

THESIS

AN ECONOMIC ASSESSMENT OF WHITE CRAPPIE (*POMOXIS ANNULARIS*)

CULTURE METHODS

Submitted by

Erik C. Larsen

Department of Agricultural and Resource Economics

In partial fulfillment of the requirements

For the Degree of Master of Science

Colorado State University

Fort Collins, Colorado

Fall 2012

Master's Committee:

Advisor: Craig Bond

Christopher Goemans
Christopher Myrick

ABSTRACT

AN ECONOMIC ASSESSMENT OF WHITE CRAPPIE (*POMOXIS ANNULARIS*) CULTURE

METHODS

Due to uncertain inputs into production, the development of a successful aquaculture operation requires meticulous business and contingency planning efforts. Given the degree of complexity involved in creating these plans, however, the culturist must often consider and carefully analyze an array of options presented to them. In an effort to assist the culturist with these complicated decisions, the goal of this work is to design a decision support tool, in which the user may explore business planning options and contingency planning scenarios. This decision support tool uses the Habitat Suitability Index (HSI) as an analytical cornerstone in order to estimate the expected output of a white crappie (*Pomoxis annularis*) culture system (in terms of both number of fish for stocking and dollars of total net revenues). To address stochasticity in production, Monte Carlo simulation tools have been incorporated into the model in an effort to facilitate meaningful economic analyses of production planning on the basis of expected habitat quality.

ACKNOWLEDGMENTS

I am grateful to many people for their contributions to this thesis. A special thank you goes out to Craig Bond, Chris Goemans, and Chris Myrick, all of whom served my thesis committee with patience and diligence. “Be the Wolf” is good thesis advice, but it is even better life advice. I am also thankful for the contributions of Brian Egloff, David Harris, and Rick Swanson of Colorado Parks and Wildlife. The efforts of Rick Hesse of Pepperdine University, who graciously aided me in developing an understanding of creating Monte Carlo simulations around triangular distributions in Microsoft Excel, should also be recognized. Thank you to the classmates who provided valuable input during the initial stages of the development of this thesis including Adam Herdrich, Nick Wilson, Quinten Haire, and Tyler Swarr. For their steadfast dedication to my education, I am also very grateful to many members the faculty and staff of the Department of Agriculture and Resource Economics at Colorado State University including Stephan Kroll, Stephen Koontz, Dana Hoag, and Bob Kling. Likewise, a special thank you goes out to Ric Holt, Linda Wilcox Young, Hassan Pirasteh, and all of the economic faculty and staff at Southern Oregon University. Who ever thought that a discussion of corn and Grateful Dead dolls would be a precursor to this thesis? Finally, thank you to my friends and family for the continued support and encouragement in all of my endeavors. Your names are too many to list, but your contributions are greatly appreciated.

También estoy agradecido por haber recibido el apoyo de mi familia, Chilena y mis compañeros hispanoparlantes. ¡Es siempre mejor ser el martillo que el yunque!

TABLE OF CONTENTS

Chapter 1: Introduction	1
1.1: Problem statement.....	1
1.2: Objectives	2
Chapter 2: Literature Review.....	4
2.1: Overview of biology literature.....	4
2.1.1: <i>Effects of variations in water quality and habitat characteristics on the reproductive wellbeing of crappies and related species</i>	5
2.1.2: <i>Use of habitat indexes to predict the state of populations of animals</i>	7
2.2: Overview of economics literature and economic principles in aquaculture literature	7
2.2.1: <i>Capital budgeting approach to aquaculture production and technology adoption</i>	8
2.2.2: <i>Demonstration of economic principles in the aquaculture literature</i>	9
Chapter 3: The Model	12
3.1: Modeling habitat quality using the white crappie HSI model.....	13
3.2: Pond output as a function of habitat quality	15
3.2.1: <i>Modeling the determinants of output and mortality at different life stages</i>	16
3.2.2: <i>The impact of habitat quality-related management decisions on output</i>	18
3.2.3: <i>Calculation of the habitat adjusted mortality rate</i>	20
3.2.4: <i>Calculation of expected output</i>	21
3.2.5: <i>The impact of the habitat adjusted mortality constant on the habitat adjusted mortality rate and expected output</i>	22
3.3: Overview of total revenues and total costs	25
3.4: Total revenues	25
3.5: Total costs	26

3.6: Application of methodology	27
3.7: Baseline scenario	27
3.8: Alternative scenario	29
3.9: Discounted total net revenues	33
Chapter 4: Addressing Stochastics.....	35
4.1: Developing distributions around user-provided input.....	36
4.2: Generating random inputs.....	37
4.3: Monte Carlo simulations.....	38
4.4: An example of Monte Carlo analysis around fecundity.....	39
Chapter 5: Populating the Model	42
5.1: Estimating the determinants of total revenues	43
<i>5.1.1: Estimating the value of the components of HSI</i>	43
<i>5.1.2: Estimating other determinants of total revenues</i>	44
5.2: Estimation of total costs.....	46
Chapter 6: Results and Conclusions.....	47
6.1: Model outputs using likely values of all parameters.....	47
6.2: Comparison of model outputs across the baseline and alternative scenarios for low, likely, and high quality habitats	48
6.3: Monte Carlo Simulations	51
<i>6.3.1: Sensitivity of results to changes in fecundity</i>	52
<i>6.3.2: Sensitivity of results to changes in mortality</i>	54
<i>6.3.3: Sensitivity of results to changes in output price</i>	56
Chapter 7: Conclusions and Discussions	58

7.1: Summary of results	58
7.2: The impacts of temperature-related habitat parameters on habitat quality and model outputs	58
7.3: Discussion of sensitivity analyses.....	60
7.4: Contributions of the model and significance of the results.....	61
7.5: Limitations of the model and areas for future research.....	64
Works Cited	66
Appendix.....	69
Appendix A: Structure of HSI Model	69
Appendix B: Application of HSI and Adjusted Mortality Rate	73
Appendix C: Comparative Statics of the Determinants of Output.....	78
Appendix D: Comparative Statics of the Habitat Adjusted Mortality Constant	81
Appendix E: Monte Carlo Distribution Assumptions	85
Appendix F: Oxygenation Capacity Calculations.....	87
Appendix G: Detailed HSI Assumptions for the Scenario for Modeling	93
Appendix H: Variables, Parameters, and Definitions	96
Appendix I: Visual Basic Code.....	104
Appendix J: An Example of the Usefulness of the General Form of the Triangular Distribution in Creating Randomly Generated Inputs for Monte Carlo Simulations.....	113

CHAPTER 1: INTRODUCTION

The motivation for this work is provided by the condition of the Colorado Parks and Wildlife warm water hatcheries. Currently, the facilities located in Pueblo and Wray are responsible for the culture of white crappie (*Pomoxis annularis*) for release into regional lakes (Swanson, 2011). However, due to problems with invasive species (*i.e.* zebra and quagga mussels), the Pueblo hatchery is somewhat limited in its ability to stock fish (Harris, 2011). As a result of the termination of crappie production in Pueblo, the state runs a higher risk of being unable to meet their stocking goals. The elevated exposure to risk faced by Colorado Parks and Wildlife demonstrates a general need for the analysis of contingency scenarios in aquaculture management decisions. In the case of state-sponsored white crappie culture in Colorado, for example, the knowledge of the potential costs associated with the inability to produce crappie at Wray (*e.g.* by contamination or loss of water supply), is an important piece of information in the decision of whether or not to safeguard against such risks.

1.1: PROBLEM STATEMENT

Elevated exposure to risk is not unfamiliar to the crappie culturist, regardless of whether they are state-sponsored or producing privately. Mechanical error, human error, disease, predation, or other exogenous factors (*e.g.* extreme weather) all represent risks that may translate directly into a loss of revenues for the culturist. Uncertainty in production adds another element of difficulty to the culturist's task. Biological factors (*e.g.* mortality and fecundity) are subject to some degree of stochasticity. The complication is that these uncertain biological parameters are important determinants of the total net revenues of a culture system.

In short, due to uncertainty in production faced by all crappie culturists, the development of a successful aquaculture operation requires meticulous business and contingency planning efforts. Given

the degree of complexity involved in creating these plans, however, the culturist must often consider and carefully analyze an array of options presented to them. In an effort to assist the culturist with these complicated decisions, the goal of this work is to design a decision support tool, through which the user may explore planning options and scenarios associated with the production of white crappie (*Pomoxis annularis*).

1.2: OBJECTIVES

A well-designed decision tool must be able to accommodate the goals of several different types of culturists and culture systems. For example, the private producer may be interested in maximizing the total net revenues yielded from a culture system. On the other hand, a public producer (e.g. Colorado Parks and Wildlife) that needs to meet annual stocking goals may be primarily concerned with the physical output of the system rather than the net revenues generated by it. In language that an economist is familiar with, the objective function of the relevant maximization problem may change across culture systems and, in some cases, there is reason to believe that this objective is constrained beyond budgetary limitations.

For these reasons, the author has developed a Microsoft Excel white crappie production model, which is designed to achieve these objectives. For the culturist that is primarily concerned with the physical output of fish for stocking, the model provides an estimation of the expected size of the harvest in terms of number of fish for stocking. For those who are interested in monitoring the habitat quality of a culture system, the model provides a summary of the estimated Habitat Suitability Index. For those interested in accounting and budgeting objectives, a tabulation of all costs and revenues are provided in a separate tab. Finally, for the economist or the private producer, the expected total net revenues of each system in the model are calculated.

Such a decision support tool must be able to predict the outcome of a system (*i.e.* both in terms of total net revenues as well as physical output of fish) on the basis of some observable characteristics of the system. In order facilitate the analysis of “what if” contingency scenarios, these observable characteristics must be able to be estimated ex-ante with some degree of ease.

To address these requirements, the model developed for this thesis uses a system of habitat variables to predict the expected outcome of a culture system. These habitat variables represent the critical water quality parameters of a culture system (*e.g.* pH, temperature, dissolved oxygen, *etc.*), which are easily observable, estimable, and important determinants of the outcome of a given system.

In order to demonstrate this tool, the decision of whether or not to adopt a mechanical aeration technology is analyzed. For simplicity, two alternatives are considered. The baseline scenario outlines the outcome of production in ponds to which no modifications have been made, and the alternative scenario describes a pond that been treated with a fine pore aerator. This scenario is developed by estimating the overall suitability of a hypothetical pond on the basis of input provided by Colorado Parks and Wildlife hatchery managers (Egloff, 2011; Harris, 2011; Swanson, 2011). The estimated outputs and total revenues associated with each these systems are calculated by developing a production function that uses a measure of habitat suitability (among other determinants of output) as a parameter. The costs associated with production are estimated on the basis of existing literature (Deisenroth & Bond, 2010). Finally, an analysis of the sensitivity of the results to value of stochastic determinants of total revenues is presented by incorporating Monte Carlo simulation tools.

CHAPTER 2: LITERATURE REVIEW

The existing literature may be classified into four broad categories. Descriptions of these four broad categories are as follows:

1. The first category addresses the effects of variations in water quality parameters and habitat characteristics on measures of the reproductive status and the general wellbeing (*e.g.* mortality rates, metabolic rate, fecundity rates, growth rates) of crappies and other Centrarchid species.
2. A second group of studies examines the use of habitat indexing tools to predict the state of populations of wild animals or culture species as a function of the quality of their habitat.
3. A third group of studies demonstrates the use of a capital budgeting approach to examine the profitability of aquaculture production and the potential gains from technology adoption in aquaculture.
4. Finally, a fourth classification views the biology literature through lens of economic theory in order to understand how a classic theoretical doctrine (*i.e.* the Law of Diminishing Marginal Productivity) manifests itself in the production of fishes.

2.1: OVERVIEW OF BIOLOGY LITERATURE

In order to describe the biology literature and its relevance to this thesis, literature from this field has been divided into two categories. The first category of biology literature addresses the physiological and behavioral responses of crappies (or related species) when critical parameters of the environment in which they live (*e.g.* temperature, pH, dissolved oxygen, turbidity, etc.) are allowed to vary. For this thesis, an understanding of the biological responses of a culture species to changes in their habitat serves three purposes. First, it highlights the need for the development of system designed to evaluate the suitability of a habitat on a species-specific level. Second, it provides a useful description of the biological

principles behind the production function that serves as the foundation of this white crappie production model. Finally, it aids the culturist in recreating spawning habitats chosen by white crappies in the wild and furthers their understanding of how changes in critical habitat parameters may affect the expected output of their culture system.

The second category of biology literature examines the relationship between the HSI and population densities of species in the wild. For this thesis, a review of this literature serves two purposes. First, it demonstrates that a general consensus about the functional form that best represents the relationship between the HSI and the state of populations of wild animals has yet to be reached. Second, it reveals two gaps in the literature in that 1) there are no previous studies that relate the HSI to the production of white crappie, and 2) an attempt to incorporate HSI tools into a simulation model has not been made in the past.

2.1.1: EFFECTS OF VARIATIONS IN WATER QUALITY AND HABITAT CHARACTERISTICS ON THE REPRODUCTIVE WELLBEING OF CRAPPIES AND RELATED SPECIES

Studying the biological responses of a culture species to habitat changes is an important part in understanding how variations in habitat quality parameters may affect the output of a culture system. Examining the cardiac responses of fishes to exhaustive exercise, for example, highlights the relationship between oxygenation and the productivity of a culture system, because essential functions (*e.g.* reproduction) may be affected by limited oxygen availability (*e.g.* due to the increased water temperature or higher energy demands of fishes after exhaustive exercise or other stress-inducing factors) (Kramer, 1987). These responses will vary across species. For example, the increased heart rate and recovery time in black crappie (*Pomoxis nigromaculatus*) after exhaustive exercise in cold water are less than that of the largemouth bass (*Micropterus salmoides*) but greater than that of the white bass (*Morone chrysops*) (Cooke, Grant, Schreer, Philipp, & Devries, 2003). The difference in cardiac responses between these

species accentuates the fact that a system designed to evaluate habitat suitability must be developed on a species-to-species basis.

Field studies are also useful in understanding the biology of white crappies and related species. A natural experiment using biotelemetry to examine the characteristics of spawning locations chosen by wild black crappie in their native bodies of water suggests that the characteristics of black crappie nesting sites are statistically different from randomly selected sites. Of note was the species affinity for sites with dense cover and protection from wind and wave turbulence (Pope & Willis, 1997). For the crappie culturist, the observation of crappie spawning behavior in the wild is helpful in recreating natural spawning habitats. It is also useful information in developing a detailed description of suitable habitat characteristics.

The information provided from studying the biology and behavior of white crappies is synthesized in a meta-analysis report that outlines the ideal habitat conditions for white crappies (Edwards, Krieger, Gebhart, & Maughan, 1982). This project is part of a series of reports designed to develop a system of “Habitat Suitability Indexes” (HSIs). These HSIs evaluate the suitability of a habitat at a species-specific level on the basis of a zero-to-one index, and the HSI model for the white crappie is a fundamental part of the production model developed in this thesis. For details about the white crappie HSI model, the reader is referred to Section 3.1.

2.1.2: USE OF HABITAT INDEXES TO PREDICT THE STATE OF POPULATIONS OF ANIMALS

In the past, field studies that are designed to examine the link between the HSI and population densities of species in the wild have been conducted. Linear regression models have been used to model the link between HSI and wild populations of brown trout (*Salmo trutta*) in Wyoming and Fishers (*Martes pennati*) in Michigan (Wesche, Goertler, & Hubert, 1987; Thomasma, Drummer, & Peterson,

1991). In the case of the American oyster (*Crassostrea virginica*), log-linear regressions have been used to link HSI and population densities in Galveston Bay, Texas (Soniat & Brody, 1988). The HSI has also been linked to the production of the Manila clam (*Tapes philippinarum*) in Italian farms by developing piecewise-linear functions on the basis of observations about population densities (Vincenzi, Caramori, Rossi, & De Leo, 2007). The differences in the methodological approaches of these studies indicate that a general consensus about functional form that best represents the relationship between HSI and population densities has yet to be reached. To the author's knowledge, there are no previous studies that relate the HSI to the production of white crappie. Furthermore, there are no previously developed simulation models that attempt to incorporate this relationship.

2.2: OVERVIEW OF ECONOMICS LITERATURE AND ECONOMIC PRINCIPLES IN AQUACULTURE LITERATURE

For this thesis, the literature addressing the economics of aquaculture production has been placed into two categories. The first category of economic studies uses a capital budgeting approach to analyze production. For the purposes of this thesis, a review of these studies are useful in that they act as methodological templates for the application of cost-benefit analysis to aquaculture production models. The second category of economic literature provides an economic interpretation of the biology literature. For the purposes of this thesis, this category helps in justifying the assumption of diminishing marginal returns to habitat quality.

2.2.1: CAPITAL BUDGETING APPROACH TO AQUACULTURE PRODUCTION AND TECHNOLOGY ADOPTION

The capital budgeting in aquaculture literature includes studies that are designed to examine the profitability of culture system and the potential change in profitability resulting from technology adoption. One such study, for example, has developed a trout production model in spreadsheet form that is aimed at “help[ing] prospective recirculating system operators examine the economics of proposed systems (Dunning, Losordo, & Hobbs, 1998).” This study uses a two-period model to estimate the “returns above variable costs.” In order to cater to a variety of users, these returns are calculated on the basis of user-provided input, which is likely to vary across different culture systems. While this model does incorporate the biology of the species by estimating output as a function of both mortality rates and feed conversion ratios, it does not attempt to incorporate HSI tools nor does it provide the user with the option to simulate the sensitivity of the results to changes in critical determinants of total revenues.

A capital budgeting approach has also been used to evaluate the potential profitability of the adoption of new technologies in aquaculture systems. A technology termed “Integrated Multi-trophic Aquaculture” (IMTA) provides an excellent example of this type of analysis. The idea behind this technology is summarized as follows:

“IMTA is the practice which combines, in the appropriate proportions, the cultivation of fed aquaculture species (*e.g.* finfish/shrimp) with organic extractive aquaculture species (*e.g.* shellfish/herbivorous fish) and inorganic extractive aquaculture species (*e.g.* seaweed) to create balanced systems for environmental sustainability (biomitigation) economic stability (product diversification and risk reduction) and social acceptability (better management practices) (Barrington, Ridler, Chopin, Robinson, & Robinson, 2008).”

Capital budgeting approaches have been used to evaluate the profitability of the integrated culture of salmon/mussels off the west coast of Scotland, salmon/mussel/seaweed culture in New Brunswick’s Bay of Fundy, and abalone/seaweed culture in South Africa (Whitmarsh, Cook, & Black, 2006; Ridler, et al., 2007; Nobre, et al., 2010). All these studies employ similar methodological approaches in that they first

examine the profitability of IMTA by tabulating total revenues less total costs and subsequently perform sensitivity analyses by allowing key parameters (*e.g.* output price, exposure to risk, rate of time preference, etc.) to vary.

In developing a white crappie production model, the author has borrowed elements from the methodological approaches of each of these studies. Similar to approach adopted by the trout production model, the white crappie production model develops results on the basis of user-provided input in order to accommodate the needs of several different types of users. From the IMTA adoption literature, the white crappie production model borrows the approach of performing sensitivity analysis on several key parameters by developing Monte Carlo simulation tools.

2.2.2: DEMONSTRATION OF ECONOMIC PRINCIPLES IN THE AQUACULTURE LITERATURE

The Law of Diminishing Marginal Productivity, which asserts that “... the marginal physical product of an input depends on how much of that input is used,” is a reoccurring theme in the aquaculture literature (Nicholson & Christopher, 2008). For this thesis, the concept of diminishing marginal benefits from habitat quality improvements (in terms of the value of biomass lost or gained) is particularly important. The concept of diminishing marginal returns to habitat quality improvements is exemplified by a study examining the effects of variable oxygen concentrations on the specific growth rate (SGR)¹ of Atlantic halibut (*Hippoglossus hippoglossus L.*) (Thorarensen, et al., 2010). The findings of their experiment suggest that the SGR of halibut maintained at 84 percent of saturation concentration was significantly higher than the group maintained at 57 percent. The SGR in the 84 percent group was, however, not significantly different from that of the groups exposed to 100-150 percent saturation. In their own words:

¹ The SGR can be conceptualized as the amount of body mass gained (or lost) by a species over a period of time; in this case, it is expressed in units of percent body mass gained (or lost) per day.

“These results suggest that the minimum oxygen levels required to support the maximum growth in halibut are higher than 84 percent and may be close to 100 percent of air saturation... [the effects of oxygen levels on growth rates and feed conversion suggest that] it may be of advantage for fish farmers to rear Atlantic halibut at 100 percent saturation if the price of oxygen is favorable (Thorarensen, et al., 2010).”

In the case of the Thorarensen study, an improvement to the habitat quality of a culture system has been made by adopting a technology (*i.e.* oxygenating the water). The marginal benefit from this habitat quality improvement is represented by the value of the change in biomass, and the marginal cost of the improvement is represented by the pecuniary cost (as well as the opportunity cost) of providing that increase in the level of oxygenation. If it is the case that the “price of oxygen is favorable,” the marginal net benefit of habitat quality improvements up to “close to 100 percent air saturation” is positive. As the cost of oxygen increases, however, the optimal level of provision tends towards 84 percent saturation, as the marginal benefit of units past this level are increasing at a diminishing rate.

The Thorarensen study is just one example of the concept of diminishing marginal returns to habitat quality improvements in the aquaculture literature, but there are many other examples including, the following: the optimal temperature at which to manage the virulence of fish pathogens, the pH level at which digestive enzymes best perform in different fish species for a given temperature, the effect of varying levels of turbidity on SGR, and the photoperiod at which the wellbeing of juvenile species is optimized (Ishiguro, et al., 1981; Hart, Hutchinson, & Purser, 1996; Hidalgo, Urea, & Sanz, 1999; Ardjosoediro & Ramnarine, 2002). In order to develop a model that is consistent with the concept of diminishing marginal returns to habitat quality improvements, which is a reoccurring theme in the aquaculture literature, it has been assumed that the culture of white crappies also exhibits this trait. Details about the incorporation of this assumption are available to the reader in Section 3.2.4 and Appendix C.

CHAPTER 3: THE MODEL

The goal of this work is to design a decision support tool, through which the user may explore business planning options (*e.g.* analysis of the expected returns to capital under alternative production technologies) and contingency planning scenarios (*e.g.* sudden changes to water quality as a result of equipment failure, unusual weather patterns, unexpected changes to a water source, or other unforeseen events) associated with the production of white crappie. A well-designed decision support tool must be able to accommodate the needs of several different types of users as their goals may vary on a case-to-case basis. The Microsoft Excel model that has been developed for this thesis provides a comparison of alternative production systems across four metrics: habitat quality conditions, the output of fish for stocking, annualized total net revenues, and non-annualized total net revenues. The model allows the user to compare across these four metrics as several production inputs and parameters (*e.g.* stocking density, mortality, fecundity, habitat quality, and input and output prices) are allowed to change.

The process of comparing these two systems begins with a comparison of their impact on habitat quality. An index number characterizing habitat quality is used as a parameter in the estimation of the expected output of a culture system and thus the expected total revenues that result from a given level of this parameter. For the purposes of estimating the total net revenues generated by each system, these measures of revenue can be compared against the cost of creating the habitat in which the white crappies are reared. The following subsections describe this process in more detail.

