REFINING MAGNETIC AMPLITUDE METHODOLOGY
FOR USE IN THE PRESENCE OF REMANENT
MAGNETIZATION

by

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science (Geophysics).

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The magnetic method is a common geophysical technique used to detect and image magnetically susceptible sources. Interpretation of magnetic data can be difficult in the presence of remanent magnetization. Remanence can alter the total magnetization to an unknown direction and hinder the interpretation of 3D susceptibility models by leading to incorrect shape and size of the sources. The magnetic amplitude method is an effective tool for interpretation of magnetic data in such settings. Amplitude data are the amplitude of the anomalous magnetic field and are weakly dependent on magnetization direction. The data are often used to bypass challenges posed by the presence of remanently magnetized materials as they not rely on known magnetization direction. 3D amplitude inversion plays a particularly important role in quantitative interpretation of magnetic data affected by remanence. Inversion of amplitude data results in an effective susceptibility model capable of delineating geologically interpretable anomalies where susceptibility modeling approaches struggle.

Challenges remain, however, with the use of amplitude method. The first challenge is that a methodology for determining the noise statistics relating to amplitude data and associated inversion has not been well developed, hindering the ability to properly estimate the data misfit associated with an inverse model and therefore the model itself. The second challenge is amplitude data, while successful at delineating remanent source bodies, do not contain direction information. This limitation makes it difficult to recover dipping structures affected by remanence. These challenges are the focus of my work.

I investigate amplitude error statistics to address the first challenge. I analytically derive the propagation of errors associated with calculating amplitude data and confirm the derivation by implementing parametric bootstrapping. The investigation reveals that the noise in amplitude data is approximately equal to that of the total-field anomaly data. Having char-
acterized the relationship between noise in total-field anomaly and amplitude data, I estimate the noise in total-field anomaly datasets. Using synthetic and field data, I demonstrate that equivalent source technique can recover accurate estimates of noise in magnetic datasets. I use this estimate for inversion of amplitude data to aid in recovering optimal models. I show that noise in magnetic datasets can be estimated by equivalent source technique and that the estimate can be used in inversion resulting in increased confidence in the recovered model. Following this work, I address the second challenge of estimating the magnetization direction. I implement a method to recover the missing direction information using an effective susceptibility model. I segment the amplitude inversion into distinct anomalous bodies and assume a constant magnetization direction for each. Assuming that the effective susceptibility provides a sufficiently accurate representation of the magnitude of the magnetization, I then solve a least-squares problem to recover the magnetization directions of multiple source bodies simultaneously. This approach is able to recover magnetization direction from datasets containing both single and multiple anomalies. The direction estimate can be used in susceptibility inversion to recover accurate dipping structures of anomalies.

My work improves amplitude method by increasing confidence in interpretation and by broadening the scope of the method. I demonstrate that a reliable noise estimate can be obtained using equivalent source technique and used to recover an optimal effective susceptibility model. Using the effective susceptibility model, magnetization direction can then be estimated for single and multiple anomaly datasets. Following my research, amplitude data can now be used to fully characterize magnetization in both magnitude and direction.
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A special thanks to my friends and family. Your love and guidance have kept me going through this entire journey. I would not be myself without you.
To B.C.
CHAPTER 1
INTRODUCTION

The magnetic method is the measurement of the Earth’s naturally occurring magnetic field to detect and characterize magnetic variations within the subsurface. Many rocks are composed of magnetic minerals and therefore possess a non-zero magnetic susceptibility. The magnetization of such geologic units produce their own magnetic field, causing the total-field to fluctuate due to the presence of geological units. Fluctuations in the Earth’s magnetic field can be measured and recorded to reveal information about the location, distribution, and magnetization of formations in the subsurface.

A common interpretation technique is to invert magnetic data to recover a model of magnetic susceptibility. A susceptibility model represents the distribution and strength of susceptible anomalies in the subsurface. One challenge currently facing magnetic data interpretation is the presence of remanent magnetization. Remanent magnetization is the presence of permanent magnetization in the subsurface independent of Earth’s current inducing magnetic field. If remanence is parallel or weak with respect to the inducing field direction, its presence is does not pose interpretation challenges. In many instances, strong remanent magnetization can be present and often has an unknown direction that can be significantly different from the direction of the inducing field. These two vector quantities, the inducing magnetization and the remanent magnetization, sum together to form the total magnetization. Traditional magnetic data processing and inversion relies on a known total magnetization direction as the quantity is needed to relate total-field anomaly data to recovered susceptibility. When remanent magnetization is present, however, the total magnetization direction is often unknown resulting in difficulties in data interpretation.

Various methodologies have been developed to overcome this challenge. One such method is to calculate the amplitude of the total-field anomaly data, to be discussed in subsequent
chapters. Shearer (2005) illustrated the success of using this method to aid in the interpretation of complex geologic environments affected by remanent magnetization without knowledge of the total magnetization direction.

My research expands on previous work by Shearer (2005) and investigates two topics relating to refining the use of amplitude data. There are aspects associated with amplitude that need improvement. One challenge is that error statistics of amplitude data are unknown. Without knowledge of the error in amplitude data, selection of an optimal model is difficult. If the error is known, the optimal model can be recovered using discrepancy principle. I use analytical derivation and monte carlo simulation to characterize the noise statistics. By doing this, I establish a relationship between the noise in amplitude data to the noise in total-field anomaly data and use this relationship to aid in recovering an optimal amplitude derived susceptibility model. Selection of an optimal model is important for increasing confidence in the recovered model. The first challenge is to characterize the propagation of noise following an amplitude calculation. The second aspect of amplitude methodology in need of improvement is estimation of magnetization direction. Knowledge of magnetization direction is necessary for recovering the dipping structure of an anomaly affected by remanence through inversion. Amplitude method, so far, has not addressed estimation of magnetization direction. Here, I show that quantity can be estimated by solving a least squares problem formulated using an effective susceptibility model obtained through inversion of amplitude data. By illustrating that magnetization direction can be estimated using amplitude method, I help in increasing the information amplitude method can extract from a magnetic dataset. The estimated direction information reveals more information about the data and can be used to recover the dip of anomalies. The first half of my research allows for increased confidence in the recovery of an optimal model calculated from amplitude data. The second part allows for the recovery of direction information. As such, amplitude data can then be used to recover both the magnitude and direction of magnetization. Both parts of the research result in improvements to amplitude method.
Chapter 2 of this thesis provides a background of the magnetic field, magnetization, remanent magnetization, and magnetic amplitude. I use a case study to illustrate the difficulties posed by working with magnetic data affected by remanence and how amplitude data and inversion can overcome these challenges where traditional modeling techniques fail.

All data have inherent noise, whether due to the interfering signals, acquisition error, or positioning error. Being able to distinguish the noise from the signal is invaluable in the interpretation of all geophysical data types. The noise characteristics of magnetic amplitude data, however, have not been previously investigated. This presents a challenge in recovering an optimal inverse model. I investigate this problem in Chapter 3 by first analytically deriving the propagation of errors equation for amplitude data. I continue by applying a bootstrapping method to characterize how error propagates from total-field anomaly data to amplitude data using synthetic data. After establishing the relationship between noise in total-field anomaly data and amplitude data, I illustrate how the equivalent source technique can be used to estimate error in total-field data. This noise estimate is then translated into an estimate of the error in amplitude data. The amplitude data are then inverted to illustrate the success of recovering an optimal amplitude model with the aid of error estimates obtained by the equivalent source method. A proper estimate of the noise in amplitude data allows for the selection of an optimal inverse model by making use of discrepancy principle and allows for the selection of an optimal regularization parameter and data misfit.

In Chapter 4, the challenge of estimating magnetization direction using an amplitude derived susceptibility model is addressed. An estimate of the total magnetization direction reveals more information about the dataset and can be used to recover the correct orientation of dipping features affected by remanent magnetization. I apply a least squares formulation to estimate the magnetization direction and illustrate that the quantity can be recovered using simple steps. This work illustrates that by use of amplitude data, both magnitude and direction information can be estimated.
In Chapter 5, I apply the developed methodology of the preceding chapters to field examples. All datasets are total-field anomaly data affected by remanence. The first dataset is from Quinghai, China over intrusive igneous formations. The next set of data is collected over a kimberlite dike from the Northwestern Territories, Canada. The last example is from Black Hills, Australia over norite deposits.

Chapter 6 is the concluding chapter. Here, I summarize the key points of this research and discuss challenges and suggestions for future work.
CHAPTER 2
REMANENT MAGNETIZATION AND THE USE OF AMPLITUDE DATA

This chapter describes remanent magnetization and illustrates how amplitude data can be used to address the challenges posed by its presence. I first start by giving a brief overview of magnetic method leading to a description of remanent magnetization. Parametric modeling and susceptibility inversion are then performed on field data examples affected by remanent magnetization to illustrate the challenges that are commonly encountered. I then introduce amplitude data and their inversion and demonstrate how their use can overcome the problems faced by traditional susceptibility inversion that assumes weak remanent magnetization and inverts for susceptibility. Inversion of amplitude data is then performed to the field examples to recover simple, geologically interpretable models where other methods struggled.

2.1 Magnetic Method

Magnetic fields are generated by an electrical current. Magnetic flux due to a small loop can be described using Biot Savart law in equation 2.1, which states that a closed of current, I, will produce a magnetic field $\vec{B}$.

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \oint d\vec{l} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|}$$  \hspace{1cm} (2.1)

Each current loop generates a small dipole moment, $\vec{m}$, equation 2.2, where $a$ is the radius of the current loop, $i$ is the current, and $\hat{n}$ is the unit normal vector. The collective density of a group of dipole moments, $\vec{m}$, is known as magnetization.

$$\vec{m} = i\pi a^2 \hat{n}$$  \hspace{1cm} (2.2)

$$\vec{M} = \frac{1}{V} \sum \vec{m}_i$$  \hspace{1cm} (2.3)

In the subsurface, magnetically susceptible rocks contain a collection of magnetic domains, each with its own dipole moment, $\vec{m}$. In the absence of an external magnetic field,
these domains are randomly oriented and collective magnetization sums to zero. When a magnetically susceptible material is exposed to an external magnetic field, the individual magnetic domains will tend to align themselves along the orientation of the external field and effectively magnetize the material (Telford et al., 1990). The magnetized material will produce its own secondary magnetic field that will superimpose on the external field. In this way, magnetization is induced, $\vec{M}_I$ and is proportionate to the total magnetic field applied to it, equation 2.6. When the external field is removed, the magnetic domains will return to a random state in most cases. Any magnetization remaining outside the influence of an external field is remanent magnetization, $\vec{M}_R$. Total magnetization is therefore the vector summation of these two fields by equation 2.4 and Figure 2.1, (Blakely, 1996).

\[ \vec{M} = \vec{M}_I + \vec{M}_R. \]  

(2.4)

Figure 2.1: Total magnetization, $M$, is the vector sum of induced magnetization, $M_i$ and remanent magnetization, $M_r$.

The magnetic field is a vector quantity and describes both length and direction. The direction of the field is expressed using inclination and declination. Inclination, $I$, is the angle between the magnetic field and its projection in the horizontal plane. Declination, $D$, is the direction between the horizontal projection and a zero reference Figure 2.2. The orthogonal
Figure 2.2: Magnetic inclination is the angle between the magnetic field vector and horizontal. Declination is the angle between the magnetic field north and true north.
components of a magnetic field, $\vec{B}$, can be expressed with respect to the orientation of the field by equation 2.5.

\[
\begin{align*}
B_x &= B_0 \cos I \cos D \\
B_y &= B_0 \cos I \sin D \\
B_z &= B_0 \sin I
\end{align*}
\] (2.5)

2.1.1 Magnetic Susceptibility

Magnetic susceptibility, $\kappa$, is the measure of how strongly a material can be magnetized in the presence of a magnetic field (Sheriff, 2002). It is a dimensionless quantity that relates the amount of magnetization of a rock or mineral to an applied magnetic field, equation 2.6. Susceptibility values for common magnetically susceptible rock types are listed in Table 2.1.

