THESIS

EVALUATION OF SAMPLING TECHNIQUES TO CHARACTERIZE
TOPOGRAPHICALLY-DEPENDENT VARIABILITY FOR SOIL MOISTURE
DOWNSCALING

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Kevin Werbylo

Department of Civil and Environmental Engineering

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Master’s Committee:

Advisor: Jeff Niemann

Tim Green
Stephanie Kampf
ABSTRACT

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Soil moisture patterns are an important consideration in many catchment-scale hydrologic applications. Unfortunately, estimating soil moisture patterns at resolutions that are appropriate for these applications (e.g., grid cells with a linear dimension of 10 to 50 m) is difficult. Downscaling methods can be used to estimate catchment-scale soil moisture patterns from coarser resolution estimates or spatial average soil moisture values. These methods usually infer the fine-scale variability in soil moisture using variations in ancillary variables like topographic attributes that have relationships to soil moisture. Previously, such relationships have been observed in catchments using soil moisture observations taken on uniform grids at hundreds of locations on multiple dates, but collecting data in this manner limits the applicability of this approach. The objective of this paper is to evaluate the effectiveness of two strategic sampling techniques for characterizing the relationships between topographic attributes and soil moisture for the purpose of constraining downscaling methods. The strategic sampling methods considered are conditioned Latin hypercube sampling (cLHS) and stratified random sampling (SRS). Each sampling method is used to select a limited number of locations and/or dates for soil moisture monitoring at three catchments with detailed soil moisture datasets (Tarrawarra, Satellite Station, and Cache la Poudre). These samples are then used to calibrate two available downscaling methods, and the effectiveness of the sampling methods is evaluated by the ability
of the downscaling methods to reproduce the known soil moisture patterns at the catchments. The results show that cLHS and SRS can characterize the relationships between soil moisture and ancillary topographic variables with many fewer locations and dates than previously used. For example, when the number of locations for soil moisture monitoring is reduced by 82-90% and these locations are only monitored on 3 dates, the explanatory power of the downscaling methods frequently only reduces by less than 50%. Furthermore, both strategic sampling methods can substantially outperform random sampling when the number of samples is limited.
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1.0 INTRODUCTION

Many hydrologic processes are influenced by patterns of volumetric water content in the soil (soil moisture). Specifically at the catchment scale, spatial patterns of soil moisture are closely related to spatial patterns of erosion (Fitzjohn et al., 1998), crop yield (Green and Erskine, 2004), and the magnitude and timing of runoff production (Western, 2001). As such, soil moisture is an important consideration in many catchment-scale hydrologic applications including flood forecasting, agricultural production, watershed and land management, and various military operations.

Accurately estimating catchment-scale soil moisture patterns at fine spatial resolutions (e.g., grid cells with a 10 to 50 m linear dimension) is difficult. Microwave-based remote sensing methods are commonly used to estimate soil moisture patterns over large areas, but the resolution of the estimated patterns is too coarse for catchment-scale applications. For example, from 2002 to 2011, the Advanced Microwave Scanning Radiometer (AMSR-E) produced global soil moisture patterns with a spatial resolution of 60 km (Njoku et al., 2003), while the Soil Moisture and Ocean Salinity (SMOS) satellite currently produces global soil moisture patterns with a spatial resolution of 30 to 50 km (Kerr et al., 2001). Even the planned Soil Moisture Active Passive (SMAP) satellite will produce soil moisture patterns with a spatial resolution of only 9 to 36 km (Entekhabi et al., 2010). Synthetic aperture radar (SAR) has been proposed to estimate soil moisture patterns at resolutions near 10 m (Ulaby et al., 1996), but the ability of these methods to accurately characterize soil moisture patterns at the catchment scale has been questioned (Western et al., 2001). Alternatively, remote sensing methods utilizing the optical and thermal range of the spectrum have been proposed to indirectly estimate soil moisture patterns at fine spatial resolutions by first estimating the components of the surface energy balance and then
using empirical relationships to estimate soil moisture (Ahmad and Bastiaanssen, 2003; Scott et al., 2003). If applied using Moderate-Resolution Imaging Spectroradiometer (MODIS) images, this method would provide soil moisture estimates at a 1 km resolution. Recently, a non-invasive ground-based method to estimate soil moisture from neutron emissions (Zreda et al., 2008) has been proposed and developed as part of the Cosmic-Ray Soil Moisture Observing System (COSMOS) project (Shuttleworth et al., 2010), but the resolutions of the estimated patterns are still relatively coarse (about 700 m).

Various methods have been proposed to downscale the coarse-resolution soil moisture estimates obtained from remote sensing to a variety of finer resolutions. When applying these methods, the desired resolution of the soil moisture estimate is important to consider because different factors control spatial and temporal variations in soil moisture at different scales (Western et al., 2002). Merlin et al. (2006) developed a method using soil temperature, atmospheric conditions, and vegetation information to downscale soil moisture patterns to a resolution of 1 km. Statistical methods, one using fractal analysis coupled with observed soil texture, vegetation, and terrain ancillary data (Kim and Barros, 2002) and another utilizing multifractal analysis (Mascaro et al., 2010; Mascaro et al., 2011), have been proposed to downscale soil moisture patterns to resolutions of 825 and 800 m, respectively. Moving to finer resolutions, Crow et al. (2000) proposed a method using surface soil dielectric values obtained from remote sensing, along with soil texture information, to obtain estimates of soil moisture at resolutions as fine as 100 m. Pellenq et al. (2003) estimated soil moisture patterns at resolutions as fine as 20 m using a method that incorporates topographic indices and soil depth observations, although the accuracy of the estimated patterns was only considered adequate at a resolution of
100 m. Furthermore, Temimi et al. (2010) used terrain and vegetation-based indices to downscale soil moisture estimates to a resolution of 90 m.

Methods have also been proposed to downscale soil moisture patterns to fine resolutions that are more suitable for catchment-scale hydrologic applications (50 m or finer). Kaheil et al. (2008) proposed a method to downscale soil moisture patterns to a 50 m resolution using sparse ground observations. Wilson et al. (2005) used fine-resolution topographic attributes, fine-resolution (10-40 m) in-situ soil moisture observations, and a single spatial average soil moisture value on each date to estimate soil moisture patterns with resolutions of 10-40 m. Similarly, Perry and Niemann (2007) and Busch et al. (2012) proposed a method using empirical orthogonal function (EOF) analysis, fine-resolution (5-15 m) topographic attributes, fine-resolution (10-40 m) in-situ soil moisture observations, and a single spatial average soil moisture value on each date to estimate soil moisture patterns with resolutions of 10-40 m. Recently, Coleman and Niemann (2013) proposed a conceptual model known as the Equilibrium Moisture from Topography (EMT) model to estimate soil moisture patterns at resolutions of 10-40 m using fine-resolution (5-15 m) topographic attributes, fine-resolution (10-40 m) soil moisture observations, and a single spatial average soil moisture value on each date.

Several of these downscaling methods infer the fine-scale variability of soil moisture from its relationship to available ancillary variables. For example, soil moisture patterns have been shown to be correlated with spatial patterns of topography, vegetation, soil texture, or a combination of these variables (Cantón et al., 2004; Gómez-Plaza et al., 2001; Gutiérrez-Jurado et al., 2006; Western et al., 1999). At the catchment-scale, topography has been a widely-used ancillary data source (Busch et al., 2012; Coleman and Niemann, 2013; Perry and Niemann, 2007; Wilson et al., 2005) because of its known influence on soil moisture patterns at this scale.
(Famiglietti et al., 1998; Western and Grayson, 1999) and its nearly global availability at fine resolutions (Welch et al., 1998). Many of these downscaling methods also use in-situ soil moisture observations to characterize the relationships between the variations in soil moisture and the variations in ancillary data. These relationships are commonly obtained through linear regression (Busch et al., 2012; Perry and Niemann, 2007; Wilson et al., 2005) or parameter calibration (Coleman and Niemann, 2013). Busch et al. (2012) found that such relationships are catchment-specific, which implies that soil moisture observations need be collected from the catchments where the downscaling method will be applied or the relationships need to be inferred from knowledge of the physical characteristics of the catchment. In the development of most catchment-scale downscaling methods, the in-situ soil moisture observations have been collected on uniform grids, which contain hundreds of points on multiple dates (Busch et al., 2012; Coleman and Niemann, 2013; Perry and Niemann, 2007; Wilson et al., 2005). Collecting data in this manner is expensive and time-consuming, which limits the applicability of such methods.