The remainder of this section is organized as follows: After a summary of the details of modeling the HSI, the use of this index number in the estimation of the expected output of a culture system is presented. The process of transforming this expected physical output into dollars of total revenues is discussed. The methods used to categorize and model the costs associated with developing the culture system are then presented. Finally, the presentation of net revenues in the model is discussed.

3.1: MODELING HABITAT QUALITY USING THE WHITE CRAPPIE HSI MODEL

Certain habitat quality parameters are critical in determining the suitability of a habitat for aquaculture production. For white crappies, these critical parameters and characteristics include temperature, dissolved oxygen (DO), pH, total dissolved solids (TDS), turbidity, percent cover, and percent littoral area (Edwards, Krieger, Gebhart, & Maughan, 1982). In the white crappie production model, the Habitat Suitability Index is the parameter through which these components of habitat quality are reflected.

This index number, which was developed by the Department of the Interior (DOI) in the 1980s, is intended to “synthesize habitat information into explicit habitat models that are useful in quantitative assessment (United States Geological Survey, 2011).” HSI models are meta-analyses that “reference numerous literature sources in an effort to consolidate scientific information on species-habitat relationships (United States Geological Survey, 2011).” These models are “some of the most influential management tools in use … [and] are applied daily by natural resource managers and decision-makers (Brooks, 1997).”

The HSI model for white crappie summarizes nine habitat quality parameters into a single HSI (Edwards, Krieger, Gebhart, & Maughan, 1982). Figure 1 illustrates the relationship between each of the nine habitat quality parameters where HSI is calculated as a function of the following life requisite indexes: food, cover, water quality, and reproduction. Each of these life requisites are in turn a function of a subset of the nine parameters. A detailed description of these relationships is presented in Appendix A.

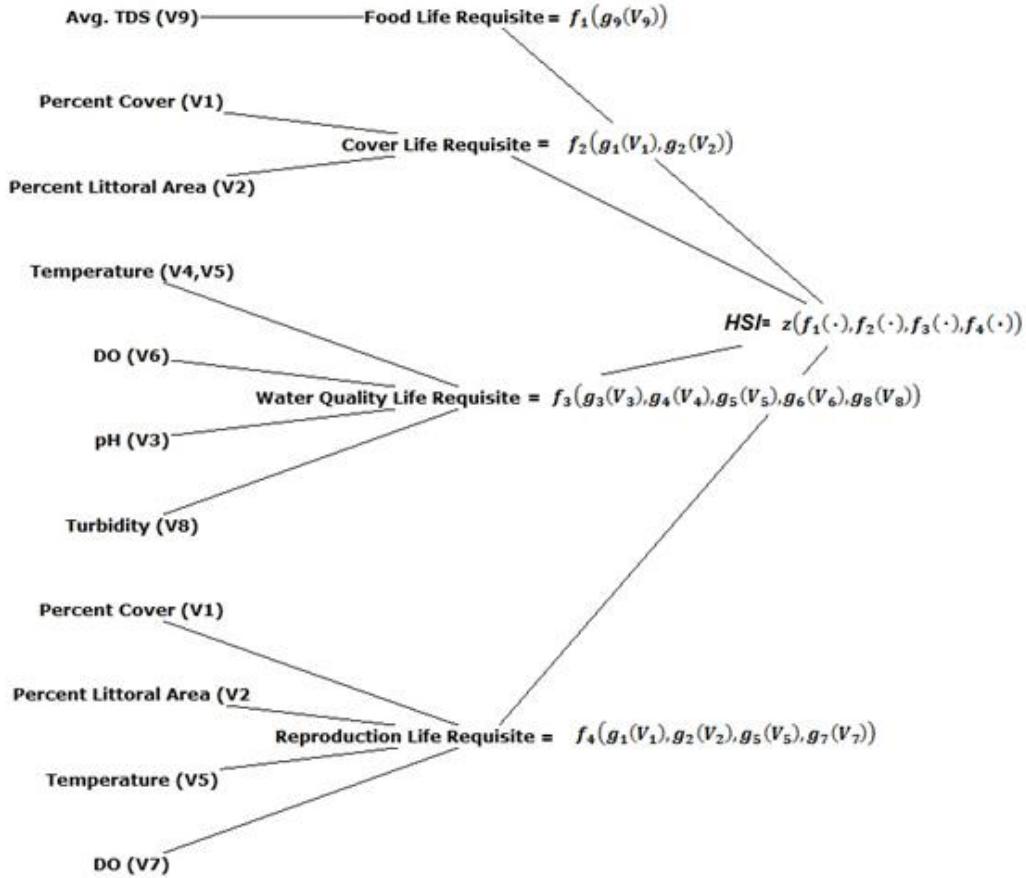


Figure 1: Functional relationships of the white crappie habitat suitability index model

Formally, the HSI can be represented by the following equation:

$$(1) \quad HSI = \begin{cases} (C_F * C_C * C_{WQ}^2 * C_R^2)^{\frac{1}{6}} & \text{if } C_{WQ} > 0.4 \text{ and } C_R > 0.4 \\ \min [C_{WQ}, C_R, (C_F * C_C * C_{WQ}^2 * C_R^2)^{\frac{1}{6}}] & \text{if } C_{WQ} \leq 0.4 \text{ or } C_R \leq 0.4 \end{cases}$$

where C_F, C_C, C_{WQ}, C_R represent the life requisite indexes for food, cover, water quality, and reproduction, respectively. For habitats with sufficiently suitable water quality and reproduction life indexes (*i.e.* $C_{WQ} > 0.4$ and $C_R > 0.4$), the HSI described by (1) is calculated by interacting the suitability of each of the life requisite indexes in a concave function that best “produce[s] an index between zero and one which is believed to have a positive relationship to the carrying capacity [of the habitat] (Edwards, Krieger,

Gebhart, & Maughan, 1982)." Alternatively, for habitats with sufficiently unsuitable water quality or reproduction life requisite indices, the index number described by (1) works in the same way as the Von Liebig's Law of the Minimum in that the overall habitat quality is critically dependent on the least suitable of each of the habitat life requisites (Liebig, 1972).

3.2: POND OUTPUT AS A FUNCTION OF HABITAT QUALITY

Habitat quality influences the productivity of a system. The productivity of a system is measured in terms of the number of fingerlings for stocking (*i.e.* the number of fish harvested). This information is particularly useful, for example, in developing stocking plans for the purposes of managing wild populations of white crappie. Additionally, the estimation of physical output is an important component in the calculation of the total revenues that are generated by a culture system.

In the model, physical output is assumed to be represented by the following equation:

$$(2) \quad E[Y] = (\theta\mu)(1 - \varphi)$$

where the expected output ($E[Y]$) is a function of the habitat adjusted post-larval mortality rate of a species (φ)², the initial size of the female broodstock (μ), and the relative fecundity of the culture species (θ). The expected output of the system is represented in (2) by an "inflow less outflow" equation. The product of fecundity and stocking density ($\theta\mu$) represents the fertility of the pond (*i.e.* the inflow of fingerlings to the system). The adjusted mortality rate (φ) represents the fraction of this total inflow that is netted out of the total expected output of the system. The fecundity and stocking density are parameters that are not impacted by the choice of technology in the culture system; however, since the habitat adjusted post-larval mortality rate is a function of HSI, this parameter is impacted by a culturist's choice of technology.

² The habitat adjusted adult mortality rate is discussed in detail in Section 3.2.2.

3.2.1: MODELING THE DETERMINANTS OF OUTPUT AND MORTALITY AT DIFFERENT LIFE STAGES

In the model, the life of the white crappie has been divided into three stages. These stages are the egg, the larval, and the post-larval stages of development. This categorization has been made in order to model the biological outflows (*e.g.* mortality rate and non-fertilization of eggs) that are subject to change as the fish progresses through these different life stages. Absent information relating the suitability of a habitat for white crappies in the egg and larval stages of development to mortality, the mortality rates in these stages of development are modeled as if they were independent of habitat quality. Underestimation of these mortality rates will result in an overestimation of the output of a culture system. The opposite is also true.

The causes of changing mortality across the different early life stages of fishes vary by species and include predation and cannibalism, starvation, increased sensitivity to temperature, siltation, low oxygen levels, unfavorable water flow rates, and human and natural perturbation (Dahlberg, 1979). For the egg stage of development, the use of fecundity data to calculate egg production may result in an overestimation of fertility, because this data does not account for egg losses from non-fertilization or incomplete egg extrusion. It has been estimated, for example, that the ratio of actual egg deposition to potential egg deposition (*i.e.* the ratio of eggs actually counted in the field to estimated egg production from fecundity figures) is as low as 15 percent in smallmouth bass (Clady, 1975). Egg mortality in white crappies is highly variable, with as few as 49 percent of eggs expected to survive to the larval stages of development. However, it is estimated that parental care was a factor in 49-94 percent of these surviving eggs (Dahlberg, 1979). A rate of forty-nine percent survival is comparatively high relative to studies on other species of freshwater fish (*e.g.* white sucker, walleye, and rainbow smelt) that report as little as 1-3 percent survival of eggs (Dahlberg, 1979). Mortality is also expected to be high among larval and juvenile fish, with previous studies indicating mortality rates as high as twenty-eight percent among juvenile white crappies of less than 80mm in length (Pine & Allen, 2001).

In the model, the expected output of a culture system in egg stage of development is represented as follows:

$$(3) \quad E[Y_{egg}] = c_1 \theta \mu * (1 - m_1)$$

where the expected output of a culture system in terms of the number of eggs surviving to the larval stages of development ($E[Y_{egg}]$), which is a function of the initial size of the female broodstock (μ), the relative fecundity of the culture species (θ), the ratio of actual egg deposition to potential egg deposition (c_1), and the egg mortality rate (m_1). Similar to (2), the expected output of the culture system in (3) is an inflow less outflow equation. In the case of (3), the product of μ and θ represent the fertility of the pond, which is deflated by a factor of c_1 in order to correct for the discrepancy between actual egg deposition and potential egg deposition (Clady, 1975). Outflows are represented by m_1 .

The expected output of a culture system in larval stage of development is represented as follows:

$$(4) \quad E[Y_{larval}] = Y_{egg} * (1 - m_2)$$

where the expected output of the culture system at the larval stage of development ($E[Y_{larval}]$) is represented by the output of the culture system in terms of the number of surviving eggs as defined by (3) less the larval mortality rate (m_3).

Finally, the expected output of a culture system in post-larval stage of development is represented as follows:

$$(5) \quad E[Y_{post\ larval}] = Y_{larval} * (1 - \varphi)$$

where the expected output of the culture system in terms of number of fish for stocking ($E[Y_{post\ larval}]$) is represented by the output of the culture system in terms of the number of surviving larval fish as defined by (4) less the habitat adjusted post-larval mortality rate (φ).

3.2.2: THE IMPACT OF HABITAT QUALITY-RELATED MANAGEMENT DECISION ON OUTPUT

The carrying capacity of a culture system is closely related to the suitability of the habitat in which the culture species is reared, and the characteristics of an optimal habitat will vary across the different life stages of the species. To capture this idea, a functional relationship between the HSI and the expected output of a culture system must be developed. HSI is used here, because it is a practical tool that may be used to estimate the expected output of a culture system on the basis of easily observable and estimable parameters without requiring a large data set with which to run regressions.

On the matter of choosing an appropriate functional form to link HSI and population densities, a wide variety of methodological approaches have been employed in the past and a general consensus has yet to be reached. In order to link HSI with the expected output of a white crappie culture system, the model uses an exponential functional form. The assumed functional relationship between HSI and the expected output of a white crappie culture system is as follows:

$$(6) \quad \varphi = m_3^{c_2 * HSI} \quad \forall HSI \in [0,1]$$

where the habitat adjusted post-larval mortality rate (φ) is a function of the minimum mortality rate of white crappies under optimal conditions (m_3), a habitat adjusted mortality rate constant (c_2), and the HSI of a culture system (HSI). The link between the HSI of a culture system and the output of that system (in terms of number of white crappies for stocking) is established in (6) by using the HSI to adjust the post-larval mortality rate of white crappies. The habitat adjusted post-larval mortality rate is intended to model the changes in the mortality of a culture species across different culture systems that may be attributed to differences in the suitability of the culture habitat. In adjusting the post-larval mortality rate of a culture system, the corresponding output of the culture system is also adjusted.

The method of adjusting the post-larval mortality rate in (6) was selected in order to develop a functional form that is both commonly used in production theory and that reflects the assumption of diminishing marginal returns to habitat quality improvements, which is consistent with the aquaculture literature as presented in Section 2.2.2. The functional form in (6) is not the only functional form that models the assumption of diminishing marginal returns to habitat quality improvements. In fact, any convex habitat adjusted mortality function will capture this assumed effect. For any given value of HSI (\hat{H}), the value of φ is critically dependent on the shape of the habitat adjusted post-larval mortality function that is selected. For a “true” habitat adjusted post-larval mortality function, $h(HSI)$, and an alternative function, $j(HSI)$, such that $\frac{\partial h}{\partial HSI} < 0$, $\frac{\partial^2 h}{\partial HSI^2} > 0$, $\frac{\partial j}{\partial HSI} < 0$, and $\frac{\partial^2 j}{\partial HSI^2} > 0$, $j(HSI)$ will underestimate φ if $j(\hat{H}) < h(\hat{H})$. Alternatively, φ will be overstated if $j(\hat{H}) > h(\hat{H})$. Overstating the mortality of the culture species will result in an understatement of the number fish for stocking and, as a result, underestimate the total benefit of a culture system. The opposite is also true.

3.2.3: CALCULATION OF THE HABITAT ADJUSTED MORTALITY RATE

In order to further discuss the habitat adjusted post-larval mortality function, it is helpful to graph it. Three examples of adjusted mortality rate schedules are graphed below:

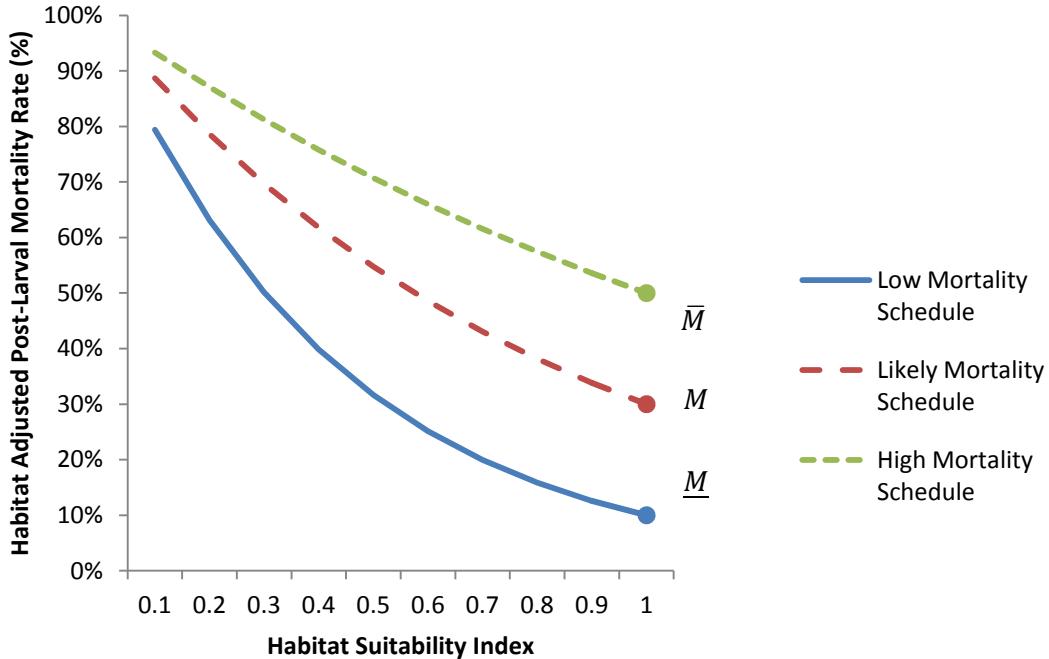


Figure 2: Habitat adjusted post-larval mortality rate schedules ($m_3 = 0.5, 0.3, \text{ and } 0.1; C_2 = 1.0$)

The habitat adjusted post-larval mortality rate schedules in Figure 2 were calculated using (6). Figure 2 illustrates an inverse relationship between HSI and φ . The three adjusted mortality rate schedules in Figure 2 differ only in that the assumed minimum mortality rates under optimal conditions (m_3) vary across each schedule. In the case of \bar{M} , the user has indicated a minimal mortality rate of fifty percent. With M and \underline{M} , the user has indicated a minimal mortality of thirty and ten percent, respectively. At the lower bound of $HSI = 0$ (*i.e.* for perfectly unsuitable habitats), all three schedules approach one hundred percent adjusted mortality. As the habitat quality is improved (*i.e.* as HSI tends towards unity), the limit of each mortality schedule approaches the respective minimum mortality value that the user has indicated.

The convexity of these functions captures the idea of diminishing marginal returns to habitat quality improvements (*i.e.* marginal increases in HSI). A detailed example of the calculation and application of the adjusted post-larval mortality rate is available in Appendix B.

SECTION 3.2.4: CALCULATION OF EXPECTED OUTPUT

In order to discuss the expected output function, it is helpful to graph it:

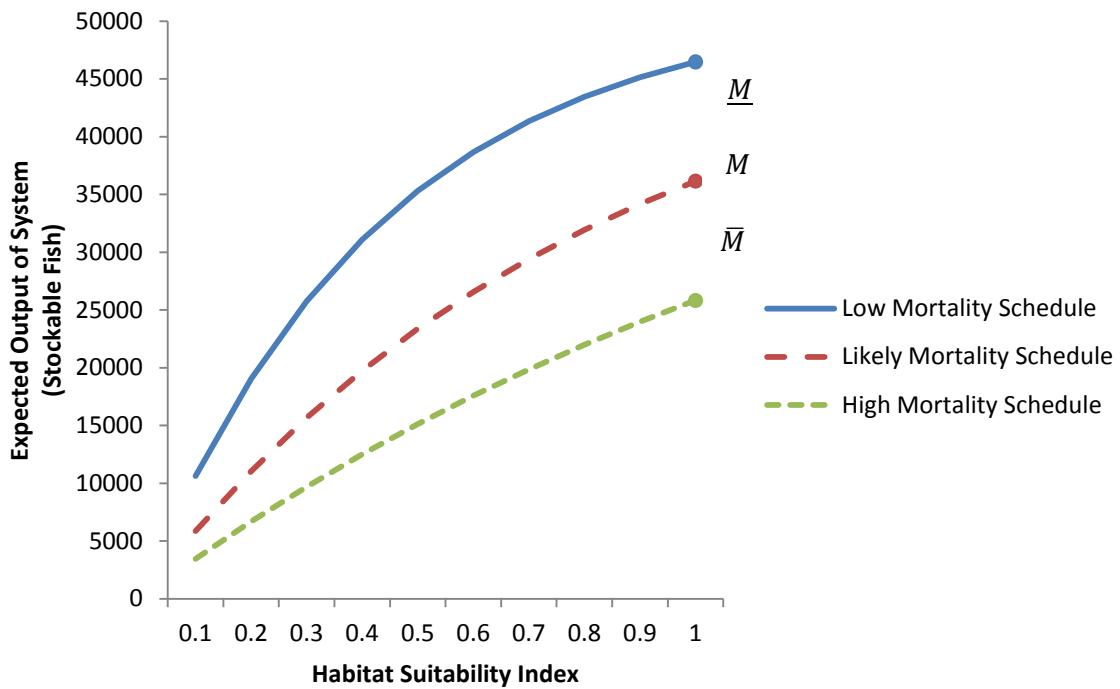


Figure 3: Expected output schedules ($m_3= 0.5, 0.3, \text{ and } 0.1; C_1=0.25, C_2=1.0, m_1=0.75, m_2=0.28$)

The expected output schedules in Figure 3 were calculated using (3), (4), and (5) and are linear transformations of the adjusted mortality rate schedules in Figure 2. Figure 3 illustrates a direct relationship between the expected output of a culture system and HSI. Changes in the expected output of a culture system are driven by changes in the HSI, which result in changes in φ as per (6). As with Figure

2, the three expected output schedules in Figure 3 differ only in that the assumed minimum mortality rate under optimal conditions (m_3) varies across each schedule³. At the lower bound of $HSI = 0$ (*i.e.* for perfectly unsuitable habitats), φ approaches one hundred percent mortality, and each of the expected output schedules approach zero. As the habitat quality is improved (*i.e.* as HSI tends towards unity), the expected output increases as a result of reductions in φ . The concavity of these functions captures the idea of diminishing marginal returns to habitat quality improvements (*i.e.* marginal increases in HSI). A detailed explanation of the calculus describing how changes in the value of assumed parameters will affect the expected output of a system is presented in Appendix C.

3.2.5: THE IMPACT OF THE HABITAT ADJUSTED MORTALITY CONSTANT ON THE HABITAT ADJUSTED MORTALITY RATE AND EXPECTED OUTPUT

For a given minimum mortality rate (m_3), the habitat adjusted mortality constant (c_2) indicates the degree to which improvements to habitat quality (*i.e.* increases in HSI) may result in reductions in φ . For simplicity and in the absence of any information detailing the exact value of c_2 , the model assumes that this constant is equal to one. If this constant is not equal to one, however, the mortality rate that is actually observed may differ from φ . A graphical representation of the effect of c_2 is presented below:

³ The assumed minimum mortality rates in Figure 3 take on the same values as in Figure 2.

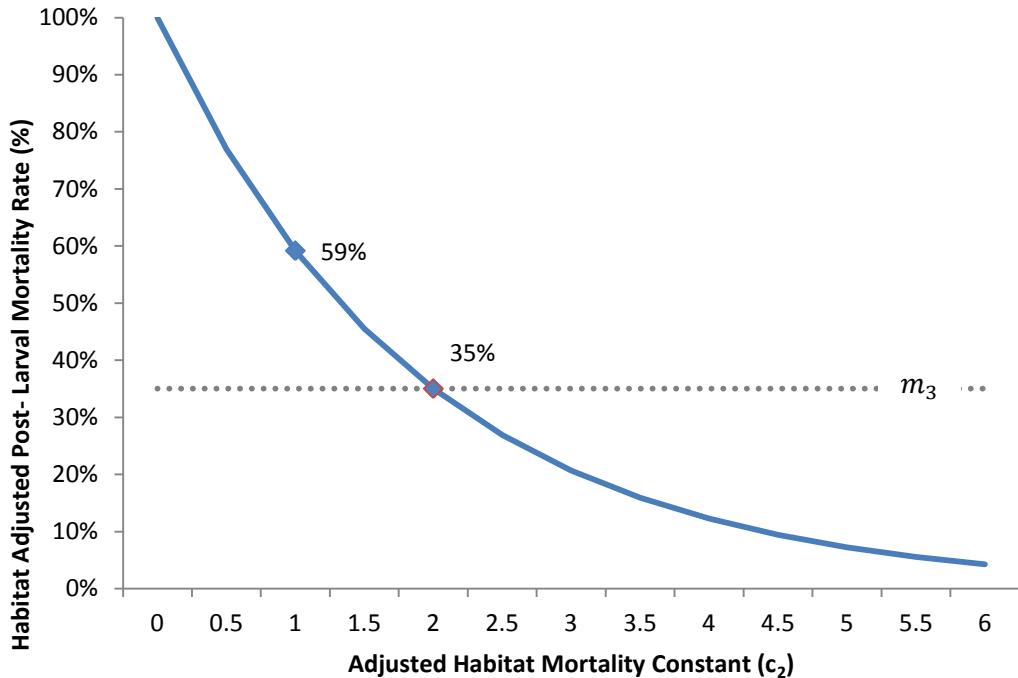


Figure 4: Habitat adjusted mortality constant schedule ($m_3 = 0.35$, HIS=0.5)

Figure 4 holds habitat quality and minimum mortality constant in order to illustrate the inverse relationship between c_2 and φ . When c_2 is zero, φ assumes a value of one hundred percent mortality regardless of the value of HSI. Since the model assumes that post-larval mortality is not independent of habitat quality (and since morality rates cannot exceed one hundred percent), the lower bound of c_2 is assumed to be zero, non-inclusive. Since m_3 is assumed to be the minimal possible mortality rate of a culture species under optimal conditions, it is also assumed that the upper bound of c_2 is such that $f(c_2; m_3, \text{HSI}) = m_3$. In the case of Figure 4, the relevant domain of $f(c_2; m_3, \text{HSI})$ is $(0, 2]$. The relevant domain of this function, however, is subject to change as the values of m_3 and HSI are allowed to change across culture systems. Increases (decreases) in m_3 will decrease (increase) the upper bound of c_2 , and increases (decreases) in HSI will decrease (increase) the upper bound of c_2 , *cet. par.*

For the purposes of this paper, an analysis of Figure 4 yields two important conclusions about the role of c_2 in the model. First, since the premise of the model is that the output of a culture system is a function habitat quality conditions, it follows that, regardless of the values of m_3 and HSI, the lower bound of c_2 will always be zero, non-inclusive; the upper bound of c_2 will vary, depending on the values of m_3 and HSI. Second, the concavity of $f(c_2; m_3, HSI)$ indicates that the elasticity of substitution between c_2 and φ ($\varepsilon_{c_2,\varphi}$)⁴ is increasing as c_2 increases from zero to m_3 . The assumption that $\frac{\partial \varepsilon_{c_2,\varphi}}{\partial c_2} > 0$ implies that increases in c_2 will have an increasingly smaller marginal effect on φ . The opposite is also true.