Magnetic susceptibility is affected by the size, composition, and magnetic domain properties of the magnetic grains contained in the rock. Larger grains will typically have greater susceptibility compared to smaller grains (R.L. Reynolds & Fishman, 1990). Minerals that are naturally ferromagnetic include iron ores, (i.e megnetite or lodestone), as well as minerals with cobalt or nickel compositions. Table 2.1 lists the magnetic susceptibility ranges of common magnetically susceptible rocks and minerals.

\[
\vec{M}_I = \kappa \vec{H}
\] (2.6)

Table 2.1: Magnetic susceptibilities of various rocks. Modified from Telford, 1990

<table>
<thead>
<tr>
<th>Rock or Mineral Type</th>
<th>Magnetic Susceptibility Range x 10^6 SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porphyry</td>
<td>20-16,700</td>
</tr>
<tr>
<td>Basalts</td>
<td>20-14,500</td>
</tr>
<tr>
<td>Diorite</td>
<td>50-10,000</td>
</tr>
<tr>
<td>Periodite</td>
<td>7,600-15,600</td>
</tr>
<tr>
<td>Hematite</td>
<td>40-3,000</td>
</tr>
<tr>
<td>Magnetite</td>
<td>$10^5$-6x$10^6$</td>
</tr>
</tbody>
</table>
2.2 Remanent Magnetization and its Challenges

Remanent magnetization, $\vec{M}_R$, is the presence of permanent magnetism of a rock or mineral independent of any external field (Merrill et al., 1996). Remanence can occur due to a number of circumstances causing the magnetic grains in the rock to permanently align themselves in a set orientation. The main causes of remanence are detailed in Table 2.2. The causes of remanence fall into two categories, primary and secondary.

Primary remanence occurs as the rock is forming and can provide information about the orientation of the earth’s external magnetic field at the time of deposition. Secondary remanence is magnetization that occurs after the rock has formed.

Table 2.2: Descriptions of remanent magnetization types. (Telford et al., 1990)

<table>
<thead>
<tr>
<th>Remanence Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical</td>
<td>Magnetic grains change in size as the result of chemical actions below the Curie point</td>
</tr>
<tr>
<td>Detrital</td>
<td>Slow deposition of fine particles in the presence of an external field</td>
</tr>
<tr>
<td>Isothermal</td>
<td>Produced by lightning strikes in very small, irregular areas.</td>
</tr>
<tr>
<td>Thermal</td>
<td>Magnetic material is cooled from the Curie point in the presence of an external field.</td>
</tr>
<tr>
<td>Viscous</td>
<td>Secondary remanence acquired over time, related to thermal agitation.</td>
</tr>
</tbody>
</table>

In cases where remanent magnetization is strong and has an orientation significantly different from the inducing magnetic field, the contribution of the remanent field will likely alter the direction of the total magnetization direction away from that of the inducing field. It is often the case that the remanent magnetization direction is not known and as a result makes the total magnetization direction unknown. It can be difficult to determine the strength and direction of the total magnetization and presents challenges in data processing and interpretation. This presents a challenge as susceptibility inversion techniques rely on a known total magnetization direction.
2.2.1 Parametric Modeling and Inversion to Illustrate the Effects of Remanent Magnetization

Here I use parametric modeling and inversion of field data to illustrate the difficulties posed with recovering an interpretable model in the presence of remanent magnetization.

The survey area is located in Qinghai Province, China in an area of known iron ore resources. The geology is considered to be complex and heavily metamorphosed. Iron deposits in the location are related to magmatic intrusions having occurred in the Ordovician - Silurian group and the early Carboniferous. Iron ores are a common source of remanent magnetization. Nonmetallic deposits in the region are often related to metamorphic crystalline basement. The deposits and host rock are non-uniformly overlain with sandstone.

Figure 2.3: Survey area over prospective magnetite site. The magnetic field direction has an $I=55.8^\circ$ and $D=0.3^\circ$. Subsets $C_3$ and $C_5$ are investigated here. Data were provided by the Qinghai Geologic Survey in China.
The data region is shown in Figure 2.3. I focus on two subsets of the data, referred to as $C_3$ and $C_5$, contained within the region. The $C_3$ dataset covers an area of 1.5 km by .72 km. The $C_5$ dataset covers an area of 1.8 km by 1.9 km. The survey data were collected with a line spacing of 100 m trending north south. Figure 2.4 and Figure 2.5 show the total-field datasets with an inclination of 55.8° and declination of 0.3°. Both datasets show magnetic responses that cannot be accounted for by induced magnetization alone, indicating the presence of remanence.

![Figure 2.4: Dataset $C_3$. Observed total-field anomaly data over prospective iron ore site.](image)

The parametric model was produced using GM-SYS along 2300 m easting from the $C_3$ dataset, Figure 2.6. The model shown was inverted for two dimensional spatial variability with a susceptibility of 1 SI. The model of closest fit is shown in Figure 2.6. The black dots correspond to the observed data, the black line represents the predicted response from the constructed model. The red line represents the difference between the observed and predicted data. The recovered model reflects the best obtained fit and shows an elongated structure at depth along the direction of the inducing field. This elongation is typical of a model recovered using total-field modeling algorithms without knowledge of the magnetization direction. The inability of parametric modeling to recover a simple structure is a strong indication of remanent magnetization and is the result of an unknown magnetization direction. If the magnetization direction were known, a compact structure, consistent with known geology, could be recovered. This example illustrates the difficulty of susceptibility
Figure 2.5: Dataset $C_5$. Observed total-field magnetic anomaly data over prospective iron ore site. The dashed red line corresponds the cross section in Figure 2.12.

Figure 2.6: Top of Figure: The black dots correspond to the observed data, the black line represents the predicted response from the constructed model. The red line represents the difference between the observed and predicted data. Bottom of Figure: Parametric model of $C_3$ data profile through 2300 m Easting. Distance is in northing. The structure shown is elongated along the direction of the inducing field. The image illustrates the difficulty in recovering a simple anomaly.
inversion to recover a logical model that reproduces data in the presence of remanence.

The second dataset, \( C_5 \) is also heavily affected by remanence. The total-field anomaly data in Figure 2.5 were inverted using a susceptibility inversion algorithm. The optimal model obtained through Tikhonov regularization is shown in Figure 2.7. The model is geologically uninterpretable. The plunging characteristics of the distributed susceptibility, shown in the cross sectional view, Figure 2.8, represents a typical inversion response when incorrect magnetization direction is used as a result of remanence. The model shows elongated structures produced to attempt to fit the data given the incorrect magnetization information.

Figure 2.7: Recovered model from \( C_5 \) total-field anomaly data. Susceptibilities below .009 cutoff.

Figure 2.8: Cross section through recovered \( C_5 \) total-field model. Profile extracted along red dashed line in Figure 2.5. Structure of anomalies are plunging and geologically unrealistic.

Failure of parametric modeling and susceptibility inversion to recover simplistic geologic structures is an indication of strong remanence. Remanence can distort the depth and
shape of recovered magnetic sources and lead to incorrect interpretation. The distribution of anomalies, especially in the $C_3$ dataset, Figure 2.5, cannot be explained by induced magnetization alone. Next, I explore the use of inversion of amplitude data to overcome these challenges.

2.3 Amplitude of the Anomalous Field

Various methodologies have been developed in order to address the problems posed by the presence of remanent magnetization to minimize the dependence on total magnetization direction. Here I focus on one method specifically, the use of amplitude data.

Nabighian (1984) showed that in 2D the vertical derivative of a magnetic field can be calculated from the horizontal derivative by applying a Hilbert transform, equation 2.7. The analytic signal is obtained by combining the horizontal and vertical derivatives, equation 2.8. Nabighian continues to show that the absolute value of the analytic signal, $|A(z)|$, the amplitude, is independent of magnetization direction. This is because the amplitude of the anomaly vector is the envelope of the two components of the vector.

$$\frac{\partial M}{\partial z} = H \left( \frac{\partial M}{\partial x} \right) \quad (2.7)$$

$$A(z) = \frac{\partial (M)}{\partial x} + i \frac{\partial (M)}{\partial z} \quad (2.8)$$

While this property does not extend to 3D problems, the amplitude of the magnetic anomaly is only weakly dependent on magnetization direction in 3D (Haney et al., 2003; Nabighian, 1984). The use of amplitude data allows for the processing and interpretation of magnetic data without the need to know the magnetization direction. This method allows for the interpretation of data with remanent magnetization. Additionally, the data can be inverted to recover the magnitude of the source strength. For complex datasets, it is often necessary to work with the 3D amplitude data in order to assess the anomaly vector and recover a geologically consistent model.
The total-field anomaly is the projection of the magnetic anomaly onto the direction of the earth’s magnetic field. Magnetic amplitude data is calculated from the magnetic anomaly through a Fourier domain calculation or equivalent source by converting to the three orthogonal components of the total-field anomaly, $B_{ax}$, $B_{ay}$, and $B_{az}$, and then calculating the magnitude of the quantities, $A$:

$$A = \sqrt{B_{ax}^2 + B_{ay}^2 + B_{az}^2}$$  \hspace{1cm} (2.9)

I now illustrate how, in the presence of remanent magnetization, inversion of magnetic amplitude data can be used to recover a model where susceptibility inversion algorithms struggle to do so.

Shearer (2005) developed an inversion algorithm to be used with amplitude data; the method was later expanded by Li et al. (2010). Inversion of amplitude data can be used in cases where magnetization direction is not fully known or is highly variable. It is also applicable in instances of multiple or complex source bodies and does not rely on additional information. To find the amplitude of the magnetic anomaly, the components of the total-field magnetic anomaly, $B_{ax}$, $B_{ay}$, and $B_{az}$, are calculated in the wavenumber domain (Pedersen, 1978) or by the equivalent source technique (Dampney, 1969). The magnitude of the quantities, $B_a$, is then calculated by equation 2.9.

Figure 2.9 and Figure 2.10 show the amplitude data calculated from the total-field data in Figure 2.4 and Figure 2.5 respectively. These datasets are the amplitude of the total-field datasets and only weakly dependent on magnetization direction. With these datasets, amplitude inversion can be performed to recover effective susceptibility models that reflect the structure of anomalies without being heavily impaired by unknown magnetization direction.

2.3.1 Amplitude Inversion Method and Field Examples

Inversion of the amplitude data was performed using the methodology of Shearer (2005) and Li et al. (2010). The algorithm minimizes an $l_2$ model objective function of the susceptibility subject to fitting the amplitude data. I have briefly outlined the general inversion
Figure 2.9: $C_3$ Amplitude data calculated using total-field data seen in Figure 2.4.

Figure 2.10: $C_5$ Amplitude data calculated using total-field data seen in Figure 2.5.
algorithm below; more detail of the specific algorithm can be found in Shearer (2005).

The recovered model is obtained through the inverse solution to the equation \( G\vec{m} = \vec{d} \) where \( \vec{m} = [p_1, p_2, p_3 \ldots p_m]^T \) are the model cells and \( \vec{d} = [d_1, d_2, d_3 \ldots d_n]^T \), the data, such that \( m \gg n \). \( G \) is the sensitivity matrix describing the geometry and physics of the problem.

The inverse solution is obtained by minimizing the total objective function, \( \phi \), through Tikhonov regularization with bound constraints given by equation 2.10 where \( p_{\text{min}} \) and \( p_{\text{max}} \) are the upper and lower bounds on the susceptibility and \( \beta \) is the regularization parameter.

\[
\begin{align*}
\min \phi &= \phi_d + \beta \phi_m \\
\text{s.t.} \quad \vec{p}_{\text{min}} \leq \vec{p} \leq \vec{p}_{\text{max}} \\
&\quad \kappa > 0.
\end{align*}
\]

The elements of \( \phi \) correspond to a measure of the data misfit between the observed and predicted data, \( \phi_d \), and a model norm term, \( \phi_m \), which contains information about the model structure. The resulting model shows effective susceptibility, \( \kappa \) which is the ratio of the magnitude of magnetization, \( |\vec{M}| \) over the strength of the inducing field, \( H_0 \).

