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GRAVEL
A METHOD FOR MATHEMATICAL MODELING AND COMPUTATION
OF HYDRAULIC PROCESSES IN RIVER BEND

By Chao-Lin Chiu*, Carl F. Nordin, Jr.,**, and Wei De Hu***

This paper presents a method for modeling and computing three-dimensional flows and shear-stress distributions in curved and straight reaches of streams and rivers, along with results from its application to flow in a river bend. The shear-stress formula is capable of including the effects of both the primary and secondary flows. Of special value is its capability to give the shear-stress distribution near the river banks where the flow is very three-dimensional and complex.

Introduction

The flow, as well as the shear stress distribution, in both the straight and curved reaches of an open channel, natural or man-made, is three-dimensional. The three-dimensional structure of flow and shear should therefore be understood in investigations of any other processes that take place in the channel because of interactions of all these processes. Since the actual measurements of the distribution of all three components of flow and shear stress are extremely difficult, mathematical models are needed for numerical computations and simulation.

There are different approaches to mathematical modeling. The mean vorticity equation in the longitudinal direction relates the secondary flow to vorticity and shear stress. However, none of the variables in the equation (shear stress, two components of secondary flow velocity, and vorticity) can be measured, estimated, or checked easily, especially in natural channels. The vorticity must be numerically calculated with secondary flow velocities. Therefore, the vorticity equation is not suitable for practical applications to natural channels, although it would certainly be useful in analyzing interactions among the secondary flow, vorticity and shear stress, if the secondary flow velocity and shear stress can be measured accurately (e.g. in a laboratory with hot-film or laser equipment). Another approach is to use momentum equations which relate the secondary flow to the primary flow and shear stress. One way to use this approach would be to measure or estimate (calculate) the shear stress and use it along with boundary conditions of flow and channel geometry to compute the primary and secondary flows. The problem of this approach is similar to the first approach: the required measurement or estimation (of shear stress, in this case) cannot be made or checked in natural channels.

*Professor of Civil Engrg., University of Pittsburgh, Pittsburgh, PA
***Grad. Res. Asst., Dept. of Civ. Engrg., University of Pittsburgh, Pittsburgh, PA
The method presented herein uses the measured or estimated primary flow velocity distribution in a shear equation, momentum equation in the longitudinal direction (which approximately represents the mean kinetic energy equation), and continuity equation to compute the shear stress and secondary flow (1,2). It is advantageous not only from the practical viewpoint but also from the viewpoint of estimation theory which emphasizes the observability and accuracy of one or more variables in the mathematical system model, as the primary flow can be measured and checked relatively easily and accurately.

Mathematical Models

1. Primary flow velocity distribution

The isovels of primary flow in all three cases in Fig. 1 in an open channel can be represented quite well by the following equation:

$$\xi = Y(1 - z) \exp(\beta_i z - Y + 1)$$  \hspace{1cm} (1)

in which

$$Y = \frac{y + \delta_y}{D + \delta_y + \varepsilon}$$  \hspace{1cm} (2)

$$Z = \frac{|z|}{B_i + \delta_i}$$  \hspace{1cm} (3)

and $\xi$ is a variable defined in a logarithmic velocity distribution law,

$$u = \frac{u_\ast}{k} \ln\left(\frac{\xi}{\xi_0}\right)$$  \hspace{1cm} (4)

$D = $ the water depth at the $y$-axis; $B_i$ for $i$ equal to either 1 or 2 = the transverse distance on the water surface between the $y$-axis and either the left or right bank of a channel cross section; $z$ = the coordinate in the transverse direction; $y$ = the coordinate in the vertical direction (the $y$-axis is selected such that it passes through the point of maximum primary flow velocity; $u_\ast$ = the mean shear velocity; $u$ = the primary flow velocity (in the $x$-direction); $\varepsilon$, $\delta_y$, $\delta_i$, $\xi_0$, $\beta_i$, and $k$ = coefficients characterizing the velocity distribution of primary flow. Fig. 1 shows the coordinates chosen along with other variables which appear in the above equations.