Several studies have considered more efficient sampling techniques to observe catchment-scale soil moisture behavior, but these studies aim to capture catchment-average conditions. In early work, Vachaud et al. (1985) suggested that locations exist within a catchment that are consistently similar to the catchment-average soil moisture. It was proposed that the temporal behavior of the catchment-average soil moisture could be monitored using observations only at these locations. Grayson and Western (1998) referred to these locations as catchment average soil moisture monitoring (CASMM) sites. Many other researchers have performed related work by either attempting to estimate a catchment-wide spatial average soil moisture using point-scale observations from a limited number of locations (Brocca et al., 2009;
Martinez-Fernandez and Ceballos, 2005) or by attempting to validate coarse-resolution remote sensing estimates by upscaling point soil moisture observations (Cosh et al., 2008; Cosh et al., 2006; Crow et al., 2012; Crow et al., 2005).

Other sampling techniques have been proposed to efficiently capture the variability of catchment conditions, but such techniques have not been applied to soil moisture. Conditioned Latin hypercube sampling (cLHS) (Minasny and McBratney, 2006) and stratified random sampling (SRS) (Avery and Burkhart, 2001) aim to determine monitoring locations for the variable of interest based on knowledge of ancillary variables. The goal of both methods is to identify sampling locations so that they include a diverse set of values for the ancillary variables. cLHS and SRS are similar in that they each divide the observed range of each ancillary variable into bins and then select the observation locations from the locations within each bin. These methods differ in how the observed ranges of the ancillary variables are divided into bins. cLHS divides the range of the ancillary variable into equally probable bins such that each bin contains the same number of observations. For SRS, different methods have been used to determine the bins (McKenzie and Ryan, 1999; Worsham et al., 2012). Here, we focus on the case where SRS divides the range of each ancillary variable into bins that cover an equal fraction of the full range, regardless of the number of observations within each bin. Both cLHS and SRS are potentially more efficient than uniform or random sampling because they aim to reduce redundancy in the information gathered at the sampling locations. Minasny and McBratney (2006) evaluated cLHS in the context of soil mapping and found that sample histograms created from cLHS better replicate the known histograms of topographic, vegetative, and land use ancillary variables than those created from random sampling and a stratified sampling method. Recently, Worsham et al. (2012) evaluated the use of cLHS and a SRS method by their ability to
improve spatial estimates of soil carbon content. Both methods outperform random sampling when sample sizes are limited, but cLHS does not consistently outperform SRS in that context. The SRS method they used stratifies the landscape into units based on soil type and land use data. Samples are then selected randomly from each spatially-contiguous unit in order to sample across the ranges of the ancillary variables as well as the spatial extent of the region. McKenzie and Ryan (1999) also used an SRS method (Brus and de Gruijter, 1997) with climate and topographic ancillary variables to make spatial predictions of soil depth, total phosphorus, and total carbon. The SRS method they used only focuses on adequately covering the ranges of the ancillary variables (not the spatial extent of the area of interest).

The objective of the present paper is to assess the effectiveness of two strategic sampling techniques at identifying the relationships between topographic attributes and soil moisture for catchment-scale downscaling applications. Two strategic sampling techniques are considered: the cLHS method proposed by Minasny and McBratney (2006) and an SRS method that is similar but not identical to the SRS technique used by McKenzie and Ryan (McKenzie and Ryan, 1999). These sampling methods are coupled with two downscaling methods: the EMT model (Coleman and Niemann, 2013) and the EOF method (Busch et al., 2012). The ancillary variables that are required by these downscaling methods (various topographic attributes) are used by the sampling techniques to identify locations where the soil moisture should be monitored. Then, the soil moisture values at the monitored locations are used to define the relationships between the topographic attributes and soil moisture in the downscaling methods. The downscaling methods are then used to produce estimates of the catchment-scale soil moisture patterns. Ultimately, the performance of the sampling methods is evaluated by the ability of the two downscaling techniques to reproduce the actual catchment-scale soil moisture
patterns at three application catchments (Tarrawarra, Satellite Station, and Cache la Poudre) when supplied with data from the sampling methods. As a secondary objective in this study, the EMT model and EOF method are compared under a variety of the data-limited conditions.
2.0 METHODOLOGY

2.1 Sampling Methods

The cLHS method proposed in Minasny and McBratney (2006) can be summarized as follows. To start, the values of the ancillary variables at all locations on the desired fine-resolution grid within the region of interest are organized into a matrix $X$ of size $N \times K$ where $N$ is the number of locations and $K$ is the number of ancillary variables observed at each location. Any row in $X$ represents a location in the catchment, and each column contains the values for a particular ancillary variable. In the present application, the ancillary variables are various topographic attributes that are required by the downscaling methods (discussed in more detail later). Using the values in each column of $X$, each of the $K$ ancillary variables is divided into $n$ bins where $n$ is the number of desired samples (i.e. locations). For a given ancillary variable, the limits for the bins are defined so that each bin contains an equal number of values of the ancillary variable. Figure 1a displays a hypothetical example where a single ancillary variable is used and the range of the ancillary variable has been divided into 3 bins in this manner. A sample of locations of size $n$ is then randomly selected from $X$ producing a matrix $x$ of size $n \times K$ that contains the values of the ancillary variables at the selected locations. A particular row of $x$ represents one of the selected locations. An associated matrix $\eta$ of size $n \times K$ is then created. An element of $\eta$ is associated with a particular bin number (1 to $n$) and a particular ancillary variable (1 to $K$). The element contains the number of locations in $x$ that occur within that bin for the associated ancillary variable. If the sample represents a perfect Latin hypercube of $X$, each element of $\eta$ would have a value of one, which would imply that the sample locations produce one observation in each of the $n$ bins for all $K$ ancillary variables. When considering multiple ancillary variables, a perfect Latin hypercube often cannot be obtained because certain
combinations of ancillary variables do not exist at any single location (e.g., a location with large contributing area and a large slope).

Minasny and McBratney (2006) developed an algorithm in MATLAB (Mathworks, 2011) to search through $X$ to find the sample that best approximates the perfect Latin hypercube of $X$. The best sample is found by minimizing an objective function that includes two terms. The first term is used to determine how well a trial set of sample locations and the associated ancillary variables represents the perfect Latin hypercube. The second term measures how well the cross-correlation of the elements in $X$ is represented by that of the elements in $x$. This ensures that the correlation structure of the ancillary variables at all locations on the fine-resolution grid is well represented by those at the sampled locations. The first term of the objective function $O_1$ is calculated by summing the absolute values of the differences between each element of $\eta$ and one:

$$O_1 = \sum_i^n \sum_{j=1}^K |\eta(q_j^i \leq x_j \leq q_{j+1}^i) - 1|$$

where $\eta(q_j^i \leq x_j \leq q_{j+1}^i)$ represents the number of values of an ancillary variable $x_j$ that fall into a bin with quartile bounds of $q_j^i$ and $q_{j+1}^i$ (from Minasny and McBratney, 2006). Thus, if $x$ represents a perfect Latin hypercube of $X$, each element of $\eta$ would be equal to one and the first term of the objective function would equal zero. The second term of the objective function $O_2$ is determined by summing the absolute values of the differences between the elements of the correlation matrix of $X$ and those of the correlation matrix of $x$:

$$O_2 = \sum_i^n \sum_{j=1}^K |c_{ij} - t_{ij}|$$
where \( c_{ij} \) represents an element of the correlation matrix of \( X \) and \( t_{ij} \) represents an element of the correlation matrix of \( x \) (from Minasny and McBratney, 2006). Thus, if the correlation of the ancillary variables in the population matches the correlation of the ancillary variables in the sample, this term will be zero. Both terms of the objective function include coefficients \( (w_1 \) and \( w_2) \) that can be specified to control the importance of that term. For the present application, the final form of the objective function is:

\[
O = w_1 O_1 + w_2 O_2
\]  

(3)

Minasny and McBratney (2006) recommended that both coefficients be set to one for general applications of cLHS. If the ancillary variables include categorical data like land-use classifications, the objective function would have a third term. However, this third term is not required in the present application. The optimization is performed with simulated annealing (Press, 1992). In each iteration, locations in \( x \) are replaced with other candidate locations from \( X \) to minimize the objective function. Minasny and McBratney (2006) stopped the algorithm after 50,000 iterations, which was also found to be sufficient for the present application.