Previous to my research, the noise characteristics of amplitude data had not been characterized. As such, selection of an optimal inverse model has been a challenge. To select the optimal model, Tikhonov curve, a heuristic approach, is often used. A Tikhonov curve is plotted for various regularization parameters with data misfit, \( \phi_d \), on the y axis and model norm, \( \phi_m \), on the x axis using logarithmic scales. The optimal solution is considered to be the regularization parameter where the curve exhibits maximum curvature. While this is an established regularization method, it is heuristic and not guaranteed to be the optimal solution. This challenge is the focus of the first portion of my research. Investigating noise statistics of amplitude data could potentially allow for the use of alternative regularization methods that could increase confidence in the optimal solution.

Effective susceptibility models for the \( C_3 \) and \( C_5 \) datasets were obtained through blind amplitude inversion using AMP3D CGEM Software (2012). The \( C_3 \) dataset contains 1527
data points and the $C_5$ dataset contains 5850 data points. Both datasets are gridded with 25 m spacing. The lower and upper bounds on susceptibility were 0.0 and 2.0 respectively to accommodate the high susceptibilities and remanent magnetization associated with magnetite ores.

A cross section through the recovered model of the $C_3$ data is shown in Figure 2.11. The cross section is taken at 2300 m Easting over the anomaly, along the same profile used for parametric modeling at 2300 m Easting. The recovered effective susceptibility model illustrates the recovery of a shallow compact structure. By contrast, the corresponding parametric model is not realistic. A drilling operation, which drilled to a depth of 367 m over the anomaly, successfully located iron ore. Magnetite deposits were found at depths of 17.84-18.76 m, 20.92-22.44 m, and 30.58-31.84 m. The structure of the recovered effective susceptibility model is consistent with the known depths of the iron ore deposits.

![Figure 2.11: Effective susceptibility model of $C_3$ data taken from 2300 m Easting.](image)

The $C_3$ effective susceptibility model recovered susceptibilities in the range of approximately 0.20 $\kappa$. An analysis of the samples recovered from the drilling operation over the deposit determined the iron content ranges from 40% to 50%. The susceptibilities for magnetite ores can range from 0.1 SI to 10 SI. (Clark & Emerson, 1991). The effective susceptibility model recovered values of approximately 0.25 $\kappa$ for the anomaly. This value falls in the range of magnetite. The effective susceptibility model recovers a compact structure of appropriate susceptibility consistent with known information.
Figure 2.12: Cross section through recovered $C_5$ effective susceptibility model. Profile extracted along red dashed line in Figure 2.5.

Figure 2.13: Plan view of $C_5$ model at 100 m depth. Model indicates presence of distinct anomalies.
Figure 2.14: Recovered $C_5$ effective susceptibility model. Depth slice at 100 m depth. Susceptibilities below .009 cutoff.

A cross sectional and plan view of the recovered effective susceptibility model for the $C_5$ dataset can be seen in Figure 2.12 and Figure 2.13. These views reflect the recovery of simplistic structures reflective of the amplitude data, Figure 2.10. A comparison of the total-field anomaly cross section views, Figure 2.8 and Figure 2.12, and the model volume images, Figure 2.7 and Figure 2.14, show that the amplitude inversion reflects a significant improvement with respect to model structure. The recovered effective susceptibility model is simplistic and geologically interpretable. The susceptibility inversion was unable to resolve a simple structure of deposits that fit the data due to the contribution of remanence to the total magnetization direction. Induced magnetization alone could not account for the orientation of the anomalies in $C_5$ resulting in difficulties of susceptibility inversion to properly recover the subsurface structure.

The recovered effective susceptibilities for this model ranged from .009 to .035 SI. This susceptibility range is consistent with what would be expected for a granite deposit (Reynolds, 1997). The model indicates that drilling at this location would likely intersect a granite body, which are also verified to be in the area based on nearby drilling operations. Drilling did not occur at the $C_5$ subset but did occur at the nearby $C_2$ dataset shown in Figure 2.3 which drilled through 40 m of aeolian sands before encountering porphyry granite. Inversion results
indicate that porphyry granite bodies are also present at the $C_5$ location. In these examples, the use of amplitude data aided, not only in the recovery of geologically interpretable compact structures, but also susceptibilities capable of discriminating iron ore from granitic bodies.

2.4 Summary

Here we have provided an overview of magnetic method and remanent magnetization. Parametric modeling of field examples illustrate the challenges posed by the presence of remanent magnetization. Amplitude data is one method that can be applied to address this challenge. Other established methodology will be discussed in subsequent chapters. Unfortunately, noise statistics associated with amplitude data have not been previously investigated thus limiting the ability to select an optimal model. Addressing this challenge will be the focus of the next chapter.
Magnetic amplitude data can be an effective tool in magnetic exploration when remanently magnetized materials are present, and 3D inversion plays an important role in the quantitative interpretation of magnetic data arisen from such applications. The error statistics of amplitude data, calculated from total-field anomaly data, must be characterized for inversion. Lack of knowledge of the error in amplitude data hinders the ability to properly estimate the data misfit associated with an inverse model and therefore the selection of the optimal model itself. To overcome these challenges, we investigate the propagation of errors from total-field anomaly to amplitude data. Using parametric bootstrapping, we show that the standard deviation of the noise in amplitude data is approximately equal to that of the noise in total-field anomaly data. We then illustrate how equivalent source method can be used to estimate the error in total-field anomaly data. The obtained noise estimate can then be applied to amplitude inversion to recover an optimal inverse model by discrepancy principle. We test this methodology using both synthetic and field data examples.

3.1 Introduction

The presence of strong remanent magnetization adversely affects the interpretation of magnetic data by altering the direction of the total magnetization. In such cases, it is no longer acceptable to assume that the total magnetization direction is parallel to the inducing magnetic field. This results in difficulties in interpretation and renders traditional magnetic inversion for magnetic susceptibility ineffective. A variety of techniques have been developed to handle instances of remanence. One approach is to estimate the total magnetization direction. Many methods are available to do so, such as Helbig analysis which calculates the magnetization direction from data (Helbig, 1962, Lourenco & Morrison, 1973, Schmidt & Clark, 1998, Phillips, 2005). Cross correlation techniques can also be implemented as
shown by Dannemiller & Li (2006) which searches for the magnetization direction from
cross-correlating the total and vertical gradient of reduced-to-pole data using trial magne-
tization directions. Gerovska et al. (2009) developed a similar method by cross correlating
the reduced-to-pole field and the magnetic amplitude anomaly with trial values of magneti-
zation direction. Once the total magnetization direction is estimated, the value can be
applied to susceptibility inversion algorithms that require a known magnetization direction.
An additional method is to invert for a distribution of the magnetization vector as those
developed by Lelièvre (2009) and Ellis et al. (2012). This approach is effective but requires
additional a priori information to counter the additional non-uniqueness introduced by the
attempt to recover the 3D functions. Similar methods used by Mueller et al. (1997) and
Foss & McKenzie (2011) invert for both magnetization vectors within caustive sources and
source geometry.

For complicated anomalies, another method that can be utilized is to invert for the
magnitude of the magnetic vector directly from the magnetic amplitude anomaly. The use of
amplitude inversion, developed by Shearer (2005), is based on the methodology of Nabighian
(1972) who used Hilbert transforms to show that the absolute value of the analytic signal,
the amplitude, is independent of magnetization direction in 2D. It was later shown that
amplitude data are only weakly dependent on magnetization direction in 3D (Nabighian,
1984, Stavrev & Gerovska, 2000, Haney et al., 2003). As magnetic amplitude data are only
weakly dependent on magnetization direction in 3D, the data provide for interpretation in
environments with remanently magnetized materials by circumventing the need to know
the total magnetization direction. The method is useful for datasets with complex geology,
involving multiple anomalies and highly variable magnetization direction with a minimum
amount of prior information. 3D inversion of amplitude data has been shown to be able
to recover appropriate structure and effective susceptibilities in magnetically challenging
environments (Shearer, 2005; Li et al., 2010; Li & Li, 2012, Santos et al., 2012).
We examine the case where magnetic amplitude data are calculated from the total-field anomaly through a Fourier domain calculation, (Pedersen, 1978) by converting to three orthogonal components of the magnetic field, $B_{ax}$, $B_{ay}$, and $B_{az}$, and calculating the amplitude as,

$$B_A = \sqrt{B_{ax}^2 + B_{ay}^2 + B_{az}^2}.$$  \hspace{1cm} (3.1)

Inverting these data using Tikhonov formalism (Tikhonov & Arsenin, 1977), leads to the minimization of an objective function

$$\min \phi = \phi_d + \beta \phi_m,$$ \hspace{1cm} (3.2)

where $\phi_d$ is a data misfit term, $\phi_m$ is a model objective function, and $\beta$ is the regularization parameter. The choice of $\beta$ often relies on an estimate of the data errors, which have not been well explored for amplitude data.

To date, the relationship between the errors associated with total-field data and those associated with amplitude data are not well characterized, hindering the ability to properly estimate the appropriateness of a particular regularization parameter and data misfit associated with an inverse amplitude model. In this paper, We investigate the error statistics of calculated amplitude data and the influences of the error on the recovered inverse models by posing two questions. First, if the error distribution in the total-field anomaly data is known, what is the error and expected data misfit for the amplitude data? Second, can the equivalent source technique (Dampney, 1969) be used to estimate the errors in total-field anomaly and thereby provide info to estimate the error and expected data misfit of the amplitude data?

We address the first question by first analytically deriving the standard deviation of the noise in amplitude data, and then verify the relationship numerically by statistically characterizing synthetic data. For the second question, We illustrate that equivalent source layers computed from total-field data can be used to estimate noise in synthetic and collected magnetic data with multiple realizations of noise. The obtained error estimate then enables
us to select an optimal regularization parameter during the amplitude inversion.

### 3.2 Analytical derivation

We investigate the first question in this section, i.e., determining the error and expected data misfit in amplitude data in relation to the error distribution of the total-field anomaly. We derive the standard deviation relationship of the amplitude and confirming through numerical simulations.

For the equation \( f(x) = \sqrt{x_1^2 + x_2^2 + x_3^2} \), the variance of \( f \) can be approximated by the Taylor series expansion as (Meyer, 1975)

\[
\text{Var}[f] = \left( \frac{df}{dx_1} \right)^2 \sigma_1^2 + \left( \frac{df}{dx_2} \right)^2 \sigma_2^2 + \left( \frac{df}{dx_3} \right)^2 \sigma_3^2 + \left( \frac{df}{dx_1} \right) \left( \frac{df}{dx_2} \right) \sigma_{12} + \left( \frac{df}{dx_1} \right) \left( \frac{df}{dx_3} \right) \sigma_{13} + \left( \frac{df}{dx_2} \right) \left( \frac{df}{dx_3} \right) \sigma_{23}
\]

(3.3)

where \( \sigma_i \) is the standard deviation associated with the term \( x_i \).

In the special case where the variance is constant and the components are independent, Equation 3.3 can then be simplified as,

\[
\text{Var}[f] = \sum_{i=1}^{n} \left( \frac{df}{dx_i} \right)^2 \sigma_i^2,
\]

(3.4)

where

\[
\frac{df}{dx_i} = \frac{x_i}{\sqrt{x_1^2 + x_2^2 + x_3^2}}.
\]

(3.5)

Furthermore, if all three components have the same standard deviation \( \sigma \),

\[
\text{Var}[f] = \sigma^2,
\]

(3.6)

Equation 3.6 states that the standard deviation of the square root of the sum of the squares of three quantities, the amplitude, is equivalent to the standard deviation of the individual components.

