Analysis, Characterization and Application of Microwave Metamaterials

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Metamaterials are an extremely fast growing and interesting field of study. The promise of new materials whose constitutive parameters are based on structure inclusions have drawn the interest of thousands and continue to inspire. Despite these promises not much in the way of applied technologies has come from the research.

This thesis is the result of numerous years of work into the characterization and application of microwave metamaterials. The aim of the research was to develop a design process, characterize fully, and apply metamaterials to some current technology.

The thesis first details the enhanced directivity of a metamaterial slab and a plausible explanation for the phenomenon. It looks into the design process of low loss metamaterials, and the use of infra-red imaging to help characterize them. Then it details how a metamaterial, the S-shaped split ring resonator, behaves with changes in polarization and angle of incidence. It details the ambiguity issues that come with constitutive parameter extraction and how to resolve them. The thesis ends with the ongoing work to explore a new metamaterial design and application to millimeter wave notch filters.
Acknowledgments

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Chapter 1: Introduction

This dissertation completed through the Physics Department at the University of Colorado Colorado Springs, directed by Dr. Anatoliy Pinchuk. This thesis is divided into chapters that either have been published or have been submitted for peer reviewed publication. Chapters two through seven are in the same order as the publications section previous to this chapter. This first chapter details the specific contributions I made to each research project as well as painting a narrative connecting one chapter to another.

The second chapter, *Focusing Effect of a Metamaterial Slab on the Radiation Pattern Produced by a Patch Antenna*, details the experimental results of transmission through a wire mesh metamaterial based on theory previously developed [1]. The metamaterial slab was tested with a patch antenna and there was a significant increase in directivity observed. The research presented in chapter two was motivated by findings from other research groups [2,3]. The other groups did not present a satisfactory explanation for the increased directivity and our work presents a unique and plausible cause. In addition to the findings, the second chapter details the design, modeling and construction of the metamaterial under test. We showed that the increased directivity could be tied to a simple idea; the focusing effect was in fact due to attenuation of the electromagnetic waves within the metamaterial. This was groundbreaking at the time because other authors had thought that the focusing effect was purely due to the permittivity of the metamaterial slab being near zero [2,3]. My contribution to the work was two-fold, I modeled the metamaterial design using a commercial software, ANSYST HFSS, and
helped take measurements in the anechoic chamber at UCCS. Although this is my smallest contribution out of all the papers presented in this thesis, it is included due to its importance in the narrative. The research presented in chapter two is not an improvement over current microwave technologies for a very apparent reason; the attenuation that causes the focusing effect is so large that the structure would not be an improvement. This motivated me to begin the search for a metamaterial design that possessed lower attenuation. The search for a less attenuating metamaterial design led to the S-shaped split ring resonator (SSRR) [4].

Chapter three, *Low-loss Negative Index Metamaterials for X, Ku, and K Microwave Bands*, details design of a low loss metamaterial slab with the S-shaped split-ring resonator (SSRR) as its base unit cell. The main goal was to develop a design process for the SSRR with specific constitutive parameters. What made this project very interesting is that although the theory for the SSRR is well defined, it remained very difficult to design and our process showed consistent results [4]. We were able to verify our results with simulations using ANSYS HFSS, a finite element equation solver. Our research also uncovered that a ratio of 0.7 between the height and width of metamaterial unit cell produced a very wide bandwidth of transmission. The bandwidth produced with this ratio was much larger than had been reported previously[4]. To estimate the index of refraction of our metamaterial slab, we used the standard transmission through a prism. In addition to the standard prism retrieval for index of refraction, it was also verified using infra-red near field imaging, which to my knowledge is the first case of this method being used.
For this project I designed each metamaterial, modeled them using Ansoft HFSS, I participated in radiation pattern measurements, and helped take IR images. The idea that we could use IR imaging to view the radiation pattern going through metamaterials in the near field was very exciting and led to a full project that is the basis for chapter four. The SSRR unit cell showed a lot of promise for use but needed further understanding as well as a full characterization. The largest downfall of the SSRR is that the design was based on a specific polarization so was thought to be highly anisotropic to changes in polarization which we fully parameterize and overcome in chapter five.

Chapter four, *Infrared Imaging of Microwave Negative Index Metamaterials*, explored a novel method for retrieving the index of refraction from a microwave metamaterial slab. Near-field infrared imaging was used to “view” the electric field intensity through a metamaterial prism. The chapter details our process for measuring acquiring the infrared pattern for the metamaterial and relate it directly to electric field intensity. The chapter also verifies that 3-D contour mapping of the infrared images are in agreement with simulated electric field intensities. Traditionally metamaterials have been characterized by observing far-field radiation patterns, whereas the infrared imaging technique allowed for an alternative near field measurement. The infrared imaging used in chapter four could be used to characterize irregular forms of metamaterials, such as gradients, prisms, and conformal surfaces all in the near field.
For this project I simulated radiation patterns and took the infrared images used in the findings. I helped design the experiment from start to finish. In addition I helped with the low pass filter to overcome dead pixels on the infrared camera.

Chapter five, *Polarization Dependent Transmission through Microwave Metamaterials*, presents experimental work with a few goals in mind; first was to characterize how the transmitted intensity through the metamaterial slab changes with adjustments in polarization and angle of incidence, second was to produce metamaterial designs based on the SSRR that would be polarization independent. We successfully document how the SSRR metamaterial behaves with changes in angle of incidence and polarization. From this documentation we found that the SSRR allows for all frequency bands to pass through when oriented perpendicular to the electric field. This finding allowed for us to create two novel geometries for the SSRR that make it polarization independent. In addition to this, several layers of the metamaterial are tested to see if transmission changes with thickness of the metamaterial. This produced a very confusing result; the transmitted power changed non-linearly with changes in thickness. This is not what one would expect from a homogeneous material, which hints that the SSRR metamaterial is inhomogeneous. If this metamaterial was indeed inhomogeneous then a modified method must be used to retrieve the constitutive parameters.

With the observation that the transmitted power changes non-linearly with thickness, it became apparent that this was in fact an inhomogeneous metamaterials. This in turn
meant that the simple Nicholson-Ross-Weir extraction method [5] we had been using to characterize the metamaterial would produce inaccurate results as it assumes homogeneity. The results from NRW extraction method produce infinite ambiguities for any constitutive parameter retrieved if the material is inhomogeneous. Chapter six, *Resolving Parameter-Extraction Ambiguities in Inhomogeneous Materials and Metamaterials*, describes the mathematical origin of this ambiguity in extracting the constitutive parameters [6] as well as describing our novel solution to this problem. We showed that a process that is commonly used in image processing could be applied to resolving the ambiguity for constitutive parameter extraction. In addition, this chapter details the process, advantages, anddownfalls of this modified NRW method. After extracting the parameters from several layers of the extended s-shaped split-ring resonator (ESRR) metamaterial, it was seen that although not negative index of refraction, specific numbers of metamaterial unit cells matched the index of refraction of air. The result from the ESRR metamaterial was very interesting and led to us exploring it further in chapter seven.

Chapter seven, *Exploiting Inhomogeneity in Metamaterials for Radome Application*, goes into exploring if the ESRR metamaterial could be used in radome design despite being inhomogeneous. It was my goal in this chapter to look at the inhomogeneity as something that we could design with, not around and make use of. Previous chapters demonstrated that the SSRR and ESRR metamaterials are not negative at specific layers and frequencies and we wanted to make use of this. In chapter seven we find that the ESRR at three layers actually produced a positive material that matched the impedance of air,
allowing for no attenuation at specific frequencies. This was a huge success, as it lent credit to using the metamaterial in radome design.

The research into the SSRR and its variants felt fairly exhausted after chapter seven and with interest rising in the millimeter wave bands we began looking at different structured materials. Chapter eight, *Millimeter Wave Tunable Band Gap Filter*, focuses on research done for a periodic liquid crystal structure. The overall goal of chapter eight was to investigate a new all dielectric metamaterial for use in millimeter wave devices. The work presented in this chapter is only preliminary findings but still show great potential. A simple Bragg reflection model is implemented to create the notch-filter and surprisingly had extremely good agreement. The chapter details some basic background information and simulated data presenting some initial key results that are being expanded on beyond the work in this thesis.

In summary, this body of work details research into metamaterials and the search for potential areas of application of this new and interesting technology.
Chapter 2: Focusing effect of a metamaterial slab on the radiation pattern produced by a patch antenna

2.1 Introduction

Meshes made from metallic wires have long been used in various branches of science and engineering. Their applications in optics, radio, and electrical engineering include diffraction gratings, antenna reflectors, radio telescope dishes, shells of shielded wires, and shield screens (e.g., microwave ovens). Since the end of the 1990’s, meshes have also been considered as a platform for designing artificial media, metamaterials, with predefined effective electromagnetic properties. It has been demonstrated that a mesh of thin metallic wires can produce an effective permittivity, of the Drude type. In this regard, the mesh responds to external electromagnetic radiation like plasma within bulk metals. For plasma, the plasma frequency (\(\omega_p\)) is defined by the concentration of free electrons [7]. For the mesh, however, the plasma frequency is defined only by its geometric parameters; the mesh periodicity, \(a\), and the wire radius, \(r\). This allows one to design mesh media with desired values of the plasma frequency, particularly \(\omega_p\) in the GHz range. Because the mesh permittivity is close to zero near the plasma frequency, and negative below the plasma frequency, mesh metamaterials can be used wherever a small index of refraction, or a transition from transparency to opacity, is a crucial point. For example, proper plasma frequency placement, within the frequency domain, can be used to improve antenna pattern directivity [2, 3].
The effect of a mesh metamaterial slab on the radiation pattern of a source placed inside the slab has been examined in [2]. The case of an external source was studied in [3]. In both of the scenarios, antenna directivity improvement was observed, with [3] being much less pronounced than that reported in [2]. Using simple geometric considerations, the antenna directivity effect has been explained in both cases, as a consequence of the refractive index of the slab being close to zero and operating close to the plasma frequency of the slab. Obviously, such an explanation that takes into account only the geometry of refraction on the slab boundaries is not valid for the case of an external source of radiation, as considered in [3]. Furthermore, accounting for the weakness of the effect in the latter case, one can ask if it is possible, in general, to improve the directivity of radiation emitted by an external source and, if so, what is the cause of such improvement?

In improving radiation directivity using a slab of metamaterial, smallness of the effective refractive index, \( n_{\text{eff}} \), of the slab is a crucial point [2]. The minimal value of \( n_{\text{eff}} \) is near the plasma frequency, \( \omega_p \), of the slab. Therefore, it is vital to know the value of \( \omega_p \) in order to properly select the proper operating frequency of the slab. The value of \( \omega_p \) can be estimated either from theoretical calculations, based on the geometrical parameters of the mesh metamaterial, or from experimentally measured data, the transmission spectrum of the slab \( T(\omega) \). In the latter case, there are some arbitrary rules for selecting the point
of the $T(\omega)$ spectrum where $\omega_p$ exists. For example, $\omega_p$ corresponds to $T = 0$ in [8] and to the maximum $T$ value in [2].

In this chapter, we experimentally examine the effect of a mesh metamaterial slab on the radiation pattern of an antenna placed near the slab. The measurements are performed within a frequency range covering the plasma frequency, $\omega_p$, of the slab. We estimate $\omega_p$ value using two methods: Theoretical calculations using mesh geometry parameters, and experimentally via measuring the transmission spectrum of the slab, $T(\omega)$. In the latter method, $\omega_p$ is not defined as a specific point of the spectrum. Rather, it is defined by approximating the experimental spectrum with a theoretical one calculated for a slab with Drude-like permittivity. The slab’s effect on the directivity of the antenna’s radiation is determined by comparing the radiation patterns of the antenna with and without the metamaterial. The pattern’s peculiarities due to the slab are explained using simple physical considerations, and by taking into account the attenuation of electromagnetic waves within the slab, at frequencies close to the plasma frequency of the metamaterial.

2.2 Metamaterial Design and Fabrication

Our sample of a mesh metamaterial has been designed based on the thin wire mesh model [1], as shown in following expression for the plasma frequency:
where $r$ is the mesh wires’ radius, $a$ is the mesh periodicity, and $c = 3 \times 10^8 \text{ m/s}$ is the free space speed of light. Eq. (2.1) is derived for a mesh of thin wires assuming the mesh periodicity $a$ is much smaller than the operating wavelength $\lambda$, and the wires’ radius $r$ is much smaller than periodicity, $\lambda >> a >> r$. It should also be mentioned that according to Eq. (2.1), $\omega_p$ is dependent only on the mesh geometrical parameters and is independent of material parameters such as concentration of free electrons, $n$, and the effective electron’s mass $m^*$, which define the plasma frequency of a bulk material.

The metamaterial sample has been shaped in the form of a slab consisting of six identical plates of sizes 195x195x1.5 mm assembled in parallel at distance of 7 mm from each other (Fig. 2.1). The total slab thickness is 35 mm. The plates are manufactured from conventional electrical boards coated with copper\(^1\). Protoboard maker ProtoMat S62 (LPKF Laser & Electronics AG) milled flat copper grids with 0.1 mm thickness and a periodicity of $a = 7.4$ mm. The wires’ had a width of $d = 0.9$ mm. These plates were stacked to construct the metamaterial slab. The copper grids are active elements of the metamaterial.

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1 FR4 single layer copper, ½ ounce.
The theoretical value of the plasma frequency calculated from Eq. (2.1) is

\[ \nu_p = \frac{\omega_p}{2\pi} = 9.6 \text{ GHz}. \]

The corresponding plasma wavelength is

\[ \lambda_p = \frac{\omega_p}{c} = \frac{2\pi \nu_p}{c} = 3.1 \text{ cm}, \]

where \( c \) is the free space speed of light. Regarding our calculation of \( \nu_p \), the relative permittivity of the electrical board’s base material was simply assumed to be 1, and the half-width of the mesh wires was considered as their radius, \( r = d/2 \). The above mentioned requirement \( r \ll a \) is satisfied in our case since \( a = 7.4 \text{ mm} \) and \( r = 0.45 \text{ mm} \). Feasibility of the \( a \ll \lambda \) requirement depends on the operating wavelength. To achieve minimal values of the slab permittivity, we used radiation with \( \lambda \approx \lambda_p \) in our experiments. At the plasma wavelength, we have \( \lambda/a \approx 4.2 \).