The SRS method used in the present study is comprised of the following steps. First, like cLHS, the values of the ancillary variables at the locations on the fine-resolution grid in the region of interest are organized into a matrix \( X \) of size \( N \) by \( K \). If the region and ancillary variables are the same, then the \( X \) matrix will be the same for the cLHS and SRS methods. Second, the ranges of the ancillary variables in \( X \) are examined to determine the appropriate limits for the bins that will be used to divide those ranges. If \( p \) bins are selected, then the limits are set so that each bin covers an equal fraction \( (1/p) \) of the observed range of the ancillary variable. Figure 1b displays a hypothetical scenario where the values of a single ancillary
variable have been divided into three bins in this manner. Next, a matrix $Y$ of size $N$ by $K$ is created. Each element of $Y$ contains a number that identifies the bin in which the associated value of the ancillary variable in $X$ falls. These numbers are referred to as individual identifiers. Notice that for any given location, it is possible that the value for one ancillary variable falls in the smallest bin for that ancillary variable, while the value of another ancillary variable falls in the largest bin for its ancillary variable. Thus, a given row in $Y$ can have different individual identifiers in each column. Next, a matrix $Z$ of size $N$ by 1 is created. This matrix contains the values of a joint identifier. The joint identifier is a number that refers to a unique combination of individual identifiers. Thus, the value in a row of $Z$ identifies the particular combination of individual identifiers that occurs at that location. Once $Z$ has been created, locations with similar combinations of ancillary variable values can be identified because they have identical joint identifiers. These locations are said to be part of the same land unit. Similar to cLHS, not all possible land units (combinations of ancillary variables) necessarily exist in a particular region of interest. The last step in the SRS method is to select the sampling locations. To do this, one location from each land unit is randomly selected, regardless of the number of locations in that unit. Thus, the number of sampling locations is equal to the number of land units that are present in the region of interest. Unlike cLHS, the number of sample locations $n$ is not specified by the user. Instead, the user specifies the number of bins for each ancillary variable $p$.

In this paper, cLHS and SRS are compared to random sampling, which is used as the control case. This approach is similar to other studies that evaluated the effectiveness of strategic sampling methods (Minasny and McBratney, 2006; Worsham et al., 2012). Random sampling was selected instead of uniform sampling because the number of sampling locations can be controlled more directly. In addition, it was determined that, at least for small numbers of
samples, uniform and random sampling yield similar results. To perform random sampling, the desired number of samples $n$ is specified by the user. Then, $n$ locations are randomly selected without replacement from the locations on the fine-resolution grid within the region of interest. No ancillary variables are used to assist in the sampling process.

2.2 Downscaling Methods

The EMT model is a conceptual soil hydrology model that estimates soil moisture patterns within a catchment by inferring the roles of vadose zone processes from topographic attributes. A detailed derivation and description of the model is provided by Coleman and Niemann (2013). The model was derived by considering the soil water balance for the land that drains through the edge of a digital elevation model (DEM) grid cell. In deriving this water balance, it was assumed that the spatial variation of soil moisture within each coarse-resolution grid cell can be inferred from equilibrium conditions (thus hysteresis is disallowed). The model includes infiltration, evapotranspiration, lateral flow, and deep drainage. These hydrologic processes are simulated using conceptual expressions that involve four topographic attributes: slope, curvature as defined by Heimsath et al. (1999), specific contributing area (SCA), and the potential solar radiation index (PSRI) on the summer solstice (Dingman, 2002). To apply the model, eleven soil, vegetation, and climatic parameters must be calibrated using local soil moisture observations from the catchment. After the parameters have been calibrated, the EMT model can be used to estimate the fine-resolution soil moisture patterns in a catchment from the spatial average soil moisture for the catchment and topographic attributes at the same fine resolution.
The EOF method estimates soil moisture patterns within a catchment by empirically relating local soil moisture values to topographic attributes. The details of the method development and application procedure are provided by Busch et al. (2012). The EOF method relies on an EOF decomposition of a soil moisture dataset that includes observations at multiple locations within the catchment on two or more dates. The decomposition produces a series of time-invariant spatial patterns of variation (EOFs), a series of expansion coefficients (ECs) that measure the importance of each EOF on each date, and the spatial average soil moisture on each date. Together, these elements can be used to reconstruct the original dataset. When downscaling with the EOF method, only the EOFs that are determined to be statistically significant patterns of variation are retained. Then, a stepwise multiple linear regression is performed to estimate the retained EOFs at any location from the topographic attributes of slope, cosine of the aspect, SCA, natural log of the SCA, wetness index (Beven and Kirkby, 1979), various curvatures, and PSRI. The curvatures include the profile, plan, and tangential curvatures, as well as the Laplace curvature (Mitasova and Hofierka, 1993). Thus, while the EMT model uses four predetermined topographic attributes to estimate the soil moisture pattern, the EOF method considers 10 topographic attributes and uses multiple linear regressions to determine the subset of attributes that are statistically relevant and thus used for a particular case. Piecewise linear relationships are used to estimate the ECs that are associated with the retained EOFs on any date from the spatial average soil moisture. After the regressions and piecewise functions have been developed, the EOFs can be estimated from the topographic attributes at any location (even if soil moisture observations are not available at that point), and the associated ECs can be estimated for any date from the spatial average soil moisture. The spatial average (or a grid of spatial averages) is always known in a downscaling application. Thus, the EOF method can be
used to estimate fine-scale soil moisture patterns in a catchment from the spatial average soil moisture and the fine-scale topographic attributes.

2.3 Catchment Descriptions

The methods of this study are applied at three catchments where large soil moisture datasets are available and topography has been shown to influence the spatial variability of soil moisture. The first catchment is Tarrawarra, which is located near Melbourne, Australia (Western and Grayson, 1999), covers an area of 10.5 ha, and has a total relief of about 30 m. The climate at Tarrawarra is temperate with an average annual precipitation of 82 cm and an average annual potential evapotranspiration (PET) of about 83 cm. The vegetation is homogeneous grasses, and the land is predominately used for grazing. The soil is fairly deep with a clay-loam A horizon extending as deep as 40 cm and a clay B horizon that extends to depths beyond 100 cm at some locations. The terrain attributes that are used in this study were calculated from an available 5 m DEM that was originally developed from a land survey (Western and Grayson, 1998). In-situ soil moisture observations are available from time-domain reflectometry (TDR) in the top 30 cm of the soil. We use only the 454 locations that are available on all 13 sampling dates, which span a total of 14 months. The observations were taken on a uniform grid with a 10 m by 20 m spacing over the entire catchment.

It is worth noting a few characteristics of the Tarrawarra soil moisture patterns that might influence the performance of the sampling methods. Among the three catchments considered, the soil moisture at Tarrawarra has the strongest dependence on topography (Busch et al., 2012). In addition, the observed soil moisture patterns at Tarrawarra are temporally unstable, meaning
that the spatial structure of the soil moisture patterns changes through time. During dry
conditions, locations on the hillslopes that are oriented away from the sun tend to be wetter than
locations on the hillslopes oriented towards the sun (Grayson et al., 1997). This structure has
been referred to as a hillslope-dependent pattern (Coleman and Niemann, 2013) and has been
shown to be correlated with PSRI (Busch et al., 2012). Figure 2a shows the histogram of PSRI
values at Tarrawarra, which is approximately uniform. Figure 2b plots the soil moisture on a date
with a hillslope-dependent pattern (28 March 1996) against PSRI. Figure 2c shows the histogram
of soil moisture on this same date. The soil moisture values are roughly normally distributed
with a slight negative skew. Together, Figures 2a-2c suggest that large values of soil moisture
usually occur at large values of PSRI, which tend to be relatively abundant in the catchment.

During wet conditions at Tarrawarra, locations in the valley bottoms are generally wetter than
locations on the hillslopes (Grayson et al., 1997). This structure has been called a valley-
dependent pattern (Coleman and Niemann, 2013) and has been shown to be correlated with the
natural log of SCA (Busch et al., 2012). Figure 2d shows the histogram of the natural log of SCA
at Tarrawarra. Most values for this ancillary variable are small, but a few values are much larger
than the rest, which produces the tail on the right side of the histogram. Figure 2e shows the
relationship between the natural log of SCA and soil moisture on a date with a valley-dependent
soil moisture pattern (22 April 1996). The locations with the rare large values of the natural log
of SCA also have large values of soil moisture. A few locations with smaller values of the
ancillary variable are also wet. Figure 2f shows the histogram of soil moisture on the same date.
This histogram also has a tail on the right, but that tail includes more locations. Together,
Figures 2d-2f suggest that wet locations tend to occur when the natural log of SCA is large, and
that such locations are relatively rare in the catchment. This property contrasts with the hillslope-
dependent case and might make observing the relationship between topography and soil moisture more difficult for valley-dependent patterns.