Let us first assume that the total-field anomaly data have a constant standard deviation of \( \sigma \). If we perform a direct conversion of the total-field anomaly to three components by using the wavenumber-domain relationships (Pedersen, 1978), the three components will each also have a standard deviation \( \sigma \). Following the result in equation 3.6, we arrive at the
conclusion that the standard deviation of the amplitude data is $\sigma$.

3.2.1 Numerical Simulations

We now proceed to simulate the noise in amplitude data to verify the conclusion in the preceding section. To numerically characterize the noise in amplitude data, we need to estimate the parameters of the noise distribution. To begin, we first verify that the noise in the total-field anomaly and amplitude data are normally distributed, Figure 3.4(a) and Figure 3.4(b). We then implement the popular bootstrapping method (Efron (1979); Nicholas Metropolis (1949)) to estimate the parameters of the noise distribution of the total-field anomaly and amplitude data, the mean, $\mu$ and standard deviation, $\sigma$. The method allows us to obtain estimates of the mean and standard deviation, as well as the bias associated with the estimate, for a given population. The parametric bootstrap method is implemented by calculating the standard deviation, $\sigma^*$ and mean, $\mu^*$ of a random realization of data of size $n$, known as estimators. This is repeated $B$ times to obtain $B$ bootstrap samples and $B$ estimates of $\sigma$ and $\mu$. The mean of the obtained $\sigma^*$ and $\mu^*$ is computed by

$$\bar{\mu} = \frac{1}{B} \sum_{i=0}^{B} \hat{\mu}_i^*$$
$$\bar{\sigma} = \frac{1}{B} \sum_{i=0}^{B} \hat{\sigma}_i^*$$

where $\bar{\mu}$ and $\bar{\sigma}$ are estimates of unknown noise parameters $\mu$ and $\sigma$. To estimate the bias or confidence in the estimators, $\hat{\mu}^*$ and $\hat{\sigma}^*$, the standard error, the standard error of the obtained values, is computed by:

$$S_\mu = \sqrt{\frac{1}{B-1} \sum_{i=0}^{B} (\hat{\mu}_i^* - \bar{\mu})^2}$$
$$S_\sigma = \sqrt{\frac{1}{B-1} \sum_{i=0}^{B} (\hat{\sigma}_i^* - \bar{\sigma})^2}$$

The calculated $\bar{\mu}$ and $\bar{\sigma}$, and the respective standard errors, $S_\mu$ and $S_\sigma$, characterize the noise in the amplitude distributions. To perform the parametric modeling, we generate a synthetic model consisting of a 100x100x100 $m^3$ cube with a susceptibility of 0.1 $\kappa$ embedded
Figure 3.1: (a) View of synthetic susceptibility model. (b) Forward modeled total-field data with inducing field inclination of 45° and declination of 45°.
Figure 3.2: (a) Total-field data with 5 nT added noise. (b) Corresponding synthetic amplitude data calculated from noisy total-field anomaly realization.
in a non-magnetic background (Figure 3.1(a)). The forward modeled total-field anomaly data are computed using an inducing field with a strength of 50,000 nT oriented $45^\circ$ inclination and $45^\circ$ declination. The remanent magnetization direction of the source has an inclination of $30^\circ$ and a declination of $135^\circ$. The data have a 25 m station spacing and a constant elevation of one meter above ground surface covering an area of $400 \times 400 \text{ m}^2$. The dataset contains 1089 data (Figure 3.1(b)).

From the forward modeled total-field data, we generate 1,000 realizations of noisy data by adding 1,000 different realizations of Gaussian random number sequences with a 0-nT mean and 5-nT standard deviation. We then compute amplitude data from the true total-field anomaly and each of the realizations through wavenumber-domain conversion of components, resulting in 1,000 realizations of noise-contaminated amplitude datasets. Example realizations can be seen in Figure 3.2(a) and Figure 3.2(b).

Figure 3.3(a) and Figure 3.3(b) show histogram plots of the noise in the total-field and amplitude realizations from Figure 3.2(a) and Figure 3.2(b) respectively. The noise is obtained from subtracting the true data from the noise-contaminated realizations. The histograms for the total-field and amplitude data both indicate that the noise is normally distributed. The Normal Q-Q plots in Figure 3.4(a) and Figure 3.4(b) further support that the data is normally distributed by the linearity and slope of the points (A. Tamhane, 2000). From these simulations, we conclude that the noise in amplitude data is also Gaussian if it is Gaussian in the total-field data.

Having established the distribution of the noise, we have confirmed that we can implement the parametric bootstrapping method. We proceed to characterize the noise distribution by calculating the standard deviation, $\sigma^*$, and mean, $\mu^*$, of the noise in each of the 1,000 realizations. These values are used to calculate $\bar{\sigma}$ and $\bar{\mu}$ and the standard errors by equations 3.7 and 3.8. We record the estimators and standard errors in Table 3.1. We then calculate the same descriptive statistics at each data location across the 1,000 realizations to gain insight into the spatial properties of the realizations and record the results in Table 3.2.
Figure 3.3: (a) Histogram of noise in total-field anomaly realization shown in Figure 3.2(a). (b) Histogram of noise in amplitude realization shown in Figure 3.2(b).
Figure 3.4: Normal Q-Q plots of total-field anomaly realization noise and amplitude realization noise. The linearity of the points suggests both datasets are normally distributed.
The standard deviation of the noise for the amplitude realizations is calculated by obtaining the noise of each realization and calculating the value at each data location according to

$$\sigma_{A_j} = \sqrt{\frac{1}{(B - 1)} \sum_{j=1}^{B}(A_j - \bar{A}_j)^2}, j = 1, \ldots, N$$

where $B$ is the number of realizations, 1,000, $\bar{A}$ is the mean, $A_j$ is the realization, and $N$ is the number of data.

Table 3.1: Descriptive Statistics of Individual Realizations

<table>
<thead>
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<th></th>
<th>Total-Field (nT)</th>
<th>Amplitude (nT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\sigma}$</td>
<td>4.99</td>
<td>4.54</td>
</tr>
<tr>
<td>$S_{\sigma}$</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>$\bar{\mu}$</td>
<td>0.01</td>
<td>1.85</td>
</tr>
<tr>
<td>$S_{\mu}$</td>
<td>0.15</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table 3.2: Descriptive Statistics of Spatial Properties of Realizations

<table>
<thead>
<tr>
<th></th>
<th>Total-Field (nT)</th>
<th>Amplitude (nT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\sigma}$</td>
<td>4.99</td>
<td>4.31</td>
</tr>
<tr>
<td>$S_{\sigma}$</td>
<td>0.11</td>
<td>0.67</td>
</tr>
<tr>
<td>$\bar{\mu}$</td>
<td>0.01</td>
<td>1.85</td>
</tr>
<tr>
<td>$S_{\mu}$</td>
<td>0.16</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Important conclusions can be derived from the information in Table 3.1 and Table 3.2. Firstly, we see that in both the instances of the individual realizations and the spatial properties, the mean is larger in the amplitude realizations. Table 3.1 shows that the 1,000 amplitude realizations have an average mean of 1.85 nT compared to 0.005 nT for the total-field anomaly realizations. The same observation can be seen in Table 3.2. The difference in mean indicates that the amplitude data contain a small bias. This bias, however, is insignificant compared to the overall magnitude of the amplitude data. While noteworthy, bias in $\bar{\mu}$ does not impact the dispersion of noise which is of primary interest for inversion purposes.
Next we examine the values of standard deviation. The total-field anomaly realizations show an average standard deviation of 4.99 nT, with a standard error of 0.11 nT, consistent with the 5 nT of assigned noise. The amplitude realizations have an average standard deviation of 4.54 nT, with a standard error of 0.13 nT. While $\bar{\sigma}$ is lower than the value expected from equation 3.6, this value is also consistent with the 5 nT of added noise, especially considering the data ranges over 500 nT. The spatial standard deviation for the total-field is 4.99 nT with a standard error of 0.11 nT. The amplitude by contrast has an average standard deviation of 4.31 with a standard error of 0.67 nT. This spatial range of standard deviation values is much greater than what is observed amongst the individual realizations.

The spatial statistics of the amplitude noise indicate that there are higher order noise contributions at play. This is attributed to a covariance between the magnetic components used to calculate the amplitude data. The covariance is a result of all magnetic components being calculated from the same total field anomaly data and would not be present if the component data were collected independently. It is important to note that while present, the covariance does not impact the first order noise statistics and is therefore not significant for inversion purposes. The noise in amplitude data is best characterized by a normal distribution and not higher order bimodal distribution that the spatial noise map reveals.

Having investigated the error propagation characteristics of the amplitude data, the remaining task is to calculate the expected data misfit of the amplitude data. This value defines the target misfit for the amplitude inversion given an estimate of the standard deviation. The misfit, $\phi_d^k$, for each amplitude realization is calculated according to the misfit function in amplitude inversion (Shearer & Li, 2004),

$$
\phi_d^k = \sum_{i=1}^{n} \left( \frac{A_i^k - A_i^{true}}{\sigma_A} \right)^2,
$$

(3.10)

where $n$ corresponds to the number of data in a realization, $A_i^k$ is the $i$’th amplitude datum in the $k$’th realization, $A_i^{true}$ is the true amplitude value, and $\sigma_A$ is the estimated standard
deviation. In this instance, $\sigma_A$ is assigned a value of 5 nT, the same standard deviation added to the total-field anomaly realizations. The resulting average misfit for all 1,000 realizations is 1049. Compared to the number of data, 1,089, this represents a difference of 3.67%. The expected misfit value estimated using the standard deviation assigned to the total-field data is close to the number of data. This calculation further supports that the noise in the amplitude data is approximately the same as that in the total field anomaly data. The results also show that a noise estimate obtained from total-field anomaly data will yield a data misfit approximately equal to the number of data for amplitude data.

Thus we have answered the first question we set out to investigate and have obtained a relationship between the noise in total-field anomaly and amplitude data, and a well-defined value of the expected data misfit for amplitude data.

### 3.3 Equivalent Source Technique

Next we explore the second part of the investigation, whether the equivalent source technique can be used to estimate the errors in total-field anomaly data and subsequently the error and expected data misfit of the amplitude data. The equivalent source technique constructs a single layer of sources that reproduces all data within a dataset (Dampney, 1969). The equivalent source technique stems from the harmonic properties of the potential fields in the source-free region. If the potential is known on the bounding surface of the region, then the field anywhere in this region is uniquely determined by the solution of a Dirichlet boundary value problem (Kellogg, 1953). Therefore, if one can construct a source distribution outside this region that reproduces the field on the boundary, then it also defines the field in the source-free region. There are infinitely many such source distributions, but only one of them need be constructed. Since it usually bears no resemblance to the true source, the term equivalent source is used to describe it.

The equivalent source layer is constructed by solving the minimization problem $\phi = \phi_d + \beta \phi_m$ where $\beta$ is a regularization parameter that balances data fit and model complexity. The optimal $\beta$ parameter would produce a data misfit, $\phi_d$, equal to the number of data, based
Figure 3.5: Tikhonov plot of equivalent source reconstructions for synthetic total-field anomaly data. The construction corresponding to the corner point is selected to estimate the noise in the total-field data. $\phi_d$ is data misfit. $\phi_m$ is model norm.

on a $\chi^2_k$ distribution assumption which considers known error statistics. For our purposes the error statistics are the quantities we are attempting to obtain so we apply an inverse formulation to estimate the noise from the difference between the observed and predicted data. Instead of implementing the traditional route of solving the minimization problem to find the optimal regularization parameters with known noise statistics, we estimate the optimal $\beta$ as a means of approximating the noise (Li, 2001; Salem et al., 2010).

