

DISSERTATION

PRODUCTION FUNCTION ESTIMATIONS AND POLICY IMPLICATIONS

Submitted by

Mohammad Abdallah Khreisat

Department of Economics

In partial fulfillment of requirements

For the Degree of Doctor of Philosophy

Colorado State University

Fort Collins, Colorado

Fall 2011

Doctoral Committee:

Advisor: Harvey Cutler

David Mushinski

Martin Shields

Stephan Davies

## ABSTRACT

### PRODUCTION FUNCTION ESTIMATIONS AND POLICY IMPLICATIONS

The main purpose of the dissertation is to provide the decision makers in local governments in Colorado with a information regarding the economic characteristics of industries and firms they need to attract to their regions to mitigate the inverse impacts of job loss, and local government revenue decreases during economic down turns. The dissertation estimates four different production functions classified in two groups; homogeneous functions, and non-homogeneous functions. The estimation is held at the industry level using firm level data for six major counties in Colorado. This is the first empirical study that explores the importance of land in the production process, in addition to the primary inputs of capital stock and labor. The study also determines the production function that best fits the data structure instead of other studies which assume in priori the type of production function. In addition, the study will explore the returns to scale and elasticity of substitutions for the three input variables (land, labor, and capital) at the industry and firm levels. Furthermore, the dissertation explores the convergence of total factor productivity within industry and among counties, and within county and among the different industries in the same county.

The main findings of the study are: (i) local governments have to attract the industries or firms with low elasticity of substitutions between labor and capital from one side; and industries or firms with low complementarity between land and labor, and capital and land; (ii) land is an important input variable in the production process,

especially in Denver County; (iii) local government has to attract firms with increasing returns to scale because of their positive impact on employment, economic growth, local government revenues, and competitiveness outside the county; (iv) local governments have to encourage firms with high  $k/l$  ratio accompanied with low elasticity of substitutions; and (v) the negative relation between total factor productivity and partial scale elasticity, leads to the conclusion that the industries or firms either substitute TFP with RTS or firms with high RTS delay applying advanced technology.

## ACKNOWLEDGEMENTS

First of all, I would like to thank God for everything. I also would like to thank all the people who helped me during this work. I am especially thankful to my advisor, Dr. Harvey Cutler for his kindness, support, guidance and encouragement during my graduate study. Also, I would like to thank Dr. David Mushinski, Dr. Martin Shields, and Dr. Stephan Davies for agreeing to be on my dissertation committee and reviewing my dissertation.

I would like also to thank my parents, brothers and sisters for their love and support while I was far away from home. Finally, I would like to thank my wife, daughters, and sons for all their love, support and encouragement over the years and while doing my dissertation.

## DEDICATION

To ...

My beloved parents, wife, sons, daughters, brothers and sisters.

## TABLE OF CONTENTS

|   |    |
|---|----|
| ABSTRACT  | ii |
| ACKNOWLEDGEMENTS  | iv |
| DEDICATION  | v  |
| Chapter One: Introduction                                     | 1  |
| Chapter Two: Literature Review                                | 8  |
| 2.1. Levels of Production Functions .....                     | 8  |
| 2.1.1. Hsing, Yu (1996) .....                                 | 8  |
| 2.1.2. Ryohei Nakamura (1985) .....                           | 10 |
| 2.1.3. Leo Sveikauskas (1975).....                            | 11 |
| 2.1.4. Chow et al (2002) .....                                | 13 |
| 2.1.5 Segal, David (1976).....                                | 14 |
| 2.1.6 Leon-Ledesma Et al (2010) .....                         | 15 |
| 2.1.7 Lynch et al (2011).....                                 | 16 |
| 2.1.8 Baldwin et al (2010) .....                              | 16 |
| 2.2. Production Function Homogeneity .....                    | 16 |
| 2.2.1. Edwin F. Ulveling and Lehman B. Fletcher (1970) .....  | 16 |
| 2.2.2. Green, Alison; and Mayes, David (1991).....            | 17 |
| 2.2.3. Vinod (1972).....                                      | 18 |
| 2.2.4. Ringstad (1974) .....                                  | 19 |
| 2.2.5. Laurits R. Christensen, Dale W. Jorgenson (1973) ..... | 20 |
| 2.3. Estimation of Unobservables .....                        | 21 |
| 2.3.1. Blundell and Bond (1998)                               | 21 |
| 2.3.2. Wooldridge, Jeffrey (2009)                             | 22 |
| 2.4. Competitiveness  | 23 |
| 2.4.1. Dobbelaere, Sabien; and Mairesse, Jacques (2008)       | 23 |
| 2.4.2. Basu and Fernald (1995)                                | 24 |
| 2.5. Data Aggregates  | 24 |
| 2.5.1. Basu et al (1997)                                      | 24 |
| 2.6. International Trade and productivity                     | 25 |
| 2.6.1. Taymaz, Erol and Yilmaz, Kamil (2007)                  | 25 |
| 2.6.2. Kasahara, Hiroyuki and Rodrigue, Joel (2008)           | 26 |
| 2.6.3. Harrigan, James (1999)                                 | 26 |
| 2.7. Theoretical Models                                       | 27 |
| 2.7.1. Mundlak (1996)   | 27 |
| 2.7.2. Klump, Rainer and Preissler, Harald (2000)             | 27 |
| Chapter Three: The Data                                       | 29 |
| 3.1 Data Availability   | 29 |
| 3.2 Output in Literature Survey                               | 30 |
| 3.3 Output Estimation   | 33 |
| Chapter Four: Exploring Model Fitness                         | 34 |
| 4.1. Prominent Production Functions                           | 34 |
| 4.1.1. Homogeneous Production Functions                       | 35 |
| 4.1.1.1. The Leontief production function                     | 35 |
| 4.1.1.2. The Cobb-Douglass Production Function                | 36 |
| 4.1.1.3. The CES production Functions                         | 37 |
| 4.1.1.3.1. Linear CES Production Functions                    | 37 |
| 4.1.1.3.2. New CES Production Function                        | 38 |

|   |     |
|---|-----|
| 4.1.2. Non-homogeneous Production Function  | 39  |
| 4.1.2.1 The Translog Function   | 39  |
| 4.2. Capital-Labor, Capital-Land, Land-Labor Ratios   | 40  |
| 4.2.1. Capital-Labor (K/L) Ratio  | 40  |
| 4.2.2. Capital-Land (K/LA) Ratio  | 41  |
| 4.2.3. Land-Labor (LA/L) Ratio  | 42  |
| 4.3. Estimating Production Functions Results  | 43  |
| 4.3.1. Cobb-Douglas   | 45  |
| 4.3.2. New CES Function   | 53  |
| 4.3.3. Non-homogeneous Cobb-Douglas Function  | 54  |
| 4.3.4. The Translog Production Function   | 61  |
| 4.4. Model Fitness Exploring  | 65  |
| 4.5. Summary  | 81  |
| Chapter Five: Exploring Returns to Scale (RTS)  | 94  |
| 5.1 Potential comparison of RTS impact  | 95  |
| 5.1.1. Within county and among industries   | 95  |
| 5.1.2. Within industry and among counties   | 96  |
| 5.2. Measuring Scale of Economies   | 97  |
| 5.3 The results of scale of economies   | 98  |
| 5.3.1 Economies of Scale within County and Among Industries                                       | 98  |
| 5.3.1. Economies of Scale within the Industry and among Counties                                  | 114 |
| 5.3.1.1. Cobb-Douglas Function  | 114 |
| 5.3.1.2. Non-homogeneous Production Function  | 118 |
| 5.3.1.2.1. The Translog Function  | 122 |
| 5.4. Summary:   | 124 |
| Chapter Six: Exploring the Elasticity of Substitutions  | 127 |
| 6.1. Introduction   | 127 |
| 6.1.1 Within County and Among Industries  | 129 |
| 6.1.2 Within Industry and Among Counties  | 129 |
| 6.2. Elasticity of Substitution   | 130 |
| 6.3. Non-homogeneous Cobb-Douglas Function  | 133 |
| 6.3.1. Arapahoe County  | 133 |
| 6.3.2. El Paso County   | 138 |
| 6.3.3. Denver County:   | 141 |
| 6.3.6. Weld County  | 149 |
| 6.4. The elasticity between Inputs by Translog Function   | 151 |
| 6.4.1. Arapahoe County:   | 152 |
| 6.4.2. El Paso County:  | 154 |
| 6.4.3. Denver County:   | 158 |
| 6.4.5. Boulder County:  | 163 |
| 6.5. Summary  | 166 |
| 6.5.1. Capital-Labor Ratio  | 169 |
| 6.6. Capital-Land Ratio   | 170 |
| 6.7. The Relation between K/L Ratio and Elasticity of Substitution between Labor and Capital: 172 |     |
| 6.7.1. Within County and among Industries   | 173 |
| 6.7.1.1. Arapahoe County:   | 173 |
| 6.7.1.2. El Paso County:  | 173 |
| 6.7.1.3. Denver County:   | 174 |

|   |     |
|---|-----|
| 6.7.2. Within Industry and Among County:  | 175 |
| 6.7.2.1. Construction Industry:   | 175 |
| 6.7.2.2. Manufacturing Industry:  | 176 |
| 6.7.2.3. Wholesale Trade Industry:  | 176 |
| 6.7.2.4. Retail Trade Industry:   | 177 |
| 6.7.2.5. Transportation and Warehousing:  | 177 |
| 6.7.2.6. Information Industry:  | 178 |
| 6.7.2.7. Finance and Insurance Industry:  | 179 |
| 6.7.2.8. Real Estate and Leasing:   | 179 |
| 6.7.2.9. Professional, Scientist, and Technical Services Industry:                      | 180 |
| 6.7.2.10. Management of Companies and Enterprises:                                      | 181 |
| 6.7.2.11. Administrative and Waste Management:  | 181 |
| 6.7.2.12. Education Services:   | 182 |
| 6.7.2.13. Health Care Services Industry:  | 182 |
| 6.7.2.14. Art, Entertainment, and Recreation Industry:                                  | 183 |
| 6.7.2.15. Lodge and Restaurants:  | 183 |
| 6.4.2.16. Other Services (Except Public Services):                                      | 184 |
| Chapter Seven: Total Factor Productivity (TFP)  | 185 |
| 7.1. Introduction   | 185 |
| 7.1.1. The Potential Impacts of Differences in TFP within a County and among Industries | 188 |
| 7.1.2. The Potential Impact of TFP within Industry and among Counties                   | 188 |
| 7.2. Exploring Convergence, Jacobs', and Porter's Effects                               | 189 |
| 7.2.1. Convergence and Jacobs' Effect   | 190 |
| 7.2.1.1. Cobb-Douglas:  | 191 |
| 7.2.1.2. Non-homogeneous Cobb-Douglas:  | 193 |
| 7.2.1.3. Translog Function:   | 196 |
| 7.2.2. Exploring Convergence and Porter's Effect  | 198 |
| 7.3.1. Construction Industry:   | 203 |
| 7.3.2. Manufacturing Industry:  | 203 |
| 7.3.3 Wholesale Trade Industry:   | 204 |
| 7.3.4. Retail Trade:  | 204 |
| 7.3.5. Transportation and Warehousing Industry:   | 205 |
| 7.3.6. Information Industry:  | 205 |
| 7.3.7. Finance and Insurance Industry:  | 206 |
| 7.3.8. Real Estate Industry:  | 206 |
| 7.3.9. Professional, Scientist, and Technical Services Industry:                        | 207 |
| 7.3.10. Management of Companies and Enterprises:  | 207 |
| 7.3.11. Administrative and Waste Management Services Industry:                          | 208 |
| 7.3.12. Education Services:   | 208 |
| 7.3.13. Health Care Services:   | 209 |
| 7.3.14. Art, Entertainment, and Recreation Services:                                    | 209 |
| 7.3.15. Lodge and Restaurants:  | 210 |
| 7.3.16 Other services (Except Public Services):   | 210 |
| 7.4. The relation between TFP and RTS   | 210 |
| 7.4.1. The Relationship between TFP and RTS in Arapahoe County                          | 214 |
| 7.4.1.1. Standard Cobb-Douglas  | 214 |
| 7.4.1.2. Non-homogeneous Cobb-Douglas Function  | 215 |
| 7.4.1.3. Translog Function  | 217 |



|  |     |
|--|-----|
| 7.4.2. The Relation between TFP and RTS in El Paso County .....        | 217 |
| 7.3.2.1. Cobb-Douglas Function.....                                    | 218 |
| 7.4.2.2. Non-homogeneous Function.....                                 | 218 |
| 7.5. Summary .....   | 219 |
| Chapter Eight: Recommendations and Policy Implications .....           | 222 |
| 8.1.Data .....   | 222 |
| 8.2.Exploring the production function fitness.....                     | 222 |
| 8.3 Estimation Results .....   | 223 |
| 8.4.Exploring the returns to scale (RTS) .....                         | 223 |
| 8.5.Exploring The Elasticity of substitutions or complementarity ..... | 225 |
| 8.5.1 Non-homogeneous Production Function .....                        | 225 |
| 8.5.1.1 Elasticity between capital and Labor .....                     | 225 |
| 8.5.1.2 Elasticity between Capital and Land.....                       | 226 |
| 8.5.1.3 The Elasticity between Labor and Land.....                     | 226 |
| 8.6 Exploring TFP convergence.....                                     | 227 |
| 8.6.1 TFP Convergence Within County and among Industries .....         | 227 |
| 8.6.1.1 Cobb-Douglas Production Function.....                          | 227 |
| 8.6.1.2 Non-homogeneous Cobb-Douglas Production Function .....         | 228 |
| 8.6.1.3 Translog Production Function.....                              | 228 |
| 8.6.2 TFP convergence Within Industry and among Counties .....         | 228 |
| 8.6.2.1 Cobb-Douglas Production Function.....                          | 228 |
| 8.6.2.2 Non-homogeneous Cobb-Douglas function:.....                    | 229 |
| 8.6.2.3 Translog Production Function.....                              | 229 |
| 8.7 Further studies.....   | 229 |
| References .....   | 230 |

## **Chapter One: Introduction**

The study estimates four different production functions at 2-digit NAICS codes for 6 counties in Colorado by using firm level data. For each firm in a sector, data is available on output, capital, labor (wages and workers), and land (values and area). The access to land data per firm is not common in the literature which allows an examination of the role of land in a production function. The number of workers and wages is obtained from Quarterly Census of Employment and Wages (QCEW); capital and land data are obtained from County Assessor's data; and output is estimated at the firm level based on literature reviewed outlined in chapter 3.

This study estimates four types of production functions for the private sector at the firm level and data for 2-digit NAICS industries for major Colorado counties; Arapahoe, El Paso, Denver, Larimer, Boulder, and Weld. The four production functions are: Cobb-Douglas, new CES as a homogeneous production function; and translog and non-homogeneous Cobb-Douglas functions. The objective is to determine which production function specification best fits the data for a group of firms in a particular 2-digit NAICS classification. The purpose of estimating four different production functions is to understand the behavior of the real economy and how the available resources are utilized. Contrary to most previous studies which assume a priori as the type of production function, this study attempts to let the data determine the proper specification of a production function for an industry. Batten et al (2009), Dobbelaere et al (2008), and Segal (1976) estimated production with Cobb-Douglas function; O'Donnell et al (1979), Sveikauskas (1975) estimate using CES function; Bairam (1989, 1991) applied new

CES; Ringstad (1974) applies the mixed model with left-hand side of new CES and right-hand side of Standard Cobb-Douglas. Other types such as non-homogeneous models are also used. In this case, Christensen et al (1973) applied translog; and Vinod (1972) applied non-homogeneous Cobb-Douglas function.

Furthermore, the study investigates the economies of scale, the elasticity of substitution between factors, and total factor productivity for each 2-digit NAICS sector to obtain important information about characteristics for each county in the study. The returns to scale in the production function reveal the relative economic efficiency of the industry. Increasing returns to scale means that doubling inputs will increase output by more than double while other industries may experience decreasing returns to scale. Decreasing returns leads to inefficiency or misuse of factors of production. These types of estimates provide the policy maker with information about the types of firms to attract in order to be more competitive in today's economy. Industries with increasing returns to scale could provide a county with a greater likelihood of reaching higher long-run growth rates. On the other hand, a county with many decreasing returns to scale industries will have to employ mark up prices at higher levels than constant return to scale in order to survive. Thus, raising the price level for a county and hurting economic growth. This could affect investment in infrastructure projects such as roads or the quality of education.

In addition, the study estimates the elasticity of substitutions for firms in a county. The elasticity of substitution accompanied by the intensity of capital-labor ratio is a priority of planners and decision makers to keep unemployment at low levels during the hard economic times. Thus, to encourage firms and industries in a county requires a full

understanding of the economic characteristics of firms and industries regarding capital intensity and elasticity of substitutions. An industry characterized by low elasticity of substitutions and high capital intensity may benefit a county because an increase in idle capital would be accompanied by a small reduction in employment. Counties dominated by high elasticity of substitutions and low capital-labor ratio industries will experience larger fluctuations in employment during the business cycle.

Another part of the study focuses on estimating total factor productivity (TFP) for industries. The differences in TFP among industries help the decision makers attract firms that enjoy higher total factor productivity because of efficient use of resources and their ability compete at the domestic and the international export levels. In addition, these firms can increase the level of county revenues because of high wages paid to the workers since wages are related to productivity. This will also provide the county with the flexibility to use its revenue more effectively. The importance of estimating the production function stems from different points of view. First, the return to scale in the production function reveals the relative economic efficiency of industries operating in the county, region, state, or nation to produce at economies of scale. Increasing returns to scale means producing more output with the same amount of input. Other industries may experience decreasing returns to scale which means using more inputs to produce the same amount of output. The decreasing returns lead to inefficiency or misuse of factors of production. In addition, the estimation of production function provides the decision maker with information about which industries are more competitive and can offer these industries incentives to stay in the state, region, or county. These highly competitive

industries can grow steadily and export to other regions, states, or internationally, which increases production and employment.

The prominent feature of this study, compared to other studies, stems from the fact that it is the first empirical study that introduces land as one of the explicit variables in determining the production function in the economic empirics. Previous studies include land in the theoretical model and estimate this variable as an omitted variable under constant return to scale, and the land variable reflects all other variables except labor and capital (Nakamura 1985).

Unlike other studies, this study is the first to estimate the production function at the county level, in particular for counties in Colorado. This study also estimates production function at the industry level using intensive cross-section data at the firm level. The literature is full of research that estimates the production function at different aggregate levels. Dobbelaere et al (2008) estimates the production function at the firm level for manufacturing industries in France. Dobbelaere et al include capital, labor, and intermediate inputs in when estimating production function. Nakamura (1985) estimates the production function at the city level in Japan to study the impact of agglomeration on labor productivity. Chow et al (2002) estimate standard production function at the national level for China. A study by Hsing (1996) conducted at the state and national level regarding manufacturing industries in USA.

Estimating different production functions that fit the data structure is highly important for the planner, decision makers, and researchers. Because the production function reflects the real economy and the extent that the production function linkages

interact or become more complicated. A summary of each chapter included in the study follows:

Chapter 2 reviews literature on production function estimates classified into five main groups. These groups are: 1) production function according to economic level, such as firm, industry, county, state, or national; 2) production functions displaying homogeneity; 3) perfect or imperfect competitiveness functions and the input elasticity impact on output and their revenue shares; 4) aggregation of data, such as the difference in estimating production function using output or value added; and 5) demonstration of some theoretical models.

Chapter 3 discusses data sources and problems. The data on land, labor and capital is available, but the data for output at the firm level is problematic or not available directly at the firm level. Thus, the research will estimate output at the firm level following previous research in this regard (Basu et al, 1996, 1997). The data on labor and wages are available from Quarterly Employment of Wages and Salaries (QCEW). Data on capital and land (value, and area) is available from each County Assessor Office. In addition, the study merges these two files to analyze and explore the hypotheses of the study.

Chapter 4 depicts the most prominent production functions in economic theory and empirical research. The study classified the research into homogeneous functions, which include Cobb-Douglas, CES, and the new CES production functions. The second group is the non-homogeneous production function such as nonhomogeneous Cobb-Douglas production function, and translog function. Results of the estimated production functions are illustrated in this chapter. The study explores the production function that

fits the data structure best. For this purpose, the study followed nonnested J-test. The main result of this test shows evidence of inconclusiveness of any model to any other model. Therefore, the analysis in the following chapters regarding returns to scale, elasticities, and total factor productivity will be based on this result.

Chapter 5 illustrates the returns to scale for the three production functions that the study continues to investigate (Cobb-Douglas, nonhomogeneous Cobb-Douglas, and translog functions). The main results show that the high services industries mostly behave as increasing returns to scale in the three functions. These industries include the NAICS industries from 51 to 56. There are other non-common industries that behave as increasing returns to scale especially in Arapahoe and El Paso Counties. In addition, the magnitude of returns to scale in nonhomogeneous Cobb-Douglas functions is higher than that of standard Cobb-Douglas function. This suggests that to double output less than double input combination is needed according to Cobb-Douglas, and less than that is needed in non-homogeneous function. Also, the proportion of the input combination is changed by using non-homogeneous function while it is fixed in standard Cobb-Douglas.

Chapter 6 explores the partial elasticity between capital and labor, capital and land, and land and labor. According to Hicks (1970) the elasticity between primary inputs, labor and capital, is a substitute. But for more than two inputs, the elasticity between the third factor of input or more is either a substitute or a complement between that input and the primary ones. Thus, the elasticity between labor and capital is a substitute and on average is greater than 1 in most of the industries in each county. The best plan for local governments is to encourage firms with low elasticity of substitution between labor and capital to limit the impact of job loss during economic down turns.

The elasticity between labor and land is small and in most industries, on average, is around zero. This means that the elasticity between these two inputs is like Leontief. Thus, these two inputs are expected to be reduced in the same proportion during economic downturns.

Chapter 7 explores the total factor productivity at the industry and firm levels in the six counties included in the study. This chapter has two main parts. The first discusses convergence of TFP according to Bernard and Jones (1996), and explores Jacobs' effect. The results show evidence that there is convergence or Jacobs' effect in TFP in different industries in the county. Knowledge spillover flows through different industries because of proximity and expected high education of the workers in Colorado. The second concept explored is convergence within industry and among counties in TFP and at the same time a test for Porter effect in TFP. The study shows that there is convergence in TFP.



## **Chapter Two: Literature Review**

This Chapter discusses the economic literature reviewed related to the research topic. This section divides the production function literature review into seven groups. The first group deals with production function according to economic level, such as firm, industry, county, state, or national. The second group of production functions displays the homogeneity of production functions. The third group depicts perfect or imperfect competitiveness functions and the input elasticity impacts on output and their revenue shares. The fourth section reviews competitiveness. The fifth part reviews the type of aggregation of data such as the difference in estimating production function by using output or value added. The sixth part reviews international trade and productivity. The seventh group demonstrates some theoretical models.

### **2.1. Levels of Production Functions**

Production functions are estimated at different levels such as firm, industry, city, or nation. Here is a partial illustration on the literature that attempts to shed light on this part.

#### **2.1.1. Hsing, Yu (1996)**

Hsing examines five different types of production functions to investigate which production function best fits the data for the manufacturing industry in the U.S. In addition, the paper explores the economies of scale at the national and state level. The author uses cross-sectional data from the Census of Manufacturers for 1987. He uses value added as a representative to output. For capital data, he uses the data from the US

Department of Commerce. For value added and number of employees, the author employs the census data.

This study applies five different production functions: Cobb-Douglas, Translog, CES of Arrous et al (1961), new CES of Bairam (1989) function, and Generalized Leontief Production function. In the estimation, the researcher uses value added, net structures and equipment, and total number of workers to represent output, capital, and labor. The data is from the 1987 Manufacturers Census. He deflated value added and structures and equipment by producer price index for manufacturing and implied deflator for structures and equipment to express these variables in real terms.

The main results of the study at the national level are: the Leontief production function estimate is inconclusive because the negative is insignificant of the term  $K^{0.5} L^{0.5}$ , while Translog and CES reveal heteroskedasticity. Therefore, the researcher used WLS to correct for heteroskedasticity for Cobb-Douglas, CES, and new CES. The CES has correct signs but the elasticity of substitution is negative. The value of return to scale which is 1.01 suggests that the economy is operating within constant return to scale under CES estimates. Also, the manufacturing sector exhibits constant return to scale within Cobb-Douglas estimates. In the new CES, at the mean, the output elasticity of labor and capital are 0.78 and 0.23. The elasticity of substitution is 1.56 which is different from unity as suggested by Cobb-Douglas.

At the state level, the economies of scale ranged between 0.85% for Washington DC and 1.12 for New Mexico. The main conclusion of the study is that the new CES production function is the most appropriate function that fits the data under study.

### 2.1.2. Ryohei Nakamura (1985)

Nakamura studies the impact of agglomeration economies productivities for manufacturing industries at a two-digit SIC level in Japan. In this study, the researcher uses a cross-section data of Japanese cities in 1979. The study focuses on the impact of urbanization and localization in agglomeration as an important shift factor. In the paper, the author adopted a firm-level production function with localization and urbanization affects. The form of the production function is:

$$v_{ij} = g_j(p_i) f_j(k_{ij}, l_{ij}, e_{ij}; V_j)$$

Where:  $v_{ij}, k_{ij}, l_{ij}, e_{ij}$  are value added, capital, labor, and land input of firm  $j$  in city  $i$ , and  $V_j$  is total value added of the industry in which firm  $j$  is included in city  $i$ . The function  $g_j(p_i)$  is the firm-specific function assumed to be independent of the production technology in firm  $j$ , and  $p_i$  is the urban population of city  $i$ . The urbanization economies assume to be in the form  $g_j(p_i) = \alpha_1 P_i^{\alpha_p}$ . The urbanization occurred if  $\alpha_p$  is positive because firms in large cities experience productive advantage.

The main assumptions of the study are that the firms in the industry are competitive and that production function is constant return to scale with regard to three inputs, namely capital, labor, and a composite input, land. The land variable includes all other inputs except labor and capital. In the estimation of the production function, the author treats land as an omitted variable. The estimated coefficient of land is computed as  $1 - \alpha_K - \alpha_L$ .

In the estimation, the author uses iterative three-stage least square technique to simultaneously estimate the translog production function and the cost share equation of labor. The instrumental variables are: city population, city total employment, city

population density, city total land area, and total tangible capital stock of all manufacturing industries in the city. The researcher estimated a translog production function and a share equation of the following forms:

$$\ln \frac{V_i}{L_i} = \frac{\alpha_0}{1-\alpha_s} + \sum_m \frac{\alpha_{cm}}{1-\alpha_s} \ln C_{im} + \frac{\alpha_p}{1-\alpha_s} \ln P_i + \frac{\alpha_k}{1-\alpha_s} \ln \frac{K_i}{L_i} + \frac{\alpha_L}{1-\alpha_s} \ln L_i + \frac{1}{2} \frac{\beta_{KK}}{1-\alpha_s} (\ln \frac{K_i}{L_i})^2 \dots\dots\dots (2.1)$$

$$M_i = \alpha_L + \beta_{KL} \ln \frac{K_i}{L_i} \dots\dots\dots (2.2)$$

Where:  $C_{im}$  are the city-specific characteristics such as density, climate, and a dummy variable to reflect the metropolitan area, and  $\alpha_s$  is the parameter of localization and if  $\alpha_s$  is positive then the industry exhibits increasing return to scale.

The main findings are that Cobb-Douglas function does not fit the data structure of Japanese industry: population density which reflects urbanization is significant in 14 out of 19 industries included in the estimation; the urbanization parameter  $\alpha_p$  is significantly greater than zero to reflect urbanization in 9 industries but its value is small (0.0336). This means that the productivity of labor increased by 3% while doubling the population size. The light industries experience more urbanization and are statistically significant in large cities (0.06); this is in furniture and fixtures, printing and publishing. While heavy industries experience more localization experience. As previously mentioned, the study assumed a constant return to scale. Thus the researcher computed the elasticity of land indirectly  $1-\alpha_k - \alpha_l$ . The results show that the elasticity of land is significant in 10 industries and it ranges between 0.186 for SIC 39 and 0.458 for SIC 30.

**2.1.3. Leo Sveikauskas (1975)**

In his article titled ‘The Productivity of Cities,’ the author explores if city size affects productivity. He argues that as a city grows in size productivity is expected to

increase. In this case, the big city permits more specialization and a greater division of labor and this will lead to higher productivity. In the study, the author uses a two-digit SIC for 14 industries covering 1967 data. He assumes that each city encounters the following CES production function of the following form:

$$Q_i = g_i(dK_i^{-t} + (1 - d)L_i^{-t})^{-1/t} e^{u_{oi}} \dots\dots\dots (2.3)$$

Where: Q=Output value, K = Capital input, L= labor input, d=distribution parameter, and the elasticity of substitution is computed as  $s= 1/ (1+t)$ ,  $g$  =Hicks-neutral productivity,  $i$ = city number.

The author assumes that  $d$  and  $t$  are the same for all cities but the cities are different in  $g_i$ ,  $e^{u_{oi}}$  is the disturbance term. The impact of city size entered the equation through the Hicks neutral productivity term. This type is expressed formally by:

$$\text{Log}(g_i) = a + b \log(\text{Pop}_i) + u_{1i} \dots\dots\dots (2.4)$$

Where:  $\text{Pop}_i$  = population of the SMSA.

The capital data is not available for SMSAs, therefore, the modified equation estimated is:

$$\text{Log}\left(\frac{wL}{V}\right)_i = s \log(1-d) + (s-1) \alpha + (s-1) b \log \text{Pop}_i + (1-s) \log w_i + \text{random term}$$

Where  $s$  is the elasticity of substitution,  $s=t / (1+t)$ .

To avoid biased estimation as a result of expected correlation between  $w_i$  and  $g_i$ , the author estimates the impact of city size in productivity directly in the following equation:

$$\text{Log}\left(\frac{V}{L}\right)_i = a + b \log \text{Pop}_i + c \log \text{Educ}_i + \text{random term.}$$

Where:  $Educ_i$  is the median years of education completed to reflect the differences in labor quality among cities.

The estimation results show population size is significant in 11 industries. This means that the city size has a great effect on productivity. The results show that as city size doubles productivity of labor increases by 6.4%. Education has a positive impact on labor productivity and is statistically significant in 6 out of 14 manufacturing industries.

The author investigates another hypothesis regarding city size's impact on the wages. He finds that if the city size is doubled the wage is increased by 4.8%. Also, the author investigates the impact of city size on the capital-labor ratio between two cities.

For this investigation, the author uses the following formula:  $\frac{(K/L)_A}{(K/L)_B} - 1 = (1 - b)^s - 1$ .

Where:  $b$  is the coefficient of population impact on wages,  $s$  is the estimated elasticity of substitutions, and  $(K/L)_A$  and  $(K/L)_B$  are capital labor ratio in city A and city B. By applying this formula, the researcher find that the higher wages paid in cities will increase  $K/L$  by about 2-6% as the city size doubles. This means that city size has an important effect on the increase in labor productivity.

#### **2.1.4. Chow et al (2002)**

The authors estimate the standard Cobb-Douglas production function for China's economies. In the regression, they add time to reflect the technical progress. The purpose of the paper is to shed light on the forecast of China's GDP up to 2010, and to estimate economies of scale for China's economy. In this paper, the authors exploit annual data that covers the period 1952-1998, excluding the data for the period 1958-1969. Data for capital stock and GDP are in real terms, while labor data are employee numbers. In this

model, the authors add time to control for technical progress. The following is the estimated production function:

$$\ln \text{GDP}_t = \alpha_0 + \alpha_1 \ln K_t + \alpha_2 \ln L_t + \alpha_3 t \dots\dots\dots (2.5)$$

The researchers use OLS statistical technique to estimate the previous model. The estimation shows that the Cobb-Douglas production function is constant return to scale in both inputs of labor and capital together. The elasticity of output with respect to capital is 0.6136 and for labor is 0.4118. The technical progress shows that there is an average annual technical progress in the Chinese economy of 2.62% for the period 1978-1998.

**2.1.5 Segal, David (1976)**

The author’s paper titled, ‘Are There Returns to Scale in City Size’ investigates the hypotheses that wages and output per worker in large cities are greater than in small cities. The estimated sample size is 73 SMSAs, which is the number of areas represented in at least 4 consecutive censuses of manufactures. The sample size was reduced to 58 SMSAs because data was lacking in several observations. OLS is estimated using the logarithmic form. The author assumes that the aggregate urban output in city *i* is determined by the following production function:

$$Q_i = AS^\gamma C_i^\delta K_i^\alpha L_i^{\sum_k \beta_k a_{ik}} \dots\dots\dots (2.6)$$

Where:

$Q_i$ = real output or value added in production

$K_i$ = the city capital stock

$L_i$ =employment

$q_{ik}$ = vector of labor quality reflected the composition of education, age, sex, and race

$C_i$ = Vector of site characteristics

$\delta$ = elasticity of  $C_i$  characteristics

$A$ = transformation coefficient

$S$ = dummy variable for size with  $\gamma$  is its elasticity

Segal finds that the largest SMSAs with populations of 2 million or more had a return to factors 8% higher than the SMSAs with populations less than 2 million. The author attributes these differences to the differences in capital/labor ratio, where this ratio increases with city size; the possible explanation of economies of scale may exist with the city size, and the possibility of differences in the constant term among the SMSAs although the difference of labor and capital coefficients are not statistically significant.

### **2.1.6 Leon-Ledesma Et al (2010)**

In this study, the authors explore different types of estimation of CES functions. These types of estimation are a single equation and system equation. This investigation is confined to US manufacturing industries. They estimate a normalized CES production by the geometric mean of output, labor and capital. The estimated model includes three equations: the output equation, marginal product to labor, and marginal product to capital. The main finding is that “the jointly modeling of the production function and first-order conditions is superior to single equation approaches in terms of robustly capturing production and technical parameters, especially when merged with normalization”.



### **2.1.7 Lynch et al (2011)**

In their study, the authors investigate whether industries zones receiving subsidies perform better than industries operating out of such zones. They test the hypotheses in Colorado enterprise zone programs (EZP). The authors use data of ES202. The main results are: EZs have no effect on monthly payroll per worker in establishments with more than 10 employees. Also, workers in small establishments have reductions in their payroll per worker. The authors attribute this to the substitute of low skill workers to high skill workers. In addition, EZs have positive and significant impact on small firms' employment. Furthermore, only manufacturing industries in EZs have positive effects on employment compared to non EZ areas in Colorado.

### **2.1.8 Baldwin et al (2010)**

In their research, the authors examine the impact of co-location on productivity of labor. In this paper, the authors follow Rosenthal and Strange's papers (2001, 2003) to measure the concentration of own industry impacts on productivity in Canada. The main findings are: productivity of labor increases with the number of plants in own industry (MAR effect) and within a nearby distance. Also, they find that the plants within 5 kilometers exert positive and significant effects on labor productivity, while the impact of farther distances is insignificant.

## **2.2. Production Function Homogeneity**

### **2.2.1. Edwin F. Ulveling and Lehman B. Fletcher (1970)**

The authors suggest in their study titled 'Cobb-Douglas Production Function with Variable Return to Scale' that different production techniques lead to different

production elasticities and scale of return. In this study, the researchers include land, labor, and capital to estimate a nonhomogeneous Cobb-Douglas production function with variable elasticity of substitution for farm land in Mexico. The estimated production function is:

$$\ln Y = \ln A + B_1(I) \ln X_1 + B_2(I) \ln X_2 + B_3(I) \ln X_3 \dots\dots\dots (2.7)$$

Where: X1=Land, X2=Labor hour, X3=Capital, I = a quantitative variable at least has two derivatives. This variable may be the size, managerial ability, type of capital, or different quality of labor. In this study I variable is capital services per hectare of land.

The main finding of the study is that the production elasticity of capital decreases as the capital-to-land ratio increases. At the same time, the elasticity of labor increases as the ratio of capital to land ratio increases. Also, as more capital intensive techniques are exploited in the production process the economies of scale increase by levels.

**2.2.2. Green, Alison; and Mayes, David (1991)**

The study tests the hypothesis of technical inefficiency in manufacturing industries in the United Kingdom. The sample includes 19,023 establishments in 151 industries from the 1977 Annual Census of Production. The study applies translog stochastic frontier production functions. The residuals were divided into two components, one to measure inefficiency and the other to measure the unobservable random effects. The estimated equation is:

$$\ln Q = a_0 + a_1 \ln L + a_2 \ln K + a_3 (\ln L)^2 + a_4 (\ln K)^2 + a_5 (\ln L \ln K) + \sum_{i=6}^m a_i X_i + e$$

Where: L=Labor, K=Capital, Q=Output,  $e=u+v$ ,  $u$ = Random effect, and  $v$ = Technical inefficiency effect.  $X$ = vector of variables reflect the structural characteristics of the establishment.

The main finding of the paper is that the measure of technical inefficiency shows 48 industries have positive skewness which means these industries experienced low inefficiency. Also, the study shows that 31 industries experienced high levels of inefficiency in UK. Other industries are not used for the inefficiency study because of lack of data.

### 2.2.3. Vinod (1972)

Under the title ‘Nonhomogeneous Production Function and Application to Telecommunication,’ Vinod (1972) applies a non-homogeneous quadratic Cobb-Douglas production function to estimate the elasticity of substitution and economies of scale for Western Electric Company. In this production function, the author adds the product of the logs of the capital and labor inputs. Thus, the expected type of production functions has variable elasticity of substitutions (VES), and variable return to scale.

The non-homogeneous production function is of the following form:

$$Y = e^{a_0} X_1^{a_1 + a_3 \ln X_2} X_2^{a_2}$$

$$\begin{aligned} \ln Y &= a_0 + (a_1 + a_3 \ln X_2) \ln X_1 + a_2 \ln X_2 \\ &= a_0 + a_1 \ln X_1 + a_2 \ln X_2 + a_3 \ln X_2 \ln X_1 \end{aligned}$$

The scale elasticity then:

$$E = a_1 + a_2 + a_3 \ln(X_2 X_1)$$

And the elasticity of substitution is:

$$\sigma = (a_1 + a_2 + a_3 \ln(X_2 X_1)) / (a_1 + a_2 + a_3 [\ln(X_2 X_1) + 2])$$

Based on 59 observations, the author conducted estimation for Western Electric after adding an engineer variable. The estimation result is the following:

$$\ln Y = -11.577 - 4.918x_1 + 1.999x_2 + 11.820x_3 + 0.976x_1x_2 + 0.583x_1x_3 + 1.93x_2x_3$$

(7.29)   (2.56)   (6.27)   (3.56)   (2.6)   (5.97)   (2.96)

Where:  $x_1$ =Capital,  $x_2$ = Labor,  $x_3$ = Engineering

Another aggregate production function is conducted for the Bell System for the period 1947-1970. In this estimation, the author includes labor, net capital, and the product of them as input variables and value added as a proxy for output. The main findings are that the elasticity of substitution varies and is not unity as in the standard Cobb-Douglas production function, and the predicted scale of return also varies and is not fixed.

#### **2.2.4. Ringstad (1974)**

Ringstad writes an article regarding the decreasing return to scale in non-homogeneous Cobb-Douglas production function. The author applies the study at the establishment level. The study includes two types of data. The first is across sectional data of 20,994 of Norwegian establishments for mining and manufacturing industries. This data is from the 1963 Census of Norwegian Mining and Manufacturing. The second set of data includes 907 large establishments in Norwegian mining and manufacturing for the period 1959-1967. The second set of data includes only large firms employing at least 100 workers.

The author is testing whether the conventional Cobb-Douglas production function fits the data under study or the non-homogeneous Cobb-Douglas production function

best fits. The researcher uses both direct and indirect cost production function. He uses a nonlinear maximum likelihood estimate method by using Cox-Box transformation for the left-hand side only, while the right-hand side is of conventional Cobb-Douglas production function. The form of the production function is:

$$\ln V_i^\lambda = \mu \ln L_i + \beta \ln (K/L)_i + u_i$$

Where: V=Value added, L=Labor, K=capital, U= Disturbance term,  $\lambda$ =Cox-Box transformation.

The study finds that the mining and manufacturing industries exhibit decreasing return to scale for the time series data. For census data, the study finds that 10 out of 15 industry groups exhibit decreasing return to scale and conventional Cobb-Douglas does not fit the data under study.

#### **2.2.5. Laurits R. Christensen, Dale W. Jorgenson (1973)**

In their article about ‘Transcendental Logarithm Production Frontier’, the authors say that “constant elasticity of substitution has to be fruitful point of departure for the analysis of production function with one output and two factors of production, as in Arrow, Chenery, Minhas, and Solow. For more than one product or more than two factors of production, constant elasticity of substitution and transformation is highly restrictive (Uzawa, 1962, and McFadin, 1963)”.

The objective of the study is to develop tests of the theory of production that don’t employ additives and homogeneity. Their purpose is to represent the production frontier quadratic in the logarithmic of the quantities of inputs and outputs. The resulting frontiers permit a greater variety of substitution and transformation patterns than frontier

pattern based on constant elasticity of substitution and transformation. Therefore, the authors apply a translog production function. The authors apply time series data for private US economy covering the period 1929-1969. The main finding of the study rejects the homogeneity and additively hypotheses in production frontier taking into account two inputs and two outputs.

### 2.3. Estimation of Unobservables

#### 2.3.1. Blundell and Bond (1998)

The authors apply GMM estimation on panel data to Cobb-Douglas production function at the firm-level. The authors apply the technique of first difference method of GMM to eliminate the unobserved specific-effects. The paper tests the Griliches and Mairesse statement regarding the application of panel data at the micro-level. In 1997 Griliches and Mairesse said “in empirical practice, the application of panel methods to micro-data produced rather unsatisfactory results: low and often insignificant results of capital coefficients and unreasonable low estimates of return to scale.”

The authors apply the following Cobb-Douglas production function:

$$Y_{it} = \beta_n n_{it} + \beta_k k_{it} + \gamma_{it} + (\eta_i + v_{it} + m_{it})$$

$$v_{it} = \rho v_{i,t-1} + e_{it} \quad |\rho| < 1$$

$$e_{it}, m_{it} \sim MA(0)$$

Where:

$Y_{it}$  = log sales of firms.

$n_{it}$  = log employment.

$k_{it}$  = log capital stock.

$\gamma_t$ = year specific intercept.

$\eta_i$ = an unobserved firms specific effects.

$v_{it}$ = a possibly autoregressive shock.

$m_{it}$ =serially uncorrelated measurement error.

The researcher used annual balanced panel data for 509 R&D-performing US manufacturing companies for the period 1982-1989. Capital stock and employment measured at the end of firm's accounting year. Sales used as a proxy to output.

The main findings are: (i) in the presence of firms-specific effects, OLS levels give a bias estimate of the coefficient in the lagged dependent variable. Also, no constant returns to scale appear under OLS; (ii) The Difference GMM reveals no constant return to scale and the  $\beta_k$  is weak and statistically significant at 10% level; and, (iii) in the system GMM,  $\beta_k$  is higher and reasonable than the difference GMM.

### **2.3.2. Wooldridge, Jeffrey (2009)**

This paper estimates firm-level production function using proxies to control for unobservables. To do this, the author suggests a theoretical generalized method of moments to be estimated instead of two step estimation for the unobservable variables as applied by Olley and Pakes (1996); and Levinshon and Pertin (2003). Olley and Pakes applied investment in their research to represent the unobservable effects, while Levinshon and Pertin apply intermediate goods. According to the author, the Generalized GMM estimation method provides robust standard errors, and eliminates serial correlation problems.

## **2.4. Competitiveness**

### **2.4.1. Dobbelaere, Sabien; and Mairesse, Jacques (2008)**

The authors write a paper titled ‘Panel Data Production function and Product and Labor Market Imperfection’. In this paper, the researchers extended Hall’s (1988) microeconomic model of estimating price cost margin. For this purpose, they used two types of imperfect labor markets. The first is the efficient bargaining model and the second is the monopsony model. The aim of the paper is to investigate if product and labor market imperfection show differences between the estimated coefficients of the production function and their related share revenue. Because in perfect competition markets of product and labor, the output elasticities of each production factor equal its revenue share.

To test the previous hypothesis, the authors used annual unbalanced panel data for 10,646 French firms in 38 manufacturing industries. The data covers the period 1978-2001. They classified this data into six groups according to the type of product and labor market competitiveness.

In their work, they estimate the Cobb-Douglas production function by imposing and non imposing constant returns to scale. They use data on output, capital, intermediate inputs, and average number of employees during the year for each firm. The estimation is conducted at the manufacturing and industry levels. The authors use different methods of estimations, namely, OLS level, OLS difference, GMM difference, GMM system, dynamic GMM difference, dynamic GMM system.

The main findings are that the OLS results show a constant return to scale, while other methods of estimate show decreasing return to scale ranging from 0.688 in GMM



difference and 0.969 for GMM system method. Furthermore, they find that the output elasticity of inputs is different than their revenue share. This means that the output market is working under imperfect competition in France.

#### **2.4.2. Basu and Fernald (1995)**

The authors investigate if imperfect competition is one way of explaining increasing return to scale and differences of input use. The authors employ data of 34 industries on US private economy at two-digit level manufacturing industries. On average, they find that a typical industry has a constant or decreasing return to scale. Also, they investigate the hypothesis for durable and nondurable industries. The main finding of Basu and Fernald is that durable manufacturing industries exhibit increasing return to scale, while non-durable industries exhibit decreasing returns to scale.

### **2.5. Data Aggregates**

#### **2.5.1. Basu et al (1997)**

They investigate return to scale in private US industries. In their paper, the authors explore the impact of different levels of aggregation on return to scale. For example, the paper tests economies of scale at the industry level, manufacturing level, and total private economy. Furthermore, the paper tests the impact of data source on economies of scale, such as using gross output, direct value added, or computed value added to reflect the bias of estimation from omitted variables when applying firm-level production function for different level of data aggregations. In this study, the authors apply a firm-level production function to estimate higher aggregate data. The firm-level production function is:

$$Y = f(K, L, M, T)$$

Where:

Y= output or value added.

K= capital, L= labor, M= intermediate inputs and energy, and T= state of technology.

The data used by the authors covers the period 1959-1989. This data cover the total private industries in US economy at 2-digit SIC, of which 20 manufacturing industries.

The data contains information regarding primary inputs (labor and capital), intermediate inputs, gross output, and direct value added data.

The researchers estimated the return to scale by using OLS and 2SLS weighted and not weighted by the relative importance industry. They estimate the return to scale at the total private economy, total manufacturing, and divide manufacturing into durable and nondurable. The main findings reveal that there is a bias in estimating return to scale from direct value added data; the return to scale for gross output is higher than the direct value added estimate, but less than computed value added which takes into account omitted variables; and the return to scale is constant for total private sector for computed value added, increasing for durable manufacturing, and decreasing for nondurable manufacturing.

## **2.6. International Trade and productivity**

### **2.6.1. Taymaz, Erol and Yilmaz, Kamil (2007)**

The authors investigate the impact of trade policy reform on productivity growth in Turkey. Turkey started trade reform in the 1980s to the early 1990s. The authors divided

the economy into three sectors: the import-competing, export oriented, and non-traded sectors. The data covered 51 four-digit SIC industries during the period 1984-2000.

The study applies Cobb-Douglas production function in log levels. The study includes labor, capital, material inputs, electricity, fuel, and productivity term observed by econometricians. The main findings are: productivity gain is largest during periods of rapid decline protection rates, and productivity gain is higher in import competing industries than in export-oriented and non-traded sectors.

### **2.6.2. Kasahara, Hiroyuki and Rodrigue, Joel (2008)**

Their paper titled, ‘Does the Use of Imported Intermediates Increase Productivity?’, explores the impact of importing intermediate goods on plant performance. The expected reason behind improving plant performance is due to technology diffusion through internationalization by adoption and imitation of imported technology. The authors used data at plant level for Chilean manufacturing panel data. They applied Cobb-Douglas production function by employing GMM method of estimation. The main finding of the study is that importing foreign intermediate inputs improve productivity in Chilean manufacturing industries.

### **2.6.3. Harrigan, James (1999)**

Based on the international trade theory about differences in TFP among countries, the author tests two hypotheses, the first one is returns to scale with country specific technology differences, while the second hypothesis is the Industry-level scale economies with identical technology in each country.

In his estimate, he used feasible generalized least squares FGLS estimator, a weighted regression given by  $\frac{1}{\hat{\sigma}_{ij}}$ . The author estimates the following production function:

$$\ln\left(\frac{Y_{cjt}}{L_{cjt}}\right) - \ln \mu_{ct} = \beta_{0c} + \beta_{1j}t + \alpha_j + \alpha_{1j} \ln\left(\frac{K_{cjt}}{L_{cjt}}\right) + \gamma_j \ln L_{cjt} + \epsilon_{cjt}$$

$\mu_{ct}$  = GDP gap = (actual GDP/ Potential GDP) which is a measure of capacity utilization.

The main findings are that constant return to scale is supported by increasing return to scale for the same technology hypothesis, and there are large and persistent TFP differences among industrialized countries in the 1980's.

## **2.7. Theoretical Models**

### **2.7.1. Mundlak (1996)**

The author introduces a statistical model to estimate the production function at the firm-level by applying the concept of duality or the indirect estimate of production function through cost function. The researcher suggests that this type of estimation will give consistent and more efficient estimates than the direct production function. The reason behind that is the input variables may be determined endogenously.

### **2.7.2. Klump, Rainer and Preissler, Harald (2000)**

The researchers' paper titled, 'CES Production Functions and Economic Growth' theoretically examines the consistency of using different CES production functions in growth models. The authors find that a higher elasticity of substitution leads to a higher steady state and possible permanent growth in the economy. It also pointed out that the effect of higher elasticity of substitution on the speed of convergence depends on the relative scarcity of the factor of production.

The authors examine the following 4 variant linear homogeneous CES production function:

1. Pitchford (1960): 
$$Y = [aK^\varphi + bL^\varphi]^{\frac{1}{\varphi}}$$
2. Arrow et al (1961) 
$$Y = C[aK^\varphi + (1 - a)L^\varphi]^{\frac{1}{\varphi}}$$
3. David and von de Klundert (1965) 
$$Y = [(BK)^\varphi + (AL)^\varphi]^{\frac{1}{\varphi}}$$
4. Barrow and Sala-i-Martin (1995) 
$$Y = C[\alpha(BK)^\varphi + (1 - \alpha)[(1 - B)L]^\varphi]^{\frac{1}{\varphi}}$$

Where: 
$$\varphi = \frac{\sigma - 1}{\sigma}$$

In equation 2, they assume substitution parameter  $\varphi$ , a restricted distribution parameter  $a$ , and efficiency parameter  $C$  which is considered neutral in the sense of Hicks. There is restriction on the technical progress. In equation 3, in order to introduce the nature of technical progress, restricted or unrestricted, David and Klundert introduce  $A$  and  $B$  for the efficiency of capital and labor. The authors find that the first model is unstable, the second model is stable and moves toward a steady state, the third moves to permanent growth, while the fourth results from the fact that with a high elasticity of factor substitution ( $\varphi > 0$  or  $\sigma > 1$ ) the marginal product of capital bounded away from zero. The existence and stability of the steady state requires that both factors are essential for production.

## **Chapter Three: The Data**

To construct a data set that permits an estimation of production function at the industry level, data for land, capital, and labor are available straight the firm level to the researcher. While determining a measure to output or proxy to output is the main challenge that faces the research. Thus, the research will discuss literature survey to estimate a measure to output that permits an estimate of the production function at the industry level for major counties in Colorado. The rest of the chapter will take into account the sources of data availability, brief review of the output definitions and their proxies in empirical work, and the processes to estimate output at the firm level for each industry in each county.

### **3.1 Data Availability**

There are two sources of data regarding land, labor, and capital. The first is the Quarterly Census of Employment and Wages (QCEW). In this census, the Colorado Department of Labor collects data on the number of workers in each firm, and the corresponding wages paid the workers. This data is collected quarterly. From theoretical point of view, every firm irrespective of its size, by law, is required to provide the Colorado Department of Labor such information.

The second source for the data is the County Assessors Office. This Office keeps records on the use of each parcel of land in the county because property taxes differ between commercial and residential properties. Each county assessor has codes that identify commercial parcels for most producing sectors and residential housing categories. In each parcel the data includes land area in square feet or acres, market

values for land, and the capital structures (machines, equipment, and fixtures) on the land. The County Assessor provides excellent data on land and capital.

These two sources collect information for different purposes. The County Assessor collects data on firms and residential areas for the purpose of property taxes for local governments. While QCEW collects data from the firm level regarding labor subjects such as wages, employment, unemployment, and unemployment insurance, etc. Theoretically, the numbers of firms in the two sources are expected to be equivalent. But when the two files were merged according to address and business names, the number of observations reflects about 20% of the QCEW observations. This is due to different business names and address registration between these two sources.

### **3.2 Output in Literature Survey**

Based on reviewing the economic literature that applies to production function, this research will use better and more comprehensive data on output compared to the data used by previous research to estimate production function. For instance, table 3.1 illustrates that most other research use value added as a proxy to output at the industry level, not cross-sectional firm level data to estimate industry production function. In addition, most of the estimated functions are confined to manufacturing industries, while this study will estimate the production function for all private sector industries.

This is the first empirical research study that incorporates land variable as one of the factors determining production function which no other empirical research includes. For instance, Nakamura (1985) conducted a study on Japan to examine the impact of agglomeration economies on productivity for manufacturing industries in Japanese cities

in 1979. As a consequence of lack information on output data, the author adopted a firm level production function using value added as a representative to output.

**Table 3.1. Summary of Output and Proxy to Output in Different Research**

|     | <b>Researcher</b>              | <b>Output and Proxies to Output</b>  | <b>Homogeneity</b>   | <b>Level of Estimation</b> |
|-----|--------------------------------|--|--|----------------------------|
| 1   | Hsing (1996)                   | Value added  | Homogeneous  | Manufacturing              |
| 2   | Ringstad (1974)                | Value added  | Nonhomogeneous   | Establishment              |
| 3   | Basu et al (1997)              | Gross output<br>$Y=f(K,L, IM, E,T)$<br>Value added<br>Computed value added | Homogeneous<br>K= capital<br>L=Labor<br>IM= Intermediate inputs<br>E= Energy | US industry                |
| 4   | Basu (1996)                    | Output   | K,L,M,E,T  | US industry                |
| 5   | Chow et al (2002)              | GDP  | Homogeneous  | National (China)           |
| 6   | Bernard and Jones (1996)       | Value added  |  | 14 OECD country            |
| 7   | Dobbelaere et al (2002)        | Output from firm statements  | Homogeneous  | French firms               |
| 8.  | Blundell and Bond              | Sales  |  |                            |
| 9.  | Cutler and Davies (2007, 2009) | $y=f(K,L, La, IM, M)$  | Homogeneous  |                            |
|     | CGE Model                      |  |  |                            |
| 10. | This study                     | $y=f(K, L, La)$  |  | Industry and Firm Levels   |

Largely due to lack of data on output, other research generally employed proxies to output in estimating production functions or productivity movement. Theoretically, the proxies to output are sales, revenues, and value added. For instance, Hsing's (1996) work estimated five different production functions for manufacturing in US at the



regional and national level using the data from the Manufacturer Census 1987. Because the output data is not available, the author uses value added as a representative to output. In this context, Ringstad (1974) estimates a non-homogeneous production function at the establishment level for 20,994 Norwegian establishments. The data in the study includes value added as a representative to output.

Another study by Basu et al (1997) estimates return to scale in US industries. The authors use gross output, value added, and computed value added to test the impact of different output aggregation on economies of scale. The data includes gross output, value added, capital stock, labor, intermediate input and energy. The paper finds computed value added, not direct value added or gross output, is the best data to reflect economies of scale for US industry. In this paper, gross output is capital stock, labor compensation, and intermediate inputs.

Also, Basu and Fernald (1996) investigate the impact of capital and labor utilization on productivity by applying production function using gross output of the following formula:  $Y_i = (Z_i K_i, C_i L_i, M_i, T_i)$ , where: Y= Gross output; K, and L are capital and labor; M=material input: and T= technology status. C and Z are the levels of capital and labor utilization. He estimates the total derivative of the previous equation by exploiting unpublished data from Dale Jorgenson. The data covers US manufacturing industries at 2-digit SIC for the period 1953-1984. The author finds that the degree of returns to scale is equal across industries and ranged between 1.09 and 1.1. In addition, Basu et al uses capital stock as representative for capital because of data unavailability in returns to capital. Also, as a result of lack of data on output, Bernard and Jones (1996)

use value added instead of output to test for the heterogeneity of TFP in six major industries for 14 OECD countries covering the period 1970-1987.

At the macroeconomic level, Chow et al (2002) studied the estimated production function for Chinese economy. The purpose of the study was to investigate the economies of scale and more importantly to predict China's economic future until 2010. In this study GDP was used as a representative of output. Furthermore, Dobbelaere et al (2008), in their NBER paper, used panel data to estimate Cobb-Douglas production function for 10,646 French firms in 38 manufacturing industries covering the period 1978-2001. The data was collected from firm accounting information. They used current output deflated by producer price index as a proxy to output. According to the authors, the output includes labor, capital, and intermediate inputs.

The CGE models used by Cutler and Davies (2007, 2009) sum intermediate inputs, value added (labor compensations and capital), taxes, and imports to be representative of output. Also, value of land is added to the output in these two studies.

### **3.3 Output Estimation**

In this study outputs is not available at the firm level in the counties studied or even a proxy to output such as sales, revenues, or value added. Therefore, the output will be estimated at the firm level by using value added of labor compensation, capital of the firm, and the value of land.

## **Chapter Four: Exploring Model Fitness**

This chapter consists of three principal parts. The first part demonstrates the main prominent production functions in the economic literature. These functions are classified into two groups: homogeneous and non-homogeneous production functions. The homogeneous group includes Leontief, Cobb-Douglas, CES, and new CES functions. The nonhomogeneous functions include the translog and non-homogeneous Cobb-Douglas functions. The second part of the chapter discusses the main features of estimating results of the four production functions at the industry level for six counties in Colorado. The third part is about hypothesis exploration regarding the model fitness. In this part, the J-test of nonnested hypothesis will be implemented to determine the direction of nesting functions, whether the two functions nested each other, neither, or one function nested the other.

### **4.1. Prominent Production Functions**

This part discusses the most prominent of the two groups of production functions in the theoretical and empirical economic literature. The homogeneous production functions main characteristics are homothetic, the elasticity of substitution and fixed returns to scale. This group includes Cobb-Douglas production function with unity elasticity of substitution, general Leontief production function with zero elasticity of substitution irrespective of change in the price ratio of the factors of production; and two types of CES functions, the CES of Arrow et al (1961) type, and the new CES of Bairam (1989, and 1991). The second group is the non-homogeneous production function. The main characteristics of this group are varying scales of economy and elasticity of substitution according to the input combinations. These functions include the non-homogeneous

Cobb-Douglas function of Vinod (1972), and Ringstad (1974), and the translog production function with varying elasticity of substitutions, and scale of economies. In this study, only Vinod (1972) nonhomogeneous Cobb-Douglas function will be subject to different tests, such as economies of scale and elasticity of substitutions.

In addition, the research will explain the different characteristics of each model. For instance, the elasticity of substitution is unity in Cobb-Douglas production function, constant in CES, and varies in translog production functions. Furthermore, the returns to scale are different in their computation among different types of production functions. The return to scale in standard Cobb-Douglas is summing the coefficients of inputs, but it is different in other types of production functions. For example, each model has its own characteristics either from economic or statistical points of view. In addition, the paper attempts to empirically estimate four production functions to test which one is more appropriate for the industries in the different counties. The following is a detailed discussion of the two different groups of production functions.

**4.1.1. Homogeneous Production Functions**

This type of production functions includes Cobb-Douglas, Leontief, CES, and new CES. The following is a detailed explanation of these production functions and their characteristics.

