A Methodology for Estimating the Parameters of a Gamma Raindrop Size Distribution Model from Polarimetric Radar Data: Application to a Squall-Line Event from the TRMM/Brazil Campaign

V. N. Bringi, Gwo-Jong Huang, and V. Chandrasekar

Colorado State University, Fort Collins, Colorado

E. Gorgucci

Istituto di Fisica dell’Atmosfera (CNR), Rome, Italy

(Manuscript received 25 June 2001, in final form 22 October 2001)

ABSTRACT

A methodology is proposed for estimating the parameters of a gamma raindrop size distribution model from radar measurements of $Z_h$, $Z_{dr}$, and $K_{dp}$ at S band. Previously developed algorithms by Gorgucci et al. are extended to cover low rain-rate events where both $Z_{dr}$ and $K_{dp}$ are noisy. Polarimetric data from the S-band Dual-Polarization Doppler Radar (S-Pol) during the Tropical Rainfall Measuring Mission (TRMM)/Brazil campaign are analyzed; specifically, the gamma parameters are retrieved for samples of convective and trailing stratiform rain during the 15 February 1999 squall-line event. Histograms of $N_w$ and $D_o$ are retrieved from radar for each rain type and compared with related statistics reported in the literature. The functional behavior of $N_w$ and $D_o$ versus rain rate retrieved from radar is compared against samples of 2D-video and RD-69 disdrometer data obtained during the campaign. The time variation of $N_w$, $D_o$, and $m$ averaged over a 5 km x 5 km area (within which a network of gauges and a profiler were situated) is shown to illustrate temporal changes associated with the gamma parameters as the squall line passed over the network. The gauge-derived areal rainfall over the network is compared against radar using the areal $F_{dp}$ method, and the concept of an effective slope of a linear axis ratio versus diameter model is shown to significantly reduce the bias in radar-derived rainfall accumulation.

1. Introduction

A long-standing goal in polarimetric radar has been the retrieval of the raindrop size distribution using measurements of reflectivity ($Z_h$), differential reflectivity ($Z_{dr}$), and specific differential phase ($K_{dp}$). Early studies focused on the estimation of $D_o$ (the median volume diameter) or $D_m$ (the mass-weighted mean diameter) using $Z_{dr}$ measurements alone (Seliga and Bringi 1976; Goddard and Cherry 1984; Aydin et al. 1987; Bringi et al. 1998). The functional relation between $D_o$ and $Z_{dr}$ is known to be dependent on the mean axis ratio versus drop diameter relation, which can deviate from the equilibrium relation due to drop oscillations [e.g., Andsager et al. (1999); see also the summary in Pruppacher and Klett (1997) and numerous reference therein]. The same is also true for the functional relation between $K_{dp}$ and rain rate. While the combined use of $K_{dp}$ and $Z_{dr}$ tends to mitigate somewhat the effects of drop oscillations on the estimation of rain rate (see Bringi and Chandrasekar 2001, chapter 7), a more satisfactory approach has been formulated in a series of papers by Gorgucci et al. (2000, 2001, 2002). The essential concept is related to the fact that drop oscillations and drop canting tend to bias the axis ratio toward sphericity, but this is generally nonlinear with respect to drop diameter. However, it is possible to define an equivalent linear model for the mean axis ratio versus $D$ relation using an effective slope ($\beta_{eq}$), which can be estimated from radar measurements of $Z_h$, $Z_{dr}$, and $K_{dp}$ and subsequently used in the estimation of the raindrop size distribution parameters. A background section is included that provides more detail on this concept.

At low rain rates, such as in stratiform rain, the polarimetric measurements of $Z_{dr}$ and $K_{dp}$ tend to be very noisy, unless the data are substantially averaged in space. In such cases, the effective $\beta$ method cannot be applied, and in this paper a method is proposed for retrieval of the drop size distribution (DSD) parameters in the gamma model. The normalized gamma DSD with parameters ($D_o$, $N_w$, and $\mu$) is suitable for inverting the radar measurements. The normalization procedure
(which defines the normalized intercept parameter $N_{\nu}$) can be found in Willis (1984) or Testud et al. (2001).

In this paper S-band Dual-Polarization Doppler Radar (S-Pol) data collected during the Tropical Rainfall Measuring Mission (TRMM)/Brazil field campaign are analyzed to provide the statistics of $D_o$ and $N_w$ in samples of convective and stratiform rain during a squall-line episode on 15 February 1999 that lasted several hours. The behavior of $D_o$ and $N_w$ versus rain rate is also analyzed and compared with 2D video (Joanneum Research) and RD-69 (Disdromet, Ltd.) disdrometer measurements of samples of convective and stratiform rain obtained during the field campaign. The hypothesis that the use of $\beta_{st}$ in polarimetric rain-rate algorithms will reduce the bias in cumulative rainfall is tested by comparing with a network of gauges for the 15 February case.

2. Background

The radar measurement set of reflectivity at horizontal polarization ($Z_a$), differential reflectivity ($Z_{dr}$), and specific differential phase ($K_{dp}$) can in the Rayleigh scattering limit be related to the microphysics of raindrops. Specifically, the $Z_a$ is related to the sixth moment of the DSD, $Z_{dr}$ is related to the reflectivity-weighted mean axis ratio, and $K_{dp}$ is related to the product of the water content and the deviation of the mass-weighted mean axis ratio from unity (Jameson 1983, 1985). If a model relating the axis ratio ($r$) of oriented oblate raindrops versus the equivolumetric spherical diameter ($D$) is selected, then $Z_{dr}$ can be related to the reflectivity-weighted mean diameter of the DSD, while $K_{dp}$ can be related to the product of $W$ and $D_o$ (the mass-weighted mean diameter of the DSD). Generally, the linear fit to the wind-tunnel data of Pruppacher and Beard (1970), $r = 1.03 - 0.062D$ (with $D$ in mm), or the numerical equilibrium shape model of Beard and Chung (1987) has been used. If the DSD is modeled as a normalized gamma form (Willis 1984; Testud et al. 2001),

