RADIATIVE ALPHA CAPTURE ON S34 AT ASTROPHYSICALLY RELEVANT ENERGIES AND DESIGN OF A SCATTERING CHAMBER FOR HIGH PRECISION ELASTIC SCATTERING MEASUREMENTS FOR THE DRAGON EXPERIMENT

by

Devin Connolly
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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Applied Physics).

Golden, Colorado
Date ______________________

Signed: ____________________
   Devin Connolly

Signed: ____________________
   Dr. Uwe Greife
   Thesis Advisor

Golden, Colorado
Date ______________________

Signed: ____________________
   Dr. Jeff Squier
   Professor and Head
   Department of Physics
ABSTRACT

Radiative $\alpha$ capture on $^{34}$S has been shown to have an impact on nucleosynthesis in hot and explosive astrophysical environments, including stellar (core and shell) oxygen burning, Type II supernovae, and Type Ia supernovae. However, there exist discrepancies in the literature for the resonance strengths of two strong resonances within the Gamow window for oxygen burning temperatures (which ranges from $E_{\text{CM}} = 2.286 - 4.080$ MeV at 2.2 GK). Previous measurements suffered from systematic uncertainties inherent in the experimental technique employed. Furthermore, there are several states in $^{38}$Ar lying within the Gamow window for oxygen burning temperatures which no $^{34}$S + $\alpha$ resonance strength/energy measurements have been performed. The strengths of these resonances could significantly impact the astrophysical reaction rate at oxygen burning temperatures. The present measurement was performed in inverse kinematics at the DRAGON electromagnetic mass separator. DRAGON’s experimental technique allows direct measurement of quantities such as stopping power and resonance energy, alleviating the need for external inputs and reducing uncertainty in many cases. The results of the present measurement agree well with existing Hauser-Feshbach statistical models of the astrophysical reaction rate for $^{34}$S($\alpha, \gamma$)$^{38}$Ar in the temperature range of interest.

Additionally, in order to increase the science reach of the DRAGON experiment, within this thesis a new type of scattering chamber was designed to allow for the high precision measurement of charged particle elastic scattering cross sections in inverse kinematics with radioactive beams. The chamber was designed and fabricated at CSM and for the first time tested in beam.
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LIST OF SYMBOLS

barn ................................................................. 1 \( b = 10^{-24} \text{ [cm}^2\text{]} \)

vacuum speed of light ........................................... \( c = 2.99792458 \times 10^{10} \text{ [cm/s]} \)

elementary charge ................................................ \( e = 4.80320425(10) \times 10^{-10} \text{ [statC]} \)

Planck constant .................................................. \( h = 4.135667516(91) \times 10^{-15} \text{ [eV} \cdot \text{s]} \)

reduced Planck constant/Dirac constant ..................... \( \hbar \equiv \frac{h}{2\pi} = 6.58211928(15) \times 10^{-16} \text{ [eV} \cdot \text{s]} \)

\( \hbar c \) ............................................................... \( \hbar c = 197.327 \text{ [MeV} \cdot \text{fm]} \)

fine structure constant .......................................... \( \alpha \equiv \frac{e^2}{\hbar c} = \frac{1}{137.035999074(44)} = 0.0072973525698(24) \)

center of mass energy ............................................ \( E_{\text{CM}} \)

lab frame energy .................................................. \( E_{\text{Lab}} \)

resonance energy .................................................. \( E_r \)

excitation energy .................................................. \( E_x \)

Boltzmann constant ............................................... \( k \)

Loschmidt constant ............................................... \( L \)

Avogadro constant ............................................... \( N_A \)

atomic mass unit (Dalton) ..................................... \( u \)

electron mass ....................................................... \( m_e \)

proton mass ....................................................... \( m_p \)

neutron mass ...................................................... \( m_n \)

mass number ....................................................... \( A \)

atomic (proton) number ........................................ \( Z \)

neutron number ................................................... \( N \)
\( m_\alpha = 4.00260325413(6) \) [u]

\( m_{^{34}\text{S}} = 33.967867004(47) \) [u]

\( m_{^{38}\text{Ar}} = 37.962732106(209) \) [u]

\( \Delta m_\alpha = 2424.91561(6) \) [keV]

\( \Delta m_{^{34}\text{S}} = -2993.693(45) \) [keV]

\( \Delta m_{^{38}\text{Ar}} = -3471.820(195) \) [keV]

J, j

J^\pi

\( \bar{Q} \)

\( \bar{R} \)

\( \sigma \)

\( \eta \)

Y

\( \omega_\gamma \)

\( \lambda_r \)

\( \hbar \equiv \frac{\lambda}{2\pi} = \frac{1}{\kappa} \)

\( M_\odot = 1.989 \times 10^{30} \) [kg]
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>analog to digital converter</td>
<td>ADC</td>
</tr>
<tr>
<td>Bismuth germanate</td>
<td>BGO</td>
</tr>
<tr>
<td>charge to digital converter</td>
<td>QDC</td>
</tr>
<tr>
<td>constant fraction discriminator</td>
<td>CFD</td>
</tr>
<tr>
<td>data acquisition</td>
<td>DAQ</td>
</tr>
<tr>
<td>Detector of Recoils and Gammas of Nuclear reactions</td>
<td>DRAGON</td>
</tr>
<tr>
<td>diamond-like carbon</td>
<td>DLC</td>
</tr>
<tr>
<td>drift tube linac</td>
<td>DTL</td>
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<tr>
<td>double-sided silicon strip detector</td>
<td>DSSSD</td>
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<tr>
<td>electromagnetic mass separator</td>
<td>EMS</td>
</tr>
<tr>
<td>electrostatic dipole</td>
<td>ED</td>
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<tr>
<td>emitter coupled logic</td>
<td>ECL</td>
</tr>
<tr>
<td>ion-implanted silicon</td>
<td>IIS</td>
</tr>
<tr>
<td>ionization chamber</td>
<td>IC</td>
</tr>
<tr>
<td>Isotope separator and accelerator-I facility</td>
<td>ISAC-I facility</td>
</tr>
<tr>
<td>liquid nitrogen</td>
<td>LN2</td>
</tr>
<tr>
<td>linear accelerator</td>
<td>linac</td>
</tr>
<tr>
<td>magnetic dipole</td>
<td>MD</td>
</tr>
<tr>
<td>microchannel plate</td>
<td>MCP</td>
</tr>
<tr>
<td>nuclear instrumentation module</td>
<td>NIM</td>
</tr>
<tr>
<td>Offline Ion Source</td>
<td>OLIS</td>
</tr>
</tbody>
</table>
particle identification .............................................. PID
radio frequency ..................................................... RF
radio frequency quadrupole ....................................... RFQ
Scattering Of Nuclei in Inverse Kinematics ....................... SONIK
sodium iodide ........................................................ NaI
sulfur hexafluoride .................................................. $\text{SF}_6$
time to digital converter ............................................. TDC
time of flight .......................................................... TOF
Wentzel-Kramers-Brillouin approximation ................................ WKB approximation
with respect to ....................................................... w.r.t.
In social situations, I am often asked what my profession is, and my answer usually prompts follow-up inquiries as to the nature of my research. After a lengthy explanation, I am often asked the question (in some variation) “how will this benefit society?”, “of what consequence is this to me?”, or sometimes more rudely “why should I care?” I struggle to answer such questions, mostly because I find that the core concepts of physics are quite foreign to a large portion of the population, but also, because I have a hard time finding a way to relate something convoluted and often esoteric to the non-scientists’ everyday life, let alone giving it a sense of meaning and consequence thereunto. Often I find myself frustrated with such questions and my answers can be hostile: “If you cannot see the value of basic scientific research, then I cannot be bothered to explain its value to you” is a reactionary response I often have to restrain myself from retorting. When I’m not hostile, my initial reaction is to resort to Vannevar Bush’s linear model of scientific research; basic research leads to applied research and development, which leads to technology, which leads to application which leads to social benefit. In this view, what I’m researching is of utmost importance because it is basic science, and basic science drives innovation, no matter how inconsequential, peripheral or esoteric it may seem. I would love for the answer to the above questions to be that simple, but that is a fairy tale that the public is not buying anymore and even if they were, I feel uncomfortable selling it because I know it’s not truly how scientific research works (most of the time), if indeed it ever really did work that way...

I often find myself contemplating Carl Sagan’s famous monologue describing the Voyager 1 portrait of the solar system [1], and its implication of our insignificance. This line of thinking often leads me down the rabbit hole of wondering of what consequence was any of the knowledge of modern physics to an ancient person (or analogously, anyone ignorant of the fundamentals of modern science), who would have had no knowledge of (and indeed, could not even have dreamed of) the existence of such things (from superclusters to quasars,
black holes and neutron stars, all the way to the other end of the scale; atoms, protons, neutrons, electrons, quarks, leptons, hadrons and everything in between). I have come to the conclusion that the answer is that nature is simultaneously of no consequence and of ultimate consequence to all inhabitants of the universe. Those that had no idea (or have no idea and continue to choose not to) of the workings of the universe go on about there lives in ignorant bliss, hardly affected at all by the earth’s location in the Orion Arm of the Milky Way, or the trillions of neutrinos traversing their viscera every second, or the metallicity of the sun and the chemical composition of the solar system. And yet, we humans, and everything about us, and everything we do are inexorably intertwined with these phenomena, with nature, whether we are aware of it or not. And although our understanding of nature has progressed immensely since the dawn of civilization, it is not hard to imagine that a future generation (or species?) may one day view our understanding as quaintly incipient, if not downright medieval.

First and foremost, I would like to thank my advisor, Uwe Greife. Like I tell all of the prospective physics graduate students that visit Mines, I cannot say enough good things about Uwe; he is the best advisor I could have asked for. He has repeatedly gone above and beyond in his role and duties as an advisor for myself and my family. There are no words that can express my gratitude towards him and I will forever be in his debt. I would also like to thank my committee members, Fred Sarazin and Lawrence Wiencke and Mark Jensen. In addition I would like to thank the two department heads, Tom Furtak and Jeff Squier who sat during my academic career at Mines, as well as the Mines physics community. I would especially like to thank Barbara Johnson for all of her help, camaraderie, support and comic relief during my academic career at Mines. I owe immense gratitude to Greg Christian for answering many questions, engaging in stimulating discussions, and providing copious amounts of free technical support, as well as Dave Hutcheon for his subtle and masterful guidance. Special thanks go to Randy Bachman for fabricating SONIK and providing invaluable design input and feedback, as well as Martin Alcorta, Arthur Firmino and Chaim
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For Aurelia and Charlie.
PART I
SCIENTIFIC RESULT: DIRECT MEASUREMENT OF RADIATIVE ALPHA CAPTURE ON S34 AT RESONANT ENERGIES WITHIN THE GAMOW WINDOW FOR HOT AND EXPLOSIVE ASTROPHYSICAL ENVIRONMENTS
Nucleosynthesis is sensitive to radiative $\alpha$ captures on $^{34}$S in hot and explosive astrophysical environments, including (core and shell) oxygen burning [2], explosive oxygen burning (type II supernovae) [3–5], and type Ia supernovae [6]. $^{34}$S and the $\alpha$-capture reaction product $^{38}$Ar are stable nuclei, with isotopic abundances of 4.25% and 0.0629% respectively. It has been shown that reaction rates involving $^{34}$S and $^{38}$Ar significantly impact final abundances of mid-mass elements ($28 \leq A \leq 62$) [3]. Furthermore, there exist discrepancies in the literature for two strong resonances within the Gamow window (for oxygen burning) that present significant uncertainties in the stellar reaction rates at these temperatures. Finally, there are several excited states in $^{38}$Ar lying within and below the Gamow window for oxygen burning temperatures ($T = 1.5 – 2.7$ GK) for which no $^{34}$S + $\alpha$ resonance strength/energy measurements have been performed [7].

$^{34}$S is synthesized mainly via $\alpha$-capture on $^{30}$Si in a secondary reaction to the heavy ion reactions that occur during oxygen burning. $^{30}$Si is synthesized by one of the seven primary reactions given Equations 1.1a - 1.1g that occur during oxygen burning. [9]

The large $Q$ value for Equation (1.1g) populates many compound states in $^{32}$S, most of which transition to lower energies via particle emission rather than $\gamma$-ray transitions, so Equation (1.1g) does not contribute significantly to the total fusion cross section at oxygen burning temperatures [3]. There are several measurements of the $^{16}$O + $^{16}$O reaction in the literature [10–15]. However, the partial cross sections (and therefore the branching ratios of the exit channels) are in poor agreement amongst these measurements. Iliadis [2] notes that on average, Equation (1.1a) and Equation (1.1b) amount to $\approx 60\%$ of the total exit

---

*Isolated narrow resonances in the energy range $[0.3E_0, E_0]$ (where $E_0$ is the Gamow peak energy) can dominate astrophysical reaction rates [2].

$^{30}$Si (and subsequently $^{34}$S) can be synthesized through a number of other reaction channels - such as 3(4) subsequent $\alpha$-captures beginning on $^{18}$O (which is synthesized in the CNOIII & IV cycles), or numerous channels in the $^{16}$O+$^{16}$O system [8] - but these are much less likely.
channel at the lowest measured energy of \( E_{\text{cm}} = 6.8 \) MeV. The individual contributions of Equation (1.1a) and Equation (1.1b) to the total fusion cross section have not been experimentally determined [5], but a theoretical calculation sets the branching ratios at 0.5 and 0.1, respectively [16]. The remaining 40\% of the total fusion cross section at \( E_{\text{cm}} = 6.8 \) MeV is split amongst Equations (1.1) to (1.1), with Equation (1.1c) and Equation (1.1f) contributing \( \approx 25\% \), Equation (1.1d) contributing \( \approx 10\% \), and Equation (1.1e) contributing \( \approx 5\% \) [2]. Figure 1.1 illustrates the main reaction channel for the production of \( ^{34}\text{S} \).

1.1 Evolution of massive stars

After the main sequence phase of stellar evolution, massive stars embark on a Red Supergiant Phase beginning with shell hydrogen burning (as is illustrated in Figure 1.2). Eventually, the inert He core becomes dense enough to contract, increasing in temperature and igniting helium burning which converts helium to carbon and oxygen and builds up an inert C-O core. When the He is exhausted, the C-O core contracts and heats up until C burning is ignited, converting C to Mg, Ne and O. When the C in the core is exhausted, the core again contracts and heats up. At this point, stars in the mass range \( 4M_\odot < M < 8M_\odot \) will not reach sufficient temperatures to ignite further burning stages. Instead, thermal pulses in the core will destabilize the outer layers (which will then be blown off by stellar winds)
Figure 1.1: Principal oxygen burning reaction leading to production of $^{34}\text{S}$ and the subsequent $^{34}\text{S}(\alpha,\gamma)^{38}\text{Ar}$ reaction.

and the core will contract into a degenerate O-Ne-Mg white dwarf. Stars more massive than $8M_\odot$ will continue the pattern of contraction and heating to ignite successive burning stages of Ne, O and finally, Si (see Figure 1.3.) The eventual product of Si burning is an inert Fe-Ni core, which will eventually succumb to gravity and collapse cataclysmically in a type II supernova, the degenerate remnant of which will be either a neutron star or black hole (depending on the star’s initial mass.) During the supernova, an additional burning stage (that of explosive oxygen burning) is of interest for radiative $\alpha$ capture on $^{34}\text{S}$. 
1.2 Oxygen burning and explosive oxygen burning

Oxygen burning occurs in $M > 8 \, M_\odot$ stars near the end of their evolution (during the blue supergiant phase.) Core oxygen burning typically occurs at temperatures of $T = 1.5 - 2.7$ GK, and at somewhat higher temperatures during shell burning [2]. The primary ashes (byproducts) of oxygen burning are $^{28}\text{Si}$ and $^{32,34}\text{S}$ [8]. A network calculation [2] predicts that the most abundant nuclides at the end of oxygen burning are $^{28}\text{Si}(X_f = 0.54)$, $^{32}\text{S}(X_f = 0.28)$,
$^{38}\text{Ar} \ (X_f = 0.084)$, $^{34}\text{S} \ (X_f = 0.044)^*$, $^{36}\text{Ar} \ (X_f = 0.027)$ and $^{40}\text{Ca} \ (X_f = 0.021)$ [2, 8].

![Figure 1.3: Shell structure of nuclear burning stages in massive evolved stars (not to scale). Note: Ne burning occurs at $T_{\text{Ne}} = 1.2 - 1.8 \text{GK} < T_{\text{O}}$](image)

Explosive oxygen burning occurs in core collapse (type II) supernovae typically at temperatures of $T = 3 - 4 \text{ GK}$. Synthesis up to the iron peak along the main line of stability ($A = 2Z$) experiences a bottleneck at Sc and Ti which quickly photodisintegrate at these temperatures because they are weakly bound relative to the magic proton number isotopes of Ca ($Z = 20$). On the other hand, nucleosynthesis can proceed along the neutron rich path $A = 2(Z+1)$, where nuclides are more tightly bound than the main line of stability [5]. Thus $^{34}\text{S} (\alpha, \gamma) ^{38}\text{Ar}$ provides an important channel for nucleosynthesis up to the iron peak.

### 1.3 Type Ia Supernovae

A recent sensitivity study showed that radiative $\alpha$ capture on $^{34}\text{S}$ could impact nucleosynthesis in Type Ia supernovae [6]. In this study, the authors varied the reaction rate for

---

*The mass fraction of $^{34}\text{S}$ at the end of oxygen burning can be as high as 0.16 in more massive ($\sim 25M_\odot$) stars [8].*
specific reactions (among a reaction network) by a factor of $10^{\pm 1}$ (the enhancement factor, $f_0$) and compared the yield (mass ejecta, $D_i$) of nuclides to a reference model. The reference model was a one-dimensional delayed detonation model DDTc [18], using a Chandrasekhar mass C-O-Ne (49.5%-49.5%-1.0% by mass) white dwarf progenitor with a deflagration-to-detonation (DDT) transition density of $\rho_{\text{DDT}} = 2.2 \times 10^7$ g cm$^{-3}$ [6]. Table 1.1 shows the results for nucleosynthetic sensitivity to radiative $\alpha$ capture. The first column gives the parent nuclide, while the second gives the mass ejecta (in $M_\odot$) processed through the channel in the reference model. The third column gives the species for which increasing (decreasing) the enhancement factor by an order of magnitude resulted in an increased (decreased) yield of species $i$ by greater than a factor of 2. Finally, the fourth column gives the species for which increasing (decreasing) the enhancement factor by an order of magnitude increased (decreased) the yield of species $i$ between $\sim 12 - 100\%$. It is evident from Table 1.1 that in the reference model, $0.029 M_\odot$ (about 2% of the total mass ejecta) were processed thorough the $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$ channel and that varying the enhancement factor by an order of magnitude resulted in the yield of $^{38}\text{Ar}$ and $^{39}\text{K}$ being altered by 12 - 100\%. It is worth noting that:

1. Only 6 (including $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$) of the 18 ($\alpha, \gamma$) reactions considered affected the yield of $^{38}\text{Ar}$, and they all altered the yield of $^{38}\text{Ar}$ in the range of 12 - 100\%

2. Only 6 (including $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$) of the 18 ($\alpha, \gamma$) reactions considered affected the yield of $^{39}\text{K}$; of those 6 reactions, only $^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$ altered the yield of $^{39}\text{K}$ by more than a factor of 2 (the rest altered the yield of $^{39}\text{K}$ in the range of 12 - 100\%)

1.4 Existing Data

There exists about a factor of 2 discrepancy in the literature for the nominal value of the resonance strength of three strong resonances within the Gamow window for oxygen burning temperatures (see Table 1.2). Additionally, the uncertainties for these resonances do not overlap, meaning that these values have a significant degree of discrepancy. This
Table 1.1: Sensitivity of nucleosynthesis to radiative $\alpha$ captures on the parent nucleus in type Ia supernovae. $^{34}\text{S}(\alpha,\gamma)^{38}\text{Ar}$ is highlighted in red. $M_{jk}(M_\odot)$ is the mass ejecta processed through the channel in the reference model, and $|D_i|$ is the logarithmic derivative of the mass ejecta of the $i^{th}$ species with respect to the enhancement factor $f_0$. The listed reactions are those that processed more than $10^{-6}M_\odot$ in the reference model and with any $\max(|D_i|) > 0.05$. Adapted from Bravo et. al. [6].

| Parent Nuclide | $M_{jk}(M_\odot)$ | $|D_i| > 0.3$ | $0.3 \geq |D_i| \geq 0.05$ |
|----------------|-----------------|----------------|----------------------------------|
| $^{28}\text{Si}$ | 0.93            | 24,25$\text{Mg}$, 26,27$\text{Al}$, 30$\text{Si}$, 35,37$\text{Cl}$, 39$\text{K}$, 41–43$\text{Ca}$, 46,47$\text{Ti}$ | $^{30}\text{Si}$, 31,33$\text{P}$, 33,34$\text{S}$, 35$\text{Cl}$, 38$\text{Ar}$, 37$\text{Cl}$ |
| $^{32}\text{Si}$ | 0.39            | 14$\text{N}$, 20,21$\text{Ne}$, 23$\text{Na}$, 26$\text{Mg}$, 28,29$\text{Si}$, 32,33$\text{P}$, 33,34$\text{S}$, 36–38$\text{Ar}$, 40$\text{Ca}$, 44$\text{Ti}$, 48,49$\text{V}$, 52,53$\text{Mn}$ |
| $^{20}\text{Ne}$ | 0.33            | 14$\text{N}$, 20,21$\text{Ne}$, 23$\text{Na}$, 24$\text{Mg}$, 27$\text{Al}$, 29$\text{Si}$, 32$\text{P}$, 33,35$\text{S}$, 45$\text{Sc}$ |
| $^{16}\text{O}$ | 0.30            | 24$\text{Mg}$, 35$\text{S}$ | 44$\text{Ti}$ |
| $^{40}\text{Ca}$ | 0.20            | 23$\text{Na}$, 25$\text{Mg}$, 26,27$\text{Al}$, 30$\text{Si}$, 31$\text{P}$, 35$\text{Cl}$, 38$\text{Ar}$, 39$\text{K}$, 41,42$\text{Ca}$, 45,46$\text{Sc}$, 46,47$\text{Ti}$ |
| $^{58}\text{Ni}$ | 0.15            | 62$\text{Ni}$, 63$\text{Cu}$, 64–66$\text{Zn}$ |
| $^{57}\text{Ni}$ | 0.090           | 61$\text{Ni}$ |
| $^{12}\text{C}$ | 0.074           | 45$\text{Sc}$ | 14$\text{N}$, 28–30$\text{Si}$, 32$\text{P}$, 37$\text{Ar}$, 39$\text{K}$, 40–42$\text{Ca}$, 44,46$\text{Ti}$, 48$\text{V}$, 52$\text{Mn}$ |
| $^{29}\text{Si}$ | 0.065           | 33$\text{S}$ |
| $^{33}\text{S}$ | 0.062           | 37$\text{Cl}$, 37$\text{Ar}$ |
| $^{30}\text{Si}$ | 0.047           | 35$\text{S}$ | 38$\text{Ar}$, 39$\text{K}$, 41,42$\text{Ca}$, 46,47$\text{Ti}$ |
| $\rightarrow^{34}\text{S}$ | 0.029           | 43$\text{Ca}$, 47$\text{Ti}$ |
| $^{41}\text{Ca}$ | 0.024           | 38$\text{Ar}$, 39$\text{K}$, 41,42$\text{Ca}$, 46,47$\text{Ti}$ |
| $^{42}\text{Ca}$ | 0.011           | 21$\text{Ne}$ |
| $^{14}\text{N}$ | $3.1 \times 10^{-4}$ | 21$\text{Ne}$ |
| $^{62}\text{Zn}$ | $1.2 \times 10^{-4}$ |
| $^{17}\text{O}$ | $1.0 \times 10^{-4}$ |
disagreement leads to a significant uncertainty in the narrow resonance reaction rate in this energy range (see Figure 2.5). Further, it is clear from transfer reactions [7] and heavy ion fusion evaporation (HIFE) reactions [19] that there exist isolated narrow resonances in $^{38}$Ar at lower energies within the Gamow window (see section 2.3) for which radiative $\alpha$-capture resonance strengths have not been measured (see Figure 1.4.) The existing data was collected from solid targets experiments. Solid target reactions suffer from systematic uncertainties due to straggling∗, stoichiometry and contamination that are difficult to quantify. Further, obtaining an absolute value for yield measurements from solid target experiments is challenging [2], so absolute yield measurements are typically only performed for one or two energies of interest, with subsequent measurements being performed relative to these absolute measurements (leading to additional uncertainties). The DRAGON experiment (utilized in this work) is capable of measuring absolute resonance strengths for each energy of interest. Additionally, in solid target experiments, one cannot reliably measure the stopping power due to the impurity and temporal variability of the target stoichiometry, so calculation of the resonance strengths obtained from solid target yield data rely on stopping power values calculated from models or simulations. DRAGON can make direct stopping power measurements.

The present work, designated (and hereafter referred to as) TRIUMF experiment S1372, measured the resonance strengths and energies of eight isolated narrow resonances in the radiative $\alpha$ capture reaction $^{34}$S($\alpha$, $\gamma$)$^{38}$Ar. All of these resonances fall within the Gamow window for oxygen burning temperatures (see Figure 1.4), so their resonance strengths could have a significant impact on the astrophysical reaction rate of $^{34}$S($\alpha$, $\gamma$)$^{38}$Ar at oxygen burning temperatures.

The states in black above the $\alpha$-separation energy in Figure 1.4 are all known from previous $^{34}$S($\alpha$, $\gamma$)$^{38}$Ar measurements [20–22]. The $2^+$ state at $E_x = 9720$ keV has been observed in the heavy ion fusion evaporation (HIFE) reaction $^{35}$Cl($\alpha$, $\gamma p$)$^{38}$Ar [19] as well

∗Interactions between the reaction products and the target or other intervening material that alter the energy and/or trajectories of the beam or recoil particles
Table 1.2: Literature resonance strengths for the $^{34}\text{S}(\alpha,\gamma)^{38}\text{Ar}$ reaction. Values highlighted in red are those for which there exists significant disagreement amongst the literature. The Gamow window for $^{34}\text{S}(\alpha,\gamma)^{38}\text{Ar}$ at $T = 2.2$ GK is $2280 - 4080$ keV in the CM frame. All CM resonance energies carry an uncertainty of $\pm 4.5$ keV unless otherwise noted.

<table>
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<th>$E_{\text{CM}}$ [keV]</th>
<th>$\omega\gamma$ [eV]$^*$</th>
<th>$E_{\text{CM}}$ [keV]</th>
<th>$\omega\gamma$ [eV]</th>
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$^*$ Uncertainty $\pm 50\%$ unless otherwise noted.

$^†$ Absolute resonance strength measurement.

as the $^{36}\text{Ar}(t,p)^{38}\text{Ar}$ transfer reaction [24], but were not observed in $^{34}\text{S}$ radiative $\alpha$ capture measurements.

There are other states in the energy region of interest for this experiment, (for example a $1^+$ state at 9431 keV known from $^{38}\text{Ar}(e,e')$ [25] and a state of unknown $J^\pi$ ($4^- : 8^-$) at $E_x = 9829$ keV, also known from HIFE [19]), but we expect not to populate these states$^*$ and they have thus been omitted from Figure 1.4 for clarity. The states highlighted in red in Figure 1.4 are those for which there exist ambiguities in the literature for the measured

$^*$The former because it is of unnatural parity for target and projectile nuclei of $j_{t,p} = 0^+$, the latter because our likely upper limit for the $\alpha$’s angular momentum is $\ell = 3$ or possibly 4 considering the Coulomb barrier height and the $\alpha$ penetration factor $P_l(E)$.
Figure 1.4: Partial level diagram of $^{38}$Ar. States highlighted in blue were measured in this work. The two states in red (also measured in this work) are those for which there exists disagreement in the literature values of the resonance strength. The dashed red line indicates the minimum energy to which prior $^{34}$S($\alpha, \gamma$) measurements probed.
resonance strength, as well as for the resonance energy of the $E_x = 9917$ state. The states highlighted in blue are those we aimed to measure during S1372. The dashed red line indicates the minimum energy to which prior $^{34}\text{S}(\alpha, \gamma)$ measurements probed.

The $J^\pi = 2^+$ state at $E_x = 9535(30)$ keV is known from both the $^{36}\text{Ar}(t,p)^{38}\text{Ar}$ [24] and the $^{35}\text{Cl}(\alpha,p\gamma)^{38}\text{Ar}$ [26] transfer reactions; Flynn et. al. [24] measured a $J^\pi = 2^+$ state at $E_x = 9530 \pm 20$ keV and Kern et. al. [26] measured a state of unknown $J^\pi$ at $E_x = 9535 \pm 30$ keV. However, no electromagnetic transitions have been observed from this state [27].

In summary, the $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$ reaction occurs in hot and explosive astrophysical environments as a secondary reaction to the primary reaction channels of oxygen burning. There exists discrepancy in the literature for the resonance strengths of two resonances in the $^{34}\text{S} + \alpha$ system that lie within the Gamow energy window for oxygen burning temperatures. Additionally, there are several low $\ell$ states in $^{38}\text{Ar}$ that lie within the Gamow window for oxygen burning temperatures that have not been studied via radiative $\alpha$ capture on $^{34}\text{S}$ whose resonance strengths and energies could significantly impact the astrophysical reaction rate in these temperature ranges. The Gamow window is an important concept in the theory of radiative capture, which is the subject to which we turn our attention next.
CHAPTER 2
THEORY OF RADIOACTIVE CAPTURE REACTIONS

In this chapter, we present the quantitative formalism relevant to radiative capture reactions. We begin with the kinematics of radiative capture reactions, and then lay out the formalism of cross section, reaction rate (nonresonant and resonant), as well as the penetration factor and selection rules, all of which are important in the description of radiative capture reactions. The kinematics formalism is presented for the non-relativistic limit, which is sufficient for the purpose of this work (as is demonstrated below). The maximum incident beam energy in this work was measured at $E_b = T_{\text{lab}} = 25.7535 \text{ MeV}$, corresponding to a center of mass (CM - see Appendix B.1) energy of $E_{\text{CM}} = 2.715 \text{ MeV}$. Taking this to be the true relativistic kinetic energy, the Einstein relation yields

$$T^{(\text{rel})}_{\text{lab}} = m c^2 (\gamma - 1) \Rightarrow \gamma = \frac{T^{(\text{rel})}_{\text{lab}}}{m c^2} + 1 \approx 1.00081 \quad (2.1)$$

where $m c^2$ is the rest mass energy of the beam ions. It is often convenient to express the rest mass in units of $\text{amu} \times 931.494061 \text{[MeV}/c^2]$ ($m_{34\text{S}} = 31640.9 \text{[MeV}/c^2]$). Hence,

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \Rightarrow \beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} \approx 0.040322 \quad (2.2)$$

The nonrelativistic kinetic energy is then

$$T^{(\text{class})}_{\text{lab}} = \frac{1}{2} m v^2 = \frac{1}{2} m \beta^2 c^2 \approx 25.7221 \text{ [MeV]} \quad (2.3)$$

where we have used the rest mass of $^{34}\text{S}$ above. Then

$$\frac{\delta T}{T^{(\text{rel})}_{\text{lab}}} = \left| \frac{T^{(\text{rel})}_{\text{lab}} - T^{(\text{class})}_{\text{lab}}}{T^{(\text{rel})}_{\text{lab}}} \right| \approx 0.00121957 \quad (2.4)$$

The nonrelativistic lab frame kinetic energy of $T^{(\text{class})}_{\text{lab}} = 25.7221 \text{ MeV}$ corresponds to a CM energy of $E^{(\text{class})}_{\text{CM}} = 2.711 \text{ MeV}$. Then the relative difference between the relativistic and classical CM energies is $\approx 0.00118$. Since this is the maximum kinetic energy measured in this work, 0.12% is the maximum error introduced by relativistic effects.
Although the above shows that the nonrelativistic approximation could introduce possibly as much as a 4 keV error in the CM resonance energy (and therefore the excitation energy), the uncertainty in beam energies measured at DRAGON’s charge slits is 0.17% [28]. Additionally, the spread in the beam energy delivered by the ISAC-I accelerators was 0.3% FWHM, which is an uncertainty of 0.19%. Added in quadrature, this gives a total uncertainty in beam energy of \( \sigma_b = 0.26\% \). At the maximum measured bombarding energy of \( E_b = 25.7535 \text{ MeV} \), this translates to a (minimum) uncertainty of \( \sigma_b = \pm 7 \text{ keV} \) in the CM frame (and the excitation energy) at the maximum measured beam energy. Clearly the error introduced by the nonrelativistic approximation is less than that of the measurement uncertainty of the beam energy, so the nonrelativistic approximation will not introduce any significant error to the measured beam energies. Thus the nonrelativistic approximation is justified.

2.1 Kinematics

Consider a radiative capture reaction in which two nuclei, say nuclei \( a \) and \( A \) (with masses \( m_a \) and \( m_A \)) collide and fuse, producing a new nucleus (nucleus \( B \), of mass \( m_B = m_a + m_A \)) and a \( \gamma \)-ray:

\[
a + A \rightarrow B + \gamma
\]

(2.5)

The energy released in such a reaction is called the \( Q \)-value, and it is given by

\[
Q_{aA} = (m_a + m_A - m_B) c^2 = \Delta m_a + \Delta m_A - \Delta m_B
\]

(2.6)

where \( \Delta m_i \) is the atomic mass defect, which is defined as the difference between the measured (atomic) mass of the nuclide and that of the sum of the masses of its constituent nucleons:

\[
\Delta m(\frac{A}{Z}X) \equiv [m(\frac{A}{Z}X) - A] c^2 \, M_u = 931.494061 \, [m(\frac{A}{Z}X) - A] c^2 \, [\text{MeV}]
\]

(2.7)

where \( m(Z^A X) \) is the measured atomic mass of the nuclide (in amu), \( Z \) and \( A \) are the atomic number and mass number (respectively) of the given nucleus and \( M_u \) is the mass of
1 atomic mass unit (1 amu = 931.494 [MeV/c^2]).

The kinematics of the collision between nuclei a and A can be easily understood in the center of mass of the two particle system (or the equivalent one-body problem). The center of mass is defined as

\[ R = \frac{\sum_i m_i r_i}{\sum_i m_i} = \frac{m_a r_a + m_A r_A}{m_a + m_A} \tag{2.8} \]

Differentiating Equation (2.8) with respect to time gives

\[ \frac{dR}{dt} = \frac{d}{dt} \left( \frac{m_a r_a + m_A r_A}{m_a + m_A} \right) = \frac{m_a v_a + m_A v_A}{m_a + m_A} \equiv V \tag{2.9} \]

The (nonrelativistic) momentum of nucleus a with respect to the center of mass is given by (see Appendix B)

\[ p_{CM}^a = \mu v \tag{2.10} \]

where \( v \equiv v_a - v_A \) is the relative velocity of the two nuclei and \( \mu \equiv \frac{m_a m_A}{m_a + m_A} \) is the reduced mass of the system. Similarly,

\[ p_{CM}^A = -\mu v \tag{2.11} \]

The total energy of the system before the collision is given by

\[ E = \frac{1}{2} m_a v_a^2 + \frac{1}{2} m_A v_A^2 \tag{2.12} \]

and can be expressed in terms of the center of mass system as

\[ E_{CM} = \frac{1}{2} (m_a + m_A) V^2 + \frac{1}{2} \mu v^2 \tag{2.13} \]

For a two body collision (such as described above), conservation of energy demands

\[ E_b = E_{CM} + (m_a + m_A) c^2 = E_\gamma + m_B c^2 = E_f \tag{2.14} \]

where \( E_{CM} \) is the kinetic energy in the center of mass frame and \( E_\gamma \) is the energy of the emitted \( \gamma \)-ray. Rearranging Equation (2.14), we find

\[ E_\gamma = E_{CM} + (m_a + m_A) c^2 - m_B c^2 \tag{2.15} \]

*1 amu is \( \frac{1}{12} \) the mass of a neutral \(^{12}\text{C} \) atom.
and using Equation (2.6), we have
\[ E_\gamma = E_{CM} + Q \] (2.16)

It is then straightforward to show (see

2.1.1 Inverse Kinematics

Experiments at DRAGON are conducted in inverse kinematics, in which a heavy ion projectile is accelerated to moderate energies (\(\sim 15 - 50\) MeV) and impinged on a stationary low mass target nucleus in a gaseous phase (see Figure 2.1). Immediately after fusion occurs, the recoil nucleus has the same momentum as the projectile nucleus (because of conservation of momentum) and is initially in an excited energy state. The recoil nucleus subsequently decays to its ground state via \(\gamma\) transition(s). This slightly alters the momentum of the recoil. For the collision in Figure 2.1, conservation of momentum demands
\[ P_i = P_f \Rightarrow m_b v_b = (m_t + m_b)v_{rec} + p_\gamma = m_{rec}v_{rec} + p_\gamma \] (2.17)
where the subscripts p, t and rec represent the projectile, target and recoil nuclei, respectively. Then
\[ p_{rec} = p_b - p_\gamma \] (2.18)
and \(p_\gamma\) is a small correction to the momentum of the recoil nucleus. Thus in experiments conducted in inverse kinematics, recoil nuclei leave the target with nominally the same momentum as the beam. This presents a challenge to the experimenter because beam intensities

\footnote{For simplicity, we have assumed that excited recoil nuclei, transition directly to the ground state by emission of a single \(\gamma\)-ray. In general, an excited recoil can transition to the ground state by emission of multiple \(\gamma\)-rays (known as a \(\gamma\)-cascade), in which case
\[ \sum_i E^{(i)}_\gamma = E_{CM} + Q \]}

\footnote{The following exposition of the quantitative formalism of inverse kinematics also applies to forward kinematics, in which a light projectile nucleus is impinged on a (stationary) heavy target nucleus (which is bound within a solid). However, in the forward kinematics scenario, the recoil nucleus is not detected because, although it does recoil in the manner detailed below, it’s energy is typically not sufficient to liberate it from its solid bonds.}

\footnote{Of course the target nuclei in a gaseous phase are not stationary, but their kinetic energies are negligible compared to that of the heavy ion projectile.}
Figure 2.1: Collision of two nuclei in inverse kinematics. Panel (a) - Before the collision; a projectile nucleus of mass $m_b$ is incident on a (stationary) target nucleus of mass $m_t \ll m_b$ Panel (b) - After the collision; the two nuclei from the previous panel fuse, creating a compound recoil nucleus of mass $m_{rec} = m_b + m_t$ in an excited state. The excited recoil nucleus subsequently decays to its ground state via emission of one (or several) $\gamma$-ray(s) and the recoil nucleus is emitted into a cone of maximum half-angle $\phi_{\text{max}}$.

are much greater than recoil intensities, making detection of recoil nuclei difficult and necessitating suppression of the beam. At DRAGON, beam suppression is achieved through the use of an electromagnetic mass separator (see section 3.3). The momentum of a photon is given by $p_\gamma = \hbar k = \frac{E_\gamma}{c}$ (where $k$ is the wave vector), so the momentum of the recoil nucleus is

$$p_{\text{rec}} = \sqrt{2m_bE_b} \left(1 \pm \frac{E_\gamma}{\sqrt{2m_b c^2 E_b}}\right)$$

(2.19)

(the insertion of the $\pm$ will be explained below.) The recoil will be emitted into a cone of maximum half angle (see Appendix B.2)

$$\phi_{\text{max}} = \tan^{-1} \left(\frac{E_\gamma}{\sqrt{2m_b c^2 E_b}}\right)$$

(2.20)

In order to more completely understand the energy dependence of $\phi_{\text{max}}$, we can recast this in terms of $E_{\text{CM}}$ by replacing $E_\gamma$ with Equation (2.16) and using ??:

$$\phi_{\text{max}} = \tan^{-1} \left(\frac{E_{\text{CM}} + Q}{\sqrt{2m_b c^2 (m_b/\mu) E_{\text{CM}}}}\right)$$

(2.21)

Equation (2.21) is minimum at $Q$. At center of mass energies greater than $Q$, $\phi_{\text{max}}$ increases
with increasing $E_{\text{CM}}$ (and therefore increasing $E_b$), but at center of mass energies less than $Q$, $\phi_{\text{max}}$ increases with decreasing $E_{\text{CM}}$ (and therefore decreasing $E_b$). All of the yield measurements for S1372 were conducted at $E_{\text{CM}}$ energies less than $Q$, so our minimum CM energy will yield the maximum possible recoil cone angle (assuming, of course, that the excited recoil nucleus transitions directly to the ground state). The minimum measured center of mass resonance energy is $E_{\text{CM}} = 2089.4 \pm 8.7$ keV. Given the uncertainty in this measurement, the minimum center of mass resonance energy that we could expect to measure is $E_{\text{min}} \approx 2080.6$ keV. Using this along with $Q = 7208$ keV, the maximum recoil cone angle is $\phi_{\text{max}} = 8.311$ mrad, which is well within DRAGON’s maximum acceptance of $\phi_{\text{max}} = 20.0$ [mrad] [29, 30].