**4.1.1.1. The Leontief production function**

The general formula for Leontief production function is as follows:

$$Q_t = \gamma_1 + \gamma_2 K_t + \gamma_3 L_t + \gamma_4 K_t^{0.5} L_t^{0.5} + \varepsilon_t \dots\dots\dots (4.1)$$

Where:

$Q_t$  = output or value added at time t.

$K_t$ =capital at time t.

$L_t$ =labor at time t.

$\varepsilon_t$ =disturbance term, which represents all other factors of production not mentioned in equation 4.1.

$\gamma'_s$ = parameters to be statistically estimated.

This type of production function concerns the minimum combination of factor inputs to produce a certain amount of output. This means that, even if a firm increases one of the inputs while the other input remains unchanged, then the output will not increase. The main drawback in such models is that they don't permit for substitutions among the factors of production even if the price ratios among these factors of production change (Nicholson, 2005). In other words the isoquants of this function are L shaped (Lau et al, 1972). In addition Basu (1996) said a reasonable ‘‘Leontief case happened when the material inputs used in strict proportion to value added’’.

#### **4.1.1.2. The Cobb-Douglass Production Function**

The Cobb-Douglass production function enjoys several advantages. This function is widely used in economic literature and econometric applications. Among these advantages, the function is flexible in the number of input variables that the researcher uses to explore their effects on the production process. In addition, scale of economies can be estimated as restricted input coefficients that sum to one or without this restriction to reflect the type of scale for the economy, industry, state, firm, and so on. The other main characteristic is that the elasticity of substitution is unity. While keeping other inputs constant, the known formula of the production function in the economic literature is:

$$Q_t = AK_t^\alpha L_t^\beta \dots\dots\dots (4.2)$$

Where: Q, K, and L, are output, capital stock, and labor. A is technical or technological level, or total factor productivity. The parameters  $\alpha$ , and  $\beta$  are the elasticities of output with respect to capital and labor, respectively. Also, these coefficients reflect the share of capital revenue in total production under perfect competition (Dobbelaere et al, 2008). In the case of estimating equation 4.2 without any restrictions, then if the sum of  $\alpha$ , and  $\beta$  coefficients equal one, it is called constant return to scale, if greater than one it is increasing, and less than one it is decreasing return to scale.

**4.1.1.3. The CES production Functions**

There are two general types of CES production functions. The first type is the production function used by Arrow et al (1961), Pitchford (1960), David and de Klundert (1965), and Barro and Sala-i-Martin (1995). The second type is the new CES production functions used by Bairam (1989, 1991). This part will discuss the characteristics of each type and its limitations.

**4.1.1.3.1. Linear CES Production Functions**

The following table shows the linear CES production function as shown in the Klump et al (2000) paper pertaining to the CES production function and economic growth.

| Table 4.1, the Linear CES Homogeneous Production Function |   |
|---|---|
| 1- Pitchford (1960):                                      | $Y=[\alpha K^\varphi + bL^\varphi]^\frac{1}{\varphi}$                         |
| 2- Arrow et al (1961)                                     | $Y=C[\alpha K^\varphi + (1 - \alpha)L^\varphi]^\frac{1}{\varphi}$             |
| 3- David and von de Klundert (1965)                       | $Y= [ (BK)^\varphi + (AL)^\varphi]^\frac{1}{\varphi}$                         |
| 4- Barrow and Sala-i-Martin (1995)                        | $Y=C[\alpha (BK)^\varphi + (1 - \alpha)[(1 - B)L]^\varphi]^\frac{1}{\varphi}$ |

Where:  $\varphi = \frac{\sigma-1}{\sigma}$

In table 4.1, equation 2 reflects the assumed substitution parameter  $\varphi$ , the restricted distribution parameter among factor inputs  $\alpha$ , the technology or efficiency parameter C. Equation 3, introduces the nature of technical progress, David and Klundert introduce A and B for the efficiency of capital and labor. Klump et al (2000) shows mathematically that the first model is unstable, while others are stable and move toward steady state.

The CES production function enjoys some unique characteristics, such as the elasticity of substitution among factors of production is fixed, and it may deviate from one. It takes any value. For example, the elasticity of substitution is unity in Cobb-Douglas, while it varies in the translog function. The main restriction to CES production function is that the researcher is restricted to two variables to be estimated at once in CES production functions (Diewert, 1971). There is some attempt to remove this restriction by Uzawa (1962), McFadden (1963), and Sato (1967). In recent work, Kemfert (1998), estimated nested CES production function by including three input variables, labor, capital, and energy. The nested CES function is of the form:

$$Q = \{[\alpha K^\rho + (1 - \alpha)E^\rho]^{\theta/\rho} + \beta L^\theta\}^{1/\theta}$$

Where: Q= output, K= accounts of capital services, E= energy, and L=labor.

**4.1.1.3.2. New CES Production Function**

The second type of CES production function is non-linear. This type is mentioned in Hsing (1996) as well as in Bairam 1989 and 1991. This function has the following form:

$$(Q_t^\theta - 1)/\lambda = \alpha_1 + \alpha_2(K_t^\lambda - 1)/\lambda + \alpha_3 (L_t^\lambda - 1)/\lambda + v_t \dots\dots\dots (4.3)$$

Where:

$Q_t$  = output or value added

$K_t$  = capital stock

$L_t$  = Labor

$\alpha_1$  = parameters to be estimated for technical progress

$\lambda$  = Box-Cox transformation Parameter for the right hand side variables.

$\theta$  = Box-Cox transformation parameter for the dependent variable data. And  $\theta$  may equal  $\lambda$  as in Hsing (1996) work.

The new CES production function is more flexible than the previous types of CES discussed regarding the number of input variables that can be freely included in the estimation of empirical work. The new CES production function is required to estimate the transformation parameter by maximum likelihood methods. Then the elasticity of substitution can be computed by the formula  $\sigma = 1 / (1 - \lambda)$ . The scale economy is the sum of estimated input coefficients. Furthermore, this function can apply Box-Cox transformation to the left-hand side of the equation and leaving the other side, as Cobb-Douglas (Ringstad, 1974).

#### **4.1.2. Non-homogeneous Production Function**

Griliches and Ringstad (1971), Berndt and Christensen (1973), and Christensen and Lau (1973) introduce the translog production function; Vinod (1972), and Ringstad (1974) introduce non-homogeneous Cobb-Douglas functions.

##### **4.1.2.1 The Translog Function**

Griliches and Ringstad (1971), Berndt and Christensen (1973), and Christensen and Lau (1973) introduce this type of production function. The main characteristics of this type of production function are that the elasticity of substitutions varies between factors



## 4.2. Capital-Labor, Capital-Land, Land-Labor Ratios

This section describes capital-labor, capital-land (area in footage), and land-labor ratios to provide in depth analysis of the estimation results regarding the independent variable coefficients that will be discussed in the following part.

### 4.2.1. Capital-Labor (K/L) Ratio

Table 4.2 shows that high services industries have the highest capital-labor ratio in all counties. The K/L ratio is highest in Arapahoe County compared to other counties. At the same time, the K/L ratio in high services industry in Arapahoe in any industry is several times that of any other county. For instance, the K/L ratio in finance and insurance is 12.3 times that of the same industry in El Paso County. The second highest K/L ratio is in the industries that operate in Denver County.

Table 4.2, Capital-Labor Ratio by Industry and County

| Weld    | Boulder | Larimer | Denver  | El Paso | Arapahoe |      |
|---------|---------|---------|---------|---------|----------|------|
| 27,500  | 68,367  | 22,923  | 23,832  | 27,649  | 79,006   | 23   |
| 42,462  | 45,077  | 51,494  | 29,358  | 25,000  | 83,333   | 31   |
| 39,063  | 84,397  | 47,589  | 43,326  | 48,468  | 105,155  | 42   |
| 36,842  | 41,121  | 50,185  | 91,056  | 48,780  | 65,897   | 44   |
| 40,005  | 33,678  | 39,678  | 69,684  | 80,477  | 88,036   | 48   |
| 157,982 | 99,398  | 23,820  | 52,304  | 43,366  | 82,053   | 51   |
| 40,737  | 54,182  | 97,293  | 141,584 | 24,748  | 304,323  | 52   |
| 81,155  | 143,868 | 117,797 | 364,583 | 191,937 | 720,730  | 53   |
| 94,872  | 98,575  | 64,748  | 135,556 | 59,681  | 395,997  | 54   |
| 52,754  | 194,446 | 58,294  | 13,068  | 41,721  | 290,525  | 55   |
| 27,283  | 40,838  | 19,081  | 21,958  | 39,577  | 141,633  | 56   |
| 39,312  | 64,519  | 38,024  | 72,315  | 5,761   | 154,197  | 61   |
| 34,682  | 38,889  | 41,182  | 57,180  | 27,132  | 137,202  | 62   |
| 13,237  | 16,043  | 30,156  | 39,522  | 39,130  | 71,452   | 71   |
| 52,808  | 40,220  | 38,922  | 37,335  | 50,752  | 93,040   | 72   |
| 52,551  | 28,678  | 105,508 | 85,003  | 55,556  | 357,470  | 81   |
| 39,907  | 57,628  | 47,601  | 55,522  | 38,112  | 174,276  | Avg. |

#### 4.2.2. Capital-Land (K/LA) Ratio

Table 4.3 shows that the capital-land ratios vary among industries and counties except in Larimer County which is stable around two dollars per square foot. The highest capital-land ratio is in Boulder County, followed by k/la ratios for industries in Arapahoe County and then Denver County.

Table 4.3, Capital-Land Ratio by Industry and County

| Weld | Boulder | Larimer | Denver | El Paso | Arapahoe | NAICS   |
|------|---------|---------|--------|---------|----------|---------|
| 1    | 95      | 2       | 17     | 3       | 61       | 23      |
| 5    | 54      | 3       | 15     | 4       | 57       | 31      |
| 2    | 93      | 2       | 14     | 7       | 62       | 42      |
| 8    | 54      | 2       | 21     | 10      | 66       | 44      |
| 3    | 67      | 2       | 7      | 1       | 34       | 48      |
| 19   | 123     | 2       | 141    | 21      | 81       | 51      |
| 11   | 111     | 2       | 32     | 12      | 87       | 52      |
| 8    | 45      | 3       | 58     | 3       | 112      | 53      |
| 6    | 116     | 2       | 44     | 10      | 74       | 54      |
| 8    | 143     | 3       | 25     | 12      | 82       | 55      |
| 4    | 93      | 2       | 23     | 3       | 77       | 56      |
| 14   | 104     | 3       | 17     | 6       | 68       | 61      |
| 14   | 94      | 3       | 28     | 15      | 80       | 62      |
| 0    | 40      | 2       | 2      | 1       | 55       | 71      |
| 20   | 67      | 2       | 32     | 5       | 98       | 72      |
| 1    | 76      | 3       | 35     | 10      | 81       | 81      |
| 2    | 79      | 2       | 19     | 5       | 79       | Average |

### 4.2.3. Land-Labor (LA/L) Ratio

The land-labor ratio reflects the area in footage requested to create a new job opportunity. The least ratio is in Boulder County where 731 square feet are required to create a new job. This may attribute to the high cost of land in Boulder, while the highest ratio is in Larimer and Weld counties. In these two counties, on average, to create a job

Table 4.4, Land-Labor Ratio by Industry and County

| Weld    | Boulder | Larimer | Denver | El Paso | Arapahoe |         |
|---------|---------|---------|--------|---------|----------|---------|
| 20,352  | 907     | 22,208  | 3,115  | 7,317   | 1,702    | 42      |
| 4,379   | 762     | 27,865  | 4,292  | 4,821   | 1,006    | 44      |
| 12,172  | 506     | 20,011  | 10,094 | 62,883  | 2,607    | 48      |
| 8,168   | 810     | 9,937   | 371    | 2,060   | 1,013    | 51      |
| 3,790   | 487     | 45,582  | 4,389  | 2,151   | 3,513    | 52      |
| 10,527  | 3,215   | 44,209  | 6,282  | 61,613  | 6,428    | 53      |
| 16,408  | 851     | 25,948  | 3,083  | 6,072   | 5,331    | 54      |
| 6,905   | 1,359   | 19,305  | 530    | 3,511   | 3,559    | 55      |
| 6,738   | 439     | 7,659   | 950    | 14,220  | 1,848    | 56      |
| 2,844   | 621     | 13,059  | 4,142  | 949     | 2,278    | 61      |
| 2,483   | 412     | 13,858  | 2,025  | 1,838   | 1,722    | 62      |
| 325,581 | 401     | 13,984  | 22,243 | 42,199  | 1,293    | 71      |
| 2,576   | 605     | 21,053  | 1,176  | 9,660   | 949      | 72      |
| 21,414  | 731     | 21,989  | 2,864  | 7,648   | 2,214    | Average |

require 22 thousand square feet.

### 4.3. Estimating Production Functions Results

The data in this study is collected at the firm level for each industry in all counties studied from two different types of data files. The first data file is the QCEW (Quarterly Census of Employment and Wages). In this file, by law, all firms in Colorado have to provide the Colorado Department of Labor with quarterly data regarding employment, their corresponding wages, unemployment insurance, address, and the name of the firm. This file also includes information about the economic activity that the firm practices through the NAICS codes. The second file is provided by County Assessors Office. The Assessors Office keeps records regarding each parcel of land and the type of use for that parcel, residential or commercial, because of different property tax rates. The data provided by such offices are the land variable (value, and area), and the structure on the parcel of land including the value of machines and equipment in the buildings, or in other words the capital value. The research merges the data from these two files together. In addition, the research estimates the data concerning output by adding wages, capital value, and land value. This is the minimum data required to estimate the production functions.

This study estimates four different types of production functions for each industry in each of the six counties. These functions are Cobb-Douglas, new CES, non-homogeneous Cobb-Douglas, and translog functions. This part introduces the first filter of choosing the production functions that fit the data structure in the six Colorado counties that are incorporated in the study. The first filtering criterion depends on the conditions if the production function realizes the non-homogeneous production functions, i.e., if the production function satisfies the conditions to be non-homogeneous Cobb-Douglas function by testing the sum of the product input variables, if it is statistically zero or different than zero ( $L*K+L*LA+K*LA$ ). When the sum of product

input variables is not different than zero, then the production function will satisfy the standard Cobb-Douglas function.

In the case of translog production function, there are two simultaneous conditions required to consider the production function is a translog model or not. These conditions are: (i) the sum of the input product variables have to be statistically significant (different than zero) or the production function will be Kementa approximation of the CES production function (Kim, 1992); and (ii) the sum of the square input values and the product variables have to be different than zero or the function will be standard Cobb-Douglas according to the second condition.

The analysis of the research depends on the following NAICS 2007 codes and their corresponding industry labels. This part will explain the results of different production functions mentioned in the previous paragraph.

Table 4.5, the NAICS Codes and Their Corresponding Industry Label

| Codes | Industry  |
|-------|---|
| 23    | Construction  |
| 31-3  | Manufacturing   |
| 42    | Wholesale Trade   |
| 44-5  | Retail Trade  |
| 48-9  | Transportation and Warehousing                                  |
| 51    | Information   |
| 52    | Finance and Insurance   |
| 53    | Real Estate and Rental and Leasing                              |
| 54    | Professional, Scientific, and Technical Services                |
| 55    | Management of Companies and Enterprises                         |
| 56    | Administrative and Support and Waste Management and Remediation |
| 61    | Educational Services  |
| 62    | Health Care and Social Assistance                               |
| 71    | Arts, Entertainment, and Recreation                             |
| 72    | Accommodation and Food Services                                 |
| 81    | Other Services (except Public Administration)                   |

#### **4.3.1. Cobb-Douglas**

In general, the Cobb-Douglas estimation results seem to fit the data structure of the industries for the six counties. The following is a brief explanation for the results of this type of estimation, specifically at the county level. In particular, the impact of input variables on output industry will be explained at the county level.

##### **Arapahoe County:**

- 1- Land: Except the real estate and construction industries which are significant at 95% and 90%, respectively, table 4.6 shows that the area of land measured in square feet is essential, and highly significant in all other industries at 99%. The elasticity of output with respect to land ranged between 0.4255 in the transportation and warehousing industry and 0.0777 in real estate and leasing. For instance, this means that if the area of land area increased on average by 1% in the transportation industry, then output in that industry is expected to increase by 0.4255%.
- 2- Capital: Except for transportation and warehousing industry which is insignificant, the capital is significant at 99% level in all other industries mentioned in table 4.6. The elasticity of output with respect to capital ranged between 0.7004 in the real estate industry and 0.3309 in management of companies and enterprises. In addition, the elasticity of output with respect to capital is almost higher than that of land and labor variables.

3- Labor: Without exception, the coefficient variable is significant at 99% level in all industries in Arapahoe County. The output elasticity of substitution with respect to labor ranged between 0.41 in constructions industry and 0.15 in education services.

**Table 4.6, Estimating Cobb-Douglas Function For Arapahoe County at the Industry Level**

|       | LK                    | LL                    | LA                    | Constant               | R2     | No. | F-test  |
|-------|-----------------------|-----------------------|-----------------------|------------------------|--------|-----|---------|
| 23    | 0.3976***<br>(0.0385) | 0.4067***<br>(0.0206) | 0.3362***<br>(0.0347) | 5.1546***<br>(0.2902)  | 0.8823 | 305 | 760.96  |
| 31-33 | 0.6217***<br>(0.0572) | 0.2797***<br>(0.0422) | 0.1044*<br>(0.0597)   | 4.6129***<br>(0.5493)  | 0.9071 | 90  | 127.43  |
| 42    | 0.4538***<br>(0.0325) | 0.3177***<br>(0.0220) | 0.2858***<br>(0.0311) | 5.0854***<br>(0.2688)  | 0.9071 | 250 | 811.81  |
| 44-45 | 0.5692***<br>(0.0267) | 0.3094***<br>(0.0231) | 0.1252***<br>(0.0273) | 5.05838***<br>(0.2777) | 0.8534 | 256 | 495.7   |
| 48-49 | 0.2141<br>(0.1564)    | 0.2715***<br>(0.0676) | 0.4255***<br>(0.1268) | 6.8217***<br>(1.0788)  | 0.8982 | 29  | 83.34   |
| 51    | 0.5098***<br>(0.0801) | 0.3103***<br>(0.0343) | 0.2445***<br>(0.0767) | 4.7609***<br>(0.5770)  | 0.9267 | 74  | 308.63  |
| 52    | 0.6355***<br>(0.0252) | 0.2388***<br>(0.0177) | 0.1149***<br>(0.0261) | 4.4741***<br>(0.1894)  | 0.9298 | 302 | 1330.85 |
| 53    | 0.7004***<br>(0.0287) | 0.2779***<br>(0.0359) | 0.0777**<br>(0.0318)  | 3.7753***<br>(0.2805)  | 0.9236 | 153 | 613.51  |
| 54    | 0.4993***<br>(0.0211) | 0.2726***<br>(0.0174) | 0.292***<br>(0.0204)  | 4.4803***<br>(0.1610)  | 0.9318 | 523 | 2376.79 |
| 55    | 0.3309***<br>(0.0302) | 0.2047***<br>(0.0353) | 0.4507***<br>(0.0419) | 5.4697***<br>(0.3574)  | 0.956  | 47  | 334.31  |
| 56    | 0.4851***<br>(0.0419) | 0.2876***<br>(0.0228) | 0.2989***<br>(0.0401) | 4.4735***<br>(0.2886)  | 0.9194 | 213 | 807.44  |
| 61    | 0.6389***<br>(0.0745) | 0.1522***<br>(0.0504) | 0.2486***<br>(0.0729) | 3.0571***<br>(6006)    | 0.9032 | 51  | 156.55  |
| 62    | 0.6124***<br>(0.0307) | 0.2269***<br>(0.0183) | 0.1741***<br>(0.0314) | 4.0882***<br>(0.2336)  | 0.9184 | 242 | 904.6   |
| 71    | 0.4645***<br>(0.0521) | 0.2074***<br>(0.0626) | 0.3462***<br>(0.0652) | 4.3777***<br>(0.6330)  | 0.8955 | 29  | 80.98   |
| 72    | 0.5696***<br>(0.0301) | 0.2136***<br>(0.0279) | 0.1919***<br>(0.0329) | 4.6060***<br>(0.3006)  | 0.873  | 164 | 374.36  |
| 81    | 0.6004***<br>(0.0389) | 0.2223***<br>(0.0351) | 0.2149***<br>(0.0414) | 3.9140***<br>(0.2996)  | 0.8935 | 166 | 462.34  |

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\* Significant at 95% level.

**El Paso County:** the analysis of the impact of variable inputs in El Paso County

industries relies on table 4.7.

**Table 4.7, Estimating Cobb-Douglas Function For El Paso County at the Industry Level**

|       | LK                    | LL                    | LA                    | Constant              | R2     | No. | F-test |
|-------|-----------------------|-----------------------|-----------------------|-----------------------|--------|-----|--------|
| 23    | 0.5767***<br>(0.0269) | 0.6315***<br>(0.0188) | 0.0511***<br>(0.0129) | 4.5503***<br>(0.3280) | 0.8458 | 297 | 542.22 |
| 31-33 | 0.4523***<br>(0.0352) | 0.4796***<br>(0.0348) | 0.1538***<br>(0.0295) | 5.2753***<br>(0.4449) | 0.9115 | 93  | 316.91 |
| 42    | 0.4867***<br>(0.0389) | 0.4107***<br>(0.0358) | 0.0741**<br>(0.0291)  | 5.8778***<br>(0.4570) | 0.8818 | 104 | 257.03 |
| 44-45 | 0.4068***<br>(0.0197) | 0.3373***<br>(0.0179) | 0.2489***<br>(0.0189) | 5.2411***<br>(0.1932) | 0.8881 | 378 | 998.59 |
| 48-49 | 0.4183***<br>(0.0454) | 0.4812***<br>(0.0512) | 0.1234***<br>(0.0338) | 5.9912***<br>(0.5256) | 0.8818 | 53  | 130.28 |
| 51    | 0.7315***<br>(0.1055) | 0.4676***<br>(0.0436) | -0.1179<br>(0.1043)   | 4.7498***<br>(0.6742) | 0.897  | 45  | 128.7  |
| 52    | 0.3897***<br>(0.0354) | 0.6388***<br>(0.0264) | 0.1275***<br>(0.0355) | 6.3181***<br>(0.3859) | 0.8898 | 125 | 334.82 |
| 53    | 0.6655***<br>(0.0266) | 0.3409***<br>(0.0273) | 0.0964***<br>(0.0209) | 3.4988***<br>(0.2622) | 0.9288 | 134 | 565.45 |
| 54    | 0.6503***<br>(0.0298) | 0.6236***<br>(0.0251) | 0.0343*<br>(0.0212)   | 3.9832***<br>(0.3185) | 0.8689 | 243 | 535.87 |
| 55    | 0.4385**<br>(0.1768)  | 0.2723<br>(0.1693)    | 0.1325<br>(0.1081)    | 6.3627*<br>(2.8755)   | 0.253  | 11  | 2.13   |
| 56    | 0.553***<br>(0.0345)  | 0.4812***<br>(0.0277) | 0.0983***<br>(0.0267) | 4.4661***<br>(0.3663) | 0.8587 | 167 | 337.24 |
| 61    | 0.4221***<br>(0.0724) | 0.6246***<br>(0.0450) | 0.1235*<br>(0.0665)   | 5.6733***<br>(0.8739) | 0.8578 | 51  | 101.53 |
| 62    | 0.3432***<br>(0.0378) | 0.4904***<br>(0.0271) | 0.1949***<br>(0.0381) | 6.3663***<br>(0.3968) | 0.5135 | 193 | 252.4  |
| 71    | 0.2893***<br>(0.0678) | 0.2458***<br>(0.0636) | 0.3869***<br>(0.0445) | 5.1663***<br>(0.7323) | 0.8659 | 35  | 74.15  |
| 72    | 0.5101***<br>(0.0234) | 0.4083***<br>(0.0239) | 0.1257***<br>(0.0236) | 4.9458***<br>(0.2536) | 0.8322 | 271 | 447.46 |
| 81    | 0.4269***<br>(0.0369) | 0.4843***<br>(0.0334) | 0.1407***<br>(0.0359) | 5.7947***<br>(0.3697) | 0.8182 | 166 | 248.59 |

\*\*\* Significant at 99% level.  
 \*\* Significant at 95% level.  
 \* Significant at 95% level



- 1- Land: Table 4.7 shows that land variable is significant at 99% level in 11 industries out of 16. The output elasticity with respect to land ranged between 0.389 in arts, entertainment, and recreation industry on one side, and 0.05 in the construction industry. The table shows that the magnitude of the land coefficient variable in El Paso County is less than that of the land variable coefficients for corresponding industries in Arapahoe County.
- 2- Capital: Except for the management of companies and enterprises industry which is significant at 95% level, the estimated coefficients of capital variable in all other industries are significant at 99% level. The elasticity of output with respect to capital ranged between 0.73 in the information industry, and 0.29 in arts, entertainment, and recreation industry.
- 3- Labor: The coefficients of labor input are significant in all industries in El Paso County at 99% level except for management of companies and enterprises which is insignificant. The elasticity of output with respect to labor ranged between 0.64 in finance and insurance industry, and 0.25 in arts, entertainment, and recreation industry.

**Denver County:** the analysis of the impact of input variables on output by industry is based on table 4.8.

**Land:** The magnitude of the land variable coefficients in most of the industries in Denver County is the highest compared to other counties included in the study. In this county, 15 out of 16 industries the land coefficient variable is significant at 99% level.

**Capital:** The coefficients of capital inputs are significant at better than 99% in all industries in Denver County.

Labor: The labor input coefficients are significant at 99% in the industries operated in Denver County.

**Table 4.8, Estimating Cobb-Douglas Function For Denver County at the Industry Level**

|       | LK                    | LL                    | LA                    | Constant              | R2     | No. | F-test |
|-------|-----------------------|-----------------------|-----------------------|-----------------------|--------|-----|--------|
| 23    | 0.2389***<br>(0.0298) | 0.4534***<br>(0.0265) | 0.2519***<br>(0.0467) | 7.381***<br>(0.4422)  | 0.8225 | 154 | 237.31 |
| 31-33 | 0.3326***<br>(0.0502) | 0.3328***<br>(0.0284) | 0.3028***<br>(0.0386) | 5.9144***<br>(0.5146) | 0.9204 | 100 | 382.6  |
| 42    | 0.4373***<br>(0.0405) | 0.3315***<br>(0.0223) | 0.1388***<br>(0.0325) | 6.2741***<br>(0.4142) | 0.9138 | 117 | 410.81 |
| 44-45 | 0.2066***<br>(0.0251) | 0.3019***<br>(0.0337) | 0.4636***<br>(0.0373) | 6.089***<br>(0.3226)  | 0.8351 | 143 | 240.67 |
| 48-49 | 0.3605***<br>(0.0495) | 0.2334***<br>(0.0373) | 0.2852***<br>(0.0477) | 5.775***<br>(0.3781)  | 0.9869 | 18  | 426.49 |
| 51    | 0.4877***<br>(0.0829) | 0.4293***<br>(0.0749) | 0.1537***<br>(0.0740) | 5.261***<br>(1.0929)  | 0.982  | 15  | 256.23 |
| 52    | 0.3519***<br>(0.0694) | 0.2902***<br>(0.0744) | 0.3826***<br>(0.0857) | 5.2952***<br>(0.7057) | 0.8818 | 29  | 62.18  |
| 53    | 0.3557***<br>(0.0563) | 0.2261***<br>(0.0817) | 0.3852***<br>(0.0977) | 5.3100***<br>(0.7751) | 0.7673 | 54  | 59.26  |
| 54    | 0.3092***<br>(0.0311) | 0.2944***<br>(0.0338) | 0.3204***<br>(0.0455) | 6.3826***<br>(0.4419) | 0.6789 | 197 | 139.15 |
| 55    | -0.0334<br>(0.1809)   | 0.1681<br>(0.4021)    | 0.6729<br>(0.6166)    | 7.8884<br>(4.9290)    | 0.4137 | 7   | 2.41   |
| 56    | 0.2167***<br>(0.0415) | 0.3122***<br>(0.0464) | 0.3259***<br>(0.0722) | 7.2423***<br>(0.7832) | 0.6339 | 84  | 48.9   |
| 61    | 0.6875***<br>(0.0757) | 0.1985***<br>(0.0463) | 0.1137<br>(0.0791)    | 3.4291***<br>(0.5269) | 0.9623 | 35  | 290.01 |
| 62    | 0.2686***<br>(0.0346) | 0.2718***<br>(0.0299) | 0.4558***<br>(0.0470) | 5.5767***<br>(0.3674) | 0.8484 | 134 | 249.09 |
| 71    | 0.4697***<br>(0.0777) | 0.0415<br>(0.0845)    | 0.2387***<br>(0.0611) | 5.3342***<br>(0.8313) | 0.9317 | 19  | 82.81  |
| 72    | 0.0949***<br>(0.0224) | 0.3694***<br>(0.0369) | 0.3944***<br>(0.0419) | 8.0288<br>(0.4348)    | 0.6308 | 137 | 78.46  |
| 81    | 0.1748***<br>(0.0326) | 0.3149***<br>(0.0455) | 0.4514***<br>(0.0534) | 6.6417***<br>(0.4617) | 0.7212 | 125 | 107.91 |

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\* Significant at 95% level

**Table 4.9, Estimating Cobb-Douglas Function For Larimer County at the Industry Level**

|       | LK                    | LL                    | LA                    | Constant              | R2     | No. | F-test  |
|-------|-----------------------|-----------------------|-----------------------|-----------------------|--------|-----|---------|
| 23    | 0.357***<br>(0.0331)  | 0.5004***<br>(0.0350) | 0.1512***<br>(0.0277) | 6.5640***<br>(0.4432) | 0.9051 | 82  | 258.6   |
| 31-33 | 0.6359***<br>(0.0471) | 0.3396***<br>(0.0342) | 0.0660***<br>(0.0353) | 4.1537***<br>(0.4395) | 0.9172 | 69  | 251.94  |
| 42    | 0.4344***<br>(0.0302) | 0.3406***<br>(0.0333) | 0.1714***<br>(0.0308) | 5.7497***<br>(0.3968) | 0.9171 | 80  | 292.27  |
| 44-45 | 0.3961***<br>(0.0188) | 0.3025***<br>(0.0222) | 0.1986***<br>(0.0181) | 6.0254***<br>(0.2202) | 0.8957 | 310 | 885.97  |
| 48-49 | 0.4442***<br>(0.0447) | 0.3393***<br>(0.0536) | 0.1652***<br>(0.0422) | 5.6089***<br>(0.5108) | 0.9492 | 30  | 181.66  |
| 51    | 0.4832***<br>(0.1077) | 0.4309***<br>(0.0599) | 0.0949<br>(0.0616)    | 5.7506***<br>(0.9911) | 0.9211 | 28  | 106.01  |
| 52    | 0.2775***<br>(0.0227) | 0.2950***<br>(0.0313) | 0.4071***<br>(0.0311) | 5.7417***<br>(0.3052) | 0.8869 | 116 | 301.63  |
| 53    | 0.6212***<br>(0.0429) | 0.1567***<br>(0.0392) | 0.1063***<br>(0.0372) | 4.3800***<br>(0.4684) | 0.86   | 72  | 146.38  |
| 54    | 0.7465***<br>(0.0382) | 0.3212***<br>(0.0249) | 0.0056<br>(0.0267)    | 3.5115***<br>(0.3684) | 0.903  | 117 | 361.03  |
| 55    | 0.4812*<br>(0.0406)   | 0.2777**<br>(0.0182)  | 0.1775<br>(0.0503)    | 5.5308<br>(0.3255)    | 0.999  | 5   | 1274.13 |
| 56    | 0.5057***<br>(0.0600) | 0.3421***<br>(0.0556) | 0.1332*<br>(0.0677)   | 5.2501***<br>(0.7453) | 0.8681 | 36  | 77.76   |
| 61    | 0.6511***<br>(0.0627) | 0.0854**<br>(0.0379)  | 0.3318***<br>(0.0763) | 1.5804**<br>(0.6805)  | 0.975  | 14  | 169.66  |
| 62    | 0.3270***<br>(0.0366) | 0.4053***<br>(0.0337) | 0.1949***<br>(0.0409) | 6.8013***<br>(0.4037) | 0.837  | 134 | 228.72  |
| 71    | 0.3654***<br>(0.0305) | 0.1997***<br>(0.0651) | 0.1868***<br>(0.0428) | 6.6942***<br>(0.6611) | 0.8492 | 33  | 61.05   |
| 72    | 0.5182***<br>(0.0204) | 0.2359***<br>(0.0186) | 0.1150***<br>(0.0169) | 5.3766***<br>(0.2474) | 0.8385 | 251 | 433.61  |
| 81    | 0.4872***<br>(0.0362) | 0.3394***<br>(0.0324) | 0.1745***<br>(0.0329) | 5.040***<br>(0.3504)  | 0.8665 | 121 | 260.55  |

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\* Significant at 95% level.

## Boulder County

1- Land: The magnitudes of the land variable coefficients in seven industries in Boulder County are significant at 90% level and higher. Also, the magnitude of the land coefficients ranged between 0.22 in retail and other services industries and 0.69 in education services.

Table 4.10, Estimating Cobb-Douglas Function For Boulder County at the Industry Level

|       | LK                    | LL                    | LA                    | Constant                | R2     | No. | F-test |
|-------|-----------------------|-----------------------|-----------------------|-------------------------|--------|-----|--------|
| 23    | 0.2314***<br>(0.0639) | 0.3119***<br>(0.0378) | 0.3997***<br>(0.0832) | 6.9010***<br>(0.6546)   | 0.5958 | 104 | 51.6   |
| 31-33 | 0.4317***<br>(0.1588) | 0.2191***<br>(0.0788) | 0.2931*<br>(5.3633)   | 5.3633***<br>(1.2588)   | 0.9132 | 27  | 92.23  |
| 42    | 0.7493<br>(0.0970)    | 0.2133***<br>(0.0404) | 0.0797<br>(0.1033)    | 3.1975***<br>(0.7600)   | 0.7953 | 78  | 100.7  |
| 44-45 | 0.0264<br>(0.0569)    | 0.3179***<br>(0.0680) | 0.2215**<br>(0.0901)  | 11.0156***<br>(0.8005)  | 0.5215 | 41  | 15.53  |
| 48-49 | 0.4426<br>(0.3345)    | 0.4926**<br>(0.0960)  | -0.194<br>(0.2022)    | 8.8775<br>(5.1019)      | 0.8279 | 6   | 9.02   |
| 51    | 0.8172**<br>(0.3169)  | 0.3789***<br>(0.0902) | -0.1817<br>(0.3602)   | 4.5415**<br>(1.8593)    | 0.6065 | 26  | 13.85  |
| 52    | 0.7664***<br>(0.1691) | 0.4319***<br>(0.0898) | -0.3561<br>(0.2451)   | 6.3076***<br>(1.4021)   | 0.7921 | 25  | 31.48  |
| 53    | 0.3131<br>(0.2046)    | 0.3085***<br>(0.0919) | 0.2504<br>(0.1682)    | 7.1376***<br>(1.5572)   | 0.8609 | 25  | 50.52  |
| 54    | 0.3131<br>(0.0559)    | 0.3085<br>(0.0274)    | 0.2504<br>(0.0671)    | 5.2213***<br>(0.4927)   | 0.6803 | 228 | 162.02 |
| 55    | 1.0007<br>(0.5454)    | 0.5452<br>(0.4722)    | -0.2896<br>(1.2255)   | 2.9032<br>(6.3359)      | 0.3861 | 7   | 2.26   |
| 56    | 0.7380***<br>(0.1411) | 0.2111***<br>(0.0392) | 0.0995<br>(0.1386)    | 3.1107***<br>(1.0103)   | 0.6659 | 71  | 47.5   |
| 61    | 0.0417<br>(0.2760)    | 0.1903*<br>(0.0989)   | 0.6939*<br>(0.3408)   | 7.1665***<br>(0.1.8423) | 0.4704 | 21  | 6.92   |
| 62    | 0.3407**<br>(0.1326)  | 0.2352***<br>(0.0447) | 0.2684**<br>(0.1308)  | 6.8330***<br>(0.9836)   | 0.6663 | 65  | 43.6   |
| 71    | 0.1729<br>(0.1680)    | 0.2683***<br>(0.0727) | 0.2634**<br>(0.1043)  | 8.8971***<br>(1.8463)   | 0.8865 | 15  | 37.46  |
| 72    | 0.4472**<br>(0.1702)  | 0.3021***<br>(0.0568) | 0.207<br>(0.1438)     | 5.5399***<br>(1.1063)   | 0.9322 | 20  | 88.02  |
| 81    | 0.3172***<br>(0.1097) | 0.3479***<br>(0.0629) | 0.2223*<br>(0.1251)   | 7.2812***<br>(1.0251)   | 0.7894 | 35  | 43.48  |

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\* Significant at 95% level.

2. Capital: The capital coefficients are significant at 95% level and higher in 8 industries.
3. Labor: The coefficients of labor variable show significance in all industries at least at 90% level (14 industries), except in industries 54 and 55.

### Weld County

- 1- Land: The magnitude of the land variable coefficients is significant in 5 industries at 90% level or better. These industries are 23, 44, 53, 56, and 81.

**Table 4.11, Estimating Cobb-Douglas Function For Weld County at the Industry Level**

|       | LK                    | LL                    | LA                    | Constant              | R2     | No. | F-test |
|-------|-----------------------|-----------------------|-----------------------|-----------------------|--------|-----|--------|
| 23    | 0.4147***<br>(0.0448) | 0.5005***<br>(0.0291) | 0.0792***<br>(0.0214) | 6.4194***<br>(0.5357) | 0.8358 | 115 | 194.47 |
| 31-33 | 0.5200***<br>(0.0408) | 0.4645***<br>(0.0307) | 0.0368<br>(0.0304)    | 5.6762***<br>(0.4239) | 0.9705 | 38  | 406.99 |
| 42    | 0.5348***<br>(0.0752) | 0.4774***<br>(0.0506) | -0.0146<br>(0.0393)   | 6.2519***<br>(0.9594) | 0.7389 | 46  | 43.45  |
| 44-45 | 0.6083***<br>(0.0619) | 0.3458***<br>(0.0569) | 0.1319**<br>(0.0578)  | 3.7172***<br>(0.7122) | 0.9413 | 31  | 161.49 |
| 48-49 | 0.5189***<br>(0.0660) | 0.5362***<br>(0.0457) | -0.0206<br>(0.0357)   | 6.175***<br>(0.7294)  | 0.8877 | 37  | 95.89  |
| 52    | 0.5916***<br>(0.0793) | 0.4528***<br>(0.0461) | 0.0283<br>(0.0601)    | 5.1381***<br>(0.8255) | 0.8886 | 27  | 70.1   |
| 53    | 0.4085***<br>(0.1116) | 0.3264***<br>(0.0905) | 0.2966***<br>(0.0845) | 4.7939***<br>(0.9517) | 0.8383 | 21  | 35.55  |
| 54    | 0.6289***<br>(0.0398) | 0.2968***<br>(0.0364) | 0.0253<br>(0.0209)    | 4.6872***<br>(0.4972) | 0.7883 | 91  | 107.96 |
| 56    | 0.4814***<br>(0.1254) | 0.3643***<br>(0.0654) | 0.0765*<br>(0.0480)   | 5.799***<br>(1.5415)  | 0.409  | 48  | 11.84  |
| 61    | 0.7156**<br>(0.1831)  | 0.2614*<br>(0.1087)   | 0.082<br>(0.1974)     | 3.1329<br>(1.8373)    | 0.9298 | 8   | 31.91  |
| 62    | 0.6125***<br>(0.0845) | 0.3254***<br>(0.0469) | -0.0214<br>(0.0806)   | 5.255***<br>(0.7838)  | 0.9255 | 27  | 108.61 |
| 71    | 0.6425*<br>(0.0786)   | 0.4607*<br>(0.0874)   | 0.0687<br>(0.0454)    | 3.8932<br>(1.3889)    | 0.9591 | 5   | 32.29  |
| 72    | 0.8020***<br>(0.1264) | 0.2575*<br>(0.0865)   | 0.0613<br>(0.1338)    | 1.9906**<br>(0.8237)  | 0.9444 | 13  | 68.98  |
| 81    | 0.6134***<br>(0.0606) | 0.3216***<br>(0.0453) | 0.0854***<br>(0.0307) | 4.2116***<br>(0.7749) | 0.8107 | 43  | 60.96  |

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\* Significant at 95% level.

2. Capital: The capital coefficients are significant at 90% or better in all industries.
3. Labor: The coefficients of labor variable show significance in all industries at least at 90% level.

#### **4.3.2. New CES Function**

The new CES function of Bairam (1989, 1991) is estimated for US manufacturing at the national and state level by Hsing (1996). The results of the study show that the New CES function best fits the data structure for the manufacturing industry for 1987 data. The study finds that the Box-Cox transformation parameter  $\lambda=0.36$  is greater than 0 and less than one. Thus, the elasticity of substitution is positive and equal to 1.56. But if  $\lambda$  is greater than 1, then the elasticity of substitution will be negative according to the formula of computing this concept ( $\sigma=\frac{1}{1-\lambda}$ ), and therefore, this will be contrary to the economic theory of production function literature.

According to equation 4.3, the transformation parameter  $\lambda$  can take any real values (Hossain, 2011), but the value of the dependent variable has to be greater than zero. According to Hsing (1996), the value of the transformation parameter has to be restricted between zero and one to satisfy the elasticity of substitutions between capital and labor (Hicks, 1970) and Hsing (1996). In addition, " Hossain and King (2003) develop a new form of model selection by using the Box-Cox transformation technique where the Box-Cox transformation parameter  $\lambda$  is restricted to be in the range [0, 1]" (Hossain 2011).

This study attempts to estimate new CES production functions for each industry at the firm level using data for 6 major counties in Colorado. The main results show that the

estimated  $\lambda$  value of Box-Cox transformation parameter either is greater than one and then the elasticity of substitution will be negative, or less than 1 but statistically insignificant for all industries in all counties without exception. Thus, the new CES function will be excluded from the research in the next steps and investigations.

### **4.3.3. Non-homogeneous Cobb-Douglas Function**

The non-homogeneous Cobb-Douglas function is characterized by varying elasticity of substitutions, and varying economies of scale. In addition, the production function is not an array from the origin, nonhomothetic. The Cobb-Douglas function is non-homogeneous if the sum of product input variables ( $L^*K$ ,  $L^*LA$ , and  $K^*LA$ ) are statistically significant, that is if  $\sum\beta_{ij}\neq 0$ , Kim (1992), and Vinod (1972). Therefore, the research will be analyzed according to Kim (1992) criteria by economic activity for the Colorado counties under study. The analyses will concentrate on the statistical significance of the product of the input variables. In this case, if the sum of the product variables is not different than zero, then production function supports the standard Cobb-Douglas function with three input variables. But if the sum of the product input variables is significantly different than zero, then the production function is non-homogeneous Cobb-Douglas function for that industry in that county. The research relies on F-test to prove the homogeneity of the production process for each industry among the counties incorporated in the study. Hereby is a glance at the results of estimations. The study uses OLS technique for estimating this type of production function. The analysis depends on table 4.12 and for more detail (look at tables 4.13-4.18, chapter 4 appendix).

**Construction:** The construction industry, in Weld, El Paso, and Arapahoe counties follow the non-homogeneous Cobb-Douglas function, while other counties, in construction industry show that this industry is not different than zero from a statistically significance point of view. For example, in Denver county, although each input product variable is significant, the sum of the product variables reveals of insignificance because the coefficients of  $K*L$  product variable are positive and equivalents to the negative magnitude of  $K*LA$  product variable, while the third product variable magnitude,  $L*LA$ , is small although it is statistically significant. Regarding the construction industry in Boulder County, there are opposite forces and high equivalents in magnitude in two product inputs,  $K*L$  coefficients is negative and approximately equal to the product variable with positive impact of  $L*LA$ .

**Manufacturing Industry:** Only the manufacturing industry in Arapahoe County shows that the data for this sector behaves as non-homogeneous Cobb-Douglas function and at 95% level significance. In other counties, data reveal that this estimation is not different than standard Cobb-Douglas function with multi-input factors of production. The main reasons for these results may be due to equivalent magnitude and opposite forces for these product variables as tables 4.10 to 4.15 reveal. For instance, the sum of product variables of  $K*L+L*LA$  with negative impact are closely equal to  $K*LA$  product variable with positive impact. Also, manufacturing in Denver County has the same behavior with difference in the product variables,  $K*LA+L*LA$  with negative



coefficients are near to the positive impact of  $K*L$ . In Larimer, Weld, and Boulder counties in the manufacturing industries, the estimation of the product variables are insignificant.

**Wholesale Trade:** The wholesale industry shows that the non-homogeneous Cobb-Douglas functions, in all counties, are not different than zero except in Arapahoe, which is significant at 90% level, table 4.12. The main reason may be due to the original estimations which show that the product variables are insignificant in Boulder, Arapahoe, Weld, and Larimer counties. El Paso and Denver counties show close opposite coefficients in the estimated product variables, tables 4.13-4.18.

**Retail Trade:** the retail trade production function, as table (4.12) shows, behaves as non-homogeneous Cobb-Douglas in both Arapahoe and El Paso counties. In other counties, the production function is not non-homogeneous Cobb-Douglas function. The main reasons for this behavior may be due to insignificance of the estimated product variables in retail industry for Larimer and Boulder counties, or approximate equal, significant and opposite in parameters sign of the product variables as in the estimated functions for Denver and Weld.

**Transportation and warehousing:** Except in Larimer County, this industry behaves as not non-homogeneous Cobb-Douglas production function because of two main reasons. The first one is that in both Arapahoe and Weld counties estimation of product variables are close in magnitude and different in their signs. While, the second reason may be attributed to the insignificance in the estimating parameters of the product

variables of the non-homogeneous Cobb-Douglas functions as in El Paso, Denver, and Boulder counties.

**Table 4.12, Non-homogeneous Cobb-Douglas Functions tests by County and Industry**

| El Paso              | Arapahoe             | Denver             | Boulder            | Weld                 | Larimer              |    |
|----------------------|----------------------|--------------------|--------------------|----------------------|----------------------|----|
| -0.209***<br>(44.03) | -0.0712***<br>(17.3) | 0.030<br>(0.78)    | 0.182<br>(2.18)    | -0.112***<br>(21.86) | -0.027<br>(0.87)     | 23 |
| 0.023<br>(0.89)      | -0.083**<br>(4.14)   | 0.008<br>(0.12)    | 0.016<br>(0.06)    | 0.002<br>(0.00)      | 0.011<br>(0.10)      | 31 |
| -0.035<br>(2.40)     | -0.026*<br>(2.75)    | 0.031<br>(2.01)    | -0.033<br>(0.20)   | -0.112<br>(2.45)     | 0.033<br>(1.92)      | 42 |
| -0.063***<br>(35.69) | -0.103***<br>(39.08) | 0.011<br>(0.17)    | -0.082<br>(0.44)   | -0.060<br>(1.35)     | 0.014<br>(1.44)      | 44 |
| -0.058<br>(2.34)     | -0.1138<br>(2.86)    | 0.003<br>(0.02)    |                    | -0.051<br>(0.90)     | 0.084***<br>(5.01)   | 48 |
| -0.033<br>(1.21)     | -0.083***<br>(13.47) | 0.300<br>(1.34)    | -0.146<br>(0.48)   |                      | -0.041<br>(0.71)     | 51 |
| -0.057***<br>(8.09)  | -0.065***<br>(45.11) | 0.073<br>(2.80)    | 0.057<br>(0.06)    | -0.085<br>(0.45)     | -0.076**<br>(5.04)   | 52 |
| -0.103***<br>(27.99) | -0.109***<br>(29.96) | 0.021<br>(0.08)    | -0.073<br>(0.49)   | -0.249**<br>(4.91)   | 0.030<br>(0.67)      | 53 |
| -0.078***<br>(7.63)  | -0.08***<br>(79.96)  | -0.001<br>(0.00)   | -0.126**<br>(4.91) | -0.303***<br>(19.96) | -0.0004<br>(0.00)    | 54 |
| 0.516<br>(0.09)      | -0.011<br>(0.21)     |                    | -4.288<br>(0.43)   |                      |                      | 55 |
| -0.104***<br>(44.64) | -0.061***<br>(19.97) | 0.115**<br>(4.74)  | -0.034<br>(0.07)   | -0.494***<br>(24.29) | 0.1476<br>(0.38)     | 56 |
| -0.181***<br>(8.25)  | -0.076**<br>(4.81)   | -0.035<br>(1.78)   | 0.766**<br>(5.79)  | 0.980<br>(6.78)      | 0.019<br>(0.07)      | 61 |
| -0.036<br>(2.29)     | -0.074***<br>(28.92) | 0.054**<br>(3.43)  | -0.188**<br>(4.79) | 0.002<br>(0.00)      | 0.006<br>(0.07)      | 62 |
| -0.107**<br>(4.43)   | -0.107<br>(2.39)     | -0.029<br>(0.46)   | 0.009<br>(0.00)    |                      | -0.151**<br>(7.16)   | 71 |
| -0.105***<br>(42.93) | -0.074***<br>(10.31) | -0.101**<br>(4.95) | -0.102<br>(0.99)   | -0.114<br>(0.66)     | 0.0008<br>(0.00)     | 72 |
| -0.042*<br>(3.74)    | -0.110***<br>(28.09) | 0.046<br>(1.54)    | 0.117<br>(1.59)    | -0.084<br>(2.00)     | -0.080***<br>(14.02) | 81 |

F-Ratios are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\* Significant at 90% level.

**Information Industry:** The information industry is significant as non-homogeneous Cobb-Douglas function in Arapahoe County. The production function is not behaving non-homogeneous Cobb-Douglas Function in other counties due to either insignificant estimated parameters as in Denver, Boulder, Larimer, and Weld counties, or significant and equivalent opposite forces of signs of the product variables coefficients as in El Paso County. In this case, the product variable of  $K*L$  which has a negative significance parameter is near the positive and significance parameter of  $K*LA$  parameter variable.

**Finance and Insurance Industry:** Table 4.12 shows that the finance industry's production function in Arapahoe, El Paso, and Larimer counties behaves as non-homogeneous Cobb-Douglas function, hence the sum of the product variables is statistically significant at better than 95%. On the other side, the sum of the product variables in other counties reveal that non-homogeneous Cobb-Douglas function is not working because of insignificant estimated coefficient of product variables as in Boulder, and Weld counties.

**Real Estate Industry:** El Paso, Arapahoe, and Weld counties behave in their production function as non-homogeneous Cobb-Douglas function because of the significance of the sum of the product variables in these counties. In Denver County, although the industry estimated parameter for each individual product variable is statistically significant, the sum impact of these variables appears to be insignificant because of opposite and equivalent forces of the variables coefficients. For instance, the sum of  $K*L + L*LA$  variables coefficients which exert negative impacts are near to positive impact of  $K*LA$  product variable coefficient with positive impact. Other

counties' production functions are not behaving as non-homogeneous production function because of insignificant estimated parameters for this industry product input variables as in Boulder and Larimer counties.

**Professional and Scientists Industry:** Arapahoe, El Paso, Weld, and Boulder counties show a significant production process of non-homogeneous Cobb-Douglas function for professional, scientists, and technical services industry. Despite statistical significance of each individual product variable parameter in this industry in Denver County, the sum effect of such variables canceled their impact because of opposite and closes impact of such variable parameters. For example,  $L*LA$  which exerts negative impact in the production process is close in its magnitude to positive  $K*LA$  variable parameter. In Larimer County, the sum of the product variable coefficients reveal that the production process is not non-homogeneous function because of opposite and equal parameters of the product variables,  $K*L+L*LA \cong K*LA$ .

**Management of Companies and Enterprise Industry:** All counties show that industry management of companies and enterprises is not behaving as non-homogeneous Cobb-Douglas function.

**Administrative and Support and Waste Management and Remediation Services Industry:** Arapahoe, El Paso, Denver, and Weld counties production functions process behave like non-homogeneous Cobb-Douglas function because of the statistically significant sum of the parameters of the product variables in the non-homogeneous function in these counties at better than 95%. The rest of the counties aren't revealing non-homogeneous production function process because of insignificance of the sum of the product variable coefficients.

**Education Services:** The non-homogeneous Cobb-Douglas production function for education industry fits the data structure for Arapahoe, El Paso, and Boulder counties, while other counties' production functions are not behaving as non-homogeneous production function in education services.

**Health Care Services:** The data structure for health services industry in Arapahoe, Denver, and Boulder counties appears to be non-homogeneous Cobb-Douglas function. Other counties' data are not behaving non-homogeneous function. There are two reasons for not behaving as non-homogeneous functions. The first is the close magnitude and opposite impact of the product parameters as in Larimer County,  $K*L+K*LA=L*LA$ , and in El Paso County ( $K*LA=K*L+L*LA$ ). The second reason is the insignificance of the estimated parameters of the product variables in health services industry for Weld County.

**Arts, Entertainment, and Recreation Industry:** This industry reveals non-homogeneous Cobb-Douglas function process in Larimer and El Paso counties at better than 95%. The production functions in this industry in other counties are not behaving as non-homogeneous Cobb-Douglas production function.

**Lodge and Restaurants:** The lodge and restaurants industry data behaves as non-homogeneous Cobb-Douglas function in Arapahoe, El Paso, and Denver counties. Boulder and Weld counties' production function in this industry are not behaving as non-homogeneous function because the sum of the product parameter variables is not significant. In Larimer County, the magnitude of the  $K*L$  coefficient variable is negative and proximate to  $K*LA$  parameter with positive effect on the production process.

**Other Services Industry:** This industry behaves as non-homogeneous Cobb-Douglas function at better than 99% significance, and at 90% level in El Paso County.

#### **4.3.4. The Translog Production Function**

The translog production function is characterized by varying elasticity of substitutions and varying economies of scale. In addition, the production function is not an array from the origin, nonhomothetic. The translog function is behaving as non-homogeneous production function if two conditions are satisfied at the same time. These conditions are: the sum of product input variables ( $L \cdot K$ ,  $L \cdot LA$  and  $K \cdot LA$ ) are statistically significant and the sum of product input estimated parameters are also significant, that is if  $\sum \beta_{ij} \neq 0$  (where  $i \neq j$ ), and  $\sum \beta_{ij} \neq 0$  Kim (1992), and Vinod (1972). If  $\sum \beta_{ij} = 0$  (where  $i \neq j$ ), then the production function is Kmenta approximation of CES (Kim, 1992). Therefore, according to Kim's (1992) criteria, this research part is analyzed by economic activity for the Colorado counties under study. The analyses concentrate on the statistical significance of the product of the input variables, and the parameters of the product and square input variables. In this case, if the sum of the product variables and the square of the input variables are not different than zero, then production function supports the standard Cobb-Douglas function with three input variables.

Table 4.19, Translog Test by County and Industry

|    | Arapahoe                     |                      | Elpaso                       |                     | Denver                       |                     | Boulder                      |                    | Larimer                      |                    | Weld                         |                    |
|----|------------------------------|----------------------|------------------------------|---------------------|------------------------------|---------------------|------------------------------|--------------------|------------------------------|--------------------|------------------------------|--------------------|
|    | $\Sigma\beta_{ij, i \neq j}$ | $\Sigma\beta_{ij}$   | $\Sigma\beta_{ij, i \neq j}$ | $\Sigma\beta_{ij}$  | $\Sigma\beta_{ij, i \neq j}$ | $\Sigma\beta_{ij}$  | $\Sigma\beta_{ij, i \neq j}$ | $\Sigma\beta_{ij}$ | $\Sigma\beta_{ij, i \neq j}$ | $\Sigma\beta_{ij}$ | $\Sigma\beta_{ij, i \neq j}$ | $\Sigma\beta_{ij}$ |
| 23 | -0.365***<br>(62.04)         | 0.025<br>(2.59)      | -0.172***<br>(52.61)         | 0.002<br>(0.00)     | -0.179***<br>(20.94)         | 0.035<br>(1.39)     | -0.055<br>(0.07)             | 0.30***<br>(8.33)  | -0.194**<br>(54.39)          | -0.029<br>(2.53)   | -0.216**<br>(111.8)          | 0.009<br>(0.08)    |
| 31 | -0.327***<br>(35.86)         | 0.028<br>(0.84)      | -0.1789**<br>(85.16)         | -0.004<br>(0.1)     | -0.249***<br>(42.55)         | 0.071***<br>(11.62) | -0.998<br>(1.90)             | 0.048<br>(0.21)    | -0.114*<br>(3.48)            | 0.053<br>(2.57)    | -0.119**<br>(7.65)           | 0.024<br>(1.35)    |
| 42 | -0.350***<br>(114.49)        | 0.019<br>(2.41)      | -0.241***<br>(42.82)         | 0.013<br>(0.38)     | -0.188***<br>(15.76)         | 0.062***<br>(11.28) | 0.043<br>(0.00)              | 0.087<br>(0.46)    | -0.168**<br>(14.22)          | -0.002<br>(0.01)   | -0.048<br>(0.71)             | 0.255**<br>(8.4)   |
| 44 | -0.236***<br>(134.26)        | -0.064***<br>(18.86) | -0.146***<br>(100.64)        | -0.036***<br>(13.1) | -0.187***<br>(37.96)         | -0.0005<br>(0.00)   | 0.045<br>(0.06)              | 0.105<br>(0.56)    | -0.172**<br>(90.71)          | 0.007<br>(0.63)    | -0.313**<br>(14.8)           | -0.028<br>(0.39)   |
| 48 | -0.811**<br>(4.6)            | -0.003<br>(0.00)     | -0.256***<br>(48.91)         | 0.008<br>(0.07)     | -0.191<br>(1.49)             | 0.015<br>(0.42)     |                              |                    | -0.098<br>(1.26)             | 0.103**<br>(8.33)  | -0.171*<br>(10.43)           | 0.077<br>(2.81)    |
| 51 | -0.358***<br>(28.59)         | -0.047***<br>(9.48)  | 0.013<br>(0.02)              | 0.022<br>(0.67)     | -1.093<br>(2.18)             | 0.042<br>(0.02)     | -2.91<br>(1.94)              | 0.086<br>(0.13)    | -0.566**<br>(9.27)           | 0.0002<br>(0.00)   |                              |                    |
| 52 | -0.194***<br>(125.29)        | -0.032<br>(11.44)    | -0.059***<br>(40.49)         | -0.037**<br>(4.61)  | -0.226<br>(2.86)             | 0.068<br>(1.66)     | 0.462<br>(0.23)              | 0.039<br>(0.02)    | -0.235**<br>(47.21)          | -0.039*<br>(2.84)  | -0.139<br>(0.53)             | 0.110<br>(0.53)    |
| 53 | -0.219***<br>(34.15)         | -0.059***<br>(4.79)  | -0.136***<br>(22.34)         | -0.007<br>(0.07)    | -0.138<br>(2.25)             | 0.145**<br>(6.1)    | -1.54***<br>(16.34)          | 0.077<br>(0.15)    | -0.173**<br>(15.31)          | 0.081**<br>(6.41)  | -0.507**<br>(13.61)          | 0.326<br>(1.63)    |
| 54 | -0.178***<br>(220.99)        | -0.031***<br>(10.89) | -0.161***<br>(24.84)         | 0.019<br>(0.61)     | -0.158***<br>(13.64)         | 0.049<br>(2.19)     | -0.615**<br>(37.86)          | -0.139**<br>(6.48) | -0.130**<br>(7.52)           | 0.041<br>(2.46)    | -0.293**<br>(21.55)          | -0.054<br>(0.37)   |
| 55 | -0.048<br>(0.03)             | -0.025<br>(1.14)     | 1.764<br>(1.01)              | 2.317<br>(2.38)     |                              |                     |                              |                    |                              |                    |                              |                    |
| 56 | -0.301***<br>(47.31)         | -0.013<br>(1.09)     | -0.149***<br>(56.25)         | -0.004<br>(0.05)    | 0.108**<br>(5.34)            | 0.187***<br>(26.06) | -0.149<br>(0.05)             | -0.037<br>(0.07)   | -0.177**<br>(4.40)           | 0.005<br>(0.01)    | -0.269*<br>(3.12)            | -0.175<br>(0.45)   |
| 61 | -0.372***<br>(13.32)         | 0.006<br>(0.03)      | -0.204***<br>(21.38)         | -0.021<br>(0.28)    | 0.083<br>(0.13)              | -0.027<br>(0.71)    | -4.680*<br>(4.24)            | 0.557**<br>(5.08)  | -0.043<br>(0.00)             | 0.002<br>(0.00)    |                              |                    |
| 62 | -0.263***<br>(27.63)         | -0.047***<br>(13.26) | -0.213***<br>(52.43)         | -0.037**<br>(4.62)  | -0.116***<br>(7.89)          | 0.086***<br>(13.44) | -1.623**<br>(19.24)          | 0.007<br>(0.01)    | -0.126**<br>(18.46)          | 0.05***<br>(7.72)  | -0.031<br>(0.05)             | -0.016<br>(0.19)   |
| 71 | -0.186<br>(0.53)             | -0.104<br>(1.16)     | -0.207***<br>(23.04)         | -0.028<br>(0.22)    | -0.154<br>(0.36)             | -0.004<br>(0.00)    | -0.095<br>(0.03)             | 0.143<br>(0.31)    | -0.176**<br>(15.47)          | -0.006<br>(0.02)   |                              |                    |
| 72 | -0.213***<br>(39.03)         | -0.03<br>(1.8)       | -0.111***<br>(72.31)         | 0.038***<br>(7.13)  | -0.196<br>(47.62)            | 0.027<br>(0.89)     | -0.318<br>(0.14)             | -0.036<br>(0.13)   | -0.132**<br>(43.08)          | -0.003<br>(0.05)   | 0.550<br>(0.85)              |                    |
| 81 | -0.273***<br>(33.38)         | -0.044**<br>(4.14)   | -0.168***<br>(31.33)         | -0.015<br>(0.57)    | -0.116<br>(9.81)             | 0.023<br>(1.00)     | -0.324<br>(2.52)             | 0.103<br>(1.68)    | -0.208**<br>(39.25)          | 0.048**<br>(4.59)  | -0.178**<br>(18.87)          | 0.041<br>(0.7)     |

F-Rtios are in parantheses.  
 \*\*\* Significant at 99% level.  
 \*\* Significant at 95% level.  
 \* Significant at 90% level.