In the absence of data detailing the actual value of c_2 , the model assumes that $c_2 = 1$. Without data to use for regression analysis, the actual value of c_2 is unknown, and further research is required in order to evaluate the validity of this assumption. However, since the mathematical properties of $f(c_2, m_4, HSI)$ are known, the consequences of erroneously imposing the assumption that $c_2 = 1$ can be discussed. In the case of Figure 4, imposing the assumption that $c_2 = 1$ will result in a φ of fifty-nine percent. If c_2 is actually less (greater) than one, the observed φ will be greater (less) than fifty-nine percent. An underestimation (overestimation) of φ will result in an overestimation (underestimation) of the output of a culture system. The marginal effect of a mistaken estimate of c_2 on the value of φ is decreasing (increasing) as the value of c_2 increases (decreases). A mathematical presentation detailing the effects of changes in the assumed value of the parameters impacting φ is available in Appendix D.

3.3: OVERVIEW OF TOTAL REVENUES AND TOTAL COSTS

The model produces a comparison of total revenues to total costs in two ways: present values and annualized values. Both present and annualized values are calculated as the discounted total net revenues

⁴ Formally, the elasticity of substitution between c_2 and φ is defined as follows:

$$\varepsilon_{c_2,\varphi} = \frac{\frac{\partial f(c_2; m_4, HSI)}{\partial c_2}}{f(c_2; m_4, HSI)} * \frac{c_2}{\frac{\% \Delta f(c_2; m_4, HSI)}{\% \Delta c_2}} = \frac{\% \Delta f(c_2; m_4, HSI)}{\% \Delta c_2} \Leftrightarrow \frac{\% \Delta \varphi}{\% \Delta c_2}$$

of a culture system. These two accounting methods differ in the way that they represent of the cost of capital goods. Under the present value accounting method, these costs are represented by the one-time purchase price of a capital good. Alternatively, under annualized value accounting, the cost of a capital good is represented by an average cost approach, in which the purchase price of the good is spread out over its useful life.

The model makes a distinction between these two accounting methods in order to more accurately represent the cost of capital goods that have a useful life of several culture seasons. For costly capital goods, the total revenues of a culture system in any one culture season may not be large enough to justify the up-front purchase price of the good. However, when the cost of the good is spread out of its useful life, the total net revenues of a system may become positive. In crappie culture, for example, the distinction between these two accounting methods may be important when considering goods such as aerators, pond liners, and filter systems and other water treatment equipment. An overview of the methods used to calculate total revenues and total costs follows.

3.4: TOTAL REVENUES

In its most general form, total revenue is represented as follows:

$$(7) \quad TR = P_Y * Y$$

where the total revenues of a culture system (TR) are the product of the output of the system (Y) and the output price (P_Y). The output price means the price received per fingerling, and the output of the system means the number of fingerlings for stocking produced by a culture system at a given level of habitat quality. The model uses the market price of crappie fingerlings (as charged by private producers) as a proxy for the output price. The total revenues received by private producers of crappies may differ from the social benefit of crappie production. In the context of the social planner's problem, the conceptually correct "output price" is the willingness to pay (WTP) for the next crappie stocked in a body of water.

This WTP may include the use value to anglers as well as other values (*e.g.* the social benefit of ecosystem services provided by crappie populations in the wild or non-use values such as bequest and existence values that are associated with the continued presence and wellbeing of crappies in the wild).

3.5: TOTAL COSTS

In the model, the user has the option to input the cost information specific to their culture system. Absent data on current costs, the model uses the methodology that is outlined in subsequent sections. In their most general form, total costs are represented as follows:

$$(8) \quad TC = FC + VC$$

where the total costs of a culture system (TC) are equal to the sum of fixed costs (FC) and variable costs (VC). Here the variable cost of the i^{th} variable input is defined as follows:

$$(9) \quad VC = \sum_i^I c_i x_i$$

where x_i is the quantity of the i^{th} input employed in white crappie production and c_i represents the marginal cost of that same input.

Optional technological improvements made to the pond are assumed to increase total costs. The increase in total cost is driven by two distinct impacts of the optional technological improvements. First, fixed costs are assumed to increase as a result of the purchase of capital goods associated with a technological improvement. Second, total variable costs of the culture system are also assumed to increase as the installation, operation, and maintenance of these technologies is assumed to require additional variable inputs in all culture ponds (*e.g.* increased feed, electricity, and labor expenditures).

3.6: APPLICATION OF METHODOLOGY

The following approach is used to estimate costs under the baseline and alternative scenarios.

Similar to IMPLAN, the entirety of total revenues is accounted for in total costs and residual categories. For a given stocking density and in the absence of an alternative technology, the baseline scenario is defined as the expected total revenue yielded from the most likely values of output price, HSI, mortality, and fecundity (*i.e.* all of the determinants of total revenues). For a given stocking density and in the presence of an alternative technology, an alternative scenario means the expected total revenue yielded from the most likely values of these same determinants.

3.7: BASELINE SCENARIO

For a given stocking density, the expected total revenues under the baseline scenario are represented as follows:

$$(10) \quad TR^* = f(\mathbf{A}^*)$$

where the expected revenues under the baseline scenario (TR^*) are a function of a vector containing the most likely values of all of the determinants of the expected total revenues of a culture system (\mathbf{A}^*).

Formally, \mathbf{A}^* may be defined as follows:

$$(11) \quad \mathbf{A}^* = \begin{bmatrix} P_Y^* \\ \theta^* \\ \varphi^* \end{bmatrix}$$

where P_Y^* is the most likely value of output price, θ^* is the most likely value of the relative fecundity of the culture species, and φ^* is the habitat adjusted post-larval mortality rate resulting from the most likely values of HSI and m_3 .

Fixed costs under the baseline scenario are represented as follows:

$$(12) \quad FC^* = \alpha TR^* \quad \{\alpha | 0 < \alpha < 1\}$$

where, absent data detailing the exact value of fixed costs, the fixed costs under the baseline scenario (FC^*) are assumed to be represented by some fraction of the total revenues under the baseline scenario (αTR^*).

Variable costs under the baseline scenario are represented as follows:

$$(13) \quad VC^* = \beta TR^* \quad \{\beta | 0 < \beta < 1\}$$

where, absent data detailing the exact value of variable costs, the variable costs under the baseline scenario (VC^*) are assumed to be represented by some fraction of the total revenues under the baseline scenario (βTR^*). The entirety of TR^* is assumed to be accounted for in the sum of VC^* and FC^* such that the following condition holds:

$$(14) \quad 1 - \alpha = \beta \Leftrightarrow TR^* = TC^*.$$

The condition defined by (14) models a long run zero profit condition.

In order to estimate α and β , the author borrows from literature on the subject of Aquacultural Suppliers of Recreational Fisheries (ASRF) in the Western United States. On average ASRFs in the Western United States gross \$330,000 annually in recreational fish sales (Deisenroth & Bond, 2010). Of these total gross annual revenues, \$120,000 is used on non-depreciated expenditures (*e.g.* feed, eggs, electricity, and gasoline) annually, \$90,000 is spent on annual labor expenditures including wages, labor taxes, and benefits, \$75,000 is used to purchase and maintain capital equipment (*e.g.* lease of buildings, fish production facilities, equipment and transportation, *etc.*), and there are \$45,000 of net revenues annually (Deisenroth & Bond, 2010). The cost structure of a white crappie production system is assumed to mimic that of an ASRF in that the share of total revenue going to each expenditure category is expected to be the same. For example, in the case of total annual expenses on non-depreciated goods, the

representative producer of white crappies (as well as the representative ASRF) is expected to spend thirty-six percent of their total revenues. The assumed cost structure of the average crappie producer is summarized in Table 1 of Section 3.8.

3.8: ALTERNATIVE SCENARIO

Total costs under the alternative scenario will differ from total costs under the baseline scenario as the former must reflect changes in a producers cost structure due to the implementation of an optional production technology. Under the net present value (NPV) accounting method, total costs under the alternative are represented as follows:

$$(15) \quad \widetilde{T\bar{C}}'_{NPV} = FC^* + \widetilde{VC} + \phi$$

where $\widetilde{T\bar{C}}'_{NPV}$ represents the total costs under the alternative scenario. The components of $\widetilde{T\bar{C}}'_{NPV}$ include the fixed costs under the baseline scenario (FC^*), the variable costs under the alternative scenario (\widetilde{VC}), and purchase price of any capital goods associated with the implementation of an alternative technology (ϕ). For reasons discussed in Section 3.5, \widetilde{VC} will be different from VC^* . It is assumed that \widetilde{VC} is defined as follows:

$$(16) \quad \widetilde{VC} = VC^* + \left[\frac{E[Y^*(\mathbf{B}^*)] - E[\tilde{Y}(\mathbf{B}^*)]}{E[Y^*(\mathbf{B}^*)]} * VC^* \right]$$

where the assumed variable costs under the alternative scenario (\widetilde{VC}) are a function of the assumed variable costs under the baseline scenario (VC^*), the expected output of the culture system under the baseline scenario resulting from a vector of the most likely determinants of output ($E[Y^*(\mathbf{B}^*)]$), and the expected output of the culture system under the alternative scenario resulting from a vector of the most likely determinants of output ($E[\tilde{Y}(\mathbf{B}^*)]$). The increase in the variable costs associated with the adoption

of an optional technological improvement is defined by (16) to be directly proportional to the expected percentage increase in output associated with this technology. Formally, \mathbf{B}^* may be defined as follows:

$$(17) \quad \mathbf{B}^* = \begin{bmatrix} \theta^* \\ \varphi^* \end{bmatrix}$$

where θ^* is the most likely value of the relative fecundity of the culture species, and φ^* is the habitat adjusted post-larval mortality rate resulting from the most likely value of HSI and m_3 .

The annualized accounting method makes use of previously developed methods from the capital budgeting literature (Walker & Kumaranayake, 2002). Under the annualized accounting method, total costs under the alternative are represented as follows:

$$(18) \quad \widetilde{T\bar{C}}'_{ANN} = FC^* + \widetilde{VC} + \left(\frac{\phi}{A_{t,r}} \right)$$

where the where $\widetilde{T\bar{C}}'_{ANN}$ represents the total costs under the alternative scenario and annualized accounting method, which are equal to the sum of the fixed costs under the baseline scenario (FC^*), the variable costs under the alternative scenario (\widetilde{VC}), and purchase price of any capital goods associated with the implementation of an alternative technology (ϕ) divided by a annuity factor ($A_{t,r}$)⁵. Formally, the annuity factor is defined as follows:

$$(19) \quad A_{t,r} = \frac{[(1+r)^{n_i} - 1]}{[r(1+r)^{n_i}]}$$

where the annuity factor ($A_{t,r}$) is a function of the discount rate (r) and the useful life of the i^{th} capital good (n_i). The advantage to this approach defined by (18) is twofold. First, adjusting by an annuity factor allows for the incorporation of the opportunity cost of capital and offers an economic measure of total annualized costs rather than an accounting measure. Second, calculating the total annualized costs allows

⁵ The ratio of $\left(\frac{\phi}{A_{t,r}} \right)$ is known as the equivalent annualized cost (EAC) of a capital good (Walker & Kumaranayake, 2002).

the user to compare across alternative capital investment opportunities even when there are differences in the useful lives of the capital goods in question (Walker & Kumaranayake, 2002). The assumed cost structures of both a representative white crappie culture system and an ASRF are tabulated in Table 1.

TABLE 1: SUMMARY OF ASSUMED TOTAL COST STRUCTURES OF A REPRESENTATIVE WHITE CRAPPIE CULTURE SYSTEM AND AN ASRF

ASRF Study			Baseline Scenario		Alternative Scenario	
	Total Revenues (dollars)	Share to Category (% of TR)	Total Revenues (dollars)	Share to Category (% of TR)	Total Revenues (dollars)	Share to Category (% of TR)
Total Revenues	\$330,000	100	TR^*	100	\widetilde{TR}	100
Variable Costs	\$210,000	63	VC^*	70	$VC^* + \left[\frac{(E[Y^*] - E[\tilde{Y}])}{E[Y^*]} * VC^* \right]$	$70 + 100 * \frac{(E[Y^*] - E[\tilde{Y}])}{E[Y^*]}$
Fixed Costs	\$75,000	23	FC^*	30	$FC^* + \phi$	$30 + 100 * \frac{\phi}{TR^*}$
Total Net Revenues	\$45,000	14	$TR^* - (VC^* + FC^*)$	0	$\widetilde{TR} - (\widetilde{VC} + \widetilde{FC})$	$100 - 100 * \left[\frac{(E[Y^*] - E[\tilde{Y}])}{E[Y^*]} + \frac{\phi}{TR^*} \right]$

3.9: DISCOUNTED TOTAL NET REVENUES

In order to adjust for time preferences, the value of the total net revenues of a culture system, which are received at the end of a culture period, must be discounted back to their present value terms. For the baseline scenario, the discounted total net revenues of a culture system are as follows:

$$(20) \quad TNR_{NPV}^* = \sum_t^T \frac{1}{(1+r)^t} * [TR^* - TC^*] = 0$$

where the NPV under the baseline scenario (TNR_{NPV}^*) is represented by the sum of the discounted net revenues resulting from the most likely determinants of total revenues (TR^*) and the assumed total cost of the culture system as defined by (12) and (13). In the model, the value TNR_{NPV}^* is always zero as per the long run zero profit condition defined in (14).

The discounted total net revenues under the alternative scenario will vary across accounting methods. Under the NPV accounting method, discounted net revenues are as follows:

$$(21) \quad \widetilde{TNR}_{NPV} = \sum_t^T \frac{1}{(1+r)^t} * [\widetilde{TR} - \widetilde{TC}_{NPV}]$$

where the NPV under the alternative scenario (\widetilde{TNR}_{NPV}) is represented by the sum of the discounted net revenues resulting from the most likely determinants of total revenues (\widetilde{TR}) and the assumed total cost of the culture system as defined by (15) and (16)⁶.

Under the annualized value approach, discounted net revenues are as follows:

$$(22) \quad \widetilde{TNR}_{ANN} = \frac{1}{(1+r)^t} * [\widetilde{TR} - \widetilde{TC}_{ANN}]$$

⁶ The value \widetilde{TNR}_{NPV} is not necessarily zero, because the total net revenues under the alternative scenario reflect the non-annualized changes in both total revenues and total costs that are associated with the introduction of an optional technological improvement.

where the NPV of the total net revenues under the alternative scenario (\widetilde{TNR}_{ANN}) is represented by the sum of the discounted net revenues resulting from the most likely determinants of total revenues (\widetilde{TR}) and the assumed total cost of the culture system as defined by (18) and (19)⁷.

⁷ The value \widetilde{TNR}_{ANN} is not necessarily zero, because the total net revenues under the alternative scenario reflect the annualized changes in both total revenues and total costs that are associated with the introduction of an optional technological improvement.

CHAPTER 4: ADDRESSING STOCHASTICS

Modeling the pond culture of white crappie involves some uncertainty. This uncertainty arises in attempting to estimate the value of several different parameters that are important to the modeling process. These parameters include, for example, the expected measure of habitat quality inputs, the average relative fecundity of a relatively small population of white crappies, the output price received at the time of harvest, and the minimum mortality rate of white crappies under optimal habitat quality conditions. As a strategy for addressing this uncertainty, this thesis adopts a Monte Carlo simulation approach. Under this method, users estimate low, likely, and high values for each uncertain parameter. A triangular probability distribution function (PDF) is built around the values that the user has indicated, and random draws can then be made from within this distribution. In the absence of detailed information concerning the distribution of stochastic inputs, a triangular distribution is used in the model because it facilitates the development of a Monte Carlo simulation without requiring large data sets. This approach allows the model to report outputs as distributions of results (rather than a series of point estimates) on the basis of easily estimable, user-provided input. Reporting the results as distributions aids the user in analyzing the outcome of the modeling exercise in accordance with their personal risk preferences. In the model, users have the option to run Monte Carlo simulations on several determinants of the total revenues of their culture systems including the relative fecundity of the pond's white crappie population, the output price, and the minimum mortality rate under optimal habitat conditions.

4.1: DEVELOPING DISTRIBUTIONS AROUND USER-PROVIDED INPUT

Similar to previous studies, the model uses a PDF as the foundation for the development of Monte Carlo simulations (Hesse, 2000). The general form for the PDF requires a lower limit, an upper limit, and a mode. For the purposes of this thesis, it is assumed that the necessary parameters of the PDF

are represented by the user-provided low, high, and likely estimates for each stochastic determinant of the total revenues of the culture system. The general form of the PDF is as follows:

$$(23) \quad f(x; a, b, c) = \begin{cases} 0 & \text{for } x < a \\ \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c < x \leq b \\ 0 & \text{for } b < x \end{cases}$$

where the PDF ($f(x; a, b, c)$) that is described by (23) is defined by its lower bound (a), upper bound (b), and mode (c). These parameters of the PDF correspond to the values of the user-provided low estimate of a stochastic determinant of total revenues (a), the user-provided high estimate (b), and the user provided likely estimate (c). The probability of drawing a true random number (x) from $f(x; a, b, c)$ on the interval $x \in (-\infty, \psi]$ is represented as follows:

$$(24) P(a \leq x \leq \psi) = \begin{cases} \int_{-\infty}^a 0 dx & \text{for } x < a \\ \int_{-\infty}^a 0 dx + \int_a^\psi \frac{2(x-a)}{(b-a)(c-a)} dx & \text{for } a \leq x \leq c \\ \int_a^c \frac{2(x-a)}{(b-a)(c-a)} dx + \int_c^\psi \frac{2(b-x)}{(b-a)(b-c)} dx & \text{for } c < x \leq b \\ \int_{-\infty}^a 0 dx + \int_a^c \frac{2(x-a)}{(b-a)(c-a)} dx + \int_c^b \frac{2(b-x)}{(b-a)(b-c)} dx + \int_b^\infty 0 dx & \text{for } b < x \end{cases}$$

An explanation of the usefulness of (24) and the potential impacts of the assumptions used in developing a triangular PDF for this thesis are presented to the reader in Appendix J.

The Monte Carlo method employed by the model also requires the use of a corresponding triangular cumulative distribution function (CDF). The general form of the CDF is as follows:

$$(25) \quad F(x; a, b, c) = \begin{cases} 0 & \text{for } x < a \\ \frac{(x-a)^2}{(b-a)(c-a)} & \text{for } a \leq x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & \text{for } c < x \leq b \\ 1 & \text{for } b < x \end{cases}$$

where the CDF ($F(x; a, b, c)$) that is described by (25) is defined by its lower bound (a), upper bound (b), and mode (c). These parameters of the CDF correspond to the values of the user-provided low estimate of a stochastic determinant of total revenues (a), the user-provided high estimate (b), and the user provided likely estimate (c). Since the user provided input dictates the shape of the PDF, the shape of the corresponding CDF is also critically dependent on the values indicated by the user. An analysis of the effect of changes in the user-provided input on the shapes of these distributions is provided in Appendix E.

4.2: GENERATING RANDOM INPUTS

The model generates uniform random numbers (URN)⁸ that corresponds to a point along the CDF. These numbers can be transformed to true random numbers (TRN)⁹ (*i.e.* numbers that are drawn from the PDF), which can then be used in Monte Carlo simulations. Recovering the TRNs from the URNs allows the model to employ generally applicable methods that will generate valid inputs to the Monte Carlo simulation independent of the value of the parameters of the relevant triangular PDF. The generality of this approach is useful for applying these methods to other stochastic inputs or culture species that may be the focus of future research questions. Any URN may be transformed into a TRN as per the following equation (Hesse, 201):

$$(26) \quad X = \begin{cases} a + \sqrt{U(b-a)(c-a)} & \text{for } 0 < U < F(c) \\ b - \sqrt{(1-U)(b-a)(c-a)} & \text{for } F(c) \leq U < 1 \end{cases}$$

⁸ A Uniform Random Number (URN) is a randomly generated number between the bounds of zero and one.

⁹ A True Random Number (TRN) is a randomly generated number between the bounds of any two previously selected rational numbers. For the purposes of this thesis, TRNs are generated between the lower and upper bound estimates of the value of each of the determinants of total revenues.

where a , b , and c are user-provided parameters that determine the shape of the PDF and CDF and U represents a randomly generated URN. For any randomly generated URN, (26) is able to convert it to a TRN by referencing the position of the URN relative to the mode of the CDF ($F(c)$). For URNs with a value less than that of the mode of the CDF, (26) will produce a TRN with a value less than that of the mode of the PDF. The opposite is also true. An example of the usefulness of (26) is presented in Section 4.4.

4.3: MONTE CARLO SIMULATIONS

The repeated generation of random inputs from URNs results in a distribution of TRNs. This distribution of TRNs is useful in creating a distribution of outputs from the model via a Monte Carlo simulation. In the model, the user has the option to generate random relative fecundities, output prices, and minimum mortality rates. For simplicity and in order to develop a *ceteris paribus* approach to analysis, randomization of only one of these stochastic determinants of total revenues may be simulated at any one time. When running a simulation, the model assumes that the value of the stochastic inputs not being randomized is fixed at the value that has been indicated as the likely value by the user. For example, if the user wishes to simulate the affects of changes in minimum mortality rate on the total revenues of their culture system, the values of fecundity, and output price are fixed at their most likely values.

