how to value individual gold deposits accurately in order to evaluate mineral prospects that are continually being presented to them for sale by others.
4. Investors and security analysts, in their quest for undervalued securities, may be interested in deposit values, since the total value of a pure gold mining company must be the sum of the value of its individual gold deposits plus any “going concern” value it possesses.
5. The U.S. government is also interested in in-situ mineral values in order to establish or reevaluate its policy regarding exploration and exploitation of gold minerals on multi-use public lands.
6. Environmental and conservation groups would like to know the mineral value in order to provide input to their cost/benefit analyses.
This thesis and research investigates the valuation of such developed gold deposits and develops a valuation model which is based on theoretical foundations, is practical to use, and provides reasonably accurate results. Most of what we discuss and analyze in this work, concerning valuation techniques, procedures, and theory, relates to all types of exhaustible minerals. We will confine our focus, however, to developed gold deposits and operating gold companies. A deposit is developed when the capital investment in mine access infrastructure, processing plants, and general infrastructure has been made and the deposit is currently capable of producing gold.

The following defined financial/economic terms will be used throughout this paper:

- **value** - the price at which an asset will trade between a willing buyer and seller, neither of which is under duress or obligation concerning the exchange; often used as a synonym for “market value”, “fair market value”, or “present value”.

- **market value** - the value observed in the marketplace, where buyers and sellers negotiate a mutually acceptable price for the asset; also known as “fair market value”.

- **present value** - the value in today’s dollars of a future payment or cash flow, discounted back to the present time at the required rate of return.

- **option value** - the additional value of a gold deposit that derives, in our case, from the ability of management to make operating decisions (i.e. management has options) about the deposit such as to alter the production rate or to altogether cease production either temporarily or permanently.

- **efficient financial market** - a market in which information is widely and cheaply available to buyers and sellers and all relevant and ascertainable information is already reflected in security prices.

- **uncertainty** - the state of economics in which the future value of a variable is not known because the probability distribution of the variable is unknown; since the distribution is not known, probabilities cannot be assigned to the various outcomes.
• *risk* - in a financial sense, risk is the variability of returns or cash flows from those that are expected of an asset, when the probability distribution of the cash flows is known; often quantified as the standard deviation or coefficient of variation.

**Currently Used Valuation Methods**

As noted by Davis (1994), there are three generally acceptable procedures by which mineral deposits are valued:

1. the Discounted Cash Flow (DCF) method
2. the Hotelling Valuation Principle (HVP), which is a special type of discounting procedure based on the famous Hotelling Principle (HP) of Mineral Economics
3. the Option Pricing Method (OPM) applied to real assets

Examples of real assets which the OPM can be applied to are machines, factories, land, and mineral deposits, as opposed to financial assets such as stocks, bonds, and other securities. Gold is unique in that it can be a real asset when considered as a mineral deposit, and it can also be a financial asset when considered as gold bullion, a monetary store of value and medium of exchange. In this work we will consider gold in the real asset case, i.e., as an in-situ mineral deposit.

In addition to these methods for valuing individual deposits, security analysts use various other financial methods for valuing the common stock of gold mining companies. Such other methods include dividend discount models, price/earnings models, and various other financial statistics per ounce of gold reserves such as market capitalization per ounce, cash costs per ounce, etc. These methods use a rule of thumb or a single financial
statistic to compare a single company across time or to compare several similar companies at a common point in time. The underlying method of computation of the statistic may itself be based on some present value technique, as are DCF and HVP. These other ad hoc methods not only aggregate the value of the individual gold deposits of the company, but also may capture the additional "going concern" value of a gold company which is related in an unquantifiable way to such things as its legal organization and current operating status, its financial capitalization structure, its historical and expected future operating results and profitability, the background and current reputation of its management, the forecast for the macroeconomy in general and/or the gold industry specifically, etc.

In Chapter 6 we will analyze this "going concern" value in more detail, and isolate and quantify it as a value of gold companies separate and distinct from the value of the gold deposits themselves.

Discounted Cash Flow Method (DCF)

The DCF method determines, on a present value basis, the difference between the annual cash inflows and cash outflows associated with the gold deposit. These cash flows are discounted at the appropriate risk-adjusted rate; such rate is often called the opportunity cost of capital or the "hurdle" rate. The summation of these resulting present values of the positive and negative annual cash flows is called the net present value (NPV) or simply the "value" of the deposit. The major variations of DCF include the net present
value (NPV), the internal rate of return (IRR), and the profitability index (PI). NPV is the variation useful in valuation problems, while IRR and PI are primarily useful in the selection of alternate projects in the capital budgeting decision. If the discount rate is varied so that the NPV becomes zero, then such rate is called the internal rate of return (IRR). The DCF model is given as

\[ NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+\mu)^t} \]  

(1.1)

where

- NPV = net present value of a stream of cash flows
- t = the time period
- N = the number of time periods, in our case in years
- CF_t = the cash flow experienced, positive or negative, during the year t
- \mu = the risk adjusted discount rate appropriate for this cash flow

It can be seen immediately that the accuracy of the DCF method in valuing a gold deposit depends on how well one can estimate future annual cash flows (in the numerator) and on which value is used for the discount rate (in the denominator).

The cash flows, CF_t, represent the results of operations and are obtained by subtracting cash expenses and the results of investing activities, which are primarily determined by capital expenditures necessary during the next N years, from cash revenues. Since revenues are determined by the product of unit gold price times the units sold, one must accurately forecast the price of gold in order to accurately forecast the revenues and thereby compute an accurate NPV. But we do not know the future gold price with
certainty and therefore DCF valuation, once made, does not account for the possibility that unanticipated price changes might cause a change in the true value of the project at the current time. This problem of uncertainty is usually dismissed in practice by stating that the values used to calculate the annual cash flows needed for the numerator of Equation (1.1) are the "expected values." The probability distribution that produces these expected values and their standard deviations are usually not given. Many, maybe most, mining industry practitioners do not even realize that the values they are estimating for revenues and costs, and therefore using in their DCF calculations, should be expected values subject to uncertainty caused by the existence of a probability distribution. The figures used, in most cases, are the "most likely value" which is a number chosen based on the analyst's judgment. The mechanics and mathematics of the calculation are usually emphasized rather than the accuracy of the inputs such as cash flows and the discount rate.

Choosing the correct discount rate also presents a problem to the valuation practitioner's task. Various texts in this area define the discount rate as the opportunity cost of capital, i.e., the rate that the valuator could earn in available alternate investments of equal risk. Most firms start by computing and using their own company-wide average cost of capital which may be higher or lower than a more accurate rate that matches the level of risk attached to the individual gold deposit in question. If the firm decides that the project currently under consideration is more (less) risky than its "average" project, it may account for this by adding (subtracting), in an ad hoc manner, an increment to its cost of
capital and thereby determine a discount rate appropriate for this more risky (less risky) project. We use the term “ad hoc” because the amount of the increment is usually not explained, supported, or otherwise justified by theoretical considerations; the increment is simply chosen by the analyst based on her experience and judgment.

To be theoretically correct, each and every input to the total cash flow during a time period should have an individual discount rate attached to it, depending on the relative uncertainty, and therefore risk, associated with it. For example, if extraction costs associated with the gold deposit are within the control of the mine operator and therefore more certain than the gold price which is totally exogenous, then costs should be discounted at a lower rate than the price discount. In practice this is rarely done.

Some firms use Monte-Carlo-based stochastic simulations to observe and understand the degree of uncertainty surrounding the expected values chosen for the DCF input variables. When using stochastic simulation, a probability distribution is attached to each input variable implicitly included in the numerator of the DCF, such as gold price, extraction cost, ore grade, recovery per cent, etc. After 500 to 1000 iterations of the NPV calculation, allowing the value of each of the input variables to vary stochastically for each iteration according to their probability distribution, a histogram of the NPVs is produced showing the probability of each NPV or NPV range. The histogram shape usually resembles the familiar bell-shaped curve of the normal probability distribution. The
standard deviation or "spread" of the histogram gives the degree of overall risk associated with the project.

A more preferred method of choosing the correct discount rate is to use the Capital Asset Pricing Model (CAPM)\(^1\) which is described in detail in all current finance textbooks (Brealey and Myers 1991). The CAPM theory links the risk and return of any investment through the following model:

\[ \mu = r_f + \beta \cdot \phi_m \]  

(1.2)

where

- \( \mu \) = the risk adjusted discount rate (RADR) appropriate for the project
- \( r_f \) = risk free rate of interest; typically measured by returns offered on U.S. Treasury Securities with a maturity date corresponding to the life of the project
- \( \beta \) = the Beta coefficient of an asset, which is a measure of the asset's nondiversifiable risk relative to the risk of the entire market of assets
- \( \phi_m \) = the risk premium of the market portfolio of assets defined as \((r_m - r_f)\)
- \( r_m \) = the expected return on a market portfolio of assets

Guzman (1991) has reported that a \( \beta \) of 0.45 for gold projects is an appropriate average, implying that gold projects are less risky (variable) on average than the overall market. Of course, any particular gold deposit (project) could be assigned a \( \beta \) higher or lower than 0.45, if it was perceived to be more risky or less risky than an "average" gold project.

\(^1\)The Capital Asset Pricing Model, known as CAPM, was developed by William Sharpe in 1964 and it is one of the primal theoretical foundations of modern finance, relating risk and return in asset market equilibrium.
In the theory of the CAPM, risk derived from initially uncertain input parameters has no direct bearing on the discount rate of a project. "Under the portfolio definition of risk, all that matters is the covariance of the NPV with movements in the market. This is taken care of by the beta term in the CAPM" (Davis 1995, 78). Thus, while DCF can handle risk in the portfolio sense by using a RADR computed from Equation (1.2), it cannot handle adequately the risk attributed to uncertainty of the value of input parameters, including in our case the price risk associated with future gold price. Practitioners generally attempt to analyze this uncertainty of input parameters by performing what is commonly referred to in DCF circles as "sensitivity analysis." In this procedure, each input variable is individually varied by discrete amounts over an acceptable range, and the NPV change is observed. In this way one can see how sensitive the NPV value is to the variable in question and one can make a ceteris paribus observation as to the riskiness of the project.

Other problems arise when using this traditional DCF method; the difficulties are caused by the simultaneous interrelations between the expected cash flow, the risk-adjusted discount rate, and the optimum management strategy. Even though the DCF method is the method of choice for most mining companies, it has recently come under increasing scrutiny in financial economics circles (Davis 1994; Dixit and Pindyck 1994;  

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2 Bhappu and Guzman (1994) report a survey they conducted, in which the percentage of mining companies using DCF in investment decisions is 95%. 
Guzman 1991). NPV techniques are useful and accurate in valuing projects with non-stochastic cash flows; they are less satisfactory in valuing projects where growth opportunities are significant, where revenues are stochastic due to the volatility of the product price, or where management has the ability to make a series of strategic decisions based on the outcome of intermediate events during the life of the project. Most operating gold deposits contain

- growth opportunities because of possible or even likely additional undiscovered mineral reserves
- stochastic revenues due to the volatility of gold prices in the 12-22% range
- significant management options in terms of varying production rates or even suspending operations

Thus, we will have to look to models other than DCF in order to capture the value (if any exists) of these characteristics of gold deposits.

The fact that DCF tends to undervalue gold investment projects may be because of implementation problems, rather than because of a shortcoming in the method itself.

Myers (1984) reports the following typical mistakes made in applying DCF that create a bias against long-lived projects (i.e., undervalue them):

- Ranking projects on IRR rather than NPV; it is easier to earn a high rate of return if the project life is short and investment is small. NPV is the only DCF calculation which gives you an accept/reject signal (accept if NPV>0) and also gives you the absolute dollar economic contribution that the project makes to shareholder value. The preferred use of IRR by industrial practitioners is validated by a survey of decision making criteria used by mining companies when evaluating mineral investments (Bhappu and

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3 Cash flows are non-stochastic when the revenues and operating costs are relatively certain given the levels of production and sales.
Guzman, 1994); they reported that 55% of their respondents used IRR as the primary tool, while only 40% used NPV as the primary tool.

- An inconsistent treatment of inflation; a surprising number of firms use high nominal discount rates, but do not adjust (inflate) their cash flows for future inflation. Thus their long-lived projects look less attractive than they really are. Bhappu and Guzman (1994) again confirm this in their survey: only 25% of the respondents use some type of escalated dollar cash flows, while 70% use constant dollar amounts (but presumably nominal discount rates).

- The use of unrealistically high discount rates, even after proper adjustment for inflation. These high rates may result from
  - an ignorance of what normal returns in capital markets are
  - premium tacked on for risks that can be easily diversified away in stockholders’ portfolios
  - premium tacked on to offset the overly optimistic biases of managers sponsoring projects.
  - some projects are very risky at inception, but only of normal risk after start up.

**Hotelling Valuation Principle (HVP)**

The HVP, as formulated by Miller and Upton (1985a), is a special type of the DCF model applied to a fixed stock of mineral resources. The HVP comes from the more generally known Hotelling Principle (HP) which was derived by Harold Hotelling (1931) in his seminal article concerning the net-price time-path of a fixed stock of optimally exploited minerals. The HP is generally considered as one of the classical works that define theoretical mineral economics (Solow 1974). The HVP says that if the HP holds, then the current value of a developed and operating gold property is equal to the units of total gold reserves times the current operating margin per unit. The technique does not
require any discounting or estimating the rate of inflation for either the price of gold or the marginal cost of mining the gold. In effect, the net operating margin (net price) is assumed to inflate at the discount rate, making the current net price the same as the discounted net price of any future period. Thus, while HVP discounts future revenues and costs, just as DCF does, it makes some special assumptions about how the revenues and costs change together in the future in the form of a net price.

Miller and Upton (1985a) found the HVP to be a reasonably accurate predictor of the market value of the oil and gas companies in its data set, and we therefore wish to consider it to value our gold deposits since it is a method specific to mineral deposit valuation (whereas DCF is not) and because of its utter simplicity.

As explained later in more detail, the HVP, and the HP from which it is derived, assume certainty in the present and future values of costs and reserves. Since ore grade and mineral recovery are subsumed in the extraction cost, these variables are also taken with certainty, which means the deposit is taken to be homogeneous. As for price, the literature surrounding Hotelling’s (1931) article and its implications for mineral exploitation have generally implied or directly stated that the price of the mineral is known with certainty. Hotelling himself did not directly state this, but he did present a downward sloping demand curve, which would fix the price, given the quantity sold and assuming market equilibrium exists. If demand or supply were to change, then disequilibrium would result and the price would presumably move to the new equilibrium point. Since costs are
known with certainty, the mine operator could adjust his output and thereby manipulate costs so that net price would continue to rise at the rate of interest. Thus price certainty is not a necessary condition for the Hotelling Principle and HVP to be operative; the spot price may go up or down as long as the mine operator is able to manipulate production quantity and thereby cause extraction costs to fall (or rise if need be) and thus keep net price rising at the proper rate.

**Option Pricing Method (OPM)**

The value of a developed, operating gold mine can also be estimated using OPM as developed by Black and Scholes (1973) and Merton (1973). The OPM is well developed and used extensively in markets for financial securities and base commodities. A financial call option gives its owner the right, but not the obligation to purchase the underlying asset at some specified time in the future and at some specified purchase price; the underlying asset in a financial option is a unit of some type of security or a unit of the base commodity. At the time of maturity of the option, if the market price of the underlying asset is greater than the fixed purchase price, the call option is exercised, otherwise the option is allowed to expire.

The “real option” analogy to financial options, in the case of a *developed* gold deposit, is to pay the unit extraction cost (the purchase or exercise price per ounce of gold) at any time and receive the underlying asset (the ounce of gold bullion) which has a value given by the price in the spot market. The use of the OPM to value real options
including mineral deposits has been suggested, developed, and analyzed by many authors (Bjerksund and Ekern 1990; Brennen and Schartz 1985; Davis 1994; Dixit and Pindyck 1994; Frimpong, Laughton, and Whiting 1991; Guzman 1991; Paddock, Siegel, and Smith 1988; Pindyck 1991; Sick 1989). These real options are known as operating options when they relate to the production of gold bullion from the in-situ, but developed, gold minerals; they are separate from the option of developing the deposit in the first place, but they still contribute to the deposit's value. Such operating options can be enumerated as follows:

1. The option to increase or decrease the rate of production
2. The option to temporarily shut down mining operations until economic conditions improve
3. The option to restart an operation that was previously shut down
4. The option to abandon a deposit that is thought to be permanently unprofitable

These operating options have value because they limit any economic downside of extracting the gold but allow any upside potential without limit. For example, if the price of gold were to drop down near or below the unit extraction cost so that the mine could not be operated profitably, the mine operator could curtail or temporarily shut down operations. If the price dropped so low that it was determined the deposit could never again be operated profitably, the deposit could be permanently abandoned. In these situations, the downside, represented by the unprofitability, would be limited by the option to shut down or abandon. If operations were curtailed or shut down and the price were to rise again into the region of profitability, the deposit could again be operated at an increased rate, and of course if the price went very high, this upside, in the form of
resource rent, could be enjoyed without limit. Thus these options give gold mine operators a defense strategy to use against the stochastic gold price changes by allowing for managerial flexibility which, in turn, produces a larger present value than if the deposit were operated at some prescribed rate until depleted without ever varying from the forecast, which is what DCF assumes in its Equation (1.1). A model which includes the managerial flexibility options, then, should be a more accurate model, since we observe this managerial flexibility in actual operations.

The option pricing model is based on the application of stochastic calculus, where the uncertain time path of gold price is modeled according to a Wiener Process; a full development of the OPM is given in later chapters. OPM thus handles uncertainty directly by assuming a model which describes the stochastic value of the affected variables; in our study only the gold price is considered stochastic, costs are considered known with certainty as are other variables. Thus input uncertainty of gold price is modeled directly, as opposed to DCF where an expected value is used, and as opposed to HVP where the gold price is subsumed in the net price, which is controlled by the mine operator so as to rise at the rate of interest through production cost manipulation.
Thesis Presentation

Purpose and Objectives

The purpose of this work in general is to investigate the valuation of developed gold deposits and more specifically to investigate the usefulness of the HVP and the OPM as alternative valuation techniques to DCF. Commonly used DCF methods, using risk-adjusted discount rates but ignoring input parameter uncertainty, appear to underestimate mineral asset values, both at the project and firm level (Davis, 1994); thus the motivation for this work is to seek a method better than DCF. HVP may do a better job than DCF and is certainly easier to apply. Miller and Upton (1985a) in their empirical test of the HVP found that the HVP is more highly correlated with imputed market values of oil and gas deposits than two other public sources of valuation which are based on the DCF method. In another test of the HVP, Miller and Upton (1985b) could not duplicate their earlier results; other investigators (Watkins 1992) also had less than validating results when testing the HVP. So does the HVP and therefore the Hotelling Principle’s μ% rule hold or not? And is the HVP equally valid for all minerals, including our mineral of interest, developed gold deposits? The HVP theory and its assumptions are not mineral specific, but all the HVP research to date has been on oil and gas which has an extractive behavior quite different from hard rock minerals.