The second application catchment is Satellite Station, which is located near Auckland, New Zealand and was part of the Maharungi River Variability Experiment (Wilson et al., 2003). Satellite Station is a 60 ha catchment with a total relief of 50 m. Similar to Tarrawarra, the catchment is mainly used for grazing. The climate is warm and humid with an average annual precipitation of 160 cm and an average annual PET of 130 cm. The soil at Satellite Station can be up to a meter deep and varies in texture from the hillslopes to the valley bottoms (Woods et al., 2001). The soil on the hillslopes is silty clay loam, while the soil in the valley bottoms is predominately clay. The terrain indices for Satellite Station were calculated from a 10 m DEM. In-situ soil moisture observations are available for the top 30 cm of the soil using a TDR probe. We use only the 322 locations that were collected on all 6 dates, which span a total of 20 months from April 1998 to May 1999. The observations are on a uniform grid that has a 40 m by 40 m spacing.

The soil moisture patterns at Satellite Station have substantial differences from Tarrawarra that might affect the performance of the sampling methods. Overall, the dependence of soil moisture on topography is weaker at this catchment (Busch et al., 2012). The soil moisture patterns at Satellite Station are temporally stable valley-dependent patterns. Figure 2g shows the histogram of the natural log of SCA at Satellite Station. Even more than at Tarrawarra, the histogram exhibits a noticeable tail on the right side. Figure 2h shows the dependence of soil moisture on the natural log of SCA for 22 November 1998. Because topography plays a weaker role in determining soil moisture variations at Satellite Station than at Tarrawarra, more scatter is seen here. Figure 2i shows the histogram of soil moisture on the
same date. Like the histogram for the ancillary variable, it has a tail on the right side. Together, Figures 2g-2i suggest that the variation in soil moisture tends to be concentrated at the few locations in the catchment with very large SCA values, similar to the valley-dependent pattern at Tarrawarra.

The third application catchment is Cache la Poudre, which is a mountain catchment located near Rustic, Colorado in the Cache la Poudre River basin (Coleman and Niemann, 2012; Lehman and Niemann, 2008). The catchment covers an area of about 8 ha and has a total relief of 124 m. The climate at Cache la Poudre is semiarid with an average annual precipitation of about 40 cm and an average annual PET of about 93 cm. The vegetation within the catchment is aspect dependent. The north-facing hillslopes are part of a coniferous forest, while the south-facing hillslopes are covered with deciduous shrubs. The soil at Cache la Poudre can be described as a sandy loam and is usually shallow as the terrain consists of fairly steep slopes with several granite outcrops. The soil on the north-facing hillslopes has more litter cover and organic matter than the soil on the south-facing hillslopes. The terrain indices used in this study were calculated from a 15 m DEM developed from a land survey. Due to the shallow soils, in-situ soil moisture observations were taken only in the top 5 cm of the soil using a TDR. The observations are available at 350 locations for all 9 sampling dates, which span 3 months from April to June 2008. The observations are on a uniform grid with a 15 m by 15 m spacing.

The soil moisture patterns at Cache la Poudre are also distinctive from the previous catchments. Overall, the dependence on topography at this site is the weakest among the three catchments considered (Busch et al., 2012). Furthermore the spatial patterns of soil moisture at Cache la Poudre are temporally stable hillslope-dependent patterns. The histogram of PSRI values observed at Cache la Poudre is shown in Figure 2j where it can be seen that they roughly
follow a uniform distribution. A plot of soil moisture on 22 April 2008 against PSRI values is shown in Figure 2k and indicates a moderately-strong negative linear relationship. Figure 2j shows that the soil moisture values on this date are approximately normally distributed. Together, Figures 2j-2l suggest that high and low values of PSRI are relatively abundant, and they tend to produce the high and low values of soil moisture, similar to the hillslope-dependent pattern at Tarrawarra.
3.0 RESULTS AND DISCUSSION

Soil moisture monitoring strategies are identified using each of the sampling methods for three different scenarios: location-limited, date-limited, and location-and-date-limited. For the location-limited scenario, the sampling methods are used to select a limited number of locations in a catchment using the topographic attributes that are used by each downscaling method as ancillary variables. Only the observed soil moisture values at the selected locations (on all sampling dates) are used to calibrate the downscaling method. For the EMT model, these observations are used to calibrate the model parameters. For the EOF method, the sampled soil moisture dataset is decomposed using EOF analysis, and the multiple linear regressions and segmented linear relationships are developed for the EOFs and ECs, respectively. For the date-limited scenario, all available locations are sampled but only on a limited number of dates. In this scenario, the spatial average soil moisture is viewed as an ancillary variable (the only ancillary variable). Different dates have different values for the spatial average soil moisture, so this ancillary variable can be used to identify sampling dates. For the combined location-and-date-limited scenario, soil moisture is sampled at a limited number of locations and those locations are only observed for a limited number of dates. In this case, both the topographic attributes and the spatial average soil moisture are considered as ancillary variables.

For cLHS, the number of samples can be specified directly, but for SRS, the number of samples is controlled indirectly by varying the number of bins. For the location-limited scenario, Table 1 shows the corresponding number of sample locations as the number of bins is varied at the three catchments. The number of locations corresponds to the number of land units that occur in each catchment. For a given number of bins, the EMT model yields many fewer sampling locations than the EOF method because the EMT model only uses 4 topographic attributes while
the EOF method considers 10. Even when the number of bins is small, the implied number of samples can be quite large for the EOF method because so many topographic attributes are considered. For the date-limited scenario, the number of bins is equal to the number of sampled dates for each downscaling model because observations are available in each bin for the number of bins considered and the same ancillary variable is used with each model. The location-and-date-limited scenario is simply a combination of the previous two scenarios in that the number of bins is specified, which determines a corresponding number of locations, while the number of dates is specified directly.

For any given number of sampling locations and/or dates, the selection process is repeated numerous times to produce multiple realizations. Then, the average performance for a given number of samples is assessed. The number of realizations is chosen so that the average performance does not substantially change with the addition of new realizations. Through trial and error it was determined that the EOF method requires about 100 realizations for the average model performance to stabilize, but the EMT model requires only about 50 trials for its average performance to stabilize. The EMT model likely requires fewer samples because it has fewer parameters (i.e. degrees of freedom). For each realization, the downscaling method is used to estimate the soil moisture value at all locations and all dates. Both sampled and unsampled locations are used in the evaluation because neither the EMT model nor EOF method necessarily reproduce the observed soil moisture value at the sampled locations. The following three subsections describe the results of the three sampling scenarios.
3.1 Location-Limited Scenario

Figure 3 presents the results for the location-limited scenario at Tarrawarra. To generate the figure, the model performance is quantitatively evaluated using the Nash-Sutcliffe Coefficient of Efficiency (NSCE) (Nash and Sutcliffe, 1970). The NSCE has a maximum possible value of one, which would imply that the downscaled pattern perfectly matches the observed pattern. Values greater than zero indicate that the downscaled pattern is a better estimate of the observed pattern than simply using the observed spatial average as the estimate. For a given number of sampled locations, the NSCE is calculated for all available dates and then averaged to produce a single average NSCE value for one realization. This process is then repeated for the 50 or 100 realizations to produce an overall average NSCE, which is displayed in Figure 3. This process was repeated using the root mean squared error (RMSE) and mean absolute error (MAE) as alternative measures of performance. These metrics yielded similar results that lead to the same conclusions.

Figure 3a shows the average performance of each of the sampling methods when coupled with the EMT model. For this part of the figure, the spatial average soil moisture values that are supplied to the EMT model were determined by averaging the soil moisture values at the sampled locations. For all three sampling methods, the average NSCE increases rapidly with the sample size until about 50 observations are collected. After this point, additional samples result in much smaller gains in performance. When the number of sampled locations is less than 38, both of the strategic sampling methods consistently outperform random sampling. Furthermore, when the number of sampled locations is less than 22, SRS slightly outperforms cLHS. For example, when 9 locations are selected, the average NSCE values for random sampling, cLHS, and SRS are 0.07, 0.11, and 0.15, respectively, indicating a clear advantage for the strategic
sampling methods. When the number of sampled locations is 38, the average NSCE values for random sampling, cLHS, and SRS are 0.26, 0.29, and 0.25, respectively, so an advantage is only seen for cLHS. In fact, when the number of sampled locations is between 38 and 162, random sampling consistently outperforms SRS. The advantage is largest when the number of sampled locations is between 38 and 80. Overall, when the number of sampled locations is greater than 38, cLHS is consistently the best performing sampling method. Eventually, the average NSCE of each of the sampling methods plateaus at a value of 0.30 at Tarrawarra, which is the average NSCE when soil moisture observations at all locations on all available dates are used to calibrate the EMT model (Coleman and Niemann, 2013). Once the NSCE values start to approach 0.30 for the EMT model at Tarrawarra, the associated average RMSE and MAE values of the estimated patterns are 0.028 v/v and 0.021 v/v, respectively.