Formally, one cannot assume \( a \ll \lambda \). Nevertheless, previously reported results obtained
with \( \lambda/a = 3.3 \) to 6.6 [1, 2, 3] give us hope to observe anticipated effects with our slab, as well.

2.3 Experimental Results

We measured the slab transmittance in the frequency range \( \nu = 7.5 \) to 12.5 GHz covering the theoretical plasma frequency, \( \nu_p = 9.6 \) GHz. Transmittance measurement experiments employed a radiating bi-ridged antenna\(^2\), and the received system employed a flared waveguide horn antenna. Both antennas were placed 3.0 m from each other, and the slab was located directly in front of the transmission antenna\(^3\).

The E-plane radiation pattern (angle distribution of the radiated power) of an antenna, with and without the slab, was measured over numerous frequencies, ranging from 9.8 to 12.0 GHz. Azimuth angle range used was \(-50^\circ < \theta < 50^\circ\). All pattern measurements were performed within a microwave anechoic chamber (Fig. 2.2). The effect of the metamaterial slab on antenna directivity is studied for a patch antenna placed at distance 3 m from radiating ridged horn antenna. Representative results of the measurements are shown in Fig. 2.3.

\(^2\) HP11966E, bi-ridged antenna.
\(^3\) The metamaterial was placed in direct contact with the face of the transmit antenna, but had no DC electrical contact.
Fig. 2.2. Experimental setup: The slab of metamaterial in front of a patch antenna (left) within an anechoic chamber. The metamaterial sets on top of a Styrofoam stand, all of which sits on a stand that is rotated 180° to obtain the antenna pattern.

As one can see from Fig. 2.3a, the slab transmittance changes substantially in the frequency range 10-12.4 GHz, from 0% to 40%. From Fig 2.3b, it can be seen that use of the slab improves the radiation pattern of the patch antenna at some frequencies making its main lobe narrower, and the side lobes less intensive than the original pattern (black dotted line). Note the radiation patterns with a half power beam width less than 20°, at 10.9 (blue curve) and 11.7 GHz (red curve). At other frequencies, an example at 11.2 GHz (green curve), did not display appreciable improvement of the radiation pattern. In fact, other frequencies displayed a significant degradation in antenna radiation pattern performance.
Fig. 2.3. (a) Experimental transmittance spectrum of the metamaterial slab. (b) Radiation patterns of the patch antenna with and without the slab at different frequencies. The black dotted line is the antenna pattern for the patch antenna without any metamaterial mounted to it which remained the same across all frequencies measured. The blue line measurement was taken at 10.9 GHz and shows the most directivity, the red line was taken at 11.7 GHz and the green (which shows near total attenuation) was taken at 11.2 GHz.

2.4 Discussion

The frequency dependence of the slab transmission, as well as the slab effect on the radiation pattern of an antenna can be explained by the frequency dependence of the effective permittivity, \( \varepsilon_{\text{eff}} \), of the slab. For a mesh of metallic wires, \( \varepsilon_{\text{eff}} \) turns out to be of Drude type [1]:

\[
\varepsilon_{\text{eff}}(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma \omega},
\]

(2.2)

where the plasma frequency, \( \omega_p \), is defined by (1), and the damping factor \( \gamma \) depends on the conductivity of the wires’ material. For a mesh of extremely conductive material, as
that normally seen in metals, $\gamma$ is of order $0.1 \omega_p$ [1]. The effective index of refraction of the slab can be defined from Eq. (2.2) in the usual way: $n_{\text{eff}} = \sqrt{\varepsilon_{\text{eff}} \cdot \mu_{\text{eff}}}$. Assuming $\mu_{\text{eff}} = 1$ (nonmagnetic slab\(^4\)), this becomes $n_{\text{eff}} = \sqrt{\varepsilon_{\text{eff}}}$. Like $\varepsilon_{\text{eff}}$, the effective index $n_{\text{eff}}$ is a complex quantity: $n_{\text{eff}} = n_{\text{eff}}' + i n_{\text{eff}}''$, where $n_{\text{eff}}'$ and $n_{\text{eff}}''$ are the real and imaginary parts of $n_{\text{eff}}$, respectively.

It follows from Eq. (2.2) that properties of the metamaterial change substantially near the plasma frequency, $\omega_p$. More specifically, an abrupt transition occurs from a strong attenuation, when $\omega < \omega_p$, to a weak attenuation, when $\omega > \omega_p$. Experimentally, the transition should show itself as a substantial increase in the slab’s transmission coefficient. We observe this within the frequency range $\nu = 10.5 - 11.5$ GHz, see Fig. 2.3a. Another consequence of Eq. (2.2) is that a mesh metamaterial possesses $n_{\text{eff}}' < 1$ near $\omega_p$. This can be used to improve directivity of electromagnetic radiation emitted by a source placed inside the metamaterial slab [2]. In order to use this effect to the maximum extent possible, one must select an operating frequency close to $\omega_p$ but still greater than $\omega_p$. This fact makes the estimation of $\omega_p$ a very important problem.

Estimation of $\omega_p$ from experimental transmission spectra, $T(\omega)$, has been performed in Refs. [1, 2]. In both references, however, $\omega_p$ corresponds to a different point within the spectra. In [1], $\omega_p$ corresponds to transition from minimal transmission to ‘significant’

\(^4\) FR-4 materials with ½ ounce copper cladding is assumed to have $\mu_{\text{eff}} = 1.0$, for our experiments.
transmission. [2] corresponds to maximal transmission. Theoretically, $\omega_p$ does not correspond to either minimum or maximum transmission, see Fig. 2.4. Therefore, at which point should we treat as corresponding to $\omega_p$, of an experimental transmission spectrum, $T(\omega)$?

Fig. 2.4. Calculated transmittance of a slab with Drude-like permittivity $\varepsilon(\omega)$ for different values of the damping factor $\gamma$ (expressed in $\omega_p$ units). The slab thickness is chosen to be $1.1 \lambda_p$, as for the real slab.

To estimate $\omega_p$ from the experimental transmission spectrum shown in Fig. 2.3a, we fitted the spectrum with a theoretical one. The theoretical spectrum and the spectrum displayed in Fig. 2.4 were calculated using the standard expression for a slab
transmission coefficient [8] by using the effective permittivity, Eq. (2.2), for the slab material. The slab thickness was assumed to be the same as our metamaterial sample, $d = 3.5$ cm. The plasma frequency $\omega_p = 2\pi\nu_p$, and the damping factor $\gamma$, of the metamaterial were used as fitting parameters. The best fitting result is shown in Fig. 2.5. The metamaterial parameters estimated from this fit are $\nu_p = 10.5$ GHz and $\gamma = 0.065\omega_p$. The $\nu_p$ value differs by about 9% from the theoretical value, $\nu_p = 9.6$ GHz, calculated from Eq. (2.1). Note that the theoretical spectrum (the solid line in Fig. 2.5) is a very good approximation for the experimental one (the dotted line in Fig. 2.5) at frequencies up to 11.5 GHz. It is in this frequency range the transition in the slab transparency occurs, enabling us to define its plasma frequency. It should also be mentioned that the slab transmission at the plasma frequency, $T(\nu_p)$, is about 5% in both the Figs. 2.4 and 2.5. This result is closer to that in [1], where $T(\nu_p) \approx 0$, whereas in [2] $T(\nu_p) = \text{max}$. 

![Graph showing transmittance vs frequency](image)
Fig. 2.5. Theoretical fit to the experimental transmission spectrum. The \( \nu_p \) vertical dotted line marks the experimentally observed plasma frequency. The value of 10.5 GHz is within 9% of the theoretically predicted value of 9.6 GHz.

Let us now analyze the angle dependence of the radiated power shown in Fig. 2.3b. In Fig. 2.3b, one can see the more directive nature of the main lobes at frequencies of 10.9 and 11.7 GHz, which are somewhat higher than the plasma frequency, \( \nu_p = 10.5 \) GHz; 1.04 \( \nu_p \) and 1.11 \( \nu_p \), respectively. Thus, this displays antenna directivity improvement (more directive). At these frequencies, the index of refraction of the slab should be less than 1. Such an improvement was reported earlier [2, 3] and was explained by a specific way in which a transparent slab with \( n_{\text{eff}} < 1 \) bends rays incident on its boundaries. That explanation is based on simple geometrical optics consideration. Note, however, that according to geometrical optics, the slab effect on radiation directivity occurs only if the source of radiation is placed inside the slab (Fig. 2.6a). It is this case that was applied in [2]. In the case of an external source, which was applied in [3], as well as in our experiments, a slab should not affect directivity of the source radiation. After leaving the slab, each source ray undergoes two consecutive refractions at each of the slab boundaries, resulting in only lateral displacement of each ray (Fig. 2.6b). This fact was not considered in [3]. What, then, is responsible for the slab effect observed in [3], and in our experiments?
Fig. 2.6. Effect of a transparent slab with index of refraction less than unity on radiation from a source placed (a) inside and (b) outside the slab. In case (a), radiation is concentrated in directions around the slab’s normal. In case (b), there is no concentration effect and ray spreading should occur.

The directivity improvement can be explained, if the following two factors are considered simultaneously: The above mentioned phenomenon of rays bending on boundaries for a slab in which $n_{\text{eff}} < 1$, and the appreciable attenuation of electromagnetic waves inside the real slab. Indeed, even at frequencies larger than the plasma frequency, $\nu > \nu_p$, the real slab is not completely transparent. This is confirmed since the slab transmission magnitude is less than 1; actually, less than 0.4 for our slab in the frequency region in
which we are interested. Then, the larger the angle a ray travels at with respect to the slab’s normal, the larger the distance it will travel inside of the slab and, thus, the more attenuated it will experience before exiting the slab, see Fig. 2.7. Stronger attenuation of the peripheral rays manifests as the beam concentration near the slab’s normal. Note also that for an opaque slab $n_{\text{eff}}^\prime \neq 0$, and we should use $n_{\text{eff}}^\prime$ instead of $n_{\text{eff}}$.

\[
\begin{align*}
n_1 &= 1 \\
n_2^\prime &< 1, \\
n_2^\prime &\neq 0
\end{align*}
\]

**Fig. 2.7.** Effect of an opaque slab with index of refraction less than unity on radiation from an external source. The arrow thickness is proportional to the intensity of radiation.

It is clear from the above, that the slab’s effect on the directivity of an external source antenna will lessen with increased slab transparency, or index of refraction. We observed both a lesser directivity, as well as a higher transmittance, as the slab’s operating frequency increases. A decrease in the operating frequency down to $\omega_p$ results in a
decrease in the slab’s index of refraction. This is advantageous for the directivity improvement, as the material attenuation increases. However, the improvement at frequencies extremely close to $\omega_p$ is hardly distinguishable, due to the very strong attenuation of radiation inside the slab. Recall that the slab’s transmittance at the plasma frequency is close to 5%. Furthermore, the directivity improvement might be influenced by the interference effects that are well known in optics of thin films, but this is not considered in this chapter, nor is the meta-antenna efficiency factor which will also play a role, but most likely in an increased directivity manner. These effects might be responsible for the absence of improved directivity, at some frequencies in the examined frequency range where the necessary conditions ($n'_{\text{eff}} < 1$ и $n''_{\text{eff}} \neq 0$) are seemingly satisfied. All of this makes the directivity effect even more evident within only a narrow frequency range close to the plasma frequency, $\omega_p$, of the mesh metamaterial.

2.5 Chapter Conclusion

We experimentally studied the transmission coefficient frequency dependence, $T(\nu)$, of a slab made from a mesh metamaterial, and the effect of the slab on antenna directivity when placed externally near the slab. Measurements were performed in a frequency range near the theoretical plasma frequency, $\omega_p$, of the metamaterial. The experimental value of the plasma frequency was estimated not by using specific points of the $T(\nu)$ curve, but by approximating the experimental curve with a theoretical one that was calculated for a slab with Drude-like permittivity. The experimental and theoretical values of $\omega_p$ differ
from each other by approximately 9%. Thus, the theoretical expression for \( \omega_p \) can be applied not only in the case of cylindrical wires, but also for flat wires. Our experiments confirm previously reported slab effects on antenna directivity at operating frequencies near the slab’s plasma frequency, \( \omega_p \). However, this effect cannot be explained merely by the fact that the effective index of refraction of the slab \( n_{eff}' \) is less than 1, with \( \omega \geq \omega_p \). We attribute the effect also to the attenuation of the electromagnetic wave inside the slab, among other issues not detailed within this chapter. We predict that the frequency range in which the effect is pronounced should be quite limited. This prediction is due to the fact that the effect vanishes in the case of both strong attenuation (at \( \omega \) close to \( \omega_p \)) and weak attenuation (at \( \omega \) far from \( \omega_p \)), again along with other issues not detailed within this chapter. Our results can be used in designing novel emitting and reception systems for which spatial or frequency discrimination is an important requirement.
Chapter Three: Low-loss Negative Index

Metamaterials for X, Ku, and K Microwave Bands

3.1 Introduction

The research presented in chapter two was interesting but the large loss defeated many potential uses. To make this technology viable, loss needed to be lowered and a design process needed to be solidified. Recent research has shown some improvement in the design of metamaterials [18]. Metamaterials that have low insertion losses, wide bandwidths, and multi-frequency responses are prime candidates for use in microwave applications. The main objective of this chapter was to develop a design process for low-loss, wide bandwidth microwave metamaterials. Another issue with microwave negative index metamaterials (NIMs) is the mismatch between theoretical and experimental responses. For example, errors in center frequency of NIM pass-band filters are often unacceptable for practical applications. The second goal of this research was to achieve a better agreement between the theory and experiment by combining near-field imaging of electromagnetic waves (EM) and standard transmission measurements of the S21 to simulations. Finally, our goal was to design several metamaterial samples in different microwave bands to validate frequency scaling of the design process.