This part of the analysis depends mainly on table 4.19, and tables 4.20-4.24 in the chapter appendix. Table represents the test of data for each industry if it follows the translog behavior or not. Here is the analysis of data behavior by industry in the counties of Colorado under study. The analysis depends mainly on table 4.19.

**Construction Industry:** Although all counties, except construction industry in Boulder County, meet the first condition of translog function, i.e., the sum of the product

variables coefficients are statistically significant at 99% level, this industry fails to meet the second condition which requests that the sum of both the product input variable and square variable input coefficients be statistically significant. Thus, this industry doesn't follow the translog production function in all counties under study. According to the previous part Weld, Arapahoe, and El Paso counties follow the non-homogeneous Cobb-Douglas function.

**Manufacturing Industry:** Only manufacturing industry in Denver County shows that the data for this sector behaves as translog production function at 99% level of significance. The manufacturing industry in Denver meets the two requested conditions at the same time for the production function to be considered as translog. Other counties' data reveal that their estimations meet the first condition but not the second condition to consider the estimation of translog function. Also, as previously shown only manufacturing industry in Arapahoe meets the criteria of non-homogeneous Cobb-Douglas production function.

**Wholesale Trade:** The wholesale industry shows that only Denver County meets the two conditions to reflect the data structure for this industry to behave as translog production function. The wholesale trade industry in other counties does not meet these conditions at the same time. As non-homogeneous Cobb-Douglas function, only Arapahoe meets this condition at 90% level.

**Retail Trade:** The retail trade data shows that production function behaves as translog function in both Arapahoe and El Paso counties. In other counties, the production function is not translog function. In addition, in these two counties, the production process is non-homogeneous Cobb-Douglas functions.

**Transportation and warehousing:** The production process in this industry is not translog function in all counties.



**Information Industry:** Only in Arapahoe County does the information industry production process follow the translog function. In other counties, the production process is standard Cobb-Douglas function.

**Finance and Insurance Industry:** The finance industry in El Paso follows the translog production function and meets the two conditions at better than 99%. Also, the production process follows translog production function at 99% for the first condition and at 90% for the second condition in Larimer County. In Boulder, Weld, and Denver counties, the production process is of standard Cobb-Douglas function with three input Variables.

**Real Estate Industry:** In real estate industry, the production process is translog in both Arapahoe and Larimer counties. The production process is of standard Cobb-Douglas function in Boulder and Denver counties.

**Professional, Scientific, and Technical Services Industry:** The production process in Arapahoe and Boulder counties follows the translog production function at better than 99%. But the production process in Larimer and Denver counties follows the standard Cobb-Douglas function. Furthermore, the production process in this industry both in Weld and El Paso counties follows the non-homogeneous Cobb-Douglas function.

**Management of Companies and Enterprises Industry:** All counties show that this industry process is not behaving as translog production function.

**Administrative and Support and Waste Management and Remediation Services Industry:** Although most counties realize the first condition of translog production function, the industry is not meeting the second condition of translog production function in all counties included in the study. Thus, the production function in this industry is not translog in all counties.

**Education Services:** Only education services in Boulder meet the two conditions simultaneously to produce translog function, while in Arapahoe and El Paso the

production process in education services is of non-homogeneous production function type. In the remaining counties, Larimer, Weld, and Denver, the production process follows the standard Cobb-Douglas function.

**Health Care Services:** The data structure for health services industry in Arapahoe, Denver, El Paso, and Larimer counties appears to be translog function. Other counties' data in Health services are not behaving translog function (Boulder and Weld Counties).

**Arts, Entertainment, and Recreation Industry:** This industry reveals that the production process of translog doesn't follow translog in any of the counties included in the study.

**Lodge and Restaurants:** The lodge and restaurants industry data behaves as translog function in El Paso County. In other counties the production process in lodge and restaurants does not meet the two conditions of translog production at the same time.

**Other Services Industry:** This industry behaves as translog production function at better than 99% level of significance in Arapahoe County, and in Larimer County at better than 95% level.

#### **4.4. Model Fitness Exploring**

This part focuses on exploring the hypotheses that will be examined in the empirical work concerning the model fitness for the firms' data structure in each private sector industry among Colorado counties under study. There are six hypotheses to be tested for each industry in each county. The hypothesis tests are does the Cobb-Douglas, non-homogeneous Cobb-Douglas, or Translog production function fit the data structure for each industry better than other models, or do all, or part, or none of the models fit the data structure.

For the purpose of fitness model, the research estimates three different production functions for each industry in each county. They are: translog, and non-homogeneous Cobb-Douglas functions; and Cobb-Douglas. The former two functions are classified as non-homogeneous production functions, while the latter function is classified as

homogeneous production functions. A comparison among the results of the three production functions will be conducted at the industry level for the six counties in Colorado to choose the function that fits the industry in each county. The new CES production estimate results are contrary to economic theory of production function, since the elasticity of substitutions is negative as a consequence of the estimated Box-Cox transformation parameter being greater than one. Therefore, the results of new CES estimation will not be displayed in this research.

The homogeneity or non-homogeneity of production function reflects the degree of complication and interrelationship among factors of production required to produce output. For instance, the homogeneous production function assumes a constant return to scale Cobb-Douglas function, then to double output the firm needs to double the proportion of inputs. But under non-homogeneous function, the relation among factors of production is more advanced, interrelated, and complicated. Therefore, perfect elasticity of substitution is not available. Thus to produce output, any factor of production has to be greater than zero. Also, to double output under non-homogeneous production function, even under constant return to scale, may not double the inputs in the same proportion.

The production function models provide an excellent portrayal of the advancement of production process in one county compared to another. Also, the production function reflects the extent of interaction and interrelationship between inputs to produce output. For example, if one homogeneous production function is prevalent in county A firms industry, while non-homogeneous function is prevalent in county B firms industry, then the production function in county A reflects the traditional relationship between factors of production. On the other hand, if the non-homogeneous function is pervasive in county B, then this reflects the advanced and complicated interrelationship between factors of production.

#### 4.4.1. Nested Models

Before investigating the comprehensive J-test, the research describes the nested relations among the different production functions; the translog, non-homogeneous Cobb-Douglas, CES, and standard Cobb-Douglas functions. According to Kim (1992), translog functions nested all other functions. For example, assume a translog function with three input variables; capital (K), labor (L), and land (LA). The following is the formula for this translog function:

$$\begin{aligned} \ln Y = & \alpha_0 + \alpha_1 \ln K + \alpha_2 \ln L + \alpha_3 \ln LA + \alpha_4 (\ln K * \ln L) + \alpha_5 (\ln K * \ln LA) + \alpha_6 (\ln L * \ln LA) \\ & + \alpha_7 \ln K^2 + \alpha_8 \ln L^2 + \alpha_9 \ln LA^2 \end{aligned}$$

In the previous function, if  $\alpha_4, \alpha_5, \alpha_6$  are statistically not different than zero, then the production function is of Kmenta (1967) CES function (Kim, 1992). In other cases, if  $\alpha_7, \alpha_8, \alpha_9$  are not statistically different than zero, then the function is non-homogeneous Cobb-Douglas function. In addition, if the coefficients of the product variables and the square of the input variables are not statistically different than zero, then the production function is standard Cobb-Douglas function.

Also, the standard Cobb-Douglas is a special case of non-homogeneous Cobb-Douglas function. Therefore, Cobb-Douglas function is nested in non-homogeneous Cobb-Douglas function and translog function. Also, the nonhomogeneous Cobb-Douglas function is nested in the translog function.

#### 4.4.2. Nonnested Tests

In this study, the comprehensive J-test will be adopted to compare different models of production function that will fit the data structure better than others based on testing nonnested hypotheses. There are different approaches to conduct this test: (i) the comprehensive approach-the J-test; (ii) Young and Kullback-leibler information criterion

(KLIC) is a measure that provides a test between the true model and the hypothesized model. This type of test depends on the likelihood ratio. The main drawback of this test is requested time series data; and (iii) Akaike information Criterion and Bayesian Information Criterion. The main drawback with this type of test is that these tests are biased toward small models (Greene (6<sup>th</sup> edition, 2007).

This part of the chapter will adopt the second filter to choose the model or models that fit the data structure for each industry in each county, the J-test. Adopting this test helps choose the best model that fits the data structure in Colorado counties, either the Cobb-Douglas, non-homogeneous Cobb-Douglas, or translog production function. The J-test is introduced by Davidson and MacKinnon (1981). According to this test, there are four possible outcomes: (i) rejecting both the null and alternative hypotheses where none of the models perfectly fit the data structure; (ii) neither of the model hypotheses, null or alternative, are rejected. This means both models are accepted to fit the data; (iii) if the null model is accepted; (iv) if the alternative model is accepted. The last two outcomes means that either the null hypotheses model or the alternative model better fits the data structure under study, but not the other way. This means the model that best fits the data structure moves in one direction but not the other (Greene 6<sup>th</sup> edition, 2007).

The two hypotheses must have the same dependent variable. The null and alternative hypotheses can be written as follows:

$H_0: y = X\beta + \varepsilon_0$ , and the alternative model hypothesis is:

$H_1: y = Z\alpha + \varepsilon_1$

The test suggests estimating the alternative hypothesis by OLS. Then estimating the null hypothesis model by including the fitted data of  $y$  from the alternative model, also by using OLS technique. If the coefficient of  $y$  is significant, then the model in the null hypothesis fits the data. In the second step, the null and alternative hypothesis must be altered. In this case if the estimated coefficient of the fitted values from the null model,

which are included in the alternative model, is significant, then the alternative model also fits the data structure. Therefore, two models have to be accepted and the results of the two models explained accordingly. In this study, 6 estimates for each industry in each county will be conducted. These estimates will reflect the direction of the hypotheses relations between Cobb-Douglas and non-homogeneous Cobb-Douglas, Cobb-Douglas and translog, and non-homogeneous Cobb-Douglas and translog production functions, and the reverse of these production function hypotheses. For example, the direction of the relation between non-homogeneous function and Cobb-Douglas hypotheses will also be tested.

Hereby the research displays the results of the model fitness for the estimated different production function models for the private sector industries in six Colorado counties. The analysis of this part will rely on tables 4.25-4.30.

**Table 4.25, Testing Fitness Model by Industry and Production Functions in Arapahoe**

|       | Between Cobb Douglas<br>and Non-homogeneous<br>CD |                     | Between Cobb Douglas<br>and Translog |                     | Between Translog<br>and Non-<br>homogeneous |                      |
|-------|---|---------------------|--------------------------------------|---------------------|---|----------------------|
|       | H0  | H1                  | H0                                   | H1                  | H0  | H1                   |
| 23    | Dropped   | 0.999***<br>(0.092) | Dropped                              | 1.000***<br>(0.053) | Dropped                                     | 0.621***<br>(0.075)  |
| 31-33 | Dropped   | 0.999***<br>(0.353) | Dropped                              | 1.000***<br>(0.089) | Dropped                                     | 0.864***<br>(0.215)  |
| 42    | Dropped   | 0.999***<br>(0.252) | Dropped                              | 1.000***<br>(0.065) | Dropped                                     | 1.000***<br>(0.0698) |
| 44-45 | Dropped   | 0.999***<br>(0.121) | Dropped                              | 1.000***<br>(0.070) | 1.088***<br>(0.096)                         | 1.000***<br>(0.097)  |
| 48-49 | Dropped   | 0.999***<br>(0.318) | Dropped                              | 1.000***<br>(0.146) | 0.802**<br>(0.306)                          | 0.999***<br>(0.209)  |
| 51    | Dropped   | 0.999***<br>(0.129) | Dropped                              | 1.000***<br>(0.059) | 1.154***<br>(0.163)                         | 0.999***<br>(0.094)  |
| 52    | Dropped   | 0.999***<br>(0.094) | Dropped                              | 1.000***<br>(0.057) | 0.915***<br>(0.081)                         | 1.000***<br>(0.086)  |
| 53    | Dropped   | 0.999***<br>(0.152) | Dropped                              | 0.999***<br>(0.114) | 0.995***<br>(0.156)                         |                      |
| 54    | Dropped   | 1.00***<br>(0.099)  | Dropped                              | 1.00***<br>(0.051)  | 0.805***<br>(0.081)                         | 1.00***<br>(0.065)   |
| 55    | Dropped   | 1.000***<br>(0.266) | Dropped                              | 1.000***<br>(0.168) | 1.114***<br>(0.250)                         | 1.000***<br>(0.260)  |
| 56    | Dropped   | 0.999***<br>(0.122) | Dropped                              | 1.000***<br>(0.068) | 0.936***<br>(0.093)                         | 1.000***<br>(0.094)  |
| 61    | Dropped   | 0.999***<br>(0.354) | Dropped                              | 0.999***<br>(0.165) | -0.317<br>(0.727)                           | 0.999***<br>(0.209)  |
| 62    | Dropped   | 0.999***<br>(0.142) | Dropped                              | 1.000***<br>(0.080) | 0.940***<br>(0.126)                         | 1.000***<br>(0.107)  |
| 71    | Dropped   | 1.000***<br>(0.206) | Dropped                              | 0.999***<br>(0.192) | 0.707<br>(0.6490)                           | 0.999<br>(0.805)     |
| 72    | Dropped   | 1.000***<br>(0.123) | Dropped                              | 1.000***<br>(0.083) | 0.761***<br>(0.110)                         | 1.000***<br>(0.137)  |
| 81    | Dropped   | 1.000***<br>(0.163) | Dropped                              | 1.000***<br>(0.075) | 0.751***<br>(0.122)                         | 1.000***<br>(0.096)  |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

• Significant at 90% level.

**Table 4.26, Testing Fitness Model by Industry and Production Functions in El Paso**

|       | Between Cobb Douglas and Non-homogeneous CD |                      | Between Cobb Douglas and Translog |                      | Between Translog and Non-homogeneous |                      |
|-------|---|----------------------|-----------------------------------|----------------------|--------------------------------------|----------------------|
|       | H0  | H1                   | H0                                | H1                   | H0                                   | H1                   |
| 23    | Dropped                                     | 1.000***<br>(0.103)  | Dropped                           | 0.999***<br>(0.049)  | 0.643***<br>(0.0702)                 | 0.999***<br>(0.0648) |
| 31-33 | Dropped                                     | 1.000***<br>(0.2088) | Dropped                           | 1.000***<br>(0.0544) | 1.932***<br>(0.1292)                 | 1.000***<br>(0.0644) |
| 42    | Dropped                                     | 1.000***<br>(0.2696) | Dropped                           | 0.999***<br>(0.1182) | 0.007<br>(0.4807)                    | 0.999***<br>(0.1425) |
| 44-45 | Dropped                                     | 1.000***<br>(0.0593) | Dropped                           | 1.000***<br>(0.0430) | 0.8596***<br>(0.0507)                | 1.000***<br>(0.0835) |
| 48-49 | Dropped                                     | 1.000***<br>(0.2663) | Dropped                           | 1.000***<br>(0.0883) | 1.609***<br>(0.1880)                 | 0.999***<br>(0.1100) |
| 51    |   |                      |                                   |                      |                                      |                      |
| 52    | dropped                                     | 0.999***<br>(0.1109) | dropped                           | 1.000***<br>(0.0652) | 0.917***<br>(0.0695)                 | 1.000***<br>(0.1057) |
| 53    | dropped                                     | 1.000***<br>(0.153)  | dropped                           | 1.000***<br>(0.0964) | 0.777***<br>(0.125)                  | 1.000***<br>(0.1447) |
| 54    | dropped                                     | 0.999***<br>(0.2075) | dropped                           | 1.000***<br>(0.0791) | 1.026***<br>(0.139)                  | 1.000<br>(0.0901)    |
| 55    |   |                      |                                   |                      |                                      |                      |
| 56    | dropped                                     | 0.999***<br>(0.1188) | dropped                           | 1.000***<br>(0.0718) | 0.773***<br>(0.0928)                 | 0.999***<br>(0.1090) |
| 61    | dropped                                     | 0.999***<br>(0.247)  | dropped                           | 1.000***<br>(0.0772) | 0.635***<br>(0.121)                  | 1.000***<br>(0.097)  |
| 62    | dropped                                     | 0.999***<br>(0.1176) | dropped                           | 1.000***<br>(0.0526) | 0.833***<br>(0.0702)                 | 1.000***<br>(0.0696) |
| 71    | dropped                                     | 0.999**<br>(0.443)   | dropped                           | 1.000***<br>(0.151)  | 0.999<br>(0.732)                     | 1.000***<br>(0.183)  |
| 72    | dropped                                     | 1.000***<br>(0.0729) | dropped                           | 1.000***<br>(0.0372) | 0.5383***<br>(0.0612)                | 0.999***<br>(0.0569) |
| 81    | dropped                                     | 0.999***<br>(0.107)  | dropped                           | 1.000***<br>(0.0623) | 1.017***<br>(0.0988)                 | 1.000***<br>(0.0953) |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.



**Table 4.27, Testing Fitness Model by Industry and Production Functions in Denver**

|       | Between Cobb Douglas and Non-homogeneous CD |                       | Between Cobb Douglas and Translog |                      | Between Translog and Non-homogeneous |                       |
|-------|---|-----------------------|-----------------------------------|----------------------|--------------------------------------|-----------------------|
|       | H0  | H1                    | H0                                | H1                   | H0                                   | H1                    |
|       | 23  | Dropped               |                                   | Dropped              | 0.999***<br>(0.0971)                 | 1.5003***<br>(0.3581) |
| 31-33 | Dropped                                     | 1.000***<br>(0.2971)  | Dropped                           | 0.999***<br>(0.1026) | 0.937***<br>(0.3138)                 | 0.999***<br>(0.1175)  |
| 42    | Dropped                                     | 1.000***<br>(0.3093)  | Dropped                           | 1.000***<br>(0.1093) | 0.999***<br>(0.294)                  | 1.000***<br>(0.124)   |
| 44-45 | Dropped                                     | 0.999***<br>(0.174)   | Dropped                           | 0.999***<br>(0.073)  | 1.161***<br>(0.197)                  | 0.999***<br>(0.0905)  |
| 48-49 |   |                       |                                   |                      |                                      |                       |
| 51    |   |                       |                                   |                      |                                      |                       |
| 52    | Dropped                                     | 1.00***<br>(0.255)    | Dropped                           | 0.999***<br>(0.169)  | 0.2440<br>(0.5878)                   | 0.999*<br>(0.493)     |
| 53    | Dropped                                     | 1.000***<br>(0.2253)  | Dropped                           | 1.000***<br>(0.102)  | 0.825<br>(0.529)                     | 0.999***<br>(0.140)   |
| 54    | Dropped                                     | 1.000***<br>(0.222)   | Dropped                           | 0.999***<br>(0.085)  | 0.864***<br>(0.249)                  | 1.000***<br>(0.097)   |
| 55    |   |                       |                                   |                      |                                      |                       |
| 56    | Dropped                                     | 1.000**<br>(0.384)    | Dropped                           | 0.999***<br>(0.086)  | 0.431<br>(0.525)                     | 0.999***<br>(0.094)   |
| 61    | Dropped                                     | 0.999<br>(0.639)      | Dropped                           | 1.000***<br>(0.295)  | -1.184<br>(1.162)                    | 1.000**<br>(0.365)    |
| 62    | Dropped                                     | 1.000***<br>(0.212)   | Dropped                           | 1.000***<br>(0.091)  | 0.512<br>(0.366)                     | 1.000***<br>(0.110)   |
| 71    |   |                       |                                   |                      |                                      |                       |
| 72    | dropped                                     | 1.000***<br>(0.0.174) | dropped                           | 1.000***<br>(0.049)  | 0.557***<br>(0.1077)                 | 1.000***<br>(0.058)   |
| 81    | dropped                                     | 1.000***<br>(0.154)   | dropped                           | 1.000***<br>(0.057)  | 0.659***<br>(0.167)                  | 0.999***<br>(0.073)   |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

• Significant at 90% level.

**Table 4.28, Testing Fitness Model by Industry and Production Functions in Larimer**

|       | Between Cobb Douglas and Non-homogeneous CD |                     | Between Cobb Douglas and Translog |                     | Between Translog and Non-homogeneous |                     |
|-------|---|---------------------|-----------------------------------|---------------------|--------------------------------------|---------------------|
|       | H0  | H1                  | H0                                | H1                  | H0                                   | H1                  |
| 23    | Dropped                                     | 1.000**<br>(0.409)  | Dropped                           | 1.000***<br>(0.076) | 1.881***<br>(0.208)                  | 1.000***<br>(0.083) |
| 31-33 | Dropped                                     | 0.999<br>(0.752)    | Dropped                           | 1.000***<br>(0.181) | Dropped                              | 1.000***<br>(0.194) |
| 42    | Dropped                                     | 1.000**<br>(0.382)  | Dropped                           | 0.999***<br>(0.134) | Dropped                              | 0.999***<br>(0.153) |
| 48-49 | Dropped                                     | 0.999***<br>(0.346) | Dropped                           | 1.000***<br>(0.064) | Dropped                              | 1.000***<br>(0.066) |
| 51    | Dropped                                     | 1.000**<br>(0.417)  | Dropped                           | 0.999***<br>(0.223) | -0.223<br>(0.800)                    | 0.999***<br>(0.313) |
| 52    | Dropped                                     | 1.000**<br>(0.423)  | Dropped                           | 0.999***<br>(0.208) | 2.261**<br>(0.822)                   | 0.999***<br>(0.287) |
| 53    | Dropped                                     | 1.000***<br>(0.334) | Dropped                           | 1.000***<br>(0.081) | 1.305***<br>(0.209)                  | 1.000***<br>(0.087) |
| 54    | Dropped                                     | 1.000**<br>(0.396)  | Dropped                           | 1.000***<br>(0.123) | 0.999***<br>(0.205)                  | 1.000***<br>(0.138) |
| 55    | Dropped                                     | 1.000***<br>(0.227) | Dropped                           | 1.000***<br>(0.131) | 1.000***<br>(0.207)                  | 0.999***<br>(0.176) |
| 56    | Dropped                                     | 0.999<br>(0.620)    | Dropped                           | 0.999***<br>(0.151) | Dropped                              | 0.999***<br>(0.171) |
| 61    |   |                     |                                   |                     |                                      |                     |
| 62    | Dropped                                     | 0.999***<br>(0.188) | Dropped                           | 1.000***<br>(0.066) | 1.156***<br>(0.174)                  | 1.000***<br>(0.079) |
| 71    | Dropped                                     | 0.999***<br>(0.322) | Dropped                           | 0.999***<br>(0.099) | 1.036***<br>(0.301)                  | 0.999***<br>(0.128) |
| 72    | Dropped                                     | 0.999***<br>(0.200) | Dropped                           | 1.000***<br>(0.063) | Dropped                              | 1.000***<br>(0.070) |
| 81    | Dropped                                     | 1.000***<br>(0.171) | Dropped                           | 1.000***<br>(0.082) | 0.849***<br>(0.134)                  | 1.000***<br>(0.108) |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

• Significant at 90% level.

**Table 4.29, Testing Fitness Model by Industry and Production Functions in Boulder**

|       | Between Cobb Douglas and Non-homogeneous CD |                     | Between Cobb Douglas and Translog |                     | Between Translog and Non-homogeneous |                     |
|-------|---|---------------------|-----------------------------------|---------------------|--------------------------------------|---------------------|
|       | H0  | H1                  | H0                                | H1                  | H0                                   | H1                  |
|       | 23  | Dropped             | 0.999***<br>(0.215)               | Dropped             | 0.999***<br>(0.107)                  | -0.189<br>(0.432)   |
| 31-33 | Dropped                                     | 1.000*<br>(0.554)   | Dropped                           | 1.000**<br>(0.367)  | 0.466<br>(0.445)                     | 1.000*<br>(0.567)   |
| 42    | Dropped                                     | 1.000<br>(0.663)    | Dropped                           | 0.999**<br>(0.409)  | -2.44<br>(2.102)                     | 0.999*<br>(0.539)   |
| 44-45 | Dropped                                     | 1.000<br>(0.785)    | Dropped                           | 0.999***<br>(0.315) | 0.578<br>(0.834)                     | 0.999**<br>(0.367)  |
| 48-49 |   |                     |                                   |                     |                                      |                     |
| 51    | Dropped                                     | 1.000**<br>(0.352)  | Dropped                           | 0.999***<br>(0.226) | 0.524<br>(0.530)                     | 0.9999**<br>(0.374) |
| 52    | Dropped                                     | 0.999<br>(0.862)    | Dropped                           | 1.000**<br>(0.453)  | 1.354<br>(1.854)                     | 1.000<br>(0.596)    |
| 53    | Dropped                                     | 1.000<br>(1.186)    | Dropped                           | 0.999***<br>(0.198) | 1.681<br>(1.707)                     | 0.499*<br>(0.269)   |
| 54    | Dropped                                     | 0.999***<br>(0.273) | Dropped                           | 0.999***<br>(0.122) | 1.425***<br>(0.267)                  | 0.999***<br>(0.141) |
| 55    |   |                     |                                   |                     |                                      |                     |
| 56    | Dropped                                     | 1.000***<br>(0.293) | Dropped                           | 1.000***<br>(0.192) | -1.451<br>(2.966)                    | 1.000***<br>(0.282) |
| 61    |   |                     |                                   |                     |                                      |                     |
| 62    | Dropped                                     | 1.000**<br>(0.421)  | Dropped                           | 0.999***<br>(0.127) | 4.766***<br>(1.212)                  | 0.999***<br>(0.143) |
| 71    |   |                     |                                   |                     |                                      |                     |
| 72    |   |                     |                                   |                     |                                      |                     |
| 81    | Dropped                                     | 0.999<br>(0.687)    | Dropped                           | 1.000***<br>(0.187) | Dropped                              | 1.000***<br>(0.212) |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

• Significant at 90% level.

**Table 4.30, Testing Fitness Model by Industry and Production Functions in Weld**

|       | Between Cobb Douglas<br>and Non-homogeneous<br>CD |                     | Between Cobb<br>Douglas<br>and Translog |                     | Between Translog<br>and Non-<br>homogeneous |                     |
|-------|---|---------------------|---|---------------------|---|---------------------|
|       | H0  | H1                  | H0                                      | H1                  | H0  | H1                  |
|       | 23  | Dropped             | 1.000***<br>(0.099)                     | Dropped             | 0.999***<br>(0.057)                         | 1.723***<br>(0.194) |
| 31-33 | Dropped   | 1.000<br>(0.971)    | Dropped                                 | 1.000***<br>(0.181) | Dropped                                     | 1.000***<br>(0.196) |
| 42    | Dropped   | 0.999***<br>(0.231) | Dropped                                 | 1.000***<br>(0.119) | 1.147***<br>(0.282)                         | 1.000***<br>(0.175) |
| 44-45 | Dropped   | 1.000***<br>(0.316) | Dropped                                 | 1.000***<br>(0.153) | 2.787***<br>(0.871)                         | 0.999***<br>(0.219) |
| 48-49 | Dropped   | 1.000***<br>(0.282) | Dropped                                 | 0.999***<br>(0.101) | 0.917***<br>(0.181)                         | 0.999***<br>(0.134) |
| 51    |   |                     |   |                     |   |                     |
| 52    | Dropped   | 0.999**<br>(0.433)  | Dropped                                 | 0.999***<br>(0.197) | 1.195**<br>(0.477)                          | 0.999***<br>(0.265) |
| 53    |   |                     |   |                     |   |                     |
| 54    | Dropped   | 0.999***<br>(0.195) | Dropped                                 | 0.999***<br>(0.132) | 1.054***<br>(0.247)                         | 0.999***<br>(0.209) |
| 55    |   |                     |   |                     |   |                     |
| 56    | Dropped   | 1.000***<br>(0.114) | Dropped                                 | 0.999***<br>(0.095) | 0.458<br>(0.288)                            | 1.000***<br>(0.304) |
| 61    |   |                     |   |                     |   |                     |
| 62    | Negative<br>Land<br>Coefficient                   |                     |   |                     |   |                     |
| 71    |   |                     |   |                     |   |                     |
| 72    |   |                     |   |                     |   |                     |
| 81    | Dropped   | 0.999***<br>(0.198) | Dropped                                 | 1.000***<br>(0.090) | -1.042<br>(0.522)                           | 1.000***<br>(0.136) |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\* Significant at 90% level.

The research starts clarifying some of the results in these estimations, in particular, the meaning of dropped terms. The first dropped term appears in the estimation because of high multicollinearity between the fitted values of Cobb-Douglas function and the translog function. In addition, the second dropped term appears because of high multicollinearity between the fitted values of Cobb-Douglas function and translog production function. Accordingly, the research accepts these relations are holds irrespective of the term dropped that appears in the different estimations.

The main results of the estimations show that the results among different hypotheses and the reverse of their directions are inconclusive. The results of testing the model fitness mostly agree with Mushinski et al (unpublished paper), and Pesaran and Deaton (1978) that is the research left without satisfactory model. The following are the main results of these estimations according to industry in each county:

**Construction Industry:** The results of the estimated direction hypotheses between Cobb-Douglas and non-homogeneous Cobb-Douglas, and its reverse direction are inconclusive at better than 99% level of significance in all counties except for Larimer County which is significant at 95% level of significance. Also, the results of the estimated direction hypotheses between Cobb-Douglas and translog production functions and their reverse direction are inconclusive at better than 99% level in all counties. In addition, the relation between non-homogeneous Cobb-Douglas function and translog functions are inconclusive and statistically significant at 99% level.

**Manufacturing Industry:** In the six tests, it appears that the model fitness test is inconclusive for different production functions in Arapahoe, Denver, and Boulder counties at better than 99% level of significance. On the other hand, the model fitness

direction is from Cobb-Douglas to non-homogeneous Cobb-Douglas function in Weld, and Larimer counties, while other tests for fitness models are inconclusive in Weld and Larimer counties at better than 99% level of significance.

**Wholesale Trade Industry:** The results show that the production function that fits the data in wholesale trade industry is inconclusive in Arapahoe, Weld, El Paso, and Denver counties at better than 99% level of significance. In Larimer County, this industry production function is inconclusive, but it is significant at better than 95% level. The production function that fits the data structure in Boulder County is clear, where the direction is from Cobb-Douglas to non-homogeneous Cobb-Douglas function but the reverse is insignificant. Also, in Boulder County, this industry direction is insignificant from translog to non-homogeneous Cobb-Douglas function. Therefore Boulder County is governed by non-homogeneous Cobb-Douglas function in wholesale trade industry.

**Retail Trade Industry:** The retail trade industry production function fitness test shows inconclusive results regarding the direction of the production process data in five counties; El Paso, Arapahoe, Denver, Larimer, and Weld. The production function fitness is obvious in Boulder County; the non-homogeneous Cobb-Douglas production function is the best fit for the data.

**Transportation and Warehousing Industry:** The direction of production function fitness in transportation and warehousing industry is inconclusive in four counties; El Paso, Arapahoe, Larimer, and Weld Counties at better than 99% level of significance. In other counties (Boulder and Denver), there are no results because of the small number of observations in this industry in these counties.

**Information Industry:** The fitness model tests results for information industry show that the direction of the fitness model is inconclusive at better than 99% of significance level in four counties: El Paso, Arapahoe, Larimer, and Weld. In Denver and Boulder counties, there are no results due to a small number of observations.

**Finance and Insurance Industry:** The test of fitness model results of this test for finance and insurance industry is inconclusive in five counties at better than 95% in Larimer and Weld counties, while at better than 99% of significance in El Paso, Arapahoe, and Denver counties. In Boulder County, the tests of fitness model show that the results do not hold in any direction.

**Real Estate Industry:** The results of the fitness model indicate are inconclusive in three counties at better than 99% of significance level (El Paso, Arapahoe, and Larimer). In Denver and Boulder counties these tests reveal that the non-homogeneous is preferred to standard Cobb-Douglas, and translog to non-homogeneous production function. That translog is preferred to any of the fitness models in real estate industry in Denver County.

**Professionals, Scientists, and Technicians Services:** The fitness model results reveal that the direction of fitness model in five counties in this industry is inconclusive at better than 99% level of significance in Denver, Arapahoe, Boulder, and Weld counties, while at better than 95% level in Larimer County. In El Paso County, the direction of fitness model is clear from standard Cobb-Douglas or translog to non-homogeneous Cobb-Douglas function.

**Management of Companies and Enterprises:** This industry shows that the fitness model tests are inconclusive in Arapahoe County for the six different estimates, while other counties show no results because of small number of observations.

**Administrative and Support and Waste Management and Remediation Services Industry:** The testing hypotheses results of model fitness show differences among counties. For example, this industry in El Paso and Arapahoe counties show inconclusive direction and significance at better than 99% level. In Boulder, Weld, and Denver counties the tests show that the direction of test is inconclusive between Cobb-Douglas and non-homogeneous Cobb-Douglas, and between translog and Cobb-Douglas. But the fitness results in these three counties obviously determine the direction from non-homogeneous to translog. The results of the test regarding Larimer County are from Cobb-Douglas to non-homogeneous Cobb-Douglas function. In other tests, the model fitness tests are inconclusive at better than 95%.

**Education services:** The results of the fitness test for this industry can be divided into three groups. The first group is the inconclusive results for El Paso and Arapahoe counties at better than 99% level of significance. The second group which includes Denver and Boulder counties shows the results from standard Cobb-Douglas to non-homogeneous Cobb-Douglas function and from Cobb-Douglas to translog production function and their reverse direction tests are inconclusive at better than 99% level. But the direction of the fitness is from non-homogeneous Cobb-Douglas to translog. Regarding the third group, the number of observations is not enough to conduct the estimations.



**Health Care Services:** The three production function models show inconclusive results to satisfy certain production functions. Thus, the three models under study can represent the data structure for health services at better than 99% level in all counties except Weld.

**Arts, Entertainment, and Recreation:** Three groups of results appeared. The first group includes the inconclusive results of fitness estimation at better than 99% of significance in this industry in El Paso and Larimer counties. The second group includes the industry in Arapahoe County where the fitness test reveals that the test from standard Cobb-Douglas to non-homogeneous Cobb-Douglas, and from standard Cobb-Douglas to translog and the reverse direction is inconclusive at better than 99% level. The translog and non-homogeneous test reveals no relation between these two functions in this industry for Arapahoe County. The third group shows no results because of the small number of observations in this industry in Weld, Denver, and Boulder counties.

**Accommodation and Food Services:** The fitness tests for this industry are inconclusive in their direction at 99% level in El Paso, Denver, Arapahoe, and Larimer counties. The other counties show no results due to small number of observations.

**Other Services (except Public Administration):** The results of model fitness for other services industry can be classified into three groups. The first group, the main group, shows inconclusive direction of the fitness test hypotheses at better than 99% level of significance in El Paso, Denver, Arapahoe, and Larimer counties. The second group, Boulder County, reveals that the result is from Cobb-Douglas to non-homogeneous Cobb-Douglas, but the reverse is insignificant. The other two parts of the test between Cobb-Douglas and translog, and translog and non-homogeneous Cobb-

Douglas functions are inconclusive at better than 99% level of significance. The third group, Weld County, shows obvious direction from non-homogeneous to translog direction, but not the other way around, while the other two parts of fitness test reveals inconclusive results.

#### **4.5. Summary**

The research estimates four different production functions. They are Cobb-Douglas, new CES, non-homogeneous Cobb-Douglas, and translog function. The new CES is excluded from any hypotheses testing due to contrary results to economic literature regarding the production function, where the elasticity of substitution is negative. Regarding the model fitness, J-test is conducted. The main conclusions to this chapter are:

- (i) Land is an important and significant variable in the production process.
- (ii) The magnitude of land variable is different among counties, but as Cobb-Douglas production function reveals, Denver County has the highest land coefficients in all industries among the counties under study.
- (iii) Model fitness tests are mostly inconclusive to determine the best production function that fits the data structure for each industry in each county. Thus, there is no satisfactory model.

## Chapter 4, Appendix

**Table 4.13, Non-homogeneous Cobb-Douglas estimation in Arapahoe County**

|    | K                     | L                     | LA                   | K*L                   | K*LA                  | L*LA                  | Cons.                  | R2   |
|----|-----------------------|-----------------------|----------------------|-----------------------|-----------------------|-----------------------|------------------------|------|
| 23 | 0.2603**<br>(0.1066)  | 2.474***<br>(0.2153)  | -0.61***<br>(0.1443) | -0.226***<br>(0.0277) | 0.0571***<br>(0.0099) | 0.0977***<br>(0.0225) | 8.591***<br>(1.278)    | 0.91 |
| 31 | 0.605**<br>(0.2827)   | 1.4747***<br>(0.4337) | 0.0195<br>(0.3820)   | -0.0726*<br>(0.0393)  | 0.0101<br>(0.0278)    | -0.0204<br>(0.0350)   | 3.8427<br>(3.6705)     | 0.82 |
| 42 | 0.2162*<br>(0.1292)   | 0.9670***<br>(0.2137) | -0.1152<br>(0.1824)  | -0.0223<br>(0.026)    | 0.0315**<br>(0.0131)  | -0.0356<br>(0.0245)   | 8.0477***<br>(1.6159)  | 0.91 |
| 44 | 0.5551***<br>(0.1249) | 1.8901***<br>(0.2075) | -0.1011<br>(0.1784)  | -0.091***<br>(0.0183) | 0.0229*<br>(0.0132)   | -0.0355**<br>(0.0147) | 4.4904***<br>(1.5378)  | 0.88 |
| 48 | 0.5285<br>(0.4300)    | 3.3252**<br>(1.219)   | -0.9330*<br>(0.5323) | -0.3639**<br>(0.1636) | 0.0652**<br>(0.0301)  | 0.1849*<br>(0.1109)   | 7.2268*<br>(4.0981)    | 0.91 |
| 51 | 0.2043<br>(0.2515)    | 1.2390***<br>(0.2803) | 0.1658<br>(0.2754)   | 0.0508<br>(0.0358)    | 0.0216<br>(0.0206)    | -0.156***<br>(0.0340) | 6.7907**<br>(3.0664)   | 0.95 |
| 52 | 0.4709***<br>(0.0769) | 1.6057***<br>(0.1438) | -0.32***<br>(0.1036) | -0.075***<br>(0.0175) | 0.0312***<br>(0.0069) | -0.0217<br>(0.0171)   | 6.5953***<br>(0.9497)  | 0.94 |
| 53 | 0.5968***<br>(0.1488) | 1.8231***<br>(0.3053) | -0.2194<br>(0.2208)  | -0.0586<br>(0.0381)   | 0.0240*<br>(0.0149)   | -0.0746**<br>(0.0362) | 4.8727**<br>(1.8858)   | 0.93 |
| 54 | 0.6560***<br>(0.0698) | 1.4546***<br>(0.1336) | 0.1984**<br>(0.1009) | -0.085***<br>(0.0158) | 0.0024<br>(0.0068)    | 0.0019<br>(0.0149)    | 2.9110***<br>(0.8707)  | 0.94 |
| 55 | -0.0678<br>(0.2861)   | 1.2333***<br>(0.3383) | -0.3106<br>(0.3198)  | -0.0791<br>(0.0613)   | 0.0504**<br>(0.0224)  | 0.0169<br>(0.0649)    | 11.3867***<br>(3.4799) | 0.96 |
| 56 | 0.4807***<br>(0.1391) | 1.4745***<br>(0.1677) | -0.1108<br>(0.1510)  | -0.083***<br>(0.0245) | 0.0252**<br>(0.0109)  | -0.0031<br>(0.0238)   | 5.0843***<br>(1.5488)  | 0.93 |
| 61 | 1.3057***<br>(0.2955) | 0.7142*<br>(0.4452)   | 0.8594**<br>(0.3929) | -0.0764<br>(0.0567)   | -0.0509*<br>(0.0281)  | 0.0508<br>(0.0621)    | -5.1388<br>(3.6721)    | 0.91 |
| 62 | 0.6576***<br>(0.1131) | 1.1762***<br>(0.1713) | 0.1405<br>(0.1526)   | -0.0344<br>(0.0212)   | 0.0058<br>(0.0107)    | -0.0458**<br>(0.0204) | 2.9865**<br>(1.3817)   | 0.93 |
| 71 | 0.4705<br>(0.3423)    | 0.1941<br>(0.5453)    | 1.3717**<br>(0.5614) | 0.1277***<br>(0.0421) | -0.0381<br>(0.0372)   | -0.197***<br>(0.0447) | -0.1343<br>(4.8527)    | 0.94 |
| 72 | 0.8469***<br>(0.1311) | 1.8502***<br>(0.2170) | -0.3109<br>(0.2036)  | -0.176***<br>(0.0257) | 0.0170<br>(0.0125)    | 0.0846***<br>(0.0279) | 3.2126***<br>(1.7129)  | 0.90 |
| 81 | 0.7099***<br>(0.1266) | 1.9029***<br>(0.2912) | 0.0575<br>(0.1785)   | -0.135***<br>(0.0378) | 0.0103<br>(0.0122)    | 0.0141<br>(0.0375)    | 2.6270**<br>(1.5299)   | 0.91 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

**Table 4.14, Non-homogeneous Cobb-Douglas estimation in El Paso County**

|    | K                     | L                     | LA                   | K*L                  | K*LA                  | L*LA                | Constant               | R2   |
|----|-----------------------|-----------------------|----------------------|----------------------|-----------------------|---------------------|------------------------|------|
| 23 | 0.6497***<br>(0.1923) | 3.5473***<br>(0.3135) | -0.3068<br>(0.2045)  | -0.23***<br>(0.0247) | 0.0307*<br>(0.0167)   | -0.0126<br>(0.0117) | 3.4571<br>(2.3355)     | 0.88 |
| 31 | -0.5549**<br>(0.2667) | 1.2313***<br>(0.2671) | -0.80***<br>(0.3024) | -0.0129<br>(0.0228)  | 0.0850***<br>(0.0229) | -0.05**<br>(0.0196) | 16.6141***<br>(3.4212) | 0.92 |
| 42 | 0.8293***<br>(0.2648) | 0.0463<br>(0.4646)    | 0.8599**<br>(0.3494) | 0.1049**<br>(0.0441) | -0.0479*<br>(0.0276)  | -0.1***<br>(0.0309) | -0.3044<br>(3.2101)    | 0.89 |
| 44 | 0.0348<br>(0.0984)    | 1.9523***<br>(0.1088) | -0.47***<br>(0.0944) | -0.11***<br>(0.0119) | 0.0575***<br>(0.0075) | -0.0107<br>(0.0119) | 9.62***<br>(1.1584)    | 0.93 |
| 48 | 0.0014<br>(0.2311)    | 1.7726***<br>(0.4884) | -0.411**<br>(0.1950) | -0.0663<br>(0.0476)  | 0.0469***<br>(0.0148) | -0.0390<br>(0.0416) | 10.5770***<br>(2.5821) | 0.90 |
| 51 | -0.3571<br>(0.4284)   | 2.2376***<br>(0.2929) | -1.31***<br>(0.3986) | -0.098**<br>(0.047)  | 0.1054***<br>(0.0319) | -0.041<br>(0.0481)  | 16.8073***<br>(4.8003) | 0.94 |
| 52 | -0.1052<br>(0.1829)   | 2.3492***<br>(0.2038) | -0.85***<br>(0.2192) | -0.14***<br>(0.0178) | 0.0762***<br>(0.0165) | 0.0008<br>(0.0140)  | 12.6684***<br>(2.3518) | 0.93 |
| 53 | 0.5355***<br>(0.1121) | 1.9366***<br>(0.2573) | -0.2002*<br>(0.1146) | -0.09***<br>(0.0259) | 0.0257***<br>(0.0086) | -0.035*<br>(0.0219) | 4.7523***<br>(1.3305)  | 0.94 |
| 54 | 0.3073*<br>(0.1920)   | 2.0333***<br>(0.3479) | -0.4171*<br>(0.2125) | -0.070**<br>(0.0350) | 0.0419**<br>(0.0165)  | -0.05*<br>(0.0268)  | 7.5028***<br>(2.3511)  | 0.87 |
| 55 | -0.7854<br>(5.6159)   | -3.3270<br>(13.5392)  | -2.0544<br>(7.2917)  | 0.0414<br>(0.6199)   | 0.1376<br>(0.5313)    | 0.3376<br>(0.723)   | 25.9487<br>(76.2492)   | 0.03 |
| 56 | 0.5796***<br>(0.1998) | 2.3114***<br>(0.2411) | -0.404**<br>(0.1938) | -0.16***<br>(0.0211) | 0.0382**<br>(0.0172)  | 0.0199<br>(0.0193)  | 4.3830<br>(2.2177)     | 0.89 |
| 61 | 1.4219**<br>(0.6291)  | 2.4026***<br>(0.4862) | 0.6009<br>(0.6331)   | -0.14***<br>(0.0367) | -0.0423<br>(0.0495)   | -0.0034<br>(0.0286) | -6.4015<br>(7.7910)    | 0.88 |
| 62 | -0.4991*<br>(0.2615)  | 2.0711***<br>(0.2000) | -0.94***<br>(0.2771) | -0.07***<br>(0.0201) | 0.0976***<br>(0.0223) | -0.06**<br>(0.0267) | 15.8161***<br>(3.1256) | 0.85 |
| 71 | 0.9409**<br>(0.4257)  | 1.1162*<br>(0.5996)   | 0.9164**<br>(0.3576) | -0.0433<br>(0.0459)  | -0.0391<br>(0.0298)   | -0.0249<br>(0.0399) | -3.5382<br>(4.9323)    | 0.87 |
| 72 | 0.1955**<br>(0.0935)  | 2.3761***<br>(0.1902) | -0.51***<br>(0.0977) | -0.12***<br>(0.0187) | 0.0581***<br>(0.0066) | -0.1***<br>(0.015)  | 7.7664***<br>(1.1423)  | 0.90 |
| 81 | -0.453***<br>(0.1449) | 1.9668***<br>(0.2472) | -0.88***<br>(0.1564) | -0.0342<br>(0.0225)  | 0.0926***<br>(0.0121) | -0.1***<br>(0.0239) | 15.2857***<br>(1.7412) | 0.88 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

**Table 4.15, Non-homogeneous Cobb-Douglas estimation in Denver County**

|    | K                    | L                     | LA                    | K*L                  | K*LA                  | L*LA                 | Constant               | R2   |
|----|----------------------|-----------------------|-----------------------|----------------------|-----------------------|----------------------|------------------------|------|
| 23 | -0.2806<br>(0.2806)  | 0.991***<br>(0.3159)  | -0.6564*<br>(0.3598)  | 0.0603**<br>(0.0294) | 0.0700**<br>(0.0297)  | 0.0205<br>(0.0341)   | 14.2276***<br>(3.3195) | 0.83 |
| 31 | 0.8852**<br>(0.3494) | -0.8219**<br>(0.3615) | 1.3186***<br>(0.4866) | 0.099***<br>(0.0321) | -0.0722**<br>(0.0360) | -0.0184<br>(0.0233)  | -1.9489<br>(4.5754)    | 0.92 |
| 42 | -0.0490<br>(0.2673)  | 0.8678**<br>(0.3425)  | -0.7640*<br>(0.3877)  | -0.08**<br>(0.0314)  | 0.0607**<br>(0.0285)  | 0.0469**<br>(0.0199) | 13.6808***<br>(3.5633) | 0.91 |
| 44 | -0.55***<br>(0.1561) | 0.952***<br>(0.2515)  | -0.4729**<br>(0.2136) | 0.0079<br>(0.0227)   | 0.0770***<br>(0.0153) | -0.074**<br>(0.0292) | 15.1412<br>(2.0638)    | 0.86 |
| 48 | -0.0030<br>(0.2127)  | 0.3287<br>(0.4806)    | 0.0908<br>(0.3027)    | 0.0446<br>(0.0478)   | 0.0267<br>(0.0224)    | -0.0681*<br>(0.0345) | 8.9847***<br>(2.4395)  | 0.98 |
| 51 | -3.2803<br>(4.527)   | 2.1625<br>(2.7003)    | -5.8838<br>(6.3442)   | -0.1301<br>(0.0837)  | 0.4496<br>(0.5125)    | -0.0194<br>(0.2276)  | 56.1791<br>(56.0281)   | 0.98 |
| 52 | -1.036**<br>(0.4472) | 1.2250**<br>(0.4894)  | -1.722***<br>(0.5895) | -0.0433<br>(0.0821)  | 0.1506***<br>(0.0431) | -0.0341<br>(0.0902)  | 24.4686***<br>(5.8184) | 0.91 |
| 53 | -1.29***<br>(0.3836) | 1.8354**<br>(0.8828)  | -1.867***<br>(0.5859) | -0.0363<br>(0.0736)  | 0.1682***<br>(0.0404) | -0.1111<br>(0.0961)  | 27.1254***<br>(5.3349) | 0.82 |
| 54 | -0.88***<br>(0.2862) | 1.462***<br>(0.4201)  | -1.137***<br>(0.3850) | -0.0034<br>(0.0346)  | 0.1254***<br>(0.0305) | -0.12***<br>(0.0441) | 20.1942***<br>(3.5631) | 0.71 |
| 55 |                      |                       |                       |                      |                       |                      |                        |      |
| 56 | -0.7486*<br>(0.4260) | 0.3783<br>(0.4779)    | -1.1367*<br>(0.5982)  | -0.0473<br>(0.0362)  | 0.1043**<br>(0.0448)  | 0.0584<br>(0.0427)   | 20.6825***<br>(5.5704) | 0.65 |
| 61 | 1.149***<br>(0.3747) | 0.2928<br>(0.5113)    | 0.5839<br>(0.4911)    | -0.0390<br>(0.0644)  | -0.0394<br>(0.0381)   | 0.0435<br>(0.0578)   | -2.1343<br>(4.4697)    | 0.97 |
| 62 | -0.95***<br>(0.2767) | 1.075***<br>(0.2982)  | -1.136***<br>(0.4131) | -0.0225<br>(0.0347)  | 0.1249***<br>(0.0290) | -0.0486<br>(0.0362)  | 20.9816***<br>(3.7483) | 0.86 |
| 71 | 1.0378<br>(0.6535)   | -0.5053<br>(1.1317)   | 1.2293<br>(0.9296)    | 0.0673<br>(0.093)    | -0.0657<br>(0.0714)   | -0.0311<br>(0.0417)  | -3.3527<br>(8.2455)    | 0.93 |
| 72 | -0.3534<br>(0.2329)  | 1.901***<br>(0.3517)  | -0.0337<br>(0.3128)   | -0.0281<br>(0.0190)  | 0.0543**<br>(0.0222)  | -0.13***<br>(0.0346) | 11.2879***<br>(3.1545) | 0.69 |
| 81 | -0.676**<br>(0.2633) | 1.3831***<br>(0.3259) | -1.212***<br>(0.3440) | -0.12***<br>(0.0311) | 0.1162***<br>(0.0258) | 0.0539<br>(0.0415)   | 19.1462***<br>(3.3287) | 0.78 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

**Table 4.16, Non-homogeneous Cobb-Douglas estimation in Larimer County**

|    | K                    | L                     | LA                  | K*L                  | K*LA                  | L*LA                 | Constant               | R2   |
|----|----------------------|-----------------------|---------------------|----------------------|-----------------------|----------------------|------------------------|------|
| 23 | 0.1263<br>(0.3088)   | 1.2935***<br>(0.3920) | -0.2815<br>(0.3586) | 0.0677**<br>(0.0283) | 0.0348<br>(0.0306)    | 0.0058<br>(0.0187)   | 9.4109**<br>(3.5825)   | 0.91 |
| 31 | 0.2940<br>(0.2822)   | 0.5451<br>(0.4692)    | -0.2924<br>(0.3359) | 0.0048<br>(0.0465)   | 0.0315<br>(0.0265)    | -0.0250<br>(0.0355)  | 8.0286**<br>(3.3833)   | 0.92 |
| 42 | 0.3727<br>(0.2495)   | -0.1645<br>(0.3017)   | 0.2087<br>(0.3082)  | 0.0545**<br>(0.0239) | -0.0005<br>(0.0239)   | -0.0208<br>(0.0238)  | 6.2496*<br>(3.1514)    | 0.92 |
| 44 | 0.0771<br>(0.1135)   | 0.4405**<br>(0.1863)  | -0.1418<br>(0.1438) | 0.0045<br>(0.0168)   | 0.0289**<br>(0.0111)  | -0.0190<br>(0.0139)  | 9.7775***<br>(1.399)   | 0.89 |
| 48 | 0.6821*<br>(0.3469)  | -1.1701<br>(0.7382)   | 0.5397<br>(0.3889)  | 0.0906<br>(0.0535)   | -0.0379<br>(0.0349)   | 0.0306<br>(0.0468)   | 3.691<br>(3.6564)      | 0.95 |
| 51 | 0.0609<br>(0.7603)   | 1.1919<br>(0.9365)    | -0.1555<br>(0.7887) | 0.0227<br>(0.0864)   | 0.0376<br>(0.0680)    | -0.1018<br>(0.0694)  | 8.7954<br>(8.1638)     | 0.92 |
| 52 | 0.0753<br>(0.1852)   | 1.3603***<br>(0.3658) | 0.2607<br>(0.2284)  | -0.0119<br>(0.0209)  | 0.0206<br>(0.0171)    | -0.0846<br>(0.0384)  | 7.0591<br>(2.3551)     | 0.89 |
| 53 | 0.0837<br>(0.2984)   | 0.6179<br>(0.4071)    | -0.6812<br>(0.3512) | -0.0586<br>(0.0364)  | 0.0573<br>(0.0266)    | 0.0316<br>(0.0415)   | 11.7195***<br>(3.8056) | 0.86 |
| 54 | 0.2373<br>(0.1722)   | 0.9008***<br>(0.3232) | -0.6***<br>(0.2192) | -0.0149<br>(0.0353)  | 0.0517***<br>(0.0158) | -0.0372*<br>(0.0236) | 9.5281<br>(2.1790)     | 0.91 |
| 55 |                      |                       |                     |                      |                       |                      |                        |      |
| 56 | 0.3557<br>(0.5091)   | 0.3435<br>(0.7362)    | -0.3223<br>(0.7052) | -0.0415<br>(0.0565)  | 0.0247<br>(0.0542)    | 0.0533<br>(0.0369)   | 8.5044<br>(6.4502)     | 0.86 |
| 61 | 1.3302<br>(0.9477)   | -0.6683<br>(1.0196)   | 0.8950<br>(1.3511)  | -0.0201<br>(0.0988)  | -0.0619<br>(0.1064)   | 0.1015<br>(0.0704)   | -4.8637<br>(11.7209)   | 0.97 |
| 62 | -0.79***<br>(0.2697) | 1.3296***<br>(0.3262) | -0.9***<br>(0.3338) | 0.0278<br>(0.0341)   | 0.1024***<br>(0.0253) | -0.12***<br>(0.0312) | 18.7245***<br>(3.2735) | 0.86 |
| 71 | 0.8156**<br>(0.3347) | 2.0805**<br>(0.9261)  | 0.4114<br>(0.3207)  | -0.0966<br>(0.0664)  | -0.0103<br>(0.0258)   | -0.0444<br>(0.0561)  | -0.4664<br>(3.2705)    | 0.87 |
| 72 | 0.0499<br>(0.1267)   | 0.9681***<br>(0.2487) | -0.6***<br>(0.1563) | -0.05***<br>(0.0195) | 0.0566<br>(0.0119)    | -0.0029<br>(0.0149)  | 11.3724***<br>(1.5710) | 0.85 |
| 81 | -0.0975<br>(0.1706)  | 2.1117***<br>(0.3267) | -0.6***<br>(0.1766) | -0.12***<br>(0.0353) | 0.0694***<br>(0.0151) | -0.0331<br>(0.0332)  | 11.7557***<br>(1.8952) | 0.89 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

**Table 4.17, Non-homogeneous Cobb-Douglas estimation in Boulder County**

|    | K                    | L                     | LA                   | K*L                   | K*LA                 | L*LA                  | Constant               | R2   |
|----|----------------------|-----------------------|----------------------|-----------------------|----------------------|-----------------------|------------------------|------|
| 23 | -0.794<br>(0.6776)   | 1.807***<br>(0.6295)  | -2.595**<br>(1.1671) | -0.302***<br>(0.0666) | 0.1987**<br>(0.0891) | 0.2853***<br>(0.0923) | 23.7296***<br>(8.6962) | 0.65 |
| 31 | 1.3576<br>(0.8432)   | -0.4927<br>(1.6125)   | 1.1486<br>(1.5378)   | -0.0465<br>(0.2314)   | -0.09422<br>(0.1036) | 0.1569<br>(0.1923)    | -3.6354<br>(10.4186)   | 0.91 |
| 42 | 1.3409*<br>(0.7393)  | 0.3617<br>(1.0626)    | 0.6754<br>(1.1108)   | -0.0833<br>(0.1007)   | -0.0600<br>(0.0909)  | 0.1098<br>(0.0750)    | -2.9632<br>(8.8465)    | 0.79 |
| 44 | 0.6377<br>(0.7547)   | -0.0514<br>(1.1274)   | 1.5276<br>(1.2272)   | 0.0703<br>(0.0962)    | -0.0916<br>(0.0955)  | -0.0612<br>(0.0922)   | 2.0884<br>(9.552)      | 0.50 |
| 48 |                      |                       |                      |                       |                      |                       |                        |      |
| 51 | 0.1962<br>(1.1589)   | 5.2908**<br>(2.2482)  | -2.0846<br>(1.7573)  | -0.6030*<br>(0.3427)  | 0.1231<br>(0.1196)   | 0.3337<br>(0.3226)    | 15.1756<br>(14.9892)   | 0.67 |
| 52 | 0.2726<br>(1.8091)   | 1.7492<br>(1.8072)    | -1.6072<br>(2.9837)  | -0.2322<br>(0.2136)   | 0.0879<br>(0.2403)   | 0.2019<br>(0.2808)    | 13.6442<br>(22.163)    | 0.77 |
| 53 | 0.8123<br>(1.1045)   | 2.0252<br>(3.0426)    | 0.4404<br>(1.5969)   | -0.2868<br>(0.4038)   | -0.0966<br>(0.1293)  | 0.2432<br>(0.3335)    | 2.3015<br>(12.7017)    | 0.84 |
| 54 | 0.9461**<br>(0.3896) | 1.8480***<br>(0.4851) | 0.5334<br>(0.6028)   | -0.171***<br>(0.0570) | -0.0337<br>(0.0479)  | 0.0781<br>(0.0608)    | 0.4633<br>(4.7534)     | 0.69 |
| 55 | 19.3846<br>(18.6239) |                       | 28.7091<br>(30.5129) | 3.2457<br>(11.711)    | -2.3027<br>(2.3491)  | -5.2313<br>(19.5464)  | -228.402<br>(241.5154) | 0.44 |
| 56 | -0.6479<br>(0.9275)  | 1.7686<br>(1.2070)    | -1.4061<br>(1.3356)  | -0.0189<br>(0.1623)   | 0.1570<br>(0.1109)   | -0.1726<br>(0.1250)   | 16.8453<br>(10.7214)   | 0.70 |
| 61 | -4.9893<br>(3.2276)  | 3.5178<br>(3.0746)    | -10.0134<br>(5.8111) | -0.7757*<br>(0.3785)  | 0.7688<br>(0.4525)   | 0.7734**<br>(0.3401)  | 78.7493<br>(41.1633)   | 0.58 |
| 62 | 1.7517**<br>(0.7416) | 0.9940<br>(0.8426)    | 2.0437*<br>(1.1683)  | -0.0982<br>(0.1043)   | -0.1512<br>(0.0913)  | 0.0609<br>(0.0998)    | -9.8590<br>(8.8881)    | 0.67 |
| 71 | 0.0023<br>(1.9947)   | 0.5388<br>(1.4667)    | -0.0633<br>(2.8306)  | -0.0366<br>(0.1020)   | 0.0242<br>(0.2390)   | 0.0216<br>(0.1005)    | 11.2265<br>(23.8010)   | 0.84 |
| 72 | 1.3925<br>(1.0396)   | 3.1246<br>(1.7888)    | -0.6398<br>(0.8964)  | -0.4293<br>(0.2756)   | 0.0076<br>(0.0751)   | 0.3196<br>(0.2222)    | -0.2484<br>(9.9917)    | 0.93 |
| 81 | -0.1087<br>(0.8793)  | 0.2811<br>(0.9591)    | -0.9541<br>(1.6551)  | -0.0765<br>(0.1111)   | 0.0694<br>(0.1235)   | 0.1247<br>(0.1113)    | 15.0508<br>(11.4961)   | 0.78 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

**Table 4.18, Non-homogeneous Cobb-Douglas estimation in Weld County**

|    | K                     | L                     | LA                    | K*L                   | K*LA                 | L*LA                  | Constant             | R2   |
|----|-----------------------|-----------------------|-----------------------|-----------------------|----------------------|-----------------------|----------------------|------|
| 23 | 0.9887***<br>(0.2015) | 1.6947***<br>(0.4248) | 0.1924<br>(0.2657)    | -0.147***<br>(0.0358) | -0.0189<br>(0.0217)  | 0.0538***<br>(0.0102) | 0.7096<br>(2.4464)   | 0.91 |
| 31 | 0.3349<br>(0.2312)    | 0.6802<br>(0.3681)    | -0.1895<br>(0.2325)   | -0.0153<br>(0.0358)   | 0.0187<br>(0.0197)   | -0.0017<br>(0.0286)   | 7.8841<br>(2.6336)   | 0.97 |
| 42 | 0.4412<br>(0.6401)    | 2.1984***<br>(0.6499) | -0.3580<br>(0.7206)   | -0.203***<br>(0.0514) | 0.0260<br>(0.0592)   | 0.0647*<br>(0.0366)   | 7.6715<br>(7.7842)   | 0.81 |
| 44 | 1.0921*<br>(0.5434)   | 1.1607<br>(0.7835)    | 0.1973<br>(0.6578)    | -0.1699**<br>(0.0737) | -0.0216<br>(0.0559)  | 0.1311***<br>(.0458)  | -0.2586<br>(6.3163)  | 0.95 |
| 48 | -0.8279<br>(0.5398)   | 2.9242***<br>(0.7712) | -1.8553**<br>(0.6909) | -0.1775**<br>(0.0718) | 0.1598**<br>(0.0588) | -0.0338<br>(0.0319)   | 21.5919<br>(6.3221)  | 0.91 |
| 51 |                       |                       |                       |                       |                      |                       |                      |      |
| 52 | 0.7728<br>(1.2525)    | 1.6845**<br>(0.7982)  | -0.0046<br>(1.7471)   | -0.1784*<br>(0.0909)  | -0.0003<br>(0.1392)  | 0.0937<br>(0.0887)    | 3.2692<br>(15.6685)  | 0.89 |
| 53 | 0.8912<br>(0.6003)    | 2.7758***<br>(0.6313) | 0.6043<br>(0.7709)    | -0.1449**<br>(0.0543) | -0.0255<br>(0.0583)  | -0.0791<br>(0.0657)   | -1.0306<br>(7.6874)  | 0.93 |
| 54 | 1.5057***<br>(0.2482) | 2.9857***<br>(0.8163) | 0.9287***<br>(0.2906) | -0.1825**<br>(0.0697) | 0.073***<br>(0.0242) | -0.0475<br>(0.0332)   | -6.1234<br>(2.9744)  | 0.82 |
| 55 |                       |                       |                       |                       |                      |                       |                      |      |
| 56 | 1.6951**<br>(0.7870)  | 5.3919***<br>(0.6473) | 0.8788<br>(0.9178)    | -0.442***<br>(0.0522) | -0.0684<br>(0.0751)  | 0.0163<br>(0.0337)    | -8.5756<br>(9.6038)  | 0.77 |
| 61 | -0.5974<br>(3.1286)   | -4.5615<br>(1.2213)   | -4.0047<br>(4.5097)   | -1.0001<br>(0.2875)   | 0.1379<br>(0.3335)   | 1.8422<br>(0.4499)    | 41.4413<br>(42.2453) | 0.98 |
| 62 | -0.2757<br>(0.6705)   | 1.5984***<br>(0.5570) | -1.3363<br>(0.8966)   | -0.0896<br>(0.0566)   | 0.1074<br>(0.0725)   | -0.0159<br>(0.0664)   | 16.0985*<br>(8.1686) | 0.93 |
| 71 |                       |                       |                       |                       |                      |                       |                      |      |
| 72 | -0.1803<br>(1.5154)   | 2.7666<br>(2.3558)    | -0.9309<br>(1.4201)   | -0.1320<br>(0.1089)   | 0.0997<br>(0.1402)   | -0.0814<br>(0.2117)   | 11.6405<br>(15.2441) | 0.93 |
| 81 | 1.3632***<br>(0.3435) | 0.7932<br>(0.6876)    | 0.7815**<br>(0.3825)  | -0.1504**<br>(0.0576) | -0.0648*<br>(0.0326) | 0.1314***<br>(0.0315) | -3.9848<br>(4.0379)  | 0.87 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.