The slope of an isovel can be expressed by the following derivative obtained from Eq. 1:

$$s_\xi = \frac{dy}{dz} = \frac{D + \delta_y}{B_i + \delta_i} \frac{YZ}{[1-Y][1-Z]}$$  \hspace{1cm} (5)

$Y$, $Z$ and $[1 - Z]$ in Eq. 5 have non-negative values. For $\varepsilon > 0$ or for $\varepsilon < 0$ with $y < D + \varepsilon$, even $[1 - Y]$ is greater than zero as $Y < 1$, so that isovel slopes are greater than or equal to zero (increasing with $z$).
Fig. 1.-Primary Flow Velocity Distribution and Coordinate Systems
For $\varepsilon < 0$ as shown in Fig. 1(a) where the maximum primary flow velocity occurs below the water surface, $Y > 1$ in the region $D + \varepsilon < y < D$ and, hence, $s_\xi < 0$, so that isovels tend to curve in towards the $y$-axis. For $\varepsilon = 0$ as shown in Fig. 1(b), the isovels are perpendicular to the water surface. A detailed procedure for estimating the coefficients in the above equations is given in an earlier paper (3).

A family of orthogonal trajectories of the isovels ($\xi$-curves) of primary flow can be derived from Eq. 1 as:

$$\eta = \pm \frac{1}{2} \left(1 - \frac{1}{Y_1}\right) \exp\left[\beta_i \left(\frac{D + \delta_i + \varepsilon}{B_i + \delta_i}\right)^2 Y_2\right] \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 
in the plane perpendicular to the $\xi$-direction; and $\sigma'$ is the normal stress (excluding the pressure) in the $x$-direction. In Eq. 10 both $u$ and $V_\xi$ are mean (time-averaged) values. The term $\frac{\partial \sigma'}{\partial x}$ is usually small relative to other quantities in Eq. 10 and, hence, may be neglected. The solution of Eq. 9 can be expressed as

$$\tau_{\xi x}(\xi, \eta) = \frac{-1}{h \eta} \int_{\xi}^{\xi_{max}(\eta)} h \xi h \eta F d\xi + \tau_{\xi x}[\xi_{max}(\eta), \eta] \quad \ldots \ldots (11)$$

in which $\xi_{max}(\eta) =$ the maximum value of $\xi$ on an $\eta$-curve, which is located at the point of maximum primary flow velocity on each $\eta$-curve; and the second term on the right side of Eq. 11 is the value of $\tau_{\xi x}$ at the point, $[\xi_{max}(\eta), \eta]$. On each $\eta$ curve, $\xi' \leq \xi \leq \xi_{max}$. An alternative form of the solution of Eq. 9 as used by Chiu and Hsiung (1) is

$$\tau_{\xi x}(\xi, \eta) = \alpha_0 + \alpha_1[\xi_{max}(\eta) - \xi] + \alpha_2[\xi_{max}(\eta) - \xi]^2 \quad \ldots \ldots (12)$$

in which

$$0 \leq [\xi_{max}(\eta) - \xi] < 1 \quad \ldots \ldots \ldots \ldots \ldots \ldots (13)$$

and $\alpha_0$, $\alpha_1$, $\alpha_2 =$ coefficients. Eq. 12 is a polynomial in $[\xi_{max}(\eta) - \xi]$, an approximate solution of Eq. 9.

The boundary conditions of $\tau_{\xi x}$ and $V_\xi$ that can be used to determine the three coefficients of Eq. 12 are: (a) $\tau_{\xi x} = 0$ on the water surface and at the point of maximum velocity where $\xi_{xe} = \xi_{max}(\eta)$; (b) the mean boundary shear is $\rho g R S_f$; (c) the secondary flow velocity $V_\xi$ is zero along the channel bed (including the banks) represented by an isovel, along which $\xi = \xi'$. $\xi'$ tends to differ somewhat from $\xi_{xo}$ in Eq. 4, although the actual distance between the $\xi'$ and $\xi_{xo}$ curves is very small because of the extremely large gradient of $\xi$ near the channel bed. The above conditions, combined with Eqs. 9 and 12, give that:

$$\alpha_0 = 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots (14)$$

$$\alpha_2 = \frac{-1}{[\xi_{max}(\eta) - \xi']f_2(\eta)} [\rho g R S_f + \frac{h \xi h \eta}{\xi_{xo} f_2(\eta)}] \quad \ldots \ldots (15)$$

in which the bars above terms signifies the mean values taken along the entire length of wetted perimeter (including the channel bed and sides); $F_0$ is the function defined by Eq. 10 without the $V_\xi$ term

$$f_2(\eta) = \frac{\xi_{max}(\eta) - \xi'}{1 - \frac{1}{h \eta} \frac{\partial h}{\partial \xi}[\xi_{max}(\eta) - \xi'] \ldots \ldots \ldots \ldots \ldots \ldots (16)$$

$$\alpha_1 = f_1(\xi = \xi', \eta) \quad \ldots \ldots \ldots \ldots \ldots \ldots (17)$$
\[
\begin{align*}
\alpha_1 = \alpha_2 [1 - \xi_D(\eta)] \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (19)
\end{align*}
\]

\(\xi_D(\eta)\) is \(\xi\) at the intersection of an \(\eta\) curve with the water surface (where \(y = D\)). The boundary shear can be computed by Eq. 12 with \(\xi = \xi_0^t\).