We used a commercial finite element method solver, ANSYS HFSS, and near-field infrared imaging, to adjust the parameters as described in Sections 3.2 and 3.5. These tools are essential in validating the propagation modes within the NIMs and their
corresponding EM fields. In this chapter, we present the experimental and theoretical results obtained for low-loss, wideband, high-fidelity NIMs for applications in the X (8-12.5 GHz), Ku (12.5-18 GHz), and K (18-27 GHz) microwave bands. Specific design constraints were introduced to improve operational bandwidth, and a dielectric with a low loss tangent was used to lower the insertion loss.

3.2. Design of Microwave Metamaterials

Currently, there are several different designs of metamaterials based on S-shaped split ring resonators (SSRR), as summarized recently by [19]. Accordingly, due to its low-pass and wideband characteristics, we selected the SSRR as a unit cell for our NIM designs. The magnetic permeability of a unit cell is a function of the geometrical parameters of the cell and is given by [4]

$$\mu_{eff} = 1 - \frac{2F+iA}{1-\frac{1}{\omega^2\mu_0\rho_{a}}\left(\frac{3}{\varepsilon}\right)B+iG},$$  \hspace{1cm} (3.1)

where:

$$A = \frac{2 \omega \mu_0 R F^2 I a b}{(\omega \mu_0 F a b)^2\left(1-\frac{1}{\omega \mu_0 F a b c \varepsilon}\right)}$$

$$B = \frac{(RI)^2}{(\omega \mu_0 F a b)^2\left(1-\frac{1}{\omega \mu_0 F a b c \varepsilon}\right)}$$

$$G = \frac{(RI)(2\omega \mu_0 F a b - \frac{2^2}{\varepsilon \omega})}{(\omega \mu_0 F a b)^2\left(1-\frac{1}{\omega \mu_0 F a b c \varepsilon}\right)}.$$
\[ C_s = \varepsilon \frac{hc}{d} + \varepsilon \frac{hc}{l-d} \]  

(3.2)

The parameters \( a, b, h, c, d \) and \( l \) are the geometrical parameters shown in Fig 3.1; \( R \) is the resistance of the metallic strips in each loop; \( F \) is the fractional area of the surface covered by the loop in the S shape.

Eqns. (3.1) & (3.2) were used to estimate the resonance frequency of the cell while minimizing its electromagnetic losses. In addition, we used geometrical constraints on the metal (copper) portion of the unit cell. The dimensions of the unit cell and the spacing and thickness of the board are important design parameters as shown in Fig. 3.1. We selected the thinnest (0.508 mm) commercially-available board for use in our final designs.

Fig 3.1. (left) Example of HFSS simulation of SSRR with E-field and H-field orientation and wave propagation (k) direction; (right) unit cell with conventional dimensions labeled.
The size constraint of the unit cell was that each dimension for both designs had to be less than one-fifth of the wavelength ($\lambda/5$). The prism materials consisted of 120 equally-spaced boards. The dimensions of the metamaterial cell are summarized in Table I. The model was extended to arrays of unit cells in order to simulate transmission in NIM flat slabs and prisms. The rationale for these constraints in the design of the K-band was to ensure that the material would be less sensitive to the orientation of incident power and for ease of manufacturing.

### Table I. Geometrical parameters of the SSRR Unit Cells

<table>
<thead>
<tr>
<th>Band</th>
<th>a</th>
<th>b</th>
<th>h</th>
<th>w</th>
<th>c</th>
<th>d</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Ku</td>
<td>5.2</td>
<td>4.0</td>
<td>5.0</td>
<td>2.8</td>
<td>0.4</td>
<td>0.51</td>
<td>2.0</td>
</tr>
<tr>
<td>K</td>
<td>3.0</td>
<td>3.0</td>
<td>2.04</td>
<td>2.04</td>
<td>0.2</td>
<td>0.51</td>
<td>1.0</td>
</tr>
</tbody>
</table>

#### 3.3. Fabrication of Prisms

The metamaterial boards were fabricated commercially by CircuitsWest, Inc. using RO4003C with uniformly-sized unit cells arranged in regular arrays on both sides of the board with 10 unit cells in the direction of propagation of the electromagnetic waves and 50 unit cells in the perpendicular direction, as shown in Fig. 3.2. The uniform-sized boards were assembled into a slab by using a holder with equally-spaced slots. The slabs of material were characterized for their transmission and reflection coefficients (S-parameters) in the near-field by holding all components stationary on a bench and using
two microwave feeds and a network analyzer. After testing the S-parameters with the uniform slabs, the boards were cut into stair-step prisms to test the refraction of the electromagnetic waves.

![Image](image1.jpg)

**Fig 3.2.** (a) Photographs of the metamaterials with S-shaped, split-ring resonators; (b) K-band board and X-Ku prism with a 8.1 degree pitch; (c) K-band prism 13.6 degree pitch. Each prism achieved a maximum length in the direction of propagation with ten S-shaped unit cells. Each prism had eight stair stepped sections.

These angle and frequency-dependent characterizations were conducted in an anechoic chamber at the U.S. Air Force Academy (USAFA). Fig. 3.3 shows the metamaterial being tested in the anechoic chamber and on the benchtop using the Agilent 8753ES network analyzer. In this setup, the tripod holding the microwave receiving antenna rotated ~ 180 degrees, while the material being tested was held in an absorbing foam window. The material was stationary on top of a Styrofoam box with floor supports above the rotating platform. Transmission measurements were recorded over a semicircle of the receiving antenna. The distance from the transmitter to the receiver was 9.14 meters, ensuring the far-field measurement conditions.
Fig 3.3. Diagram of the anechoic chamber (left) and photo of the test setup (right), which was used to test the metamaterials. The left side shows a sketch of the anechoic chamber with the rotating pedestal on which the sample was mounted. The distance between the transmitting antenna and the sample was 9.14 m. The distance, d, between the sample and the receiving antenna was 0.76 m.

3.4. Experimental and Theoretical Characterization of the Metamaterials

Fig. 3.4 shows far-field experimental and simulated transmission spectra ($S_{21}$), vertically-polarized incident electromagnetic wave through the uniform slab of metamaterial. Within the target frequency (12-13 GHz), the slab of metamaterial had a high transmission band with less than 5 dB attenuation. The HFSS simulation results compared fairly well with the experimental transmission spectra, with the transmission band around 11.5-13 GHz, with the negative index beginning about 12.5 GHz and
occurring at higher frequencies. The model and experimental result on the high-frequency (negative index) side of the pass-band indicate good agreement.

![Fig. 3.4. Comparison of the experimental measurements and HFSS theoretical simulation (dotted curve) of the transmission $S_{21}$ through the homogeneous slab of metamaterial designed for the X-Ku band. The vertical dashed line indicates the positive and negative index of refraction sides for the transmission; the dashed vertical line delineates the positive and negative indices of refraction.](image)

After measurements of the transmission through the slab, several ‘stair-step’ prisms were modeled to estimate index of refraction for selected frequencies. Accordingly, a X-Ku uniform slab was cut into a prism to prove directly the negative refraction of a plane electromagnetic wave transmitting through the prism. An example of a simulation of transmission through prism is shown in Fig. 3.5, with a diagram of the prism.
Fig 3.5. An HFSS simulation (left) at 12.5 GHz through an extended material array transmission through a prism (right) is shown. The ‘stair-step’ prism was created by eliminating one cell depth per step. Transmitted energy through the prism is considered positive or negative depending on which side of normal surface of the prism. At 12.5 GHz, as shown the index transitions to negative.

An 8.1° prism was next constructed and was inserted into a window of microwave-absorbing foam and held stationary while the receiver was rotated in a semicircle, as depicted in Fig. 3.3 of the anechoic chamber. This process was repeated at different frequencies across the X-Ku band (8-18 GHz) to generate spectral-spatial measurements, as shown in Fig. 3.6. Negative angles indicate negative indices of refraction.
Fig 3.6. Experimentally-measured intensity of the transmitted wave as a function of the frequency and angle. The angle is measured relative to the normal of the prism so negative angles correspond to a negative index of refraction whereas the positive angles correspond to a positive index of refraction.

Fig. 3.7 shows the experimentally-measured power of the electromagnetic wave refracted at the back surface of the prism at different angles relative to the normal to the surface. The measurements were performed at different frequencies in the X-Ku range. Negative angles of refraction correspond to negative indices of refraction according to the Snell’s law, i.e., $n_m \sin \theta_m = n_a \sin \theta_a$, where $n_m$ is the index of the metamaterial, $n_a = 1.0$ is the index of air, $\theta_m$ is the angle of incidence inside the prism (which is equal to the angle of the prism), and $\theta_a$ is the measured angle of refraction, which corresponds to the measured angle of peak transmission plus the angle of incidence inside the prism. The measurement of power-angle transmission with the most pronounced negative refraction...
occurred between 12.5 and 15 GHz. For instance, at 13.75 GHz the data indicated a peak transmission power at about $-22^\circ$, as shown below.

Fig 3.7. Experimentally-measured power as a function of the angle of refraction of a vertically-polarized electromagnetic wave (the frequency 13.75 GHz,) by the prism. The peak at -22 degrees relative to the normal to the surface of the prism indicates negative refraction by the prism.

Fig. 3.8 shows the negative index of refraction of the X-Ku prism calculated by using Snell’s law. For instance, the index of refraction was $n_m \approx -3.0$ at 13.5 GHz with a negative index well over a 2-GHz band. Further, the insertion loss measured for the X-Ku material was 1.69 dB/cm or, equivalently, 0.42 dB/cell, which was comparable to previous SSRR designs. Overall, the results of the X-Ku measurement showed a negative index with modest insertion loss. The positive indices below frequencies of 12.5 GHz are indicated by the blue line with pronounced peak prior to sign transition; the negative indices are indicated by the red line.
Fig 3.8. Experimentally-determined indices of refraction of the metamaterial fabricated for the X-Ku band (blue line shows positive indices; red line shows negative indices). This was determined by taking the maximum intensity observed at a given frequency and assuming that is the refracted beam.

3.5 Experimental and theoretical results for the K-band slab

The specific design resonance frequency of 18-20 GHz had a target negative index sub-band of 20.25 to 21.25 GHz. The sub-band corresponded to an existing satellite signal of interest, which may be explored in future investigations. As stated previously, a slightly different design method was used in the K-band to impose symmetry on the unit cell and uniform active area constraints. As seen in Fig 3.9, the simulated and experimental data showed somewhat different bandwidths but better agreement than had been seen for the X-Ku band material in Fig 3.4 or from other authors [4]. The simulated data predicted a band pass region (when attenuation less than -3dB) from 19.6 GHz to 21.2 GHz. The
experimental results gave much wider band pass region of 18.7 GHz to 21GHz, this result is very welcomed but an oddity as to why they do not agree.

Fig 3.9. Comparison of the HFSS simulation (dotted blue line) and experimental (red) transmission through K-band (20 GHz) metamaterial: The positive to negative index transition is indicated with a vertical dashed line near 20 GHz.

Fig. 3.10 shows a predominantly positive index material for frequencies under 20 GHz and becomes predominately negative index between 20.25 to 21.25 GHz. A fundamental transition occurred near that frequency. The power that was transmitted through positive angles at lower frequencies shifted to negative angles; the material became a predominantly negative-index material.
Fig. 3.10. The experimentally-measured intensity of the transmitted electromagnetic wave is shown as a function of the frequency and angle relative to the normal surface of the prism.

Fig. 3.11 shows the transmission measurements at 21 GHz (K-band). At this frequency, the data indicated that the peak transmission power occurred at about -40°. The insertion loss for the K-band was measured to be 0.95 dB/cm or, equivalently, 0.285 dB/cell. We attributed the modest reduction in insertion loss over similar SSRR unit to the increased homogeneity due to unit cell symmetry of the design, but further analysis is needed to confirm this assumption. The insertion loss, is nearly one dB less than had been previously observed [4].

Fig. 3.12 shows that the index values that were measured for a 12.5° prism. In the figure, the positive index of refraction in this dataset is on the lower frequency side of the center frequency, while the negative index is on the high frequency side.
Fig 3.11. Experimentally-measured absolute power as a function of the angle of refraction of the vertically-polarized electromagnetic wave (frequency = 21 GHz, K-band) by the metamaterial prism.

The peak at -40 degrees relative to the normal to the surface of the prism indicates negative refraction by the prism.

Fig 3.12. Experimentally-determined index of refraction of the metamaterial fabricated for the K band: Positive indices in blue; negative indices in red. This was calculated by assuming the maximum power transmitted power at any frequency corresponds to the refracted beam.
3.6. Infrared Imaging of the Transmitted EM Waves

Infrared imaging is a proven experimental tool for studying EM waves.[20] Imaging the EM fields as they exit the prism is an additional means of comparing the output of models and experimental measurements, which might well reveal microwave modes, as in the waveguides. The equipment setup shown in Fig 3.13 uses an IR camera (3-5 microns) to capture images projected on Kapton™ HN film at graduated distances of 10, 20, 30, 40, and 50 mm.