**Table 4.20, Estimation of Translog Production Function by Industry in Arapahoe County**

|    | LK                    | L                     | LA                    | LK2                   | L2                    | LA2                   | LK*L                  | LK*LA                  | L*LA                 | R2   |
|----|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|----------------------|------|
| 23 | -2.256***<br>(0.2933) | 1.5372***<br>(0.1872) | 1.5106***<br>(0.2488) | 0.1854***<br>(0.0225) | 0.1008***<br>(0.0094) | 0.1039***<br>(0.0257) | -0.084***<br>(0.0252) | -0.0230***<br>(0.0453) | -0.051**<br>(0.0224) | 0.94 |
| 31 | -1.946***<br>(0.5333) | 1.2747***<br>(0.3177) | 0.8119*<br>(0.4318)   | 0.1676***<br>(0.0198) | 0.0612***<br>(0.0179) | 0.0127***<br>(0.0422) | -0.0351<br>(0.0273)   | -0.2145***<br>(0.0433) | -0.78***<br>(0.0276) | 0.91 |
| 42 | -1.577***<br>(0.2393) | 1.0568***<br>(0.1599) | 1.3569***<br>(0.2429) | 0.1618***<br>(0.0149) | 0.0878***<br>(0.0093) | 0.1197***<br>(0.0227) | -0.0126<br>(0.0201)   | -0.2395***<br>(0.0311) | -0.10***<br>(0.0208) | 0.95 |
| 44 | -1.285***<br>(0.2368) | 2.0577***<br>(0.1825) | 0.9151***<br>(0.2270) | 0.1102***<br>(0.0114) | 0.0328**<br>(0.0132)  | 0.0288**<br>(0.0123)  | -0.138***<br>(0.0169) | -0.0931***<br>(0.0177) | -0.0051<br>(0.0139)  | 0.92 |
| 48 | -4.4965**<br>(2.0309) | 2.6662**<br>(1.0204)  | 3.0576<br>(1.8797)    | 0.4216*<br>(0.2063)   | 0.1180***<br>(0.0373) | 0.2693<br>(0.1962)    | -0.2455*<br>(0.1423)  | -0.6039<br>(0.3962)    | 0.0379<br>(0.1095)   | 0.95 |
| 51 | -0.4126<br>(0.3863)   | 1.4308***<br>(0.2024) | 1.2603***<br>(0.3808) | 0.1173***<br>(0.0308) | 0.0721***<br>(0.0076) | 0.1220***<br>(0.0411) | -0.0433<br>(0.0307)   | -0.2336***<br>(0.0647) | -0.08***<br>(0.0298) | 0.98 |
| 52 | -0.942***<br>(0.1642) | 1.4704***<br>(0.1294) | 0.9398***<br>(0.1708) | 0.0921***<br>(0.0100) | 0.0445***<br>(0.0069) | 0.0261**<br>(0.0105)  | -0.081***<br>(0.0155) | -0.0917***<br>(0.0182) | -0.0213<br>(0.0143)  | 0.96 |
| 53 | -0.4004<br>(0.3166)   | 1.8137***<br>(0.2857) | 0.3186<br>(0.2756)    | 0.0762***<br>(0.0153) | 0.02471<br>(0.0259)   | 0.0592**<br>(0.0272)  | -0.108***<br>(0.0386) | -0.0949***<br>(0.0349) | -0.0154<br>(0.0394)  | 0.94 |
| 54 | -0.405***<br>(0.1453) | 1.1709***<br>(0.1186) | 0.7777***<br>(0.1427) | 0.0595***<br>(0.0048) | 0.0636***<br>(0.0096) | 0.0239**<br>(0.0108)  | -0.0083<br>(0.0146)   | -0.0651***<br>(0.0104) | -0.10***<br>(0.0144) | 0.96 |
| 55 | -0.9758*<br>(0.5074)  | 1.3745***<br>(0.3083) | 0.6763<br>(0.6156)    | 0.0324***<br>(0.0089) | 0.0242<br>(0.0199)    | -0.0717<br>(0.0585)   | -0.0999<br>(0.0755)   | 0.0703*<br>(0.0440)    | 0.0195<br>(0.0851)   | 0.97 |
| 56 | -0.169***<br>(0.313)  | 1.3812***<br>(0.1386) | 1.6874***<br>(0.2706) | 0.1565***<br>(0.0239) | 0.0654***<br>(0.0093) | 0.0661***<br>(0.0251) | -0.104***<br>(0.0212) | -0.1979***<br>(0.044)  | 0.0008<br>(0.0219)   | 0.95 |
| 61 | -0.4146<br>(0.9493)   | 0.5749<br>(0.4181)    | 1.4003**<br>(0.6041)  | 0.1461***<br>(0.0421) | 0.0584**<br>(0.0252)  | 0.1738**<br>(0.0854)  | -0.0183<br>(0.0568)   | -0.3159***<br>(0.1042) | -0.0384<br>(0.0565)  | 0.93 |
| 62 | -0.879***<br>(0.2248) | 1.1060***<br>(0.1486) | 1.4158***<br>(0.2087) | 0.1157***<br>(0.0150) | 0.0372***<br>(0.0098) | 0.06320**<br>(0.0286) | -0.0312*<br>(0.0189)  | -0.1712***<br>(0.0391) | -0.06***<br>(0.0189) | 0.94 |
| 71 | 0.5405<br>(0.7361)    | 0.6811<br>(0.7467)    | 0.9704<br>(0.8914)    | 0.0341<br>(0.0294)    | -0.0086<br>(0.0403)   | 0.0569<br>(0.1054)    | 0.0479<br>(0.0816)    | -0.1085<br>(0.1127)    | -0.1264<br>(0.0829)  | 0.93 |
| 72 | -0.4806*<br>(0.2738)  | 1.4072***<br>(0.2036) | 0.7828***<br>(0.2490) | 0.0861***<br>(0.0129) | 0.0462***<br>(0.0149) | 0.0503***<br>(0.0159) | -0.115***<br>(0.0241) | -0.1158***<br>(0.0247) | 0.0178<br>(0.0266)   | 0.93 |
| 81 | -1.684***<br>(0.2736) | 1.4289***<br>(0.2319) | 1.8918***<br>(0.2480) | 0.1396***<br>(0.0146) | 0.0616***<br>(0.0212) | 0.0272<br>(0.0239)    | -0.0711**<br>(0.0302) | -0.1531***<br>(0.0316) | -0.0488*<br>(0.0304) | 0.94 |

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

Table 4.21, Estimation of Translog Production Function by Industry in El Paso County

|    | LK                   | L                     | LA                    | LK2                   | L2                    | LA2                   | LK*L                  | LK*LA                 | L*LA                  | constant               | R2   |
|----|----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|------|
| 23 | -1.86***<br>(0.3410) | 2.2814***<br>(0.2491) | 0.8072***<br>(0.2038) | 0.1032***<br>(0.134)  | 0.1086***<br>(0.0091) | -0.037***<br>(0.0056) | -0.158***<br>(0.0190) | 0.0076<br>(0.0128)    | -0.0225*<br>(0.0087)  | 14.7204***<br>(2.4765) | 0.93 |
| 31 | -1.53***<br>(0.1885) | 2.3785***<br>(0.1591) | 0.4605**<br>(0.1932)  | 0.0895***<br>(0.0078) | 0.1042***<br>(0.0099) | -0.0188**<br>(0.0076) | -0.171***<br>(0.0179) | 0.0074<br>(0.0144)    | -0.0157<br>(0.0147)   | 14.1376***<br>(1.8687) | 0.98 |
| 42 | -0.7507<br>(0.6707)  | 2.0372***<br>(0.5202) | 0.0066<br>(0.4134)    | 0.0664**<br>(0.0256)  | 0.1497***<br>(0.0253) | 0.0375**<br>(0.0167)  | -0.0295<br>(0.0464)   | -0.0299<br>(0.0234)   | -0.18***<br>(0.0311)  | 12.0267***<br>(4.2876) | 0.92 |
| 44 | -0.38***<br>(0.1150) | 1.6782***<br>(0.0991) | 0.5778***<br>(0.1388) | 0.0494***<br>(0.0064) | 0.0674***<br>(0.0092) | -0.0060<br>(0.0066)   | -0.108***<br>(0.0103) | -0.0164<br>(0.0265)   | -0.021**<br>(0.0113)  | 6.8110***<br>(1.0200)  | 0.95 |
| 48 | -2.23***<br>(0.3759) | 2.8525***<br>(0.3333) | 0.4232**<br>(0.1739)  | 0.1331***<br>(0.0179) | 0.1363***<br>(0.0297) | -0.0045<br>(0.0119)   | -0.219***<br>(0.0343) | -0.0199<br>(0.0210)   | -0.0180<br>(0.0263)   | 18.6004***<br>(2.1144) | 0.96 |
| 51 | -0.4333<br>(0.8626)  | 1.8063***<br>(0.2713) | -1.0167<br>(0.7301)   | -0.0226<br>(0.0610)   | 0.0816***<br>(0.0170) | -0.0498<br>(0.0434)   | -0.0548<br>(0.0393)   | 0.1675<br>(0.1009)    | -0.099**<br>(0.0400)  | 16.0970***<br>(4.4116) | 0.96 |
| 52 | -1.06***<br>(0.2313) | 2.1533***<br>(0.1632) | 0.5915*<br>(0.3094)   | 0.0687***<br>(0.0137) | 0.0751***<br>(0.0089) | -0.0224**<br>(0.0112) | -0.132***<br>(0.0139) | 0.0048<br>(0.0208)    | -0.03***<br>(0.0116)  | 11.0956***<br>(1.8534) | 0.96 |
| 53 | -1.19***<br>(0.2836) | 1.5061***<br>(0.2429) | 0.4835***<br>(0.1502) | 0.0842***<br>(0.0155) | 0.0459***<br>(0.0172) | -0.0014<br>(0.0103)   | -0.078***<br>(0.0225) | -0.0239<br>(0.0227)   | -0.0338*<br>(0.0189)  | 12.9302***<br>(1.7043) | 0.95 |
| 54 | -0.71**<br>(0.3403)  | 2.0866***<br>(0.2837) | 0.1899<br>(0.3135)    | 0.0603***<br>(0.0195) | 0.1289***<br>(0.0120) | -0.0082<br>(0.0105)   | -0.155***<br>(0.0296) | 0.0028<br>(0.0252)    | -0.0091<br>(0.0223)   | 10.9139***<br>(2.0627) | 0.91 |
| 55 | -2.0788<br>(28.956)  | -14.6479<br>(10.005)  | -9.5018<br>(30.4206)  | -0.0844<br>(1.3329)   | 0.4970<br>(0.3166)    | 0.1405<br>(0.8393)    | 0.4084<br>(1.5093)    | 0.4574<br>(0.9406)    | 0.8981<br>(1.9307)    | 76.19<br>(82.06)       | 0.61 |
| 56 | -0.5512*<br>(0.3394) | 1.7883***<br>(0.2145) | 0.3012<br>(0.2001)    | 0.0634***<br>(0.0132) | 0.0876***<br>(0.0118) | -0.0051<br>(0.0095)   | -0.165***<br>(0.0179) | -0.0112<br>(0.0188)   | 0.0256*<br>(0.0158)   | 8.7523***<br>(2.3487)  | 0.93 |
| 61 | 0.0544<br>(0.4894)   | 1.5251***<br>(0.2917) | 0.5276<br>(0.4092)    | 0.0597***<br>(0.0144) | 0.0998***<br>(0.0108) | 0.0242<br>(0.0213)    | -0.091***<br>(0.0213) | -0.0682*<br>(0.0365)  | -0.045**<br>(0.01720) | 3.7467<br>(4.5955)     | 0.96 |
| 62 | -0.84***<br>(0.2601) | 1.7249***<br>(0.1454) | 1.1087***<br>(0.2878) | 0.0848***<br>(0.0097) | 0.0969***<br>(0.0099) | -0.0059<br>(0.0186)   | -0.086***<br>(0.0141) | -0.0582**<br>(0.0279) | -0.07***<br>(0.0186)  | 7.1473***<br>(2.3004)  | 0.92 |
| 71 | -1.2926<br>(0.9464)  | 1.1976**<br>(0.5066)  | 1.1149**<br>(0.45890) | 0.1318***<br>(0.0345) | 0.0123<br>(0.0418)    | 0.0345**<br>(0.0163)  | -0.0656<br>(0.0464)   | -0.129***<br>(0.0283) | -0.0127<br>(0.0405)   | 9.6331*<br>(5.6241)    | 0.93 |
| 72 | -0.99***<br>(0.1263) | 1.2793***<br>(0.1456) | 0.4519***<br>(0.1077) | 0.0617***<br>(0.0066) | 0.1059***<br>(0.0078) | -0.018***<br>(0.0048) | -0.093***<br>(0.0137) | 0.0122<br>(0.0104)    | -0.03***<br>(0.0115)  | 11.8383***<br>(0.8369) | 0.95 |
| 81 | -1.67***<br>(0.2023) | 2.0013***<br>(0.1944) | 0.9901***<br>(0.2784) | 0.0986***<br>(0.0148) | 0.0907***<br>(0.0127) | -0.0366**<br>(0.0155) | -0.133***<br>(0.0203) | -0.0044<br>(0.0253)   | -0.0309<br>(0.0201)   | 12.6473***<br>(1.3923) | 0.92 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

**Table 4.22, Estimation of Translog Production Function by Industry in Denver County**

|    | LK                    | L                     | LA                    | LK2                    | L2                    | LA2                   | LK*L                 | LK*LA                 | L*LA                  | R2     |
|----|-----------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|----------------------|-----------------------|-----------------------|--------|
| 23 | -0.4622<br>(0.3246)   | 1.4862***<br>(0.3547) | -0.0217<br>(0.4371)   | 0.0558***<br>(0.0145)  | 0.1145***<br>(0.0192) | 0.0446*<br>(0.0239)   | -0.06**<br>(0.0319)  | 0.0427<br>(0.0323)    | -0.072**<br>(0.0292)  | 0.8920 |
| 31 | -1.872**<br>(0.7503)  | 0.3535<br>(0.4722)    | 1.2368***<br>(0.4138) | 0.1652***<br>(0.0271)  | 0.0669***<br>(0.0189) | 0.0885**<br>(0.0334)  | 0.0388<br>(0.0494)   | -0.202***<br>(0.0505) | -0.09***<br>(0.0264)  | 0.9575 |
| 42 | -2.26***<br>(0.6638)  | 2.0871***<br>(0.3298) | -0.1788<br>(0.5587)   | 0.1242***<br>(0.0404)  | 0.0984***<br>(0.0141) | 0.0275*<br>(0.0166)   | -0.15***<br>(0.0276) | -0.0235<br>(0.0511)   | -0.0158<br>(0.0187)   | 0.9479 |
| 44 | -0.574***<br>(0.1703) | 1.1050***<br>(0.1881) | 0.6298**<br>(0.2768)  | 0.0845***<br>(0.0081)  | 0.0519***<br>(0.0184) | 0.0504**<br>(0.0201)  | -0.06***<br>(0.0181) | -0.103***<br>(0.0244) | -0.0183<br>(0.0242)   | 0.927  |
| 48 | 0.0911<br>(1.0894)    | 1.4569*<br>(0.7581)   | -0.7343<br>(0.4744)   | 0.0051<br>(0.0748)     | 0.120***<br>(0.0315)  | 0.0810<br>(0.0839)    | 0.0589<br>(0.1066)   | -0.0049<br>(0.1398)   | -0.244**<br>(0.0884)  | 0.9942 |
| 51 | -5.3509<br>(5.9351)   | 9.5533<br>(4.5891)    | -14.8645<br>(9.7784)  | 0.2469<br>(0.2436)     | 0.0121<br>(0.1161)    | 0.8759<br>(0.5182)    | 0.1234<br>(0.3410)   | -0.0214<br>(0.5537)   | -1.1947<br>(0.8014)   | 0.9852 |
| 52 | -0.8326<br>(0.7922)   | 1.714*<br>(0.8455)    | -0.4202<br>(1.0122)   | 0.0628<br>(0.0513)     | 0.1658<br>(0.0975)    | 0.0655<br>(0.0734)    | -0.0144<br>(0.1233)  | -0.0367<br>(0.1272)   | -0.1750<br>(0.1065)   | 0.9289 |
| 53 | -1.413***<br>(0.4088) | 1.3031*<br>(0.7131)   | -1.5398<br>(0.9886)   | 0.0784***<br>(0.0124)  | 0.1123***<br>(0.0384) | 0.0922<br>(0.0713)    | -0.0453<br>(0.0538)  | -0.0096<br>(0.0542)   | -0.0828<br>(0.0879)   | 0.9106 |
| 54 | -1.264***<br>(0.3259) | 1.2626***<br>(0.3449) | -0.0131<br>(0.4826)   | 0.0847***<br>(0.0088)  | 0.0813***<br>(0.0297) | 0.0416<br>(0.0335)    | -0.0351<br>(0.0288)  | -0.0394**<br>(0.0385) | -0.0839<br>(0.0385)   | 0.8072 |
| 55 |                       |                       |                       |                        |                       |                       |                      |                       |                       |        |
| 56 | -3.067***<br>(0.4256) | 0.4536<br>(0.3417)    | -0.4904<br>(0.5967)   | 0.1013***<br>(0.0111)  | 0.0365<br>(0.0224)    | -0.0593<br>(0.0379)   | -0.0481*<br>(0.0259) | 0.1205***<br>(0.0337) | 0.0356<br>(0.0361)    | 0.8700 |
| 61 | -1.3607<br>(1.3347)   | 0.0363<br>(0.5631)    | 2.3539**<br>(0.9310)  | 0.0968<br>(0.1028)     | -0.0832<br>(0.0556)   | -0.1242<br>(0.1264)   | -0.1555<br>(0.0974)  | -0.0206<br>(0.1967)   | 0.2591*<br>(0.1287)   | 0.9661 |
| 62 | -1.338***<br>(0.3227) | 1.2562***<br>(0.2467) | -0.5815<br>(0.3813)   | 0.0944***<br>(0.01146) | 0.0536***<br>(0.0137) | 0.0553*<br>(0.0299)   | -0.12***<br>(0.0358) | -0.0360<br>(0.0401)   | 0.0453<br>(0.0387)    | 0.9181 |
| 71 | 0.3416<br>(1.3973)    | -0.7348<br>(2.4153)   | 1.6845<br>(1.3046)    | 0.0665<br>(0.0698)     | 0.0246<br>(0.2483)    | 0.0596<br>(0.0528)    | 0.1377<br>(0.1286)   | -0.1796<br>(0.1104)   | -0.1126<br>(0.2358)   | 0.9368 |
| 72 | -0.647***<br>(0.2003) | 1.0596***<br>(0.2047) | 0.3907*<br>(0.2195)   | 0.0798***<br>(0.0061)  | 0.0862***<br>(0.0159) | 0.0568***<br>(0.0144) | -0.04***<br>(0.0109) | -0.087***<br>(0.0178) | -0.063***<br>(0.0198) | 0.9057 |
| 81 | -1.075***<br>(0.1979) | 0.9122***<br>(0.2308) | 0.6654**<br>(0.2983)  | 0.0840***<br>(0.0064)  | 0.0454***<br>(0.0191) | 0.0101<br>(0.0183)    | -0.05***<br>(0.0209) | -0.0497**<br>(0.0247) | -0.0118<br>(0.0340)   | 0.9160 |

Standard Errors in Parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

**Table 4.23, Estimation of Translog Production Function by Industry in Larimer County**

|    | LK                    | L                      | LA                    | LK2                   | L2                    | LA2                   | LK*L                 | LK*LA                | L*LA                 | constant               | R2   |
|----|-----------------------|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|----------------------|----------------------|----------------------|------------------------|------|
| 23 | -1.393***<br>(0.2534) | 2.4329***<br>(0.26947) | 0.8342***<br>(0.2642) | 0.0868***<br>(0.0116) | 0.1119***<br>(0.0170) | -0.034***<br>(0.0079) | -0.18***<br>(0.0195) | 0.0008<br>(0.0198)   | -0.0104<br>(0.0175)  | 11.7441***<br>(2.2731) | 0.96 |
| 31 | -1.4128**<br>(0.5599) | 1.4212***<br>(0.4376)  | 0.5321<br>(0.4482)    | 0.0908**<br>(0.0373)  | 0.0957***<br>(0.0238) | -0.0203<br>(0.0186)   | -0.14***<br>(0.0488) | -0.0059<br>(0.0538)  | 0.0267<br>(0.0320)   | 13.8572***<br>(3.2364) | 0.93 |
| 42 | -1.371***<br>(0.4939) | 1.8923***<br>(0.4418)  | 0.4516*<br>(0.2861)   | 0.1021***<br>(0.0179) | 0.0605**<br>(0.0258)  | 0.0039<br>(0.0183)    | -0.16***<br>(0.0464) | -0.0392<br>(0.0279)  | 0.0273<br>(0.0326)   | 14.0373***<br>(3.4214) | 0.94 |
| 44 | -0.867***<br>(0.1179) | 1.3205***<br>(0.1696)  | 0.7868***<br>(0.1503) | 0.0842***<br>(0.0064) | 0.0889***<br>(0.0126) | 0.0059<br>(0.0069)    | -0.09***<br>(0.0152) | -0.06***<br>(0.0121) | -0.031**<br>(0.0141) | 9.3665***<br>(1.0749)  | 0.94 |
| 48 | -1.0079<br>(0.6513)   | 0.2613<br>(0.9363)     | 0.9722*<br>(0.5519)   | 0.0989**<br>(0.0434)  | 0.1078**<br>(0.0435)  | -0.0058<br>(0.0292)   | -0.089<br>(0.0834)   | -0.0716<br>(0.0622)  | 0.0634<br>(0.0454)   | 10.3217**<br>(4.1566)  | 0.96 |
| 51 | -5.8889**<br>2.3494)  | 2.6948**<br>(0.9804)   | 3.1089**<br>(1.5569)  | 0.4148**<br>(0.1563)  | 0.0621*<br>(0.0357)   | 0.0900*<br>(0.0431)   | -0.2298*<br>(0.1096) | -0.3788*<br>(0.1826) | 0.0419<br>(0.0800)   | 28.7997**<br>(10.3229) | 0.94 |
| 52 | -0.3025<br>(0.2631)   | 1.7751***<br>(0.2853)  | 0.5106**<br>(0.2058)  | 0.0663***<br>(0.0074) | 0.0830***<br>(0.0157) | 0.0460**<br>(0.0201)  | -0.07***<br>(0.0155) | -0.08***<br>(0.0263) | -0.08***<br>(0.0305) | 7.2587***<br>(1.8129)  | 0.95 |
| 53 | -1.0483**<br>(0.4444) | 1.6886***<br>(0.3465)  | -0.2880<br>(0.3705)   | 0.0932***<br>(0.0228) | 0.1215***<br>(0.0248) | 0.0392***<br>(0.0146) | -0.19***<br>(0.0350) | -0.0472<br>(0.0307)  | 0.0734**<br>(0.0332) | 16.4066***<br>(3.1495) | 0.92 |
| 54 | -0.6977<br>(0.6560)   | 1.4877***<br>(0.3091)  | -0.0564<br>(0.3940)   | 0.0655*<br>(0.0383)   | 0.0941***<br>(0.0168) | 0.0116<br>(0.0121)    | -0.10***<br>(0.0352) | -0.0131<br>(0.0400)  | -0.0146<br>(0.0214)  | 12.4373***<br>(3.0863) | 0.93 |
| 56 | -1.1092**<br>(0.4839) | 1.3919**<br>(0.6626)   | 0.9821<br>(0.8418)    | 0.1037***<br>(0.0274) | 0.0794**<br>(0.0368)  | -0.0008<br>(0.050)    | -0.113**<br>(0.0430) | -0.0673<br>(0.0592)  | 0.0031<br>(0.0612)   | 9.3379*<br>(4.9107)    | 0.93 |
| 61 | -0.2058<br>(3.6729)   | 0.0739<br>(3.4479)     | 1.8024<br>(5.0654)    | 0.0725<br>(0.2094)    | 0.0173<br>(0.0725)    | -0.0447<br>(0.5546)   | -0.1161<br>(0.2135)  | -0.0747<br>(0.5099)  | 0.1482<br>(0.1481)   | -0.1101<br>(18.514)    | 0.96 |
| 62 | -1.159***<br>(0.2708) | 1.5378***<br>(0.2321)  | -0.2011<br>(0.2415)   | 0.0622***<br>(0.0077) | 0.1041***<br>(0.0135) | 0.0096<br>(0.0183)    | -0.056**<br>(0.0261) | 0.0172<br>(0.0295)   | -0.09***<br>(0.0222) | 16.687***<br>(2.3940)  | 0.93 |
| 71 | -0.873***<br>(0.3009) | 2.1561***<br>(0.6266)  | -0.6145*<br>(0.3623)  | 0.0634***<br>(0.0094) | 0.0715**<br>(0.0315)  | 0.0347**<br>(0.0151)  | -0.107**<br>(0.0387) | 0.0062<br>(0.0157)   | -0.0749<br>(0.0455)  | 15.6687***<br>(3.1064) | 0.95 |
| 72 | -1.426***<br>(0.1632) | 1.8660***<br>(0.2006)  | 0.5109***<br>(0.1612) | 0.0806***<br>(0.0089) | 0.0800***<br>(0.0097) | -0.033***<br>(0.0050) | -0.14***<br>(0.0173) | 0.0229**<br>(0.0120) | -0.0144<br>(0.0116)  | 13.5378***<br>(1.2346) | 0.91 |
| 81 | -1.269***<br>(0.3243) | 1.7948***<br>(0.2837)  | 0.6436**<br>(0.2746)  | 0.1069***<br>(0.0201) | 0.1343***<br>(0.0184) | 0.0152<br>(0.1610)    | -0.17***<br>(0.0294) | -0.068**<br>(0.0323) | 0.0300<br>(0.0268)   | 12.6397***<br>(1.6179) | 0.93 |

Standard errors are in parentheses.

\*\*\*Significant at 99% level.

\*\*Significant at 95% level.

\*Significant at 90% level.

Table 4.24, Estimation of Translog Production Function by Industry in Larimer County

|    | LK                     | L                      | LA                    | LK2                   | L2                    | LA2                    | LK*L                   | LK*LA                 | L*LA                   | constant               | R2     |
|----|------------------------|------------------------|-----------------------|-----------------------|-----------------------|------------------------|------------------------|-----------------------|------------------------|------------------------|--------|
| 23 | -1.3932***<br>(0.2534) | 2.4329***<br>(0.26947) | 0.8342***<br>(0.2642) | 0.0868***<br>(0.0116) | 0.1119***<br>(0.0170) | -0.0338***<br>(0.0079) | -0.1846***<br>(0.0195) | 0.0008<br>(0.0198)    | -0.0104<br>(0.0175)    | 11.7441***<br>(2.2731) | 0.9680 |
| 31 | -1.4128**<br>(0.5599)  | 1.4212***<br>(0.4376)  | 0.5321<br>(0.4482)    | 0.0908**<br>(0.0373)  | 0.0957***<br>(0.0238) | -0.0203<br>(0.0186)    | -0.1345***<br>(0.0488) | -0.0059<br>(0.0538)   | 0.0267<br>(0.0320)     | 13.8572***<br>(3.2364) | 0.9381 |
| 42 | -1.3713***<br>(0.4939) | 1.8923***<br>(0.4418)  | 0.4516*<br>(0.2861)   | 0.1021***<br>(0.0179) | 0.0605**<br>(0.0258)  | 0.0039<br>(0.0183)     | -0.1565***<br>(0.0464) | -0.0392<br>(0.0279)   | 0.0273<br>(0.0326)     | 14.0373***<br>(3.4214) | 0.9482 |
| 44 | -0.8673***<br>(0.1179) | 1.3205***<br>(0.1696)  | 0.7868***<br>(0.1503) | 0.0842***<br>(0.0064) | 0.0889***<br>(0.0126) | 0.0059<br>(0.0069)     | -0.0847***<br>(0.0152) | -0.056***<br>(0.0121) | -0.0309**<br>(0.0141)  | 9.3665***<br>(1.0749)  | 0.9406 |
| 48 | -1.0079<br>(0.6513)    | 0.2613<br>(0.9363)     | 0.9722*<br>(0.5519)   | 0.0989**<br>(0.0434)  | 0.1078**<br>(0.0435)  | -0.0058<br>(0.0292)    | -0.089<br>(0.0834)     | -0.0716<br>(0.0622)   | 0.0634<br>(0.0454)     | 10.3217**<br>(4.1566)  | 0.9633 |
| 51 | -5.8889**<br>(2.3494)  | 2.6948**<br>(0.9804)   | 3.1089**<br>(1.5569)  | 0.4148**<br>(0.1563)  | 0.0621*<br>(0.0357)   | 0.0900*<br>(0.0431)    | -0.2298*<br>(0.1096)   | -0.3788*<br>(0.1826)  | 0.0419<br>(0.0800)     | 28.7997**<br>(10.3229) | 0.9473 |
| 52 | -0.3025<br>(0.2631)    | 1.7751***<br>(0.2853)  | 0.5106**<br>(0.2058)  | 0.0663***<br>(0.0074) | 0.0830***<br>(0.0157) | 0.0460**<br>(0.0201)   | -0.0712***<br>(0.0155) | -0.083***<br>(0.0263) | -0.0807***<br>(0.0305) | 7.2587***<br>(1.8129)  | 0.9500 |
| 53 | -1.0483**<br>(0.4444)  | 1.6886***<br>(0.3465)  | -0.2880<br>(0.3705)   | 0.0932***<br>(0.0228) | 0.1215***<br>(0.0248) | 0.0392***<br>(0.0146)  | -0.1993***<br>(0.0350) | -0.0472<br>(0.0307)   | 0.0734**<br>(0.0332)   | 16.4066***<br>(3.1495) | 0.9226 |
| 54 | -.6977<br>(0.6560)     | 1.4877***<br>(0.3091)  | -0.0564<br>(0.3940)   | 0.0655*<br>(0.0383)   | 0.0941***<br>(0.0168) | 0.0116<br>(0.0121)     | -0.1024***<br>(0.0352) | -0.0131<br>(0.0400)   | -0.0146<br>(0.0214)    | 12.4373***<br>(3.0863) | 0.9325 |
| 55 |                        |                        |                       |                       |                       |                        |                        |                       |                        |                        |        |
| 56 | -1.1092**<br>(0.4839)  | 1.3919**<br>(0.6626)   | 0.9821<br>(0.8418)    | 0.1037***<br>(0.0274) | 0.0794**<br>(0.0368)  | -0.0008<br>(0.050)     | -0.1130**<br>(0.0430)  | -0.0673<br>(0.0592)   | 0.0031<br>(0.0612)     | 9.3379*<br>(4.9107)    | 0.9327 |
| 61 | -0.2058<br>(3.6729)    | 0.0739<br>(3.4479)     | 1.8024<br>(5.0654)    | 0.0725<br>(0.2094)    | 0.0173<br>(0.0725)    | -0.0447<br>(0.5546)    | -0.1161<br>(0.2135)    | -0.0747<br>(0.5099)   | 0.1482<br>(0.1481)     | -0.1101<br>(18.514)    | 0.9603 |
| 62 | -1.1591***<br>(0.2708) | 1.5378***<br>(0.2321)  | -0.2011<br>(0.2415)   | 0.0622***<br>(0.0077) | 0.1041***<br>(0.0135) | 0.0096<br>(0.0183)     | -0.0559**<br>(0.0261)  | 0.0172<br>(0.0295)    | -0.0871***<br>(0.0222) | 16.687***<br>(2.3940)  | 0.9378 |
| 71 | -0.8726***<br>(0.3009) | 2.1561***<br>(0.6266)  | -0.6145*<br>(0.3623)  | 0.0634***<br>(0.0094) | 0.0715**<br>(0.0315)  | 0.0347**<br>(0.0151)   | -0.1071**<br>(0.0387)  | 0.0062<br>(0.0157)    | -0.0749<br>(0.0455)    | 15.6687***<br>(3.1064) | 0.9586 |
| 72 | -1.4263***<br>(0.1632) | 1.8660***<br>(0.2006)  | 0.5109***<br>(0.1612) | 0.0806***<br>(0.0089) | 0.0800***<br>(0.0097) | -0.0325***<br>(0.0050) | -0.1400***<br>(0.0173) | 0.0229**<br>(0.0120)  | -0.0144<br>(0.0116)    | 13.5378***<br>(1.2346) | 0.9181 |
| 81 | -1.2685***<br>(0.3243) | 1.7948***<br>(0.2837)  | 0.6436**<br>(0.2746)  | 0.1069***<br>(0.0201) | 0.1343***<br>(0.0184) | 0.0152<br>(0.1610)     | -0.1709***<br>(0.0294) | -0.0677**<br>(0.0323) | 0.0300<br>(0.0268)     | 12.6397***<br>(1.6179) | 0.9380 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

Table 4.25 , Estimation of Translog Production Function by Industry in Boulder County

|    | LK                     | L                    | LA                   | LK2                  | L2                   | LA2                  | LK*L                  | LK*LA                  | L*LA                 | constant               | R2    |
|----|------------------------|----------------------|----------------------|----------------------|----------------------|----------------------|-----------------------|------------------------|----------------------|------------------------|-------|
| 23 | -4.709***<br>(0.9623)  | 0.4554<br>(0.5512)   | 0.4927<br>(1.1212)   | 0.235***<br>(0.0461) | 0.115***<br>(0.0325) | 0.0044<br>(0.1331)   | -0.0928<br>(0.0818)   | -0.0518<br>(0.1569)    | 0.0873<br>(0.1224)   | 35.98***<br>(7.6058)   | 0.771 |
| 31 | -3.1109<br>(5.8214)    | -0.5545<br>(2.5521)  | 4.3999<br>(3.0250)   | 0.4159<br>(0.2953)   | 0.1644<br>(0.1456)   | 0.4666<br>(0.4449)   | 0.2627<br>(0.4484)    | -0.8885<br>(0.6517)    | -0.3726<br>(0.5249)  | 10.9392<br>(31.6266)   | 0.912 |
| 42 | -3.2726<br>(2.8181)    | -0.8032<br>(1.2381)  | 3.4175<br>(2.9906)   | 0.1892<br>(0.2504)   | 0.0019<br>(0.0590)   | -0.1459<br>(0.3519)  | -0.0151<br>(0.1396)   | -0.0916<br>(0.6106)    | 0.1493<br>(0.1677)   | 15.695<br>(13.8533)    | 0.794 |
| 44 | -1.5237<br>(1.1911)    | -0.4685<br>(1.1837)  | 0.8843<br>(1.2753)   | 0.0626<br>(0.0307)   | 0.0435<br>(0.0629)   | -0.0469<br>(0.1081)  | 0.0732<br>(0.1406)    | 0.0091<br>(0.1352)     | -0.0367<br>(0.1611)  | 17.4756<br>(11.3824)   | 0.553 |
| 51 | -20.7422*<br>(10.1506) | 2.7758<br>(2.8051)   | 17.7066*<br>(9.3801) | 1.7449*<br>(0.9302)  | 0.1719*<br>(0.0940)  | 1.0803<br>(1.2096)   | -0.4506<br>(0.3324)   | -2.8027<br>(1.9899)    | 0.3432<br>(0.3477)   | 69.4335**<br>(31.9336) | 0.720 |
| 52 | -3.5851<br>(4.0679)    | 2.3697<br>(3.2424)   | 2.2899<br>(7.4155)   | 0.0515<br>(0.3135)   | 0.0624<br>(0.1159)   | -0.5376<br>(0.7075)  | -0.5120<br>(0.2901)   | 0.4393<br>(0.8020)     | 0.5353<br>(0.5799)   | 21.9749<br>(25.9817)   | 0.765 |
| 53 | -13.19304<br>(9.6649)  | 3.4035<br>(3.4568)   | 8.4382<br>(3.7881)   | 0.9662<br>(0.3974)   | 0.1527<br>(0.0822)   | 0.4995<br>(0.2697)   | -0.3444<br>(0.4377)   | -1.3010<br>(0.4753)    | 0.1041<br>(0.3965)   | 57.5648<br>(53.1270)   | 0.914 |
| 54 | -4.0064***<br>(1.3691) | 2.634***<br>(0.4942) | 3.914***<br>(1.0397) | 0.303***<br>(0.0567) | 0.123***<br>(0.0259) | 0.0485<br>(0.0863)   | -0.1382**<br>(0.0581) | -0.3651***<br>(0.0829) | -0.1123*<br>(0.0701) | 18.007***<br>(6.5599)  | 0.748 |
| 56 | -2.2917<br>(3.7898)    | 1.4587<br>(1.2433)   | 2.0408<br>(4.1712)   | 0.1127<br>(0.3365)   | 0.122***<br>(0.0359) | -0.1231<br>(0.3416)  | -0.0627<br>(0.1551)   | 0.0202<br>(0.6805)     | -0.1066<br>(0.1241)  | 13.8706<br>(13.5375)   | 0.740 |
| 61 | -18.357***<br>(4.2188) | 1.9087<br>(2.2188)   | 11.8618*<br>(6.6082) | 2.2516**<br>(0.7100) | 0.0984<br>(0.0987)   | 2.8868*<br>(1.5301)  | -0.3872<br>(0.2982)   | -4.6332*<br>(2.2283)   | 0.3402<br>(0.3036)   | 74.3374**<br>(31.5825) | 0.805 |
| 62 | -8.7977**<br>(3.3948)  | 0.0662<br>(0.6833)   | 9.740***<br>(2.4772) | 0.848***<br>(0.2341) | 0.145***<br>(0.0293) | 0.631***<br>(0.1789) | 0.0598<br>(0.0939)    | -1.545***<br>(0.3752)  | -0.11381<br>(0.0933) | 26.6211*<br>(13.9819)  | 0.817 |
| 71 | -12.8854<br>(12.3738)  | 4.5865<br>(2.9077)   | 2.5193<br>(3.0536)   | 0.4665<br>(0.6281)   | 0.0721<br>(0.0769)   | -0.3002<br>(0.1832)  | -0.4638<br>(0.3301)   | 0.2256<br>(0.4392)     | 0.1431<br>(0.1974)   | 78.0764<br>(68.077)    | 0.917 |
| 72 | -0.2949<br>(5.7356)    | 4.7767**<br>(1.7915) | -0.8665<br>(5.3512)  | 0.1029<br>(0.5038)   | 0.1418**<br>(0.0557) | 0.0376<br>(0.3776)   | -0.618**<br>(0.2586)  | -0.0399<br>(0.8977)    | 0.3398<br>(0.1976)   | 9.2344<br>(17.8000)    | 0.947 |
| 81 | -4.7873***<br>(1.5509) | 2.1629**<br>(0.9087) | 1.6097<br>(1.6277)   | 0.279***<br>(0.0762) | 0.1133**<br>(0.0417) | 0.0345<br>(0.1626)   | -0.2401*<br>(0.1175)  | -0.1707<br>(0.1941)    | 0.087<br>(0.1202)    | 31.416***<br>(9.9845)  | 0.865 |

Standard errors are in parentheses.

\*\*\* Significant at 99% level.

\*\* Significant at 95% level.

\*Significant at 90% level.

## **Chapter Five: Exploring Returns to Scale (RTS)**

In this chapter, economies of scale will be measured based on three different estimated production functions. These production functions are: the standard Cobb-Douglas, the non-homogeneous Cobb-Douglas, and the translog functions. The main feature of the first function is that the economies of scales are fixed, while the latter two functions are characterized by varying economies of scale according to the input combination. This measure will be conducted at the industry and firm level for each industry in each county studied.

The returns to scale is an important indicator of measuring the efficiency of exploiting input factors in the production at the firm level for the private sector industries within the same county, and within industries across different counties. Thus, this measure is essential for decision makers to assist them in choosing the type of industries or firms to attract in their regions. Furthermore, the RTS test explores the relationship between the changes in input and output. Furthermore, it will test if the returns to scale are fixed or vary by the input combinations. In this trajectory, Nicholson (2005) mentioned that Adam Smith identified two counter forces to determine the level of scale for the firm. The division and specialization of labor, and this is expected to lead to higher labor productivity and more output, while the firm size is will offset efficiency. In this case the manager will face difficulties as the firm size increases. Therefore, which of these forces dominate will affect the results of economies of scales of the firm . In this regard, local governments will encourage industries that exhibit increasing returns to scale which may increase the potential of exports as efficiency increases, and thereby

increasing production, and then raising the employment level. In addition, the hypotheses will be tested within the same county among different industries and tested within industry among different counties.

The rest of the chapter will discuss the potential impact of RTS within the county among different industries and within the same industry among different counties. In addition, this chapter displays the estimated results by the type of estimated aforementioned production functions.

## **5.1 Potential comparison of RTS impact**

Here are the potential impacts of RTS within the county and among industries, and within industry and different counties:

### **5.1.1. Within county and among industries**

Suppose there are two industries in county A, X and Y. Industry X exhibit increasing returns to scale (IRTS), while industry Y exhibits decreasing RTS (DRTS). To maximize the industries profits, this requires that the mark-up price is slightly higher than the average cost because of efficient use of resources by industry X. In industry Y, in order to continue in the market and realize profits, it must charge higher mark-up prices than industry X. Therefore, industry Y is expected to initiate higher price level and cause higher inflation as a result of inefficient use of economic resources (Basu et al, 1996). Industry Y is expected to exert inverse impacts on the poor and the county by increasing expenses of local governments to the entitlement purposes instead of directing the money to investment projects such as roads, education, and so on. As a consequence, counties



have to attract industries and firms that exhibit increasing RTS or firms that exhibit constant returns to scale.

### **5.1.2. Within industry and among counties**

Comparing the impact of RTS within the same industry and among different counties, suppose the state or nation consists of two counties A and B and X is the common industry. Assume in county A, X industry exhibits IRTS, while industry X exhibits DRTS in county B. In order to maximize profits in the industry in the two counties, industry X in county A requires slight mark-up prices over the average cost compared to industry X in county B. Thus, the price level in county B is expected to be higher than county A and this may lead county A to export to county B if the difference in prices has a relative advantage for county A or B. In addition, output, employment, and local government income will increase in A, relative to county B. As a result there will be improvement in county A investments in infrastructure, education, and so on, while these services are expected to stay the same or deteriorate in county B.

The two explicit hypotheses tested are if the industry exhibits fixed returns to scale or the economy of scale varies. This hypothesis is compatible with the studies of U.S. economy. For instance, Hsing (1996) finds that the US manufacturing industry enjoys constant return to scale. The output elasticities of labor and capital are 0.78 and 0.23, respectively. At the same time he estimates the return to scale for the manufacturing industry in 50 states and Washington DC by using new CES production function. He finds differences in economies of scale among the states in manufacturing industry. He finds that New Mexico exhibits increasing return to scale. The sum of labor and capital coefficients is 1.12. While Washington DC exhibits decreasing return (0.85), and other states reveal constant return to scale. The author attributed the difference in these results

to the capital labor ratio. The states with high capital labor ratios (0.55) are more efficient than states with low capital labor ratios (0.30). Another reason, as suggested by the author, is the declining average cost of production.

## 5.2. Measuring Scale of Economies

This section discusses whether or not the return to scale is fixed as in Cobb-Douglas or varies as in non-homogeneous functions of translog function, and non-homogeneous Cobb-Douglas production functions. As known in the literature, the measurement of economies of scale can be calculated based on the following formula in regard to production functions incorporated in the study:

$$\sum_i \frac{\partial \ln y}{\partial \ln x_i} \dots\dots\dots (5.1)$$

For Cobb-Douglas function, the economies of scale are the sum of the input parameters. For non-homogeneous Cobb-Douglas function model with three input variables the economies of scale are computed by summing the partial economies of scale as follows:

The scale elasticity of  $X_1$  (*Capital*)  $\varepsilon_1 = \alpha_1 + \alpha_4 \ln X_2 + \alpha_5 \ln X_3 \dots\dots\dots (5.2)$

The scale elasticity of  $X_2$  (*Labor*)  $\varepsilon_2 = \alpha_2 + \alpha_4 \ln X_1 + \alpha_6 \ln X_3 \dots\dots\dots (5.3)$

The scale elasticity of  $X_3$  (*Land*)  $\varepsilon_3 = \alpha_3 + \alpha_5 \ln X_1 + \alpha_6 \ln X_2 \dots\dots\dots (5.4)$

And the total scale elasticity is:

$$\varepsilon_N = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \dots\dots\dots (5.5)$$

Also, the economies of scale for translog production function with three input variables will be computed. The scale economy for translog is the sum of the partial scale elasticities. The following are the formulas for computing the economies of scale for the three input variables.

The scale elasticity of  $X_1$  (*Capital*)  $\varepsilon_4 = \alpha_1 + \alpha_4 \ln X_2 + \alpha_5 \ln X_3 + 2 * \alpha_7 \dots\dots\dots (5.6)$

The scale elasticity of  $X_2$  (*Labor*)  $\varepsilon_5 = \alpha_2 + \alpha_4 \ln X_1 + \alpha_6 \ln X_3 + 2 * \alpha_8 \dots (5.7)$

The scale elasticity of  $X_3$  (*Land*)  $\varepsilon_6 = \alpha_3 + \alpha_5 \ln X_1 + \alpha_6 \ln X_2 + 2 * \alpha_9 \dots (5.8)$

And the total scale elasticity is:

$$\varepsilon_T = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 \dots (5.9)$$

Therefore, the scale elasticity varies according to the combination of input variables. This research will extend the method of computing non-homogeneous functions to include three variable inputs; land, labor, and capital. The return to scale for translog function will be computed for each firm in the industry.

### 5.3 The results of scale of economies

This part is divided into two subparts. The first discusses results of economies of scale within the county among different production functions and industries. The second subpart will examine the economies of scale within industry but among different counties.

#### 5.3.1 Economies of Scale within County and Among Industries

This section compares the economies of scale results of the three different estimations of production functions; standard Cobb-Douglas function, non-homogeneous Cobb-Douglas, and translog functions. These comparisons are shown in tables 5.1 and 5.1A to 5.6A.

**Arapahoe County:** Table 5.1A shows the economies of scale of the three production functions for different industries in Arapahoe County. The main results are:

**1- Cobb-Douglas:** Five industries in Arapahoe County show evidence of increasing returns to scale. Construction industry has the highest scale with 1.14. The other

industries with increasing returns to scale are high service industries such as information, real estate, professional, scientist, and technical services, and administrative, and waste management industries. The scales in these industries are close and equal to 1.06. The industries that exhibit increasing returns to scale are contributing around 32.7% of total estimated output (table 5.1).

**Table 5, The Distributions of Relative Importance of Output by Industry in Major Counties in Colorado**

|       |         |         |        | El    |          |          |
|-------|---------|---------|--------|-------|----------|----------|
| Weld  | Boulder | Larimer | Denver | Paso  | Arapahoe | Industry |
| 18.2  | 8.1     | 4.2     | 5.7    | 4.5   | 3.7      | 23       |
| 21.5  | 8.1     | 3.3     | 10.7   | 6.1   | 2.2      | 31       |
| 6.5   | 7.6     | 4.4     | 9.5    | 2.8   | 5.3      | 42       |
| 9.3   | 5.3     | 25.3    | 8.6    | 17.0  | 6.2      | 44       |
| 4.1   | 0.7     | 1.9     | 2.4    | 2.0   | 0.5      | 48       |
| 0.2   | 6.1     | 3.9     | 5.6    | 3.8   | 7.3      | 51       |
| 3.9   | 3.1     | 8.5     | 2.3    | 7.8   | 20.3     | 52       |
| 3.0   | 6.3     | 4.7     | 5.7    | 5.0   | 5.7      | 53       |
| 8.8   | 23.7    | 6.0     | 7.0    | 7.4   | 17.7     | 54       |
| 2.1   | 0.7     | 1.0     | 0.8    | 0.1   | 3.8      | 55       |
| 5.7   | 5.3     | 2.8     | 3.7    | 5.7   | 8.4      | 56       |
| 1.8   | 2.0     | 0.6     | 5.0    | 7.6   | 1.8      | 61       |
| 6.4   | 8.6     | 11.2    | 16.0   | 9.6   | 7.8      | 62       |
| 0.8   | 3.4     | 2.7     | 1.9    | 1.6   | 0.5      | 71       |
| 2.4   | 4.1     | 14.4    | 6.5    | 13.0  | 4.2      | 72       |
| 5.4   | 6.7     | 5.2     | 8.5    | 5.8   | 4.5      | 81       |
| 100.0 | 100.0   | 100.0   | 100.0  | 100.0 | 100.0    | Total    |

Also, table 5.1A shows evidence that the other industries exhibit constant returns to scale. In 1996 Basu et al found that private sector revealed constant returns to scale, while manufacturing durable goods exhibited increasing returns to scale, and manufacturing nondurables exhibited decreasing returns to scale.

Also, the table illustrates that economies of scale in two industries are not revealed in table 5.1A. The main reason is negative partial elasticity of one of the input variables. The two industries are education services and other services.

**Table 5.1 A, Economies of Scale by Production Function Type**

In Arapahoe County.

| Translog | Nonhomogeneous<br>Cobb-Douglas | Cobb-Douglas |    |
|----------|--------------------------------|--------------|----|
| 1.01     | 1.12                           | 1.14 (I)     | 23 |
| 0.94     | 1.00                           | 1.01 (C)     | 31 |
| 0.98     | 1.07                           | 1.06 (C)     | 42 |
| 1.01     | 1.06                           | 0.90 (C)     | 44 |
|          |                                | 0.91 (C)     | 48 |
| 1.04     | 1.23                           | 1.06 (I)     | 51 |
| 1.01     | 1.05                           | 0.99 (C)     | 52 |
| 1.04     | 1.19                           | 1.06 (I)     | 53 |
| 1.09     | 1.13                           | 1.06 (I)     | 54 |
|          | 1.01                           | 0.99 (C)     | 55 |
| 1.02     | 1.12                           | 1.07 (I)     | 56 |
| 1.07     | 1.09                           | 1.04 (C)     | 61 |
| 1.03     | 1.06                           | 1.01 (C)     | 62 |
|          |                                | 1.02 (C)     | 71 |
| 0.95     | 0.92                           | 0.98 (C)     | 72 |
| 1.05     | 1.12                           | 1.04 (C)     | 81 |

(I) Increasing Returns to Scale.

(C) Constant Returns to Scale.

(D) Decreasing Returns to Scale.

- 2. Non-homogeneous Cobb-Douglas:** In this function, the economies of scale are computed at the firm level for each industry in each county. The results in table 5.1A reveal the arithmetic average of the firms' scale of economy in the industry operating in Arapahoe County. In this table, the non-homogeneous function magnitude in scale of economy for the industries in Arapahoe is higher than the Cobb-Douglas estimation function. For instance, the economy of scale in information industry is 1.23 according to non-homogeneous estimation function compared to 1.06 with regard to standard Cobb-

Douglas function. The main differences in scale of economy are due to higher magnitude of partial scales that pertain to both capital and labor beyond the decrease in land partial economies of scale.

Also, two industry results of economies of scale are not revealed in table 5.1A. The main reason is negative partial elasticity of one of the variables. The two industries are transportation and warehousing, and lodge and restaurants.

Table 5.1B provides information on the distribution of the economies of scale in non-homogeneous functions in each industry in Arapahoe County. Although, the range of the distribution of economies of scale (measured as the difference between X firm maximum scale and Y firm minimum scale) is the highest in retail industry (1.07), followed by professional, scientist, and technical services industry (1.05), the standard deviation in these two sectors is high, but the ratio of range relative to the corresponding standard deviation is still wide. This means that to attract or keep the efficient firms operating in Arapahoe County has a high cost to the county.