Conversely, in Equation (B.26), $\phi = 0$ when $\theta = n \pi$, where $n \in \mathbb{Z}$ (or when $\theta = 0^\circ, 180^\circ$) and the photon is emitted either parallel or antiparallel to the beam axis. Then the momentum of the recoil is decreased (increased) slightly if the photon is emitted parallel (antiparallel) to the beam axis. From Equation (B.19a), we find

$$p_{\text{rec}} = \begin{cases} \sqrt{2 m_b E_b} \left( 1 - \frac{E_{\gamma}}{\sqrt{2 m_b c^2 E_b}} \right) & \theta = 0^\circ \\ \sqrt{2 m_b E_b} \left( 1 + \frac{E_{\gamma}}{\sqrt{2 m_b c^2 E_b}} \right) & \theta = 180^\circ \end{cases} (2.22a)$$

and we now see the origin of the $\pm$ sign in Equation (B.17). Like $\phi_{\text{max}}$, the correction to the recoil’s momentum $\pm \frac{E_{\gamma}}{\sqrt{2 m_b c^2 E_b}}$ increases with decreasing beam energy when $E_{\text{CM}} < Q$, so the greatest spread in recoil momentum will correspond to our lowest resonance energy. The minimum beam energy measured during S1372 was that of the outgoing beam energy during yield measurement 10 (see section 5.11): $E_b = 19.5942$ MeV. At this energy, we find that the momentum spread is $\sim \pm 0.833\%$. The spread in the recoil energy can then be calculated using $E_{\text{rec}} = p_{\text{rec}}^2 / 2m_{\text{rec}}$:

$$E_{\text{rec}} = \frac{m_b}{m_{\text{rec}}} E_b \left( 1 + \frac{E_{\gamma}^2}{2 m_b c^2 E_b} \right) \pm \frac{2 E_{\gamma}}{\sqrt{2 m_b c^2 E_b}} (2.23)$$
At the minimum beam energy of $E_b = 19.594$ keV, the (classical) recoil energy is $E_{\text{rec}} = 17.532^{+0.293}_{-0.291}$ MeV, giving an energy spread of $(^{+1.67}_{-1.66})\%$, which is well within DRAGON’s energy acceptance of $\pm 4\%$ for recoil trajectories (anti)parallel to the beam axis [31].

### 2.2 Cross Section

The cross section is a quantitative description of the probability of an interaction (elastic scattering or reaction) occurring. It is formally defined as

$$\sigma \equiv \frac{\text{number of interactions per unit time}}{(\text{num. incident nuclei per area per time})(\text{num. target nuclei within beam area})}$$

$$= \frac{N_t/t}{[N_{\text{inc}}/(At)](N_{\text{tgt}})}$$  \hspace{1cm} (2.24)

$$= \frac{N_e/t}{[N_{\text{inc}}/(At)](N_{\text{tgt}})}$$  \hspace{1cm} (2.25)

where we have replaced the number of interactions with the number of ejected interaction products $N_e$. Then the differential interaction cross section* is

$$\frac{d\sigma}{d\Omega} = \frac{N^d\Omega / t}{(N_{\text{tgt}})(N_{\text{inc}} / t)} \frac{A}{d\Omega}$$  \hspace{1cm} (2.26)

where $N^d\Omega$ is the number of interaction products emitted into a detector of area $dA = r^2 d\Omega$, where $r$ is the distance from the detector to the point where the interaction occurred. We can recast the total cross section and differential cross section in terms of (particle) current densities as

$$\sigma = \frac{N_e/t}{j_{\text{inc}}N_{\text{tgt}}}$$  \hspace{1cm} (2.27)

$$\frac{d\sigma}{d\Omega} = \frac{j_e dA}{j_{\text{inc}} d\Omega} = \frac{j_e r^2 d\Omega}{j_{\text{inc}} d\Omega} = \frac{j_e r^2}{j_{\text{inc}}}$$  \hspace{1cm} (2.28)

where

*The differential cross section is related to the total cross section by $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$, where $d\Omega$ is the element of differential solid angle ($d\Omega = \sin \theta d\theta d\phi$ in spherical polar coordinates).
\[ j_{\text{inc}} \equiv \frac{N_{\text{inc}} / t}{A} \]  
\[ j_e \equiv \frac{N_e^d d\Omega / t}{N_{\text{tgt}} dA} \]  

The total interaction cross section \( \sigma_T \) can be expressed as the sum of the elastic scattering cross section \( \sigma_e \) and the reaction cross section \( \sigma_r \):

\[ \sigma_{\text{Tot}} = \sigma_e + \sigma_r \]  

2.3 Reaction Rate

The reaction rate for reactions in the stellar interior is formally defined as the number of reactions occurring per unit time \( t \) and unit volume \( V \)

\[ \frac{N_{\text{Rxn}}}{V t} = \frac{N_{\text{inc}} N_{\text{tgt}}}{V A t} \sigma = \frac{N_{\text{inc}}}{V} N_{\text{tgt}} v \sigma \]  

where we have used \( j_{\text{inc}} \equiv \frac{N_{\text{inc}}}{A t} = v \frac{N_{\text{inc}}}{V} \), and \( v \) is the relative velocity between the two particle species. For particle induced reactions (such as a radiative capture reaction) the above can be rewritten as

\[ r_{01} = N_0 N_1 v \sigma(v) \]  

where we have defined \( r_{01} \equiv \frac{N_{\text{Rxn}}}{V t} \) and \( N_0 \equiv \frac{N_{\text{inc}}}{V} \) and \( N_1 \equiv \frac{N_{\text{tgt}}}{V} \) are the number densities of the particles participating in the reaction. The (relative) velocity of ions in the stellar plasma is described by a distribution of velocities, so the probability of an ion having a velocity between \( v \) and \( v + dv \) is \( P(v) dv \). We are interested in the most probable velocity for ions in the stellar plasma, hence the reaction rate is described by the expectation value of the velocity distribution, namely

*For identical particles, the rate given in Equation (2.33) double counts the number of particle pairs \( N_0^2 \); so the rate is generally given by \( r_{01} = \frac{N_0 N_1 v \sigma(v)}{1 + N_{\text{tgt}}} \)

†The expectation value of a probability distribution \( P(x) \) is generally defined as \( \langle x \rangle = \int x P(x) dx \), where the integration is over the entire domain of the distribution.
\[ r_{01} = N_0 N_1 \int_0^{\infty} \int_0^{\infty} v P(v_0) P(v_1) \sigma(v) \, d^3v_0 \, d^3v_1 = N_0 N_1 \langle \sigma v \rangle \]  

(2.34)

Most stellar environments can be reasonably well approximated as an ideal gas, so the velocity distribution of ions in the stellar plasma can be described by the Maxwell-Boltzmann velocity distribution, so Equation (2.34) becomes (see Appendix C)

\[ P_0(v_0) d^3v_0 P_1(v_1) d^3v_1 = \frac{(m_0 m_1)^{3/2}}{(2\pi k T)^3} \exp \left( - \frac{m_0 v_0^2 + m_1 v_1^2}{2kT} \right) d^3v_0 d^3v_1 \]  

(2.35)

This is expressed in terms of the CM energy as (see Appendix C)

\[ N_0 N_1 \langle \sigma v \rangle = N_0 N_1 \sqrt{\frac{8}{\pi \mu (k T)^3}} \int_0^{\infty} e^{-E/(k T)} E \sigma(E) \, dE \]  

(2.36)

The reaction rate is frequently tabulated in terms of the matter density \( \rho \) and mass fraction \( X_i \) or mole fraction \( Y_i \) of nuclei in the stellar plasma. These values are related to the number density thusly

\[ N_i = N_A \frac{\rho X_i}{A_i} = \rho N_A Y_i \]  

(2.37)

where \( N_A \) is Avogadro’s number and \( A_i \) is the mass number of nuclear species \( i \) in atomic mass units. Then Equation (2.34) becomes

\[ r_{01} = Y_0 Y_1 \rho^2 N_A^2 \langle \sigma v \rangle = Y_0 Y_1 \rho^2 N_A^2 \sqrt{\frac{8}{\pi \mu (k T)^3}} \int_0^{\infty} e^{-E/(k T)} E \sigma(E) \, dE \]  

(2.38)

The reaction rate is often expressed in terms of the *stellar reaction rate*:

\[ N_A \langle \sigma v \rangle = N_A \sqrt{\frac{8}{\pi \mu (k T)^3}} \int_0^{\infty} E \sigma(E) e^{-E/kT} \, dE \]  

(2.39)

### 2.3.1 Nonresonant Reaction Rate

The reaction rate for particle induced reactions is given by Equation (2.39). For nonresonant reactions, the reaction cross section may be replaced by

\[ \sigma(E) \equiv \frac{1}{E} e^{-2\pi \eta S(E)} \]  

(2.40)
where $S(E)$ is the so-called astrophysical S-factor and $e^{-2\pi \eta}$ is the Gamow factor (see Figure 2.2) defined by

$$P_{\ell=0}(E) \approx \exp \left( -2\pi \frac{Z_0 Z_1 e^2}{\hbar} \sqrt{\frac{\mu}{2E}} \right) \equiv e^{-2\pi \eta} \quad (2.41)$$

where $Z_i$ are the atomic number of the nuclei in question and $\eta \equiv \frac{Z_0 Z_1 e^2}{\hbar} \sqrt{\frac{\mu}{2E}}$ is the Sommerfeld parameter. The (non-resonant) reaction rate is then given by

$$N_A \langle \sigma v \rangle = N_A \sqrt{\frac{8}{\pi \mu (kT)^3}} \int_0^\infty S(E) e^{-E/kT} e^{-2\pi \eta} dE \quad (2.42)$$

Assuming $S(E) = S_0$ is constant, we can bring it outside the integral:

$$N_A \langle \sigma v \rangle = N_A S_0 \sqrt{\frac{8}{\pi \mu (kT)^3}} \int_0^\infty e^{-E/kT} e^{-2\pi \eta} dE \quad (2.43)$$

The integrand in Equation (2.43) is known as the Gamow peak, and it can be well approximated by a Gaussian distribution with identical maximum and concavity (see Figure 2.3):
Figure 2.3: Gamow window (vertical blue lines) and Gaussian approximation of the Gamow peak (dashed blue curve.)

\[ G(E) = C \exp \left( -\frac{(E - E_0)^2}{2 \Delta^2} \right) \]  \hspace{1cm} (2.44)

where \( C, E_0, \) and \( \Delta \) are determined by matching the respective extrema and concavity values of the Gamow peak and Gaussian distribution. Doing so (see Appendix C.1), we find that the Gamow peak is given by

\[ E_0 = \left( \frac{\pi Z_0 Z_1 e^2 kT}{\hbar} \sqrt{\frac{\Pi}{2}} \right)^{2/3} \]  \hspace{1cm} (2.45)

and the width of the Gamow window is

\[ \Delta = \frac{4}{\sqrt{3}} \sqrt{E_0 kT} \]  \hspace{1cm} (2.46)

The Gaussian approximation of the Gamow peak is then

\[ G(E) = e^{\frac{3E_0}{kT}} \exp \left( -\frac{3(E - E_0)^2}{4E_0 kT} \right) \]  \hspace{1cm} (2.47)
The region between $E_0 - \Delta/2$ and $E_0 + \Delta/2$ is known as the Gamow window. For $^{34}\text{S} + \alpha$ at a temperature of $T = 2.2$ GK (typical for core oxygen burning), the Gamow window ranges from 2280 - 4080 keV ($E_0 = 3180 \pm 900$ keV) in the center of mass frame. For nonresonant reactions, the Gamow window is the most probable energy range for thermonuclear reactions to occur at a given temperature. For explosive oxygen burning ($T = 3.5$ GK), the Gamow window ranges from 3000 - 5660 keV ($E_0 = 4340 \pm 1320$ keV).

The Gaussian approximation of the integrand in Equation (2.43) gives a rough idea of the energy region that will most significantly impact reaction rates of primary interest to nuclear astrophysics. Additionally, Iliadis [2] notes that at elevated temperatures, isolated narrow resonances below the Gamow peak could have the most significant contribution to the astrophysical reaction rate, in which case the effective energy window would more appropriately be $[0.3E_0, E_0]$, rather than the oft quoted $E_0 \pm \Delta/2$. Following this approach, the effective energy window for oxygen burning would be $[955, 3180]$ keV, and the effective energy window for explosive oxygen burning would be $[1300, 4340]$ keV (in the center of mass frame).

### 2.3.2 Narrow Resonance Reaction Rate

The cross section for an isolated narrow resonance is generally described by the one-level Breit-Wigner formula

$$\sigma_{\text{BW}}(E) = \frac{\lambda^2}{4\pi(2j_i + 1)(2j_p + 1)} \frac{(1 + \delta_{tp})(\Gamma_a\Gamma_b)}{(E_r - E)^2 + \Gamma^2/4}$$

(2.48)

where $\lambda$ is the de Broglie wavelength of the reaction, $j_i$ are the spins of the target and projectile, $J$ is the spin of the excited state in the recoil nucleus, $E_r$ is the energy of the resonance, $\Gamma_i$ are the partial widths of the entrance and exit channels of the resonance, $\Gamma$ is the total width of the resonance, and $\delta_{tp}$ is the Kronecker delta ($\delta_{tp} = 0$ for $^{34}\text{S} + \alpha$.)

Equation (2.48) is a distribution of Lorentzian shape (see Figure 2.4.) We are concerned with radiative $\alpha$ capture reactions in which we assume that only the $\gamma$ and $\alpha$ channels are
open (in which case $\Gamma = \Gamma_\gamma + \Gamma_\alpha$), so we may replace $\Gamma_i$ with $\Gamma_\gamma$ and $\Gamma_\alpha$ and Equation (2.48) becomes

$$\sigma_{BW}(E) = \frac{\lambda^2}{4\pi} \omega \frac{\Gamma_\alpha \Gamma_\gamma}{(E_r - E)^2 + \Gamma^2/4}$$

where we have defined*

$$\omega \equiv \frac{(2J + 1)}{(2j_t + 1)(2j_p + 1)}$$

Inserting Equation (2.49) into Equation (2.39), we find that the narrow resonance reaction rate is (see Appendix C.1.1)

$$N_A \langle \sigma v \rangle = N_A \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \omega_\gamma e^{-E_r/kT}$$

from which it follows that the reaction rate due to multiple narrow resonances is (numerically) given by [2]

*Note: $\omega = 2J + 1$ for $^{34}$S + $^\alpha$ because both nuclei have a ground state spin and parity of $J^\pi = 0^+$. 

25
\[ N_A \langle \sigma v \rangle = 1.5399 \times 10^{11} (\mu T_9)^{-3/2} \sum_i (\omega \gamma)_i e^{-11.605 E_{r,i}/T_9} \quad [\text{cm}^3\text{mol}^{-1}\text{s}^{-1}] \quad (2.52) \]

where \( T_9 \) is the temperature expressed in units of GK. It is common practice to compare reaction rates obtained from laboratory measurements to that of the Hauser-Feshbach statistical model [32–34], given by

\[ N_A \langle \sigma v \rangle = \exp \left( a_0 + a_1 T_9^{-1} + a_2 T_9^{-1/3} + a_3 T_9^{1/3} + a_4 T_9 + a_5 T_9^{5/3} + a_6 \ln T_9 \right) \quad (2.53) \]

where \( a_i \) are reaction dependent parameters. Table 2.1 lists the parameters \( a_i \) given by Rauscher and Thielemann for radiative \( \alpha \) capture on \( ^{34}\text{S} \) [32].

**Table 2.1:** Parameters of the Hauser-Feshbach Statistical model for the \( ^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar} \) reaction rate. Set 1 are the fit parameters for the NONSMOKER code [32], while set 2 are the recommended fit parameters from REACLIB [35]. See text for explanation of the differences between the parameter sets.

<table>
<thead>
<tr>
<th>Set</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( a_6 )</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>1.29700</td>
<td>-408.32</td>
<td>706.73</td>
<td>-48.690</td>
<td>3.04950</td>
<td>-308.000</td>
</tr>
<tr>
<td>2</td>
<td>55.3373</td>
<td>0.0</td>
<td>-65.4106</td>
<td>-7.70521</td>
<td>6.43820</td>
<td>-2.88918</td>
<td>-6.66667</td>
</tr>
<tr>
<td></td>
<td>90.3440</td>
<td>-6.34146</td>
<td>0.0</td>
<td>-62.1734</td>
<td>9.25995</td>
<td>-0.666665</td>
<td>-1.500000</td>
</tr>
</tbody>
</table>

Additionally, the REACLIB reaction rate database includes experimental input to the Hauser-Feshbach statistical model. The REACLIB fit is the same fit used by Rauscher and Thielemann [32] (Equation (2.53)), but allows for multiple instances of Equation (2.53) with differing fit parameters sets \( a_i \) to be summed in order to properly fit rates with contributions from non-resonant charged particle induced reactions, non-resonant neutron induced reactions, and narrow resonance reactions. The REACLIB database gives the two sets of parameters \( a_i \) listed in Table 2.1. The two sets of parameters are inserted into Equation (2.53) and the two instances are summed to obtain the total REACLIB reaction rate. Figure 2.5 shows the uncertainty spread in the narrow resonance reaction rate due to the literature discrepancies for the two strong known resonances in \( ^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar} \) assuming the factor 2
uncertainty for both, and compares the narrow resonance reaction rate to the theoretical Hauser-Feshbach statistical model calculated using Equation (2.53) [32, 33].

2.4 Coulomb Barrier Penetration Factor

In order for a radiative capture reaction to occur, an incident particle must penetrate the Coulomb and centripetal barriers. The probability of penetration is described by the penetration factor, $P_\ell(E)$, given by

$$ P_\ell(E) = \left( \frac{|\chi_\ell(\infty)|^2}{|\chi_\ell(R_0)|^2} \right) = \frac{1}{|F_\ell(\eta, \rho)|^2 + |G_\ell(\eta, \rho)|^2} \bigg|_{\rho = ER_C} $$

(2.54)

where $F_\ell(\eta, \rho)$ and $G_\ell(\eta, \rho)$ are the regular and irregular Coulomb wave functions, respec-

Figure 2.5: Uncertainties in narrow resonance contribution to astrophysical reaction rate arising from discrepancies in the literature and comparison to the Hauser-Feshbach statistical model [32, 35]. Vertical lines denote typical temperature ranges for oxygen burning (1.7-2.5 GK) and explosive oxygen burning (3-4 GK).
Figure 2.6: Total resonant reaction rates calculated using Equation (2.52) and resonance strengths and energies from Sinha et. al. (red) [22] and Erne and Van Der Leun (blue) [21] compared to the NONSMOKER [32] and REACLIB [35] Hauser-Feshbach statistical models. The REACLIB rate is plotted with uncertainty bands of $\pm 5 \times \text{rate}$ and $\pm 10^2 \times \text{rate}$. Vertical lines denote typical temperature ranges for oxygen burning (1.7-2.5 GK) and explosive oxygen burning (3-4 GK).

The penetration factor can be approximated using the WKB approximation [9]:

$$P_\ell(E) = \frac{|\Psi_{11}(\infty)|^2}{|\Psi_{11}(R_0)|^2} \approx \left[\frac{V_B - E}{E}\right]^{1/2} \exp\left(-\frac{2\sqrt{2}E}{\hbar} \int_{R_0}^{R_c} \sqrt{V_\ell(r') - E} \, dr'\right)$$  \hspace{1cm} (2.55)$$

where $R_0 \equiv 1.25(A_0^{1/3} + A_1^{1/3})$, $V_\ell(r)$ is the Coulomb plus centrifugal potential barrier (see Figure 2.7), given by

$$V_\ell(r) = \frac{Z_0 Z_1 e^2}{r} + \frac{\hbar^2 \ell(\ell + 1)}{2\mu r^2}$$  \hspace{1cm} (2.56)$$

$V_B$ is the potential barrier height at the nuclear radius ($V_B = V_\ell(R_0)$) and $R_c$ is the classical turning point. For an incident ion of energy $E$, the classical turning point is
Figure 2.7: Coulomb plus centripetal barrier penetration for radiative α-capture. $R_0$ is the nuclear radius (approximated here as a square well for $r < R_0$) and $R_C$ is the classical turning point, and $E_\alpha$ is the (center of mass) incident α-particle energy.

$$R_c = \frac{Z_0 Z_1 e^2 \mu \pm \sqrt{(Z_0 Z_1 e^2 \mu)^2 + 2 \mu \hbar^2 (\ell + 1) E}}{2 \mu E} \tag{2.57}$$

Humblet et al. [36] note that the WKB approximation for the penetration factor should be accurate as long as $E < 0.8 V_B$. For $^{34}\text{S} + \alpha$, $V_{(\ell=0)}(R_0) = V_C \approx 7640$ keV is the minimum (i.e. $\ell = 0$) potential barrier height. Our maximum measured center of mass resonance energy was $E_r = 2706$ keV $\approx 0.35 V_C$, so the WKB approximation should give accurate results. Equation (2.55) can be evaluated numerically, but is often approximated as [9]

$$P_\ell(E) \approx \left[\frac{V_B - E}{E}\right]^\frac{1}{2} \exp \left(-\frac{a}{\sqrt{E}} \left[1 + \frac{2}{3 \pi} \left(\frac{E}{V_C}\right)^{\frac{3}{2}} \right] + 4b - \frac{2}{b} \left(\ell + \frac{1}{2}\right)^2\right) \tag{2.58}$$

where

$$a \equiv \pi Z_0 Z_1 e^2 \sqrt{\frac{2 \mu}{\hbar^2}} \tag{2.59}$$
and

\[ b \equiv \sqrt{\frac{2\mu}{\hbar^2} Z_0 Z_1 e^2 R_0} \]  \hspace{1cm} (2.60)

By evaluating Equation (2.58) at varying values of angular momentum for a given energy, we can estimate the angular momentum limit of states that will be populated in $^{38}\text{Ar}$. Evaluating the penetration factor at the maximum CM resonance energy measured during S1372 $E_\text{r}^{\text{max}} = 2709$ keV, we find that $P_3/P_0 \approx 0.031$, $P_4/P_0 \approx 0.0030$ and $P_5/P_0 \approx 1.7 \times 10^{-4}$. $P_\ell(E)$ decreases as $E_\text{CM}$ decreases, so the probability of populating higher spin states also decreases with decreasing energy (e.g. $P_0(2085 \text{ keV})/P_0(2706 \text{ keV}) \approx 0.009$). Thus we anticipate being limited to populating states with a maximum angular momentum of $\ell = 3$ or possibly $\ell = 4$.

### 2.5 Selection Rules

Given that $^{34}\text{S}$ and $^4\text{He}$ are both even-even nuclei, they both have a ground state spin and parity of $J^\pi = 0^+$. For a channel containing two nuclei (1 & 2) with spins $j_i$ and parities $\pi_i$, conservation of total angular momentum $J$ and total parity $\Pi$ demand

\[ J = \ell + j_1 + j_2 = \ell + s \]  \hspace{1cm} (2.61)

\[ \Pi = \pi_1 \pi_2 (-1)^\ell \]  \hspace{1cm} (2.62)

where $s = j_1 + j_2$ and $s = |j_1 - j_2|, ..., j_1 + j_2$. Inserting $j_\alpha = j_{34}\text{S} = 0$ and $\pi_\alpha = \pi_{34}\text{S} = +1$, we have $s = |0 - 0|, ..., 0 + 0 = 0$ and

\[ J_\pi = \ell_\alpha \]  \hspace{1cm} (2.63)

\[ \pi_\pi = (-1)^{\ell_\alpha} \]  \hspace{1cm} (2.64)

Thus the spins and parities of resonances in $^{38}\text{Ar}$ populated by radiative $\alpha$ capture on $^{34}\text{S}$ are uniquely determined by the orbital angular momentum $\ell_\alpha$. Then the allowed quantum numbers for $\ell_\alpha = 0, 1, 2, ..., \text{etc.}$ are $J_\pi^{\pi_\pi} = 0^+, 1^-, 2^+, 3^-, 4^+, \text{etc.}$ We expect to populate
such states in $^{38}\text{Ar}$ which are said to have natural parity. On the other hand, we expect not to populate states of unnatural parity (i.e. those with $J_r^{\pi} = 0^-, 1^+, 2^-, 3^+, ...$ etc.), such as the $J^{\pi} = 1^+$ state at $E_x = 9431$ keV.

The above only concerns half of the reaction of interest, namely the $\alpha$-capture. We must also consider the selection rules for radiative transition of the excited $^{38}\text{Ar}$ recoil nuclei. For a channel containing only 1 nucleus and a photon, Equation (2.61) becomes

$$J_r = L + J_f$$  \hspace{1cm} (2.65)

$$\Pi_E = \pi_1(-1)^L$$ \hspace{1cm} for electric (E) multipole radiation \hspace{1cm} (2.66)

$$\Pi_M = \pi_1(-1)^{L+1}$$ \hspace{1cm} for magnetic (M) multipole radiation \hspace{1cm} (2.67)

where $L = 1, 2, 3, ...$ etc. is the multipolarity of the electromagnetic radiation, $J_r$ is the total angular momentum of the resonance (initial) state and $J_f$ is the total angular momentum of the final state. We only expect to populate states with a maximum angular momentum $l_\alpha = 4 \Rightarrow J = 4$ (see section 2.4), so we will only consider states with $J^{\pi} = 0^+, 1^-, 2^+, 3^-, 4^+$. According to the selection rules, $|J - J_f| \leq L \leq J_r + J_f$. Consider transitions from a $J^{\pi} = 1^-$ resonance in $^{38}\text{Ar}$. Then a transition to a $J = 0$ state, $|0 - 1| = 1 \leq L \leq 0 + 1 = 1 \Rightarrow L = 1$ ($\Rightarrow$ dipole radiation) and $\pi_0 = \pi_r(-1)^1 = (-1)(-1) = 1$ for electric multipole radiation and $\pi_0 = \pi_r(-1)^{1+1} = (-1)(-1)^2 = -1$ for magnetic multipole radiation, so transitions to the $J^{\pi} = 0^+$ ground state may proceed via emission of electric dipole (E1) radiation or transitions to a $J^{\pi} = 0^-$ state may proceed via emission of magnetic dipole (M1) radiation. Similarly, a transition from a $J^{\pi} = 4^+$ state to a $J = 3$ state, $|4 - 3| = 1 \leq L \leq 0 + 1 = 1 \Rightarrow L = 1$ ($\Rightarrow$ dipole radiation) and $\pi_0 = \pi_r(-1)^1 = (-1)(-1) = 1$ for electric multipole radiation and $\pi_0 = \pi_r(-1)^{1+1} = (-1)(-1)^2 = -1$ for magnetic multipole radiation, so transitions to the $J^{\pi} = 0^+$ ground state may proceed via emission of electric dipole (E1) radiation or transitions to a $J^{\pi} = 0^-$ state may proceed via emission of magnetic dipole (M1) radiation.
the character and polarity for various allowed electromagnetic transitions from states of $J_i^\pi$ to $J_f^\pi$ relevant to $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$.

Table 2.2: Character and multipolarity of allowed EM transitions from states of $J_i^\pi$ to $J_f^\pi$. Only states of natural parity with respect to the $^{34}\text{S} + \alpha$ system are given for $J_f^\pi$. Note that transitions with multipoles greater than sextupole ($L > 3$) are unlikely.

<table>
<thead>
<tr>
<th>Final State $J^\pi$</th>
<th>Initial State $J^\pi$</th>
<th>0$^+$</th>
<th>1$^-$</th>
<th>2$^+$</th>
<th>3$^-$</th>
<th>4$^+$</th>
</tr>
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<tr>
<td></td>
<td></td>
<td>E1</td>
<td>E2</td>
<td>E3</td>
<td>E4</td>
<td></td>
</tr>
<tr>
<td>0$^+$</td>
<td>—*</td>
<td>M1</td>
<td>M2</td>
<td>M3</td>
<td>M4</td>
<td></td>
</tr>
<tr>
<td>1$^+$</td>
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<td>M1,E2</td>
<td>M1,E2,M3</td>
<td>M2,E3,M4</td>
<td>M3,E4,M5</td>
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</tr>
<tr>
<td>1$^-$</td>
<td>E1</td>
<td>E1,M2</td>
<td>E1,M2,E3</td>
<td>E2,M3,E4</td>
<td>E3,M4,E5</td>
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</tr>
<tr>
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<td>M1,E2,M3,E4</td>
<td>E1,M2,...,E5</td>
<td>E2,M3,...,E6</td>
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<td>M1,E2,M3</td>
<td>E1,M2,E3,M4</td>
<td>M1,E2,...,M5</td>
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<tr>
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<td>E2,M3,E4</td>
<td>E1,M2,...,E5</td>
<td>M1,E2,...,E6</td>
<td>E1,M2,...,E7</td>
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</tr>
<tr>
<td>4$^+$</td>
<td>M4</td>
<td>E3,M4,E5</td>
<td>E2,M3,...,E6</td>
<td>E1,M2,...,E7</td>
<td>M1,E2,...,E8</td>
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<tr>
<td>4$^-$</td>
<td>E4</td>
<td>M3,E4,M5</td>
<td>M2,E3,...,M6</td>
<td>M1,E2,...,M7</td>
<td>E1,M2,...,M8</td>
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<tr>
<td>5$^+$</td>
<td>M5</td>
<td>M4,E5,M6</td>
<td>M3,E4,...,M7</td>
<td>M2,E3,...,M8</td>
<td>M1,E2,...,E9</td>
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<tr>
<td>5$^-$</td>
<td>E5</td>
<td>E4,M5,E6</td>
<td>E3,M4,...,E7</td>
<td>E2,M3,...,E8</td>
<td>E1,M2,...,M9</td>
<td></td>
</tr>
</tbody>
</table>

* The $J_i \rightarrow J_f = 0 \rightarrow 0$ transition is forbidden because $\{ L \in \mathbb{I} | L > 0 \}$, so the photon must carry away at least $\hbar$ of angular momentum.
CHAPTER 3
EXPERIMENTAL PROCEDURE

Figure 3.1: Schematic of ISAC-I experimental hall at TRIUMF in Vancouver BC, Canada circa 2007. LTNO and $8\pi$ have been decommissioned. $8\pi$ has since been replaced by GRIFFIN.

The experiment was conducted in inverse kinematics at the Detector of Recoils And Gammas Of Nuclear reactions (DRAGON) which is located in the Isotope Separator and ACcelerator-I (ISAC-I) experimental hall (see Figure 3.1) at TRIUMF, Canada’s national laboratory for nuclear and particle physics. An isotopically pure beam of $^{34}$S ions was generated by the Supernanogan [37] electron cyclotron resonance (ECR) plasma source - part of ISAC’s OffLine Ion Source (OLIS) - using enriched SF$_6$ gas. After magnetic separation, the beam was accelerated through the Radio-Frequency Quadrupole (RFQ) and the Drift-Tube Linac (DTL) to energies between 596 and 758 keV/u. The beam was delivered to DRAGON’s gas target at an average intensity of $2.74 \times 10^9$ s$^{-1}$ in the 7+ charge state.
Figure 3.2: Schematic of the ISAC-I RFQ and DTL accelerators [38].

(satisfying ISAC-I’s $A/q$ requirements [38]). Use of the post-accelerator bunching station (consisting of a low-$\beta$ 11.8 MHz triple-gap structure and high-$\beta$ 35.4 MHz spiral resonator - see Figure 3.2) allowed the beam to be delivered to DRAGON with an energy spread of $\Delta E/E < 0.3\%$ at full width at half maximum (FWHM).

3.1 DRAGON Overview

DRAGON consists of three main components: the head, the electromagnetic mass separator (EMS) and the tail (see Figure 3.3). The head is comprised of DRAGON’s windowless, high-density gas target and an array of 30 Bismuth Germanate (BGO) scintillation crystals. The EMS is comprised of DRAGON’s optical (magnetic and electrostatic) separation and
focusing elements, as well as various beam diagnostics. The tail is comprised of DRAGON’s heavy ion detectors: a dual micro-channel plate (MCP) time of flight (TOF) system and either a double-sided silicon strip detector (DSSSD) [39] or ionization chamber (IC) [40]. For this measurement, the ionization chamber was selected because some $Z$ identification capability was expected for this recoil mass and energy range [40].
3.2 DRAGON Head: The windowless gas target and BGO $\gamma$-ray array

DRAGON’s high-density windowless gas target is capable of maintaining constant pressures between 0.2 - 10 Torr of $\text{H}_2$, $\text{He}$ or $^3\text{He}$ gas within its central gas volume, which has an cm effective length of $12.3 \pm 0.4$ [29]. The gas streaming out of the gas flow limiting apertures is collected in the first pumping stage by a series of roots blowers and recirculated through a liquid nitrogen (LN2) cooled zeolite cleaning trap in order to ensure gas purity (see Figure 3.6). Two ion-implanted silicon (IIS) charged particle detectors mounted inside the target box at angles of $30^\circ$ and $57^\circ$ (with respect to the beamline) continuously monitor the beam intensity by detecting elastically scattered target nuclei. This information is then used for normalization purposes. The gas target is further differentially pumped via a series of seven turbomolecular pumps attached to pumping boxes immediately upstream and downstream of the target box (see Figure 3.6). The turbos maintain a beamline vacuum on

![Figure 3.4: DRAGON’s windowless gas target.](image-url)
the order of $< 10^{-6}$ Torr. This is particularly critical on the downstream side of the target, where charge changing reactions outside the target (due to poor beam line vacuum) could degrade the EMS’s suppression factor [29].

![Figure 3.5: Schematic of DRAGON’s pumping tubes and apertures.](image)

DRAGON’s $\gamma$-ray detection array consists of 30 Bismuth Germanate (BGO) scintillation crystals of hexagonal cross section packed in a tight geometry surrounding the gas target box. The $\gamma$-detector array detects prompt $\gamma$-ray emission from excited recoil nuclei resulting from radiative capture reactions occurring inside the gas target volume. These data are used for coincidence tagging with heavy ion events in DRAGON’s end detectors (MCPs and IC/DSSSD). The total efficiency of the $\gamma$-detector array depends on the geometric coverage (89–92%) and $\gamma$ energy(ies) ($E_\gamma$), which depend(s) on the decay scheme of the recoil nucleus. Monte Carlo simulations of the $\gamma$-detector array predict an efficiency of $\sim 45–60\%$ for 1–10 MeV $\gamma$-rays [41, 42].

### 3.3 Electromagnetic mass separator (EMS)

Traditionally, radiative capture experiments using accelerated ion beams were performed by impinging a light projectile nucleus (beam particle - such as protons or $\alpha$ particles) on heavy target nuclei in solid form. This method has the advantage of simplifying the detection mechanism as well as the kinematic formalism because the target and recoil nuclei
are assumed to have negligible momentum and experimental yields rely on the detection of
the light reaction products (\(\gamma\)-rays in the case of radiative capture experiments.) However,
this setup introduces systematic uncertainties due to beam energy loss and straggling, as well
as target stoichiometry (which changes over the course of bombardment) and contamination
which are difficult to quantify.

Radiative capture reactions at DRAGON are performed in inverse kinematics, in which
a heavy projectile ($^{34}$S in our case) is impinged on a light (He) target. Recoil nuclei exit the target with the same momentum as the beam but have different energies (see section 2.1.1):

\begin{align}
    p_{\text{rec}} &\approx p_{\text{beam}} \\
    T_{\text{rec}} &= \frac{m_{\text{beam}}}{m_{\text{rec}}} T_{\text{beam}}
\end{align}

Detection of the recoil presents a challenge to the experimenter because the reaction cross sections for radiative capture reactions are small, making the intensity (or flux) of recoil nuclei many orders of magnitude smaller than the beam intensity. Therefore, the beam must be suppressed in order to detect the recoil. DRAGON achieves this utilizing two electromagnetic mass separators (EMS) arranged in series. DRAGON’s EMS consists
of two each of magnetic (M) and electrostatic (E) dipoles (for separation), ten magnetic quadrupoles (Q - for focusing) and four magnetic sextupoles (S - for focusing) arranged in the order (QQMSQQQSE)(QQSMQSEQQ). Additionally, the EMS consists of various beam diagnostics - Faraday cups (FC), beam centering monitors (BCM), a CCD camera and steering magnets (for tuning - see Figure 3.3.) Recoils can be separated from beam particles by exploiting the laws that govern charged particles in electric and magnetic fields, namely the Lorentz force law:

\[ \mathbf{F} = \frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \] (3.3)

Recoil separation is achieved by transmitting a recoil tune through the separator and steering beam particles into slits located at the focal planes of the selection stages. Recoils are separated from beam particles by selecting for charge at the (first) magnetic dipole; an
ion’s trajectory in a (uniform) magnetic field depends on its magnetic rigidity:

\[ R_M \equiv r |\mathbf{B}| = \frac{p}{q} \]  

(3.4)

Because recoil and beam particles exit the target with (nominally) the same momentum (see section 2.1.1), their trajectories in the MD depend only on their charge state \( q \). Ions that do not have the “selected” charge are steered into slits at the charge focal plane (focal plane Q in Figure 3.3.)

The next stage of selection is for mass at the (first) electric dipole (see Figure 3.9). Mass selection is achieved by exploiting the dependence of an ion’s trajectory in an electric field on electric rigidity:

\[ R_E \equiv r |\mathbf{E}| = \frac{mv^2}{q} \propto \frac{T}{q} \]  

(3.5)

Inserting Equation (3.1) into Equation (3.5), we find

\[ r_{\text{rec}} |\mathbf{E}| \propto \frac{m_{\text{beam}}}{m_{\text{rec}}} \frac{T_{\text{beam}}}{q} \] \hspace{1cm} (3.6)

whereas

\[ r_{\text{beam}} |\mathbf{E}| \propto \frac{T_{\text{beam}}}{q} \] \hspace{1cm} (3.7)

Because the overwhelming majority of ions making it to the ED have the same charge state (having been selected for charge at the MD), their trajectories in the EDs depend only on their energy, and hence, their mass. Because recoil ions are more massive than beam ions, their trajectories have a larger radius of curvature in the EDs than do beam ions. Ions that do not have the “selected” mass are steered into slits at the mass focal plane (focal plane M in Figure 3.3.) DRAGON’s sequence of two such charge and mass selections (via a second set of magnetic and electrostatic dipoles) gives it unparalleled beam suppression capabilities. Together with vetoes provided by coincidence matching between DRAGON’s \( \gamma \)-ray array and heavy ion detectors, as well as veto criteria specific to each detector, DRAGON

\*Note that recoil particles differing in charge from the selected charge state will also be steered into the charge slits; this introduces a systematic uncertainty to the yield measurement, which must be corrected by measuring the charge state distribution of the recoil species at the energies of interest.
Figure 3.9: Schematic of DRAGON’s electrostatic dipole.

has demonstrated beam suppression factors of $10^{10} - 10^{14}$ [28, 43].

3.4 DRAGON’s heavy ion detectors: microchannel plate (MCP) detector and ionization chamber (IC)

DRAGON employs a dual microchannel plate (MCP) detector to make local time of flight (TOF) measurements as one method of particle identification. DRAGON’s MCP is a transmission detector: it interacts minimally with heavy ions, allowing subsequent measurements. Each MCP detector setup consists of a diamond-like carbon (DLC) foil which produces secondary electrons, an electrostatic mirror (a grid of charged Au plated W wires)
that redirects secondary $e^-$ toward the MCP, and of course, the MCP itself (see Figure 3.10).

Generally, a microchannel plate is a slab of highly resistive material containing millions of microscopic ($\phi \sim 10 \mu m$) channels connecting the opposing faces of the slab. A sufficient bias is applied across the slab so that each channel acts as a continuous dynode electron multiplier [44]. Each of DRAGON’s MCP detector setups actually contains two MCPs in the well known Chevron configuration. Additionally, DRAGON’s MCP0 (the upstream MCP) is coupled to a resistive encoded anode (REA), which provides position data (see Figure 3.11). DRAGON’s MCP has a geometric transmission of $76.9 \pm 0.6\%$, and a detection efficiency $\sim 100\%$ for $A > 20@ \geq 500A$ keV [40]. However, it was recently discovered that during S1372 (and other recent DRAGON experiments), the electrostatic mirror wires were slightly thicker than the previously reported value of $20 \mu m$, leading to a geometric transmission of
**Figure 3.11:** MCP position coordinate hit pattern in the plane perpendicular to the beam axis at MCP0 as read out by the resistive encoded anode. Blue squares represent singles events, red triangles are coincident events, and green asterisks are events that passed a cut condition in the MCP TAC vs Separator TOF 2-D pulse height spectrum.

66.4 ± 1.0%.