Our analysis indicated that a thermal contrast range of 5.24 °C was possible in the infrared based on a power of 2 W, an area of 12 in², and Kapton film-specific heat specifications. Fig 3.14 shows a few calibration images (21-26 °C) through the X-Ku metamaterial that show lobes of energy for 12.5 GHz. The maximum thermal contrast

**Fig 3.13. Test setup for IR imaging of EM fields, Kapton HN Film is not conductive enough to perturb the field.**

...
apparent in these images was about 5 °C, which was slightly less than the expected value of 5.24 °C. The reasons for the lower measured contrast might be power loss in the cable-antenna interface and/or unaccounted-for propagation losses between the antenna and the Kapton film. In the closest IR frames (bottom, incremental distances), the dominant heat profile indicated a negative refraction. However, farther away from the prism, the dominance of the negative index side energy is not as clear.

![Graph showing IR images](image)

**Fig 3.14.** IR images are perpendicular to propagating wave taken with IR imaging. The IR images were taken at distances of 0, 10, 20, and 30 mm from the surface of the metamaterial using Kapton film.

A higher resolution simulation for the incremental distances is shown in Fig. 3.15 for the 20 mm distance from the prism. The IR image for the same distance is below the HFSS simulation. The thermal contrast (21-26 °C) translates linearly to the E-Field scale (1200-3500 V/m). At this resolution, the dual transmission lobes were evident in both the simulated and measured results; as a result, new spatial dimensions for validating simulations and measurements are possible.
Fig 3.15. HFSS E field intensity (top) with lobes compared to an IR image (bottom) at (1 cm) from the prism

3.7. Chapter Conclusion

Microwave-domain EM metamaterials are fairly well-developed. However, issues remain that must be resolved in order for industry to fully embrace these materials as viable solutions for issues associated with other devices. NIMs still exhibit high insertion loss and difficulty of design; in addition, uses of the devices traditionally have been difficult due to the inhomogeneity of the unit cells. To help address these remaining deficiencies, this chapter reports the results of a design process using specific constraints on the SSRR
and high-grade materials designed for the microwave domain to keep insertion loss low. These constraints and materials lead to us producing much wider bandwidth than expected from theory. Additionally, this wider bandwidth was close to agreement with what was seen in the simulation which had not been observed previous work [4].

Table II reports the size constraints of the SSRR unit cell and the board for both designs. The K-band design required the most stringent constraints due to the symmetry of its unit cell to enhance homogeneity and electrically-large, overall configuration to avoid diffraction. The K-band design added the following constraints to the design of the SSRR unit cell parameters: \( w/b \) ratio < 0.7, \( h/a \) ratio < 0.7, \( a/b = w/h = 1 \). The 0.7 ratio was chosen because it produced a large band pass region in simulations which was seen to be even greater than expected in experimental results.

**Table II. Design of Unit Cell SSRR and Material Constraints**

<table>
<thead>
<tr>
<th>Band</th>
<th>( \lambda ) (mm)</th>
<th>( \lambda/x )</th>
<th>S-Shape SSR Cell Details</th>
<th>Board Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Ku</td>
<td>25</td>
<td>0.77</td>
<td>0.7</td>
<td>x = 5.9</td>
</tr>
<tr>
<td>K</td>
<td>15</td>
<td>1</td>
<td>0.67</td>
<td>x = 10.0</td>
</tr>
</tbody>
</table>

It is important to note that the SSRR design offers a large pass-band, but only a portion (slightly less than half the pass-band) represents the dominant negative-index properties. That portion is located on the high-frequency side of the pass-band, as presented in Table III, along with the insertion loss. Since the raw materials in both the X/Ku and K-band were the same, the design approach using unit cell symmetry (in the K-band design)
differentiates it from other similar designs, which could be a possible explanation for more predictable agreement between simulation and test results.

Table III. Negative Sub-bands and Insertion Loss

<table>
<thead>
<tr>
<th>Band</th>
<th>Res. Freq. (GHz)</th>
<th>Negative Subband (GHz)</th>
<th>Insertion Loss dB/cm</th>
<th>Insertion Loss dB/cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Ku</td>
<td>12.5</td>
<td>13.5 (+/-) 0.75</td>
<td>1.69</td>
<td>0.422</td>
</tr>
<tr>
<td>K</td>
<td>20</td>
<td>20.75 (+/-) 0.5</td>
<td>0.95</td>
<td>0.285</td>
</tr>
</tbody>
</table>

The approximation of the exact symmetry in the unit cell design correlated well with simulation-test fidelity and may facilitate future 3-D prototypes. Furthermore, metamaterials based on symmetrical unit cells are easier to fabricate in bulk. When combined with direct far-field microwave measurements, IR imaging enhances the investigator’s ability to compare EM fields in high-resolution 3-D to verify negative indices within frequency ranges. As a result, new spatial dimensions for validating simulation and measurement are possible.
Chapter Four: Infrared Imaging of Microwave Negative Index Metamaterials

4.1 Introduction:

The Infrared imaging conducted in chapter three was extremely promising and we wanted to fully develop the technique as applied to metamaterials. Infrared imaging is a proven experimental tool for studying electromagnetic waves [20], however, to our knowledge, its utility for electromagnetic research of metamaterials has not been explored previously to our work. Although infrared imaging is an indirect measurement of the electric field intensity, these measurements are highly correlated for the direct microwave transmission measurement, as a result, the image analysis of these material properties afford precise mapping in higher dimensions without the graduated positioning that direct measurements might require; for example, microwave transmission can be projected in three dimensions (3-D) through uniform as well as irregular-shaped materials rendering a spatial projection that is difficult to achieve with direct microwave measurements.

Since the electric field incident on heat-sensitive films transduce a proportional response, heat contrast creates an image using commercially available infrared cameras, translating electric field (V/m) to temperature (degrees). The high spatial resolution of detector elements in the camera and the calibrated temperature sensitivity
of the film, enable an accurate digital image of the electric field. In the near-field, microwave wave on the order of 1Watt have sufficient power to radiate through a metamaterial slab or prism (ten cells thick) and produce transmission that can be imaged when the entire field-of-view of the camera and exposed film are carefully aligned.

4.2 Theory

As laid out in previous chapters, there are many excellent sources devoted to the theory of metamaterials and their applications [15,21,22]. For this reason, only a brief introduction is appropriate in order to focus on the infrared imaging measurements and analysis for metamaterials. In short, metamaterials exhibit a number of unique properties, such as negative index of refraction, when their permittivity ($\varepsilon$) and permeability ($\mu$) are both negative at the same frequency. The index of refraction ($n$) is related to the dielectric permittivity and magnetic permeability of a medium by Eq. 4.1.

$$n = \pm \sqrt{\varepsilon_r \mu_r}$$  \hspace{1cm} (4.1)

In addition, the index of refraction of a material is defined as the ratio of the speed of wave propagation ($c$) in a vacuum to the speed in the medium($v_m$) as

$$n_m = c / v_m$$  \hspace{1cm} (4.2)
The theory of the negative index of refraction was first suggested by Veselago [9]. This theoretical work evaluated the possibility of materials with negative values of permittivity and permeability and proposed that such materials were consistent with Maxwell’s equations for electromagnetism. Three decades later, a renewed interest in metamaterials was generated [1,11,12] which proposed that resonators could provide negative permeability, while a wire mesh could provide negative permittivity for a particular matching frequency. All of this research supports the variation of Snell’s law (Eq.4.3) where the index of refraction ($n$) of an engineered material can be negative at some frequencies.

$$n \sin(\theta) = n \sin(\theta_m)$$

Eq.4.3

Research done in [18], explained a phenomena described as “enhanced diffraction” witnessed in experiments of gratings on surfaces of metamaterials. This is a modification of Snell’s law for this particular form of the material, to account for a minor additional beam of energy measured in the transmission through a ‘stair-stepped’ prism surface.

$$\sin(\theta) = \frac{m \lambda}{d} + n \sin(\theta_m)$$

Eq.4.4

The additional term, $\nu_m$, is considered relevant when the ratio of those terms is on the same order; in the case of microwave metamaterial design, that occurrence is not unusual. Note that the additional term is not a function of angle, and thus can be discriminated. Because of issues such as the ‘enhanced diffraction’ that was observed in Smith’s 2000 paper but not reported and not understood until 2004, the analysis of
metamaterials is not a simple, straight-forward task. Reconciling data from similar designs and experimental methods is sometimes complex, inviting new approaches to study the electromagnetic properties more completely.

The use of infrared imaging to measure electric field intensity is not new; its theory and utility in are well-established, nonetheless its use in metamaterial research is novel. The conversion of EM field strength onto a heat-sensitive film projects a thermal contrast on an image plane that can be imaged by an infrared camera, creating a high-resolution 2-D image representing the electric field. When a series of images at incremental distances are collected, a 3-D rendition of the electric field intensity is captured. When electromagnetic waves within the design frequencies are transmitted through the metamaterial, transmission occurs on the negative side (as shown), otherwise transmission is dominant on the positive side of the normal midsection line, as depicted in Fig. 4.1.

![Image of negative refraction measurements](image-url)

Fig 4.1. A sketch of the negative refraction measurements based on the near-field 3-D Infrared Imaging. Each ‘image plane’ is a sheet of Kapton film which is not conductive enough to perturb the field but would still allow for some conduction losses on its’s surface.
With image processing techniques, the array element (pixels) are translated into corresponding radial units that can be mapped to a microwave horn’s primary beam 3 dB point. When a material (in the form of a prism) is introduced for testing, the baseline thermal contrast due to the primary beam, is over-laid onto the refracted image and a straightforward comparison is made to measure the angle of refraction.

The calculation of the index of refraction for each image plane is as follows: 1) Align 3 dB point to image, by pixel assignment, make conversion factor, 2) Identify prominent peak (if any) on ‘negative side’ of image, 3) Measure the refracted beam peak from image center (primary beam), 3) Translate pixel unit offset to radial units and 4) calculate index by Eq. 4.5.

\[ n_m = \frac{\sin(\theta_{measured})}{\sin(\theta_{prism})} \]  

(4.5)

The procedure above is repeated for a series of image plane if a trend in indices are desired. A stack of images can be generated to illustrate the high resolution measurement of 3-D mapping of electric field intensity. This data set enables contours, indices gradients, etc. for detailed analysis.

### 4.3. METAMATERIAL DESIGN

There are several different metamaterial designs based on SSRRs, as described in chapter 3. Accordingly, we selected the SSRR as a unit cell for our NIM designs for its low-loss and wideband characteristics. The reader is referred to [23] for a detailed
development of the SSSR; sufficient to say the prototype developed for this chapter was designed for low-loss transmission in the 12-15 GHz range. The metamaterial construction process is the same as described in chapter 3.

After testing the S-parameters with the uniform slabs, the boards were cut into stair-step prisms to test the refraction of the electromagnetic waves as shown in Fig. 4.2. Because the material is made of discrete unit cells, much like a crystal, the prism construction results in single side stepping slope. The design in this study had a slope of 8°.

Fig 4.2. A picture of the prism manufactured with NIMs and diagram indicating step sizes in the prism. The step size for this prism was on the order of a wavelength, ie: \( d \sim \lambda \) causing the phenomenon of “enhanced diffraction”

According to Smiths’ derivation, when \( d << \lambda \) in Eq. 4.4 no diffracted beams are expected, but when \( d \) is on the order of the wavelength \(( d \sim \lambda \) ) diffracted beams due to
the stair step prism should be expected. In our design, there were 120 boards divided into 8 steps, or 15 boards per step. Each board has a 2mm spacing, resulting in a prism step size of 30mm. At 12-15 GHz, the wavelength is 25-20mm; so, the step size and wavelength are indeed on the same order; meaning that the ‘enhanced diffraction’ beam described by Smith is expected in our measurements. It is essential to recognize the enhanced diffraction lobe during analysis, but it will effectively be ignored in the evaluation of the index of refraction because it is associated with the irregular stepping in the prism and is not evident in uniform slabs.

4.4. NEAR-FIELD INFRARED MEASUREMENTS

Fig. 4.3 shows the experimental setup to measure the near-field distribution of an electromagnetic wave propagating through a metamaterial prism. A generator is connected to a power amplifier to provide a sinusoidal signal of 2 W power at 12.5 GHz (within the 12-15 GHz design band). A mid-IR (3-5 micron) camera (Merlin-Indigo) is positioned at 1.5 m distance from the surface of the heat-sensitive Kapton film mounted on a microwave-transparent Styrofoam. The camera has a 320x256 InSb detector array with 25 mK temperature sensitivity based on 12-bit ADC at 60 Hz frame rate [24].
Fig 4.3. A sketch of the experimental setup for IR imaging of electric field intensity.

To establish the baseline, an image was captured as shown schematically in Fig. 4.1 without the metamaterial prism. As with most dense IR detector arrays based upon InSb, individual detectors can fail due to detector-readout electronic disconnects or other reasons, resulting in a ‘dead’ pixel. To mitigate the ‘dead pixels’ a 2-D median spatial low pass filter was quite effective in removing the fixed values to render the image free of obvious detector failures. In this case, two passes with a 7x7 median filter was used. The result, shown in fig 4.4 shows the corrected image is 21-27 degrees Celsius. The process requires that the image be cropped (trimmed) by the filter size (in pixels) around the border of the image.
Fig 4.4. IR image of microwave horn with no metamaterial prism set in front of it. This serves as the base measurement to compare and see how the metamaterial perturbs the field. A low pass filter was applied to account for any dead pixels in the original image.

Having sampled a baseline image, the metamaterial prism was inserted into the experimental setup. A median-filtered image (21-26 degree) is shown in Fig.4.5 (a) at a 20-level contour. The contour plot clearly outlines two lobes of unequal intensity. The ‘primary’ or 1st order lobe has peak temperature near 26 degrees, while the 2nd order lobe peak is lower. These lobes and their relative intensities are consistent with Smith’s description of negative index of fraction (1st order) and enhanced diffraction (2nd order) lobes [25]. The 2nd order lobe shall be ignored for the purposes of retrieving the index of refraction because it is an artifact of the prism step size (d ~ λ).
Fig 4.5. (a) Low pass filtered infrared image taken of the horn with a metamaterial prism in front of it. (b) 20-level contour taken of (a) in order to move to apply this method to a 3D plot.