**Table 5.1B, The Distribution of Scale for Non-homogeneous**

Cobb-Douglas Function in Arapahoe County

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 6.2       | 0.76  | 0.12 | 0.82 | 1.57 | 23 |
| 4.3       | 0.73  | 0.17 | 0.59 | 1.31 | 31 |
| 5.2       | 0.39  | 0.07 | 0.78 | 1.17 | 42 |
| 4.8       | 1.07  | 0.22 | 0.49 | 1.56 | 44 |
| 4.0       | 0.83  | 0.21 | 0.72 | 1.55 | 51 |
| 5.1       | 0.83  | 0.16 | 0.49 | 1.32 | 52 |
| 3.7       | 0.68  | 0.18 | 0.79 | 1.46 | 53 |
| 5.5       | 1.05  | 0.19 | 0.54 | 1.59 | 54 |
| 4.8       | 0.48  | 0.10 | 0.84 | 1.31 | 55 |
| 5.4       | 0.79  | 0.15 | 0.56 | 1.35 | 56 |
| 3.9       | 0.79  | 0.20 | 0.59 | 1.38 | 61 |
| 4.3       | 0.71  | 0.16 | 0.65 | 1.36 | 62 |
| 4.8       | 0.78  | 0.16 | 0.59 | 1.37 | 72 |
| 4.8       | 0.85  | 0.18 | 0.69 | 1.54 | 81 |

Table 5.1B shows that there are tremendous differences in the economies of scale in each industry in Arapahoe County. The maximum scale column shows evidence that part of the firms are operating with increasing returns to scale, while the minimum scale for the same industry shows evidence that other firms are operating within decreasing returns to scale. This means that there is heterogeneity in the firms' economies of scale operating within the same industry. Furthermore, these differences in economies of scale appear among different industries. For instance, the highest scale of economy for maximum scale is 1.59 in professional, scientist, and technical services industry, while the lowest maximum scale is 1.17 in wholesale trade industry. The main implication of this result is that more firms that exhibit increasing returns to scale need to be attracted

**Table 5.1C, The Distribution of economies of scale for Translog Function in Arapahoe County**

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 5.9       | 0.41  | 0.07 | 0.87 | 1.28 | 23 |
| 5.5       | 0.33  | 0.06 | 0.81 | 1.14 | 31 |
| 5.0       | 0.30  | 0.06 | 0.8  | 1.1  | 42 |
| 4.9       | 0.73  | 0.15 | 0.55 | 1.28 | 44 |
|           |       |      |      |      | 48 |
| 2.9       | 0.52  | 0.18 | 0.78 | 1.3  | 51 |
| 4.8       | 0.43  | 0.09 | 0.83 | 1.26 | 52 |
| 5.0       | 0.55  | 0.11 | 0.68 | 1.23 | 53 |
| 4.0       | 0.52  | 0.13 | 0.78 | 1.3  | 54 |
|           |       |      |      |      | 55 |
| 4.3       | 0.34  | 0.08 | 0.84 | 1.18 | 56 |
| 4.3       | 0.34  | 0.08 | 0.92 | 1.26 | 61 |
| 4.0       | 0.52  | 0.13 | 0.74 | 1.26 | 62 |
|           |       |      |      |      | 71 |
| 5.1       | 0.36  | 0.07 | 0.78 | 1.14 | 72 |
| 4.2       | 0.59  | 0.14 | 0.71 | 1.3  | 81 |

**3-Translog function:** The economies of scale in translog function are computed at the firm level. The results in the table are the arithmetic average of economies of scale for

the firms operating in the industry. Except for the professional, scientist, and technical services, and health care services industries which exhibit increasing returns to scale, other industries exhibit constant returns to scale (table, 5.1A). In addition, the magnitude of scale of economies in translog function is less than that of Cobb-Douglas function estimates.

Concerning the distribution of firms scale economies within the industry, table 5.1C reveals substantial differences among firms within an industry. Table 5.1C has two indicators that show evidence of scale differences. The first indicator is that the maximum scale of firms shows evidence of increasing returns to scale in all industries. While the firms minimum scale shows evidence that part of the firms in each industry are operating within decreasing returns to scale. The second indicator is the standard deviation (SD). The table shows evidence of homogeneity and heterogeneity of this indicator in the distribution within industry. For instance, manufacturing industry and wholesale trade industry show evidence of small differences among their firms' standard deviation, 0.06. Thus, firms operating in such industries are highly homogeneous. Other industries show that their firms' scales are heterogeneous. In these industries, the standard deviation is high as in information industry (0.18). In addition, the table shows a wide distribution as measured by the range relative to standard deviation. This means that local governments have to pay attention to attract efficient firms.

**El Paso County:** The analysis of the economies of scale in the industries and firms in El Paso County is based on tables 5.2A, 5.2B, and 5.2C. Following are the types of economies of scale that firms exhibits within each industry, and among three types of production functions.



**Table 5.2A, Economies of Scale by Production Function Type in El Paso County**

| Translog | Non-homogeneous Cobb-Douglas | Cobb-Douglas |    |
|----------|------------------------------|--------------|----|
|          | 1.36                         | 1.2593 (I)   | 23 |
|          | 1.09                         | 1.0857 (I)   | 31 |
|          | 1.05                         | 0.9684 (D)   | 42 |
|          | 1.05                         | 0.993 (C)    | 44 |
|          | 1.04                         | 1.0229 (C)   | 48 |
|          | 1.20                         | 1.081(N/A)   | 51 |
|          | 1.23                         | 1.1561 (I)   | 52 |
|          | 1.14                         | 1.1028 (I)   | 53 |
|          | 1.40                         | 1.3082 (I)   | 54 |
|          |                              | 0.8433 (D)   | 55 |
|          | 1.28                         | 1.1324 (I)   | 56 |
|          | 1.48                         | 1.1701 (I)   | 61 |
|          | 1.10                         | 1.0285 (C)   | 62 |
|          | 1.05                         | 0.9219 (C)   | 71 |
|          | 0.99                         | 1.0441 (C)   | 72 |
|          | 1.13                         | 1.052 (C)    | 81 |

(I) Increasing Returns to Scale.

(C) Constant Returns to Scale.

(D) Decreasing Returns to Scale

**1- Cobb-Douglas:** In El Paso County, table 5.2A shows three different groups of industries in the economies of scale according to Cobb-Douglas estimates. The first group is the increasing returns to scale industries which consist of 7 out of 16 industries, of which 4 are within high service industries. The industry with highest scale is the professional, scientist, and technical services (1.308), followed by construction with 1.259 of scale economies. The second group of six industries is the constant returns to scale; retail trade, transportation, health services, lodges and restaurants, arts and recreation, and other services (except public). The third group of industries exhibit decreasing returns to scale and include wholesale trade, and management of companies and enterprises.

**2- Non-homogeneous Cobb-Douglas:** Table 5.2A reveals that the economies of scale in this type of functions are greater than that of corresponding industries of Cobb-Douglas. For example, the economies of scale for professional industry is 1.40 in non-homogeneous Cobb-Douglas function compared to 1.308 in standard Cobb-Douglas estimates.

The economies of scale in non-homogeneous Cobb-Douglas are computed at the firm level. Therefore, the distribution of economies of scale reflects the firms' behavior in the industry. Table 5.2B shows a remarkable heterogeneity among firms in each industry. The SD of firms' scale economy is high and the range between maximum and minimum scale in each industry is high. The maximum economies of scale shows evidence that part of the firms are exhibiting increasing returns to scale in all industries. The highest maximum scale of economies is in education services (1.94), followed by construction (1.93). The minimum scales show evidence that part of the firms are operating within decreasing returns to scale in all industries.

**Table 5.2B, The Distribution Of Non-homogeneous Cobb-Douglas Function in El Paso County**

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 5.2       | 1.41  | 0.27 | 0.52 | 1.93 | 23 |
| 5.8       | 0.58  | 0.10 | 0.86 | 1.43 | 31 |
| 5.8       | 0.89  | 0.15 | 0.52 | 1.41 | 42 |
| 5.7       | 0.92  | 0.16 | 0.58 | 1.50 | 44 |
|           |       | 0.15 | 0.68 | 1.28 | 48 |
| 3.6       | 0.79  | 0.22 | 0.74 | 1.53 | 51 |
| 5.9       | 1.00  | 0.17 | 0.62 | 1.62 | 52 |
| 5.0       | 0.78  | 0.16 | 0.62 | 1.40 | 53 |
| 5.9       | 0.61  | 0.10 | 0.93 | 1.54 | 54 |
| 6.4       | 1.31  | 0.21 | 0.54 | 1.86 | 56 |
| 3.6       | 1.21  | 0.34 | 0.73 | 1.94 | 61 |
| 5.1       | 0.94  | 0.19 | 0.61 | 1.54 | 62 |
|           |       | 0.26 | 0.39 | 1.47 | 71 |
| 5.4       | 1.01  | 0.19 | 0.48 | 1.49 | 72 |
| 5.3       | 0.66  | 0.12 | 0.76 | 1.41 | 81 |

**3- Translog function:** The translog estimation is not applied to all industries in El Paso County because of negative partial elasticities of the firms in each industry.

**Denver County:** The analysis of economies of scale with different types of production functions will be explored in Denver County. The analysis will rely on tables 5.3A, 5.3B, and 5.3C. The first table provides a glance at the economies of scale according to different production functions, while the last two tables provide the distribution of the economies of scale in each industry for the non-homogeneous group of production functions. Here is a brief analysis of the economies of scales by production function.

**1- Cobb-Douglas:** In Denver County, within Cobb-Douglas production function, none of the industries exhibit increasing returns to scale. Also, this table shows evidence of decreasing returns to scale in five industries; wholesale trade, transportation and warehousing, lodge and restaurants, administrative and waste management, and art, entertainment, and recreation industries. Other industries in Denver County operate within constant returns to scale.

**Table 5.10 A, Economies of Scale by Production Function Type in Denver County**

| Translog | Non-homogeneous<br>Cobb-Douglas | Cobb-Douglas |    |
|----------|---------------------------------|--------------|----|
| 0.93     | 0.95                            | 0.9442 ( C ) | 23 |
| 0.97     | 1.00                            | 0.9682 ( C ) | 31 |
| 0.89     | 0.89                            | 0.9076 ( D ) | 42 |
| 0.99     | 0.93                            | 0.9721 ( C ) | 44 |
|          |                                 | 0.8791 (D)   | 48 |
|          |                                 | 1.0709 ( C ) | 51 |
|          | 0.84                            | 1.0247 ( C ) | 52 |
|          | 0.80                            | 0.967 ( C )  | 53 |
| 0.83     | 0.90                            | 0.9241 ( C ) | 54 |
|          |                                 | 0.808 (N/A)  | 55 |
|          | 0.72                            | 0.8549 (D)   | 56 |
|          | 1.04                            | 0.9997 ( C ) | 61 |
| 0.97     | 0.97                            | 0.9962 ( C ) | 62 |
|          |                                 | 0.7499 (D)   | 71 |
| 0.94     | 0.84                            | 0.8588 (D)   | 72 |
| 0.96     | 0.90                            | 0.9411 ( C ) | 81 |

(I) Increasing Returns to Scale.

(C) Constant Returns to Scale.

(D) Decreasing Returns to Scale

**2- Non-homogeneous Cobb-Douglas Function:** All the applied 12 industries within this estimation show evidence of non-homogeneous Cobb-Douglas function.

The economies of scale in the 12 industries are either constant or decreasing return to scale. The other four industries don't apply to this type of function due to small sample or the partial scale of the input variables is negative.

**3- Translog function:** The translog function applies only for 8 out of 16 industries included in the study; economies of scale for these 8 industries are constant or decreasing returns to scale. Other industries are not applied either because of the small number of firms in the industry (small sample) or of negative partial scale of the input variables.

The distribution of firms' economies of scale operating in Denver County indicates there is a wide distribution, as measured by the ratio of range relative to SD which shows that the range is more than four times the standard deviation. This means that the county has to implement policies to keep or attract firms which exhibit increasing returns to scale.

**Table 5.3C, The Distribution Of Translog Function in Denver County**

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 4.2       | 0.40  | 0.10 | 0.80 | 1.20 | 23 |
| 4.4       | 0.62  | 0.14 | 0.64 | 1.26 | 31 |
| 4.6       | 0.57  | 0.12 | 0.63 | 1.19 | 42 |
| 5.1       | 0.11  | 0.02 | 0.93 | 1.04 | 44 |
| 7.6       | 0.65  | 0.09 | 0.53 | 1.18 | 54 |
| 4.4       | 0.77  | 0.17 | 0.72 | 1.49 | 62 |
| 4.7       | 0.30  | 0.06 | 0.78 | 1.08 | 72 |
| 6.2       | 0.38  | 0.06 | 0.81 | 1.18 | 81 |

**Larimer County:** The analysis of economies of scale and the firms' scale distributions for Larimer County depends on tables 5.4A, 5.4B, and 5.4C. Here is a summary of the analysis of the scale of economies for industries operating in Larimer County by the type of production function.

**1. Cobb-Douglas:** Only professional, scientist, and technical services industry show increasing returns to scale (1.07). The other industries are distributed between constant returns to scale (9 industries), and decreasing returns to scale (6 industries).

**2. Non-homogeneous Cobb-Douglas:** The magnitude of economies of scale in non-homogeneous functions is close to the standard Cobb-Douglas in most of the industries. The non-homogeneous estimation shows the same industry of professional, scientist, and

**Table 5.4 A, Economies of Scale by Production Function Type in Larimer County**

| Translog | Non-homogeneous<br>Cobb-Douglas | Cobb-Douglas |    |
|----------|---------------------------------|--------------|----|
| 1.03     | 1.01                            | 1.0088 ( C ) | 23 |
| 1.03     | 1.04                            | 1.0415 ( C ) | 31 |
| 1.02     | 0.95                            | 0.9464 (D)   | 42 |
| 0.95     | 0.88                            | 0.8972 (D)   | 44 |
|          | 0.91                            | 0.9487 ( C ) | 48 |
|          |                                 | 1.009 ( C )  | 51 |
| 1.01     | 0.98                            | 0.9796 ( C ) | 52 |
| 0.97     | 0.89                            | 0.8842 (D)   | 53 |
|          | 1.13                            | 1.0733 (I)   | 54 |
|          |                                 | 0.9364 ( C ) | 55 |
|          | 1.00                            | 0.981 ( C )  | 56 |
|          |                                 | 1.0684 ( C ) | 61 |
| 0.89     | 0.94                            | 0.9272 (D)   | 62 |
|          | 0.88                            | 0.752 (D)    | 71 |
| 0.97     | 0.86                            | 0.8691(D)    | 72 |
| 1.00     | 0.92                            | 1.0011 ( C ) | 81 |

(I) Increasing Returns to Scale.

( C ) Constant Returns to Scale.

(D) Decreasing Returns to Scale

technical services exhibit increasing returns to scale with slightly higher magnitude (1.13). Table 5.4A also shows evidence of three industries that are not applied to non-homogeneous function either because of small number of observations or negative partial scale of elasticities. The non applied non-homogeneous industries include

information, management of companies and enterprises, and education services industries.

The statistical distribution of the firms in this function as appear in table 5.4B indicates that there is high homogeneity among firms in 5 industries; the SD is less than 0.10. At the same time, according to SD standards, the table illustrates a high heterogeneity in scale economies in art, entertainment, and recreation, with SD 0.32. Additionally, the range (the difference between maximum and minimum scale in the industry firms) is at least 4.4 time of SD for the same industry. This means that the county has to adopt policies to attract efficient firms in different industries to operate in Larimer County.

**Table 5.4B, The Distribution Of Non-homogeneous Cobb-Douglas Function in Larimer County**

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 5.3       | 0.41  | 0.08 | 0.85 | 1.26 | 23 |
| 5.0       | 0.19  | 0.04 | 0.97 | 1.16 | 31 |
| 7.4       | 0.54  | 0.07 | 0.60 | 1.14 | 42 |
| 8.0       | 0.38  | 0.05 | 0.68 | 1.06 | 44 |
|           |       | 0.18 | 0.60 | 1.29 | 48 |
|           |       |      |      |      | 51 |
| 4.8       | 0.66  | 0.14 | 0.58 | 1.24 | 52 |
| 4.8       | 0.57  | 0.12 | 0.67 | 1.24 | 53 |
| 5.2       | 0.24  | 0.05 | 1.03 | 1.28 | 54 |
|           |       |      |      |      | 55 |
| 4.4       | 0.42  | 0.09 | 0.81 | 1.23 | 56 |
|           |       |      |      |      | 61 |
| 5.9       | 0.60  | 0.10 | 0.74 | 1.34 | 62 |
|           |       | 0.32 | 0.33 | 1.34 | 71 |
| 5.9       | 0.59  | 0.10 | 0.66 | 1.25 | 72 |
| 5.5       | 0.75  | 0.14 | 0.52 | 1.26 | 81 |

**3. Translog Function:** Only 9 industries pass the test of behaving translog functions, of which 8 industries are operating within constant returns to scale. The translog function is not applied to 7 industries due to small sample size or negative partial scale of economies for the firms operating in that industry.

Statistical firms' distribution for translog function (table 5.4C) shows evidence of heterogeneity except for wholesale trade industry. The range between maximum and minimum scale is 0.04 in the wholesale trade industry. To attract or keep the efficient firms in these industries, the county has to provide incentives for firms that are operating within increasing returns to scale.

**Table 5.4C, The Distribution of Translog**

**Function in Larimer County**

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 4.5       | 0.34  | 0.08 | 0.87 | 1.21 | 23 |
| 4.0       | 0.42  | 0.11 | 0.82 | 1.24 | 31 |
| 4.9       | 0.05  | 0.01 | 1.00 | 1.04 | 42 |
| 6.8       | 0.57  | 0.08 | 0.64 | 1.20 | 44 |
| 5.1       | 0.41  | 0.08 | 0.83 | 1.24 | 52 |
| 3.4       | 0.43  | 0.13 | 0.80 | 1.24 | 53 |
| 4.1       | 0.37  | 0.09 | 0.73 | 1.10 | 62 |
| 5.7       | 0.32  | 0.06 | 0.80 | 1.11 | 72 |
| 5.4       | 0.52  | 0.10 | 0.80 | 1.32 | 81 |

**Boulder County:** The analysis of economies of scale in Boulder County depends on tables 5.5A. The firms' statistical scale distribution analysis relies on table 5.5B for non-homogeneous Cobb-Douglas function, and table 5.5C for the translog function. Here is a description of the economies of scale according to the type of estimated production function.

**Table 5.5 A, Economies of Scale by Production Function Type In Boulder County**

| Translog | Non-homogeneous<br>Cobb-Douglas | Cobb-Douglas    |    |
|----------|---------------------------------|-----------------|----|
| 0.94     | 0.95                            | 0.943 ( C )     | 23 |
| 0.90     | 0.99                            | 0.9445 ( C )    | 31 |
| 0.93     | 1.03                            | 1.0423 ( C )    | 42 |
| 0.93     | 0.71                            | 0.5658 ( D )    | 44 |
|          |                                 | 0.7412 ( N/A )  | 48 |
|          |                                 | 1.0144 ( N/A )  | 51 |
| 0.94     |                                 | 0.84229 ( N/A ) | 52 |
|          | 0.97                            | 0.872 ( C )     | 53 |
| 0.93     | 1.05                            | 1.0181 ( C )    | 54 |
|          |                                 | 1.2563 ( N/A )  | 55 |
|          | 1.09                            | 1.0494 ( C )    | 56 |
|          |                                 | 0.926 ( C )     | 61 |
| 0.93     | 0.94                            | 0.8443 ( D )    | 62 |
|          |                                 | 0.7046 ( D )    | 71 |
| 0.92     |                                 | 0.9563 ( C )    | 72 |
| 0.94     | 0.90                            | 0.8874 ( C )    | 81 |

(I) Increasing Returns to Scale.

( C ) Constant Returns to Scale.

(D) Decreasing Returns to Scale

**1. Cobb-Douglas:** According to economies of scales, the industries in Boulder County can be divided into three groups. The first one includes 9 industries operating within constant returns to scale; the second group includes 3 industries operating within decreasing returns to scale. And the third group of industries which are not applied because of negative partial scale of one of the input variables.



**2. Non-homogeneous Cobb-Douglas:** This type of production function is applied only to 9 industries in Boulder County, table 5.5A. On average, the economies of scale for these industries are either constant (7 industries) or decreasing (2 industries, retail trade (0.71), and other services, (0.90)).

Although the average scale economy shows constant or decreasing returns to scale, the distribution of firms scale economies within industry shows a high heterogeneity of scales among firms (table 5.5B). For instance, except for the firms of retail trade industry which operates within decreasing returns to scale, the maximum economies of scale show evidence that part of the firms are operating with high levels of increasing scale of economies in other industries. For example, the highest firms' maximum economies of scale are in construction (1.69), and the lowest maximum scale is in manufacturing and wholesale trade industries, 1.14 for each. Therefore, for long run planning and raising the advantage of the industry in the county, Boulder has to submit incentives for the firms that are operating within increasing returns to scale, or at least the firms of constant returns to scale.

**Table 5.5B, The Distribution Of Non-homogeneous Cobb-Douglas Function in Boulder County**

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 5.2       | 1.12  | 0.21 | 0.57 | 1.69 | 23 |
| 3.1       | 0.36  | 0.12 | 0.77 | 1.14 | 31 |
| 4.6       | 0.20  | 0.04 | 0.94 | 1.14 | 42 |
| 4.7       | 0.45  | 0.10 | 0.49 | 0.94 | 44 |
| 4.6       | 0.58  | 0.13 | 0.71 | 1.29 | 53 |
| 6.6       | 0.91  | 0.14 | 0.54 | 1.46 | 54 |
| 4.1       | 0.73  | 0.18 | 0.74 | 1.48 | 56 |
| 4.5       | 1.00  | 0.22 | 0.37 | 1.37 | 62 |
| 3.4       | 0.76  | 0.23 | 0.60 | 1.36 | 81 |

**3. Translog Function:** The translog production function is applied to 9 industries in Boulder County, table 5.5A. The firms operating in Boulder County

industry under such function reveal constant economies of scale. But the statistical distributions as measured by the difference between maximum and minimum scales depict high heterogeneity in economies of scale within industry among the firms. The maximum scale points out increasing returns in all industries, while the minimum scales indicate decreasing scale for the other of the firms. The scale range is 0.85 in finance and insurance industry, and 0.42 in health services, table 5.5C.

**Table 5.5C, The Distribution Of Translog Function in Boulder County**

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 5.9       | 0.74  | 0.13 | 0.53 | 1.27 | 23 |
| 5.2       | 0.54  | 0.11 | 0.63 | 1.17 | 31 |
| 4.5       | 0.49  | 0.11 | 0.73 | 1.21 | 42 |
| 4.8       | 0.50  | 0.10 | 0.69 | 1.19 | 44 |
| 4.1       | 0.85  | 0.21 | 0.64 | 1.49 | 52 |
| 5.3       | 0.63  | 0.12 | 0.63 | 1.26 | 54 |
| 4.6       | 0.42  | 0.09 | 0.74 | 1.17 | 62 |
| 4.5       | 0.51  | 0.11 | 0.68 | 1.19 | 72 |
| 5.5       | 0.56  | 0.10 | 0.62 | 1.18 | 81 |

**Weld County:** This part describes the results of three different production functions conducted to estimate the firm data in Weld County regarding the economies of scale.

**1. Cobb-Douglas:** Table 5.6A shows evidence which indicates that the industries in Weld County are behaving as constant returns to scale in 11 industries. The estimation is not applied to information, and management of companies and enterprises industries due to small number of observations. The other 4 industries are not applied as a result of negative partial scale of economies of one of the input variables.

**2. Non-homogeneous Cobb-Douglas:** This type of function is applied to 7 out of 16 industries. The magnitude of scale of economy in these 7 industries is higher than the corresponding Cobb-Douglas estimation.

The statistical distribution of the firms in these industries shows high heterogeneity, table 5.6B. The maximum scale indicates that part of the firms operate within increasing returns to scale, except manufacturing industry (1.08). The highest maximum scale is 1.69 in retail, while the lowest maximum is 1.52 in construction industry. In addition the minimum scale shows evidence of decreasing returns to scale in all industries that apply to non-homogeneous Cobb-Douglas function. Thus, the county has to attract or keep efficient firms by providing incentives to such firms.

**Table 5.6B, The Distribution Of Non-homogeneous Cobb-Douglas Function in Weld County**

| Range/ SD | Range | SD   | Min  | Max  |    |
|-----------|-------|------|------|------|----|
| 5.8       | 0.93  | 0.16 | 0.59 | 1.52 | 23 |
| 4.3       | 0.12  | 0.03 | 0.97 | 1.08 | 31 |
| 4.4       | 0.91  | 0.21 | 0.77 | 1.69 | 44 |
| 3.5       | 0.78  | 0.22 | 0.81 | 1.59 | 48 |
| 4.7       | 0.80  | 0.17 | 0.79 | 1.59 | 54 |
| 4.0       | 1.31  | 0.33 | 0.33 | 1.64 | 56 |
| 3.9       | 0.76  | 0.19 | 0.82 | 1.58 | 81 |

### 5.3.1. Economies of Scale within the Industry and among Counties

The analysis of this part will display the economies of scale within industry and among counties for the three different estimated production functions.

#### 5.3.1.1. Cobb-Douglas Function

The literature is vast in estimating the Cobb-Douglas function either at the national, state, industry, or firm level for the purpose of measuring economies of scale.

For example, Basu et al (1996, 1997) conduct studies at the level of US manufacturing industries, Dobbelaere et al (2008) applies a study for 38 manufacturing industries at the firm level, while Chow et al (2002) commissioned a study at the macroeconomic level to China's economy. This part will talk about the results regarding the estimation of standard Cobb-Douglas function in the six counties. This part will be divided into two subparts to explore the economies of scale and comparing these results among counties within the same industry, and within the county among different industries.

The analysis also, will depend on table 5.7 and tables 5.1-5.6. Here is the description of scale of economies for each industry among all counties. The results of Cobb-Douglas function by industry and among the counties under study will be analyzed.

**Table 5.7, Return to Scale by Industry and Major Counties in Colorado**

| Weld | Boulder   | Larimer | Denver    | El Paso | Arapahoe |    |
|------|-----------|---------|-----------|---------|----------|----|
| CRTS | CRTS      | CRTS    | CRTS      | IRTS    | CRTS     | 31 |
| N/A  | CRTS      | DRTS    | DRTS      | DRTS    | CRTS     | 42 |
| CRTS | DRTS      | DRTS    | CRTS      | CRTS    | CRTS     | 44 |
| N/A  | Not Apply | CRTS    | DRTS      | CRTS    | CRTS     | 48 |
|      | Not Apply | CRTS    | CRTS      | N/A     | IRTS     | 51 |
| CRTS | Not Apply | CRTS    | CRTS      | IRTS    | CRTS     | 52 |
| CRTS | CRTS      | DRTS    | CRTS      | IRTS    | IRTS     | 53 |
| CRTS | CRTS      | IRTS    | CRTS      | IRTS    | IRTS     | 54 |
|      | Not Apply | CRTS    | Not Apply | DRTS    | CRTS     | 55 |
| CRTS | CRTS      | CRTS    | DRTS      | IRTS    | IRTS     | 56 |
| CRTS | CRTS      | CRTS    | CRTS      | IRTS    | CRTS     | 61 |
| N/A  | DRTS      | DRTS    | CRTS      | CRTS    | CRTS     | 62 |
| CRTS | DRTS      | DRTS    | DRTS      | CRTS    | CRTS     | 71 |
| CRTS | CRTS      | DRTS    | DRTS      | CRTS    | CRTS     | 72 |
| CRTS | CRTS      | CRTS    | CRTS      | CRTS    | CRTS     | 81 |

CRTS= Constant Return to Scale.

IRTS =Increasing Return to Scale.

DRTS= Decreasing Return to Scale.

**Construction Industry:** The construction industry is efficient in Arapahoe and El Paso counties. In these two counties, the production process exerts increasing returns to scale. While in other counties, construction is exerting constant returns to scale. This

designates that construction in these counties can produce for the local economy and export to other counties at competitive prices because the expected markup output prices are lower compared to other counties. In addition, it is expected that this industry can grow and employ more in Arapahoe and El Paso counties compared to other counties included in the study.

**Manufacturing Industry:** Only manufacturing industry in El Paso shows evidence of increasing return to scale, while in other counties this industry produces at constant return to scale. This means that this industry is efficient in El Paso compared to other counties incorporated in the study.

**Wholesale trade:** Not one county shows evidence which indicates that this industry is operating within increasing return to scale. The wholesale industry shows constant return to scale in two counties, Arapahoe and Boulder. In other counties, this industry operates within decreasing return to scale.

**Retail Trade Industry:** This industry behaves as constant return to scale in four counties; Arapahoe, El Paso, Denver and Weld. On the other hand, the retail trade industry behaves as decreasing return to scale in Larimer and Boulder counties.

**Transportation and Warehousing Industry:** This industry behaves as constant return to scale in two counties, Arapahoe and El Paso, while decreasing returns to scale in Denver and Larimer.

**Information Industry:** In Arapahoe, information industry is efficient. It is characterized by increasing return to scale. Thus, Arapahoe County is expected to produce and export to other counties or even to international markets at competing prices. The industry can also grow naturally and expand employment in this sector. In addition, the local government can introduce projects with high quality to the area

because of expected high local government revenues as a consequence of high local economy income.

**Finance and Insurance:** This industry is highly efficient in El Paso County. In this county the industry production behaves as increasing returns to scale. In other counties, the finance and insurance industry is behaves as constant returns to scale, which is the least requirement to use inputs in the production process without waste.

**Real Estate and Leasing Industry:** This industry scale of economies varies among the counties included in the study. For example, the real estate industry behaves as constant return to scale in three counties (Denver, Boulder, and Weld). In addition, it exhibits increasing return to scale in Arapahoe and El Paso counties. The last county is Larimer where this industry is inefficient in its production process and behaves as decreasing returns to scale.

**Professional, Scientist, and Technical Services:** This industry, in three counties, behaves as increasing returns to scale; Arapahoe, El Paso, and Larimer. The industry in the other three counties is behaving as constant returns to scale. Therefore, the local governments in Denver, Boulder, and Weld counties have to facilitate this type of industry for the reasons mentioned previously in the introduction of the subsection.

**Management of Companies and Enterprises:** On average, this industry is behaving constant return to scale in Arapahoe and Larimer counties, while decreasing return to scale in El Paso County, but local government has to encourage such industries because of expected high wages of the employees in these industries. To raise the local county income, local governments have to attract efficient firms to operate in their areas.

**Administrative and Waste Management Services:** In Arapahoe and El Paso counties, this industry shows evidence of increasing return to scale. In Larimer, Boulder, and Weld this industry exhibits constant returns to scale, while in Denver County, the industry exhibits decreasing returns to scale.

**Education Services:** In all counties, the education services industry shows constant return to scale except in El Paso County, where the education services reveals increasing returns to scale.

**Health Care services:** It appears that this industry exhibits constant returns to scale in three counties (Arapahoe, El Paso, and Denver), while this industry shows decreasing returns to scale in Larimer and Boulder counties.

**Art, Entertainment, and Recreation Services:** In this industry, there are three different types of economies of scale that the production process followed. For instance, the production function reveals constant returns to scale in 3 counties; Arapahoe, El Paso, and Weld. In addition the production function follows decreasing returns to scale in Denver, Larimer, and Boulder counties.

**Lodge and Restaurants:** Two types of economies of scale production process appear in this industry. The economies of scale are either constant return to scale as in Arapahoe, El Paso, Boulder, and Weld or decreasing return to scale as in Denver and Larimer.

**Other Services (Except Public Services):** In all counties without exception, the production process in this industry follows the constant returns to scale.

#### **5.3.1.2. Non-homogeneous Production Function**

This part is divided into two parts. The first discusses non-homogeneous Cobb-Douglas function, while the second part reviews the translog function. The main

characteristics of these two parts are that they have varies economies of scale and varies elasticities of substitutions. In addition, the economies of scale are computed at the firm level. This type of production function reflects the complexity and advances in the economy.

### **Non-homogeneous Cobb-Douglas function**

The analysis of this subsection depends on the arithmetic means of the economies of scale for the firms operating in the industry. As an example, Vinod (1972) examines economies of scale for a certain firms. The researcher computes non-homogeneous Cobb-Douglas function for telecommunication. He finds that economies of scale are cyclical and increasing over time. Here, the economies of scale are computed at the firm level in each industry (the arithmetic means of firms operating in the industry).

**Construction Industry:** This industry is operating within increasing returns to scale in El Paso and Arapahoe (1.36, and 1.12, respectively), while it is constant returns to scale in the rest of the counties studied.

**Manufacturing Industry:** In El Paso County only is this industry operating as increasing returns to scale, while in the rest of the counties the manufacturing industry is operating within constant returns to scale.

**Wholesale Trade:** In Weld County, this industry shows no evidence of economies of scale because of negative partial economies of scale. In Arapahoe, this industry is operating within increasing returns to scale, while it is operating within decreasing returns to scale in Denver County. Otherwise, it is constant RTS in other counties.

**Retail Trade:** It shows increasing returns to scale in Weld and Arapahoe counties, constant in El Paso, and decreasing return to scales of economy in Denver, Boulder, and Larimer counties.



**Transportation and Warehousing:** This industry shows three different results for economies of scale. For instance, it is increasing returns to scale in Weld County, constant returns to scale in El Paso, and decreasing returns to scale in Larimer County.

**Information Industry:** The information industry, on average, follows the increasing returns to scale in Arapahoe and El Paso counties, while it is not applied to other counties due to small number of observations or negative partial scale of economies.

**Finance and Insurance:** This industry exerts increasing returns to scale in El Paso County, constant in Larimer and Arapahoe counties, and decreasing returns to scale in Denver County.

**Real Estate and Leasing:** Two groups of economies of scale appear in this industry, the increasing returns to scale in Arapahoe and El Paso counties, and decreasing return to scale in Larimer and Denver counties.

**Professional, Scientist, and Technical Services:** This industry, on average, is behaving as increasing returns to scale in El Paso, 1.40. In Weld, Arapahoe, and Larimer this industry is behaving as increasing returns to scale by 1.19, 1.13, and 1.13, respectively. In Boulder and Denver counties, this industry is behaving as decreasing returns to scale.

**Management of Companies and Enterprises:** This industry is constant returns to scale in Arapahoe County. No results show for other counties because of small number of observations or negative partial economies of scale.

**Administrative and Waste Management:** This industry behaves as increasing returns to scale in El Paso and Arapahoe counties with 1.28 and 1.12,

respectively. Also, this industry behaves as decreasing returns to scale in Weld and Denver, while it is constant returns to scale in Larimer County.

**Education Services:** The education services industry shows three different groups. The first group of efficient education services is in El Paso and Arapahoe counties where this industry is behaving as increasing returns to scale. In the second group, the economies of scale are constant in Denver County. The third group is not shown because of either small number of observations or negative partial elasticities of one of the estimated coefficients.

**Health Care Services:** This industry is operating at increasing returns to scale in El Paso and Arapahoe counties with 1.10 and 1.06, respectively. In other counties, this industry is behaving as constant returns to scale.

**Art, Entertainment, and Recreation Services:** Although this industry shows increasing returns to scale under SCD estimations for Denver County, non-homogeneous function for this county is invalid. Also, the non-homogeneous function is decreasing in Larimer County, while it is constant returns to scale in El Paso County.

**Lodge and Restaurants:** Within non-homogeneous Cobb-Douglas function, this industry is constant returns to scale in El Paso County, while it is decreasing returns to scale in Arapahoe, Denver, and Larimer counties. In other counties, this function is not working, because of negative partial economies of scale.

**Other Services (Except Public Services):** This industry shows increasing returns to scale in El Paso and Arapahoe counties with 1.13 and 1.12, respectively. In

Weld County it is constant returns to scale, while in the rest of the counties it shows decreasing returns to scale.

#### **5.3.1.2.1. The Translog Function**

Economies of scale for translog production function will be analyzed by industry but among different counties.

**Construction Industry:** Construction industry shows evidence of IRTS only in Weld County. This industry exhibits CRTS in Larimer and Arapahoe, while DRTS in Denver and Boulder Counties.

**Manufacturing Industry:** All counties in this industry are exerting constant returns to scale in manufacturing. The economies of scale are between 0.94 in Arapahoe and 1.03 in Larimer County. The economy of scale is not applied to Weld County because of negative partial economies of scale.

**Wholesale Trade:** The translog estimation is not applied to Weld County in the wholesale trade industry. But it is constant returns to scale in Larimer and Arapahoe counties, while decreasing return to scale in Denver and Boulder County.

**Retail Trade:** This industry is working within constant returns to scale in Arapahoe (1.01), Denver (0.99), Larimer (0.95), and Boulder (0.93). It is working within increasing returns to scale in Weld County with 1.13.

**Transportation and Warehousing Services:** The translog function is not applied in any of the counties for transportation and warehousing industry because of negative partial economies of scale in this industry, or a small number of observations.

**Information Industry:** The translog function is applied only to this industry in Arapahoe County with constant returns to scale, 1.04.

**Finance and Insurance:** On average, this industry is working within constant returns to scale in Larimer and Arapahoe counties with 1.01 for each, and 0.94 in Boulder. But this type of functions is not applied to other counties in finance and insurance industry.

**Real Estate and Leasing Services:** This type of function, on average, exhibits constant returns to scale in Arapahoe (1.04), and Larimer (0.97). Otherwise, the translog function is not applied to other counties.

**Professional, Scientist, and Technical Services:** This industry exhibits increasing returns to scale in Arapahoe County (1.09), but it exerts decreasing returns to scale in Boulder (0.93) and Denver (0.83). In other counties, this type of functions is not applied because of negative computed partial economies of scales.

**Management of Companies and Enterprises:** The economies of scale regarding translog function are not applied to this industry in any of the counties under study.

**Administrative and Waste Management:** The economies of scale regarding translog function are applied only to Arapahoe county with constant returns to scale. In other counties, the economies of scale of this industry are not applied.

**Education Services:** In Arapahoe, the education services exert increasing returns to scale (1.07), while this type of function is not working for other counties.

**Health Care Services:** On average, the translog is exerting constant returns to scale in Arapahoe (1.03), Denver (0.97), and Boulder (0.93). Regarding Larimer County, this function exhibits decreasing returns to scale (0.89).

**Lodge and Restaurants:** Only in El Paso County, is this industry operating within constant returns to scale (0.99), while in other counties the industry is decreasing return to scale.

**Other Services (Except Public Services):** In this industry, on average, the economies of scale for translog estimation appear to be constant returns to scale. The economies of scale range between 0.94 in Boulder and 1.05 in Arapahoe.

#### **5.4. Summary:**

Table 5.8 shows that industries with increasing returns to scale appear only in 3 counties. These counties are: Arapahoe County with 5 industries according to standard Cobb-Douglas, and 2 more industries in non-homogeneous Cobb-Douglas function; In El Paso county 7 industries exhibit increasing returns to scale within standard Cobb-Douglas function, and 3 more industries according to non-homogeneous Cobb-Douglas function; Larimer County with one industry operating within increasing returns to scale; and in other counties, on average, there is no evidence of increasing returns to scale at the industry level. But at the firm level there is evidence which shows that part of the firms exhibit increasing returns to scale, tables 5.1-5.6.

In addition, table 5.8, shows only the industries that exhibit increasing return to scale in the counties included in the study. This table reveals that, in general, although the partial scale of land in non-homogeneous function in each industry is less than the corresponding partial scale in standard Cobb-Douglas and as appears in table 5.8, the total economies of scale in most of the industries is higher in non-homogeneous function compared to conventional Cobb-Douglas. The higher scale of economy in non-homogeneous function may be attributed to the increase in partial scale of capital and labor more than the reduction in land scale. Thus, 1% increase in the three input variables leads to more than 1% in output according to Cobb-Douglas production

function. Also, the growth in output as a consequence of 1% growth in the three inputs is higher in non-homogeneous Cobb-Douglas compared to standard Cobb-Douglas. Therefore, a 1% growth in output required less than 1% growth in appeared inputs in the estimated Cobb-Douglas equation. And on average, the 1% growth in output needs less growth in input in non-homogeneous than the standard Cobb-Douglas. Also part of the firms as revealed in previous tables need much smaller growth than the industry average. This means that economies of scale in some industries vary with input combinations.

**Table 5.8, Comparison of Increasing Returns to Scale Between Cobb-Douglas and Non-homogeneous Cobb-Douglas Among Industries**

| Non-homogeneous Cobb-Douglas |                 |              |              | Cobb-Douglas |                 |              |              | NAICS | County   |
|------------------------------|-----------------|--------------|--------------|--------------|-----------------|--------------|--------------|-------|----------|
| $\epsilon_T$                 | $\epsilon_{LA}$ | $\epsilon_L$ | $\epsilon_K$ | $\epsilon_T$ | $\epsilon_{LA}$ | $\epsilon_L$ | $\epsilon_K$ |       |          |
| 1.12                         | 0.23            | 0.44         | 0.44         | 1.14         | 0.34            | 0.41         | 0.40         | 23    | Arapahoe |
| 1.23                         | 0.32            | 0.45         | 0.46         | 1.06         | 0.24            | 0.31         | 0.51         | 51    |          |
| 1.19                         | 0.07            | 0.35         | 0.78         | 1.06         | 0.08            | 0.28         | 0.70         | 53    |          |
| 1.13                         | 0.23            | 0.33         | 0.58         | 1.06         | 0.29            | 0.27         | 0.50         | 54    |          |
| 1.12                         | 0.22            | 0.33         | 0.56         | 1.07         | 0.30            | 0.29         | 0.49         | 56    |          |
| 1.09                         | 0.27            | 0.16         | 0.66         |              |                 |              |              | 61    |          |
| 1.03                         | 0.21            | 0.21         | 0.61         |              |                 |              |              | 81    |          |
| 1.36                         | 0.06            | 0.61         | 0.69         | 1.26         | 0.05            | 0.63         | 0.58         | 23    | El Paso  |
| 1.09                         | 0.21            | 0.52         | 0.35         | 1.09         | 0.15            | 0.48         | 0.45         | 31    |          |
| 1.20                         | 0.13            | 0.38         | 0.69         |              |                 |              | N/A          | 51    |          |
| 1.23                         | 0.16            | 0.57         | 0.50         | 1.16         | 0.13            | 0.64         | 0.39         | 52    |          |
| 1.14                         | 0.10            | 0.33         | 0.71         | 1.10         | 0.10            | 0.34         | 0.67         | 53    |          |
| 1.40                         | 0.08            | 0.64         | 0.68         | 1.31         | 0.03            | 0.62         | 0.65         | 54    |          |
| 1.28                         | 0.10            | 0.51         | 0.66         | 1.13         | 0.10            | 0.48         | 0.55         | 56    |          |
| 1.48                         | 0.07            | 0.70         | 0.71         | 1.17         | 0.12            | 0.62         | 0.42         | 61    |          |
| 1.10                         | 0.22            | 0.50         | 0.38         |              |                 |              |              | 62    |          |
| 1.13                         | 0.18            | 0.48         | 0.47         |              |                 |              |              | 81    |          |
| 1.13                         | 0.05            | 0.30         | 0.77         | 1.07         | 0.01            | 0.32         | 0.75         | 54    | Larimer  |

In addition, table 5.8 shows that some industries are increasing returns to scale in non-homogeneous but not applied to Cobb-Douglas such as information industry in El Paso County. Also there are industries that are, on average, increasing returns to scale in non-homogeneous function, while these industries are either constant or

decreasing returns to scale as in education services, and other services in Arapahoe County; and health care services and other services in El Paso County.

## **Chapter Six: Exploring the Elasticity of Substitutions**

This study has data on capital, labor, and land at the firm level for all industries in six counties of Colorado; Arapahoe, El Paso, Denver, Larimer, Boulder, and Weld. Chapter six will explore the type of elasticity among these input variables at the industry and firm levels. In the production literature, the elasticity between labor and capital is substitute. But for more than two input variables, as in this study, the type of the elasticity will be either substitute or complement (Hicks, 1970). This is the first time elasticity for land will be investigated to determine whether it is substitute or complementary with capital and labor. Previous studies have found that the elasticity between capital and energy is complementary, as in Thompson et al (2001), and Olson et al (2002). Also a study conducted by Henderson (2009) finds the elasticity between capital and skill labor is complementary.

The rest of the chapter will discuss the following sections. The first section reviews the potential impact of elasticity of substitutions within a county and among industries, and within an industry among counties. The second part discusses the theoretical and display parts of the empirical studies regarding elasticity of substitution. The third and fourth parts discuss the results of partial elasticities at the industry and firm levels. The final section will be the conclusions.

### **6.1. Introduction**

One of the greatest advantages of the production function is how easy it is to substitute one factor of production for another factor by keeping the output level unchanged. Nicholson (2005) said that “if the rate of transformation is not changed as a result of changes in the ratio of the factors of production, then the two factors are easy to substitute”. This means that the higher the elasticity of substitutions the easier to replace



one input factor of production for another, i.e., replace capital for labor. This may help local governments adopt different economic policies during economic down turns to encourage industries that are characterized by low substitution between factors of production. But if the level of output changes, then the degree of capital intensity and low elasticity of substitution best determine the extent to which job loss in an industry/firm is a result of reducing output during reverse economic periods.

Theoretically, the elasticity of substitution is different between homogeneous and non-homogeneous production functions. In homogeneous functions, the elasticity of substitution is unity in standard Cobb-Douglas function; fixed with constant elasticity of substitution (CES), and new CES. In the latter two functions, the elasticity of substitution may deviate from unity, Kim (1992) and Hsing (1996). In the non-homogeneous functions like translog, and non-homogeneous Cobb-Douglas functions, the elasticity of substitutions varies according to the combinations of the input variables.

In this chapter, two groups of production functions are estimated. The first group is the homogeneous function which includes the standard Cobb-Douglas, and new CES of Bairam (1989, 1991). The second group is the non-homogeneous functions which include the translog, and non-homogeneous Cobb-Douglas functions of Vinod (1972). The standard Cobb-Douglas and new CES include three input variables; capital, labor as number employed, and land in square feet. The non-homogeneous Cobb-Douglas function includes the above three variables and their product variables (L\*K, L\*LA and K\*LA), while the translog function is:

$$\ln Q_t = \alpha_0 + \sum_{i=1}^n \alpha_i x_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \beta_{ij} \ln x_i \ln x_j \dots\dots\dots (6.1)$$

This function includes the three input variables, their products, and the square of the input variables.

To support the above argument, the potential importance of the elasticity of substitutions accompanied by capital intensity are displayed in two ways. The first is within the county and among different industries; and the second hypothesis will be within an industry among different counties. Here is a potential description of the two ways:

### **6.1.1 Within County and Among Industries**

Assume there are two industries X and Y operating in county A. X industry has low elasticity of substitutions and a high degree of capital intensity (K/L ratio) compared to industry Y. Suppose the economy experiences a slowdown in economic activity, then output in both industries is expected to drop back from its previous level. As a result of the drop in output, the two industries lay off workers because of the positive relation between output and jobs. The number of jobs lost will be different in the two industries as a consequence of the difference in elasticity of substitution and capital-labor ratio. For instance, the number losing jobs in industry X will be less than that of industry Y because of the high cost that will be assumed by the firms in industry X as a result of huge idle capital and low loss of jobs related to the output shrinking. Thus, firms in industry X will try not to be that far from the output level by replenishing the inventories. On the other hand, industry Y is expected to lose more jobs due to little idle capital. Therefore, firms in industry Y will benefit by minimizing the cost of labor through high layoffs.

### **6.1.2 Within Industry and Among Counties**

Assume there are two counties A and B. There is a common industry in the two counties called X. Industry X's elasticity of substitution between labor and capital is low in county A while it is high in county B. Also, suppose industry X in county A is more capital intensive than industry X operating in county B. Now, assume the economy in both counties is in a down turn period, and therefore, both counties are expected to suffer

from unemployment as a result of reduction in output in industry X. In this case, county A suffers less job loss because of the high cost burden the firms' can assume as a consequence of large amounts of idle capital. Therefore, the firms in industry X operating in county A will attempt to minimize the reduction in output as a result of high capital cost. On the other side, firms in county B can lose more jobs and minimize their costs of production. Therefore, county B will suffer a higher unemployment rate compared to county A. Thus, county B will direct more of its revenues toward current expenditures compared to county A. This may lead to deterioration of the local government's investment even in already existing projects. As a consequence, counties need to attract firms that are characterized by high capital intensity.

The rest of the chapter is organized as follows, section two discusses elasticity of substitution from Hicks' (1970) point of view; the third section will demonstrate the elasticity of substitution in non-homogeneous Cobb-Douglas among industries and firms in each county; the fourth part illustrates the type of elasticity between factors of production according to translog estimations of production functions; the fifth part examines the capital land ratio; the sixth part discusses land-capital ratio; the seventh section is covers the elasticities in the capital labor ratio; and, the last section is the summary.

## **6.2. Elasticity of Substitution**

Measuring the elasticity of substitutions for more than two variables in a translog or non-homogeneous production function is a tedious process. Hicks (1970) is the economist who discusses the measurement of elasticity of substitutions in a production function with more than two factor input variables in the production function. In this paper, Hicks wants to complete the idea of elasticity of substitutions between two input variables discussed in

his doctrine in the theory of wages (1932). In the 1970 paper, Hicks extends the concept of elasticity of substitution to include three or more input variables, and investigates whether the third input variable is a substitute or complement. According to Hicks, if the partial elasticity between the third input and the first input is positive, then the two inputs are substitutes. But if this partial elasticity is negative, then the relation between the two inputs is complementary.

This study computes the partial elasticities among capital, labor, and land variables in private sector industries in six counties in Colorado at the different industry levels for non-homogeneous production function. The computed elasticity of substitution is computed at the mean, maximum, minimum, and standard deviations. The formula used to calculate this concept is from Sato et al (1973). The general formula is:

$$\sigma_{ij} = \frac{\sum X_k f_k}{X_i X_j} * \frac{F_{ij}}{F} \dots\dots\dots (6.2)$$

Where:

$\sigma_{ij}$  = Allen Partial elasticity of substitution (AES) between inputs  $i$  and  $j$ .

$f_k$  = Marginal product of input  $k$ .

$F_{ij}$  = The cofactor determinant of Hessian matrix for inputs  $i$  and  $j$ .

$F$  = The border determinant of the Hessian matrix.

To compute the partial elasticity of substitution for different inputs, Hessian matrix is needed. The following is the derivative of the Hessian matrix components according to non-homogeneous Cobb-Douglas and translog functions:

1- The non-homogeneous components in Hessian Matrix:

$$F_1 = \frac{\partial y}{\partial x_1} = \frac{Y}{X_1} [\alpha_1 + \alpha_4 \ln X_2 + \alpha_5 \ln X_3] \dots\dots\dots (6.3)$$

$$\epsilon_1 = \alpha_1 + \alpha_4 \ln X_2 + \alpha_5 \ln X_3 \dots\dots\dots (6.4)$$

$$F_2 = \frac{\partial y}{\partial x_2} = \frac{Y}{X_2} [\alpha_2 + \alpha_4 \ln X_1 + \alpha_6 \ln X_3] \dots\dots\dots (6.5)$$

$$\epsilon_2 = \alpha_2 + \alpha_4 \ln X_1 + \alpha_6 \ln X_3 \dots\dots\dots (6.6)$$

$$F_3 = \frac{\partial y}{\partial x_3} = \frac{Y}{X_3} [\alpha_3 + \alpha_5 \ln X_1 + \alpha_6 \ln X_2] \dots\dots\dots (6.7)$$

$$\epsilon_3 = \alpha_3 + \alpha_5 \ln X_1 + \alpha_6 \ln X_2 \dots\dots\dots (6.8)$$

$$F_{11} = \frac{\partial^2 y}{\partial x_1^2} = \frac{Y\epsilon_1[\epsilon_1-1]}{X_1^2} \dots\dots\dots (6.9)$$

$$F_{22} = \frac{\partial^2 y}{\partial x_2^2} = \frac{Y\epsilon_2[\epsilon_2-1]}{X_2^2} \dots\dots\dots (6.10)$$

$$F_{33} = \frac{\partial^2 y}{\partial x_3^2} = \frac{Y\epsilon_3[\epsilon_3-1]}{X_3^2} \dots\dots\dots (6.11)$$

$$F_{12} = \frac{\partial^2 y}{\partial x_1 \partial x_2} = \frac{Y[\alpha_4 + \epsilon_1 \epsilon_2]}{X_1 X_2} \dots\dots\dots (6.12)$$

$$F_{13} = \frac{\partial^2 y}{\partial x_1 \partial x_3} = \frac{Y[\alpha_5 + \epsilon_1 \epsilon_3]}{X_1 X_3} \dots\dots\dots (6.13)$$

$$F_{23} = \frac{\partial^2 y}{\partial x_2 \partial x_3} = \frac{Y[\alpha_6 + \epsilon_2 \epsilon_3]}{X_2 X_3} \dots\dots\dots (6.14)$$

2- The translog components in Hessian Matrix:

$$F_1 = \frac{\partial y}{\partial x_1} = \frac{Y}{X_1} [\alpha_1 + \alpha_4 \ln X_2 + \alpha_5 \ln X_3 + 2\alpha_7] \dots\dots\dots (6.15)$$

$$\epsilon_4 = \alpha_1 + \alpha_4 \ln X_2 + \alpha_5 \ln X_3 + 2\alpha_7 \dots\dots\dots (6.16)$$

$$F_2 = \frac{\partial y}{\partial x_2} = \frac{Y}{X_2} [\alpha_2 + \alpha_4 \ln X_1 + \alpha_6 \ln X_3 + 2\alpha_8] \dots\dots\dots (6.17)$$

$$\epsilon_5 = \alpha_2 + \alpha_4 \ln X_1 + \alpha_6 \ln X_3 + 2\alpha_8 \dots\dots\dots (6.18)$$

$$F_3 = \frac{\partial y}{\partial x_3} = \frac{Y}{X_3} [\alpha_3 + \alpha_5 \ln X_1 + \alpha_6 \ln X_2 + 2\alpha_9] \dots\dots\dots (6.19)$$

$$\epsilon_6 = \alpha_3 + \alpha_5 \ln X_1 + \alpha_6 \ln X_2 + 2\alpha_9 \dots\dots\dots (6.20)$$

$$F_{11} = \frac{\partial^2 y}{\partial x_1^2} = \frac{Y\epsilon_4[\epsilon_4-1]}{X_1^2} \dots\dots\dots (6.21)$$

$$F_{22} = \frac{\partial^2 y}{\partial x_2^2} = \frac{Y\epsilon_5[\epsilon_5-1]}{X_2^2} \dots\dots\dots (6.22)$$

$$F_{33} = \frac{\partial^2 y}{\partial x_3^2} = \frac{Y\epsilon_6[\epsilon_6-1]}{X_3^2} \dots\dots\dots (6.23)$$

$$F_{12} = \frac{\partial^2 y}{\partial X_1 \partial X_2} = \frac{Y[\alpha_4 + \epsilon_4 \epsilon_5]}{X_1 X_2} \dots\dots\dots (6.24)$$

$$F_{13} = \frac{\partial^2 y}{\partial X_1 \partial X_3} = \frac{Y[\alpha_5 + \epsilon_4 \epsilon_6]}{X_1 X_3} \dots\dots\dots (6.25)$$

$$F_{23} = \frac{\partial^2 y}{\partial X_2 \partial X_3} = \frac{Y[\alpha_6 + \epsilon_5 \epsilon_6]}{X_2 X_3} \dots\dots\dots (6.26)$$

In this part, the research explains the results of the computed partial elasticities of substitutions for different private sector industries in the counties of Colorado studied.

### **6.3. Non-homogeneous Cobb-Douglas Function**

The partial elasticities are computed at three levels for each industry in each county. These levels are: (i) capital and labor ( $\sigma_{K,L}$ ); (ii) capital and land area ( $\sigma_{K,LA}$ ) and (iii) labor and land area ( $\sigma_{L,LA}$ ). This part depicts the type of elasticities at the county level. Also, the distribution of the partial elasticities will be explained.

#### **6.3.1. Arapahoe County**

Table 6.1A shows the average of partial elasticity of substitutions at the industry level for non-homogeneous Cobb-Douglas estimation. The partial elasticity of substitutions between labor and capital is computed at the firm level. But at the industry level it reflects the arithmetic average partial elasticities for the firms in the industry. According to Hicks (1970), the partial elasticity of substitution for the third input and higher may behave as substitute or complement. Hicks's study determines the substitution or complement between input variables by the price or quantity. In the price criteria, if the partial elasticity is negative (positive), then the relation between the inputs is substitute (complement). In the quantity criteria, if the value of elasticity is negative (positive), then the partial elasticity between these two inputs is complement (substitute). This study takes into account only the quantity side because of lack of data at the firm level concerning the

prices of inputs, in particular the capital input. The prices of inputs at the firm level can be calculated for labor and land, but not for capital.

According to the previous information, table 6.1A shows evidence that the partial elasticity between the primary inputs of labor and capital ( $\sigma_{K,L}$ ) is substitute which is in accord with the production economic literature on this point. The partial elasticity of substitution between labor and capital is different among industries. For instance, it is inelastic in information (0.51); constant in wholesale trade (1.07); real estate industries (1.05); and health care services (1.01). At the optimal point, this means that if the capital use is reduced by 10%, *ceteris paribus*, then jobs are expected to decrease by 5.1% in information industry, and 10.1% in health services. In other industries the elasticity of substitution between labor and capital is greater than 1, and it reaches about 1.82 in education services.

Table 6.1B shows the distribution of the elasticity of substitutions between labor and capital among firms in the same industry. The table shows a remarkable difference in the elasticity of substitutions among firms. For instance, the highest range in the elasticity of substitution between labor and capital is in the health services industry (2.41). In the health services industry, the minimum elasticity reveals that part of the firms have zero elasticity of substitutions, and the maximum  $\sigma_{K,L}$  is 2.41 which reveals that other firms, have high elasticity of substitution, on average, for the whole industry 1.01 is the unit of elasticity of substitution between labor and capital inputs.

The distribution of the  $\sigma_{K,L}$  among firms as measured by the ratio of range relative to standard deviation shows that the narrowest ratios are in art, entertainment, and recreation (1.78), followed by lodging and restaurants (2.42), and in administrative and waste management industries (3.97). This ratio is wide in other industries, which means that to

attract or keep firms with low elasticity of substitutions, in order to stabilize jobs in reverse economic periods, local government needs long-run planning and introduction of incentives for these firms.

**Table 6.1A, Average Partial Elasticity of Substitution in Arapahoe County by Non-homogeneous Function**

| $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{L,LA}$ | NAICS2 |
|----------------|-----------------|-----------------|--------|
| 1.99           | 0.83            | -0.92           | 23     |
| 1.21           | -0.16           | -0.17           | 31     |
| 1.07           | 0.01            | 0.19            | 42     |
| 1.35           | -0.61           | -0.88           | 44     |
| 0.51           | 0.30            | -0.51           | 51     |
| 1.21           | -0.65           | -0.61           | 52     |
| 1.05           | -0.60           | -0.88           | 53     |
| 1.46           | 0.13            | 0.35            | 54     |
| 1.79           | -0.07           | -0.54           | 55     |
| 1.45           | 0.05            | 0.39            | 56     |
| 1.82           | 0.41            | -0.62           | 61     |
| 1.01           | -0.54           | -0.62           | 62     |
| 1.13           | -0.06           | -0.01           | 71     |
| 1.66           | 0.98            | -0.25           | 72     |
| 1.59           | 0.04            | 0.23            | 81     |

**Table 6.1B , The distribution of the Elasticity of Substitutions Between Labor and Capital in Arapahoe County**

| Range/SD | Range | SD   | Min  | Max  | NAICS2 |
|----------|-------|------|------|------|--------|
| 5.65     | 2.27  | 0.40 | 0.23 | 2.50 | 23     |
| 9.28     | 1.56  | 0.17 | 0.01 | 1.57 | 31     |
| 9.11     | 0.98  | 0.11 | 0.73 | 1.72 | 42     |
| 9.63     | 2.41  | 0.25 | 0.00 | 2.42 | 44     |
| 4.21     | 1.60  | 0.38 | 0.01 | 1.62 | 51     |
| 10.09    | 2.35  | 0.23 | 0.11 | 2.45 | 52     |
| 4.30     | 2.41  | 0.56 | 0.01 | 2.41 | 53     |
| 4.24     | 1.41  | 0.33 | 0.05 | 1.46 | 54     |
| 4.24     | 1.90  | 0.45 | 0.49 | 2.39 | 55     |
| 3.97     | 1.33  | 0.33 | 0.13 | 1.45 | 56     |
| 5.85     | 1.71  | 0.29 | 0.78 | 2.49 | 61     |
| 7.11     | 2.41  | 0.34 | 0.00 | 2.41 | 62     |
| 1.78     | 1.13  | 0.63 | 0.00 | 1.13 | 71     |
| 2.42     | 1.59  | 0.66 | 0.07 | 1.66 | 72     |
| 4.74     | 2.37  | 0.50 | 0.04 | 2.41 | 81     |



## 2- The Partial Elasticity between Capital and Land

Tables 6.1A and 6.1C show that the relation of elasticity between capital and land is mixed between complement and substitute. In all industries without exception, the minimum elasticity shows that some firms are operating within complement elasticity between capital and land variables. This means to increase (decrease) output, both jobs and capital have to move in the same proportion and direction of output.