$$N(D) = N_{\nu}f(\mu) \left(\frac{D}{D_o}\right)^{\alpha} e^{-\left(3.67 + \mu XD_o\right)}$$

with

$$f(\mu) = \frac{6}{(3.67)^4} \left(\frac{3.67 + \mu}{\Gamma(\mu + 4)}\right)^{4+\mu}$$

where $N_{\nu}$ is the normalized intercept parameter of an equivalent exponential DSD that has the same water content and median volume diameter ($D_o$) as the gamma DSD, then it follows that $Z_a = N_{\nu}F_1(\mu)D_o^5$, $Z_{dr} = F_2(\mu, D_o)$, while $K_{dp} = N_{\nu}F_3(\mu, D_o)$ where $F$ represents a functional form. Thus, in principle, estimates of $D_o$, $N_w$, and $\mu$ (and $W$ or rain rate $R$) can be obtained from the radar measurement set of ($Z_a$, $Z_{dr}$, and $K_{dp}$). Note that $D_o$ and $D_m$ are related by $D_o/D_m = (3.67 + \mu)/(4 + \mu)$ (Ulbrich and Atlas 1998).

The effects of raindrop shape oscillations (either due to resonance maintained by vortex shedding or due to collisions) and raindrop canting (due to turbulence) will bias the retrieval of the gamma DSD parameters under the above model assumptions (e.g., see Bringi and Chandrasekar 2001, chapter 7 and references therein). Both drop oscillations and canting angle distributions tend on average to drive the effective axis ratio toward sphericity relative to equilibrium axis ratios and perfect orientation. The rain microphysics model can be improved by accounting for drop oscillations using the axis ratio versus $D$ fit proposed by Andsager et al. (1999) and by using a Gaussian canting angle distribution with a mean of 0° and a standard deviation ($\sigma$) in the range 5°–10° (Bringi and Chandrasekar 2001). For example, if the axis ratio versus $D$ relation is assumed to be linear with a slope of ($\beta$), $r = 1 - \beta D$, then $K_{dp}$ is modified as (see Bringi and Chandrasekar 2001)

$$K_{dp} = \beta N_{\nu}F_1(\mu, D_o) \exp(-2\sigma^2)$$

$$= \beta_{st} N_{\nu}F_1(\mu, D_o).$$

Gorgucci et al. (2000) recognized that drop canting and oscillations could be incorporated into an “effective” slope parameter ($\beta_{st}$) and proceeded to develop an algorithm to estimate $\beta_{st}$ from the radar measurement set ($Z_a$, $Z_{dr}$, $K_{dp}$). It is important to recognize that even if drop axis ratio is in fact a nonlinear function of $D$, it is possible to define an equivalent linear model with a slope of $\beta_{st}$ such that it results in the same $K_{dp}$ (for a given value of the product $WD_o$) as the nonlinear form. In subsequent articles, Gorgucci et al. (2001, 2002) developed algorithms (see summary in the appendix) for retrieving rain rate ($R$) as well as $D_o$, $N_{\nu}$, and $\mu$ using $\beta_{st}$ in combination with the measurement pair ($Z_a$, $Z_{dr}$). They show via comparisons with disdrometer DSD measurements that $D_o$ and $N_{\nu}$ can be retrieved with excellent accuracy (ranging from 4%–8% for $D_o$ and log$_{10}N_{\nu}$). Simulations were also used to study the effects of radar measurement error in the retrieval of $D_o$ and $N_{\nu}$, and it was found that the accuracy was still quite good (ranging from 5%–20% for $D_o$ and log$_{10}N_{\nu}$) and, more importantly, the estimates were nearly unbiased. The $\mu$ estimator was found to be less accurate, though it may be possible to distinguish between certain ranges of $\mu$, for example, $-1 \leq \mu \leq 2$ versus $\mu > 5$, which may be sufficient in practice.

The concept of an “effective” slope ($\beta_{st}$) of the mean axis ratio versus $D$ relation is an important one since drop oscillations or canting are likely to be different in, for example, tropical rain versus rain in the midlatitudes. Oscillations/canting may be suppressed when rain is formed via melting of graupel or tiny hail as compared with warm rain formation. Gorgucci et al. (2001) applied the $\beta_{st}$ concept to an unusual tropical-like flash flood-producing storm in Colorado and showed that rain-rate estimators based on $\beta_{st}$, $Z_a$, and $Z_{dr}$ resulted in better agreement with gauge data as compared with
the standard $R(K_{dp})$ algorithm (see, also, Petersen et al. 1999). May et al. (1999) also found that use of the Pruppacher and Beard equilibrium shape model ($\beta$ fixed at 0.062 mm$^{-1}$) resulted in a systematic underestimate in rainfall when using $R(K_{dp})$, as compared with a dense gauge network in the Tropics, and attributed this bias to drop oscillations causing an upward shift in mean axis ratio (toward sphericity). More recently, Fulton et al. (1999) have suggested an empirical adjustment to the $R(K_{dp})$ algorithm using a multiplicative bias correction factor, $B_i(\langle Z_{dr} \rangle)$, which they found reduced the temporal bias in rain accumulation. This correction factor, though empirical, tends to account for the tendency of drop oscillations/canting to cause an upward shift in mean axis ratio. Thus, there appears to be sufficient evidence to warrant further application of the effective $\beta$ concept to retrieve $D_o$, $N_w$, and $\mu$, as well as rain rate, and this is the principal objective of this paper.