DRAGON’s ionization chamber detects heavy ions that transit the separator. The IC detection volume is 25 cm in length and filled with isobutane (typically 10-15 Torr). It is separated from the beam line by a thin mylar entrance window. Ions that enter the IC lose energy in the detection volume due to interaction with the fill gas [40]. A non-relativistic ion’s stopping power is classically described by the Bethe formula:[45]

\[
\frac{-dE}{dx} \approx N Z \frac{4 \pi e^4 q^2}{m_e v^2} \ln \left( \frac{2m_e v^2}{I} \right)
\]  

(3.8)

where \( m_e \) is the rest mass of the electron, \( e \) is the elementary charge, \( N \) and \( Z \) are the respective number density and atomic number of the absorber, \( q \) and \( v \) are the charge state and velocity of the incident ion, and \( I \) is the (usually empirically determined) average excitation energy and ionization potential of the absorber. Note that the stopping power
does not explicitly depend on an incident ion’s mass, but as we have already seen, unreacted beam particles and recoils differ in energy by the ratio of their masses, so an ion’s stopping power implicitly depends on $A$ (because it depends on $v$.) Additionally, beam and recoil nuclei differ in $Z$ (by $\Delta Z = 2$ in the case of $\alpha$ capture and $\Delta Z = 1$ in the case of proton capture). Thus unreacted beam nuclei and recoil nuclei will deposit different amounts of energy in the IC and follow different path lengths. A modest electric field of $E = 50$ V/cm separates the electron-ion pairs created by the incident ion’s interaction with the gas, and is kept uniform by field shaping wires surrounding the detection volume. A Frisch grid is held at an equipotential between the segmented anodes and the cathode, which allows the anode pulse amplitude to depend only on the number of electron-ion pairs created by an incident ion’s interaction with the detection volume (rather than depending on the location - or distance from the anode - of the electron-ion pair production.)

### 3.5 Data Acquisition

DRAGON’s data acquisition (DAQ) system was recently replaced with a custom, state-of-the-art timestamp based DAQ [46] that was commissioned in the spring of 2013. Experiment S1372 was the first experiment run on the new DAQ following its commissioning, so there were still a few bugs in the system, as will be evident in the following sections on the analysis.
DRAGON’s DAQ consists of two independent DAQ systems: one for the BGO γ-ray array (colloquially known as the head) and one for DRAGON’s heavy-ion detectors (including the IIS elastic scattering monitors - colloquially known as the tail). This design enables triggering on singles events from either detector system, while coincidence events are identified in software via timestamps during the analysis phase. This method of coincidence matching reduces dead time in the DAQ and also alleviates the need for error-prone hardware coincidence gates and delays. The heart of each DAQ is the IO32, a general purpose VERSAmodule Eurocard (VME) designed and manufactured at TRIUMF [48]. Each IO32 consists of an Altera-Cyclone field-programmable gate array (FPGA) with sixteen nuclear instrumentation module (NIM) and sixteen emitter-coupled logic (ECL) input channels, sixteen NIM output channels, and a 20 MHz quartz oscillator crystal accurate to 20 parts per
Figure 3.13 illustrates the trigger logic of DRAGON’s head and tail DAQ systems. The signals from the BGO $\gamma$-ray array’s photomultiplier tubes (PMTs) are divided into logic and analog branches. The logic branch signals are sent through a pair of CAEN V812 constant fraction discriminators (CFD). The CFD signals are then sent to a CAEN V1190 time to digital converter (TDC) and a channel by channel OR is sent to the IO32 ECL inputs 0 and 1 to create the system trigger. The analog branch is sent to a CAEN V792 charge to digital converter (QDC) after being delayed (in accordance with the QDC’s specifications - see [46]). The signals of DRAGON’s heavy-ion detectors are sent through a combination
Figure 3.15: Schematic of the coincidence matching algorithm used by DRAGON’s DAQ. The analysis buffer is filled from separate head and tail events. The coincidence matching window is user programmable and must be set large enough to recognize coincident events arriving in the queue at significantly different times.

of amplifiers, shapers and discriminators specific to each detector in order to create analog and logic signals suitable for measurement by the DAQ. The analog signals are sent to a CAEN V785 analog to digital converter (ADC) after being delayed (if necessary) to put the signal inside the system trigger gate. The logic signals are sent to another CAEN V1190 TDC and to ECL inputs 0 - 7 of the IO32 to create the system trigger. Copies of both system triggers are sent to the other DAQ system in order to make a second measurement of the separator TOF. Additionally, a copy of the ISAC-I 11.8 MHz radio frequency quadrupole (RFQ) accelerator signal is sent to both systems’ TDC as an additional timing reference. The RFQ signal is gated by the system trigger in order to avoid flooding the TDC buffers
with rF pulses.

Control of DRAGON’s DAQ and storage of the data collected by it is implemented through the Maximum Integrated Data Acquisition Software (MIDAS) [49]. The MIDAS software code stores event based data in banks of classes in a MIDAS (.mid) file format, which can be converted into ROOT [50, 51] trees with the DRAGON Analyzer software package [52, 53]. The data can then be viewed and analyzed in ROOT. Additionally, TRIUMF has implemented the Experimental Physics and Industrial Control System (EPICS) software [54] developed by Argonne National Laboratory and Los Alamos National Laboratory throughout the lab for control of many beamline elements and apparatuses, including DRAGON. DRAGON’s DAQ records a vast amount of EPICS data, including (but not limited to) the field, current and voltage settings of DRAGON’s optical elements, position settings and current readings of DRAGON’s beam diagnostics, as well as the target pressure and temperature.

Matching of coincident events between the head and tail DAQ systems is performed both online and offline. MIDAS places events into a local buffer on the frontend to be pre-analyzed (online) for coincidences before transferring them to the backend (the offline analysis buffer) via the network. The method used for coincidence matching differs for online and offline analysis, and we will only concern ourselves with the method used for offline analysis of the data (for details concerning online coincidence matching, see [53]). A schematic of DRAGON’s offline coincidence matching algorithm is given in Figure 3.15. The backend analysis buffer is filled from banks of separate head and tail events. This introduces the possibility that coincident events could arrive at the analysis buffer separated by a large number of events in the queue. Thus the (user programmable) coincidence matching time window must be set large enough to recognize coincident events arriving in the queue at significantly different times. For further details on DRAGON’s DAQ, the reader is referred to [46, 53, 55] and the references therein.
CHAPTER 4
ANALYSIS METHODS

During experiment S1372, ten separate yield measurements were performed for incident beam energies ranging from \( E_b = 757.5 \text{ keV/u} \) to \( E_b = 595.1 \text{ keV/u} \) (corresponding to center of mass energies of \( E_{CM} = 2715 \text{ keV} \) to \( E_{CM} = 2133 \text{ keV} \)). Relevant run parameters for these ten yield measurements are given in Table 5.1. The following sections detail the methods used to analyze the data collected during experiment S1372.

4.1 Yield

Laboratory experiments of radiative capture reactions attempt to measure the yield of the reaction, which is defined as

\[
Y = \frac{N_{rxn}}{N_b}
\]  

where \( N_{rxn} \) is the number of reactions occurring during the yield measurement and \( N_b \) is the total number of beam particles incident on the target volume. DRAGON measures the number of recoils from reactions occurring within the target volume (via its heavy ion and \( \gamma \)-ray detectors) and then infers the number of reactions based on the systematics of the experimental setup. Thus DRAGON’s yield is given by

\[
Y = \frac{N_{rec}}{\eta_{rec} N_b}
\]  

where \( \eta_{rec} \) is the total recoil detection efficiency (see Appendix D for a more complete exposition of DRAGON’s yield).

In order to relate the experimental yield to the resonance strength, consider a beam of energy \( E_b \) incident on a target of thickness \( \Delta x \), which we can divide into slices of width \( \Delta x_i \). Assuming that the energy lost \( \Delta E_i \) by beam ions across the slice is small and that the number density of target nuclei within the slice \( N_i \) is constant (or that the stopping power \( \epsilon_i \) and cross section \( \sigma_i \) are constant over the width of the slice), the yield can be written...
\[ \Delta Y_i = \frac{N_{\text{rxn},i}}{N_{\text{b},i}} = \frac{\sigma_i N_i}{A} = \sigma_i N_i \Delta x_i \] (4.3)

where we have used Equation (2.24). Then the total yield is obtained by integrating over the thickness of the target

\[ Y(E_0) = \int_{\Delta x} N \sigma(x) \, dx \] (4.4)

where we have assumed that the number density of target nuclei is constant over the width of the target. We can recast this in terms of the energy using the stopping power; for an absorption medium of number density \( N \), the stopping power is given by

\[ S_N(E) \equiv -\frac{1}{N} \frac{dE}{dx} \equiv -\epsilon(E) \] (4.5)

where the stopping power is expressed in units of energy per atomic number density (MeV·cm\(^2\) / \(10^{15}\) atoms is common, and the unit). If the incident ions only lose a fraction of their energy over the width of the absorption medium, then the thickness of the absorption medium \( \Delta x \) is given by

\[ \Delta x = \int_{E_0-\Delta E}^{E_0} \frac{dE}{E_0} \frac{\Delta E}{dE/\,dx} \] (4.6)

where \( \Delta E \) is the total energy loss of the ions across the width of the medium and we have used the linear stopping power \( S_L \equiv dE/\,dx \) (expressed in units of energy per length). If the stopping power is constant across the width of the absorption medium, then arrive at

\[ \Delta x = \frac{\Delta E}{(dE/\,dx)_{E_0}} \] (4.7)

where \((dE/\,dx)_{E_0}\) is the stopping power evaluated at \( E_0 \). Multiplying by \( N / N \) and rearranging, we have

\[ N \Delta x = \frac{\Delta E}{\left(\frac{1}{N} \frac{dE}{dx}\right)_{E_0}} = \frac{\Delta E}{\epsilon} \] (4.8)

Then we can rewrite Equation (4.4) as

\[ Y(E_0) = \int_{\Delta x} N \sigma(x) \, dx \frac{dE}{dx} \frac{1}{dE/\,dx} = \int_{E_0-\Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)} \, dE \] (4.9)
4.1.1 Resonant Yield

For isolated narrow resonances, we can replace $\sigma$ in Equation (4.9) with the one level Breit-Wigner formula (Equation (2.48)), which gives [56]

$$Y(E_0) = \int_{E_0 - \Delta E}^{E_0} \frac{1}{\epsilon(E)} \frac{\lambda^2 (2J + 1)(1 + \delta_{01})}{4\pi (2j_0 + 1)(2j_1 + 1)} \frac{\Gamma_a \Gamma_b}{(E_r - E)^2 + \Gamma^2/4} dE$$

(4.10)

If the width of the resonance is small compared to the width of the target (i.e. $\Gamma \ll \Delta E$), and the energy dependence of $\Gamma_a$, $\Gamma_b$, $\lambda$ and $\epsilon$ are small over the width of the resonance, then we can replace them with their values at $E_0$ and we arrive at

$$Y(E_0) = \frac{\lambda^2}{2\pi} \omega \gamma \frac{m_p + m_t}{m_t} \frac{1}{\epsilon_r} \int_{E_0 - \Delta E}^{E_0} \frac{\Gamma/2}{(E_r - E)^2 + \Gamma^2/4} dE$$

(4.11)

where the transformation factor $\frac{m_p + m_t}{m_t}$ has been introduced because the stopping power is typically measured in the lab frame (and we are extracting the center of mass frame resonance strength). Equation (4.11) can be integrated following the procedure in section 2.3.2 (replacing the appropriate limits), which yields

$$Y(E_0) = \frac{\lambda^2}{2\pi} \omega \gamma \frac{m_p + m_t}{m_t} \frac{1}{\epsilon_r} \left[ \tan^{-1}\left( \frac{E_0 - E_r}{\Gamma/2} \right) - \tan^{-1}\left( \frac{E_0 - E_r - \Delta E}{\Gamma/2} \right) \right]$$

(4.12)

Equation (4.12) is maximum when

$$\frac{d}{dE_0} \left[ \tan^{-1}\left( \frac{E_0 - E_r}{\Gamma/2} \right) - \tan^{-1}\left( \frac{E_0 - E_r - \Delta E}{\Gamma/2} \right) \right] = 0$$

(4.13)

$$\Rightarrow \frac{2/\Gamma}{1 + \left( \frac{E_0 - E_r}{\Gamma/2} \right)^2} = \frac{2/\Gamma}{1 + \left( \frac{E_0 - E_r - \Delta E}{\Gamma/2} \right)^2}$$

(4.14)

$$\Rightarrow \left( \frac{E_0 - E_r}{\Gamma/2} \right)^2 - \left( \frac{E_0 - E_r - \Delta E}{\Gamma/2} \right)^2 = 0$$

(4.15)

$$\Rightarrow (E_0 - E_r)^2 - (E_0 - E_r - \Delta E)^2 = 0$$

(4.16)

$$\Rightarrow [(E_0 - E_r) - (E_0 - E_r - \Delta E)] [(E_0 - E_r) + (E_0 - E_r - \Delta E)] = 0$$

(4.17)

The first term in brackets on the left hand side gives the trivial solution ($\Delta E = 0$) and the
second term gives

$$E_0^{(\text{max})} = E_r + \frac{\Delta E}{2}$$

(4.18)

Inserting this back into Equation (4.12), we find

$$Y_{\text{max}} = \frac{\lambda_r^2}{\pi} \omega \gamma \frac{m_p + m_t}{m_t} \frac{1}{\epsilon_r} \tan^{-1}\left(\frac{\Delta E}{\Gamma}\right)$$

(4.19)

In the limit that $\Delta E \to \infty$ (or $\Delta E \gg \Gamma$) we recover the thick target yield for a narrow Breit-Wigner resonance [57]

$$Y_\infty = \frac{\lambda_r^2}{2} \omega \gamma \frac{m_p + m_t}{m_t} \frac{1}{\epsilon_r}$$

(4.20)

where $\omega \gamma$ is the resonance strength, $m_p$ and $m_t$ are the masses of the projectile and target nuclei (respectively), $\epsilon_r$ is the lab frame stopping power at the resonance energy, and $\lambda_r$ is the (center of mass) de Broglie wavelength of the resonance. We are interested in the resonance strength, so solving for $\omega \gamma$ gives:

$$\omega \gamma = \frac{2Y_\infty}{\lambda_r^2} \frac{m_t}{m_p + m_t} \epsilon_r$$

(4.21)

Thus calculating the resonance strength requires experimentally determined knowledge of the (thick target) reaction yield (which in turn requires knowledge of the number of incident beam particles, the number of observed recoils and the recoil detection efficiency) and the stopping power. Fortunately, all of these quantities (with the exception of the efficiency of DRAGON’s BGO $\gamma$-ray array $\eta_{\text{BGO}}$) can be measured directly with DRAGON. We turn to the methods used to ascertain these values in the following sections.

4.1.2 Effects of the Energy Distribution of the Beam and Straggling

The above treatment of the thick target yield for an isolated narrow resonance makes two key assumptions:

1. The beam is monoenergetic.

2. The energy spread in the beam due to straggling in the target is negligible.
In general, these factors must be taken into account by convoluting the yield curve with the distribution (usually approximated as Gaussian) of the beam energy and straggling:

\[ Y(E_0) = \int_{E_0 - \Delta E}^{E_0} dE'' \int_0^{\infty} dE' \int_0^{E'} \frac{\sigma(E'')}{\epsilon(E'')} g(E_0, E') \eta(E, E', E'') dE \]  \hspace{1cm} (4.22)

However, if the energy spread of the beam is small compared to the total beam energy, and if the energy loss due to straggling is small compared to the total energy loss across the target and the stopping power is constant across the target, then these factors may be neglected. Additionally, in the case of an infinitely thick target (or equivalently, \( \Gamma \ll \Delta E \)), and if \( g(E_0, E') \rightarrow g(E_0 - E') \) and \( \eta(E, E', E'') \rightarrow \eta(E' - E, E'') \) (i.e. - the spreads of \( g \) and \( \eta \) are independent of their respective means) then Equation (4.22) becomes

\[ Y_{\infty}(E_0) = \frac{1}{\epsilon r} \int_0^{E_0} dE'' \int_0^{\infty} dE' \int_0^{E'} \sigma(E'') g(E_0 - E') \eta(E' - E, E'') dE \]  \hspace{1cm} (4.23)

where we have assumed that the stopping power is constant across the width of the target.

The probability of a particle losing any amount of energy \( E' - E \) due to straggling in the target is unity (i.e. - \( \eta \) is normalized over the domain of \( \{0, E_0\} \), so we have

\[ Y_{\infty}(E_0) = \frac{1}{\epsilon r} \int_0^{\infty} dE' \int_0^{E'} \sigma(E'') g(E_0 - E') dE \]  \hspace{1cm} (4.24)

Therefore, as long as the target can be approximated as being infinitely thick, the yield is independent of the energy loss due to straggling. In the case that \( E' \ll E_0 \) (i.e. - the energy spread is small compared to the beam energy), we recover the expression for the thick target yield:

\[ Y_{\infty}(E_0) = \frac{1}{\epsilon r} \int_0^{\infty} \sigma(E'') dE \]  \hspace{1cm} (4.25)

For an infinitely thick target, the yield only depends on the cross section and stopping power and is independent of the beam resolution, beam straggling and resonance width. In the case of a target of finite thickness, it can be shown [2] that the area under the yield curve is independent of beam resolution, straggling, target thickness, stopping power and resonance width, and that
\[ A_Y = \int_{0}^{\infty} Y(E_0) dE_0 = Y_\infty \Delta E = \frac{\Delta E \lambda^2}{\epsilon_r 2 \omega \gamma} \] \quad (4.26)

However, for a target of finite thickness, beam resolution and straggling do influence the shape of the yield curve, and therefore must be taken into account when computing \( A_Y \).

In most cases, DRAGON’s gas target is approximated as an infinitely thick target (i.e. it is assumed that the target thickness \( \Delta E \) is much greater than the total width of the resonance \( \Gamma \)). In the case of S1372 measurements, total resonance widths were not available for any of the resonances measured, so they were assumed to be isolated and narrow. The \( Z \)-distributions of highest energy \( \gamma \) per event for resonances measured during S1372 (see for example Figure 4.16) illustrates that this was a reasonable assumption.

For S1372 measurements, use of ISAC-I’s post-accelerator bunching station (consisting of a low-\( \beta \) 11.8 MHz triple-gap structure and high-\( \beta \) 35.4 MHz spiral resonator [38]) enabled a narrow energy spread of \( \Delta E_b/E_b < 0.3\% \) (FWHM). The fractional uncertainty in the incident beam energy was then

\[ \frac{\sigma_b}{E_b} = \frac{\Delta E_b}{2\sqrt{2 \ln 2} E_b} \leq 0.0013 \] \quad (4.27)

The energy spread* of the beam due to straggling as it traverses the target can be described by the Bohr straggling model [58]:

\[ \delta_{\text{stragg}} = 2\sqrt{2 \ln(2)} \sqrt{4\pi Z_p^2 Z_t^2 e^4 N \Delta x} \] \quad (4.28)

where \( Z_p \) and \( Z_t \) are the respective atomic numbers of the projectile and target, \( N \) is the number density of target atoms, and \( \Delta x \) is the thickness of the target. Table 4.1 lists the uncertainty due to straggling \( \sigma_{\text{stragg}} \) for each yield measurement and the parameters used to compute \( \sigma_{\text{stragg}} \).

Note that although \( \sigma_{\text{stragg}} \) is insignificant compared to the total beam energy \( (\sigma_{\text{stragg}}/E_b \lesssim 0.1\%) \) is a fairly significant fraction of the energy thickness of the target, so we must take

*The term “spread” in this document is used to indicate the FWHM of the distribution (i.e. - FWHM = \( 2\sqrt{2 \ln(2)} \sigma \) for a Gaussian distribution).
Table 4.1: Parameters used to calculate the beam energy spread due to straggling in the target using Equation (4.28). The effective length of DRAGON’s gas target is $\Delta x = 12.3$ cm, and $\sigma_{\text{stragg}} = \delta_{\text{stragg}}/(2\sqrt{2\ln2})$ is the standard deviation of the energy distribution of the beam due to straggling. A global average temperature of $T_{\text{tgt}} = 28.16 \pm 2.82$ °C was used in the computation of $N$.

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care in our assumption of an infinitely thick target. If the resonance width for any of the resonances measured is not sufficiently narrow compared to the target thickness to justify the infinite thickness approximation, then we must measure the yield curve and include straggling and beam resolution into our calculation of $\omega\gamma$. On the other hand, if the resonance width is large compared to the energy thickness of DRAGON’s gas target, the narrow resonance formalism is invalid. In this case it would be necessary to use the broad resonance formalism in addition to altering DRAGON’s measurement procedure (to that of scanning the beam energy in small steps over the energy width of the resonance). Therefore, as long as the total resonance width is small compared to the energy thickness of the target (which we have taken as an assumption), we may neglect the energy spread in the beam due to straggling within the target. Since (as demonstrated above) we may also treat the beam as being monoenergetic, the thick target yield (Equation (5.2)) sufficiently describes the relationship between the experimental yield and the resonance strength.
4.2 Beam Normalization

In order to ascertain the total number of nuclei incident on the DRAGON gas target ($N_b$), the beam was continuously monitored during data collection by detecting elastically scattered $^4$He nuclei with a pair of ion-implanted silicon (IIS) charged particle detectors (mounted at angles of $30^\circ$ and $57^\circ$ with respect to the beamline) inside the gas target box. These data were normalized to regular Faraday cup (FC) readings of the beam current via a FC immediately upstream of the target (FC4 - see Figure 3.3). FC readings were taken at the beginning and end of each hour long run. The relationship between beam current (FC4 current readings) and number of elastically scattered target nuclei yields a normalization coefficient $R$, which can be calculated for a given run as:

$$R = \frac{I \Delta t P}{q e N_\alpha E^2 \epsilon_{tgt.}}$$  \hspace{1cm} (4.29)

where $N_\alpha$ is the number of $^4$He nuclei scattered into the IIS detector within a time window $\Delta t$, $I$ is the current reading on FC4, $e$ is the elementary charge, $q$ is the beam charge state (7+), $E$ is the incident beam energy (keV/u), $P$ is the target pressure (Torr), and $\epsilon_{tgt.}$ is the beam transmission through an empty target. The factor of $\frac{P}{E^2}$ makes $R$ invariant with respect to the target pressure $P$ and beam energy $E$ [59]. The data from S1372 seemed to (mostly) adhere to this invariance (see Figure 4.1). An average normalization coefficient over all runs (for given $E$) can then be computed by fitting a constant $\bar{R}$ to the normalization data (see Figure 4.1), and the total number of beam particles incident on the gas target volume can then be calculated as

$$N_b = \bar{R} N_\alpha \frac{E^2}{P}$$  \hspace{1cm} (4.30)

where $N_\alpha$ is the total number $\alpha$ particles scattered into the IIS detectors during the yield measurement, and $E$ and $P$ are the beam energy and (average) pressure during the yield measurement. The normalization data from S1372 yielded a normalization coefficient of $\bar{R} = (32.6 \pm 4.20) \times 10^3$. The uncertainty in $\bar{R}$ ($\sim 13\%$) is dominated by the statistical fluctuation in $R_i$. 

57
Figure 4.1: Normalization coefficient $R_i$ vs (arbitrary) run number for both IIS detectors for all S1372 runs. The solid line is the average, $\overline{R}$, the dashed lines are the statistical uncertainty of $\overline{R}$, and the dotted lines are the total uncertainty in $\overline{R}$.

4.3 Stopping power

Often, laboratory measurements of nuclear yields cannot effectively measure the stopping power (due to a variety of factors) [2, 60], so one must rely on semi-empirical model calculations of the stopping power in order to calculate resonance strengths. Typically, experimental values of stopping power differ from that of model calculations by $< 5\%$, but at energies near the stopping power maximum, these differences can be significantly larger because it is often the case that little experimental data exists in this range [60]. DRAGON is able to make direct stopping power measurements by varying the target pressure and then measuring the magnetic field strength required to center the beam on the charge slits. The target pressure can be converted into a target thickness (number density of gas atoms/molecules) via

$$N = \nu L \frac{P}{760 \text{[Torr]}} \frac{273 \text{[K]}}{T} \quad (4.31)$$
Figure 4.2: Determination of stopping power from MD1 B-field (beam energy) vs target pressure (target thickness) for yield measurements 2 - 6. Panel (a) - $E_b = 758$ [keV/u]. Panel (b) - $E_b = 706$ [keV/u]. Panel (c) - $E_b = 678$ [keV/u]. Panel (d) - $E_b = 659$ [keV/u] (yield measurements 5 and 6).

where $L = 2.68677 \times 10^{19}$ cm$^{-3}$ is the Loschmidt constant and $\nu = 1$ (for He) is the number of atoms per molecule. The beam energy is calculated from the magnetic field strength via the relationship

$$E_{\text{beam}} = c_{\text{mag}} \frac{(qB_{\text{MD1}})^2}{A} \quad (4.32)$$

where $B_{\text{MD1}}$ is the magnetic field in MD1, $q$ is the beam charge state, and a value of $c_{\text{mag}} = 48.15(7)$ [MeV T$^{-2}$] was used (as recommended in [62]). Plotting the beam energy vs the target thickness yields a linear relationship (see Figure 4.2 and Figure 4.3), of which the slope is the stopping power. These measurements were performed with the $^{34}$S ion beam at
Figure 4.3: Determination of stopping power from MD1 B-field (beam energy) vs target pressure (target thickness) for yield measurements 7 - 10. Panel (a) - $E_b = 641$ [keV/u]. Panel (b) - $E_b = 629$ [keV/u]. Panel (c) - $E_b = 611$ [keV/u]. Panel (d) - $E_b = 596$ [keV/u].

all beam energies for which yield data was taken, as well as with the $^{40}$Ar ion beam at the three beam energies for which charge state distribution (CSD - see section 4.4.2) data were taken. The stopping power measurement can then be used in Equation (5.2) to calculate the resonance strength with a greater degree of confidence than would be possible using model calculations of the stopping power. Figure 4.4 compares the stopping power measurements of S1372 to that of the Stopping and Range of Ions in Matter (SRIM) model calculation [61]. It is apparent that the experimentally measured stopping power differs from the model calculation by $\sim 10 - 15\%$, which agrees well with previous comparisons to the SRIM code [60].
4.4 Recoil Detection efficiency

In order to ascertain the reaction yield, we must first have knowledge of the recoil detection efficiency. The definition of DRAGON’s detection efficiency varies depending on whether one is analyzing the data collected from DRAGON’s detectors in singles or in coincidence (and can often vary depending on which detectors were used to identify recoils). The coincidence detection efficiency for S1372 data is given by

\[
\eta_{\text{rec}}^{\text{coinc}} = \eta_{\text{BGO}} \eta_{\text{CSF}} \eta_{\text{sep}} \eta_{\text{MCP}} \eta_{\text{MCP}} \eta_{\text{live}}
\]  \hspace{1cm} (4.33)

whereas the singles detection efficiency for S1372 data is given by

\[
\eta_{\text{rec}}^{\text{sing}} = \eta_{\text{IC}} \eta_{\text{CSF}} \eta_{\text{sep}} \eta_{\text{MCP}} \eta_{\text{MCP}} \eta_{\text{live}}
\]  \hspace{1cm} (4.34)

where \( \eta_{\text{BGO}} \) is the detection efficiency of DRAGON’s BGO \( \gamma \)-ray array, \( \eta_{\text{IC}} \) is the detec-
tion efficiency of DRAGON’s ionization chamber, $\eta_{\text{CSF}}$ is the fraction of ions in the charge state selected by MD1, $\eta_{\text{sep}}$ is the transmission of ions through the separator, $\eta_{\text{trans}}^{\text{MCP}}$ is the geometric transmission of DRAGON’s MCP, $\eta_{\text{det}}^{\text{MCP}}$ is the detection efficiency of DRAGON’s MCP detectors, and $\eta_{\text{live}}$ is the appropriate (singles or coincident) live time of DRAGON’s DAQ. Inserting Equation (4.33) and Equation (4.34) into Equation (4.2), we obtain the full expressions for DRAGON’s coincident and singles yields:

$$Y_{\text{coinc}} = \frac{c}{N_b \eta_{\text{BGO}} \eta_{\text{CSF}} \eta_{\text{sep}} \eta_{\text{trans}}^{\text{MCP}} \eta_{\text{det}}^{\text{MCP}} \eta_{\text{live}}^{\text{coinc}}}$$  \hspace{1cm} (4.35)$$

$$Y_{\text{sing}} = \frac{s}{N_b \eta_{\text{IC}} \eta_{\text{CSF}} \eta_{\text{sep}} \eta_{\text{trans}}^{\text{MCP}} \eta_{\text{det}}^{\text{MCP}} \eta_{\text{live}}^{\text{sing}}}$$ \hspace{1cm} (4.36)$$

where $c$ is the number of signal events identified as recoils detected in coincidence between DRAGON’s BGO $\gamma$-ray array and the MCPs and $s$ is the number of signal events identified as recoils detected in DRAGON’s IC (see Appendix D for derivations of Equation (4.33), Equation (4.34) and Equation (4.35)).

The separator transmission $\eta_{\text{sep}}$ is primarily governed by DRAGON’s acceptance of recoils into a cone of half angle $\phi \leq 20$ mrad [29, 30]. This value is primarily limited by the downstream aperture of DRAGON’s windowless gas target. As discussed previously (see section 2.1.1), the maximum recoil cone angle for S1372 measurements was $\phi_{\text{max}} = 8.317$ mrad, well within DRAGON’s acceptance. Thus, for the purposes of this document, the canonical value [63] of $\eta_{\text{sep}} = 0.999^{+0.001}_{-0.002}$ was adopted.

The MCP transmission is primarily governed by geometric factors [64] (i.e. - the frame on which MCP0’s DLC foil is mounted as well as the Au plated W wires of both MCPs electrostatic mirrors). It was discovered in the spring of 2014 (during a separate measurement) that the MCP transmission efficiency $\eta_{\text{trans}}^{\text{MCP}}$ was less than the canonical value of $76.9 \pm 0.6\%$ [64]. This was due to the grid wires constituting the MCPs’ electrostatic mirrors having a diameter greater than their specifications. The MCP transmission with the over-spec electrostatic mirror wires was measured to be $66.4 \pm 1.0\%$ [65].
The MCP detection efficiency $\eta_{\text{MCP}}^{\text{det}}$ varies with yield measurement and was determined from attenuated beam runs, as well as from recoil data in cases where recoil events were abundant. These data are displayed in Table 5.2. The MCP detection efficiency was determined in the normal way, i.e. - by summing the number of events that produce a signal in both the MCP and IC and dividing that by the sum of events that produce a signal in the IC:

$$
\eta_{\text{MCP}}^{\text{det}} = \frac{N_{\text{IC},\text{MCP}}}{N_{\text{IC}}} \quad (4.37)
$$

where $N_{\text{IC},\text{MCP}}$ is the number of events in the signal peak of the IC vs MCP TAC 2-D pulse height spectrum and $N_{\text{IC}}$ is the total number of events in the IC signal peak.

The IC detection efficiency $\eta_{\text{IC}}$ also varies with yield measurement and was determined from attenuated beam runs as well as from recoil data in cases where recoil events were abundant. The IC detection efficiency was determined in a manner similar to that of the MCP efficiency, namely from by summing the number of events that produce a signal in both the MCP and IC and dividing that by the sum of events that produce a signal in the MCP:

$$
\eta_{\text{IC}} = \frac{N_{\text{IC},\text{MCP}}}{N_{\text{MCP}}} \quad (4.38)
$$

where $N_{\text{IC},\text{MCP}}$ is defined as above and $N_{\text{MCP}}$ is the total number of events in the MCP TAC signal peak. These data are displayed in Table 5.2 (see Table 5.14 for resonance strengths determined from singles yields).

The coincident and singles livetime of DRAGON’s DAQ vary by yield measurement. For each of DRAGON’s DAQs (head and tail), the singles live time for Poisson events is defined as [46]

$$
L = 1 - \frac{\tau}{T} \quad (4.39)
$$

where $\tau$ is the sum of all busy times for a given run and $T$ is the total runtime. To calculate the coincident live time, DRAGON’s coincidence matching algorithm sorts through the start and stop times for all events from both DAQs and sums the total time when either DAQ
was busy (τ^h_i \parallel τ^t_i), removing times for which the busy times of both DAQs overlapped (τ^h_i \&\& τ^t_i):

\[ \tau_{\text{coinc}} = \sum_i (\tau^h_i \parallel \tau^t_i) - (\tau^h_i \&\& \tau^t_i) \]  

(4.40)

The live time is then calculated using Equation (4.39).

### 4.4.1 Efficiency of DRAGON’s BGO γ-ray Array

Because of the nature of radiative capture experiments conducted at DRAGON, the efficiency of DRAGON’s BGO γ-ray array varies with varying experimental conditions. Thus the efficiency of the BGO γ-ray array cannot be calibrated via an independent measurement. If there is good discrimination between recoil and unreacted beam (or “leaky” beam) events in DRAGON’s heavy-ion detectors for a given yield measurement of a given reaction, then the BGO efficiency can be measured by comparing the number of singles recoil events and coincidence recoil events, but this is not always possible.

In order to ascertain the efficiency of the BGO γ-ray array, GEANT3 Monte Carlo simulations of the response of the BGO γ-ray array were performed. The GEANT3 model of DRAGON’s BGO γ-ray array included the geometry of the BGO array, DRAGON’s windowless gas target pumping box and gas cell, as well as all materials and all possible interactions of γ-rays with the materials of DRAGON’s head [41, 42]. When known, the γ-decay scheme of the state of interest, including branching ratios and γ-energies was also included in the simulation. When the γ-decay scheme of the state was not previously known, several simulations were performed using a variety of decay schemes constrained by the spin and parity of the state, as well as the observed data. The detection efficiency of the BGO array is dependent primarily on two parameters: the number of γ’s emitted in the decay (above the threshold of the individual detectors) and the γ energy (and therefore the branching ratio). For all yield measurements, several simulations were performed varying values for the number of γ’s in the cascade and branching ratios in order to quantify the uncertainty in the simulated detection efficiency.
Although a γ background calibration of the BGO thresholds was performed immediately preceding S1372 data collection, a background calibration was not performed at the end of S1372 data collection. This step is standard procedure to determine the proper threshold value on the BGO’s photomultiplier tubes. This is because the high voltage (HV) settings on the PMTs tend to drift slightly (possibly due to the presence of the nearby magnetic field of DRAGON’s first quadrupole magnet), causing the threshold values on the PMTs to change slightly over long periods of data collection. However, we can circumvent this problem by applying a “software threshold” to our data by making an energy cut on the spectrum of highest energy γ per event (γ_0). We set our software threshold at 1.5 MeV (well above the hardware threshold of 750 keV) in order to ensure that the hardware threshold has no effect on the (gated) data. Table 5.2 lists (for each energy) the simulated BGO efficiencies as well as the measured efficiency obtained by comparing coincidence to singles yields. Table 5.14 lists the resonance strengths and energies determined from coincidence yields. More details on the various γ spectra are given in the following resonance energy sections and the decay schemes used for GEANT3 simulations are given in the corresponding sections below for a given yield measurement.

The measured BGO detection efficiency is just the ratio of the number of recoils detected by the BGO N^det_{rec} to the number of reactions occurring in the target N_{rxn}:

$$\eta_{\text{BGO}}^{\text{meas}} = \frac{N_{\text{det}}^{\text{rec}}}{N_{\text{rxn}}}$$

(4.41)

The number of recoils detected by the BGO is not simply the number of coincidences; this number must be corrected for the fraction of recoils that are selected for charge at MD1, the fraction of recoils that get transmitted to DRAGON’s end detectors, the fraction of recoils transmitted through and detected by the MCP, and the fraction of recoils that are recorded by DRAGON’s DAQ. Thus the number of detected recoils can be related to the number of coincidences by:

$$N_{\text{rec}}^{\text{det}} = \frac{c}{\eta_{\text{CSF}} \eta_{\text{sep}} \eta_{\text{MCP}} \eta_{\text{MCP}} \eta_{\text{live}}}$$

(4.42)
On the other hand, the number of reactions occurring in the target is just the number of detected singles events divided by the singles detection efficiency:

\[
\mathcal{N}_{\text{rxn}} = \frac{s}{\eta_{\text{sing}}} = \frac{s}{\eta_{\text{CSF}} \eta_{\text{sep}} \eta_{\text{MCP}} \eta_{\text{det}} \eta_{\text{IC}} \eta_{\text{live}}} = \frac{c \eta_{\text{BGO}}}{\eta_{\text{coinc}}}
\]

Inserting Equation (4.42) and Equation (4.43) into Equation (4.41), we have

\[
\eta_{\text{meas}} = \frac{c \eta_{\text{IC}} \eta_{\text{sing}}^{\text{live}}}{s \eta_{\text{coinc}}^{\text{live}}}
\]

The measured BGO detection efficiency can only be estimated in cases in which there is good discrimination between recoil events and leaky beam events in DRAGON’s end detectors. It should also be noted that the BGO detection efficiency obtained from GEANT3 simulation is used to calculate the coincident yield, and the measured BGO efficiency is simply a sanity check that was possible in S1372 data because of good discrimination between recoils and leaky beam in the singles particle identification (PID) spectra. Measured BGO efficiencies for S1372 data were calculated using Equation (4.44) and are compared to BGO efficiencies obtained from GEANT3 simulations in Table 5.2.

### 4.4.2 Charge State Distributions

Ions (recoil and beam ions) traversing DRAGON’s gas target exit the target in a distribution of charge states. Some fraction of the recoils produced in reactions in DRAGON’s gas target will be directed into the charge slits at the charge selection stage because they are not in the selected charge state, and therefore do not possess the required magnetic rigidity to be focused onto the focal plane at the charge slits. Consequently, yield measurements at DRAGON have to correct for the charge state fraction (CSF) of recoil nuclei that are transmitted to the charge focal plane. Ideally, a direct measurement of a beam of the recoil species is performed. The reader should note that it is not necessary to make CSD measurements of a beam of the specific (possibly radioactive) recoil isotope because the average equilibrium charge state (Equation (4.46)) does not depend on the mass (or baryon number
Figure 4.5: S1372 charge state distribution measurements of $^{40}\text{Ar}$ in He gas at a bombarding energy of $E_b = 540$ keV/u compared to CSD obtained form the semi-empirical relationship obtained by Liu et. al. [66].

A) of incident ions (see [58, 67–70] and the references therein). This simplifies the CSD measurement because a stable beam of the recoil species of interest may be used.

CSD data for S1372 were collected at three $^{40}\text{Ar}$ bombarding energies ($E_b = \{610, 540, 484\}$ keV/u) and at three pressures ($P \approx \{7.0, 8.0, 9.0\}$ Torr) for each energy. Concerning the S1372 data, at two out of the three energies at which $^{40}\text{Ar}$ CSDs were measured (the 610 keV/u and 484 keV/u measurements) only three charge states had a magnetic rigidity sufficient to center the beam on the charge slits (see Figure 4.6). As such, the Gaussian fits obtained for CSDs at those energies had zero degrees of freedom. Thus there is no way of knowing how well S1372 measurements described the actual CSD. Additionally, examination of Figure 4.6 reveals that the measurements at the 484 keV/u beam energy appear to not be normalized. There are a couple possible causes for this: either the charge slits were too
Figure 4.6: All S1372 charge state distribution measurements of $^{40}$Ar in He gas.

narrow to allow the full (charge selected) beam to be deposited on the charge Faraday cup (FCCH) or FCCH was malfunctioning at the time of these measurements. Therefore only the CSD measurements obtained for the $^{40}$Ar bombarding energy of $E_b = 610$ keV/u may be considered valid measurements and the rest must be discarded.

There are several methods in the literature for fitting CSD data in order to find the relationship between the average equilibrium charge state $\bar{q}$ and ion energy $E$ [58, 67–70]. Liu et. al. [66, 71] studied the charge state distributions of many species of heavy ions of interest to DRAGON passing through H and He gas and found that the CSDs of the ions were well described by a Gaussian distribution with mean $\bar{q}$ and standard deviation

\[
d = d_1 Z_p^w\tag{4.45}
\]

where $d_1 = 0.23675$ and $w = 0.54772$. Liu et. al. found that the following semi-empirical relationship was the best fit to describe the relationship between $\bar{q}$ and $E$ for CSD data
relevant to DRAGON’s commissioning:

\[ \bar{q} = Z_p \left[ 1 - \exp \left( -\frac{c_1}{Z_p^2} \sqrt{\frac{E}{E'}} + c_2 \right) \right] \tag{4.46} \]

where \( Z_p \) is the atomic number of the projectile ion with energy \( E \), \( E' = 0.067635 \text{ MeV/u} \) is the energy corresponding to the (modified) reduced velocity \( v' = 3.6 \times 10^6 \text{ m/s} \), and \( c_i \) and \( \gamma \) are free parameters of the fit.

Figure 4.7 and Figure 4.8 compare the valid S1372 CSD data to the fit obtained by Liu et al. [66]. It is evident from Figure 4.7 and Figure 4.8 that S1372 data agrees with the semi-empirical formula obtained by Liu. Since it is impossible to obtain a relationship between \( E_b \) and \( \bar{q} \) with only one data point, the yield values obtained for S1372 data in this thesis uses the semi-empirical formula obtained by [66] to estimate the CSF for a given yield
measurement with a generous uncertainty of 5 percentage points. Using Equation (4.46) along with Equation (4.45) one can interpolate the value of $\bar{q}$ for a given recoil energy, we can ascertain the average equilibrium CSD for a given energy and evaluate it at the selected charge state $q$ in order to ascertain $\eta_{CSF}$. The values obtained for $\bar{q}$ by evaluating Equation (4.46) and subsequent values obtained for $F_8$ and $F_7$ by evaluating the resulting gaussian distribution for S1372 recoil energies are given in Table 4.2. It is recommended that the CSD of $^{40}$Ar be measured again with greater care at a later date.