**Table 6.1C , The distribution of the Elasticity of Substitutions  
Between Land Area and Capital in Arapahoe County**

| Range/SD | Range | SD   | Min    | Max   | naics2 |
|----------|-------|------|--------|-------|--------|
| 6.25     | 2.99  | 0.48 | -0.20  | 2.79  | 23     |
| 4.56     | 0.75  | 0.16 | -0.67  | 0.08  | 31     |
| 6.19     | 0.49  | 0.08 | -0.37  | 0.12  | 42     |
| 5.86     | 4.00  | 0.68 | -4.01  | 0.00  | 44     |
| 4.53     | 4.05  | 0.89 | -1.58  | 2.46  | 51     |
| 10.62    | 18.85 | 1.78 | -12.56 | 6.30  | 52     |
| 5.85     | 14.88 | 2.54 | -8.91  | 5.98  | 53     |
| 3.05     | 0.49  | 0.16 | -0.36  | 0.13  | 54     |
| 4.44     | 0.52  | 0.12 | -0.40  | 0.12  | 55     |
| 2.82     | 0.34  | 0.12 | -0.29  | 0.05  | 56     |
| 5.60     | 1.11  | 0.20 | 0.20   | 1.31  | 61     |
| 6.99     | 8.69  | 1.24 | -6.78  | 1.91  | 62     |
| 1.44     | 1.34  | 0.93 | -1.40  | -0.06 | 71     |
| 1.88     | 3.06  | 1.63 | -2.08  | 0.98  | 72     |
| 5.89     | 1.23  | 0.21 | -0.87  | 0.36  | 81     |

Table (6.1C) shows that the maximum elasticity is positive in all industries except 71 (around zero elasticity of substitution). This means that the relation between capital and land is substitute. The maximum elasticity is 2.79 in the construction industry. While the average elasticity values in table 6.1A are mixed among industries. For instance, on average, the  $\sigma_{K,LA}$  is positive in construction, information, professional, scientists, and technical services, lodging and restaurants, and administrative and waste management. This result reflects that elasticity between

capital and land in the previous industries is substitute. While, on average, the elasticity between capital and land in the other industries is complement.

In addition, the lowest range as table 6.1C reflects is in wholesale trade (0.49), followed by manufacturing (0.75), and high services industries. The range of distribution in high services is very narrow which indicates that the policy maker in the county has to follow the same policies for attracting firms working in industries 56, 54, and 55. Other industries with high range in their elasticity, and with high ratio of range relative to standard deviation, require the local government of Arapahoe County to offer incentives for the firms with low elasticity of substitutions or complements to attract them to the area to mitigate the impact of job loss during reverse economic periods.

### **3- The Partial Elasticity between Labor and Land**

This part explains the partial elasticity between labor and land according to the results in table 6.1D. The table shows evidence of mixed results of substitution and complementary. For instance, the minimum elasticities show complementarities between labor and land in all industries. While the positive maximum values show evidence of substitution between labor and capital. Thus, the local government in Arapahoe County has to attract firms with low complement and substitute elasticities between land and labor to mitigate the side effects during reverse economic periods.

The distribution of the partial elasticity between land and labor is wide in all industries (except industry 71). This means that policy makers have to attract firms by submitting incentives to the firms that have a low complement between labor and land to decrease job loss during economic down turns in Arapahoe County.

**Table 6.1D , The distribution of the Elasticity of Substitutions  
Between Land Area and Labor in Arapahoe County**

| Range/SD | Range | SD   | Min    | Max   | NAICS2 |
|----------|-------|------|--------|-------|--------|
| 5.64     | 12.06 | 2.14 | -9.61  | 2.46  | 23     |
| 7.36     | 8.45  | 1.15 | -6.22  | 2.24  | 31     |
| 8.34     | 13.18 | 1.58 | -10.89 | 2.29  | 42     |
| 5.92     | 10.65 | 1.80 | -10.65 | 0.00  | 44     |
| 6.25     | 12.68 | 2.03 | -10.99 | 1.70  | 51     |
| 7.06     | 12.63 | 1.79 | -10.54 | 2.09  | 52     |
| 5.68     | 10.81 | 1.90 | -8.86  | 1.95  | 53     |
| 3.90     | 2.64  | 0.68 | -2.29  | 0.35  | 54     |
| 4.58     | 8.39  | 1.83 | -6.51  | 1.88  | 55     |
| 6.33     | 5.29  | 0.84 | -4.90  | 0.39  | 56     |
| 4.66     | 6.42  | 1.38 | -5.79  | 0.64  | 61     |
| 7.49     | 12.92 | 1.72 | -10.65 | 2.26  | 62     |
| 1.86     | 0.03  | 0.02 | -0.04  | -0.01 | 71     |
| 5.02     | 7.39  | 1.47 | -7.65  | -0.25 | 72     |
| 9.57     | 10.30 | 1.08 | -7.91  | 2.38  | 81     |

### 6.3.2. El Paso County

**1- The Partial Elasticity between Capital and Labor:** On average, table 6.2A, shows that all industries, except wholesale trade (0.72), have high elasticity of substitutions between labor and capital. But the minimum partial elasticity between these two inputs (table 6.2B) shows evidence that in most industries the elasticity is around zero, or a type of Leontief production function. This means to decrease output, the firm has to reduce the use of labor and capital in the production process in the same proportion. While the maximum partial elasticity between labor and capital shows evidence of more than 2 in all industries, table 6.2B. This means that substitution of capital to labor is easy in some firms in El Paso County. Thus, the local government in this county has to attract firms with low elasticity between labor and capital for ease of countering job issues during reverse economic periods.

Also, the range to standard deviation ratio shows evidence of wide distribution among the firms' elasticity of substitution between labor and capital, table 6.2B. Thus, the local government has to offer incentives for firms with low elasticity of substitutions to attract them to locate in El Paso County.

**Table 6.2A , Average Partial Elasticity of Substitution  
In El Paso County by Non-homogeneous Function**

| $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{L,LA}$ | NAICS2 |
|----------------|-----------------|-----------------|--------|
| 1.65           | -1.10           | -0.06           | 23     |
| 1.25           | -0.54           | 0.00            | 31     |
| 0.72           | 0.51            | 0.05            | 42     |
| 1.56           | -0.27           | -0.03           | 44     |
| 1.38           | -0.98           | -0.02           | 48     |
| 1.11           | 0.81            | 0.01            | 51     |
| 1.55           | -0.87           | -0.02           | 52     |
| 1.23           | -0.49           | -0.10           | 53     |
| 1.24           | -0.94           | -0.16           | 54     |
| 1.44           | 0.15            | 0.04            | 56     |
| 1.50           | -0.85           | -0.05           | 62     |
| 1.55           | -0.91           | -0.04           | 72     |
| 1.25           | -0.56           | -0.04           | 81     |

**Table 6.2B , The distribution of the Elasticity of Substitutions  
Between Labor and Capital in El Paso County**

| Range/SD | Range | SD   | Min  | Max  | NAICS2 |
|----------|-------|------|------|------|--------|
| 7.60     | 2.23  | 0.29 | 0.22 | 2.45 | 23     |
| 7.31     | 2.40  | 0.33 | 0.05 | 2.45 | 31     |
| 6.92     | 1.45  | 0.21 | 0.01 | 1.46 | 42     |
| 6.86     | 2.44  | 0.36 | 0.00 | 2.45 | 44     |
| 6.64     | 1.73  | 0.26 | 0.38 | 2.11 | 48     |
| 5.20     | 2.44  | 0.47 | 0.01 | 2.45 | 51     |
| 7.50     | 2.46  | 0.33 | 0.04 | 2.50 | 52     |
| 8.13     | 2.34  | 0.29 | 0.08 | 2.42 | 53     |
| 8.03     | 1.67  | 0.21 | 0.58 | 2.25 | 54     |
| 9.25     | 2.22  | 0.24 | 0.09 | 2.32 | 56     |
| 6.50     | 2.49  | 0.38 | 0.01 | 2.50 | 62     |
| 6.44     | 2.41  | 0.37 | 0.01 | 2.42 | 72     |
| 5.36     | 2.43  | 0.45 | 0.02 | 2.45 | 81     |

**2-Partial Elasticity between Land and Capital:** The analysis of this part will be based on two tables, 6.2A and 6.2C. On average, table 6.2A shows evidence that all industries in El Paso County have negative elasticity between land and capital.

Thus, the elasticity between these two factors at the industry level is complementary, and less than 1. This means that El Paso needs to encourage industries with low complementarities to stabilize jobs during reverse economic situations. Table 6.2C shows the elasticity between land and capital at the firm level. The table shows that the maximum elasticity is positive which means that there is elasticity of substitution between land and capital among part of the firms. Thus, if there is a low elasticity of substitution between labor and capital, the local government has to encourage the firms that have low elasticity of substitutions to keep jobs during reverse economic periods. Furthermore, the minimum partial elasticities between these two factors show evidence of complementarities among these two factors of production. As a consequence, authorities in El Paso County have to offer incentives to the industries with low complement elasticity to keep jobs in reverse economic periods.

**Table 6.2C, The distribution of the Elasticity of Substitutions  
Between Land Area and Capital in El Paso County**

| Range/SD | Range | SD   | Min    | Max   | NAICS2 |
|----------|-------|------|--------|-------|--------|
| 5.95     | 8.68  | 1.46 | -8.74  | -0.06 | 23     |
| 9.49     | 12.10 | 1.28 | -7.62  | 4.49  | 31     |
| 10.87    | 14.67 | 1.35 | -9.83  | 4.84  | 42     |
| 8.69     | 2.57  | 0.30 | -2.44  | 0.13  | 44     |
|          |       | 1.58 | -6.64  | 4.44  | 48     |
| 5.07     | 13.44 | 2.65 | -8.74  | 4.69  | 51     |
| 8.41     | 12.11 | 1.44 | -8.52  | 3.59  | 52     |
| 12.88    | 14.53 | 1.13 | -10.99 | 3.54  | 53     |
| 8.17     | 18.45 | 2.26 | -13.48 | 4.97  | 54     |
| 7.90     | 1.84  | 0.23 | -1.34  | 0.50  | 56     |
| 8.88     | 9.15  | 1.03 | -4.85  | 4.30  | 62     |
| 7.03     | 11.79 | 1.68 | -6.87  | 4.92  | 72     |
| 6.14     | 10.99 | 1.79 | -5.99  | 5.00  | 81     |

**3-Partial Elasticity between Labor and Land:** The analysis of the elasticity between labor and land is based on table 6.2A and table 6.2D. On average, table 6.2A shows evidence that the elasticity between labor and land is too small and

concentrated around zero, or Leontief elasticity of either substitute or complementarities. This situation at the industry level shows a satisfactory partial elasticity in El Paso County. On the other hand, Table 6.2D shows that the maximum elasticity at the firm level is around zero in 7 industries, and more than one in 4 industries. The distribution of the elasticity of substitution, according to ratio of range relative to standard deviation is very wide, which means that the county has to attract firms or industries with low complementarities or substitution of elasticities to mitigate the loss in jobs and local government revenues.

**Table 6.2D, The distribution of the Elasticity of Substitutions  
Between Land Area and Labor in El Paso County**

| Range/SD | Range | SD   | Min   | Max  | NAICS2 |
|----------|-------|------|-------|------|--------|
| 8.58     | 1.65  | 0.19 | -1.65 | 0.00 | 23     |
| 12.14    | 1.51  | 0.12 | -0.30 | 1.22 | 31     |
| 7.53     | 1.26  | 0.17 | -0.19 | 1.07 | 42     |
| 19.75    | 6.88  | 0.35 | -4.42 | 2.46 | 44     |
|          |       | 0.06 | -0.41 | 0.00 | 48     |
| 8.82     | 1.07  | 0.12 | -0.33 | 0.74 | 51     |
| 15.58    | 3.94  | 0.25 | -1.69 | 2.24 | 52     |
| 10.16    | 3.77  | 0.37 | -3.75 | 0.01 | 53     |
| 9.51     | 4.82  | 0.51 | -4.69 | 0.13 | 54     |
| 12.62    | 1.97  | 0.16 | -0.35 | 1.62 | 56     |
| 7.36     | 1.16  | 0.16 | -1.16 | 0.00 | 62     |
| 14.07    | 4.25  | 0.30 | -3.85 | 0.40 | 72     |
| 6.84     | 1.36  | 0.20 | -1.34 | 0.02 | 81     |

### 6.3.3. Denver County:

In Denver County, the partial elasticities discussion is based on tables 6.3A to 6.3D. The following sections discuss these elasticities in Denver County.

**1- The Partial Elasticity between Capital and Labor:** On average, at the industry level, partial elasticity of substitution in Denver County is more than 1 (table 6.3A), except for 3 industries; manufacturing (0.52), transportation and warehousing (0.65),

and information (0.68). While at the firm level, as shown in table 6.3B, the maximum and minimum partial elasticity is positive for all industries. This means that the partial elasticity even at the firm level between labor and capital is substitute. Thus, because of the wide range in elasticities between maximum and minimum values, the local government has to encourage firms with low elasticity of substitutions such as manufacturing at the firm level.

**Table 6.3A, Average Partial Elasticity of Substitution in Denver County by Non-homogeneous Function**

| $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{L,LA}$ | NAICS2 |
|----------------|-----------------|-----------------|--------|
| 1.46           | -0.29           | -0.21           | 23     |
| 0.52           | 0.81            | 0.01            | 31     |
| 1.35           | 0.10            | -0.05           | 42     |
| 1.22           | -0.31           | -0.24           | 44     |
| 0.65           | 0.02            | 0.07            | 48     |
| 0.68           | 0.23            | 0.03            | 51     |
| 1.40           | -0.73           | -0.16           | 52     |
| 1.15           | -0.10           | -0.11           | 53     |
| 1.46           | -0.76           | -0.56           | 54     |
| 1.79           | -0.70           | -0.07           | 56     |
| 1.39           | 0.68            | -0.21           | 61     |
| 1.49           | -0.36           | -0.15           | 62     |
| 2.01           | -0.80           | -0.11           | 72     |
| 2.01           | -0.43           | -0.17           | 81     |

Table 6.3B shows the distribution of the elasticities of substitutions between labor and capital at the firm level. The lowest elasticity is in transportation and warehousing (0.72), followed by the manufacturing industry (0.80). The ratio (range to standard deviation) shows evidence that there is a wide distribution between firms in the same industry and among industries in the elasticity between labor and capital inputs. The highest ratio is in the wholesale trade industry, while the lowest ratio is in real estate (3.07). The policy implication to the wide distribution suggests that Denver County has

to adopt selective policies for attracting firms and industries to its region characterized by low partial elasticity of substitutions between labor and capital.

**Table 6.3B, The distribution of the Elasticity of Substitutions Between Labor and Capital in Denver County**

| Range/SD | Range | SD   | Min  | Max  | NAICS2 |
|----------|-------|------|------|------|--------|
| 6.78     | 1.45  | 0.21 | 0.91 | 2.36 | 23     |
| 4.17     | 0.80  | 0.19 | 0.04 | 0.85 | 31     |
| 9.27     | 1.65  | 0.18 | 0.17 | 1.82 | 42     |
| 6.35     | 2.19  | 0.35 | 0.25 | 2.44 | 44     |
| 4.23     | 0.72  | 0.17 | 0.38 | 1.10 | 48     |
| 4.28     | 1.25  | 0.29 | 0.28 | 1.53 | 51     |
| 3.36     | 1.90  | 0.57 | 0.24 | 2.15 | 52     |
| 3.07     | 2.11  | 0.69 | 0.12 | 2.24 | 53     |
| 5.96     | 2.48  | 0.42 | 0.02 | 2.49 | 54     |
| 4.61     | 1.39  | 0.30 | 1.04 | 2.43 | 56     |
| 6.32     | 1.51  | 0.24 | 0.22 | 1.73 | 61     |
| 5.14     | 2.09  | 0.41 | 0.36 | 2.45 | 62     |
| 5.19     | 2.38  | 0.46 | 0.13 | 2.51 | 72     |
| 3.52     | 0.83  | 0.23 | 1.57 | 2.40 | 81     |

**2-Partial Elasticity between Land and Capital:** The analysis in this part depends on table 6.3A, and table 6.3C. On average, at the industry level, table 6.3A shows that in 9 industries the partial elasticity between land and capital is complementary and the value of elasticity is less than 1 in absolute value. Also, the table shows that the partial elasticity is positive (substitute) in 5 industries. While table 6.3C, which represents the partial elasticity between land and capital at the firm level, shows that the maximum value is about zero in other service industries, and positive less than 1 in three industries (construction, transportation and warehousing, and information). On the other hand, the minimum value of elasticity is positive and less than unity in 2 industries; manufacturing with (0.41), and education services (0.08). While in other industries, the firms' minimum partial elasticity is negative. Such results suggest that the local government in Denver County need to attract firms with low partial elasticity between land and capital in absolute values to preserve jobs during economic down turns.



The distribution of the partial elasticity of firms between land and capital in Denver County is very wide according to the ratio of range to standard deviation, table 6.3C. The highest ratio is in construction (11.92), followed by retail trade and wholesale trade, 10.37 and 8.82, respectively. The lowest ratio is in finance and insurance, 2.51. This means there is a high heterogeneity in partial elasticity between land and capital within firms in the same industry and among industries. This wide distribution requires that the county adopt selective policies to attract firms with low partial elasticity, in absolute value, between land and capital to minimize job loss in its region during economic down turns.

**Table 6.3C, The Distribution of the Elasticity of Substitutions  
Between Land Area and Capital in Denver County**

| Range/SD | Range | SD   | Min   | Max  | NAICS2 |
|----------|-------|------|-------|------|--------|
| 11.92    | 5.70  | 0.48 | -5.26 | 0.44 | 23     |
| 6.04     | 1.51  | 0.25 | 0.41  | 1.92 | 31     |
| 8.82     | 5.35  | 0.61 | -4.00 | 1.34 | 42     |
| 10.37    | 4.83  | 0.47 | -3.73 | 1.10 | 44     |
| 3.69     | 0.59  | 0.16 | -0.33 | 0.26 | 48     |
| 3.86     | 1.55  | 0.40 | -0.82 | 0.74 | 51     |
| 2.51     | 3.26  | 1.30 | -1.95 | 1.31 | 52     |
| 4.32     | 6.89  | 1.59 | -4.50 | 2.38 | 53     |
| 8.38     | 5.15  | 0.61 | -2.88 | 2.27 | 54     |
| 5.54     | 3.23  | 0.58 | -2.20 | 1.04 | 56     |
| 4.62     | 1.14  | 0.25 | 0.08  | 1.22 | 61     |
| 8.79     | 4.74  | 0.54 | -2.58 | 2.17 | 62     |
| 7.77     | 4.60  | 0.59 | -2.19 | 2.41 | 72     |
| 3.69     | 1.29  | 0.35 | -1.24 | 0.05 | 81     |

**3-Partial Elasticity between Land and Labor:** The analysis of this part relies on table 6.3A and table 6.3D. Table 6.3A, reflects the average partial elasticity and shows that the partial elasticity between labor and land is too small in absolute value in all industries except 'professional, scientist, and technical services (-0.56). This means that to reduce output during economic down turns requires proportional loss of jobs as more land in the firms become idle. Thus, these values of partial elasticity are suitable for policy makers at the county level because they will attract firms at low cost.

On the other side, table 6.3D reveals that the maximum partial elasticity between labor and land is around zero in 7 industries. Also, the minimum  $\sigma_{L,LA}$  is around zero in

different 3 industries. Thus, the county has to adopt long-run planning to select the firms with low elasticity of substitutions in absolute value to the expected benefit during economic down turns.

**Table 6.3D, The distribution of the Elasticity of Substitutions  
Between Land Area and Labor in Denver County**

| Range/SD | Range | SD   | Min   | Max  | NAICS2 |
|----------|-------|------|-------|------|--------|
| 7.91     | 4.26  | 0.54 | -3.87 | 0.40 | 23     |
| 6.73     | 0.27  | 0.04 | 0.00  | 0.27 | 31     |
| 8.19     | 2.36  | 0.29 | -2.09 | 0.27 | 42     |
| 8.12     | 6.18  | 0.76 | -5.20 | 0.98 | 44     |
| 3.79     | 0.56  | 0.15 | -0.16 | 0.40 | 48     |
| 3.36     | 0.17  | 0.05 | -0.01 | 0.16 | 51     |
| 4.00     | 1.44  | 0.36 | -1.40 | 0.05 | 52     |
| 5.02     | 2.06  | 0.41 | -1.55 | 0.51 | 53     |
| 5.31     | 6.56  | 1.23 | -6.46 | 0.11 | 54     |
| 6.79     | 2.85  | 0.42 | -2.85 | 0.00 | 56     |
| 6.98     | 7.80  | 1.12 | -6.04 | 1.76 | 61     |
| 7.66     | 3.87  | 0.51 | -3.87 | 0.00 | 62     |
| 7.13     | 3.46  | 0.49 | -3.46 | 0.00 | 72     |
| 5.51     | 3.37  | 0.61 | -3.24 | 0.13 | 81     |

**6.3.4. Larimer County:** The following discusses the partial elasticities in Larimer County at the industry and firm level.

**1- Partial Elasticity between Labor and Capital:** The partial elasticity between labor and capital is positive and less than unity as shown in table 6.4A. This means that these industries are different than standard Cobb-Douglas function.

**Table 6.4A, Average Partial Elasticity of Substitution  
In Larimer County by Non-homogeneous  
Function**

| $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{L,LA}$ | NAICS2 |
|----------------|-----------------|-----------------|--------|
| 0.63           | -0.37           | 0.00            | 62     |
| 0.61           | 0.05            | 0.03            | 72     |
| 0.84           | -0.10           | -0.87           | 81     |

**Table 6.4B, The distribution of the Elasticity of Substitutions  
Between Labor and Capital in Larimer County**

| Range/SD | Range | SD   | Min  | Max  | NAICS2 |
|----------|-------|------|------|------|--------|
| 4.14     | 1.47  | 0.36 | 0.00 | 1.47 | 62     |
| 6.84     | 2.01  | 0.29 | 0.00 | 2.01 | 72     |
| 6.02     | 2.50  | 0.42 | 0.00 | 2.50 | 81     |

**2-Partial Elasticity between Land and Capital:** Table 6.4A shows that the partial elasticity between land and capital is small in absolute value. But table 6.4C shows that this elasticity is widely distributed among firms in the industry. Thus, the county has to attract firms with small value of elasticity between land and capital. This is expected to help the county during down turns in the economy by saving jobs.

**Table 6.4C, The distribution of the Elasticity of Substitutions  
Between Land Area and Capital in Larimer County**

| Range/SD | Range | SD   | Min   | Max  | NAICS2 |
|----------|-------|------|-------|------|--------|
| 6.31     | 3.93  | 0.62 | -2.35 | 1.58 | 62     |
| 18.90    | 2.67  | 0.14 | -1.21 | 1.46 | 72     |
| 8.89     | 1.23  | 0.14 | -0.67 | 0.55 | 81     |

**3-Partial Elasticity between Land and Capital:** Table 6.4A, at the industry level, shows that the elasticity between labor and land is inelastic in education and lodging and restaurants industries. Thus the county has to concentrate on such industries to keep the level of jobs during down turns.

**Table 6.4D, The distribution of the Elasticity of Substitutions  
Between Land Area and Labor in Larimer County**

| Range/SD | Range  | SD    | Min     | Max   | NAICS2 |
|----------|--------|-------|---------|-------|--------|
| 10.91    | 0.36   | 0.03  | -0.15   | 0.21  | 62     |
| 15.61    | 5.20   | 0.33  | -0.64   | 4.56  | 72     |
| 12.79    | 156.37 | 12.23 | -130.62 | 25.75 | 81     |

**6.3.5. Boulder County:** The partial elasticities among the three input variables will be illustrated in the following discussions.

**1- Partial Elasticity between Capital and Labor:** Table 6.5A shows that the average partial elasticity between labor and capital is different than unity as supposed by Cobb-Douglas in three industries. While it is around unity in two industries: construction, and manufacturing. In addition, table 6.5B shows a wide distribution in the elasticity between labor and capital among firms in the industry. The highest range is 2.28 in other services, followed by 2.22 in manufacturing industry. Therefore, the county has to adopt selective policies to attract firms with low partial elasticity of substitutions between labor and capital.

**Table 6.5A, Average Partial Elasticity of Substitution  
In Boulder County by Non-homogeneous Function**

| $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{L,LA}$ | NAICS2 |
|----------------|-----------------|-----------------|--------|
| 0.97           | -2.06           | 1.04            | 23     |
| 1.10           | 0.92            | 0.06            | 31     |
| 1.95           | 0.67            | -0.40           | 54     |
| 0.74           | -2.54           | -1.00           | 56     |
| 1.80           | 0.56            | 0.01            | 62     |
| 1.33           | -0.44           | 0.37            | 81     |

**Table 6.5B, The distribution of the Elasticity of Substitutions  
Between Labor and Capital in Boulder County**

| Range/SD | Range | SD   | Min  | Max  | NAICS2 |
|----------|-------|------|------|------|--------|
| 2.95     | 1.51  | 0.51 | 0.21 | 1.71 | 23     |
| 4.41     | 2.22  | 0.50 | 0.21 | 2.44 | 31     |
| 5.87     | 1.93  | 0.33 | 0.55 | 2.48 | 54     |
| 2.49     | 1.51  | 0.61 | 0.05 | 1.56 | 56     |
| 4.07     | 2.20  | 0.54 | 0.31 | 2.51 | 62     |
| 4.97     | 2.28  | 0.46 | 0.22 | 2.50 | 81     |

**Table 6.5C, The distribution of the Elasticity of  
Substitutions**

**Between Land Area and Capital in Boulder County**

| Range/SD | Range | SD   | Min   | Max   | NAICS2 |
|----------|-------|------|-------|-------|--------|
| 2.82     | 2.41  | 0.85 | -3.24 | -0.82 | 23     |
| 4.80     | 3.77  | 0.79 | -1.35 | 2.42  | 31     |
| 3.88     | 1.42  | 0.36 | -0.06 | 1.36  | 54     |
| 4.00     | 14.98 | 3.75 | -13.6 | 1.43  | 56     |
| 4.78     | 5.81  | 1.21 | -3.60 | 2.21  | 62     |
| 4.24     | 2.97  | 0.70 | -2.27 | 0.69  | 81     |

**2-Partial Elasticity between Land and Capital:** The following table shows that the partial elasticity between land and capital is wide in all industries except construction. Thus, Boulder County has to adopt long-run planning to keep firms with low elasticity, in absolute value between land and capital.

**3-Partial Elasticity between Land and Labor:** Table 6.5A shows that the average elasticities are substitutes in construction and other services; zero in manufacturing and health services; and complementarities in administrative and waste management industry. On the other side, table 6.5D shows a wide range among firms.

This suggests that the county has to adopt different policies to attract firms to the county.

**Table 6.5D, The distribution of the Elasticity of Substitutions  
Between Land Area and Labor in Boulder County**

| Range/SD | Range | SD   | Min   | Max  | NAICS2 |
|----------|-------|------|-------|------|--------|
| 2.62     | 2.35  | 0.90 | -0.01 | 2.34 | 23     |
| 3.10     | 0.40  | 0.13 | 0.00  | 0.40 | 31     |
| 4.77     | 4.49  | 0.94 | -2.36 | 2.13 | 54     |
| 2.83     | 2.77  | 0.98 | -2.41 | 0.36 | 56     |
| 4.22     | 4.37  | 1.03 | -2.34 | 2.03 | 62     |
| 4.31     | 3.78  | 0.88 | -1.33 | 2.45 | 81     |

### 6.3.6. Weld County

**1-Partial Elasticity between Labor and Capital:** The elasticity of substitution is less than one in all estimations for Weld County. Thus, the elasticity of substitution is deviated from Cobb-Douglas. Table 6.6B shows that the range between maximum and minimum elasticities is remarkable. Therefore, the local government has to adopt selective policies regarding the firms that operate in its region.

**Table 6.6A, Average Partial Elasticity of  
Substitution  
In Weld County by Non-homogeneous Function**

| $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{L,LA}$ | NAICS2 |
|----------------|-----------------|-----------------|--------|
| 0.74           | -0.38           | -0.01           | 23     |
| 0.88           | -1.31           | 0.03            | 48     |
| 0.74           | -0.32           | 0.05            | 54     |
| 0.41           | -0.10           | -0.06           | 56     |
| 0.47           | -0.18           | 0.00            | 81     |

**Table 6.6B, The Distribution of the Elasticity of Substitutions  
Between Labor and Capital in Weld County**

| Range/SD | Range | SD   | Min  | Max  | NAICS2 |
|----------|-------|------|------|------|--------|
| 8.27     | 1.27  | 0.15 | 0.13 | 1.40 | 23     |
| 4.38     | 2.19  | 0.50 | 0.10 | 2.29 | 48     |
| 9.23     | 1.51  | 0.16 | 0.09 | 1.60 | 54     |
| 5.07     | 1.21  | 0.24 | 0.02 | 1.23 | 56     |
| 3.25     | 1.17  | 0.36 | 0.01 | 1.18 | 81     |

**2-Partial Elasticity between Land and Capital:** Table 6.6A shows that the partial elasticity between land and capital, on average, is negative and less than unity in all industries as appeared in the regression of non-homogeneous Cobb-Douglas for Weld County. Thus, the elasticity between these two variables is complement. Therefore, to attract such industries or firms, the county has to choose the industry with low elasticity between capital and land to lessen the job losses during reverse economic periods. Also, table 6.6C supports this direction.

**Table 6.6C, The distribution of the Elasticity of Substitutions  
Between Land Area and Capital in Weld County**

| Range/SD | Range | SD   | Min    | Max  | NAICS2 |
|----------|-------|------|--------|------|--------|
| 5.90     | 10.97 | 1.86 | -8.42  | 2.55 | 23     |
| 4.30     | 14.99 | 3.49 | -12.65 | 2.34 | 48     |
| 6.67     | 6.36  | 0.95 | -3.99  | 2.36 | 54     |
| 4.58     | 1.49  | 0.33 | -1.09  | 0.41 | 56     |
| 3.46     | 4.56  | 1.32 | -2.96  | 1.60 | 81     |

### 3-Partial Elasticity between Land and Labor

The distribution of the elasticity of substitution, according to the ratio of range relative to standard deviation, is very wide which means that the county has to attract the firms or industries with low complementarities or substitution of elasticities to limit the inverse effects on employment and local government revenues.

**Table 6.6D, The distribution of the Elasticity of Substitutions**

| <b>Between Land Area and Labor in Weld County</b> |       |      |       |      |        |
|---|-------|------|-------|------|--------|
| Range/SD  | Range | SD   | Min   | Max  | NAICS2 |
| 8.90  | 0.53  | 0.06 | -0.51 | 0.02 | 23     |
| 4.83  | 0.51  | 0.11 | 0.00  | 0.51 | 48     |
| 6.34  | 0.94  | 0.15 | -0.14 | 0.80 | 54     |
| 7.24  | 2.72  | 0.38 | -1.78 | 0.94 | 56     |
| 5.82  | 0.12  | 0.02 | -0.08 | 0.04 | 81     |

#### 6.4. The elasticity between Inputs by Translog Function

This part will display the type of elasticity at the industry and firm levels in the six counties under study by applying translog production function. At the industry level, the arithmetic partial elasticity between labor and capital is computed. The second part will discuss the analysis at the firm level by taking the maximum and minimum partial elasticities. The third part examines the distribution of the partial elasticity within and among different industries according to NAICS (2007) classification system. The aforementioned analysis will be examined at the county level.



#### **6.4.1. Arapahoe County:**

The analysis of the type of elasticities in Arapahoe County will rely on tables 6.7A to 6.7D. The partial elasticities are: partial elasticity between the primary inputs of labor and capital; partial elasticity between capital and land and whether this elasticity is substitute or complement; and the partial elasticity between labor and land and whether it is substitute or complement. The reason for such division is to suggest policies that help the county in the long run to increase its revenue and provide best services for its community, and reduce the job loss during economic down turns.

**1- Partial Elasticity between Capital and Labor:** Table 6.6A shows that only information industry exhibits as Cobb-Douglas with unity elasticity of substitution. Otherwise, the partial elasticity is deviated from unity and ranges between 0.66 in professional, scientist, and technical services industry to 1.27 in transportation and warehousing industry. This means that if the cost of labor doubled in professional industry then the reduction in jobs will be less than 66% as a result of replacing the labor by capital. If the wages in transportation industry, on average, increased by 10%, then loss in jobs will be estimated to be around 12.7 as a result of raising the use of capital.

Table 6.7B shows the maximum partial elasticities between labor and capital varies substantially. Also, the minimum values of these elasticities reveal that part of the firms are operating as in Leontief production function as a consequence of small elasticity of substitutions. On the other side, the maximum elasticity values reveal that other firms have high elasticity of substitutions of greater than 1. Where, the maximum elasticity ranges between 1.36 in other services industry and 3.3 in manufacturing industry.

The distribution of the partial elasticity between labor and capital, as measured by the range relative to standard deviation, is remarkably high. To keep the industries with

lowest elasticity of substitution in the counties regions have to introduce incentives for these industries in order to mitigate the impacts of economic down turns and their negative impact on jobs.

**2-Partial Elasticity between Capital and Land:** On average, according to translog function, in Arapahoe County, the partial elasticity between land and capital is complementarities. This means as capital uses increase, the area of land usage will increase. The partial elasticity in all these industries is less than unity. This suggests that the Cobb-Douglas function will not work. This elasticity ranges between 0.61 in other services to 0.90 in professional and scientist industry.

The distribution of the elasticity between land and capital shows evidence that part of the firms are working under Leontief production functions. Also, part of the firms are working with high elasticity of complementarities. In addition, the ratio of range to Standard deviation shows heterogeneity among industries. For example, this ratio between 2.76 in other services, which require little intervention by the community to encourage such firms in the industry, and 10.20 in professional and scientist industry, which requires intervention in the market to attract the firms with low elasticities.

**3-Partial elasticity between Land and Labor:** By the same pattern, this part depicts the partial elasticity between labor and land. Table 6.7A shows that, on average, the elasticity between land and labor is substitute in all industries without exception. Also, the elasticity is less than unity in all industries ranges between 0.61 in other services to 0.90 in' professional and scientist industry.

At the firm level data, table 6.7D shows that there is a tremendous difference between maximum elasticity for a firm and minimum value of elasticity in the same

industry. The minimum values indicate that some of the industries are working as complement because of negative elasticity in all industries. While the maximum elasticity shows that part of the industries are operating as substitutes between labor and land. Thus the county has to introduce incentives to the firms with low elasticity of substitutions and complementarities to stabilize the number of jobs and local governments' revenues in the region.

#### **6.4.2. El Paso County:**

In El Paso County, the analysis is based on tables 6.8A to 6.8D. The analysis includes the three partial elasticities in the study as mentioned in Arapahoe County. Also, the distribution of elasticities among firms and industries will be discussed.

**1- Partial Elasticity between Capital and Labor:** On average, the elasticity of substitutions between capital and labor is less than unity in all industries. This elasticity ranges between 0.47 in manufacturing to 0.77 in education services. Table 6.8.B shows that there is substantial heterogeneity between maximum and minimum values of elasticity of substitution among the firms within the same industry. For example, the minimum value of elasticity indicates that some firms are operating within Leontief function with zero or around zero elasticity of substitution between labor and capital. On the other side, the maximum value of elasticity shows evidence that the elasticity is greater than unity. Thus, to preserve the firms with low elasticity of substitutions to save jobs during economic down turns requires the county to introduce incentives for such firms.

**Table 6.8A, Translog Elasticity by Industry in  
El Paso County.**

| $\sigma_{L,LA}$ | $\sigma_{K,LA}$ | $\sigma_{K,L}$ | NAICS |
|-----------------|-----------------|----------------|-------|
| 0.23            | 0.25            | 0.64           | 23    |
| -1.08           | 0.76            | 0.47           | 31    |
| -0.46           | 0.26            | 0.74           | 44    |
| -0.99           | 0.59            | 0.49           | 48    |
| -0.73           | 0.59            | 0.49           | 52    |
| -1.35           | 0.81            | 0.68           | 53    |
| 0.19            | -0.55           | 0.56           | 54    |
| -1.13           | 0.89            | 0.64           | 56    |
| -0.61           | 0.82            | 0.77           | 61    |
| 0.47            | 0.42            | 0.62           | 62    |
| -1.33           | 0.95            | 0.61           | 72    |
| 0.19            | 0.36            | 0.64           | 81    |

The distribution of the firms in each industry as measured by the ratio of range (the difference between maximum and minimum value) is high. This supports the previous policy that the county has to pay attention to the firms with low elasticities of substitutions.

**Table 6.8B, Elasticity of Substitution between Capital and  
Labor in El Paso County**

| Range/SD | Range | SD   | Min  | Max  | NAICS |
|----------|-------|------|------|------|-------|
| 10.05    | 2.26  | 0.23 | 0.02 | 2.29 | 23    |
| 4.83     | 1.20  | 0.25 | 0.01 | 1.21 | 31    |
| 3.74     | 3.08  | 0.82 | 0.00 | 3.08 | 44    |
| 3.57     | 1.16  | 0.32 | 0.06 | 1.22 | 48    |
| 6.43     | 2.29  | 0.36 | 0.00 | 2.29 | 52    |
| 4.61     | 0.98  | 0.21 | 0.40 | 1.37 | 53    |
| 4.22     | 1.46  | 0.35 | 0.00 | 1.46 | 54    |
| 5.70     | 2.76  | 0.48 | 0.00 | 2.76 | 56    |
| 3.83     | 1.62  | 0.42 | 0.00 | 1.62 | 61    |
| 7.53     | 1.35  | 0.18 | 0.00 | 1.36 | 62    |
| 9.38     | 1.55  | 0.17 | 0.15 | 1.70 | 72    |
| 11.09    | 2.10  | 0.19 | 0.00 | 2.11 | 81    |

**2-Partial Elasticity between Capital and Land:** Table 6.8A, on average, shows there is substitute elasticity between land and capital in all industries except for professional and scientist industry, which are complement. Also, there is heterogeneity in elasticity among industries. The lowest elasticity is in construction (0.25), while the maximum in lodge and restaurants is 0.95.

**Table 6.8C, Elasticity between Capital and Land in El Paso County**

| Range/SD | Range | SD   | Min   | Max  | NAICS |
|----------|-------|------|-------|------|-------|
| 9.88     | 4.79  | 0.49 | -1.95 | 2.84 | 23    |
| 5.18     | 3.70  | 0.71 | -1.10 | 2.60 | 31    |
| 5.96     | 4.98  | 0.84 | -2.65 | 2.33 | 44    |
| 3.29     | 2.05  | 0.62 | -0.24 | 1.80 | 48    |
| 6.82     | 5.30  | 0.78 | -2.44 | 2.87 | 52    |
| 4.56     | 3.76  | 0.83 | -1.27 | 2.49 | 53    |
| 4.89     | 5.76  | 1.18 | -2.92 | 2.85 | 54    |
| 6.05     | 6.50  | 1.07 | -2.65 | 3.85 | 56    |
| 6.51     | 3.54  | 0.54 | -1.01 | 2.53 | 61    |
| 6.92     | 2.59  | 0.37 | -0.18 | 2.41 | 62    |
| 6.40     | 2.85  | 0.45 | -0.25 | 2.60 | 72    |
| 8.09     | 2.57  | 0.32 | -0.73 | 1.84 | 81    |

Table 6.8C reveals the distribution firms within the same industry. This table shows that there is substantial heterogeneity in the firms' distributions of elasticities. For example, the minimum elasticity values indicate, in all industries, that part of the firms' have complement effect in elasticity, while the maximum values point to the elasticity of substitution for some firms in the same industry. Also, the term ratio to standard deviation provides an indication of wide distribution among firms. Thus, the county has to keep firms with low elasticity of substitutions or complementarities to stabilize jobs in the region.

**3-Partial Elasticity between Labor and Land:** Table 6.8A shows that the partial elasticity between labor and land among industries is mixed. In this case, construction,

education services, and other services industries reveal that the elasticity between labor and land is complement, while in other industries the elasticity is substitute.

**Table 6.8D, Elasticity between Labor and Land  
in El Paso County**

| Range/SD | Range | SD   | Min   | Max  | NAICS |
|----------|-------|------|-------|------|-------|
| 6.84     | 7.54  | 1.10 | -3.62 | 3.92 | 23    |
| 4.99     | 6.58  | 1.32 | -3.85 | 2.73 | 31    |
| 6.34     | 7.22  | 1.14 | -3.28 | 3.94 | 44    |
| 3.63     | 7.90  | 2.18 | -3.90 | 4.00 | 48    |
| 6.80     | 6.85  | 1.01 | -3.38 | 3.47 | 52    |
| 3.39     | 7.86  | 2.32 | -3.89 | 3.97 | 53    |
| 6.13     | 5.40  | 0.88 | -2.09 | 3.31 | 54    |
| 6.49     | 6.21  | 0.96 | -3.53 | 2.68 | 56    |
| 5.98     | 5.59  | 0.93 | -3.52 | 2.07 | 61    |
| 4.70     | 7.58  | 1.61 | -3.91 | 3.67 | 62    |
| 10.84    | 6.37  | 0.59 | -3.63 | 2.74 | 72    |
| 7.74     | 7.12  | 0.92 | -3.46 | 3.66 | 81    |

Table 6.8D shows evidence of heterogeneity in the firms' elasticity within the industry. For instance, the minimum values of elasticities are negative in all industries. This indicates that the elasticity among part of the firms is complement. On the other hand, the elasticity appears to be substitute between land and labor when the maximum values of elasticity are taken into consideration. Thus, the distribution of elasticities among firms is very wide which requires attractive policies for firms with low elasticity of substitution or complementarities.

The distribution of the firms' elasticity as shown in table 6.8D is very wide, as measured by the ratio of the range relative to the standard deviations. In addition, there

is substantial heterogeneity among industries in the previous ratio. This ratio ranges between 3.39 in real estate and 10.84 in lodge and restaurants industry.

### 6.4.3. Denver County:

The analysis of the elasticity whether substitutes or complement is discussed for industries and firms in Denver County. The partial elasticities between labor and capital, capital and land, and land and labor will be depicted.

**1-Partial Elasticity between Labor and Capital:** The partial elasticity between the primary inputs, labor and capital, is substitute. The data in table 6.9A supports this direction. In addition, the table shows that this partial elasticity deviates from unity. Thus, the elasticity is not of standard Cobb-Douglas function. Also the table shows that there is heterogeneity in this elasticity among industries. In all industries, the elasticity of substitution between labor and capital is less than unity except for health services (1.43).

**Table 6.9A, Translog Elasticity by Industry  
in Denver County.**

| NAICS | $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{LLA}$ |
|-------|----------------|-----------------|----------------|
| 23    | 0.71           | 0.82            | -0.87          |
| 31    | 0.46           | 0.84            | -2.1           |
| 42    | 0.96           | 0.86            | -0.53          |
| 44    | 0.94           | 0.83            | -0.82          |
| 54    | 0.6            | 0.89            | -1.58          |
| 62    | 1.43           | 0.78            | 0.25           |
| 72    | 0.62           | 0.81            | -1.35          |
| 81    | 0.85           | 0.62            | 0.59           |

**Table 6.9B, Elasticity of Substitution between Capital and Labor in Denver County**

|    |      |      |      |      |       |
|----|------|------|------|------|-------|
| 23 | 1.68 | 0.41 | 0.14 | 1.27 | 9.39  |
| 31 | 2.7  | 0.03 | 0.36 | 2.67 | 7.33  |
| 42 | 1.56 | 0.65 | 0.11 | 0.9  | 7.93  |
| 44 | 2.81 | 0.14 | 0.25 | 2.67 | 10.73 |
| 54 | 1.56 | 0.04 | 0.16 | 1.52 | 9.77  |
| 72 | 2.19 | 0    | 0.33 | 2.19 | 6.72  |
| 81 | 2.09 | 0.71 | 0.16 | 1.38 | 8.42  |

**Table 6.9C, Elasticity between Capital and Land in Denver County**

| NAICS | Max  | Min   | SD   | Range | Range/SD |
|-------|------|-------|------|-------|----------|
| 23    | 1.23 | -0.07 | 0.32 | 1.3   | 4.13     |
| 31    | 2.17 | -0.03 | 0.4  | 2.2   | 5.51     |
| 42    | 1.88 | -0.03 | 0.34 | 1.91  | 5.69     |
| 44    | 2.87 | -0.05 | 0.4  | 2.92  | 7.38     |
| 54    | 1.89 | -0.02 | 0.35 | 1.91  | 5.47     |
| 62    | 2.47 | -0.2  | 0.37 | 2.66  | 7.27     |
| 72    | 1.21 | -0.14 | 0.33 | 1.35  | 4.11     |
| 81    | 1.94 | 0     | 0.33 | 1.94  | 5.86     |

**2-Partial Elasticity between Land and Capital:** Table 6.9A shows, on average, that the partial elasticity between land and capital is positive and less than 1. This means that the elasticity, in general, in all industries is substitute. On other side, table 6.9C shows that the minimum elasticity between these inputs is negative but near zero. Thus, part of the firms is operating within Leontief's assumption of no substitution between land and capital inputs. While the maximum values of elasticity show that the elasticity is substitute in all industries with elasticity greater than unity.

The distribution of these elasticities is substantially large as measured by the ratio of range to standard deviation. Therefore, the county has to attract firms with small elasticity in absolute value to lessen job losses during economic down turns.



**3-Partial Elasticity between Land and Labor:** Table 6.9A, on average, shows that elasticity between land and labor is substitute. While table 6.9D shows that part of the firms operate with negative elasticity and the maximum values are positive (means that part of the firms are operating within positive elasticities).

**Table 6.9D, Elasticity between Labor and Land  
in Denver County**

| Range/SD | Range | SD   | Min   | Max  | NAICS |
|----------|-------|------|-------|------|-------|
| 19.24    | 5.41  | 0.28 | -1.49 | 3.92 | 23    |
| 3.77     | 4.05  | 1.08 | -3.77 | 0.28 | 31    |
| 9.12     | 6.16  | 0.68 | -3.82 | 2.34 | 42    |
| 11.31    | 6.52  | 0.58 | -3.88 | 2.65 | 44    |
| 10.71    | 6.21  | 0.58 | -3.36 | 2.85 | 54    |
| 3.93     | 2.86  | 0.73 | -2.31 | 0.55 | 62    |
| 9.09     | 6.02  | 0.66 | -2.74 | 3.29 | 72    |
| 2.97     | 5.05  | 1.70 | -3.93 | 1.12 | 81    |

The distribution of the elasticity among firms in the same industry is remarkable as shown by the ratio of range relative to standard deviations. The highest value in this ratio is 19.24 in construction while the smallest value is 2.97 in other services industry. These huge differences in the ratio for the firms within the same industry or among industries requires adoption of long range plans from the county to attract firms with small elasticity in absolute value.

**6.4.4. Larimer County:** Tables 6.10A to 6.10D depict the type of elasticity between the three input variables; capital, labor, and land. The partial elasticity among these inputs, at the firm level or industry level, is expected to be either substitute or complement (Hicks, 1970). These partial elasticities are illustrated below.

**Table 6.10A, Translog Elasticity by Industry in Larimer County.**

| NAICS | $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{L,LA}$ |
|-------|----------------|-----------------|-----------------|
| 23    | 0.41           | 0.59            | -0.22           |
| 31    | 0.78           | 0.89            | -0.53           |
| 42    | 0.86           | 0.78            | -0.81           |
| 44    | 0.81           | 0.64            | 1.44            |
| 51    | 1.16           | 0.75            | 0.32            |
| 52    | 0.73           | 0.83            | -0.8            |
| 53    | 1.42           | 0.64            | 0.64            |
| 62    | 0.63           | 1.01            | -1.5            |
| 71    | 0.83           | 0.97            | -0.67           |
| 72    | 0.74           | 0.67            | -0.74           |
| 81    | 0.94           | 0.61            | 1.06            |

**1-Partial Elasticity between Capital and Labor:** Hicks (1932, 1970) said that the relation between primary inputs, labor and capital, is substitute. With the third or more input, the elasticity may be substitute or complement. On average, in Larimer County, the partial elasticity is substitute among all industries, and the elasticity is deviated from unity. The table shows drastic differences in the elasticity among industries. This elasticity ranges between 0.41 in construction industry and 1.42 in real estate industry. While table 6.10B, depicts the minimum elasticity is near zero at the firm level in three industries; construction, education services, and lodge and restaurants. On the other hand, the maximum values show evidence that part of the firms enjoy a high elasticity of substitution. This high elasticity is expected to exacerbate job loss during economic down turns. Thus, for the benefit of the region, the county has to attract the firms with low elasticity of substitution between labor and capital.

**Table 6.10B, Elasticity of Substitution between Capital and Labor in Larimer County.**

| Range/SD | Range | SD   | Min  | Max  | NAICS |
|----------|-------|------|------|------|-------|
| 5.20     | 1.43  | 0.28 | 0.04 | 1.47 | 23    |
| 6.45     | 2.20  | 0.34 | 0.45 | 2.65 | 31    |
| 6.31     | 0.65  | 0.10 | 0.45 | 1.10 | 42    |
| 5.66     | 1.09  | 0.19 | 0.37 | 1.46 | 44    |
| 4.07     | 1.07  | 0.26 | 0.54 | 1.61 | 51    |
| 2.60     | 0.42  | 0.16 | 0.58 | 1.00 | 52    |
| 4.55     | 2.36  | 0.52 | 0.22 | 2.58 | 53    |
| 5.29     | 2.92  | 0.55 | 0.07 | 2.99 | 62    |
| 4.90     | 0.28  | 0.06 | 0.73 | 1.01 | 71    |
| 4.61     | 2.59  | 0.56 | 0.01 | 2.61 | 72    |
| 7.94     | 1.07  | 0.13 | 0.37 | 1.44 | 81    |

**2-Partial Elasticity between Capital and Land:** On average, table 6.10A shows that the elasticity between land and capital is substitute in all industries in Larimer County. While at the firm level, table 6.10C shows that the elasticity type within the same industry is mixed. For instance, the minimum values of elasticity demonstrate that part of the firms have negative elasticity (complementarities), while the maximum value provides substitute elasticity. Thus, there is heterogeneity in the elasticity between the firms within the same industry. Therefore, to attract the firms with low elasticity in absolute value, the county has to offer incentives to mitigate inside effects of job losses during economic down turns.

**Table 6.10C, Elasticity between Capital and Land in El Paso County.**

| Range/SD | Range | SD   | Min   | Max  | NAICS |
|----------|-------|------|-------|------|-------|
| 6.04     | 2.00  | 0.33 | 0.00  | 2.00 | 23    |
| 6.25     | 3.21  | 0.51 | 0.00  | 3.21 | 31    |
| 3.02     | 1.80  | 0.60 | -0.31 | 1.49 | 42    |
| 8.59     | 3.99  | 0.46 | -0.43 | 3.56 | 44    |
| 2.71     | 0.94  | 0.35 | 0.00  | 0.93 | 51    |
| 4.30     | 1.16  | 0.27 | 0.00  | 1.15 | 52    |
| 3.86     | 1.20  | 0.31 | -0.18 | 1.01 | 53    |
| 5.25     | 2.37  | 0.45 | -0.64 | 1.72 | 62    |
| 5.58     | 1.78  | 0.32 | 0.00  | 1.78 | 71    |
| 10.38    | 5.07  | 0.49 | -1.61 | 3.45 | 72    |
| 7.73     | 2.47  | 0.32 | -0.47 | 2.00 | 81    |

In addition, table 6.10C shows the distribution of the partial elasticity between land and capital at the firm level. According to the concept of range to standard deviation, there is a tremendous heterogeneity between firms in the same industry and among industries. In this context, the relative range to standard deviation is between 2.7 in information industry and 10.38 in lodge and restaurants industry.

**3-Partial Elasticity between Land and Labor:** The results of partial elasticity between labor and land are mixed either at industry or firm level. For example, at the industry level part of the industries has negative values, while other industries have positive elasticities. The industries with positive elasticities are: retail trade (1.44), information (0.32), real estate (0.64), and other services (1.06) industries. At the firm level, table 6.10D shows that some firms have negative elasticity between labor and land in all industries, while the maximum values show positive elasticities. Thus the county has to be aware of which firms or industries to attract to Larimer County.

**Table 6.10D, Elasticity between Labor and Land in Larimer County.**

| Range/SD | Range | SD   | Min   | Max   | NAICS |
|----------|-------|------|-------|-------|-------|
| 5.89     | 3.84  | 0.65 | -2.85 | 0.99  | 23    |
| 6.88     | 2.67  | 0.39 | -2.11 | 0.55  | 31    |
| 3.43     | 8.82  | 2.57 | -3.82 | 5.00  | 42    |
| 3.67     | 5.62  | 1.53 | -0.77 | 4.85  | 44    |
| 4.13     | 5.70  | 1.38 | -3.05 | 2.65  | 51    |
| 3.90     | 1.21  | 0.31 | -1.22 | -0.01 | 52    |
| 5.16     | 7.52  | 1.46 | -3.45 | 4.07  | 53    |
| 4.57     | 8.58  | 1.88 | -3.62 | 4.96  | 62    |
| 5.41     | 3.75  | 0.69 | -1.01 | 2.74  | 71    |
| 4.56     | 4.93  | 1.08 | -3.92 | 1.01  | 72    |
| 5.67     | 7.41  | 1.31 | -2.66 | 4.76  | 81    |

#### **6.4.5. Boulder County:**

Tables 6.11A to 6.11D show that the elasticity between labor and capital is substitute. The values of elasticity at the industry level are greater than 1 in health

services and other services industries. The minimum value of elasticity of substitutions between labor and capital is less than 1. In addition, the elasticity between land and capital is positive and less than 1, and it is zero in other services. Table 6.11C shows that the minimum elasticity between land and capital is zero. Thus, the change in capital requires the same proportion change in land for either expanding or shrinking the project. To attract such firms, the local government has to provide incentives for industries with low elasticity of substitutions.

**Table 6.11A, Translog Elasticity by Industry in Boulder County.**

| NAICS | $\sigma_{K,L}$ | $\sigma_{K,LA}$ | $\sigma_{L,LA}$ |
|-------|----------------|-----------------|-----------------|
| 54    | 0.54           | 0.88            | -4.18           |
| 62    | 2.21           | 0.72            | 1.39            |
| 81    | 1.78           | 0               | 0               |

Table 6.11A and table 6.11D show evidence that some firms are operating at high elasticity of substitutions or complementarities. Thus, local governments have to attract firms with low elasticity of substitutions or complementarities.

**Table 6.11B, Elasticity of Substitution between Capital and Labor in Boulder County.**

| NAICS | Max  | Min  | SD   | Range | Range/SD |
|-------|------|------|------|-------|----------|
| 54    | 1.67 | 0.16 | 0.15 | 1.51  | 10.02    |
| 62    | 3.16 | 0.6  | 0.82 | 2.55  | 3.11     |
| 81    | 2.81 | 0.75 | 1.46 | 2.06  | 1.41     |

**Table 6.11C, Elasticity between Capital and Land  
in Boulder County.**

| NAICS | Max  | Min | SD   | Range | Range/SD |
|-------|------|-----|------|-------|----------|
| 54    | 2.15 | 0   | 0.35 | 2.15  | 6.11     |
| 62    | 0.93 | 0   | 0.39 | 0.93  | 2.41     |

**Table 6.11D, Elasticity between Labor and Land  
in Boulder County.**

| NAICS | Max  | Min   | SD   | Range | Range/SD |
|-------|------|-------|------|-------|----------|
| 54    | 2.29 | -5.99 | 1.42 | 8.28  | 5.82     |
| 62    | 2.86 | 0     | 1.05 | 2.86  | 2.72     |
| 81    | 0    | -0.01 | 0    | 0.01  | 1.41     |

**6.4.6. Weld County:** In Weld County 4 industries are under study for the elasticity of substitutions as they appear in tables 6.12A-6.12D. On average, the elasticity of substitution is less than 1 and zero in wholesale trade. Also the elasticity between capital and land is negative in all industries, table 6.12A.

**Table 6.12A, Translog Elasticity by Industry in  
Weld County.**

| $\sigma_{L,LA}$ | $\sigma_{K,LA}$ | $\sigma_{K,L}$ | NAICS |
|-----------------|-----------------|----------------|-------|
| 0.00            | -0.17           | 0.13           | 23    |
| 0.06            | -0.18           | 0.52           | 31    |
| -0.03           | -0.05           | 0.01           | 42    |
| 0.01            | -0.25           | 0.48           | 48    |

**Table 6.12B, Elasticity of Substitution between Capital and  
Labor in Weld County.**

| Range/SD | Range | SD   | Min   | Max  | NAICS |
|----------|-------|------|-------|------|-------|
| 11.77    | 0.26  | 0.02 | -0.09 | 0.18 | 23    |
| 4.85     | 0.94  | 0.19 | 0.02  | 0.96 | 31    |
| 3340.64  | 1.28  | 0.00 | -1.27 | 0.01 | 42    |
| 5.22     | 0.98  | 0.19 | 0.00  | 0.98 | 48    |

The distribution of the elasticities among firms in the same industry is very wide, tables 6.12B-6.12D. Thus, the counties have to attract firms with low elasticity whether substitute or complementarities.

**Table 6.12C, Elasticity between Capital and Land in Weld County.**

| Range/SD | Range | SD   | Min   | Max  | NAICS |
|----------|-------|------|-------|------|-------|
| 10.68    | 2.36  | 0.22 | -2.15 | 0.21 | 23    |
| 4.19     | 0.55  | 0.13 | -0.39 | 0.16 | 31    |
| 7.65     | 0.27  | 0.04 | -0.21 | 0.06 | 42    |
| 5.17     | 2.13  | 0.41 | -2.11 | 0.02 | 48    |

**Table 6.12D, Elasticity between Labor and Land in Weld County.**

| Range/SD | Range | SD   | Min   | Max  | NAICS |
|----------|-------|------|-------|------|-------|
| 8.69     | 0.88  | 0.10 | -0.09 | 0.79 | 23    |
| 3.65     | 0.11  | 0.03 | 0.02  | 0.13 | 31    |
| 8.32     | 1.70  | 0.20 | -1.27 | 0.44 | 42    |
| 5.28     | 0.10  | 0.02 | 0.00  | 0.10 | 48    |

## 6.5. Summary

In this chapter, the study computes partial elasticity of substitutions for three input variables according to Hicks (1932, 1970). The partial elasticities are computed at the firm level in each industry for each county included in the study. The summary and policy implications only reflect the average of the partial elasticity for the firms in the industry. These partial elasticities include; the partial elasticity between labor and capital, the partial elasticity between land and capital, and partial elasticity of land and labor. These elasticities will be illustrated according to non-homogeneous production functions because the elasticity is unity in standard Cobb-Douglas production function.

