the amplitude, offset or shape can be modified to the desired levels at the test section through the use of the pump. Thus, this device fulfills the design goals of providing a pressure source for small blood vessel experiments, but may be applied to a variety of experimental systems as well.

REFERENCES


Correction to "Reduced Order Kalman Filtering for the Enhancement of Respiratory Sounds"

S. Charleston and M. R. Azimi-Sadjadi

In the above paper1 a portion of Section II was omitted. The omitted text is the paragraph beginning "Under these assumptions..." For clarity, Section II is printed here in its entirety. We apologize to the authors and readers for this omission.

II. MODELS FOR THE HEART AND RESPIRATORY SIGNALS

Signal estimation using ROKF requires a mathematical model for the signal to be estimated (desired) as well as for the observation process. In this paper, the heart sounds are considered as the desired signals to be estimated while the respiratory sounds are assumed to be additive colored noise. Three assumptions are made based upon the properties of these contributing signals for the modeling and cancellation purposes.

1) The interaction between the heart and respiratory sounds is additive [5], [6].
2) The Signals are considered to be mutually uncorrelated processes as they are generated from independent sources, while they are correlated themselves [5], [6].
3) Prior and subsequent heart sounds are linearly related to the heart sounds corrupted by the respiratory signal.

Under these assumptions the observation equation can be written as

\[ z(k) = x(k) + v(k) \]  

where \( z(k) \) is the acquired signal, \( x(k) \) represents the heart signal, and \( v(k) \) corresponds to the respiratory signal. The dynamics of the heart signal is modeled by an \( M \)-th order AR model driven by a white Gaussian noise process, i.e.,

\[ x(k) = - \sum_{n=1}^{M} a_n x(k-n) + \omega(k) \]  

where \( a_n \) is the model coefficient, \( n \in [1, M] \), and \( \omega(k) \) is a zero-mean white Gaussian noise with variance \( \sigma^2_\omega \). The AR model fits the spectral characteristics of the heart sounds since its power spectral density (PSD) possesses distinctive peaks. This model is arrived at by using the heart information present in the manually extracted sections of an acquired signal that are free of respiratory sounds.

To represent the dynamical model (2) in state equation, we define a state vector that contains the current and past values of the heart signal, i.e., \( \mathbf{x}(k) = [x(k-M+1) x(k-M+2) \ldots x(k-1) x(k)]^T \). Using this state assignment and the AR model in (2), the following state equation can be obtained:

\[ \mathbf{x}(k) = \mathbf{F} \mathbf{x}(k-1) + \mathbf{G} u(k) \]  

where

\[
\mathbf{F} = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
-\alpha_2 & -\alpha_3 & \ldots & -\alpha_M & 1 \\
-\alpha_1 & 0 & \ldots & 0 & 0
\end{bmatrix}
\]

and

\[
\mathbf{G} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1
\end{bmatrix}
\]

The observation equation (1) can now be expressed in terms of the state vector \( \mathbf{x}(k) \) as

\[ z(k) = \mathbf{H} \mathbf{x}(k) + v(k) \]  

where \( \mathbf{H} = [0 \ 0 \ \cdots \ 0] \). Note that in the above equations, even though the driving process \( u(k) \) is a white process, \( v(k) \) is a colored process owing to its band-limited behavior. Thus the standard Kalman filter can not be applied. This calls for the ROKF which is reviewed briefly in the next section.
Communications

Reduced Order Kalman Filtering for the Enhancement of Respiratory Sounds

S. Charleston and M. R. Azimi-Sadjadi,*

Abstract—In the processing and analysis of respiratory sounds, heart sounds present the main source of interference. This paper is concerned with the problem of cancellation of the heart sounds using a reduced-order Kalman filter (ROKF). To facilitate the estimation of the respiratory sounds, an autoregressive (AR) model is fitted to heart signal information present in the segments of the acquired signal which are free of respiratory sounds. The state-space equations necessary for the ROKF are then established considering the respiratory sound as a colored additive process in the observation equation. This scheme does not require a time alignment procedure as with the adaptive filtering-based schemes. The scheme is applied to several synthesized signals with different signal-to-interference ratios (SIR) and the results are presented.

I. INTRODUCTION

Interferences are usually present in the acquisition and processing of biomedical signals; the respiratory sounds are good examples of such cases. Respiratory or breathing sounds have always been of interest to physicians because of the information that they carry about the lung’s condition [1]–[3]. One of the main problems in their processing and analysis, however, is the presence of interference due to heart sounds which occur in most of the chest and neck sites. Additionally, the acquired signals contain other disturbances caused by muscle contraction, ambient noise, skin, and hair. Although, the effects of these interferences could be significantly reduced by using a sound proof room and a firm microphone placement, the effects of heart sounds cannot be removed easily. Respiratory and heart signals overlap not only temporally but also spectrally [1]–[3]. In addition, frequency contents could change or exhibit a shift due to several factors such as the inherent variability of biological systems, conditions during the signal acquisition, and cardiac disorders. Highpass filtering has traditionally been used as an ad-hoc solution to this separation problem. However, variability inherent to any biological system as well as the variability among subjects limits the effectiveness of the standard filtering schemes owing to the loss of spectral information [3], [4].