Results when the EOF method is used in the same analysis are shown in Figure 3b. In this case, the average NSCE increases rapidly with the sample size until reaching about 55 locations. The strategic sampling methods significantly outperform random sampling when the number of sampled locations is less than 55. The strategic sampling methods continue to slightly, but consistently, outperform random sampling until the number of sampled locations increases to about 105. In fact, these results suggest that when the number of sampled locations is less than about 55, the only feasible way to apply the EOF method is to use the strategic sampling methods. With 15 sampling locations, for example, the average NSCE value for cLHS is 0.12, but the average NSCE value for random sampling is -2.22, indicating that the estimated patterns are unrealistic. An average NSCE value is not given for SRS because the binning cannot yield fewer than 43 locations when coupled with the EOF method. Overall, the average performance of cLHS and SRS are nearly identical in Figure 3b. Eventually, the average NSCE of each of the
sampling methods plateau at a value of 0.35 at Tarrawarra, which is the average NSCE value when soil moisture observations at all locations and dates are used in the construction of the EOF method (Busch et al., 2012). Once the NSCE values start to approach 0.35 for the EOF method at Tarrawarra, the associated average RMSE and MAE values of the estimated patterns are 0.027 v/v and 0.020 v/v, respectively.

Comparing the results in Figures 3a and 3b allows a comparison of the EMT model and EOF method when they are calibrated with soil moisture observations from limited locations. The random sampling case allows the most direct comparison because the same random locations were used in both cases (the other sampling methods are based on different sets of topographic attributes). It can be seen that the EMT model significantly outperforms the EOF method when the number of sampled locations is less than 55, while the EOF method outperforms the EMT model when the number of sampled locations is greater than 55. For small datasets, the EMT model is likely superior because it is based on a physical description of vadose zone hydrology that helps constrain the model behavior. For large datasets, the EOF model is likely superior because its empirical approach allows it to better reproduce the observed behavior.

In the above analysis, the spatial average soil moisture was estimated from the soil moisture values at the sampled locations. However, because the context for this analysis is downscaling, it is possible that the spatial average is known from other sources (the coarse grid input would be equivalent to a grid of spatial averages). To consider this situation, the performance of the sampling methods is assessed when the true spatial average soil moisture value is used as the input to the downscaling method. Specifically, the spatial average soil moisture value was determined by averaging all of the available soil moisture values in the
catchment (not just the values at the sampled locations). The results for the EMT model are shown in Figure 3c. For every number of sampled locations, the performance of the sampling methods increases when the downscaling method is supplied with the true spatial average. The increase is largest when the number of sampled locations is very small. For example, when 9 locations are sampled, the average NSCE values for random sampling, cLHS, and SRS are 0.17, 0.19, and 0.25, respectively. This is an increase in average NSCE of 0.10, 0.08, and 0.10, respectively, from the results when the spatial average was estimated (Figure 3a). Additionally, the performance for SRS still becomes worse than that of random sampling when the number of sampled locations is between 38 and 80 if the true spatial average is supplied to the EMT model, but this difference is substantially reduced compared to the case when the spatial average is estimated.

Figure 3d shows the equivalent analysis when the EOF method is used. Here, the increase in model performance is less than what was seen with the EMT model. For example, when 15 locations are sampled using cLHS, the average NSCE of the EOF method is 0.16, an increase of only 0.04 from when the spatial average was estimated from the sampled locations. For SRS and random sampling, the increase in model performance is even smaller. These results indicate that when selecting locations to monitor soil moisture for the purpose of calibrating the EMT model, it is more important to select locations that will yield an accurate estimate of the spatial average soil moisture than when selecting locations to construct the EOF method. Comparing the EMT model (Figure 3c) and EOF method (Figure 3d) when random sampling is used and the true spatial average is provided to each, it can be seen that the EMT model performs best when the number of sampled locations is less than 70, and the EOF method performs best when the number of sampled locations is greater than 70.
When the true spatial average soil moisture is provided to the downscaling methods, it changes the requirements for successful sampling. The sampling methods no longer need to provide a reliable estimate of the spatial average; they only need to characterize the relationships between the topographic attributes and soil moisture. Figures 4a-4b evaluate how well the sampling methods estimate the spatial average soil moisture. Specifically, for each realization, the relative error in the estimated spatial average soil moisture was calculated on all dates and then averaged to get the mean relative error for that realization. Then, the average mean relative error was calculated from all the realizations for a given number of samples. These values are plotted in Figure 4 for each sampling method.

Figure 4a shows the average mean relative error when the topographic attributes from the EMT model are used as ancillary variables in the sampling methods. In general, the estimated spatial averages are accurate as the average mean relative error is less than 0.025, or 2.5%, for all numbers of sampled locations. In considering volumetric soil moisture values, this means that on average each of the sampling methods estimates the spatial average soil moisture with an accuracy of greater than 0.012 v/v for all numbers of sampled locations. In nearly every instance, the estimate from cLHS is more accurate than the estimate from random sampling, and the estimate from random sampling is more accurate than the one from SRS. SRS performs poorest because it samples approximately uniformly across the entire range of each topographic attribute, regardless of how the observed values are distributed across that range. The problem with this approach can be seen in Figure 2e. For the case shown in that figure, SRS would obtain samples at large values for the natural log of SCA in order to characterize the relationship between this variable and soil moisture. However, these large values are rare (see Figure 2d). Thus, when this sample is used to estimate the spatial average soil moisture, it can produce noticeable errors. This
problem does not occur at the smaller samples sizes in Figure 4 because the number of bins is not large enough to force sampling at the extremes of the histogram. It also does not occur with random sampling or cLHS because these methods respect the distribution of the topographic attributes and thus better represent the distribution of soil moisture. These results also explain why the performance of SRS lags behind that of cLHS and random sampling at intermediate numbers of sampled locations when the spatial average is estimated but does less so when the true spatial average is supplied. It is also interesting to note that such small errors in the estimate of the mean can have a substantial negative influence on model performance. Nearly identical results are shown in Figure 4b for the sampling methods when the topographic attributes from the EOF method are used as ancillary variables.

Figure 4c-4d evaluate how well the observations at the sampled locations estimate the value of the spatial variance in soil moisture. It is important to note that here we are considering the estimated value of the variance and not the spatial variability of the soil moisture pattern as is considered throughout this study. It is possible that for some hydrologic applications (e.g., a hydrologic model input), that the structure of the soil moisture pattern cannot be specified but the variance of the pattern can be specified. In this case it would be useful to know if the results shown here also pertain to the estimated variance. As such, the mean relative error in the estimate of the value of the spatial variance was evaluated using the same methods that the error in the value of the estimated spatial average was evaluated. In general, it can be seen in Figure 4c-4d that the cLHS method provides the most accurate estimate of the spatial variance, followed by the random sampling and SRS methods. These results agree with those shown in Figure 4a-4b, although the mean relative error of the estimated variance is always much larger than that of the estimated spatial average. These results are potentially useful in other applications, but
because the value of the variance in soil moisture is not needed in the EMT model or EOF method, they will not be expanded on further.

Next, the consistency in the performance of the sampling methods is evaluated. Ideally, a sampling method provides not only good average performance for a given number of samples but also consistent performance between individual realizations. The consistency of performance is analyzed by calculating the variance among the 50 or 100 average NSCE values for each selected number of samples (Figure 5). Figures 5a and 5c display the results of this analysis when the sampling methods are coupled with the EMT model and the estimated and true spatial averages are used, respectively. Figures 5b and 5d display the equivalent results when the EOF method is used. In every case shown in Figure 5, the performance of the strategic sampling methods is less variable (i.e. more consistent) than the performance of random sampling. This difference is most pronounced for the EOF method where random sampling produces very unreliable performance when small numbers of locations are sampled. These results show that the inclusion of topographic ancillary variables in the sampling method helps avoid samples that produce unrealistic soil moisture patterns. Overall, cLHS and SRS provide similar consistency in model performance. However, if fewer than about 40 locations are selected, the EMT model performance is more reliable if locations are chosen using SRS. If the number of sampled locations is greater than 40, the performance is more reliable if the locations are selected using cLHS. Furthermore, the EMT model performance is much more reliable than the EOF method performance. This reliability likely arises from the physically-based constraints that are included in the EMT model. For example, the EMT model limits the soil moisture estimates to be less than or equal to the calibrated porosity, but the EOF method includes no such limitation. Thus,
the EOF method can produce obviously inaccurate soil moisture values if the sampled locations are not representative of the range of conditions observed at the catchment.