4.4: AN EXAMPLE OF MONTE CARLO ANALYSIS AROUND FECUNDITY

An example helps to illustrate the method by which the PDF is constructed on the basis of user-provided input. The relative fecundity of white crappies is subject to some variance. A study conducted in Pennsylvania's Susquehanna River, for example, found that the relative fecundity of 273 wild-caught white crappies was between 177 and 304 ova per gram of fish (Mathur, McCreight, & Nardacci, 1979).

Correspondingly, estimating the fertility of their pond, the user may choose to input 177 ova per gram as a low estimate for fecundity and 304 ova per gram as a high estimate. The likely estimate may be represented by 241 ova per gram, which is the arithmetic mean of 177 and 304. The values that the user inputs into the model (*e.g.* 177, 241, 304 ova per gram of fish, in this case) serve as the lower limit, mode, and upper limit of the corresponding triangular PDF. This function is represented graphically in Figure 5:

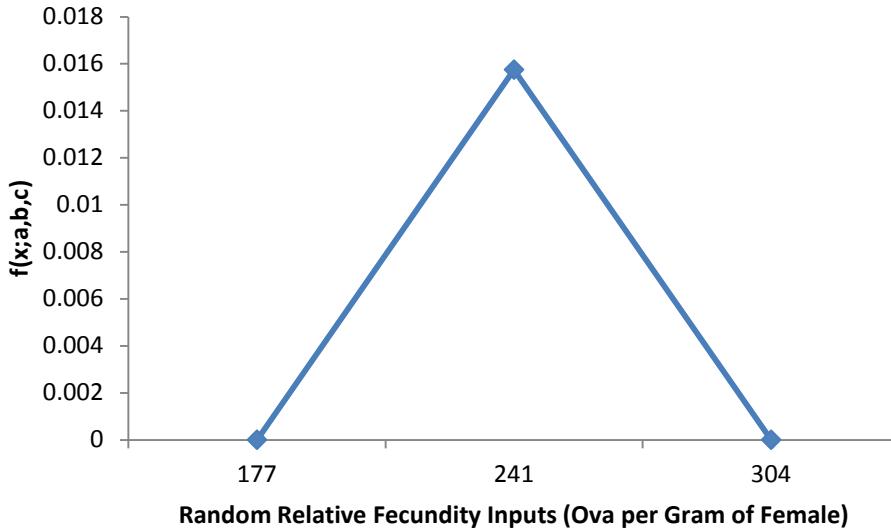


Figure 5: Triangular PDF for fecundity inputs ($a=177$, $b=304$, $c=241$)

In this case, the mode of this PDF corresponds to the midpoint of the CDF since the user has chosen to input the likely value of the distribution as the arithmetic mean of its two extremes. For any URL, the corresponding TRN is generated as follows:

$$(27) \quad X = \begin{cases} 177 + \sqrt{U * 8,320} & \text{for } 0 < U < 0.5 \\ 304 - \sqrt{(1 - U) * 8,320} & \text{for } 0.5 \leq U < 1 \end{cases}$$

A URL below of the midpoint of the CDF (*e.g.* 0.234) generates a TRN that is below the mode of the PDF:

$$(28) \quad X = 177 + \sqrt{0.234 * 8,320} \approx 221 \text{ ova per gram} < 241 \text{ ova per gram}$$

The opposite is also true:

$$(29) \quad X = 177 + \sqrt{0.741 * 8,320} \approx 255 \text{ ova per gram} > 241 \text{ ova per gram}$$

Iterating this process creates a distribution of randomly generated fertility inputs:

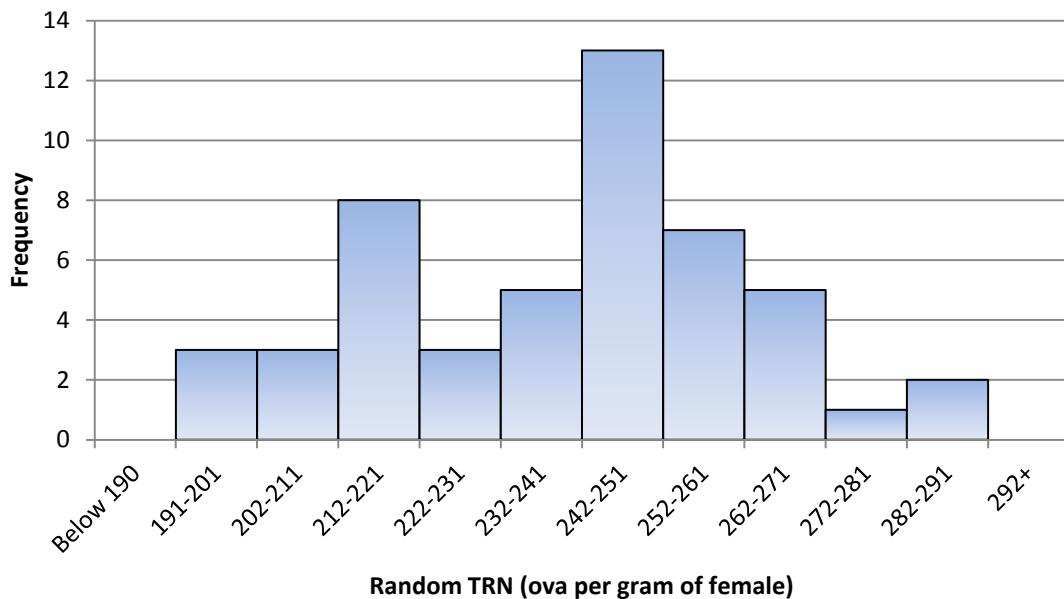


Figure 6: Distribution of fecundity inputs from randomly generated TRNs

The distribution of randomly generated fecundity inputs is useful in creating a distribution of modeling outputs (*i.e.* a distribution of randomly generated total net revenues or outputs of the culture system). This distribution of outputs demonstrates the sensitivity of the model's results to changes in fecundity. An analysis of this information aids the user in analyzing the outcome of the modeling exercise in accordance with their personal risk preferences.

CHAPTER 5: POPULATING THE MODEL

To demonstrate the usefulness of the model, it is necessary to establish scenario for modeling. For the purposes of demonstration, suppose that a firm is endowed with ten ponds available for crappie production. Each pond is assumed to measure one acre-foot in volume and has a surface area of 0.16 acres, and each spawning event is assumed to last 4-6 weeks. Further assume that the length of the spawning season is 8-12 weeks, so that the culturist is able employ all ten ponds twice annually (Egloff, 2011). The culturist is considering two alternative oxygenation technologies. These alternatives are as follows:

- The baseline scenario outlines the outcome of production in ponds to which no modifications have been made. In other words, the baseline scenario assumes that the culturist chooses to fill their ponds with water from their source, add broodstock, and allow the brood to spawn without making any additional changes to their ponds.
- The alternative scenario describes a pond that been treated with a fine pore aerator. It is assumed that the culturist will choose to employ an RA-1 aerator, which is available through Kasco Marine, Inc.

Further development of this scenario requires estimating the value of the determinants of both total revenues and total costs. In order to provide appropriate estimates in the absence of data, the author draws from several sources include phone interviews and e-mail correspondences with Colorado Parks and Wildlife hatchery managers, information provided by Kasco Marine Inc., and a published articles. The estimates for these parameters are established in the subsequent sections.

The remainder of this section is organized as follows: After a summary of the assumptions and estimations that allow all of components of HSI to be valued, the remaining determinants of total

revenues are estimated. Finally, the estimation of the total costs of the alternative oxygenation technology is presented using both present value and annualized accounting methods.

5.1: ESTIMATING THE DETERMINANTS OF TOTAL REVENUES

The estimation of total revenues of the culture system under the baseline scenario requires values for each component of HSI, stocking density, mortality rate, fecundity rate, and output price. Estimates for stocking densities, and temperature and dissolved oxygen (DO) variables are made based on the best available information from Colorado Parks and Wildlife. Estimates for mortality and fecundity rates are made based on the literature, and estimates for output price are made based on the observed market price for crappie fingerlings.

5.1.1: ESTIMATING THE VALUE OF THE COMPONENTS OF HSI

The temperature of the crappie spawning ponds at Colorado Park and Wildlife's Wray Hatchery has been estimated to be between 60-62°F (*i.e.* approximately 15.5-16.6°C) (Egloff, 2011). There are two dissolved oxygen variables in the HSI model. Assuming away any natural oxygenation of the culture pond (*e.g.* wind agitating the surface of the pond), it is assumed that DO levels are homogenous throughout the pond since the determinants of oxygenation (*i.e.* temperature, salinity, and atmospheric pressure) are also assumed to be homogenous throughout the pond. Under the baseline scenario, the pond is assumed to be at forty-nine percent saturation, and under the action alternative, the pond is assumed to be 69-71 percent saturation (Colt, 1984; Creswell, 1991; Kasco Marine Inc., n.d.; Kepenyes & Varadi, n.d.; Parsons & Sylvester, 1992). For a detailed description of the calculations behind these estimates, the reader is referred to Appendix F.

For each of the remaining habitat variables in the HSI model (*i.e.* percent cover, percent littoral area, turbidity, TDS, and pH), it assumed the that lower bound estimates yield an HSI value as close to 0.5 as possible, the likely estimates yield an HSI value as close to 0.85 as possible, and the upper bound estimates yield an HSI of 1.0. For example, the optimal provision of cover for white crappies is between 15-75 percent of the pond bottom (Edwards, Krieger, Gebhart, & Maughan, 1982). As per the above assumption, this scenario will assume that the high estimate of this parameter is forty-five percent cover (*i.e.* the mean of the optimal range). It will further assume that the lower estimate is ten percent cover and the likely estimate is twenty percent cover, because these estimates correspond to HSI values of 0.52 and 0.84, respectively.

Table 2 summarizes the assumed HSI values for both the baseline and alternative scenarios. For a more detailed tabulation of the assumed value of each of the components of HSI, the reader is referred to Appendix G.

TABLE 2: SUMMARY OF THE ASSUMED HSI VALUES

	Assumed Lower Bound of HSI	Assumed Likely Value of HSI	Assumed Upper Bound of HSI
Comprehensive HSI of Baseline Scenario	0.1	0.36	0.73
Comprehensive HSI of Alternative Scenario	0.30	0.36	0.92

5.1.2: ESTIMATING OTHER DETERMINANTS OF TOTAL REVENUES

It is assumed that the output price of white crappies is equal to the market price of crappie fingerlings. In the author's experience, \$0.80, \$1.00, and \$1.40 represent an appropriate lower bound, likely value, and upper bound for this parameter (Beemer Fisheries, 2011; Rainbowhead Farms, 2011; Aquatic Environmental Services, Inc., 2011; Dunn's Fish Farm, 2011).

Regarding stocking density, Colorado Parks and Wildlife stocks about 125 fish per acre of pond, and the average size of a fish stocked is 0.75-1 pound per fish (Egloff, 2011). It is assumed that the mean expected size of fish stocked is assumed to be 0.875 pounds (*i.e.* the mean of 0.75 and 1 pounds), and the sex ratio of the model pond is 1:1. The stocking density of a model pond is able to be estimated as follows:

$$(30) 125 \frac{\text{fish}}{\text{acre}} * 0.16 \frac{\text{acres}}{\text{pond}} = 20 \frac{\text{fish}}{\text{pond}} * 0.875 \frac{\text{pounds}}{\text{fish}} \approx 18 \frac{\text{pounds}}{\text{pond}} \rightarrow 9 \frac{\text{pounds of females}}{\text{pond}}$$

The relative fecundity of the species represents the potential fertility of the pond. it is assumed that the lower bound, likely, and upper bound estimates for the potential fertility of the pond are 177, 241, and 304 ova per gram of female fish, respectively (Mathur, McCreight, & Nardacci, 1979). In order to adjust for the discrepancy between the potential fertility of the pond and the actual fertility of the pond, it is assumed that the actual fertility of the pond is twenty-five percent of the potential fertility of the pond as indicated by the relative fecundity of the species (Clady, 1975).

Estimates for biological outflows at the early stages of life (*i.e.* the mortality of crappies in the egg and larval stages) were arrived at after consulting the literature. An egg mortality rate of seventy-five percent is assumed, and a larval mortality rate of twenty-eight percent is also assumed (Dahlberg, 1979; Pine & Allen, 2001). It's assumed that the low, likely, and high estimates of post-larval mortality take on values of thirty percent, thirty-five percent, and forty percent, respectively (Egloff, 2011).

5.2: ESTIMATION OF TOTAL COSTS

Under the action alternative, the crappie culturist will incur an additional cost associated with aerating the pond¹⁰. The RA-1 aeration unit and all the necessary hardware is available for purchase at a

¹⁰ Kasco recommends the installation of a model RA-1 fine pore aerator for ponds with surface areas less than 1.5 acres (Kasco Marine Inc., n.d.).

cost of approximately \$1,200.00 (Kasco Marine Inc., n.d.). Assuming a constant depreciation schedule, zero resale value, no additional benefits from the purchase of a technology (*e.g.* tax rebates), a discount rate of four percent, and a useful lifespan of five years, the equivalent annualized cost associated with the aerator is estimated below¹¹:

$$(31) EAC = \frac{\$1,200}{A_{5,0.04}} = \frac{\$1,200}{\frac{[(1.04)^5 - 1]}{[0.04(1.04)^5]}} = \$269.55$$

¹¹ Refer to (18) and (19) for an explanation of the estimation of the EAC.

CHAPTER 6: RESULTS AND CONCLUSIONS

The results of the modeling exercise described in Chapter 5 are presented in the following sections. The remainder of this section is organized as follows: After a summary of the model outputs for the baseline (i.e. the culture system without aeration) and alternative (i.e. the culture system with aeration) scenarios resulting from fixing all parameters and determinants of physical output and total net revenues at their most likely value, HSI is allowed to vary across low, likely, and high habitat quality scenarios, and a comparison of the results across the baseline and alternative scenarios for each of the three habitat quality scenarios is made. A detailed explanation of the results and a discussion of their significance are presented to the reader in the subsequent chapter.

6.1: MODEL OUTPUTS USING LIKELY VALUES OF ALL PARAMETERS

This section describes the results of the “likely scenario,” in which the value of all parameters and determinants of physical output and total net revenues are fixed at their likely values. The results of the likely scenario are tabulated below:

TABLE 3: RESULTS OF LIKELY SCENARIO

	Baseline Scenario (without aeration)	Alternative Scenario (with aeration)
HSI (Index Number)	0.36	0.36
Physical Output (Fish for Stocking)	278,680	278,680
Change in Total Net Revenues Relative to the Baseline (Non-annualized Dollars)	\$0	\$ (1,154)
Change in Total Net Revenues Relative to the Baseline (Annualized Dollars)	\$0	\$ (259)

In the case presented in Table 3, the adoption of an RA-1 aeration system has no impact on either the HSI or the physical output of the culture system (see Section 7.2 for more details). The aeration of culture system results in a loss of total net revenues of \$1,154 under the NPV accounting method and \$259 under the annualized accounting method.

6.2: COMPARISON OF MODEL OUTPUTS ACROSS THE BASELINE AND ALTERNATIVE SCENARIOS FOR LOW, LIKELY, AND HIGH HABITAT QUALITY HABITATS

This section compares the model outputs across the baseline and alternative scenarios and reports the changes in total net revenues relative to baseline levels for the low, likely, and high habitat quality scenarios. The value of all determinants and parameters of physical output and total net revenues (except for HSI) have been fixed at their likely values. The model output for both the baseline and alternative scenarios are tabulated in Table 4:

TABLE 4: COMPARISON OF MODEL OUTPUTS ACROSS THE BASELINE AND ALTERNATIVE SCENARIOS FOR THE LOW, LIKELY, AND HIGH HABITAT QUALITY SCENARIOS

	Baseline Scenario (without aeration)			Alternative Scenario (with aeration)		
	Low Estimate	Likely Estimate	High Estimate	Low Estimate	Likely Estimate	High Estimate
HSI (Index Number)	0.30	0.36	0.73	0.30	0.36	0.92
Physical Output (Fish for Stocking)	236,380	278,680	474,120	236,380	278,680	549,540
Percentage Change in Physical Output Relative to Baseline Scenario (%)	-----	-----	-----	0%	0%	16%
Change in Total Net Revenues Relative to the Baseline Scenario (Non-annualized Dollars)	-----	-----	-----	\$(1,154)	\$(1,154)	\$20,602
Total Net Revenues Relative to the Baseline Scenario (Annualized Dollars)	-----	-----	-----	\$(259)	\$(259)	\$21,497

In the case presented in Table 4, the largest marginal change in HSI within the baseline and alternative system occurs across the likely-high habitat quality margin. Comparing across the baseline and alternative scenarios, the overall habitat quality of the system is not impacted by aeration in the case of low and likely quality habitats as the HSI remains unchanged across the baseline and alternative scenarios for these habitat qualities. Aeration has a positive impact on habitat quality across the likely-high habitat quality margin, however, as the HSI does vary across the baseline and alternative scenarios for high quality habitats.

This simulation suggests that the physical output of the culture system (in terms of number of fish for stocking) within the baseline and alternative scenarios increases across each of the habitat quality improvement margins. The largest marginal change in physical output within the baseline and alternative scenarios occurs across the likely-high habitat quality margin. Since the output price is assumed to be unity, the same result holds for total revenues. Comparing across the baseline and alternative scenarios, aeration has no marginal impact on output across the low-likely habitat quality margin. Aeration has a positive impact on output across the likely-high habitat quality scenario. For high quality habitats, the introduction of an aerator results in a 16 percent increase in the physical output of the culture system.

By assumption, the introduction of an aerator increases (decreases) the variable costs of production only in cases where aeration results in an increase (decrease) in physical output. For low and likely quality habitats, aeration does not result in an increase in physical output, and the entirety of the marginal cost of aeration can be attributed to the fixed cost associated with the purchase of the capital good. For high quality habitats, a 16 percent increase in output is assumed to correspond to a 16 percent increase in variable production costs. Therefore, the total marginal cost of aeration under high quality habitats is equal to the sum of increase in variable costs associated with increased output and the increase in fixed costs associated with the optional technological improvement. For both low and likely quality scenarios, aeration results in a loss of total net revenues of \$1,154 under the NPV accounting method and \$259 under the annualized accounting method. For high quality habitats, aeration results in a gain of total

net revenues of \$20,602 under the NPV accounting method and \$21,497 under the annualized accounting method.

6.3: MONTE CARLO SIMULATIONS

The results of sensitivity analyses around two biological parameters (*i.e.* relative fecundity and the minimum post-larval mortality rate of the culture species under optimal conditions) and one economic parameter (*i.e.* output price) have been reported in the subsequent sections. Generally, the aeration of low and likely quality habitat results in a loss of total net revenue regardless of the value of these stochastic inputs; high quality habitats result in a gain of total net revenues, and the size of this gain depends critically on the value of these biological and economic parameters. Comparing across the baseline and alternative scenarios, the results of the fecundity and mortality Monte Carlo simulations suggest that the output of the culture system is sensitive to change in biological parameters only in the case of high quality habitats. For low and likely quality habitats, variations in these parameters result in changes within the baseline and alternative scenarios but not across these scenarios. As a result, the total net revenues of the culture system are always equal to the present value of the fixed cost associated with the aerator regardless of the value of these biological parameters. Similarly, the results of the output price Monte Carlo simulation suggest that a comparison of the total net revenues of the culture system across the baseline and alternative scenarios results in sensitivity to price only for high quality habitats. For low and likely quality habitats, there is no variation in physical output across the baseline and alternative scenarios. As a result, the total net revenues of the culture system are always equal to the present value of the fixed cost associated with the aerator regardless of the value of output price. The results of these Monte Carlo simulations are discussed in detail in Section 7.3, and a discussion of the significance of the results is presented in Section 7.4.

6.3.1: SENSITIVITY OF RESULTS TO CHANGES IN FECUNDITY

The distribution of the random relative fecundity inputs used in the fecundity Monte Carlo simulation was created using estimates that were derived from previous studies (Mathur, McCreight, & Nardacci, 1979). For more details, the reader is referred to Section 5.1.2. This distribution of random fecundity inputs is summarized by the CDF in Figure 7:

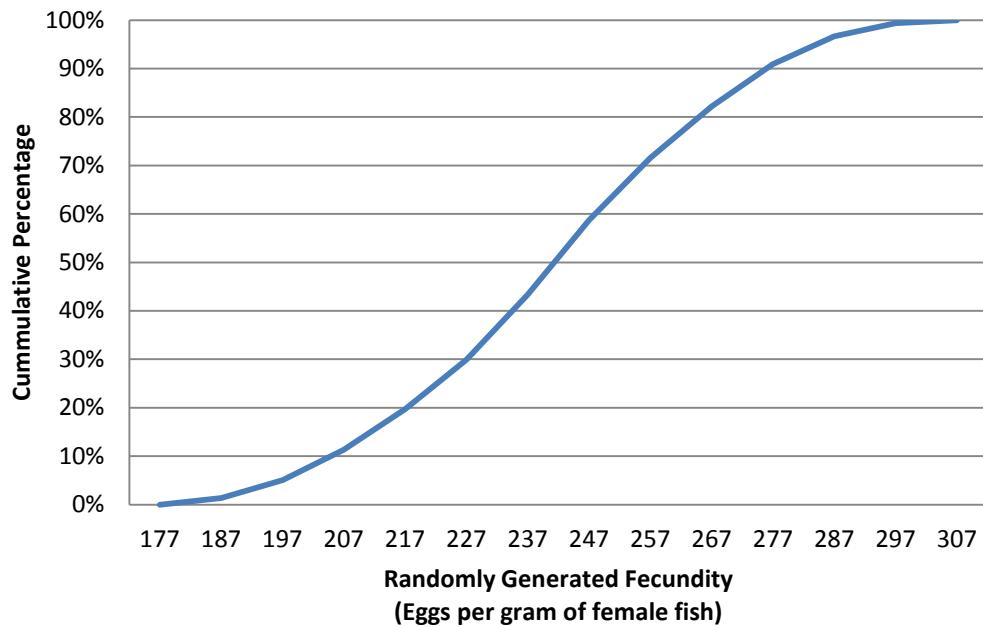


Figure 7: Cumulative Distribution Function of randomly generated relative fecundity inputs (n=10,000)

A tabulation of the summary statistics describing the distributions of resulting from this Monte Carlo simulation is presented in Table 5:

TABLE 5: SUMMARY STATISTICS FOR THE DISTRIBUTIONS OF THE PHYSICAL OUTPUT OF THE CULTURE SYSTEMS RESULTING FROM RANDOM FECUNDITY INPUTS (N=10,000)

High Habitat Quality Scenario						
	Minimum	1 st Quartile	Median	3 rd Quartile	Maximum	Standard Deviation
Baseline Output (number of fish for stocking)	349,689	432,725	474,353	511,390	596,911	51,201
Alternative Output (number of fish for stocking)	405,306	507,344	549,799	592,726	691,849	59,344
Alternative Output (Non-annualized dollars)	14,890	18,929	20,609	22,308	26,232	2,349
Alternative Output (Annualized dollars)	15,784	19,823	21,504	23,203	27,127	2,349

For a given relative fecundity, the physical output of the culture system remains unchanged across the baseline and alternative scenarios in both the low and likely habitat quality scenarios¹². In the case of low and likely quality habitats, aeration of the culture system results in a loss of total net revenues equal to \$1,154 under the NPV accounting method and \$259 under the annualized accounting method. In the high habitat quality scenario, however, the physical output of the culture system does differ across the baseline and alternative scenarios. Similar to results reported in Table 4, aeration results in positive net revenues only in the case of high quality habitats. The size of the gain in total net revenues is critically dependent on the assumed value of the relative fecundity parameter.