3. **Secondary flow**

Eq. 9 is equivalent to:

\[
V_\xi = (\rho \frac{\partial u}{\partial \xi})^{-1} \left[ \frac{\partial \tau_{\xi x}}{\partial \xi} + \frac{1}{h} \frac{\partial h}{\partial \xi} \tau_{\xi x} - h F_0 \right] \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (20)
\]

which gives the \(\xi\)-component of secondary flow velocity. The continuity equation gives the \(\eta\)-component of velocity as

\[
v_\eta = -\frac{1}{h} \int_{\eta_0^*}^{\eta} \left[ h \frac{\partial h}{\partial \eta} \frac{\partial V_\xi}{\partial \xi} + \frac{\partial V_\xi}{\partial \xi} \right] d\eta + V_{\eta_0^*} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (21)
\]

in which \(V_\eta = V_\eta\) at a boundary point on a \(\xi\) curve, where \(\eta = \eta_0^*\). After computing \(V_\eta\) at grid points in the \(\xi-\eta\) coordinate network, using Eq. 12 for \(\tau_{\xi x}\) and Eq. 4 for \(u\), Eq. 21 gives \(V_\eta\). The integration of Eq. 21 is executed along each \(\xi\) curve starting from a boundary point where \(\eta = \eta_0^*\) and \(V_{\eta_0^*}\) is known (computed). The most convenient boundary point along a \(\xi\) curve is on the water surface where the \(y\) (vertical) component of velocity is zero which, along with \(V_\xi\) given by Eq. 20, can be used to compute \(V_\eta\) and the \(z\) component of velocity, \(w\), by the transformation rule

\[
V_\xi = h_\xi (v \frac{\partial \xi}{\partial y} + w \frac{\partial \xi}{\partial z}) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (22)
\]

\[
V_\eta = h_\eta (v \frac{\partial \eta}{\partial y} + w \frac{\partial \eta}{\partial z}) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (23)
\]

Eqs. 22 and 23 can also be used to transform \(V_\xi\) and \(V_\eta\) computed in the \(\xi-\eta\) coordinate system into \(v\) and \(w\) in the Cartesian, \(y-z\) system.
The modeling and computational method presented above have been developed and tested (2,3) with data from bends of laboratory channels (5,6). The boundary shear distributions measured (with a Preston tube) agreed quite well with the computed. The mathematical model of primary flow isovels was also found capable of closely simulating the primary flow velocity distributions and channel cross sections. The general pattern of computed secondary flow circulation agreed quite well with the measured, although the measured and computed magnitudes of secondary flow velocity did not come close enough at many points. Since the exact accuracy of measured secondary flow velocity is unknown a conclusion cannot be drawn from any disagreements between the measured and computed magnitudes of secondary flow velocity. The small magnitude of secondary flow velocities, relative to the primary flow velocities and other interacting variables in the mathematical model, seems to make it unrealistic to expect a high level of accuracy in secondary flow velocity, computed or measured. When applying the model presented, one should, therefore, be aware of such a reality.

Application to East Fork River

Field data were collected during June 24 through June 28, 1980, on a bend of the East Fork River near Boulder, Wyoming. Six cross sections were established along the reach shown in Fig. 2, tag lines were strung at each section, and two field crews operated from boats to collect the velocity data. One crew measured velocities with a Marsh-McBirney Model 512 electromagnetic flow meter (EMF) which measures simultaneously two components of velocity; the second crew used an Ott-type current meter described by Smoot and Novak (4)*. This is a cosine meter that senses the component of velocity along the axis of the meter. By measuring two components of the velocity at any point, the velocity vector can be determined by vector addition and resolved into an x component perpendicular to the section and a z component parallel to the section. The EMF measures x and z components directly; and by rotating the meter 90° on its mount, it can measure also the vertical y component. All three components were measured only at section 3 on June 28, 1980. On all other observations, only x and z components were recorded. The East Fork River is fed by snow-melt so there are strong diurnal fluctuations in the flow. Measurements were made at all six cross sections on June 24 and 26 but only on the four downstream sections on June 28. Only the data from June 26 are used in this paper.

