# A CCURACY OF DISCHARGE DETERMINATIONS

by

W. T. Dickinson

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COLORADO STATE UNIVERSITY
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Relation of Hydrology Paper No. 20 to Research Program:
"Hydrology of Weather Modification"

The present study is part of a more comprehensive project, whose objective is the development of techniques of evaluation of weather modification attainments, based on streamflow. The difficulty in this evaluation can be traced to two main causes:

(1) The natural variability in the hydrologic cycle far exceeds the expected range of the increase induced by man, and

(2) The inaccuracy of the flow measurements may be of the same order of magnitude as the induced change.

Relatively little, at least in the restricted and more stringent context of weather modification evaluation, has been done to date with respect to the reliability of flow measurements. The present paper offers an initial objective approach towards the estimation of the accuracy of discharge records. As a result it contributes to the development of techniques for evaluation. Its value extends, however, beyond the scope of weather modification evaluation.

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# ABSTRACT

The objective of this study was to analyze the errors that may be incurred in discharge determinations made on mountain streams. The possible sources of error were carefully considered, and a classification of these sources, including notations on the nature of the resulting errors, was prepared. A mathematical error model for a single discharge measurement has been hypothesized, and methodology presented for the evaluation of daily, monthly, and annual discharge estimates.

An exhaustive literature review was undertaken regarding both the qualitative and quantitative aspects of the topic. This material was sorted in an attempt to divide the total error in a discharge determination into various component errors. Each component was analyzed separately, and with respect to the others, in order to yield information about the random or systematic nature of the error, and about possible functional relationships which might be involved. This information has been summarized in the form of a classification of errors.

Upon the foundation developed in the first phase, a hypothetical error model was developed for a single discharge measurement. No attempt has been made to render this model a practical working tool. Rather, it was essentially a qualitative undertaking to reveal the manner of combination of the various component errors, and to clarify the nature of some of the errors. The expected value and variance of the model were studied in order that inferences could be made regarding the significant error terms.

Finally, consideration was given to the errors arising from use of an estimated rating curve. A mathematical representation was given to the stage-discharge relationship and found to account for virtually all the variability in sample data for nine mountain stream-gaging stations in Colorado. The concept of a divisive discharge value was introduced to separate the rating curve into two portions: one along which the relative error was virtually constant; and the other along which the absolute error remained constant. Both confidence and tolerance limits were established for the estimated curves, and used for inferences regarding the error bounds on daily discharge estimates and future discharge measurements. After consideration was given to the correlation between errors in single discharge estimates, conclusions were drawn regarding the magnitude of the error bounds on monthly and annual discharge estimates.

# ACCURACY OF DISCHARGE DETERMINATIONS

by W. T. Dickinson[1]

## CHAPTER I

## INTRODUCTION

### 1.1 Subject Matter

Since the beginning of the twentieth century, published volumes of streamflow data have served extensively as a basis for planning the comprehensive utilization of rivers and for various research studies. The value of these hydrological investigations is largely determined by the degree of accuracy of the determinations of runoff from natural sources. In recent years, due to the increased demand for water, more attention has been focused on the quality of hydrometric work.

What is the degree of accuracy available? In an attempt to delineate the error in its streamflow records, the U. S. Geological Survey has classified the accuracy of single streamflow measurements to be within two to five percent depending on whether the measurements are excellent or good. The average error of published daily flows has been estimated by the same agency to be less than five to ten percent for the better stations. It must be noted, however, that these values have been obtained from subjective evaluations of stage-discharge relationships.

The immediate problem is not just to insure correct measurement of discharge, but rather to be able to estimate as objectively as possible what is the error in a set of discharge estimates. It would be of great value if the error in a particular period of record for a given gaging station could be readily determined for selected degrees of confidence. This estimation requires an understanding of the potential sources of error in the data acquisition system and knowledge of the relative magnitudes of the component errors. Then possible modifications, if required, may be advanced for improved streamflow records.

### 1.2 Approach

For a consideration of errors, two basic approaches can be taken: one is analytical, and the other experimental. The analytical approach considers in detail the potential sources of error, and analyzes the nature of the component errors involved, by use of previous research results and theoretical considerations. On the other hand, the experimental approach involves extensive comparative field studies, conducted by several groups of individuals employing various methods of gaging at the same site. If river reaches are used where the discharge can be controlled, the experimental approach affords an evaluation of the global or total error in discharge measurements. A combination of the two approaches, involving experimentation based on analytical results, would yield the maximum information. For this preliminary study, only the analytical approach has been considered.

### 1.3 Purpose

The purpose of this study was to: (i) consider in detail the potential sources of error in the present methodology of determining streamflow, (ii) prepare a classification of errors, (iii) derive a hypothetical mathematical error model for a single discharge measurement, and (iv) present an objective procedure for evaluating discharge estimates.

### 1.4 Scope

The prime concern of the study was to gain knowledge of errors in streamflow records to be used for the evaluation of weather modification projects. Therefore, characteristics of only those records of gaging stations which might be used in such a program were considered in detail. The immediate interest was in river stations in mountainous watersheds, and primarily in the smaller upper basins.

The advantage of considering these records is that the gaging stations are characterized by a stable control section for relatively long periods of time. Since the bed is usually composed of coarse gravels and boulders, a true stage-discharge relationship tends to exist between occurrences of high flow.

In the discussion of the potential sources of error in streamflow records, those sources which are minimized or magnified for a mountain river-gaging station have been noted. Only those of major importance were considered for the mathematical model. The records from the above-characterized stations should be of relatively high accuracy when compared generally with those from stations in much less stable regimes. Therefore, this study considered the accuracy of the better streamflow records.

The methodology studied is that employed by the U. S. Geological Survey. It has not been outlined in this presentation because it has become very familiar to workers in all aspects of hydrology and has been considered in numerous references. The most authoritative descriptions have been given by Grover and Harrington (1943), and by Corbett et al. (1961) in the U. S. Geological Survey Water Supply Paper 888.

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## CHAPTER II

## SOURCES OF ERROR

### 2.1 Possible Sources

An evaluation of record accuracy involves a consideration of errors and the nature of such errors. This section is devoted to a discussion of possible error sources, includes information contained in the literature, and summarizes the topic with the author's classification of errors.

Errors may be incurred during three phases of the development of a record of streamflow at a river station. Initially, there are errors introduced by measurement instrumentation and technique in a single stream-gaging observation. The establishment of a stage-discharge relationship involves those errors which cause the relationship to be non-unique in nature. The last phase, involving the use of the rating curve, incorporates stilling-well and stage-recorder errors, and those introduced by the methodology of calculating daily discharge values. The various sources within these categories are outlined below in an attempt to separate the areas giving rise to systematic and random components of error.

### 2.2 A Single Stream-Gaging Measurement

#### 2.2.1 A single discharge measurement

Errors inherent in a single discharge measurement at a stream-gaging station affect each point plotted for the establishment of a rating curve. Each source contributing to this error, such as the velocity meter, flow turbulence, and the sampling of velocity in both time and space, must be considered regarding the nature of this component of error.

The use of a current meter introduces a number of potential sources of error into the determination of the flow velocity. The precision of the meter itself is the most basic and straight forward of these sources. Velocity measurements close to the water surface, the bed, and the banks, have revealed that boundary effects influence the meter. Also, the three-dimensional and unsteady nature of the flow gives rise to particular conditions affecting the meter registration. Considerable attention has been given to the effect of turbulence on current meters. Variations in the direction of the velocity vector are considered under the topic obliquity of flow, whereas variations in velocity magnitude are referred to as pulsations in the flow regime. The effects of flow obliquity on the accuracy of measurement are considered with respect to the meter.

Other sources of error in a single discharge measurement are independent of the meter. The effect of the above-defined pulsations will be considered as a problem of sampling velocity in time. There is also the effect of sampling velocity in the vertical and horizontal directions.

(a) The current-meter still-water calibration curve

The experimentally determined mean still-water calibration curve for a particular current meter defines a relationship between the number of revolutions of the meter rotor per second and the corresponding velocity of the meter through still water. Certain characteristics of this curve describe the meter's precision. These characteristics are: (i) the starting velocity $V_s$, determined by the velocity at which the meter begins to rotate, (ii) the measuring range, determined by the starting velocity and the maximum value of the velocity at which the meter may operate without damage to its measuring qualities, and (iii) the shape of the curve in the measuring range and its accuracy of indication.

The above characteristics of the meter-calibration curve depend on a number of factors. Firstly, the magnitude of the starting velocity varies directly with the magnitude of mechanical resistances of the meter. An upper limit of the measuring range is established by the type and the shape of the meter rotor, and the accuracy of indication of the curve is determined by the precision available for rating the meter.

Variation of a meter's calibration curve, with time, may be caused by accidental changes in the rotor shape or by variations in the mechanical resistances of the instrument. Many researchers have observed that a slight bend in the blade or cup of a meter rotor has a marked effect on the meter-calibration curve. The constancy of mechanical resistances depends on the degree of wear experienced by the instrument, which in turn depends on the meter design, construction materials, and the machining and assembly of the components.

It has been suggested by Hogan (1922) that the ideal current meter might operate such that the meter rotor would turn through the same number of revolutions per unit length of water irrespective of the velocity of the meter relative to the water. The calibration curve for such a meter would be of the form,

$$V_c = C_1 \omega_t$$

where $\omega_t$ is the time rate of the meter, in revolutions per second of the rotor; $V_c$ is the velocity of the calibration car; and $C_1$ is a meter constant. The same curve could also be expressed as,

$$\omega_l = C_2$$

where $\omega_l$ is the distance rate of the meter, in revolutions of the rotor per foot of water; and $C_2$ is a constant.

In practice, however, current meters experience the effects of fluid friction on their blades and of bearing friction, causing considerable slippage at low velocities. The amount of slip decreases as the velocity increases, and at a certain value of velocity the revolutions per unit distance of flow become a constant and independent of velocity. For accurate velocity measurements, the number of revolutions of the meter rotor per foot of fluid should be very close to this constant value for the velocity measured.