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Oil and gas are fluids, which are pumped or otherwise made to flow from underground reservoirs. A distinguishing feature of oil and gas production is that production generally shows an exponential decline over time with the decline rate given as a stated percent for each time period.
The OPM paradigm is worth looking at for four reasons:

1. The assumptions of underlying stochastic behavior of output prices used in OPM seem to accurately describe natural resource investment projects and thus the use of OPM may accurately model the stochastic behavior of any value associated with it.
2. Since managerial flexibility is observed in gold mining operations, this flexibility can be modeled as operating options and any value associated with it can be captured with OPM.
3. As we shall see later, HVP does not work well when the net price of gold approaches zero while at the same time the option to close the mine achieves its largest value at this point, and thus from a model building point of view, the OPM may work well as an “add on” valuation factor to HVP.
4. The OPM is a popular topic in financial economic circles at this time, with much research and journal space being taken up by the subject.

The HVP and OPM techniques will therefore be theoretically presented and analytically developed as separate models. The HVP model and the OPM model will then be empirically tested as stand-alone models using the publicly available financial and operating data of 23 North American gold mining companies. Any deviations of the empirical test results from the HVP theory will be discussed and analyzed and the two stand-alone model results will be compared. We will then develop a combined model using both the HVP and OPM models, being careful to delete any redundant terms, to see if such a combined model can explain the empirical results better than an individual HVP or OPM model could.

Our primary objective is to determine whether these techniques provide useful and accurate valuations of developed gold assets by comparing the valuation so determined against the valuation given by the financial markets. We assume here that the market-
determined value is the correct or true value, following from the efficient market hypothesis. A secondary objective is to provide a relatively simple methodology that industrial practitioners can use, either as a supplement or as an alternative to their present DCF models.

As mentioned above and explained in detail in Chapter 3, the data we use for the empirical testing of HVP and OPM are taken in part from publicly available data, including the market value of the common stock equity of operating gold companies. Since we are accepting the efficient market hypothesis, at least in a semi-strong form, then these market equity values will contain the value of all publicly available information including the "going concern" value of the gold firm. We will isolate and measure this value that operating gold mining firms have over and above their gold deposits by themselves.

Organization

What follows is a detailed theoretical development and discussion of the HVP and OPM as applied to the valuation of real assets, specifically developed gold reserves. The data sources and transformations are then described along with the econometric methods of empirically testing the HVP and OPM using the data. Following this, the econometric and financial analysis results are presented and a combined HVP + OPM valuation model of gold deposits is formulated. The "going concern" value of gold companies is analyzed, measured, and formulated as a value distinct from the value of the in-situ deposits themselves. Conclusions and recommendations for further work follow.
Chapter 2

VALUATION MODELS

Two valuation alternatives to DCF, the HVP and the OPM, are developed and analyzed in this section. The use of the DCF method in valuing gold projects/deposits is not discussed further in this work since it is well documented elsewhere (Gentry and O’Neil 1984; Stermole and Stermole 1994)

The Hotelling Principle and the HVP

Hotelling Principle (HP)

Hotelling (1931) introduced the concept that, under certain simplifying assumptions, the net prices of minerals will rise over time at the rate of interest. This has come to be known as the “Hotelling Principle” or the “r% rule”. In this paper we use $\mu$ as the risk adjusted rate of interest, not $r$, so we will refer to the HP as the “$\mu$% rule”; we earlier use $r$ as the symbol for the risk free interest rate and the market interest rate. The same basic problem, analysis, and conclusions of Hotelling were introduced by Gray (1914). Gray defined the problem from the standpoint of a single mine operator facing a horizontal demand curve (i.e., perfect competition) while Hotelling approached the
problem from the level of industry facing a downward sloping industry demand curve. Both authors assumed that the demand curve was fixed, i.e., did not shift, and that the market was in equilibrium.

Hotelling himself did not define or otherwise discuss the rate of interest. He definitely did not refer to the rate of interest as risk adjusted in the way of modern financial theory; the Capital Asset Pricing Model was not presented until some 33 years later. Current acceptable financial theory now tells us that "the" interest rate must be a risk adjusted rate; since most of the input parameters are assumed known with certainty by the HP (as we shall soon discuss), then the proper interest rate to use with the HP is the risk free rate. More on this later. Thus the HP assumes the interest rate to be certain and constant and exogenous to the process, i.e., the rate is determined by the general market for assets.

For net price to rise in Gray's model of an individual mine facing a horizontal demand curve and therefore constant price, the marginal cost must fall over time, necessitating a decreasing quantity extracted over time, ceteris paribus. The decreasing quantity over time will allow the marginal cost to decrease as it moves down the upward sloping marginal cost curve, until marginal cost equals average cost in the final time period when the mineral is completely exhausted.

Hotelling took the viewpoint of a social planning agency that had as its goal the maximization of social welfare from the production of minerals; the social welfare was
defined as the present value of the total consumer and producer surplus over the time
periods of extraction. Herfindahl (1955) gives a concise summary of the Gray and
Hotelling assumptions and conclusions.

Both Gray and Hotelling however, arrived at the same condition for the efficient
extraction of a mineral: the present value of a unit of a homogeneous but finite stock of
the mineral in the ground must be constant regardless of when it is extracted. This, of
course, is the same as saying that the current unit value of the mineral (the net price) must
grow in the future at the rate of interest.

The quantity of the mineral extracted during this time of Hotelling rising prices will
conform to this rising net price scenario so that the basic economic law of demand is not
violated, i.e. if the spot price rises, the quantity consumed, and therefore produced, will
fall or vice versa. As previously stated, an individual price-taking mine operator facing a
horizontal demand curve can only keep net prices rising by adjusting production and
thereby reducing marginal costs. If all operators are doing this independently but
simultaneously, then supply must be falling and the industry supply curve will shift to the
left, producing a new equilibrium at a lower q and a higher P. Thus the horizontal demand
curve of an individual operator will eventually move upward and the operator will adjust
production again to keep net price rising at μ%.

This “Hotelling Principle” is derived simply from the assumption that individual
mine operators, like other business managers, seek to maximize profits, and therefore to
maximize the present value of the mineral property under their control. The HP can also be explained as a requirement of equilibrium in asset markets where all assets of the same risk class must necessarily increase in value at the same rate over time; if not, an arbitrage opportunity would exist and mine owners could earn a riskless profit by either selling or buying the gold reserves as the case might be. If the net price of gold was expected to rise by less than the risk-adjusted rate of interest, the owner would extract the gold immediately, sell the bullion, and invest the proceeds in a similar risk asset that would earn the higher market rate. If the net price was expected to rise by more than the risk-adjusted rate of interest, the owner would extract none of the gold, because it would be appreciating faster in the ground than would the invested cash profit obtained by producing and selling it. Hence, the value-optimizing owner will extract the gold at a rate such that the net price will rise at μ%.

Hotelling Theory’s basic assumptions for the μ% rule to hold are these:

- The owner of an exhaustible mineral wants to exploit it so as to maximize NPV.
- The market for the mineral is perfectly competitive; the individual owner cannot influence the price.
- Future costs are known with certainty.
- Costs will not fall over time due to technological innovations.
- Costs will not rise over time due to lower grades or less accessible reserves; i.e., the resource stock is homogenous.
- The resource stock will not increase due to exploration success; i.e., it is fixed and known with certainty.
Notice that we do not list output prices as known with certainty as most of the HP literature does. Our HP gold prices would be known with certainty if costs, quantity produced, and demand were known with certainty themselves, and this is what most authors assume. We, however, will assume that demand, and therefore the gold price, is not known with certainty, but that the HP and HVP can still be operative as long as operators can manipulate the net price through cost control so that it still rises at μ%. Our analysis reaches the same conclusion as that of Pindyck (1980). Pindyck found that with constant extraction costs and risk-neutral mineral producers, demand uncertainty does not affect the expected price dynamics in competitive markets and Hotelling's μ% rule still applies. He modeled demand uncertainty by assuming that the market demand function shifts randomly but continuously through time according to a stochastic process.

Given these assumptions, we can derive the HP and its famous "μ% rule" by considering the question facing the profit-maximizing, price-taking owner of a gold-bearing, exhaustible mineral deposit: "Should we extract the reserves in the current period or should we wait until any of the next N periods?"

Since we are assuming that the individual mining firm seeks to maximize the present value (PV) of profits generated over the time of production, a production profile over time must be chosen that maximizes the difference in total revenue and total costs, all discounted to the present. The general objective function to maximize, then, is given by
\[
\max_{q_t} \quad V_0 = \sum_{t=0}^{N} \left[ \frac{P_t \cdot q_t - C_t(q_t, Q_t)}{(1 + \mu)^t} \right]
\]

subject to

\[
\sum_{t=0}^{N} q_t = R_0
\]

where when considering any time \( t \) in the future:

- \( V_0 \) = the current fair market value of the in-situ gold reserves
- \( P_t \) = the exogenously determined spot price of gold at time \( t \) in the future
- \( q_t \) = production of gold in ounces during time \( t \) in the future
- \( Q_t \) = cumulative production from the present through time \( t \)
- \( C_t(q_t, Q_t) \) = total extraction cost of the production of gold during time \( t \) in the future
- \( \mu \) = the market rate of return for assets of risk comparable to gold mining; assumed to be constant during all \( t \)
- \( N \) = the number of time periods until the gold reserves are depleted
- \( R_0 \) = the current amount of gold reserves in ounces

Using the methods of constrained optimization, the first order condition for profit maximization in any period is given by (Miller and Upton 1985a, Eq. 3):

\[
\left( \frac{P_t - c_{q,t}}{(1 + \mu)^t} \right) = \sum_{t=0}^{N} \left( \frac{\partial C_t}{\partial Q_t} \right) \cdot \left( \frac{1}{1 + \mu} \right) = \lambda
\]

where

- \( c_{q,t} = \frac{\partial C_t}{\partial q_t} \) = marginal extraction cost at any time \( t \)
- \( \lambda \) = the Lagrangian multiplier, interpreted in this case as the increase in \( V \) if \( R_0 \) were increased by one more ounce of gold; also known as the "user cost" or "shadow price" of in-situ reserves
\[ \frac{\partial C_s}{\partial Q_s} = \text{the rate of change of total costs with cumulative extraction at time } s \geq t \]

As to the properties of \( C_s \), we know \( \frac{\partial C_s}{\partial Q_s} > 0 \) since total extraction costs in period \( t \) will rise with the amount extracted; \( \frac{\partial C_s}{\partial Q_s} \geq 0 \) and will be >0 if additional reserves are increasingly costly to extract. Reserves might be increasingly costly to extract if the deposit is non-homogeneous and the less costly reserves are extracted first. Another explanation of extraction cost increasing over time is that the extraction takes place at increasingly deeper levels within the deposit which would increase the unit cost even if the deposit were homogeneous.

Hotelling himself introduced and discussed the case where \( \frac{\partial C_s}{\partial Q_s} > 0 \), although Equation (2.3) shows that it does not lead to the famous \( \mu \% \) rule. If \( \frac{\partial C_s}{\partial Q_s} > 0 \), then the net price will rise at a rate less than \( \mu \% \). For our present discussion, let us assume for now that the production function of the gold mining company allows the unit extraction cost to be independent of cumulative production \( Q \), so that \( \frac{\partial C_s}{\partial Q_s} = 0 \). Then the first-order condition of Eq. (2.3) reduces to

\[ \text{...} \]

\[ ^5 \text{Ceteris paribus, one would always extract the least costly reserves first, if maximization of the present value of future profits were the objective.} \]
\[
\frac{(P_i - c_{g,t})}{(1 + \mu)^t} = \lambda
\]  

Equation (2.4) tells us that in an optimal production program, working under the above HP assumptions, the present value of the net price per unit of output must be the same regardless of when it is produced. Using the method of difference equations (Dowling 1992), we can solve the system of equations (2.4) at various times \( t \) and, after dropping the \( q \) subscript on the marginal extraction cost for simplicity of exposition, we obtain the familiar Hotelling Principle:

\[
(P_i - c_t) = (P_0 - c_0) \cdot (1 + \mu)^t
\]  

In this form we can see that the net price in the future is simply the current net price compounded at the appropriate rate of interest. It can be seen from Equation (2.5) and from the above discussion of costs that the famous Hotelling "\( \mu \% \) rule" is actually a very special limiting case where \( \frac{\partial C_i}{\partial Q_s} = 0 \).

If we do not make the simplifying assumption of \( \frac{\partial C_i}{\partial Q_s} = 0 \), then we can manipulate and rearrange Equation (2.3) (Levhari and Liviatan, 1977) and present the HP as

\[
MR_0 = MC_0 + \lambda + MC_d
\]  

where, in this presentation

\[
MR_0 = \text{the marginal revenue of the mineral unit; the same as the current spot price in this competitive gold market}
\]
\[ MC_0 = \frac{\partial C_0}{\partial q_0}, \] the standard (or normal or regular) marginal cost of producing the mineral unit

\[ \lambda = \text{the present value of the net price of the unit of mineral in the terminal period } T; \text{ this is also called the "user cost" and it represents the opportunity cost of producing the unit today instead of waiting until the terminal time } T \text{ in the future; this opportunity cost is unique to the production of exhaustible minerals} \]

\[ MC_d = \text{the present value of all future additional costs of production caused by producing a unit now rather than later; caused by } \frac{\partial C}{\partial q} > 0; \text{ this cost component is often called the "degradation" cost} \]

Thus under the HP, optimal output does not take place where marginal revenue equals marginal cost as it does in classical microeconomics, but rather where marginal revenue equals "full" marginal cost. "Full" marginal cost, in this case, is the sum of regular marginal cost + user cost + degradation cost; this requires that \( q_{HP} \) be less than the \( q^* \) given by classical microeconomics.

**Hotelling Valuation Principle (HVP)**

The HVP, as formulated by Miller and Upton (1985a), is less well known than the basic Hotelling "\( \mu \)% rule", but using certain *additional* simplifying assumptions, it is derived directly from the Hotelling concept. The HVP says that where mineral output prices, *net of mineral extraction costs*, obey the basic Hotelling Principle of net prices rising over time at \( \mu \)%, the value of the total reserves in any mineral property depends

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6 These additional assumptions of the HVP, separate from the basic assumptions of the Hotelling Principal, are discussed in the next several paragraphs.
solely on the current spot price per unit of the mineral, net of current extraction costs.

The units of production deferred to future years will rise in value at the risk-adjusted rate of interest, \( \mu \), to higher net prices, but the growth of the net price will be exactly offset by the discounting factor, \( \mu \), used to compute the current present value; therefore the current net price is the in-situ unit value without any adjustments or discounting of future cash flows. An important point is that it does not matter which interest rate and discount rate are being used by the HVP: risk free, risk adjusted, nominal, real, etc. As long as the two rates are numerically equal, then HVP must hold.

The HVP result, as stated above, can be quantified using Equations (2.1) - (2.5) and making some additional assumptions. Keeping the assumption for the time being that

\[
\frac{\partial c_0}{\partial Q_s} = 0
\]

and therefore that Equation (2.4) holds, let us also assume that the gold deposit also enjoys constant returns-to-scale in current extraction as well as cumulative extraction. The current marginal cost \( c_0 \) is constant and the same as average cost, \( \bar{c}_0 \), so that the substitution of Equation. (2.5) into Equation. (2.1) yields as the present value of the total reserves:

\[
V_0 = (P_0 - \bar{c}_0) \cdot \sum_{t=0}^{N} q_t = (P_0 - \bar{c}_0) \cdot R_0
\]  

(2.7)

Equation (2.7) can be rearranged by dividing both sides by \( R_0 \) (Miller and Upton 1985a, 5-6) so that the HVP is stated per unit of current reserves:
\[ \frac{V_0}{R_0} = (P_0 - \tilde{c}_0) \]  \tag{2.8}

Miller and Upton extend the analysis of the HVP to non-constant returns-to-scale, to extraction costs increasing with cumulative production, to risky outcomes, and to the presence of income taxes. The effect of these relaxations of the basic assumptions is discussed in the next section.

**Empirically Testing the HVP**

In the manner of Miller and Upton (1985a), we can test the implication of the HVP on gold reserves by regressing the observed current market values per unit of current reserves, \( \frac{V_0}{R_0} \), of any particular gold company on the corresponding HVP value \( (P_0 - \tilde{c}_0) \) of that company's reserves. This method gives us the simple linear regression equation

\[ \frac{V_0^i}{R_0^i} = \beta_0 + \beta_1 \cdot (P_0^i - \tilde{c}_0^i) \]  \tag{3.1}

which comes from Equation (2.8) and where \( i \) indexes companies and the \( 0 \) subscript on the variables represents the then current values of spot price, average cost, market value, and reserves at the sample date \( t \). The test of the HVP then hinges on the values of the regression coefficients. Under the constant returns-to-scale assumption, the HVP implies that \( \beta_0 = 0 \) and \( \beta_1 = 1 \) and also implies that additional variables such as interest rates or
projected future mineral prices will contribute nothing to the explanatory power of Equation (3.1); i.e., net price is the only state variable.

If we relax the assumption of constant returns-to-scale (marginal extraction cost = average extraction cost), the effect will be found in the intercept term and is likely to be small and of ambiguous sign (Miller and Upton 1985a, 6-8). A second variation, extraction costs increasing with cumulative extraction and therefore incurring degradation costs, would also affect the intercept and not the slope of unity. This degradation cost effect on the intercept term would again be small but probably negative. Miller and Upton conclude that the combined effect of these relaxed assumptions concerning returns-to-scale of current and cumulative production is found solely in the intercept term and is unlikely to move it far from the zero value that the HVP theory predicts.

Miller and Upton (1985a) further explain that risk adjustments would be of little consequence because any effect they had on the expected change in net prices (the numerator) would have offsetting impacts on the discount rate (the denominator). As stated previously, if the rates are the same, the HVP holds no matter what rate is used. We will explore a possible difference in the two rates when we analyze our empirical testing results in a later section.