Since $\beta_{eff}$ is estimated from the measurement set ($Z_a$, $Z_{dr}$, $K_{dp}$), and $K_{dp}$ at long wavelengths (such as S band) is known to be very noisy at low rain rates, it follows that the retrieval of the DSD parameters is only practical when the rain rate is sufficiently high (typical threshold of $Z_a \geq 35$ dBZ). At low rain rates, such as in stratiform rain, the $Z_{dr}$ also tends to be noisy so that a large areal average is necessary to reduce the measurement fluctuations. Thus, it is necessary to extend the retrieval of DSD parameters ($D_o$ and $N_w$) at low rain rates ($Z_a < 35$ dBZ), at which $K_{dp}$ and at times $Z_{dr}$ are generally too noisy to be useful; this is another goal of this paper. Finally, the last goal is to compare radar-derived rain rates against a network of gauges to illustrate the application of $\beta_{eff}$ in reducing the rainfall accumulation bias. The data sources used are the S-Pol radar, 2D-video and RD-69 disdrometers, and a network of gauges deployed for the TRMM/Brazil field campaign held in 1999 in Amazonia. Data from a squall line on 15 February 1999 are used for analysis.

3. Data analysis methods

a. S-Pol radar

The S-Pol radar is a dual-polarized radar operating at a frequency near 2.8 GHz (S band). It uses a mechanical polarization switch and two separate receivers to measure the polarimetric covariance matrix (Randall et al. 1997). The datastream used here consists of $Z_a$, $Z_{dr}$, and $\Phi_{dp}$ (differential propagation phase), which are available every 150 m in range. For each beam of data, a “good” data mask is generated based on the standard deviation of $\Phi_{dp}$ over 10 consecutive gates (<10°), the copolar correlation coefficient ($\rho_{dp} \approx 0.9$), and the signal-to-noise ratio (SNR $\geq 3$ dB). These thresholds tend to eliminate nearly all nonmeteorological echoes. The $\Phi_{dp}$ range profile is filtered according to Hubbert and Brugi (1995). Once the filtered $\Phi_{dp}$ range profile is obtained, $K_{dp}$ is calculated based on the slope of a least squares fit line to the filtered $\Phi_{dp}$ profile in an adaptive manner (30 consecutive range samples are used in the linear fit for $Z_a < 35$ dBZ; 20 for $35 < Z_a \leq 45$ dBZ; and 10 for $Z_a > 45$ dBZ). The $Z_a$ is corrected for attenuation using the algorithm of Testud et al. (2000) adapted for S band, while $Z_{dr}$ is corrected for differential attenuation using a self-consistent, constraint-based algorithm described by Bringi et al. (2001b). Corrections are significant only when $\Phi_{dp} \approx 50^\circ$. The corrected $Z_a$ and $Z_{dr}$ range profiles are averaged in range using uniform block averaging for the different $Z_a$ ranges described earlier. The effective $\beta$ is calculated based on the averaged $Z_a$, $Z_{dr}$, and $K_{dp}$ data, and $D_o(\beta_{eff}, Z_a, Z_{dr})$, $N_w(\beta_{eff}, Z_a, Z_{dr})$, and $R(\beta_{eff}, Z_a, Z_{dr})$ are calculated using the algorithms given in the appendix. The threshold for computing $\beta_{eff}$ is based on $Z_a \approx 35$ dBZ, $Z_{dr} \approx 0.2$ dB, and $K_{dp} \approx 0.3$ km$^{-1}$.

When the $Z_a < 35$ dBZ, which occurs for light rainfall (e.g., stratiform rain), a different retrieval method, which is based on disdrometer measurements, is proposed for $D_o$ and $N_w$.

b. Disdrometer

A 2D-video disdrometer (Schönhuber et al. 1995) and a RD-69 disdrometer (Joss and Waldvogel 1967) were available during the TRMM/Brazil field campaign. These two instruments were sited close to each other and near the National Oceanic and Atmospheric Administration (NOAA) profilers. (For a description of the 2D-video instrument refer to the Web site http://www.disdrometer.at.) Because of technical difficulties the 2D-video disdrometer was not operating continuously through the field campaign. However, it is believed that representative samples of DSD measurements were made in convective rain (164 2-min-averaged DSD samples) and stratiform rain (49 2-min-averaged samples). The stratification of rain types was based on manual examination of profiler reflectivity/velocity images, for example, absence or presence of a “bright band.” RD-69 disdrometer data were available more or less throughout the field campaign. In this study, the RD-69 DSD data were selected during those times when the 2D-video was operational. The 2D-video disdrometer has a large sample volume relative to the RD-69 disdrometer (Tokay et al. 1999). Intercomparisons between these two instruments are available in Tokay et al. (1999) and Williams et al. (2000). The latter study demonstrates the underestimation of small drops (<1.5 mm) by the RD-69 at higher rain rates (reflectivity $\geq 40$ dBZ), which causes the mass-weighted mean diameter ($D_o$) and $R$ to be biased low relative to the 2D-video disdrometer. At low rain rates the $D_o$ and $R$ from both instruments are in very good agreement. The underestimation of small drops by the RD-69 also tends

1 A detailed description of the instrumentation can be found online at http://radarmet.atmos.colostate.edu/lab_trmm.
This method separates the estimation of \( N_w \) (mm) are calculated first, after which the Bringi et al. (2001a). In short, the water content (using a method previously described in the appendix of shape, from the normalizing parameters \( D_m \) between log normalized DSD is constructed as

\[
N = \frac{Z_h}{D_o^2} \text{mm}^2
\]

for Rayleigh scattering by spherical drops. The exponent theoretically expected value of 1/7 is slightly smaller because the drops are oblate. It is important to note that the exponent is accurately determined from the plot of \( Z_h/N_w \) versus \( D_o \) as compared to the determination of both the multiplicative coefficient and the exponent from a plot of \( Z_h \) versus \( D_o \), which displays much more scatter. The disdrometer analysis in Fig. 1 shows that

\[
D_o = 1.513 (N_w)^{0.136} Z_h^{0.136} = \gamma Z_h^{0.136} \tag{3}
\]

(note that \( Z_h \) here is in mm\(^6\) m\(^{-3}\)). This fit will be used to retrieve \( D_o \) from \( Z_h \) for light rain rates when the measurement of \( Z_h \) falls below the threshold of 0.2 dB.