The response of current meters at low velocities has been considered often in the literature with regard to either the starting velocity of the meter or the lower limit of accurate velocity measurement. Murphy (1904) concluded, from his experiments at the Cornell Hydraulic Laboratory, that the smallest velocity that meters could measure "with a fair degree of accuracy" was 0.3 fps for both a large Price meter and a Haskell meter, and 0.22 fps for a small Price meter. Barrows was 0.3 fps for both a large Price meter. Barrows (1905) found the small Price meter to be unreliable below 0.4 fps and Hoyt (1910) felt that errors were negligible for velocities greater than 0.5 fps. Troskolanski (1960) has suggested that modern types of meters begin to rotate at flow velocities of approximately 0.2 fps and that an approximate relationship between the lower limit of accurate velocity, $V_{min}$, and the starting velocity, $V_s$, might be $V_{min} \approx 3V_s$. The research of Fortier and Hoff(1920) is one of the best substantiations of most subjective and experimental observations. Figure 1 reveals a number of comparative curves and illustrates clearly the advantages of the small Price meter in having both a low starting velocity and a very short interval in which to reach a constant distance rate.

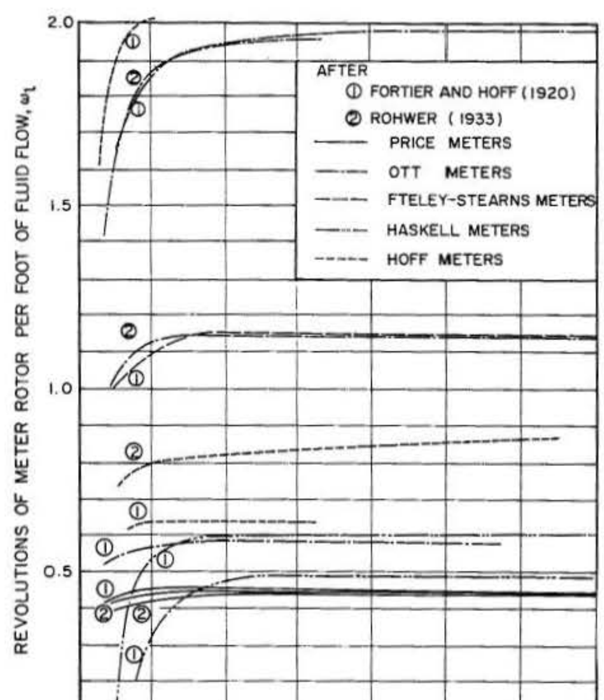


Fig. 1 Comparative current-meter calibration curves

The upper limit of the meter measuring range has not concerned stream-gaging researchers, as very little stream-gaging work approaches that end of the range. Troskolanski (1960) suggested that the measuring range lay within limits of $V_{min}$ = 0.6 fps and $V_{max}$ = 18 fps.

There have been many subjective estimates of the precision and variability of a current-meter calibration curve in the literature, but relatively few concrete values have been computed. For example, Murphy (1902) recorded that there was an uncertainty in observed velocity of one to two percent due to changes in the meter constants; both Rumpf (1914) and Groat (1914-1915) concluded from more than 1100 runs that the calibrations of both a Fteley and Stearns meter and a Haskell meter were more consistent than those of a large Price meter. Wood (1944) noted that calibrations made by the National Bureau of Standards for small Price meters were accurate to within one percent. In order to further check the accuracy of some experimental meters, Anderson (1961) calibrated them at the David Taylor Model Basin and at Colorado State University in addition to the original calibrations at the National Bureau of Standards. He observed the deviations for any one meter to be less than one percent. From the same data, Carter and Anderson (1963) computed that the maximum difference in velocity given by the National Bureau of Standard's equation and another calibration was 0.7 percent and the mean difference for 16 values was 0.34 percent. Troskolanski (1960) has suggested that the relative errors of vane and helical current meters, within the limits of their measuring range, ought not to exceed the values expressed by,

$$v = 3.0 - \frac{2V}{V_{max}}$$

and those of cup-type current meters,

$$v = 3.5 - \frac{2V}{V_{max}}$$

where

$$v = \frac{V - V_c}{V_c} \times 100\%$$

is the percentage error,

$V$ is the true velocity,

$V_c$ is the velocity of the calibration car,

$V_{max}$ is the upper limit of the measurement range of the meter.

In order to establish an error distribution due to the nature of the current-meter calibration curve, data from Hogan (1922), and Carter and Anderson (1963) have been plotted along with the suggestions of Troskolanski (1960) in fig. 2. A suggested distribution is here proposed. From the previous discussion, it seems reasonable to assume that the greatest percentage errors are experienced at the lower velocities in the region where the meter revolutions per distance of travel have not reached a constant value. However, as the velocity increases, the percentage error decreases rapidly and then levels off, perhaps approaching some minimum value asymptotically. There is little data to support this error distribution, and further investigation is necessary to adequately define it.

(b) Properties of the fluid

Characteristics of the water, such as density and temperature, have been found to have no appreciable effect on the current meter and its calibration curve. Schubauer and Mason (1937) found that densities greater than that of water have little effect on the

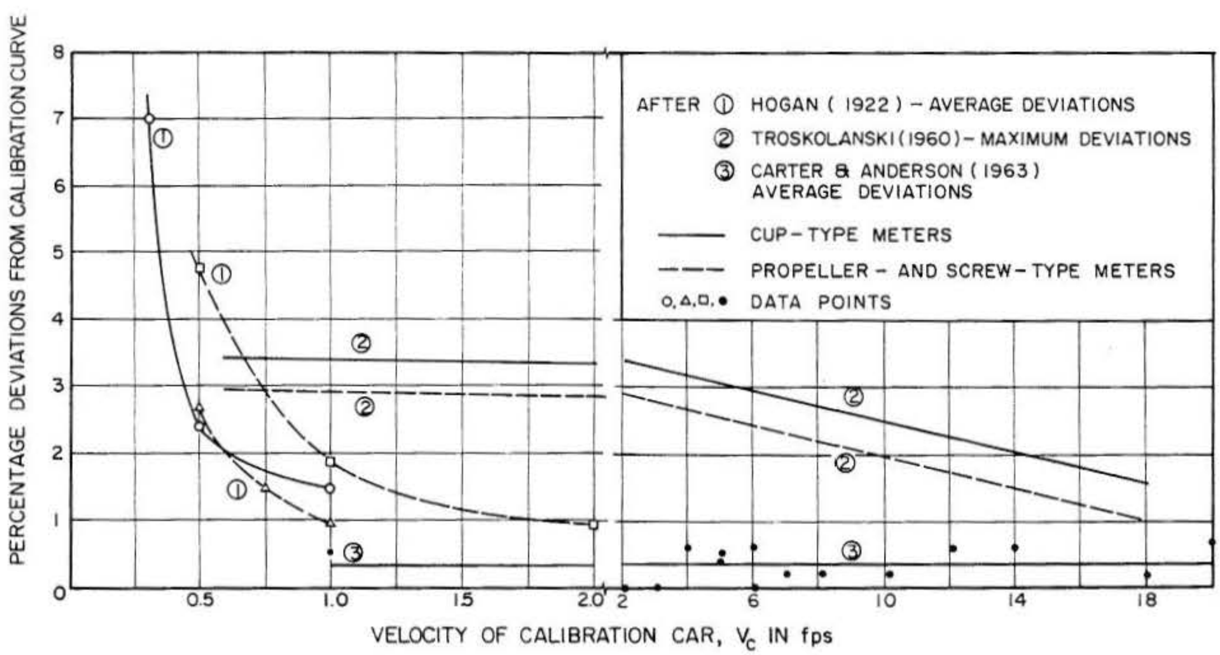


Fig. 2 Error distributions for current-meter calibration curves

meter performance, except at very low velocities. They concluded that changes of density occurring in field use could cause no appreciable error, except where very large sediment loads hampered meter operation. With regard to temperature effects, Robson (1954) found that within the limits of accuracy of the ordinary current-meter calibration tables, that is fps and rps to two decimal places, there was no indication of change in calibration with variation in water temperature over a range of 36.9 to 62.5° Fahrenheit, with a velocity range of 0.15 to 7.0 fps. Therefore, it may be concluded that neither temperature nor density alter the field performance of a current meter.

(c) Boundary effects

The geometric boundaries of streamflow cause a current-velocity meter to respond differently than its still-water calibration curve would indicate. Evidence of such boundary effects has been gathered in the literature by comparing, for a single meter, the calibration curves which have been developed at different regions of a stream cross section.

The effect of the water surface on a current meter has appeared to be a function of the type of meter, the distance from the measuring point to the surface, and the velocity. Rumpf (1914) observed that a Fteley and Stearns meter increased its registration while a Price meter decreased when both were rated within six inches of the surface. Disagreeing with Rumpf, Scobey (1914) found a cup-type meter to react slightly faster for all velocities at a depth of one foot. Pierce (1941) obtained results similar to those of Rumpf for a cup-type meter, and the action of the meter was retarded in the vicinity of the water surface. Rohwer (1933) found the effect to be a function of velocity, with the meter tending to under-register at velocities greater than five feet per second. Oltman (1954), and Chamberlain and Ham (1958) made similar observations at natural stream sites and in still-water tanks, particularly for flow depths less than one foot.

The effect of the channel sides and bottom has been a function of the above-mentioned parameters and of the roughness. Near the sides, Rumpf (1914) found a Fteley and Stearns meter to increase its registration, while the Price meter increased on one side and decreased on the other. The cup-type meter used by Scobey (1914) registered correctly when close to the one-to-one sloping concrete sides, but was 0.06 fps slow over the entire velocity range near the bottom. Pierce (1941) obtained variations in the calibration curves of a cup-type meter when placed in the proximity of the bed. The action seemed to be affected by the distance to the bed and by irregularities in the boundary surface. Pierce's table of coefficients for adjusting measurements made in depths less than one foot are used as standard practice by many agencies. Godfrey (1958) found the proximity to the floor of a flume to have only minor influence on the calibration.

