An Areal Rainfall Estimator Using Differential Propagation Phase: Evaluation Using a C-Band Radar and a Dense Gauge Network in the Tropics

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ABSTRACT

An areal rainfall estimator based on differential propagation phase is proposed and evaluated using the Bureau of Meteorology Research Centre (BMRC) C-POL radar and a dense gauge network located near Darwin, Northern Territory, Australia. Twelve storm events during the summer rainy season (December 1998-March 1999) are analyzed and radar-gauge comparisons are evaluated in terms of normalized error and normalized bias. The areal rainfall algorithm proposed herein results in normalized error of 14% and normalized bias of 5.6% for storm total accumulation over an area of around 100 km². Both radar measurement error and gauge sampling error are minimized substantially in the areal accumulation comparisons. The high accuracy of the radar-based method appears to validate the physical assumptions about the rain model used in the algorithm, primarily a gamma form of the drop size distribution model, an axis ratio model that accounts for transverse oscillations for $D \leq 4$ mm and equilibrium shapes for $D > 4$ mm, and a Gaussian canting angle distribution model with zero mean and standard deviation $10^\circ$. These assumptions appear to be valid for tropical rainfall.

1. Introduction

The differential propagation phase ($\Phi_{dp}$) between horizontal and vertical polarization due to rain at microwave frequencies is now well known to be an important radar measurement, in particular for estimating rain amounts (Seliga and Bringi 1978; Sachidananda and Zrnić 1987). In particular, $\Phi_{dp}$-based methods offer many practical advantages over power-based methods, for example, immune to radar system gain variations, attenuation effects, beam blockage (Zrnić and Ryzhkov 1996). The $\Phi_{dp}$ field can be naturally expressed in polar coordinates ($r, \theta$), where $r$ is the radar range and $\theta$ is the azimuth angle, when the radar scans the rain area at low elevation angle in the usual plan position indicator (PPI) mode. It was recognized by Raghavan and Chandrasekar (1994) in the context of area-time integral methods, that the azimuthal sweep of $\Phi_{dp}$ across the rain area can be viewed as an areal integration of the instantaneous rain-rate field. Thus, to calculate the mean areal rain rate ($\overline{R}$), it is not necessary to know the specific differential phase [$K_{dp} = (1/2)\Delta \Phi_{dp}/dr$], which is a “noisy” measure and involves substantial smoothing of the $\Phi_{dp}$ field (e.g., Hubbert and Bringi 1995). In particular, large gradients of reflectivity can cause the estimated $K_{dp}$ to be biased (Gorgucci et al. 1999). The areal rainfall method using $\Phi_{dp}$ for estimating $\overline{R}$ does not involve the prior estimation of $K_{dp}$. Therefore, this method preserves all the practical advantages of the $\Phi_{dp}$ measurement and avoids the major disadvantage of computing $K_{dp}$, the only trade-off being that an areal estimate of $\overline{R}$ is available.

Another advantage of the areal $\Phi_{dp}$ method is related to validation using a dense network of gauges. It is well known that the usual method of comparing radar rain rates over single gauges is fraught with large uncertainty, and that a significant portion of the variance may be due to gauge sampling error, that is, point gauge estimates cannot accurately represent rainfall over typical radar pixel sizes (e.g., $2 \text{ km} \times 2 \text{ km}$), see, for example, Anagnostou et al. (1999). However, the gauge sampling error can be substantially reduced if a dense network of gauges is used, and, hence, the mean areal rainfall from the network can be used to validate areal $\Phi_{dp}$ algorithms more robustly as compared to individual radar-gauge comparisons. It follows that the physical basis of areal $\Phi_{dp}$ algorithms can be evaluated by intercomparison with a dense gauge network. In particular, since a parametric form is often used to convert from areal $\Phi_{dp}$ to $\overline{R}$, the parameterization errors (arising from
drop size distribution fluctuations and choice of drop shape models) are likely to dominate the variance between the radar and gauge comparisons.