The application of adaptive filtering to this problem was suggested in [5] and [6] where the electrocardiogram (ECG) signal and a particular filtered version of the acquired signal were used as reference signals. In [5] a reference signal, the “augmented ECG,” was generated by adding a delayed version of the original signal to the acquired ECG. However, the approach cannot follow the time variations between the first and second heart sounds resulting in a sound reduction performance between 50% and 80%. To avoid the acquisition of an additional signal, in [6] an elaborate scheme was applied to get the reference from the acquired sounds. The acquired signal is first low-pass filtered, squared, and then smoothed by another low-pass filter. The idea was to produce a spike whenever the heart sounds occur in the acquired signal. The experimental results showed moderate heart sounds reduction between 24% and 49% [6].

The results in [5] and [6] pointed out the important fact that the temporal alignment between the reference and interference signal procedure plays a crucial role in the performance of the adaptive filtering based methods. For the cases where the signal-to-interference (SIR) (heart) ratio is low, an automatic alignment procedure based on the maximum value of the cross correlation between the heart sounds and the acquired signal could be used to determine the locations of the heart sounds prior to filtering. However, an accurate time alignment can not readily be achieved in those cases where the SIR is moderately high or when the subjects suffer combined heart and respiratory diseases that produce alterations in the heart sound’s morphology and cardiac frequency. Consequently, methods that rely on a simple time alignment procedure do not perform satisfactorily. In addition, the selection of a reference signal can not be made easily due to the possibility of more than one heart sound inside the breathing section.

More recently, the analysis of respiratory sounds on dogs was performed by monitoring the ventilation during anesthesia where at least two microphones were used and a strain gauge monitored the inspiration/expiration/rest periods [7]. The adaptive heart sounds cancellation was achieved using the phonocardiogram (PCG) signal present in the rest periods assuming that the neighboring PCG beats were correlated. The reference signals were formed with the PCG beats just before the inhalation and just after the exhalation periods. The detection of the PCG signals was determined by the cross-correlation method. It was observed that, even after the adaptive filtering, PCG interference posed some problems especially in the lower frequency range.

The purpose of this paper is to investigate the potential application of Kalman filtering [8] to this signal separation problem using only the acquired sounds. A reduced-order Kalman filter (ROKF) is used which does not rely on the time alignment procedure required by the adaptive filtering based schemes. In the state-space formulations for ROKF, the heart sounds are considered as the desired signal to be estimated and the respiratory sounds are treated as additive colored interference. An AR model is fitted to the heart signals manually segmented from the portions of an acquired signal which are free of respiratory sounds. A low-order model is used to represent the colored respiratory sounds. The state variables associated with this model are then augmented into the state vector. This augmentation leads to a noise-free observation model for which an ROKF can be derived [8]. The scheme is then applied to a number of synthesized cases with different SIR that are formed by adding acquired heart sounds to synthesized respiratory sounds.

II. MODELS FOR THE HEART AND RESPIRATORY SIGNALS

Signal estimation using ROKF requires a mathematical model for the signal to be estimated (desired) as well as for the observation process. In this paper, the heart sounds are considered as the desired signals to be estimated while the respiratory sounds are assumed to be additive colored noise. Three assumptions are made based upon the properties of these contributing signals for the modeling and cancellation purposes.
1) The interaction between the heart and the respiratory sounds is additive [5], [6].
2) The signals are considered to be mutually uncorrelated processes as they are generated from independent sources, while they are correlated themselves [5], [6].
3) Prior and subsequent heart sounds are linearly related to the heart sounds corrupted by the respiratory signal.

Under these assumptions the observation equation can be written as

To represent the dynamical model (2) in state equation we define a state vector that contains the current and past values of the heart signal, i.e., \( \mathbf{x}(k) = [x(k-M+1)x(k-M+2)\ldots x(k-1)x(k)]^T \).

Using this state assignment and the AR model in (2), the following state equation can be obtained

\[
\mathbf{x}(k) = F \mathbf{x}(k-1) + G \mathbf{u}(k) \\
\text{where} \]

\[
F = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a_{M-1} & \cdots & 0 & \cdots & a_1 \\
0 & \cdots & a_{M-1} & \cdots & -a_1
\end{bmatrix}
\quad \text{and} \quad
G = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

The observation equation (1) can now be expressed in terms of the state vector \( \mathbf{x}(k) \) as

\[
z(k) = H \mathbf{x}(k) + v(k)
\]

where \( H = [0\ 0\ \cdots \ 1] \). Note that in the above equations, even though the driving process \( u(k) \) is a white process, \( v(k) \) is a colored process owing to its band-limited behavior. Thus, the standard Kalman filter cannot be applied. This calls for the ROKF which is reviewed briefly in the next section.