Hogan (1922), in comparing calibrations made in the Froude Tank with those made at Imperial College, observed discrepancies particularly arising at high velocities when there was a correspondence between the velocity of translation of disturbance waves in the channel and the velocity of towing the meter. He assumed the calibrations in the larger channel to reflect the true effect, and observed that the interference effects caused by the nearness of one wall or by wave action always resulted in negative error and too low a velocity. Rohwer (1933) obtained results which could be explained in the same way. Using both a long rectangular tank and a circular tank for calibrating, he found meters to run more slowly in the circular tank. Also replicate calibrations of Price meters made at the rotary station did not agree as closely as those at the tangent station.

From these experimental findings, it is apparent that the flow boundaries can exhibit effects on a current-meter registration. Further, the largest effects occur in instances where the velocity distribution is rapidly changing in the space over which the current meter is integrating. Therefore, near very rough boundaries and in flows of shallow depth, the meter may be expected to respond erroneously, but no general qualitative or quantitative statements can be made regarding the errors. In most mountain stream-gaging, the velocity is normally sampled at points sufficiently distant from the boundaries to avoid effects. An exception to this statement occurs during winter flows.

(d) Oblique currents

For flow either vertically or horizontally oblique to the axis of the hydrometric section, the absolute value of the local velocity, $V_o$, is of no concern, but rather the component in the direction of the channel axis, $V = V_o \cos \alpha$ . Since a determination of the rate of flow consists of computing the volume of the solid of velocities, limited by the envelope of axial velocity components, current meters are required which measure accurately in these oblique flows over a large variation of the angle $\alpha$ . As the characteristics of the general types of current meters, cup-, screw-, and propeller-type, are important in a study of this effect, the discussion will be centered around each type.

Cup-type meters, including primarily Price meters and their variations, tend to register the magnitude of the maximum velocity vector regardless to what direction they are oriented in the flow. However, the position of the meter yoke, particularly with respect to obliqueness in a vertical plane, may cause additional discrepancies. Rumpf (1914) observed different velocity registrations for horizontal inclinations to the right and to the left of the channel axis, but noted that the mean value was virtually the maximum velocity at the point. The differences in registrations were attributable to the effect of the yoke of the meter on the flow. Brown and Nagler (1914-15) found that vertical oblique flows had more effect than horizontal obliquities. For a large Price meter tilted above or below the horizontal, the revolutions increased to a maximum when the angle of tilt corresponded to the angle of the cups. Better results were presented for a small Price meter. Rohwer (1933), Addison ( 1949), Kolupaila (1957), and Troskolanski (1960), to mention only a few, agree that except where the frame or yoke interferes, cup-type meters consistently give large results whenever oblique currents are present in the channel. A number of the experimental results have been summarized in fig. 3 and Table 1.

On the other hand, screw- and propeller-type meters tend to record a maximum number of revolutions when the axis of the meter is parallel to the flow direction and the number constantly decreases as the angle of obliquity increases. Expressing it algebraically,

$$V_c = C_o V_o$$

where

| | |
|---|---|
| $V_c$ | is the velocity registered by the meter, |
| $C_o = \phi(\alpha)$ | is a coefficient dependent on the obliquity of the flow, and decreases from unity as the angle $\alpha$ increases, |
| $V_o$ | is the true local velocity. |

Unfortunately, $\phi(\alpha)$ does not always correspond to the desired $\cos \alpha$ , but rather underestimates the cosine component of the local velocity. This underestimation has been observed by Stearns (1883), Rumpf (1914), Brown and Nagler (1914-15), Rohwer (1933), Addison (1949), Kolupaila (1957), and Troskolanski (1960). However, some of the more modern screw-type meters do assure the correct component up to a certain critical obliqueness. For example, Kolupaila (1957) referred to the auto-component Ott meter which

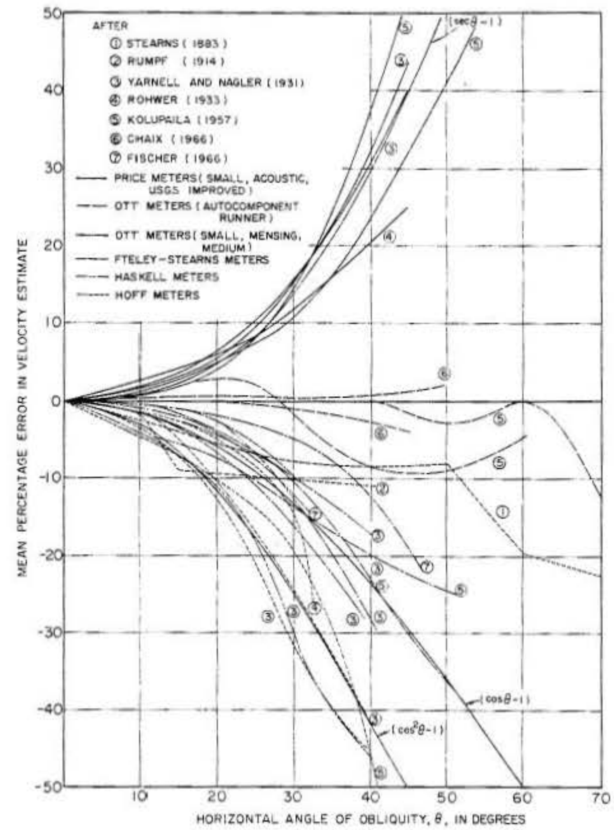


Fig. 3 The effect of oblique currents on current-meter registration

registered correct axial components for angles up to 45 degrees in either a vertical or horizontal direction.

If the mean oblique angle during the period of a single measurement can be accurately measured, a correction corresponding to the cosine of the angle can be applied to the results of a cup-type meter. This procedure is recommended by most organizations involved with stream-gaging work. However, it is both difficult and awkward to obtain a measurement of horizontal obliquity and virtually impossible to get a measure of vertical obliquity.

In summary, the experimental results have indicated that for currents of small obliquity, it is possible to register the correct axial component of flow with a screw-type meter. Use of a cup-type in such flows invariably leads to a systematic overestimation of the flow which is proportional to the oblique angle; use of a propeller-type leads to an underestimation of the axial flow. Three error functions, suggested by Kolopaila (1957), have been superimposed on fig. 3. These functions are subsequently used in the error model to describe the effect of obliquity.

(e) Microscale turbulence

Even for velocity measurements where the mean flow is in the axial direction of the channel, there are minor relatively rapid random variations in both the magnitude and direction of a point velocity

TABLE 1. THE EFFECT OF OBLIQUE CURRENTS ON VELOCITY MEASUREMENTS

| Author | Meter Type | Results Noted In The Literature |
|---|---|---|
| Stearns (1883) | Fteley and Stearns | The correct cosine component in the horizontal plane was underestimated. |
| Rumpf (1914) | Price | Different velocities were registered for left and right inclinations, but the mean value was close to the maximum velocity existing. Stream lines through the yoke of the meter had an effect. |
| | Fteley and Stearns | The cosine component was underestimated. |
| Brown and Nagler (1914-15) | Small Price | Except where the frame interfered, the maximum velocity was registered. |
| | Ordinary Price | When tilted above or below the horizontal, the meter yielded an error which increased to a maximum when the angle of tilt corresponded to the angle of the buckets, the positive bias reaching 25 percent. |
| | Fteley and Stearns | The error (negative) at 40 percent obliquity was about 11 percent. |
| Hogan (1922) | Stoppani | The maximum negative error was five percent at 30 degree obliquity; the maximum positive error was three percent at 45 degree obliquity. |
| | Amsler | The meter registration was in error -24 percent at 30 degrees, and -13 percent at 45 degrees. |
| | Sendtner | The error was -25 percent at 30 degrees, and -75 percent at 45 degrees. |
| Rohwer (1933) | Cup | All meters over-registered the cosine component. |
| | Propeller | All meters under-registered the cosine component. |
| Addison (1949) | Propeller | The meters under-estimated the cosine component. The error increased as the angle increased, was greatly influenced by the shape and type of propeller, and was sometimes affected by the velocity. |
| Kolupaila (1949) | Ott (autocomponent) | The correct cosine component was assured for angles up to 45 degrees. |
| Troskolanski (1960) | Screw | The discrepancies increased negatively as the angle of deviation increased. |

vector during a measurement. These variations about the mean microscale condition might be termed microscale or general turbulence. Researchers have observed that such turbulence often causes a current meter to respond differently than it normally would in either a steady flow regime or a still-water rating tank.

Turbulent effects may arise from the failure of the meter to respond instantaneously to sudden changes in the velocity vector and to continuously present the correct velocity component in the axial flow direction. The first cause relates to the inertial influence of the current-meter rotor, and the second to the meter response to oblique flows in the microscale.

Failure of a current meter to respond to sudden changes in the magnitude of velocity is generally considered to be of minor importance, particularly with modern-day instruments. Yarnell and Nagler (1931), conducting experiments at the hydraulic laboratory of the State University of Iowa, found that disturbances caused by general turbulence were not caused to any marked degree by the inertial influence of the meter rotor.

The second effect of general turbulence appears when a current meter fails to present the correct mean axial component of velocity from an integration of the instantaneous axial component in time. As might be expected, this effect parallels that of macro-scale obliquity and is highly dependent on the meter type. The majority of the literature on the subject reflects that a cup-type meter shows a tendency to over-register in turbulent water while a propeller-type meter tends to under-register. Further, the over-registration of a cup-type meter is invariably greater than the under-registration of a propeller-type. Researchers such as Murphy (1902), Groat (1913), Horton (1916), Yarnell and Nagler (1931), and Kerr (1935) have observed this effect.