In this paper, an areal $\Phi_{dp}$ estimator is proposed that is philosophically somewhat different from Ryzhkov et al. (2000). In order to estimate the mean areal rain rate from the azimuthal sweep of $\Phi_{dp}$, a linear $R-K_{dp}$ relation is assumed to be valid locally ($R = cK_{dp}$). However, from physical considerations, the relation between $R$ and $K_{dp}$ at long wavelengths is somewhat nonlinear (Sachidananda and Zrnić 1987). This nonlinearity is accounted for by adopting a piecewise linear fit to specify the $R-K_{dp}$ relation. On the other hand, the areal $\Phi_{dp}$ estimator of Ryzhkov et al. (2000) preserves the nonlinear form for $R-K_{dp}$, but assumes that $K_{dp}$ is constant along radials intercepting the area of interest. Model simulations are used to compare these two estimators using various range profiles of $K_{dp}$.

Validation of the areal $\Phi_{dp}$ algorithm developed in this paper is based on comparison with a dense gauge network located near Darwin, Northern Territory, Australia. The Bureau of Meteorology Research Center (BMRC) C-POL radar (frequency near 5.5 GHz) located near Darwin provided the $\Phi_{dp}$ data (Keenan et al. 1998). Twelve storm events are analyzed from the summer rainy season (December 1998–March 1999), which included a variety of rainfall types.

This paper is organized as follows. Background material is provided in section 2 on the two areal rainfall estimators, and model simulations are used to understand the differences between these two estimators. Radar data processing details are dealt with in section 3, together with a brief discussion of the radar–gauge comparison methodology. In section 4 the result of the radar–gauge comparisons is discussed, while section 5 provides a short summary and discussion of results.

### 2. Background

The areal rainfall $AR$ can be defined as

$$AR = \iint R(x, y) \, dx \, dy,$$

(1)

where $R(x, y)$ is the instantaneous rain-rate field. The mean areal rain rate $\overline{R}$ is defined as $AR$ divided by the corresponding area. The use of polar coordinates is suitable for low-elevation angle radar data acquired in the conventional PPI scan mode. If $r$ is the range and $\theta$ is the azimuth angle, the areal rainfall in polar coordinates is

$$AR = \int_{r_1}^{r_2} \int_{\theta_1}^{\theta_2} R(r, \theta) \, dr \, d\theta.$$

(2)

If a linear relationship between $R$ and $K_{dp}$ is assumed of the form $R = cK_{dp}$, and using $K_{dp} = 1/2(d/dr)(\Phi_{dp})$, (2) can be expressed as

$$AR = \frac{c}{2} \int_{\theta_1}^{\theta_2} d\theta \int_{r_1}^{r_2} \frac{d}{dr} \Phi_{dp}(r, \theta) \, dr$$

(3)

$$= \frac{c}{2} \int_{\theta_1}^{\theta_2} d\theta \int_{r_1}^{r_2} r \, d\Phi_{dp}(r, \theta).$$

(4)

Integrating by parts results in

$$AR = \frac{c}{2} \int_{\theta_1}^{\theta_2} \left\{ \left[ r \Phi_{dp}(r, \theta) \right]^{r_2}_{r_1} - \int_{r_1}^{r_2} \Phi_{dp}(r, \theta) \, dr \right\} \, d\theta.$$

(5)

In the above formula, for a given beam with constant $\theta$, $AR$ depends on its boundary values at $r_1$ and $r_2$ as well as on the area under the $\Phi_{dp}$ versus range profile. As the azimuthal angle changes from $\theta_1$ to $\theta_2$, an areal sweep of $\Phi_{dp}$ over the rain region occurs naturally performing a spatial integration of the rainfall. Thus, it is not necessary to estimate $K_{dp}(r)$, which is a noisy field because it is obtained as one-half of the range derivative of $\Phi_{dp}(r)$. On the other hand, the $\Phi_{dp}(r)$ is easily smoothed in range (Hubbert and Brini 1995) and an accurate estimate of $AR$ is readily available. However, some error is introduced since the $R-K_{dp}$ relation is somewhat nonlinear, that is, at long wavelengths, $R = aK_{dp}^b$ where $b = 0.85$ (Sachidananda and Zrnić 1987; Chandrasekar et al. 1990). To reduce this error, a piecewise linear fit is proposed as illustrated in Fig. 1. The data points are based on 2-min averaged drop size distributions (dSD) from a disdrometer (Joss and Waldvogel 1967) located in Darwin, Northern Territory, Australia (details are provided in the appendix). These data are representative of an entire rainy season in Darwin. The $K_{dp}$ calculations are performed at a frequency of 5.5 GHz (C band) and assuming that raindrop axis ratios (for $1 \leq D \leq 4$ mm) obey the relation given in Andsager et al. (1999), and for $D < 1$ or $D > 4$ mm the relation given in Beard and Chuang (1987). In addition, a Gaussian canting angle distribution is assumed with zero mean and standard deviation of 10°. This model is believed to be applicable for tropical rainfall [see chapter 7 of Brinch and Chandrasekar (2001)]. The multiplicative coefficient $c$ in (5) is selected from the piecewise fit based on the average $K_{dp}$ value in the range interval $r_1$ to $r_2$ for any given beam. The areal rainfall estimate based on (5) and the piecewise linear fit in Fig. 1 will be termed the Colorado State University (CSU) estimate. When $AR$ in (5) is divided by the corresponding area, it will...
be termed the $\overline{R}_{\text{csu}}$ algorithm or simply the CSU algorithm.