A word on the CSD study by Liu et al. [66, 71]. Because DRAGON’s experimental yield (and consequently, the resonance strength) is inversely proportional to the CSF, it is important that this value be known a high degree of accuracy, as experimental yields are sensitive to variations of the CSF. However, Engel notes in [63] that:
With the presented results, it becomes obvious that deviations of the measured resonance strength especially in $^{21}\text{Ne}(p,\gamma)^{22}\text{Na}$ at $E_{\text{cm}} = 258.6$ keV might be linked to assumptions made on the charge state probability. It can be concluded that the study to predict charge state distributions offers room for further improvements. In the meantime, for reliable results, the charge state fraction has to be measured and analyzed for each experiment individually.

Additionally, Liu et al. [66] note that “The distribution width is a very sensitive parameter and no theoretical prediction is available yet.” Furthermore, Liu et al. give no uncertainties for the fit parameters they obtained, nor do they recommend any value for the systematic uncertainty of charge state fractions calculated using their semi-empirical formula. Close examination of their semi-empirical formula reveals a possible flaw: rearranging Equation (4.46), we find

$$\ln(1 - \frac{q}{Z_p}) = -\frac{c_1}{Z_p} \sqrt{\frac{E}{E'}} + c_2$$  

(4.47)

which has the form of a line with slope $-c_1$ and intercept $c_2$ (where $y \rightarrow \ln(1 - \frac{q}{Z_p})$ and $x \rightarrow \frac{1}{Z_p} \sqrt{\frac{E}{E'}}$). Examining the left hand side of Equation (4.47), we expect that it can never be $> 0$, otherwise $\frac{q}{Z_p}$ would have to be negative (which is physically impossible because $Z_p \in \mathbb{Z}^+$ by definition and the probability of $q < 0$ is vanishingly small). Continuing on to solve Equation (4.47) for $c_2$, we obtain

$$c_2 = \ln(1 - \frac{q}{Z_p}) + \frac{c_1}{Z_p^2} \sqrt{\frac{E}{E'}}$$

(4.48)

As incident ion energy $E \rightarrow 0$, we expect $q \rightarrow 0$, in which case $c_2 \rightarrow \ln(1) = 0$. Otherwise, for nonzero energies, we always expect that $q < Z_p$, in which case $\ln(1 - \frac{q}{Z_p})$ is always $< 0$. Thus, $\ln(1 - \frac{q}{Z_p})$ (and, by proxy, $C_2$) is physically constrained to always be $\leq 0$. Yet Liu et al. give positive values for $c_2$ (see figures 8 (a) and (b) in [66]), which, it would seem, are not physically realizable.
Table 4.2: Average equilibrium charge states ($\bar{q}$) and resulting charge state fractions ($\eta_{CSF}$) obtained by applying the Liu semi-empirical formula [66] to $^{38}$Ar ($Z = 18$) recoil energies ($E_{rec}$) corresponding to $^{34}$S($\alpha, \gamma$)$^{38}$Ar resonance energies $E_r$.

<table>
<thead>
<tr>
<th>$E_r^{meas}$ [MeV]</th>
<th>$E_{rec}^{lab}$ [MeV/u]</th>
<th>$\bar{q}$</th>
<th>$\eta_{8+}$</th>
<th>$\eta_{7+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2696$^{a,*}$</td>
<td>0.647</td>
<td>8.34</td>
<td>0.355</td>
<td>0.188</td>
</tr>
<tr>
<td>2477$^{a}$</td>
<td>0.594</td>
<td>7.95</td>
<td>0.371</td>
<td>0.265</td>
</tr>
<tr>
<td>2391$^{a}$</td>
<td>0.573</td>
<td>7.78</td>
<td>0.365</td>
<td>0.295</td>
</tr>
<tr>
<td>2299$^{a,b}$</td>
<td>0.551</td>
<td>7.60</td>
<td>0.350</td>
<td>0.324</td>
</tr>
<tr>
<td>2249$^{b}$</td>
<td>0.539</td>
<td>7.50</td>
<td>0.338</td>
<td>0.338</td>
</tr>
<tr>
<td>2218$^{b}$</td>
<td>0.532</td>
<td>7.43</td>
<td>0.329</td>
<td>0.346</td>
</tr>
<tr>
<td>2164$^{b}$</td>
<td>0.519</td>
<td>7.32</td>
<td>0.312</td>
<td>0.357</td>
</tr>
<tr>
<td>2089$^{b,*}$</td>
<td>0.501</td>
<td>7.16</td>
<td>0.285</td>
<td>0.368</td>
</tr>
</tbody>
</table>

- $^a$ Tuned to the $8^+$ recoil charge state during this yield measurement.
- $^b$ Tuned to the $7^+$ recoil charge state during this yield measurement.
- $^*$ See section 5.2 for details concerning this value of $E_r^{meas}$.
- $^\dagger$ See section 5.11 for details concerning this value of $E_r^{meas}$.

Finally, DRAGON has a backlog of CSD data from years of radiative capture experiments. For these reasons, it is recommended that the CSD data from [66] as well as subsequent CSD data collected at DRAGON be reexamined with the aim of a more accurate and complete understanding of charge state distributions of heavy ions passing through H and He gas.

4.5 Particle Identification

Particle identification (PID) was conducted via analyses of two data sets; singles data (events that had detector signatures only in DRAGON’s heavy ion detectors) and coincident data (events that had detector signatures in DRAGON’s BGO γ-ray array in coincidence with detector signatures in the heavy ion detectors.) The following sections detail the methods utilized in analyzing singles and coincidence data.

4.5.1 PID in Coincidence

Particle identification of coincident events was achieved by plotting a 2-D pulse height spectrum of the time to amplitude converter of the MCP signals (MCP TAC) vs the separator
Figure 4.9: MCP TAC vs separator TOF for (coincident) events during yield measurement number 6 (at $E_b = 2359$ keV tuned to the 8+ recoil charge state - see Table 5.1). Gray triangles are coincident events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

TOF (TAC-TOF) for heavy ion events occurring in coincidence with detection of a $\gamma$-ray in the BGO array (that passed the threshold and pileup gates on the BGO $E_\gamma^{(0)}$ spectra, see Figure 5.24). The TAC-TOF spectra for data taken during S1372 were largely free of background, making identification of a recoil signal remarkably obvious, and negating the necessity of background estimation via sideband analysis.

A word on the relative “cleanliness” of the TOF-TOF spectra: the probability of a random $\gamma$-ray occurring in coincidence with a leaky beam event that falls within the MCP TOF ROI is a Poisson process. For a BGO threshold setting of 750 keV (the threshold setting we ran at for the duration of S1372 data collection), the BGO trigger rate ($\lambda$) is roughly 200 Hz. Thus, the probability of a random $\gamma$-ray occurring in coincidence with a leaky beam event
Figure 4.10: rF period as measured by the TDC during attenuated beam run 1034.

within the $\tau = 20 \mu s$ coincidence window is

$$P(k = 1) = \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!} = (200[Hz] \times 20[\mu s])e^{-200[Hz] \times 20[\mu s]} \approx 0.004 \quad (4.49)$$

Thus, if we see $\sim 1000$ leaky beam events over the course of a yield measurement, we should expect on the order of a few (or, more precisely $1000 \times 0.004 = 4$) accidental coincidences within the $20\mu s$ coincidence window.

We can perform a simple sideband analysis in order to ascertain the expected background in our defined signal region (the golden recoil gate). Using the “noisiest” MCP TAC vs separator TOF spectrum (i.e. - the TAC-TOF spectrum for yield measurement number 6 - see Table 5.1) as an example, we can estimate the expected background as follows. The rF period measured by the TDC during this yield measurement was $\tau_{rF} = 86.8$ ns
(see Figure 4.10), so we define our signal region to be $2080 \pm 86.8$ ns.\(^*\) This allows room for $\tau = 224$ shifted background regions within the default $\pm 10\,\mu s$ coincidence window (from $-136\tau_{rF}$ to $88\tau_{rF}$). There are $n = 3$ events that fall within the MCP TAC recoil gate, but outside the separator TOF recoil gate (see Figure 5.24). This gives a (negligible) background rate of $r = n/\tau = 0.013$.

### 4.5.2 PID in Singles

![MCP TAC spectrum of yield measurement 4](image)

**Figure 4.11:** MCP TAC spectrum of yield measurement 4. The orange filled histogram shows events that occurred in coincidence with a $\gamma$ signal in the BGO array and the gold-lined histogram are coincident events in the “golden” recoil gate. Vertical lines denote a software gate used to identify recoil (signal) events. Note the presence of jitter in the MCP TAC signal and the large “leaky beam” shoulder on the signal peak.

\(^*\)The expected separator TOF for $^{38}\text{Ar}$ recoils at the resonance energy of 2327 keV/u (in the CM frame) is 2065 ns, so accounting for energy loss across the target we expect $^{38}\text{Ar}$ recoils to have a separator TOF of 2080 ns.
Figure 4.12: 2D pulse height spectrum of IC anode(1) vs IC anode(0) singles data (colormap) for yield measurement 4. Gray circles are data from a preceding attenuated beam run. Panel (a) - No gates applied. Panel (b) - Gate from Figure 4.11 applied illustrating the inadequacy of recoil/leaky (signal/background) discrimination.

Unfortunately, the MCP TOF resolution was not sufficient to distinguish recoil candidates from leaky beam events in singles (see Figure 4.12). However, good energy separation between recoil candidates and leaky beam events within DRAGON’s ionization chamber made analysis of singles events possible. The following procedure was used to identify singles events for all resonance energies:

1. From the MCP TAC vs separator TOF histogram, we can define a “golden” coincidence recoil gate in order to highlight ROIs in singles histograms (in the IC and MCP - and possibly in “RF TOF” as well).

2. Make a (narrow) cut on the local (MCP) TAC on the region(s) where “golden” recoils events populate (see Figure 4.11).

From here, we could plot a 2-D pulse height spectrum of IC anodes and draw another narrow gate around the ROI where the “golden” recoil events populate (as in Figure 4.13(a)), but this does not give us a good idea of how many leaky beam (background) events are leaking into our signal region.∗ Furthermore, close examination of Figure 4.13(a) reveals that there

∗Note that a simple sideband analysis would not work here because the background is not uniform.
Figure 4.13: Two methods of particle identification in DRAGON’s IC. 2-D pulse height spectra of ion chamber anode(1) vs anode(0) for yield measurement number 4. This Plot is an overlay of an attenuated beam run (gray circles), singles events that passed the MCP TAC gate, (colormap), and coincident events that passed the MCP TAC vs separator TOF 2-D “golden” recoil gate (gold stars).

are “golden” recoil events that bleed into the region of leaky beam events, hence if we make an appropriate cut that includes the “bleed” region, we will inadvertently overestimate the number of singles recoils. On the other hand, we could project the 2-D spectrum of IC anode[1] vs IC anode[0] onto the diagonal and attempt to identify recoils in the resulting 1-D pulse height spectrum. For the S1372 data, IC anodes 0 and 1 exhibit good separation between leaky beam and recoil events, so we can proceed with recoil identification as follows:

3. Plot IC anode[1] vs IC anode[0] and project this 2-D pulse height spectrum onto the diagonal using the rotation matrix

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

(4.50)

4. Plot the resulting projected 1-D pulse height spectrum with events in the “golden” recoil gate overlaid (see Figure 4.14). This reveals the same structure that we expected from the 2-D histogram of IC anode[1] vs anode[0], namely, that there is a marked peak where the “golden” recoil events populate that seems to bleed into the leaky beam region, as well as a leaky beam peak that appears to also bleed into the signal region.
5. This distribution is well modeled by a double Gaussian (see Figure 4.14):

\[
    f(x) = c_0 \exp\left( -\frac{(x - c_1)^2}{2c_2^2} \right) + c_3 \exp\left( -\frac{(x - c_4)^2}{2c_5^2} \right)
\]  

(4.51)

6. We can choose a signal gate that includes all of the “golden” recoils, count the number of events in the signal region and then subtract the estimated number of background counts in the signal region by integrating the first Gaussian over the signal region.

7. One can then estimate the uncertainty using a number of different methods: adding the uncertainties in the number of counts in the signal and background regions in quadrature, or use ROOT’s TRolke [72] class to model the background(s) and efficiency(ies) as Gaussian or Poisson distributions.∗

∗addition of uncertainties in quadrature was sufficient for the purposes of this work.
Figure 4.15: Same as Figure 4.14 fit with a double Gaussian.

4.5.3 \( E_{r}^{(\text{lab})} \) Measurement

By analyzing the hit pattern of \( \gamma \)-ray signals in the BGO array, it is possible to estimate the location along the beamline (in \( z \)) of the resonance within the target. Because beam ions lose energy (as a result of interactions with the target material) as they traverse the target, the \( z \)-position (i.e. - the position along the beam axis with respect to the target center) of the resonance can be used to estimate the energy of the resonance \( E_r \). The measured resonance energies for S1372 data were calculated from the measured arithmetic mean of the \( z \)-position distribution of of the highest energy \( \gamma \) per event using equations (3) - (5) from [62].

\[^{*}\text{Our calculations exclude the geometric correction, i.e. - the systematic offset of 0.57 cm (due to the ‘less efficient two upstream BGO counters’) from eqn (3) because the two aforementioned detectors were in their normal position for stable beam measurements (i.e. - the BGO array was symmetric with respect to the \( z \)-axis).}\]
Figure 4.16: Spectrum of BGO Z-position of the highest energy $\gamma$ per event for yield measurement 4 (see Table 5.1).

\[ z_{\text{BGO}} = 0.79 z_{\text{true}} \]

\[ f = \frac{0.5z}{L_{\text{eff}}} \tag{4.52} \]

\[ E_f = (1 - f)E_i + fE_o - f(1 - f) \frac{(E_i - E_o)^2}{E_i + E_o} \mathcal{R} \]

Where possible (i.e. - when the outgoing beam energy from the gas target was approximately equal to or overlapped with the next measurement's incoming beam energy), $\mathcal{R}$ was calculated using measured values for $E$, $S$, $\Delta E$ and $\Delta S$. Otherwise it was calculated using interpolated values from the SRIM calculation scaled by a factor of 0.9.
CHAPTER 5
RESULTS

The following sections describe the results of the analysis of S1372 data in detail. Table 5.1 displays various physical values relevant to analysis of the S1372 data, and Table 5.2 displays various efficiency values associated with DRAGON, also necessary for analysis of S1372 data. We begin discussion of the results with a word on jitter.

5.1 Jitter

As described in section 3.5, every timing signal processed through DRAGON’s DAQ contains jitter. Although in most cases the effect is small, jitter does affect data analysis as illustrated below. Specifically, “ringing” in the timing signals produced by DRAGON’s MCPs give rise to a “jitter peak” in the MCP spectra (see Figure 4.11). These jitter events can also be seen in the 2-D MCP TAC vs separator TOF pulse height spectra (the spectra on which the principal cut is made for identifying recoils in coincidence - see Figure 5.15). Taking yield measurement number 4 as an example (because out of all of the yield measurements it has the most events in the jitter region), we can define a golden recoil gate on the MCP TAC vs separator TOF spectrum that includes only these events in order to determine whether they should be included in the analysis or discarded.

Figure 5.1 displays various BGO spectra useful for identifying recoils populated with events in the jitter recoil gate superimposed on the identical spectrum for golden recoil events. Figure 5.2 displays a 2-D pulse height spectrum of IC anode[1] vs IC anode[0] of the same jitter events superimposed over the same spectrum for golden recoil events. It is evident from Figure 5.3 and Figure 5.2 that the jitter events satisfy all other criteria for golden recoils - namely - they are coincident events that have the correct separator TOF occurring in the central region of the target with $\gamma$-ray energies between 1.5 and 11.0 MeV. Therefore the events in the jitter region cannot be vetoed using any criteria apart from being
Figure 5.1: BGO spectra for events in the jitter region of the MCP TAC vs separator TOF spectrum for yield measurement 4. Panel (a) BGO z-position of highest energy $\gamma$ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy $\gamma$ per event ($\gamma_0$). Panel (d) - 2-D spectrum of 2nd highest energy $\gamma$-ray per event vs highest energy $\gamma$-ray per event ($\gamma_1$ vs $\gamma_0$).

jitter events. Apart from being in the jitter region of the MCP TAC spectrum, these events would make convincing recoil candidates (for example if one were to use a golden recoil gate on just the separator TOF, these events would be counted as recoils). On the other hand, if one were unaware of the phenomenon of jitter or not careful in analyzing the MCP TAC spectrum, these events would be missed entirely. For these reasons, the jitter events have been taken as a systematic uncertainty of the yield measurement (or, more specifically of the measurement of $N_{\text{rec}}$) and are added in quadrature with the statistical uncertainty in order to obtain the total uncertainty in $N_{\text{rec}}$.  

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Figure 5.2: 2D pulse height spectrum of IC anode(1) vs IC anode(0) jitter events (orange diamonds) superimposed on coincident events that pass the golden recoil gate (navy circles) illustrating the lack of a reasonable veto criteria for the jitter events.

Figure 5.3: Same as Figure 5.1 but superimposed on identical spectra for events in the golden recoil gate during yield measurement 4 to illustrate the lack of a reasonable veto criteria for the jitter events.
Table 5.1: S1372 measured resonance energies (in the CM frame) compared to literature values [27] and other relevant run data. The measured excitation energy of $^{38}\text{Ar}$ $E_x$ was calculated using the literature value of $Q = 7208.04$ keV [73, 74] for the $^{34}\text{S} (\alpha, \gamma)^{38}\text{Ar}$ reaction. All energies are in units of keV. $N_\alpha$ are the number of $\alpha$ particles detected in the elastic scattering monitors during the given yield measurement (not corrected for deadtime). Note: see section 5.2.1 for discussion regarding the literature value of $E_x = 9917$ keV.

<table>
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<tr>
<th>Measurement Number</th>
<th>Runs</th>
<th>$E_{CM}$ range [keV]</th>
<th>$E_r$ [keV]</th>
<th>$E_x^{\text{calc.}}$ [keV]</th>
<th>$E_x^{\text{lit}}$ [keV]</th>
<th>$P_{\text{avg}}$ [Torr]</th>
<th>$N_\alpha$ (raw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$^a$</td>
<td>967-979</td>
<td>2689-2715</td>
<td>2706(9)</td>
<td>9914(9)</td>
<td>9917</td>
<td>3.27(10)</td>
<td>10715</td>
</tr>
<tr>
<td>2$^a$</td>
<td>985-994</td>
<td>2661-2715</td>
<td>2696(8)</td>
<td>9904(8)</td>
<td>9917</td>
<td>6.97(35)$^c$</td>
<td>21920</td>
</tr>
<tr>
<td>3$^a$</td>
<td>1000-1008</td>
<td>2457-2528</td>
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<td>9685(10)</td>
<td>9689</td>
<td>8.98(19)</td>
<td>16907</td>
</tr>
<tr>
<td>4$^a$</td>
<td>1018, 1021-1033</td>
<td>2354-2428</td>
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<td>9599(7)</td>
<td>9597</td>
<td>9.03(21)</td>
<td>55424</td>
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<tr>
<td>5$^a$</td>
<td>1035-1044</td>
<td>2287-2359</td>
<td>2299(8)$^d$</td>
<td>9507(8)</td>
<td>9535</td>
<td>9.04(20)</td>
<td>263288</td>
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<tr>
<td>6$^b$</td>
<td>1047-1058</td>
<td>2289-2359</td>
<td>2300(8)$^d$</td>
<td>9508(8)</td>
<td>9535</td>
<td>8.75(19)</td>
<td>156758</td>
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<td>2227-2289</td>
<td>2249(10)</td>
<td>9457(10)</td>
<td>9460</td>
<td>9.00(18)</td>
<td>91370</td>
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<tr>
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<td>2180-2252</td>
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<td>9426(8)</td>
<td>9431</td>
<td>9.13(19)</td>
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<tr>
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<td>2127-2187</td>
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<td>9372(8)</td>
<td>9374</td>
<td>7.89(19)</td>
<td>235743</td>
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<td>10$^b$</td>
<td>1110-1122</td>
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<td>2089(9)</td>
<td>9297(9)</td>
<td>9300</td>
<td>8.57(18)</td>
<td>170144</td>
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</tbody>
</table>

$^a$ separator was tuned to the 8+ recoil charge state.
$^b$ separator was tuned to the 7+ recoil charge state.
$^c$ EPICS pressure data was only recorded for runs 992-994, so a generous 5% uncertainty has been adopted.
$^d$ The measured resonance energy for both yield measurements at $E_b = 659$ keV/u (corresponding to $E_{CM} = 2359$ keV) was $E_r = 2299(7)$ keV.
Table 5.2: Measured DRAGON efficiency parameters for S1372 singles and coincidence data. All values for $\eta_{\text{CSF}}$ have an uncertainty of ±5%. The recoil cone angle for $90^\circ \gamma$’s at the lowest measured resonance energy is 8.31 mrad is well within DRAGON’s acceptance, so we adopt the canonical value of $\eta_{\text{DRA}} = 99.9^{+0.1}_{-0.2}$% [59] as the separator transmission for all yield measurements.

<table>
<thead>
<tr>
<th>Measurement Number</th>
<th>$\eta_{\text{CSF}}$</th>
<th>$\eta_{\text{IC}}$</th>
<th>$\eta_{\text{MCP}}$</th>
<th>$\eta_{\text{sim.}}_{\text{BGO}}$</th>
<th>$\eta_{\text{meas}}_{\text{BGO}}$</th>
<th>$\eta_{\text{singles}}$</th>
<th>$\eta_{\text{coinc.}}$</th>
<th>$\eta_{\text{coinc}}_{\text{rec}}$</th>
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<td>99.6(5)</td>
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<td>99.9</td>
<td>95.7</td>
<td>15.6 ± 2.9</td>
<td>19.1 ± 2.4</td>
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<td>99.9</td>
<td>95.6</td>
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<td>90(3)</td>
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<td>99.9</td>
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<td>86.5(9)</td>
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<td>71(11)</td>
<td>99.7</td>
<td>95.5</td>
<td>13.7 ± 3.7</td>
<td>14.9 ± 2.3</td>
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<td>7$^b$</td>
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<td>43(8)</td>
<td>89(3)</td>
<td>83.6$^{+8.8}_{-11.0}$</td>
<td>73(25)</td>
<td>98.4</td>
<td>94.6</td>
<td>15.9 ± 2.7</td>
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<td>92$^{+8}_{-18}$</td>
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<td>15.2 ± 1.9</td>
</tr>
<tr>
<td>10$^b$</td>
<td>36.8</td>
<td>79(5)</td>
<td>88(5)</td>
<td>85.1$^{+8.8}_{-11.7}$</td>
<td>70(15)</td>
<td>99.8</td>
<td>95.7</td>
<td>17.4 ± 3.4</td>
<td>16.9 ± 2.5</td>
</tr>
</tbody>
</table>

* See section 5.2.2 concerning data for yield measurement 1.

$^a$ Separator was tuned to the 8+ recoil charge state.

$^b$ Separator was tuned to the 7+ recoil charge state.

$^c$ The discrepancy between coincident and singles detection efficiencies for yield measurement 7 is due to the poor IC detection efficiency for this yield measurement, see section 5.8.
5.2 Yield Measurements 1 and 2: $E_x^{(\text{lit})} = 2709$ keV

Yield measurements at an incident beam energy of $E_b = 2715$ keV were taken at two different pressures. This was done because online analysis of the data revealed that the count rate for recoil candidates was much lower than expected, so it was apparent that the resonance near $E_{CM} = 2709$ keV was not in the target. After increasing the target pressure and touching up the tune, the count rate increased to expected levels. We begin with a discussion on apparent inconsistencies in the literature value of the excitation energy of this state $E_x^{(\text{lit})} = 9917$ keV.

5.2.1 $E_x^{(\text{lit})} = 9917$ keV state

According to Cameron [27] and the NNDC [75], the $J^\pi = 1^-$ state at $E_x = 9.917$ MeV is attributable solely to $^{34}\text{S}(\alpha,\gamma)^{38}\text{Ar}$ measurements. There are 3 sources [20–22] in the literature that claim to have measured the energy of a $1^-$ resonance near $E_x = 9.913$ MeV in $^{34}\text{S}(\alpha,\gamma)^{38}\text{Ar}$. Table 5.3 displays the various literature values of the excitation energy ($E_x$) of this state. The values of $E_{CM}$ given for the NNDC and Endt are the center of mass resonance energy calculated from the respective adopted values of $E_x$ and $Q$ in these sources. The values given for the three literature sources [20–22] are the measured $\alpha$ energy values and the respective values of $E_x$ and $Q$ adopted in those works. The values of $E_{CM}$ for the sources in Table 5.3 are calculated by transforming the $\alpha$ energy to a CM energy using the most recent values of $m_{\alpha}^{4\text{He}}$, $m_{34\text{S}}$ [73, 74]. The values given in parenthesis for $E_x$ are the “updated” values, calculated using the most recent value of $Q = 7.20804$ MeV (calculated from the most recent values of $\Delta m_{\alpha}^{4\text{He}}$, $\Delta m_{34\text{S}}$ and $\Delta m_{38\text{Ar}}$ [74]) and the calculated values for $E_{CM}$. Note that the values for $E_x$ given in the three sources in Table 5.3 are erroneous because the $Q$-values used to calculate them are obsolete. The NNDC’s average value for the excitation energy of this state $E_x = 9.917$ MeV is similarly obsolete. It is recommended that the excitation energy of this state be updated to $E_x = 9.912$ MeV.
5.2.2 3.38 Torr runs

Data for yield measurement 1 (see Table 5.1) were collected at an average target pressure of 3.38 ± 0.05 Torr. The incoming beam energy was measured at $E_b = 757.5$ keV/u (corresponding to $E_{CM} = 2715$ keV) and the outgoing beam energy was measured at $E_{out} = 750.57$ keV/u (corresponding to $E_{CM} = 2690$ keV). For this target thickness, only the $1^{-}$ state near $E_x^{lit} = 9.917$ MeV should have been in the target. However, an unexpectedly low yield was measured during these runs compared to the later measurement (see section 5.2.3), when the target pressure was increased to 6.97 Torr and both the $J^\pi = 1^-$ state at $E_x^{lit} = 9.917$ MeV and the $J^\pi = 2^+$ state at $E_x = 9.894$ MeV were clearly in the target. This is quite puzzling as the yield should have increased only slightly when the much weaker $^*$ $2^+$ resonance was included in the target. Additionally, because the state near $E_x^{lit} = 9.917$ MeV has a spin and parity of $J^\pi = 1^-$, we would expect to see a strong transition directly to the ground state (owing to the E1 character and polarity of the emitted $\gamma$) as in Figure 5.9(c). However, we see no such evidence of a strong ground state transition in the yield measurement 1 data (see Figure 5.5(d)). This suggests that the $E_x^{lit} = 9.917$ MeV state was not in the target during this yield measurement.

Table 5.3: Discrepancies in the literature for the value of the excitation energy $E_x$ of the $1^-$ state near 9.913 MeV in $^{38}$Ar. The values in parentheses in column 2 are the values of $E_x$ obtained using the most recent (from reference [74]) $Q_\alpha$ - value of 7.20804 MeV for the $^{34}$S($\alpha, \gamma$)$^{38}$Ar reaction. All units are in MeV.

<table>
<thead>
<tr>
<th>Source</th>
<th>$E_x$</th>
<th>$Q_\alpha$</th>
<th>$E_{CM}$</th>
<th>$E_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNDC [27, 75]</td>
<td>9.917</td>
<td>7.2077</td>
<td>2.709</td>
<td>-</td>
</tr>
<tr>
<td>Endt [7, 76]</td>
<td>9.912</td>
<td>7.2078</td>
<td>2.704</td>
<td>-</td>
</tr>
<tr>
<td>Sinha [22]</td>
<td>9.914(9.911)</td>
<td>7.2102</td>
<td>2.703</td>
<td>3.022</td>
</tr>
</tbody>
</table>

$^*$Sinha et. al. [22] measured a resonance strength of $\omega \gamma = 0.6 \pm 0.3$ eV for the $E_x = 9.894$ MeV state compared to $\omega \gamma = 2.62 \pm 0.56$ eV for the $E_x^{lit} = 9.917$ MeV state.
Figure 5.4: Local (MCP) TOF vs separator TOF for (coincident) events during yield measurement 1. Gray triangles are coincident events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

There are a total of 10 events in the MCP TAC vs separator TOF (TAC-TOF) golden recoil gate for yield measurement 1 (see Figure 5.4), and a total of 14 events in the jitter region. Because there are more events in the jitter peak than the golden recoil gate and because the total number of events in the separator TOF signal region is relatively low, the histograms in this section are overlaid with histograms including events from both the golden recoil gate and the jitter peak.

One possible explanation for the discrepancy is the possibility of a mistune during the data collection at the 3.38 Torr pressure. Table 5.4 displays various parameters for DRAGON’s separation elements that are indicative of the integrity of a tune, along with the
Figure 5.5: BGO spectra for events in the separator TOF signal region for yield measurement 1. Panel (a) - Spectrum of BGO Z-position of the highest energy $\gamma$ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy $\gamma$ per event. Panel (d) - 2-D spectrum of 2nd highest energy $\gamma$-ray per event vs highest energy $\gamma$-ray per event ($\gamma_1$ vs $\gamma_0$). The overlaid orange histograms result when the golden recoil gate is expanded to include the jitter peak.

resulting “measured”* atomic mass to charge ratio $A/q$ and the percent error between the “measured” and theoretical values. The mass to charge ratio “measured” by DRAGON is calculated as

$$\frac{A}{q} = 2.468 \times 10^{-5} \frac{B_{MD1}^2}{V_{ED1}}$$  \hspace{1cm} (5.1)

where $B_{MD1}$ is given in gauss and $V_{ED1}$ is the ED1 set point voltage in kV. Although

*i.e. - the atomic mass to charge ratio to which DRAGON’s optical elements are tuned
Table 5.4: Tune parameters for DRAGON’s separation elements during S1372 data collection. The direction of the arrow in column 14 indicates a relative increase (↑) or decrease (↓) in the “measured” mass to charge ratio.

<table>
<thead>
<tr>
<th>( E_b ) [MeV]</th>
<th>( q_{tune} )</th>
<th>MD1 Field [g]</th>
<th>ED1 [V]</th>
<th>Field Ratios (F/MD1)</th>
<th>ED2 [V]</th>
<th>Q1</th>
<th>Q2</th>
<th>A/q</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.754 Torr</td>
<td>8</td>
<td>5304.49</td>
<td>147.3</td>
<td>1232.0</td>
<td>146.4</td>
<td>117.1</td>
<td>116.4</td>
<td>0.707</td>
<td>4.72</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>5293.55</td>
<td>146.2</td>
<td>1232.0</td>
<td>145.4</td>
<td>116.3</td>
<td>115.6</td>
<td>0.706</td>
<td>4.73</td>
</tr>
<tr>
<td>23.985</td>
<td>8</td>
<td>5073.11</td>
<td>148.9</td>
<td>1229.0</td>
<td>148.1</td>
<td>118.4</td>
<td>117.7</td>
<td>0.705</td>
<td>4.26</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>5703.25</td>
<td>133.3</td>
<td>1229.0</td>
<td>132.5</td>
<td>106.0</td>
<td>105.3</td>
<td>0.707</td>
<td>4.77</td>
</tr>
<tr>
<td>23.036</td>
<td>8</td>
<td>4966.67</td>
<td>142.3</td>
<td>1229.0</td>
<td>141.5</td>
<td>113.5</td>
<td>112.8</td>
<td>0.705</td>
<td>4.28</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>4966.85</td>
<td>127.4</td>
<td>1229.0</td>
<td>126.7</td>
<td>101.5</td>
<td>100.9</td>
<td>0.705</td>
<td>4.78</td>
</tr>
<tr>
<td>22.377 8+ rec.</td>
<td>8</td>
<td>4893.36</td>
<td>137.7</td>
<td>1228.0</td>
<td>137.0</td>
<td>110.1</td>
<td>109.5</td>
<td>0.704</td>
<td>4.29</td>
</tr>
<tr>
<td>21.789 7+ rec.</td>
<td>7</td>
<td>5595.47</td>
<td>141.1</td>
<td>1228.0</td>
<td>140.4</td>
<td>112.8</td>
<td>112.1</td>
<td>0.658</td>
<td>5.47</td>
</tr>
<tr>
<td>21.364</td>
<td>7</td>
<td>5517.32</td>
<td>153.4</td>
<td>1229.0</td>
<td>152.6</td>
<td>122.5</td>
<td>121.8</td>
<td>0.707</td>
<td>4.90</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>5517.36</td>
<td>137.3</td>
<td>1229.0</td>
<td>136.5</td>
<td>109.6</td>
<td>109.0</td>
<td>0.707</td>
<td>4.74</td>
</tr>
<tr>
<td>20.744 8+ rec.</td>
<td>7</td>
<td>5378.08</td>
<td>145.8</td>
<td>1228.0</td>
<td>145.0</td>
<td>116.4</td>
<td>115.7</td>
<td>0.705</td>
<td>4.90</td>
</tr>
<tr>
<td>20.234 7+ rec.</td>
<td>7</td>
<td>5389.09</td>
<td>132.0</td>
<td>1228.0</td>
<td>131.3</td>
<td>105.4</td>
<td>104.8</td>
<td>0.705</td>
<td>5.428</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>5315.97</td>
<td>142.5</td>
<td>1228.0</td>
<td>141.7</td>
<td>113.6</td>
<td>112.9</td>
<td>0.705</td>
<td>4.89</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>5316.15</td>
<td>127.5</td>
<td>1228.0</td>
<td>126.8</td>
<td>101.6</td>
<td>101.1</td>
<td>0.705</td>
<td>5.47</td>
</tr>
</tbody>
</table>
discrepancies of up to $\sim$ 1% can often be tolerated by DRAGON, an interesting feature of the S1372 data is that for almost all of the resonances measured, the percent error did not change drastically (or at all in many cases) between the attenuated beam run and the recoil runs (sc. - the systematic error in the mass to charge ratio was consistent between attenuated beam and recoil runs for a given energy). For example, the attenuated beam run for the 2478 keV resonance (which was taken in the 8+ charge state) DRAGON was tuned to a “measured” mass to charge ratio of $4.26 - 0.34\%$ higher than the actual value of $A/q = 34/8 = 4.25$. Similarly, during the recoil runs for the 2478 keV resonance, DRAGON was tuned to a “measured” mass to charge ratio of $4.77 - 0.34\%$ higher than the actual value of $A/q = 38/8 = 4.75$. On the other hand, during the recoil runs at a pressure of 3.38 Torr for the 2706 keV resonance, DRAGON was tuned to a mass to charge ratio of $4.72 - 0.73\%$ lower than the actual value of 4.75, whereas the attenuated beam run was tuned to a mass to charge ratio of $3.785 - 0.19\%$ higher than the actual value of 3.778. Although this is not compelling evidence that DRAGON was mistuned during the 3.38 Torr runs at the 2706 keV resonance, it is suggestive that a mistune was the possible reason for the puzzling data collected at this target pressure. The lack of a competing explanation for the puzzling data collected during yield measurement 1 makes a mistune the likely explanation for the discrepancy, and since nothing reliable can be stated about the recoil energy and charge state in the event of a mistune, the data for yield measurement number 1 should be discarded.

5.2.3 6.97 Torr Runs

Data for yield measurement 2 were collected at an average target pressure of $6.97 \pm 0.35$ Torr. However, EPICS data is missing from the midas/root files for runs 988 - 991, and the pressure reading from runs 985 - 987 seems to be have been recorded erroneously, having a singular value of 3.17 Torr (which conflicts with the runlog comment stating 7.08 Torr) over the span of these three runs (approximately 2 hours), so we have adopted the average pressure value over runs 992 - 994 of 6.97 Torr with a generous 5% uncertainty.
The incoming beam energy remained unchanged at $E_b = 757.5$ keV/u (corresponding to $E_{\text{CM}} = 2715$ keV), but the increased target pressure resulted in an outgoing beam energy of $E_{\text{out}} = 742.0$ keV/u, corresponding to a CM energy of $E_{\text{CM}} = 2659$ keV. At this bombarding energy and target thickness, both the $J^\pi = 1^-$ state at $E_x^{\text{lit}} = 9.917$ MeV ($E_{\text{CM}}^{\text{lit}} = 2709$ keV) and the $J^\pi = 2^+$ state at $E_x = 9.894$ MeV ($E_{\text{CM}} = 2686$ keV) were roughly centered in the target (see Figure 5.8). During yield measurement 2, a total of 10,715 $\alpha$ particles were detected in the IIS signal peaks, resulting in a total integrated beam on target of $N_b = (5.81 \pm 0.81) \times 10^{13}$ incident $^{34}\text{S}^7+$ ions.

5.2.4 Coincidence Analysis of Yield Measurement 2

There are a total of $c \pm \sigma_c^{\text{stat}} \pm \sigma_c^{\text{sys}} = 863 \pm 29.4 \pm 9$ events in the MCP TAC vs separator TOF (TAC-TOF) golden recoil gate for these runs (see Figure 5.6). We can estimate the background rate using a simple sideband analysis, with $n = 2$, $\tau = 225$ (corresponding to an RF period of $\tau_{\text{RF}} = 86.8$ ns measured by the TDC) and $r = 0.0089$. This rate is insignificant considering the number of detected recoils. Of the 863 events in the expected signal region, 745 events have a valid signal in the IC, suggesting an IC detection efficiency of $\eta_{\text{IC}} = 86.3 + / - 4.3\%$. This agrees with the IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 81.5 \pm 0.6\%$ measured during the attenuated beam run for this energy.

The only parameter that is missing from our calculation of the coincidence yield (and subsequently the resonance strength) then is the efficiency of the BGO $\gamma$-ray array. This was ascertained by performing a GEANT3 simulation [42] of the BGO $\gamma$-ray array. Figure 5.9(a) shows the spectrum of highest energy $\gamma$ per event in the BGO array, and compares the measured spectra to the spectra obtained from GEANT3 simulations. Cameron and the NNDC [27, 75] only give the intensity of one electromagnetic transition from the $E_x^{\text{lit}} = 9.917$ MeV state (the ground state transition), but list a total of five transitions four of which have no transition intensities listed. S1372 data clearly shows (weaker) transitions

*Note that the IC detection efficiency is only used in calculation of the singles yield.
**Figure 5.6:** MCP TAC vs separator TOF for (coincident) events during yield measurement.

Gray triangles are coincident events, orange diamonds are jitter events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

The BGO efficiency was determined using the (known [27, 75]) $\gamma$-decay scheme in Figure 5.7 in conjunction with the (unknown) branching ratios given in column 4 of Table 5.5. The uncertainty on the BGO efficiency was determined by running the simulation with altered decay schemes (also given in Table 5.5); the lower limit on the BGO efficiency was determined by forcing a branching ratio of 100% for the direct to ground state transition, whereas the upper limit was determined by splitting the branching ratio evenly between a direct to ground state transition and a three (above threshold) $\gamma$ cascade through the $E_x = 3810$, keV and $E_x = 2168$ keV states. Figure 5.9(a) compares the spectra obtained...
Figure 5.7: (Abbreviated) $\gamma$-decay scheme used in GEANT3 simulations of DRAGON’s BGO array for the $E_x^{\text{lit}} = 9.917$ MeV state. Note: for the lower lying states, only the strongest transition is shown.

from GEANT3 simulations to the observed spectrum of highest energy $\gamma$ per event for yield measurement 2. In this figure, the orange filled histogram are the observed spectra, the blue dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 4 of Table 5.5, the green dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 5 of Table 5.5, and the red dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 6 of Table 5.5. The simulation was performed with the known spin and parity assignment of $J^\pi = 1^-$. It should be stressed that the branching ratios given in
Table 5.5: Branching ratios (in %) for electromagnetic transitions from the $E_x^{\text{lit}} = 9.917$ MeV state used in GEANT3 simulations of DRAGON’s BGO array. Labeling corresponds to that in Figure 5.7. Note: for the lower lying states, only the strongest transition is shown.

<table>
<thead>
<tr>
<th>Branch</th>
<th>$E_{\gamma}$ [keV]</th>
<th>$E_x^{(f)}$ [keV]</th>
<th>Nominal</th>
<th>Upper</th>
<th>Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>9913</td>
<td>0</td>
<td>71.5</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>b</td>
<td>7745</td>
<td>2168</td>
<td>3.0</td>
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<td>0</td>
</tr>
<tr>
<td>c</td>
<td>6535</td>
<td>3378</td>
<td>6.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>6103</td>
<td>3810</td>
<td>10.0</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>e</td>
<td>5977</td>
<td>3936</td>
<td>3.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>5348</td>
<td>4565</td>
<td>6.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>g</td>
<td>2397</td>
<td>2168</td>
<td>93.81</td>
<td>0</td>
<td>0</td>
</tr>
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<td>2168</td>
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<td>2168</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

Column 3 of Table 5.5 represent a possible set of branching ratios used for the purposes of simulation of the BGO array’s detection efficiency. DRAGON’s BGO $\gamma$-ray array does not have sufficient resolution to accurately measure the intensities of the electromagnetic transitions of excited recoil nuclei.