Selection of the regularization parameter, $\beta$ is accomplished by the use of a Tikhonov curve also refered to as an L-Curve. It is observed that a log-log plot of the norm of the least squares solution, $\phi_m$, as a function of the data misfit, $\phi_d$, displays a characteristic corner point. The regularization parameter corresponding to the corner point best minimizes both the $\phi_d$ and $\phi_m$ terms and is therefore representative of the optimum solution.

A Tikhonov plot is constructed by the use of regularized parameterization by constructing a series of source layers for a range of $\beta$ parameters. The $\phi_m$ and $\phi_d$ from each construction is plotted on a log scale and an optimal source layer is chosen. The optimal model is located at
the point of maximum curvature of the L-Curve plot. The optimization solution is considered
to be the best balance between fitting the data and having the simplest equivalent source,
the resulting difference between observed and predicted data is treated as the estimated noise
in the data.

3.4 Demonstration

Figure 3.2(a) shows one of the 1,000 realizations of noisy total-field anomaly generated
during the numerical simulations. We treat this data set as an example of the observed
total-field anomaly and use the data to illustrate that the equivalent source method can
be used to estimate noise. Equivalent source reconstructions are calculated with the use of
parameterized regularization. The equivalent source layer consists of 25 $m^3$ cells with 64
cells extending in the easting, 64 cells in the northing, and 1 cell in the depth direction. The
$\phi_d$ and $\phi_m$ values corresponding to the reconstructions are plotted in an L-Curve and the
model corresponding to the the corner point or point of maximum curvature is selected to
estimate the noise in the total-field anomaly, Figure 3.5. The standard deviation calculated
from the residual of the selected model is 4.6 nT. This value directly corresponds to the true
value of 5 nT noise added to the total-field anomaly realization. This example illustrates
that equivalent source technique can successfully estimate the noise in total-field data.

Figure 3.6: View of recovered amplitude model from synthetic data. Compare to true model
in Figure 3.1(a).
Having used equivalent source technique to successfully estimate noise, we now perform a blind inversion of synthetic amplitude data. The amplitude realization calculated from the total-field anomaly realization used for the equivalent source noise estimation, Figure 3.2(b), is inverted using AMP3D CGEM CGEM (2012). The error value used in the inversion is 4.6 nT, the estimate obtained by the equivalent source technique. The mesh is composed of 25 $m^3$ cells with 32 cells in the easting, 32 cells in the northing, and 12 cells in depth and added padding cells.

Figure 3.6 shows the recovered amplitude inversion model. A visual comparison to Figure 3.1(a), the true model, shows that the correct structure is recovered. The data misfit, $\phi_d$, of the recovered model is 1064 and corresponds directly with the number of data, 1089. The $\phi_d$ and recovered structure of the model both support that the inversion model appropriately reflects the data as well as the true model.

Thus we have answered the second question We set out to investigate and have shown that equivalent source technique can successfully estimate noise in total-field data. This noise estimate also applies to amplitude data and can be used in amplitude inversion to obtain an expected data misfit equivalent to the number of data.

The synthetic example has demonstrated the success of using equivalent source technique to estimate the noise in total-field anomaly data and also that this approximation can be used as an estimate of the noise in amplitude data, to recover an optimal model using discrepancy principle. This method is applied to two case study examples in Chapter 5.

3.5 Summary

We have established that the noise in amplitude data is normally distributed and approximately equal to the noise in total-field data. Analytical derivation and parametric bootstrapping simulations support this conclusion. We have illustrated that the noise in total-field data can indeed be well approximated by the equivalent source technique and that this estimate can be applied to amplitude inversion. Both synthetic and field case examples were inverted to show that an appropriate noise approximation for amplitude data...
results in an expected data misfit value equal to the number of data, which is consistent with the commonly used target misfit based on a $\chi^2_k$ distribution assumption. This allows for increased confidence in selecting an amplitude inverse model.

### 3.6 Higher Order Noise Effects

The noise in amplitude data is normally distributed to the first order as supported by Normal Q-Q plots of the data. However, the noise show higher order spatial correlation. An image of the spatial standard deviation of the noise at each data location for the amplitude realizations is shown in Figure 3.8 and shows that the noise is not normally distributed spatially. We will now investigate the cause of this spatial distribution to determine its cause and to better understand its influence, if any, on individual amplitude realizations.

To find the cause of the spatial correlation seen in the amplitude noise map, Figure 3.8, we revisit the amplitude noise equation, equation 3.3, and examine the individual terms to investigate their impact on the spatial distribution of the amplitude noise. We numerically simulate the six covariance terms using a set of amplitude, $B_x,B_y,$ and $B_z$ data all derived from the same total-field anomaly realization. We calculate the variance and covariance $\sigma_{ij}^2$ values between all magnetic components at each data location according to the following equation:

$$cov(i,j) = \sigma_{ij} = E(ij) - \mu_i\mu_j$$  \hspace{1cm} (3.11)

where $E$ is the expected value, and $\mu_i$ and $\mu_j$ are the means of the components.

Previously, we made the assumption that the components of the total-field anomaly are independent. This assumption holds for this investigation as the contribution of covariance to individual realizations is low. In reality, however, all components were calculated from the same total-field data and therefore there is dependence among them. For this reason, we calculate the covariance between terms.

The calculated covariance terms are shown in Figure 3.9. The plots in (a), (d), and (g) show the theoretical spatial noise distribution due to the $B_x,B_y,$ and $B_z$ components
Figure 3.7: (a) Noise in a single total-field realization. (b) Noise in a single amplitude realization.
respectively. The plots in (b), (c), and (e) show the theoretical spatial noise distribution due to the covariance between terms. The plot in (f) shows the original standard deviation map which initiated the investigation.

Adding the \( \left( \frac{\partial A}{\partial B_x} \right) \sigma_{xx}^2 \), \( \left( \frac{\partial A}{\partial B_y} \right) \sigma_{yy}^2 \), and \( \left( \frac{\partial A}{\partial B_z} \right) \sigma_{zz}^2 \) terms together results in Figure 3.9(b). The maps in Figure 3.9(b) and Figure 3.9(f) contain the same pattern. This verifies that the higher order spatially correlated noise is the result of propagation of errors in calculating the amplitude data, equation 3.3.

![Figure 3.8: Standard deviation map at each data location over 1,000 amplitude realizations.](image)

The spatial variance at each data location is reflective of the distribution of noise. The amplitude standard deviation map, therefore, shows which locations will tend to be noisier. This effect is not significant or distinguishable to the first order, however. The noise distribution of a single amplitude realization more closely resembles a normal distribution than the bimodal distribution of the spatial correlation map. Figure 3.7(a) and Figure 3.7(b) show images of the noise in a single total-field and amplitude realization. In the case of inversion, we are dealing only with a single amplitude dataset and as such the spatial variance information is negligible to the first order for our purposes, namely inversion. Inversions performed using the variable amplitude standard deviation map did not produce improved inversion
Figure 3.9: Error calculation applied to a single realization set. (a) \( \frac{\partial A}{\partial B_x} \sigma_{xx}^2 \), (b) \( \left( \frac{\partial A}{\partial B_x} \right) \left( \frac{\partial A}{\partial B_y} \right) \sigma_{xy}^2 \), (c) \( \left( \frac{\partial A}{\partial B_x} \right) \sigma_{xz}^2 \), (d) \( \left( \frac{\partial A}{\partial B_y} \right) \sigma_{yy}^2 \), (e) \( \left( \frac{\partial A}{\partial B_y} \right) \left( \frac{\partial A}{\partial B_z} \right) \sigma_{yz}^2 \), (f) \( \sigma_A^2 \), (g) \( \left( \frac{\partial A}{\partial B_z} \right) \sigma_{zz}^2 \). All units are in \( nT^2 \).
result compared to using a uniform standard deviation value of 5 nT. Rather, the inversions used the areas of higher noise to elongate the recovered body, effectively distributing higher susceptibilities where there is none. The spatial variance map is not representative of the distribution of noise in an amplitude realization. A normal distribution is the most accurate distribution to characterize noise in amplitude data.
CHAPTER 4
RECOVERY OF MAGNETIZATION DIRECTION BASED ON INVERTED EFFECTIVE SUSCEPTIBILITY

Interpretation of magnetic anomalies relies on a known total magnetization direction. The total magnetization direction is often assumed to be in the same direction as the inducing field. In the presence of remanent magnetization, however, the total magnetization direction is altered and often unknown. Unknown magnetization direction can hinder the interpretation of inverse models by leading to incorrect shape and size of the sources. Amplitude data, weakly dependent on total magnetization direction, are often used to bypass this challenge. While successful at delineating remanent source bodies, amplitude data do not contain direction information. To mitigate this drawback, we propose a method to recover the missing direction information with the use of amplitude inversion. We segment the amplitude inversion into distinct anomalous bodies and assume a constant magnetization direction for each. Assuming the effective susceptibility from the amplitude inversion provides a sufficiently accurate representation of the magnitude of the magnetization, we then solve a least-squares problem to recover the magnetization directions of multiple source bodies simultaneously. This approach is able to recover magnetization direction from datasets containing both single and multiple anomalies.

Unknown magnetization direction can result in inaccurate interpretation and renders susceptibility inversion ineffective at properly characterizing sources. A variety of techniques have been developed to overcome the challenges posed by remanent magnetization. One approach to addressing remanence is the use of inversion (Lelièvre & Oldenburg, 2009; Foss & McKenzie, 2011; Shearer, 2005) discussed in previous chapters. Here we focus on the methodology developed by Shearer (2005) who developed an inversion algorithm to recover a susceptibility distribution from the amplitude of the magnetic vector. Amplitude inversion
is based on the methodology of Nabighian (1972) who used Hilbert transforms to show that the absolute value of the analytic signal, the amplitude, was independent of magnetization direction in 2D. Nabighian (1984) and Haney et al. (2003) later extended this relationship to show that the amplitude are only weakly dependent on magnetization direction in 3D. Use of amplitude data is advantageous as it can be calculated from datasets with complex geology, including datasets with multiple anomalies and cases of highly variable magnetization direction.

Magnetic amplitude data can be calculated in the Fourier domain or by equivalent source technique by converting to the three orthogonal components of the total-field anomaly, $B_{ax}$, $B_{ay}$, and $B_{az}$, and computing the magnitude of the values as shown in equation 4.1:

$$B_a = \sqrt{B_{ax}^2 + B_{ay}^2 + B_{az}^2}. \quad (4.1)$$

3D inversion of amplitude data has been shown to recover a first order approximation of magnitude of magnetization and can be used to delineate magnetic sources in magnetically challenging environments (Shearer, 2005). Amplitude data do not contain phase information, however, and the ability to infer the dip of the source bodies is limited. A subsequent inversion with magnetization direction may help refine the result. Furthermore, the amplitude inversion does not recover total magnetization direction, which is valuable additional information. Therefore, there is a need to improve the amplitude inversion in this respect.

We have developed a two step process to estimate magnetization direction. We first segment the result of amplitude inversion into distinct source bodies, and then solve a least-squares problem to recover a magnetization direction in each source region. In the following, we first present the formulation and apply it to a synthetic single anomaly case. The estimated magnetization direction is then used in a susceptibility inversion algorithm to recover the appropriate dipping structure of the anomaly. We then expand the estimation method to a multiple source problem and show the method can estimate magnetization directions for multiple source bodies simultaneously.
4.1 Methodology

We begin by assuming we have carried out an amplitude inversion and have identified a number of source bodies. Let the magnitude of magnetization (represented by effective susceptibility) in a single body be $|M(r)|$. We further assume that the magnetization in the source body has a constant magnetization direction $\hat{M}$. The total-field magnetic anomaly produced by this source body is given by:

$$\Delta \mathbf{T}_i = \hat{B}_0^T \mathbf{G}(r_i)^T \hat{M}, \quad i = 1, \ldots, N \quad (4.2)$$

where $\hat{B}_0$ is the inducing field direction, $\mathbf{G}$ is a dyadic Greens tensor defined by

$$\mathbf{G}(r_i) = \frac{\mu_0}{4\pi} \Delta \int_{V} |\mathbf{M}(r')| \nabla \nabla \frac{1}{|\mathbf{r} - \mathbf{r}'|} \, dv, \quad (4.3)$$

and $\hat{M}$ is the unknown magnetization direction we wish to recover.