![Figure 1](image1.png)

**Fig. 1.** Scatterplot of \( Z_h/N_w \) vs \( D_o \) based on gamma fits to 2D-video (in convective and stratiform rain) and RD-69 (in stratiform rain) disdrometer data obtained during the TRMM/Brazil field campaign. Each data point (+) refers to a 2-min-averaged DSD to which a gamma DSD is fitted. There are 164 2-min samples of convective rain from the 2D-video and 152 2-min samples of stratiform rain from the 2D-video and RD-69. The power law fit is also shown.

![Figure 2](image2.png)

**Fig. 2.** As in Fig. 1, except \( Z_h \) vs \( D_o \). Power law fit is based on rain rates exceeding 2 mm h\(^{-1}\).
However, the estimate of $\gamma$ will be obtained in a manner to be described later [see (6b)].

Figure 2 shows a plot of $Z_a$ versus $D_o$. The power law fit to these data result in,

$$D_o = 1.81(Z_a)^{0.486}.$$  \hfill (4)

For radar measurements with $Z_a < 35$ dBZ and $Z_d \geq 0.2$ dB, the $D_o$ is retrieved using (4), and $N_w$ is retrieved from (3), which is expressed as

$$N_w = \frac{21Z_a}{D_o^{0.353}} \text{ mm}^{-1} \text{ m}^{-3},$$  \hfill (5)

where $Z_a$ is in mm$^6$ m$^{-3}$.

For radar measurements with $Z_a < 35$ dBZ and $Z_d < 0.2$ dB the following method is proposed. Using (3) and (4), $D_o$ can be eliminated to obtain a relation between $Z_a$ and $Z_d$ of the form $Z_a = \alpha Z_d^\delta$ where $\delta$ is the ratio of the exponents in (3) and (4) given by $\delta = 0.136/0.486 = 0.28$. The coefficient $\alpha$ can be determined, in practice, from all radar measurements of $Z_a$ with corresponding $Z_d < 35$ dBZ. The estimate $\hat{\alpha}$ is easily determined as $\hat{\alpha} = (Z_a)/(Z_d^{0.35})$ where angle brackets denote a spatial average; note that $Z_a$ is in dB and $Z_d$ in mm$^6$ m$^{-3}$. Figure 3 shows a scatterplot of $Z_a$ versus $Z_d$ for radar data in stratiform rain from 15 February 1999 as well as the power law fit with $\hat{\alpha} = 0.0741$ (this value is close to that obtained from disdrometer analysis, 0.0842). The essential hypothesis is that even though $Z_a$ measurements are noisy at low reflectivities and tend on average to near 0 dB at very light rain rates, scattering simulations based on disdrometer DSD samples and a rain model with $B_{model}$ as in (A5) indicate that the mean relation should follow a power law of form $Z_a = \alpha Z_d^{0.38}$. Thus, a method exists for retrieving $D_o$, even if the individual resolution volumes have $Z_d < 0.2$ dB (the prespecified threshold). First, $\hat{\alpha}$ is determined from the data, which includes all $Z_d$ values with $Z_a < 35$ dBZ (the lower bound of $Z_a$ is set to 0 dBZ here). Next, (4) is used with $Z_d = \hat{\alpha}(Z_a)^{0.28}$ to arrive at

$$D_o = 1.81(\hat{\alpha})^{0.468}(Z_a)^{0.136}; \text{ mm}^{-1} \text{ m}^{-3}$$  \hfill (6a)

where $\hat{\gamma} = 1.81(\hat{\alpha})^{0.468}$. Subsequently, $N_w$ is obtained from (3) as

$$N_w = (1.513/\hat{\gamma})^{7.35}; \text{ mm}^{-1} \text{ m}^{-3}$$  \hfill (7)

Note that this retrieval $N_w$ can be interpreted as an estimate of the expected value of $N_w$, since $\hat{\alpha}$ is an estimate of the expected value of $\alpha$. For example, the expected value of $N_w$ for the stratiform rain data in Fig. 3 is 2920 mm$^{-1}$ m$^{-3}$. If $\sigma_w$ is the standard deviation of $\hat{\alpha}$ then a range of $N_w$ values is to be expected, and as a first approximation $N_w$ may be assumed to be uniformly distributed between $[N_w, N_w]$ where the lower and upper values correspond to using $\hat{\alpha} + \sigma_w/2$ and $\hat{\alpha} - \sigma_w/2$ in (7) and noting that $\hat{\gamma} = 1.81(\hat{\alpha})^{0.468}$. Analysis of a large number of volumes of radar measurements of $(Z_a, Z_d)$ pairs in both convective and stratiform rain types on 15 February 1999 suggests that $\sigma_w$ is around 0.015, which puts the 1/2 standard deviation bounds of $N_w$ in the range $2100-4300$ mm$^{-1}$ m$^{-3}$. To demonstrate that these assumptions are reasonable, Fig. 4 shows a histogram of $\log_{10}(N_w)$ from the combined set of 2D-video and RD-69 data in stratiform rain (composite from different days during TRMM/Brazil). Note from Fig. 4 that the $N_w$ values in stratiform rain range between 250 and 6000 mm$^{-1}$ m$^{-3}$ with the mode being 2500 mm$^{-1}$ m$^{-3}$.