¹² The physical output (in terms of number of fish for stocking) within the baseline and alternative scenarios is dependent on the value of the randomly generated fecundity. For low quality habitats the minimum output is 174,366 fingerlings, the median output is 236,488, the maximum value is 297,588, and the standard deviation is 25,526. For likely quality habitats the minimum output is 205,539 fingerlings, the median output is 278,815, the maximum value is 350,851, and the standard deviation is 30,095. For a given value of fecundity, no variation across the two systems exists for low or likely quality habitats.

6.3.2: SENSITIVITY OF RESULTS TO CHANGES IN MORTALITY

The distribution of the randomly generated inputs used in the mortality Monte Carlo simulation was created using input provided by Colorado Parks and Wildlife (Egloff, 2011). The distribution of randomly generated mortality inputs used in this simulation is summarized by the CDF in Figure 8:

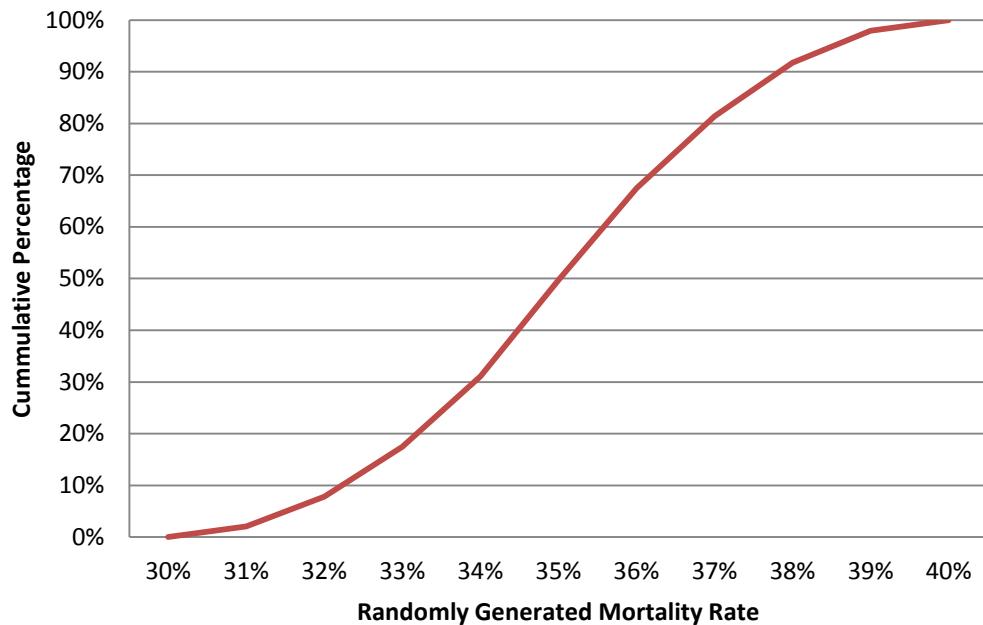


Figure 8: Cumulative Distribution Function of randomly generated relative post-larval mortality inputs (n=10,000)

A tabulation of the summary statistics describing the distributions of resulting from this Monte Carlo simulation is presented in Table 6:

TABLE 6. SUMMARY STATISTICS FOR THE DISTRIBUTIONS OF THE PHYSICAL OUTPUT OF THE CULTURE SYSTEMS RESULTING FROM RANDOM MINIMUM MORTALITY INPUTS (N=10,000)

High Habitat Quality Scenario						
	Minimum	1 st Quartile	Median	3 rd Quartile	Maximum	Standard Deviation
Baseline Output (number of fish for stocking)	432,130	461,418	474,025	486,310	517,179	17,544
Alternative Output (number of fish for stocking)	505,608	536,362	549,431	562,064	593,351	18,107
Alternative Output (Non-annualized dollars)	20,042	20,465	20,598	20,698	20,819	169
Alternative Output (Annualized dollars)	20,936	21,359	21,493	21,593	21,714	169

For a given mortality rate, the physical output of the culture system remains unchanged across the baseline and alternative scenarios in both the low and likely habitat quality scenarios¹³. In the case of low and likely quality habitats, aeration of the culture system results in a loss of total net revenues equal to \$1,154 under the NPV accounting method and \$259 under the annualized accounting method. In the high habitat quality scenario, however, the physical output of the culture system does differ across the baseline and alternative scenarios. Similar to results reported in Table 4, aeration results in positive net revenues only in the case of high quality habitats. The size of the gain in total net revenues is critically dependent on the assumed value of the mortality parameter.

¹³ The physical output (in terms of number of fish for stocking) within the baseline and alternative scenarios is dependent on the value of the randomly generated mortality. For low quality habitats the minimum output is 210,306 fingerlings, the median output is 236,310, the maximum value is 264,799, and the standard deviation is 11,26. For likely quality habitats the minimum output is 248,895 fingerlings, the median output is 278,607, the maximum value is 310,865, and the standard deviation is 12,769. For a given value of fecundity, no variation across the two systems exists for low or likely quality habitats.

6.3.3: SENSITIVITY OF RESULTS TO CHANGES IN OUTPUT PRICE

The distribution of the randomly generated inputs used in the output price Monte Carlo simulation was created using the market price of white crappie fingerlings as advertised by private producers (Beemer Fisheries, 2011; Rainbowhead Farms, 2011; Aquatic Environmental Services, Inc., 2011; Dunn's Fish Farm, 2011). The distribution of random output prices used in this simulation is summarized by the CDF below:

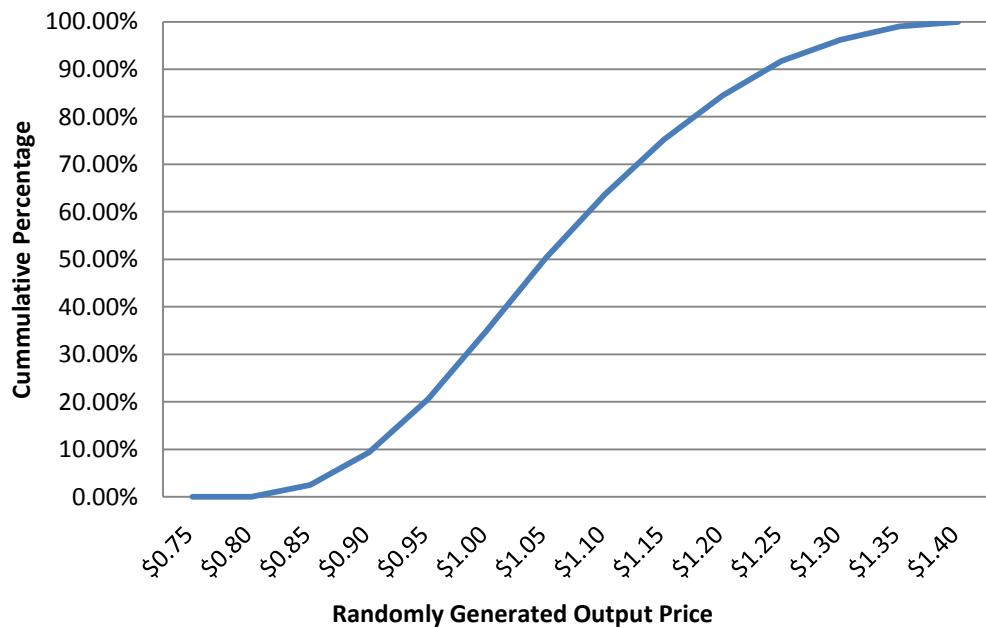


Figure 9: Cumulative Distribution Function of randomly generated output prices (n=10,000)

A tabulation of the summary statistics describing the distributions of resulting from this Monte Carlo simulation is presented in Table 7:

TABLE 7: SUMMARY STATISTICS FOR THE DISTRIBUTIONS OF THE PHYSICAL OUTPUT OF THE CULTURE SYSTEMS RESULTING FROM RANDOM OUTPUT PRICES (N=10,000)

High Habitat Quality Scenario						
	Minimum	1 st Quartile	Median	3 rd Quartile	Maximum	Standard Deviation
Baseline Output (number of fish for stocking)	474,125	-----	-----	-----	-----	0
Alternative Output (number of fish for stocking)	549,534	-----	-----	-----	-----	0
Alternative Output (Non-annualized dollars)	16,343	20,001	21,767	23,951	29,165	2,714
Alternative Output (Annualized dollars)	17,238	20,895	22,662	24,846	30,059	2,714

Similar to the results presented in Table 4, the aeration of low and likely quality habitat results in a loss of total net revenues. In the case of low and likely quality habitats, aeration of the culture system results in a loss of total net revenues equal to \$1,154 under the NPV accounting method and \$259 under the annualized accounting method. The aeration of high quality habitats results in a gain of total net revenues, which is critically dependent on the value of output price and varies across accounting methods. The physical output of the culture system within the baseline and alternative scenarios is independent of the value of output price and varies across the baseline and alternative scenarios only in the case of high quality habitats¹⁴.

¹⁴ For low quality habitats, the physical output of the culture system is 236,374 fingerlings for both the baseline and alternative scenarios. For likely quality habitats, the physical output of the system is 278,680 fingerlings for both scenarios.

CHAPTER 7: CONCLUSIONS AND DISCUSSIONS

7.1: SUMMARY OF RESULTS

Comparing across the baseline and alternative scenarios, the results of this simulation suggest that the mechanical aeration of culture systems with low and likely quality habitats will result in a loss of total net revenues. The simulation also suggests that aeration does not increase the physical output of the culture system (*i.e.* the output in terms of number of fish for stocking) for low and likely quality habitats. For high quality habitats, aeration results in positive total net revenues and increased physical output. The magnitude of the potential gain in total net revenues depends on the assumed value of key biological and economic parameters.

7.2 THE IMPACTS OF TEMPERATURE-RELATED HABITAT PARAMETERS ON HABITAT QUALITY AND MODEL OUTPUTS

The results presented in Chapter 6 hold in large part due to the binding nature of the temperature-related habitat parameters. In the case of the simulation results presented in Table 4, the HSI remains unchanged across the baseline and alternative scenarios for low and likely quality habitats, because the estimates for the both the temperature of the littoral areas (*i.e.* V_4) and the temperature of the epilimnion (*i.e.* V_5) act as binding constraints to the overall habitat quality of the system. As a direct result of the lack of variation in HSI across the baseline and alternative scenarios for low and likely quality habitats, there is no variation in the physical output of the culture system across the baseline and alternative scenarios despite the addition of mechanical aeration under the alternative scenario. As a consequence, the variable costs associated with white crappie production do not change across these scenarios, and the marginal cost of aeration is exactly equal to the fixed cost associated with the purchase of the aerator. Correspondingly, the loss in total net revenues relative to baseline levels under these habitat quality

scenarios is exactly equal to present value of the fixed cost associated with the aerator. Under the low and likely habitat quality scenarios, the losses in total net revenues resulting from aeration are minimized under the annualized accounting method, because the fixed cost associated with aeration is spread out over the useful life of the aerator under this accounting method.

In the case of high quality habitats, the sharp marginal increase in HSI may be attributed to the following two impacts:

- First, the improved suitability of most of the habitat quality parameters (*i.e.* all parameters except for the oxygen-related parameters, which are held constant within the baseline and alternative scenarios in order to develop a *ceteris paribus* approach) is cause for an increase in the overall HSI within each system.
- Second, the temperature-related parameters that had acted as binding constraints to HSI under the low and likely quality scenarios are improved to the point that they are no longer binding factors to overall habitat suitability. As a consequence, the HSI under the high habitat quality scenario is calculated by interacting all of the habitat parameters.

For high quality habitats, the HSI will vary across the baseline and alternative scenarios, because the oxygen-related parameters are relatively more suitable under the alternative scenario (*i.e.* the system with mechanical aeration) than the baseline scenario (*i.e.* the system without mechanical aeration). This variation across the baseline and alternative scenarios is cause of a 16 percent increase in the physical output of the culture system. As a direct result of the increased output of the culture system, the variable costs of production under the alternative scenario are also increased by 16 percent. The increase in fixed costs across the baseline and alternative scenario is equal to the purchase price associated with the aerator. In this case, the marginal change in total revenues associated with increased production due to aeration outweighs the marginal increase in total costs, and total net revenues under the alternative scenario are positive for high quality habitats. The gain in total net revenues is maximized under the annualized

accounting method, as the fixed costs associated with aeration are spread out over the useful life of the aerator under this accounting method.

7.3: DISCUSSION OF SENSITIVITY ANALYSES

The results of the Monte Carlo simulations presented in Sections 6.1.1 through 6.1.3 suggest that the sensitivity of the model outputs to changes in key parameters depends on the habitat quality scenario being modeled. For low and likely quality habitats, the physical output of a culture system does not change across the baseline and alternative scenarios, because the HSI of the culture system remains unchanged across these two scenarios (see Section 7.2 for more details). In this case, changes in biological parameters have the effect of scaling the physical output up or down within the baseline and alternative scenario, but output remains unchanged across the two scenarios. As a result, the loss in total net revenues of the culture system is exactly equal to present value of the fixed cost associated with the aerator regardless of the assumed value of the stochastic biological parameter. For high quality habitats, aeration results in a change in HSI across the baseline and alternative systems. As a result, variations in biological parameters have the effect of impacting physical output both within and across the baseline and alternative scenarios. In this case, the gain in total net revenues associated with aeration is critically dependent on the assumed value of the biological parameters¹⁵. The sensitivity of the results to changes in output price is also dependent on the habitat quality scenario. For low and likely quality habitats, the only factor impacting changes in total net revenues across the baseline and alternative systems is the purchase of the aerator under the alternative scenario. Since all other determinants of total net revenue (*i.e.* output and price) remain unchanged across the two scenarios, the loss in total net revenues of the culture system is exactly equal to present value of the fixed cost associated with the aerator regardless of the assumed value of output price. For high quality habitats, the physical output of the culture system varies across the

¹⁵ For a discussion of the comparative statics associated with changes in key biological parameters, the reader is referred to Appendices C and D.

baseline and alternative systems. In this case, the change in output price scales the total net revenues of the system up or down within the baseline and alternative scenarios. The difference in total net revenues across the baseline and alternative scenarios, however, is always equal to \$895, which represents the difference in the value of the aerator as calculated by the different accounting methods (*i.e.* the savings in the fixed cost of the aerator associated with being able to spread this cost over the useful life of the capital good). By assumption, the physical output of the culture system within the baseline and alternative scenarios is independent of output price.

7.4: CONTRIBUTIONS OF THE MODEL AND SIGNIFICANCE OF THE RESULTS

The decision support tool developed in this thesis is designed to aid the user in the analysis of business planning decisions and contingency scenarios. Although this thesis uses mechanical aeration as a case study for illustrative purposes, the usefulness of the model extends far beyond the analysis of this particular technology adoption decision. Generally, the model is useful under varying biological or habitat quality conditions for the analysis of any capital improvement decision, technology adoption decision, or contingency scenario. In the same sense, the general framework developed in this thesis may be adapted to model the culture of any species for which an HSI model has been developed. Other useful applications for business planning analyses include, but are not limited to the adoption of pond liners, buffering systems, filtration systems, chemical water treatment, pond fertilization, and disease treatment and prevention technologies. Other useful contingency planning analyses including, but are not limited to changes to habitat quality or biological conditions due to critical equipment failure, compromised quality of the water source, inclement weather, and disease. Other useful species for analysis include, but are not limited to Channel Catfish (*Ictalurus punctatus*), Common Carp (*Cyprinus carpio*), Rainbow Trout (*Oncorhynchus mykiss*), Smallmouth Bass (*Micropterus dolomieu*), Walleye (*Sander vitreus*), Yellow Perch (*Perca flavescens*), Northern Pike (*Esox lucius*), Arctic Grayling (*Thymallus arcticus*), and Bluegill (*Lepomis macrochirus*).

In real world applications, culturists experience heterogeneity in habitat quality endowments. This heterogeneity may be a result of several contributing factors including, for example, differences in the suitability of their water source, variations in naturally occurring weather patterns that may impact the suitability of their culture habitat, dissimilarities in the characteristics of the interaction between their soils and their pond, etc. Culturists may also experience heterogeneity in information endowments in that some may have better access to or knowledge of the exact value of the biological determinants of the output of a culture system or the economic determinants of total net revenues.

Differences in the model outcomes across the low, likely, and high habitat quality scenarios illustrate the impact of heterogeneity in habitat quality endowments across different culture systems. In the case of the aeration simulation, the net benefit associated with the aerator is negative and remains unchanged across the low-likely habitat quality margin. As the habitat endowment is improved from likely to high quality, however, the simulation produces a positive change in the marginal net benefit. In short, the marginal impact of habitat quality improvements is non-constant, and the net impact of technology adoption or capital improvements will depend critically on the habitat quality endowment of the culture system in question.

Comparing the variances of model outcomes across modeling scenarios (i.e. a comparison of the change in the variances of the results across Tables 3 and 4) provides an illustration of the impact of heterogeneity in information. The case of a perfect information endowment is illustrated by the “likely scenario,” which is summarized in Table 3. Here it is assumed that the value of all parameters and determinants of the model outcomes are fixed at their most likely value. As a result of assuming that all parameters are fixed at their most likely value, there is no variance in the model outcomes of the scenario. The assumption of a perfect information endowment is repealed in Table 4, which allows HSI to vary across three potential values. In this case, uncertainty around the exact value of HSI results in three potential values for each model outcome, and the imperfect information endowment results in a larger variance in model outcomes. In short, the efficiency of the model outcomes as estimators is critically

dependent on the information endowment of the user. Information endowments may vary significantly from user to user, and in many cases, the estimation of these parameters for modeling purposes may involve uncertainty. The model provides tools that help the user address this uncertainty by using the following two approaches:

- First, to obtain a point estimate of the impacts of changes in each of these parameters, the user may manually change the value of these determinants and rerun the simulation. This approach provides estimates for model outcomes (both in terms of dollars of total net revenues and well as number of fish for stocking) under plausible biological or habitat scenarios (e.g. worst case and best case fecundity scenarios). These plausible outcomes are useful in that they aid the user in decision making in accordance with their own risk preferences and culture objectives.
- Second, users also have the option to make use of the Monte Carlo simulation tools that have been built into the model. These tools may be used to generate a distribution of potential outcomes as a function of randomly generated biological and economic parameters, which are determined on the basis of user-provided input. The resulting distribution provides a more detailed sensitivity analysis in that it generates probability information in addition to point estimate value information. This information may be useful in expected value types of questions that the user may have.

7.5: LIMITATIONS OF THE MODEL AND AREAS FOR FUTURE RESEARCH

The modeling framework developed in this thesis is a simplification of a complex biological process. As such, the model and resulting analyses are subject to several important limitations. First, the value of many of the parameters used to estimate the habitat quality, physical output, and total net revenues of the culture systems were estimated on the basis of existing literature and the best judgment of the author. In order to more accurately simulate white crappie production, data detailing the exact value of

these parameters in the culture system of interest is needed for regression analysis. The model assumes that the habitat quality of the culture system is unaffected by exogenous factors (*e.g.* natural aeration by wind waves or seasonal fluctuations in temperature) or any impacts that changes in habitat quality parameters may have on other organisms living in a culture pond. The incorporation of quantitative study outlining the expected impact that these factors may have on HSI would aid in the further development of the model. In the model, it is assumed that the output price received for white crappies is equal to the market value of fingerlings as charged by private producers. It is further assumed that the value received for crappies is unchanged throughout the course of culture season. Future research projects will benefit from WTP information that may be used to create a better proxy for the true social value of white crappie populations in local waters. Likewise, an understanding of how this WTP may change in response to factors that may impact the relative scarcity of the species in the wild (*e.g.* predation, availability of food, disease, pollution, *etc.*) is also useful information. In the model, oxygen-related and pH-related habitat parameters use piecewise functions to assess the overall suitability of habitat characteristics. As a result, there is no additional marginal benefit from habitat quality improvements within each of these blocks. For example, an oxygen-related parameter that assumes a “category 1” value corresponds to culture systems with oxygen saturation concentrations of 50 percent or more. In this case, improving the saturation concentration of oxygen from a base value that is above 50 percent saturation (*e.g.* 60 percent) to an alternative value that is also above 50 percent (*e.g.* 75 percent) yields no additional benefits as these oxygen-related variables are both considered to yield category 1 values. Future research efforts may benefit from the continued refinement of the curves that determine the value of HSI. This is particularly true of habitat parameters that make use of piecewise HSI functions. Uncertainty surrounding the true value of the HSI of a culture system further limits the model. Since the HSI of a culture system is a function of several different habitat quality parameters, obtaining an accurate estimation of the true value of HSI involves a heavy burden of information. Monte Carlo simulation tools have been used to address the stochasticity of other uncertain parameters in the model. However, the development a similar tool for HSI requires that the value of each of the randomly selected habitat quality parameters be held

constant across each pond in order to develop a *ceteris paribus* approach. Future HSI-based simulation models may incorporate Monte Carlo tools around habitat quality by building distributions around each habitat quality parameter. Finally, a logical extension of this research is the development of similar HSI-based simulation models that may be useful in the culture of other species.

WORKS CITED

- Aquatic Environmental Services, Inc. (2011). *Aquatic Environmental Services, Inc.: Black Crappie (Pomoxis nigromaculatus)*. Retrieved 2011, from <http://lakework.com/site/black-crappie.php>
- Ardjosoediro, I., & Ramnarine, I. W. (2002). The Influence of Turbitidy on Growth, Feed Conversion, and Survivorship of the Jamaica Red Tilapia Strain. *Aquaculture* , 159-165.
- Barrington, K., Ridler, N., Chopin, T., Robinson, S., & Robinson, B. (2008). Social aspects of the sustainability of integrated multi-trophic aquaculture. *Aquaculture International (published online on 13 December 2008: doi:10.1007/s10499-008-9236-0)* .
- Beemer Fisheries. (2011). *Beemer Fisheries: Products*. Retrieved 2011, from <http://www.beemerp fisheries.com/products.htm>
- Brooks, R. P. (1997). Improving Habitat Suitability Index Models. *Wildlife Society Bulletin, Vol. 25, No. 1* , 163-167.
- Clady, M. (1975). Early Survival and Recruitment of Smallmouth Bass in Northern Michigan. *Journal of Wildlife Management* , 194-200.
- Colt, J. (1984). Computations of dissolved gas in concentrations in water as functions of temperature, salinity, and pressure. *American Fisheries Society, Bethesda, MD*.
- Cooke, S. J., Grant, E. C., Schreer, J. F., Philipp, D. P., & Devries, A. L. (2003). Low temperature cardiac response to exhaustive exercise in fish with different levels of winter quiescence. *Comparative Biochemistry and Physiology Part A* , 157-165.
- Creswell, L. (1991). Aquaculture Desk Reference, Table 4-33: Typical Standardized Aerator Efficiency (SAE) and Equivalent Aeration Depths for Aerators used in Aquaculture. *Provided by Kenneth Rust of Kasco Marine Inc. via E-mail to the author on 12 December, 2011* .
- Dahlberg, M. (1979). A Review of Survival Rates of Fish Eggs and Larvae in Relation to Impact Assessments. *Marine Fisheries Review* , 1-12.
- Deisenroth, D., & Bond, C. A. (2010). The Economic Contribution of the Private, Recreation-Based Aquaculture Industry in the Western United States. . *Ft Collins, CO: Colorado State University*. Available at: <http://dare.colostate.edu/tools/aquaculture.aspx>.
- Dunning, R. D., Losordo, T. M., & Hobbs, A. O. (1998). The Economics of Recirculating Tank Systems: A Spreadsheet for Individual Analysis. *Souther Regional Aquaculture Center* .
- Dunn's Fish Farm. (2011). *Dun's Fish Farm Fingerling Fish*. Retrieved 2011, from http://www.dunnsfishfarm.com/fingerling_fish_59_ctg.htm
- Edwards, E. A., Krieger, D. A., Gebhart, G., & Maughan, O. E. (1982). *HABITAT SUITABILITY INDEX MODELS: WHITE CRAPPIE report FWS/OBS-82/10.7*. Washington, D.C.: Western Energy and Land Use Team of the Office of Biological Services, U.S. Fish and Wildlife Service, U.S. Department of the Interior.
- Egloff, B. (2011). Telephone Interview with author, August 24, 2011.
- Harris, D. (2011). Crappie Culture Research. E-mail to the author. 23 June, 2011.