The GEANT3 simulation yielded an efficiency of $\eta_{\text{BGO}} = 70 \pm 9\%$. Combining this with the other relevant coincidence efficiencies, we obtain a total coincident recoil detection efficiency of $\eta_{\text{coinc}}^{\text{rec}} = 15.61^{+2.91}_{-2.94}\%$. We then use this along with the aforementioned values for $c$ and $N_b$ to calculate the coincident yield, obtaining $Y_{\text{coinc}} = 9.80^{+2.24}_{-2.25} \times 10^{-11}$. Then, using the thick target yield (Equation (4.20)), we calculate a resonance strength of $\omega_{\gamma} = 4.37^{+1.04}_{-1.05}$ eV. Note that for the calculated resonance strength, the asymmetric uncertainties propagated from the BGO detection efficiency are so close in value that it does not add any meaning to the value. Thus we report the resonance strength with the maximum uncertainty: $\omega_{\gamma} = 4.37 \pm 1.05$ eV.

The Z-distribution of highest energy $\gamma$ per event for events passing the MCP TAC vs separator TOF golden recoil gate has an arithmetic mean of -1.564 cm (see Figure 5.8). This
results in a measured resonance energy (using the method described by Hutcheon et. al. [62]) of $E_r = 2696 \pm 8$ keV. Some discussion is warranted here.

It is clear from Figure 5.8 that the presence of the $E_x = 9.894$ MeV ($E_{CM} = 2686$ keV) in the target shifts the arithmetic mean of the $E_{\gamma}^0 z$-distribution downstream of the distribution’s peak, causing an erroneous measurement of the resonance energy of the state near $E_x = 9.917$ MeV. Examining Figure 5.8, we see that the distribution is peaked near $z = -3$ cm (corresponding to a center of mass energy of $E_{CM} = 2704$ keV) and that the distribution has a definite shoulder near the expected location (at the target center, $z = 0$) of the $E_{CM} = 2686$ keV resonance. Hence we measure a resonance energy nearly exactly between the $E_{CM} = 2686$ keV resonance and the $E_{CM} = 2704$ keV resonance.

Because of the presence of the $E_r = 2686$ keV resonance in the target during this yield
Figure 5.9: BGO spectra for (coincident) events passing all recoil cuts for the yield measurement number 2. Panel (a) - spectrum of highest energy $\gamma$ per event with spectrum from Geant3 simulations overlaid - see text for further explanation. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - 2-D spectrum of 2nd highest energy $\gamma$-ray per event vs highest energy $\gamma$-ray per event ($\gamma_1$ vs $\gamma_0$).
measurement the only measured value we can report is the combined resonance strength of the 2705 keV and 2686 keV states. On the other hand, Sinha et. al. measured the resonance strength of the $E_{CM} = 2686$ keV ($E_\alpha = 3002$ keV) resonance to be $0.6 \pm 0.3$ eV. Using Equation (4.12), we can estimate the $E_{CM} = 2686$ keV resonance’s contribution to the thick target yield and subtract it from our combined yield value in order to estimate the yield (and subsequently the resonance strength) of the $E_{CM} = 2705$ keV resonance. Doing so, we find that the expected yield from the $E_{CM} = 2686$ keV resonance is $Y_{2686} = (1.30 \pm 0.65) \times 10^{-11}$, which gives a yield of $Y_{2705} = (8.20 \pm 2.35) \times 10^{-11}$ and a resonance strength of $\omega \gamma = 3.77 \pm 1.09$ eV. Unfortunately, the resonance strength obtained from S1372 yield measurements does nothing to resolve the ambiguity in the literature values for the strength of the resonance near $E_{CM} = 2706$ keV, as it agrees with both canonical values (within uncertainties). In order to resolve the discrepancy, it would be necessary to measure both the $E_{CM} = 2706$ and the $E_{CM} = 2686$ resonances again with greater care. This could possibly be done at a later date in combination with a planned $^{34}$S($p, \gamma$)$^{35}$Cl beam time.

Comparing the combined resonance strength of the $E_r = 2706$ keV and the $E_r = 2686$ keV resonances obtained from S1372 measurements to the combined literature values of the resonance strengths for the two resonances would be misleading because such a comparison would not take into account the effect that the value of the corresponding resonance energy has on the astrophysical reaction rate. However, we can compare our measured values to the literature values by calculating the narrow resonance reaction rates for these resonance strengths and energies using Equation (C.28) and Equation (2.52).

Table 5.6 compares the values for the narrow resonance reaction rate calculated from resonance strengths and energies obtained in this work to those obtained by Sinha et. al. [22] and Erne and Van Der Leun [21] for a temperature of $T = 2.2$ GK. The reaction rate calculated using the resonance strength and energy measured in this work agrees much more closely with that of the reaction rate calculated using the resonance strength and energy measured by Erne and Van Der Leun [21] than for the combined resonance strengths and
Table 5.6: Narrow resonance reaction rates calculated from resonance strengths and energies obtained in this work compared to those obtained by Sinha et. al. [22] and Erne and Van Der Leun [21].

<table>
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<td>2686(4.5)</td>
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energies measured by Sinha et. al. [22]. The values of the resonance strengths of these states remain doubt. However, given that the total resonant reaction rate obtained by combining S172 observations with existing data agrees so well with Hauser-Feshbach statistical models, any future measurements of these resonances would likely be less valuable than the pursuit of lower energy resonances (of which there are several - see reference [27]).

5.2.5 Singles Analysis of Yield Measurement 2

Employing the method outlined in section 4.5.2 for identifying recoil events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.10(a) and the fit shown in Figure 5.10(b). There are a total of $N_{\text{sig}} = 1114$ events in the signal region in Figure 5.14(a), but we estimate the background via the fit in Figure 5.14(b) to be $b = 26.7$, giving a total of $s = 1088 \pm 34$ singles recoil events. Combining this with the singles livetime and IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 81.5 \pm 0.5\%$, we calculate the measured BGO detection efficiency to be $\eta_{\text{BGO}}^{\text{meas}} = 67 \pm 3\%$. This value agrees well with the value of $\eta_{\text{BGO}} = 70 \pm 9\%$ obtained from the GEANT3 simulation of the BGO array for the $E_r = 2705$ keV resonance. Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of $\eta_{\text{sing}} = 19.1 \pm 2.4\%$, giving a singles yield of $Y_{\text{sing}} = (9.80 \pm 1.95) \times 10^{-11}$. This agrees well with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power of $\epsilon = 186 \pm 6$ eV/(10¹⁵ ⁴He cm³) for this bombarding energy and the de Broglie wavelength
Figure 5.10: IC signal spectra obtained by employing the method outlined in section 4.5.2. Panel (a) - 1-D pulse height spectrum of IC anode[1] vs IC anode[0] projected onto the diagonal overlaid with the same spectrum for all events in the golden recoil gate and signal region identification. Panel (b) - Double Gaussian fit to spectrum in panel (a).

of the resonance ($\lambda^2 = 0.855$ barns for $E_r = 2696$ keV), we obtain a singles resonance strength of $\omega\gamma_{\text{sing}} = 4.51 \pm 0.91$ eV. This is of course in good agreement with the coincident resonance strength obtained in the previous section.

5.3 Yield Measurement 3 ($E_{r}^{\text{lit}} = 2481$ keV)

Data for yield measurement number 3 were collected at an average target pressure of 8.98(19) Torr. The incoming beam energy was measured at $E_b = 706.1$ keV/u (corresponding to $E_{CM} = 2528$ keV) and the outgoing beam energy was measured at $E_{out} = 686.1$ keV/u (corresponding to $E_{CM} = 2456$ keV). At this bombarding energy and target thickness, the $J^\pi = 1^-$ state at $E_x^{\text{lit}} = 9.689$ MeV was roughly centered in the target. From the $Z$-distribution of highest energy $\gamma$ per event (see Figure 5.12(a)), we measured a resonance energy of $E_r = 2477 \pm 10$ keV, which is in good agreement with the literature values of $E_{r}^{\text{lit}} = 2480 \pm 4.5$ keV [21] and $E_{r}^{\text{lit}} = 2482 \pm 4.5$ keV [22]. During yield measurement 3, a total of 16,907 $\alpha$ particles were detected in the IIS signal peaks, resulting in a total integrated beam on target of $N_b = 3.07(42) \times 10^{13}$ incident $^{34}$S$^7+$ ions.
5.3.1 Coincidence Analysis of Yield Measurement 3

There are a total of $c \pm \sigma_c^{\text{stat}} \pm \sigma_c^{\text{sys}} = 205 \pm 14.3 \pm 4$ events in the MCP TAC vs separator TOF golden recoil gate for the yield measurement at $E_b = 2528$ keV. Figure 5.11 shows that there was virtually no background in the MCP TAC vs separator TOF spectrum, and very few jitter events in the MCP TAC. Of the 205 golden recoil events in the TAC-TOF spectrum, 161 events have a valid signal in the IC, suggesting an IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 78.5 \pm 8.3\%$. This is significantly higher than the IC detection efficiency of $61.9 \pm 3.1\%$ measured during the attenuated beam run for this energy, but is in good agreement with the IC detection efficiency of $\eta_{\text{IC}} = 71.9 \pm 3.9^\star$ measured with singles recoil events.

![Figure 5.11](image.png)

**Figure 5.11:** MCP TAC vs separator TOF for events passing the BGO threshold gate during yield measurement 3. Gray triangles are coincident events, orange diamonds are events in the jitter region, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

> $^\star$This quantity was measured using the method in section 4.4.
Figure 5.12: BGO spectra for events within the golden recoil gate for yield measurement 3. Panel (a) - BGO $z$-position spectrum for highest energy $\gamma$ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - BGO spectrum of highest energy $\gamma$ per event (see text for further explanation). Panel (d) - 2-D spectrum of 2nd highest energy $\gamma$ per event vs highest energy $\gamma$ per event ($\gamma_1$ vs $\gamma_0$).

Figure 5.12(c) shows the spectrum of highest energy $\gamma$ per event in the BGO array, and compares the measured spectra to the spectra obtained from Geant3 simulations. The Geant3 simulation yielded an efficiency of $\eta_{\text{BGO}} = 71^{+9}_{-10}$. The BGO efficiency was determined using the $\gamma$-decay scheme in Figure 5.13 with the branching ratios given in Table 5.7. The uncertainty on the BGO efficiency was determined by running the simulation with altered decay schemes (also given in Table 5.7); the lower limit on the BGO efficiency was determined by forcing a branching ratio of 100% for the direct to ground state transition,
Figure 5.13: (Abbreviated) $\gamma$-decay scheme used in GEANT3 simulations of DRAGON’s BGO array for the $E_x^{\text{lit}} = 9.689$ MeV state. Note: for the lower lying states, only the strongest transition is shown.

whereas the upper limit was determined by splitting the decay probability equally between the direct to ground state transition and the transition to the 3936 keV state. Figure 5.12(c) compares the spectra obtained from GEANT3 simulations to the observed spectrum of highest energy $\gamma$ per event for yield measurement 3. In this figure, the orange filled histogram are the observed spectra, the blue dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 4 of Table 5.7, the green dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 5 of Table 5.7, and the red dashed histogram is the spectrum obtained from the GEANT3
Table 5.7: Branching ratios (in %) for electromagnetic transitions from the $E_{ix}^{\text{lit}} = 9.689$ MeV state used in GEANT3 simulations of DRAGON’s BGO array. Labeling corresponds to that in Figure 5.13. Note: for the lower lying states, only the strongest transition is shown.

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<th>$E_{x}^{(f)}$ [keV]</th>
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The observed distribution in Figure 5.12(c) does not match the GEANT3 simulation using the literature values for the branching ratios for $\gamma$-transitions from this excited state. The literature assignment of a spin and parity of $J^\pi = 1^-$ leads one to expect a strong ground state transition. Although the observed spectrum in Figure 5.12(c) does exhibit some direct to ground state transition, it is significantly suppressed in contrast to Figure 5.9(a) and Figure 5.16(c). Further, the literature branching ratios (given in Table 5.7) show that the strongest observed electromagnetic transition is to the $J^\pi = 2^+$ state at 3936 keV (the fourth excited state). This fact, coupled with the observation of a significant (but suppressed) direct to ground state transition suggest that the spin and parity of this state could be $J^\pi = 2^+$ rather than the literature assignment of $J^\pi = 1^-$. 

Combining the BGO detection efficiency with the other relevant coincidence efficiencies for this yield measurement (see Table 5.2), we obtain a total coincidence detection efficiency of $\eta_{\text{coinc}} = 14.1 \pm 2.7\%$. Along with the number of golden recoils and the total integrated beam on target, we calculate a coincident yield of $Y_{\text{coinc}} = (4.74 \pm 1.16) \times 10^{-11}$. Then, using Equation (5.2) along with the measured stopping power of $\epsilon = (192 \pm 11) [\text{eV}/(10^{15} \text{ 4He cm}^{-2})]$ and the de Broglie wavelength of $\lambda = 0.930$ barns (for this energy), we calculate a
resonance strength of $\omega\gamma = 2.07 \pm 0.52$ eV. This value agrees with the canonical values of $\omega\gamma = 1.3 \pm 0.3$ eV and $\omega\gamma = 1.5 \pm 0.5$ obtained by Sinha et. al. and Erne and Van Der Leun, respectively.

### 5.3.2 Singles Analysis of Yield Measurement 3

Employing the method outlined in section 4.5.2 for identifying recoil events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.14(a) and the fit shown in Figure 5.14(b). There are a total of $N_{\text{sig}} = 302$ events in the signal region in Figure 5.14(a). Integration of the background fit over the signal region results in an estimated $b = 88.3$ background events in the signal region, giving a total of $s = 214 \pm 20$ singles recoil events. Combining this with the singles livetime and IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 78.5 \pm 8.3\%$, we calculate the measured BGO detection efficiency to be $\eta_{\text{BGO}}^{\text{meas}} = 72 \pm 12\%$. This value agrees well with the value of $\eta_{\text{BGO}} = 71^{\pm 9}_{-10}\%$ obtained from the Geant3 simulation of the BGO array for the $E_r = 2481$ keV resonance. Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of $\eta_{\text{sing}} = 14.9 \pm 2.7\%$, giving a singles yield of $Y_{\text{sing}} = (4.67 \pm 1.05) \times 10^{-11}$. This is in good agreement with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power of $\epsilon = 192 \pm 11$ eV/(10$^{15}$ $^4$He cm$^3$) for this bombarding energy and the de Broglie wavelength of the resonance ($\lambda_r^2 = 0.930$ barns for $E_r = 2477$keV), we obtain a singles resonance strength of $\omega\gamma_{\text{sing}} = 2.03 \pm 0.47$ eV. This is of course in good agreement with the coincident resonance strength obtained in the previous section.

### 5.4 Yield Measurement 4: $E_r^{(\text{fit})} = 2389$ keV

Data for yield measurement number 4 were collected at an average target pressure of $P_{\text{avg}} = 9.03 \pm 0.21$ Torr. The incoming beam energy was measured at $E_b = 678.2$ keV/u (corresponding to $E_{\text{CM}} = 2428$ keV) and the outgoing beam energy was measured at $E_{\text{out}} = 657.5$
Figure 5.14: IC signal spectra obtained by employing the method outlined in section 4.5.2. Panel (a) - 1-D pulse height spectrum of IC anode[1] vs IC anode[0] projected onto the diagonal overlaid with the same spectrum for all events in the golden recoil gate and signal region identification. Panel (b) - Double Gaussian fit to spectrum in panel (a).

keV/u (corresponding to $E_{CM} = 2354$ keV). At this bombarding energy and target thickness, the $J^\pi = 1^-$ state at $E_{litt} = 9.597$ MeV was roughly centered in the target. From the $Z$-distribution of highest energy $\gamma$ per event (see Figure 5.16(a)), we measured a resonance energy of $E_r = 2391.1 \pm 7.2$ keV, which is in good agreement with the literature values of $E_{r}^{lit} = 2389 \pm 4.5$ keV [21] and $E_{r}^{lit} = 2388 \pm 4.5$ keV [22]. During yield measurement number 4, a total of 55,424 $\alpha$ particles were detected in the IIS signal peaks, resulting in a total integrated beam on target of $N_b = 9.22 \pm 1.21 \times 10^{13}$ incident $^{34}$S$^{7+}$ ions.

5.4.1 Coincidence Analysis of Yield Measurement 4

There are a total of $c \pm \delta c_{stat} \pm \delta c_{sys} = 590 \pm 24.3 \pm 25$ events in the MCP TAC vs separator TOF golden recoil gate for the yield measurement 3 at $E_b = 2428$ keV. The TAC-TOF 2-D pulse height spectrum has virtually no background, but does include a significant number of jitter events ($N_{jitt} = 25$). Of the 590 events in the golden recoil gate, 492 events have a valid signal in the IC, suggesting an IC detection efficiency of $\eta_{IC} = 83.4 \pm 5.1\%$. This is in good agreement with the IC detection efficiency of $\eta_{IC}^{AB} = 79.6 \pm 2.7\%$ measured during the attenuated beam run for this energy.
Figure 5.15: MCP TAC vs separator TOF for events during yield measurement 3. Gray triangles are coincident events, orange diamonds are events in the jitter region of the MCP TAC spectrum, open red triangles are events in the background region (i.e. - in the MCP TAC signal region but outside separator TOF signal region), and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

Figure 5.16(c) shows the spectrum of highest energy $\gamma$ per event in the BGO array, and compares the measured spectra to the spectra obtained from GEANT3 simulations. The GEANT3 simulation yielded an efficiency of $\eta_{\text{BGO}} = 69^{+8}_{-9}$. Cameron and the NNDC [27, 75] report branching ratios for only one electromagnetic transition from the $E_{x}^{\text{lit}} = 9.597$ MeV state (the ground state transition). However, the $\gamma_0$ and the $\gamma_1$ vs $\gamma_0$ spectra (Figure 5.16(c) and Figure 5.16(d), respectively) suggest that there exist transitions of significant intensity to other lower lying states that transition to the ground state in a cascade through the first excited state at $E_x = 2168$ keV. The BGO efficiency was determined using the $\gamma$-decay scheme in Figure 5.17 with the branching ratios given in Table 5.8.

The uncertainty on the BGO efficiency was determined by performing the simulation with
Figure 5.16: BGO spectra of events within the golden recoil gate for the yield measurement 4. Panel (a) - $z$-position spectrum of highest energy $\gamma$ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy $\gamma$ per event (see text for further explanation). Panel (d) - 2-D spectrum of 2nd highest energy $\gamma$ per event vs highest energy $\gamma$ per event ($\gamma_1$ vs $\gamma_0$).

altered decay schemes (also given in Table 5.8); the lower limit on the BGO efficiency was determined by forcing a branching ratio of 100% for the direct to ground state transition, whereas the upper limit was determined by splitting the decay’s branching ratio evenly between a direct to ground state transition and a three (above threshold) $\gamma$ cascade through the $E_x = 3810$, keV and $E_x = 2168$ keV states. Figure 5.16(c) compares the spectra obtained from GEANT3 simulations to the observed spectrum of highest energy $\gamma$ per event for yield measurement 4. In this figure, the orange filled histogram are the observed spectra, the blue
Figure 5.17: $\gamma$-decay scheme used in GEANT3 simulations of DRAGON’s BGO array for the $E_{\text{ex}}^{\text{lit}} = 9.597$ MeV state. Note: for the lower lying states, only the strongest transition is shown.

dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 4 of Table 5.8, the green dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 5 of Table 5.8, and the red dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 6 of Table 5.8. The simulation was performed with the known spin and parity assignment of $J^\pi = 1^-$. Combining the BGO detection efficiency with the other relevant coincidence efficiencies for this yield measurement (see Table 5.2), we obtain a total coincidence detection efficiency
Table 5.8: Branching ratios (in %) for electromagnetic transitions from the $E_x^{\text{lit}} = 9.597$ MeV state used in GEANT3 simulations of DRAGON’s BGO array. Labeling corresponds to the that in Figure 5.17. Note: for the lower lying states, only the strongest transition is shown.

<table>
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<th>Branch</th>
<th>$E_\gamma$ [keV]</th>
<th>$E_x^{(f)}$ [keV]</th>
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of $\eta_{\text{coinc}} = 14.4 \pm 3.3\%$. Along with the number of golden recoils and the total integrated beam on target, we calculate a coincident yield of $Y_{\text{coinc}} = (4.44 \pm 1.04) \times 10^{-11}$. Then, using Equation (5.2) along with the measured stopping power of $\epsilon = (197 \pm 9) \text{[eV/(10^{15} \text{He cm}^{-2})]}$ and the de Broglie wavelength of $\lambda^2 = 0.964$ barns (for this energy), we calculate a resonance strength of $\omega \gamma = 1.92 \pm 0.46$ eV. This value agrees well with the value of $\omega \gamma = 1.71 \pm 0.34$ eV obtained by Sinha et.al.

5.4.2 Singles Analysis of Yield Measurement

Employing the method outlined in section 4.5.2 for identifying recoil events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.18(a) and the fit shown in Figure 5.18(b). There are a total of $\mathcal{N}_{\text{sig}} = 957$ events in the signal region in Figure 5.18(a). Integration of the background fit over the signal region resulted in an estimated background of $b = 261$, giving a total of $s = 698 \pm 35$ singles recoil events. Combining this with the singles livetime and IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 79.6 \pm 2.7\%$, we calculate
the measured BGO detection efficiency to be \( \eta_{\text{BGO}}^{\text{meas}} = 70 \pm 7\% \). This value agrees well with the value of \( \eta_{\text{BGO}} = 69^{+8}_{-9}\% \) obtained from the GEANT3 simulation of the BGO array for the \( E_r = 2388 \) keV resonance. Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of \( \eta_{\text{sing}} = 17.3 \pm 2.5\% \), giving a singles yield of \( Y_{\text{sing}} = (4.38 \pm 0.88) \times 10^{-11} \). This is in good agreement with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power \( \epsilon = 197 \pm 9 \text{ eV/(10}^{15} \text{ } ^4\text{He cm}^3) \) for this bombarding energy and the de Broglie wavelength of the resonance \( (\lambda_{2r}^2 = 0.964 \text{ barns for } E_r = 2391 \text{ keV}) \), we obtain a singles resonance strength of \( \omega \gamma_{\text{sing}} = 1.89 \pm 0.39 \text{ eV} \). This is of course in good agreement with the coincident resonance strength obtained in the previous section.

5.5 Yield Measurements 5 and 6: \( E_{r}^{(\text{lit})} = 2327 \text{ keV tuned to 7+ and 8+ recoils} \)

The separator was tuned to 2 different recoil charge states (7+ and 8+) for yield measurements 5 and 6 (at an incoming beam energy of \( E_{\text{CM}} = 2359 \text{ keV} \)) with the intention of comparing the yield data from these two measurements to the CSF measurements. This
Figure 5.19: Spectrum of BGO Z-position of the highest energy $\gamma$ per event for yield measurements 5 and 6 (see Table 5.1).

section details the analysis of the coincident data of the combined 7+ and 8+ data sets.

From the BGO $z$-position spectra of $\gamma_0$ events in the golden recoil gate, we measure center of mass resonance energies of $E_r = 2297.9 \pm 9.7$ keV for the 8+ charge state and $E_r = 2300.5 \pm 10.0$ keV for the 7+ charge state. (see Figure 5.25(a) and Figure 5.28(a)). However, plotting the BGO $z$-position spectra of $\gamma_0$ events in the golden recoil gate for runs 1035-1044 and runs 1047-1058 (i.e. - all golden recoils from yield measurements 5 and 6), we measure a center of mass resonance energy of $E_r = 2298.5 \pm 7.3$ keV (see Figure 5.19.) This corresponds to an excitation energy of $E_x = 9506$ keV. Although this is significantly lower than the excitation energies of $E_x^{\text{lit}} = 9537.2(4)$ keV and $E_x^{\text{lit}} = 9535(30)$ keV of the doublet of $J^\pi = (8^+)$ and $J^\pi = 2^+$ states, this measurement does agree with previous measurements of the excitation energy of the $J^\pi = 2^+$ state at $E_x = 9535(30)$ keV due to the large uncertainty
in the canonical value of this state’s excitation energy [24, 26, 27].

The $\gamma_1$ vs $\gamma_0$ spectrum (Figure 5.20(d)) of all recoil events for the yield measurements at $E_b = 658$ keV/u does not exhibit a strong transition directly to the ground state* (as would be expected for a $J^\pi = 1^-$ resonance). However, there does seem to be a strong transition from the first excited state (the $J^\pi = 2^+$ state at $E_x = 2168$ keV) to the ground state, so it is likely that there is a transition to the first excited state or a $\gamma$ cascade that runs

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* A strong direct to ground transition would be indicated by the presence of a peak near $E_x = 9506$ keV (or generally $E_x = E_r + Q$) in both the $\gamma_0$ and the $\gamma_1$ vs $\gamma_0$ spectra, as illustrated in ?? and Figure 5.16(d).
Table 5.9: Branching ratios (in %) for electromagnetic transitions from the $E_x^{\text{meas}} = 9506$ keV state used in GEANT3 simulations of DRAGON’s BGO array. Labeling corresponds to the that in Figure 5.21.

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<td>9507</td>
<td>0</td>
<td>16.67</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

through the first excited state. The lack of a strong direct to ground transition makes a spin and parity of $J^\pi = 1^-$ highly improbable, and the presence of at least one $\gamma_0$ of $E_\gamma^{(0)} \approx 9$ MeV makes spin and parity of $J^\pi = 0^+$ equally improbable. Thus the most probable spin and parity assignment to this state is that of $J^\pi = 2^+$. 

No electromagnetic transitions have been previously observed from this state [27]. As such, a hypothetical $\gamma$-decay scheme for this state was constructed from the information available. In the nomenclature of the NNDC, there are strong arguments that suggest the spin and parity of the state at $E_x = 9535(30)$ keV is $J^\pi = 2^+$. Using this and Table 2.2, we can hypothesize a decay scheme such as the one given in Figure 5.21. The GANT simulation that yielded the nominal value for the BGO efficiency used the $\gamma$ decay scheme given in Figure 5.21 and the branching ratios given in Table 5.9. The GEANT3 simulation of DRAGON’s BGO array for this resonance yielded a detection efficiency of $\eta_{\text{BGO}}^{\text{meas}} = 77.0^{+11.5}_{-14.8}\%$. 

The uncertainty on the BGO detection efficiency was estimated as follows: the lower limit for the detection efficiency was estimated by forcing a 100% branching ratio to the ground state. The upper limit for the detection efficiency was estimated by a $\gamma$ cascade beginning with a transition to the $E_x = 4586$ keV with a 100% branching ratio. The branching ratios used for each of these simulations are given in Table 5.9. Figure 5.20(c) compares the spectra obtained from GEANT3 simulations to the observed spectrum of highest energy $\gamma$ per event.
Figure 5.21: Hypothetical $\gamma$ decay scheme of the $E_{\text{meas}} = 9506$ keV resonance.
for yield measurements 5 and 6. In this figure, the orange filled histogram are the observed spectra, the blue dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios given in column 4 of Table 5.9, the green dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios given in column 5 of Table 5.9, and the red dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios given in column 6 of Table 5.9. The simulation was performed with a spin and parity assignment of $J^\pi = 2^+$, as this is likely spin and parity of this state. The fact that we were able to populate this resonance through radiative $\alpha$ capture is a strong indication that this state has a spin and parity of $J^\pi = 2^+$.

5.6 Yield Measurement 5: $E_r^{(lit)} = 2327$ keV tuned to $8^+$

Data for yield measurement number 5 were collected at an average target pressure of $P_{\text{avg}} = 9.04 \pm 0.20$ Torr. The incoming beam energy was measured at $E_b = 658.8$ keV/u (corresponding to $E_{\text{CM}} = 2359$ keV) and the outgoing beam energy was measured at $E_{\text{out}} = 638.7$ keV/u (corresponding to $E_{\text{CM}} = 2287$ keV). At this bombarding energy and target thickness, the $J^\pi = 2^+$ state at $E_{x}^{\text{lit}} = 9.535$ MeV was just downstream of the center of the target.

![Figure 5.22](image)

**Figure 5.22:** Spectra of elastic scattering monitors during run 1037. Panel (a) - IIS$_{30}$ spectrum. Panel (b) - IIS$_{57}$. In both figures, the orange shaded spectrum is the signal region and the navy filled spectrum denotes the background region.
During one of the hour-long runs (run 1037) for yield measurement number 5, there was a sudden burst of events in the background region of the elastic scattering monitors (see Figure 5.22). This was likely caused by a momentary (transient) fluctuation of the beam intensity related to the problems with the ISAC-I mass analyzer magnet’s NMR probe feedback loop. In the IIS mounted at an angle of 57° (hereafter referred to as IIS$_{57}$), the increased background had virtually no effect on the signal region (see Figure 5.22(b)). However, in the IIS mounted at an angle of 30° (hereafter referred to as IIS$_{30}$), the increased background overlapped significantly with the signal region. Thus ascertaining the number of $\alpha$ particles scattered into the elastic scattering monitors required subtracting the underlying background on IIS$_{30}$ (see Figure 5.22(a)). The total number of $\alpha$ particles scattered into the elastic scattering monitors during yield measurement number 5 was then $N_{\alpha} = 262,857$, resulting in a total integrated beam on target of $N_b = 4.11E+014$ incident $^{34}\text{S}^{7+}$ ions.

![Figure 5.23: Spectra of IIS$_{30}$ during run 1037 with background fit.](image)
Figure 5.24: MCP TAC vs separator TOF for (coincident) events during yield measurement 5 (at $E_b = 2359$ keV tuned to the 8+ recoil charge state - see Table 5.1). Gray triangles are coincident events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

5.6.1 Coincidence Analysis of Yield Measurement 5

There are a total of $c \pm \sigma_{c}^{\text{stat}} \pm \sigma_{c}^{\text{sys}} = 123 \pm 11.1 \pm 1$ events within the golden recoil gate of the TAC-TOF spectrum for the yield measurement 4 (at $E_b = 2359$ keV tuned to the 8+ recoil charge state - see Figure 5.24). Like most of the other TAC-TOF spectra, Figure 5.24 exhibits virtually no background in the MCP TAC vs separator TOF spectrum, and very few jitter events in the MCP TAC. Of the 123 events in the golden recoil gate, 103 events have a valid signal in the IC, suggesting an IC detection efficiency of $\eta_{\text{IC}}^{\text{coinc}} = 83.7 \pm 11.2\%$. This is in good agreement with the IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 80.1 \pm 0.9\%$ measured during the attenuated beam run for this energy.

Combining the BGO detection efficiency with the other relevant coincidence efficiencies
for this yield measurement (see Table 5.2), we obtain a total coincidence detection efficiency of $\eta_{\text{coinc}} = 14.6 \pm 3.8\%$. Along with the number of golden recoils and the total integrated beam on target, we calculate a coincident yield of $Y_{\text{coinc}} = (2.04 \pm 0.60) \times 10^{-12}$. Then, using Equation (5.2) along with the measured stopping power of $\epsilon = (191 \pm 9) \text{[eV/(10^{15} \text{He cm}^{-2})]}$ and the de Broglie wavelength of $\lambda^2 = 1.002$ barns (corresponding to the resonance energy of $E_r = 2298 \text{ keV}$ measured during this yield measurement), we calculate a resonance strength of $\omega \gamma = 0.082 \pm 0.024 \text{ eV}$. 

\textbf{Figure 5.25:} BGO spectra for events within the golden recoil gate for yield measurement 5. Panel (a) - $z$-position spectrum of highest energy $\gamma$ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy $\gamma$ per event (see text for further explanation). Panel (d) - 2-D spectrum of 2nd highest energy $\gamma$ per event vs highest energy $\gamma$ per event ($\gamma_1$ vs $\gamma_0$).
5.6.2 Singles Analysis of Yield measurement

Employing the method outlined in section 4.5.2 for identifying recoil Events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.26(a) and the fit shown in Figure 5.26(b). There are a total of $N_{\text{sig}} = 2204$ events in the signal region in Figure 5.26(a). Integration of the background fit over the signal region resulted in an estimated background of $b = 2051$ leaky beam events, giving a total of $s = 153 \pm 65$ singles recoil events. Combining this with the singles livetime and IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 80.1 \pm 0.9\%$, we calculate the measured BGO detection efficiency to be $\eta_{\text{BGO}}^{\text{meas}} = 67 \pm 43\%$. Although this value does agree with the values of $\eta_{\text{BGO}}^{\text{sim}} = 77^{+12}_{-15}\%$ from the GEANT3 simulation of the BGO array for the $E_r = 2298$ keV resonance, the large background in the IC for this (singles) yield measurement leads to a large uncertainty on the number of singles recoils, making the value of the measured BGO efficiency virtually meaningless. Fortunately, the subsequent yield measurement, which was made at the same bombarding energy but with the separator tuned to a different recoil charge state (the 7+ charge state), had a substantially increased beam suppression. The increased beam suppression led to a much smaller background in the IC signal region, making a more meaningful comparison of the measured and simulated BGO detection efficiencies possible.

Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of $\eta_{\text{sing}} = 15.9 \pm 2.3\%$, giving a singles yield of $Y_{\text{sing}} = (2.3 \pm 1.1) \times 10^{-12}$. This agrees well with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power $\epsilon = 191 \pm 9$ eV/(10$^{15}$ 4He cm$^3$) for this bombarding energy and the de Broglie wavelength of the resonance ($\lambda_r^2 = 1.002$ barns for $E_r = 2299$ keV), we obtain a singles resonance strength of $\omega \gamma_{\text{sing}} = 0.234 \pm 0.110$ eV which is in good agreement with the coincident resonance strength obtained in the previous section.
Figure 5.26: IC signal spectra for yield measurement 6 obtained by employing the method outlined in section 4.5.2. Panel (a) - 1-D pulse height spectrum of IC anode[1] vs IC anode[0] projected onto the diagonal overlaid with the same spectrum for all events in the golden recoil gate and signal region identification. Panel (b) - Double Gaussian fit to spectrum in panel (a).

5.7 Yield Measurement 6: $E_{\text{lit}} = 2327$ keV tuned to $7^+$ recoils

Data for yield measurement 6 were collected at an average target pressure of $P_{\text{avg}} = 8.75 \pm 0.19$ Torr. The incident beam energy did not change from yield measurement 5, but the outgoing beam energy increased slightly (due to the slight decrease in average target pressure) to a measured outgoing energy of $E_{\text{out}} = 639.4$ keV/u (corresponding to $E_{\text{CM}} = 2289$ keV). At this bombarding energy and target thickness, the $J^\pi = 2^+$ state at $E_x = 9.535$ MeV was just downstream of the center of the target. During yield measurement 7, a total of 156,758 $\alpha$ particles were detected in the IIS signal peaks, resulting in a total integrated beam on target of $N_b = (2.54 \pm 0.35) \times 10^{13}$ incident $^{34}$S$^{7+}$ ions.

5.7.1 Coincidence Analysis of Yield Measurement 6

There are $c \pm \sigma_{c}^\text{stat} \pm \sigma_{c}^\text{sys} = 74 \pm 8.6 \pm 6$ events within the golden recoil gate of the TAC-TOF spectrum for the yield measurement 5 (at $E_b = 2359$ keV tuned to the $7^+$ recoil charge state see Figure 5.27). Figure 5.27 exhibits virtually no background in the MCP TAC vs separator TOF spectrum, and very few jitter events in the MCP TAC. Of the 74 events
Figure 5.27: MCP TAC vs separator TOF for (coincident) events during yield measurement 6 (at $E_b = 2359$ keV tuned to the 7+ recoil charge state - see Table 5.1). Gray triangles are coincident events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

In the golden recoil gate, 73 events have a valid signal in the IC, suggesting an IC detection efficiency of $\eta_{IC} = 98.6^{+1.4}_{-1.6}\%$. Although this is much higher than the IC detection efficiency of $\eta_{IC}^{AB} = 80.1 \pm 0.9\%$ measured during the attenuated beam run for this energy, the large uncertainty on this efficiency (due to the low counting statistics) only disagrees slightly within the uncertainties of the two efficiencies (the efficiency measured during the attenuated beam run has an upper limit of $\eta_{IC}^{AB} = 82\%$ while the efficiency suggested by golden recoils has a lower limit of $\eta_{IC}^{coinc} = 82.3\%$). The BGO detection efficiency of $\eta_{BGO}^{sim} = 77^{+12}_{-15}\%$ for this yield measurement was taken from the simulation for the previous yield measurement.

Combining the BGO detection efficiency with the other relevant coincidence efficiencies for this yield measurement (see Table 5.2), we obtain a total coincidence detection efficiency
Figure 5.28: BGO spectra for events within the golden recoil gate for yield measurement 6. Panel (a) - z-position spectrum of highest energy $\gamma$ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy $\gamma$ per event (see text for further explanation). Panel (d) - 2-D spectrum of 2nd highest energy $\gamma$ per event vs highest energy $\gamma$ per event ($\gamma_1$ vs $\gamma_0$).

of $\eta_{\text{coinc}} = 13.7 \pm 3.7\%$. Along with the number of golden recoils and the total integrated beam on target, we calculate a coincident yield of $Y_{\text{coinc}} = (2.12 \pm 0.65) \times 10^{-12}$. Then, using Equation (5.2) along with the measured stopping power of $\epsilon = (191 \pm 9) \text{[eV}/(10^{15} \text{He cm}^{-2})]$ and the de Broglie wavelength of $\lambda^2 = 1.002$ barns (for this energy), we calculate a resonance strength of $\omega\gamma = 0.085 \pm 0.026$ eV. This agrees with the coincident resonance strength obtained in the previous yield measurement (tuned to the 8+ recoil charge state).
5.7.2 Singles Analysis of Yield measurement 6

Employing the method outlined in section 4.5.2 for identifying recoil Events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.29(a) and the fit shown in Figure 5.29(b). There are a total of \( N_{\text{sig}} = 87 \) events in the signal region in Figure 5.29(a). As one can see in Figure 5.37(b), there is virtually no overlap between the signal and background regions in the IC spectrum, so integration of the background fit over the signal region resulted in an estimated background of \( b = 0 \), giving a total of \( s = 87 \pm 9.3 \) singles recoil events. The intensity and energy of the unreacted, down-scattered (“leaky”) beam was significantly reduced when the separator was scaled to the 7+ recoil charge state for this and subsequent yield measurements as is illustrated in Figure 5.29(a) (cf. Figure 5.26(a)). Combining the number of detected singles recoils with the singles livetime and IC detection efficiency of \( \eta_{\text{IC}}^{\text{AB}} = 80.1 \pm 0.9\% \), we calculate the measured BGO detection efficiency to be \( \eta_{\text{BGO}}^{\text{meas}} = 71 \pm 16\% \). This value agrees with the values of \( \eta_{\text{BGO}}^{\text{sim}} = 77_{-15}^{+12}\% \) from the GEANT3 simulation of the BGO array for the \( E_r = 2298 \) keV resonance.

Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of \( \eta_{\text{sing}} = 14.9 \pm 2.3\% \), giving a singles yield of \( Y_{\text{sing}} = (2.30 \pm 0.54) \times 10^{-12} \). This agrees well with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power \( \epsilon = 191 \pm 9 \) eV/(10^{15} \(^4\)He cm\(^3\)) for this bombarding energy and the de Broglie wavelength of the resonance \( (\lambda_\gamma^2 = 1.002 \) barns for \( E_r = 2300 \) keV), we obtain a singles resonance strength of \( \omega \gamma_{\text{sing}} = 0.230 \pm 0.055 \) eV which is in good agreement with the coincident resonance strength obtained in the previous section, as well as the coincident and singles resonance strengths obtained for yield measurement 5 (tuned to the 8+ recoil charge state).

5.8 Yield Measurement 7: \( E_r^{(\text{lit})} = 2252 \) keV

Data for yield measurement number 4 were collected at an average target pressure of \( P_{\text{avg}} = 9.00 \pm 0.18 \) Torr. The incoming beam energy was measured at \( E_b = 641.5 \) keV/u
Figure 5.29: IC signal spectra for yield measurement 6 obtained by employing the method outlined in section 4.5.2. Panel (a) - 1-D pulse height spectrum of IC anode[1] vs IC anode[0] projected onto the diagonal overlaid with the same spectrum for all events in the golden recoil gate and signal region identification. Panel (b) - Double Gaussian fit to spectrum in panel (a).

(corresponding to $E_{CM} = 2297$ keV) and the outgoing beam energy was measured at $E_{out} = 621.9$ keV/u (corresponding to $E_{CM} = 2227$ keV). At this bombarding energy and target thickness, the state of unknown $J^\pi = (3^- : 7^-)$ at $E_{lit} = 9.460$ MeV should have been roughly centered in the target. However, Figure 5.31(a) shows no definitive evidence of the presence of an isolated narrow resonance in the target. From the $Z$-distribution of highest energy $\gamma$ per event (see Figure 5.31(a)), we measured a resonance energy of $E_r = 2249 \pm 10$ keV, which does agree with the literature values of $E_{lit} = 2252 \pm 2$ keV. During yield measurement number 4, a total of 91,730 $\alpha$ particles were detected in the IIS signal peaks, resulting in a total integrated beam on target of $N_b = (1.39 \pm 0.19) \times 10^{14}$ incident $^{34}$S$^{7+}$ ions.