Figure 6 displays soil moisture patterns produced by the EMT model from the most and least accurate samples when it is calibrated using 9 and 38 locations from each of the sampling methods. Here, the most and least accurate samples refer to the sets of both 9 and 38 locations from each sampling method that produce the highest and lowest average NSCE for all available dates. The soil moisture pattern from 27 September 1995 is shown for each case as an example (the observed pattern is in Figure 6b) with the associated sample locations marked on each pattern. Comparing the patterns from the most accurate samples in Figures 6c, 6d, and 6e, the estimated patterns all appear very similar. Also similar are the NSCE values of the shown patterns, which are 0.52, 0.46, and 0.53 for random sampling, SRS, and cLHS, respectively. However, when examining the patterns from the least accurate set of 9 locations in Figures 6f, 6g, and 6h, the estimated patterns have noticeable differences. The NSCE values of these patterns are -0.42, 0.20, and -0.01 for random sampling, SRS, and cLHS, respectively. These results are consistent with the earlier figures, which showed that SRS was the most accurate and reliable method when the number of samples was very small (see Figures 3a and 5a). These results suggest that the difference in performance between the sampling methods is not due to differences in the best estimates from the sampling methods but in the worst estimates. Similarly, it suggests that the variability in the performance of the sampling methods is also associated with variations in the poorest performing cases. When the number of sampled locations is increased to 38, the NSCE of the least accurate estimates for each of the sampling methods is more similar. Specifically, the NSCE values of the patterns shown in Figures 6l, 6m, and 6n are 0.51, 0.48, and 0.58 for random sampling, SRS, and cLHS respectively.
To assess whether the results obtained thus far are applicable to other catchments, the sampling methods are applied to the Satellite Station and Cache la Poudre catchments. Figures 7a-7b show the results for Satellite Station when the spatial average soil moisture is estimated from the samples. The case where the true spatial average is used is not shown because its results are equivalent to those at Tarrawarra. Overall, for the EMT model, the average NSCE increases rapidly with sample size until reaching about 50 sampling locations; after this point additional samples produce modest improvements in performance. For the EOF method, the increase in performance remains more steady at large sample sizes. The strategic sampling methods perform better than random sampling when the number of locations is small to intermediate (less than 70 for the EMT model and less than 135 for the EOF method). For small numbers of sampled locations (36 or less for the EMT model and 39 for the EOF method), SRS outperforms cLHS. Generally, these results are consistent with the results shown in Figure 3 for Tarrawarra. The exception is that at Satellite Station cLHS does not outperform random sampling if the number of sampled locations is very small. When the EMT model is used and 22 locations are selected using random sampling, cLHS, and SRS, the average NSCE values are 0.04, 0.01, and 0.10, respectively. This difference from Tarrawarra can be understood by recalling that the soil moisture patterns at Satellite Station are temporally stable valley-dependent patterns and by recalling the relationship between the natural log of the SCA and soil moisture shown in Figure 2e. If the number of sampled locations is very small, SRS still ensures that locations with a high SCA are sampled because each bin covers an equal fraction of the observed range of the attribute. However, cLHS does not capture these large values because the bins are sized to contain an equal number of observations. Notice that the EMT model actually uses SCA rather than the natural log of SCA, which is shown in Figure 2e, and the large SCA values are even
more separated from the most common values than what is shown in Figure 2e. As the number of sampled locations is increased, cLHS becomes more effective at selecting the necessary locations to characterize the observed relationship, and its average performance becomes slightly but consistently higher than the SRS method for intermediate to larger numbers of sampled locations. For all of the sampling methods considered, as the number of sampled locations approaches the total number of locations in the catchment, the average NSCE value approaches 0.17 for the EMT model (Coleman and Niemann, 2013) and 0.24 for the EOF method (Busch et al., 2012). Once the NSCE values start to approach 0.17 for the EMT model and 0.24 for the EOF method at Tarrawarra, the associated average RMSE and MAE values of the estimated patterns are 0.048 v/v and 0.028 v/v, respectively, for the EMT model and 0.047 v/v and 0.027 v/v, respectively, for the EOF method. As was the case at Tarrawarra, the EMT model performs best at Satellite Station when the number of sampled locations is limited (less than 130), but the EOF method performs best when the number of sampled locations is large.

Figure 7 also shows the average NSCE values at Cache la Poudre for each of the sampling methods when coupled with the EMT model (Figure 7c) and EOF method (Figure 7d). For this catchment, the average NSCE increases steadily until reaching about 100 sampling locations, after which only modest improvements are seen. Only cLHS is advantageous as this method outperforms SRS and random sampling for all numbers of samples considered. Also, when the number of sampled locations is small to intermediate, SRS generally has weak performance when compared to the other sampling methods and what was observed at the other catchments. For example, when 27 locations are sampled to calibrate the EMT model, the average performance of random sampling, cLHS, and SRS is -0.01, 0.02, and -0.01, respectively. When the number of selected locations is increased to 47, these values increase to 0.04, 0.05, and
0.02, respectively. Furthermore, the difference in model performance and the advantage of strategic sampling are smallest at Cache la Poudre. This distinct behavior can be explained by recalling that the soil moisture patterns are temporally stable hillslope-dependent patterns and by recalling the relationship between PSRI and soil moisture shown in Figure 2k. The relative abundance of low and high PSRI values and approximate linearity of the relationship shown in Figure 2k imply that strategic sampling is not essential to characterize the relationship. cLHS performs best because it ensures that the relationship is observed while still capturing a representative distribution of values (unlike SRS). The advantage of the strategic sampling methods is also smaller at Cache la Poudre because the soil moisture patterns are most weakly related to topography at this catchment. For all of the sampling methods, as the number of sampled locations approaches the total number of locations in the catchment, the average NSCE approaches 0.08 for the EMT model (Coleman and Niemann, 2013) and 0.11 for the EOF method (Busch et al., 2012). Once the NSCE values start to approach 0.08 for the EMT model and 0.11 for the EOF method at Tarrawarra, the associated average RMSE and MAE values of the estimated patterns are 0.030 v/v and 0.022 v/v, respectively, for the EMT model and 0.029 v/v and 0.021 v/v, respectively, for the EOF method. As was the case at Tarrawarra and Satellite Station, the EMT model performs best when the number of sampled locations is small (less than 47), but the EOF method performs best when the number of sampled locations is large.

The accuracy of the estimated spatial average soil moisture at Satellite Station and Cache la Poudre is shown in Figure 8. The average mean relative errors at Satellite Station (Figure 8a and 8b) are less than 4.5%, while the average mean relative errors at Cache la Poudre (Figure 8c and 8d) are less than 8%. When converted to volumetric soil moisture values, these results show that the sampling methods can estimate the average with an accuracy greater than 0.021 v/v at
Satellite Station, and an accuracy greater than 0.015 v/v at Cache la Poudre. These values are greater than what was observed at Tarrawarra but still relatively small. A very slight advantage is usually seen for cLHS at Satellite Station and Cache la Poudre. However, this advantage is not as large or consistent as what was observed for Tarrawarra (Figure 4), likely because the dependence of soil moisture on topography is weaker at these catchments. SRS performs poorest at Satellite Station when the topographic attributes from the EOF method are used and the number of sampled locations is small. This behavior occurs because SRS yields many unique land units when applied with the 10 topographic attributes of the EOF method. Many of these land units are located in the valley bottoms where the soil moisture is generally wetter than the spatial average. When the sample emphasizes such locations, it causes errors in the estimated spatial average. SRS does not have this problem when applied with the 4 topographic attributes of the EMT model because not as many land units are created. Furthermore, the disadvantage of SRS is not observed at Cache la Poudre because a temporally stable hillslope-dependent pattern occurs, so the valley bottoms are not particularly wet.

The variance of the average NSCE values for each number of samples is plotted in Figure 9 for Satellite Station and Cache la Poudre. At Satellite Station, for almost every number of sampled locations, the strategic sampling methods are more reliable than random sampling. These differences are fairly small for the EMT model (Figure 9a) but rather large for the EOF method (Figure 9b). The difference between these cases is most notable for smaller numbers of sampled locations, where SRS generally provides more reliable performance than cLHS. This behavior is consistent with what was observed at Tarrawarra and further suggests that the use of ancillary variables in the sampling methods and the physical constraints in the EMT model limit the likelihood of producing soil moisture estimates that are very inaccurate. At Cache la Poudre,
where cLHS consistently produces the most accurate soil moisture estimates (Figure 8), it also has the most stable model performance for almost all numbers of sampled locations. However, the relative lack of difference in the variance for each of the sampling methods at Cache la Poudre suggests that the strategic sampling methods less consistently outperform random sampling when the relationships between topography and soil moisture are weaker.