Groat (1913) further suggested that it was fair to suspect that the still-water rating curve for a cup meter was a line of the minimum number of revolutions for such a meter. When the water was disturbed to any degree, the number of revolutions for a particular velocity was always increased. Similarly, he suggested that a still-water rating curve for a propeller meter was a line of the maximum number of revolutions for such a meter. From extensive tests with Price and Haskell meters on hydraulic turbines of the Saint Lawrence River Power Company at Messena, New York, Groat (1913) observed a Price meter to be affected to the extent of six percent while a Haskell meter was affected by less than one percent. Either meter, however, gave uniform records in equal times provided the times were sufficiently long. When the meters were run simultaneously, the disparity between the velocities determined from the still-water calibrations were considered as a basis for correcting the discrepant velocities. The two meters were then rated in diverse conditions of turbulence, and the Haskell meter never varied by more than one percent in any individual observation, and its curves for different calibrations differed by only 0.2 or 0.3 percent at the greatest. The Price meter exhibited a much greater range of variations, it being five to six percent up to velocities of five feet per second. Finally, several types of meters were used simultaneously on a network of mountain streams. In all cases of turbulent water, the cup meters were accelerated considerably while propeller meters were retarded slightly. Again the errors of the cup meters, based on still-water calibration curves, were from three to six times greater than those of the propeller type and in the contrary sense.

The only reference in the literature which did not observe a turbulent effect was the paper by Schubauer and Mason (1937). They calibrated two small Price current meters in the National Bureau of Standards' calibration tank and in a wind tunnel. Two degrees of turbulence were used in the wind tunnel, and evaluated by the relationship,

$$\frac{100\sqrt{\overline{V'^2}}}{\overline{V}}$$

where $\sqrt{\overline{V'^2}}$ was the turbulence expressed in terms of the root-mean-square value of the velocity fluctuations; and $\overline{V}$ was the mean velocity of the fluid. It was observed that the rates of the two meters were the same for both degrees of turbulence in the wind tunnel and for the flow in the flume, suggesting no turbulent effect. Although these results do not correspond to the rest of the literature, they have been used by Anderson (1961), and Carter and Anderson (1963) to infer that the error due to stream turbulence at measurement sites is small.

Recent studies performed by the International Current Meter Group [ Fischer (1966), Bonnafoux (1966), Vahs (1966), Morel (1966)] have emphasized that the effect of turbulence on the registration of a meter is particular to the meter and cannot be generalized. Further, it was pointed out, that a proper means of describing and measuring turbulence is required before the effect can be adequately determined.

It may be concluded that turbulence can have a significant effect on a current-meter registration, particularly if the meter is of the cup type. Further, there is strong reason to expect that such an effect is systematically positive in nature. In mountain streams, where there is often considerable turbulence, this source of error must be recognized as a major potential contributor to bias in a single discharge measurement. However, until more research has been conducted on both the measurement of turbulence and the relationship of turbulence to meter response, it is difficult to make meaningful quantitative estimates of the error caused by turbulence.

(f) Pulsations in the flow regime

Large scale velocity pulsations, determined by the dimensions and geometry of the streambed upstream of the metering section, contribute another possible source of error. These pulsations are to be differentiated from small-scale pulsations, determined by the viscosity of the fluid and fluctuations of much shorter duration than the velocity measurement at a point. The existence of pulsations presents a problem of sampling velocity in time. Questions are raised regarding the length of time required for measuring the velocity at a point in order to obtain a representative mean velocity, and ultimately the length of time required for measuring at each point in a stream cross section to obtain a representative discharge value.

The order of magnitude of the period of large-scale pulsations must be taken into account in determining the optimum duration of velocity measurements. Henry (1871) observed fluctuations having periods of five to ten minutes; Proskuryakov* (1953), noted twelve minutes; Linford (1949), five to ten minutes; and Dement'ev (1962) found a distinguishing property of mountain streams to be the presence of pulsation waves with periods from 1.5 to 3.0 minutes to several tens of minutes.

The first general conclusions regarding the nature of velocity pulsations in rivers of the plains were drawn by Garlyakher* (1881), on the Elbe River, and by Lauda* (1897), on the Danube. These investigations established that: (i) pulsations in the same vertical increased with depth and were greatest near the bottom; (ii) in the transverse profile, velocity pulsations increased from midstream to the banks; (iii) pulsations in the vertical increased with an increase in velocity; and (iv) pulsations increased with an increase in roughness. Many investigations, primarily by Russian scientists, have generally confirmed the above conclusions, and are ably summarized by Dement'ev (1962). His paper has been used extensively for the preparation of Tables 2 and 3, which serve to summarize some of the results presented in the literature.

*Reference taken from Dement'ev (1962)

TABLE 2. EFFECTS ATTRIBUTED TO VELOCITY PULSATIONS IN THE LITERATURE

| Author | Length of Measurement | Type of Measurement | Position of Measurement | Mean Variation (% of longer period mean) | Maximum Variation (% of longer period mean) | Additional Comments |
|---|---|---|---|---|---|---|
| Unwin (1882) on the Thames River | 1600 revs. of the meter rotor | Velocity at a point | 0.5 metre depth | -- | +8.3 to -6.0 | -- |
| | | | 6 metres depth | -- | +16.1 to -37.4 | |
| | 2100 revs. of the meter rotor | Velocity at a point | 0.5 metre depth | -- | 12.0 | |
| | | | 6 metres depth | -- | 36.7 | |
| Mackenzie (1884) on the Mississippi River | -- | Simultaneous observation 5 meters | 0.09D | -- | +3.6 to -6.9 | Relative depth was measured from the surface. |
| | | | 0.83D | -- | +9.9 to -11.1 | |
| | 2 consecutive 1 minute periods | " | 0.09D | -- | 6 | |
| | | | 0.83D | -- | 12.5 | |
| Hogan (1922) on speculation | 60 seconds | Velocity at a point | | 2 to 4 | -- | -- |
| Sokolov* (1909) and Shafalovich* (1909) on the Zee River | 2 minutes | Velocity at a point | Near surface, at midstream | 0.8 | -- | Variation was expressed as the average error of the velocity measurement. |
| | | | Near bed, at midstream | 4.3 | -- | |
| | | | Near surface, by bank | 1.2 to 1.3 | -- | |
| | | | Near bed, by bank | 5.3 to 6.0 | -- | |
| Bliznyak and Ziring* (1911) on the Yenisey River | 2 minutes | Velocity at a point | Near surface, at midstream | 2.4 | 3.8 | The standard duration was selected as 12 minutes. The relative depth was measured from the surface. |
| | | | 0.2D at midstream | 1.1 | 2.1 | |
| | | | 0.6D at midstream | 1.5 | 3.6 | |
| | | | Near bed, at midstream | 3.3 | 7.1 | |
| Sokolnikov* (1932) -on mountain streams | 80 seconds | Total discharge | -- | 2 | 5-6 | The variation was expressed as probable error. |
| -on the Neva River | 2 minutes | Total discharge | -- | 2 | -- | |
| | 1.5 minutes | Total discharge | -- | 2.2 | -- | |
| Mexheraup* (1933) on the Luga River | 5 minutes | Velocity in a vertical | At midstream | 5 to 8 | -- | The deviation was the mean variation from a number of runs. |
| | | | By the bank | 6 to 12 | -- | |
| Kalinske* (1943) on the Mississippi River | -- | Velocity at a point | -- | -- | 7-24 | -- |
| Prochazka* (1955) on the Danube, Vaga, and Vltava Rivers | 2 minutes | Velocity at a point | 0.80 to 0.95D | -- | 4.6 to 9.6 | Relative depth was measured from the surface. |
| Koplan-Diks* (1957) on the Polomet River | 100 and 120 seconds | Velocity at a point | -- | 3 | -- | These were deviations of the average point velocity. |
| | | | Near bottom | -- | 8 | This was the greatest deviation. |

*Reference taken from Dement'ev (1962)

TABLE 3. METER EXPOSURE TIME REQUIRED TO OBTAIN A PRESCRIBED ACCURACY FOR A POINT VELOCITY MEASUREMENT

| Author | Exposure Time, in Minutes, Required for 2% Accuracy | | | | |
|---|---|---|---|---|---|
| | Surface | 0.2D | 0.6D | 0.8D | Bottom |
| Sokolov* (1909), and Shafalovich* (1909) on the Zee River, in open water | -- | 2 | -- | 8 | -- |
| Bliznyak and Ziring* (1911), on the Yenisey River, in open water | -- | 1 | -- | 2 | -- |
| Moyseyenko* (1911), -on the Chusova River, in open water | -- | 2-3 | -- | 5.6 | -- |
| -on the Sylva River, in open water | -- | 2.5 | -- | 9 | -- |
| Kolupaila* (1914-16), -on the Western Dirna River, in open water | 1.5 | 1.5 | 2 | 4 | 7 |
| -on the Western Dirna River, in ice conditions | 3 | 1.5 | 1.5 | 2 | 5 |
| Sokolnikov* (1932), on the Neva River in open water | 1.5 | 2 | 4 | -- | 6 |
| Dement'ev (1962), -on mountain streams with stone beds -in mid stream | 1.5 | 1.5 | 3 | 5 | 10 |
| -at bankside | 2 | 3 | 6 | 10 | > 10 |
| -on large plains rivers, -at low water | 1 | 1 | 1 | 2 | 3 |
| -at high water | 1.5 | 1.5 | 2 | 3 | 4 |

*References taken from Dement'ev (1962)

In addition to the above observations, Kolupaila* (1925) found pulsations to decrease from the surface to 0.2 depth and then to increase with depth to reach a maximum near the bottom. With ice cover, the increase from 0.2 depth to the surface was usually more pronounced, as is reflected in Table 3. Further, Proskuryakov* (1953) found velocity pulsations to correspond to water level pulsations with a slight unclear shift of phase. Regarding the nature of the pulsations, Blumberg* (1933) observed on the Neva River that the pulsation at a point was of a random nature and followed Gauss' law of normal distribution. Kalinske (1945) and Dement'ev (1962) noted the same distribution and sought to solve the sampling problem.