5.8.1 Coincidence Analysis of Yield Measurement 7

There are $c \pm \sigma_c^{stat} \pm \sigma_c^{sys} = 31 \pm 5.6 \pm 1$ events within the golden recoil gate of the TAC-TOF spectrum for yield measurement 7 (see Figure 5.30). The 2-D pulse height spectrum used for recoil identification during this yield measurement exhibits virtually no background, and very few jitter events in the MCP TAC. Of the 31 events in the golden recoil gate, only 15
Figure 5.30: MCP TAC vs separator TOF for (coincident) events during yield measurement number 7. Gray triangles are coincident events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

have a valid signal in the IC, suggesting an IC detection efficiency of $\eta_{\text{IC}}^{\text{coinc}} = 48.4 \pm 15.2\%$. This is much lower than the IC detection efficiency of $\eta_{\text{IC}} = 78.3 \pm 2.9\%$ measured during the attenuated beam run for this energy but is in good agreement with the IC detection efficiency of $\eta_{\text{IC}}^{\text{sing}} = 43.2 \pm 7.5\%$ measured with singles recoils (using the method in section 4.4). This suggests that the recoils may have been poorly focused at the end focal plane during this yield measurement, but the large uncertainty on this efficiency (due to the low counting statistics) makes it difficult to make any definitive statements about the IC detection efficiency for this yield measurement. The poor focusing on the final focal plane may be correlated with the lack of a discernible resonance in the target, but the extent of such a correlation is unclear. Additional calculations shed some light on the subject.
Figure 5.31: BGO spectra for events within the golden recoil gate for yield measurement 7. Panel (a) - z-position spectrum of highest energy γ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy γ per event (see text for further explanation). Panel (d) - 2-D spectrum of 2nd highest energy γ per event vs highest energy γ per event (γ₁ vs γ₀).

As above, we can estimate the BGO detection efficiency using Equation (4.44). If we use the IC detection efficiency of $\eta_{IC} = 78.3 \pm 2.9\%$ measured during the attenuated beam run, we obtain a measured BGO efficiency of $\eta_{IC} = 133 \pm 39\%$. On the other hand, if we use the IC detection efficiency of $\eta_{IC}^{\text{sing}} = 43.2 \pm 7.5\%$ measured with singles recoils, we obtain a measured BGO efficiency of $\eta_{IC} = 74 \pm 25\%$. If we use the IC detection efficiency of $\eta_{IC}^{\text{coinc}} = 48.4 \pm 15.2\%$ suggested by the coincidence data, we obtain a measured BGO detection efficiency of $\eta_{BGO}^{\text{meas}} = 82 \pm 35\%$. The latter two efficiencies agree quite well with the
Figure 5.32: Known $\gamma$-decay scheme used for GEANT3 simulations of DRAGON’s BGO array for the $E^\text{lit}_x = 9.460$ MeV state.

simulated BGO efficiency of $\eta^\text{sim}_{\text{BGO}} = 84^{+9}_{-11}$% (see below), albeit with large uncertainties. The BGO efficiency calculated using the IC detection efficiency obtained during the attenuated beam run is obviously an absurd value, so it may be concluded that the recoils during this measurement were indeed poorly focused on the final focal plane, and therefore that the IC detection efficiency measured with the attenuated beam run drastically overestimates the IC detection efficiency for this yield measurement. For these reasons, the IC detection efficiency measured with singles recoils was used to calculate the singles yield.
Table 5.10: Branching ratios (in %) for electromagnetic transitions from the $E_{x}^\text{lit} = 9.450$ MeV state used in GEANT3 simulations of DRAGON’s BGO array. Labeling corresponds to that in Figure 5.32.

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<th>$E_{x}^{(f)}$ [keV]</th>
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<th>Lower</th>
</tr>
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<td>0</td>
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<td>0</td>
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* Not shown in Figure 5.32.

GEANT3 simulation of DRAGON’s BGO array yielded a detection efficiency of $\eta_{\text{BGO}}^{\text{meas}} = 84^{+9}_{-11}$ %. The only known electromagnetic transition from the $E_{x}^\text{lit} = 9.460$ MeV state is to the $J^\pi = 5^-$ (hence the NNDC’s assignment of “weak arguments suggest that the spin and parity has a value between $3^-$ and $7^-$”) state at $E_x = 4.586$ MeV. The resulting $\gamma$-decay scheme used for the GEANT3 simulation is given in Figure 5.32. The uncertainty on the BGO detection efficiency was estimated as follows: the lower limit for the detection efficiency was estimated by forcing a 100% branching ratio to the ground state. The upper limit for the detection efficiency was estimated by forcing a 100% branching ratio through a 3 (above threshold) $\gamma$ cascade through the 4586 keV state and the first excited state. Figure 5.31(c) compares the spectra obtained from GEANT3 simulations to the observed spectrum of highest energy $\gamma$ per event for yield measurement 7. In this figure, the orange filled histogram are the observed spectra, the blue dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 4 of Table 5.10, the green dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 5 of Table 5.10, and the red dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 6 of Table 5.10. The branching ratios used...
for each of these simulations are given in Table 5.10. The simulation was performed with a spin and parity assignment of $J^p = 3^-$, as this is the likely upper limit for the channel spin due to the $\alpha$ particle's low penetrability at this energy and angular momentum. However, the lack of definitive evidence of the presence of a resonance in the target during this yield measurement may suggest a higher value of $J$ for this state.

Combining the BGO detection efficiency with the other relevant coincidence efficiencies for this yield measurement (see Table 5.2), we obtain a total coincidence detection efficiency of $\eta_{\text{coinc}} = 15.9 \pm 2.7\%$. Along with the number of golden recoils and the total integrated beam on target, we calculate a coincident yield of $Y_{\text{coinc}} = (1.41 \pm 0.41) \times 10^{-12}$. Then, using Equation (5.2) along with the measured stopping power of $\epsilon = (188 \pm 10) \text{ [eV/(10^{15} \text{ He cm}^{-2})]}$ and the de Broglie wavelength of $\lambda^2 = 1.025 \text{ barns}$ (for this energy), we calculate a resonance strength of $\omega \gamma = 0.054 \pm 0.016 \text{ eV}$.

### 5.8.2 Singles Analysis of Yield Measurement

Employing the method outlined in section 4.5.2 for identifying recoil events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.33(a) and the fit shown in Figure 5.33(b). There are a total of $N_{\text{sig}} = 19$ events in the signal region in Figure 5.33(a). Integration of the background fit over the signal region resulted in an estimated background of $b = 0.1$, resulting in a total of $s = 18.9 \pm 4.4$ singles recoil events. As noted in the previous section, the measured BGO efficiency is $\eta_{\text{IC}} = 73 \pm 25\%$ (corresponding to the IC detection efficiency measured with singles recoils of $\eta_{\text{IC}}^{\text{sing}} = 42.9 \pm 7.5\%$). This value agrees with the values of $\eta_{\text{BGO}}^{\text{sim}} = 84^{+9}_{-11}\%$ obtained from the GEANT3 simulations of the BGO array for this resonance energy.

Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of $\eta_{\text{sing}} = 8.5 \pm 1.8\%$, giving a singles yield of $Y_{\text{sing}} = (1.61 \pm 0.56) \times 10^{-12}$. This agrees well with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power $\epsilon =
Figure 5.33: IC signal spectra for yield measurement 7 obtained by employing the method outlined in section 4.5.2. Panel (a) - 1-D pulse height spectrum of IC anode[1] vs IC anode[0] projected onto the diagonal overlaid with the same spectrum for all events in the golden recoil gate and signal region identification. Panel (b) - Double Gaussian fit to spectrum in panel (a).

188 ± 10 eV/(10^{15} \text{^4}\text{He cm}^3) for this bombarding energy and the de Broglie wavelength of the resonance (λ_2^2 = 1.025 barns for E_r = 2249 keV), we obtain a singles resonance strength of ω_γ\text{sing} = 0.062 ± 0.022 eV which is in good agreement with the coincident resonance strength obtained in the previous section.

5.9 Yield Measurement 8: \(E_r^{\text{lit}} = 2229\) keV

Data for yield measurement number 8 were collected at an average target pressure of \(P_{\text{avg}} = 9.13 ± 0.19\) Torr. The incoming beam energy was measured at \(E_b = 629.0\) keV/u (corresponding to \(E_{\text{CM}} = 2085\) keV) and the outgoing beam energy was measured at \(E_{\text{out}} = 608.8\) keV/u (corresponding to \(E_{\text{CM}} = 2180\) keV). At this bombarding energy and target thickness, the he state of unknown \(J^\pi = (3^- : 7^-)\) at \(E_x^{\text{lit}} = 9.437\) MeV was roughly centered in the target (see Figure 5.35(a)). The arithmetic mean of the \(Z\)-distribution of highest energy \(\gamma\) per event was (see Figure 5.35(a)), giving a measured a resonance energy of \(E_r = 2218 ± 8\) keV, which is in good agreement with the literature value of \(E_r^{\text{lit}} = 2229 ± 2\) keV. During yield measurement number 4, a total of 127,131 α particles were detected in the IIS.
Figure 5.34: MCP TAC vs separator TOF for (coincident) events during yield measurement number 8. Gray triangles are coincident events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).

signal peaks, resulting in a total integrated beam on target of $N_b = (1.83 \pm 0.24) \times 10^{14}$ incident $^{34}\text{S}^{7+}$ ions.

5.9.1 Coincidence Analysis of Yield Measurement 8

There are a total of $c \pm \sigma_c^{\text{stat}} \pm \sigma_c^{\text{sys}} = 77 \pm 8.8 \pm 17$ events that fall within the golden recoil gate of the TAC-TOF spectrum for the yield measurement 8. The 2-D pulse height spectrum used for recoil identification during this yield measurement exhibits virtually no background, but does have a significant amount of jitter events ($N_{\text{jit}} = 17$). Of the 77 events in the golden recoil gate, 60 events have a valid signal in the IC, suggesting an IC detection efficiency of $\eta_{\text{IC}} = 77.9 \pm 13.4\%$. This is in good agreement with the IC detection efficiency
Figure 5.35: BGO spectra for events within the golden recoil gate for yield measurement 8. Panel (a) - z-position spectrum of highest energy $\gamma$ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy $\gamma$ per event (see text for further explanation). Panel (d) - 2-D spectrum of 2nd highest energy $\gamma$ per event vs highest energy $\gamma$ per event ($\gamma_1$ vs $\gamma_0$).

of $\eta_{IC}^{AB} = 79.8 \pm 8.1\%$ measured during the attenuated beam run for this energy.

GEANT3 simulation of DRAGON’s BGO array yielded a detection efficiency of $\eta_{BGO}^{\text{meas}} = 84^{+9}_{-11}\%$. The only known electromagnetic transition from the $E_x^{\text{lit}} = 9.437$ MeV state is to the $J^\pi = 5^-$ state at $E_x = 5.659$ MeV. The resulting $\gamma$-decay scheme used for the GEANT3 simulation is given in Figure 5.36. The uncertainty on the BGO detection efficiency was estimated as follows: the lower limit for the detection efficiency was estimated by forcing a 100% branching ratio to the ground state. The upper limit for the detection efficiency was
Figure 5.36: Known $\gamma$ decay scheme of the 2229 keV resonance.
estimated by forcing a 100% branching ratio through a 3 (above threshold) \( \gamma \) cascade through the 5.659 MeV state and the first excited state. The branching ratios used for each of these simulations can be calculated from the intensities given in Figure 5.36. Figure 5.35(c) compares the spectra obtained from GEANT3 simulations to the observed spectrum of highest energy \( \gamma \) per event for yield measurement 8. In this figure, the orange filled histogram are the observed spectra, the blue dashed histogram is the spectrum obtained from the GEANT3 simulation using branching ratios calculated from the intensities given in Figure 5.36, the green dashed histogram is the spectrum obtained from the GEANT3 simulation for the 100% 3 \( \gamma \) cascade, and the red dashed histogram is the spectrum obtained from the GEANT3 simulation for the 100% direct to ground state transition. The simulation was performed with a spin and parity assignment of \( J^\pi = 3^- \) to this state, as this is the likely upper limit for the channel spin due to the low penetrability of the \( \alpha \) particle at this energy and angular momentum. The fact that we were able to populate this resonance through radiative \( \alpha \) capture is a strong indication that this state has a spin and parity of \( J^\pi = 3^- \).

Combining the BGO detection efficiency with the other relevant coincidence efficiencies for this yield measurement (see Table 5.2), we obtain a total coincidence detection efficiency of \( \eta_{\text{coinc}} = 15.8 \pm 3.5\% \). Along with the number of golden recoils and the total integrated beam on target, we calculate a coincident yield of \( Y_{\text{coinc}} = (2.68 \pm 0.75) \times 10^{-12} \). Then, using Equation (5.2) along with the measured stopping power of

\[
\epsilon = (190 \pm 5) \, [\text{eV}/(10^{15} \text{He cm}^{-2})]
\]

and the de Broglie wavelength of \( \lambda^2 = 1.00 \) barns (for this energy), we calculate a resonance strength of \( \omega_G = 0.103 \pm 0.027 \) eV.

### 5.9.2 Singles Analysis of Yield Measurement 8

Employing the method outlined in section 4.5.2 for identifying recoil events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.37(a) and the fit shown in Figure 5.37(b). There are a total of \( N_{\text{sig}} = 70 \) events in the signal region in Figure 5.37(a). As one can see in Figure 5.37(b), there is virtually no overlap between
Figure 5.37: IC signal spectra for yield measurement 8 obtained by employing the method outlined in section 4.5.2. Panel (a) - 1-D pulse height spectrum of IC anode[1] vs IC anode[0] projected onto the diagonal overlaid with the same spectrum for all events in the golden recoil gate and signal region identification. Panel (b) - Double Gaussian fit to spectrum in panel (a).

the signal and background regions in the IC spectrum, so integration of the background fit over the signal region resulted in an estimated background of $b = 0$, giving a total of $s = 70 \pm 8.4$ singles recoil events. Combining this with the singles livetime and IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 79.8 \pm 8.1\%$, we calculate the measured BGO detection efficiency to be $\eta_{\text{BGO}}^{\text{meas}} = 92 \pm 15\%$. This value agrees with the values of $\eta_{\text{BGO}}^{\text{sim}} = 85^{+9}_{-11}\%$ obtained from the GEANT3 simulations of the BGO array for this resonance energy.

Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of $\eta_{\text{sing}} = 15.5 \pm 3.4\%$, giving a singles yield of $Y_{\text{sing}} = (2.47 \pm 0.66) \times 10^{-12}$. This agrees well with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power $\epsilon = 190 \pm 5$ eV/(10$^{15}$ 4He cm$^3$) for this bombarding energy and the de Broglie wavelength of the resonance ($\lambda = 1.039$ barns for $E_r = 2218$ keV), we obtain a singles resonance strength of $\omega_{\gamma_{\text{sing}}} = 0.095 \pm 0.025$ eV which is in good agreement with the coincident resonance strength obtained in the previous section.
5.10 Yield Measurement 9: $E_r^{(lit)} = 2166$ keV

Data for yield measurement number 9 were collected at an average target pressure of $P_{\text{avg}} = 7.98 \pm 0.19$ Torr. The incoming beam energy was measured at $E_b = 610.7 \text{ keV/u}$ (corresponding to $E_{\text{CM}} = 2187$ keV) and the outgoing beam energy was measured at $E_{\text{out}} = 594.0 \text{ keV/u}$ (corresponding to $E_{\text{CM}} = 2121$ keV). At this bombarding energy and target thickness, the he state of unknown $J^{\pi} = (3^{-} : 7^{-})$ at $E_x^{\text{lit}} = 9.374 \text{ MeV}$ was roughly centered in the target (see Figure 5.39(a)). The arithmetic mean of the $Z$-distribution of highest energy $\gamma$ per event was (see Figure 5.39(a)), giving a measured a resonance energy of $E_r = 2164 \pm 8$ keV, which is in good agreement with the literature value of $E_r^{\text{lit}} = 2166 \pm 2$ keV. During yield measurement number 9, a total of 235,743 $\alpha$ particles were detected in the IIS.

![Figure 5.38: MCP TAC vs separator TOF for (coincident) events during yield measurement number 9. Gray triangles are coincident events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).](image-url)
signal peaks, resulting in a total integrated beam on target of \( N_b = (3.70 \pm 0.53) \times 10^{14} \) incident \(^{34}\text{S}^{7+}\) ions.

### 5.10.1 Coincidence Analysis of Yield Measurement 9

There are a total of \( c \pm \sigma_c^{\text{stat}} \pm \sigma_c^{\text{sys}} = 56 \pm 7.5 \pm 1 \) events that fall within the golden recoil gate of the TAC-TOF spectrum for yield measurement 9. Like other S1372 yield measurements, Figure 5.38 exhibits virtually no background, as well as only one jitter event. Of the 56 events in the golden recoil gate, 49 events have a valid signal in the IC, suggesting an IC detection efficiency of \( \eta_{\text{IC}} = 87.5 \pm 17.1\% \). This agrees with the IC detection efficiency of \( \eta_{\text{IC}}^{\text{AB}} = 79.2 \pm 1.2\% \) measured during the attenuated beam run for this energy.

GEANTE3 simulation of DRAGON’s BGO array yielded a detection efficiency of \( \eta_{\text{BGO}}^{\text{meas}} = 84^{+9}_{-11}\% \). The only known electromagnetic transition from the \( E_{\gamma}^{\text{lit}} = 9.374 \) MeV state is to the \( J^\pi = 5^- \) state at \( E_x = 4.586 \) MeV. The resulting \( \gamma \)-decay scheme used for the GEANTE3 simulation is given in Figure 5.40. The uncertainty on the BGO detection efficiency was estimated as follows: the lower limit for the detection efficiency was estimated by forcing a 100% branching ratio to the ground state. The upper limit for the detection efficiency

### Table 5.11: Branching ratios (in %) for electromagnetic transitions from the \( E_{\gamma}^{\text{lit}} = 9.374 \) MeV state used in GEANTE3 simulations of DRAGON’s BGO array. Labeling corresponds to the that in Figure 5.40.

<table>
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<th>( E_\gamma ) [keV]</th>
<th>( E_{\gamma}^{(f)} ) [keV]</th>
<th>Nominal</th>
<th>Upper</th>
<th>Lower</th>
</tr>
</thead>
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<td>100</td>
<td>0</td>
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<td>2168</td>
<td>0.4</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>776</td>
<td>3810</td>
<td>9.9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>106</td>
<td>4480</td>
<td>89.7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>e</td>
<td>670</td>
<td>3810</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>f</td>
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<td>2168</td>
<td>99.92</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>h</td>
<td>2168</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(g.s.*)</td>
<td>9374</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

* Not shown in Figure 5.40.
Figure 5.39: BGO spectra for events within the golden recoil gate for yield measurement 9. Panel (a) - z-position spectrum of highest energy $\gamma$ per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy $\gamma$ per event. Panel (d) - 2-D spectrum of 2nd highest energy $\gamma$-ray per event vs highest energy $\gamma$-ray per event ($\gamma_1$ vs $\gamma_0$).

was estimated by forcing a 100% branching ratio through a 3 (above threshold) $\gamma$ cascade through the 4586 keV state and the first excited state. The branching ratios used for each of these simulations are given in Table 5.11. Figure 5.39(c) compares the spectra obtained from GEANT3 simulations to the observed spectrum of highest energy $\gamma$ per event for yield measurement 9. In this figure, the orange filled histogram are the observed spectra, the blue dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 4 of Table 5.11, the green dashed histogram is the spectrum
Figure 5.40: Known γ-decay scheme used for GEANT3 simulations of DRAGON’s BGO array for the $E_x^{\text{lit}} = 9.374$ MeV state.

obtained from the GEANT3 simulation using the branching ratios in column 5 of Table 5.11, and the green dashed histogram is the spectrum obtained from the GEANT3 simulation using the branching ratios in column 6 of Table 5.11. The simulation was performed with a spin and parity assignment of $J^\pi = 3^-$, as this is the likely upper limit for the channel spin due to the low penetrability of the α particle for this energy and angular momentum. The fact that we were able to populate this resonance through radiative α capture is a strong indication that this state has a spin and parity of $J^\pi = 3^-$. Combining the BGO detection efficiency with the other relevant coincidence efficiencies
for this yield measurement (see Table 5.2), we obtain a total coincidence detection efficiency of $\eta_{\text{coinc}} = 15.3 \pm 2.8\%$, and using Equation (5.2) along with the measured stopping power of $\epsilon = (182 \pm 4) \text{eV/(10}^{15}\text{ } ^4\text{He cm}^{-2})$ and the de Broglie wavelength of $\lambda^2 = 1.06$ barns (for this energy), we calculate a resonance strength of $\omega \gamma = 0.035 \pm 0.01$ eV.

5.10.2 Singles Analysis of Yield Measurement 9

Employing the method outlined in section 4.5.2 for identifying recoil Events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.41(a) and the fit shown in Figure 5.41(b). There are a total of $N_{\text{sig}} = 62$ events in the signal region in Figure 5.41(a). Integration of the background fit over the signal region resulted in an estimated background of $b = 1.9$, resulting in a total of $s = 60.1$ singles recoil events. Combining this with the singles livetime and IC detection efficiency of $\eta_{\text{AB}}^{\text{IC}} = 79.2 \pm 1.2\%$, we calculate the measured BGO detection efficiency to be $\eta_{\text{BGO}}^{\text{meas}} = 70 \pm 14\%$. This value agrees with the values of $\eta_{\text{BGO}}^{\text{sim}} = 84^{+9}_{-11}\%$ from the GEANT3 simulation of the BGO array for the $E_r = 2166$ keV resonance.

**Figure 5.41:** IC signal spectra for yield measurement 9 obtained by employing the method outlined in section 4.5.2. Panel (a) - 1-D pulse height spectrum of IC anode[1] vs IC anode[0] projected onto the diagonal overlaid with the same spectrum for all events in the golden recoil gate and signal region identification. Panel (b) - Double Gaussian fit to spectrum in panel (a).
Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of $\eta_{\text{sing}} = 15.2 \pm 1.9\%$, giving a singles yield of $Y_{\text{sing}} = (1.07 \pm 0.25) \times 10^{-12}$. This agrees well with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power $\epsilon = 182 \pm 4$ eV/(10$^{15}$ 4He cm$^3$) for this bombarding energy and the de Broglie wavelength of the resonance ($\lambda_2^2 = 1.065$ barns for $E_r = 2164$ keV), we obtain a singles resonance strength of $\omega_\gamma_{\text{sing}} = 0.039 \pm 0.009$ eV. This is of course in good agreement with the coincident resonance strength obtained in the previous section.

5.11 Yield Measurement 10: $E_{\text{cm}}^{(\text{in})} = 2133$ keV

Data for yield measurement number 10 were collected at an average target pressure of $P_{\text{avg}} = 8.57 \pm 0.18$ Torr. The incoming beam energy was measured at $E_b = 595.7$ keV/u (corresponding to $E_{\text{CM}} = 2133$ keV) and the outgoing beam energy was measured at $E_{\text{out}} = 576.8$ keV/u (corresponding to $E_{\text{CM}} = 2065$ keV). At these beam energies, there were possibly as many as 3 resonances in the target during this yield measurement: the state of unknown spin and parity $J^\pi = (3^- : 7^-)$ at $E_{\text{lit}} = 9293$ keV (corresponding to a center of mass energy of $E_{\text{CM}} = 2085$ keV), the state of unknown spin and parity $J^\pi = (0^+ : 4^+)$ at $E_{\text{lit}}^2 = 9300$ keV (corresponding to a CM energy of $E_{\text{CM}} = 2092$ keV), and the state of unknown spin and parity $J^\pi = (4^+ : 8^+)$ state at $E_{\text{lit}} = 9330$ keV (corresponding to a CM energy of $E_{\text{CM}} = 2122$ keV - although it is not likely that we populated this state as the $\alpha$ penetration factor is quite small for $\ell \geq 4$ - see section 2.4). The known $\gamma$-decay scheme for the $E_r = 2085$ keV state is given in Figure 5.44. The $(0^+ : 4^+)$ state at 9300 has only one known transition: to the first excited state).

The arithmetic mean of the $z$-distribution of highest energy $\gamma$ per event was (see Figure 5.43(a)), giving a measured resonance energy of $E_r = 2089 \pm 9$ keV. This is roughly directly in between the resonance energies of the $E_x = 9.300$ MeV and $E_x = 9.293$ MeV states. Given that this doublet of states are close in energy, it is unlikely that they would
be able to be resolved individually in DRAGON’s gas target. On the other hand, the combined resonance strength of this doublet is quite small, and the individual contributions of these states to the total resonant astrophysical reaction rate would likely yield no new information, so any future attempts to measure isolated narrow resonances in $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$ should prioritize measuring states below this energy (such as the $J^{\pi} = (0^+: 4^+)$ states at $E_x = 9.260, 9.204$ and 9.158 MeV, the $J^{\pi} = (1^+, 2^+)$ states at $E_x = 9.100$ MeV, the $J^{\pi} = (1^-)$ state at $E_x = 8.233$ [27], among others) over measuring these resonances again. During yield measurement number 10, a total of 170,144 $\alpha$ particles were detected in the IIS signal peaks, resulting in a total integrated beam on target of $N_0 = 3.31 \pm 0.43 \times 10^{14}$ incident $^{34}\text{S}^7+$ ions.

**Figure 5.42:** MCP TAC vs separator TOF for (coincident) events during yield measurement number 10. Gray triangles are coincident events, open red triangles are events in the background region, and filled red triangles are events that passed the “golden” TAC-TOF recoil gate (inset).
5.11.1 Coincidence Analysis of Yield Measurement 10

There are \( c \pm \sigma_c^{\text{stat}} \pm \sigma_c^{\text{sys}} = 42 \pm 6.5 \pm 1 \) events that fall within the golden recoil gate of the TAC-TOF spectrum for the yield measurement number 10 (see Figure 5.42). Once again, the 2-D pulse height spectrum used for recoil identification exhibits virtually no background, but does have a significant amount of jitter events \( (N_{\text{jit}} = 9) \). Of the 42 events in the golden recoil gate, all 42 events have a valid signal in the IC, suggesting an IC detection efficiency of \( \eta_{\text{IC}} = 100^{+0}_{-22}\% \). This agrees with the IC detection efficiency of \( \eta_{\text{IC}}^{\text{AB}} = 79.4 \pm 4.6\% \) measured during the attenuated beam run for this energy.

![Figure 5.43: BGO spectra for events within the golden recoil gate for yield measurement 10. Panel (a) - z-position spectrum of highest energy \( \gamma \) per event. Panel (b) - sum of energy spectra of all BGO detectors. Panel (c) - spectrum of highest energy \( \gamma \) per event. Panel (d) - 2-D spectrum of 2nd highest energy \( \gamma \) per event vs highest energy \( \gamma \) per event (\( \gamma_1 \) vs \( \gamma_0 \)).](image-url)
From the arithmetic mean of the $Z$-distribution of $\gamma_0$ for each event, we calculate a resonance energy of $E_r = 2089.4 \pm 8.7$ keV. Given that the measured resonance energy of $E_r = 2089.4$ keV is roughly midway between the CM energies of the $E_{r}^{\text{lit}} = 2085$ keV $E_{r}^{\text{lit}} = 2092$ keV states and given that the measured resonance energy agrees (within uncertainty) the CM resonance energy of both states, it is likely that there was significant contribution to the measured yield from both resonances. Further study - either performing this measurement with greater care at a later date or perhaps exploration via simulation (i.e. - extension of DRAGON’s Geant3 simulation to include multiple resonances in the target) - is needed to be able to state anything more than a combined resonance strength with any degree of confidence. On the other hand, the states in this doublet are so close in energy that it may not be possible to resolve each resonance individually in DRAGON’s gas target, so the combined resonance strength may be the best that can be done with DRAGON.

Geant3 simulation of DRAGON’s BGO array yielded a detection efficiency of $\eta_{\text{BGO}} = 85^{+9}_{-12}$%. The only known electromagnetic transition from the $E_{x}^{\text{lit}} = 9.293$ MeV state is to the $J^{\pi} = 5^{-}$ state at $E_{x} = 4.586$ MeV. The resulting $\gamma$-decay scheme used for the Geant3 simulation is given in Figure 5.44. The uncertainty on the BGO detection efficiency was estimated as follows: the lower limit for the detection efficiency was estimated by forcing a 100% branching ratio to the ground state. The upper limit for the detection efficiency was estimated by forcing a 100% branching ratio through a 3 (above threshold) $\gamma$ cascade through the 4586 keV state and the first excited state. The branching ratios used for each of these simulations are given in Table 5.12. Figure 5.43(c) compares the spectra obtained from Geant3 simulations to the observed spectrum of highest energy $\gamma$ per event for yield measurement 10. In this figure, the orange filled histogram are the observed spectra, the blue dashed histogram is the spectrum obtained from the Geant3 simulation using the branching ratios in column 4 of Table 5.12, the red dashed histogram is the spectrum obtained from the Geant3 simulation using the branching ratios in column 5 of Table 5.12, and the green dashed histogram is the spectrum obtained from the Geant3 simulation using the branching
Figure 5.44: Known $\gamma$-decay scheme used in GEANT3 simulations of DRAGON’s BGO array for the $E_x^{\text{lit}} = 9.293$ MeV state.

ratios in column 6 of Table 5.12. The simulation was performed with a spin and parity assignment of $J^\pi = 3^-$, as this is the likely upper limit for the channel spin due to the low penetrability of the $\alpha$ particle at this energy and angular momentum. The fact that we were able to populate this resonance through radiative $\alpha$ capture is a strong indication that this state has a spin and parity of $J^\pi = 3^-$. Combining the BGO detection efficiency with the other relevant coincidence efficiencies for this yield measurement (see Table 5.2), we obtain a total coincidence detection efficiency of $\eta_{\text{coinc}} = 15.3 \pm 2.8\%$. Along with the number of golden recoils and the total
Table 5.12: Branching ratios (in %) for electromagnetic transitions from the $E_x^{\text{fit}} = 9.293$ MeV state used in GEANT3 simulations of DRAGON’s BGO array. Labeling corresponds to the that in Figure 5.44.

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* Not shown in Figure 5.44.

integrated beam on target, we calculate a coincident yield of $Y_{\text{coinc}} = (7.30 \pm 2.05) \times 10^{-13}$. Then, using Equation (5.2) along with the measured stopping power of $\epsilon = (189 \pm 4)$ [eV/(10$^{15}$ $^4$He cm$^{-2}$)] and the de Broglie wavelength of $\lambda^2 = 1.10$ barns (for this energy), we calculate a resonance strength of $\omega_\gamma = 0.026 \pm 0.007$ eV.

5.11.2 Singles Analysis of Yield Measurement 10

Employing the method outlined in section 4.5.2 for identifying recoil events in singles with DRAGON’s heavy ion detectors, we obtain the spectrum in Figure 5.45(a) and the fit shown in Figure 5.45(b). There are a total of $N_{\text{sig}} = 50$ events in the signal region in Figure 5.45(a). As one can see in Figure 5.45(b), there is virtually no overlap between the signal and background regions in the IC spectrum, so integration of the background fit over the signal region resulted in an estimated background of $b = 0$, giving a total of $s = 50$ singles recoil events. Combining this with the singles livetime and IC detection efficiency of $\eta_{\text{IC}}^{\text{AB}} = 79.4 \pm 4.6\%$, we calculate the measured BGO detection efficiency to be $\eta_{\text{BGO}}^{\text{meas}} = 69.5 \pm 15.1\%$. This value agrees with the values of $\eta_{\text{BGO}}^{2085} = 85.1^{+7.5}_{-10.8}\%$ and
Figure 5.45: IC signal spectra for yield measurement 10 obtained by employing the method outlined in section 4.5.2. Panel (a) - 1-D pulse height spectrum of IC anode[1] vs IC anode[0] projected onto the diagonal overlaid with the same spectrum for all events in the golden recoil gate and signal region identification. Panel (b) - Double Gaussian fit to spectrum in panel (a).

\[ \eta_{\text{BGO}}^{2092} = 77.0 \pm 6.1\% \] obtained from the respective GEANT3 simulations of the BGO array for the \( E_r = 2085 \) keV and \( E_r = 2092 \) keV resonances. Further, the measured BGO detection efficiency agrees remarkably well with the total simulated efficiency of \( \eta_{\text{sim}}^{\text{BGO}} = 65.5^{+6.3}_{-8.1}\% \) (which is just taken to be the product of the simulated efficiencies of the individual resonances, assuming detection of \( \gamma \)-rays from the two resonances by the BGO array are independent events).

Using the relevant efficiencies for this yield measurement (see Table 5.2), we calculate a total singles detection efficiency of \( \eta_{\text{sing}} = 17.4 \pm 3.4\% \), giving a singles yield of \( Y_{\text{sing}} = (8.93 \pm 2.18) \times 10^{-13} \). This agrees well with the coincident yield obtained in the previous section. Inserting this into Equation (5.2) along with the measured stopping power \( \epsilon = 189 \pm 4 \text{ eV}/(10^{15} \text{ 4He cm}^3) \) for this bombarding energy and the de Broglie wavelength of the resonance \( (\lambda_r^2 = 1.103 \text{ barns for } E_r = 2089 \text{ keV}) \), we obtain a singles resonance strength of \( \omega_{\gamma_{\text{sing}}} = 0.032 \pm 0.008 \text{ eV} \). This is of course in good agreement with the coincident resonance strength obtained in the previous section.
5.12 Resonance strengths and energies

Resonance strengths were calculated from the experimentally measured yields and resonance energies using the expression for a thick target yield (see section 4.1.1):

\[
\omega \gamma = \frac{2Y}{\lambda^2} \frac{M_t}{M_p + M_t} \epsilon(E)
\]  

(5.2)

where \(\epsilon(E)\) is the (lab frame) stopping power. The parameter values used to calculate the resonance strength for each yield measurement are tabulated in Table 5.13, and Table 5.14 compares the values of the measured resonance strengths and energies to available literature values. Figure 5.46 plots the resonance strengths obtained from S1372 measurements vs energy and compares the S1372 results to the literature values.

Figure 5.47 shows a plot of the resonant reaction rates resulting from S1372 measurements as a function of \(T_9\) along with the NONSMOKER and REACLIB Hauser-Feshbach statistical models. It is clear from Figure 5.47 that the resonance at \(E_{\text{CM}} = 2389\) has the strongest contribution to the total resonant reaction rate over the range of oxygen burning temperatures. Figure 5.48 shows a plot of the total resonant reaction rate and compares the result to the theoretical (NONSMOKER) [32] and the experimental (REACLIB) [35] Hauser-Feshbach statistical models. The REACLIB rate is plotted with factor of 5 and factor of 100 uncertainty bands. It is apparent from Figure 5.48 that the total reaction rate calculated from S1372 measurements along with the canonical values for higher energies agrees well with both the theoretical (NONSMOKER) [32] and the experimental (REACLIB) [35] Hauser-Feshbach statistical models.

5.13 Part 1 Summary

In summary, TRIUMF experiment S1372 measured the resonance strengths and energies of eight resonances in \(^{34}\text{S}(\alpha,\gamma)^{38}\text{Ar}\) in the energy range relevant to hot and explosive astrophysical environments. Of the eight resonances measured, six had not been previously studied via radiative \(\alpha\) capture on \(^{34}\text{S}\). The two previously measured resonances had
significant discrepancies in the literature values for the resonance strength. The S1372 measurements resolved the discrepancy for the $E_{\text{CM}}^{\text{lit}} = 2389$ keV resonance, but the value of the resonance strength of the $E_{\text{CM}}^{\text{lit}} = 2709$ keV resonance remains in doubt. Further study is needed here. The total resonant reaction rate calculated using S1372 data (along with existing literature values) agrees well with the theoretical (NONSMOKER) and experimental (REACLIB) Hauser-Feshbach statistical model calculations. This is an example in which the Hauser-Feshbach statistical model appears to be confirmed by the experimental data.

The discrepancy amongst the measured resonance strengths for the $E_{\text{CM}}^{\text{lit}} = 2709$ keV state was not adequately resolved. However, given that the total resonant reaction rate obtained by combining S172 observations with existing data agrees so well with Hauser-Feshbach statistical models, any future measurements of this resonance would likely be
Table 5.13: Parameters used to calculate $\omega \gamma$ from Equation (5.2).

<table>
<thead>
<tr>
<th>$E_r$ [keV]</th>
<th>$\lambda^2 \text{[b]}$</th>
<th>$Y_{\text{coinc}} \left[ \frac{\text{rxns}}{10^{12} \text{S}^{34}\text{S}^{\text{ion}}} \right]$</th>
<th>$Y_{\text{sing}} \left[ \frac{\text{rxns}}{10^{11} \text{S}^{34}\text{S}^{\text{ion}}} \right]$</th>
<th>$\epsilon \left[ \frac{eV}{10^{15} \text{He cm}^{-2}} \right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2696(8)</td>
<td>0.8548</td>
<td>9.51 ± 2.25</td>
<td>9.80 ± 1.95</td>
<td>186(6)</td>
</tr>
<tr>
<td>2477(10)</td>
<td>0.9301</td>
<td>4.74 ± 1.13</td>
<td>4.67 ± 1.05</td>
<td>192(11)</td>
</tr>
<tr>
<td>2391(7)</td>
<td>0.9637</td>
<td>4.44 ± 1.04</td>
<td>4.38 ± 0.88</td>
<td>197(9)</td>
</tr>
<tr>
<td>2299(8)$^a$</td>
<td>1.002</td>
<td>0.204 ± 0.050</td>
<td>0.234 ± 0.110</td>
<td>191(9)</td>
</tr>
<tr>
<td>2300(8)$^b$</td>
<td>1.002</td>
<td>0.212 ± 0.065</td>
<td>0.230 ± 0.055</td>
<td>191(9)</td>
</tr>
<tr>
<td>2249(10)</td>
<td>1.025</td>
<td>0.141 ± 0.041</td>
<td>0.161 ± 0.056</td>
<td>188(10)</td>
</tr>
<tr>
<td>2218(8)</td>
<td>1.039</td>
<td>0.268 ± 0.071</td>
<td>0.247 ± 0.066</td>
<td>190(5)</td>
</tr>
<tr>
<td>2164(8)</td>
<td>1.065</td>
<td>0.099 ± 0.025</td>
<td>0.107 ± 0.025</td>
<td>182(4)</td>
</tr>
<tr>
<td>2089(9)</td>
<td>1.103</td>
<td>0.073 ± 0.019</td>
<td>0.089 ± 0.022</td>
<td>189(4)</td>
</tr>
</tbody>
</table>

$^a$ separator was tuned to the 8+ recoil charge state
$^b$ separator was tuned to the 7+ recoil charge state

less valuable than the pursuit of lower energy resonances. The possibility for further study of radiative $\alpha$ capture on $^{34}$S at DRAGON exists, as there are several states in $^{38}$Ar with excitation energies that correspond to CM energies within the range $[0.3 E_0, E_0]$ for both oxygen burning and explosive oxygen burning that have not been measured via $^{34}$S ($\alpha, \gamma$)$^{38}$Ar. Of particular interest is a $J^\pi = (1^-)$ state at $E_x = 8.233$ MeV (corresponding to a CM energy of $E_{CM} = 1025$ keV. This corresponds to a lab energy of $E_b = 286$ keV/u, which is well within the energy capabilities of TRIUMF’s ISAC-I accelerators.
Table 5.14: Measured resonance strengths and energies from analysis of S1372 data.