## **1-Non-homogeneous Production Function**

### **a- Elasticity between Capital and Labor**

On average, the elasticity between capital and labor is substitute in all industries in each county under study. The results show that the elasticity between labor and capital deviates from unity, i.e., is not standard Cobb-Douglas function. In addition, the elasticity between these two inputs is greater than 1. This means that if the wage rate is increased by 10%, the capital will substitute labor by more than 10%, and in some cases it will be double, 20% as in construction in Arapahoe County.

Furthermore, there is heterogeneity in the elasticity of substitutions between capital and labor among industries in the same county. For instance, in Arapahoe, the elasticity ranges between 1.01 in health services and 1.99 in construction. While in Denver, it ranges between 0.52 in manufacturing and 2.01 in other services, and lodge and restaurants industries.

### **b- Elasticity between Capital and Land**

The elasticity between land and capital is complement in most industries among the counties, except in wholesale trade, information, and education services which is substitute. This means that the employment of these two factors of production in most industries moves in the same direction. Also, the elasticity is less than unity, which means that the elasticity between these two factors is deviated from standard Cobb-Douglas function.



Furthermore, there is heterogeneity in the elasticity between these two factors of production in the same county among different industries. For instance, the elasticity in most of the industries is less in Arapahoe County, and it moves around zero in 7 industries in Arapahoe County. This means that the elasticity between land and capital is Leontief in these industries. Thus, the policy implication recommends that the counties have to encourage such industries to mitigate the economic fluctuations during economic down turns because of the relation between land and labor as it appears in the next section.

#### c- The Elasticity between Labor and Land

On average, the elasticity between labor and land is zero in all industries in El Paso and Denver counties. This means that there is Leontief relation between these two inputs. That is to reduce the usage of labor during economic down turns, the industries have to reduce the use of land in the same proportion. The main policy implication is to encourage such industries to mitigate job reduction during tough economic periods. In addition, the elasticity between land and labor in Arapahoe County is complement and less than unity. This elasticity, in Arapahoe County, range is -0.01 in arts, recreation, and entertainment and -0.92 in construction. Thus, Arapahoe County has to encourage firms and industries with low elasticity of substitution or complements to reduce the fluctuations in jobs during economic down turns, for instance, the firms that are operating in manufacturing, wholesale trade, art, recreation, and entertainment, lodge and restaurants, and other services (except public services).

### 6.5.1. Capital-Labor Ratio

In general, table 6.13A shows that the capital labor ratio in Arapahoe County is the highest in all industries compared to the corresponding industries in other counties. Also, the table shows that there are substantial differences among industries within the same county, especially in the high services industry. These industries include information, finance and insurance, real estate and releasing, professional and scientist, and technical services industry. These industries are capital intensive. Thus, on average, it is expected that it is too costly for firms working in these industries to terminate their labor force. Therefore, these industries are expected to possess the most stable jobs, in particular during inverse economic periods. Other industries, like construction, manufacturing and wholesale industries show evidence that these industries are not capital intensive.

**Table 6.13A, Capital-Labor Ratio by County and Industry.**

| Weld     | Boulder  | Larimer  | Denver   | El Paso  | Arapahoe |       |
|----------|----------|----------|----------|----------|----------|-------|
| 27500.0  | 68367.3  | 22922.8  | 23831.5  | 27648.8  | 79005.5  | 23    |
| 42462.3  | 45076.6  | 51494.3  | 29358.4  | 25000.0  | 83333.3  | 31    |
| 39062.5  | 84397.2  | 47589.4  | 43325.8  | 48467.8  | 105155.2 | 42    |
| 36842.1  | 41121.5  | 50184.8  | 91055.6  | 48780.5  | 65897.1  | 44    |
| 40004.6  | 33677.8  | 39678.0  | 69684.2  | 80476.7  | 88036.1  | 48    |
| 157981.8 | 99397.6  | 23819.6  | 52304.1  | 43366.3  | 82053.0  | 51    |
| 40737.2  | 54182.0  | 97293.2  | 141584.2 | 24748.3  | 304323.1 | 52    |
| 81155.4  | 143867.9 | 117796.6 | 364583.3 | 191936.9 | 720730.4 | 53    |
| 94871.8  | 98575.1  | 64747.6  | 135555.6 | 59680.8  | 395996.5 | 54    |
| 52754.5  | 194446.2 | 58293.8  | 13067.9  | 41721.2  | 290524.7 | 55    |
| 27283.2  | 40837.7  | 19081.0  | 21957.5  | 39576.6  | 141633.1 | 56    |
| 39311.9  | 64519.4  | 38024.0  | 72315.0  | 5761.0   | 154197.1 | 61    |
| 34682.1  | 38888.9  | 41182.4  | 57180.0  | 27131.8  | 137201.6 | 62    |
| 13237.4  | 16042.6  | 30156.3  | 39522.1  | 39130.4  | 71452.4  | 71    |
| 52807.6  | 40220.4  | 38922.0  | 37335.2  | 50751.7  | 93040.3  | 72    |
| 52551.0  | 28677.8  | 105508.5 | 85003.5  | 55555.6  | 357469.5 | 81    |
| 38722.8  | 57932.6  | 47442.6  | 55547.6  | 38135.1  | 176844.4 | Total |

**Table 6.13B, Capital-Labor Ratio by County and Industry**  
**(Standardized by Total Average of Each County)**

| Weld | Boulder | Larimer | Denver | El Paso | Arapahoe |       |
|------|---------|---------|--------|---------|----------|-------|
| 0.7  | 1.2     | 0.5     | 0.4    | 0.7     | 0.4      | 23    |
| 1.1  | 0.8     | 1.1     | 0.5    | 0.7     | 0.5      | 31    |
| 1.0  | 1.5     | 1.0     | 0.8    | 1.3     | 0.6      | 42    |
| 1.0  | 0.7     | 1.1     | 1.6    | 1.3     | 0.4      | 44    |
| 1.0  | 0.6     | 0.8     | 1.3    | 2.1     | 0.5      | 48    |
| 4.1  | 1.7     | 0.5     | 0.9    | 1.1     | 0.5      | 51    |
| 1.1  | 0.9     | 2.1     | 2.5    | 0.6     | 1.7      | 52    |
| 2.1  | 2.5     | 2.5     | 6.6    | 5.0     | 4.1      | 53    |
| 2.5  | 1.7     | 1.4     | 2.4    | 1.6     | 2.2      | 54    |
| 1.4  | 3.4     | 1.2     | 0.2    | 1.1     | 1.6      | 55    |
| 0.7  | 0.7     | 0.4     | 0.4    | 1.0     | 0.8      | 56    |
| 1.0  | 1.1     | 0.8     | 1.3    | 0.2     | 0.9      | 61    |
| 0.9  | 0.7     | 0.9     | 1.0    | 0.7     | 0.8      | 62    |
| 0.3  | 0.3     | 0.6     | 0.7    | 1.0     | 0.4      | 71    |
| 1.4  | 0.7     | 0.8     | 0.7    | 1.3     | 0.5      | 72    |
| 1.4  | 0.5     | 2.2     | 1.5    | 1.5     | 2.0      | 81    |
| 1.0  | 1.0     | 1.0     | 1.0    | 1.0     | 1.0      | Total |

### **6.6. Capital-Land Ratio**

Capital-land intensity measures the amount of capital in dollars relative to area of land by square feet. The higher this indicator is the more capital needed per square foot to invest in and the most intensive investment sector. In general, table 6.14A indicates that high services industry like information, finance and insurance, real estate and leasing, professional, scientist, and technical services, and management of companies and enterprises industries have the highest ratio in all counties.

**Table 6.14A, Capital-Land Ratio by County and Industry.**

| Weld | Boulder | Larimer | Denver | El Paso | Arapahoe |       |
|------|---------|---------|--------|---------|----------|-------|
| 0.9  | 95.4    | 3.0     | 17.1   | 2.7     | 61.2     | 23    |
| 5.2  | 54.5    | 6.9     | 14.6   | 3.6     | 57.1     | 31    |
| 1.9  | 93.0    | 6.9     | 13.9   | 6.6     | 61.8     | 42    |
| 8.4  | 53.9    | 8.2     | 21.2   | 10.1    | 65.5     | 44    |
| 3.3  | 66.6    | 5.3     | 6.9    | 1.3     | 33.8     | 48    |
| 19.3 | 122.7   | 14.2    | 140.9  | 21.1    | 81.0     | 51    |
| 10.7 | 111.4   | 16.7    | 32.3   | 11.5    | 86.6     | 52    |
| 7.7  | 44.8    | 10.5    | 58.0   | 3.1     | 112.1    | 53    |
| 5.8  | 115.8   | 9.2     | 44.0   | 9.8     | 74.3     | 54    |
| 7.6  | 143.1   | 25.4    | 24.7   | 11.9    | 81.6     | 55    |
| 4.0  | 93.1    | 11.1    | 23.1   | 2.8     | 76.6     | 56    |
| 13.8 | 103.8   | 18.4    | 17.5   | 6.1     | 67.7     | 61    |
| 14.0 | 94.5    | 15.1    | 28.2   | 14.8    | 79.7     | 62    |
| 0.0  | 40.0    | 1.4     | 1.8    | 0.9     | 55.3     | 71    |
| 20.5 | 66.5    | 6.2     | 31.8   | 5.3     | 98.0     | 72    |
| 1.0  | 75.6    | 7.3     | 35.0   | 10.1    | 80.8     | 81    |
| 1.7  | 79.2    | 6.6     | 19.4   | 4.2     | 78.7     | Total |

**Table 6.14B, Capital-Land Ratio by County and Industry.**  
(Standardized by Total Average of Each County)

| Weld | Boulder | Larimer | Denver | El Paso | Arapahoe |       |
|------|---------|---------|--------|---------|----------|-------|
| 0.6  | 1.2     | 0.5     | 0.9    | 0.6     | 0.8      | 23    |
| 3.1  | 0.7     | 1.0     | 0.8    | 0.8     | 0.7      | 31    |
| 1.2  | 1.2     | 1.1     | 0.7    | 1.6     | 0.8      | 42    |
| 5.1  | 0.7     | 1.2     | 1.1    | 2.4     | 0.8      | 44    |
| 2.0  | 0.8     | 0.8     | 0.4    | 0.3     | 0.4      | 48    |
| 11.7 | 1.5     | 2.2     | 7.3    | 5.0     | 1.0      | 51    |
| 6.5  | 1.4     | 2.5     | 1.7    | 2.7     | 1.1      | 52    |
| 4.6  | 0.6     | 1.6     | 3.0    | 0.7     | 1.4      | 53    |
| 3.5  | 1.5     | 1.4     | 2.3    | 2.3     | 0.9      | 54    |
| 4.6  | 1.8     | 3.9     | 1.3    | 2.8     | 1.0      | 55    |
| 2.4  | 1.2     | 1.7     | 1.2    | 0.7     | 1.0      | 56    |
| 8.3  | 1.3     | 2.8     | 0.9    | 1.4     | 0.9      | 61    |
| 8.4  | 1.2     | 2.3     | 1.5    | 3.5     | 1.0      | 62    |
| 0.0  | 0.5     | 0.2     | 0.1    | 0.2     | 0.7      | 71    |
| 12.3 | 0.8     | 0.9     | 1.6    | 1.2     | 1.2      | 72    |
| 0.6  | 1.0     | 1.1     | 1.8    | 2.4     | 1.0      | 81    |
| 1.0  | 1.0     | 1.0     | 1.0    | 1.0     | 1.0      | Total |

### 6.7. The Relation between K/L Ratio and Elasticity of Substitution between

#### Labor and Capital:

The analysis of capital-labor ratio and varies elasticity of substitutions are confined for three counties. The elasticity of substitutions in all tables reflects the average elasticities of the firms in each industry. As theoretically known, the higher the capital-labor ratio is, the less job loss is expected at the firm level or at the industry level, on average. In particular, when a high capital-labor ratio is accompanied by low elasticity of substitution, the loss in jobs will be less during economic down turns (Nicholson, 2005). In addition, the loss in local government is expected to be less under the previous circumstances. The analysis of this issue will be by county at the industry level, and within industry among counties.

## 6.7.1. Within County and among Industries

### 6.7.1.1. Arapahoe County:

This part is based on table 6.15A. The table shows that high services industry characterized by high K/L ratio; 52, 53, 54, and 55. But the corresponding elasticities are greater than one according to non-homogeneous Cobb-Douglas function, and inelastic according to translog production function. Thus, the county has to estimate the appropriate production function for each firm, and then select the firms that are characterized by low elasticity of substitution between capital and labor accompanied by high capital-labor ratio.

**Table 6.15A, Elasticity and K/L Ratio in Arapahoe County.**

| (K/L) Ratio<br>(Standardized*) | Elasticities of Substitution |                |    |
|--------------------------------|------------------------------|----------------|----|
|                                | Translog                     | Nonhomogeneous |    |
| 0.45                           | 0.86                         | 1.99           | 23 |
| 47                             | 0.78                         | 1.21           | 31 |
| 0.59                           | 0.81                         | 1.07           | 42 |
| 0.37                           | 0.89                         | 1.35           | 44 |
| 0.50                           | 1.27                         |                | 48 |
| 0.46                           | 1.07                         | 0.51           | 51 |
| 1.72                           | 0.83                         | 1.21           | 52 |
| 4.08                           | 0.94                         | 1.05           | 53 |
| 2.24                           | 0.66                         | 1.46           | 54 |
| 1.64                           |                              | 1.79           | 55 |
| 0.80                           | 0.94                         | 1.45           | 56 |
| 0.87                           | 0.90                         | 1.82           | 61 |
| 0.78                           | 0.76                         | 1.01           | 62 |
| 0.40                           |                              | 1.13           | 71 |
| 0.53                           | 1.24                         | 1.66           | 72 |
| 2.02                           | 0.91                         | 1.59           | 81 |

\* Standardized by dividing the average of industry K/L ratio by Average total industries K/L ratio in the county.

### 6.7.1.2. El Paso County:

Table 6.15B shows that the elasticity of substitutions computed by non-homogeneous Cobb-Douglas and translog production functions. Also, the table shows

the corresponding K/L ratio at the industry level. In El Paso County, most of the industries are characterized by high capital-labor ratio. The corresponding computed elasticities of substitution are inelastic according to translog function. But by non-homogeneous function, the elasticities of substitution, on average, are greater than unity. The exception is in wholesale trade industry which has high K/L ratio and inelastic elasticity of substitutions between labor and capital.

**Table 6.15B, Elasticity and K/L Ratio in El Paso County.**

| (K/L) Ratio<br>(Standardized*) | Elasticities of Substitution |                |    |
|--------------------------------|------------------------------|----------------|----|
|                                | Translog                     | Nonhomogeneous |    |
| 0.73                           | 0.64                         | 1.65           | 23 |
| 0.66                           | 0.47                         | 1.25           | 31 |
| 1.27                           |                              | 0.72           | 42 |
| 1.28                           | 0.74                         | 1.56           | 44 |
| 2.11                           | 0.49                         | 1.38           | 48 |
| 1.14                           |                              | 1.11           | 51 |
| 0.65                           | 0.49                         | 1.55           | 52 |
| 5.03                           | 0.68                         | 1.23           | 53 |
| 1.56                           | 0.56                         | 1.24           | 54 |
| 1.09                           |                              |                | 55 |
| 1.04                           | 0.64                         | 1.44           | 56 |
| 0.15                           | 0.77                         |                | 61 |
| 0.71                           | 0.62                         | 1.50           | 62 |
| 1.03                           |                              |                | 71 |
| 1.33                           | 0.61                         | 1.55           | 72 |
| 1.46                           | 0.64                         | 1.25           | 81 |

\* Standardized by dividing the average of industry K/L ratio by Average total industries K/L ratio in the county.

### 6.7.1.3. Denver County:

On average, in eight industries, the K/L ratio intensity is high. These industries include most of the high services industries (real estate, professional, scientist, and technical services, and finance and insurance). In addition, retail trade, transportation, education services, and other services have intensive capital –labor ratio. On the other

hand, on average, the elasticities of substitutions between labor and capital for such industries are elastic except in transportation industry. Thus, to attract firms that are characterized by high K/L ratio and low elasticities of substitution between labor and capital, the local government in El Paso County has to introduce incentives for such firms at the beginning of their operations.

**Table 6.15C, Elasticity and K/L Ratio in Denver County.**

| (K/L) Ratio<br>(Standardized*) | Elasticities of Substitution |                |    |
|--------------------------------|------------------------------|----------------|----|
|                                | Translog                     | Nonhomogeneous |    |
| 0.43                           | 0.71                         | 1.46           | 23 |
| 0.53                           | 0.46                         | 0.52           | 31 |
| 0.78                           | 0.96                         | 1.35           | 42 |
| 1.64                           | 0.94                         | 1.22           | 44 |
| 1.25                           |                              | 0.65           | 48 |
| 0.94                           |                              | 0.68           | 51 |
| 2.55                           |                              | 1.40           | 52 |
| 6.56                           |                              | 1.15           | 53 |
| 2.44                           | 0.60                         | 1.46           | 54 |
| 0.24                           |                              |                | 55 |
| 0.40                           |                              | 1.79           | 56 |
| 1.30                           |                              | 1.39           | 61 |
| 1.03                           | 1.43                         | 1.49           | 62 |
| 0.71                           |                              |                | 71 |
| 0.67                           | 0.62                         | 2.01           | 72 |
| 1.53                           | 0.85                         | 2.01           | 81 |

\* Standardized by dividing the average of industry K/L ratio by Average total industries K/L ratio in the county.

### **6.7.2. Within Industry and Among County:**

In this part, the analysis of elasticity of substitution and capital intensity will be based on the level of industry and among 3 counties; Arapahoe, El Paso, and Denver.

#### **6.7.2.1. Construction Industry:**

The following table shows that capital-labor ratio is less than the average of total private sector industries in each county. Also, the table shows that the elasticity of



substitution is greater than unity as in homogeneous Cobb-Douglas. This means that during reverse economic periods this industry will highly impact the jobs in all the three regions. As a result, the local governments have to follow selective policies to adopt the firms with low elasticity of substitutions with high K/L ratios.

**Table 6.16, the Average Elasticity of Substitution or Complementarity And K/L ratio in Construction Industry by County**

| Nonhomogeneous | Translog | K/L Ratio | Counties |
|----------------|----------|-----------|----------|
| 1.99           | 0.86     | 0.45      | Arapahoe |
| 1.65           | 0.64     | 0.73      | El Paso  |
| 1.46           | 0.71     | 0.43      | Denver   |

#### **6.7.2.2. Manufacturing Industry:**

In this industry, the K/L ratio, on average, is low, while the elasticities of substitutions between capital and labor are low in the translog, and elastic in non-homogeneous Cobb-Douglas. Thus, counties have to adopt selective policies to attract firms to operate in their regions for the expected benefit from the firms that are characterized by low elasticity of substitution and high intensive K/L ratio.

**Table 6.17, the Average Elasticity of Substitution or Complementarity And K/L ratio in Manufacturing Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Nonhomogeneous       | Translog | K/L Ratio | Counties |
| 1.21                 | 0.78     | 0.47      | Arapahoe |
| 1.25                 | 0.47     | 0.66      | El Paso  |
| 0.52                 | 0.46     | 0.53      | Denver   |

#### **6.7.2.3. Wholesale Trade Industry:**

In El Paso County, the capital-labor ratio is high, while the elasticity of substitutions among these input variables is less than unity. Thus, the wholesale trade

in El Paso County is expected to have small negative impact on job loss and local government revenues. Therefore, it is to the benefit of El Paso County to encourage such industry to operate in its region.

**Table 6.18, the Average Elasticity of Substitution or Complementarity And K/L ratio in Wholesale Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Nonhomogeneous       | Translog | K/L Ratio | Counties |
| 1.07                 | 0.81     | 0.59      | Arapahoe |
| 0.72                 |          | 1.27      | El Paso  |
| 1.35                 | 0.96     | 0.78      | Denver   |

**6.7.2.4. Retail Trade Industry:**

In this industry, the K/L ratio is highly intensive in El Paso and Denver Counties but with high elasticity of substitutions. On the other hand, in Arapahoe, the K/L ratio is low with high elasticity of substitution.

**Table 6.18, the Average Elasticity of Substitution or Complementarity And K/L ratio in Retail Trade Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Nonhomogeneous       | Translog | K/L Ratio | Counties |
| 1.35                 | 0.89     | 0.37      | Arapahoe |
| 1.56                 | 0.74     | 1.28      | El Paso  |
| 1.22                 | 0.94     | 1.64      | Denver   |

**6.7.2.5. Transportation and Warehousing:**

The following table shows that the K/L ratio is high for transportation and warehousing industry in El Paso and Denver Counties. But the elasticity of substitution

between labor and capital is low in Denver according to non-homogeneous function. On the other hand, the elasticity of substitution is low in El Paso County according to translog function. Therefore, these two counties have to adopt selective policies for firms operating in their regions to mitigate job loss and local government revenues.

**Table 6.19, the Average Elasticity of Substitution or Complementarity And K/L ratio in Transportation Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Nonhomogeneous       | Translog | K/L Ratio | Counties |
|                      | 1.27     | 0.50      | Arapahoe |
| 1.38                 | 0.49     | 2.11      | El Paso  |
| 0.65                 |          | 1.25      | Denver   |

**6.7.2.6. Information Industry:**

For El Paso County, the K/L ratio and elasticity are high. While in Denver, the K/L ratio is unitary with low elasticity of substitution in information industry. Thus, this industry in Denver County is better than in El Paso County because of less inverse impacts expected during economic down turns.

**Table 6.20, the Average Elasticity of Substitution or Complementarity And K/L ratio in Information Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 0.51                 | 1.07     | 0.46      | Arapahoe |
| 1.11                 |          | 1.14      | El Paso  |
| 0.68                 |          | 0.94      | Denver   |

### 6.7.2.7. Finance and Insurance Industry:

The K/L ratio is highly intensive in Arapahoe and Denver Counties. The corresponding elasticity of substitution is low according to translog function, but greater than unity in non-homogeneous function. Therefore, Arapahoe and Denver Counties have to adopt selective policies to attract firms with low elasticities and high capital-labor intensity to mitigate job loss during inverse economic periods.

**Table 6.21, the Average Elasticity of Substitution or Complementarity  
And K/L ratio in Financial Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Nonhomogeneous       | Translog | K/L Ratio | Counties |
| 1.21                 | 0.83     | 1.72      | Arapahoe |
| 1.55                 | 0.49     | 0.65      | El Paso  |
| 1.40                 |          | 2.55      | Denver   |

### 6.7.2.8. Real Estate and Leasing:

This industry operates with high intensity of K/L ratio in all counties. Also, the elasticities of substitutions, on average, are around unity. This type of industry is good for the region with these economic characteristics to lessen the inverse impact of economic down turns periods in jobs and local governments' revenues.

**Table 6.22, the Average Elasticity of Substitution or Complementarity  
And K/L ratio in Real Estate Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.05                 | 0.94     | 4.08      | Arapahoe |
| 1.23                 | 0.68     | 5.03      | El Paso  |
| 1.15                 |          | 6.56      | Denver   |

**6.7.2.9: Professional, Scientist, and Technical Services Industry:**

In this industry K/L ratio is highly intensive in the three counties but the elasticity of substitution between labor and capital is high and greater than one according to non-homogeneous Cobb-Douglas function. While the elasticity between labor and capital is low according to translog function. Therefore, the counties have to study each firm's production function. If translog is the dominant function, then the local government has to work toward keeping such firms operating in their regions.

**Table 6.23, the Average Elasticity of Substitution or Complementarity  
And K/L ratio in Professional and scientist Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.46                 | 0.66     | 2.24      | Arapahoe |
| 1.24                 | 0.56     | 1.56      | El Paso  |
| 1.46                 | 0.60     | 2.44      | Denver   |

### 6.7.2.10. Management of Companies and Enterprises:

In this industry, only Arapahoe County is different than standard Cobb-Douglas. Also, the K/L ratio and the elasticity of substitution are high as shown in the following table.

**Table 6.24, the Average Elasticity of Substitution or Complementarity And K/L ratio in management of Companies Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.79                 |          | 1.64      | Arapahoe |
|                      |          | 1.09      | El Paso  |
|                      |          | 0.24      | Denver   |

### 6.7.2.11. Administrative and Waste Management:

The K/L ratio is around unity in Arapahoe and El Paso Counties, while it's low in Denver County. Furthermore, according to translog function, the elasticity of substitution is inelastic for El Paso County and near unity in Arapahoe. But, according to non-homogeneous functions, the elasticity of substitution is high. Thus, the counties have to adopt selective policies based on the firms' production function and the corresponding K/L ratio.

**Table 6.25, the Average Elasticity of Substitution or Complementarity**

**And K/L ratio in Administration and Waste Management Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.45                 | 0.94     | 0.80      | Arapahoe |
| 1.44                 | 0.64     | 1.04      | El Paso  |
| 1.79                 |          | 0.40      | Denver   |

### 6.7.2.12. Education Services:

Because of the purpose of mitigating job loss during reverse economic periods, the education services in Denver seem to be better than in the other two counties. The main reason for that is the high capital-labor ratio and high elasticity of substitutions, according to non-homogeneous function, between capital and labor in this county. On the other hand, in other counties, the elasticity of substitution is high while the K/L ratio is lower than in Arapahoe.

**Table 6.26, the Average Elasticity of Substitution or Complementarity And K/L ratio in Education Services Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.82                 | 0.90     | 0.87      | Arapahoe |
|                      | 0.77     | 0.15      | El Paso  |
| 1.39                 |          | 1.30      | Denver   |

### 6.7.2.13. Health Care Services Industry:

According to translog function, the elasticity of substitutions is low between labor and capital, but the K/L ratio is low. On the other hand, the elasticity of substitutions is high according to non-homogeneous function. Thus, each county has to study the type of production function for each firm to submit incentives for firms with high capital intensity and low elasticity of substitutions between capital and labor to save jobs in the region.

**Table 6.27, the Average Elasticity of Substitution or Complementarity And K/L ratio in Health Care Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.01                 | 0.76     | 0.78      | Arapahoe |
| 1.50                 | 0.62     | 0.71      | El Paso  |
| 1.49                 | 1.43     | 1.03      | Denver   |

#### 6.7.2.14. Art, Entertainment, and Recreation Industry:

In this industry, standard Cobb-Douglas is applied for El Paso and Denver Counties. In addition, the industry has low K/L ratio with high elasticity of substitution. This situation will lead to more job loss during reverse economic periods.

**Table 6.28, the Average Elasticity of Substitution or Complementarity And K/L ratio in Recreation Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.13                 |          | 0.40      | Arapahoe |
|                      |          | 1.03      | El Paso  |
|                      |          | 0.71      | Denver   |

#### 6.7.2.15. Lodge and Restaurants:

The following table shows that the lodge and restaurants industry is characterized as follows: In El Paso County, high K/L ratio with high elasticity of substitutions between labor and capital, according to non-homogeneous Cobb-Douglas function. On the other hand, the K/L ratio is low and the elasticity is high, according to non-homogeneous function. The latter result is not desirable because of expected high job loss in reverse economic periods.

**Table 6.29, the Average Elasticity of Substitution or Complementarity And K/L ratio in Lodge Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.66                 | 1.24     | 0.53      | Arapahoe |
| 1.55                 | 0.61     | 1.33      | El Paso  |
| 2.01                 | 0.62     | 0.67      | Denver   |



The translog function is highly substituted in Arapahoe County with low K/L ratio, which is not desirable for the reasons mentioned previously. While in El Paso County the K/L ratio is high with low elasticity of substitution. This result is preferred to other results because it helps to mitigate job loss during reverse economic periods.

**6.4.2.16. Other Services (Except Public Services):**

The following table shows high K/L ratio with low elasticity of substitution according to translog function. However, the elasticity of substitution is high according to non-homogeneous Cobb-Douglas function. Thus, any county has to select which production function that best fits the data structure for the firms. And then accordingly can attract the firms with high K/L ratio with low elasticity of substitutions such as in translog functions.

**Table 6.30, the Average Elasticity of Substitution or Complementarity And K/L ratio in Other Services Industry by County**

| Average Elasticities |          |           |          |
|----------------------|----------|-----------|----------|
| Non-homogeneous      | Translog | K/L Ratio | Counties |
| 1.59                 | 0.91     | 2.02      | Arapahoe |
| 1.25                 | 0.64     | 1.46      | El Paso  |
| 2.01                 | 0.85     | 1.53      | Denver   |

## **Chapter Seven: Total Factor Productivity (TFP)**

The purpose of this chapter is to explore the differences in TFP among industries in the same county, and within industries among the counties. The main reasons regarding this approach are the importance of total factor productivity in identifying the technological progress, the impact of TFP on economic growth at the macroeconomic level, and the output growth at the firm and industry levels. In addition, the chapter will quantify TFP and examine whether the differences in TFP are statistically significant to provide policy advice for decision makers regarding which industry to attract to operate in their regions. Also, TFP affects the revenue of local governments and the spending behavior for the revenue. In addition, it is expected to affect employment and output growth through competition and the ability to export, in particular during economic down turns. In this chapter both effects of Porter (1989), and Jacobs (1969) regarding TFP will be explored. Also, the convergence of TFP within industry and among counties, and within counties and among industries will be explored (Bernard and Jones, 1996).

### **7.1. Introduction**

In the economic literature, there are three types of total factor productivity. The first type is the Hicks total factor productivity. In this type, as a result of technological progress, the whole production function shifts up, increasing the productivity of all factors of production in the same proportion. The second type of TFP is related to labor savings or increasing the labor productivity. In economic literature, this is referred to as the Harrod's neutral effect. This means that to produce the same amount of output less labor is needed. The third type is concerns the

changes in productivity as related to the change in capital productivity. This type is prominent in the literature as Solow technical change. In addition, the economic literature has many studies on the effect of externality factors on productivity such as MAR effect (Marshall, Arrow, and Romer), Porter's effect, and Jacobs's effect.

The importance of studying total factor productivity or technical progress stems from its impact on economic growth (Chow et al, 2002). Also, many studies tackle the impact of technological progress on business cycle; most studies focus on the impact of technology in business cycles (Kydland and Prescott, 1982). In their study, they show that productivity shock can produce business cycle compatible to U.S. economy. In addition, studying TFP is important in the regional and country economies in explaining the growth and catch up in the economy between poor areas and rich areas (Bernard and Jones, 1996).

In this chapter, the total factor productivity is assumed Hicks neutral. The study of Bernard and Jones (1996) will be taken as a benchmark to compute TFP. The Bernard and Jones study assumes constant returns to scale in a standard Cobb-Douglas function. The study includes 6 broad economic sectors for 14 OECD countries. In their paper, the authors argue that there is convergence in technological progress, measured as TFP. This convergence is examined within industry among the 14 OECD countries, and within country among different industries.

In the study, they find substantial heterogeneity of productivity movement across sectors in the same country. In the movement within industry and among countries, they find convergence in TFP in services and utility sectors, but not in manufacturing. The technological convergence is expected to happen because of technological transfer from advanced to less advanced countries. Also, it is expected that international trade fosters the transfer of technology, as a consequence,

manufacturing has to be more convergence (Bernard and Jones 1996). The authors rely on the following general formula to compute TFP in country  $i$ , industry  $j$  at time  $t$ .

$$TFP_{ijt} = \frac{Y}{L_{ijt}^\alpha K_{ijt}^{1-\alpha}} \dots \dots \dots (7.1)$$

For computing the differences among countries in the same economic sector, they calculate TFP according to the following formula:

$$\hat{A}_{ijt} = \frac{A_{1jt}}{A_{ijt}} \dots \dots \dots (7.2)$$

Where:  $\hat{A}_{ijt}$ : Ratio indicator of total factor productivity of country 1 ( the benchmark country with the highest TFP ) to the TFP of another country  $i$ . Or in the log form:

$$\hat{A}_{ijt} = A_{1jt} - A_{ijt} \dots \dots \dots (7.3)$$

In this chapter, formula (7.3) will be used to compute the significance of TFP differences among industries in the counties incorporated in this study, and in the same industries among counties.

As a result of economic externalities, and of competition within the economic sector, it is expected that Porter's effect (1989) will dominate within the same industry, i.e., the TFP could be assumed equal among different industries in a specific county from one side, and assumes the same TFP for the same industry in different counties in Colorado. This can be due to easy mobility of labor and proximity of the counties in Colorado, and therefore, easy dissemination of knowledge in the same industry. The main argument is that a higher TFP in an industry means producing the same level of output by fewer inputs, or more production and export by the same level of inputs. Under competition assumption, the wage rates reflect the increase in productivity of labor. Thus, higher income for labor eventually will be reflected in the

expenditures and tax revenues for the city or county. Also, Jacobs' effect will be explored; knowledge spillover among different industries. Thus TFP is expected to be the same among different industries in each county. This happens in cities because of proximity and easy moving of labor among economic sectors. In addition, this chapter attempts to explain the possible impact of different TFP among industries within a county, and the difference in TFP within an industry among counties.

#### **7.1.1. The Potential Impacts of Differences in TFP within a County and among Industries**

Assume county A with two producing industries X and Y, and that TFP for industry X is twice that of industry Y. Also, assume all other factors are the same, such as the production function and input variable coefficients. In this case, with the same amount of inputs, the output will be expected to be twice as much in industry X than in industry Y. This means that industry X is more efficient in using resources than industry Y, highly productive both in labor and capital, is expected to export to other counties or regions, and may increase employment. On the other hand, Y industry needs twice the amount of inputs to produce the same as X industry output; the mark-up prices may be higher, wages are expected to be less in Y industry than X industry, and accordingly, local government income is expected to be less from industry Y production.

#### **7.1.2. The Potential Impact of TFP within Industry and among Counties**

Assume two counties A and B with industry X common in both county economies. Leaving all other factors the same in both counties, TFP in industry X in county A is double that of industry X in county B. Then county A produces twice

that of industry X in county B using the same amount of factors of production. This means that county A is more efficient in producing X than B. This provides relative advantage for county A and may export to county B. Also, wages in county A are expected to be higher because of high labor productivity compared to B. Then, the local government income will be higher in A than B which allows for A to raise the quality of existing projects and execute new ones.

The rest of the chapter is organized as follows: the second section examines the convergence, Jacobs' effect hypotheses. Also, convergence and Porter's effect will be explored. This effect expects that there are no differences in TFP within the same industry among the counties under study in Colorado. The third section illustrates the relation between TFP and RTS. This section depicts this relation in six counties under study. The fourth section will be summary and conclusions.

## **7.2. Exploring Convergence, Jacobs', and Porter's Effects**

Dekle (2002) studied the impact of dynamic externalities of Jacobs and Porter, through TFP, on the growth of TFP in Japanese prefectures. The study includes the nine main industries at one-digit level. The Jacobs' effect concerns the prediction that TFP among industries located in a region and highly diversified should grow faster. In this regard, Dekle examines the growth in TFP for all industries for all prefectures. The author finds a negative and significant coefficient for the initial TFP, which concludes the presence of  $\beta$  convergence. At the industry level, the author finds the absence of Jacobs' effect in all industries.

While, Porter argues that specialization and competition within the same industry enhances innovation and the dissemination of ideas and knowledge spillover. The knowledge spillover is a result of proximity, free labor movement among jobs and industries, and openness. As a consequence, the knowledge and

innovations will spread among industries. The knowledge spillover will affect TFP in the industries. Thus, as a result of dissemination of this information, the TFP within the industry is expected to be the same among industry in the same county.

In Dekle's study, the author divides dynamic changes in TFP into 4 parts as in the following equation:

$$gr(A_{t-1,t}) = g(\rho_0, D_0, C_0, (IC)) \dots\dots\dots (7.4)$$

Where:  $\rho_0$ = Concentration,  $D_0$  = Diversity,  $C_0$ =Competition, and  $IC$ = Initial Conditions.

The main results are: In finance, wholesale and retail trade, and services industries the Jacobs' effect do not exist.

On the other side, Porter's effect (represented by  $C_0$ ) shows no existence in finance industry. But in services, and wholesale and retail trade industries some Porter's effect is evident.

In addition, the analysis will rely on Bernard's and Jones' (1996) paper for computing the TFP. But due to the proximate magnitude and insignificant difference of TFP in both cases, either estimated statistically or computed arithmetically there is no difference. The analysis of TFP in these parts depends on the estimated constants for different production functions.

**7.2.1. Convergence and Jacobs' Effect**

In this section, convergence and the Jacobs' effect hypotheses regarding TFP are explored among industries in the same county. Jacobs' effect expects that total factor productivity is not statistically different in the same county and among different industries due to knowledge spillover among counties. Also, exploring the convergence hypothesis of TFP relies on the significance of differences in TFP between the construction industry TFP and other industries' TFP in the same

county. This difference in TFP is expected to be statistically insignificant because of easy mobility of labor force and technology transfer. Bernard and Jones (1996) test stationary TFP between any two sectors to explore convergence hypothesis. According to the authors, it is expected that the differences among industries in TFP is stationary to be convergence. But this study depends on snapshot data. Therefore, it is expected that the difference in TFP is insignificant among industries to test either convergence or Porter's effect hypotheses. The investigation of this hypothesis is examined according to the three different types of production functions; namely, Cobb-Douglas, non-homogeneous Cobb-Douglas, and translog production functions.

The Jacobs' effect analysis on this part depends on tables 7.1 Cobb-Douglas estimation, 7.2 non-homogeneous Cobb-Douglas, and 7.3 translog function. In each county, TFP of construction industry is chosen as a benchmark to examine the significance of differences in TFP in any industry in the county, i.e.,  $\ln TFP_i - \ln TFP_c$ , and the TFP in the construction industry. The TFP of construction industry is chosen as benchmark because of its estimated significance in all counties and in all types of production functions studied.

#### **7.2.1.1. Cobb-Douglas:**

Table 7.1 raises several points. In general, the data in this table shows evidence of supporting Jacobs' effect and TFP convergence in most of the industries in each county. This means, that TFP differences in most industries are not statistically different than TFP of construction industry. In addition, although, the table shows there are common industries which have TFP deviating significantly from TFP of construction industry, such as professional, scientist, and technical services; other



services (except government); and education services, but the table reveals that there is an industry effect in each county. The following demonstrates the industry specific in each county. The study relies on t-ratio to test for the significance in TFP differences.

**Arapahoe County:** The results in Arapahoe County are mixed. On the one hand, there is support to convergence and Jacobs' effect in 10 industries, which show no significant differences in each industry TFP compared to construction industry TFP. On the other hand, there are five industry TFP's that deviate significantly from the construction TFP. For instance, the education services, and other services TFP are lower than construction TFP and this difference is significant at 99% level. While TFP in professional, scientist, and technical services, and health care services are significant at 95% level. This means that there is divergence in these industries and deviation from Jacobs' effect.

**El Paso County:** The TFP in all industries in El Paso county shows no significant differences compared to TFP in construction industry. Thus, this is highly supportive of convergence and Jacobs' effect in all industries in El Paso without exception. That is the difference in TFP between industries and construction is not statistically significant.

**Denver County:** Table 7.1 shows that there is convergence and Jacobs' effect is partially applied for TFP in 5 industries. On other side, the TFP in 7 industries deviates significantly from TFP of construction.

**Larimer County:** Five industries show significant differences in TFP in Larimer County compared to the construction industry TFP. Other industries TFP show evidence of convergence and support Jacobs' effect.

**Boulder County:** In Boulder County, the TFP shows significant differences in 5 industries. Among them are the wholesale, administrative and waste management industries. The Jacobs' effect for TFP is supported in 10 industries in Boulder County, because the TFP in these industries is not statistically different than the TFP in construction industry.

**Weld County:** In Weld County, 9 industries TFP are not statistically different from the benchmark TFP of construction. The 4 industries that are statistically different in their TFP are the retail trade, lodge and restaurants, other services, and professional, scientist, and technical services industries.

#### **7.2.1.2. Non-homogeneous Cobb-Douglas:**

Health care services TFP deviates significantly from Jacobs' effect and diverge in four counties compared to TFP in construction industry. Also, Table 7.2 reveals that there is industry specific effect of TFP among industries in each county. The following part depicts such differences.

**Boulder County:** The TFP in Boulder County industries does not support Jacobs' effect in 4 industries, of which 3 are at 95% level. These industries are health care services, professional, scientist, and technical services, and wholesale trade. Otherwise, there is support of Jacobs' effect and convergence in TFP in the rest of the industries in Boulder County.

**Arapahoe County:** The TFP in non-homogeneous Cobb-Douglas function shows that 9 industries are not statistically different from TFP of construction

industry. Thus, there is support of convergence and the Jacobs' effect in TFP among the Arapahoe industries.

**Table 7.1, Estimated Total Factor productivity by County and Industry  
by Cobb-Douglas**

| County Industry | Arapahoe  | Denver    | El Paso  | Larimer  | Boulder  | Weld       |
|-----------------|-----------|-----------|----------|----------|----------|------------|
| 23              |           |           |          |          |          |            |
| 31-33           | -0.5417   | -1.4666   | 0.725    | -2.410** | -1.5377  | -0.7432    |
| t-ratio         | (-0.872)  | (-2.162)  | (1.312)  | (-3.862) | (-1.084) | (-1.087)   |
| 42              | -0.0692   | -1.1069   |          | -0.8143  | -3.70*** | -0.1675    |
| t-ratio         | (-0.175)  | (-1.826)  |          | (-1.368) | (-3.692) | (-0.152)   |
| 44-45           | -0.09622  | -1.292**  | 0.6908   | -0.5386  | 4.1146** | -2.7022*** |
| t-ratio         | (-0.239)  | (-2.3604) | (1.814)  | (-1.088) | (3.979)  | (-3.032)   |
| 48-49           | 1.6671    | -1.606**  | 1.4409*  | -0.9551  | 1.9765   | -0.2444    |
| t-ratio         | (1.492)   | (-2.760)  | (2.325)  | (-1.412) | (0.384)  | (-0.270)   |
| 51              | -0.3937   | -2.12     | 0.1995   | -0.8134  | -2.3595  |            |
| t-ratio         | (-0.609)  | (-1.798)  | (0.266)  | (-0.749) | (-1.197) |            |
| 52              | -0.6805*  | -2.086**  | 1.7678   | -0.8223  | -0.5934  | -1.2813    |
| t-ratio         | (-1.964)  | (-2.504)  |          | (-1.528) | (-0.383) | (-1.302)   |
| 53              | -1.3793   | -2.071    | -1.0515  | -2.184   | 0.2366   | -1.6255    |
| t-ratio         | (-0.298)  | (-0.347)  | (-0.199) | (-0.522) | (0.042)  | (-0.282)   |
| 54              | -0.674**  | -0.9984   | -0.5671  | -3.05*** | -1.6797* | -1.7322**  |
| t-ratio         | (-2.032)  | (-1.597)  |          | (-5.296) | (-2.050) | (-2.370)   |
| 55              | 0.3151    | 0.5074    | 1.8124   | -1.0332  | -3.9978  |            |
| t-ratio         | (0.684)   | (0.1025)  | (0.626)  | (-1.879) | (-0.627) |            |
| 56              | -0.6811   | -0.1387   | -0.0842  | -1.3139  | -3.79*** | -0.6204    |
| t-ratio         | (-1.664)  | (-0.154)  | (-0.171) | (-1.515) | (-3.148) | (-0.380)   |
| 61              | -2.09***  | -3.95***  | 1.123    | -4.98*** | 0.2655   | -3.2865    |
| t-ratio         | (-3.144)  | (-5.7451) | (1.203)  | (-6.136) | (0.1358) | (-1.717)   |
| 62              | -1.066**  | -1.80***  | 1.816    | 0.2373   | -0.068   | -1.1644    |
| t-ratio         | (-2.862)  | (-3.138)  |          | (0.3958) | (-0.057) | (-1.226)   |
| 71              | -0.7769   | -2.047**  | 0.616    | 0.1302   | 1.9961   | -2.5262    |
| t-ratio         | (-1.115)  | (-2.173)  |          | (0.1635) | (1.018)  | (-1.697)   |
| 72              | -0.5486   | 0.6478    | 0.3955   | -1.187** | -1.3611  | -4.4288*** |
| t-ratio         | (-1.3129) | (1.044)   |          | (-2.339) | (-1.058) | (-4.507)   |
| 81              | -1.24***  | -0.7393   | 1.2444   | -1.524** | 0.3802   | -2.2078**  |
| t-ratio         | (-2.974)  | (-1.156)  |          | (-2.697) | (0.312)  | (-2.344)   |

t-ratio is in Parentheses

\*\*\* Significant at 99%

\*\* significant at 95%

\* significant at 90%

**Table 7.2, Estimated Non-homogeneous Cobb-Douglas Function of Total Factor Productivity By County and Industry**

| County Industry | Arapahoe  | Denver    | El Paso    | Larimer  | Boulder  | Weld     |
|-----------------|-----------|-----------|------------|----------|----------|----------|
| 23              |           | -         |            |          |          |          |
| 31-33           | -4.7483   | 16.1765** | 13.157***  | -1.3823  | -27.365* | 7.1745   |
| t-ratio         | (-1.221)  | (-2.861)  | (3.176)    | (-0.281) | (-2.016) | (1.995)  |
| 42              | -0.5433   | -0.5468   | -3.7615    | -3.1613  | -26.69** | 6.9619   |
| t-ratio         | (-0.2637) | (-0.112)  | (-0.947)   | (-0.663) | (-2.152) | (0.853)  |
| 44-45           | -4.1006   | 0.9136    | 6.1629**   | 0.3666   | -21.6412 | -0.9682  |
| t-ratio         | (-2.051)  | (0.234)   | (2.364)    | (0.095)  | (-1.675) | (-0.143) |
| 48-49           | -1.3642   | -5.2429   | 7.1199     | -5.7199  |          | 20.8823  |
| t-ratio         | -0.3178   | -1.2727   | 2.044986   | -1.1174  |          | 3.080474 |
| 51              | -1.8003   | 41.9515   | 13.35**    | -0.6155  | -8.554   |          |
| t-ratio         | (-0.542)  | (0.747)   | (2.501)    | (-0.069) | (-0.493) |          |
| 52              | -1.9957   | 10.241    | 9.2113***  | -2.3518  | -10.0854 |          |
| t-ratio         | (-1.254)  | (1.528)   | (2.779)    | (-0.548) | (-0.424) |          |
| 53              | -3.7183   | 12.8978** | 1.2952     | 2.3086   | -21.4281 |          |
| t-ratio         | (-1.632)  | (2.053)   | (0.482)    | (0.442)  | (-1.392) |          |
| 54              | -5.68***  | 5.9666    | 4.0457     | 0.1172   | -23.27** | -6.833   |
| t-ratio         | (-3.673)  | (1.225)   | (1.221)    | (0.028)  | (-2.347) | (-1.774) |
| 55              | 2.7957    |           | 22.4916    |          | -252.132 |          |
| t-ratio         | (0.754)   |           | (0.2948)   |          | (-1.043) |          |
| 56              | -3.5067   | 6.4549    | 0.9259     | -0.9065  | -6.8843  | -9.2852  |
| t-ratio         | (-1.746)  | (0.995)   | (0.287)    | (-0.122) | (-0.498) | (-0.936) |
| 61              | -13.73*** | -16.36*** | -9.8586    | -14.274  | 55.0197  | 40.7317  |
| t-ratio         | (-3.531)  | (-2.938)  | (-1.21)    | (-1.165) | (1.307)  | (0.963)  |
| 62              | -5.605*** | 6.754     | 12.359***  | 9.3136*  | -33.59** | 15.3889  |
| t-ratio         | (-2.978)  | (1.348)   | (3.167)    | (1.919)  | (-2.701) | (1.805)  |
| 71              | -8.7253   | -17.5803* | -6.9953    | -9.877*  | -12.5031 |          |
| t-ratio         | (-1.738)  | (-1.977)  | (-1.281)   | (-2.036) | (-0.493) |          |
| 72              | -5.3784** | -2.9397   | 4.3093     | 1.9615   | -23.978  | 10.9309  |
| t-ratio         | (-2.516)  | (-0.641)  | (1.657)    | (0.501)  | (-1.810) | (0.708)  |
| 81              | -5.964*** | 4.9186    | 11.8286*** | 2.3448   | -8.6788  | -4.6944  |
| t-ratio         | (-2.992)  | (1.046)   | (4.060)    | (0.578)  | (-0.602) | (-0.994) |

T-ratio is in Parentheses.

\*\*\* Significant at 99%.

\*\* Significant at 95%.

\* Significant at 90%.

**El Paso County:** In 7 industries the TFP does not support convergence and Jacobs' effect in El Paso County. Among these industries are manufacturing, finance and insurance, health services, and other services (except public services).

**Denver County:** The table shows evidence of 4 industries' TFP that varies from construction TFP. Otherwise, Jacobs' effect supports other industries in Denver County. The industries TFP that deviate from Jacobs' effect are education services at 99% level, manufacturing and real estate industries at 95% level, while art, entertainment, and recreation services industry is at 90% level.

**Larimer County:** At 90% level of significant, TFP does not support convergence and Jacobs' effect in two industries in Larimer County. These industries are health care services, and art, entertainment, and recreation services.

**Weld County:** The TFP shows evidence of none significant differences in all industries compared to TFP of construction. This simply means that Jacobs' effect of TFP in Weld County is applied for all non-homogeneous estimations. Also, these results support the convergence of TFP among industries.

#### **7.2.1.3. Translog Function:**

According to translog estimation, there is convergence in TFP and Jacobs' effect in El Paso, Larimer, Boulder, and Weld Counties. Also, the TFP supports the convergence and Jacobs' effect in the previous counties. In these counties, the difference in TFP

among industries compared to TFP in construction industry is statistically insignificant.

The TFP differences will be discussed in Arapahoe and Denver Counties.

**Table 7.3, Estimated Translog Total Factor Productivity by County and Among Industries**

| County Industry | Arapahoe  | Denver    | El Paso  | Larimer  | Boulder  | Weld     |
|-----------------|-----------|-----------|----------|----------|----------|----------|
| 23              |           |           |          |          |          |          |
| 31-33           | 1.5568    | 3.5681    | -0.5828  | 2.1131   | -25.0408 | -3.8718  |
| t-ratio         | (0.445)   | (0.623)   | (-0.189) | (0.534)  | (-0.769) | (-0.79)  |
| 42              | -3.489*   | 12.507**  | -2.6937  | 2.2932   | -20.285  | 36.96*   |
| t-ratio         | (-1.831)  | (2.834)   | (-0.544) | (0.558)  | (-1.283) | (2.570)  |
| 44-45           | -4.433**  | -2.5719   | -7.909** | -2.3776  | -18.5044 | -10.336  |
| t-ratio         | (-2.160)  | (-0.793)  | (-2.953) | (-0.945) | (-1.352) | (-1.12)  |
| 48-49           | 5.6222    | 0.0943    | 3.88     | -1.4224  |          | -2.425   |
| t-ratio         | (0.916)   | (0.016)   | (1.192)  | (-0.300) |          | (-0.36)  |
| 51              | -11.14*** | 94.8645   | 1.3766   | 17.0556  | 33.4535  |          |
| t-ratio         | (-4.468)  | (1.461)   | (0.272)  | (1.613)  | (1.019)  |          |
| 52              | -5.9343   | 4.89      | -3.6248  | -4.4854  | -14.0051 | 3.4363   |
| t-ratio         | (-3.613)  | (0.729)   | (-1.172) | (-1.542) | (-0.517) | (0.188)  |
| 53              | -7.113*** | 14.711*** | -1.7902  | 4.6625   | 21.5848  | -5.413   |
| t-ratio         | (-2.845)  | (2.751)   | (-0.595) | (1.200)  | (0.402)  | (-0.46)  |
| 54              | -8.873*** | 5.4975    | -3.8065  | 0.6932   | -17.9753 | -8.98    |
| t-ratio         | (-5.482)  | (1.361)   | (-1.181) | (0.181)  | (-1.789) | (-1.07)  |
| 56              | -5.113**  | 19.712*** | -5.9681  | -2.4062  | -22.1094 | -23.947  |
| t-ratio         | (-2.487)  | (4.178)   | (-1.749) | (-0.444) | (-1.424) | (-1.142) |
| 61              | -11.879*  | -5.3461   | -10.974* | -11.8542 | 38.3574  |          |
| t-ratio         | (-2.087)  | (-0.794)  | (-2.102) | (-0.635) | (1.181)  |          |
| 62              | -8.759*** | 8.507*    | -7.573*  | 4.9429   | -9.3589  | -2.7467  |
| t-ratio         | (-4.527)  | (2.066)   | (-2.241) | (1.497)  | (-0.588) | (-0.375) |
| 71              | -16.210** | -12.5649  | -5.0873  | 3.9246   | 42.0964  |          |
| t-ratio         | (-2.752)  | (-1.199)  | (-0.827) | (1.019)  | (0.614)  |          |
| 72              | -8.578*** | -0.9657   | -2.8821  | 1.7937   | -26.7456 | 9.803    |
| t-ratio         | (-3.798)  | (-0.290)  | (-1.102) | (0.693)  | (-1.382) | (0.453)  |
| 81              | -5.758*   | 0.7025    | -2.0731  | 0.8956   | -4.5633  | -11.91*  |
| t-ratio         | (-2.880)  | (0.198)   | (-0.729) | (0.321)  | (-0.363) | (-1.96)  |

t-ratio is in Parentheses.

\*\*\* Significant at 99%.

\*\* Significant at 95%.

\* Significant at 90%.

**Arapahoe County:** In Arapahoe County, according to translog estimation, the TFP in 9 industries deviates significantly from supporting Jacobs' effect, table 7.3. This means that different production functions reveal different results regarding the equality of TFP or testing Jacobs' effect or convergence in TFP among industries.

**Denver County:** The difference in TFP is significant at 99% level in two industries in Denver County; real estate, and administrative and waste management. Wholesale trade deviates from Jacobs' effect in TFP at 95% level. Otherwise, the TFP supports Jacobs' effect and convergence in other industries in Denver County

### **7.2.2. Exploring Convergence and Porter's Effect**

In this part convergence and Porter's effect hypotheses will be examined for this study. The convergence in TFP is considered across industries in the same county. Bernard and Jones (1996) find that there is heterogeneity in TFP among industries in OECD Countries. Also, the Porter's effect concerns spillover of knowledge within the same industry. That is TFP within industry and among proximate counties is expected to be the same. To examine convergence and Porter's effect in this section, the study assumes that the TFP of the industries in Arapahoe County to be the benchmark measurement of significance or insignificance differences in TFP within the same industry among different counties. In this case, if the difference in TFP is not statistically significant. This result supports convergence and Porter's effect in TFP.

In general, table 7.4 shows evidence of the Glaeser et al (1992) effect regarding the impact of city density in TFP, where density is expected to positively affect the TFP. In this regard, the following table provides evidence that 10 industries in Denver County have higher TFP compared to Arapahoe County industries. This difference in TFP is positive and statistically significant at least at 95% level except for real estate

industry. In El Paso County only 4 industries reveal that the TFP is higher, positive and significant, compared to TFP in Arapahoe County.

**Table 7.4, Population Density in Major Colorado**

**Counties**

| Density | Area | Population |          |
|---------|------|------------|----------|
| 712     | 803  | 572,003    | Arapahoe |
| 293     | 2126 | 622,263    | El Paso  |
| 3914    | 153  | 600,158    | Denver   |
| 115     | 2601 | 299,630    | Larimer  |
| 397     | 742  | 294,567    | Boulder  |
| 63      | 3992 | 252,825    | Weld     |

Source: Counties sites.

The rest of the section will explore the significant difference in TFP within the industry and among counties. Also this discussion will take into consideration the estimated three production functions. In the analysis, tables 7.5, 7.6, and 7.7 mainly will be used to explore convergence and Porter's effect hypotheses.



**Table 7.5, Estimated Total Factor productivity by County and Industry  
by Cobb-Douglas**

|         | Arapahoe | Denver   | El Paso   | Larimer  | Boulder   | Weld     |
|---------|----------|----------|-----------|----------|-----------|----------|
| 23      |          | 2.226*** | -0.6043   | 1.409**  | 1.7464**  | 1.265**  |
| t-ratio |          | (4.209)  | -1.37984  | (2.660)  | (2.438)   | (2.076)  |
| 31-33   |          | 1.302*   | 0.6624    | -0.4592  | 0.7504    | 1.0633   |
| t-ratio |          | (1.729)  | (0.937)   | (-0.653) | (0.5464)  | (1.532)  |
| 42      |          | 1.1887** | 0.7924    | 0.6643   | -1.8879** | 1.1665   |
| t-ratio |          | (2.407)  | (1.494)   | (1.3860) | (-2.342)  | (1.171)  |
| 44-45   |          | 1.031**  | 0.18272   | 0.967**  | 5.957***  | -1.342   |
| t-ratio |          | (2.421)  | (0.540)   | (2.728)  | (7.031)   | -1.754   |
| 48-49   |          | -1.0467  | -0.8305   | -1.2128  | 2.0558    | -0.6467  |
| t-ratio |          | (-0.915) | (-0.692)  | (-1.016) | (0.394)   | (-0.496) |
| 51      |          | 0.5001   | -0.0111   | 0.9897   | -0.2194   |          |
| t-ratio |          | (0.404)  | (-0.0125) | (0.863)  | (-0.1127) |          |
| 52      |          | 0.8211   | 1.844***  | 1.268*** | 1.8335    | 0.664    |
| t-ratio |          | (1.1238) | (4.289)   | (3.529)  | (1.296)   | (0.784)  |
| 53      |          | 1.5347*  | -0.2765   | 0.6047   | 3.3623**  | 1.0186   |
| t-ratio |          | (1.861)  | (-0.720)  | (1.107)  | (2.124)   | (1.027)  |
| 54      |          | 1.902*** | -0.4971   | -0.968** | 0.741     | 0.2069   |
| t-ratio |          | (4.045)  | (-1.393)  | (-2.409) | (1.429)   | (0.396)  |
| 55      |          | 2.4187   | 0.893     | 0.0611   | -2.5665   |          |
| t-ratio |          | (0.489)  | (0.308)   | (0.126)  | (-0.404)  |          |
| 56      |          | 2.769*** | -0.0074   | 0.7766   | -1.3628   | 1.3255   |
| t-ratio |          | (3.317)  | (-0.0159) | (0.972)  | (-1.297)  | (0.8451) |
| 61      |          | 0.372    | 2.616***  | -1.4767  | 4.1094*   | 0.0758   |
| t-ratio |          | (0.465)  | (2.467)   | (-1.626) | (2.121)   | (0.039)  |
| 62      |          | 1.488*** | 2.278***  | 2.713*** | 2.7448**  | 1.1668   |
| t-ratio |          | (3.418)  | (4.947)   | (5.816)  | (2.715)   | (1.427)  |
| 71      |          | 0.9565   | 0.7886    | 2.3165** | 4.5194**  | -0.4845  |
| t-ratio |          | (0.915)  | (0.815)   | (2.531)  | (2.315)   | (-0.317) |
| 72      |          | 3.423*** | 0.3398    | 0.7706*  | 0.9339    | -2.615** |
| t-ratio |          | (6.475)  | (0.864)   | (1.979)  | (0.814)   | (-2.982) |
| 81      |          | 2.728*** | 1.881***  | 1.126**  | 3.367***  | 0.2976   |
| t-ratio |          | (4.956)  | (3.952)   | (2.442)  | (3.153)   | (0.358)  |

t-ratio is in Parentheses

\*\*\* Significant at 99%

\*\* significant at 95%

\* significant at 90%

**Table 7.6, Estimated Non-homogeneous Cobb-Douglas Function of Total Factor Productivity By Industry and Among Counties**

| County   | Arapahoe | Denver     | El Paso   | Larimer   | Boulder  | Weld     |
|----------|----------|------------|-----------|-----------|----------|----------|
| Industry |          |            |           |           |          |          |
| 23       |          | 5.6366     | -5.1339*  | 0.8199    | 15.1386  | -7.881** |
| t-ratio  |          | (1.584)    | (-1.928)  | (0.215)   | (1.722)  | (-2.856) |
| 31-33    |          | -5.7916    | 12.7714** | 4.1859    | -7.4781  | 4.0414   |
| t-ratio  |          | (-0.987)   | (2.545)   | (0.838)   | (-0.677) | (0.894)  |
| 42       |          | 5.6331     | -8.352**  | -1.7981   | -11.0109 | -0.3762  |
| t-ratio  |          | (1.439)    | (-2.324)  | (-0.508)  | (-1.224) | (-0.047) |
| 44-45    |          | 10.6508*** | 5.1296**  | 5.2871*   | -2.402   | -4.749   |
| t-ratio  |          | (4.138)    | (2.664)   | (2.543)   | (-0.248) | (-0.731) |
| 48-49    |          | 1.7579     | 3.3502    | -3.5358   |          | 14.3651* |
| t-ratio  |          | (0.368)    | (0.692)   | (-0.644)  |          | (1.906)  |
| 51       |          | 49.3884    | 10.0166   | 2.0047    | 8.3849   |          |
| t-ratio  |          | (0.880)    | (1.758)   | (0.229)   | (0.548)  |          |
| 52       |          | 17.873***  | 6.0731**  | 0.4638    | 7.0489   |          |
| t-ratio  |          | (3.0317)   | (2.394)   | (0.183)   | (0.317)  |          |
| 53       |          | 22.2527    | -0.1204   | 6.8468    | -2.5712  |          |
| t-ratio  |          | (3.933)    | (-0.052)  | (1.612)   | (-0.200) |          |
| 54       |          | 17.2832*** | 4.5918    | 6.617**   | -2.4477  | -9.034*  |
| t-ratio  |          | (4.712)    | (1.831)   | (2.819)   | (-0.506) | (-2.915) |
| 55       |          |            | 14.562    |           | -239.789 |          |
| t-ratio  |          |            | (0.191)   |           | (-0.993) |          |
| 56       |          | 15.598**   | -0.7013   | 3.4201    | 11.761   | -13.6599 |
| t-ratio  |          | (2.698)    | (-0.259)  | (0.515)   | (1.085)  | (-1.404) |
| 61       |          | 3.0045     | -1.2627   | 0.2751    |          |          |
| t-ratio  |          | (0.5194)   | (-0.147)  | (0.022)   |          |          |
| 62       |          | 17.995***  | 12.829*** | 15.738*** | -12.8455 | 13.112   |
| t-ratio  |          | (4.504)    | (3.754)   | (4.429)   | (-1.428) | (1.583)  |
| 71       |          | -3.2184    | -3.4039   | -0.3321   | 11.3608  |          |
| t-ratio  |          | (-0.336)   | (-0.492)  | (-0.056)  | (0.468)  |          |
| 72       |          | 8.0753**   | 4.554**   | 8.1598*** | -3.461   | 8.4279   |
| t-ratio  |          | (2.249)    | (2.212)   | (3.511)   | (-0.341) | (0.549)  |
| 81       |          | 16.519***  | 12.659*** | 9.129***  | 12.4238  | -6.6118  |
| t-ratio  |          | (4.509)    | (5.461)   | (3.7479)  | (1.071)  | (-1.531) |

t-ratio is in Parentheses.