By extension of the 2-D image data collected by each image plane, a 3-D representation can be constructed with a series of images at incremental equidistant planes. Once mapped, an image stake can be translated back to (V/m) units in electric field to compare to simulations. By this process, the theory and experimental results can be analyzed. Fig. 4.6 shows a series of images captured in steady state conditions; the distance from the prism plane is indicated below each image.

Figure 4.6. Infrared Images at incremental distances from the NIM surface.

Each of these images are broken into a 3D contour and then combined to give a 3D map of the near field intensity.
4.5. ANALYSIS OF THEORETICAL AND EXPERIMENTAL RESULTS

Our analysis indicates that a thermal contrast range of 5.24 °C was possible in the infrared based on a power of 2 W, an area of 12 in² (Microwave Horn Area) and Kapton film-specific heat specifications. Fig 4.5 shows a few calibration images (21-26 °C) through the X-Ku metamaterial that show lobes of energy for 12.5 GHz. The maximum thermal contrast apparent in these images is about 5 °C, which was slightly less than the expected value of 5.24 °C. The reasons for the errors in contrast might be power loss in the cable-antenna interface and/or unaccounted-for propagation losses between the antenna and the Kapton film.

In addition, HFSS simulation of the prism experiment were run to compare the simulated electric field to the measured temperature contrast relationship as well as spatial relationship of the radiation lobes transmitted through the NIM prism. Fig 4.7 depicts a comparison of the HFSS simulation (above) and the IR image (below) for the nearest image plane parallel to the prism surface. The comparison translates a range of electric field units (~1200-3500 v/m) to approximately the thermal contrast (21-26 deg. Celsius). Furthermore, the 1st order and 2nd order lobes are evident in both with the expected relative intensities. At this resolution, the dual transmission lobes were evident in both
the simulation and measurement results. As a result, new spatial dimensions for validating simulation and measurement are possible.

Fig 4.7. Top: HFSS simulation of Electric field intensity at 1 cm from the face of the metamaterial prism. Bottom: Infrared Image taken at 1 cm from the face of the metamaterial prism. The two together show the expected direct mapping of electric field intensity in simulation to thermal intensity from measurements.
Fig 4.8. An image slice is taken from the IR image of the horn antenna alone and one with the metamaterial prism. This is done to see how the metamaterial prism perturbs the field from which we can extract index of refraction or view near field interactions.

With the baseline image (microwave horn) and the metamaterial image, an analysis of the index of refraction is possible. By taking image slice (same pixel row) of both images, as shown in Fig 4.8, a direct comparison may be constructed to evaluate the index of refraction. Once the image slices are overlaid, the beam width (-3dB points) is determined in terms of pixels from image centerline center-line. That is, the beam width can be compared to the image to match as close as possible to the one half points (or amplitude drop to 0.707 of the original amplitude. In Fig 4.9, this is 22 degrees. With that information, a conversion factor can be calculated to translate pixel offset to degree offset. Once the 1st order peak is determined, the degree offset (34 degree, in fig 4.9) is introduced into Eq. 4.5, and the index of refraction based upon the image is
calculated. See Fig 4.9, to identify key parameters; in this case, the magnitude for the index of refraction is:

\[
\sin \left( \frac{34}{180} \right) = 4.0
\]

\[
\frac{\sin \left( \frac{34}{180} \right)}{\sin \left( \frac{8.1}{180} \right)} = 4.0
\]

(4.5)

the peak location on corresponds to a ‘negative’ index, the value becomes \(-4.0\).

Fig 4.9. An Image Slide Analysis

The single image slice analysis described above is one step in the overall process. In the previous example, the image centerline was selected for analysis. To extend this analysis to better represent the entire image taken at a given frequency, a series of image slices are taken, and their indices of refraction calculated and plotted. This approach was taken to make Fig 4.10; with this data set the range of values of the indices of refraction for this material are \(-3.8\) to \(-4.5\), as shown; recall that the microwave frequency used to generate the EM field was 12.5 GHz. According to the design, the
material was intended to have negative indices between 12-15 GHz. In fact, the direct microwave measurements taken in chapter 3, indicate negative values within that band, as seen in Fig 4.10 (b)’ note that the particular index of refraction at 12.5 GHz is $\sim 4.2$, which was measured in the far-field in an anechoic chamber using the identical metamaterial prism.

Fig 4.10. (a) Image slices are taken from the 12.5GHz IR measurement and the index of refraction is calculated for each slice. The average index of refraction for the entire image is found to be $n=-4.1$  (b) Index of refraction measurement by direct microwave methods on the same metamaterial prism. The value of -4.2 at 12.5 GHz is very similar to what was found in the near field using the IR image technique.

Comparing the average index extracted using the IR method described in this chapter for 12.5 GHz gives and index of refraction of -4.2. This value is very close to the value measured in chapter 3 of $n=-4$. Although only used in this chapter to extract the index of refraction, using thermal imaging to view near field interactions
could have many uses in metamaterial research. Overall infrared imaging of the electric field offers additional capability of high spatial-resolution to construct 3-D data sets. This in turn may be used to advance modeling of electric fields in metamaterials possibly even analyzing the field within a metamaterial slab. However, there are clear advantages of direct microwave measurements: when precise transmission loss in required and when the full S-parameter matrix (transmission, reflection and phase) are required. Together direct microwave measurement and IR imaging of the electric field offer a complementary set of tools.

4.6 Chapter Conclusion
Since many proposed applications of metamaterials require conformal or other irregular forms, existing test and analysis methods may prove inadequate to fully characterize their electromagnetic properties. For that reason, novel methods involving infrared imaging were considered in this chapter to explore 3-D analysis. Infrared imaging of an electric field is a technique that can be used in combination with direct microwave measurements to gain a better understanding of the constitutive parameters of any metamaterial. We have described a method of measurement and analysis to estimate the index of refraction based upon thermal contrasts due to the intensity of the electric field on heat-sensitive film. Although only used here to retrieve the index of refraction, this method could be applied to any use in which the near field needs to be examined. Only a modest amount of power is needed to boost the microwave signal on the order of a few Watts, creating images
with a commercial mid-wave infrared camera. Although some initial processing is required to mitigate defective detector in the image array, the application is simple.

Images taken at equidistant increments facilitate staking of images to render 3-D data sets with good spatial resolution. In addition, a further dimension frequency is another variable that might be considered to more fully characterize metamaterials. In fact, beyond the single frequency analysis performed here, this concept could be extended to create high spatial resolution 3-D data sets for a range of frequencies.
Chapter Five: Polarization-Dependent Transmission through Microwave Metamaterials

5.1 Introduction

In previous chapters, and most metamaterial research, considerable interest has been devoted to the transmission characteristics [13,15,18,23]. However, nearly all the experimental results are reported where the electric field is strictly aligned with the metamaterial [13,15,18,23]. As a result, there has been no research on the transmission response when the material and polarization are misaligned to some degree. The purpose of this chapter is to answer that question as well as propose and investigate two geometries of superimposed S-shaped split ring resonator (SSRR) metamaterials. This new geometry allows for circularly polarized signals to propagate through the material without significant additional loss.

Fig 5.1. A sketch of the SSRR unit cell with E-field and H-field orientation and wave propagation (k) direction (left) and unit cell with conventional dimensions labeled (right).
5.2 Design & Manufacturing

The metamaterial unit cell design was finalized using HFSS as in previous chapters. One SSRR metamaterial slab used Rogers Duroid 4003 as the dielectric. After the boards were printed they were cut into varying cell depths. The cell depth thickness is the number of S-shapes, as defined in Fig 5.1, in the direction of wave propagation. The metamaterial slabs were made to be 6-7cm wide for each cell depth. To keep spacing uniform, Duroid 4003 dielectric at the ends of each as is seen in Fig 5.2. The cell dimensions were designed for specific frequencies and are detailed in Table 5.1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Frequency</th>
<th>ε</th>
<th>a</th>
<th>b</th>
<th>h</th>
<th>w</th>
<th>c</th>
<th>d</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSRR</td>
<td>20 GHz</td>
<td>3.52</td>
<td>3.0</td>
<td>3.0</td>
<td>2.04</td>
<td>2.04</td>
<td>0.2</td>
<td>0.51</td>
<td>1.0</td>
</tr>
<tr>
<td>SSRR</td>
<td>24 GHz</td>
<td>4.5</td>
<td>3.0</td>
<td>3.0</td>
<td>2.04</td>
<td>2.04</td>
<td>0.2</td>
<td>0.51</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 5.1 All lengths as described in Fig1. The relative permittivity is listed for Rogers Duroid 4003 (ε=3.52) and Rogers TMM2 (ε=4.5) at 20GHz.

A total of sixty-five boards wide were used per slab with varying cell depths. We created a one, two and three-layer slab for testing. The second metamaterial, designed for 24 GHz, was printed on Rogers TMM2 dielectric (ε=4.5) and was also used in the construction of our cross design to be described later.
In an effort to create a polarization independent SSRR, two interlocking unit cells were inserted perpendicular to one another in the same plane. This design makes the metamaterial polarization independent but puts further limitations on the parameters of the SSRR, ie: The spacing between the boards, must be equal to the height parameter, $a$, to allow for us to place one board perpendicular to another in the same plane. The final result can be seen in figure 4. To interlock the boards, we used the dimensions are described in Table 5.2, demonstrating the added restriction that $a=b$ and $h=w$.

<table>
<thead>
<tr>
<th>Cross SSRR Design Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Cross SSRR</td>
</tr>
</tbody>
</table>

Table 5.2: All lengths as described in Fig1. The relative permittivity is listed for Rogers Duroid 4003 and Rogers TMM2 at the target frequency.

After cutting the boards between the S-shapes to insert them perpendicular to one another, there was an issues of the boards bending and uniform spacing being unachievable. To correct this problem Styrofoam balls were inserted and provide a uniform 3mm spacing between the boards. Styrofoam has nearly the same permittivity as...
air in the microwave domain, so it made an ideal candidate for spacing. The Styrofoam balls were inserted with tweezers in order to not disturb the material spacing as is seen in Fig 5.3. After being inserted the SSRR cells were extremely stable and the metamaterial could be moved around freely.

Fig. 5.3: Left: The cross sectional design using TMM2 dielectric. Each Styrofoam ball was inserted to keep equal spacing between the S-shapes. The boards are cut and set perpendicular to one another in the same plane unlike the design depicted to the right. Right: the metamaterial slabs placed perpendicular on one another but not in the same plane. This design is essentially two metamaterial slabs where one is set on top of the other but perpendicular in orientation. Both of these designs allow for isotropic transmission of circular polarized waves.

5.3 Experimental Setup

A Styrofoam turn table was marked to rotate in fifteen degree increments for both changes to incident polarization and changes in angle of incidence. These rotations are referred to as axial (changes in incident polarization) and lateral (changes to angle of incidence) rotations and can be seen in Fig. 5.4.
Fig 5.4: A sketch of the angle dependent transmission measurements with the rotational stage.

The measurement set up can be seen in Fig. 5.5. The two ETS 3106-09 horn antennas were placed 25cm apart from the metamaterial to guarantee that we were in the far field. A transmission spectrum was taken with only the horn antennas as a source to normalize the curves to.

Fig. 5.5: Experimental set up for measuring the metamaterials. The Styrofoam was cut to rotate and lock in place for 15 degree increments.
5.4 Results

The transmission spectra, $S_{21}$, with respect to the polarization of the incident electromagnetic waves is shown in Figs 5.6-5.8. It is clear that the material exhibits an anisotropic response with respect to changes in polarization. In addition, the response of the single, double, and triple layer produce radically different results from one another. This effect is not what one would expect if the material were homogeneous, making one believe this metamaterial may be inhomogeneous.

![Graph showing transmission coefficient ($S_{21}$) of the single layer Duroid 4003 metamaterial as a function of the changes to incident polarization.](image)

**Fig 5.6.** Transmission coefficient ($S_{21}$) of the single layer Duroid 4003 metamaterial as a function of the changes to incident polarization.
Fig 5.7. Transmission coefficient ($S_{21}$) of the two-layered Duroid 4003 metamaterial as a function of the changes in incident polarization.

Fig 5.8. Transmission coefficient ($S_{21}$) of the three-layered Duroid 4003 metamaterial as a function of changes in incident polarization.
The results shown in Figs 5.6-5.8 demonstrate that when the S-shapes are oriented perpendicular to the electric field, the metamaterial does not attenuate any frequency acting as an all-pass filter. This is what allows for us to create our cross design and would also allow for stacking of one metamaterial slab on top of another as in Fig 5.3 to affect any incident polarization. This effect is demonstrated in Fig. 5.9, where a single axis polarization board is tested and then the perpendicular boards are inserted into it allowing for all polarizations to be effected by the metamaterial.

Fig 5.9. Left: Change in planar angle of 6 layer TMM2 metamaterial slab before the insertion of perpendicular SSRRs. Right: Perpendicular SSRRs were inserted to create the cross design. This allows for the metamaterial response to be unchanged with changes to polarization.

The interlocking design worked but it was overly complicated to assemble and imposed additional restrictions in the unit cell design. Instead, two slabs may just be stacked perpendicular to one another and produce the same polarization independent response,
as in Fig 5.9. This works again because when the boards are perpendicular to the electric field they do not perturb the field at all as can be seen in Fig 5.10.

![Graph showing transmission characteristics](image)

**Fig 5.10.** Two Duroid 4003 metamaterial slabs were stacked perpendicular to one another in the direction of propagation. This is a simple design that allows for the metamaterial to affect any polarization of incident radiation just as the “cross” design did but with far fewer constraints on manufacturing.