Rearranging equation 4.2 by the components of the magnetization direction, the total-field anomaly can be expressed as a weighted sum of the total-magnetic anomalies produced by the magnetization if it were in the x-, y-, and z-directions:

$$\Delta T_i = T_{xi} \hat{M}_x + T_{yi} \hat{M}_y + T_{zi} \hat{M}_z, \quad (4.4)$$

where $T_{xi} = \hat{B}_0 \cdot \mathbf{G}(r_i) \cdot \hat{x}$, and so on. This equation can be represented in matrix notation as

$$\Delta \mathbf{T} = \mathbf{A} \hat{\mathbf{M}}, \quad (4.5)$$

where $\mathbf{A}$ is the sensitivity matrix:

$$\mathbf{A} = \begin{pmatrix} T_{x1} & T_{y1} & T_{z1} \\ \vdots & \vdots & \vdots \\ T_{xN} & T_{yN} & T_{zN} \end{pmatrix}, \quad (4.6)$$

and

$$\Delta \mathbf{T} = (\Delta T_1, \ldots, \Delta T_N)^T \quad (4.7)$$

where N is the number of data.
Thus, we have an overdetermined system of $N$ equations with three unknown components of $\hat{M}$. The solution is given by:

$$\hat{M} = (A^T A)^{-1} A^T \Delta \hat{T}. \quad (4.8)$$

The inclination, $I_m$, and declination, $D_m$, of the magnetization direction can be expressed in terms of the components of the unit vector, $\hat{M}$ by:

$$I_m = \arcsin \left( \frac{\hat{M}_z}{|\hat{M}|} \right),$$
$$D_m = \arccos \left( \frac{\hat{M}_y}{\sqrt{\hat{M}_x^2 + \hat{M}_y^2}} \right). \quad (4.9)$$

The inclination and declination are an intuitive way of expressing the magnetization direction. Such information provides more insight into the data and can later be used in susceptibility inversion to recover the appropriate dip of the source body.

The method can be expanded for datasets that contain multiple remanent source bodies as well. By considering that the total-field anomaly is the summation of the responses from all the sources, we can describe $\Delta \hat{T}$ by the following:

$$\Delta \hat{T} = \Delta \hat{T}^1 + \Delta \hat{T}^2 + ... + \Delta \hat{T}^L. \quad (4.10)$$

Equation 4.5 can be extended to

$$\Delta \hat{T} = A^1 \hat{M}^1 + A^2 \hat{M}^2 + ... + A^L \hat{M}^L, \quad (4.11)$$

where $L$ is the number of sources. By concatenating each of the $A_i$ matrices into a single ($N \times L$) matrix $A$, we can solve the least-squares problem similar to equation 4.5.

### 4.2 Single Source

We begin by constructing a synthetic model shown in Figure 4.1. The model consists of a dike dipping at $45^\circ$ to the east. The dike is of 0.1 SI embedded in a non-magnetic background. The feature extends 120 m in the easting and northing and is 60 m thick. The total magnetization direction of the dike has an $I_m = 30^\circ$ and a $D_m = -30^\circ$. The inducing field direction has an $I = 60^\circ$, a $D = 50^\circ$, and a total-field strength of 50,000 nT. The total-field anomaly is computed at 25 m station spacing at a constant elevation of one meter.
above ground surface and covers an area of 500x500 m$^2$, Figure 4.2(a).

Gaussian noise with zero mean and 5 nT standard deviation is added to the total-field anomaly data. The amplitude data are then calculated from the noise contaminated total-field data in the Fourier domain, Figure 4.2(b). The synthetic amplitude data are inverted using (CGEM, 2012). The inversion algorithm follows that of Li & Oldenburg (1996) and Li & Oldenburg (2003). More details can be found in Shearer (2005). The mesh used is composed of 20 m$^3$ cells with 20 in the easting and northing directions, 15 cells in depth, with additional padding cells. The optimal model, selected using discrepancy principle is shown in Figure 4.3. The model reflects a simple compact structure. The location and distribution of the modeled source is consistent with the true model but lacks the dipping characteristic of the true model as is expected with effective susceptibility models which lack direction information. The model was used to forward model the $\Delta T_x$, $\Delta T_y$, and $\Delta T_z$ components needed to formulate the least squares problem in equation 4.8, Figure 4.4.

Figure 4.1: Synthetic dike of $\kappa_e = 0.1$ SI embedded in a non-susceptible background. Dike dips at an angle of 45$^\circ$.

The $\Delta T_x$, $\Delta T_y$, and $\Delta T_z$, and noisy total-field anomaly data, $\Delta T$, are used to formulate the least-squares problem in equation 4.5. The solution to the system of equations is given
Figure 4.2: (a) Noise contaminated total-field data with inducing field direction of I=60°, D=50° and magnetization direction of $I_m=30^\circ$ and $D_m=-30^\circ$ (b) Corresponding amplitude data.
in Table 4.1. The difference in the estimated magnetization direction from the true direction is calculated by:

$$\Delta \Theta = \arccos(\hat{M}_t \cdot \hat{M}_e),$$

(4.12)

where $\Delta \Theta$ is the difference in the two directions given in degrees, $\hat{M}_t$ is the true magnetization direction, and $\hat{M}_e$ is the estimated magnetization direction.

Table 4.1: True and estimated magnetization direction for synthetic dike

<table>
<thead>
<tr>
<th>Source</th>
<th>True $I_m$ and $D_m$ (30°, -30°)</th>
<th>Estimated $I_m$ and $D_m$ (34.08°, -14.4°)</th>
<th>Error 13.81°</th>
</tr>
</thead>
</table>

The estimated direction is consistent with the true total magnetization vector.

The method estimated $I_m=34.08°$ and $D_m=-14.4°$. These values represent a magnetization estimate 13.8° within the true value. This estimate is valuable information and can now be used with the observed total-field data using a standard inversion algorithm to recover the dipping feature of the dike.
Figure 4.4: Total-field anomaly components forward modeled from effective susceptibility model in Figure 4.1. (a) $\Delta T_x$, (b) $\Delta T_y$, (c) $\Delta T_z$
Figure 4.5: Comparison of susceptibility inversion models with and without total magnetization direction information to true model. All cross sections taken from 0 m Northing. (a) True synthetic dike model with magnetization direction of $I=30, D=-30$. (b) Susceptibility model from Inversion 1 incorporating estimated direction information. (c) Susceptibility model from Inversion 2 inverted assuming purely induced magnetization.
Two inversions were performed on the total-field anomaly data in Figure 4.2(a) using the same parameters with the exception of magnetization direction. Inversion 1 uses the estimated values of $I_m=34.08$ and $D_m=-14.4$. Inversion 2 assumes the magnetization to be parallel to the inducing field with $I_m=60$ and $D_m=-30$, effectively neglecting the contribution of remanence. Optimal models of both inversions were selected by discrepancy principle. The data misfits for the models are $\phi_d=2680$ and $\phi_d=2634$ for Inversions A and B respectively. Both $\phi_d$ values are close to the 2601 number of data. Cross sections of the true, and recovered models are shown in Figure 4.5 along 0 m Northing. The model corresponding to Inversion 1 which incorporated estimated magnetization direction is showing a significant improvement in the recovery of structure and dip in comparison to the resulting model from Inversion 2. Failure to account for the contribution of remanence, as in Inversion 2, has resulted in an inability to properly recover the shape and dip of the anomaly. Use of the magnetization direction estimate obtained by use of amplitude method, has successfully aided in recovering appropriate shape and dip of a dipping source body affected by remanent magnetization.

Having demonstrated that the magnetization direction can be successfully recovered in the case of a single source, we illustrate the method for a multiple source problem. The demonstration has illustrated that this method can be used to reasonably estimate the magnetization direction in the case of a single source.

### 4.3 Multiple Source Model

A data area containing multiple source bodies of differing magnetization directions poses an additional challenge as the problem requires the magnetization direction of all sources to be estimated simultaneously. Multiple remanent sources cannot simply be isolated from the dataset as the total-field data is the summation of the signals from all of the sources requiring an expansion of the least squares problem in equation 4.5. In the case of $L$ number of anomalies, $L$ number of magnetization directions will need to be estimated giving equation 4.11. In this case, sensitivity matrices, $A$ are calculated $L$ times for each anomaly. To accomplish this, the recovered effective susceptibility model is segmented to separate the
delineated anomalies and $\Delta T_x$, $\Delta T_y$, and $\Delta T_z$ are forward modeled over the data area for each anomaly.

To begin, we take a synthetic model of 50 m$^3$ cells consisting of two sources embedded in a non-magnetic background, Figure 4.6. The rectangular anomaly, Anomaly A, to the upper-left of the model contains a magnetization direction of $I_m = 45^\circ$ and $D_m = 75^\circ$. The second anomaly, Anomaly B, a dike located to the lower right, has a magnetization direction of $I_m = 75^\circ$ and $D_m = -45^\circ$. The inducing field direction is $I = 65^\circ$ and $D = -25^\circ$. The total-field anomaly data forward modelled from the model are shown in Figure 4.7(a). The total-field anomaly data is contaminated with noise of zero mean and 2 nT standard deviation. The amplitude data are calculated and shown in Figure 4.7(b).

![Figure 4.6: Synthetic model consisting of two sources. $\kappa_e=0.04$ SI and 0.05 SI. Sources are separated by a distance of 1200 m.](image)

The amplitude data are inverted with a mesh consisting of 50 m$^3$ cells with 30 cells in the easting and northing directions, 10 cells in depth, and additional padding cells. The optimal model is selected by discrepancy principle and shown in Figure 4.8. The model delineates and recovers the structure of the two sources but, as expected, does not recover the dip of Anomaly B. From this model, the $\Delta T_x$, $\Delta T_y$, and $\Delta T_z$ components are calculated for each of the sources and used to solve the least-squares problem in equation 4.11, as shown in Figure 4.9 and Figure 4.10. The estimated magnetization directions for the source bodies
Figure 4.7: (a) Total-field anomaly data with inducing field direction of \(I=65^\circ, D=-25^\circ\). Both sources contain remanent magnetization. Anomaly A to the north west has a magnetization direction of \(I_m = 45^\circ\). Anomaly B has a magnetization direction of \(I_m = 75^\circ\). (b) Amplitude data corresponding to synthetic total-field data.
are recorded in Table 4.2.

Table 4.2: True and estimated magnetization direction for synthetic two source problem.

<table>
<thead>
<tr>
<th></th>
<th>True $I_m$ and $D_m$</th>
<th>Estimated $I_m$ and $D_m$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anomaly A</td>
<td>$(45^\circ,75^\circ)$</td>
<td>$(33.46^\circ,54.09^\circ)$</td>
<td>$27.98^\circ$</td>
</tr>
<tr>
<td>Anomaly B</td>
<td>$(75^\circ-45^\circ)$</td>
<td>$(60.87^\circ,-20.2^\circ)$</td>
<td>$16.63^\circ$</td>
</tr>
</tbody>
</table>

Figure 4.8: Effective susceptibility model recovered from amplitude inversion in SI units. Anomaly A is to the left and Anomaly B is the the right.

The estimated magnetization direction for Anomaly A is within $27.98^\circ$ of the true direction and the estimate for Anomaly B is within $16.63^\circ$ of the dipping dike. It is important to consider that a number of factors can contribute to the ability of this method in recovering accurate estimates. As the magnetization directions of multiple anomalies interfere, accurate recovery of direction information becomes increasingly difficult. Factors such as proximity of the sources, strength of the remanent field, and direction of the fields impact the estimation.