Perhaps the most important conclusions regarding velocity pulsations in mountain streams have been drawn by Sokolnikov* (1936) and Dement'ev (1962). Dement'ev (1962) observed that the velocity pulsations in small- and medium-sized mountain rivers with stable beds and swift currents were considerably more clearly expressed than, and in magnitude exceeded, the pulsations observed in plains rivers. Although Sokolnikov* (1936) had recommended an exposure time at each point of two minutes, Dement'ev (1962) concluded that on mountain rivers a 100 to 200 second exposure time did not ensure two percent accuracy of measurement either for the individual point velocity or for the average velocity in the vertical. With this time exposure, the error of measurement of point velocities could reach five to ten percent, and the error of the mean velocity in a vertical could be four to six percent, as illustrated in fig. 4.

Anderson (1961), upon evaluating a sample of twenty-three streams, found the error in total discharge, for individual velocity measurements over forty-five seconds at the 0.2 and 0.8 depth points at n verticals in the section, to be equal to $4.3/n$ percent. This relationship, presented in fig. 5, tends to yield error estimates which are less than values obtained from the results of Dement'ev (1962), applying simplified assumptions to the data for large rivers.

(g) Velocity distribution in the vertical

A velocity-area approach to the computation of discharge involves the sampling of velocity in area in an attempt to define the distribution of velocity in the channel. The area is sampled at one point or several points in each of a number of verticals in order to determine the velocity distributions in both the vertical and horizontal directions. In this section, the possible error incurred in the estimation of each mean velocity term from a sample of points in the vertical will be considered.

Geometric curves have been fitted by researchers to velocity data distributed in the vertical. Murphy (1904), Pardoe (1916), Vanoni (1941), Troskolanski (1960), Kolupaila (1964), and Matalas and Conover (1965), have studied the goodness of fit of parabolic, hyperbolic, elliptic, and logarithmic curves. If one such curve is assumed to be representative of the distribution, then the accuracy of estimation of the mean velocity by different point velocity sampling methods can be evaluated. Studies of the parabolic distribution by several investigations led to the adoption of the 0.6 depth, and 0.2 and 0.8 depths sampling methods.

Another approach used for the evaluation of vertical sampling techniques has involved laboratory and field comparisons among the different techniques and with weir measurements. A few of the results presented in the literature are given in Table 4. It must be stressed that the error terms in the tables attributed to mean velocity in a vertical and to total discharge may in some instances include more than just the vertical velocity distribution effect. However, if the other effects are assumed to be constant, the comparisons are an aid to obtaining an appreciation of the possible variations.

Regardless of which theoretical curve is considered to best represent the vertical distribution of velocity, it appears doubtful that one curve may be

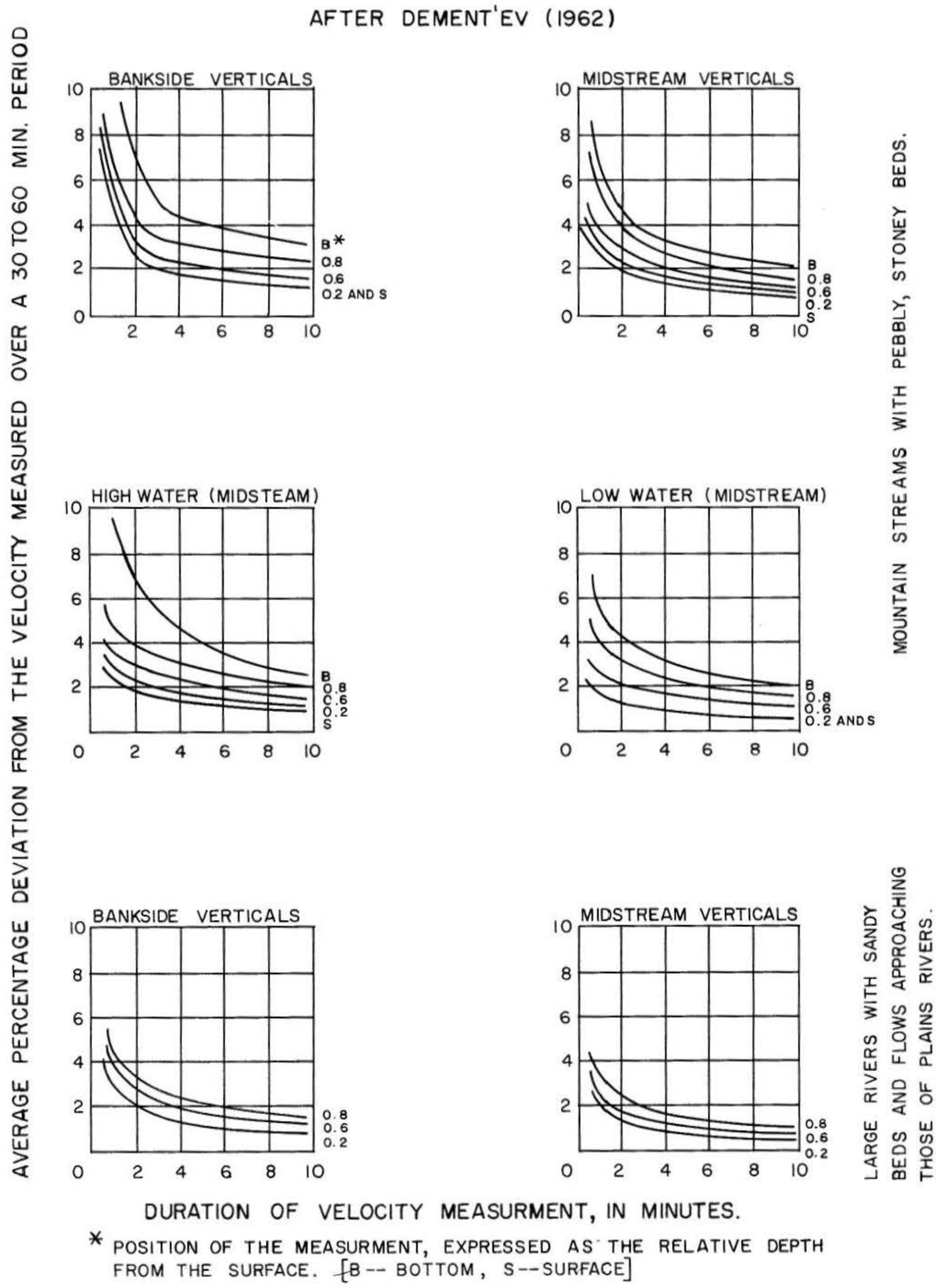


Fig. 4 The average error in a velocity measurement as a function of the duration and position of the measurement

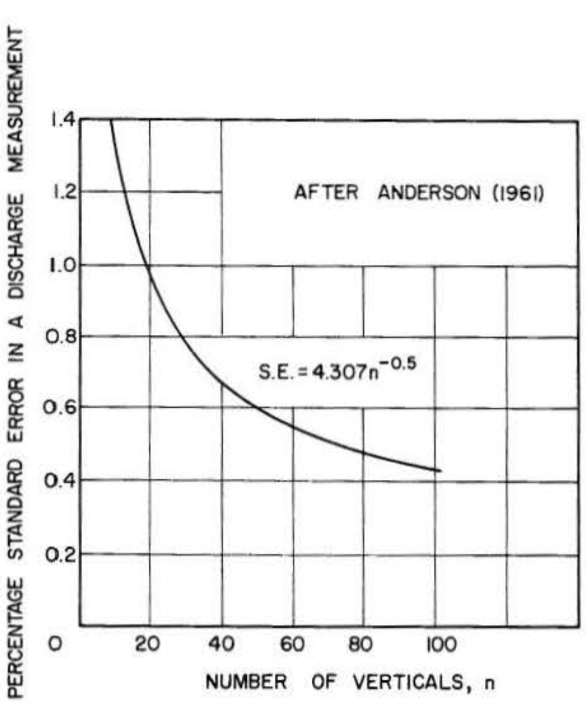


Fig. 5 The standard error in a discharge measurement due to velocity pulsations, as a function of the number of vertical sections

valid across the entire gaging section. The distribution has been observed in the literature to be a function of the ratio of depth of flow to stream width, roughness of bed and obstructions, the slope of the bed, and the surface wind direction. Hence, studies of large samples in the field, such as that undertaken by Anderson (1961) of 100 streams, best reflect the degree of variation.

(h) A single depth measurement

Measurements of flow depth are made across a stream-gaging section in order to define the cross-sectional area through which flow is passing. Each depth measurement is subject to error and hence has an effect on the accuracy of computed discharge.

Two sources of systematic errors in a depth measurement may lead to an overestimation of discharge. If the channel bottom is soft, the measuring instrument is easily pushed into the bottom rather than resting on the bottom. In instances where the bottom is obscured by sediment load, this type of error is difficult to avoid. A second source of error is that introduced by the effect of surface tension on the shaft of the measuring device to yield a reading that is too high. For a gaging section having solid boundaries, a careful evaluation of the depth measurement reduces errors resulting from the above-described sources to a negligible level.

Random errors may result from sampling depth on a rock or irregular channel bottom, from fluctuations of the water surface, and from velocity fluctuations when cable measurements are made. Further, scour may occur at the base of the stand or around a weight to cause a shift in the meter position. These errors can be minimized by careful selection of the gaging section, and good technique. However, they must be acknowledged as possible sources of inaccuracies in mountain stations.

(i) The horizontal distributions of velocity and depth

The depth measurements and mean velocities in a number of verticals across a section are used as horizontal samples of depth and velocity. In practice, a distribution is assumed to adequately fit these points, or groups of these points, and the discharge is computed from the corresponding formula. A procedure involving the sketching of equal velocity lines for each discharge measurement is employed extensively outside the United States. This latter technique generally yields the best estimate of discharge, but has the disadvantage of requiring more computation time than the use of formulas.

Formulas for the computation of stream discharge from a sample of depth and sectional mean velocity values may be classified as either rectilinear or curvilinear. In the rectilinear formulas, the depth and mean velocities are considered in consecutive groups of two or three ordinates each, based on the assumption that the cross section of the stream and the horizontal velocity distribution each describe the perimeter of a polygon. The curvilinear formulas treat the depths and velocities in groups of three under the assumption that the bed of the stream and the velocity distribution consist of a series of parabolic arcs. Many such formulas have been outlined and used in the literature by Stearns (1883), Murphy (1904), Barrows (1905), Stevens (1908), Young (1950), U. S. Bureau of Reclamation (1953), Troskolanski (1960), and Colby (1964).