The formula for AR proposed and evaluated by Ryzhkov et al. (2000), henceforth referred to as RZF, is based on a nonlinear relation $R = aK_{dp}^b$ (here, $a = 32.4$, $b = 0.83$ from scattering simulations described above). It is assumed that $K_{dp}(r, \theta)$ is constant for a given $\theta$. It follows that (2) can be simplified as

$$AR_{\text{ref}} = \int_{r_1}^{r_2} d\theta \int_{r_1}^{r_2} aK_{dp}^b(r, \theta)dr$$

$$= \frac{a}{2}(r_2^2 - r_1^2) \int_{\theta_1}^{\theta_2} K_{dp}^b(\theta) d\theta$$

$$= \frac{a}{2}(r_2 + r_1)(r_2 - r_1)$$

$$\times \int_{\theta_1}^{\theta_2} d\theta \left[ \Phi_{dp}(r_2, \theta) - \Phi_{dp}(r_1, \theta) \right]^b$$

$$= \frac{a}{2} \left( \frac{r_2 + r_1}{2} \right) \left[ 2(r_2 - r_1) \right]^{1-b}$$

$$\times \int_{\theta_1}^{\theta_2} \left[ \Phi_{dp}(r_2, \theta) - \Phi_{dp}(r_1, \theta) \right]^b d\theta. \quad (9)$$

When the above AR is divided by the corresponding area it will be termed the $\overline{R}_{\text{csu}}$ algorithm or simply the RZF algorithm. In this formula, only the boundary values of $\Phi_{dp}$ occur for each beam; thus it is simpler to implement as compared with (5). However, the range-weighting, which is exact in (5), is constant in (9); that is, the range-weighting is constant at $(r_2 + r_1)/2$. If $(r_2 - r_1)$ is small, then (9) becomes more exact, but the accuracy of the approximation depends not only on $(r_2 - r_1)$ but also on how different the actual $K_{dp}(r)$ profile is from being a constant. For both areal $\Phi_{dp}$ estimators, the measurement error is virtually negligible because of the areal integration and prior smoothing of the $\Phi_{dp}$ range profiles [see the appendix of Ryzhkov et al. (2000)].

To investigate the differences between (5) and (9), several model $K_{dp}(r)$ profiles are chosen as illustrated in Figs. 2a,b and simulations are used (assuming $\theta = \text{constant}$) to compare $AR_{\text{csu}}$ and $AR_{\text{ref}}$ against the “exact” value using (2) with $R = 32.4K_{dp}^{0.83}$. Figure 3 shows the percentage error in $AR_{\text{csu}}$ and $AR_{\text{ref}}$ versus the $K_{dp}$ profile number (each number from 1 to 6 corresponds to a particular profile in Fig. 2). It is clear that $AR_{\text{csu}}$,
even with the piecewise linear fit, has small error (≤10%) while AR_{r zf} can have large error (e.g., profile 5), especially when the $K_{dp}$ profile is asymmetrically located relative to the center $(r_1 + r_2)/2$, with its peak value closer to $r_1$. Moreover, from the results of Fig. 3, the error in AR_{csu} appears to fluctuate from −10% to +10% depending on the shape of the $K_{dp}$ profile whereas the error in AR_{r zf} appears to be one-sided. In practice, this implies that the error in AR_{csu} should tend to balance out as the actual $K_{dp}$ profiles will tend to vary more or less randomly in shape.