To summarize the retrieval of $D_o$ and $N_w$ from radar measurements, if the measurement set $(Z_a, Z_d, K_{dp})$ exceeds the thresholds of 35 dBZ, 0.2 dB, and 0.3° km$^{-1}$, respectively, then the algorithms using $B_{model}$ as described in the appendix are used. If $Z_a < 35$ dBZ and $Z_d \geq 0.2$ dB, then $D_o$ and $N_w$ are retrieved via (4) and (5), re-
respectively. If $Z_h < 35$ dBZ and $Z_{dr} < 0.2$ dB, then $D_o$ is retrieved using (6) and $N_w$ using (7), with the added provision of distributing $N_w$ uniformly in a prescribed range. The DSD shape parameter ($\mu$) is not retrieved in this case and is set to zero. The rain rate is derived assuming $\mu = 0$ (exponential shape) and using the retrieved $N_w$ and $D_o$ and the terminal velocity relation $v = 3.78D^{0.67}$ from Atlas and Ulbrich (1977).

4. Results from 15 February 1999

a. Statistics of $D_o$ and $N_w$

On 15 February 1999 of the TRMM/Brazil field campaign, a squall line formed to the east of the S-Pol radar and moved westward, crossing the measurement area over a period of 4 hours (0300–0800 UTC; all times henceforth will be UTC). The squall line was well organized as a north–south line during the early phase (0300–0400) but became disorganized past 0430 with a number of strong cells embedded within a large area of weak echo. Past 0700, there was a transition from convective to primarily stratiform rain. Figure 5 shows the squall line as a PPI of $Z_h$ (in dBZ) at 0350. The polar area marked in the figure refers to the area where retrieval of $D_o$ and $N_w$ was performed, and is presumed to be representative of strong convective rain within the squall line. Figure 6 shows histograms of radar-derived (a) $D_o$ and (b) $\log_{10} N_w$ for rain rates $R < 10$ (mm h$^{-1}$), while similar histograms for $R \geq 10$ (mm h$^{-1}$) are shown in Fig. 7. Figure 6a shows that the spread in $D_o$ is relatively large at the lower rain rates compared to Fig. 7a, while the modal $N_w$ in Fig. 6b is near 1000 mm$^{-2}$ m$^{-3}$ as compared to 20 000 mm$^{-2}$ m$^{-3}$ in Fig. 7b. The lower concentration of larger drops for $R < 10$ mm h$^{-1}$ likely represents nonequilibrium distributions similar to data from positive $Z_{dr}$ columns (Caylor and Illingworth 1987; Bringi et al. 1991), whereas the $D_o$, $N_w$ histograms for $R > 10$ mm h$^{-1}$ reflect data from more mature rainshafts (narrower spread in $D_o$). Figure 8 shows $D_o$ and $N_w$ versus rain rate; also plotted are

---

2 S-Pol radar images for this day at 10-min intervals can be viewed online at http://www.atd.ucar.edu/rs/TRMM-LBA/quicklook/990215.
data from the 2D-video disdrometer in convective rain from all times during the TRMM/Brazil project during which it was operational. The radar retrievals do not show any obvious functional dependence of either $D_o$ or $N_w$ on rain rate, except possibly for $D_o$ at the very lowest rates. The disdrometer $D_o$ does show an increasing trend with $R$ (for $R \leq 5$ mm h$^{-1}$) in agreement with the radar-retrieved $D_o$. There is general agreement between the disdrometer and radar retrievals with regard to the spread of $D_o$ and $N_w$, even though the radar retrievals are from a specific convective area of the 15 February squall line, whereas the 2D-video data are from a small sample of different types of convective rain in the same region.

Figure 9 shows the histogram of $D_o$ and $\log_{10}N_w$ from an area of stratiform rain at 0756 on 15 February. Vertical sections of $Z_h$ and $Z_{dr}$ (not shown here) indicated a “brightband” feature, and there was no convection that could be interpreted from the images. The spread of $D_o$ around its mode is now significantly smaller as compared to Fig. 6a. The spread of $N_w$ around its mode (modal value $\approx 2000$ mm$^{-1}$ m$^{-3}$) is also smaller in stratiform rain compared with Fig. 6b. The $D_o$ and $N_w$ histograms in stratiform rain are generally comparable to those derived from airborne imaging probes by Testud et al. (2001) for stratiform rain during Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE) (see their Fig. 3). In particular, their mean and standard deviation of $D_o$ (1.21 and 0.28 mm, respectively) can be compared with Fig. 9a (corresponding values of 1.34 and 0.24 mm). Similarly, for the mean and standard deviation of $\log_{10}N_w$, Testud et al. (2001) obtained 3.48 and 0.5 versus 3.31 and 0.28 from Fig. 9b. It is, in fact, remarkable that these statistics for a sample of stratiform rain from Brazil derived by polarimetric radar generally agree with a more complete ensemble of stratiform rain from TOGA COARE, despite large sample volume differences. Of course, it is well known that convective rain characteristics over land and ocean are very different, and the histograms for convective rain in Fig. 6 do not agree with the Testud et al. (2001) analysis of convective rain from TOGA COARE for $R < 10$ mm h$^{-1}$. However, the statistics for $R > 30$ mm h$^{-1}$ are in good agreement, as summarized in Tables 1 and 2.

One possible reason for the general agreement of the statistics in stratiform rain from these two climatic regimes may be due to similarity in the dominant microphysical processes leading to rain formation (i.e., rain formed from melting aggregates). As for the agreement in convective rain in the higher rain-rate regime ($R > 30$ mm h$^{-1}$), it may be that the microphysical processes leading to an equilibrium-type DSD are similar in the two regimes (e.g., Hu and Srivastava 1995).
The biggest difference in Tables 1 and 2 is related to the case of convective rain with $R < 10 \text{ mm h}^{-1}$, which from the radar perspective is characteristic of a lower concentration of relatively larger drops as compared with the Testud et al. (2001) results and with the stratiform rain results. This is not surprising, given that warm rain processes as well as drop sorting (Carbone and Nelson 1978; Atlas et al. 1999) are likely to be dominant in the updraft area of the convective portion of the squall line leading to nonequilibrium-type DSD spectra, similar to those found in positive $Z_{dr}$ columns. Stronger updraft over land versus ocean could be one reason why the statistics of $D_o$ and $N_w$ in the low rain-rate category ($R < 10 \text{ mm h}^{-1}$) from the radar retrievals are so different from the convective TOGA COARE statistics derived by Testud et al. (2001).