\footnote{Constant returns-to-scale would be characterized by a range of output, \( q \), where the long run average cost curve would have a flat spot at the bottom of its "U" shape. This horizontal, or nearly so, area must have the marginal cost equal to average cost, otherwise the average cost would rise or fall.}
Income taxes can be important in empirical testing, even though the various theoretical analyses of the HP for the most part ignore the effect of income taxes on the "\( \mu \% \) rule." Real world observations of market values, however, can be presumed to take income taxes into account, and we must therefore make some theoretical assumption as to how taxes enter into the predicted values. The "\( \mu \% \) rule" could be interpreted as "net prices, after income tax, grow at the after-tax interest rate," and the HVP would imply a slope value of 1.0 minus the tax rate. Even so, this would only imply a lower bound for the slope since the stated tax rate\(^8\) neglects the many offsets (deductions) available under the tax laws. Miller and Upton deduced that income taxes would push the slope below 1.0, "but not by much." Our data set does not include income taxes in the extraction costs; neither does the Miller and Upton data. Other investigators (Watkins 1992) do include income taxes in their cost data; more about other investigations later.

In testing the hypothesis and explaining any deviations from the theory, we will utilize four Hotelling models. The models are of increasing complexity and they differ in their assumptions concerning the extraction cost function and in their assumptions concerning the fixed stock of gold resources. All of the models assume that the price of

\(^{8}\) During the time period of our data, the U.S. Federal corporate income tax rate averaged 34\%. Estimating average state income taxes at 3\%, a maximum income tax rate of 36\% would be operative on our data. It must be remembered that income taxes are computed on net income after all expenses including selling, general, and administrative expenses; (P - C) does not include SG & A expense and the effective tax rate on the (P - C) margin would be less than 34\%. Also gold producers are allowed a special percentage depletion tax allowance which shields 15\% of their gross revenues from taxation. The state of Nevada has no state income tax.
gold is exogenous; i.e., the gold mining firm is a competitive price taker; as previously stated, the price may be stochastic or not, only net price is taken to be certain.

**Hotelling Model 1**

Hotelling Model 1 (HM1) is the model of Miller and Upton (1985a) and it assumes

\[
\frac{\partial C_t}{\partial Q_t} = 0 \quad \text{and} \quad \frac{\partial C_t}{\partial q_t} = \bar{c}_t \quad \text{so that the second term on the LHS of Equation (2.3) drops out.}
\]

Since costs are constant at \( \bar{c}_t \), \( C_t \) can be represented by \( \bar{c}_t \cdot q_t \) and Equation (2.1) becomes

\[
V_0 = \sum_{t=0}^{N} \left[ \frac{P_t - \bar{c}_t}{(1 + \mu)^t} \cdot q_t \right]
\]

But, from Equation (2.5) we can see that

\[
\frac{P_t - \bar{c}_t}{(1 + \mu)^t} = (P_0 - \bar{c}_0)
\]

so Equation (2.1) now becomes \( V_0 = (P_0 - \bar{c}_0) \cdot \sum_{t=0}^{N} q_t \). Substituting from Equation (2.2) for the \( \sum_{t=0}^{N} q_t \) term, we get

\[
V_0 = (P_0 - \bar{c}_0) \cdot R_0 \quad \text{or} \quad \frac{V_0}{R_0} = (P_0 - \bar{c}_0)
\]

which is the same as Equation (2.8).

Since from Equation (2.5) the net price, \( (P_0 - \bar{c}) \), is rising at the risk-adjusted rate of interest when \( \frac{\partial C_t}{\partial Q_t} = 0 \), then the PV of the net price of any unit must be the same
regardless of when extracted. Because costs do not increase with cumulative extraction then the numerator of equation (2.1) becomes \((P_t - \bar{c}) \cdot q_t\), and the growth of the net price in the numerators of the terms in the summation of Equation (2.1) will be exactly offset by the discount factors in the denominators. Therefore the value of an ounce of gold at any time in the future is equal to its net price at the current time (t=0); this is the "Hotelling Valuation Principle" that our empirical test is based upon. When the current net price per unit is multiplied by the current reserves, \(R_0\), the result is the value of the entire deposit \((V_0)\). This simple form immediately suggests empirical testing by a simple linear regression model, where it is hypothesized that the slope = 1.0 and the intercept = 0.0. Since the assumptions of the model allow us to use average cost instead of marginal cost, abundant data is readily available for empirical testing; in general, industrial marginal cost data is not available.

**Hotelling Model 2**

Hotelling Model 2 (HM2) assumes \(\frac{\partial C}{\partial Q_t} = 0\) as does HM1, but it does allow \(\frac{\partial^2 C}{\partial q_t^2} > 0\) because of decreasing returns to scale. Fundamental microeconomics tells us that when decreasing returns to scale are present, the marginal cost curve lies above the average cost curve and the average cost is therefore rising with an increasing output rate. Miller and Upton (1985a, 6-9) showed that this particular relaxation of one of the HM1
assumptions would be impounded in the intercept which we can no longer presume to be 0. The sign of the intercept is ambiguous, but they claim any departure is likely to be small. Therefore empirical testing of this model can still use average cost in the HVP regression data and the slope is presumably still 1.0. If the intercept is found different from 0, then it can be explained as a deviation caused by differences in marginal cost and average cost, i.e. \( \frac{\partial C_i}{\partial q_i} \neq 0 \).

**Hotelling Model 3**

Hotelling Model 3 (HM3) assumes \( \frac{\partial C_i}{\partial Q_i} > 0 \) and \( \frac{\partial^2 C_i}{\partial q_i^2} > 0 \), so that extraction cost now increases with cumulative production and with current production. The concept of production costs increasing with increasing cumulative extraction (or increasing with decreasing reserves) goes back to Ricardo (1817) and Mill (1848). They predicted that the higher grade (i.e., lower cost per unit of mineral produced) deposits of an exhaustible resource are exploited first, and as deeper and thinner deposits are exploited, cost will rise. Miller and Upton (1985a) again showed that this additional relaxation of an HM1 and HM2 assumption would also be impounded in the intercept; therefore average costs can still be used in the regression data and the slope of the linear regression would presumably still be 1.0. Any intercept different from 0.0 will be explained in terms of rising marginal costs and/or in terms of costs rising with cumulative extraction.
Hotelling Model 4

Hotelling Model 4 (HM4) starts with HM3 and further generalizes it by allowing exploration simultaneously with extraction. Hartwick (1991) took Pindyck's (1978) model of the optimal exploitation of a mineral deposit when exploration is allowed, and reworked it so as to show that a pure µ% rule can also govern an exploring/extracting firm. Exploration reduces the total in-situ value by adding an additional cost, while at the same time increasing the in-situ value by reducing future extraction cost due to increasing reserves. Hartwick shows that if exploration is adding reserves at the same rate that the reserves are being depleted, the HVP model can be easily modified to include an average exploration cost in addition to the average extraction cost, and the regression intercept and slope should still be 0.0 and 1.0 respectively. This model is not tested in this thesis.

Results of Previous Tests of the Hotelling Principal

There exists a large and well-developed literature based on the theoretical framework introduced by Hotelling. Nonetheless, the ability of the theory to describe and predict the actual behavior of resource markets remains largely unproved; see Halvorsen and Smith (1986) for a summary.

The principal problem of empirical tests of the Hotelling Principal has been data availability. The implications of the theory for economic behavior are found in the time

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9 This would not be an unusual case, since after a reserve build up in the early life of a mine, management will try to find new reserves on an annual basis equal to the reserves depleted due to annual production.
path of the unextracted resource's net price. The net price is also referred to in various places in the literature as the in-situ price of the resource, the shadow price of the resource, and the scarcity rent of the resource. Throughout this work, we refer to it as the net price and represent it as (P - c).

Hotelling's simplest model calls for a steady rise in the net price of non-renewable resources, with interest rate being the central determinant of the growth rate. Hotelling's μ% rule has also been interpreted as a statement on resource scarcity; the rise in rent is indicative of rising scarcity, which is the empirical fact needing testing. Because of vertical integration, however, in natural resource industries, market transactions generally occur only after a resource has been extracted and processed, hence the base commodity in-situ, net-price time path may be nearly impossible to observe. Another problem is that some studies investigate the path of prices and not in-situ net prices; net price in the ground is what the theory predicts, not the spot price of the produced mineral. The spot price may go up or down at any rate; the HP would still be intact if net prices behave properly.

In the following list, we summarize several empirical investigations of the Hotelling Principal to date. First we look at five studies that investigated the price of the produced mineral; we then describe two studies of the in-situ net price.

- Barnett and Morse (1963) examined the hypothesis of increasing economic scarcity of natural resources, rather than testing the consistency of the HP rising prices. Their study was based on a purely Ricardian view, implying that scarcity arises from increases in extraction cost rather than exhaustibility. They use two alternative measures for Ricardian scarcity: (1) absolute productivity of labor and capital, and (2) productivity in
extractive sectors relative to productivity in manufacturing sectors. Within the long-run focus of their study which was based on U.S. data for 1870-1957, they did not find any statistical support for the hypothesis of rising scarcity. The study was later updated (Barnett 1974) to include non-U.S. data; again he could not find any statistically significant trends to validate the HP.

- Smith (1979) studied a wide range of resource prices while controlling for such important factors as extraction costs, new discoveries, and changes in market structure. He concluded that for the period 1900-1973 the trend in mineral prices was negative with the rate of decline decreasing over time in absolute magnitude. He suggested that simple time series models, rather than the HP, have the best predictive power of mineral prices.

- Slade (1982) concluded, from a study of twelve major metals and fuels, that the price paths for exhaustible natural resources were U-shaped. She argued that the declining portion of the price-path can be explained by technological change and that the scarcity component will eventually become dominant and cause the price to rise.

- Heal and Barrow (1980) found that interest rate changes, but not levels, were significantly related to metal prices. They thus conclude that resource pricing is affected by arbitrage in asset markets, i.e. capital theoretic considerations are important in the formation of resource prices.

- Smith (1981) extended the Heal and Barrow analysis to a wider range of natural resource products and examined long-term rather than short-term price movements. He tested and rejected a Hotelling-type model in which extraction costs were assumed to be zero, while finding that Heal-Barrow type models incorporating changes in interest rates do have some predictable power.

The following two published studies tested the theory of exhaustible resources using time series estimates of in-situ “net” prices of resources, and they reached contrasting conclusions:

- Stollery (1983), used annual data for the International Nickel Company for 1952-1973 to estimate a log-linear demand function and a Cobb-Douglas production function. The estimation results were used to calculate the price of the resource in-situ as the difference between marginal revenue and marginal cost. The null hypothesis that the estimated time path of in-situ
prices was consistent with the time path implied by Equation (2.3) was accepted, with the estimated rate of discount being 15%.

- Farrow (1985), estimated a cost function for a U.S. metal mining firm using monthly data for 1975-1981. The in-situ value was calculated as the difference between the product price and the estimated marginal extraction cost. His estimation of a number of alternative specifications of test equations based on Equation (2.3) yielded results inconsistent with the Hotelling Principle. In fact, he found significantly negative estimates of the rate of discount, which he concluded would follow from net prices decreasing over time rather than increasing.

We thus can see that empirical studies have not verified that the HP and the μ% rule hold, and in some cases have given contradicting results.

**Results of Previous Tests of the HVP**

Miller and Upton (1985a) used Equation (3.1) to test the HVP on 94 pooled cross-section time-series data points from 39 oil and gas companies covering the time period 1979-1981. Nineteen companies had 3 points, 17 companies had 2 points, and 3 companies had 1 point. Their estimated parameters were strongly supportive of the HVP. While the explanatory power of their results, depicted by the $R^2$ statistic, was not high, this is not unusual with cross sectional data. Additional regressions with their data, breaking the independent variable $(P_o - \bar{c}_o)$ into its components $P_o$ and $\bar{c}_o$ yielded results consistent with the basic regression: the coefficients of P and c were both 1.0 or a "little less" and the cost coefficient was negative. See row 1 of Table 1 for econometric results of Miller and Upton’s (1985a) test. Miller and Upton (1985b) repeated the same empirical test of the HVP on a second set of time-series cross-section oil and gas data.
Table 1 Results of Previous Empirical Tests of the HVP

<table>
<thead>
<tr>
<th>Previous Test</th>
<th>$\beta_0$</th>
<th>$t$-Stat for $H_0: \beta_0=0$</th>
<th>$\beta_1$</th>
<th>$t$-Stat for $H_0: \beta_1=1$</th>
<th>$R^2$</th>
<th>No. Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miller and Upton 1985a (Oil and Gas)</td>
<td>-2.24</td>
<td>-2.2</td>
<td>0.91</td>
<td>-1.0</td>
<td>0.42</td>
<td>94</td>
</tr>
<tr>
<td>Miller and Upton 1985b (Oil and Gas)</td>
<td>-0.47</td>
<td>-0.3</td>
<td>0.47</td>
<td>-4.7</td>
<td>0.15</td>
<td>98</td>
</tr>
<tr>
<td>Watkins 1992 (Oil and Gas)</td>
<td>-12.8</td>
<td>-0.8</td>
<td>0.61</td>
<td>-5.1</td>
<td>0.71</td>
<td>27</td>
</tr>
<tr>
<td>White 1993 (Gold)</td>
<td>78.9</td>
<td>3.2</td>
<td>0.47</td>
<td>-4.7</td>
<td>0.70</td>
<td>306</td>
</tr>
</tbody>
</table>

covering the later time period of 1981-1983. This second test, shown as row 2 of Table 1, was still *somewhat* supportive of the HVP in that the scatter plot of $\frac{V_t}{R_t}$ vs. $(P_t - \bar{c}_t)$ continued to show a pervasive linear relation of the variables. The explanatory power dropped considerably however, and the estimate of the coefficient $\beta_1$ was 0.47 or only 47% of the theoretical value of 1.0, and the t-statistic showed the slope to be significantly different from 1.0. The authors attributed the difference in results between the second test and the first test to the different price history of the two sample periods, and, therefore, the independent variable $(P_t - \bar{c}_t)$ had rather wide swings. The first period was characterized by a relatively sharp movement in oil and gas prices; by contrast, the second period was one of relatively quiet oil and gas prices. Because the data is subject to
considerable measurement error, this difference is important. The lower "signal to noise" ratio during the second period reduces the precision and biases the slope coefficient, $\beta_1$, towards zero.

Watkins (1992) tested the HVP using a data set of 27 actual sales transactions of developed Canadian oil and gas reserves during the period 1989-1991. While Miller and Upton (1985a and 1985b) inferred the market value of in-situ reserves by using market values of the debt and equity of the oil and gas companies and combining this with financial accounting data, Watkins used the actual reserve sales transactions where the values are not proxies and are thus uncontaminated by other asset values and unaffected by financial accounting principles. His method was not to use Equation (3.1) as the regression model but to compute the following ratio for each sales transaction:

$$\frac{V_0}{\left(P_{oil} \cdot R_{oil} \right) + \left(P_{gas} \cdot R_{gas}\right)}$$

(3.3)

where

- $V_0$ = the total value of the deposit as given by the actual sales price
- $P_{oil}$ = the net price of oil
- $P_{gas}$ = the net price of gas
- $R_{oil}$ = the in-situ reserves of oil sold
- $R_{gas}$ = the in-situ reserves of gas sold

We took Watkins's actual data points and regressed them according to Equation (3.2) to facilitate the comparison to Miller and Upton's tests; the results are shown in row 3 of
Table 1. We again find that the results are not strongly supportive of the HVP; the negative intercept and the slope of 0.61 place Watkins's results very close to Miller and Upton's second test. Watkins suggests that the HVP is a special case of a model by Adelman (1990), and he rejects it as a useful model for valuing oil and gas reserves because a statistical test of the mean ratio of 0.54, computed using Equation (3.3), rejects the null hypothesis that the ratio is 1.0.

Adelman (1990) did not perform an empirical test of the HVP, but he did offer an analysis and explanation of the Miller and Upton's (1985a, 1985b) results. He concludes that the assumption of a fixed stock of exhaustible resource to divide between two or more extracting periods is an irrelevant, non-binding constraint. He notes that in the long run, practically all mineral prices have declined; what actually exists are flows from unknown resources into a reserve inventory. There are good reasons to expect reserve replenishment to show diminishing returns over time and thus by standard microeconomics, marginal costs and prices would rise even if ultimate depletion were very distant. "There exists an endless tug-of-war between diminishing returns and increasing technology and knowledge" (Adelman 1990, 2). He suggested that the HVP of Equation (2.8) be modified as follows:

$$\frac{V_0}{R_0} = -\frac{K_0}{R_0} + \left(\frac{\alpha}{\alpha + \mu}\right)(P_0 - \bar{c}_0) \quad (3.4)$$

where
\[ V_0 = \text{the total value of the deposit as given by the sales price} \]
\[ R_0 = \text{the mineral reserves at time zero} \]
\[ K_0 = \text{the total capital cost needed to develop the reserves} \]
\[ \alpha = \text{the depletion/decline rate, as a percentage per year} \]
\[ (P_t - \bar{c}_t) = \text{the net price as previously defined} \]
\[ \mu = \text{the discount rate as previously defined} \]

In our study, the gold deposits are currently developed and producing and therefore \( K_0 \) is zero; thus the first term on the RHS of Adelman’s Equation (3.4), the intercept, is zero and Equation (3.4) therefore takes on the form of the HVP model. If \( \left( \frac{\alpha}{\alpha + \mu} \right) = 1.0 \), then Adelman’s model would be the same as the HVP model. Adelman points out that in the United States, \( \alpha \) and \( \mu \) have tended toward equality which would make the slope coefficient equal to 0.5; this is supported by Miller and Upton’s (1985b) results of 0.47 and Watkins’ results of 0.61. Even if the depletion rate, \( \alpha \), approached 1.0, the impossible case of instantaneous, total production of all oil and gas reserves, the slope of Equation (3.4) would be 0.87, assuming \( \mu \) to be a standard oil industry rule-of-thumb 15% rate of return. Adelman compared actual sales of oil leases to the theoretical value given by Equation (3.4) and found that the actual price was always greater than the theoretical value. He attributed the positive increment to the “option value” of possible additional reserves in the deposits.

Adelman (1993) used the same data as Watkins (1992) for further analysis and conclusions. He again proposed Equation (3.4) as the correct model, and substituting
known values of $a$, $\mu$, and $c$, he computed the price of oil (gas) and compared it to the actual price of oil (gas) at the time of the data. He attributed any difference, i.e., the failing of Equation (3.4), to the buyers' price expectations (for oil and gas) at the time of purchase. A comparison of the computed, expected prices with actual subsequent price movements showed an accurate predictive power.