III. REDUCED-ORDER KALMAN FILTER (ROKF)

A generalization of the Kalman filter theory deals with the cases where the noise terms \( u(k) \) and/or \( v(k) \) may not be white processes. In these circumstances, it is necessary to find additional models for these colored processes. Typically, a low-order model, e.g., a second-order AR model, driven by a white noise process is used. This model is arranged into state equations and then augmented with that of the original process. In our case, the augmented state-space equations, assuming a second order AR model for \( v(k) \), become

\[
\mathbf{x}_a(k) = F_1 \mathbf{x}_a(k-1) + G_1 \mathbf{w}(k) \\
z(k) = H_1 \mathbf{x}_a(k)
\]

where

\[
\mathbf{x}_a(k) = [\mathbf{z}(k) \mathbf{y}(k)]^T, \quad \mathbf{w}(k) = [u(k)\eta(k)]^T
\]

\[
\mathbf{v}(k) = \begin{bmatrix}
v(k-1) \\
v(k)
\end{bmatrix}, \quad F_1 = \begin{bmatrix}
F & 0 \\
0 & F_v
\end{bmatrix}
\]

\[
F_v = \begin{bmatrix}
0 & 1 \\
-b_2 & -b_1
\end{bmatrix}, \quad H_1 = [H \ \mathbf{H}_v], \quad \mathbf{H}_v = [0 \ 1]
\]

\[
G_1 = \begin{bmatrix}
G & 0 \\
0 & G_v
\end{bmatrix}, \quad G_v = \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

and \( b_1 \) and \( b_2 \) are the coefficients for the AR model associated with \( v(k) \) with a white driving process \( \eta(k) \). Note that the measurement equation in (5) does not contain an observation noise term. This leads to numerical problems in the implementation of the Kalman filter which can be resolved using ROKF [8]. The goal is to estimate a reduced-order state vector for which a new observation equation is generated. The concept of a reduced-order state vector is based on the fact that a set of "perfect" measurements reduce the number of states that have to be estimated. This new reduced-order state vector of dimension \((M+1) \times 1\), \( \mathbf{p}(k) \), is assumed to be a linear transformation of \( \mathbf{x}_a(k) \) of the form

\[
\mathbf{p}(k) = C \mathbf{x}_a(k).
\]

Augmenting the observation equation in (5) with this new vector yields

\[
\begin{bmatrix}
z(k) \\
\mathbf{p}(k)
\end{bmatrix} = \begin{bmatrix}
H_1 \\
C
\end{bmatrix} \mathbf{x}_a(k).
\]

The only condition on \( C \) is that \( [H_1 \ C]^T \) must be invertible. Note that since there are multiple choices for \( C \), this step does not have a unique solution. Now using \(([H_1 \ C]^T)^{-1} = [L_1 \ L_2] \) where \( L_1 \) and \( L_2 \) are matrices of dimensions \((M+2) \times 1\) and \((M+2) \times (M+1)\), respectively, \( \mathbf{x}_a(k) \) can be expressed as

\[
\mathbf{x}_a(k) = L_1 z(k) + L_2 \mathbf{p}(k).
\]

To obtain the filtered estimate \( \tilde{z}_a(k|k) \), the estimate \( \tilde{p}(k) \) is needed. This can be generated using the recursive predictor implementation of ROKF, the details of which are given in [8].

IV. RESULTS AND DISCUSSION

The performance of the ROKF scheme was tested on a number of synthesized signals formed by summing the heart sounds extracted from a breathing free segment of a real acquired signal and a simulated respiratory sound at different SIR's. The simulated breathing signal was generated by filtering a Gaussian noise sequence covering the same spectral range that is present in the real signals and allowing spectral overlap between heart and breathing frequency. Additionally, to mimic the breathing-like shape of the signal, the filtered sequence was envelope modulated by a Hamming window in the time domain. This procedure generates the simulated breathing sound for just one phase of the respiratory cycle.

The final prediction error (FPE) criterion was applied to select the AR model-order for the heart sounds. The optimum order was found to be 15. Having determined the order of the AR model, the coefficients and the variance \( \sigma_v^2 \) of the driving process were calculated using the Burg approach [9]. The statistical information needed to identify these parameters was extracted from several cardiac cycles in order to consider possible changes in the heart sounds morphology. For the synthesized respiratory sound a second-order AR model was chosen for which the coefficients were also computed using the Burg approach.