To further illustrate the error caused by constant range-weighting in AR_{r zf}, Fig. 4 shows an idealized Gaussian profile of $K_{dp}$ centered near 50 km; note that $r_1 = 40$ km is fixed whereas $r_2$ is allowed to vary from 60 to 100 km. Figure 5 shows the percentage error in AR_{csu} and AR_{r zf} versus $(r_2 − r_1)$. These idealized simulations show that the error in AR_{csu} is bounded to ≤10% whereas the error in AR_{r zf} increases with $(r_2 − r_1)$ and does not appear to be bounded. Note that these model simulations are based on a constant $r_1$. The error in AR_{r zf}, in general, will depend on both $(r_2 − r_1)$ as well as the mean range, $(r_2 + r_1)/2$.

3. Data sources and processing

This study uses data from the C-POL radar (available online at www.bom.gov.au/bmrc/meso/darwin/darwins.htm) located near Darwin, Australia, and operated by the Bureau of Meteorology Research Center (Keenan et al. 1998). The gauge network consists of 20 gauges within a 100 km² area located about 40 km southeast of the radar as illustrated in Fig. 6. The polar area used in the estimate of areal rainfall is also shown in this figure. The gauges are 203-mm-diameter tipping-bucket type and the time of accumulation of 0.2 mm of rainfall is recorded. The gauges are routinely calibrated and strict data quality control procedures were used to reject faulty gauge data (May et al. 1999). For each gauge, 1-min rain rates ($R_g$) were available as a time series. Raindrop size distribution data were also available from a disdrometer (Joss and Waldvogel 1967) located in this network; over 2000 2-min averaged $N(D)$ were available for analysis representing a variety of rain types occurring in this region (i.e., thunderstorms and continental and oceanic squall lines).

The C-POL radar data stream consists of $Z_s$, $Z_{dr}$, and $\Phi_{dp}$ at range increments of 300 m. The $\Phi_{dp}$ data are filtered in range using an adaptive filtering algorithm that eliminates local scattering-induced differential phase excursions, while retaining the monotonic increasing differential propagation phase component (Hubbert and Bringi 1995). The reflectivity is corrected
for attenuation effects using a self-consistent, constraint-based method (Bringi et al. 2001).

A threshold in $\Delta \Phi_{dp} = \Phi_{dp}(r_z) - \Phi_{dp}(r_r) > 2^a$ is applied for each beam for application of the formulas in (5) and (9). Because the $\Phi_{dp}$ is filtered in range, the fluctuations in measured $\Phi_{dp}$ are reduced considerably to $< 1^\circ$. Below this threshold value of $\Delta \Phi_{dp}$, a $Z - R$ relation is used to determine the rain rate; the coefficient and exponent of the power law are determined from disdrometer data resulting in $Z_a = 305R^{1.36}$. The piecewise linear fit shown in Fig. 1 is used to determine the value of $c$ to be used in (5) based on the average $K_{dp}$ value for the beam. The coefficient $a$ and exponent $b$ used in (9) are based on a nonlinear fit to disdrometer-based scattering simulations at C band, which results in $R = 32.4(K_{dp})^{0.53}$.

Radar data from the lowest available elevation (0.5°) tilt, or sweep, were used, and within the polar area in Fig. 6 a total of 12–15 beams per sweep were generally available for the azimuthal integration. The low-elevation angle sweep data were available every 10 min; that is, the radar sampling interval was 10 min. The areal rainfall in (5) and (9) obtained for each sweep was divided by the polar area in Fig. 6 resulting in a time series of mean areal rain rate ($\overline{R}_{csu}$ or $\overline{R}_{sf}$) spaced every 10 min.

As mentioned earlier, a time series of 1-min averaged rain rate was available from each gauge in the network. Let $t_o$ be the radar sampling time defined here as the center time for each radar sweep. The mean areal rain rate from the gauge network at $t_o$, $\overline{R}_g(t_o)$, is estimated as follows. A time window corresponding to $t_o \pm 1$ min is defined and all gauge rain rates in this window are averaged to obtain the first estimate of $\overline{R}_g(t_o)$. Next, a time delay is introduced by sliding the time window forward in 1-min increments, and an optimal delay time is found by minimizing the absolute deviation between the radar-estimated $\overline{R}_{csu}$ and $\overline{R}_g$. In practice, the average optimal delay was around 1 min. The optimal delays based on $\overline{R}_{csu}$ were similar to those based on $\overline{R}_{sf}$, which is not surprising since the algorithms are similar. Here, the optimal delays based on $\overline{R}_{csu}$ are used. It is standard procedure to introduce time delays before comparing radar- and gauge-based estimates, since the radar resolution volume is always at some finite height above the surface. It constitutes one component of the variance between radar and gauge estimates, which, in practice, can be minimized.