\*\*\* significant at 99%.

\*\* significant at 95%.

\* significant at 90%.

**Table 7.7, Estimated Translog Total Factor Productivity by Industry and Among Counties**

|         | Arapahoe | Denver    | El Paso  | Larimer  | Boulder  | Weld     |
|---------|----------|-----------|----------|----------|----------|----------|
| 23      |          | -4.9894   | -1.6336  | -4.6099  | 19.626** | 2.3812   |
| t-ratio |          | (-1.596)  | (-0.575) | (-1.731) | (2.538)  | (0.568)  |
| 31-33   |          | -2.9781   | -3.7732  | -4.0536  | -6.9716  | -3.0474  |
| t-ratio |          | (-0.502)  | (-1.016) | (-0.889) | (-0.219) | (-0.715) |
| 42      |          | 11.007*** | -0.8383  | 1.1723   | 2.83     | 42.8***  |
| t-ratio |          | -3.013    | (-0.187) | (0.320)  | (0.203)  | (3.084)  |
| 44-45   |          | -3.1279   | 5.109*** | -2.5541  | 5.555    | -3.516   |
| t-ratio |          | (-1.405)  | (-2.803) | (-1.377) | (0.483)  | (-0.417) |
| 48-49   |          | -10.5173  | -3.3758  | 11.654** |          | -5.6664  |
| t-ratio |          | (-1.308)  | (-0.584) | (-2.499) |          | (-1.056) |
| 51      |          | 101.0246  | 10.8925  | 23.595** | 64.229   |          |
| t-ratio |          | (1.556)   | (0.167)  | (2.102)  | (1.913)  |          |
| 52      |          | 5.8349    | 0.6759   | -3.161   | 11.5552  | 11.7518  |
| t-ratio |          | (0.949)   | (0.329)  | (-1.569) | (0.444)  | (0.658)  |
| 53      |          | 16.835*** | 3.6891   | 7.1655   | 48.324   | 4.0807   |
| t-ratio |          | (3.361)   | (1.372)  | (1.898)  | (0.908)  | (0.365)  |
| 54      |          | 9.381**   | 3.4331   | 4.9565   | 10.5239  | 2.2742   |
| t-ratio |          | (3.097)   | (1.544)  | (1.551)  | (1.591)  | (0.308)  |
| 56      |          | 19.836*** | -2.4884  | -1.9028  | 2.6299   | -16.442  |
| t-ratio |          | (4.851)   | (-0.890) | (-0.370) | (0.193)  | (-0.797) |
| 61      |          | 1.5435    | -0.7283  | -4.5851  | 69.8624* |          |
| t-ratio |          | (0.187)   | (-0.101) | (-0.237) | (2.179)  |          |
| 62      |          | 12.277*** | -0.4477  | 9.092*** | 19.0261  | 8.3935   |
| t-ratio |          | (3.714)   | (-0.168) | (3.309)  | (1.354)  | (1.329)  |
| 71      |          | -1.344    | 9.4894   | 15.525** | 77.9327  |          |
| t-ratio |          | (-0.115)  | (1.183)  | (2.384)  | (1.141)  |          |
| 72      |          | 2.6231    | 4.063**  | 5.762**  | 1.4586   | 20.7624  |
| t-ratio |          | (1.036)   | (2.064)  | (2.658)  | (0.081)  | (0.974)  |
| 81      |          | 1.4712    | 2.0514   | 2.0438   | 20.821*  | -3.7742  |
| t-ratio |          | (0.563)   | (1.025)  | (0.944)  | (2.064)  | (-0.781) |

t-ratio is in Parentheses.

\*\*\* Significant at 99%.

\*\* Significant at 95%.

\* Significant at 90%.

### **7.3.1. Construction Industry:**

The differences in TFP in construction industry will be explored through the estimated production functions among different counties.

**1- Cobb-Douglas:** The difference in TFP does not support convergence and Porter's effect hypotheses in construction industry. For instance table 7.4 reveals that the TFP in Denver County for construction industry is higher than TFP in construction industry in Arapahoe County by 2.226 points. This difference in TFP is significant at 99% level. In other counties TFP is significant at 95% level except for El Paso County which shows no significant differences in TFP in construction industry.

**2- Non-homogeneous Cobb-Douglas:** The difference in TFP in construction industry is significant and lower than the benchmark TFP in El Paso and Weld Counties. Other counties have no significance differences in TFP compared to benchmark TFP. In other words, the difference in TFP supports both convergence and Porter's effect hypotheses in Arapahoe, Denver, Larimer, and Boulder Counties.

**3- Translog Function:** Except for Boulder County, the difference in TFP among counties shows evidence of convergence and supports Porter's effect hypotheses.

### **7.3.2. Manufacturing Industry:**

In this industry different production functions provide different results in the significance of TFP among the counties under study. The following discussions according to production functions show such differences.

**1- Cobb-Douglas:** Only in Denver the difference in TFP is positive and significant at 90% level. Other counties, the difference in TFP is not significant. The results support convergence and TFP in Porter's effect hypotheses in Arapahoe County manufacturing industries.

**2- Non-homogeneous Cobb-Douglas:** In El Paso County only this industry reveals positive and significant differences in TFP compared to Arapahoe County TFP in manufacturing industry.

**3- Translog Function:** The translog function shows evidence that none of the counties have significant difference in TFP compared to Arapahoe County manufacturing industry. Thus, this industry TFP assumes convergence and Porter's effect according to translog estimation.

### **7.3.3 Wholesale Trade Industry:**

**1- Cobb-Douglas:** The TFP difference is positive and significant in Denver County compared to Arapahoe County. On the other hand, a TFP difference in Boulder County is significant but less than that in Arapahoe County. In other counties the difference in TFP in wholesale trade industry is statistically insignificant. Therefore, the difference in TFP in El Paso, Larimer, and Weld support convergence and Porter's effect hypotheses.

**2- Non-homogeneous Cobb-Douglas:** Only in El Paso County, the difference in TFP is significant and less than that of wholesale trade TFP in Arapahoe County. In other counties the difference in TFP is not statistically significant. Therefore, this result supports convergence and Porter's effect hypotheses in all counties except El Paso.

**3- Translog Function:** Only in Denver County, the difference in TFP is positive and significant. Otherwise, the TFP differences are not significant which supports the convergence and Porter's effect in these counties except Denver.

### **7.3.4. Retail Trade:**

**1- Cobb-Douglas:** In three counties (Denver, Larimer, and Boulder), the difference in TFP reveals positive and statistically significant compared to TFP in this industry in Arapahoe County.

**2- Non-homogeneous Cobb-Douglas:** In three counties (Denver, El Paso, and Larimer) there is evidence of positive and significant differences in TFP compared to TFP in retail industry in Arapahoe County. Thus, the convergence and Porter's effect hypotheses in TFP are not applied in this industry among the counties.

**3- Translog Function:** In this industry, only El Paso County shows negative significance in TFP compared to TFP of retail trade in Arapahoe County. Thus it is expected that the TFP in this industry supports convergence and Porter's effect hypotheses.

#### **7.3.5. Transportation and Warehousing Industry:**

**1- Cobb-Douglas:** The difference in TFP in this industry is insignificant in all counties. This suggests convergence and Porter's effect dominate in this industry among the counties.

**2- Non-homogeneous Cobb-Douglas:** The results support convergence and Porter's effect hypotheses in TFP among all counties in this industry because the differences in TFP in any county are not significant compared to TFP in this industry in Arapahoe County.

**3- Translog Function:** The difference in TFP is negative and significant in Larimer County. Otherwise, the results support convergence and Porter's effect hypotheses in TFP in other counties for this industry.

#### **7.3.6. Information Industry:**

**1- Cobb-Douglas:** Porter's effect is applied for all counties in this industry regarding the difference in TFP.

**2- Non-homogeneous Cobb-Douglas:** The results support both convergence and Porter's effect hypotheses for all counties in this industry regarding the difference in TFP.

**3- Translog Function:** Only in Larimer County, the difference in TFP is positive and significant. Otherwise, the results support convergence and Porter's effect hypotheses in TFP in this industry in other counties.

### **7.3.7. Finance and Insurance Industry:**

**1- Cobb-Douglas:** In this industry, the difference in TFP is positive and significant in El Paso and Larimer Counties. The other four counties show insignificant difference in TFP. Thus, the results in these counties support convergence and Porter's effect hypotheses in TFP.

**2- Non-homogeneous Cobb-Douglas:** The difference in TFP is significant and positive in both Denver and El Paso Counties.

**3-Translog Function:** According to translog function, the results support convergence and Porter's effect hypotheses in finance and insurance industry.

### **7.3.8. Real Estate Industry:**

**1- Cobb-Douglas:** The difference in TFP is positive and significant in Denver and Boulder Counties. While in other counties, the difference in TFP is insignificant. Therefore, the results support both convergence and Porter's effect hypotheses in real estate industry.

**2- Non-homogeneous Cobb-Douglas:** The results support convergence and Porter's effect hypotheses in TFP in all counties in real estate industry. This is because the difference in TFP is not significant in this industry in all counties.

**3- Translog Function:** Only in Denver County, this industry shows evidence of positive and significance in TFP differences. In other counties, the results support convergence and Porter's effect hypotheses.

#### **7.8.9. Professional, Scientist, and Technical Services Industry:**

**1- Cobb-Douglas:** The difference in TFP in this industry shows positive and significant in Denver County, while the difference in TFP in Larimer County is negative and significant. In other counties, the difference in TFP is not statistically significant. Thus, the results support convergence and Porter's hypotheses in El Paso, Boulder, Weld, and Arapahoe Counties.

**2- Non-homogeneous Cobb-Douglas:** Except in El Paso County, the difference in TFP is significant and positive in Denver and Larimer, while the difference in TFP is negative and significant in Boulder and Weld Counties. Thus, the results show no support of convergence and Porter's effect hypotheses in TFP in this industry and among counties.

**3- Translog Function:** The difference in TFP is positive and significant only in Denver County. Otherwise, the results support convergence and Porter's effect hypotheses in this function in other counties.

#### **7.3.10. Management of Companies and Enterprises:**

**1- Cobb-Douglas:** In this industry, the difference in TFP is statistically insignificant among all counties. Thus, Porter's effect is expected to dominate within this industry and among all counties.

**2- Non-homogeneous Cobb-Douglas:** Non-homogeneous estimation is not applied for discussion because of already insignificant estimation regarding this industry and among these counties.



**3- Translog Function:** Translog estimation is not applied for discussion because of already insignificant estimation regarding this industry and among these counties.

#### **7.3.11. Administrative and Waste Management Services Industry:**

**1- Cobb-Douglas:** Only in Denver the difference in TFP is significant and positive. While in other counties, the TFP is expected to support convergence and Porter's effect hypotheses.

**2- Non-homogeneous Cobb-Douglas:** The difference in TFP is significant and positive in Denver County. Otherwise, TFP supports convergence and Porter's effect hypotheses.

**3- Translog Function:** The difference in TFP is significant and positive in Denver County. Otherwise, the difference in TFP supports convergence and Porter's effect hypotheses.

#### **7.3.12. Education Services:**

**1- Cobb-Douglas:** The difference in TFP is significant and positive in Boulder and El Paso Counties compared to the TFP in education services in Arapahoe County. The other counties are expected to have no significant difference in TFP. Thus, these counties TFP supports both convergence and Porter's Effect hypotheses (Denver, Larimer, Boulder, and Arapahoe Counties).

**2- Non-homogeneous Cobb-Douglas:** The difference in TFP is insignificant for Denver, El Paso, and Larimer. Therefore, TFP is expected to be similar in these counties. As a consequence, the results support convergence and Porter's effect hypotheses.

**3- Translog Function:** None of the counties' TFP is significantly different from the TFP of Arapahoe County. Thus, the results support the convergence and Porter's effect hypotheses in the counties of Arapahoe, Denver, El Paso, and Larimer.

#### **7.3.13: Health Care Services:**

**1- Cobb-Douglas:** The difference in TFP is significant and positive in 4 counties; Denver, El Paso, Larimer, and Boulder. As a result, the Porter's effect does not support this industry.

**2- Non-homogeneous Cobb-Douglas:** The difference in TFP is positive and significant in 3 counties; Denver, El Paso, and Larimer. Thus, within the non-homogeneous estimation, TFP is deviated from convergence and Porter's effect hypotheses.

**3- Translog Function:** In Denver and Larimer Counties, the difference in TFP is positive and significant. Otherwise, the TFP differences, in other counties, are not significant.

#### **7.3.14. Art, Entertainment, and Recreation Services:**

**1- Cobb-Douglas:** In Larimer and Boulder County, the TFP is significantly higher than the TFP in Arapahoe County, while in other counties the TFP is not significantly different from that of Arapahoe County. Therefore, the results support the convergence and Porter's effect hypotheses in Denver, El Paso, Arapahoe, and Weld.

**2- Non-homogeneous Cobb-Douglas:** Based on the estimation of non-homogeneous function, the results support both the convergence and Porter's effect hypotheses of TFP in the counties studied.

**3- Translog Function:** Only Larimer County shows positive and statistical difference in TFP compared to TFP in Arapahoe County. Otherwise, the difference in

TFP is not significant. Thus, these results support both convergence and Porter's effect hypotheses of TFP in other counties.

#### **7.3.15. Lodge and Restaurants:**

**1- Cobb-Douglas:** TFP is statistically different from that of Arapahoe in three Counties (Denver, Larimer, and Weld).

**2- Nonhomogeneous Cobb-Douglas:** In three counties (Denver, El Paso, and Larimer), the TFP is statistically higher than that of TFP in Arapahoe County in lodge and restaurant industry.

**3- Translog Function:** In Larimer and El Paso, the TFP is significantly higher than the TFP in Arapahoe County concerning the lodge and restaurant industry. In the other 4 counties, the results support the convergence and Porter's effect hypotheses of TFP.

#### **7.3.16 Other services (Except Public Services):**

**1- Cobb-Douglas:** In four counties (Denver, El Paso, Larimer, and Boulder), the TFP is significantly higher than the TFP in Arapahoe County. **2- Non-homogeneous Cobb-Douglas:** In three counties (Denver, El Paso, and Larimer), the TFP in other services industry is statistically significant and higher than TFP in Arapahoe County.

**3- Translog Function:** Only in Boulder County this industry shows that the TFP is significantly higher than TFP in Arapahoe County. Therefore, the results support the convergence and Porter's effect hypotheses in TFP.

#### **7.4. The relation between TFP and RTS**

The study in this section compares and explores the relation between total factor productivity and economies of scale at the county level and among different industries. Also, the comparison will be held at the three different production functions that are

accepted in the estimation in chapter 4. The analysis will be confined to the two counties that have increasing returns to scale, Arapahoe, and El Paso. But, the general regression that the analysis of this section is based on is the following equation:

$$TFP_i = \alpha_0 + \alpha_1 RTS_i + e_i \dots\dots\dots (7.5)$$

The expected sign of  $\alpha_1$  is indeterministic. The reason for this expectation stems from Oh et al (2009). In this paper they decompose TFP into technical progress and RTS.

The TFP equation is:

$$\dot{Y} - \sum_j s_j \dot{X}_j = \frac{\dot{Y}}{Y} + (RTS - 1) \sum_j s_j \dot{X}_j \dots\dots\dots (7.6)$$

Where dot over the variable means growth,  $Y$ = output,  $X_j$ = input variables, and  $s_j$ =cost share of input  $j$ .

This study regresses the results of the two variables, TFP and RTS, to explore the relationship between these two variables. The regression employed 94 observations which include all RTS and TFP under Cobb-Douglas estimation for all industries in all the counties under study. OLS technique is applied without taking the fixed effect of each county into considerations. The estimation results as appeared in equation 7.7 show a negative and significant result between these two variables. This result may due to the small number of industries that exhibit increasing RTS. This means, on average, if the elasticity is increased by 10%, then the TFP will decrease by 9.3%. This result is in accord with 7.6. The equation 7.6 means that if RTS is increasing then it will affect TFP positively, if RTS is constant then the effect is zero to TFP, and if RTS is decreasing then the impact of RTS into TFP is negative. Therefore, because of small number of observations with increasing returns to scale (15 observations) then the estimated result in 7.7 is negative.

$$TFP = 14.606*** - 9.298*** RTS \dots\dots\dots (7.7)$$

SE (0.936) (0.939)

For more analysis, table 7.8 shows there is high and negative correlation between TFP and partial scale elasticity of capital, (-0.8). But the correlation between TFP and labor and land is positive with amount of 0.17 and 0.27, respectively. Also, there is high and negative correlation between partial economies of scale of capital and land. This means that as capital use increase, less land is needed, or the production will expand using land vertically rather than horizontally.

**Table 7.8, the Correlation between TFP and Partial Economies of Scale for Capital, Land, and Labor in Cobb-Douglas**

---

| land | Labor   | Capital | TFP     |         |
|------|---------|---------|---------|---------|
|      |         |         | 1       | TFP     |
|      |         | 1       | -0.8019 | Capital |
|      | 1       | 0.0979  | 0.1708  | Labor   |
| 1    | -0.4853 | -0.7704 | 0.2721  | land    |

---

To explore the impact of the partial scale of economies on TFP, the research regresses TFP including all input partial scale of economies; capital, labor, and land.

Table 7.9 shows the estimated results. The results provide evidence that an increase in partial economies of scale of capital exerts an important negative and significant impact in TFP. Also, the partial scale of economies for land and labor exert negative and significant impact in TFP, but by less magnitude than capital. Table 7.9 indicates that an increase in capital scale of economy by 10%, on average, will led to a decrease in TFP by 12.5%. This may be attributed to adopt less advanced technology as partial scales of economies increase.

**Table 7.9, Regression results between TFP  
and Scale of Economies of Input variables**

---

|           |                |
|-----------|----------------|
| -12.52*** | Capital        |
| (0.324)   | <i>SE</i>      |
| -2.135*** | Labor          |
| (0.368)   | <i>SE</i>      |
| -8.812*** | Land           |
| (0.398)   | <i>SE</i>      |
| 13.580*** | Constant       |
| (0.305)   | <i>SE</i>      |
| 0.9527    | R <sup>2</sup> |

---

Standard Errors are in Parentheses.

\*\*\* Significant at 99%.

\*\* Significant at 95%.

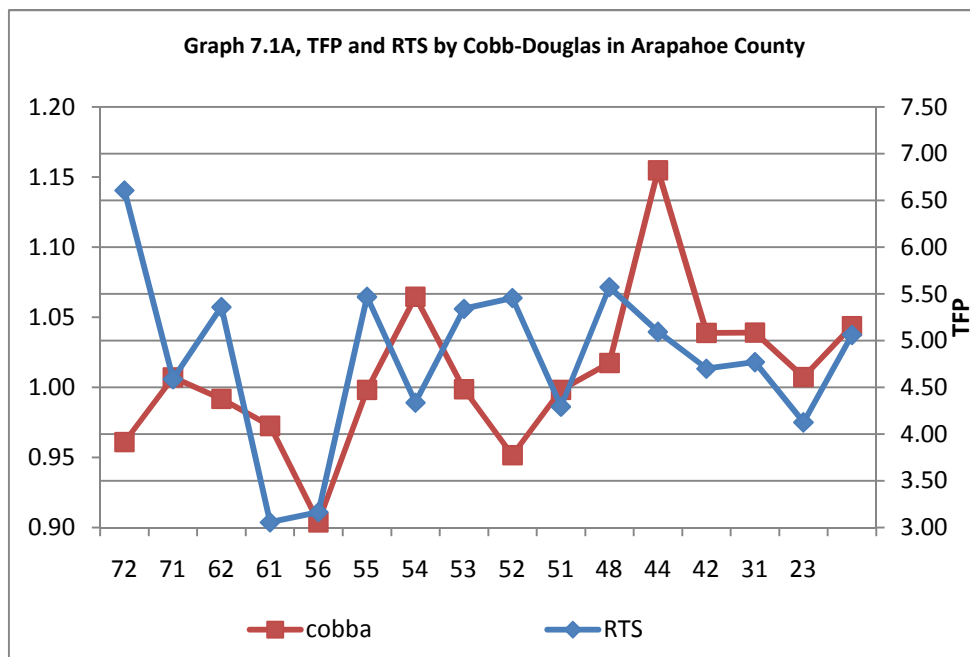
\* Significant at 90%.

### 7.4.1. The Relationship between TFP and RTS in Arapahoe County

The discussion of return to scale and total factor productivity in Arapahoe County will be based on three different production functions, namely; Standard Cobb-Douglas, Non-homogeneous Cobb-Douglas, and translog functions.

#### 7.4.1.1. Standard Cobb-Douglas

Table 7.8A and graph 7.1A show that there is a mixed relation between total factor productivity and economies of scale among the industries in Arapahoe County. For instance, the industries that exhibit constant returns to scale as transportations and warehousing, and the professional, scientist, and technical services have the highest TFP with 6.82 and 5.47 respectively. At the same time, other industries, such as education services and other services except government, have the lowest TFP with 3.06, and 3.91 respectively, but they exhibit constant returns to scale. On the other hand, the industries that are working with increasing return to scale have high rank in TFP. For example, construction with the highest IRTS in Arapahoe County operates with TFP around 5.15.



**Table 7.8A, TFP and RTS by Cobb-Douglas**

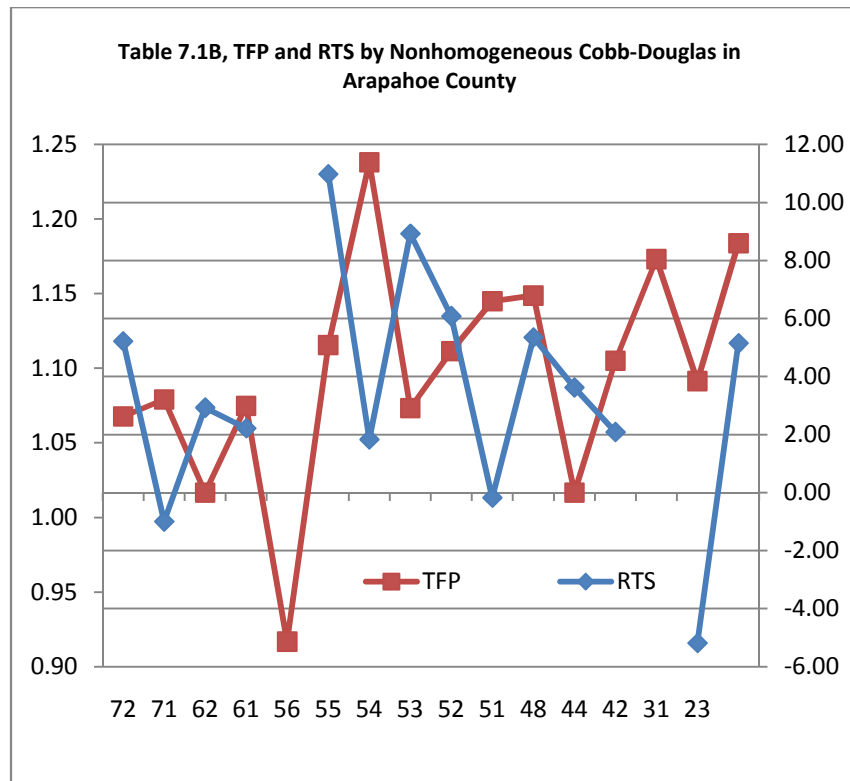
**in Arapahoe County**

| RTS  | TFP  | NAICS2 |
|------|------|--------|
| 1.14 | 5.15 | 23     |
| 1.01 | 4.61 | 31     |
| 1.06 | 5.09 | 42     |
| 0.90 | 5.08 | 44     |
| 0.91 | 6.82 | 48     |
| 1.06 | 4.76 | 51     |
| 0.99 | 4.47 | 52     |
| 1.06 | 3.78 | 53     |
| 1.06 | 4.48 | 54     |
| 0.99 | 5.47 | 55     |
| 1.07 | 4.47 | 56     |
| 1.04 | 3.06 | 61     |
| 1.01 | 4.09 | 62     |
| 1.02 | 4.38 | 71     |
| 0.98 | 4.61 | 72     |
| 1.04 | 3.91 | 81     |

**7.4.1.2. Non-homogeneous Cobb-Douglas Function**

Table 7.1B and Graph 7.1B show that the relation between economies of scale and total factor productivity is inconclusive. In this alignment, management of companies and enterprises has the highest magnitude of TFP (11.39), but this industry behaves as constant returns to scale. The second industry is the construction with TFP of 8.59. But this industry is operating within increasing returns to scale.





Estimation is conducted to reflect the relation between TFP and increasing returns to scale. For this purpose, the study applies a dummy variable for the RTS by applying 1 to the industries that perform within increasing returns to scale, and 0 elsewhere. According to equation 7.8, results show that there is a negative insignificant relation between increasing returns to scale and total factor productivity.

**Table 7.8B, TFP and RTS by Non-homogeneous Cobb-Douglas in Arapahoe County**

| RTS  | TFP   | NAICS2 |
|------|-------|--------|
| 1.12 | 8.59  | 23     |
| 1.00 | 3.84  | 31     |
| 1.07 | 8.05  | 42     |
| 1.06 | 4.55  | 44     |
| N/A  | 7.23  | 48     |
| 1.23 | 6.79  | 51     |
| 1.05 | 6.60  | 52     |
| 1.19 | 4.87  | 53     |
| 1.13 | 2.91  | 54     |
| 1.01 | 11.39 | 55     |
| 1.12 | 5.08  | 56     |
| 1.09 | -5.14 | 61     |
| 1.06 | 2.99  | 62     |
| N/A  | -0.13 | 71     |
| 0.92 | 3.21  | 72     |
| 1.12 | 2.63  | 81     |

### 7.4.1.3. Translog Function

According to translog estimation, only professional, scientist, and technical services industry is operating within increasing returns to scale. Also, table 7.1C, and graph 7.1C show inconclusive results between TFP and RTS indicators.

**Table 7.8C, TFP and RTS by Translog Function  
in Arapahoe County**

| RTS  | TFP   | NAICS2 |
|------|-------|--------|
| 1.01 | 16.71 | 23     |
| 0.94 | 17.91 | 31     |
| 0.98 | 12.87 | 42     |
| 1.01 | 11.92 | 44     |
|      | 21.98 | 48     |
| 1.04 | 5.20  | 51     |
| 1.01 | 10.42 | 52     |
| 1.04 | 9.24  | 53     |
| 1.09 | 7.48  | 54     |
|      | 12.42 | 55     |
| 1.02 | 11.24 | 56     |
| 1.07 | 4.48  | 61     |
| 1.03 | 7.60  | 62     |
|      | 0.14  | 71     |
| 0.95 | 7.78  | 72     |
| 1.05 | 10.60 | 81     |

### 7.4.2. The Relation between TFP and RTS in El Paso County

The analysis of the relation between economies of scale and TFP among industries in El Paso County hinges on the type of production function. The type of production functions are Cobb-Douglas, and non-homogeneous Cobb-Douglas. The translog functions are not applied to El Paso County because of negative partial scale of economies that appears at the firm level.

### 7.3.2.1. Cobb-Douglas Function

Table 7.9A and graph 7.2A show that the relation between RTS and TFP is negative. The highest TFP corresponds to management of companies and enterprises which operates under decreasing returns to scale. Then the high TFP is followed by the industries that work within constant returns to scale. For example health services, transportation and warehousing, wholesale trade, and other services industries have TFP of 6.37, 5.99, 5.88, and 5.79 respectively. While the industries with increasing return to scale have lower magnitude of TFP. In this trajectory, construction with 1.26 RTS has a magnitude of 4.55.

### 7.4.2.2. Non-homogeneous Function

Table 7.9B and graph 7.2B reveal mixed results between TFP and economies of scale. For instance, for constant returns to scale, TFP is negative in wholesale trade (-0.3), and art, entertainment, and recreation services, while the TFP is positive in the other industries 3 industries that operate within constant returns to scale. In addition, the industries with increasing returns to scale have high magnitude in TFP.

**Table 7.9B, TFP and RTS By Non-homogeneous Cobb-Douglas in El Paso County**

| RTS  | TFP   | naics2 |
|------|-------|--------|
| 1.36 | 3.46  | 23     |
| 1.09 | 16.61 | 31     |
| 1.05 | -0.30 | 42     |
| 1.05 | 9.62  | 44     |
| 1.04 | 10.58 | 48     |
| 1.20 | 16.81 | 51     |
| 1.23 | 12.67 | 52     |
| 1.14 | 4.75  | 53     |
| 1.40 | 7.50  | 54     |
| N/A  | 25.95 | 55     |
| 1.28 | 4.38  | 56     |
| 1.48 | -6.40 | 61     |
| 1.10 | 15.86 | 62     |
| 1.05 | -3.54 | 71     |
| 0.99 | 7.77  | 72     |
| 1.13 | 15.29 | 81     |

## **7.5. Summary**

This summary takes into consideration the TFP convergence within county and among industries (or Jacobs's effect), and within industry and among counties (Porter's effect). This summary will be taken by the estimated TFP according to the three different production functions, namely: Cobb-Douglas, non-homogeneous Cobb-Douglas, and translog function. The convergence in TFP among industries and within the county, or among counties within the same industries indicates that there is a highly educated workforce in the region to facilitate transmission of ideas, technology, and alike.

From the policy implication point of view, the county is should encourage firms and industries with higher TFP for the efficient use of available resources, high potential to export, rise in competitiveness of the firm, expected increase in employment, and expected loss of fewer jobs during economic down turns, and increase in local government revenues.

### **A- TFP Convergence Within County and among Industries**

- **Cobb-Douglas Production Function**

- 1- There is TFP convergence in information and real estate industries in all counties without exception compared to the TFP of construction industry in each county.
- 2- TFP convergence in 13 industries in Arapahoe County is not statistically different from that of TFP in construction industry. This is followed by 11 industries TFP convergence in Boulder County.
- 3- There is high homogeneity in TFP in the county and among the industries in each county under study. In this context, there is no significant difference in TFP between industries and TFP of construction in the same county.

- **Non-homogeneous Cobb-Douglas Production Function**

- 1- In general, in each county, there is homogeneity in TFP compared to TFP in construction industry in the same county. For example, the differences in TFP's are industry, followed by 10 industries TFP in each of Arapahoe, Denver, Boulder, and Weld.
- 2- In all industries, convergence occurred in administrative and waste management industry.

- **Translog Production Function**

- 1- The common industries that have convergence in TFP compared to TFP in construction industry are manufacturing, and finance and insurance industries.
- 2- Except TFP in Arapahoe, there is high homogeneity in TFP of construction in all counties compared to TFP in construction industry.

#### **B- TFP convergence Within Industry and among Counties**

- **Cobb-Douglas Production Function**

- 1- In transportation and warehousing, there is convergence in TFP in this industry and among all counties compared to transportation industry in Arapahoe County.
- 2- In three industries, there is convergence in TFP among 4 counties compared to their corresponding industries in Arapahoe. These industries are: manufacturing, management of companies and enterprises, and administrative and waste management industries.

- **Non-homogeneous Cobb-Douglas function:**

In non-homogeneous Cobb-Douglas function, there is convergence in TFP in 5 industries in all counties compared to their corresponding TFP in Arapahoe County. These industries are: manufacturing, wholesale trade, information, real estate, and administrative and waste management industries.

- **Translog Production Function**

- 1- In general, there is high convergence with industry within industry and among counties. In particular, there is convergence in manufacturing, and finance and insurance industries among all counties.
- 2- In 5 industries, TFP tends to converge in 4 counties compared to their corresponding TFP in Arapahoe county industries. These industries are: construction, retail trade, real estate, professional, scientist, and technical services, and administrative and waste management industries.

## **Chapter Eight: Recommendations and Policy Implications**

The study estimates three different production functions at the industry level using data at the firm level for six counties in Colorado. These counties are: Arapahoe, El Paso, Denver, Larimer, Boulder, and Weld. The estimated production functions are classified into homogeneous production functions such as the standard Cobb-Douglas function, and non-homogeneous functions such as translog and non-homogeneous Cobb-Douglas functions. The estimated production function includes three input variables: capital, labor, and land. The land variable is used for the first time explicitly in pure economic empirical work.

The study explores four different hypotheses that coincide with the production function. These hypotheses are: the production function that best fits the data structure, returns to scale (RTS), the partial elasticities of substitutions or complementarities, and total factor productivity (TFP). Here are the main results of the study and the policy implications.

### **8.1.Data**

In this study, the data are collected from two different sources; QCEW which IS collected by the Labor Department of Colorado, and the County Assessors Office. Each source of the data collects the information for different purposes. Therefore, the number of observations when merged reflect between 20% to 60% of total observations, although the data in both sources are mostly the same. Thus, to minimize the number of losing observations, the study suggests that the Department of Labor in Colorado and the County Assessors Office coordinate and agree on a specific code for each firm to improve and foster the benefit of using the merged data in economic research.

### **8.2.Exploring the production function fitness**

For the purpose of choosing the production function that best fits the data structure, the study adopted the nonnested J-test to investigate the production function that best fits

the data. The main results are that the production function that fits the data better is inconclusive. But the translog function nested the other two functions, and the non-homogeneous Cobb-Douglas function nested only the standard Cobb-Douglas. As a consequence, the non-homogeneous functions are expected to fit the data structure in Arapahoe and El Paso counties better.

### **8.3 Estimation Results**

The estimation results show that the land input is an important variable in the production function. The results show that the coefficient of land variable, at the industry level, is the highest in Denver County for almost all industries compared to other counties industries included in the study. This result may reflect the scarce of land in Denver County. this result needs more investigations by conducting more studies in this regard in the future.

### **8.4.Exploring the returns to scale (RTS)**

The returns to scale are explored at the industry and firm level for the three different production function. For Cobb-Douglas, it is the sum of the elasticities of the three inputs (labor, land, and capital). While in non-homogeneous function, the returns to scale is computed at the firm level, and at the industry level by taking the averages of the firms returns to scale in the industry. The main results are:

Table 5.8 shows that industries with increasing returns to scale appear only in 3 counties. These counties are: Arapahoe County with 5 industries according to standard Cobb-Douglas, and 2 more industries in non-homogeneous Cobb-Douglas function; in El Paso county with 7 industries exhibits increasing returns to scale within standard Cobb-Douglas function, and 3 more industries according to non-homogeneous Cobb-Douglas function; Larimer County with one industry operating within increasing returns to scale; and in other counties, on average, there is no evidence of increasing returns to scale at the



industry level. But at the firms' level there is evidence that part of the firms are operating within increasing returns to scale, tables 5.1-5.6.

In addition, the partial scale of land in non-homogeneous function in each industry is less than the corresponding partial scale in standard Cobb-Douglas, but, as appears in table 5.8, the total economies of scale in most of the industries is higher in non-homogeneous function compared to conventional Cobb-Douglas. The higher scale of economy in non-homogeneous function may be attributed to the increase in partial scale of capital and labor more than the reduction in land scale. Thus, 1% increase in the three input variables leads to more than 1% in output according to Cobb-Douglas production function. Also, the growth in output as a consequence of 1% growth in the three inputs is higher in non-homogeneous Cobb-Douglas compared to standard Cobb-Douglas. Therefore, a 1% growth in output required less than 1% growth in appeared inputs in the estimated Cobb-Douglas equation. Therefore, to mitigate the impact of economic down turns and increase the competitiveness in the region, the local government has to encourage the firms and industries that exhibit increasing returns to scale.

Furthermore, table 5.8 shows that some industries are increasing returns to scale in non-homogeneous but not applied in Cobb-Douglas such as information industry in El Paso County. Also, there are industries that are, on average, increasing returns to scale in non-homogeneous function, while these industries are either constant or decreasing returns to scale such as education services, and other services in Arapahoe County; and health care, services and other services in El Paso County.

## **8.5.Exploring The Elasticity of substitutions or complementarity**

In this chapter, the study computed partial elasticity of substitutions for three input variables according to Hicks (1932, 1970). The partial elasticities are computed at the firm level in each industry for each county included in the study. The summary and policy implications only reflect the average of the partial elasticity for the firms in the industry. These partial elasticities include the partial elasticity between labor and capital, the partial elasticity between land and capital, and partial elasticity of land and labor. These elasticities will be illustrated according to non-homogeneous production functions because the elasticity is unity in standard Cobb-Douglas production function.

### **8.5.1 Non-homogeneous Production Function**

#### **8.5.1.1 Elasticity between capital and Labor**

On average, the elasticity between capital and labor is substitute in all industries in each county understudy. The results show that the elasticity between labor and capital is deviated from unity, i.e., is not standard Cobb-Douglas function. In addition, the elasticity between these two inputs is greater than 1. This means that if the wage rate is increased by 10% the capital will substitute labor by more than 10%, and in some cases it will be double, 20% as in construction in Arapahoe County.

Furthermore, there is heterogeneity in the elasticity of substitutions between capital and labor among industries in the same county. For instance, in Arapahoe, the elasticity ranged between 1.01 in health services, and 1.99 in construction. While in Denver, it ranged between 0.52 in manufacturing and 2.01 in other services, and lodge and restaurants industries.

### **8.5.1.2 Elasticity between Capital and Land**

The elasticity between land and capital is complement in most industries among the counties, except in wholesale trade, information, and education services where it is substitute. This means that the employment of these two factors of production in most industries moves in the same direction. Also, the elasticity is less than unity, which means that the elasticity between these two factors is deviated from standard Cobb-Douglas function.

Furthermore, there is heterogeneity in the elasticity between these two factors of production in the same county among different industries. For instance, the elasticity in most of the industries is less in Arapahoe County, and it moved around zero in 7 industries in Arapahoe County. This means that the elasticity between land and capital is Leontief in these industries. Thus, the policy implication recommend that the counties have to encourage such industries to mitigate job loss and minimize loss in local government revenues during economic down turns.

### **8.5.1.3 The Elasticity between Labor and Land**

On average, the elasticity between labor and land is zero in all industries in El Paso and Denver counties. This means that there is Leontief relation between these two inputs. That is to reduce the usage of labor during economic down turns; the industries have to reduce the use of land in the same proportion. The main policy implication is to encourage such industries to mitigate job reduction during tough economic periods. In addition, the elasticity between land and labor in Arapahoe County is complement and less than unity. This elasticity, in Arapahoe County, ranges between -0.01 in arts, recreation, and entertainment and -0.92 in construction. In this case, Arapahoe County has to encourage the firms and industries with low elasticity of substitution or complements to reduce job loss during economic down turns. For example, the firms in manufacturing,

wholesale trade, art, recreation, and entertainment, lodge and restaurants, and other services (except public services).

## **8.6 Exploring TFP convergence**

The TFP convergence is investigated according to two groups: (i) within county and among industries (Jacobs's effect); and (ii) within industry and among counties (Porter effect). The estimated TFP will be explored according to the three different production functions, namely: Cobb-Douglas, Non-homogeneous Cobb-Douglas, and translog function. The convergence in TFP among industries and within the county, or among counties within the same industries indicates that there is a highly educated workforce in the region to facilitate transmission of ideas, technology, and alike.

From a policy implication point of view, it is recommended that the county encourage firms and industries with higher TFP for the efficiency use of available resources, because of expected high potential to export, raising competitiveness of the firm, expected increase employment, and expected loss of fewer jobs during economic down turns, and increasing local government revenues.

### **8.6.1 TFP Convergence Within County and among Industries**

#### **8.6.1.1 Cobb-Douglas Production Function**

- (i) There is TFP convergence in information, and real estate industries in all counties without exception compared to the TFP of construction industry in each county.
- (ii) TFP converge in 13 industries in Arapahoe County are not statistically different than that of TFP in construction industry. This is followed by 11 industries TFP convergence in Boulder County.

- (iii) There is high homogeneity in TFP in the county and among the industries in each county under study. In this context, there is no significant difference in TFP between industries and TFP of construction in the same county.

#### **8.6.1.2 Non-homogeneous Cobb-Douglas Production Function**

- (i) In general, in each county, there is homogeneity in TFP compared to TFP in construction industry in the same county. For example, the differences in TFPs are industry, followed by 10 industries TFP in each of Arapahoe, Denver, Boulder, and Weld.
- (ii) In all industries, convergence occurred in administrative and waste management industry.

#### **8.6.1.3 Translog Production Function**

- (i) The common industries that have convergence in TFP compared to TFP in construction industry are manufacturing, and finance and insurance industries.
- (ii) Except TFP in Arapahoe, there is high homogeneity in TFP of construction in all counties compared to TFP in construction industry.

### **8.6.2 TFP convergence Within Industry and among Counties**

#### **8.6.2.1 Cobb-Douglas Production Function**

- (i) In transportation and warehousing, there is convergence in TFP in this industry and among all counties compared to transportation industry in Arapahoe County.
- (ii) In three industries, there is convergence in TFP among 4 counties compared to their corresponding industries in Arapahoe. These industries are: manufacturing, management of companies and enterprises, and administrative and waste management industries.

### **8.6.2.2 Non-homogeneous Cobb-Douglas function:**

In non-homogeneous Cobb-Douglas function, there is convergence in TFP in 5 industries in all counties compared to their corresponding TFP in Arapahoe County. These industries are: manufacturing, wholesale trade, information, real estate, and administrative and waste management industries.

### **8.6.2.3 Translog Production Function**

- (i) In general, there is high convergence with industry within industry and among counties. In particular, there is convergence in manufacturing, and finance and insurance industries among all counties.
- (ii) In addition, there are 5 industries that the TFP tends to converge in 4 counties compared to their corresponding TFP in Arapahoe County industries. These industries are: construction, retail trade, real estate, professional, scientist, and technical services, and administrative and waste management industries.

## **8.7 Further studies**

Further studies are recommended such as studying each sector of economy in a panel data. A more depth study at the level firm.

## References

**Allen, R. G. D. (1934)** “A Comparison Between Different Definitions of Complementary and Competitive Goods”, *Econometrica*, Vol. 2, No. 2 (Apr. 1934), PP. 168-175.

**Bairam E.I. (1989)**, “Functional Form and the Elasticity of Substitution: a New CES production Function”, *Economic Discussion Papers, No. 8904*, University of Otago, New Zealand.

**Bairam E.I. (1991)**, “Functional Form and New Production Functions: Some Comments and a new VES Function”, *Applied Economic*, Vol. 23, PP 871-879.

**Baldwin, John R., Brown, W. Mark, and Rigby, David L. (2010)**, “Agglomeration Economics: Microdata Panel Estimates from Canadian Manufacturing”, *Journal of Regional Science*, Vol. 50, No. 5 (December 2010), PP. 915-934.

**Basu, Susanto, and Fernald, G. John (1995)**, “Return to Scale in US Production: Estimates and Implication”, *Board of Governors of the Federal Reserve System International Discussion Papers*, November 546/ March 1995.

**Basu, Susanto, (1996)**, “Procyclical Productivity: Increasing Returns or Cyclical Utilization?,” *The Quarterly Journal of Economics*, Vol. 111, No. 3 (Aug. 1996), PP. 719-751.

**Basu, Susanto and Fernald, John G. (1997)**, “Returns to Scale in U.S. production: Estimates and Implications,” *The Journal of Political Economy*, Vol. 105, No.2 (April 1997), PP 249-283.

**Batten, Azizul and et al (2009)** “Technical Efficiency in Stochastic Frontier Production Model: an Application to the Manufacturing Industry in Bangladesh,” *Australian Journal of Basic and Applied Science*, Vol. 3, No. 2, PP. 1160-1169.

**Bernard, Andrew B. and Jones, Charles I. (1996)**, “Productivity across Industries and Countries: Time Series Theory and Evidence”, *The Review of Economics and Statistics*, Vol. 78, No. 1 (Feb., 1996), PP. 135-146.

**Berndt, Ernst R., and Christensen, Laurits R. (1973)** “The Translog Function and the Substitution of Equipment, Structures, and Labor in US Manufacturing,” *Journal of Econometrics* (March 1973), PP. 81-113.

**Berndt, E. R. and Wood, D. O. (1975)**, “Technology, Prices, and the Derived Demand for Energy,” *Review of Economics and Statistics*, Vol. 57, No. 3 (1975), PP. 259-68.

**Berndt, Ernst R. (1976)** “Reconciling Alternative Estimates of the Elasticity of substitution,” *The Review of Economics and Statistics*, Vol. 58, No. 1 (Feb. 1976), PP. 59-68.

**Blackorby, C. and Russell, R. (1976)**, “Functional structure and the Allen partial elasticities of substitution: an application of duality theory,” *Review of Economic Studies*, 43, pp. 285-92.

**Blackorby, C. and Russell, R.R. (1981)**, “The Morishima elasticity of substitution: symmetry, constancy, separability and its relationship to the Hicks and Allen elasticities,” *Review of Economic Studies*, Vol. XLVIII, pp. 147-58.

**Blundell, Richard and Bond, Steve (1998)**, “GMM Estimation with Persistent Panel Data: An Application to Production Functions,” *The Institute for Fiscal Studies (IFS) Working Paper Series No. W99/4*, September 1998.

**Blundell, R., (1988)**, “Consumer Behavior: Theory and Empirical Evidence - A Survey,” *Economic Journal*, Vol., No. 98, PP. 16-65.

**Blundell, R. and Lewbel, A. (1991)**, “The Information Content of Equivalence Scales,” *Journal of Econometrics*, Vol. 50, PP. 49-68.

**Blundell, R, Preston, I. and Walker, I. (eds) (1994)**, *The Measurement of Household Welfare*, Cambridge: Cambridge University Press.

**Box, G.E. P., and Cox, D. R., (1964)** “An Analysis of Transformations,” *Journal of The Royal Society*, B, 26, PP. 211-252.

**Chang, H. S. (1980)**, “Functional Form and the Demand for meat in the Unites States,” *The Review of Economics and Statistics*, Vol. 59, No. 3, PP. 355-359.

**Chang, K. P. (1994)**, “Capital-Energy Substitution and Multilevel CES Production Function,” *Energy Economics*, Vol. 16, No. 1, PP. 22-26.

**Chow, Gregory and Li, Kui-Wai, (2002)** “China’s Economic Growth: 1952-2010,” *Economic Development and Cultural Change*, Vol. 51, No. 1(October 2002), PP. 472-256.

**Christensen, Laurits R., and Jorgenson, Dale W. (1973)** “Transcendental Logarithm Production Frontier,” *The Review of Economics and Statistics*, Vol. 55, No. 1 (Feb. 1973), PP. 28-45.

**Clarke, Kevin A. (2001)**, “Testing Nonnested Models of International Relations: Reevaluating Realism,” *American Journal of Political Science*, Vol. 45, PP. 724–744.

**Clarke, Kevin A. (2003)**, “Nonparametric Model Discrimination in International Relations,” *Journal of Conflict Resolution*, Vol. 47, PP. 72–93.

**Cox, David R. (1961)**, “Tests of separate families of hypotheses,” *Proceedings of the Fourth Berkeley Symposium I*, PP. 105–123.

**Cutler, Harvey and Davies, Stephen (2008)** “The Economic Consequences of Productivity Changes: A Computable General equilibrium (CGE) Analysis,” *Regional Studies*, Vol. 44, No. 10 (available online: 28 April 2008), PP. 1415-1426.



**Cutler, Harvey and Davies, Stephen (2007)** “The Impact of Specific-Sector Changes in Employment on Economic Growth, Labor Market Performance and Migration,” *Journal of Regional Science*, Vol. 47, No. 5 (2007), PP. 935-963.

**Davidson, Russell, and MacKinnon, James G. (1993)**, “Estimation and Inference in Econometrics,” *Oxford: Oxford University Press*.

**Davidson, R. and Mackinnon J.G (1981)** “Several Tests for Model Specification in The Presence of Alternative Hypothesis,” *Econometrica*, Vol. 49, No. 3 (1981), PP. 781-793.

**Duffy, Neal, (1988)** “Return to Scale Behavior and Manufacturing Agglomeration Economies in US Urban Areas,” review of *Regional Studies*, Vol. 18, PP. 47-54.

**Dekle, Robert, and Eaton, Jonathan (1999)** “Agglomeration and Land Rents: Evidence from Prefectures,” *Journal of Urban Economics*, Vol. 46, No. 2 (Sept., 1999), PP. 200-214.

**Diewert, W. E. (1971)**, “An Application of the Shepherds Duality Theorem: A Generalized Leontief Production Function,” *The Journal of Political Economy*, Vol. 79, No. 3 (May-Jun., 1971), PP. 481-507.

**Kydland, Finn E., and Prescott, Edward E. (1982)** “Time to Build an Aggregate Fluctuations,” *Econometrica*, Vol. 50, No. 6 (Nov., 1982), PP. 1345-1370.

**Ulveling, Edwin F. and Fletcher, Lehman B. (1970)** “Cobb-Douglas Production Function with Variable Return to Scale,” *American Journal of Agricultural Economics*, Vol. 52, No. 2 (May, 1970), PP. 322-326.

**Freeman, Richard B. and Medoff, James L. (1982)** “Substitution Between Production Labor and Other Inputs in Unionized and Nonunionized Manufacturing,” *The Review of Economics and Statistics*, Vol. 64, No. 2 (May, 1982), PP. 220-233.

**Green, Alison, and Mayes, David (1991)** “Technical Inefficiency in Manufacturing Industries,” *The Economic Journal*, Vol. 101, No. 406 (May 1991), PP. 523-538.

**Griliches, Z., and Ringstad V. (1971)** “Economies of Scale and the Form of the Production Function,” *Amsterdam, North Holland*.

**Hicks, J. R. (1932)** *Theory of Wages*, London: Macmillan (1932).

**Hicks, J. R. (1970)** “Elasticity of Substitution Again: Substitutes and Complements,” *Oxford Economic Papers*, Vol. 22 (1970), PP. 289-296.

**Hossain, Mohammad Zakir (2011)** “The Use of Box-Cox Transformation Technique in Economic and Statistical Analysis,” *Journal of Emerging Trends in Economics and Management Science*, Vol. 2, No. 1, PP. 32-39.

**Hsing, Yu (1996)**, “An Empirical Estimation of Regional Production Functions for the US Manufacturing Industry,” *the Analysis of Regional Science*, (1996), Vol. 30, PP. 351-358.

**Harrigan, James (1999)** “Estimation of Cross Country Differences in Industry Production Function,” *Journal of International Economics* 47(1999), PP. 267-293.

**Henderson, Daniel J. (2009)** “A Non-Parametric Examination of Capital-Skill Complementarity,” *Oxford Bulletin of Economics and Statistics*, Vol. 71, No. 4 (Aug. 2009), PP. 519-538.

**Greene, William (2007)** *Econometric Analysis*, Prentice Hall, New Jersey, sixth edition.

**Kasahara, H. and Rodrigue, J. (2008)** “Does the Use of Imported Intermediates Increase Productivity?,” *Journal of Development Economics*, Vol. 87,(2008), PP. 106-118.

**Kemfert, C. (1998)** “Estimated Substitution Elasticities of a Nested CES Production Function Approach for Germany,” *Energy Economics*, Vol. 20, No. 3 (1998), PP. 249-264.

**Kermfert, C. (2002)** “An integrated assessment model of economy energy-climate the model WIAGEM ,” *Integrated Assessment*, Vol. 3, No. 4, PP. 281–98.

**Khazzoom, J. D. (1980)** “Economic implications of mandated efficiency in standards for household appliances,” *Energy Journal*, Vol. 1, No. 4, PP. 21-40.

**Kim, H. Youn (1992)** “The Translog Production Function and Variable Returns to Scale,” *The Review of Economics and Statistics*, Vol. 74, No 3 (Aug., 1992), PP. 546-552.

**Klump, Rainer and Preissler, Harald (2000)** “CES Production Functions and Economic Growth,” *Scand. J. of Economics* 102(1) 2000, PP. 41-56.

**Klump, Rainer, McAdam, Peter, and Willman, Alpo (2011)** “The Normalized CES Production Function-Theory and Empirics,” *European Central Bank, Working Paper Series*: 1294 (2011), PP. 50 page.

**Kydland, Finn E., and Prescott, Edward E. (1982)**, ' Time to Build an Aggregate Fluctuations', *Econometrica*, Vol. 50, No. 6, PP. 1345-1370.

**Leon-Ledesma, Miguel A, McAdam, Peter, and Willman, Alpo (2010)** “Identifying The Elasticity of Substitution with Biased Technical Change,” *American Economic Review*, Vol. 100, No. 4 (September 2010), PP. 1330-57.

**Lynch, Devon and Zax, Jeffery S (2011)** “Incidence and Substitution in Enterprise Zone Programs: The Case of Colorado,” *Public Finance Review*, Vol. 39, No. 2 (March 2011), PP. 226-55.

**Lau, Lawrence J, and Tamura, Shuji (1972)** “Economies of Scale, Technical Progress, and the Nonhomothetic Leontief Production Function: An Application to the Japanese Petrochemical Processing Industry,” *The Journal of Political Economy*, Vol. 80, No. 6 (Nov.-Dec., 1972), PP. 1167-1187.

**Mundlak, Yair (1996)** “Production Function Estimation: Reviving the Primal,” *Econometrica*, Vol.64, No. 2 (Mar. 1996), PP. 431-438.

**Mushinski, David, Weiler, Stephen, and Widner, Benjamin,** “Inferring Geographic Interdependencies from Retail Base Spectra,” *Unpublished Paper*.

**Nakamura, Ryohei (1985)** “Agglomeration Economies in Urban Manufacturing Industries: A Case of Japan Cities,” *Journal of Urban Economics*, Vol. 17, (1985), PP. 108-124.

**Nicholson, Walter (2005),** *Microeconomic Theory: Basic, Principles, and Extension* (2005), Ninth Edition.

**O’Donnell, A. T., and Swales J. K. (1979)** “Factor Substitution, the CES Production Function and UK Regional Economics,” *Oxford Economic Papers*, Vol. 31, No. 3 (Nov. 1979), PP. 460-476.

**Oh, Donghyun, Heshmati, Almas, and Loof, Hans (2009),** “Technical change and Total Factor Productivity Growth for Swedish Manufacturing and Services Industries,” *CESIS Electronic Working Paper Series, Paper No. 193*, September 2009.

**Olson, Lars J., and Roy, Santanu (2002),** “The Economics of Controlling a Stochastic Biological Invasion,” *American Journal of Agriculture Economics*, Vol. 84, No. 5 (Dec. 2002), PP. 1311-1316.

**Rietveld, Piet (1989)** “Infrastructure and Regional Development a Survey of Multiregional Models,” *the Annals of Regional Science*, 1989, 23: PP. 255-274.

**Ringstad, Vidor (1974)** “Some Empirical Evidence on the Decreasing Scale Elasticity,” *Econometrica*, Vol. 42, No. 1 (Jan. 1974), PP. 87-102.

**Sato, K. (1967)** “A Two-Level Constant-Elasticity-of-Substitution production function,” *The Review of Economic Studies*, Vol. 34, No. 2 (Apr. 1967), PP. 201-218.

**Sato, Ryuzo; and Koizumi, Tetsunori (1973)** “On The Elasticity of Substitution and Complementarity,” *Oxford Economic Papers* (1973), Vol. 25, No. 1 (1973), PP. 44-56.

**Segal, David (1976)** “Are There Returns to Scale in City Size,” *The Review of Economic and Statistics*, Vol. 58, No. 3 (Aug., 1976), PP. 339-350.

**Sirmans, C.F., Kau, James B., and Lee, Cheng F. (1979)** “The elasticity of substitution in urban housing production: A VES approach,” *Journal of Urban Economics* (1979), vol. 6, PP.407-415.

**Stern, David I. (2004)** “Elasticities of Substitutions and Complementarity,” *Rensselaer Working Papers in Economics*, Number 0403.

**Sveikasuskas, Leo (1975)** “The Productivity of Cities,” *The Quarterly Journal of Economics*, Vol. 89, No. 3, (Aug. 1975), PP. 393-413.

**Taymaz, Erol, and Yilmaz, Kamil (2007)** “Productivity and Trade Orientation: Turkish Manufacturing Industry Before and After the Customs Union,” *The Journal of International Trade and Diplomacy*, Vol.1, No. 1, Spring 2007, PP 127-154.

**Thompson, H. (1997)** “Substitution Elasticities With Many Inputs,” *Appl. Math. Lett.*, Vol. 10, No. 3 (1997), PP. 123-127.

**Vinod, H. D. (1972)** “Nonhomogeneous Production Function and Applications to Telecommunications,” *The Bell Journal of Economics and Management Science*, Vol. 3, No. 2 (Autumn 1972), PP. 531-543.

**Wooldridge, Jeffrey M. (2009)** “On Estimating Firm-level Production Functions Using Proxy Variables to Control for Unobservables,” *Economic Letters*, Vol. 104 (2009), PP. 112-114.