White (1993) tested the HVP on 306 cross-section time-series data points of 36 gold companies covering the period January 1991 through May 1993. The data were generally taken at calendar quarter dates. The results of this test are presented in row 4, Table 1, and are interesting compared to the other three tests, since the other three are all oil and gas data, and all have negative intercepts. White's results show a slope of 0.47 which is significantly different from 1.0 and an intercept of 78.9 and significantly different from 0. White also segmented his data (not shown in the table) between "large" and "small" companies and ran separate regressions on each segment; he arbitrarily defined a large company as one with reserves of 3,000,000 ounces or more. He reported that a Chow test showed the regressions of the large and small companies came from separate populations. White used a market value per ounce of reserves computed from market values of equity and debt of the company, but he did not remove the value of non-gold assets due to insufficient data. Thus his market values should be somewhat high; perhaps this is one reason for his large intercept value of 78.9, but this does not explain his slope of less than 1.0.
Summarizing the four tests and Table 1, we can say that the original Miller and Upton (1985a) test confirmed the HVP theory in that the slope was not significantly different than 1.0 at the 95% confidence level, and the intercept, while not zero, was only “slightly less” than zero as the theory would predict if either decreasing returns-to-scale were present or if costs were an increasing function of cumulative output. Miller and Upton did not define or otherwise quantify the term “slightly”. The discussion of HVP Model 2 and HVP Model 3 presented earlier suggested that decreasing returns-to-scale or increasing costs with cumulative extraction will cause the intercept to be less than 1.0.

The other three tests (Miller and Upton 1985b; Watkins 1992; White 1993) generally could not support the HVP since all had slopes significantly different than 1.0 in the 0.47 - 0.61 range. A slope of less than 1.0 suggests that the net price is rising at less than the rate of interest the HP μ% rule predicts; there can be several reasons for this and we will discuss this in more detail later when we analyze the results of our current empirical test.

Miller and Upton (1985b) and Watkins (1992) had negative intercepts but they were not significantly different than zero. White’s (1993) intercept was positive and was significantly different than zero. A negative intercept where none is expected can indicate that some variable is decreasing the overall market value of the deposit; Miller and Upton suggest that it could be the additional development cost needed to produce the reserves since such cost is not part of the extraction cost in oil and gas production. A positive
intercept where none is expected can indicate that some variable is increasing the overall market value of the deposit; we suggest that it may be an option value of suspending production where the net price of gold approaches zero.

In the oil and gas data tests, the slopes were less than 1.0 and the intercepts were less than 0.0, so the theoretical HVP line was everywhere above the actual regression lines, and we can say that the HVP tends to overvalue oil and gas reserves. In White's gold case however, one must be careful with this conclusion, since the positive intercept means that the actual regression line is not everywhere below the theoretical HVP line. The point where White's regression line crosses the theoretical HVP line is \( (P - c) = 150 \), so White's line is above the HVP line at values less than this point and is below the HVP line at values greater than 150. An option value at small values of net price may again explain this observation of White's test of the HVP.

McDonald (1994) attributes the slopes of less than unity in oil and gas HVP tests to the long-prevailing system of regulating oil-well spacing and extraction rates in the United States and Canada. This constraint on oil operators' freedom to adjust production, which is designed to deal with the common pool problem of oil reservoirs, prevents them from adjusting production so as to keep the net price rising at \( \mu \)%.

*The Option Pricing Method Applied to Real Assets*

We can attempt to value a gold deposit using the Hotelling Valuation Principle (HVP) as discussed in the last several sections. The primary requirement of the HVP is
that the underlying Hotelling Principle’s “μ% rule” is operative and that the rate of net price appreciation is the same as the rate of discount used to compute the present value.

As an alternative valuation method to the HVP, a developed gold deposit can also be modeled as an option and we can therefore use the OPM to value the deposit. The deposit’s value is a derivative of the value of the underlying asset, which in this case is an ounce of gold in the ground. The exercise price of the option is the cost of extracting an ounce of gold from the deposit thereby obtaining an ounce of gold bullion, the underlying asset whose value fluctuates stochastically. The mining firm has an option on each such ounce of gold in the deposit; it can pay this extraction cost to obtain the ounce of gold above ground. This option to acquire real assets, in this case the gold bullion of a developed gold deposit, is called a “real option,” as noted previously. These options can be further classified as an “investment” option of when and/or whether to develop the gold deposit in the first place and “operating” options to shut down or abandon a currently operating deposit, to re-start a currently shut down deposit, and/or to vary the production rate of the operating deposit without totally shutting it down. Many authors have proposed OPM in the capital budgeting decision and the valuation of real assets associated with mineral investments (Bjerksund and Ekern 1990; Brennen and Schwartz 1985; Dixit and Pindyck 1994; Myers and Majd 1990; Paddock, Siegel, and Smith 1988; Palm, Pearson, and Read 1986; Tregeorgis 1990). Some of these authors emphasized the “investment option,” others emphasized the “operating options.” Since our data set
comes from operating gold companies, we are interested only in accessing the value of the "operating options"; the option to invest has already been exercised.

The operating option value derives from the uncertainty in the output price of gold and from management's decision choices concerning production in response to the uncertain gold price. When a firm extracts the ounce of gold from the deposit, the option on that ounce of gold is exercised and it is not available in the future. This lost option value is an opportunity cost that must be included as a cost of extraction. Thus the NPV of the developed deposit using conventional DCF might be "positive," but we observe many such deposits not being mined,\textsuperscript{10} because if mined, the option value would be lost, certainly decreasing the value, and perhaps even making the value negative. Thus we observe in practice that the conventional NPV must be $>>0$ before the deposit is developed as well as observing mineral properties selling well above the NPV; OPM gives us a theoretical explanation for these anomalies since it defines an opportunity cost not considered by conventional DCF methods.

Using OPM to Value the Deposit: Two Techniques

A developed gold deposit has value because it can provide a stream of cash flows in the future from producing and selling the gold. These cash flows are affected by price uncertainty and by management decisions made by the producer in response to output

\textsuperscript{10}The deposit could either be temporarily shut down or it could be operating but at a rate of production far below its capacity and thus "saving" its reserves for future extraction.
price changes and one must consider this uncertainty and discretionary decision making when valuing the deposit.

There are two mathematical techniques of OPM that can value the gold deposit and are also capable of handling the above mentioned future price uncertainty and management decisions in valuing the deposit:

1. dynamic programming (DP)
2. contingent claims analysis (CCA)

Dynamic programming is a very general tool for dynamic optimization, and is particularly useful in treating uncertainty. It breaks a whole sequence of decisions into just two components: the immediate decision, and a valuation function that encapsulates the consequences of all subsequent decisions, starting with the position that results from the immediate decision (Dixit and Pindyck 1994). The equation\textsuperscript{11} of dynamic programming has an interpretation in terms of asset value and the willingness of investors to hold the asset.

CCA builds upon financial economics: a gold deposit is defined by a stream of benefits and costs that vary through time and depend upon the unfolding of events. A modern economy has markets for all kinds of assets, and based upon the Capital Asset Pricing Model, assets of equal risk should have equal rates of return. If the gold deposit

---
\textsuperscript{11}The Bellman equation or the fundamental equation of optimality consists of two terms, the immediate profit and the continuation value. The optimum action this period is the one that maximizes the sum of these two components over multiple periods.
were a directly traded asset, then we would know its value or market price. Since deposits are not directly traded,\textsuperscript{12} we can compute their value by relating them to assets that are traded. One would choose a portfolio of traded assets that will exactly replicate the pattern of risk and returns from the deposit, at every future date and for every future uncertainty. Then the value of the deposit must equal the total value of the portfolio, because any discrepancy would present an arbitrage opportunity.

The methods of DP and CCA are closely related to each other and usually lead to identical results; however, they make different assumptions about financial markets and the discount rates that firms use to value future cash flows. The value function of dynamic programming and the asset value in contingent claims analysis satisfy very similar partial differential equations. The boundary conditions in the contingent claims approach are based on the idea that investors want to choose the option exercise date optimally to maximize the value of their assets.

There are, however, some differences between the two methods. The DP approach starts by exogenously specifying the discount rate, $\mu$, as a part of the objective function. In the CCA approach the required rate of return on the asset is derived as an implication of the overall equilibrium in capital markets. Only the riskless rate of return,

\textsuperscript{12}Actually, gold deposits are traded, but the market is thin and the conditions of the sale, for the most part, are not generally known to the public.
r, is taken to be exogenous; thus the CCA approach offers a better treatment of the discount rate.

Balancing this consideration, the CCA approach requires the existence of a sufficiently rich set of markets in risky assets. The crucial requirement is that the stochastic component of the return of the asset we are trying to value (the gold deposit) be exactly replicated by the stochastic component of the return on some market traded asset (such as gold bullion). This can be quite demanding; we require not only that the stochastic components obey the same probability law, but also that they are perfectly correlated, namely that each and every path (realization) of one process is replicated by the other. Dynamic programming makes no such demand; if risk cannot be traded in markets, the objective function can simply reflect the decision maker’s subjective valuation of risk, i.e., μ is exogenously specified. The objective function is usually assumed to have the form of the present value of a flow “utility” function calculated using a constant discount rate, μ.

Thus we see that the two methods have offsetting advantages and disadvantages, and together they can handle quite a large variety of applications. In specific applications one may be more convenient in practice than the other, and different valuers may develop a better feel for one rather than the other, but there is no difference of principle between the two on their common ground. We will use the CCA approach in this work to develop the option value of the gold deposit, since the underlying asset, gold, has a
complete set of spot, future, and option markets and therefore the primary requirement of CCA is easily met.

In our CCA model of developed gold deposits, we consider ore grade, gold recovery, mineral reserves, production rate, and extraction costs to be known with certainty, and thus the basic uncertainty in the value of the deposit is over the demand for gold, which is directly related to the price of gold by the law of demand (Dixit and Pindyck 1994, 176). The gold output price is exogenous, and we determine the value of the project, $V$, in terms of the gold price $P$; i.e., we analyze and compute $V(P)$ and take $P$ itself as the only stochastic variable.

The developed deposit generates a flow of operating profit $(P_t - \bar{c}_t)$ where $P_t$ is the unit price of gold and $\bar{c}_t$ is the average unit operating cost at time $t$. We will assume that $P_t$ is described by a stochastic process (Hull, 1993, chapter 10), namely a simple generalized type of a Wiener Process known as Geometric Brownian Motion (GBM):

$$dP = \alpha P \, dt + \sigma P \, dz$$ (4.1)

where

- $\alpha$ = the constant drift rate parameter, usually referred to as the expected capital gain rate of return for the asset in question
- $\alpha P$ = the instantaneous expected drift rate
- $\sigma$ = the square root of the constant variance parameter, usually referred to as the price volatility
- $\sigma^2 P^2$ = the instantaneous variance rate
- $dz$ = an increment of a Wiener Process
The basic Wiener process is a particular type of Markov stochastic process and the behavior of the variable $z$, which follows a Wiener process is given by $dz = \epsilon \sqrt{dt}$ where $\epsilon$ is a normally distributed random variable with a mean of zero and standard deviation of 1. Thus a Wiener process $dz$ is a process describing the evolution of a normally distributed random variable with zero drift, a variance rate of 1 per unit time, and a standard deviation of $\sqrt{t}$.

If we rearrange (4.1) by dividing by $P$, we find the proportional rate of return, $\frac{dP}{P}$, in any small interval of time is normally distributed with mean $\alpha$ and standard deviation (volatility) $\sigma$. The absolute change in $P$, $dP$, follows a lognormal probability distribution.

**Developing the OPM Model for Gold Deposits Using CCA**

Consider a gold deposit with $R_0$ of initial “in-situ” reserves that produces $q_t$ ounces of gold per year where $\sum_{t=0}^{T} q_t = R_0$. The currently estimated deposit life $T$, in years, is given by $T = \frac{R_0}{q_0}$, assuming that $R_0$ is fixed and that the current production rate, $q_0$, will be maintained. At the end of $T$ years, the deposit has no gold reserves left in the ground and thus stops operating. The price of gold is stochastic and is given by the Ito process of GBM (Equation 4.1). The value of a deposit can be represented by the general function:
\[ V(t) = v[P(t), UC(t), R(t), q(t)] \]  \hspace{1cm} (4.2)

where

\begin{align*}
V &= \text{the total value of the developed gold deposit} \\
P &= \text{the spot price of gold} \\
UC &= \text{the unit extraction cost for gold in the deposit} \\
R &= \text{the inground reserves of gold in the deposit} \\
q &= \text{the production rate of gold from the deposit} \\
t &= \text{time}
\end{align*}

If \( q(t) \) is assumed constant throughout the life of the deposit at the current rate \( q(0) \), then \( R(t) \) is given by:

\[ R(t) = R(0) - t \cdot q(0) \]  \hspace{1cm} (4.3)

i.e. \( R(t) \) is a simple linear function of \( t \), and \( R(0) \) and \( q(0) \) are given constants.

Therefore Equation (4.2) simplifies to

\[ V(t) = v(P(t), UC(t), R(t), \bar{q}) \]  \hspace{1cm} (4.4)

where the variables with the “hat” represent known, constant values. We assume the unit cost of extracting gold, \( UC(t) \), is also a constant and is given by the average cost \( \bar{c} \);

therefore Equation (4.4) simplifies further to:

\[ V(t) = v(P(t), \bar{c}, R(t), \bar{q}) \]  \hspace{1cm} (4.5)

Equation (4.5) gives the value of the deposit as a function of \( P \) and \( R \), given that costs and the production rate are constant and exogenously given for this deposit. Since \( P \) is stochastic and follows the process as given by Equation (4.1), then \( V \) is a derivative asset.
and its price is a function of the stochastic variables underlying it. From Ito's Lemma (Ito 1951), it follows that the change in the value of a derivative asset like (4.5) is given by

\[ dV = \frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial R} dR + \frac{\partial^2 V}{\partial P^2} (dP)^2 \] (4.6)

Squaring Equation (4.1) and allowing the \(dt\) terms raised to the 3/2 and 2 powers to go to zero, we have

\[ (dP)^2 = \sigma^2 P^2 dt \] (4.7)

Substituting (4.1) and (4.7) into (4.6), we get

\[ dV = \left( \alpha PV_p' + \frac{1}{2} \sigma^2 P^2 V_p'' \right) dt + V_R' dR + \sigma PV_p' dz \] (4.8)

Since the value of the deposit is spanned by the value of gold in financial markets, let us use CCA techniques described earlier and construct a riskless portfolio consisting of long one developed gold deposit and short \(n\) units of gold, where \(n\) will be chosen to make the portfolio riskless. The dividend values of this portfolio are given by

\begin{align*}
q \cdot (P - \bar{c}) dt & \quad \text{(long the deposit)} \quad (4.9) \\
-n \delta P dt & \quad \text{(short the \(n\) ounces of gold)} \quad (4.10) \\
q \cdot (P - \bar{c}) dt - n \delta P dt & \quad \text{(the total portfolio)} \quad (4.11)
\end{align*}

where \(\delta\) is the convenience yield for holding gold bullion and is given by \(\delta = \mu - \alpha\). The convenience yield, \(\delta\), of a storable commodity represents the value of physically holding the commodity in inventory; this value is due to the availability of certain and smooth production and/or the availability of profit potential from the sale of the commodity in a market with temporary shortages. This convenience yield value, which comes from the
flow of the above named benefits, is net of the carrying costs of the commodity. Carrying costs include storage cost, insurance cost, transportation cost, and financing cost.

The capital gain values of the portfolio are given by

\[
\begin{align*}
    dV & \quad \text{(long the deposit)} \quad (4.12) \\
    -n\delta P & \quad \text{(short the } n \text{ ounce of gold)} \quad (4.13) \\
    dV - ndP & \quad \text{(the total portfolio)} \quad (4.14)
\end{align*}
\]

Combining Equations (4.11) and (4.14), the total return\(^{13}\) on the portfolio is given by

\[
q \cdot (P - \delta) dt - n\delta P dt + dV - ndP \quad (4.15)
\]

Substituting Equation (4.1) for \(dP\) and Equation (4.8) for \(dV\) into Equation (4.15) and noting from Equation (4.3) that \(dR = -q(0) = -\bar{q}\), we get the following partial differential equation for the total return on the portfolio:

\[
[q \cdot (P - \delta) - n\delta P + (V'_p - n)\alpha P - \bar{q} \cdot V'_p + \frac{1}{2} \sigma^2 P^2 V''_p] dt + (V'_p - n)\sigma P dz \quad (4.16)
\]

If we choose \(n = V'_p\) so that the third and the last terms in (4.16) disappear, then the portfolio becomes riskless since the \(dz\) term is the cause of the uncertainty in the stochastic process (4.1). After (4.16) becomes riskless by letting \(n = V'_p\), it will earn the risk free rate of return \(r\) times the value of the portfolio:

\[
r(V - nP) \cdot dt = r(V - V'_p P) \cdot dt \quad (4.17)
\]

\(^{13}\)The total return of an asset equals the dividend return plus the capital gain return.
Simplifying (4.16) by dropping the third and last terms which go to zero because \( n = V_p' \), and setting this simplified riskless return equal to the riskless return of the portfolio (4.17) and collecting terms, we get the following partial differential equation:

\[
\frac{1}{2} \sigma^2 P^2 V''_p + (r - \delta) PV'_p - \bar{q} \cdot V'_R - rV + q \cdot (P - \bar{c}) = 0
\]  

(4.18)

again where \( \delta = \mu - \alpha \) is the rate of return shortfall, commonly interpreted as the convenience yield of the underlying asset and the replication asset. If \( V \) represented the price of a share of common stock, \( \delta \) would be the dividend rate on the stock. If the dividend rate is positive, there is an opportunity cost of keeping the option alive rather than exercising it. In our case, \( \delta \) represents the opportunity cost of not producing the ounce of gold, i.e. its convenience yield and it must be positive\(^{14} \). If \( \delta \) were not positive, the gold would never be mined since the rate of return in the ground would be the same as its rate of return after it is mined and converted into bullion above the ground.

Gold is considered by most authors as a commodity with a convenience yield equal to zero, because stocks of gold in the world far exceed commercial consumption needs, and the bulk of the gold is held by individuals and central banks for investment purposes. This means that investment gold is an asset like a share of common stock, except that it does not pay a dividend as a share does (or could).

\(^{14}\)Since \( \delta = \mu - \alpha \), if \( \delta \) is to be positive, then \( \alpha \) must be \( < \mu \).
Because of no dividend, no reported convenience yield, and negligible storage and insurance costs, an oz of gold deliverable in the future is equal to the current spot price and this is borne out by gold future prices, which are almost exactly equal to the spot price compounded forward at the interest rate. Therefore, if the current cost of extraction is constant or rising at less than the rate of interest, yet the spot price is compounding forward at the rate of interest, financial theory tells us that one should deter extraction of the gold reserve. Nevertheless, we observe that in practice most gold companies do extract the gold, indicating that there is a convenience yield to holding gold bullion rather than gold reserves. We suggest an explanation for this observation, in that a gold company's management tries to maximize a present value sum of future share prices, and extracting the gold currently signals to the potential investors that the reserve and reported cost are "latent assets" as defined by Brennan (1990). He defines latent assets as those assets that are not fully reflected in share prices because of the asymmetry of information between corporate insiders and investors; gold reserves certainly fit this definition because of grade, tonnage, extraction costs, etc. This signaling by management in the form of current production of the reserve, implies a profitability for the gold company and positively affects the share price.