The gauge density of the Darwin network (see Fig. 6) is high, about 5 km$^2$ per gauge. According to Silverman et al. (1981), the sampling error for storm total rainfall is “primarily a function of the number of gauges per raincell and secondarily, but importantly, a function of the spatial precipitation gradient.” For the Darwin network, the sampling error is estimated to be around 5%–7%, assuming the raincell area is around 100 km$^2$ and typical spatial gradient values adapted from Silverman et al. [1981; see their Eq. (2) with $G = 1.4$ and gauges per raincell, GPR of 20].

The radar–gauge data used in this study were obtained during the summer rainy season in Darwin (December 1998–March 1999). Twelve convective rain events were available for analysis. A variety of rain types are represented in this dataset, for example, continental and oceanic squall lines, but no attempt was made here to distinguish between rain types. As mentioned earlier, the threshold value for $\Delta \Phi_{dp}$ of $2^a$ was selected for application of (5) and (9); otherwise, the rain rate was based on $Z_a = 305R^{1.36}$ obtained from disdrometer-measured drop size spectra. This $\Delta \Phi_{dp}$ threshold corresponds to a rain-rate threshold of about 5 mm h$^{-1}$. On average, the number of beams in the polar area where $\Delta \Phi_{dp}$ threshold was exceeded was around 70% of the total number of beams for the entire event.

4. Radar–gauge comparisons

A typical time series of $\overline{R}_{csu}$ for one event (18 February 1999) is shown in Fig. 7 where the samples are spaced 10 min apart. Standard error bars for the radar-based estimate of areal rain rate are also shown. The fluctuation of the error in the $R(K_{dp})$ estimator about the true rain rate $R$ is due to both the parameterization error ($\epsilon_p$) as well as the radar measurement error ($\epsilon_r$). The parametric error is due to the form of the $R-K_{dp}$ relation, for example, $R = cK_{dp}^m$ and is based on simulations using the gamma drop size distribution model whose parameters ($N_m, D_m, \mu$) are widely varied (see appendix). Most of the error is due to $\epsilon_r$ (Scarchilli et al. 1993; Bringi and Chandrasekar 2001). The standard error due to parameterization $\sigma(\epsilon_p)$ decreases with increasing $R$ and is around 35% for $R$ around 20 mm h$^{-1}$. 
The measurement error component is estimated from the appendix of RZF (it is negligible compared with the parameterization error). The standard error bars in Fig. 7 also account for the fact that the radar estimates the mean areal rain rate, that is, the variance of the parameterization error has been reduced by \( M \), where \( M \) is the number of uncorrelated samples. Here, \( M \) is estimated as \((10/3)^2 \approx 11\) (10 km \( \times \) 10 km is the area, while 3.0 km is a typical decorrelation distance for convective rain cells in this region (Maki et al. 1999)).

Figure 8 shows \( \bar{R}_{\text{csu}} \) versus \( R_g \) for all of the 12 events. The normalized error (NE) is defined here as

\[
\text{NE} = \frac{\left( \frac{1}{N} \sum_{i=1}^{N} (\bar{R}_{\text{csu}} - R_g) \right)}{\left( \frac{1}{N} \sum_{i=1}^{N} R_g \right)}
\]

and the normalized bias as

\[
\text{NB} = \frac{\left( \frac{1}{N} \sum_{i=1}^{N} (\bar{R}_{\text{csu}} - R_g) \right)}{\left( \frac{1}{N} \sum_{i=1}^{N} R_g \right)}.
\]

For the data shown in Fig. 8, the NE is 37% while the NB is 5%. Under ideal circumstances, the parameterization error from simulations is expected to be around 10% (assuming 11 physically uncorrelated samples in the area estimate, i.e., \( 0.35/\sqrt{11} \approx 0.10 \)). Hence, the residual error component is around 27%. Note that these error estimates correspond to areal rain rates over 10 km \( \times \) 10 km area at 2-min resolution. Possible sources of error that can account for the 27% are (i) gauge measurement error, (ii) sampling error of the gauge network, and (iii) mismatched radar–gauge sample volumes. Some of these errors will reduce when rain accumulations over the duration of the precipitation event are compared. For example, the sampling error of the gauge network for storm total rainfall is expected to be around 5%–7% (Silverman et al. 1981).