The performance of the ROKF method was compared with another interference cancellation scheme using adaptive filtering based on the standard recursive least squares (RLS) learning algorithm [9]. A reference signal was formed from one of the heart sounds cycles used in the simulation. An adaptive filter of order \( N = 32 \) was implemented and time alignment between the reference and synthesized signal was carried out with the aid of the cross correlation approach, which is a common method for time alignment [7]. The performances of the methods were evaluated in terms of the squared error (SE) defined as the difference between the actual signal and the estimated one squared. SE is a common parameter used to measure the similarity between two time sequences. This index was chosen over the SIR since the purpose was to measure possible distortion of the known heart and synthesized respiratory sounds generated as a consequence of the estimation process. In addition, it is always possible to get a very large SIR at the output without really enhancing the desired signal. The SE between the known heart signal and the estimated
Fig. 1. (a) Original heart sounds, (b) synthesized signal SIR = -6 dB, (c) heart sounds estimated by the ROKF approach, (d) respiratory sound estimated by the ROKF approach, (e) heart sounds estimated by the standard RLS approach, and (f) respiratory sound estimated by the standard RLS approach.

one at the output of the ROKF was calculated for seven synthesized cases. A similar process was repeated to the spectra of these signals in the frequency domain in order to have a measure of distortion in the frequency domain. This is particularly important for the evaluation of the goodness of the respiratory sound estimates.

Table I shows the SE values, which are normalized by the energies of the known heart sounds and synthesized breathing, for seven simulated cases with SIR ranging from -6 to 3 dB. Note that here SIR calculations are performed differently, as in our formulations of ROKF the heart signal is the one to be estimated and the respiratory signal is considered as the colored noise or interference. The table also shows the results of the standard RLS-based adaptive filtering scheme. Columns 2, 3, 5, and 6 of Table I show the SE values for the heart sounds estimates in the time (SE_{TD}) and frequency (SE_{FD}) domains using the ROKF and standard RLS schemes, respectively. Columns 4 and 7, on the other hand, show the SE values for the respiratory sounds estimates in the frequency domain (SE_{FDR}) for these two schemes. These values clearly indicate the better
performance of the ROKF over the standard RLS approach. As can be seen from the results in columns 2 and 3, the SE values for heart sounds estimates using the ROKF in both domains are approximately one third of those of the standard RLS approach in columns 5 and 6. The improvement is also seen for the respiratory sound estimates as shown in columns 4 and 7. The increase in the SE values in columns 4 and 7 is due to the reduction in the respiratory signal information as the SIR increases.

Fig. 1(a) and 1(b) shows the original heart sounds used in the simulations and the synthesized signal, respectively, for one case in the table where SIR = -6 dB. Fig. 1(c) and (d) presents the estimates for the heart sound and respiratory signal, respectively, using the ROKF method. Fig. 1(e) and (f) shows the estimates of the heart and respiratory sounds, respectively, using the standard RLS method. As can be seen in these results and their comparison with the original heart signal, the quality of signal estimation and separation using the standard RLS-based adaptive filter is very poor. This is attributed to the fact that the accuracy of the cross-correlation method for time alignment is dependent on two factors: SIR in the data and morphological characteristics of the reference and the heart signal buried in the respiratory signal. Thus, even if the reference signal is aligned with the desired signal according to the cross-correlation results, the procedure does not account for possible alterations in the time lag between the first and second heart sounds in the reference and the heart signal buried in the respiratory signal. In the contrary, the ROKF method generated heart signal estimates that possess close similarity in morphology to the original heart signals in Fig. 1(a). The reason for the presence of some artifacts in the results of Fig. 1(c) has to do with the fact that the ROKF approach provides the optimal estimates based on the statistical information of the signals without requiring any knowledge about the position of the heart sounds. The adaptive scheme, on the other hand, uses a "conventional reference" signal that contains exclusively heart sounds information and further the output of the transversal filter turns off when the input is zero.

**V. CONCLUSIONS**

The purpose of this paper was to investigate the potential application of the Kalman filter for the separation of heart and respiratory sounds. The problem was formulated in the context of signal estimation where the heart signal was the one to be estimated from synthesized signals. The estimated heart signal was then used to isolate the respiratory signal. The superior performance of the ROKF over that of the standard RLS approach was demonstrated in terms of the SE index in both the time and frequency domains. In addition, it was shown that the ROKF method produced heart sounds estimates which preserved the morphology of the original heart sounds. However, the drawback of the ROKF approach lies in the necessity to establish a model for the heart and respiratory sounds and the computational complexity of the algorithm which is $O(12M^3)$, where $M$ is the order of the AR model vs that of the standard RLS method which is $O(N^2)$, where $N$ is the order of the adaptive filter. Moreover, the adaptive filter approach does not require any a priori assumption about the model for the signals under consideration, although there is a need for a more elaborate time alignment procedure.

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**REFERENCES**