Changes to angle of incidence, as seen in Figs 5.11-5.13, demonstrate that the SSRR does not change transmission characteristics much with changes to angle of incidence up to 45°. Changes beyond 45° could not be demonstrated with the experimental set up so were excluded from the results. What is more interesting, is that the single, double, and
tripple cell depth SSRR again do not show a similar response to one another. Although
the one and two layer shows somewhat of the expected SSRR resonant response at the
target frequency it is not nearly as robust as the three layer slab. These results also
suggest that the SSRR metamaterial is inhomogeneous.

![Graph](image)

**Fig 5.11. Transmission coefficient (S21) of the One-layer Duroid 4003 metamaterial as a function of
the Lateral rotation (Angle of Incidence).**
Fig 5.12. Transmission coefficient ($S_{21}$) of the Two-layer Duroid 4003 metamaterial as a function of the Lateral rotation (Angle of Incidence).

Fig 5.13. Transmission coefficient ($S_{21}$) of the One-layer Duroid 4003 metamaterial as a function of the Lateral rotation (Angle of Incidence).
5.4 Chapter Conclusion

Several variations of the SSRR metamaterial were investigated. Their resistance to angle of incidence and polarization was characterized. It is seen that with a SSRR that during axial rotation we see that it allows for complete transmission of all frequencies when oriented perpendicular to the incident electric field as expected. Two novel designs for the SSRR were presented that allowed for transmission of circularly polarized radiation. The interlocking model and the perpendicular stacked metamaterial slabs show that the signal can be broken into its corresponding vector quantities and allow for the metamaterial to affect any polarization beyond what it was originally designed for. It was also shown that the SSRR metamaterials are far more resistant to changes to angle of incidence than was anticipated, showing no significant change up to 45°. We also answered the question as to if the SSRR would retain its unique properties even with a single layer. We showed that it does not in fact retain the same response as its multi-layered counterparts, meaning that the material is in fact inhomogeneous.
Chapter 6: Resolving Parameter-Extraction Ambiguities in Inhomogeneous Materials and Metamaterials

6.1 INTRODUCTION

The results seen in chapter 5 made apparent the importance of accurate constitutive-parameter extraction techniques when faced with an inhomogeneous material [5]. Currently, the Nicholson Ross Weir (NRW) extraction method is the predominant algorithm used to extract the permeability, $\varepsilon$, and permittivity, $\mu$, of the material under test [5,18,27,28]. As mathematically described later in this chapter, the NRW method uses the relative transmitted and reflected powers to extract these constitutive parameters. This extraction process includes the logarithm of a complex function that can have many complex branch-cut ambiguities, as shown in Fig. 6.1. Each branch cut can lead to unique constitutive properties that produce the same value for the logarithm [29]. If the material is homogeneous, the NRW extraction method correctly assumes the primary branch cut. However, if the material is inhomogeneous, the primary branch cut is not always the correct choice, due to multiple reflections from discontinuities.

The correct constitutive material parameters are of interest to metamaterial design, due to the precision required for applications such as cloaking, impedance matching, and absorbing materials [16]. Effective parameters are often extracted from a simulated sample that is thin enough to be considered homogenous [27]. Due to the rapidly changing parameter values near resonance, electrical thickness cannot always be
accurately predicted. Furthermore, experimentation shows that at least some metamaterials are inhomogeneous [6]. Lorentz and Drude dispersion models are useful for characterizing inhomogeneous materials such as these.

This chapter investigates the parameter extraction of several Lorentz and Drude dispersive materials. Additionally, parameter-extraction techniques are applied to simulated and measured material, consisting of extended S-shaped split ring resonators (ESRR). These examples illustrate where the NRW method fails to extract the correct constitutive parameters. This shortfall in NRW is corrected by using an unconventional recurrent phase-unwrapping algorithm [29] to ensure that the correct branch-cut ambiguity is chosen.

6.2 BRANCH CUT AMBIGUITY

The following derivation illustrates the origin of these branch-cut ambiguities, as well as the phase-unwrapping technique for resolving them. Starting with the intrinsic impedance of the material

\[ \eta = \sqrt{\frac{\mu_r \mu_0}{\mathcal{E}_r \mathcal{E}_0}} \]  

(6.1)

where \( \eta_0 = \sqrt{\frac{\mu_0}{\mathcal{E}_0}} \) is the that of free space, the normalized impedance is
Fig. 6.1. HFSS-simulated extracted permittivity with ambiguous branch choices. The blue solid line represents the primary branch, while the dotted curves show the ambiguous values from other branch cuts.

Then the reflection coefficient at the air/metamaterial boundary can be written as

\[
\Gamma = \frac{\eta - \eta_s}{\eta + \eta_s} = \frac{z - 1}{z + 1}
\]  

(6.3)

The wave will propagate through the material with the propagation factor
\[ P = e^{-\gamma d} = e^{-jknd} \]  

(6.4)

where \( \gamma = \gamma' + j\gamma'' \) is complex. The relative permittivity can be found by dividing the index of refraction, \( n = \sqrt{\mu_r \varepsilon_r} \) by the normalized impedance, (6.2)

\[ \frac{n}{z} = \frac{\sqrt{\mu_r \varepsilon_r}}{\sqrt{\mu_r / \varepsilon_r}} = \varepsilon_r \]  

(6.5)

The permeability can be found by multiplying these two quantities,

\[ n_z = \sqrt{\mu_r \varepsilon_r} \sqrt{\mu_r / \varepsilon_r} = \mu_r \]  

(6.6)

These two constitutive parameters can be expressed in terms \( \Gamma \) and \( P \) by manipulating (3) and (4) as

\[ z = \frac{1 + \Gamma}{1 - \Gamma} \quad \text{and} \quad n = -\frac{\ln P}{jkd} \]  

(6.7)

Substituting (6.7) into (6.5), the permittivity becomes
\[
\varepsilon_r = \frac{n}{z} = j \ln \frac{P}{kd} \cdot \frac{1 - \Gamma}{1 + \Gamma}
\]  

Likewise, by substituting (6.7) into (6.6), the permeability becomes

\[
\mu_r = nz = j \ln \frac{P}{kd} \cdot \frac{1 + \Gamma}{1 - \Gamma}
\]  

These parameters can be written in terms of the easily measured scattering-parameters matrix,

\[
\begin{bmatrix}
E_1^{\text{out}} \\
E_2^{\text{out}}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
E_1^{\text{in}} \\
E_2^{\text{in}}
\end{bmatrix}
\]

The NWR \[5\] extraction techniques are used to find the total reflection, \(\Gamma_{\text{total}}\) and transmission, \(\Gamma_{\tau}\) coefficients, that result from the two boundaries, i.e., the initial air-metamaterial boundary and the second metamaterial-air boundary. These multiple-boundary coefficients simply become the measured S-parameters,

\[
S_{11} = S_{22} = \Gamma_{\text{Total}} = \frac{\Gamma(1 - P^2)}{(1 - \Gamma^2 P^2)}
\]  

and
\[ S_{21} = S_{12} = T_{\text{Total}} = \frac{(1 - \Gamma^2)P}{1 - \Gamma^2 P^2} \]  \hspace{1cm} (6.11)

To write \( P \) in terms of these \( S \) parameters, we must solve for \( P^2 \) from (6.10) and \( P \) from eq. 11

\[ P^2 = \frac{\Gamma - S_{11}}{\Gamma(1 - \Gamma S_{11})} \]  \hspace{1cm} (6.12)

\[ P = \frac{S_{21}(1 - \Gamma^2 P^2)}{(1 - \Gamma^2)} \]  \hspace{1cm} (6.13)

Combining (6.12) & (6.13) results in

\[ P = \frac{S_{21}}{(1 - \Gamma S_{11})} \]  \hspace{1cm} (6.14)

Next, (6.12) can be equated with the square of (6.14) to find an expression for \( \Gamma \), i.e.,
Further algebraic manipulation of (6.15) results in
\[
\frac{\Gamma - S_{11}}{\Gamma(1 - \Gamma S_{11})} = \left[ \frac{S_{21}}{(1 - \Gamma S_{11})} \right]^2
\]  
(6.15)

where

\[
M = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}
\]

It would seem that the sign in front of the square root would produce an ambiguity in itself; however, this is not the case for passive material. The sign can be chosen by limiting \(|\Gamma| < 1\) to obey conservation of energy. The branch cut ambiguity actually comes from taking the natural logarithm of the complex propagation term, \(P\) [30]. The argument of \(P\), \(\arg(P)\) contains the ambiguity \(\arg(P) = \text{Arg}(P) + 2\pi m\) where \(m\) is the branch number of the argument, i.e., an integer multiple of \(2\pi\), and \(\text{Arg}(P)\) is the principle branch argument of \(P\), i.e., \(\text{Arg}(P) = -\gamma d\). If \(\ln(P)\) represents the principle branch of the logarithm of \(P\), then, (6.8) and (6.9) can be written as
\[
\varepsilon_r = \frac{j}{kd} \cdot \frac{1-\Gamma}{1+\Gamma} \left[ \text{Arg}(P) + 2\pi m - j \ln(P) \right]
\] (6.17)

and

\[
\mu_r = \frac{j}{kd} \cdot \frac{1+\Gamma}{1-\Gamma} \left[ \text{Arg}P + 2\pi m - j \ln(P) \right]
\] (6.18)

Clearly, these two equations are ambiguous with infinite choices of \( m \) that can drastically change the values of these two constitutive properties. More specifically, (6.17) and (6.18) can be rewritten as

\[
\varepsilon_r = \frac{1}{kd} \cdot \frac{1-\Gamma}{1+\Gamma} \left( \text{Im}[\ln(P)] + 2\pi m - \text{Re}[\ln(P)] \right)
\] (6.19)

and

\[
\mu_r = \frac{1}{kd} \cdot \frac{1+\Gamma}{1-\Gamma} \left( \text{Im}[\ln(P)] + 2\pi m - j \text{Re}[\ln(P)] \right)
\] (6.20)

Substituting in the complex value of \( \gamma = \gamma' + j\gamma'' \) from (6.4) into (6.19) and (6.20), the parameters can be explicitly expressed as
It is evident in (6.19) and (6.20) that the ambiguity is found in the real terms. It is also evident that an additional technique is required in order to resolve these ambiguities. Therefore, a phase-unwrapping technique will be used to determine the branch number, $m$ at any given frequency.

### 6.3 Phase Unwrapping

Many phase unwrapping algorithms have been proposed and are currently useful for image phase retrieval [31]. Some algorithms work by minimizing the change in phase, from one frequency to the next, by adding or subtracting $2\pi$ whenever the phase difference exceeds $\pi$[31]. For the purpose of parameter retrieval, several techniques can be used to resolve the branch ambiguities, based on physical constraints [6]. The proposed method uses a mathematically based, modified discontinuity-detection algorithm to resolve these ambiguities [29]. This algorithm declares the phase angle, $\phi_n$ to be the $n$th element of $\ln(P)$, in the frequency domain. Then it takes the tangent of the phase difference.
\[
\tan(\phi_n - \phi_{n-1}) = \frac{\tan(\phi_n) - \tan(\phi_{n-1})}{1 + \tan(\phi_n)\tan(\phi_{n-1})}
\] 
(6.23)

If the following functions are defined for convenience,

\[
X = \tan(\phi_n) = \frac{\text{Im}(P_n)}{\text{Re}(P_n)}
\]

And

\[
Y = \tan(\phi_{n-1}) = \frac{\text{Im}(P_{n-1})}{\text{Re}(P_{n-1})}
\]

then this change in angle can be solved by combining (23) and (24)

\[
\Delta\phi_n = \phi_n - \phi_{n-1} = \tan^{-1}\left(\frac{X - Y}{1 + XY}\right)
\] 
(6.25)

From (6.25), it is apparent that \(\Delta\phi_n\) is limited to values in the interval \((-\pi/2, \pi/2)\). To ensure that the actual phase is recovered, this limitation is removed by using \(X\) and \(Y\) for discontinuity detection. This is determined though several cases [29]:

1) \( XY > -1 \)

\[
\Delta \phi_n = \tan^{-1}\left( \frac{X - Y}{1 + XY} \right)
\]

2) \( XY < -1 \) & \( X > 0 \)

\[
\Delta \phi_n = \pi + \tan^{-1}\left( \frac{X - Y}{1 + XY} \right)
\]

3) \( XY < -1 \) & \( X < 0 \)

\[
\Delta \phi_n = -\pi + \tan^{-1}\left( \frac{X - Y}{1 + XY} \right)
\]

4) \( XY = -1 \) & \( X > Y \)

\[
\Delta \phi_n = \pi / 2
\]

5) \( XY = -1 \) & \( X < Y \)

\[
\Delta \phi_n = -\pi / 2
\]

This algorithm acts recursively for \( \Delta \phi_n \) and then finds the unwrapped value for \( \phi_n \) by solving

\[
\phi_n = \phi_{n-1} + \Delta \phi_n = \phi_o + \sum_{i=1}^{n} \phi_i
\]

(6.26)

where \( \phi_o \) is the initial sampled point.
When using this algorithm, care must be taken to sufficiently sample in frequency. If undersampled, a phase discontinuity could easily be missed. The following simulations and measurements relied on a sampling rate of three thousand-samples per gigahertz. Furthermore, a root-detection algorithm ensured that no discontinuities were missed. Additionally, the initial sampling must guarantee that $\phi_n \in (-\pi/2, \pi/2)$. Using (6.26) and $\phi_n = \ln(P)$, (6.17) and (6.18) can be written as

$$\varepsilon_r = j \frac{\phi_n}{kd} \cdot \frac{1-\Gamma}{1+\Gamma} = j \left( \frac{\phi_n + \sum_i \phi_i}{kd} \right) \cdot \frac{1-\Gamma}{1+\Gamma}$$

(6.27)

and

$$\mu_r = j \frac{\phi_n}{kd} \cdot \frac{1+\Gamma}{1-\Gamma} = j \left( \frac{\phi_n + \sum_i \phi_i}{kd} \right) \cdot \frac{1+\Gamma}{1-\Gamma}$$

(6.28)