Figure 9 compares the rain accumulation (based on samples of radar \( \bar{R} \) and \( R_g \) spaced 10 min apart) for the 12 events; the normalized error is 14.1% and normalized bias is 5.6% for the CSU estimator. Note that the gauge-based accumulation is based on \( R_g \) sampled at the radar sampling interval of 10 min. Since the expected sampling error of the gauge network itself is around 5%–7%, the results of Fig. 8 show that the radar estimation of storm total accumulation over the 10 km \( \times \) 10 km area is very accurate using the \( \bar{R}_{\text{csu}} \) algorithm. Comparable values for the normalized error and normalized bias when using the RZF algorithm (\( \bar{R}_{\text{rf}} \)) are 21% and 11.4%, respectively. Corresponding error and bias values for the \( Z_{\text{dn}}-R \) algorithm are 51% and −50.8%. Comparing the error/bias results for the \( \bar{R}_{\text{csu}} \) and \( \bar{R}_{\text{rf}} \) algorithms, it appears that the model approximations used in deriving the \( \bar{R}_{\text{csu}} \) algorithm [see (5) and Fig. 1] lead to less error than those used in deriving the \( \bar{R}_{\text{rf}} \) algorithm [see (9)], which was also demonstrated through simulations (see Figs. 3 and 5). However, both algorithms significantly outperform the disdrometer-based \( Z_{\text{dn}}-R \) algorithm, which is seriously biased (underestimate of 50%).

5. Summary and discussion

A new area rainfall algorithm [see (5)] is proposed based on differential propagation phase. It is philosophically somewhat different from the areal rainfall algorithm proposed by Ryzhkov et al. (2000) in that a linear relation between \( R \) and \( K_{\text{dp}} \) is assumed to be valid.
locally ($R = cK_{dp}$) to arrive at (5) but the coefficient $c$
selected based on a piecewise linear fit to the non-
linear $R$–$K_{dp}$ relation. Disdrometer-measured drop size
distributions from an entire rainy season together with
scattering simulations are used to determine the piece-
wise linear fit (see Fig. 1) for the different $K_{dp}$ ranges.
The $c$ value used in the algorithm is based on the average
$K_{dp}$ for the particular beam, where this average is simply
computed as $\langle \Phi_{dp}(r_2) - \Phi_{dp}(r_1) \rangle / 2(r_2 - r_1)$. In contrast
the areal rainfall algorithm of Ryzhkov et al. (2000) 
assumes a nonlinear relation $R = aK_{dp}^b$, with $K_{dp}$ constant
for a particular beam to arrive at (9). The constant $K_{dp}$
assumption leads to uniform range weighting, which can
lead to error if the rain cell is not centered at the mid-
point of $(r_1, r_2)$ (see Fig. 5). Model simulations with
six different assumed $K_{dp}$ range profiles show that the
CSU algorithm in (5) appears to result in less error when
compared with the RZF algorithm in (9) when $r_2 - r_1$
was fixed at 20 km (which is generally comparable with
the Darwin gauge network, see Fig. 6).

Disdrometer-based scattering simulations at C band
(frequency of 5.5 GHz) were used to determine the co-
efficient $c$ of the piecewise linear fit, and the coefficient/
coefficient of the nonlinear relation $R = aK_{dp}^b$. It is known
that $c$ and $a$ depend on the assumed axis ratio versus
drop diameter relation. Here, the Andersager et al. (1999)
fit (which accounts for transverse drop oscillations) is
used for $1 \leq D \leq 4$ mm, whereas the equilibrium model
of Beard and Chuang (1987) is used for $D < 1$ or $D >
4$ mm. The canting angle model is assumed to be Gauss-
ian with mean zero and standard deviation $10^\circ$ (refer
to chapter 7 of Bringi and Chandrasekar 2001 for justifi-
cation of these values for tropical rainfall). The nonlin-
ear relation $R = 32.4K_{dp}^{0.83}$ was obtained for use in (9),
which is close to $R = 34.6K_{dp}^{0.83}$ used by May et al. (1999)
based on disdrometer-measured drop size distributions
from the Maritime Continental Thunderstorm Experi-
ment (MCTEX), which was conducted in the Tiwi is-
lands north of Darwin, and an empirical axis ratio versus
$D$ relation (Keenan et al. 2001). Their empirical relation
was based on minimizing the bias error between $R(K_{dp})$
and gauge data from MCTEX. The low value of nor-
malized bias (5%–6%) evident in the radar–gauge com-
parisons in Figs. 8 and 9 suggest that the axis ratio model
adapted herein and used in the algorithm (see piecewise
linear fit in Fig. 1) is valid for convective tropical rain
in the Darwin area, and generally consistent with Keen-
an et al. (2001).