3.2 Date-Limited Scenario

Next, the sampling methods are used to determine a limited number of sampling dates. Figure 10 presents the results of the date-limited scenario at each of the catchments. Again, the average NSCE among the different realizations was used as the metric to evaluate the effectiveness of each of the sampling methods. For this analysis, after the downscaling methods are calibrated using every soil moisture observation from the selected sampling dates, the true spatial average is supplied to each downscaling method as input. The true spatial average must be used in this analysis because soil moisture samples are not collected on all dates.

Overall, the results in Figure 10 show that little improvement in performance occurs after 3 to 4 dates have been observed, depending on the sampling method used. The largest advantage of strategic sampling is observed at Tarrawarra when the EMT model is used (Figure 10a) and less than 5 dates are selected. The difference is quite large when the number of sampled dates is very small. For example, when 2 dates are selected, the average NSCE for random sampling, cLHS, and SRS is -0.03, 0.12, and 0.18, respectively. Also, when 3 or more dates are selected, the average performance for SRS and cLHS is nearly identical for every number of sampling dates considered. The advantage of strategic sampling is not observed when applying the EOF method at Tarrawarra (Figure 10b) and not as great for both downscaling methods at Satellite.
Station (Figure 10c and 10d) and Cache la Poudre (Figure 10e and 10f). The differences are likely smaller at Satellite Station and Cache la Poudre because those catchments exhibit temporally stable patterns. When selecting dates at Tarrawarra, it is important that dry, intermediate, and wet conditions are represented among the sampled dates because the soil moisture patterns are different for each condition. The strategic sampling methods ensure their inclusion. Comparing the EMT model and EOF method, it can be seen that for almost every number of sampling dates, the EOF method outperforms the EMT model. This suggests that the EMT model is more sensitive to limiting the number dates than the EOF method.

3.3 Location-and-Date-Limited Scenario

Finally, we consider the scenario where both the locations and dates for soil moisture monitoring are restricted. For brevity, we consider only the strategic sampling methods. In addition, we focus on the cases where 50 locations on 3 dates are selected from a catchment using cLHS and as close to 50 locations as possible on 3 dates are selected using SRS. Because the sampling dates are limited, the downscaling methods must be provided with the true spatial average soil moisture. These numbers of sampling locations and dates are chosen because they are near the elbows of the strategic sampling method curves in the previous figures.

Table 2 displays the results of the location-and-date-limited scenario at all three catchments. Focusing on cLHS, which is shown in the top half of the table, the results show that when locations and dates for soil moisture monitoring are selected using strategic sampling methods, a large percentage of the variation in soil moisture can be explained with observations at relatively few locations on relatively few dates. Specifically, when 11 to 16% of the available locations are sampled and used for soil moisture monitoring on 23 to 50% of the available dates,
the downscaling models perform at 50 to 83% of their maximum performance. The percentage of maximum performance that is explained is greatest when the relationship between soil moisture and topography is strongest. Additionally, for the cLHS cases considered, the EMT model generally outperforms the EOF method. The one exception is when the EOF method slightly outperforms the EMT model at Cache la Poudre when considering the NSCE values and performs equally well when considering the percentage of maximum performance.

Results for the SRS method are shown in the bottom half of Table 2. The results are similar to those of cLHS in that the downscaling methods are able to explain a substantial portion of the variation in the soil moisture patterns when calibrated with observations at relatively few locations on relatively few dates. When considering the EMT model attributes the results of cLHS and SRS are very consistent. However, the numbers of sampled locations yielded from SRS are 60 (5 bins), 58 (5 bins), and 47 (4 bins) at Tarrawarra, Satellite Station, and Cache la Poudre. Thus, for the cases considered, with the exception of Cache la Poudre, cLHS performs the same as SRS when selecting slightly fewer locations on an equal number of dates. This is in agreement with previous results for similar numbers of samples in the location-limited and date-limited scenarios. When considering the EOF method attributes, the results of cLHS and SRS are similar but vary slightly. Here, the numbers of sampled locations yielded from SRS are 43 (2 bins), 39 (2 bins), and 48 (2 bins) at Tarrawarra, Satellite Station, and Cache la Poudre. At Tarrawarra, even though the number of sampled locations decreased from 50 to 43 for cLHS to SRS the average model performance increased from 0.19 to 0.21. This increase is very small and when considering the standard deviations of 0.10 and 0.07 for cLHS and SRS, respectively, the performance is about equal. The smaller number of sampled locations explains the drop in performance for the EOF method at Satellite Station from an NSCE of 0.12 for the cLHS method.
to 0.07 for the SRS method. Furthermore, the drop in NSCE of 0.06 for the cLHS method to 0.03 for the SRS is seen at Cache la Poudre because, as shown previously, the SRS method tends to lag behind cLHS here. Overall, these results suggest that the strategic sampling methods perform similarly when used to selected about 50 locations and 3 dates for soil moisture monitoring.
The following conclusions can be made from the analyses of this study:

1. Relatively accurate soil moisture patterns can be generated by the EMT model and EOF method when soil moisture is monitored at relatively few locations and/or relatively few dates within a catchment of interest. In many cases, sampling beyond about 50 to 100 locations provides only modest improvements in the ability to reproduce the observed soil moisture patterns. Similarly, monitoring more than 3 to 4 dates also usually provides only small gains. Even when only 10-18% of the locations and 23-50% of the dates are strategically sampled, the downscaling methods can achieve 25-83% of their maximum performance at the three catchments considered. This conclusion suggests that much smaller datasets than Tarrawarra, Satellite Station, and Cache la Poudre could be collected in the future, which might allow monitoring of a larger number of different catchments in the future.

2. The strategic sampling methods generally outperform the random sampling method when the number of sampled locations and dates are small. When selecting a limited number of locations for monitoring, SRS and cLHS ensure that a range of topographic attribute values are represented, which helps define the relationships between the attributes and soil moisture. When selecting a limited number of dates, SRS and cLHS ensure that different soil moisture conditions (and potentially different pattern types) are represented. Choosing sampling locations and dates by SRS and cLHS also reduces the risk that a set of observations is unrepresentative of the catchment conditions.

3. When the number of sampled locations is less than about 30, SRS generally performs better than cLHS. However, when the number of sampled locations is greater than about
30, cLHS almost always outperforms SRS. cLHS ensures that a representative range of attribute values is sampled while still respecting the probability distribution of the attribute values in the catchment. The specific thresholds that determine when one sampling method outperforms another are catchment specific and depend on a number of factors including the level of influence of topography on the soil moisture pattern, the spatial-structure of the soil moisture pattern, and the temporal stability of the soil moisture pattern.

4. The advantage of cLHS over SRS decreases if the true spatial average soil moisture is known and provided to the downscaling methods. The observations collected by SRS do not reproduce the probability distributions of the topographic attributes, so its estimates of the spatial average usually have more error. The advantage of providing the true spatial average can be quite large when the number of sampling locations is small. It is worth noting that cLHS also has a practical disadvantage compared to SRS because it specifies the exact locations where sampling must be done. SRS allows randomly-selected locations within the specified land units (unique combinations of topographic attribute values).

5. Overall, the EMT model performs better than the EOF method when a small number of locations on an abundance of dates are used in calibration, while the EOF method performs better when an abundance of locations on a limited number of dates are used in calibration. For the case when both locations and dates were limited, the EMT model performed better than the EOF method at two of three catchments. The EMT model performs better with smaller datasets because it includes physically-based considerations about soil moisture, while the EOF method is entirely empirical.
Additional research would be beneficial in several areas. The evaluation could be expanded to include other sampling and downscaling methods. In particular, downscaling methods that utilize other kinds of ancillary variables (e.g., soil texture and/or vegetation) could be considered. Also, the topographic attributes that are considered by the EOF method could be reduced, which would result in a smaller number of land units and potentially different results. Future research could also examine how these results pertain to scales that differ from the multi-hectare catchments and sub 40 m resolutions considered here.
Table 1: The numbers of sampling locations obtained when different numbers of bins are used in the SRS method at Tarrawarra, Satellite Station, and Cache la Poudre. The numbers of samples are shown when the binning is applied to the 4 topographic attributes used in the EMT model and the 10 topographic attributes used in the EOF method.