Hart, P. R., Hutchinson, W. G., & Purser, G. J. (1996). Effects of Photoperiod, Temperature, and Salinity on Hatchery-Reared Larvae of the Greenback Flounder (*Rhombosolea tapirina* Gunther, 1862). *Aquaculture*, 303-311.

Hesse, R. (2000). Triangular Distribution: Mathematica Link for Excel. *Decision Line*, 12-14.

Hesse, R. (2011). Personal correspondence with author via email, August 02, 2011.

C., Urea, E., & Sanz, A. (1999). Comparative Study of Digestive Enzymes in Fish with Different Nutritional Habits. Proteolytic and Amylase Activities. *Aquaculture*, 267-283.

Ishiguro, E. E., Kay, W. W., Ainsworth, T., Chamberlain, J. B., Austen, R. A., Buckley, J., et al. (1981). Loss of Virulence during Culture of *Aeromonas salmonicida* at High Temperature. *Journal of Bacteriology*, 333-340.

Kasco Marine Inc. (2011). *Electrical Cost Calculator*. Retrieved 2011, from www.kascomarine.com/electrical_calculator.html

Kasco Marine Inc. (n.d.). *Kasco Aeration: Robust-Aire Systems 1,2,&3*. Retrieved 2011, from http://www.kascomarine.com/pdfs/robust_specs.pdf

Kepenyes, J., & Varadi, L. (n.d.). *Chapter 21: Aeration and Oxygenation in Aquaculture*. Retrieved 2011, from FAO Corporate Documentary Repository: <http://www.fao.org/docrep/X5744E/x5744e0m.htm>

Kramer, D. L. (1987). Dissolved oxygen and fish behavior. *Environmental Biology of Fishes*, 81-92.

Liebig, Justus, Freiherr vonn (1972). The natural laws of husbandry. Arno Press. New York.

Mathur, D., McCreight, P. L., & Nardacci, G. A. (1979). Variations in Fecundity of White Crappie in Conowingo Pond, Pennsylvania. *Transactions of the American Fisheries Society*, 548-554.

McCloskey, J. T., & Oris, J. T. (1991). Effect of water temperature and dissolved oxygen concentration on the photo-induced toxicity of anthracene to juvenile bluegill sunfish (*Lepomis machrochirus*). *Aquatic Toxicology*, 145-156.

Nicholson, W., & Christopher, M. (2008). *Microeconomic theory: basic principles and extentions*. South-Western Publishers.

Nobre, A. M., & et al. (2010). Ecological-economic assessment of aquaculture options: Comparison between abalone monoculture and integrated multi-trophic aquaculture of abalone and seaweeds. *Aquaculture*, 116-126.

Parsons, G. R., & Sylvester, J. L. (1992). Swimming Efficiency of the White Crappie, *Pomoxis annularis*. *Copeia*, 1033-1038.

Pine , W., & Allen, M. (2001). Differential Growth and Survival of Weekly Age-0 Black Crappie Cohorts in a Florida Lake. *Transactions of the American Fisheries Society*, 80-91.

Pope, K. L., & Willis, D. W. (1997). Environmental characteristics of black crappie (*Pomoxis nigromaculatus*) nesting sites in two South Dakota waters. *Ecology of Freshwater Fish*, 183-189.

Rainbowhead Farms. (2011). *Rainbowhead Farms: Crappie*. Retrieved 2011, from <http://www.rainbowheadfarms.com/crappie.html>

Ridler, N., Wowchuck, M., Robinson, B., Barrinton, K., Chopin, T., Robinson, S., et al. (2007). Integrated Multi-trophic Aqauculture (IMTA): A Potential Strategic Choice for Farmers. *Aquaculture Economics & Management*, 99-110.

Soniat, T. M., & Brody, M. S. (1988). Field Validvation of a Habitat Suitability Index Model for the American Oyster. *Estuaries and Coasts*, 87-95.

Suski, C. D., Killen, S. S., Kieffer, J. D., & Tufts, B. L. (2006). The influence of environmental temperature and oxygen concentration on the recovery of largemouth bass from exercise: implications for live-release angling tournaments. *Journal of Fish Biology*, 120-136.

Swanson, R. (2011). Crappie Culture Research. E-mail to the author. 24 June, 2011.

Thomasma, L. E., Drummer, T. D., & Peterson, R. O. (1991). Testing the Habitat Suitability Index Model for the Fisher. *Wildlife Soceity Bulletin*, 291-297.

Thorarensen, H., Gustavsson, A., Mallya, Y., Gunnarsson, S., Arnason, J., Arnason, I., et al. (2010). The Effect of Oxygen Saturation on the Growth and Feed Conversion of Atlantic Halibut (*Hippoglossus hippoglossus* L.). *Aquaculture*, 96-102.

United States Geological Survey. (2011). *National Wetlands Research Center*. Retrieved November 25, 2011, from USGS National Wetlands Research Center Digital Library: Habitat Suitability Index Models: <http://www.nwrc.usgs.gov/wdb/pub/hs1/hs1intro.htm>

Vincenzi, S., Caramori, G., Rossi, R., & De Leo, G. A. (2007). A Comparative Analysis of Three Habitat Suitability Models for Commrcial Yield Estimation of *Tapes philippinarum* in a North Adriatic Coastal Lagoon (Sacca di Goro, Italy). *Marine Pollution Bulletin*, 579-590.

Vincenzi, S., Caramori, G., Rossi, R., & De Leo, G. A. (2006). A GIS-based habitat suitability model for commercial yield of estimatino of *Tapes philippinarum* in a Mediterranean coastal lagoon (Sacca di Goro,Italy). *Ecological Modeling*, 90-104.

Walker, D., & Kumaranayake, L. (2002). Allowing for differential timing in cost analyses: discounting and annualization. *Health Policy and Planning*, 112-118.

Wesche, T. A., Goertler, C. M., & Hubert, W. A. (1987). Modified Habitat Suitability Index Model for Brown Trout in Southeastern Wyoming. *North American Journal of Fisheries Management*, 232-237.

Whitmarsh, D. J., Cook, E. J., & Black, K. D. (2006). Searching for sustainability in aquaculture: An investigation into the economic prospects for an integrated salmon-mussel production system. *Marine Policy*, 293-298.

APPENDIX A: STRUCTURE OF HSI MODEL

The Habitat Suitability Index has several component parts. For white crappies (*Poxomis annularis*), the lacustrine HSI model includes nine habitat quality variables (Edwards, Krieger, Gebhart, & Maughan, 1982). These variables are as follows:

- Percent cover (vegetation, brush, debris, standing timber, etc.) during summer within riverine pools and lacustrine littoral areas (V_1)
- Percent littoral area (V_2)
- pH range during year (V_3)
- Average water temperature within epilimnion during midsummer (July-August) (V_4)
- Average water temperature within littoral areas during midsummer (V_5)
- Minimum dissolved oxygen levels during midsummer (V_6)
- Dissolved oxygen levels within littoral areas during spawning (March to July) (V_7)
- Maximum monthly average turbidity during summer (May to August) (V_8)
- Average TDS during midsummer (V_9)

The value of each of the habitat quality variables is measured in terms of their respective units (*e.g.* temperature parameters are measured in degrees Celsius). The model uses a variable-specific suitability function (VSSF) to transform these units into a zero-to-one HSI number. In its most general form, a VSSF may be represented by a function of the habitat variables:

$$(32) \quad VSSF_i = g_i(V_i) \quad \forall i = [1,9]$$

where the VSSF for the i^{th} variable ($VSSF_i$) is a function of that variable itself (V_i). The VSSF representing the average temperature of the epilimnion or back waters during midsummer (*i.e.* $g_4(V_4)$) is pictured below:

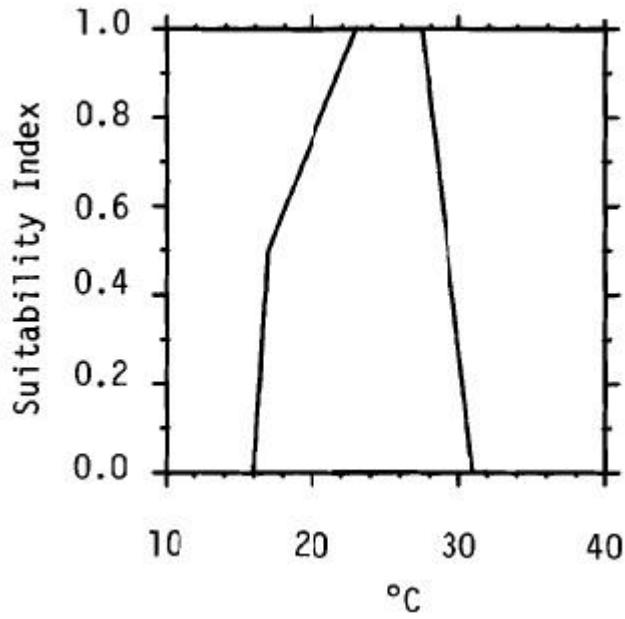


Figure 10: Example of a Variable-Specific Suitability Function

Figure 10 illustrates a VSSF. In the case of V_4 , the independent variable of the VSSF is measured in terms of degrees Celsius. The VSSF outputs a HSI number between zero and one that indicates the suitability of that particular habitat variable. In addition to the VSSF pictured above, there are eight other VSSFs in the white crappie HSI model. Each of these VSSFs corresponds to one of the eight remaining habitat variables in the model. The shapes of each of these VSSFs are unique. Changes in the shapes of the VSSFs reflect the fact that the optimal range of suitability of each habitat variable varies on a case-to-case basis.

The model aggregates VSSFs into groups corresponding to four different life requisites. For white crappies, these life requisites are as follows:

- Food Life Requisite= $C_F = f(g_9(V_9))$
- Cover Life Requisite = $C_C = f(g_1(V_1), g_2(V_2))$
- Water Quality Life Requisite = $C_{WQ} = f(g_3(V_3), g_4(V_4), g_5(V_5), g_6(V_6), g_8(V_8))$
- Reproduction Life Requisite = $C_R = f(g_1(V_1), g_2(V_2), g_5(V_5), g_7(V_7))$

The HSI resulting from a given VSSF is transformed into a life requisite index number via a life requisite function (LRF). Since the life requisite index numbers (LRIs) address the suitability of categorical habitat variables (*e.g.* the provision of food or cover), the LRFs may reference the output of more than one VSSF. For example, the water quality life requisite index is a function of the suitability of the temperature, dissolved oxygen, pH, and turbidity variables. The equation describing the water quality LRI is as follows:

$$(33) \quad C_{WQ} = f(g(*)) = \frac{g_3(V_3) + 2[(g_4(V_4) \times g_5(V_5))] + 2g_6(V_6) + g_8(V_8)}{6}$$

where the water quality LRI (C_{WQ}) is a function of several VSSFs (*i.e.* $C_{WQ} = f(g_i(V_i)) \forall i \in (3,4,5,6,8)$). If $(g_4(V_4) \times g_5(V_5))^{\frac{1}{2}} \leq 0.4$ or $g_6(V_6) \leq 0.4$ then (33) assumes a value equal to the minimum of $(g_4(V_4) \times g_5(V_5))^{\frac{1}{2}}$, $g_6(V_6)$, or (33). In the case where there are no binding constraints to the suitability of water quality, (33) interacts each of the habitat variables that affect water quality (*i.e.* pH, temperature, oxygen and turbidity) in order to produce an overall indication of the suitability water quality. Simply put, when temperature or oxygen are relatively unsuitable, (33) defaults to the minimum value of the most binding habitat variable. The method of interacting the terms in (33) has been developed by the USGS, and is intended to create an index that has a “positive relationship to the carrying capacity” of a habitat on the basis of the findings of previous studies (Edwards, Krieger, Gebhart, & Maughan, 1982).

Finally, these four LRIs are used as inputs into a single HSI, the equation for which is pictured below:

$$(34) \quad HSI = z(*) = (C_F \times C_C \times C_{WQ}^2 \times C_R^2)^{\frac{1}{6}}$$

where C_i represents the i^{th} LRI. If $C_{WQ} \leq 0.4$ or $C_R \leq 0.4$, (34) assumes a value equal to the minimum of C_{WQ} , C_R , or (34). Similar to (33), the overall suitability of a habitat is calculated by interacting all of the LRIs if the suitability of the water quality or reproduction requisites do not act as a binding constraint. Alternatively, if the suitability of the water quality or reproduction requisites do act as a binding constraints, (34) assumes the minimum value of the most binding factor.

APPENDIX B: APPLICATION OF HSI AND ADJUSTED MORTALITY RATE

In order to illustrate the application of HSI and adjusted mortality rate, it is useful to use an example.

Consider two ponds with the following characteristics:

- Pond A has been filled with water from the culturist's source, which has suboptimal pH characteristics. No additional changes have been made to this pond.
- Pond B has been filled with the same water. Additionally, it has been treated with a buffering system in order to maintain an appropriate pH level.
- The water quality variables for these two hypothetical ponds are tabulated in Table 8.

TABLE 8: SUMMARY OF WATER QUALITY VARIABLES IN HYPOTHETICAL PONDS

Habitat Variable	Pond A		Pond B	
	(Habitat Variable)	$g_i(V_i)$	(Habitat Variable)	$g_i(V_i)$
Percent Cover (%)	50	1.0	50	1.0
Percent Littoral Area (%)	50	1.0	50	1.0
pH	Category 3	0.1	Category 1	1.0
Average Temperature of Epilimnion (°C)	25	1.0	25	1.0
Average Temperature of Littoral Area (°C)	25	1.0	25	1.0
Minimum Dissolved Oxygen During Midsummer	Category 1	1.0	Category 1	1.0
Minimum Dissolved Oxygen in Littoral Area During Spawning	Category 1	1.0	Category 1	1.0
Minimum Average Turbidity (JTU)	25	1.0	25	1.0
Average Total Dissolved Solids During Midsummer (ppm)	225	1.0	225	1.0

Note that the pH and dissolved oxygen variables warrant categorical responses. These responses, as defined by the DOI are as follows:

- For pH:
 - Category 1 corresponds to a pH that is “stable and ranges between 6.5-8.5.”
 - Category 2 is reserved for pH levels that “never go below 5.5 or above 9.0, with moderate fluctuations.”
 - Category 3 represents pH levels that are “frequently less than 5.5 or greater than 9.0, with great fluctuations.”
- For minimum dissolved oxygen during midsummer¹⁶:
 - Category 1 corresponds to levels that are “always above 5 mg/l.”
 - Category 2 is reserved for levels that are “usually above 3 mg/l but below 5 mg/l.”
 - Category 3 represents levels that are “frequently below 3 mg/l.”
- The interpretation of the categorical responses for minimum dissolved oxygen in littoral area during spawning are the same as those indicated above for the minimum dissolved oxygen during midsummer.

In this case, the two ponds are assumed to be identical in every regard, except for the expected characteristics of pH. Recall that the expected difference in this variable across the two ponds may be attributed to the installation of a buffering system in pond B.

¹⁶ The saturation concentration of oxygen in fresh water under one atmosphere of pressure and at 16°C is about 10 mg/l (Colt, 1984). Correspondingly, the interpretation of the categorical responses for the oxygen variables in the HSI model is taken to be as follows:

- Category 1 corresponds to levels that are always above 50% saturation
- Category 2 represents levels that are usually above 33% saturation but below 50% saturation
- Category 3 is reserved for levels that are frequently below 33% saturation

The output of the VSSF for pH takes on a much lower value under pond A, which is indicative of less suitable pH characteristics of this pond. A summary of the LRI and HSI values assumed both systems follows:

TABLE 9: SUMMARY OF HYPOTHETICAL LRI AND HSI VALUES

Life Requisite Variable	Pond A	Pond B
	$f(g(*))$	$f(g(*))$
Food	1.0	1.0
Cover	1.0	1.0
Water Quality	0.85	1.0
Reproduction	1.0	1.0

Comprehensive HSI	Pond A	Pond B
	$z(f(*))$	$z(f(*))$
Comprehensive HSI	0.95	1.0

The installation of a buffering system in pond B has impacted the water quality LRI. As a consequence, the HSI varies across systems, with pond A being a less suitable habitat for crappie culture than pond B. The differences in habitat quality between the two systems can be used to calculate the habitat adjusted post-larval mortality rate associated with each system. These adjusted mortality rates are calculated below:

$$(35) \quad \varphi_A = 0.30^{1*0.95} \approx 32\% \text{ mortality}$$

$$(36) \quad \varphi_B = 0.30^{1*1.0} = 30\% \text{ mortality.}$$

where the habitat adjusted mortality rate of the i^{th} system (φ_i) as defined by (6) is a function of the minimum mortality rate of the culture species under optimal conditions (m_3), the habitat adjusted mortality rate constant (c_2), and the HSI of the i^{th} system (HSI_i). The increased adjusted mortality rate in system A reflects the fact that the pH characteristics of pond A are less suitable for crappie spawning than they are in pond B. These adjusted mortality rates may be used to estimate the expected output, total revenues, and total net revenues of each system. Further research and the use of data detailing the observed relationship between HSI and φ_i is required to verify that the relationship defined in (35) and (36) actually holds. However, since the mathematical properties of the assumed relationship between HSI and φ_i are known, the consequences of erroneously estimating the determinants of φ_i can be discussed. A summary of these consequences are presented in Appendices C and D. Generally, overstating (understating) mortality will result in an understatement (overstatement) of the output of a culture system.

APPENDIX C: COMPARATIVE STATICS OF THE DETERMINANTS OF OUTPUT

Let $E[Y] = v - vm_3^{c_2*HSI}$ where $v = [c_1\theta\mu * (1 - m_1)] * (1 - m_2)$. The first and second order derivatives, taken with respect to HSI, are presented below:

$$(37) \quad \frac{\partial Y}{\partial HSI} = \begin{cases} -vc_2 \log(m_3) m_3^{c_2 HSI} > 0 & \text{for } m_3 \in (0,1) \\ -vc_2 \log(m_3) m_3^{c_2 HSI} = 0 & \text{for } m_3 = 1 \end{cases}$$

$$(38) \quad \frac{\partial^2 Y}{\partial HSI^2} = \begin{cases} -vc_2^2 \log^2(m_3) m_3^{c_2 HSI} < 0 & \text{for } m_3 \in (0,1) \\ -vc_2 \log(m_3) m_3^{c_2 HSI} = 0 & \text{for } m_3 = 1 \end{cases}$$

These conditions in (37) and (38) define a direct relationship between output and habitat quality that exhibits diminishing marginal returns for all values of between zero and one, non-inclusive. In the special case of “perfect mortality” (*i.e.* $m_3 = 1$), changes in output are independent of changes in habitat quality. Here the minimum mortality of the culture species under optimal conditions is one hundred percent, and the output of a culture system will always be zero regardless of the habitat suitability of the culture system. If HSI is underestimated (overestimated), estimations of output will be less (greater) than the observed output of a culture system. The size of this underestimation (overestimation) is diminishing on the margin.

The derivatives with respect to the habitat adjusted mortality constant are as follows:

$$(39) \quad \frac{\partial Y}{\partial c_2} = \begin{cases} -vHSI \log(m_3) m_3^{c_2 HSI} > 0 & \text{for } m_3 \in (0,1) \\ -vHSI \log(m_3) m_3^{c_2 HSI} = 0 & \text{for } m_3 = 1 \end{cases}$$

$$(40) \quad \frac{\partial^2 Y}{\partial c_2^2} = \begin{cases} -vHSI^2 \log^2(m_3) m_3^{c_2 HSI} < 0 & \text{for } m_3 \in (0,1) \\ -vHSI^2 \log(m_3) m_3^{c_2 HSI} = 0 & \text{for } m_3 = 1 \end{cases}$$

The interpretation of the effect of an underestimation (overestimation) in this parameter is similar to the interpretation of (37) and (38).

The derivatives with respect to the minimum mortality rate are as follows:

$$(41) \quad \frac{\partial Y}{\partial m_3} = -vc_2HSIm_3^{(c_2HSI-1)} < 0$$

$$(42) \quad \frac{\partial^2 Y}{\partial m_3^2} = \begin{cases} -vc_2HSI(c_2HSI - 1)m_3^{(c_2HSI-2)} > 0 & \text{for } c_2HSI < 1 \\ -vc_2HSI(c_2HSI - 1)m_3^{(c_2HSI-2)} < 0 & \text{for } c_2HSI > 1 \\ -vc_2HSI(c_2HSI - 1)m_3^{(c_2HSI-2)} = 0 & \text{for } c_2HSI = 1 \end{cases}$$

(41) indicates that the minimum mortality of the culture species and the output of the culture system are always inversely related. As a result, an underestimation (overestimation) of the minimum mortality rate will result in model outputs that are greater (less) than the observed output of the culture system. The size of this underestimation (overestimation) at the margin will depend on the magnitude of c_2HSI relative to one. The three possible scenarios are described below:

- 1) If the term $c_2HSI < 1$, then $\frac{\partial^2 Y}{\partial m_3^2} > 0$. Here the effect of an error in the estimation of the minimum mortality rate on the estimated output of the culture system is increasing on the margin.
- 2) If the term $c_2HSI > 1$, then $\frac{\partial^2 Y}{\partial m_3^2} < 0$. Here the effect of an error in the estimation of the minimum mortality rate on the estimated output of the culture system is diminishing on the margin.

If the term $c_2HSI = 1$, then $\frac{\partial^2 Y}{\partial m_3^2} = 0$. Here the effect of an error in the estimation of the minimum mortality rate on the estimated output of the culture system is constant on the margin.

Since the model assumes that $c_2 = 1$, the following must hold:

$$(43) \quad \frac{\partial^2 Y}{\partial m_3^2} = \begin{cases} -vc_2HSI(c_2HSI - 1)m_3^{(c_2HSI-2)} > 0 & \text{for } HSI < 1 \\ -vc_2HSI(c_2HSI - 1)m_3^{(c_2HSI-2)} = 0 & \text{for } HSI = 1 \end{cases}$$

An interpretation of (43) reveals that, given the assumptions of the model, the effect of an error in the estimation of the minimum mortality rate on the output of a culture system is always non-decreasing on the margin. For all cases where $HSI \neq 1$, the effect of an error in the estimation of the minimum mortality rate on the output of a culture system will be increasing on the margin. In the special case of a perfectly suitable habitat (*i.e.* $HSI = 1$), a change in the minimum mortality rate of a culture species under optimal conditions will have a linear effect on the output of a culture system, because the habitat adjusted mortality rate will always equal the minimum mortality rate under optimal conditions (*i.e.* $\varphi = m_3$).