Twelve storms in the Darwin area during the summer
season (December 1998–March 1999) were analyzed
using C-POL radar measurements and data from the
Darwin D-scale gauge network. While the primary al-
gorithm to be evaluated was (5), the RZF algorithm in
(9) as well as a $Z_{dp}$–$R$ algorithm were used for compar-
ison. The coefficient/constant of the $Z_{dp}$–$R$ relation
was obtained from a nonlinear fit to disdrometer-measured
drop size distributions as $Z_{dp} = 305R^{0.36}$. The radar-based
rain accumulation values for the 12 storms when com-
pared against the gauge network values resulted in nor-
malized bias of 5.6%, 11.4%, and –50.8% for the CSU
algorithm, the RZF algorithm, and the $Z_{dp}$–$R$ algorithm,
respectively, and corresponding normalized error [see
(10a)] of 14%, 21%, and 51%. Previous areal rain ac-
cumulation results by Ryzhkov et al. (2000) based on
20 Oklahoma storms using an S-band radar and a net-
work of 42 gauges and their algorithm in (9) gave a
normalized bias of –8.2% and fractional standard error
of 18.3%. May et al. (1999) used their $R(K_{dp})$ algorithm
($R = 34.6K_{dp}^{0.83}$) and C-POL radar data with a network
of gauges during MCTEX (rainfall types similar to the
Darwin area), and in the four storm events analyzed,
the fractional standard error was 21% and normalized
bias around 14%. This latter study did not use the areal
rainfall algorithm, rather the $R(K_{dp})$ algorithm was used
in a conventional manner. In general terms, the current
results for storm total accumulation over an area are
consistent with the two earlier studies of Ryzhkov et al.
(2000) and May et al. (1999), that is, normalized error
(or, fractional standard error) in the range 15%–20%.
In contrast, $Z_{dp}$–$R$ relations based on disdrometer data
from the region used here and in the May et al. (1999)
study gave corresponding normalized error (or, frac-
tional standard error) of around 50%.

Two major conclusions can be drawn from this paper.
First, among the two assumptions needed to derive an
areal rainfall estimator based on $\Phi_{dp}$, that is, a piecewise
linear approximation to $R$–$K_{dp}$, which enables a proper
range-weighting versus a constant $K_{dp}$ approximation
that enables use of a nonlinear $R$–$K_{dp}$ relation but with
uniform range-weighting, it appears that the former ap-
proximation leads to smaller error as demonstrated by
the data. Second, the small bias (around 5%–6%) be-
 tween the CSU areal rain-rate estimator and the gauge
data appears to validate the assumptions used herein for
the axis ratio model for tropical rain, in general agree-
ment with the empirical model proposed by Keenan et
al. (2001).

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APPENDIX

Simulations Using Disdrometer Data

This appendix describes the scattering simulations
based on measured drop size distributions that are used
to arrive at the piecewise linear $R$–$K_{dp}$ fit in Fig. 1 [see,
also, chapter 7 of Bringi and Chandrasekar (2001)].