### 4.4 Summary

Having shown magnetization direction can be recovered for single and multiple source datasets, I have now addressed the second task in improving amplitude method. This methodology, as well as noise estimation methods of Chapter 3 are now applied to field examples affected by strong remanent magnetization.
Figure 4.9: Total-field anomaly components forward modeled from the upper source of the effective susceptibility model in Figure 4.8. (a) $\Delta T_{x1}$, (b) $\Delta T_{y1}$, (b) $\Delta T_{z1}$
Figure 4.10: Total-field anomaly components forward modeled from the lower source of the effective susceptibility model in Figure 4.8. (a) $\Delta T_{x2}$, (b) $\Delta T_{y2}$, (b) $\Delta T_{z2s}$
Total magnetic direction information can be obtained from an amplitude derived susceptibility model for single anomaly and multiple anomaly datasets. While amplitude data do not contain dip information, the direction estimate obtained by this method can be used in susceptibility inversion to recover the dipping structure of an anomaly.

\[
\begin{pmatrix}
\Delta T_1 \\
\vdots \\
\Delta T_N
\end{pmatrix}
= 
\begin{pmatrix}
\Delta T_{x1} & \Delta T_{y1} & \Delta T_{z1} \\
\vdots & \vdots & \vdots \\
\Delta T_{xN} & \Delta T_{yN} & \Delta T_{zN}
\end{pmatrix}
\begin{pmatrix}
\hat{M}_x \\
\hat{M}_y \\
\hat{M}_z
\end{pmatrix}
\]  

(4.13)

\[
\Delta T = \left( A^1, A^2, \ldots, A^L \right)
\begin{pmatrix}
\hat{M}^1 \\
\hat{M}^2 \\
\vdots \\
\hat{M}^L
\end{pmatrix}
\]  

(4.14)
This chapter highlights the use of amplitude data on three field examples. The following examples are affected by remanent magnetization and each demonstrate different circumstances in which the use of amplitude data has been valuable. The first example is from Qinghai, China and is collected over an area of known iron ore deposits. Amplitude data is used in this example to discriminate lithology and determine if the anomalies in the dataset are iron ore or granitic intrusions. The next example is from Victoria Island in Northwest Territory, Canada over a kimberlite dike affected strongly by remanent magnetization. Amplitude data is used in this example to recover the structure of the dike and also calculate the magnetization direction of the feature. The last example is over norite intrusions in Black Hill, Australia. The data contains multiple anomalies of varying magnetization direction. In this instance, amplitude data is able to recover the structures of all anomalies in the dataset simply and simultaneously. The resulting model is then used to estimate the magnetization directions for individual anomalies in the dataset. The three examples are used to illustrate the use of amplitude data over a range of circumstances and applications.

5.1 Qinghai, China $C_2$ Example

The first example is of ground magnetic data collected in Qinghai, China for the purpose of iron ore exploration. The ground magnetic data, shown in Figure 5.1(a), have an induced magnetization direction of $I=55.8^\circ$ and $D=0.4^\circ$. The geology in the area is considered complex and heavily metamorphosed but is known to host both magnetite deposits and also granitic intrusions. The iron ore deposits are related to magmatic intrusions dating to the Ordovician and Silurian and again in the Carboniferous period. The granite intrusions are related to the metamorphic crystalline basement. Both the ore deposits and metamorphic intrusions are overlain by sandstone. Both the ore deposits and granitic intrusive units in
the region contain strong remanent magnetization. Goals of interpretation of the magnetic data include modeling the structure of the anomaly and determining if the anomaly is iron ore or a granitic intrusion.

Initial inversion of the total-field anomaly data, assuming purely induced magnetization, produced the result shown Figure 5.2(a) through a cross-section over the anomaly. Although the susceptibility model appears simple and geologically interpretable, it is not consistent with the known geology. The presence of remanent magnetization is the main cause of the discrepancy.

It is then reasonable to interpret this dataset using amplitude inversion. The amplitude data, Figure 5.1(b), are calculated using a Fourier domain approach. The total-field anomaly and amplitude datasets consist of 5082 data points with 25 m station spacing and covers an area of 3.04 km$^2$.

The equivalent source technique is first applied to estimate the noise in the total-field data by the methodology previously discussed. The constructed layer consisted of 25 m$^3$ cells with 76 cells extending in the easting, 64 cells in the northing, and 1 cell in the depth direction. A Tikhonov plot of the parameterized equivalent sources is shown in Figure 5.3. The point corresponding to maximum curvature of the plot is used to estimate the noise in the data. The noise in the total-field anomaly data is estimated to be approximately 2.29 nT by the equivalent source approach.

An amplitude inversion is performed from the data in Figure 5.1(b) using the noise estimate of 2.29 nT. The mesh consists of 25 m$^3$ cells with 88 cells extending in the easting, 80 cells in the northing, and 26 cells in the depth direction including padding cells. The data misfit, $\phi_d$, of the model is 5092 corresponding directly to the number of data, 5082. A cross section through the resulting model can be seen in Figure 5.2(b). The structure of the recovered model is consistent with the information known about the area and drilling performed at the site.
Figure 5.1: (a) Total-field magnetic data with inclination of $55.8^\circ$ and declination of $0.3^\circ$. Parametric modeling confirms presence of remanent magnetization. (b) Amplitude magnetic data.
Figure 5.2: (a) A cross-section through the anomaly recovered from the inversion of total-field data in Figure 5.1(a). (b) A cross-section through the anomaly recovered from the inversion of amplitude data in Figure 5.1(b). The structure recovered in the effective susceptibility model is geologically consistent with drilling information.

Figure 5.3: Tikhonov plot of Qinghai data equivalent source reconstructions.
Drilling occurred over the anomaly in this dataset. The first 40 m of ground drilled consisted of eolian sand bodies and beyond the 40 m, porphyry granite was found. The effective susceptibility model is able to resolve a structure that is increasingly consistent with known geology than the susceptibility model. The effective susceptibility model gives a more accurate indication of the depth of the magnetic source. The susceptibility model shows the magnetic sources to be significantly deeper than drilling indicates. This plunging feature is very typical of susceptibility inversion algorithms in the presence of remanence.

The additional objective is to determine if the anomaly is from magnetite or iron ore. The anomaly effective susceptibilities recovered from the effective susceptibility model range from .009 to .023 \( \kappa \). The susceptibility range for magnetite ores ranges from .1 SI to 10 SI (Clark & Emerson, 1991). The recovered effective susceptibilities are significantly lower than the range expected for magnetite. These values are consistent with the porphyry granite intrusions in the area.

Amplitude data was able to aid in accurately modeling sources in the dataset and discriminate between the presence of ore or granite. The resulting model was verified to be an accurate representation of the geology by drilling operations. The drilling information indicates the effective susceptibility model to be a significantly better representation of the area compared to the susceptibility model.

5.2 Northwest Territory Kimberlite Example

The second dataset is of aeromagnetic data. The data were acquired by TeckCominco and Diamonds North over kimberlites on Victoria Island in Northwest Territory, Canada for diamond exploration. Visual inspection of the data indicates the presence of strong remanent magnetization. The inclination of the inducing magnetic field is near vertical. The expected magnetic response at this inclination is a positive peak and the data reflect a negative depression. The objectives in processing this data is to image the kimberlite dike and estimate the total magnetization direction of the dike.
The geology in the area is characterized by an Archean granitic basement overlain by a Proterozoic sedimentary sequence with minor volcanics, capped by flat-lying Cambrian-to-Devonian carbonate rocks. The host rocks are largely nonmagnetic, and the kimberlite body stands out in the magnetic data as a distinct, sharp anomaly indicative of shallow bodies, compared with the more rounded anomalies caused by deep features in the basement or within the sedimentary rocks. The negative anomaly is associated with kimberlite intrusions.

The ages of the kimberlite intrusions range from 250 to 300 million years (J. Lajoie, personal communication, 2004). The orientation of the anomaly in the data is produced by magnetization that is dominated by strong remanence with variable direction. The total-field data are shown in Figure 5.4(a). The inducing field of the total-field data has an inclination of $86.7\,^\circ$ and declination of $26.3\,^\circ$. The anomaly is characterized by a strong negative. Given the inducing field direction, a strong positive would be expected if the magnetization was dominantly induced. For this reason, the data are a good candidate for amplitude inversion.

The equivalent source technique is applied to estimate the noise in the total-field anomaly data. Equivalent source constructions were calculated over a range of regularization parameters. Both the total-field and corresponding amplitude data, Figure 5.4(b), consist of 6232 data points with 10 m station spacing and covers an area of 0.61 km$^2$. The constructed layer consisted of $10 \times 10 \times 10$ m$^3$ cells with 161 cells extending in the easting, 155 cells in the northing, and 1 cell in the depth direction. The $\phi_m$ and $\phi_d$ of each construction is plotted in the Tikhonov curve in Figure 5.5. The predicted noise in the total-field dataset is estimated to be approximately 0.025 nT by the equivalent source approach. The total-field data has undergone processing and ranges over less than 10 nT making this a reasonable noise estimate for the dataset.

An inversion is performed on the amplitude data using a noise estimation of 0.025 nT. The mesh consists of $10 \times 10 \times 10$ m$^3$ cells with 111 cells extending in the easting, 105 cells in the northing, and 40 cells in the depth direction including padding cells. The data misfit, $\phi_d$, of the model is 6161 which is approximately the same as the number of data,
Figure 5.4: (a) Total-field magnetic data of kimberlite dike with inclination of 86.7° and declination of 26.3°. Anomaly pattern indicates strong remanent magnetization. (b) Corresponding amplitude data.
Figure 5.5: Tikhonov plot of equivalent source reconstructions for Victoria Island data.

A cross section through the model can be seen in Figure 5.6(b). The cross section reflects an appropriate near-surface structure consistent with a dipping dike. The 3D view, Figure 5.6(a), shows the model structure is consistent with the data shown in Figure 5.4(a).

The orientation of the anomaly in the data indicates that the signal is dominated by strong remanence, Figure 5.4(a). For this reason, it is desirable to calculate the magnetization direction for the data.

The recovered effective susceptibility model in Figure 5.6(a) is used to calculate the terms needed to solve for magnetization direction, Figure 5.7. The forward modeled $\Delta T_x$, $\Delta T_y$, and $\Delta T_z$ terms are then used to solve the least squares problem given by the following equations:

$$\Delta \vec{T}_i = \vec{T}_{xi} M_x + \vec{T}_{yi} M_y + \vec{T}_{zi} M_z, \quad (5.1)$$

$$\hat{M} = (A^T A)^{-1} A^T \Delta \vec{T}. \quad (5.2)$$

The total magnetization direction estimated by this method is $I_m = -82.2^\circ$ and $D_m = -101.5^\circ$.

In a previous study, Li et al. (2010) estimated the magnetization direction for this dataset using Helbig’s method, Wavelet method, and cross-correlation shown in Table 5.1. The mag-
Figure 5.6: (a) Recovered effective susceptibility model corresponding to kimberlite data. Minimum cutoff of 0.00015. Model structure is reflective of amplitude data in Figure 5.4(b). (b) Cross section through model at 260 m Northing.
Figure 5.7: Victoria kimberlite total-field anomaly components forward modeled from effective susceptibility model. (a)$T_x$, (b)$T_y$, (c)$T_z$. 
netization estimated from the effective susceptibility is recorded in the last row of the table. The magnetization direction estimated by this method is consistent with estimates made by established methodology with respect to inclination. Considering that as the inclination approached vertical, the role of declination is of little impact, we can confidently say that the magnetization estimates are consistent with one another.