Figure 10 shows radar-retrieved $D_o$ and $N_w$ versus $R$ for stratiform rain; also plotted are the 2D-video and RD-69 data from samples of stratiform rain during times when the 2D-video was operational. Good agreement may be noted with regard to the range of $D_o$ and $N_w$ predicted by radar and disdrometer at very low rain rates ($R < 2 \text{ mm h}^{-1}$). There appears to be a functional relation between $D_o$ and $R$ at these low rain rates but for $R > 5 \text{ mm h}^{-1}$, if there is any correlation between $D_o$ and $R$ and between $N_w$ and $R$, it is very weak.

### Table 1. Statistics of $D_o$

<table>
<thead>
<tr>
<th>Rain type</th>
<th>Mean and (std dev) of $D_o$ from S-Pol radar</th>
<th>Mean and (std dev) of $D_o$ from Testud et al. (2001)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convective, with $R &gt; 30 \text{ mm h}^{-1}$</td>
<td>1.75 mm (0.34)</td>
<td>1.66 mm (0.33)</td>
</tr>
<tr>
<td>Stratiform</td>
<td>1.34 mm (0.24)</td>
<td>1.21 mm (0.26)</td>
</tr>
<tr>
<td>Convective, with $R &lt; 10 \text{ mm h}^{-1}$</td>
<td>1.76 mm (0.42)</td>
<td>1.13 mm (0.27)</td>
</tr>
</tbody>
</table>

### Table 2. Statistics of $\log_{10}N_w$

<table>
<thead>
<tr>
<th>Rain type</th>
<th>Mean and (std dev) of $\log_{10}N_w$ from S-Pol radar</th>
<th>Mean and (std dev) of $\log_{10}N_w$ from Testud et al. (2001)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convective, with $R &gt; 30 \text{ mm h}^{-1}$</td>
<td>4.22 (0.34)</td>
<td>4.20 (0.33)</td>
</tr>
<tr>
<td>Stratiform</td>
<td>3.31 (0.28)</td>
<td>3.48 (0.52)</td>
</tr>
<tr>
<td>Convective, with $R &lt; 10 \text{ mm h}^{-1}$</td>
<td>2.81 (0.42)</td>
<td>4.00 (0.52)</td>
</tr>
</tbody>
</table>
Fig. 11. (a) Polar area shows the gauge network. The 2D-video and RD-69 disdrometers and the NOAA profiler were located at the Ji Parana airport. (b) Time–height profile of reflectivity from the NOAA 915-MHz vertically pointing Doppler radar (or profiler).
the gauge network. Three convective rain cells can be identified in the time profile (abscissa is time in fractions of an hour; e.g., 4.5 means 0430 UTC) with D₀'s in the range 1.4–1.5 mm, N₀ around 15 000 mm⁻¹ m⁻³, and μ around 3–5 near the rain cell peaks. In the stratiform rain between the rain cell peaks, the D₀'s are around 0.7 mm, N₀ is in the range 1500–2500 mm⁻¹ m⁻³, and μ is around 1–2 (e.g., centered around times at 0515 UTC, or 5.25 and 7.125 h). Vertical profiles of the reflectivity from the

NOAA profiler showed a bright band between the rain cell peaks (see Fig. 11b). It is emphasized that the radar-derived D₀, N₀, and μ values are area-averaged (over the polar area shown in Fig. 11a).

To illustrate the use of the effective β in reducing the bias in rainfall accumulation, the areal R (or AR) is derived using differential propagation phase (Φₒ) using the algorithm proposed and evaluated by Bringi et al. (2001a):

\[
AR = \frac{c}{2} \int_{r₁}^{r₂} \left[ r₂Φₒ(r₂, θ) - r₁Φₒ(r₁, θ) \right] - \int_{r₁}^{r₂} Φₒ(r, θ) dr \, dθ
\]  

(8)

In the above, \(r₁, r₂, θ₁,\) and \(θ₂\) describe the limits of the polar area (see Fig. 11). For a given beam with constant azimuth angle θ, AR depends on the boundary values of \(Φₒ\) as well as the area under the \(Φₒ\) range profile. As the azimuthal angle changes from \(θ₁\) to \(θ₂\), an areal sweep of \(Φₒ\) over the rain region occurs naturally, performing a spatial integration of the rainfall. A linear relation between \(R\) and \(Kₒ\) of the form \(R = cKₒ\) is assumed to be valid locally to derive (8). Since the actual relation is nonlinear, a piecewise linear approximation is proposed, as shown in Fig. 13. The rain model used in the simulations is described in relation to (A5) in the appendix. Because a sufficiently large database of disdrometer data was not available from TRMM/Brazil, the simulations in Fig. 13 are based on an entire season of rain DSD measurements made with the RD-69 disdrometer near Darwin, Australia. When AR is divided by the polar area it will be termed the mean areal rain rate (\(R\)). The multiplicative coefficient \(c\) in (8) is selected based on the average \(Kₒ\) along a specific beam according to the piecewise linear fit in Fig. 13. This approach avoids the necessity of assuming a priori that \(Kₒ\) is constant along the various beams (Ryzhkov et al. 2000). The cumulative rainfall using the fixed rain model with \(β_{\text{model}}\) as in (A5) results in a bias (overestimate) of around 20% when compared with the gauge network accumulation, as illustrated in Fig. 14. Simulations performed by Gorgucci et al. (2001) show that the \(R(Kₒ)\) estimator varies as \(R = cKₒβ_{\text{eff}}\). To correct for changing \(β_{\text{eff}}\) relative to \(β_{\text{model}}\) in (A5), the modal value of \(β_{\text{eff}}\) is first computed over the polar area in Fig. 11a from radar measurements of \(Zₒ, Zₕₒ,\) and \(Kₒ\) (see appendix), and this is done as a function of time. Next, the modal value of \(Zₒ\) (\(Zₒ\) in linear scale) is used to calculate \(β_{\text{model}}\) using (A5). The areal rain rate is then adjusted by the factor \(β_{\text{model}}/β_{\text{eff}}\). During stratiform rain periods, the \(Zₒ, Kₒ,\) and \(Zₕₒ\) generally fall below the threshold required for the calculation of \(βₒ\) and, thus, no adjustment is done during these periods. Figure 14 also shows the rain accumulation after correcting for \(β_{\text{eff}}\), and the accumulation bias has now been reduced to <10%. The main advantage of using the \(β_{\text{eff}}\)-based correction is that the effects of drop oscillations and/or drop canting is implicitly accounted for in the rain-rate algorithm. One simply starts with theoretical rain model or disdrometer DSD data for the regime (or, similar to the regime under consideration) to arrive at a first approximation for the \(R-Kₒ\) relation, and then the radar data are used to correct for deviations of \(β_{\text{eff}}\) from the assumed model value \([β_{\text{model}}] \) in a relatively straightforward manner. The results shown here suggest that the use of disdrometer data from Darwin to arrive at the piecewise linear fit in Fig. 13 is valid for setting the initial \(R-Kₒ\) relation, and, in fact, demonstrates the power of the effective β method. Such an approach avoids the use of empirical-based methods suggested by Fulton et al. (1999).