### 6.4 PARAMETER EXTRACTION TEST

A Lorentz-dispersion model was chosen to test the modified extraction method. The Lorentzian model was chosen because of its previous use in verifying extraction parameters [32]. Using this model the permeability and permittivity were defined as

$$\mu_r = 1 - \frac{A^2}{\omega^2 - \omega_{\mu,\mu}^2 - j\Gamma \omega}$$

and

$$\varepsilon_r = 1 - \frac{A^2}{\omega^2 - \omega_{\epsilon,\epsilon}^2 - j\Gamma \omega}$$

(6.29)
where \( A = 2\pi \times 4.3\, \text{GHz} \), \( \Gamma = 2.0\, \text{GHz} \), \( \omega_{\mu} = 2\pi \times 7.0\, \text{GHz} \), and \( \omega_{\varepsilon} = 2\pi \times 12\, \text{GHz} \). Fig. 6.2 verifies that both the NRW method and the proposed method correctly extracted the constitutive parameters, when applied to an electrically thin (0.5mm) material. However, Figs. 6.3-6.5 shows disagreement between the two methods, when both are applied to an electrically large (12.5mm) material. These results show good agreement between the input parameters and the proposed phase-unwrapping method, but poor agreement with the NRW method. The proposed algorithm retrieves the correct parameters even when the resonances completely overlap, i.e., at \( \omega_{\varepsilon} = \omega_{\mu} = 2\pi (12\, \text{GHz}) \), as observed in Fig. 6.5.
In addition to the Lorentz-dispersion models, a Drude-dispersion model was also used. The Drude model defines the permittivity as

$$
\varepsilon_r = 1 - \frac{\omega_p^2}{\omega(\omega - j\Gamma)}
$$

(6.30)

where $\omega_p = 2\pi(12\text{GHz})$ is the plasma frequency, and $\Gamma = 5\text{GHz}$. The Lorentz model (6.29) was still used for the permeability, in order to simulate the sharp resonance commonly found in metamaterials. As in the case of the Lorentz model, the proposed algorithm was able to retrieve the correct constitutive parameters, using the Drude model. Fig. 6.6 shows these parameters for a 15mm sample.
Fig. 6.3. A Lorentz dispersive material in both permittivity and permeability was simulated to be 12.5mm thick. From left to right: The actual input for the material; The NRW retrieved permittivity showing both the unwrapped and wrapped phase results; The NRW retrieved permeability showing both the unwrapped and wrapped phase results.

Fig. 6.4. A Lorentz dispersive material in permeability only with permittivity kept constant with a value of 1. The simulated material was made to be 12.5mm thick. From left to right: The actual input for the material; The NRW retrieved permittivity showing both the unwrapped and wrapped phase results; The NRW retrieved permeability showing both the unwrapped and wrapped phase results.
Fig 6.5. A Lorentz-dispersive material where the resonances are at the same frequency 12GHz. The material was simulated to be 12.5mm thick. From left to right: The actual input for the material; The NRW retrieved permittivity showing both the unwrapped and wrapped phase results; The NRW retrieved permeability showing both the unwrapped and wrapped phase results.

Fig. 6.6. A Drude dispersion model was used to create the permittivity and a Lorentz dispersion model was used for the permeability. The material was simulated to be 15mm long to ensure it was electrically large. The *unwrap* method retrieves the correct parameters where NRW fails. The far left image represents the input parameters and the other two represent what was extracted using the modified (unwrap) and unmodified NRW method.
Next an ESRR metamaterial was tested. The ESRR, shown in Fig. 6.7, was designed using the theoretical framework specified in [4]. The dimensions of the ESRR are $A=5.0\text{mm}$ (5.2 at edge of material), $B=4.0\text{mm}$, $H=5.0\text{mm}$, $W=2.8\text{mm}$, and $C=0.4\text{mm}$. In addition, the dielectric thickness, $D=0.5\text{mm}$, and the spatial period of the boards, $L=1.5\text{mm}$. Rogers Duroid 4003 was used for the dielectric, which has a relative permittivity of 3.3-3.4 at 9-12GHz.

To demonstrate the effect of inhomogeneity using the NRW method, the phase-unwrapping and NRW methods were each applied to simulated ESRR of various thicknesses, using Ansoft HFSS. The extracted parameters are compared in Fig. 6.8, indicating significant differences between the two techniques in the inhomogeneous cases. It can be seen in Fig. 6.8 that the constituent parameters are not double negative, when considering more than a single-cell thickness. This has previously been observed with other S-shaped metamaterials in chapter 5 and in [6] when they are not electrically small.
6.5 DESIGN AND EXPERIMENTAL SET UP

This simulated ESRR design was manufactured, cutting each board to 15 cm long. The same three cases were constructed: one cell, two cells, and three cells thick, as shown in Fig. 6.9. These three cases were assembled by placing multiple boards in a parallel periodic arrangement, as shown in Fig. 6.10, with periodic spacing, $L$. A total of sixty boards were used for each of the three cases, in order to completely cover the aperture of a transmitting horn antenna. The receive horn antenna was separated from the material by 50 cm.

If significant noise is present in measurements, the proposed algorithm could produce false changes in $\phi$. These false changes would drastically change the retrieved parameters. In order to avoid these false phase changes, measurements were taken in an anechoic chamber and calibration was performed using a thru, reflection, match (TRM)
method. The TRM calibration is a standard and has been thoroughly tested and used for several decades and believed to be a valid method to reduce error [33].

When performing a TRM calibration, the thru and match measurements were easily accomplished; however, the multiple-thicknesses of the ESSR made the reflection measurement problematic. The thru measurement was performed by measuring the S21 between the two antennas in free space and with no material between them. The match measurement was performed using single-port S11 and S22 measurements in the anechoic chamber. The reflection measurement required extra care in order to exactly match the size of the material under test. A visual representation of these measurements can be seen in Fig. 6.11.

6.6 EXPERIMENTAL RESULTS

In homogeneous materials, the transparent frequency band does not change with small variations in thickness. So if the ESRR is homogeneous, the transmitted power should decrease linearly as additional layers add attenuation proportionally. However, Fig. 6.12 shows that the transmitted power does not decrease linearly with additional thickness. Rather, attenuation decreases across nearly all frequencies, with additional thickness. Additionally, the transmission band changes frequencies, when addition layers are added. This is not what one would expect to see if the material were homogenous, suggesting that a modified parameter-extraction method must be used, such as the phase-unwrapping algorithm. Fig. 6.13 shows the errors that occur when the NRW method is used in inhomogeneous material. In contrast, the proposed phase-unwrapping method correctly
retrieved the constitutive parameters, even from this multiple-layered inhomogeneous metamaterial.

Fig. 6.8. Constituent parameters of the single(A), two (B), and three(C) layer of ESRR using Unwrap as compared to not considering branch ambiguity.
Fig. 6.9. One, two, and three cell thickness ESRR strips.

Fig. 6.10. The metamaterial holder with 60 boards.

Fig. 6.11. The three steps involved with TRM calibration. Only a single thickness of metal plate need be used but all measurements must take into account the variation in thickness from the metal plate thickness as in Appendix A.
Fig. 6.12. The relative power transmitted [dB] of the ESRR as a function of frequency and thickness. It is easily seen that the transmitted power for the ESRR increases with increasing layers which implies that the material is in fact inhomogeneous and produces different constitutive parameters for different thicknesses.
Fig. 6.13. Constituent parameters of the one(A), two(B), and three(C) layer of ESRR measured in free space using unwrap algorithm as compared to not considering branch ambiguity.
6.7 Limitations

The proposed method is not without limitations. It was previously mentioned that sufficient frequency samples must be taken to avoid missing abrupt phase shifts. If the initial phase sample is not within \((-\pi/2, \pi/2)\) the wrong branch ambiguity will be chosen, as shown in Fig. 6.14. To reduce this potential error, a large frequency range should be chosen, starting at the lowest frequency possible, where the constituent parameters are not close to resonance [4].

![Graphs showing input frequency dependent response, retrieved permittivity, retrieved permeability, and unwrapped vs. wrapped data.](image)

Fig. 6.14. From left to right: The input frequency dependent response of the permittivity and permeability; The retrieved permittivity; The retrieved permeability. The algorithm for resolving branch ambiguity no longer correctly predicts the constituent parameters.

6.8 Chapter Conclusion

A novel recurrent phase-unwrapping algorithm was presented as a method for resolving the branch ambiguities that result when the NRW extraction method is performed on inhomogeneous material. Several Lorentz and Drude dispersive models were used to test
this proposed method. It was shown that the proposed phase-unwrapping algorithm correctly retrieved the constitutive parameters in both homogeneous and inhomogeneous material.

In addition, the ESRR metamaterial was simulated and measured as a test case. If the ESRR metamaterial were homogenous, attenuation would increase linearly with thickness. However, it did not increase linearly with added layers. These results suggest that the bulk material properties of the ESRR are heavily dependent on the mutual interaction with surrounding ESRR cells. Therefore, the specific number of layers must be included when determining the effective permittivity and permeability of the ESRR.

Limitations of the proposed method were presented. The algorithm will incorrectly predict the correct branch cut, if noise is not minimized, the spectrum is under sampled, or if the initial sampling does not meet the requirement of $\phi_o \in (-\pi/2, \pi/2)$. Precautions were suggested to avoid these erroneous results.
Chapter Seven: Exploiting Inhomogeneity in Metamaterials for Radome Application

7.1 Introduction

Now that we verified the SSRR and ESRR are in fact inhomogeneous, as demonstrated in previous chapters, there was a question if they could still be useful. We wanted to see if the inhomogeneity itself could be exploited to maximize power transmission, reduce reflections, and to change the bandpass region of the metamaterial.

We decided to experiment on the extended S-shaped split-ring resonator (ESRR) metamaterial from chapter 6 due to its large band-pass region [34]. Additionally, this chapter demonstrates that the inhomogeneity provides a design technique for achieving a material with a refraction index near or equal to one.

7.2 Design and Experimental Set Up

As in previous chapters the ESRR metamaterial was designed for a bandpass region, centered at 12.5 GHz. The dimensions of the ESRR, shown in Fig. 7.1 are A=5.0mm (5.2 at edge of material), B=4.0mm, H=5.0mm, W=2.8mm, and C=0.4mm. In addition, the dielectric thickness, D=0.5mm, and the spatial period of the boards, L=1.5 mm. These dimensions define a single cell thickness of the metamaterial. Rogers Duroid 4003 was used for the dielectric, which has a relative permittivity of approximately 3.3-3.4 at 12
GHz. Each ESRR board was cut to 15 cm tall, and then cut into one of three widths: one cell, two cells, and three cells wide. Then three slabs (one for each width) of material were assembled by placing multiple boards in a parallel periodic arrangement, with periodic spacing, L as shown in Fig. 7.2. A total of sixty boards were used for each of the three cases, in order to completely cover the aperture of a transmitting horn antenna. The receive horn antenna was separated from the material by 50 cm.

![Fig 7.1. Dimensions for ESRR with the inverted pattern on the opposite side of the dielectric (shown by lighter shading)](image)

Multiple cell layers of the ESRR were simulated using Ansoft HFSS, and measured in an anechoic chamber, using a thru, reflection, match (TRM) calibration method [33]. Then a parameter extraction was performed. Fig. 7.3 compares the simulated and experimentally measured transmission through multiple cell layers and indicates that the transmission band changes with different thicknesses. Fig. 7.4 shows this index calculated from the measured permittivity and permeability.
With homogeneous materials, the frequency at which power transmits through the material should not change with small variations in thickness. So if the ESRR were homogeneous, the transmitted power would decrease linearly as additional layers were added, with no significant changes in the bandpass frequency. However, the measured graph in Fig. 7.3 shows that the transmitted power increased across nearly all frequencies, as more layers were added. Additionally, the transmission band changed frequencies when addition layers were added.

Fig. 7.2. One, two, and three cell thickness ESRR strips (left) set into 60 board metamaterial slab
This property can be exploited for radome design by maximizing the transmission bandwidth and by minimizing attenuation and reflections. Fig. 7.5 shows that by choosing the appropriate thickness of ESRR material, a refraction index near one can be achieved. In this case, the three-layer thickness demonstrates that the real component of the refraction index over the frequency band of 12GHz to 14GHz is 0.71 to 1.27. this corresponds to a reflection coefficient of 0.17 and -0.12 respectively.

7.3 Chapter Conclusion

If the ESRR metamaterial were homogenous, the frequency at which attenuation occurs would not depend on the thickness. However, it was shown that these frequencies are highly dependent on the thickness, i.e., number of ESRR layers. Furthermore, the attenuation does not increase linearly with additional layers but rather can actually

---

Fig. 7.3. Simulated (left) and experimentally measured (right) transmitted power of two and three cell layers. The measured power (right) was calibrated to transmission in free space, in an anechoic chamber, using a TRM calibration method.
decrease. These results suggest that the bulk material properties are heavily dependent on the mutual interaction with surrounding ESRR cells. It was shown that this property can be used when designing radomes to maximize transmitted power and achieve a refraction index near one over a large bandwidth.