Fig. 3 shows the primary flow isovels (including the channel boundaries) of the six cross sections, based on measured data and Eq. 1 (with rotations of coordinates on both sides of the y-axis). The location of the y-axis in each cross section is indicated by z = 0. The page limitation does not allow presentation of the measured primary flow velocity.

Fig. 4 shows the shear distributions (lines of equal shear in the flow and boundary shear distribution) computed with Eq. 12 which

*Use of trade names is for identification only and does not imply endorsement by the U.S. Geological Survey.
Fig. 2.- A Bend of East Fork River near Boulder, Wyoming, Showing Six Measured Cross Sections
Fig. 3. - Primary Flow Isovels and Six Cross Sections of East Fork River, near Boulder, Wyoming

(Flow Velocity in m/s; circled points shown represent measured cross sections)
Fig. 4. Computed Shear Stress Distribution
East Fork River, near Boulder, Wyoming
correspond to the primary flow velocity distributions in Fig. 3. The numerical values shown on the computed lines of equal shear are the ratios of shear stress to the mean boundary shear, $τ_0 = \rho g R \frac{e}{e}$, at each cross section. The computed result gives a continuous boundary shear distribution curve covering the entire boundary, including the corners and walls and indicates that the peak boundary shear does not necessarily occur below the point of maximum primary flow velocity, although the boundary shear there is often found to be high if not maximum. Rises in boundary shear on the channel bottom tend to occur near the corners and walls, and create two peaks of different magnitude. One of these two peaks may become maximum.

Fig. 5 shows the computed secondary flow velocity in the six cross sections for which the distributions of primary flow velocity and shear stress have been presented in Figs. 4 and 6 respectively. The arrows in Fig. 5 show both the direction and the magnitude. Fig. 6 shows measured transverse component of secondary flow velocity. The disagreement between the computed and measured secondary flow velocity is considerable at some of the cross sections. In viewing Figs. 5 and 6 it must be kept in mind that the flow is strongly non-uniform and, therefore, the continuity equation is satisfied only in the three-dimensional space so that the circular, secondary flow "cells" may not be visible in some parts of a cross section. In fact the computed pattern at some locations tends to mislead one to vision somethings like "sinks" and "sources" which do not exist. When the direction of $V_t$ is consistently negative (i.e. downward and perpendicular to the primary flow isovels) along an η curve, a high value of boundary shear results at the intersection of the η curve and the channel bed, as supported by the theory of Eq. 11 (1,2).

Summary and Conclusion

Along the bend of East Fork River (when the Froude number $N_F < 0.2$), the points of maximum boundary shear (computed) and maximum flow depth both swing swiftly outward (not in phase) toward the outer bank from the region near the inner bank at the entrance. The point of maximum primary flow velocity also moves outward but at a much slower rate. At the middle of the bend (Section 1) the three points (the points of maximum primary flow velocity, maximum depth and maximum boundary shear) are distinctly separated. After that, toward the outlet of the bend, they tend to come close together again. This pattern is similar to that observed by Yen (6) in a curved, laboratory channel ($N_F < 0.3$) of sand bed and of geometrical features of cross sections and their variations similar to those of the present river bend studied. The present river bend generally has sand bed at the shallower side of channel cross section and the gravel bed on the deeper side. The outward movement of the three points is in contrast with results observed in another study (3,5) in a curved, laboratory channel ($N_F < 0.8$) of rigid cross sectional boundaries which did not allow cross sectional geometry to deform along the bend. In that study the points of maximum boundary shear and maximum primary flow velocity were found to stay close to the inner bank throughout the bend, while they are situated about in the middle of cross sections at the bend.
Fig. 5.—Computed Secondary Flow Velocity
East Fork River, near Boulder, Wyoming
Fig. 6. - Measured Transverse Component of Secondary Flow Velocity (w) East Fork River, near Boulder, Wyoming
entrance and outlet. Combined results of these studies describe the interaction among the channel cross section (bed material, cross sectional shape), primary flow velocity distribution, secondary flow and shear stress distribution.

Tested by laboratory data (2,3,5,6) the modeling and computational method used seems to be capable of simulating the three dimensional structure of flow, shear and channel cross sections and their variations along the river bend. Especially valuable is the ability of the mathematical model of primary flow isovels (Eq. 1 with rotations of coordinates) to simulate the irregular and asymmetrical channel cross sections, in addition to isovels. This feature in turn enables the inclusion of the effect of irregular and asymmetrical cross sectional shape and primary flow velocity distribution in computing the secondary flow and shear stress distribution.

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References