5. Summary and conclusions

A method is proposed for retrieving the parameters \(Dₒ\) and \(Nₒ\) of a normalized gamma DSD using radar measurements of \(Zₒ, Zₕₒ,\) and \(Kₒ\) at S band (frequency near 3 GHz). The algorithms based on the effective β concept derived by Gorgucci et al. (2001, 2002) have been extended to low rain rates where both \(Kₒ\) and \(Zₕₒ\) tend to be noisy and preclude an accurate estimate of \(β_{\text{eff}}\). Disdrometer data and scattering simulations are used to retrieve the \(Dₒ\) and \(Nₒ\) at low rain rates. Thus, the combined method retrieves \(Dₒ\) and \(Nₒ\) over the full range of rain rates detectable by radar.

Statistics of \(Dₒ\) and \(Nₒ\) in the form of histograms were presented for convective and stratiform samples of rain data in one squall-line event from the TRMM/Brazil field campaign using the S-Pol radar. The mean and standard deviation of \(Dₒ\) and \(log_{₁₀}Nₒ\) in stratiform rain compared favorably with similar statistics presented by Testud et al. (2001) based on airborne measurements during TOGA COARE. In convective rain, the statistics

\[Zₒ = \frac{1}{2} \int_{r₁}^{r₂} \left[ r₂Φₒ(r₂, θ) - r₁Φₒ(r₁, θ) \right] - \int_{r₁}^{r₂} Φₒ(r, θ) dr \, dθ
\]  

(8)

Vertical profiles of reflectivity from the NOAA profiler can be viewed online at http://www.aff.noaa.gov/WWW/HWD/pubdocs/TropDyn/trmm/td/td082000105/1999046.gif.
for the higher rain-rate category ($R \gtrsim 30 \text{ mm h}^{-1}$) also agreed with Testud et al. (2001). This agreement, which is based on a very limited sample of radar data, may suggest that the dominant microphysical processes are similar, for example, melting of snow to form stratiform rain or the tendency to equilibrium-like distributions in heavier convective rain. The statistics were, however, quite different in lighter convective rain ($R < 10 \text{ mm h}^{-1}$) with much larger mean $D_o$ over land as compared with TOGA COARE and, correspondingly, much lower values of $N_w$. It will be necessary to repeat the radar retrievals before firmer conclusions can be made, but the methodology proposed herein is an important step and makes it possible now to proceed with such studies in different climatic regimes where high quality dual-polarized radar data are available: for example, Darwin,

Australia; the South China Sea Monsoon Experiment (SCSMEX); the Texas–Florida Underflight Experiment (TEFLUN-B) in Florida; and the Severe Thunderstorm and Electrification Project (STEPS) in Colorado.

The functional behavior of the retrieved $D_o$ and $N_w$ with rain rate in samples of stratiform and convective rain was studied and compared with samples of 2D-video and RD-69 disdrometer measurements in similar rain types during TRMM/Brazil. The agreement, in terms of the range of $D_o$ and $N_w$ values, was good. At low rain rates ($R < 5 \text{ mm h}^{-1}$), there appeared to be a correlation between $D_o$ and $R$. Weak correlation was found between $D_o$ and $R$ or $N_w$ and $R$ in both the radar retrievals as well as the disdrometer data for $R < 5 \text{ mm h}^{-1}$. Generally, these results are supportive of the Testud et al. (2001) analysis of airborne DSD data from TOGA COARE.

The 15 February squall-line event analyzed in this paper also shows how the $\beta_{\text{eff}}$ estimate was used to remove the bias in accumulated rainfall when using the areal rain-rate estimator based on differential propagation phase via comparison with a gauge network deployed over a 5 km $\times$ 5 km area. The time profile of areally averaged $D_o$, $N_w$, and $\mu$ was determined as three consecutive convective rain cells moved over the gauge network area with periods of stratiform rain in between. Within the convective rain cells the $D_o$ values ranged from 1.4 to 1.5 mm, with $N_w$ around 15 000 mm$^{-1}$ m$^{-3}$ and $\mu$ around 3–5. In stratiform rain the corresponding ranges were 0.6–0.7 mm, $N_w$ around 1500–2500 mm$^{-1}$ m$^{-3}$, and $\mu$ around 1–2. These ranges are in general agreement with past studies (e.g., Cifelli et al. 2000; Tokay and Short 1996). Future research will be directed toward comparison with DSD retrievals from profilers (e.g., Williams et al. 2000).