A theoretical comparison of the formulas was undertaken by Stevens (1908) in an attempt to reveal possible systematic errors. His standard or so-called "exact" formula was a prismoidal formula based on the assumption that the depth and the velocity averaged over the depth varied linearly from $\overline{D}_i$ to $\overline{D}_{i+1}$. The formula was,

$$Q = \frac{W}{6}\left\{\left[\overline{D}_1\overline{V}_1 + (\overline{D}_1 + \overline{D}_2)(\overline{V}_1 + \overline{V}_2) + \overline{D}_2\overline{V}_2\right] + \right.$$

$$\left. + \left[\overline{D}_2\overline{V}_2 + (\overline{D}_2 + \overline{D}_3)(\overline{V}_2 + \overline{V}_3) + \overline{D}_3\overline{V}_4\right] + \ldots\right\}$$

where

- $Q$ was the total discharge,
- $\overline{D}_i$ was the mean depth of the ith vertical, $i = 1, 2, \ldots, n$,
- $\overline{V}_i$ was the mean velocity in the ith vertical,
- $W$ was the constant interval between adjacent verticals.

The formulas which Stevens (1908) compared with his standard formula are presented in Table 5. He concluded that under good conditions, that is where the bed was smooth and regular, and velocities were uniform and undistrubed, all formulas gave satisfactory results; under less favorable conditions, the formulas subscripted as C, F, and G, gave estimates which were too large. Limited field comparisons revealed that the three-ordinate methods could yield results with eleven percent discrepancies due entirely to the manner of grouping the data. Methods B and D gave the smallest errors both theoretically and in the field,

TABLE 4. ERROR IN THE MEAN VELOCITY IN A VERTICAL ATTRIBUTED TO SAMPLING OF THE VERTICAL VELOCITY DISTRIBUTION

| Author | Type of Section | Measurement Method | Error in the Mean Velocity | Error in the Total Discharge Estimate |
|---|---|---|---|---|
| Stearns (1883) | Flume | 2, 3, and 4 points | -- | ± 1%, as compared with a weir value |
| | | Integrating, at a rate of less than 5% of the flow velocity | -- | ± 1%, as compared with a weir value |
| Murphy (1902) | Canals | 0.6D | -- | Mean error was 3.5%; greatest departure was 2.6%; maximum error was 5.7%; range was 4.7% |
| | Flume A | Top and bottom | -2.2 to 30.6% | Approximately 2%, if the velocity > 1.5 fps. |
| | Flume B | Top and bottom | -- | -1.13 to -1.88% |
| | | 0.6D | -- | ± 2 to 6% |
| | | 0.64D | -- | Maximum error of ± 2.5% |
| | | Integration | -- | ± 1 to 9% |
| Murphy (1904) | Broad, shallow streams | Top and bottom | +2 to 8% | |
| | Mountain streams, 378 measurements | 0.6D | -- | ± a permissible amount |
| Stevens (1911) | Streams with depth < 1 ft | 0.6D | ± 1.8%, as compared with 5 point value | -- |
| Harding (1915) | Canal, 96 measurements | 0.2 and 0.8D | +0.73%, as compared with 6 point value | -- |
| | | 0.6D | +4.80%, as compared with 6 point value | -- |
| | Canal, 55 measurements | Integration | +0.76%, as compared with 6 point value | -- |
| Rohwer (1933) | Flume | 0.6D | -- | Consistently positive |
| | | Integration | -- | Consistently negative |
| | | 0.2 and 0.8D | -- | Consistently good results |
| Pierce (1941) | Flume with artificial roughness | 0.6D versus 0.2 and 0.8D | -- | 0.6 D results were better |
| Rouse (1949) | "Speculation" | 0.6D | -- | ± 5% |
| | | 0.2 and 0.8D | -- | ± 2% |
| Anderson (1961) | 100 streams | 0.2 and 0.8D | -- | ± 1.5%, as compared with 11 point value |

the error of D of being twice that of B and of opposite sign. The largest error induced by method D was 1.8 percent and by B was 0.9 percent.

Young (1950) compared the commonly termed mean- and mid-section methods, that is B and D, respectively, by considering 213 field discharge measurements. For his estimate of the true discharge at each station, a value was computed from data for four times the usual number of verticals in the section. Both methods B and D were applied, and velocity measurements were made using cable, bridge, and wading procedures. Young's results are presented in Table 6. The majority of the measurements yielded discharges which were smaller than the assumed true discharge, and the average error for the mid-section method was smaller and nearer zero than that of the mean-section method. It was suggested that the cable measurements seemed most accurate because the cableways were situated at the best cross sections. Further, Young observed that the mid-section method had a positive difference for the velocity component and a negative difference for area; while the mean-section method had negative differences for both. Consequently, the components tended to compensate in the mid-section approach, but increased negatively in the mean-section one. It was also statistically

## TABLE 5. A COMPARISON OF DISCHARGE FORMULAS

| Formula | Description of Formula | Difference Between Formulas | Comments - (Assuming "Exact Formula" is Exact) |
|---|---|---|---|
| "Exact formula" [Stevens (1911)] $Q_A = \frac{W}{6}\left[D_1V_1 + (D_1+D_2)(V_1+V_2) + D_2V_2 + \ldots\right]$ | A prismoidal formula applied to each prismoid in which the mid-area is obtained by averaging homologous dimensions of end areas. | | |
| $Q_B = W\left[\left(\frac{D_1+D_2}{2}\right)\left(\frac{V_1+V_2}{2}\right) + \left(\frac{D_2+D_3}{2}\right)\left(\frac{V_2+V_3}{2}\right) + \ldots\right]$ | A right prism of length W and cross-sectional area equal to the mid-area of the prismoid is substituted for each prismoid. (Mean-Section Method) | $Q_A - Q_B = \frac{W}{12}\left[(D_1-D_2)(V_1-V_2) + \ldots\right]$ | It usually gives too small results, (since the difference is positive as long as the factors have the same sign). |
| $Q_C = 2W\left[\left(\frac{D_1+2D_2+D_3}{4}\right)\left(\frac{V_1+2V_2+V_3}{4}\right) + \ldots\right]$ | Two consecutive prismoids are considered together, and for their actual volume there is substituted a right prism of length 2W, whose cross-sectional area is the product of the averages of the homologous dimensions of the computed mid-areas of each prismoid. | $Q_A - Q_C = \frac{W}{12}(D_1-D_2)(V_1-V_2) + \frac{W}{12}(D_2-D_3)(V_2-V_3) + \frac{W}{8}(D_1-D_3)(V_1-V_3) + \ldots$ | It usually gives smaller results than $Q_B$, but the result is also dependent on the order in which the notes are considered. |
| $Q_D = W\left[\frac{D_1V_1}{2} + D_2V_2 + \ldots + \frac{D_nV_n}{2}\right]$ | Average end-area or Mid-Section Method. | $Q_A - Q_D = -\left[\frac{W}{6}(D_1-D_2)(V_1-V_2) + \ldots\right]$ | The result is too large; the error being twice the error in $Q_A$ but of opposite sign. |
| $Q_E = \frac{W}{8}\left[(D_1+6D_2+D_3)V_2 + (D_2+6D_3+D_4)V_3 + \ldots\right]$ | Each measured velocity is assumed to apply as a mean to a partial area extending a distance W/2 on either side of the measuring vertical. | No usable relationship | The answers tend to be too large or too small indiscriminantly. |
| $Q_F = 2W\left[\left(\frac{D_1+4D_2+D_3}{6}\right)\left(\frac{V_1+4V_2+V_3}{6}\right) + \ldots\right]$ | | $Q_A - Q_F = \frac{W}{18}\left[(D_1-D_2)(V_1-V_2) + (D_2-D_3)(V_2-V_3) + (D_1-D_3)(V_1-V_3) + 3(D_1V_1 - 2D_2V_2 + D_3V_3) + \ldots\right]$ | Different results can be obtained from the same set of field notes. For continual concavity of both depth and velocity curves, the results are larger than $Q_A$ yields; for irregular conditions, the results are too large for one manner of grouping and too small for the other, the range of error being quite large |
| $Q_G = \frac{2W}{6}\left[(D_1V_1 + 4D_2V_2 + D_3V_3) + \ldots\right]$ | A section of width 2W is considered as a single prismoid in which the measured area at the middle of the double section is assumed to be the mid-area of the prismoid. | $Q_A - Q_G = -\frac{W}{6}\left[(D_1-D_2)(V_1-V_2) + (D_2-D_3)(V_2-V_3) - (D_1V_1 - 2D_2V_2 + D_3V_3) + \ldots\right]$ | |

TABLE 6. A COMPARISON OF DIFFERENCES BETWEEN COMPUTED AND "TRUE" DISCHARGE VALUES. [AFTER YOUNG (1950)]

| Type of Velocity Measurement | Number of Measurements | Average % Difference (Without Regard to Sign) | | Average % Difference (With Regard to Sign) | |
|---|---|---|---|---|---|
| | | Mean-Section | Mid-Section | Mean-Section | Mid-Section |
| Bridge | 63 | 1.46 | 1.22 | -0.97 | -0.37 |
| Wading | 80 | 1.38 | 1.24 | -0.66 | -0.04 |
| Cable | 70 | 0.98 | 0.82 | -0.54 | -0.03 |
| Average | 213 | 1.27 | 1.09 | -0.72 | -0.15 |

verified that the difference in percentage error between the two methods was significantly different from zero at the 99 percent confidence level; and that when one method was in error, the other contributed an error of like sign.

There is a conflict between the theoretical results of Stevens (1908) and the field comparisons of Young (1950). Relative to the values computed by both the mid-section and mean-section methods, Young's "true" discharge values appear larger than Steven's "true" values. The linear assumptions underlying the derivation of Steven's "true" discharge formula may account for this difference. In reality, the assumptions are valid only as the number of verticals approaches infinity; and, when used with a standard number of sections, they may render a formula not significantly more accurate than those being tested. Young's findings would seem more valid, as all formulas gave virtually the same discharge value when applied to his increased number of verticals at each site.