Table 5.1: Estimated magnetization direction by various methods for Victoria Island data

<table>
<thead>
<tr>
<th>Method</th>
<th>Estimated $I_m$</th>
<th>Estimated $D_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helbig’s method</td>
<td>$-84.7^\circ$</td>
<td>$70.0^\circ$</td>
</tr>
<tr>
<td>Wavelet method</td>
<td>$-89.3^\circ$</td>
<td>$1.8^\circ$</td>
</tr>
<tr>
<td>Crosscorrelation</td>
<td>$-87.4^\circ$</td>
<td>$26^\circ$</td>
</tr>
<tr>
<td>FCM Method</td>
<td>$-81^\circ$</td>
<td>$2^\circ$</td>
</tr>
<tr>
<td>this study</td>
<td>$-82.2^\circ$</td>
<td>$-101.5^\circ$</td>
</tr>
</tbody>
</table>

Consistency in the magnetization direction estimated from effective susceptibility with other established methods further reinforces confidence in the method. Direction information has been successfully recovered for a field data example.

In this example, amplitude data have been used to recover the structure of the kimberlite dike and also estimate the magnetization direction. The estimated direction is consistent with other established direction estimation methods re-enforcing confidence in the estimate.

5.3 Black Hill Complex

The third field example is aeromagnetic collected over norite intrusions in Black Hill, Australia located near Adelaide, Australia. The total-field anomaly data is shown in Figure 5.8(a). The direction of the inducing magnetic field is $I = -65^\circ$ and $D = 8^\circ$. The line spacing of aeromagnetic data is 1500 m with 55 m station spacing (Rajagopalan et al., 1993). The dataset contains 3135 data locations. Inspection of the magnetic anomalies in Figure 5.8(a) indicates the presence of strong remanent magnetization. The orientation of the anomalies cannot be explained by induced magnetization alone. It is expected that the anomalies should have a magnetic high to the north and a low to the south at the given
inducing field. Instead, the anomalies are characterized by a large positive to the south-west and negative to the north-east. The orientation of the anomalies is a strong indication that the deposits are affected by remanent magnetization. Here amplitude inversion is applied to produce a model that delineates the subsurface sources. The resulting effective susceptibility model is used to estimate the magnetization directions for individual sources in the dataset.

The Black Hill Norite is composed of mafic gabroic intrusions emplaced during the Ordovician period 487 Ma ago (Turner, 1991). The intrusions intersected the Kanmantoo Group sediments during the Delamerian Orogeny occurring between 515-493 Ma ago (Burtt & Purvis, 2002). Exposure of the basement rock is seen only at the Black Hill Norite, denoted as Anomaly B in Figure 5.9. Previous studies of the Black Hill Norite complex have been performed. Rajagopalan et al. (1993) have performed a paleomagnetic study. The study reports the strength of the remanent magnetization to be 4.9 A/m and its direction to have an inclination of 7.6° and a declination of 221.1°. Foss & McKenzie (2006) inverted the magnetization of the sources in the dataset and compiled a comparison of estimates with other methodology.

Figure 5.9 shows labels for the major anomalies in the dataset. Anomaly A, the southernmost of the anomalies, is named the Central Pluton. Anomaly B, located to the north-east of Anomaly A is the Black Hill Norite. Anomaly C is located in the upper left of the dataset and is named the Cambrai Pluton (Kennedy, 1989). Drill holes intersected gabbroic rocks at the locations of Anomalies B and C (Rajagopalan et al., 1993). Anomaly D is the positive anomaly located immediately to the south-east of Anomaly C.

The amplitude data are calculated from the total-field data and shown in Figure 5.8(b). The data were inverted to obtain the recovered model shown in Figure 5.10. The model, in Figure 5.10, shows the four delineated anomalies identified in the total-field anomaly data. The four sources are regionally separated in the model and \( \Delta T_x, \Delta T_y, \) and \( \Delta T_z \) are forward modeled from each of the regions. \( \Delta T_x, \Delta T_y, \) and \( \Delta T_z \) data for each of the four anomalies are shown in Figure 5.11, Figure 5.12, Figure 5.13 and Figure 5.14. The magnetization direction
Figure 5.8: (a) Total-field anomaly data with inducing field direction of $I=-65^\circ$, $D=8^\circ$. Data is over remanently affected norite deposits. (b) Amplitude data.
Figure 5.9: Total-field anomaly data with major sources circled and labeled.

Figure 5.10: Effective susceptibility model of Black Hill data in SI units. Depth slice is taken at 1272 m depth.
Figure 5.11: Total-field anomaly components forward modeled from Anomaly C region of effective susceptibility model. (a) $T_{x1}$, (b) $T_{y1}$, (c) $T_{z1}$. 
Figure 5.12: Total-field anomaly components forward modeled from Anomaly D region of effective susceptibility model. (a) $T_{x2}$, (b) $T_{y2}$, (c) $T_{z2}$. 
Figure 5.13: Total-field anomaly components forward modeled from Anomaly B region of effective susceptibility model. (a) $T_{x3}$, (b) $T_{y3}$, (c) $T_{z3}$. 
Figure 5.14: Total-field anomaly components forward modeled from Anomaly A region of effective susceptibility model. (a)$T_x$, (b)$T_y$, (c)$T_z$. 

is solved for each of the sources by equation 4.11. The estimated magnetization directions are recorded in Table 5.2. The obtained directions have similar declinations and more variation in inclination. Measurements by Wake-Dyster (1974) found the mean magnetization direction in the data area is $I_m = 23^\circ$ and $D_m = -148^\circ$. A study of rock samples conducted by Rajagopalan et al. (1993) measured the overall rock magnetization to have $I_m = 8^\circ$ and $D_m = -139^\circ$. This rock measurement correlates with the direction estimates obtained in this investigation.

Table 5.2: Estimated magnetization direction for Black Hill Norite sources.

<table>
<thead>
<tr>
<th>Anomaly</th>
<th>Estimated $I_m$ and $D_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anomaly A</td>
<td>$(8^\circ, -138^\circ)$</td>
</tr>
<tr>
<td>Anomaly B</td>
<td>$(16^\circ, -117^\circ)$</td>
</tr>
<tr>
<td>Anomaly C</td>
<td>$(41^\circ, -178^\circ)$</td>
</tr>
<tr>
<td>Anomaly D</td>
<td>$(-38^\circ, -146^\circ)$</td>
</tr>
</tbody>
</table>

Table 5.3, Table 5.4, Table 5.5 contain estimates of the magnetization directions for Anomalies A, B, and C respectively. The estimates in this investigation are consistent with previous work indicating the method is able to successfully estimate the magnetization direction. No previous magnetization estimates for Anomaly D were found.

Table 5.3: Anomaly A estimated magnetization directions

<table>
<thead>
<tr>
<th>Anomaly</th>
<th>Estimated $I_m$ and $D_m$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anomaly A</td>
<td>$(-11^\circ, -128^\circ)$</td>
<td>Helbig Scan (Phillips, 2005)</td>
</tr>
<tr>
<td>Anomaly A</td>
<td>$(25^\circ, -130^\circ)$</td>
<td>Helbig analysis (Foss &amp; McKenzie, 2006)</td>
</tr>
<tr>
<td>Anomaly A</td>
<td>$(24^\circ, -135^\circ)$</td>
<td>Magnetization inversion (Foss &amp; McKenzie, 2006)</td>
</tr>
<tr>
<td>Anomaly A</td>
<td>$(8^\circ, -138^\circ)$</td>
<td>(this study)</td>
</tr>
</tbody>
</table>

5.4 Summary

The above examples illustrate the use of amplitude data for various applications. Inversion of amplitude data can aid in the recovery of geologically consistent structures in the presence of complicated anomalies and aid in lithology discrimination. By applying simple
Table 5.4: Anomaly B estimated magnetization directions

<table>
<thead>
<tr>
<th>Anomaly</th>
<th>Estimated $I_m$ and $D_m$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anomaly B</td>
<td>$(-4^\circ, -126^\circ)$</td>
<td>Helbig Scan (Phillips, 2005)</td>
</tr>
<tr>
<td>Anomaly B</td>
<td>$(30^\circ, -123^\circ)$</td>
<td>Helbig analysis (Foss &amp; McKenzie, 2006)</td>
</tr>
<tr>
<td>Anomaly B</td>
<td>$(28^\circ, -122^\circ)$</td>
<td>Magnetization inversion (Foss &amp; McKenzie, 2006)</td>
</tr>
<tr>
<td>Anomaly B</td>
<td>$(16^\circ, -117^\circ)$</td>
<td>(this study)</td>
</tr>
</tbody>
</table>

Table 5.5: Anomaly C estimated magnetization directions

<table>
<thead>
<tr>
<th>Anomaly</th>
<th>Estimated $I_m$ and $D_m$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anomaly C</td>
<td>$(28^\circ, -87^\circ)$</td>
<td>Helbig Scan (Phillips, 2005)</td>
</tr>
<tr>
<td>Anomaly C</td>
<td>$(12^\circ, -127^\circ)$</td>
<td>Helbig analysis (Foss &amp; McKenzie, 2006)</td>
</tr>
<tr>
<td>Anomaly C</td>
<td>$(8^\circ, -128^\circ)$</td>
<td>Magnetization inversion (Foss &amp; McKenzie, 2006)</td>
</tr>
<tr>
<td>Anomaly C</td>
<td>$(41^\circ, -178^\circ)$</td>
<td>(this study)</td>
</tr>
</tbody>
</table>

additional steps, the direction of magnetization can be recovered. Amplitude data allows for the recovery of the magnitude of magnetization as well as the magnetization direction. This brings the method full circle, helping to optimize the information available from the data.
CHAPTER 6
CONCLUSIONS AND DISCUSSIONS

Remanent magnetization presents difficulties in interpretation of magnetic data. In recent years, various methodology has been developed to address the challenge including those of Lourenco & Morrison, 1973; Schmidt & Clark, 1998; Phillips, 2005; Lelièvre, 2009; Ellis et al., 2012; Mueller et al., 1997; and Foss & McKenzie, 2011 to name a few. While all methods have their strengths and weaknesses, amplitude method is beneficial as it can interpret multiple anomalies in geologically complex settings without much prior information.

Previous work with amplitude method has focused on the development of a 3D inversion algorithm to model remanent sources, (Shearer, 2005). This thesis has built upon that work by addressing the need to understand the noise statistics. The standard deviation of noise in amplitude data is approximately the same as the standard deviation of the noise in total-field data. Knowledge of the error statistics in amplitude data allow for reliable determination of regularization parameter, data misfit and therefore the optimal effective susceptibility model. This work has created increased confidence in the selection of an optimal model.

Amplitude method is useful as it is weakly dependent on magnetization direction. By-passing the need to know total magnetization direction is valuable but failure to estimate the magnetization direction has been a weakness of the method. The direction estimation method described here has addressed this issue. This simple least squares approximation has allowed for a reliable means of estimating total magnetization direction and has added value to amplitude method as an interpretation tool for magnetic data affected by remanence.

6.1 Future Work

The development of the magnetization direction estimation method has helped add value to amplitude method by extracting additional information from magentic datasets affected by remanence. The magnetization direction work has also introduced areas for future work.
Most notably, a total-field anomaly inversion algorithm capable of inverting over an area of variable total magnetization direction would be valuable. The direction estimation method is able to estimate magnetization direction for multiple anomalies within a dataset. However, we are currently unable to incorporate the estimated direction information into a total-field inversion algorithm in datasets containing multiple sources. A total-field inversion algorithm capable of inverting over an area of variable magnetization direction can solve this problem and allow for accurate modeling of multiple dipping anomalies.

Also consider that the direction estimation method assumes that the anomaly has an overall total magnetization direction. This assumption may be poor if anomalies are elongated with variable total magnetization direction along the length of the body. In this case, the estimation method would do a poor job of reflecting the magnetization direction of the body. This problem can be overcome by estimating the magnetization direction on a cell level as opposed to for each anomaly. Such an inversion would be computationally expensive, however.
REFERENCES CITED


CGEM. 2012. *AMP3D Software package for the amplitude inversion of magnetic amplitude data.*