Thus gold must have some sort of convenience yield (however small) or it would not be mined. We used the constant amount of $\delta = 0.0017$ because this value was reported by Brennan (1992) as a result of a regression analysis for gold data over the 1976
- 1984 time period. We assume it to be constant over time and for all levels of gold inventories. We determine later in the thesis (see Chapter 6) that a gold company’s production capability can be used as a proxy for “going concern” value and thereby adds value to the value of the company over and above the gold reserve value; this supports the idea of gold reserves as “latent assets” that must be mined to reveal their full value.

The HVP assumes the price of the reserve constrained mineral rises at the “interest rate”; it does not (nor does the HP) discuss in depth what interest rate. We can thus reconcile the HVP theory and the convenience yield theory by applying the HVP and using the market rate of interest less the convenience yield; this also makes more acceptable our combining an option term with the HVP variable later in our discussion of results and conclusions.

From equation (4.18) we see that the value of the deposit can change for two reasons: a different initial value of the stochastic price and a decrease in the reserves due to production at a rate of \( \bar{q} \). We note that \( \frac{\mathcal{N}}{\mathcal{R}} = V'_R \) is just the HP’s user cost of reserves which could be held and not mined until time \( T = \frac{R(0)}{\bar{q}} \). The present value of this shadow price of reserves is given by \( (Pe^{-\alpha T} - \bar{c}e^{-rT}) \) and therefore

\[
-\bar{q} \cdot V'_R = -\bar{q} \cdot (Pe^{-\alpha T} - \bar{c}e^{-rT})
\]  
(4.19)

Substituting (4.19) into (4.18) and collecting terms in \( P \) and \( c \), we have the second order ordinary linear differential equation in \( V(P) \):
\[
\frac{1}{2} \sigma^2 P^2 V''_p + (r - \delta) PV'_p - rV + q \cdot \left[ P \left(1 - e^{-\delta T}\right) - \bar{c} \left(1 - e^{-rT}\right) \right] = 0
\]

(4.20)

It can be seen from above, that in going from Equation (4.2) to Equation (4.18), we used the CCA technique to get a partial differential Equation (4.18) which is still a function of \((P, R)\) and which in general cannot be solved explicitly. We then noted that the change in \(V\) with respect to \(R\) is due only to the depletion (HP's user cost) and is captured with a discounted profit flow term given by Equation (4.19). Substituting Equation (4.19) into Equation (4.18) and collecting terms containing \(P\) and \(c\), we arrive at Equation (4.20), an ordinary differential equation for \(V(P)\) which can be solved explicitly.

Equation (4.20) is analogous to the ordinary differential equation given in Dixit and Pindyck (1994, 187) for an operating project. They too had a partial differential equation, in \(P\) and \(t\), and their assumption was to produce one unit of product per year forever; thus they assumed away the \(V'_t\) term in their partial differential equation by producing forever. Since they produce one unit per year forever, their profit function is \(\left(\frac{P}{\delta} - \frac{\bar{c}}{r}\right)\), ours is \(\left[ \frac{1 - e^{-\delta T}}{\delta} \right] P - \left[ \frac{1 - e^{-rT}}{r} \right] \bar{c}\) since we will produce one ounce of gold per year for only \(T\) years. Equation (4.20) is also analogous to equation A1 in Bjerksund and Ekern (1990, 82). They developed OPM models for several scenarios; their equation
A1 is for a developed and operating oil field\textsuperscript{15} just like our developed and operating gold deposit.

In general, other investigators also make simplifying assumptions in order to get rid of the $V'_{t'}$ or $V'_{R}$ term in their partial differential equation similar to (4.18). Brennan and Schwartz (1985) start with five independent variables $(P, R, c, t, \phi)$, where $\phi$ represents the operating policy depicting whether the mine is open or closed and the other variables are defined like ours. In their model $q$ was a decision variable to be optimized and the reserves, $R$, remains as a variable that influences $V$. They get rid of the $V'_{t'}$ term by deflating the $c$ and $\delta$ variables and thereby separate the value from time. Since they still have two variables, $P$ and $R$, they assume away the $R$ variable by letting $R$ be infinite and thus they simplify to an ordinary differential equation which can be solved analytically. This infinite reserves assumption is not acceptable for a single deposit, although it may have some merit for valuing an entire gold company, since the company can replace reserves by exploration or by outright purchase. Frimpong, Laughton, and Whiting (1991, 9) investigate the valuation of mineral deposits and they simply drop the $V'_{t'}$ term by saying that the parameters of the equation and its boundary conditions are independent of time. Bjerksund and Ekern (1990, 82) cast the dependence of the value on time in terms of $R$, much the same as Brennan and Schwartz (1985) and as we do. Rather than

\textsuperscript{15}They refer to a developed deposit as an “asset in place”.
assuming infinite reserves, however, they use an assumed production profile over time as given by $\gamma$, the constant $\%$ of the remaining reserves produced each year. We used somewhat the same concept as Bjørksund and Ekern when we developed Equations (4.18) and (4.19) except of course we assume that $q$ is constant rather than an exponential declining function of the remaining reserves. The assumption of constant $q$ is a reasonable assumption for gold mining operations. Oil extraction, due to its extraction method of pumping of liquids, can easily vary its production rate; mining operations, with their technical linkage to an optimum size processing plant and capacity constraint, must operate within a narrow range of output for economic viability. We will discuss this capacity constraint of gold mining operations in more detail in a later section.

Equation (4.20) is non-homogeneous. In order to solve it, we find it necessary only to solve the homogeneous equation\textsuperscript{16} given by

$$\frac{1}{2} \sigma^2 P^2 V_p'' + (r - \delta) PV_p' - rV = 0$$

(4.21)

Equation (4.21) is a form of Euler equation which suggests a solution of the form $V(P) = A P^\beta$. Substituting this form for $V$, $V_p'$, and $V_p''$ into (4.21), we find the power form works as a solution as long as $\beta$ is a root of the quadratic:

$$\beta_{1,2} = \frac{1}{2} \frac{(r - \delta)}{\sigma^2} \pm \sqrt{\left(\frac{(r - \delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$$

(4.22)

\textsuperscript{16}The homogeneous equation is also called the reduced or complementary equation.
and $\beta_1 > 0$ and $\beta_2 < 0$. We can now use $V(P) = A_3 P^{\beta_1} + A_4 P^{\beta_2}$ as a general solution to the homogeneous part of Equation (4.20) since (4.20) is linear in the dependent variable $V$ and its derivatives and therefore its solution can be expressed as a linear combination of any two independent solutions. A solution of the entire equation is called a particular solution, denoted here by $V_{PS}$. Thus the general solution to the full Equation (4.20) is given by

$$V = V_{PS} + A_3 P^{\beta_1} + A_4 P^{\beta_2} \tag{4.23}$$

where $V_{PS}$ is a particular solution. We will first try to find a particular solution by inspection, i.e., by guessing a form of solution and then using the method of undetermined coefficients to find the actual solution.

We repeat Equation (4.20) here again for ease of use but move the deposit’s operating profit flow to the RHS and now assume that the deposit produces one ounce of gold per year for $T$ years (since $R/q = T$ and $q$ is now = 1, we have $R = T$ numerically):

$$\frac{1}{2} \sigma^2 P^2 V''_P + (r - \delta) PV'_P - rV = -P(1 - e^{-\delta T}) + \hat{c}(1 - e^{-rT})$$

Looking at the RHS, we guess that the particular solution is of the form

$$V_{PS} = KP + L\hat{c} \tag{4.24}$$

where $K$ and $L$ are constants. Hence, we know $V'_P = K$ and $V''_P = 0$. We now substitute Equation (4.24) and its two derivatives into (4.20) and solve for the value of $K$ and $L$ by equating the coefficients of the $P$'s and the constants. The results are:
\[ K = \frac{(1 - e^{-\delta T})}{\delta} \]  \hspace{1cm} (4.25)

\[ L = -\frac{(1 - e^{-rT})}{r} \]  \hspace{1cm} (4.26)

The particular solution can be found by substituting \((4.25)\) and \((4.26)\) into \((4.24)\) giving the following particular solution:

\[ V_{ps} = \left[ \frac{(1 - e^{-\delta T})}{\delta} \right] P - \left[ \frac{(1 - e^{-rT})}{r} \right] \bar{c} \]  \hspace{1cm} (4.27)

and we can now state the general solution of \((4.20)\) by combining \((4.23)\) and \((4.27)\) into

\[ V(P) = A_1 P^{\beta_1} + A_4 P^{\beta_3} + \left[ \frac{(1 - e^{-\delta T})}{\delta} \right] P - \left[ \frac{(1 - e^{-rT})}{r} \right] \bar{c} \]  \hspace{1cm} (4.28)

The operation of the project entails a revenue flow \(P\) and a cost flow \(\bar{c}\), for the one ounce produced per year. We assume that the operating deposit can be temporarily and costlessly shut down when \(P < c\), and costlessly started back up later when \(P > c\).

Therefore, at any instant the annual profit flow from the deposit is given by

\[ \pi(P) = \max[q \cdot (P - \bar{c}), 0] \]  \hspace{1cm} (4.29)

and thus the fundamental or intrinsic value function represented by the last two terms of Equation \((4.28)\) is defined differently when \(P < c\), and when \(P > c\). Equation \((4.28)\) represents the region \(P > c\). In the region where \(P < c\), we would get an equation of the
same form as the homogeneous part of Equation (4.28); such equation would have the
form \( V = A_1 P^{\beta_1} + A_2 P^{\beta_2} \) where \( \beta_1 \) and \( \beta_2 \) are as before but the coefficients \( A_1 \) and \( A_2 \) are
different from \( A_3 \) and \( A_4 \). If \( P < c \), there would be no profit flow since the deposit would
be shut down and therefore the last two terms of Equation (4.28) would be missing;
without the last two terms, the equation would be homogeneous.

We know from previous discussions of Equation (4.22) that \( \beta_1 > 0 \) and \( \beta_2 < 0 \).
Using this and the boundary conditions of the two regions, we can determine the constants
\( A_3 \) and \( A_4 \) of Equation (4.28) and constants \( A_1 \) and \( A_2 \) of the homogeneous equation
similar to Equation (4.28) for the region \( P < c \). If \( P \) becomes very small in the region \( P < c \), then it is unlikely it will ever rise above \( c \), where the deposit could be operated again to
produce a profit flow. The value, \( V(P) \), would then approach zero, but could not do so if
a power term in \( P \) with \( \beta_2 < 0 \) were present. Therefore the constant \( A_2 \) must be zero and
we would have the following equation for the value in the region where \( P < c \):

\[
V(P) = A_1 P^{\beta_1} \quad (4.30)
\]
Since there is no profit flow available in this region, this value must represents the option
to restart operation of the deposit should \( P \) rise above \( c \) at some time in the future.

As \( P \) becomes very large in the region where \( P > c \), the suspension option is
unlikely to be exercised except in the distant future, so its value should approach zero.
This would rule out the term in Equation (4.28) with \( \beta_1 > 0 \) as a power of \( P \) and therefore
the constant $A_3$ must be zero. Then in the region where $P > c$, Equation (4.28) simplifies to

$$V(P) = A_4 P^{\beta_1} + \left[ \frac{(1-e^{-\delta T})}{\delta} \right] P - \left[ \frac{(1-e^{-\gamma T})}{\gamma} \right] \bar{c} \quad (4.31)$$

Equations (4.30) and (4.31) still have two constants, $A_1$ and $A_4$, which we need to evaluate; to do this we consider the point $P = c$ where the two regions meet. Since the GBM of $P$ must be able to diffuse freely across this boundary, the value function cannot abruptly change across the boundary, and the solution $V(P)$ must be continuously differentiable across $P = c$. Setting the values and the derivatives equal at this point\textsuperscript{17}:

$$A_1 P^{\beta_1} = A_4 P^{\beta_2} + U_1 P - U_2 \bar{c}$$

$$\beta_1 A_1 P^{\beta_1-1} = \beta_2 A_4 P^{\beta_2-1} + U_1$$

where

$$U_1 = \frac{(1-e^{-\delta T})}{\delta} \quad (4.32)$$

$$U_2 = \frac{(1-e^{-\gamma T})}{\gamma} \quad (4.33)$$

Solving these two above linear simultaneous equations for the unknowns $A_1$ and $A_4$\textsuperscript{18} we get

\textsuperscript{17}These equalities are known as the “value matching” and “smooth pasting” conditions, respectively.

\textsuperscript{18}Since we are evaluating at the point $P = c$, we can replace $P$ in the equations with $c$, thereby simplifying the solution equations (4.34) and (4.35).
\[ A_i = c^{1-\beta_i} \left[ \frac{U_1 - \beta_i U_1 + \beta_i U_2}{\beta_1 - \beta_2} \right] + U_1 - U_2 \] (4.34)

\[ A_4 = c^{1-\beta_2} \left[ \frac{U_1 - \beta_1 U_1 + \beta_1 U_2}{\beta_1 - \beta_2} \right] \] (4.35)

**Empirically Testing the OPM**

Designing an empirical test of the OPM is a challenge; no generally acceptable method exists. The standard presentation (which is *not* an empirical test) of most investigators is to show a plot of the total project value including options on the vertical axis vs. the spot price of the output or underlying asset on the horizontal axis. The total value with options, given by our Equations (4.22) and (4.30) through (4.35), could be plotted by assuming values for \( c, r, \delta, \) and \( \sigma, \) but allowing \( P \) to vary. If \( \sigma \) were set to 0, the curve would be the familiar “hockey stick” shape which is often presented as the value of a call option at expiration (Hull 1993), and called the intrinsic value. The vertical distance between the two curves\(^{19} \) represents the value of the options alone. When using such a plot in two dimensions, the values of \( \delta, \sigma, r, \) and \( c \) must be kept constant, or multiple plots can be presented showing the comparative statics of varying \( \delta, \sigma, \) or \( r, \) one at a time. The cost \( c, \) however, is always kept constant in such presentations, otherwise the multiple plots would become confusing to read and interpret. The dependent variable

\(^{19} \)One curve would be the “hockey stick” where \( \sigma = 0, \) and the other curve would be the value including options where \( \sigma > 0. \)
of such presentations is usually the value, including options, of an undeveloped project and it shows the value of the option alone is greatest when \( P = c \). This conforms to standard financial option theory which says the option will have its highest value, ceteris paribus, when the underlying asset is "at the money".

As previously discussed, our case is not one of an undeveloped deposit, but that of a developed, operating gold mine. Thus the variable on the vertical axis is the value of the operating deposit plus the value of two operating options: (1) the option to close a currently open deposit, and (2) the option to re-start a currently closed deposit. We present such a plot in Figure 1, using the above named equations to calculate the total value including options of a deposit containing 9.77 ounces of gold and producing 1 ounce per year. In Figure 1 we allow \( P \) to vary and use the following representative constants:

\[
\delta = 0.0017, \quad r = 0.036, \quad \sigma = 0.13, \quad \bar{c} = 237; \quad \text{these values are the median values for the respective variables in the data set except for } \delta \text{ which is a constant throughout.}
\]

Dixit and Pindyck (1994, 190, fig. 6.1) presented a similar plot, however ours is different in a very important respect. In their example they conveniently choose

\[
\delta = r = 4\%, \quad \text{so that in the case when } \sigma = 0 \text{ (no volatility, the deterministic case), the}
\]

"hockey stick" intercepts the horizontal axis where \( P = C \). In our data however \( \delta \ll r \), and therefore \( U_1 \), the coefficient of \( P \), is larger than \( U_2 \), the coefficient of \( C \) and therefore the deposit has an operating value at values of \( P < C \) because \( P \) is rising faster than \( C \). Thus in Figure 1, the hockey stick does not intercept the horizontal axis until \( P = 200 \), which is
Figure 1
Value (including Options) of a Developed Gold Deposit Containing 9,777 oz. of Reserves and Producing 1.0 oz. per Year

Variables Held Constant

$\delta = 0.0017$
$m = 0.1300$
$r = 0.0360$
$C = $237

Total $\$$ Value of the Deposit

Price of Gold ($/per Oz.)
below $237, the constant cost.

Occasionally investigators (Frimpong, Laughton, and Whiting, 1991) will show the value of the option value alone with a resulting “tent” shaped plot that has a peak at $P = c$, falling toward zero on either side of this point. If the deposit is undeveloped, the options include the operating options that we have previously discussed, plus the option of waiting to develop the deposit. Since our data comes from developed deposits and only the operating options are still available, the option to develop has already been exercised. In general, operating options are smaller in value than the option to develop and do not completely explain the difference in value between market prices and the intrinsic value computed from DCF (Davis, 1994).

To empirically test the OPM, we will calculate the value of the deposit using Equations (4.22) and (4.31) through (4.35) and then regress the actual market value per ounce of reserves against the calculated OPM model total value per ounce of reserves. We could also plot total value vs. $P$ and option value only vs. $P$ to see if the familiar curves discussed above are recognizable. Our cross-sectional, time-series data presents a problem, however, to plotting this standard OPM presentation of Value vs. $P$. If we look at any particular cross-section of the data, the price of gold is the same and the cost varies, which is the opposite of what the presentation requires. If we look at any time series of a single company the spot price varies but the cost also varies,\(^\text{20}\) again making the standard

\(^{20}\) Other variables which are normally held constant in this type of presentation such as $\delta, \sigma, \text{ and } r$, also vary across time periods.
presentation unobtainable. Thus empirical data preclude a plot of the form of Figure 1, since the ceteris paribus assumption of such plots is only obtainable in theoretical expositions.

We propose a hybrid method of presenting these standard plots: we will prepare plots of the total value per ounce with options and the option premium only per ounce, both against \((P - \bar{c})\). We suggest this procedure for the following reasons.

1. We are using \((P - \bar{c})\) as a proxy for \(P\) with \(\bar{c}\) fixed, since our data have costs varying at each point; it is this difference between \(P\) and \(c\) that causes the option value to vary.
2. \((P - \bar{c})\) is a measure of where “in the money” the deposit is, with \((P - \bar{c}) = 0\) being “at the money” where the option value should be highest.
3. Our primary investigation of the HVP makes extensive use of the \((P - \bar{c})\) variable and therefore the analysis and comparisons of OPM with HVP are facilitated.
4. Transforming the total option value to a per ounce of reserves number facilitates comparison among different deposits by placing the value on a standard per unit basis; since \((P - \bar{c})\) can also be interpreted as a per unit of reserves value, this method shows the dependence of the option value on how far the deposit is “in the money”.