The success of the proposed methodology in provid-
ing for unbiased estimates of \(D_o\), \(N_w\), or \(\mu\) relies on accurate calibration of the radar; specifically, the accuracy in \(Z_h\) should be 1 dB or better and for \(Z_k\) it should be 0.1 dB or better. Also, to retrieve the gamma DSD parameters at low rain rates, disdrometer DSD samples in convective and stratiform rain should be available for setting algorithm coefficients/exponents, specifically in the power laws \(D_o = aZ_h^b\) and \(Z_o/N_w = cD_h^{3.35}\). Correction for attenuation effects will be important at C band and higher frequencies. Even at S band, correction of \(Z_h\) and \(Z_k\) data is important when the differential propagation phase becomes large (\(\approx 50^\circ\)). Techniques to correct for attenuation are now available (Testud et al. 2000; Bringi et al. 2001b; Smyth and Illingworth 1998).

**Acknowledgments.** This research was supported by the NASA/TRMM Grants NAG5-7717 and NAG5-7876. VNB also acknowledges support from the National Science Foundation via ATM-9612519 for analysis of the 2D-video disdrometer data. Drs. John Hub-

bert, John Beaver, and Steve Bolen of CSU were instrumental in supporting the operation of the NASA 2D-video disdrometer during TRMM/Brazil. Dr. C. R. Williams provided the profiler data used to construct Fig. 11.

**APPENDIX**

**Retrieval Algorithm for \(D_o\), \(N_w\), and \(\mu\)**

The method of retrieving \(D_o\), \(N_w\), and \(\mu\) from \(Z_h\), \(Z_k\), and \(K_{dp}\) is summarized here from Gorgucci et al. (2001, 2002). A gamma DSD model is assumed with the following ranges for the parameters:

\[
0.5 \leq D_o \leq 3.5 \text{ mm} \quad (A1)
\]

\[
3 \leq \log_{10} N_w \leq 5 \quad (A2)
\]

\[
-1 \leq \mu \leq 5 \quad (A3)
\]

with the additional constraint that \(R < 300 \text{ mm h}^{-1}\). The parameters \(D_o\), \(\log_{10}N_w\), and \(\mu\) are varied uniformly over their respective ranges to form a large table of \(D_o\), \(N_w\), and \(\mu\). Scattering calculations are performed at 2.8 GHz over a range of \(\beta_{dp}\), and nonlinear regression is used to develop an algorithm for \(\beta\) (henceforth, the subscript “eff” will be dropped) in terms of \(Z_h\), \(Z_k\), and \(K_{dp}\):

\[
\beta = 2.08Z_h^{-0.365}K_{dp}^{0.38}Z_h^{0.965} \quad (A4)
\]

where \(Z_h\) is in mm\(^6\) m\(^{-3}\), \(K_{dp}\) in \(^\circ\) km\(^{-1}\), and \(\xi_h\) is the differential reflectivity expressed as a ratio (\(Z_{bh} = 10 \log_{10} \xi_h\)).

Simulations using gamma fits to measured drop size distributions (see section 3b) and scattering calculations at 2.8 GHz of \(Z_h\), \(Z_k\), and \(K_{dp}\), assuming (i) mean axis ratio versus \(D\) fit of Andsager et al. (1999) for \(1 \leq D \leq 4\) mm and Beard and Chuang (1987) for \(D < 1\) and \(D > 4\) mm, (ii) Gaussian canting angle distribution with mean 0\(^\circ\) and \(\sigma = 10^\circ\), and (iii) size integration up to \(D_{max} = 2.5D_m\), show that \(\beta_{model}\) using (A4) is generally clustered around 0.045–0.0475 mm\(^{-1}\) but is a nonlinear function of \(D_o\) (or equivalently \(\xi_h\)). A nonlinear fit to the simulations yields

\[
\beta_{model} = 0.0049(\xi_h)^2 - 0.0043(\xi_h) + 0.0433; \quad \xi_h > 1. \quad (A5)
\]

The median volume diameter is then derived as

\[
D_o = aZ_h^b(\xi_h)^c \quad (A6)
\]

where,

\[
a = 0.56, \quad (A7)
\]

\[
b = 0.064, \quad (A8)
\]

\[
c = 0.024\beta^{-1.42}. \quad (A9)
\]

The \(N_w\) is derived as

\[
\log_{10} N_w = aZ_h^b(\xi_h)^c \quad (A10)
\]

where now

\[
a = 3.29, \quad (A11)
\]

\[
b = 0.058, \quad (A12)
\]

\[
c = -0.023\beta^{-1.389}. \quad (A13)
\]

and \(\mu\) is derived as

\[
\mu = \frac{aD_o^c}{(\xi_h - 1) - c(\xi_h)^d} \quad (A14)
\]

where

\[
a = 200\beta^{1.89}, \quad (A15)
\]

\[
b = 2.23\beta^{0.039}, \quad (A16)
\]

\[
c = 3.16\beta^{-0.046}, \quad (A17)
\]

\[
d = 0.374\beta^{-0.355}. \quad (A18)
\]

The rain rate is derived as

\[
R = 0.105\beta^{0.865}Z_h^{0.93}(\xi_h)^c \quad (A19)
\]

where

\[
c = -0.585\beta^{-0.703}. \quad (A20)
\]

In this paper, the thresholds used are \(Z_h \geq 35\) dBZ, \(Z_k \geq 0.2\) dB, and \(K_{dp} \geq 0.3^\circ\) km\(^{-1}\) for retrieval of \(D_o\), \(N_w\), \(\mu\), and \(R\) using the above algorithms.

**REFERENCES**


———, R. C. Srivastava, and R. S. Sekhon, 1973: Doppler radar char-