<table>
<thead>
<tr>
<th>Bins</th>
<th>Tarrawarra</th>
<th>Satellite Station</th>
<th>Cache la Poudre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EMT</td>
<td>EOF</td>
<td>EMT</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>43</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>114</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>38</td>
<td>182</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
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<td>58</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
<td>-</td>
<td>70</td>
</tr>
<tr>
<td>7</td>
<td>105</td>
<td>-</td>
<td>98</td>
</tr>
<tr>
<td>8</td>
<td>130</td>
<td>-</td>
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<td>-</td>
<td>134</td>
</tr>
<tr>
<td>10</td>
<td>188</td>
<td>-</td>
<td>148</td>
</tr>
<tr>
<td>12</td>
<td>228</td>
<td>-</td>
<td>182</td>
</tr>
</tbody>
</table>

Table 2: Summary of the amount of data provided and the average performance by the EMT and EOF downscaling methods when 50 locations on 3 dates are selected by the cLHS method and as close to 50 locations as possible on 3 dates are selected by the SRS method for calibration at Tarrawarra, Satellite Station, and Cache la Poudre.

<table>
<thead>
<tr>
<th>Downscaling Method</th>
<th>Number &amp; (%) of Locations</th>
<th>Number &amp; (%) of Dates</th>
<th>Avg. &amp; (Std. Dev.) of NSCE</th>
<th>Comparison to Maximum Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>cLHS</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tarrawarra</td>
<td>EMT 50 (11)</td>
<td>EMT 50 (16)</td>
<td>0.25 (0.03) 0.19 (0.10)</td>
<td>83% 54%</td>
</tr>
<tr>
<td></td>
<td>EOF 50 (14)</td>
<td>EOF 50 (16)</td>
<td>0.13 (0.04) 0.12 (0.07)</td>
<td>76% 50%</td>
</tr>
<tr>
<td>Satellite Station</td>
<td>EMT 50 (11)</td>
<td>EMT 50 (16)</td>
<td>0.04 (0.04) 0.06 (0.06)</td>
<td>50% 50%</td>
</tr>
<tr>
<td></td>
<td>EOF 50 (14)</td>
<td>EOF 50 (16)</td>
<td>0.13 (0.04) 0.12 (0.07)</td>
<td>76% 50%</td>
</tr>
<tr>
<td>Cache la Poudre</td>
<td>EMT 50 (11)</td>
<td>EMT 50 (16)</td>
<td>0.25 (0.04) 0.21 (0.07)</td>
<td>83% 60%</td>
</tr>
<tr>
<td></td>
<td>EOF 50 (14)</td>
<td>EOF 50 (16)</td>
<td>0.04 (0.08) 0.03 (0.05)</td>
<td>50% 25%</td>
</tr>
<tr>
<td><strong>SRS</strong></td>
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<td></td>
</tr>
<tr>
<td>Tarrawarra</td>
<td>EMT 60 (13) 43 (10)</td>
<td>EMT 58 (18) 39 (12)</td>
<td>0.25 (0.04) 0.21 (0.07)</td>
<td>83% 60%</td>
</tr>
<tr>
<td></td>
<td>EOF 47 (13) 48 (14)</td>
<td>EOF 58 (18) 39 (12)</td>
<td>0.13 (0.02) 0.07 (0.07)</td>
<td>76% 29%</td>
</tr>
<tr>
<td>Satellite Station</td>
<td>EMT 60 (13) 43 (10)</td>
<td>EMT 58 (18) 39 (12)</td>
<td>0.04 (0.08) 0.03 (0.05)</td>
<td>50% 25%</td>
</tr>
<tr>
<td></td>
<td>EOF 47 (13) 48 (14)</td>
<td>EOF 58 (18) 39 (12)</td>
<td>0.13 (0.02) 0.07 (0.07)</td>
<td>76% 29%</td>
</tr>
<tr>
<td>Cache la Poudre</td>
<td>EMT 60 (13) 43 (10)</td>
<td>EMT 58 (18) 39 (12)</td>
<td>0.25 (0.04) 0.21 (0.07)</td>
<td>83% 60%</td>
</tr>
<tr>
<td></td>
<td>EOF 47 (13) 48 (14)</td>
<td>EOF 58 (18) 39 (12)</td>
<td>0.04 (0.08) 0.03 (0.05)</td>
<td>50% 25%</td>
</tr>
</tbody>
</table>
Figure 1: Application of the cLHS and SRS binning methods to a hypothetical relationship between an ancillary variable and soil moisture.
Figure 2: The left column shows histograms of topographic attributes that are related to soil moisture variability at the three catchments (Tarrawarra, Satellite Station, and Cache la Poudre). The center column shows the observed relationship between the selected topographic attribute and soil moisture on an example date. Also displayed are correlation coefficients (R) relating the ancillary variable values and soil moisture values for each date shown. The right column shows the histogram of soil moisture on the same example date. Two dates are shown for Tarrawarra because the soil moisture patterns are time unstable. Thus, the important topographic attributes change through time.
Figure 3: Average performance of the downscaling methods at Tarrawarra when the number of sampled locations and the sampling method (Random, cLHS, and SRS) are varied. Average performance is measured by the average of the NSCEs calculated from all available dates and all realizations for a given number of samples and sampling method. The left column considers the EMT model, while the right column considers the EOF method. In the top row, the spatial average soil moisture that is supplied to the downscaling methods is estimated from the samples, while in the bottom row, the true spatial average soil moisture is supplied.
Figure 4: Average mean relative error in the estimated spatial average soil moisture and estimated spatial variance of the soil moisture for Tarrawarra obtained from sampled locations when the number of samples and sampling methods are varied. The left column considers the EMT model and its 4 topographic attributes as ancillary variables, while the right column considers the EOF method and its 10 topographic attributes as ancillary variables. The top row considers the estimated spatial average while the bottom row considers the estimated spatial variance.
Figure 5: Variability in the performance of the downscaling methods at Tarrawarra when the number of sampled locations and the sampling method (Random, cLHS, and SRS) are varied. The vertical axis shows the variance among the different realizations of the average NSCE that is calculated from all available dates. The left column considers the EMT model, while the right column considers the EOF method. In the top row, the spatial average soil moisture that is supplied to the downscaling methods is estimated from the samples, while in the bottom row, the true spatial average soil moisture is supplied.
Figure 6: (a) Map of elevation at the Tarrawarra catchment, and (b) the observed soil moisture pattern on 27 September 1995. (c,d,e) show the estimated patterns on 27 September 1995 when the most accurate set of 9 samples is used in the three sampling methods in the EMT model. Accuracy is judged based on all available dates, not just the one shown. (f,g,h) show the estimated patterns on 27 September 1995 when the least accurate set of 9 samples is used in the three sampling methods. Similarly, (i,j,k) show the pattern on the same date from the most accurate and (l,m,n) show the pattern from the least accurate sample of 38 locations for the EMT model at Tarrawarra. The sampling locations associated with each estimated are marked with an ‘x’.
Figure 7: Average performance of the downscaling methods at Satellite Station (top row) and Cache la Poudre (bottom row) when the number of sampled locations and the sampling method (Random, cLHS, and SRS) are varied. Average performance is measured by the average of the NSCEs calculated from all available dates and all realizations for a given number of samples and sampling method. The left column considers the EMT model, while the right column considers the EOF method.
Figure 8: Average mean relative error in the estimated spatial average soil moisture for Satellite Station (top row) and Cache la Poudre (bottom row) obtained from sampled locations when the number of samples and sampling methods are varied. In the left column, the 4 topographic attributes used by the EMT model are used as ancillary variables in the sampling methods. In the right column, the 10 topographic attributes used by the EOF method are used.
Figure 9: Variability in the performance of the downscaling methods at Satellite Station (top row) and Cache la Poudre (bottom row) when the number of sampled locations and the sampling method (Random, cLHS, and SRS) are varied. The vertical axis shows the variance among the different realizations of the average NSCE that is calculated from all available dates. The left column considers the EMT model, while the right column considers the EOF method.
Figure 10: Average performance of the downscaling methods when the number of sampled dates and the sampling method (Random, cLHS, and SRS) are varied. Average performance is measured by the average of the NSCEs calculated from all available dates and all realizations for a given number of samples and sampling method. The left column considers the EMT model, while the right column considers the EOF method. The top row considers Tarrawarra, the middle row considers Satellite Station, and the bottom row considers Cache la Poudre.


Mathworks, 2011. MATLAB, Natick, MA.