Since our data contains points only in the region where \(P > c\), we will take Equation (4.31) and compute the total value of the deposit. The power term, \(A_4 P^{\beta_1}\), represents the operating option to suspend operations and it may also capture the option to vary production although we have not specifically modeled it. We will then plot this suspension option value per ounce of reserves vs. \((P - \bar{c})\) and observe whether or not the
“tent shaped” plot is present.\textsuperscript{21} We will also plot the total value per ounce and see if it suggests a curve similar to upper curve in Figure 1. If the correct forms are observed, we will use a curve fitting program to get an ad hoc estimate of the equation of the operating option as a function of \((P - \bar{c})\). We will also attempt to explain this option value in terms of the HVP theory and the HVP empirical test results.

\textsuperscript{21} Actually only the RHS of the symmetric “tent” should be present, since all our data have \((P - \bar{c}) > 0\).
Chapter 3

THE GOLD DATA AND DATABASE

The data used in our empirical test consists of pooled cross-section, time-series accounting, financial, and operating data of 23 North American gold mining companies and the time series data of spot gold prices, gold spot price volatility, and risk-free interest rates for the time period covered by the company data. The data comes from several sources. The majority of the accounting and financial data comes from the Compu-Stat financial database located in the Department of Finance at the Daniels School of Business at Denver University. The operating data was gleaned from the annual report and 10-K filings of the various gold companies, located in the libraries of the Colorado School of Mines and Newmont Gold Company. We also contacted two gold mining company analysts in the Denver, Colorado, area and they supplied some of the operating data from their private databases.

The risk-free interest rate was taken as the rate for a treasury note with an average maturity of 8 years as given in the Survey of Current Business published by the Department of Commerce. The inflation rate was averaged over the data period as taken from the 1994 Economic Report of the President. The volatility of gold was calculated using daily gold spot price data supplied by a Denver, Colorado, gold analyst. We used an
average of the A.M. and P.M. London daily fix and chose this average price on the first
day of each month, thereby using a month as our time period to observe price changes,
ignoring the difference in the number of days in various months. To compute an annual
volatility from the past 12 months (i.e., 12 monthly observations of the spot gold price) we
use the following formulas (Hull 1993):

\[ U_i = \ln \left( \frac{P_i}{P_{i-1}} \right) \]

for \( i = 1, 2, ..., 12 \) months, where \( U_i \) is the continuously compounded return (not
annualized) in the \( i^{th} \) month. The annual volatility, or standard deviation of the \( U_i \)'s is
given by

\[ \sigma_{\text{annual}} = \sqrt{\frac{\sum_{i=1}^{n} U_i^2 - \left( \sum_{i=1}^{n} U_i \right)^2}{n-1 - \frac{n \cdot (n-1)}{\tau}}} \]

where \( n = 12 \) and \( \tau \) is the fraction of a year between observations, which in our case is
1/12 or 0.083333.

The accounting data consists of balance sheet and income statement data for the
period 1982 through 30 June 1993, although not all years are represented for all
companies since many companies were formed in the late 1980s or were purchased or
merged in the early 1990s. Our balance sheet database record has 33 fields, breaking the
balance sheet of each company for each date into 33 asset/liability/owner's equity
classifications such as cash, receivables, inventory, total current assets, etc. Our income statement database is similarly designed with 20 fields in each record containing standard income statement and dividend per share data for the previous 12 months. The raw data for the Compu-Stat database itself comes from company annual reports and 10-K filings, according to their information.

The financial data consists of share data\(^{22}\) and additional dividend data for each available year. In addition to the financial data transferred directly from Compu-Stat, each record of this database also contains 4 fields calculated by our computer program from the original data: (1) total annual cashflow, (2) accounting return on assets (ROA), (3) accounting return on equity (ROE), and (4) the total market value of developed gold assets (TVG). The total gold value (TGV) data are calculated from the basic accounting identity: Assets = Liabilities + Owners Equity. TGV is represented in the accounting balance sheet as the gold property, plant, and equipment of the firm,\(^{23}\) The TGV can be calculated as a residual market value using a rearrangement of the accounting equation:

\[
\text{TGV} = \text{Liabilities} + \text{Owners Equity} - \text{Current assets} - \text{Other assets}
\]

TGV also includes any “going concern” value the gold company may have reflected in the market value of its common stock; we discuss and estimate this “going concern” value in Chapter 6.

\(^{22}\) The database contained the outstanding number of common shares and the per share price at the fiscal year end.

\(^{23}\) The property, plant, and equipment dollar amounts on the balance sheet are recorded at cost; however, market value is what we require in this analysis.
The market value of liabilities can be closely approximated by the balance sheet liability amounts. Owners equity market value can be exactly calculated by multiplying the number of outstanding shares by the market price per share.\textsuperscript{24} Assets other than gold property, plant, and equipment can be subdivided into two classifications: (1) current assets and (2) other assets. Current assets are accurately represented by their balance sheet amounts. "Other" assets' market value can be significantly different from the balance sheet amount; fortunately most of the companies in our data set do not have a significant amount of these other assets and we used the balance sheet dollar amounts as proxies for the market value. The ROA and ROE amounts are calculated using the standard accounting definitions.\textsuperscript{25}

The operating data consist of year-end proven and probable gold reserves in recoverable ounces, annual gold production in ounces, and average annual "cash costs." "Cash costs" are a special gold industry cost classification which approximates the extraction costs other than depreciation, depletion, and amortization per ounce. We computed extraction costs, i.e. we did not use the operating data's "cash costs", by taking the cost of goods sold from the income statement and subtracting the amount of depreciation, depletion, and amortization (DDA) included. This annual "adjusted total

\textsuperscript{24} The number of shares outstanding times the market price per share is an amount known in financial circles as the market capitalization.

\textsuperscript{25} Financial analysts use the book values from the financial statements rather than market values for the total assets dollars and the total owner equity dollars in their calculations; they do this to provide consistency and comparability in their analyses. Market values are not subject to any acceptable standards, accounting values are subject to Generally Accepted Accounting Principles.
operating cost" was divided by the ounces of gold sold during the year to arrive at the average cost per ounce which we used as $\bar{c}_o$ in the HVP Equation (2.8). Income taxes are not included in the extraction costs; severance taxes are included.

The data were manipulated mechanically by using a micro-computer and the following computer software: Microsoft FoxPro Database, Microsoft Word word-processing, Microsoft Excel spreadsheet, Mathematica math package, and Soritec Econometric & Statistical package.
Chapter 4

EMPIRICAL TEST OF THE HVP USING GOLD DEPOSIT DATA

Data for the HVP Test

A database query of the five previously described database tables was performed, matching gold companies and dates; an ASCII file of 105 data records was extracted containing the following 15 data fields for each record:

- company number
- date
- spot gold price (P)
- gold reserves in ounces; proven and probable (RES)
- gold production/sales in ounces for the previous 12 months (PROD)
- calculated gold extraction cost per ounce of annual production (c)
- calculated depreciation, depletion, and amortization cost per ounce of annual production (DDA)
- market value of gold per ounce of in-situ reserves (MKT) = TGV + RES
- gold price volatility for the previous 12 months (SIG)
- risk free interest rate for the previous 12 months (RFR)
- annual cash flow (CF)
- accounting return on assets (ROA)
- accounting return on equity (ROE)
- additional ounce of gold reserves added during the year (DISCV)
- total asset value (TAV)
The ASCII file was read into the SORITEC econometric software, and the following additional data was calculated from the above imported data and added to each of the 105 data records as additional data fields:

- \((P_t - \hat{c}_t)\), the HVP variable calculated as the difference in the spot price of gold and the average annual extraction cost \((\text{HOTEL})\)
- annual cash flow per ounce of reserves, calculated as annual cash flow divided by the ounce of proven and probable reserves \((\text{CFPOR})\)
- annual cash flow per ounce of annual production, calculated as annual cash flow divided by the annual ounces of production \((\text{CFPOP})\)

**Econometric Methods of the HVP Test**

As previously described, the empirical test of the HVP consists initially of an OLS regression of \(\text{MKT}\) on \(\text{HOTEL}\). The regression Equation (3.1) is repeated here along with its null hypotheses:

\[
\frac{V'_o}{R'_o} = \beta_0 + \beta_1 \cdot (P'_0 - \hat{c}'_0)
\]

\(H_0:\) \quad \beta_0 = 0.0
\quad \beta_1 = 1.0

The results of the regression were analyzed using the standard technique of examining the following items:

- the signs of the estimated coefficients
- the value of the estimated coefficients
- the t-statistics of the estimated coefficients at a 95% confidence level
- the presence of autocorrelation using the Durbin Watson statistic
- the presence of heteroscedasticity using the Park test
- the \(R^2\) of the overall regression
A slope (estimate of $\beta_1$) "near" 1.0 and an intercept (estimate of $\beta_0$) "near" 0.0 would validate the HVP; values different from this will have to be explained in terms of the model theory and assumptions such as the lack of a fixed stock of reserves and the simplifying cost assumptions of the HVP.

Results of the HVP Tests

Table 2 presents the regression results of the various HVP tests, and Figure 2 presents the scatter plot and basic OLS regression line of MKT on HOTEL for the 105 data points of Test 1. Table 2 presents the regression results of each HVP tests as separate rows in the table; the tests’ regressions are numbered 1-23. Column 1 of Table 2 lists the dependent variable of each regression and Columns 3-9 list the various independent variables of the regressions. Test 1 is plotted in Figure 2; the other tests 2-23,\(^{26}\) some of which are plotted and presented later, were made in order to analyze and explain the results of the basic Test 1. Table 2’s cells of Columns 2-9 show the OLS estimates of the regression coefficients ($\beta_0$ or $\beta_1$) with the t-statistic in parentheses under the estimate. The t-statistic is computed at a 95% confidence level in comparing the estimate to the null hypothesis, $H_0$. The slopes of the independent variables ($P - \bar{c}_0$),

\(^{26}\)We actually performed over 60 regression tests and analyses; only 23 are presented in Table 2. Some of the tests not shown in Table 2 were to test specific ideas we had, the results of which were not particularly interesting; other tests were strictly "data mining" where we hoped to find something interesting through the process of serendipity.
Table 2  Regression Results for the HVP Testing

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant</th>
<th>Hotel (P - c)</th>
<th>(P - cs)</th>
<th>P</th>
<th>c</th>
<th>RES</th>
<th>PROD</th>
<th>CF</th>
<th>POR</th>
<th>Adj. R²</th>
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</thead>
<tbody>
<tr>
<td>1. MKT</td>
<td>40.8</td>
<td>0.73</td>
<td>(-2.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.27</td>
</tr>
<tr>
<td>2. void</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. MKT (thru origin)</td>
<td>N/A</td>
<td>0.96</td>
<td>(-0.8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.73</td>
</tr>
<tr>
<td>4. MKT</td>
<td>32.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.28</td>
</tr>
<tr>
<td>5. MKT (P-c)&lt;45</td>
<td>157.5</td>
<td>-1.46</td>
<td>(-2.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td>6. MKT (P-c)&gt;45</td>
<td>-10.5</td>
<td>1.00</td>
<td>(0.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>7. MKT (P-c)&lt;152</td>
<td>129.1</td>
<td>-0.30</td>
<td>(-6.60)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>8. MKT (P-c)&gt;152</td>
<td>-183.4</td>
<td>1.79</td>
<td>(2.64)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.42</td>
</tr>
<tr>
<td>9. MKT (P-c)&lt;125</td>
<td>131.5</td>
<td>-0.36</td>
<td>(-4.86)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>10. MKT (P-c):125-250</td>
<td>-128.6</td>
<td>1.54</td>
<td>(1.50)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>11. MKT (P-c)&gt;250</td>
<td>-206.4</td>
<td>1.85</td>
<td>(0.86)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.24</td>
</tr>
<tr>
<td>12. MKT (P-c):100-250</td>
<td>-67.2</td>
<td>1.22</td>
<td>(0.78)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.21</td>
</tr>
<tr>
<td>13. MKT (P-c):75-250</td>
<td>-15.5</td>
<td>0.94</td>
<td>(-0.25)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.18</td>
</tr>
<tr>
<td>14. MKT</td>
<td>-321.8</td>
<td>1.53</td>
<td>(-2.2)</td>
<td>-0.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>15. MKT (thru origin)</td>
<td>N/A</td>
<td>0.79</td>
<td>(-2.7)</td>
<td>-0.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.74</td>
</tr>
<tr>
<td>16. Cost (per Ounce)</td>
<td>230.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.05E-06</td>
<td>(1.09)</td>
<td>0.00</td>
</tr>
<tr>
<td>17. Cost (per Ounce)</td>
<td>224.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.87E-05</td>
<td>(2.1)</td>
<td>0.03</td>
</tr>
<tr>
<td>18. MKT</td>
<td>136.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.20E-06</td>
<td>(0.74)</td>
<td>0.01</td>
</tr>
<tr>
<td>19. MKT</td>
<td>130.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.19E-05</td>
<td>(1.4)</td>
<td>0.02</td>
</tr>
<tr>
<td>20. MKT</td>
<td>83.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.59</td>
<td>(6.3)</td>
<td>0.27</td>
</tr>
<tr>
<td>21. (P - c)</td>
<td>146.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-7.57E-7</td>
<td>(-0.66)</td>
<td>0.00</td>
</tr>
<tr>
<td>22. (P - c)</td>
<td>156.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-3.30E-5</td>
<td>(-2.0)</td>
<td>0.04</td>
</tr>
<tr>
<td>23. (P - c)</td>
<td>113.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.01</td>
<td>(3.6)</td>
<td>0.11</td>
</tr>
</tbody>
</table>
(P - CS), P, and $\bar{c}_0$ have parameter $H_0$'s of 1.0, 1.0, 1.0, and -1.0 respectively; thus the t-statistics are telling us whether or not the estimated coefficient is significantly different from 1.0 for these variables. The constant, RES, PROD, and CFPOR variables all have parameter $H_0$'s of 0.0. The last column of the table presents the adjusted $R^2$ statistic for each of the 23 regressions.

In looking at Test 1, the primary test of the HVP,\textsuperscript{27} we see that the constant is +40.8 and is significantly different from 0.0. The slope is 0.73 and significantly different from 1.0. The $R^2$ is relatively low at 0.27 indicating considerable noise in the data, the Durbin-Watson (DW) indicates positive autocorrelation,\textsuperscript{28} and the Park test showed slight heteroscedasticity. The data points in Figure 2 shows a definite linear trend, but the scatter around the regression line is also evident. Such scatter is common in cross-sectional data.

Because of the considerable “noise” in the data, we analyzed the time series of extraction costs within each company. We found considerable variation from year to year, which we cannot explain with any technical or economic reason. We assume that these variations are due to financial accounting procedures such as writing off previously capitalized development costs in a single year or capitalizing previously expensed development costs in a single year. We therefore smoothed each company’s cost data

\textsuperscript{27} Test 1 uses the Hotelling Model 1 which we previously defined and discussed.

\textsuperscript{28} Although not shown in the table, the DW statistic was about 0.9 in all regressions on the cross-sectional, time-series 105 data points tested; see a more detailed discussion of this autocorrelation later.
using a centered moving average with length (period) of 3. For companies with less than 3 observations, we kept the observed cost, and we also kept the observed first and last values for those companies with 3 or more observations, where an average was substituted for the middle values. We found that smoothing the cost data, and therefore \((P - \bar{c})\), improved the regression results only slightly and we used the raw cost data for the remainder of our regression tests. The results are shown in Table 2 as Test 4, where \(c_s\) represents the smoothed cost data.

With a constant of 40.8, yet a slope of 0.73, the Test 1 regression line is above the HVP line when \((P_0 - \bar{c}_0) < 152\) and below when \((P_0 - \bar{c}_0) > 152\), thus making an explanation of the deviation from HVP theory somewhat complicated. As previously discussed, Miller and Upton (1985a) contend that a cost structure differing from the HVP assumptions of constant returns-to-scale and no change in cost with cumulative extraction will affect the intercept of the line, but not the slope. If the intercept were other than 0.0, but the slope not significantly different than 1.0, then costs could be the sole explanation. Even though our slope was other than 1.0, we tested the constant returns-to-scale cost assumption with tests 16 and 17 of Table 2. In test 16 we regressed unit costs\(^{29}\) on ounces of reserves and in test 17 we regressed unit costs on annual ounces of production. Both tests confirmed the reasonableness of the constant returns to scale assumption: in

\(^{29}\) The unit cost in this test was computed without any return-to-capital component, i.e. it is an accounting cost, not an economic cost.
test 16 the slope was not significantly different from 0.0; in test 17 the slope estimate was significant but would add only an average of $13 to the unit cost. Since the average cost was $237, this represents about 5% of the average cost. Notice the constants of $231 and $225 respectively for test 16 and 17; both are very close to the $237 average cost thereby supporting the conclusion that constant returns to scale are present or nearly so.

Even if we consider the small effect of decreasing returns to scale, as shown by test 17 (i.e. the slope is significant), on the constant of the HVP base regression, it is not clear whether the effect on the intercept would be positive or negative. Miller and Upton (1985a) suggest that it would/could be negative, but their mathematical analysis leaves the question generally ambiguous. If \( \frac{\partial^2 C}{\partial q \partial a} > 0 \), then the effect of the decreasing returns to scale, as shown by test 17, would be to make the intercept positive, as opposed to negative as they conclude.

It is not possible to directly test the second cost assumption of the HVP, \( \frac{\partial C}{\partial Q} = 0 \), since our data is averaged across companies and therefore it cannot tell us if the cumulative extraction is coming from the old deposits or from new deposits which have

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30 Larger annual production in test 17 would indicate larger scale plants while larger reserves in test 16 would indicate a larger scale company; these are tests of whether or not Hotelling Model 2, previously defined and discussed, is operable.

31 Decreasing returns to scale produce average costs that rise with increasing output because if true, the marginal cost curve is above the average cost curve in a plot of costs vs. quantity of output.
been found through exploration or purchased outright. If \( \frac{dc}{dQ} > 0 \)\(^{32}\), then, ceteris peribus, this would cause the intercept to be negative. Since costs could increase with cumulative extraction because of lower grade and less accessible reserves, we can look at the time trend of reserves and costs to see whether or not the assumption, as it is affected by the level of reserves,\(^{33}\) appears correct. We find that the average annual ounce added to reserves is 977,000 while the average annual production is 445,000. Reserves, therefore, would seem to be increasing over time; Figure 3, a scatter plot of reserves vs. time confirms that this is true and therefore, ceteris paribus, costs should not increase with cumulative production in this data set. Test 16 is additional confirmation of the hypothesis: the slope of the regression of cost on reserves is not significantly different from zero.