Drop size distributions (dsd) were measured with a
Joss–Waldvogel disdrometer, which was located within the Darwin gauge network shown in Fig. 6. Over 2000 2-min-averaged size distributions were available representing nearly an entire season of rainfall from the Darwin area. Each 2-min dsd was fitted to a gamma dsd form as follows [other methods are given in Willis (1984) and Ulbrich and Atlas (1998)]. The gamma dsd may be expressed as (Willis 1984; Testud et al. 2000),

$$N(D) = N_w f(\mu) \left( \frac{D}{D_m} \right)^\mu \exp\left[-(4 + \mu) \frac{D}{D_m}\right], \quad (A.1)$$

where $N_w$ is a generalized “intercept” parameter defined as

$$N_w = \frac{4^4}{\pi} \left( \frac{10^4 W}{D_m^3} \right); \quad \text{mm}^{-1} \text{m}^{-3}, \quad (A.2)$$

with $W$ the rainwater content (in g m$^{-3}$) and $D_m$ the mass-weighted mean diameter (in mm). Note that $N_w$ is the intercept parameter of an equivalent exponential dsd ($\mu = 0$ case), which has the same $W$ and $D_m$ as the gamma dsd. The $f(\mu)$ is defined as

$$f(\mu) = \frac{6}{(4 + \mu)^4} \frac{(4 + \mu)^{4+\mu}}{\Gamma(4 + \mu)}. \quad (A.3)$$

The form of the gamma dsd in (A.1) emphasizes two features, that is, the normalizing of diameter by $D_m$, and the scaling of concentration by $N_w$. The fitting of a measured dsd [$N_{\text{meas}}(D)$] to the gamma form in (A.1) follows the following simple steps.

1) Calculate $W$ and $D_m$ for the 2-min averaged measured dsd, and, hence, $N_w$ using (A.2).

2) Scale/normalize the measured dsd by constructing $N_{\text{meas}}(x) = N_{\text{meas}}(D/D_m)/N_w$.

3) Find $\mu$ by minimizing the following error function:

$$\text{Error} = \sum_{-3 \leq x_i \leq 15} \times \left[ \log_{10} N_{\text{meas}}(x_i) - \log_{10} [f(\mu)x_i \exp\left[-(4 + \mu)x_i\right]] \right],$$

$$\quad (A.4)$$

where $x_i = D_i/D_m$, and $D_i$ is the center diameter of the disdrometer sizing bins.

The above fitting method tends to separate out the “shape” $\mu$ of the gamma fit from the scaling/normalizing parameters $D_m$ and $N_w$, which is philosophically related to the method proposed by Semper-Torres et al. (1994).

For the Darwin measurements, a table of over 2000 triplets of $(N_w, D_m, \mu)$ was constructed that represents fits to each of the 2-min averaged measured dsds. For each triplet $(N_w, D_m, \mu)$, the still-air rain rate, and the specific differential phase (at a frequency of 5.5 GHz) are computed. The raindrops are assumed to be oblate with axis ratio as given by Andsager et al. (1999) for $1 \leq D \leq 4$ mm, and as given by Beard and Chuang for $D < 1$ or $D > 4$ mm. The canting angle distribution is assumed to be Gaussian with zero mean and standard deviation $10^\circ$. It is hypothesized that these assumptions are representative of tropical rainfall (Bringi and Chandrasekar 2001). Size integration is performed up to $D_{\text{max}} = 2.5$ mm. While it is recognized that the Joss disdrometer does not have sufficient sample volume to estimate the concentration of the largest drops ($D > 5$ mm), the proposed fitting method tends to compensate for this in the sense that rain rate and $K_{\text{dp}}$ are much less sensitive to this problem (Zrnić et al. 2000) as compared to $Z_\theta$ or $Z_p$. It is also recognized that at high rain rates ($R > 50$ mm h$^{-1}$ for the Darwin data) the Joss disdrometer undercounts tiny drops, which tends to bias the $\mu$ estimate too high. However, the impact on the piecewise linear fit to $R-K_{\text{dp}}$, shown in Fig. 1 is expected to be minimal. A nonlinear fit to the $R-K_{\text{dp}}$ data points results in $R = 32.4 K_{\text{dp}}^{0.83}$.

Each data point in Fig. 1 corresponds to a specific triplet $(N_w, D_m, \mu)$. Further, the equivalent reflectivity factor ($Z_\theta$) is also computed and a $Z-R$ relation is obtained by a nonlinear fit ($Z_\theta = a R^{b}$) to the data resulting in $Z_\theta = 305 R^{1.06}$.

REFERENCES


Beard, K. V., and C. Chuang, 1987: A new model for the equilibrium linear fit to $R-K_{\text{dp}}$, shown in Fig. 1 is expected to be minimal.