Other investigations to study measurement accuracy as a function of the number of verticals used across a channel, have been conducted by Harding (1915), Eisenhuth and Odell (1937), and Anderson (1961). A summary of their results is given in fig. 6. Harding (1915) considered independently eight verticals and four verticals with respect to sixteen verticals for a number of lined and earth canals. Eisenhuth and Odell, (1937), using typical field notes, recomputed discharges using approximately one half and one quarter of the original information. Anderson (1961) evaluated the error by comparing the discharge value computed from 100 verticals per cross section with that computed from the regular number of verticals. From fig. 6, it appears that Anderson's results portray the most optimistic view.

From the literature, there would appear to be some chance of a systematically negative error in the discharge value due to the use of the common mid- and mean-section computation techniques. In addition, there is certainly a random error. Both types of error are functions of the number of measurement verticals taken across the stream channel.

2.2.2 <u>A single stage observation</u>

(a) Variation in stage during a discharge measurement

During a discharge measurement, the stream stage at the measuring section seldom remains constant. When the change in stage is 0.1 foot or less, it is customary for the arithmetic mean of the stage observations to be considered as the mean gage height for the observed discharge. If the change of stage is greater than 0.1 foot, the observed gage-height readings are adjusted to yield a weighted mean stage.

The most common adjustment technique, Corbett et al. (1962), makes use of the mean gage heights during periods of constant slope on the gage-height graph and the corresponding partial discharges. The computational formula is of the form,

$$\overline{H}' = \frac{1}{\widehat{Q}} \sum_{i=1}^{\eta} \widehat{Q}_i \widehat{H}_i = \frac{\widehat{Q}_1 \widehat{H}_1 + \widehat{Q}_2 \widehat{H}_2 + \dots + \widehat{Q}_\eta \widehat{H}_\eta}{\widehat{Q}}$$

where,

$\overline{H}'$ is the weighted mean gage height,

$\widehat{H}_i$ is the average gage height during the ith time interval, $i = 1, 2, \dots, \eta$,

$\widehat{Q}_i$ is the measured discharge during the ith time interval,

$\widehat{Q}$ is the total measured discharge, i.e.,

$$\widehat{Q} = \sum_{i=1}^{\eta} \widehat{Q}_i .$$

This weighting technique is based upon the following assumptions, and the accuracy of the weighted mean stage is dependent upon how closely they are satisfied. It is assumed that: (a) the stage-discharge relationship for the section is linear over the range involved; (b) the stage, and the discharge, increases or decreases at a constant rate; (c) the horizontal distributuion of discharge is symmetrical with respect to the center line of the stream; (d) the horizontal distribution of discharge is invariant over the range of stage and discharge involved. The magnitude of possible errors arising from deviations from the above conditions appears to be unknown.

Another technique, suggested by Eisenlohr (1937), involves the adjustment of the partial discharges in order to obtain a computed discharge corresponding to a selected stage. In this method, the

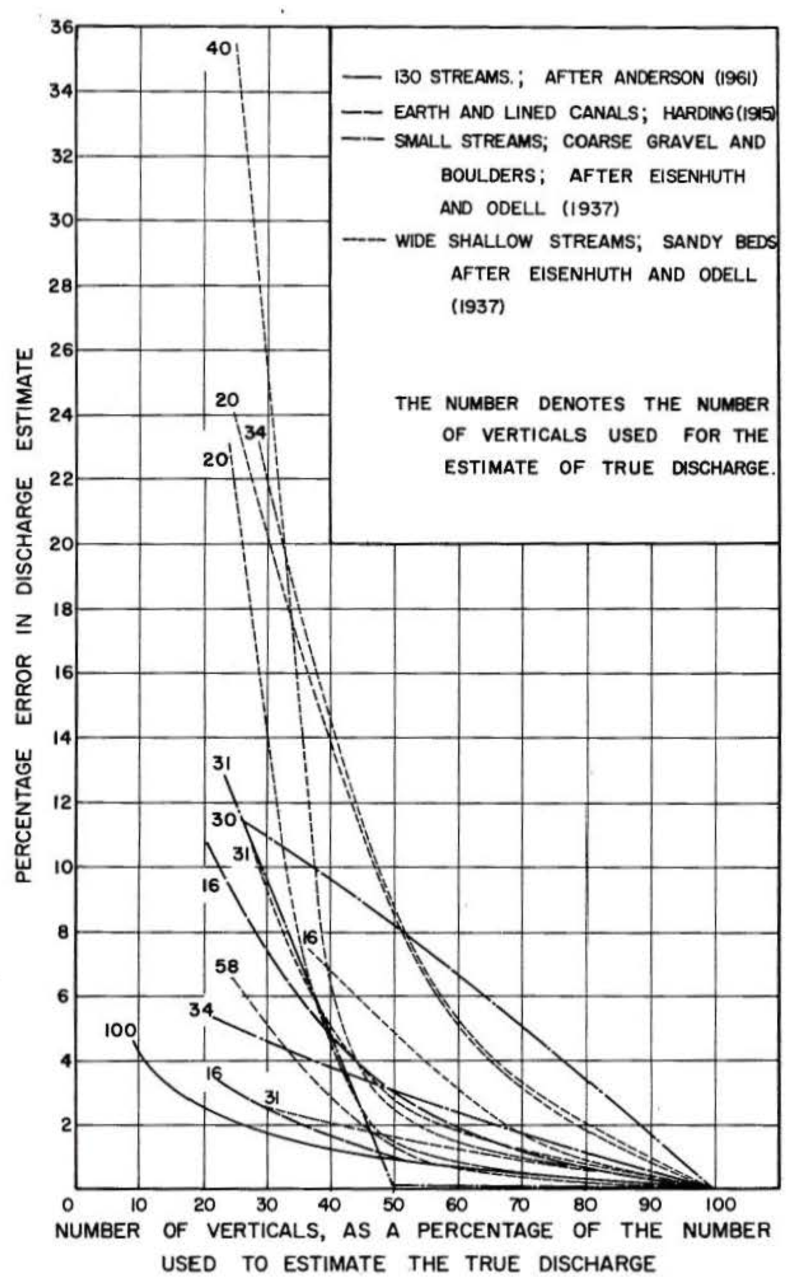


Fig. 6 The error in a discharge measurement as a function of the number of vertical sections

discharge is multiplied by the ratio of the total discharge at the selected stage to the total discharge corresponding to the stage of the partial discharge.

$$\hat{Q}^1 = \sum_{i=1}^{\eta} \hat{Q}_i^1 = \hat{Q}_1^1 + \hat{Q}_2^1 + \dots \hat{Q}_\eta^1 = \overset{*}{Q} \sum_{i=1}^{\eta} \hat{Q}_i / \overset{*}{Q}_i =$$

$$\overset{*}{Q}\left[\frac{\hat{Q}_1}{\overset{*}{Q}_1} + \frac{\hat{Q}_2}{\overset{*}{Q}_2} + \dots + \frac{\hat{Q}_\eta}{\overset{*}{Q}_\eta}\right]$$

where

$\hat{Q}^1$ is the weighted total discharge,

$\hat{Q}_i^1$ is the weighted partial discharge, $i = 1, 2, \dots, \eta$,

$\overset{*}{Q}$ is the discharge from an assumed or previously obtained rating curve corresponding to the selected stage,

$\overset{*}{Q}_i$ is the discharge from the same rating curve corresponding to the observed gage height in the ith time interval,

$\hat{Q}_i$ is the partial discharge measured in the ith time interval.

Besides the limiting conditions placed on the first technique mentioned above, this method presupposes the rating curve which the measurement is meant to define. Such an assumed curve may be an additional source of error whose magnitude has not been evaluated.

In many stream channels, a given percentage increase or decrease in the total discharge is not accompanied by the same percentage change in each partial discharge, and the horizontal distribution of velocity is not always symmetrical about the center line of the channel. Boyer (1937) suggested that the most precise method of adjustment involved an analysis of the distribution of discharge across the section and of the percentage changes in partial discharges per unit change of stage for a series of partial widths. Although this type of analysis is impractical for routine stream-gaging work, it is required in order to properly estimate the accuracy of the mean-stage determination.

(b) Stability of the control and datum

Provided that both the gage datum and the control section of a gaging station remain stable during a discharge measurement, a gage reading is indicative of a true stage-discharge relationship for the existing conditions. However, if instability in either of the above exists during the measurement, an error is introduced into the stage value.

The staff gage or reference used as a basis for all measurements at a station is seldom prone to instability. It is usually an easy task to have it established firmly, and the datum can be easily checked periodically with some other bench marks. If the stability of the gage datum is virtually ensured, no error is introduced into the record.

A more difficult problem is involved if the control section of the stream is unstable, and a true stage-discharge relationship does not exist for the particular gage. A measure of the error introduced into a stage value by such instability alone is not possible. However, in rocky stream-channels, characteristic of upper mountain watersheds where the channel and control are usually very stable, it may again be assumed that the error introduced by this source is random and relatively small.

(c) The sensitivity of the station

The refinement of a stage reading affects the data to a degree dependent on the sensitivity of the station. This sensitivity is indicated by the magnitude of the change in stage accompanying a given change in discharge; that is, the slope of the stage-discharge relationship. Grover and Hoyt (1916) suggested that the limiting requirement should be a change in stage that is readable for a change of one percent in discharge. This criterion has been adopted by the U. S. Geological Survey. The errors introduced by lack of reading refinement will usually be compensating, but may be cumulative when the stage shows very small fluctuations during an extended period.

## 2.3 The Establishment of a Stage-Discharge Relationship

### 2.3.1 The natural stage-discharge relationship

A basic premise for the development of a river gaging station is that there is a unique relationship between stage and discharge. This uniqueness is seldom realized in practice since nature is invariably altering conditions at the gaging site. However, a high degree of stability in the relationship is sought in establishing gage sites, since discrepancies from a truly singular relationship introduce inaccuracies in the flow record. The instability of the natural rating curve may be caused by the following: (a) instability of control, (b) conditions counteracting the effect of control, (c) hysteresis effects caused by the rising and falling stages of a flood wave, and (d) hysteresis effects caused by changing bed characteristics. Each of these topics will be discussed separately below.