The effect of reserves increasing over time deserves further discussion. The existence of a fixed stock of resources is basic to mineral economics; otherwise the resource is not exhaustible and mineral economics would not exist as a field distinct from economics in general. The Hotelling Principle and HVP call for the net price to rise over time because the mineral stock is known and fixed and the resource therefore is

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\(^{32}\)If costs increase with cumulative extraction, then Hotelling Model 3, previously defined and discussed, would be operable.

\(^{33}\)If reserves were decreasing over time, it is presumed that costs would rise, since the operator would be forced to use lower grade, higher cost reserves to maintain steady production.
Figure 3
Reported Proven and Probable Gold Reserves vs. Date

Gold Reserves (oz.) of Operating Companies

- 3/29/86
- 8/11/87
- 12/23/88
- 5/7/90
- 9/19/91
- 1/3/93
- 6/15/94

- 30,000,000
- 25,000,000
- 20,000,000
- 15,000,000
- 10,000,000
- 5,000,000
exhaustible. Figure 4, a scatter plot of \((P - c)\) vs. time, shows that the net price has \textit{decreased} over time, not increased. Our data and investigation also prove conclusively that the \(R_0\) we are using, the proven and probable reserves, is not fixed since it rises over time even with current production being positive (again see Figure 3). Thus our \(R_0\) is not necessarily the same as the fixed stock assumed by the HVP. In other words, the HVP may be true, but it may need the ultimate reserves amount rather than the proxy we are using. The effect of this on our conclusions about the empirical test of the HVP depends on our assumptions about which reserve amount the various players are using. We would define and rank reserves in the following ascending order of quantities:

1. Proven and probable reserves reported by the companies in their annual reports, which are the figures we are using in our data
2. Reserves (unquantifiable) used by the market in valuing the firm's stock
3. The ultimate reserves that the current deposits will produce; an amount that the theory presumably applies to

If the above list is ranked correctly, then the HVP could be operative but our regression of MKT on \((P - c)\) would give a slope less than 1.0 because the market is, for some reason, perhaps conservatism, using less reserves to value. This is complicated by the fact that we are computing the MKT as a per ounce figure by using reserve type #1 above, which is less than #2 which the market presumably uses; this would make our MKT value per ounce higher, but yet we get a slope < 1.0. The final effect on our empirical test is ambiguous but I would predict that the final slope would be less than 1.0 because the effect of the difference going from reserves #1 to #2 is less than going from #2 to #3.
The prediction of the net price rising over time is predicated on the fact that the mineral becomes more scarce, and as supply dwindles and user cost rises, the net price will rise accordingly. If, in fact, the supply\textsuperscript{34} is not declining, but rising as Figure 2 shows, the slope will be less than 1.0.

If the intercept were 0.0 or less, a slope of less than 1.0 would indicate the market is valuing the in-situ ounce of reserves at less than the HVP would predict. Since our regressions produced a positive intercept and a slope less than 1.0, we cannot reach such an unequivocal conclusion. The intercept of 40.8 with the 0.73 slope indicates some type of residual value at low values of \((P - c)\). If this residual value were present throughout the entire range of \((P - c)\), then the slope would still be 1.0 even with the positive intercept; in other words the line would just shift upwards parallel to, but above, the theoretical HVP line.

In test 14 we regressed MKT on \(P\) and on \(c\) as separate variables in a multiple regression. HVP theory says that both would have slopes of 1.0, with the sign of \(c\) being negative. Since test 1 gave the slope of HOTEL as 0.73, we expected 0.73 for both variables instead of 1.0, with of course opposite signs. The results show the correct signs, but with the coefficient of \(P\) being \(+0.73\) at 1.53 and the coefficient of \(c\) being \(-0.73\) at -0.47. The constant is -322, but it no longer has an intercept interpretation since this is a multiple regression test. The slope of HOTEL at 0.73 is less than the theoretical 1.0, yet

\textsuperscript{34}Here the supply or potential supply is represented by reserves in the ground.
the slopes of the individual P and C are both greater than 1.0 and -1.0 respectively. This would seem to indicate that gold price has a greater effect than gold cost on the HOTEL variable and this is supported by Table 3's demonstration of the time trends of HOTEL, P, and c. HOTEL rises and falls with P, regardless of how c changes.

Looking at Table 2 again, Test 3 forced the regression constant to zero in a MKT vs. HOTEL test and test 15 did likewise in a multiple regression test with P and c as separate explanatory variables. Since our HVP theory tells us that the constant is absent, we ran these tests to observe the results. In both tests, the results predictably improve the slope coefficients (i.e., the estimated parameters moved closer to 1.0); the $R^2$ statistic also substantially improves. A significant regression constant, where one is not expected, can be interpreted as data noise; therefore a regression through the origin is customary for comparison purposes. Another reason for a constant where theory predicts none, is model misspecification. To force the regression through the origin when the data statistically tells us\textsuperscript{35} that the intercept exists is also model misspecification, therefore one must be careful interpreting the results of this procedure. We draw no explanatory conclusion from tests 3 and 15, but they did nothing to disprove our slope Null hypotheses of 1.0, either.

\textsuperscript{35} The null hypothesis of the intercept is that it is zero; if the t-stat is $> 2.0$ then the intercept is statistically significant and different from zero.
Although the HVP principle makes no distinction among low vs. high values of (P-c) (i.e., the HVP value is = (P-c) with no adjustment for the relative size of (P-c) ), the results of the basic test suggest that the market may give higher relative value to low HOTEL values than the HVP principle suggests. We therefore divided the data into nine groups of (P-c) values and then regressed MKT on HOTEL for the nine data sub-sets; the results are shown as tests 5-13 in Table 2. The boundaries of the groups were chosen for various reasons:

- The regression of MKT on HOTEL, where HOTEL > $45, gives results statistically equal to the HVP; the complement is where HOTEL < $45
- $152 is where the regression line of test 1 crosses the HVP theoretical line; above $152, test 1 regression line is below HVP, below $152 test 1 is above HVP
- the other mid-ranges were attempts at coming close to the HVP coefficient values by excluding very high and very low HOTEL values

At low HOTEL values represented by tests 5, 7, and 9, the HVP breaks down completely giving meaningless results; the intercept is relatively high and significant, the slope is zero or negative and not significant. At HOTEL>45, represented by test 6, the results strongly support the HVP with the intercept not significantly different than 0.0 and the slope not significantly different from 1.0. We interpret this result to mean that some fundamental variable is giving value to the reserves at low values of (P - c), but as (P - c) rises the effect of this variable diminishes. This description immediately suggests an option value which theoretically would behave in this manner. We empirically test the OPM model in the next chapter and pursue this idea further.
Tests 18-23 are attempts at explaining the behavior of MKT and HOTEL on structural differences of the companies. Structural differences encompasses many variables, most of which cannot be quantified with our data. Three variables we can quantify and statistically test are reserves, annual production, and annual cash flow per ounce of reserves; these variables can be used as proxies for asset strength, operational capability, and profitability, respectively. In tests 18-20 we regress MKT on these three variables with the results shown in table 2. Tests 21-23 are analogous to 18-20 except HOTEL is the dependent variable instead of MKT. Neither MKT nor HOTEL showed any correlation with reserves or production as shown by tests 18-19, and 21-22. Tests 20 and 23 show that MKT and HOTEL have linear dependence on CFPOR, with MKT being more influenced and having a higher t-statistic. This is further proof that the market value of a company is influenced by various things and events other than the net price of the reserves\(^{36}\) and helps to explain the deviation of the data from the HVP theory. We explore this idea further in Chapter 6, where we measure the "going concern value of the gold company."

Table 3 gives the results of the cross-sectional regressions of MKT on HOTEL; the years shown in column 1 contain single observations for each company with the

\(^{36}\)We have previously pointed out that the behavior of the market value at small values of \((P - c)\) indicates the presence of an operating option value in addition to the HVP value.
Table 3  Results of Regression of MKT on (P - c):  Cross Sectional Data

<table>
<thead>
<tr>
<th>Cross Section</th>
<th>No. of Data Pts.</th>
<th>MKT</th>
<th>(P - c)</th>
<th>P</th>
<th>Prod. X 10^5</th>
<th>Reserve X 10^5</th>
<th>c</th>
<th>ROA</th>
<th>Constant H0: 0</th>
<th>Slope H0: 1</th>
<th>Adj. R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>3</td>
<td>234</td>
<td>190</td>
<td>390</td>
<td>427</td>
<td>4.8</td>
<td>200</td>
<td>15%</td>
<td>-409 (-1.27)</td>
<td>3.38 (2.06)</td>
<td>0.62</td>
</tr>
<tr>
<td>1987</td>
<td>9</td>
<td>279</td>
<td>238</td>
<td>446</td>
<td>361</td>
<td>6.4</td>
<td>208</td>
<td>21%</td>
<td>-58 (-0.45)</td>
<td>1.42 (2.75)</td>
<td>0.45</td>
</tr>
<tr>
<td>1988</td>
<td>12</td>
<td>202</td>
<td>185</td>
<td>419</td>
<td>376</td>
<td>6.4</td>
<td>234</td>
<td>14%</td>
<td>48 (0.51)</td>
<td>0.83 (1.80)</td>
<td>0.17</td>
</tr>
<tr>
<td>1989</td>
<td>13</td>
<td>186</td>
<td>163</td>
<td>401</td>
<td>542</td>
<td>8.7</td>
<td>237</td>
<td>9%</td>
<td>129 (2.05)</td>
<td>0.34 (1.00)</td>
<td>0.00</td>
</tr>
<tr>
<td>1990</td>
<td>19</td>
<td>134</td>
<td>144</td>
<td>384</td>
<td>467</td>
<td>6.2</td>
<td>240</td>
<td>2%</td>
<td>134 (2.84)</td>
<td>0.01 (0.02)</td>
<td>0.00</td>
</tr>
<tr>
<td>1991</td>
<td>23</td>
<td>98</td>
<td>114</td>
<td>355</td>
<td>426</td>
<td>5.7</td>
<td>241</td>
<td>-1%</td>
<td>91 (3.66)</td>
<td>0.06 (0.31)</td>
<td>0.00</td>
</tr>
<tr>
<td>1992</td>
<td>26</td>
<td>87</td>
<td>93</td>
<td>340</td>
<td>460</td>
<td>5.5</td>
<td>248</td>
<td>-9%</td>
<td>84 (3.91)</td>
<td>0.03 (0.16)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The table shows the results of regression analysis for MKT on (P - c) for different years. The parameters represent the mean values of the parameters for each cross-section, thus the time trend of the parameter can be observed by scanning down the appropriate column. Columns 8-10 contain the estimated parameters, t-statistics, and R^2 of each regression with the time trends again being observable down the column. MKT, (P-c), and P all decline during the time period. Cost increases by 17% 1986 to 1988, then increases very slowly thereafter. One could say that the net price decreases during the period and the MKT follows it; the decrease in net price is primarily attributable to the decrease in P. This conclusion is verified by Test 14 in
Table 2; the coefficient of Price is much stronger than the coefficient of Cost. The ROA also follows the decreasing P, as it should from its definition.

Notice that the regression coefficients progress from negative intercept and high slope to positive intercept and no slope through the time period. This change can be observed to follow the decreasing trend of (P-c); as we observed above, at (P-c) < $45-$150, HVP seems to break down.

Discussion of Autocorrelation Detected in the Regressions

As mentioned earlier, the Durbin-Watson statistic indicated that positive autocorrelation was present when running the various regressions summarized in Table 2. Autocorrelation is defined as correlation between the members of the series of observations ordered in time (as in time-series data) or space (as in cross-sectional data). The classical linear regression model assumes that such autocorrelation does not exist; put simply it assumes that the error term relating to any observation is not influenced by the error term of any other observation.

One possible explanation of autocorrelation in our case is that it is caused by the influence of a variable excluded from our HVP model, i.e., a specification bias. Such an explanation is very plausible because of the very simple form of the HVP; intuitively we know that many things affect the value of our dependent variable, MKT, other than the net price. The excluded variable's effect will be picked up in the error term, and to this extent the error terms will reflect a systematic pattern, thus creating (false) auto correlation. The
misspecification could also be from the wrong functional form instead of an excluded variable. The result would be the same: false autocorrelation readings.

In the presence of autocorrelation, the OLS estimators \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \), are unbiased and consistent, but they are not BLUE and are therefore inefficient. This loss of efficiency is masked by the fact that the estimates of the standard errors will be smaller than the true standard errors. Since the estimators are unbiased, but the standard errors are biased downward, this will lead to the conclusion that the estimates are more precise than they actually are and there will be a tendency to reject the null hypothesis when in fact it should not be.

One normally corrects for autocorrelation by using the Cochrane-Orcutt Procedure, the Hildreth-Lu Procedure, or the Maximum Likelihood Method. These techniques use iterative procedures to estimate, \( \rho \), the coefficient of autocovariance which is then used in a generalized least squares (GLS) methods to compute \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \), which are then BLUE. The Cochrane-Orcutt and the Hildreth-Lu procedures, however, drop the first observation. When we used these two procedures, the results improved dramatically giving intercepts of 22 and not significantly different from 0.0, and a slope of 0.85 and not significantly different from 1.0.

Using the Cochrane-Orcutt or Hildreth-Lu procedure on our entire data set of 105 points is not a valid procedure because the logic of either method breaks down when going from the last observation of one company to the first observation of the next
company; i.e. the first observation in the entire data set will be dropped, but not the first observation of each company as required. Although it is possible to instruct the SORITEC software as to the location of each company’s time series in our pooled data set, such a transformation would cause the loss of 24 of the 105 data points (the first data point of each of the 24 companies) and this was not deemed acceptable.

Because of the above problems our pooled data causes when using the Cochrane-Orcutt method, we used a maximum-likelihood method (ML), which uses the Prais-Winsten estimator to compute the coefficient of autocovariance of the first observation of each company; thereby not requiring the first observation to be discarded. The results of this correction gave $\beta_0 = 48$, $\beta_1 = 0.65$, and $R^2 = 0.54$, which are close to our basic results in test 1 except the $R^2$ is much higher.

Our conclusion was to ignore the autocorrelation and take the regression results at their face value. We did this based on several facts:

1. Since OLS estimates are unbiased and consistent, they will be reliable estimates in large sample sizes which we have.
2. The biggest problem with autocorrelation is rejection of a null hypothesis when in fact it is true; based on our results and conclusions, this is not a serious problem.
3. We ran regressions on each company’s time series, and the DW statistic detected no autocorrelation leading us to conclude that the autocorrelation was false and caused by missing variables or by structural differences in the companies. Sample sizes of the companies’ time series ranged from 3-9 observations.
Chapter 5
EMPIRICAL TEST OF THE OPM USING GOLD DEPOSIT DATA

The OPM model was tested as a "stand alone" model and as an "add-on" model to the HVP. The stand-alone model is given by the entire equation \(4.31\) and the add-on model is created by adding the first term of Equation \(4.31\)\(^{37}\) to the HVP variable of \(P - \bar{c}\) from Equation \(3.2\). The "add-on" model is an attempt to account for the breakdown of HVP at small \((P - \bar{c})\) values as described earlier.

Data for the OPM Test

The raw data used to empirically test both of the OPM models described above is the same database query data set used in the HVP test: 105 data records with 15 data fields for each record. As before, the ASCII query file was imported into SORITEC Econometric and Statistical software and the following values were calculated for each of the 105 imported data points and added to each data record as additional data fields:

- \((P_i - \bar{c}_i)\), the HVP variable calculated as the difference in the spot price of gold and the extraction cost (HOTEL)
- the life of the deposit, \(T\), calculated using \(T = \frac{R_0}{q_0}\) (LIFE)

\(^{37}\)The first term of Equation \(4.31\) represents the operating option of shutting down a currently developed and producing gold deposit.
• the following parameters of the OPM
  1. \( \beta_1 \) and \( \beta_2 \), the powers of \( P \), using Equation (4.22)
  2. \( A_1 \) and \( A_4 \), the coefficients of the power terms
     using Equations (4.34) and (4.35)
  3. \( U_1 \) and \( U_2 \), the coefficients of \( P \) and \( c \) in the intrinsic value terms,
     using Equations (4.32) and (4.33)
• CCA2, the total OPM value given by Equation (4.31) using the above calculated OPM parameters
• OPM2, the operating option given by the power term of Equation (4.31)

Since our OPM model for gold deposits was defined in terms of annual production of 1.0 ounce per year for \( T \) years, we made the following transformations on the OPM parameters in order to make them testable using our data set:

1. We multiplied the data point value obtained from Equation (4.31) by PROD, the annual ounces of production for that data point. This adjusted the value from a deposit producing 1.0 ounce per year for \( T \) years to a deposit producing PROD ounce per year for \( T \) years.
2. We divided the value from 1. above by RES, the proven and probable in-situ reserves associated with each data point. This placed the option value on a per ounce of reserves basis, which is necessary in order to analyze the OPM model using the variables MKT, \((P-c)\), HVP, etc.

Making the transformation above is the same as dividing the values obtained from Equation (4.31) by \( T \), the life of the deposit. Since we previously noted that \( R \) and \( T \) are numerically equal because \( q = 1 \), what we have done above is to take the total option value of the deposit producing one ounce per year for \( T \) years and divide it by the reserves to get the value per ounce of reserves.
Econometric Methods of the OPM Test

Although the stand-alone OPM, as defined by Equations (4.22) and (4.30) through (4.35), is not a linear model, we compute CCA2, the value given by Equation (4.31) and then use OLS linear regression on a data set of MKT vs. CCA2. The technique is much the same as in the HVP tests in that the actual market value per ounce of reserves is compared to the model (the OPM model in the present case, the HVP model in the former case) value per ounce of reserves. If our OPM model is a good model, the linear regression coefficients will be 0.0 and 1.0 for the intercept and slope respectively. We also look at the plots of both CCA2 and OPM2, the operating option alone, vs. (P-c) to see if we can observe the familiar option model curves.\(^\text{38}\) This can be done via scattergrams of CCA2 vs. \(P - \tilde{c}\) and OPM2 \((A_x P^{\beta_1})\) vs. \(P - \tilde{c}\) with a non-linear curve-fitting of this second scattergram data to obtain an empirical equation (model) of the operating option in terms of \(P - \tilde{c}\). We will use this model to see if we can explain the breakdown of HVP where \((P - \tilde{c}_o) < \$45\).