<table>
<thead>
<tr>
<th>(E_{\text{CM}}^{(\text{lit})}) [keV]</th>
<th>(E_{\text{CM}}^{(\text{meas})}) [keV]</th>
<th>(\omega \gamma_{\text{coinc}}) [eV]</th>
<th>(\omega \gamma_{\text{sing}}) [eV]</th>
<th>(\omega \gamma) [eV] [22]</th>
<th>(\omega \gamma) [eV] [21]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2709(^a)</td>
<td>2696(8)(^b)</td>
<td>4.37(1.05)(^c)</td>
<td>4.51(91)(^c)</td>
<td>2.62(56)(^d)</td>
<td>4.5(1.35)(^d)</td>
</tr>
<tr>
<td>2481</td>
<td>2477(10)</td>
<td>2.07(52)</td>
<td>2.03(47)</td>
<td>1.3(3)</td>
<td>1.5(5)</td>
</tr>
<tr>
<td>2389</td>
<td>2391(7)</td>
<td>1.92(46)</td>
<td>1.89(39)</td>
<td>1.71(34)</td>
<td>4.0(1.2)</td>
</tr>
<tr>
<td>2327</td>
<td>2299(8)(^e)</td>
<td>0.082(24)</td>
<td>0.094(44)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2327</td>
<td>2300(8)(^f)</td>
<td>0.085(26)</td>
<td>0.093(22)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2252</td>
<td>2249(10)</td>
<td>0.054(16)</td>
<td>0.062(22)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2229</td>
<td>2218(8)</td>
<td>0.103(29)</td>
<td>0.095(25)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2166</td>
<td>2164(8)</td>
<td>0.035(10)</td>
<td>0.039(9)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2085/2092</td>
<td>2089(9)(^g)</td>
<td>0.026(7)(^h)</td>
<td>0.032(8)(^h)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^a\) See section 5.2.1 for discussion of the literature value of \(E_x\) for this state.
\(^b\) Calculated from \(\gamma_0\) Z-distribution weighted mean of 2706 keV and 2686 keV resonances.
\(^c\) Combined \(\omega \gamma\) of 2706 keV and 2686 keV resonances.
\(^d\) See Table 5.6 for proper comparison of S1372 resonance strength to literature values.
\(^e\) Separator was tuned to the 8+ recoil charge state.
\(^f\) Separator was tuned to the 7+ recoil charge state.
\(^g\) Calculated from \(\gamma_0\) Z-distribution weighted mean of 2092 keV and 2085 keV resonances
\(^h\) Combined \(\omega \gamma\) of 2092 keV and 2085 keV resonances.
Figure 5.47: Resonant reaction rates calculated from S1372 results and literature values compared to the NONSMOKER and REACLIB Hauser-Feshbach statistical models. Vertical lines denote typical temperature ranges for oxygen burning (1.7-2.5 GK) and explosive oxygen burning (3-4 GK).
Figure 5.48: Total resonant reaction rate calculated from S1372 results and literature values compared to the NONSMOKER and REACLIB Hauser-Feshbach statistical models. The REACLIB rate is plotted with uncertainty bands of ±5×rate and ±10²×rate. Vertical lines denote typical temperature ranges for oxygen burning (1.7-2.5 GK) and explosive oxygen burning (3-4 GK).
PART II

IMPROVEMENTS TO DRAGON: SCATTERING OF NUCLEI IN INVERSE KINEMATICS AT DRAGON (SONIK DRAGON)
Figure 6.1: SolidWorks® 3-D model of the SONIK scattering chamber

In order to exploit TRIUMF’s radioactive ion beam capabilities for elastic scattering
experiments, a new type of scattering chamber was designed to fit in place of DRAGON’s existing gas target and utilize the existing DRAGON infrastructure in order to make high precision measurements charged particle elastic scattering cross sections in inverse kinematics. The Scattering Of Nuclei in Inverse Kinematics (SONIK) chamber was designed and fabricated at Colorado School of Mines and commissioned at TRIUMF in July of 2015. In this chapter, we will discuss the motivation for designing SONIK, details of SONIK’s design and the commissioning experiment.

6.1 SONIK Motivation

The $^7$Be($p,\gamma$)$^8$B reaction is of interest to solar neutrino physics in relation to the solar neutrino problem. Although the solar neutrino problem has been largely explained by neutrino oscillations, the focus has shifted to constraining neutrino mixing parameters. Recent measurements [77–80] of $^7$Be($p,\gamma$)$^8$B have increased the uncertainty of the extrapolation of the $S_{17}(0)$ astrophysical S-factor to solar energies, making more stringent constraints on the $^7$Be($p,\gamma$)$^8$B cross section desirable.

Solar neutrinos are primarily produced in reactions in the p-p chain. Figure 6.2 illustrates the branching ratios of the various reactions of the p-p chain. The vast majority of the neutrino flux is from $p + p \rightarrow ^2\text{H} + e^+ + \nu_e$. However, these neutrinos are low in energy ($E_{\text{Max}} = 0.425$ MeV - see Figure 6.3), which makes them difficult to detect. Neutrinos produced by the proton-electron-proton (pep) reaction, as well as those produced by $^7$Be electron capture are also low energy ($\sim 1$MeV). Therefore, the solar neutrino signal of the $^8\text{B} \rightarrow ^7\text{Be} + e^+ + \nu_e$ ($\beta$-decay of $^8\text{B}$ is relatively easy to detect. The neutrino flux from $^8\text{B}$ decay is proportional to the solar abundance of $^8\text{B}$, which is dependent on the $^7$Be($p,\gamma$)$^8$B reaction cross section - or astrophysical S-factor $S_{17}(0)$. One method for constraining the $^7$Be($p,\gamma$)$^8$B cross section (S-factor $S_{17}(0)$) is via high-precision measurements of the elastic scattering cross section of $^7$Be($p,p$)$^7$Be. We turn to this topic in the next section.
**Figure 6.2:** Branching ratios of the p-p chain [81].

### 6.1.1 Relationship between $\sigma_{el}$ and $S_{17}(0)$

Consider a nucleus with an excited resonant state at energy $E_x$ that can be populated through radiative capture $X(a,\gamma)Y$ (such as the 0.7695 MeV state in $^8\text{B}$ that can be populated through resonant proton capture on $^7\text{Be}$ illustrated in Figure 6.4). If the total width $\Gamma$ of this resonance is sufficiently large, the resonance will have a low energy tail through which the $X + a$ reaction may still proceed, albeit at a much lower rate than the on-resonance reaction. The reaction cross section at low energies for such a resonance can be estimated via the astrophysical $S$-factor by combining Equation (2.40) and Equation (2.48)

$$S(E) = \frac{\lambda^2}{4\pi} \frac{\Gamma_{a,\ell}(E)\Gamma_\gamma}{(E_r - E)^2 + \Gamma^2/4} E e^{2\pi\eta}$$

(6.1)

where $\Gamma_{a,\ell}(E)$ is the single-particle partial width given by [84]
\[ \Gamma_{a,\ell}(E, R_n) = \frac{2\hbar}{R_n} \left( \frac{2E}{\mu} \right)^{1/2} P_{\ell}(E, R_n) \theta_{\ell}^2 \] (6.2)

where \( P_{\ell}(E, R_n) \) is the Coulomb barrier penetration factor (see section 2.4) and \( \theta_{\ell}^2 \) is the single-particle reduced width. If the astrophysical \( S \)-factor (cross section) cannot be measured directly, extrapolation of the \( S \)-factor to low energies requires knowledge of the single particle partial width. On the other hand, the elastic scattering cross section for resonance scattering (i.e. - a reaction \( a(X,X)a \)) is described by the one level Breit-Wigner formula

\[ \sigma(E) = \frac{\lambda^2}{4\pi} \omega \frac{\Gamma_{a}^2(E)}{(E_r - E)^2 + \Gamma^2 / 4} \] (6.3)

Because both the \( X(a,\gamma)Y \) and the \( X(a,a)X \) reactions have the same entrance channel, the particle partial widths for these reactions are equivalent. Therefore a measurement of the

Figure 6.3: Standard Solar Model BS05(OP) [82] neutrino energy spectra [83].
elastic scattering cross section at a given energy (which can be used to extract the particle partial width via an \( R \)-matrix fit) enables calculation of the astrophysical \( S \)-factor at that energy, which can be used to constrain its extrapolation to low energies. The extraction of the particle partial width from elastic scattering measurements is achieved through fitting elastic scattering data with the phenomenological \( R \)-matrix, to which we turn our attention next.

### 6.1.2 \( R \)-Matrix

The \( R \)-matrix was developed in order to solve coupled channel Schödinger equations for systems of particles [85]. The \( R \)-matrix is the inverse of the logarithmic derivative of the wave function at the nuclear boundary. The multichannel \( R \)-matrix [85] is given by

\[
R_{cc'}(E) = \sum_n \frac{\gamma_{nc} \gamma_{nc'}}{E_n - E}
\]  

(6.4)

For a single reaction channel, the \( R \)-matrix for the \( \ell \)th partial wave and total angular momentum \( J \) is given by

\[
R_{\ell J}(E) = \sum_{n=1}^{N} \frac{\gamma_{n \ell J}^2}{E_{n \ell J} - E}
\]

(6.5)

The positive(negative) poles of Equation (6.5) describe scattering(bound) states, and the (real) parameters \( \gamma_{n \ell J} \) are the reduced width amplitudes [85], given by

\[
\Gamma_c = 2P_{c,\ell}(E, R_n)\gamma_{nc}^2
\]

(6.6)

Extraction of \( R \)-matrix parameters is simplified further in the case of isolated narrow resonances. However, \(^7\text{Be}\) has a spin and parity of \( J^\pi = 3/2^- \), so radiative proton capture can have channel spins of \( s = \{1, 2\} \). Thus mixing of partial waves can contribute significantly to the amplitudes \( \gamma_{n \ell J} \) of Equation (6.6), so Equation (6.5) is the appropriate tool. Additionally, there have been some observations [77–80] which suggest significant contributions from other reaction channels, so an accurate fit may require the use of Equation (6.4).

A full exposition of the phenomenological \( R \)-matrix formalism (including the function
Figure 6.4: Level diagram of $^8\text{B}$.

of the collision matrix) is beyond the scope of this document. For more information, the reader is referred to [85–88] and the references therein. Additionally, Barker et. al. [89–91] have developed an application of the phenomenological $\mathcal{R}$-matrix formalism for the express
purpose of extrapolating the $S$-factors of radiative capture reactions to low energies that may be of interest to the reader.

6.2 SONIK Design/Construction Overview

In order to exploit TRIUMF’s radioactive ion beam capabilities for elastic scattering experiments, the SONIK scattering chamber was designed to fit in place of DRAGON’s existing gas target and utilize the existing pumping and gas recirculation system (see Figure 3.6), with just one additional roots blower (see Figure 6.10). It consists of 30 ion implanted silicon (IIS) charged particle detectors mounted on collimated telescopes (see Figure 6.7) a distance of 17 cm from the center of the beamline at observation angles in the range $\theta_{\text{lab}} = \{22.5^\circ, 135^\circ\}$. Nine of the angles have a three-fold redundancy and there are four unique angles (see Table 6.1). Each set of nine redundant telescopes observes a different interaction region within the gas target volume (see Figure 6.5 and Figure 6.6). Because each observation point has a different $z$-coordinate, the bombarding energy and therefore the scattering energy varies slightly by observation point. The IIS detectors detect elastically scattered H$_2$ or He nuclei depending on the measurement of interest. The telescopes are collimated by a 2.0 mm wide slit aperture at the telescopes’ interface with the gas volume and a 1 mm diameter circular aperture immediately in front of the IIS detector (see Figure 6.7). Additionally, the chamber’s beamline collimators were designed as a relatively simple assembly that can be removed. This is useful for working with radioactive beam species, when unreacted beam can be inadvertently deposited on the collimators. In the event that the beamline collimators become contaminated, they may be easily disposed of. In order to build the chamber in a cost-effective manner, the chamber and all of its components are constructed of 6061-T6 aluminum alloy.

Unfortunately, due to a design flaw that was overlooked, the chamber was perforated

\*Like many SONIK’s components, the circular apertures were designed to be manufactured inexpensively, allowing for the flexibility to easily alter the diameter or the geometry of the aperture to suit varying experimental needs.
Figure 6.5: SONIK design details; all dimensions are given in mm.
Figure 6.6: SONIK outer pumping details.
Figure 6.7: SONIK detector telescope assembly details; dimensions are in mm.
Table 6.1: Angles subtended by detectors according to interaction point. See text for explanation of strikethroughs.

<table>
<thead>
<tr>
<th>Observation points</th>
<th>Angles Subtended</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20°</td>
</tr>
<tr>
<td></td>
<td>22.5°</td>
</tr>
<tr>
<td></td>
<td>25°</td>
</tr>
<tr>
<td></td>
<td>30°</td>
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<tr>
<td></td>
<td>120°</td>
</tr>
<tr>
<td></td>
<td>135°</td>
</tr>
</tbody>
</table>

during fabrication while milling the port for the 22.5° telescope for observation point 3. The location of the perforation necessitated filling and welding the 22.5° telescope for observation point 3 as well as welding an aluminum block over the corresponding 30° telescope port. Further, since the 30° ports for observation points 1 and 2 had already been machined, it was realized that machining the corresponding 20° ports called for in the design would similarly perforate the chamber. Thus one 30° port as well as all three of the planned 20° ports were lost.

6.3 SONIK Timeline

The design work and fabrication of the SONIK scattering chamber were completed during the summer of 2014 at Colorado School of Mines. The chamber was vacuum tested at CSM during this time and was subsequently vacuum tested at TRIUMF in August 2014 (see Figure 6.8.) At that time, an attempt was made to fit the chamber into the existing beamline in the location of DRAGON’s gas target. However, it was discovered that the pumping boxes immediately upstream of the target area were slightly misaligned, causing the downstream end of the chamber to be deflected off-axis by approximately 2 mm. The original design of the fittings to the existing DRAGON target beamline area were rigid, and relied on the expectation that the pumping boxes on either side of the target were fairly precisely aligned. This not being the case, the beam time had to be postponed until a flexible bellows fitting between the chamber and beam line could be manufactured. The chamber was vacuum tested using half (the downstream half) of DRAGON’s pumping/gas recirculation
system. This was achieved by attaching the chamber to the downstream pumping boxes and blanking off the upstream end of the chamber. Initially, using only the existing pumps available, the chamber was able to maintain a gas pressure of approximately 6 Torr before the maximum load of 400 W on TP3 was reached. When a 6th roots blower was attached in parallel (with the existing roots blowers) to the inner pumping ports (see Figure 6.5 and Figure 6.6) of the scattering chamber, a central cell pressure of 9.7 Torr was achieved before the max load was reached on TP3. These tests gave reasonable confidence that the SONIK scattering chamber would be ready for commissioning during the subsequent TRIUMF beam schedule.
6.4 SONIK Commissioning

The SONIK scattering chamber was commissioned between July 23rd - August 4th, 2015 in conjunction with TRIUMF experiment S1025 measurements. The chamber was commissioned with two separate measurements: elastic $^3\text{He} - \alpha$ scattering ($^3\text{He}(\alpha,\alpha)^3\text{He}$) at bombarding energies of $E_b = \{4.45, 3.58, 2.70, 1.80, 0.90\}$ MeV and elastic proton scattering off $^7\text{Li} (^7\text{Li}(p,p)^7\text{Li})$ at bombarding energies of $E_b = \{2.56, 2.18, 1.80, 1.85, 1.49\}$ MeV Both measurements were made in inverse kinematics. All scattering measurements were collected at an average target pressure of $P_{\text{He}} = 5.10 \pm 0.04$ [Torr] He (for the $^3\text{He}(\alpha,\alpha)^3\text{He}$ measurement) and $P_{\text{H}_2} = 5.22 \pm 0.09$ [Torr] H$_2$ (for the $^7\text{Li}(p,p)^7\text{Li}$ measurement). Because the volume of the gas cell of the SONIK scattering chamber is considerably larger than that of the DRAGON gas target, an additional roots blower and a modified pumping scheme (see Figure 6.10) were necessary to achieve differential pumping comparable to DRAGON’s typical operation. In order to ascertain the number of incident beam particles, the beam was dumped on FC1 (immediately downstream of the target) and the current on FC1
was continuously monitored. The FC1 current could then be normalized to regular (hourly) beam current readings on FC4.

The SONIK commissioning experiment can overall be considered a success, as it provided data suitable for characterizing the apparatus. However, there were some difficulties encountered during the experiment and there several aspects were identified in which improvements can be made. Owing to the limited availability of electronics and detectors (mainly due to budgetary constraints), the $^3\text{He}(\alpha,\alpha)^3\text{He}$ was made with only 19 detectors and the $^7\text{Li}(p,p)^7\text{Li}$ was made with only 8 detectors. Further, the suite of detectors that were used in the commissioning experiment were a mixed bag. Ten of the detectors were brand new Canberra Passivated Implanted Planar Silicon (PIPS®) detectors purchased by Colorado School of Mines. The PIPS® detectors were all model PD 150-14-100 AM partially depleted Si charged particle detectors, which have a 150 mm$^2$ active area and a 100 mm active thickness. The other nine detectors were a mix of Si charged particle detectors,
Figure 6.11: SONIK IIS calibrated energy spectrum of 45° detector observing interaction regions 2 (orange fill) and 3 (blue) for all runs at bombarding energy of $E_b = 2696$ keV.

of known and unknown make, model, active area and active thicknesses, many of which had unknown* origins. Additionally, the electronics (preamplifiers and shaping amplifiers) used during SONIK’s commissioning consisted of a borrowed elements; a mesytec MSI-8 8 channel preamplifier, shaper and timing filter amplifier, a mesytec MSCF-16 16 channel shaper and timing filter amplifier with constant fraction discriminators and an RAL preamplifier. The shaped signals from the IIS detectors were used to create (individual) logic pulses (which were sent into a logic fan-out which was subsequently used to create a system trigger) and the shaped signals were sent to the tail CAEN V785 ADC inputs normally used for the DSSSD. An extension to the DRAGON analyzer package was written in order to unpack MIDAS event banks from these MIDAS files into a SONIK class ROOT tree [52, 53, 92].

*After data collection began, it became apparent that one of the detectors (that was initially thought to be functional) showed signs that it had clearly been damaged, and another (also thought to be functional) showed signs that it had possibly been contaminated.
The Si detectors were energy calibrated in a separate chamber with a \(^{239}\text{Pu} - ^{241}\text{Am} - ^{244}\text{Cm}\) triple - \(\alpha\) source. The electronics lines used for the calibration and their respective settings were preserved when the detectors were mounted on the chamber for data collection during the \(^3\text{He}(\alpha, \alpha)^3\text{He}\) measurement. This procedure, although serviceable, proved to be cumbersome and led to problems during the \(^7\text{Li}(p, p)^7\text{Li}\) measurement, at which time the mesytec MSCF-16 unit became quite noisy, severely reducing the livetime of the DAQ and requiring its removal (along with the 8 detectors whose signals it was shaping) from the trigger. Over the course of the \(^7\text{Li}(p, p)^7\text{Li}\) measurement, several different configurations of 8 detectors were used with the mesytec MSI-8 to collect data. Because the electronics setup for nearly all of the detectors changed during this measurement, their respective energy calibrations were effectively destroyed. The potentiality of similar situations arising in the future make devising a method for insertion or installation of a calibration source desirable.
Figure 6.13: SONIK IIS calibrated energy spectrum of 75° detector (observing interaction region 3). Orange fill - all runs at bombarding energy of $E_b = 3576$ keV. Blue fill - all runs at bombarding energy of $E_b = 2696$ keV.

Data from the SONIK commissioning experiment are still being analyzed at TRIUMF and Ohio University, but preliminary results suggest that the chamber performs as expected. Figure 6.11 to Figure 6.14 show various spectra for $^3\text{He}(\alpha, \alpha)^3\text{He}$ data collected during SONIK’s commissioning. Figure 6.15 displays the spectrum of the 30° detector* observing interaction region 2 for all runs at a bombarding energy of $E_b = 2561$ keV for the $^7\text{Li}(p, p)^7\text{Li}$ measurement.

Although the data collected will be sufficient to adequately characterize the apparatus, there exists the potential for significant improvements in resolution and signal to noise ratio with the procurement of state of the art equipment. Thus it is of utmost importance

*This was the only detector for which the energy calibration was preserved when the switch was made to the $^7\text{Li}(p, p)^7\text{Li}$ measurement.
Figure 6.14: SONIK IIS calibrated energy spectrum of the 22.5° detector (observing interaction region 3). Orange fill - all runs at bombarding energy of $E_b = 1802$ MeV. Blue fill - all runs at bombarding energy of $E_b = 901$ keV.

for the future success of SONIK to acquire a full suite of new detectors (ideally matching Canberra PIPS® PD 150-14-100 AM detectors) and complementary electronics (two 16 channel shaping amplifier, preamplifier and discriminator units would be ideal).

It should be noted that the ISAC-I accelerators are capable of delivering beams of up to 1.9 MeV/u. At this energy, protons scattering from $^7$Be at a lab angle of $\theta_{\text{lab}} = 22.5^\circ$ will have an energy of 4.95 MeV. According to SRIM [61], 5 MeV protons have a range of $\sim 215 \mu$m in elemental Si. Thus it will be necessary to obtain detectors with an active thickness of 300 $\mu$m (ideally the Canberra PIPS® PD 150-14-300 AM). There are future plans to measure $^7$Be(p,p)$^7$Be in the ISAC-II facility at bombarding energies at up to 5 MeV/u. If SONIK is to be used for these measurements, it may be necessary to acquire detectors with up to 1000 $\mu$m active thickness for the shallow angles. Otherwise detectors of 300 $\mu$m active
Figure 6.15: SONIK IIS calibrated energy spectrum of the $30^\circ$ detector (observing interaction region 3) during the $^7$Li(p,p)$^7$Li measurement for all runs at a bombarding energy of $E_b = 2561$ keV.

thickness should suffice for most conceivable measurements at the DRAGON gas target location as the range of protons in Si does not exceed 300 $\mu$m unless the proton energy exceeds 6 MeV [61].

Like DRAGON’s high density, windowless gas target, SONIK will be able to make direct measurements of the stopping power. Measurement of the stopping power with DRAGON’s gas target or SONIK is achieved varying the pressure in the target and then measuring the beam energy. The stopping power can then be calculated as in section 4.3 (by linear fitting of beam energy vs target thickness data). This method of measuring the stopping power requires precise knowledge of the effective target length, which (along with the target pressure and temperature) is used to calculate the target thickness via Equation (4.31). The effective length of the target is a difficult quantity to measure. However, if we know the
stopping power of a given ion in a given medium precisely, we can use Equation (4.8) to ascertain the effective target length $\Delta x$:

$$\Delta x = \frac{\Delta E}{N \epsilon} \tag{6.7}$$

where $N$ is the number density of target nuclei (which can be calculated for a given target pressure and temperature using Equation (4.31)). Note that Equation (6.7) is only valid if the target pressure profile is (approximately) constant and the stopping power is (approximately) constant over the energy thickness of the target.

Table 6.2 and Table 6.3 display the effective target lengths obtained from measurements taken during SONIK commissioning experiment, along with the parameters needed to calculate them. The stopping power of He ions in He gas has been measured many times and is well described by the SRIM model (to within a 4.4% mean error - see Figure 6.17), so the measurements of SONIK’s effective length with the $^3$He beam should be reliable.

The SRIM model has a mean error of 13.2% with the stopping power data for Li ions in H$_2$, so the measurements of SONIK’s effective length with the $^7$Li beam are decidedly less reliable. Further, there is marked disagreement in the Li data in the range of the stopping power maximum ($\sim 100 - 400$ keV/u), which is precisely the energy range for which the $^7$Li(p,p)$^7$Li measurements taken. On the other hand, the fact that the effective length measurements from the $^7$Li(p,p)$^7$Li data agree with the measurements from the $^3$He(\(\alpha\),\(\alpha\))$^3$He data to within $\sim 3.3\%$ is encouraging. The anomalous value of $l_{\text{eff}} = 36.64$ cm calculated for the bombarding energy of $E_b = 4.45$ MeV was likely due to an erroneous energy measurement (either $E_b$ or $E_{\text{out}}$) due to some steering of the beam by DRAGON’s first quadrupole magnet. Because these effective lengths from the SONIK commissioning experiment were calculated from stopping power values obtained from the SRIM model [61], they should be treated with care until another measurement can verify them.

Such an independent measurement is planned for a future test beam time for the SONIK chamber. This independent measurement will utilize the method employed by Engel in
Figure 6.16: Existing Data for stopping power of Li ions in H\(_2\) gas and comparison to SRIM model [61].

[63, 93]. Stopping power measurements at a given bombarding energy will be made with the chamber’s standard entrance and exit apertures (\(\varnothing_{\text{std}} = 6 \text{ mm}\)) and then repeat the stopping power measurements (at the same bombarding energy) using entrance and exit apertures with diameters of \(\varnothing_{\text{lim}} = 1 - 1.5 \text{ mm}\). The entrance and exit apertures with the
Figure 6.17: Existing Data for stopping power of He ions in He gas and comparison to SRIM model [61].

Reduced diameter will limit the gas flow out of the central target volume, thereby decreasing the effective length of the target so that it is nearly the geometric length of the target. Taking stopping power measurements in this manner allows one to estimate the energy
Table 6.2: Parameters used for calculating SONIK’s effective length during the $^3$He - α scattering measurement (i.e. - with He gas). All energies are in units of MeV.

<table>
<thead>
<tr>
<th>$E_{CM}$</th>
<th>$E_b$</th>
<th>$E_{out}$</th>
<th>$\frac{dE}{dx}$ [61] [eV/(10$^{15}$ at/cm$^2$)]</th>
<th>$T$ [°C]</th>
<th>$\Delta E$</th>
<th>$l_{eff}$ [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.537</td>
<td>4.449</td>
<td>4.415</td>
<td>5.74</td>
<td>29.0</td>
<td>0.0345</td>
<td>36.64*</td>
</tr>
<tr>
<td>2.039</td>
<td>3.576</td>
<td>3.552</td>
<td>6.78</td>
<td>29.0</td>
<td>0.0241</td>
<td>21.53</td>
</tr>
<tr>
<td>1.537</td>
<td>2.696</td>
<td>2.665</td>
<td>8.34</td>
<td>29.0</td>
<td>0.0315</td>
<td>22.84</td>
</tr>
<tr>
<td>1.028</td>
<td>1.802</td>
<td>1.762</td>
<td>10.94</td>
<td>29.0</td>
<td>0.0397</td>
<td>21.93</td>
</tr>
<tr>
<td>0.515</td>
<td>0.904</td>
<td>0.849</td>
<td>15.76</td>
<td>29.0</td>
<td>0.0544</td>
<td>20.88</td>
</tr>
<tr>
<td>0.514</td>
<td>0.901</td>
<td>0.844</td>
<td>15.76</td>
<td>29.0</td>
<td>0.0571</td>
<td>21.91</td>
</tr>
<tr>
<td>0.514</td>
<td>0.901</td>
<td>0.841</td>
<td>15.76</td>
<td>27.1</td>
<td>0.0597</td>
<td>22.88</td>
</tr>
<tr>
<td>0.514</td>
<td>0.901</td>
<td>0.842</td>
<td>15.76</td>
<td>29.0</td>
<td>0.0587</td>
<td>22.08</td>
</tr>
</tbody>
</table>

$l_{eff}^{(avg)} \pm \sigma_l = 22.00 \pm 0.70$

* See text concerning the anomalousness of this value of $l_{eff}$.

lost in the central volume of the gas cell ($\Delta E_c$) as well as the energy loss in the gas leaking out of the entrance and exit collimators ($\Delta E_l$). As Engel explains [63],

Though it may not be assumed that $\Delta E_c$ is independent of the aperture, it can be argued that the effective width of the energy loss in the inner target cell is. This means that the steepness of the edges in...[the stopping power curves] changes with the size of the confining collimators; however, the effective length does not. Thus, the total energy loss remains the sum of both, the energy loss within the central target material plus contributions from the gas leaking further into the differential pumping system. The first, $\Delta E_c$, is the same in either set-up, while the latter, $\Delta E_l$, is proportional to the open area of the aperture, in first order approximation.

Then the effective length can be calculated by scaling the physical target length by the ratio of the stopping powers obtained in the previous two measurements:

$$l_{eff} = \frac{\epsilon_{std}}{\epsilon_{lim}} l_0$$  \hspace{1cm} (6.8)
Table 6.3: Parameters used for calculating SONIK’s effective length during the $^7\text{Li}$ - proton scattering measurement (i.e. - with H$_2$ gas). All energies are in units of MeV.

<table>
<thead>
<tr>
<th>$E_{\text{CM}}$</th>
<th>$E_b$</th>
<th>$E_{\text{out}}$</th>
<th>$\frac{dE}{dx}$ [61] $[\text{eV}/(10^{15} \text{at/cm}^2)]$</th>
<th>$T$ [C]</th>
<th>$\Delta E$</th>
<th>$l_{\text{eff}}$ [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.322</td>
<td>2.56</td>
<td>2.44</td>
<td>17.78</td>
<td>30.0</td>
<td>0.1192</td>
<td>19.95</td>
</tr>
<tr>
<td>0.322</td>
<td>2.56</td>
<td>2.43</td>
<td>17.78</td>
<td>30.0</td>
<td>0.1274</td>
<td>22.04</td>
</tr>
<tr>
<td>0.322</td>
<td>2.56</td>
<td>2.43</td>
<td>17.78</td>
<td>30.0</td>
<td>0.1317</td>
<td>22.02</td>
</tr>
<tr>
<td>0.274</td>
<td>2.18</td>
<td>2.04</td>
<td>19.20</td>
<td>30.0</td>
<td>0.1422</td>
<td>21.95</td>
</tr>
<tr>
<td>0.226</td>
<td>1.80</td>
<td>1.68</td>
<td>19.69</td>
<td>30.0</td>
<td>0.1220</td>
<td>17.85</td>
</tr>
<tr>
<td>0.232</td>
<td>1.85</td>
<td>1.70</td>
<td>19.69</td>
<td>30.0</td>
<td>0.1496</td>
<td>23.05</td>
</tr>
<tr>
<td>0.232</td>
<td>1.85</td>
<td>1.71</td>
<td>19.69</td>
<td>30.0</td>
<td>0.1419</td>
<td>21.43</td>
</tr>
<tr>
<td>0.232</td>
<td>1.85</td>
<td>1.74</td>
<td>19.69</td>
<td>30.0</td>
<td>0.1076</td>
<td>21.41</td>
</tr>
<tr>
<td>0.232</td>
<td>1.85</td>
<td>1.74</td>
<td>19.69</td>
<td>30.0</td>
<td>0.1068</td>
<td>21.24</td>
</tr>
<tr>
<td>0.187</td>
<td>1.49</td>
<td>1.34</td>
<td>20.36</td>
<td>31.5</td>
<td>0.1500</td>
<td>21.73</td>
</tr>
<tr>
<td>0.187</td>
<td>1.49</td>
<td>1.35</td>
<td>20.36</td>
<td>32.0</td>
<td>0.1461</td>
<td>21.20</td>
</tr>
<tr>
<td>0.187</td>
<td>1.49</td>
<td>1.34</td>
<td>20.36</td>
<td>32.5</td>
<td>0.1483</td>
<td>21.75</td>
</tr>
<tr>
<td>0.142</td>
<td>1.13</td>
<td>0.98</td>
<td>20.67</td>
<td>32.5</td>
<td>0.1476</td>
<td>21.04</td>
</tr>
</tbody>
</table>

$l_{\text{eff}}^{(\text{avg})} \pm \sigma_l = 21.28 \pm 1.25$

where $\epsilon_{\text{std}}$ is the stopping power measured using the standard entrance and exit apertures, $\epsilon_{\text{lim}}$ is the stopping power measured using the small diameter entrance and exit apertures and $l_0$ is the physical distance between the centers of the apertures. From the machining tolerances the physical target length is $l_0 = 16.82 \text{ cm}$ (see Figure 6.5 and Figure 6.6). Small diameter entrance and exit apertures will need to be machined before these measurements can be made.

However, because the SONIK chamber has two pumping stages on either side of the central gas target volume - separated by two (each) gas flow limiting entrance and exit apertures (see Figure 6.5 and Figure 6.6) - this method will only give a first order approximation to the effective length of SONIK’s central gas volume. In order to ascertain an accurate measurement of the effective length of SONIK’s central gas cell, it will be necessary to measure the contribution to the beam energy loss from the gas leaking into the outer pumping stages. Therefore, it will be necessary to machine small diameter gas flow limiting beamline apertures for the outer pumping stages as well.
6.5 SONIK Outlook

The SONIK scattering chamber has already generated some interest in the nuclear astrophysics community. The possibility of making high precision elastic scattering cross section measurements in inverse kinematics with a gaseous target in order to constrain reaction cross sections (astrophysical S-factors) is compelling. Several experiments have already been approved by the TRIUMF Subatomic Physics Experiments Evaluation Committee that will utilize the SONIK scattering chamber (see Table 6.4).

Table 6.4: Experiments approved by the TRIUMF SAP-EEC that will utilize the SONIK scattering chamber. Note that $^3\text{He}(\alpha,\alpha)^3\text{He}$ and the $^7\text{Li}(p,p)^7\text{Li}$ were measured during SONIK’s commissioning experiment in July 2015. $^7\text{Be}(p,p)^7\text{Be}$ will be measured at a later date.

<table>
<thead>
<tr>
<th>TRIUMF Experiment Number</th>
<th>Reaction(s) Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1025</td>
<td>$^3\text{He}(\alpha,\alpha)^3\text{He}$</td>
</tr>
<tr>
<td></td>
<td>$^7\text{Li}(p,p)^7\text{Li}$</td>
</tr>
<tr>
<td></td>
<td>$^7\text{Be}(p,p)^7\text{Be}$</td>
</tr>
<tr>
<td>S1425</td>
<td>$^{18}\text{F}(p,\gamma)^{19}\text{Ne}$ via $^{15}\text{O}(\alpha,\gamma)^{19}\text{F}$ and $^{15}\text{O}(\alpha,\gamma)^{15}\text{O}$</td>
</tr>
<tr>
<td>S1452*</td>
<td>$^7\text{Be}(\alpha,\alpha)^7\text{Be}$</td>
</tr>
</tbody>
</table>

* Letter of Intent

Although there were several difficulties encountered during the SONIK commissioning experiment, the data collected will enable adequate characterization of the apparatus and therefore should overall be considered a success. However, there are several issues that need to be addressed and improvements that need to be made before the next measurement with SONIK is performed. These are itemized below in order of priority:

1. As noted previously, SONIK was only partially instrumented during its commissioning.

Full instrumentation with state of the art detectors and electronics will likely dramati-
cally increase SONIK’s performance. It is of utmost importance to acquire a full suite of new detectors and complementary electronics.

2. The 30° and 22.5° slit apertures collide when they are fully inserted into their respective ports. These need to machined so that they do not collide. This should be relatively simple and can be achieved without altering the relevant geometry of the apparatus.

3. Small diameter (ø = 1.5 mm) beamline apertures need to be fabricated in order to measure the effective target length.

4. During the commissioning experiment, pressure was read from a manometer attached to one of the telescope ports. This setup is not ideal, as it removes a detector from the apparatus and, in the case of the commissioning experiment, the port chosen was directly adjacent to and pointed towards the gas inlet port. Thus the pressure readings during SONIK’s commissioning experiment carry an additional, unknown uncertainty. A port normal to the beamline with a standard connection (KF-16) needs to be machined on the central portion of the chamber for a proper pressure reading.

5. As mentioned previously, calibration of the IIS detectors was cumbersome, time consuming, and error prone. It will be necessary to devise a method for easy insertion of a calibration source.

6. The integrity of SONIK’s observations is critically dependent on the trajectory of beam ions through the chamber (i.e. - it is critical that the beam be well aligned as it passes through the chamber). The misalignment of the upstream and downstream pumping boxes complicates the beam alignment. It has been suggested that irises be installed in the upstream and downstream beam boxes to more reliably ensure beam alignment.

7. Installation of the chamber in the beamline is time consuming and cumbersome as it requires removal of the turbo pumps that are attached to the beam boxes adjacent
to DRAGON’s gas target. Simplifying the beamline installation of SONIK would drastically ease the burden of pre-run preparations.
CHAPTER 7
SUMMARY AND CONCLUSION

In summary, TRIUMF experiment S1372 measured the resonance strengths and energies of eight resonances in $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$ in the energy range relevant to hot and explosive astrophysical environments.

A direct measurement of the $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$ reaction at several resonant energies within the Gamow window for hot and explosive astrophysical environments was approved (TRIUMF experiment S1372) and conducted at the Detector of Recoils And Gammas Of Nuclear reactions at TRIUMF in Vancouver, BC Canada. The resonance strengths and energies of eight isolated narrow resonances were measured. Of the eight resonances measured, six had not been previously studied via radiative $\alpha$ capture on $^{34}\text{S}$. The two (previously measured) resonances had significant discrepancies in the literature values for the resonance strength. The S1372 measurements resolved the discrepancy for the $E_{\text{CM}}^{\text{lit}} = 2389$ keV resonance, but the value of the resonance strength of the $E_{\text{CM}}^{\text{lit}} = 2709$ keV resonance remains in doubt. Further study is needed to resolve this discrepancy. The total resonant reaction rate calculated using S1372 data (along with existing literature values) agrees well with the NONSMOKER and REACLIB Hauser-Feshbach statistical model calculations. This is an example in which the Hauser-Feshbach statistical model appears to be confirmed by the empirical data.

The possibility for further study of radiative $\alpha$ capture on $^{34}\text{S}$ at DRAGON exists. The discrepancy amongst the measured resonance strengths for the $E_{\text{CM}}^{\text{lit}} = 2709$ keV state was not adequately resolved, and there are several states within the energy range $[0.3 E_0, E_0]$ (at still lower energies than the present work probed) for both oxygen burning and explosive oxygen burning that have not been measured via $^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}$. Of particular interest is a $J^\pi = (1^-)$ state at $E_x = 8.233$ MeV (corresponding to a CM energy of $E_{\text{CM}} = 1025$ keV. This corresponds to a lab energy of $E_b = 286$ keV/u, which is well within the energy capabilities of TRIUMF’s ISAC-I accelerators.
Additionally, a new apparatus was designed and delivered that will significantly extend the scientific reach of the Detector of Recoils And Gammas of Nuclear reactions (DRAGON) experiment. The SONIK scattering chamber was designed and fabricated at CSM, and subsequently tested and commissioned at TRIUMF, Canada’s national laboratory for nuclear and particle physics. SONIK was designed to fit in place of DRAGON’s existing gas target and utilize the existing pumping and gas recirculation systems in order to make high precision elastic scattering cross section measurements, with the explicit intent to extract the $^7\text{Be}(p,\gamma)^8\text{B}$ cross section. In conjunction with TRIUMF experiment S1025, the scattering chamber was commissioned at TRIUMF between July 23rd and August 4th, 2015 with two separate scattering measurements: elastic $^3\text{He}$ - $\alpha$ scattering ($^3\text{He}(\alpha,\alpha)^3\text{He}$) and elastic proton scattering from $^7\text{Li}$ ($^7\text{Li}(p,p)^7\text{Li}$). Data from these measurements are still being analyzed, but preliminary results show that the chamber performs as expected.

Although the SONIK commissioning experiment was successful, there are issues that need to be addressed and improvements that need to be made before subsequent measurements can commence. Of utmost importance is the acquisition of a full suite of Si charged particle detectors as well as a full complement of state of the art electronics. Other issues that need to be addressed include machining of additional parts as well as some minor alterations to the existing chamber.

The SONIK scattering chamber has a great potential for making future measurements of elastic scattering cross sections of interest to nuclear physics and astrophysics. Several experiments which intend to utilize SONIK have already been approved by TRIUMF’s Sub-atomic Physics Experiments Evaluation Committee, including $^7\text{Li}(p,p)^7\text{Li}$, $^7\text{Be}(\alpha,\alpha)^7\text{Be}$ and $^{15}\text{O}(\alpha,\alpha)^{15}\text{O}$. 
REFERENCES CITED


APPENDIX A - UNITS AND PHYSICAL CONSTANTS

The most convenient expression for most formulae relevant to nuclear astrophysics is in the CGS (centimeters, grams, seconds) unit system, which is the system of units predominantly adopted throughout this text (unless otherwise noted) but with preference for listing energies in the convenient nuclear units of electronvolts (eV) \( (1 \text{ [eV]} = 1.6022 \times 10^{-19} \text{ [J]} = 1.6022 \times 10^{-12} \text{ [erg]}, \text{ kilo-electronvolts (keV)}, \text{ or mega-electronvolts (MeV)}, \text{ distances in units of fm} \) \( (1 \text{ [fm]} = 10^{-13} \text{ [cm]} \text{ or } 10^{-15} \text{ m}), \text{ cross-sections in units of barns} \) \( (1 \text{ [b]} = 10^{-24} \text{ [cm}^2]) \), and masses in atomic mass units, u \( (1 \text{ u} = 1.660538921(73) \times 10^{-24} \text{ [g]} = 931.494061(21) \text{ [MeV/c}^2]) \), or rest-mass \( (\text{MeV/c}^2)\).