Fig. 7.4. The extracted index of refraction from experimental results
Chapter 8: Millimeter-Wave Tunable Notch Filter

8.1 Introduction

Moving on from the S-shaped split ring resonators we wanted to look into new areas where metamaterial devices could fill a need. As the need for high data rates increases the use of the millimeter wave bands is becoming a reality and seemed like a place metamaterials could impact [35]. This interest is fueling a need for millimeter wave devices. However, the new devices might not resemble their lower frequency equivalents. In the millimeter wave range MEMS devices would have to become smaller which is not easily accomplished and ferroelectric materials show too high of losses to be of use in most of the millimeter wave bands [36]. In the search for alternatives, nematic liquid crystal was recently used in millimeter wave phase delay lines and shows great promise and can used in periodic metamaterials[37]. Liquid crystal has real potential for use in the millimeter wave range due to low insertion losses and already well-established theory. Liquid crystal devices work by reorienting the liquid crystal molecules with an applied electric field. This reorientation of the liquid crystal molecules produces a change in the permittivity across all frequency bands. By restricting the orientation of the liquid crystal to only be along the direction of propagation we can write a simplified model for the orientation angle as:

\[
\theta = \begin{cases} 
\frac{\pi}{2} - 2\tan^{-1}\left( \exp\left( \frac{V - V_c}{V_o} \right) \right) & V > V_c \\
0 & V < V_c \end{cases} 
\]  

(8.1)

where \( V_c \) is the threshold voltage needed to initially start orienting 1micron thickness of the liquid. \( V \) is the applied voltage per micron and \( V_o \) is a constant as described in [38]. From this we can determine the effective permittivity to be:
\[
\varepsilon(v) = \frac{\varepsilon_p \varepsilon_t}{\cos(\theta)^2 \varepsilon_t + \sin(\theta)^2 \varepsilon_p}
\]  (8.2)

where \(\varepsilon_p\) and \(\varepsilon_t\) are the permittivity of the liquid crystal medium when \(\theta\) is 0° and 90° respectfully. When used in a microstrip line with thickness \(d\), and microstrip width, \(w\), the permittivity is defined as [39]:

\[
\varepsilon_m(v) = \frac{\varepsilon(v) + 1}{2} + \frac{\varepsilon(v) - 1}{2} \left( \frac{1}{\sqrt{1 + d / w}} \right)
\]  (8.3)

One could make use of this change in permittivity by applying a DC bias at periodic points on a microstrip line, creating resonant cavities. These resonant cavities will in turn act as a band gap filter as visualized in Fig 8.1.

---

**Fig 8.1.** Liquid crystal polarization depicted with applied voltage at periodic sections. \(K\) and \(E\) are direction of propagation and electric field direction respectfully. The green box is some non-tunable dielectric with high permittivity acting as a buffer between the dc voltage lines. \(L_1\) and \(L_2\) are the lengths where the liquid crystal dielectric is active and inactive to create resonant cavities.
With the change in dielectric well defined, a simple Bragg reflection model can be used to estimate what frequencies $f$ will be reflected by each resonant cavity:

$$f(v) = \frac{(m - 1/2) \cdot c}{2\sqrt{\varepsilon_m(v)} \cdot L}$$

(8.4)

where $m$ is an integer defining the order of the reflection, $c$ is the speed of light and $L$ is the length of the resonant cavity. After the Bragg model the reflection from multiple resonant cavities is estimated using the transfer matrix method [40].

Liquid crystal has the extra benefit of allowing for the filter to not only be tuned but also turned on and off with a DC voltage bias. There will be no resonant cavities when the permittivity along the microstrip is unchanged as is seen in Fig 8.2.

![Fig 8.2: The response of a 15 resonant cavity filter when an applied DC voltage is applied to the liquid crystal and when it is off.](image-url)
8.2 Results

Using (8.4) and values for liquid crystal permittivity from [37] a microstrip loaded with liquid crystal resonant cavities was simulated using Ansoft HFSS. The parameters were: 

d=50microns, w=100microns, L1=0.5866mm, L2=0.7311mm, \( \varepsilon_p = 4.25 \), and \( \varepsilon_\tau = 6.85 \). The amount of power reflected compared with number of resonant cavities was tested and the reflection was shown to increase linearly with the amount of cavities added. As can be seen in Fig 8.3, a minimum of ten resonant cavities must be used to see decrease in transmission greater than 5dB.

The response of higher order (m=1,2,3…) reflections were tested. Two examples, 1\textsuperscript{st} order and 2\textsuperscript{nd} order reflections are shown in Fig. 8.4 and produce a difference in bandwidth of ~6GHz but the 2\textsuperscript{nd} order has an increased rejection of 5dB. This implies that the bandwidth could be tunable given the ability to change the length of the resonant cavities by changing the area that the DC bias covers.
A second microstrip was simulated with \( d = 50 \) microns, \( w = 100 \) microns, \( L_1 = 2.9 \) mm, \( L_2 = 3.6 \) mm, \( \varepsilon_r = 4.25 \), \( \varepsilon_i = 6.85 \), and 20 resonant cavities. This microstrip was to produce a band gap at \(~12\) GHz, 35 GHz, and 57 GHz. Small changes in the liquid crystal dielectric were simulated to further characterize the response at multiple orders of reflection. As can be seen in Fig 8.5, the point of maximum reflection was shown to change by 0.4 GHz at the first order reflection (11 GHz), 1.5 GHz at the 2\(^{nd}\) order reflection (35 GHz), and 2 GHz at the 3\(^{rd}\) order reflection (57 GHz). Further tuning could be achieved but as is seen in Fig 8.5 the tradeoff is a significant decrease in rejected power.

---

**Fig 8.3:** Increasing the number of resonant cells. It is apparent that 10 are needed to get a band rejection of at least 5dB.
8.3 Chapter Conclusion

A novel design was presented for a millimeter wave tunable band gap filter using liquid crystal. It was shown how the bandgap filter response changes with variations to permittivity, number of resonant cells, and the effect of higher order reflections on the bandgap region.

Fig 8.4: from (4) changing m, and consequently L1 & L2 produces a significant difference in rejection and bandwidth. For m=1, L1=0.5866mm, L2=0.7311mm; for m=2, L1=1.72mm, L2=2.18mm
Fig 8.5: The values for $\varepsilon$ were changed for all the dielectrics in the L1 cavities, while the permittivity remained unchanged in the L2 cavities.
References


Appendix A: Matlab Parameter extraction for modified NRW Lorentz Model

%%James Vedral
% Lorentz dispersion model with extraction
clear all
clc
clf
% Units in mm, Ghz, sec
% constants needed
co=299.792458; % speed of light in mm/sec
A=2*pi*4.3;
gam=2;
we=2*pi*12;
wm=2*pi*7;
d=12.5;

% Declare crequency limits in GHz for area of interest
FTop=20;
FBot=5;
w=FBot;
% number of samples 3000 points per Ghz Taken
nn=((FTop-FBot)*10);

for i=1:nn
    w=w+(1/10)*2*pi;
    freq(i)=(w/2*pi)/10;
    Eps(i)=1-(A^2)/((w^2)-(we^2)-(1j*gam*w));
    %Eps(i)=1;
    MuR(i)=1-(A^2)/((w^2)-(wm^2)-(1j*gam*w));
    %MuR(i)=1;
    P=exp(-1j*sqrt(Eps(i))*sqrt(MuR(i)))*d*w/(co);
    z=sqrt(MuR(i))/sqrt(Eps(i));
    R=(z-1)/(z+1);
    s11(i)=R*(1-P^2)/(1-(R^2)*(P^2));
    s21(i)=P*(1-R^2)/(1-R^2*P^2);
    s22(i)=s11(i);
    s12(i)=s21(i);
end

% figure(3)
% hold on
% plot(freq,abs(s21))
% plot(freq,abs(s11),'r')
% hold off

% input the gap between the MUT faces and Reflector faces.
% L1 : length between cal. plane to port 1 side and MUT's surface[mm]
% L2 : length between cal. plane to port 2 side and MUT's surface[mm]
% D  : the thickness of MUT       [mm]
% positive L1, L2 : the reflector's face is closer than MUT's face to
% antenna
% negative L1, L2 : the MUT's face is closer than Reflector's face to
% antenna
%
%   |          |-------------|              |
%   |----------|             |--------------|
%   |                                       |
%   |----------|             |--------------|
%   |          |-------------|              |
% %
%       L1           D              L2
%       ko           k              ko
%       Zo           Z              Zo
%

L1=0;D=d;L2=0;

% check the multiple index will be considered(1) or not(0)
Ind_c=0;

% normalize the frequency unit.
if max(freq)>1e3
    freq=freq/1e9;
end

% form the measured/simulated S parameter, make the S parameter with only MUT
for m=1:length(freq)
    ko=2*pi*freq(m)/co;
    s11(m)=s11(m)*exp(2j*ko*L1);
    s22(m)=s22(m)*exp(2j*ko*L2);
    s21(m)=s21(m)*exp(1j*ko*(L1+L2));
    s12(m)=s12(m)*exp(1j*ko*(L1+L2));
end
% conjugate load S parameter
s11=conj(s11);  s22=conj(s22);  s21=conj(s21);

% calculates T and Z using NRW method of FREESPACE
for m=1:length(freq)
    ko=2*pi*freq(m)/co;

    zp(m)=sqrt(((1+s11(m))^2-s21(m)^2)/((1-s11(m))^2-s21(m)^2));
    zn(m)=-zp(m);
    expjnkd(m)=(1-s11(m)^2+s21(m)^2)/(2*s21(m))+2*s11(m)/(zp(m)-1/zp(m))*s21(m));
    expjnkdn(m)=(1-s11(m)^2+s21(m)^2)/(2*s21(m))+2*s11(m)/(zn(m)-1/zn(m))*s21(m));
%
    if abs(real(zp(m)))>=0.005 & real(zp(m))>0
        expjnkd(m)=expjnkd(m);
        z(m)=zp(m);
    end
    if abs(real(zp(m)))>=0.005 & real(zp(m))<0
        expjnkd(m)=expjnkdn(m);
        z(m)=zn(m);
    end
    if abs(real(zp(m)))<0.005 & abs(expjnkd(m))<=1
        expjnkd(m)=expjnkd(m);
        z(m)=zp(m);
    end
    if abs(real(zp(m)))<0.005 & abs(expjnkd(m))>1
        expjnkd(m)=expjnkdn(m);
        z(m)=zn(m);
    end

    qq(m)=(imag(log(expjnkd(m))));
    qqn(m)=qq(m);
end

% calculate a multiple index m to resolve the ambiguity.
index=(unwrap(qq)-qq)/(2*pi);
for i=2:length(freq);
    X=atan(qq(i));
    Y=atan(qq(i-1));
    PH1(i)=(X-Y)/(1+X*Y);
end
if (X*Y>-1)
dqq=qq(i)-qq(i-1); %atan(PH1(i));
elseif (X*Y<-1) && (X>0)
dqq=pi+qq(i)-qq(i-1); %atan(PH1(i));
nnn=qq(i)-qq(i-1)-dqq
elseif (X*Y<-1) && (X<0)
dqq=-pi+qq(i)-qq(i-1);
nnnn=qq(i)-qq(i-1)-dqq
elseif (X*Y==-1) && (X>Y)
dqq=pi/2;
zzzz=33333
elseif (X*Y==-1) && (X<Y)
dqq=-pi/2;
qq(i)=qq(i-1)+dqq;
zzzzz=44444
else
zzzzz=888888888888888888888888888888888888888
end
qq(i)=qq(i-1)+dqq;
end
n5=qqn(1)-qqn(2)-atan(PH1(2))
figure(888)
hold on
plot(freq,qqn)
plot(freq,qq,'k')
plot(freq,unwrap(qqn),'r')
hold off

% calculate relative permittivity, permeability, normalized impedance and refractive index
for m=1:length(freq)
ko=2*pi*freq(m)/co;
n(m)=1/(ko*D)*(qq(m)-1j*real(log(expjnkd(m))));
n1(m)=1/(ko*D)*(qqn(m)+2*pi*0*index(m)-1j*real(log(expjnkd(m))));
mu(m)=-n(m)*z(m);
mu1(m)=-n1(m)*z(m);
er(m)=-n(m)/z(m);
er1(m)=-n1(m)/z(m);
end

% plotng results...
% figure(1)
% plot(freq,'r','LineWidth',2); hold on;
% plot(freq,real(s21),'b','LineWidth',2); hold off;
% legend('S11','S21');
% ylabel('S11 & S21 [dB]');
% xlabel('freq [GHz]');
% axis([min(freq) max(freq) -50 0]);
% grid;

% figure(21)
% plot(freq,real(n),'r','LineWidth',2); hold on;
% plot(freq,imag(n),'b','LineWidth',2);
% hold off;
% legend('real(n)','imag(n)');
% xlabel('freq [GHz]');
% axis([min(freq) max(freq) -5 5]);
% axis([18 22 -5 5]);
% grid;

% figure(3)
% plot(freq,real(z),'r','LineWidth',2); hold on;
% plot(freq,imag(z),'b','LineWidth',2); hold off;
% legend('real(z)','imag(z)');
% xlabel('freq [GHz]');
% axis([-3 4]);
% grid;

% figure(44)
plot(freq,real(er),'k','LineWidth',2); hold on;
plot(freq,real(er1),'r--','LineWidth',2); hold off;
legend('Unwrapped','Wrapped');
ylabel('Permittivity');
xlabel('freq [GHz]');
axis([-10 10]);
grid;

% figure(55)
plot(freq,real(mu),'k','LineWidth',2); hold on;
plot(freq,real(mu1),'r--','LineWidth',2); hold off;
legend('Unwrapped','Wrapped');
ylabel('Permiability');
xlabel('freq [GHz]');
axis([min(freq) max(freq) -10 10]);
grid;

figure(6);
plot(freq,real(er),'b','LineWidth',2); hold on;
plot(freq,imag(er),'b--','LineWidth',2);
plot(freq,real(mu),'r','LineWidth',2);
plot(freq,imag(mu),'r--','Linewidth',2); hold off;
legend('real(er)','imag(er)','real(mu)','imag(mu)');
ylabel('permittivity & permeability');
xlabel('freq [GHz]');
axis([min(freq) max(freq) -5 5]);
grid;

figure(1)
hold on;
plot(freq,real(Eps),'k')
plot(freq,real(MuR),'r')
hold off

figure(2)
hold on
plot(freq,real(20*log10(s21)))
plot(freq,real(20*log10(s11)),'r')
hold off

figure(4)
hold on
plot(freq,abs(s21))
plot(freq,abs(s11),'r')
hold off