Results of the OPM Test

Table 4 and Figures 5, 6, 7 present the results of the OPM tests. The OPM model, CCA2 as given by Equation (4.31), contains a suspension option embodied in the first term, and an intrinsic (or fundamental) value embodied in terms 2 and 3. The intrinsic term

\(^{38}\)The familiar curves, remember, are the "hockey stick" and the "tent."
Table 4  Results of the OPM Model Regressions

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant</th>
<th>CCA2</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24. MKT</td>
<td>16.0</td>
<td>0.68</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0.6)</td>
<td>(-2.5)</td>
<td></td>
</tr>
<tr>
<td>25. CCA2</td>
<td>63.1</td>
<td>0.89</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>(10.9)</td>
<td>(-3.1)</td>
<td></td>
</tr>
<tr>
<td>26. OPM2</td>
<td>N/A</td>
<td>Non</td>
<td>Linear</td>
</tr>
<tr>
<td>27. MKT w/o Option</td>
<td>33.1</td>
<td>0.77</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>(1.8)</td>
<td>(-2.00)</td>
<td></td>
</tr>
</tbody>
</table>

contains both a positive Price variable and a negative Cost variable as does HVP. The coefficients of price and cost in the CCA2 model, U1 and U2, are not only different from 1.0 as in the HVP model, but also are not equal. Using the option parameters of our data $(\sigma, r, \delta)$, U1 is always larger than U2 which gives the in-situ ounce a value even when price falls below cost; this was discussed previously concerning the interpretation of Figure 1. This result of the option model, i.e. $U1 > U2$, is in agreement with our multiple regression of MKT on the gold price, $P$, and gold extraction cost, $C$, as separate variables. The results are shown as Test 14 of Table 2. In the following discussion we use the following variable names, some defined here again for clarity, to simplify the narrative:

CCA2 = the total option value of Equation (4.31)
OPM2 = the suspension option value given by the 1st term of CCA2
FUND = the fundamental or intrinsic value of the operating profit given by the last two terms of CCA2

JW_MOD = value per ounce of reserves of a developed and operating gold mine as suggested by this research. It is a linear combination of the HVP and the OPM as follows: value = $(P - C) + \text{operating option term obtained from curve fitting of OPM2 vs.} (P - C)$. 
Figure 5
Market Value vs. CCA2, the OPM Value
Figure 7
Option to Suspend Production vs. \( (P - c) \)
(All per OZ of Reserves)
Table 4 shows the regression results of four OPM tests, shown as rows 24, 25, 26, and 27 in the table. Test 24 represents the "stand-alone" OPM test, referred to earlier, and is plotted as Figure 5. It regress MKT on CCA2, in the same manner of the basic HVP test shown in Figure 2. The regression results of test 24 in Table 4 show that the intercept is not significantly different from 0.0, but the slope is 0.68, whereas we were expecting 1.0. The OPM is not a linear model, but a regression of MKT on CCA2 should be linear with an intercept of 0.0 and a slope of 1.0 if CCA2 is accurately estimating the value.

Figure 6 regresses CCA2 on \((P - \tilde{c}_0)\); the regression results are shown as test 25 in Table 4. The R^2 statistic is quite high, but this is not unexpected since both dependent and independent variables are linear in P and c. The linear regression is rather non-instructional, however, since CCA2 should differ from \((P - \tilde{c}_0)\) at small values since it has a power term that increases in value in this region as \((P - \tilde{c}_0)\) goes to zero; to this extent, the regression upholds the a priori theory in that a linear regression line should show a positive intercept and a slope is less than 1.0. What we are looking for in Figure 6 is the shape of a "hockey stick" option curve as shown in the theoretical curve of Figure 1. While we do observe some additional vertical scatter at small \((P - \tilde{c}_0)\) compared to larger \((P - \tilde{c}_0)\), we could not confirm this statistically using a least squares non-linear regression. The shape we are looking for may be present, but we need data at \((p - \tilde{c}) < 0\)
to confirm it; all our data is for \((p - \bar{c}) \geq 0\).

Figure 7 shows the scatter plot of OPM2 vs. \((p - \bar{c}_0)\) and the non-linear regression line of Test 26 through it. The familiar "tent shape" is clearly visible indicating that by using the actual, real-world, parameters of our data set and the option pricing theory and equations we developed from it, we can generate a scattergram of option values vs. \(P - c\) which conforms to that which the theory predicts: the options will approach a maximum at \(P = c\), and will diminish as \(P > c\). Above net prices of $200, OPM2 is zero. Between $50 - $200, OPM2 begins to create value, but it is still below $5 at a net price of $50. As the net price goes from $50 to zero, the OPM2 rises quickly to about $30. This amount represents about 74% of the HVP intercept of 40.8 from Test 1, and certainly explains the large values of MKT at relatively low \((P-c)\) which are evident in Figure 2. In order to minimize the sum of the squares of the error terms, our HVP OLS linear regression line from Figure 2 had to rotate clockwise around the point \((P - \bar{c}_0) = 152\), thereby reducing the slope and increasing the intercept. In effect we can say that the section of the regression line near \((P - \bar{c}_0) = 0\) was pulled upward because of the option values adding to the MKT variable in this region. Since our HVP model is linear, the increased MKT value at low values of \((P - c)\), increased the intercept but decreased the slope from 1.0 since the option value diminished to zero above 150.
Using a non-linear curve fitting function found in Mathematica software, we determined the non-linear regression line plotted through the scattergram of Figure 7 has an equation given by

\[ OPM_{-JW} = 32.6[0.96]^{(P-\bar{c}_0)} \]  

(5.1)

Equation (5.1) gives a value of 32.6 at \((P - \bar{c}_0) = 0\), and falls to a value of 4.2 at \((P - \bar{c}_0) = 50\) and 0.6 at \((P - \bar{c}_0) = 100\). We defined JW_MOD as \((P - \bar{c}_0) + OPM_{-JW}\), a linear combination of the HVP model from Equation (3.2) and the operating option to shut down from the curve fitting of Figure 7 and Test 26 of Table 4, which is shown as Equation (5.1). To test JW_MOD however, we regressed (MKT - OPM2) on \((P - \bar{c}_0)\) with the results shown as Figure 8 and Test 27 of Table 4. We removed OPM2 from MKT and regressed this on \((P - \bar{c}_0)\) in order to facilitate comparisons with our prior HVP tests. We should obtain the same results if we regressed MKT on \((P - \bar{c}_0) + OPM2\).39

The results of Test 27 show an intercept of 33.1 but not significantly different from 0.0 and a slope of 0.77 but is significantly different from 1.0. When these results are compared to Test 1 of Table 2, the pure HVP model, we find that the results are superior in that the estimates are closer to the null hypotheses and the errors are smaller.

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39 Just to make sure of this statement, we performed the appropriate data conversions and regression and indeed, got the same results.
Combining OPM with HVP in the Combined Model (JW_MOD)

OPM and HVP come from different worlds, so can we simply join them in a linear fashion as we suggest above? Consider the following:

- The OPM as given by equation (4.31) contains an option term and an intrinsic term, which is linear in \( (P - \bar{c}_0) \).
- The HVP does not consider options, but it does not preclude a stochastic gold price, which the OPM is built upon.
- The HVP behaved quite well empirically if we removed low \( (P - \bar{c}_0) \) values (see Test 6 of Table 2); low \( (P - \bar{c}_0) \) values are where the option term of Equation (4.31) begins to add value.
- A combination of HVP and OPM as given by JW_MOD can be thought of in either of two ways:
  1. It is the HVP model with OPM2 “tacked on” to explain the breakdown of HVP at low \( (P - \bar{c}_0) \).
  2. It is the OPM model of Equation (4.31) with the intrinsic terms replaced by the very similar, but more simple, HVP term \( (P - \bar{c}_0) \).
- As previously discussed, the OPM model of Equation 4.31 has the theory of a convenience yield built into its derivation, and a convenience yield with its two different interest rates is not incompatible with the HVP.

Based on this common ground, we feel justified in combining the OPM and HVP into a single linear model.
Chapter 6
GOING CONCERN VALUE OF GOLD PRODUCERS

It is evident from Figure 2 that there is much scatter in our data, suggesting that there are variables which affect the value of our companies other than \((P - \bar{c})\). As previously described on page 82, we smoothed the cost data, and our regression results improved slightly as shown by Test 4 of Table 2. Since we calculated MKT, the market value of an ounce of developed reserves, by using the equity value of gold companies, we felt that there could be some additional variable affecting the value of MKT, other than the value of the in-situ developed gold deposits themselves. We will call this value the “going concern” value of the gold company and we suggested several reasons for it on page 4. For example, if you compare the value of two gold companies: one with 5,000,000 ounces of developed gold reserves and costs of $250 per ounce but no annual production, and the other company with 5,000,000 ounces of developed gold reserves and the same cost structure but producing 500,000 ounces per year, most industry observers would say the producing company is more valuable. Thus the currently producing company is more valuable even though the HVP value, as given by \((P - \bar{c}_0)\) or JW_MOD, places the same value on the two companies.
In an effort to measure the effect of these often mentioned but rarely quantified "going concern" values, we computed the following variables and performed OLS and ML regressions to test the null hypotheses that market value per oz of reserves was not being affected by "going concern" values and/or by investor anticipation of improved financial results.

The additional variables which we computed and their definitions are:

- **OV** the amount that MKT differs from \((P - \bar{c})\) by other than the OPM2 value; computed as \[\text{MKT} - (P - \bar{c}) - \text{OPM2}\]
- **CS** the extraction costs of each company, smoothed using a moving average procedure; previously discussed on page 82
- **GO_CERN** the "going concern" value of an operating gold company, computed as \[0.000062 \times \text{ounces of annual production}\]
- **GMV** the value of the gold assets per oz of reserves, after removing the option value (OPM2) and the "going concern" value as measured in the annual production (PROD) variable; computed as \[\text{MKT} - \text{OPM2} - \text{GO_CERN}\]

We decided to test the hypothesis that the production strength of a company creates a value in a gold company, if not a gold deposit, by regressing OV on PROD. OV is the value of an ounce of reserves over and above \((P - \bar{c}_0)\) and OPM2, and PROD is the annual ounces of production of the company. The results of this test and other tests yet to be described are reported below in Table 5 and referenced as Tests 28-32. The results from Test 1, Test 4, and Test 27 are repeated here for ease of comparison:
Table 5 - Results of "Going Concern" Regressions

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$ (t stat)</th>
<th>$\beta_1$ (t stat)</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1 Regress MKT on (P-C)</td>
<td>40.8 (2.2)</td>
<td>0.73 (-2.3)</td>
<td>0.27</td>
</tr>
<tr>
<td>Test 4 Regress MKT on (P-CS)</td>
<td>32.4 (1.7)</td>
<td>0.78 (-1.8)</td>
<td>0.28</td>
</tr>
<tr>
<td>Test 27 Regress (MKT-OPM2) on (P-C)</td>
<td>33.1 (1.8)</td>
<td>0.77 (-2.0)</td>
<td>0.29</td>
</tr>
<tr>
<td>Test 28 Regress OV on (P-C)</td>
<td>40.8 (2.2)</td>
<td>-0.27 (-2.3)</td>
<td>0.04</td>
</tr>
<tr>
<td>Test 29 Regress OV on PROD</td>
<td>-26.0 (-2.1)</td>
<td>0.000002 (3.4)</td>
<td>0.10</td>
</tr>
<tr>
<td>Test 30 Regress OV on RES</td>
<td>-9.2 (-0.7)</td>
<td>0.000002 (1.4)</td>
<td>0.01</td>
</tr>
<tr>
<td>Test 31 Regress GMV on (P-C)</td>
<td>-5.0 (-0.3)</td>
<td>0.84 (-1.4)</td>
<td>0.36</td>
</tr>
<tr>
<td>Test 32 Regress GMV on (P-CS)</td>
<td>-13.5 (-0.7)</td>
<td>0.89 (1.0)</td>
<td>0.36</td>
</tr>
</tbody>
</table>

It can be seen from Test 28 and 29 of Table 5 that the amount of OV is significantly influenced by the value of $(P - \bar{c})$ and by the amount of annual production; the amount of reserves does not have a significant impact as shown by Test 30. We interpret annual production to be a "going concern" value with the estimated regression slope coefficient being $6.20$ per 100,000 oz of annual production. In other words, each 100,000 oz of annual production adds $6.20$ to the market value per oz of reserves, over and above the $(P - \bar{c})$ value and the OPM2 value. The fact that reserves are not significant to the OV value is perfectly logical; the reserve value is already captured in the
\((P - \bar{c})\) amount itself. OV is negatively related (significantly) to \((P - \bar{c})\), which is in agreement of our earlier finding that an option value is present at low \((P - \bar{c})\), but goes to zero as \((P - \bar{c})\) approaches 150.

In Test 31, we regressed GMV on \((P - \bar{c})\) and compared the results to Tests 1, 4, and 27. GMV is MKT with the option value and the "going concern" value removed. The results of Test 31 are superior to Tests 1, 4, and 27, in that the estimated coefficients are nearer to 0.0 and 1.0 respectively. Test 31 also has better t-statistics and its residual sum of squares, standard errors of the estimate, and standard error of the regression are all smaller than those of Tests 1, 4, and 27; its \(R^2\) value also improved. In test 32, we regressed GMV on the cost smoothed \((P - \bar{c})\) and the results of the slope coefficient improved again.

We thus find that the HVP may hold if the influences of other variables on the market value of gold companies are accounted for. We have identified and quantified two of these variables, option value and "going concern" value. Another way of stating this is to say that the HVP term, \((P - \bar{c})\), might be a good way to value an individual developed gold deposit, but it is only one of several terms that bring value to a gold company. Since our MKT variable is derived using the market value of the equity of an operating gold company, it includes the efficient market's value using all available information including the "going concern" value and option value. A private communication with one of the United States' largest gold companies found that they were using the HVP (without these
additional value adjustments) as described by White (1995) to identify under/over valued public gold companies. They reported “surprising accuracy” in its use.
Chapter 7

CONCLUSIONS

The theoretical HVP was not upheld in an empirical test using our gold data. The model does well at moderate to high values of \((P - \tilde{c}_0)\), but predicts low at values of \((P - \tilde{c}_0) < \$125\). In this range, the market value is substantially higher than HVP would predict, with the deviation increasing as \((P - \tilde{c}_0) \Rightarrow 0\). This indicates that some alternative or additional model is active in this region. The option to suspend operations explains 74% of this deviation. This suspension option can be modeled by the following:

\[
OMM_{-JW} = 32.6[0.96]^{(P-\tilde{c}_0)}
\] (5.2)

The OPM model, given by Equation (4.31), overvalues the gold deposit overall, but gives a close fit with theoretical expectations when only the suspension option term (OPM2) is considered. This suggests that the fundamental value terms of Equation (4.31) are overvaluing the deposit, but the option term is correct.

When Equation (5.2) is combined with Equation (3.2), we get a combination of the HVP and the OPM which we have called JW_MOD. Regressions of MKT on JW_MOD give results which show that JW_MOD will predict the actual value of the gold deposit more accurately than will \((P - \tilde{c}_0)\) alone (see Test 27 and Figure 8).
The effect of the option value rising as \((P - \tilde{c}_0) \Rightarrow 0\) gives support to our analysis and explanation of the test deviation of HVP from theory. The HVP model can be used when \(50 < (P - \tilde{c}_0) < 250\), but at values of \((P - \tilde{c}_0) < 50\) Equation (5.2) should be added to \((P - \tilde{c}_0)\) to pick up the value of the operating option to shut down.

We found and measured a "going concern" value in producing gold companies; the value adds $6.20 to an ounce of gold reserves for each 100,000 ounces of annual production. Adjusting our data for these option and "going concern" values, we found the HVP does predict the value of the in-situ reserves (see Tests 31 and 32). As a result of our research we would suggest the following two equations to estimate the value per ounce of reserves of developed gold deposits and gold companies respectively:

\[
\text{Value Deposit} = (P - \tilde{c}) + 32.6[0.96]^{(P - \tilde{c})} \tag{5.3}
\]

\[
\text{Value Company} = (P - \tilde{c}) + 32.6[0.96]^{(P - \tilde{c})} + (0.000062 \cdot PROD) \tag{5.4}
\]

**Does the μ% Rule Hold in Developed Gold Deposits?**

Even when Equation (5.4) is used to adjust the HVP to account for a shut-down option at low \((P - \tilde{c}_0)\) and for "going concern" values, we report a slope of less than 1.0 (see Tests 31 and 32); all previously mentioned empirical HVP researchers got similar results, in the range of 0.5 - 0.9. We feel that one of the major reasons for this sub-optimal behavior, in the case of gold operators, is the capacity constraint (see Cairns and Davis 1995). Gold operators cannot profitably reduce the output of plants which have
been designed and constructed to optimally produce at about 90% of design capacity. Thus the operators of gold mines cannot adjust production so as to make \((P - \hat{c}_0)\) rise at \(\mu\)% or perhaps not rise at all\(^{40}\).

Without the ability to lower marginal costs by lowering output as HP theory suggests, then the net price will rise in the numerator of Equation (1.1) at a rate less than the risk-adjusted discount rate in the denominator, as given by CAPM. Thus the slope will be less than 1.0 and this result has been found in all empirical studies. Figure 9 shows the relation between the slope of the HVP line and the mine life when the difference in the rise in MKT and the interest rate is a given amount. We have run data for \(\Delta r = 1\% - 5\%\) at intervals of 1%. The \(\Delta r\) is all that matters, not the absolute values of the two \(r\) values; 18% and 20% values will give the same \(\Delta r = 0.02\) line as 2% and 4%. Notice the independent variable is the mine life since a longer life will cause more present value difference in the numerator inflating and denominator discounting values and hence a lower slope.

\(^{40}\) A glance at Table 3 shows that \((P - \hat{c}_0)\) is not rising at all, but is in fact falling.
Figure 9
Effect of Interest Rate Differential and Mine Life on HVP Slope
Suggestions for Further Research

Our study has answered some questions that we asked in the introduction, but it also has raised other questions which should be the subject of future research:

1. Test 14 in Table 2 showed the estimated coefficients of the gold and cost variables were quite different when the variables were separated in a regression; this could be a very interesting topic since Miller and Upton found the two coefficients to be the same magnitude (and of course opposite in sign) in their work with oil and gas.

2. The OPM model for gold valuation in general and the parameters of Equation (4.31) in particular need to studied further; for instance could we use Equation (4.31) to fit a 0.0 intercept, 1.0 slope line to market values by varying δ and r and thereby find the implied convenience yield of gold.

3. Why does the OPM model as theoretically developed and given by Equation (4.31) not give better results when used as a stand alone model. Test 24 and Figure 5 were somewhat disappointing with their slope of 0.68.

4. We must develop an adjusted CCA2 model that includes the stochasticity of the reserves. Reserves and annual production determine T, the life of the mine, and T is an important variable in the computation of U1 and U2, the coefficients of price and cost in the Equation (4.31) intrinsic term. Pindyck (1980) previously modeled reserves as a stochastic variable and perhaps this could be incorporated into our CCA2 model to make the intrinsic term more accurate.
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