In CGS units, Coulomb’s law is written as

\[
F = \frac{q_1 q_2}{r^2}
\]  

(A.1)

The (derived) unit of electric charge in the CGS unit system is the statcoulomb (statC). It is defined as the amount of charge required for a pair of equal charges \( q_i \) separated by a distance of 1 [cm] to exert/experience a mutual force of 1 [dyne]. In order to reconcile the units on both sides of Equation (A.1), the charges \( q_i \) must have units of \( \text{cm} \sqrt{\text{dyne}} = \text{cm} \sqrt{\text{erg} \cdot \text{cm}} = g^{1/2} \text{cm}^{3/2} \text{s}^{-1} \Rightarrow 1 \text{ [statC]} = 1 g^{1/2} \text{cm}^{3/2} \text{s}^{-1} \). Thus one cannot make a direct conversion of electric charge between the CGS unit system and the SI unit system (in which electric charge has the unit of Coulombs which is derived from the base units of Amperes and seconds.) In the CGS system, the elementary charge has a value of \( e = 4.80320425(10) \times 10^{-10} \text{ [statC]} \), which, for convenience, is often expressed as

\[
e^2 = \left(4.80320425(10) \times 10^{-10} \text{[statC]} \right)^2
\]  

\[= \left(4.80320425(10) \times 10^{-10} \left[\frac{1 \text{[eV]}}{1.602176655(35) \times 10^{-12} \text{[erg]}} \cdot 10^{-6} \frac{\text{[MeV]}}{\text{[eV]}} \cdot 10^{13} \frac{\text{[fm]}}{\text{[cm]}} \right] \right)^2
\]  

\[= 1.4399764 \text{ [MeV} \cdot \text{fm]}.
\]
The kinematics of the collision between nuclei $a$ and $A$ described in section 2.1 can be easily understood in the center of mass of the two particle system (or the equivalent one-body problem). The center of mass is given by

$$R = \frac{\sum_i m_i r_i}{\sum_i m_i} = \frac{m_a r_a + m_A r_A}{m_a + m_A} \quad (B.1)$$

Differentiating Equation (B.1) with respect to time gives

$$\frac{dR}{dt} = \frac{d}{dt} \left( \frac{m_a r_a + m_A r_A}{m_a + m_A} \right) = \frac{m_a v_a + m_A v_A}{m_a + m_A} \equiv V \quad (B.2)$$

The momentum of nucleus $a$ with respect to the center of mass is given by

$$p_a^{CM} = m_a (v_a - V) = m_a v_a - m_a \frac{m_a v_a + m_A v_A}{m_a + m_A}$$

$$= \frac{m_a v_a (m_a + m_A) - m_a^2 v_a - m_a m_A v_A}{m_a + m_A}$$

$$= \frac{m_a m_A}{m_a + m_A} (v_a - v_A) = \mu v \quad (B.3)$$

where $v \equiv v_a - v_A$ is the relative velocity of the two nuclei and $\mu \equiv \frac{m_a m_A}{m_a + m_A}$ is the reduced mass of the system. Similarly, we find for $p_A^{CM}$

$$p_A^{CM} = m_A (v_A - V) = \frac{m_a m_A}{m_a + m_A} (v_A - v_a) = -\mu v \quad (B.4)$$

Equation (B.3) and Equation (B.4) can be rearranged to yield

$$v_a = V + \frac{m_A}{m_a + m_A} v \quad (B.5)$$

$$v_A = V - \frac{m_a}{m_a + m_A} v \quad (B.6)$$

The total energy of the system before the collision is given by

$$E = \frac{1}{2} m_a v_a^2 + \frac{1}{2} m_A v_A^2 \quad (B.7)$$
We can express this in terms of the center of mass system by inserting Equation (B.5) and Equation (B.6):

\[
E = \frac{1}{2} m_a \left( \mathbf{V} + \frac{m_A}{m_a + m_A} \mathbf{v} \right)^2 + \frac{1}{2} m_A \left( \mathbf{V} - \frac{m_a}{m_a + m_A} \mathbf{v} \right)^2
\]

\[
= \frac{1}{2} (m_a + m_A) V^2 + \frac{m_a m_A^2}{2(m_a + m_A)^2} v^2 + \frac{m_A m_a^2}{2(m_a + m_A)^2} v^2
\]

\[
= \frac{1}{2} (m_a + m_A) V^2 + \frac{m_a m_A (m_A + m_a)}{2(m_a + m_A)^2} v^2
\]

\[
\Rightarrow E_{CM} = \frac{1}{2} (m_a + m_A) V^2 + \frac{1}{2} \mu v^2 \tag{B.8}
\]

B.1 Transformation Between Laboratory and Center of Mass Coordinates

We can find the energy of the center of mass system in terms of the bombarding energy by using Equation (B.2) and setting the target velocity to zero:

\[
\mathbf{V} = \frac{m_b}{m_t + m_b} \mathbf{v}_b \tag{B.9}
\]

Then the projectile and target velocities with respect to the center of mass are

\[
\mathbf{v}_b^{CM} = \mathbf{v}_b - \mathbf{V} = \left(1 - \frac{m_b}{m_t + m_b}\right) \mathbf{v}_b = \frac{m_t}{m_t + m_b} \mathbf{v}_b \tag{B.10}
\]

\[
\mathbf{v}_t^{CM} = \mathbf{v}_t - \mathbf{V} = -\frac{m_b}{m_t + m_b} \mathbf{v}_b \tag{B.11}
\]

which corresponds to kinetic energies of

\[
E_{b}^{CM} = \frac{1}{2} m_b \left( \frac{m_t}{m_t + m_b} \right)^2 v_b^2 = \left( \frac{m_t}{m_t + m_b} \right)^2 E_b \tag{B.12}
\]

\[
E_{t}^{CM} = \frac{1}{2} m_t \left( \frac{m_b}{m_t + m_b} \right)^2 v_b^2 = \frac{m_t m_b}{(m_t + m_b)^2} E_b \tag{B.13}
\]

The total kinetic energy in the center of mass frame is then

\[
E_{CM} = E_{b}^{CM} + E_{t}^{CM} = \left( \frac{m_t}{m_t + m_b} \right)^2 E_b + \frac{m_t m_b}{(m_t + m_b)^2} E_b = \frac{m_t}{m_t + m_b} E_b \tag{B.14}
\]
B.2 Inverse Kinematics

For the collision in Figure 2.1, conservation of momentum demands

\[ \mathbf{P}_i = \mathbf{P}_f \Rightarrow m_b \mathbf{v}_b = (m_t + m_b) \mathbf{v}_{\text{rec}} + \mathbf{p}_\gamma = m_{\text{rec}} \mathbf{v}_{\text{rec}} + \mathbf{p}_\gamma \]  

(B.15)

where the subscripts p, t and rec represent the projectile, target and recoil nuclei, respectively.

Then

\[ \mathbf{p}_{\text{rec}} = \mathbf{p}_b - \mathbf{p}_\gamma \]  

(B.16)

and \( \mathbf{p}_\gamma \) is a small correction to the momentum of the recoil nucleus. The momentum of a photon is given by \( \mathbf{p}_\gamma = \hbar \mathbf{k} = \frac{E_\gamma}{c} \) (where \( \mathbf{k} \) is the wave vector), so we have for the recoil momentum

\[ p_{\text{rec}} = \sqrt{2m_bE_b} \left( 1 \pm \frac{E_\gamma}{\sqrt{2m_b c^2 E_b}} \right) \]  

(B.17)

(the insertion of the \( \pm \) will be explained below.) We can break Equation (B.15) into horizontal and vertical components, and employing conservation of energy and linear momentum, we have the following relationships:

\[
\begin{align*}
  m_b v_b & = m_{\text{rec}} v_{\text{rec}} \cos \phi + \frac{E_\gamma}{c} \cos \theta & (B.18a) \\
  0 & = m_{\text{rec}} v_{\text{rec}} \sin \phi - \frac{E_\gamma}{c} \sin \theta & (B.18b) \\
  E_b + (m_t + m_b)c^2 & = E_{\text{rec}} + m_{\text{rec}} c^2 + E_\gamma & (B.18c)
\end{align*}
\]

where we have used \( m_t + m_b = m_{\text{rec}} \). Making the replacement \( m_i v_i = p_i \rightarrow \sqrt{2m_i E_i} \), and using \( Q = (m_t + m_b - m_{\text{rec}})c^2 \), we have (with a little rearranging)

\[
\begin{align*}
  \sqrt{2m_{\text{rec}} E_{\text{rec}}} \cos \phi & = \sqrt{2m_b E_b} - \frac{E_\gamma}{c} \cos \theta & (B.19a) \\
  \sqrt{2m_{\text{rec}} E_{\text{rec}}} \sin \phi & = \frac{E_\gamma}{c} \sin \theta & (B.19b) \\
  Q & = E_{\text{rec}} + E_\gamma - E_b & (B.19c)
\end{align*}
\]
Squaring Equation (B.19a) and Equation (B.19b) and adding them, we have

\[
\begin{cases}
2m_{\text{rec}} E_{\text{rec}} &= 2m_b E_b - 2\sqrt{2m_b E_b} \frac{E_{\gamma}}{c} \cos \theta + \frac{E_{\gamma}^2}{c^2} \\
Q &= E_{\text{rec}} + E_{\gamma} - E_b
\end{cases}
\]

We can divide Equation (B.20a) by \(2m_{\text{rec}}\) to obtain the recoil energy:

\[
E_{\text{rec}} = \frac{m_b}{m_{\text{rec}}} E_b - \frac{\sqrt{2m_b E_b}}{m_{\text{rec}}} \frac{E_{\gamma}}{c} \cos \theta + \frac{E_{\gamma}^2}{2m_{\text{rec}} c^2}
\]

On the other hand, we can eliminate \(E_{\text{rec}}\) by dividing Equation (B.20a) by \(2m_{\text{rec}}\) and adding it to Equation (B.20b), which yields

\[
Q = m_b E_b - \frac{\sqrt{2m_b E_b}}{m_{\text{rec}}} \frac{E_{\gamma}}{c} \cos \theta - \frac{E_{\gamma}^2}{2m_{\text{rec}} c^2} + E_{\gamma} - E_b
\]

\[
\Rightarrow E_{\gamma} = Q + E_b \left(1 - \frac{m_b}{m_{\text{rec}}} \right) + \frac{\sqrt{2m_b E_b}}{m_{\text{rec}}} \frac{E_{\gamma}}{c} \cos \theta - \frac{E_{\gamma}^2}{2m_{\text{rec}} c^2}
\]

Inserting \(m_t + m_b = m_{\text{rec}}\) along with \(\sqrt{2m_b E_b} = m_b v_b\), we have

\[
E_{\gamma} = Q + \frac{m_t}{m_t + m_b} E_b + v_b \frac{E_{\gamma}}{c} \cos \theta - \frac{E_{\gamma}^2}{2m_{\text{rec}} c^2}
\]

where we have used \(v_b = \frac{m_t}{m_b} v_b\). We can find the relationship between the recoil angle \(\phi\) and the photon emission angle \(\theta\) by dividing Equation (B.19b) by Equation (B.19a):

\[
\tan \phi = \frac{E_{\gamma} \sin \theta}{\sqrt{2m_b E_b} - E_{\gamma} \cos \theta} = \frac{E_{\gamma} \sin \theta}{\sqrt{2m_b c^2 E_b} - E_{\gamma} \cos \theta}
\]

\[
\Rightarrow \phi = \tan^{-1} \left(\frac{E_{\gamma} \sin \theta}{\sqrt{2m_b c^2 E_b} - E_{\gamma} \cos \theta}\right)
\]

The recoil cone angle \(\phi\) is maximum when the \(\gamma\) is emitted normal to the beam axis (i.e. \(-\theta = 90^\circ\)):

\[
\phi_{\text{max}} = \tan^{-1} \left(\frac{E_{\gamma}}{\sqrt{2m_b c^2 E_b}}\right)
\]

Thus the recoil nucleus is emitted into a cone of maximum half angle \(\phi_{\text{max}}\).
APPENDIX C - REACTION RATE

Most stellar environments can be reasonably well approximated as an ideal gas, so the velocity distribution of ions in the stellar plasma can be described by the Maxwell-Boltzmann velocity distribution:

\[
P_i(v_i)d^3v_i = \left(\frac{m_i}{2\pi kT}\right)^{3/2} e^{-m_i v_i^2/(2kT)}d^3v_i
\]

(C.1)

\[
\Rightarrow P_0(v_0)d^3v_0 P_1(v_1)d^3v_1 = \left(\frac{m_0 m_1}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m_0 v_0^2 + m_1 v_1^2}{2kT}\right)d^3v_0d^3v_1
\]

(C.2)

Since \(\frac{1}{2}(m_0v_0^2 + m_1v_1^2) = E\), we can replace it with Equation (B.8), and multiplying by \(\left(\frac{m_0 + m_1}{m_0 + m_1}\right)^{3/2}\) yields

\[
P_0(v_0)d^3v_0 P_1(v_1)d^3v_1 = \left(\frac{m_0 + m_1}{2\pi kT}\right)^{3/2} \left(\frac{\mu}{2\pi kT}\right)^{3/2} \exp\left(-\frac{(m_0 + m_1)V^2 + \mu v^2}{2kT}\right)d^3v_0d^3v_1
\]

(C.3)

We can recast \(d^3v_0d^3v_1\) in terms of the center of mass velocity and relative velocity using the Jacobian determinant

\[
\frac{dv_{0_x}dv_{1_x}}{dV_x dv_x} = \begin{vmatrix}
\frac{\partial v_{0_x}}{\partial V_x} & \frac{\partial v_{0_x}}{\partial v_x} \\
\frac{\partial v_{1_x}}{\partial V_x} & \frac{\partial v_{1_x}}{\partial v_x}
\end{vmatrix} = \begin{vmatrix}
\frac{m_1}{m_0 + m_1} \\
\frac{-m_0}{m_0 + m_1}
\end{vmatrix} = 1
\]

(C.4)

and similarly for \(dv_{0_y}dv_{1_y}\) and \(dv_{0_z}dv_{1_z}\). Hence \(d^3v_0d^3v_1 = d^3Vd^3v\) and Equation (C.2) becomes

\[
P(V)d^3V P(v)d^3v = \left(\frac{m_0 + m_1}{2\pi kT}\right)^{3/2} \left(\frac{\mu}{2\pi kT}\right)^{3/2} \exp\left(-\frac{(m_0 + m_1)V^2 + \mu v^2}{2kT}\right)d^3Vd^3v
\]

(C.5)

Since \(P(V)d^3V\) is normalized to unity, we can integrate it immediately upon insertion into Equation (2.34), which becomes

\[
N_0 N_1 \langle \sigma v \rangle = N_0 N_1 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int_0^{\infty} v^3 \sigma(v) \exp\left(-\frac{\mu v^2}{2kT}\right) dv
\]

(C.6)
where we have used \( d^3v = 4\pi v^2 dv \). This can be recast in terms of \( E \) by making the substitutions \( v \rightarrow \sqrt{\frac{2E}{\mu}}, \frac{1}{2} \mu v^2 \rightarrow E \) and \( dv \rightarrow \frac{1}{\sqrt{2\mu E}} dE \):

\[
N_0 N_1 \langle \sigma v \rangle = N_0 N_1 \sqrt{\frac{8}{\pi \mu (kT)^3}} \int_0^\infty e^{-E/(kT)} E \sigma(E) dE \tag{C.7}
\]

### C.1 Gamow Peak

The integrand in Equation (2.43) is known as the Gamow peak, and it can be well approximated by a Gaussian distribution with identical maximum and concavity (see Figure 2.3):

\[
G(E) = C \exp \left( -\frac{(E - E_0)^2}{2\sigma^2} \right) \tag{C.8}
\]

where \( C, E_0, \) and \( \sigma \) are determined by matching them to the respective extrema and concavity values of the Gamow peak and Gaussian distribution. The maximum of the Gamow Peak is found by differentiating with respect to \( E \) and setting the result equal to zero:

\[
\frac{d}{dE} \left[ \exp \left( -2\pi \frac{Z_0 Z_1 e^2}{\hbar} \sqrt{\frac{\mu}{2E}} - \frac{E}{kT} \right) \right] = \left( \frac{\pi Z_0 Z_1 e^2}{\hbar} \sqrt{\frac{\mu}{2 \ E^{3/2}}} - \frac{1}{kT} \right) e^{-2\pi \eta - \frac{E}{kT}} = 0 \tag{C.9}
\]

Solving for \( E \) gives

\[
E_0 = \left( \frac{\pi Z_0 Z_1 e^2 kT}{\hbar} \sqrt{\frac{\mu}{2}} \right)^{2/3} \tag{C.10}
\]

where \( E_0 \) is known as the Gamow energy (see Figure 2.2). We can then rewrite the Gamow peak in terms of the Gamow energy:

\[
\exp \left( -2\pi \frac{Z_0 Z_1 e^2}{\hbar} \sqrt{\frac{\mu}{2E}} - \frac{E}{kT} \right) = \exp \left( -\frac{2E_0^{3/2}}{\sqrt{E_0} kT} - \frac{E}{kT} \right) \tag{C.11}
\]

Evaluating Equation (C.11) at \( E = E_0 \) we find

\[
e^{-2\pi \eta \bigg|_{E=E_0}} = \exp \left( -\frac{2E_0^{3/2}}{\sqrt{E_0} kT} - \frac{E_0}{kT} \right) = \exp \left( -3 \frac{E_0}{kT} \right) = C \tag{C.12}
\]

We match the concavities of the Gaussian distribution and the Gamow peak by matching the second derivatives at \( E_0 \); for the Gamow peak, we find
\[
\frac{d^2}{dE^2} \left[ \exp \left( -\frac{2E_0^{3/2}}{\sqrt{E^3} kT} - \frac{E}{kT} \right) \right]_{E=E_0} = \frac{d}{dE} \left[ \left( \frac{E_0^{3/2}}{E^{3/2} kT} - \frac{1}{kT} \right) e^{-2\pi \eta - \frac{E}{kT}} \right]_{E=E_0}
\]

\[
= \left[ \left( -\frac{3}{2} \frac{E_0^{3/2}}{E^{5/2} kT} \right) e^{-2\pi \eta - \frac{E}{kT}} + \left( -\frac{E_0^{3/2}}{E^{3/2} kT} - \frac{1}{kT} \right)^2 e^{-2\pi \eta - \frac{E}{kT}} \right]_{E=E_0}
\]

\[
= -\frac{3}{2} \frac{E_0}{E^{5/2} kT} e^{-\frac{3E_0}{kT}}
\]

For the Gaussian approximation of the Gamow peak, we find

\[
\frac{d^2}{dE^2} \left[ e^{-\frac{3E_0}{kT}} \exp \left( -\frac{(E - E_0)^2}{2\sigma^2} \right) \right]_{E_0} = \frac{d}{dE} \left[ \left( -\frac{E - E_0}{\sigma^2} \right) e^{-\frac{3E_0}{kT}} \exp \left( -\frac{(E - E_0)^2}{2\sigma^2} \right) \right]_{E_0}
\]

\[
= e^{-\frac{3E_0}{kT}} \left[ \left( -\frac{1}{\sigma^2} + \frac{(E - E_0)^2}{\sigma^2} \right) \exp \left( -\frac{(E - E_0)^2}{2\sigma^2} \right) \right]_{E_0}
\]

\[
= -\frac{1}{\sigma^2} e^{-\frac{3E_0}{kT}}
\]

Set equal to the result for the Gamow Peak, we arrive at

\[
\left( -\frac{3}{2} \frac{E_0}{kT} \right) e^{-\frac{3E_0}{kT}} = -\frac{1}{\sigma^2} e^{-\frac{3E_0}{kT}}
\]

\[
\Rightarrow \sigma = \sqrt{\frac{2}{3} E_0 kT} \tag{C.13}
\]

Then the Gaussian approximation of the Gamow peak is

\[
G(E) = e^{-\frac{3E_0}{kT}} \exp \left( -\frac{3(E - E_0)^2}{4E_0 kT} \right) \tag{C.14}
\]

The Gamow window is defined by the e-folding width of Equation (C.14). We can find the e-folding width by setting Equation (C.14) equal to \(1/e\) of its maximum value and solving for \(E\):

\[
e^{-\frac{3E_0}{kT}} \exp \left( -\frac{3(E - E_0)^2}{4E_0 kT} \right) = e^{-\frac{3E_0}{kT}} \tag{C.15}
\]

\[
\Rightarrow e^{-\frac{3E_0}{kT}} \exp \left( -\frac{3(E - E_0)^2}{4E_0 kT} \right) = e^{-\frac{3E_0}{kT}} - 1
\]
⇒ \exp \left( -\frac{3(E - E_0)^2}{4E_0 kT} \right) = e \tag{C.16}

⇒ -\frac{3(E - E_0)^2}{4E_0 kT} = -1 \tag{C.17}

⇒ (E - E_0)^2 = \frac{4}{3} E_0 kT \tag{C.18}

⇒ E = E_0 \pm \frac{2}{\sqrt{3}} \sqrt{E_0 kT} = E_0 \pm \frac{\Delta}{2} \tag{C.19}

where the e-folding width \( \Delta \) is the full width of the Gamow window defined by the region between \( E_0 - \Delta/2 \) and \( E_0 + \Delta/2 \). For nonresonant reactions, the Gamow window is the most probable energy range for thermonuclear reactions to occur at a given temperature.

**C.1.1 Resonant Reaction Rate**

Inserting Equation (2.49) into Equation (2.39), we have

\[
N_A \langle \sigma v \rangle = N_A \sqrt{\frac{8}{\pi \mu (kT)^3}} \int_0^\infty E \sigma_{BW}(E) e^{-E/kT} dE
\tag{C.20}
\]

\[
= N_A \sqrt{\frac{8}{\pi \mu (kT)^3}} \int_0^\infty E \frac{\lambda^2}{4\pi} \frac{\Gamma_\alpha \Gamma_\gamma}{(E_r - E)^2 + \Gamma^2/4} e^{-E/kT} dE \tag{C.21}
\]

Replacing the de Broglie wavelength with its definition \( \lambda \equiv \frac{2\pi \hbar}{\sqrt{2\mu E}} \), we have

\[
N_A \langle \sigma v \rangle = N_A \sqrt{\frac{8}{\pi \mu (kT)^3}} \frac{\pi \hbar^2}{2\mu} \omega \int_0^\infty \frac{\Gamma_\alpha \Gamma_\gamma}{(E_r - E)^2 + \Gamma^2/4} e^{-E/kT} dE \tag{C.22}
\]

\[
= N_A \frac{\sqrt{2\pi \hbar^2}}{(\mu kT)^{3/2}} \omega \int_0^\infty \frac{\Gamma_\alpha \Gamma_\gamma}{(E_r - E)^2 + \Gamma^2/4} e^{-E/kT} dE \tag{C.23}
\]

If the resonance is sufficiently narrow, then \( e^{-E/kT} \) and \( \Gamma_i \) are approximately constant over the integration, so they may be replaced by their values at \( E_r \), giving

\[
N_A \langle \sigma v \rangle = N_A \frac{\sqrt{2\pi \hbar^2}}{(\mu kT)^{3/2}} e^{-E_r/kT} \omega \frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma} 2 \int_0^\infty \frac{\Gamma/2}{(E_r - E)^2 + \Gamma^2/4} dE \tag{C.24}
\]

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We can evaluate the integral in Equation (C.24) by letting $a \equiv \Gamma / 2$ and $x = (E_r - E)$ \implies $dE = -dx$ and $\{x_0, x_f\} = \{E_r, -\infty\}$. Then we have

$$\int_0^\infty \frac{\Gamma/2}{(E_r - E)^2 + \Gamma^2/4} dE = -\int_{E_r}^{-\infty} \frac{a}{x^2 + a^2} dx$$  \hspace{1cm} (C.25)

We can evaluate this integral by making a trigonometric substitution:

Let $x = a \tan \theta$

$$\Rightarrow \begin{cases} \frac{dx}{d\theta} = a \sec^2 \theta \implies dx = a \sec^2 \theta d\theta \\ \theta_0 = \tan^{-1} \left( \frac{E_r}{a} \right) \approx \frac{\pi}{2} \\ \theta_f = \tan^{-1} \left( \frac{\infty}{a} \right) = -\frac{\pi}{2} \end{cases}$$ \hspace{1cm} (C.26)

where we have assumed $\Gamma \ll 1$.

$$\Rightarrow \int_0^\infty \frac{\Gamma/2}{(E_r - E)^2 + \Gamma^2/4} dE = \int_{-\pi/2}^{\pi/2} \frac{a^2 \sec^2 \theta}{a^2 \tan^2 \theta + a^2} d\theta = \int_{-\pi/2}^{\pi/2} d\theta = \pi$$

so Equation (C.24) becomes

$$N_A \langle \sigma v \rangle = N_A \sqrt{2\pi} \frac{\hbar^2}{(\mu kT)^{3/2}} \gamma E_r / kT \frac{\Gamma_{\alpha} \Gamma_{\gamma}}{\Gamma} \sqrt{\frac{2\pi}{\mu kT}}$$ \hspace{1cm} (C.27)

Defining $\omega \gamma \equiv \omega \frac{\Gamma_{\alpha} \Gamma_{\gamma}}{\Gamma}$, we have

$$N_A \langle \sigma v \rangle = N_A \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \omega \gamma e^{-E_r / kT}$$ \hspace{1cm} (C.28)
APPENDIX D - STATISTICAL ANALYSIS AND UNCERTAINTY ESTIMATION

The yields measured by DRAGON are best estimates of the actual number of reactions occurring during a given yield measurement in DRAGON’s gas target. The estimates are derived by maximizing a probability model of DRAGON’s detection method (i.e. - the summing of events of interest in DRAGON’s various detectors \( \pi_i \) in the presence of background(s) - \( \theta_i \) - where the background(s) is/are characteristic of the physical properties of each detector - subject to the respective efficiencies of each detector) in the frequentist approach. Bishop [94] derived a probability model for DRAGON and a maximum likelihood estimator for DRAGON’s yield. The derivation of a probability model in this document differs slightly, owing to the addition of detectors (the MCPs and the IC), the replacement of DRAGON’s DAQ and a slightly different analytical method employed for particle identification. In the following sections, we generalize the probability model to include nuisance parameters (background).

D.0.1 Reaction and Detection Probability

Suppose that \( N_{\text{rxn}} \) is the true number of fusion events occurring in DRAGON’s high density gas target during a given yield measurement. The probability that \( N_{\text{rxn}} \) reactions occur within the target given \( N_b \) incident beam particles is described by a Poisson distribution

\[
P(N_{\text{rxn}} | N_b, \mathcal{Y}) = \frac{(N_b \mathcal{Y})^{N_{\text{rxn}}}}{N_{\text{rxn}}!} e^{-N_b \mathcal{Y}}
\]  

(D.1)

where \( \mathcal{Y} \) is the true mean reaction rate. However, we cannot directly observe the true number of reactions that occur in the target. Instead, DRAGON detects the reaction products (prompt \( \gamma \)-ray emissions from excited recoil nuclei and the recoil nuclei themselves). Thus our probability model must be convoluted with the detection probabilities of DRAGON’s various detectors (which themselves depend on their independent efficiencies as well as DRAGON’s transmission probabilities). The probability that a reaction product is detected by a radiation detector is described by a binomial distribution:
The binomial distribution describes the probability $P(x|n,p)$ of $x$ successes (that have a success probability of $p$) out of $n$ trials. In general, the convolution of a Poisson distribution with a binomial distribution is

$$\sum_{\xi=x}^{\infty} P_{\text{poiss}}(\xi|\nu, \rho) P_{\text{bin}}(x|\xi, p) = \sum_{\xi=x}^{\infty} \left( \frac{(\nu \rho)^\xi}{\xi!} e^{-\nu \rho} \right) \left( \frac{\xi!}{x!(\xi-x)!} p^x (1-p)^{\xi-x} \right)$$

$$= e^{-\nu \rho} \sum_{\xi=x}^{\infty} \frac{(\nu \rho)^\xi}{x!(\xi-x)!} p^x (1-p)^{\xi-x}$$

We can change the limits of the summation by defining $y = \xi - x$, then

$$\sum_{\xi=x}^{\infty} P_{\text{poiss}}(\xi|\nu, \rho) P_{\text{bin}}(x|\xi, p) = \frac{(\nu \rho p)^x}{x!} e^{-\nu \rho} \sum_{y=0}^{\infty} \frac{(\nu \rho)^y}{y!} (1-p)^y$$

$$= \frac{(\nu \rho p)^x}{x!} e^{-\nu \rho} e^{\nu \rho (1-p)}$$

$$= \frac{(\nu \rho p)^x}{x!} e^{-\nu \rho p} = P_{\text{poiss}}(x|\nu, \rho p)$$

It follows immediately that convolution of Equation (D.5) with an additional binomial distribution, say $P_{\text{bin}}(\chi|x, \eta)$ will similarly yield a Poisson distribution $P_{\text{poiss}}(\chi|\nu, \rho p \eta)$. Because the data for S1372 was analyzed in both singles and coincidence, we will derive probability models for both singles and coincident yields. We begin with the coincident yield.

**D.0.2 Probability Density Function (pdf) for the Coincident Yield $P_{\text{coinc}}$**

The probability of observing $g$ events in DRAGON’s BGO $\gamma$-ray array (assuming efficiency $\eta_{\text{BGO}}$ and $N_{rxn}$ reactions occur in the target) follows a binomial distribution:
Provided that the recoil cone angle of the reaction is within DRAGON’s acceptance, and that the tune is good, a recoil for each $\gamma^*$ detected in DRAGON’s BGO array will be transmitted to MD1. However, only the recoils that emerge from DRAGON’s gas target in the charge state selected at MD1 will be transmitted to the focal plane at DRAGON’s charge slits. The probability that $q$ out of $g$ recoils are presented at the charge focal plane is
\[
P(q | g, \eta_{\text{CSF}}) = \binom{g}{q} \eta_{\text{CSF}}^q (1 - \eta_{\text{CSF}})^{g-q}
\] (D.7)
where, as usual, $\eta_{\text{CSF}}$ is the fraction of recoils in the given charge state. Continuing on in this fashion, we find that of the $m$ recoils that are transmitted to the charge focal plane, the probability that $d$ recoils will be transmitted through the separator to the MCP is
\[
P(d | q, \eta_{\text{DRA}})\text{ where } \eta_{\text{DRA}}\text{ is the separator transmission (}\eta_{\text{DRA}} = 0.999^{+0.001}_{-0.002}\text{ for the purposes of this document). We then find that of the } d \text{ recoils that are transmitted to the MCP, the probability that } \nu \text{ recoils will be transmitted through the MCP for detection is}
\[
P(\nu | s, \eta_{\text{MCP}}) \text{ where } \eta_{\text{DRA}} \text{ is the MCP transmission (}\eta_{\text{DRA}} = 0.664 \pm 0.010 \text{ for the purposes of this document). Similarly, the probability that out of the } \nu \text{ recoils that are transmitted through the MCP, the probability that } c \text{ coincident recoils are detected by the MCP is}
\[
P(m | \nu, \eta_{\text{det}}^{\text{MCP}})\text{. Finally, the probability that } c \text{ signals out of } m \text{ events detected by the MCP arrive when DRAGON’s DAQs are not busy recording a previous event in either the head or tail systems is}
\[
P(c | m, \eta_{\text{live}}^{\text{coinc}}), \text{ where } \eta_{\text{live}}^{\text{coinc}} \text{ is the coincident live time of DRAGON’s DAQ. Then, using Equation (D.1), Equation (D.4) and Equation (D.5), the probability model for detection of recoils in coincidence with the BGO array and the MCPs is}
\[
P(g | N_{\text{rxn}}, \eta_{\text{BGO}}) = \binom{N_{\text{rxn}}}{g} \eta_{\text{BGO}}^g (1 - \eta_{\text{BGO}})^{N_{\text{rxn}}-g}
\] (D.6)

\textit{N.B. that in general, the coincident detection probability should also include the detection probability of DRAGON’s end detector (either the IC or DSSSD). The probability of detection in DRAGON’s end detector was not included in this analysis because signals from DRAGON’s IC were not used in identifying coincidence events (because it did not provide any further discrimination).}
\[ P(c | \mathcal{Y}) = \sum_{c=m} P(c | m, \eta_{\text{live}}^{\text{coinc}}) \sum_{m=\nu} P(m | \nu, \eta_{\text{MCP}}^{\text{det}}) \sum_{\nu=d} P(\nu | d, \eta_{\text{MCP}}) \sum_{d=q} P(d | q, \eta_{\text{DRA}}) \]

\[ \sum_{g=q} P(g | \eta_{\text{CSF}}) \sum_{N_{\text{rxn}}=g} P_{\text{Poiss}}(N_{\text{rxn}} | N_{\text{BGO}}, \mathcal{Y}) P(g | N_{\text{rxn}}, \eta_{\text{BGO}}) \]

\[ = \left( \eta_{\text{coinc}}^{\text{MCP}} \eta_{\text{MCP}}^{\text{det}} \eta_{\text{DRA}}^{\text{trans}} \eta_{\text{CSF}}^{\text{CSF}} \eta_{\text{BGO}}^{\text{BGO}} N_{\text{BGO}}^{\mathcal{Y}} \right)^c \]

\[ \times \exp \left( -\eta_{\text{coinc}}^{\text{MCP}} \eta_{\text{MCP}}^{\text{det}} \eta_{\text{DRA}}^{\text{trans}} \eta_{\text{CSF}}^{\text{CSF}} \eta_{\text{BGO}}^{\text{BGO}} N_{\text{BGO}}^{\mathcal{Y}} \right) \]

\[ = \left( \eta_{\text{coinc}}^{\text{MCP}} \eta_{\text{BGO}}^{\mathcal{Y}} \right)^c \frac{1}{c!} e^{-\eta_{\text{coinc}}^{\text{MCP}} N_{\text{BGO}}^{\mathcal{Y}}} \tag{D.8} \]

where \( \eta_{\text{coinc}} \equiv \eta_{\text{live}}^{\text{MCP}} \eta_{\text{MCP}}^{\text{det}} \eta_{\text{DRA}}^{\text{trans}} \eta_{\text{CSF}}^{\text{CSF}} \eta_{\text{BGO}} \) is the total coincident detection efficiency.

**D.0.3 pdf for the Singles Yield \( P_{\text{sing}} \)**

The probability model for singles events is the probability of detecting a heavy ion signal in DRAGON’s heavy ion detectors. Thus the detection probability does not depend on the detection of a \( \gamma \)-ray in DRAGON’s BGO array. It follows from the above discussion that the probability of detecting a recoil event in DRAGON’s ionization chamber is initially dependent on the transmission of recoils through MD1, through the separator, and through DRAGON’s MCP to the final focal plane. Then the probability of \( s \) signals being recorded by the DAQ out of \( i \) singles recoil events being detected by the IC is

\[ P(s | \mathcal{Y}) = \sum_{s=i}^{} P(s | i, \eta_{\text{live}}^{\text{sing}}) \sum_{i=r}^{} P(i | r, \eta_{\text{IC}}) \sum_{r=\nu}^{} P(r | \nu, \eta_{\text{MCP}}^{\text{det}}) \sum_{\nu=d}^{} P(\nu | d, \eta_{\text{MCP}}) \]

\[ \sum_{d=q}^{} P(d | q, \eta_{\text{DRA}}) \sum_{N_{\text{rxn}}=q}^{} P_{\text{Poiss}}(N_{\text{rxn}} | N_{\text{BGO}}, \mathcal{Y}) P(q | N_{\text{rxn}}, \eta_{\text{CSF}}) \]

\[ = \left( \eta_{\text{sing}}^{\text{MCP}} \eta_{\text{IC}}^{\text{IC}} \eta_{\text{MCP}}^{\text{det}} \eta_{\text{DRA}}^{\text{trans}} \eta_{\text{CSF}} \right)^s \frac{1}{s!} e^{-\eta_{\text{sing}}^{\text{MCP}} N_{\text{BGO}}^{\mathcal{Y}}} \tag{D.9} \]

where \( \eta_{\text{sing}} \equiv \eta_{\text{live}}^{\text{IC}} \eta_{\text{MCP}}^{\text{IC}} \eta_{\text{MCP}}^{\text{det}} \eta_{\text{DRA}}^{\text{trans}} \eta_{\text{CSF}} \) is the total singles detection efficiency.
The method of maximum likelihood [95] is a method of estimating parameters (such as best estimates and confidence intervals) from a statistical model using the frequentist approach. The statistical likelihood that a probability model describes a set of data can be tested using the method of maximum likelihood. This is achieved through the likelihood function, which is given by

\[ L(\pi, \theta | X) = \prod_{i=1}^{N} f(X_i | \pi, \theta) \]  \hspace{1cm} (D.10)

where \( \pi = (\pi_1, ..., \pi_k) \) are parameters of interest, \( \theta = (\theta_1, ..., \theta_l) \) are nuisance parameters, \( X = (X_1, ..., X_N) \) are independent observations and \( f(X_i | \pi, \theta) \) is the probability density function. Neglecting nuisance parameters for the moment (i.e. \( \theta \to 0 \)), the likelihood function for DRAGON’s coincident yield is given by

\[ L(Y) = \prod_{i=1}^{N} P(c_i | Y) = \prod_{i=1}^{N} \frac{(\eta_{\text{coinc}}N_{\theta}Y)^{c_i}}{c_i!} e^{\eta_{\text{coinc}}N_{\theta}Y} \]  \hspace{1cm} (D.11)

Similarly, the likelihood function for DRAGON’s singles yield is given by

\[ L(Y) = \prod_{i=1}^{N} P(s_i | Y) = \prod_{i=1}^{N} \frac{(\eta_{\text{coinc}}N_{\theta}Y)^{s_i}}{s_i!} e^{\eta_{\text{sing}}N_{\theta}Y} \]  \hspace{1cm} (D.12)

The maximum likelihood estimator for a given parameter of interest \( \pi_i \) can be found by finding the supremum of \( L \) with respect to \( \pi_i \). Thus, to find the maximum likelihood estimator for DRAGON’s coincident yield, we maximize its likelihood function with respect to \( Y \). However, it is easier to work with the log likelihood function, the maximization of which is an identical problem:*

\[ \frac{\partial L}{\partial \pi_0} = 0 \]

On the other hand, maximizing the log-likelihood yields

\[ \frac{\partial}{\partial \pi_0} \ln L = \frac{1}{L} \frac{\partial L}{\partial \pi_0} = 0 \Rightarrow \frac{\partial L}{\partial \pi_0} = 0 \]

Therefore, we see that maximizing the log likelihood is an identical problem to maximizing the likelihood.

---

*To maximize \( L \), we differentiate it with respect to the parameter we wish to maximize (say \( \pi_0 \), set the result equal to 0 and solve for the parameter:
\[
\ln \mathcal{L}(Y) = \ln \prod_{i=1}^{N} P(c_i|Y) = \sum_{i=1}^{N} \ln P(c_i|Y) = \sum_{i=1}^{N} c_i \ln(\eta_{\text{coinc}} N_b Y) - \ln(c_i!) - \eta_{\text{coinc}} N_b Y 
\]  
(D.13)

We maximize Equation (D.14) by differentiating with respect to \(Y\), set the result equal to 0 and solve for \(Y\):

\[
\frac{\partial}{\partial Y} \ln \mathcal{L}(Y) = \frac{\partial}{\partial Y} \sum_{i=1}^{N} c_i \ln(\eta_{\text{coinc}} N_b Y) - \ln(c_i!) - \eta_{\text{coinc}} N_b Y = 0 
\]  
(D.15)

\[
\Rightarrow \sum_{i=1}^{N} \frac{c_i}{Y_{\text{max}}} - \eta_{\text{coinc}} N_b = 0 
\]  
(D.16)

In order for the left hand side of Equation (D.16) to vanish, each term in the sum must be identically equal to 0:

\[
\frac{c_i}{Y_{\text{max}}} - \eta_{\text{coinc}} N_b = 0
\]  
(D.17)

\[
\Rightarrow Y_{\text{max}} = \frac{c_i}{\eta_{\text{coinc}} N_b}
\]  
(D.18)

This is just the maximum likelihood estimator for one independent observation \(X_i\), so the maximum likelihood estimator for the total coincident yield is

\[
Y_{\text{coinc}} = \frac{c}{\eta_{\text{coinc}} N_b}
\]  
(D.19)

where \(c = \sum_i c_i\). A similar procedure will show that the maximum likelihood estimator for the total singles yield is given by

\[
Y_{\text{sing}} = \frac{s}{\eta_{\text{sing}} N_b}
\]  
(D.20)

**D.0.5 Nuisance Parameters (Background Estimation)**

Because the coincident spectra used to identify recoil events in DRAGON’s detectors had negligible backgrounds, the following discussion for including nuisance parameters in
the estimation of the yield in the method of maximum likelihood will only be applied to the singles yield.

D.0.6 Confidence Intervals in the Presence of Nuisance Parameters

Rolke, Lopez and Conrad [72] have shown that the estimation of confidence intervals using the profile likelihood results in good interval coverage, even in the presence of (several) nuisance parameters. The estimation of confidence intervals in the method of maximum likelihood utilizes the profile likelihood method:

\[ \lambda(\pi_0|X) = \frac{\sup\{L(\pi_0,\theta|X);\theta\}}{\sup\{L(\pi,\theta|X);\pi,\theta\}} \]  

(D.21)

where \( \sup\{L(\pi_0,\theta|X);\theta\} \) is the likelihood function evaluated at the maximum likelihood estimator \( \theta \) and \( \sup\{L(\pi,\theta|X);\pi,\theta\} \) is the likelihood function evaluated at the maximum likelihood estimators \( \pi \) and \( \theta \). A 100(1 − \( \alpha \)) confidence interval is then determined by evaluating \(-2 \log \lambda\) at its minimum (the maximum likelihood estimators) and then finding where the function increases by \( \alpha\% \). In the case of normally distributed backgrounds and efficiencies, the confidence interval simply reduces to the simple Gaussian confidence intervals, and, consequently, independent uncertainties may simply be added in quadrature.
D.1 SONIK Technical Drawings

This section contains the SolidWorks® technical drawings of the various components of the SONIK scattering chamber.
Figure D.1: SONIK main cylinder.
Figure D.2: SONIK main cylinder.
Figure D.3: Removable $\varnothing = 5$ mm circular aperture.
Figure D.4: Variable sizes for removable circular aperture. All dimensions are given in cm. SONIK was commissioned using $\varnothing = 1$ mm apertures on all telescopes.
Figure D.5: 45° slit aperture.
Figure D.6: Differences in dimensions between slit apertures.
Figure D.7: 45° circular aperture.
Figure D.8: Differences in dimensions between circular apertures.
Figure D.9: 20° circular aperture (reduced clearance).
Figure D.10: Differences in dimensions between reduced clearance circular apertures.
Figure D.11: 60° slit aperture.
Figure D.12: 65° slit aperture.
Figure D.13: 75° slit aperture.
Figure D.14: 90° slit aperture.