APPENDIX D: COMPARATIVE STATICS OF THE HABITAT ADJUSTED MORTALITY CONSTANT

Let $E[Y] = v - \nu m_3^{c_2 * HSI}$ where $\nu = [c_1 \theta \mu * (1 - m_1)] * (1 - m_2)$. Here, the habitat adjusted mortality of fish in the culture system is expressed by the term $m_3^{c_2 * HSI}$. An isomortality line is defined as $c_3 = m^{c_1 * HSI}$, where c_3 is a mortality rate such that $c_3 \in [0,1]$. This equation can be used to define a set of three implicit functions (*i.e.* $m_3(c_2, HSI; c_3)$, $c_2(m_3, HSI; c_3)$, and $HSI(m_3, c_2; c_3)$). These functions describe the isomortality surface as a function of all the determinants of mortality, $f(m_3, c_1, HSI; c_2) = c_3 - m^{c_1 * HSI} = 0$. The signs of the derivatives of this function are evaluated below:

$$(44) \quad \frac{\partial f}{\partial HSI} = \begin{cases} c_2 \log(m_3) m_3^{c_2 HSI} < 0 \text{ for } m_3 \in (0,1) \\ c_2 \log(m_3) m_3^{c_2 HSI} = 0 \text{ for } m_3 = 1 \end{cases}$$

$$(45) \quad \frac{\partial f}{\partial c_2} = \begin{cases} HSI \log(m_3) m_3^{c_2 HSI} < 0 \text{ for } m_3 \in (0,1) \\ HSI \log(m_3) m_3^{c_2 HSI} = 0 \text{ for } m_3 = 1 \end{cases}$$

$$(46) \quad \frac{\partial f}{\partial m_3} = c_2 HSI m_3^{(c_2 HSI - 1)} > 0$$

The Implicit Function Theorem may be used to evaluate the sign of the derivate of the i^{th} determinant of habitat adjusted mortality with respect to the j^{th} determinant of habitat adjusted mortality. The signs of these derivatives are summarized in the following matrix¹⁷:

$$(47) \quad \begin{bmatrix} HSI_{HSI} & m_3_{HSI} & c_2_{HSI} \\ HSI_{m_3} & m_3_{m_3} & c_2_{m_3} \\ HSI_{c_2} & m_3_{c_2} & c_2_{c_2} \end{bmatrix} = \begin{bmatrix} -\frac{f_{HSI}}{f_{HSI}} & -\frac{f_{HSI}}{f_{m_3}} & -\frac{f_{HSI}}{f_{c_2}} \\ -\frac{f_{m_3}}{f_{HSI}} & -\frac{f_{m_3}}{f_{m_3}} & -\frac{f_{m_3}}{f_{c_2}} \\ -\frac{f_{c_2}}{f_{HSI}} & -\frac{f_{c_2}}{f_{m_3}} & -\frac{f_{c_2}}{f_{c_2}} \end{bmatrix} = \begin{bmatrix} -1 & + & - \\ + & -1 & + \\ - & + & -1 \end{bmatrix} \forall m_3 \neq 1$$

¹⁷ For the ease of the reader, it will be useful to notate the derivative of $f(m_3, c_1, HSI; c_2)$ with respect to the i^{th} determinant of mortality as f_i . Similarly, the derivate of the i^{th} determinant of mortality with respect to the j^{th} determinant of mortality will be represented as i_j .

The signs of the derivatives in (47) suggest several conclusions, including the following:

- An interpretation of the diagonals of matrix in (47) suggest that, in order to remain on the same isomortality line, a change in the i^{th} determinant of habitat adjusted mortality must be met by a change in that same determinant of equal magnitude and opposite direction. In other words, holding all else constant, habitat adjusted adjusted mortality will remain constant if a change in the i^{th} determinant “undoes” itself.
- The symmetry of (47) implies that if a change in the i^{th} determinant of habitat adjusted mortality may be exactly “undone” by a change in the j^{th} parameter, it must also be true that a change in the j^{th} determinant of habitat adjusted mortality may also be exactly “undone” by a change in the i^{th} determinant.
- Finally, for a change in the i^{th} determinant of habitat adjusted mortality, an interpretation of each of the elements of (47) defines how the j^{th} mortality must change in order to remain on the isomortality line. To illustrate this conclusion, an example is useful. Suppose, for example, that the estimated parameters of the habitat adjusted mortality function assume following values:

$$\begin{aligned} m_3 &= 0.5 \\ HSI &= 0.5 \\ c_2 &= 1.0 \end{aligned}$$

in this case, c_3 is calculated as follows:

$$(48) \quad c_3 = 0.5^{0.5*1} = 70.2 \% \text{ mortality.}$$

The elements of (47) indicate that, for a reduction in the minimum mortality rate (m_3) (e.g. from 0.5 to 0.3), the adjusted mortality rate may remain constant if and only if one of the following three conditions is met:

- a) HSI also decreases (*i.e.* $-\frac{f_{HSI}}{f_{m_3}} > 0$).
- b) c_2 also decreases (*i.e.* $-\frac{f_{c_2}}{f_{m_3}} > 0$).
- c) Some combination of both condition a) and b)

Condition a) is illustrated numerically below:

$$(49) \quad c_3 = 0.3^{0.294*1} = 70.2 \% \text{ mortality}$$

Condition b) is illustrated numerically below:

$$(50) \quad c_3 = 0.5^{0.5*0.588} = 70.2 \% \text{ mortality}$$

Condition c) is illustrated numerically below:

$$(51) \quad c_3 = 0.5^{0.4*0.735} = 70.2 \% \text{ mortality}$$

By implication, overestimation (underestimation) of the minimum mortality rate will cause the habitat adjusted mortality rate to overstate (understate) the true mortality rate unless this error is corrected for by combined effects of changes in the HSI or habitat adjusted mortality constant in the appropriate direction.

As in Appendix C, the special case of “perfect minimum mortality” (*i.e.* $m_3 = 1$) warrants a unique interpretation of (47). Note that in the case of $m_3 = 1$ many of the signs of the determinants in (47) take on undefined values. If $m_3 = 1$, for example, then the signs of $\frac{\partial HSI}{\partial m_3}$ and $\frac{\partial c_2}{\partial HSI}$ take on the following undetermined value:

$$(52) \quad HSI_{m_3} = -\frac{f_{m_3}}{f_{HSI}} = -\frac{+}{0}$$

$$(53) \quad c_{2HSI} = -\frac{f_{HSI}}{f_{c_2}} = -\frac{0}{0}$$

Other elements of (47) assume values of zero in the special case of $m_3 = 1$. The case of $\frac{\partial m_3}{\partial HSI}$ provides an interesting example:

$$(54) \quad m_{3HSI} = -\frac{f_{HSI}}{f_{m_3}} = -\frac{0}{+}$$

In the case of (54), $\frac{\partial m_3}{\partial HSI} = 0$, and yet, in the case of (52), $\frac{\partial HSI}{\partial m_3}$ is undefined. These derivatives violate the symmetry of (47), and a conventional interpretation of (47) contradicts the interpretations of (52) and (54). On one hand, (54) suggest that, for a change in HSI, m_3 must remain unchanged in order to keep the habitat adjusted mortality rate constant. On the other hand, (52) indicates that a change in m_3 must be accompanied by an infinite change in HSI in order to remain on the isomortality line. This contradiction (as well as other contradictions presented by the special case of perfect mortality) is explained away by the logical checkmate that is presented when considering the intuitive meaning of the $m_3 = 1$ condition. Since mortality may not exceed one-hundred percent, fixing the minimum mortality rate under optimal conditions at this level contradicts the effects of changes in the determinants of mortality as these effects may neither either decrease the habitat adjusted mortality rate (by mathematical definition) nor increase it (by the intuitive meaning of mortality).

APPENDIX E: MONTE CARLO DISTRIBUTION ASSUMPTIONS

In its general form, the point along the Cumulative Distribution Function (CDF) corresponding to the mode of the Probability Density Function (PDF) is represented as follows:

$$(55) \quad F(a, b, c) = \frac{(c-a)}{(b-a)}$$

In cases where the mode of the CDF is represented by the arithmetic mean of the limits of the distribution, the probability of drawing a URN that is less than or equal to the mode of the distribution is fifty percent. The calculation of this probability is presented below:

$$(56) \quad P(U \leq c) = \frac{\left[\frac{b+a}{2} - a\right]}{(b-a)} = \frac{b-a}{2(b-a)} = 0.5$$

As the mode of the CDF approaches the lower limit of the distribution, the probability of drawing a URN that is at least as great as the mode of the distribution approaches zero:

$$(57) \quad P(U \leq c) = \lim_{c \rightarrow a} \frac{(c-a)}{(b-a)} = \frac{0}{(b-a)}$$

Conversely, as the mode of the CDF approaches the upper limit of the distribution, the probability of drawing a URN that is at least as great as the mode of the distribution approaches one:

$$(58) \quad P(U \leq c) = \lim_{c \rightarrow b} \frac{(c-a)}{(b-a)} = 1$$

In the model, the user may enter the parameters that determine the shape of the CDFs (*i.e.* low, likely, and high estimates for the value of each stochastic determinant of total revenues). In the absence of detailed data, it is assumed that the likely value of each of the stochastic determinants of total revenues is represented by the mean of its low and high estimates (*i.e.* $c = \frac{a+b}{2}$ by assumption). Since the value of a

randomly generated TRN is critically dependent on the magnitude of a randomly generated URN relative to the value of $F(c)$, this assumption has some repercussions in the distributions of outcomes. These are summarized by the following scenarios:

- 1) The distribution of TRNs resulting from randomly generated URNs will be skewed negatively if it is the case that $F(c) > 0.5$ (*i.e.* the likely value of the stochastic input is greater than mean of its limits). In this case, imposing the normality assumption on the distribution of TRNs will result in an underrepresentation of TRNs with values greater than the mode.
- 2) The distribution of TRNs resulting from randomly generated URNs will be skewed positively if it is the case that $F(c) < 0.5$ (*i.e.* the likely value of the stochastic input is less than mean of its limits). In this case, imposing the normality assumption on the distribution of TRNs will result in an underrepresentation of TRNs with values less than the mode.

These potential discrepancies between the estimated distribution of the TRNs and the observed distributions of TRNs will impact the distributions of the expected output of a culture system. This direction of this impact will depend on the direction of the relationship between the expected output and the stochastic input in question. For inputs with direct relationships to the expected output of a system, an underrepresentation of TRN with values greater than the mode will result in an underrepresentation of total outputs greater than the mode. For inputs with indirect relationships to the expected output of a system, an underrepresentation of TRN with values greater than the mode will result in an overrepresentation of total outputs greater than the mode.

APPENDIX F: OXYGENATION CAPACITY CALCULATIONS

The RA-1 aerator draws 3.6 amps of 115v current per hour (Kasco Marine Inc., n.d.). In order to estimate for the standard aeration capacity of the RA-1 aerator, it is useful to first convert the power usage of the aerator to kilowatts:

$$(59) \quad 3.6 \text{amps} * \frac{115 \text{volts}}{1,000} = 0.41 \text{ kw}$$

The standard aeration efficiency of the system (SAE) is 1.2-20 (Creswell, 1991). It is possible to estimate a lower bound for the standard oxygenation capacity of the RA-1 aerator using the lower bound of this range of SAE values.

$$(60) \quad OC_{std,lower} = 0.41 \text{kw} * 1.2 = 0.492 \text{ kg O}_2 \text{ hr}^{-1}$$

Similarly, it is possible to estimate a upper bound for the standard oxygenation capacity of the RA-1 aerator using the upper bound of this range of SAE values.

$$(61) \quad OC_{std,upper} = .41 \text{kw} * 2.0 = 0.82 \text{ mg O}_2 \text{ hr}^{-1}$$

An estimate for the field oxygenation capacity of the aerator may be arrived at using the following equation:

$$(62) \quad OC_{field} = OC_{std} * \left(\frac{\beta}{9.077} \right) * 1.024^{T-20} * \alpha$$

where $\beta = C_{field}^* - C_{field}$ and $\alpha = \frac{K_{LA}Field}{K_{LA}STD}$.

It is assumed that the crappie rearing pond is about 16°C. The likely minimum DO level of the pond without aeration is assumed to be 8.5 mg l⁻¹, and the $C_{field}^* = 9.858 \text{ mg l}^{-1}$ at 16°C (Colt, 1984; Egloff, 2011). At 10°C, $\alpha = 0.830$ (Kepenyes & Varadi, n.d.). Using a pond temperature of 10°C, lower

and upper bound estimates for the field oxygenation capacity of the aerator can be arrived at as follows:

$$(63) \quad OC_{FIELD,LOWER} = 0.492 \text{ mg O}_2 \text{ hr}^{-1} * \left(\frac{11.277 \text{ mg l}^{-1} - 8.5 \text{ mg l}^{-1}}{9.077} \right) * 1.024^{10^\circ\text{C}-20^\circ\text{C}} * 0.830$$

$$(64) \quad OC_{FIELD,UPPER} = 0.82 \text{ mg O}_2 \text{ hr}^{-1} * \left(\frac{11.277 \text{ mg l}^{-1} - 8.5 \text{ mg l}^{-1}}{9.077} \right) * 1.024^{10^\circ\text{C}-20^\circ\text{C}} * 0.830$$

$$(65) \quad OC_{FIELD,LOWER} = 0.099 \text{ kg O}_2 \text{ hr}^{-1} = 99,000 \text{ mg O}_2 \text{ hr}^{-1}$$

$$(66) \quad OC_{FIELD,UPPER} = 0.164 \text{ kg O}_2 \text{ hr}^{-1} = 164,000 \text{ mg O}_2 \text{ hr}^{-1}$$

Similarly, using a pond temperature of 16°C, alternative lower and upper bound estimates for the field oxygenation capacity of the aerator can be arrived at as follows:

$$(67) \quad OC_{FIELD,LOWER} = 0.492 \text{ mg O}_2 \text{ hr}^{-1} * \left(\frac{9.858 \text{ mg l}^{-1} - 8.5 \text{ mg l}^{-1}}{9.077} \right) * 1.024^{16^\circ\text{C}-20^\circ\text{C}} * 0.830$$

$$(68) \quad OC_{FIELD,UPPER} = 0.82 \text{ mg O}_2 \text{ hr}^{-1} * \left(\frac{9.858 \text{ mg l}^{-1} - 8.5 \text{ mg l}^{-1}}{9.077} \right) * 1.024^{16^\circ\text{C}-20^\circ\text{C}} * 0.830$$

$$(69) \quad OC_{FIELD,LOWER} = 0.056 \text{ kg O}_2 \text{ hr}^{-1} = 56,000 \text{ mg O}_2 \text{ hr}^{-1}$$

$$(70) \quad OC_{FIELD,UPPER} = 0.093 \text{ kg O}_2 \text{ hr}^{-1} = 93,000 \text{ mg O}_2 \text{ hr}^{-1}$$

In calculating the anticipated DO levels of the pond, it is important to consider of the oxygen demand of both the benthos and the culture species. The pond bottom is expected to consume 1-3 grams of oxygen per square meter of pond bottom per day (Kepenyes & Varadi, n.d.). Assuming an oxygen consumption rate of 2 grams of oxygen per square meter per day and a pond bottom $2,213.54 \text{ m}^2$ yields an oxygen demand of $4,427,080 \text{ mg O}_2 \text{ pond}^{-1} \text{ d}^{-1}$. The oxygen consumption of white crappies while at rest is assumed to be 121.5 mg of oxygen per kg of fish per hour (Parsons & Sylvester, 1992). Assuming 7.95 kg of fish per pond, the oxygen demand of the fish at rest can be calculated as follows:

$$(71) \quad 7.94 \frac{\text{kg of fish}}{\text{pond}} * 121.5 \frac{\text{mg of O}_2}{\text{kg of fish}} * \frac{1}{\text{hr}} * 24 \text{ hr} = 23,153.04 \frac{\text{mg of O}_2}{\text{pond}} * \text{d}^{-1}$$

Multiplying by a factor of three allows for increased oxygen consumption of the fish due to feeding and stress factors:

$$(72) \quad 23,153.04 \frac{\text{mg of } O_2}{\text{pond}} * \text{day}^{-1} * 3 = 69,459.12 \frac{\text{mg of } O_2}{\text{pond}} * \text{day}^{-1}$$

The total daily oxygen demand of the pond is the sum of the oxygen demands of the pond bottom and the culture species:

$$(73) \quad 4,427,080 \frac{\text{mg of } O_2}{\text{pond}} * d^{-1} + 69,459.12 \frac{\text{mg of } O_2}{\text{pond}} * d^{-1} = 4,496,539.12 \frac{\text{mg of } O_2}{\text{pond}} * d^{-1}$$

It is assumed that the initial concentration of oxygen in the pond is $10,484,595.64 \text{ mgO}_2 \text{d}^{-1}$ (Egloff, 2011). Assuming a pond temperature of 10°C , lower and upper bound estimates for milligrams of oxygen in solution after 24 hours of aeration with zero oxygen demand may be arrived at as follows:

$$(74) \text{ Lower Bound} = 10,484,595.64 + (99,000 \text{ mgO}_2 \text{hr}^{-1} * 24 \text{hrs}) = 12,860,596 \text{ mgO}_2 \text{d}^{-1}$$

$$(75) \text{ Upper Bound} = 10,484,595.64 + (164,000 \text{ mgO}_2 \text{hr}^{-1} * 24 \text{hrs}) = 14,420,596 \text{ mgO}_2 \text{d}^{-1}$$

Alternatively, assuming a temperature of 16°C yields the following calculations:

$$(76) \text{ Lower Bound} = 10,484,595.64 + (56,000 \text{ mgO}_2 \text{hr}^{-1} * 24 \text{hrs}) = 11,828,596 \text{ mgO}_2 \text{d}^{-1}$$

$$(77) \text{ Upper Bound} = 10,484,595.64 + (93,000 \text{ mgO}_2 \text{hr}^{-1} * 24 \text{hrs}) = 12,716,596 \text{ mgO}_2 \text{d}^{-1}$$

Netting out the total oxygen demand of the pond estimates the total oxygen in solution after 24 hours of aeration at 10°C :

$$(78) \quad \text{Lower Bound} = 12,860,596 \text{ mgO}_2 \text{d}^{-1} - 4,496,539.12 \frac{\text{mg of } O_2}{\text{pond}} * d^{-1}$$

$$= 8,364,056.88 \frac{\text{mg of } O_2}{\text{pond}} * d^{-1}$$

$$(79) \quad \text{Upper Bound} = 14,420,596 \text{ mgO}_2 \text{d}^{-1} - 4,496,539.12 \frac{\text{mg of } O_2}{\text{pond}} * d^{-1}$$

$$= 9,924,056.88 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}$$

By the same method, alternative estimates may be arrived at for a pond kept at 16°C:

$$(80) \quad \text{Lower Bound} = 11,828,596 \text{ mgO}_2 d^{-1} \text{ mgO}_2 d^{-1} - 4,496,539.12 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}$$

$$= 7,332,057.88 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}$$

$$(81) \quad \text{Upper Bound} = 12,716,596 \text{ mgO}_2 d^{-1} - 4,496,539.12 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}$$

$$= 8,220,056.88 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}$$

Assuming a pond volume of 1 acre foot (*i.e.* 1,233,481 liters), allows for the conversion of these DO levels to concentration levels after 24 hours of aeration. The calculation for pond kept at 10°C follows:

$$(82) \quad \text{Concentration lower} = \frac{8,364,056.88 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}}{1,233,481 \frac{\text{liters}}{\text{pond}}} = 6.78 \frac{\text{mg O}_2}{\text{liter of pond volume}} * d^{-1}$$

$$(83) \quad \text{Concentration upper} = \frac{9,924,056.88 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}}{1,233,481 \frac{\text{liters}}{\text{pond}}} = 8.04 \frac{\text{mg O}_2}{\text{liter of pond volume}} * d^{-1}$$

By the same method, the estimates for the 16°C pond are as follows:

$$(84) \quad \text{Concentration lower} = \frac{7,332,057.88 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}}{1,233,481 \frac{\text{liters}}{\text{pond}}} = 5.94 \frac{\text{mg O}_2}{\text{liter of pond volume}} * d^{-1}$$

$$(85) \quad \text{Concentration upper} = \frac{8,220,056.88 \frac{\text{mg of O}_2}{\text{pond}} * d^{-1}}{1,233,481 \frac{\text{liters}}{\text{pond}}} = 6.66 \frac{\text{mg O}_2}{\text{liter of pond volume}} * d^{-1}$$

The saturation concentration of oxygen in fresh water (*i.e.* salinity at 0 ppm) is 9.86 in 16°C and 11.28 in 10°C (Colt, 1984). Use of this information allows for the estimation of saturation concentrations after 24 hours of aeration. For the 10°C pond, the calculations are as follows:

$$(86) \quad \text{Saturation Concentration lower} = \frac{6.78 \frac{\text{mg O}_2}{\text{litter of pond volume}} * d^{-1}}{11.28 \text{ mg l}^{-1}} = 60 \% \text{ saturation}$$

$$(87) \quad \text{Saturation Concentration upper} = \frac{8.04 \frac{\text{mg O}_2}{\text{litter of pond volume}} * d^{-1}}{11.28 \text{ mg l}^{-1}} = 71 \% \text{ saturation}$$

For the 16°C pond, the calculations are as follows:

$$(88) \quad \text{Saturation Concentration lower} = \frac{5.94 \frac{\text{mg O}_2}{\text{litter of pond volume}} * d^{-1}}{9.86 \text{ mg l}^{-1}} = 60 \% \text{ saturation}$$

$$(89) \quad \text{Saturation Concentration upper} = \frac{6.66 \frac{\text{mg O}_2}{\text{litter of pond volume}} * d^{-1}}{9.86 \text{ mg l}^{-1}} = 68 \% \text{ saturation}$$

Under the baseline scenario, it's assumed that both the initial concentration of oxygen and the oxygen demand of the pond do not change. An estimate for the saturation concentration of oxygen after 24 hours in the absence of aeration can be arrived at using similar methods. The saturation concentration of the baseline scenario pond is estimated below:

$$(90) \quad 10,484,595.64 \text{ mg O}_2 \text{ pond}^{-1} - 4,496,539.12 \text{ mg O}_2 \text{ pond}^{-1} * d^{-1}$$

$$= 5,988,057 \text{ mg O}_2 \text{ pond}^{-1} d^{-1}$$

$$(91) \quad \frac{5,988,057 \text{ mg O}_2 \text{ pond}^{-1} d^{-1}}{1,233481 \frac{\text{liters}}{\text{pond}}} = 4.85 \frac{\text{mg O}_2}{\text{litter of pond volume}} * d^{-1}$$

$$(92) \quad \frac{4.85 \frac{\text{mg O}_2}{\text{litter of pond volume}} * d^{-1}}{9.86 \text{ mg l}^{-1}} = 49 \% \text{ saturation}$$

APPENDIX G: DETAILED HSI ASSUMPTIONS FOR THE SCENARIO FOR MODELING

The following table summarizes all the assumed values for each of the nine habitat variables, the four life requisite variables, and the comprehensive HSI under both the baseline and the alternative scenarios:

TABLE 10: ASSUMED VALUES OF HABITAT VARIABLES, LRIS, AND HSIS FOR BASELINE AND ALTERNATIVE SCENARIOS

Habitat Variable	Low Value		Likely Value		High Value	
	Baseline	Alternative	Baseline	Alternative	Baseline	Alternative
Percent Cover (%)	10	10	20	20	50	50
Percent Littoral Area (%)	8	8	13	13	50	50
pH	Category 3	Category 3	Category 2	Category 2	Category 1	Category 1
Average Temperature of Epilimnion (°C)	15.5	15.5	16.1	16.1	16.6	16.6

Average Temperature of Littoral Area (°C)	15.5	15.5	16.1	16.1	16.6	16.6
Minimum Dissolved Oxygen During Midsummer	Category 2	Category 2	Category 2	Category 1	Category 1	Category 1
Minimum Dissolved Oxygen in Littoral Area During Spawning	Category 2	Category 2	Category 2	Category 1	Category 1	Category 1
Maximum Monthly Average Turbidity During Summer (JTU)	144	144	78	78	50	50
Average Total Dissolved Solids During Midsummer (ppm)	50	50	85	85	225	225

Life Requisite Variable	Low Value		Likely Value		High Value	
	Baseline	Alternative	Baseline	Alternative	Baseline	Alternative
Food (HSI)	0.50	0.50	0.85	0.85	1.00	1.00
Cover (HSI)	0.53	0.53	0.85	0.85	1.00	1.00
Water Quality (HSI)	0.30	0.30	0.36	0.36	0.64	0.80
Reproduction (HSI)	0.45	0.57	0.56	0.70	0.61	0.77
Comprehensive HSI	0.30	0.30	0.36	0.36	0.73	0.92

APPENDIX H: VARIABLES, DEFINITIONS, AND DEFINITIONS

Variable	Description	Equation	Low HSI Value (Index Number)	Likely HSI Value (Index Number)	High HSI Value (Index Number)	Source
HSI Variables						
V_1	Zero-to-one suitability index describing the overall suitability of the provision of cover	Refer to Edwards et al 1982	0.52	0.84	1.0	
V_2	Zero-to-one suitability index describing the overall suitability of the provision of littoral area	Refer to Edwards et al 1982	0.53	0.86	1.0	
V_3	Zero-to-one suitability index describing the overall suitability of pH	Refer to Edwards et al 1982	0.1	0.5	1.0	
V_4	Zero-to-one suitability index describing the overall suitability of the temperature of the epilimnion during midsummer (July-August)	Refer to Edwards et al 1982	0.25	0.31	0.36	(Egloff, 2011)

V_5	Zero-to-one suitability index describing the overall suitability of the temperature of the littoral areas during midsummer (July-August)	Refer to Edwards et al 1982	0.35	0.41	0.46	(Egloff, 2011)
V_6	Zero-to-one suitability index describing the overall suitability of the minimum dissolved oxygen levels during midsummer (March-July)	Refer to Edwards et al 1982	0.5 ^{18*} 1.0**	0.5* 1.0**	0.5* 1.0**	(Colt, 1984; Creswell, 1991; Kepenyes & Varadi, n.d.; Kramer, 1987)
V_7	Zero-to-one suitability index describing the overall suitability of the minimum dissolved oxygen levels of the littoral area during spawning (March-July)	Refer to Edwards et al 1982	0.5* 1.0**	0.5* 1.0**	0.5* 1.0**	(Colt, 1984; Creswell, 1991; Kepenyes & Varadi, n.d.; Kramer, 1987)
V_8	Zero-to-one suitability index describing the overall suitability of the maximum monthly average turbidity during summer (May-August)	Refer to Edwards et al 1982	0.5	0.85	1.0	

¹⁸* Indicates values under the baseline scenario and ** indicates values under the alternative scenario.

V_9	Zero-to-one suitability index describing the overall suitability of the average total dissolved solids during midsummer	Refer to Edwards et al 1982	0.5	0.85	1.0
C_F	Zero-to-one suitability index describing the overall suitability of the provision of food	Refer to Edwards et al 1982	0.5	0.85	1.0
C_c	Zero-to-one suitability index describing the overall suitability of the provision of cover	Refer to Edwards et al 1982	0.53	0.85	1.0
C_{wq}	Zero-to-one suitability index describing the overall suitability of water quality	Refer to Edwards et al 1982	0.3	0.36	0.64* 0.80**
C_R	Zero-to-one suitability index describing the overall suitability for reproduction	Refer to Edwards et al 1982	0.45* 0.57**	0.56* 0.70**	0.61* 0.77**
HSI	Zero-to-one suitability index describing the overall suitability of the culture system	(1)	0.3	0.36	0.73* 0.92**

Variable	Description	Equation	Low HSI Value ¹⁹	Likely HSI Value	High HSI Value	Source
Other Variables						
$E[Y]$	Generalized form of the expected output of a culture system (number of fish for stocking)	(2)				
$E[Y_{egg}]$	Expected output of a culture system (number of eggs surviving to larval stage of development)	(3)	61,491	61,491	61,491	(Clady, 1975; Dahlberg, 1979; Egloff, 2011; Mathur, McCreigh, & Nardacci, 1979)
$E[Y_{larval}]$	Expected output of a culture system (number of larval fish surviving to post-larval stage of development)	(4)	44,273	44,273	44,273	(Pine & Allen, 2001)
$E[Y_{post larval}]$	Expected output of a culture system (number of surviving post-larval fish for stocking)	(5)	236,380*	278,680*	474,120*	
θ	Relative fecundity of the culture species (eggs per gram of female fish)	(2)	177	241	304	(Mathur, McCreigh, & Nardacci, 1979)
μ	Stocking Density (Fish stocked per acre of pond surface area)	(2), (33)	125	125	125	(Egloff, 2011)

¹⁹ The low, likely, and high values reported herein are calculated by using the low, likely, and high values of HSI. All other all determinants of total physical output and total net revenues have been fixed at their likely values

φ	Post-larval habitat adjusted mortality rate (percent mortality)	(6)	0.73	0.69	0.46*	
c_1	Fecundity deflation factor (ratio of potential fecundity to observed fecundity)	(3)	0.25	0.25	0.25	(Clady, 1975)
c_2	Post-larval habitat adjusted mortality rate constant	(6)	1	1	1	
m_1	Egg mortality rate (percent mortality)	(3)	0.75	0.75	0.75	(Dahlberg, 1979)
m_2	Larval mortality rate (percent mortality)	(4)	0.28	0.28	0.28	(Pine & Allen, 2001)
m_3	Post-larval mortality rate (percent mortality)	(6)	0.30	0.35	0.40	(Egloff, 2011)
TR^*	Total gross annual revenues of the culture system under the baseline alternative (non-annualized dollars)	(10)	236,380	278,680	474,120	
FC^*	Total fixed costs of the culture system under the baseline alternative (non-annualized dollars)	(12)	70,914	83,604	142,236	

VC^*	Total variable costs of the culture system under the baseline alternative (non-annualized dollars)	(13)	165,466	195,076	331,884	
A^*	Vector of parameters containing the likely value of all determinants of the total revenues (<i>i.e.</i> TR^*) of the culture system under the baseline scenario	(11)				
B^*	Vector of parameters containing the likely value of all determinants of the total physical output of the culture system	(17)				
α	Share of total revenues going to fixed costs under the baseline scenario	(12)	0.30	0.30	0.30	(Deisenroth & Bond, 2010)
β	Share of total revenues going to variable costs under the baseline scenario	(13)	0.70	0.70	0.70	(Deisenroth & Bond, 2010)
\widetilde{TC}'_{NPV}	Total costs under the alternative scenario (non-annualized dollars)	(15)	237,580	279,880	528,114	

\widetilde{TC}'_{ANN}	Total costs under the alternative scenario (annualized dollars)	(18)	236,650	278,950	527,184	
ϕ	Purchase price of optional technological improvement (non-annualized dollars)	(15)	1,200	1,200	1,200	(Kasco Marine Inc., n.d.)
\widetilde{VC}	Variable costs of the culture system under the alternative scenario (non-annualized dollars)	(16)	165,466	195,076	384,678	
t	Useful lifespan of the optional technological improvement (years)	(19)	5	5	5	
r	Discount rate (percent)	(19)	0.04	0.04	0.04	
$A_{t,r}$	Annuity factor	(19)	4.45	4.45	4.45	(Walker & Kumaranayake, 2002)
TNR^*_{NPV}	Total net revenues under the baseline scenario (non-annualized present value dollars)	(20)	0	0	0	
\widetilde{TNR}_{NPV}	Total net revenues under the alternative scenario (non-annualized present value dollars)	(21)	-1,154	-1,154	20,602	
\widetilde{TNR}_{ANN}	Total net revenues under the alternative scenario (annualized present value dollars)	(22)	-259	-259	21,497	

$f(x; a, b, c)$	Triangular probability density function (general form)	(23)
$F(x; a, b, c)$	Triangular cumulative distribution function (general form)	(28)

APPENDIX I: VISUAL BASIC CODE

'this sub generates random URNs to be used in the Monte Carlo simulation tools of the Model'

```
Sub generate()
    Application.ScreenUpdating = False
    'this code creates a new set of uniform random number for HSI in the baseline scenario (i.e. the system without the change)'
    Range("AB15").Select
    ActiveCell.FormulaR1C1 = "=RAND()"
    Selection.AutoFill Destination:=Range("AB15:AB2014"), Type:=xlFillDefault
    'this code copy pastes the URN on top of itself so that the values will not change when new commands are executed'
    Range("AB15:AB2014").Select
    Selection.Copy
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    Range("AC15").Select
    Range(Selection, Selection.End(xlDown)).Select
    Application.CutCopyMode = False
    Selection.Copy
    Range("B4").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    'this code creates a new set of uniform random number for HSI in the alternative scenario (i.e. the system with the change)'
    Range("AG15").Select
    ActiveCell.FormulaR1C1 = "=RAND()"
    Selection.AutoFill Destination:=Range("AG15:AG2014"), Type:=xlFillDefault
    'this code copy pastes the URN on top of itself so that the values will not change when new commands are executed'
    Range("AG15:AG2014").Select
    Selection.Copy
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    Range("AH15").Select
    Range(Selection, Selection.End(xlDown)).Select
    Application.CutCopyMode = False
    Selection.Copy
    Range("C4").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    'this code creates a new set of uniform random number for mortality rate'
    Range("AK15").Select
    ActiveCell.FormulaR1C1 = "=RAND()"
    Selection.AutoFill Destination:=Range("AK15:AK2014"), Type:=xlFillDefault
    'this code copy pastes the URN on top of itself so that the values will not change when new commands are executed'
    Range("AK15:AK2014").Select
```

```

Selection.Copy
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Range("AL15").Select
Range(Selection, Selection.End(xlDown)).Select
Application.CutCopyMode = False
Selection.Copy
Range("D4").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
'this code creates a new set of uniform random number for fecundity'
Range("AP15").Select
ActiveCell.FormulaR1C1 = "=RAND()"
Selection.AutoFill Destination:=Range("AP15:AP2014"), Type:=xlFillDefault
'this code copy pastes the URN on top of itself so that the values will not change when new commands are executed'
Range("AP15:AP2014").Select
Selection.Copy
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Range("AQ15").Select
Range(Selection, Selection.End(xlDown)).Select
Application.CutCopyMode = False
Selection.Copy
Range("E4").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
'this code creates a new set of uniform random number for output price'
Range("AU15").Select
ActiveCell.FormulaR1C1 = "=RAND()"
Selection.AutoFill Destination:=Range("AU15:AU2014"), Type:=xlFillDefault
'this code copy pastes the URN on top of itself so that the values will not change when new commands are executed'
Range("AU15:AU2014").Select
Selection.Copy
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Range("AV15").Select
Range(Selection, Selection.End(xlDown)).Select
Application.CutCopyMode = False
Selection.Copy
Range("F4").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
'this code brings the user back to the top of the page so that they do not have to scroll back to the top manually after the random URN values have been generated'.
Range("A1").Select
End Sub

```

'this sub runs the Monte Carlo simulation around HSI'

Sub HSI()

```

Application.ScreenUpdating = False
Dim i As Integer
For i = 1 To 2000
    'this code rounds the randomly generated HSI values to two digits'
    Sheets("Monte Carlo Simulation").Select
    Range("T4").Select
    Range(Selection, Selection.End(xlDown)).Select
    Selection.Copy
    Range("B4").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    Sheets("Monte Carlo Simulation").Select
    Range("U4").Select
    Range(Selection, Selection.End(xlDown)).Select
    Selection.Copy
    Range("C4").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    'This code will copy paste the result of the random draw for HSI in to the model for the baseline system'
    Sheets("Monte Carlo Simulation").Select
    Cells(3 + i, 2).Copy
    Sheets("Results HSI").Select
    Cells(3 + i, 2).Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    Sheets("Results HSI").Select
    Cells(3 + i, 2).Copy
    Sheets("Habitat Suitability Index").Select
    Range("C34").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    Sheets("Estimated Output of Crappie").Select
    Range("D40").Select
    ActiveCell.Copy
    Sheets("Results HSI").Select
    Cells(3 + i, 3).Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    Sheets("Estimated Output of Crappie").Select
    Range("D28").Select
    ActiveCell.Copy
    Sheets("Results HSI").Select
    Cells(3 + i, 4).Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
    'This code will copy paste the result of the random draw for HSI in to the model for the alternative
    'scenario system'
    Sheets("Monte Carlo Simulation").Select
    Cells(3 + i, 3).Copy
    Sheets("Results HSI").Select
    Cells(3 + i, 5).Select

```

```

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _  

    :=False, Transpose:=False  

Sheets("Habitat Suitability Index").Select  

Range("C79").Select  

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _  

    :=False, Transpose:=False  

Sheets("Estimated Output of Crappie").Select  

Range("D105").Select  

ActiveCell.Copy  

Sheets("Results HSI").Select  

Cells(3 + i, 6).Select  

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _  

    :=False, Transpose:=False  

Sheets("Estimated Output of Crappie").Select  

Range("D100").Select  

ActiveCell.Copy  

Sheets("Results HSI").Select  

Cells(3 + i, 7).Select  

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _  

    :=False, Transpose:=False  

Sheets("Estimated Output of Crappie").Select  

Range("D81").Select  

ActiveCell.Copy  

Sheets("Results HSI").Select  

Cells(3 + i, 8).Select  

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _  

    :=False, Transpose:=False  

Next  

'this section replaces the original values in the cells of the model that were replaced by random values while the simulation was running'  

Sheets("Habitat Suitability Index").Select  

Range("C34").Select  

ActiveCell.FormulaR1C1 = "=ROUND(R[-25]C[222],2)"  

Range("C79").Select  

ActiveCell.FormulaR1C1 = "=ROUND(R[-66]C[222],2)"  

'this code brings the user back to the results tab'  

Sheets("Results HSI").Select  

Application.ScreenUpdating = True  

'this code brings up an alert that the simulation has been completed'  

Beep  

MsgBox "Simulation Complete!"  

End Sub

```

'this sub runs the Monte Carlo simulation around minimum mortality rate'

```

Sub Mort()
Application.ScreenUpdating = False
'this code saves the original value of mortality to be replaced in its original cell after the simulation is completed'
Sheets("Estimated Output of Crappie").Select
Range("C9").Select

```

```

ActiveCell.Copy
Range("F9").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
'This code will copy paste the result of the random draw for mortality in to the model for the baseline system'
Dim i As Integer
For i = 1 To 2000
Sheets("Monte Carlo Simulation").Select
Cells(3 + i, 4).Copy
Sheets("Results Mortality").Select
Cells(3 + i, 2).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("C9").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D40").Select
ActiveCell.Copy
Sheets("Results Mortality").Select
Cells(3 + i, 3).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D28").Select
ActiveCell.Copy
Sheets("Results Mortality").Select
Cells(3 + i, 4).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
'This code will copy paste the result of the random draw for mortality in to the model for the alternative system'
Sheets("Estimated Output of Crappie").Select
Range("D105").Select
ActiveCell.Copy
Sheets("Results Mortality").Select
Cells(3 + i, 5).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D100").Select
ActiveCell.Copy
Sheets("Results Mortality").Select
Cells(3 + i, 6).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D81").Select
ActiveCell.Copy

```

```

Sheets("Results Mortality").Select
Cells(3 + i, 7).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Next
'this code replaces the original value of the cells that were changes during the simulation'
Sheets("Estimated Output of Crappie").Select
Range("F9").Select
ActiveCell.Copy
Range("C9").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
'this code brings the user back to the results tab'
Sheets("Results Mortality").Select
Application.ScreenUpdating = True
'this code brings up an alert that the simulation has been completed'
Beep
MsgBox "Simulation Complete!"
End Sub

```

```

'this sub runs the Monte Carlo simulation around fecundity'
Sub fec()
Application.ScreenUpdating = False
'this code saves the original value of fecundity to be replaced in its original cell after the simulation is completed'
Sheets("Estimated Output of Crappie").Select
Range("C15").Select
ActiveCell.Copy
Range("F15").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
'This code will copy paste the result of the random draw for fecundity in to the model for the baseline system'
Dim i As Integer
For i = 1 To 2000
Sheets("Monte Carlo Simulation").Select
Cells(3 + i, 5).Copy
Sheets("Results Fecundity").Select
Cells(3 + i, 2).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("C15").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
    :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D40").Select
ActiveCell.Copy
Sheets("Results Fecundity").Select
Cells(3 + i, 3).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks =

```

```

        :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D28").Select
ActiveCell.Copy

Sheets("Results Fecundity").Select
Cells(3 + i, 4).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
'This code will copy paste the result of the random draw for fecundity in to the model for the alternative system'
Sheets("Estimated Output of Crappie").Select
Range("D105").Select
ActiveCell.Copy
Sheets("Results Fecundity").Select
Cells(3 + i, 5).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D100").Select
ActiveCell.Copy
Sheets("Results Fecundity").Select
Cells(3 + i, 6).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D81").Select
ActiveCell.Copy
Sheets("Results Fecundity").Select
Cells(3 + i, 7).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
Next
'this code replaces the original value of the cells that were changes during the simulation'
Sheets("Estimated Output of Crappie").Select
Range("F15").Select
ActiveCell.Copy
Range("C15").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
        :=False, Transpose:=False
'this code brings the user back to the results tab'
Sheets("Results Mortality").Select
Application.ScreenUpdating = True
'this code brings up an alert that the simulation has been completed'
Beep
MsgBox "Simulation Complete!"
End Sub

```

'this sub runs the Monte Carlo simulation around output price'

```

Sub Price()
Application.ScreenUpdating = False

```

'this code saves the original value of price to be replaced in its original cell after the simulation is completed'

```

Sheets("Estimated Output of Crappie").Select
Range("C22").Select
ActiveCell.Copy

Range("F22").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
:=False, Transpose:=False
'This code will copy paste the result of the random draw for price in to the model for the baselinesystem'
Dim i As Integer
For i = 1 To 2000
Sheets("Monte Carlo Simulation").Select
Cells(3 + i, 6).Copy
Sheets("Results Output Price").Select
Cells(3 + i, 2).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
:=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("C22").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
:=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D40").Select
ActiveCell.Copy
Sheets("Results Output Price").Select
Cells(3 + i, 3).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
:=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D28").Select
ActiveCell.Copy
Sheets("Results Output Price").Select
Cells(3 + i, 4).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
:=False, Transpose:=False
'This code will copy paste the result of the random draw for price in to the model for the alternative system'
Sheets("Estimated Output of Crappie").Select
Range("D105").Select
ActiveCell.Copy

Sheets("Results Output Price").Select
Cells(3 + i, 5).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=
:=False, Transpose:=False
Sheets("Estimated Output of Crappie").Select
Range("D100").Select
ActiveCell.Copy
Sheets("Results Output Price").Select
Cells(3 + i, 6).Select

```

```

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _  

    :=False, Transpose:=False  

Sheets("Estimated Output of Crappie").Select  

Range("D81").Select  

ActiveCell.Copy

Sheets("Results Output Price").Select  

Cells(3 + i, 7).Select  

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _  

    :=False, Transpose:=False  

Next  

'this code replaces the original value of the cells that were changes during the simulation'  

Sheets("Estimated Output of Crappie").Select  

Range("F22").Select  

ActiveCell.Copy  

Range("C22").Select  

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _  

    :=False, Transpose:=False  

'this code brings the user back to the results tab'  

Sheets("Results Output Price").Select  

Application.ScreenUpdating = True  

'this code brings up an alert that the simulation has been completed'  

Beep  

MsgBox "Simulation Complete!"  

End Sub

```

APPENDIX J: AN EXAMPLE OF THE USEFULNESS OF THE GENERAL FORM OF THE TRIANGULAR DISTRIBUTION IN CREATING RANDOMLY GENERATED INPUTS FOR MONTE CARLO SIMULATIONS

In order to explain (24), it is helpful to examine the special case where $\psi = c$. In this case, the probability of drawing a TRN from $f(x; a, b, c)$ on the interval $x \in (-\infty, c]$ is defined below:

$$(93) \quad P(a \leq x \leq c) = \int_{-\infty}^a 0 \, dx + \int_a^c \frac{2(x-a)}{(b-a)(c-a)} \, dx = \frac{c^2 - 2ac + a^2}{(b-a)(c-a)} + \Phi$$

where Φ is the constant of integration. In the special case of $c = \frac{a+b}{2}$ (*i.e.* in which the mode of the PDF is equal to the mean of the lower and upper bounds of the distribution), $P(a \leq x \leq c)$ is always equal to fifty percent. A numerical example helps to illustrate this point. Consider the case where $(a, b, c) = (4, 8, 6)$:

$$(94) \quad P(a \leq x \leq c) = \frac{36 - 48 + 16}{8} = \frac{52 - 48}{8} = 0.5$$

In the case of $< \frac{a+b}{2}$, $P(a \leq x \leq c)$ is always less than fifty percent. Consider the case where $(a, b, c) = (4, 8, 5)$:

$$(95) \quad P(a \leq x \leq c) = \frac{25 - 40 + 16}{4} = \frac{41 - 40}{4} = 0.25$$

Finally, in the case of $> \frac{a+b}{2}$, $P(a \leq x \leq c)$ is always greater than fifty percent. Consider the case where $(a, b, c) = (4, 8, 7)$:

$$(96) \quad P(a \leq x \leq c) = \frac{49 - 56 + 16}{12} = \frac{65 - 56}{12} = 0.75.$$

In the model, the user has option to input the values of a , b , and c for each stochastic determinant of total revenues. Absent information on the value of these parameters, estimates for a and b for each stochastic determinant of total revenues are obtained by consulting the literature. The model imposes the

assumption that $c = \frac{a+b}{2}$. As per the case presented in (26), $P(a \leq x \leq c) = 0.5$ for each stochastic determinant as a result of this assumption. For the i^{th} stochastic determinant of total revenues (λ_i), if it is the case that the actual value of $c_i > \frac{a_i+b_i}{2}$ then $P(a \leq x \leq c)$ has been understated (and $1 - P(a \leq x \leq c)$ has been overstated). The opposite is also true. The effect of an error in the estimation of $P(a \leq x \leq c)$ on the results of a Monte Carlo simulation will depend on the sign of $\frac{\partial TR}{\partial \lambda_i}$. If $\frac{\partial TR}{\partial \lambda_i} > 0$, than an understatement (overstatement) of $P(a \leq x \leq c)$ will result in an understatement (overstatement) of the probability of low total revenues. The opposite is also true.