(a) Instability of control

The control section at a gaging station may be stable, or may be unstable in either a random or a systematic manner. A stable section is one which does not change during a relatively long period of time; such as during the interval between one flood and a subsequent flood which might again cause some changes in the channel. On the other hand, the control section may vary in time as silting and scour alter the channel characteristics in a seemingly random fashion, or as either progressive silting or scour cause a systematic change. For a station with a stable control, the accuracy of the rating curve is dependent upon the accuracy of the individual discharge and stage measurements; for unstable control conditions, additional variation is introduced depending upon the degree and nature of the instability. Only an increased frequency of discharge measurements can better define the instability.

If the stream channel and its control section are stable, ample opportunity is provided for obtaining discharge measurements at various stages and a well-defined rating curve may be obtained. A single rating curve which tends to average the measurements is assumed to be more accurate than any individual discharge measurement itself. Spacing of the measurements on the rating curve, and the extent to which the curve is actually defined by measurements, are some of the details to which the engineer must give consideration in assigning the final accuracy. The probable error of the rating curve, reflecting measurement errors, can then be defined by a least squares approach, if a suitable mathematical expression can be found to represent the curve.

The accuracy of the discharge record for a station with an unstable control section is less than that for a stable site and is considerably more difficult to define. If it can be shown that the instability is of a random nature, then the probable error of the rating curve, reflecting both measurement and instability errors, could be computed as above. However, if there is a systematic variation of the control, it must be defined, and either a shifting rating curve or a time-dependent correction term developed. Three such approaches will be referred to under the section dealing with conditions counteracting the effect of control.

Artificial controls tend to improve accuracy, but it is not correct to assume that records for stations having such structures are always more accurate than records for stations having only natural controls.

(b) Conditions counteracting the effect of control

Stream-gaging stations are often affected by backwater caused by vegetative growth or ice formation. Although the control section may be quite stable, these effects may introduce a systematic variation into the stage-discharge relationship. These conditions are usually seasonal, and a number of techniques have been developed in an attempt to define the systematic nature of the rating curve variation. Liddell (1927), and the U. S. Geological Survey (1936) have outlined the following as methods of shifting ratings.

The Stout method regards the rating curve as only approximate. The differences between gage heights, as actually observed and as found from the rating curve to correspond to measured discharges, are plotted on a graph as ordinates and the times as abscissas. A smooth curve is drawn through the plotted points and gage-height corrections for intervening days are read from the graph. These corrections are applied to the observed gage heights, and the discharge values are obtained from the rating curve.

For the Bolster method, one or more standard rating curves are drawn through points plotted in chronologically consecutive order. The measurements may extend over an entire year and there may be several groups of points which will define their own rating curves. These curves will be used for all gage readings made between the first and last days whose gage heights were used in constructing the curve. For intervening days, the position of the curve may be found by joining the points representing consecutive measurements by a line, and by dividing the line into as many equal intervals as there are days intervening. The rating curve may then be raised or lowered parallel to itself until it passes through the point of division of the line.

When a correction may be justified by winter conditions, a factor may be applied to the discharge value rather than to the gage height. The open-water rating is first applied to the gage height and then the value is corrected by applying a coefficient which changes during the seasons.

It is important to realize that the Stout and Bolster methods assume individual measurements to be more accurate than the standard or average rating curve. For stations where the error, introduced by conditions counteracting the effect of control, is much larger than the error in an individual discharge measurement, this assumption may be justified. However, these techniques should not be employed at stations where the control is relatively stable and conditions counteracting the effect of this control are minor. In these latter conditions, the average rating curve remains better than any single measured point.

(c) Hysteresis effects caused by rising and falling flood-wave stages

The discharge past a stream section is not only dependent on the depth of flow and the shape of the section, but also on the water surface slope and the frictional nature of the section. It has already been argued that the degree of stability of the shape of the section has considerable effect on the accuracy of the rating curve. In this section the effect of changing slope is considered, whereas that of a variable friction factor is left for the next topic.

Under natural conditions, the slope of the water surface at any section does not remain constant when the discharge undergoes rapid variation. In the case of a rapidly rising river, the slope is steeper than the steady-flow slope, and the actual discharge will exceed the steady-flow discharge at any given stage; while on a rapidly falling river, the slope is less than that for steady flow, and the actual discharge will be less than that for steady flow. A continuously changing slope, as occurs during the passing of a flood wave, causes the corresponding stage-discharge relationship to describe a hysteresis loop rather than a single curved line. The loop is characterized by the rate of change of the water surface slope. Since this rate of change is a function of the particular flood hydrograph, no single hysteresis loop may be used to describe all unsteady flow situations at a given site. Rather, each flood wave results in a particular effect.

The errors arising from the hysteresis effect are systematic in nature but they tend to cancel each other if the rating curve is defined everywhere for the steady-flow slope. In practice, there is a tendency to obtain more of the points defining the rating curve on the recession limbs of flood waves than on the rising limbs. There is, therefore, a large chance that the rating curve is defined for a slope or slopes less than that for steady flow. Use of such a curve would systematically underestimate discharge.

In order to take the hysteresis effect into account, one must have a record of the water surface slope. This necessitates the use of two, rather than one, gaging stations along a reach of the channel. If such information is available, the technique outlined by Jones (1915) may be utilized to study the possible error. An approximate correction method has been outlined by Ionides (1934) which does not depend on a second gage in the reach. As the Jones' method is rather well known, it is not outlined here. However, Ionides' approach is briefly considered below.

Ionides (1934) suggested the following approximate simple method for finding the steady-flow rating curve. If a series of stage and corresponding discharge observations on a rise and fall are plotted versus time, they are out of phase, with the discharge leading the stage. If the discharge is plotted against a stage observed somewhat later, the lag is to some extent compensated for. Inaccuracies will occur, for example, at the peak discharge which is usually plotted against too low a stage. If the rate of change of stage, $dH/dt$, is estimated for the time of observation, the discharge may be plotted against $H_T + T_L\, dH/dt$, where $H_T$ is the gage reading at the time of observation, and $T_L$ is a time lag factor which may be assumed to be constant.

When the river is steady, $dH/dt$ is zero, and the discharge is plotted against the actual gage reading at the time of observation. If it is rising, $dH/dt$ is positive and the amount of the correction $T_L\, dH/dt$ depends upon the rate of change of stage. A rapid rise gives a large positive correction. When the river is falling, $dH/dt$ is negative and the observed gage reading is subject to a negative correction.

A trial and error procedure has been suggested by Ionides (1934) for determining $T_L$. When discharges are plotted against uncorrected gage readings, the rising and falling stage points tend to be grouped, the former closer to the discharge axis, as has been mentioned previously. A few well-defined rising and falling points should be selected and various time factors used until the best $T_L$ is found. This factor should then be applied to all observations. If it is correct, there should be no tendency for the above-mentioned grouping.

One method of evaluating the relative magnitude of the hysteresis effect caused by unsteady flow, involves the computing of the term, $\Delta S/S$, where $\Delta S$ is the maximum difference between the rising and falling slopes, and S is the channel slope. If this term tends to be negligible, no significant errors due to unsteady flow are incurred in the record. If, on the other hand, this term is large, one of the above techniques should be employed to reduce the variation in the stage-discharge data. For mountain watersheds, where the bottom slope is particularly large, and the difference in slope is not correspondingly great, the hysteresis effect due to unsteady flow has been found to be negligible.

(d) Hysteresis effects caused by changing bed characteristics

The recent studies of Simons and Richardson (1961) have revealed that the form of an alluvial channel stage-discharge relationship is closely related to (i) the regime of flow, (ii) the form of bed roughness, and (iii) the rate of change of discharge with time. In the range of shear, where ripples and dunes develop on the bed, the stage-discharge curve for the rising stage is usually quite different from that for a falling stage, presenting a special type of hysteresis effect. Like the hysteresis effect caused by unsteady flow, this effect gives rise to curves that are valid only for conditions upon which they are based and no general solution is possible. On the other hand, in the range of shear which develops plane bed, standing sand and standing water waves that are in phase, and antidunes, the rising and falling stage curves coincide and hold for all values of discharge associated with these forms of bed roughness. When the entire range of bed forms may exist in a section as the discharge varies, a discontinuity in the stage-discharge relationship occurs when the dunes wash out. At this point there is a large reduction in resistance to flow and a resultant reduction in depth even though discharge is increasing.

It is interesting to note that Boyer (1936) observed the above effect but could not adequately explain the phenomenon. He suggested that the rating curve above a certain point, which he called the "point of divergence," did not shift on sandy channels except under the influence of major floods; the curve below this point varied within well-defined limits in such a manner that a series of curves could be drawn, assuming a fan-shaped appearance. It was, therefore, termed a "fantail rating."

It is not yet possible either to predict this hysteresis effect for an actual gaging station or to define it for a flood occurrence other than by on-the-spot, continuous gaging. Further, as found by Simons and Richardson (1961), even a determined effect at a site cannot be generalized for other occurrences. This effect, is, therefore, a major source of error in records of alluvial streams. However, in upper mountain watersheds, it can generally be neglected.

(e) Summary of variations in the stage-discharge relationship

The sources of variation in the natural stage-discharge relationship can be seen to give rise to errors that are very difficult to specify. A qualitative description, as has been offered, is relatively easy; however, a quantitative description is very difficult. Fortunately, on many streams, the errors arising from such sources are random in nature. As a result, there may be a large scatter of plotted points about the rating curve, making a good plot of the stage-discharge relationship difficult. When the errors are definitely systematic in nature, some attempt to adjust the discharge or stage values is necessary if large errors in the records are to be avoided.

Gaging stations in upper mountain watersheds largely escape variations from the steady-flow rating curve. In other words, the assumption of a unique stage-discharge relationship, at least for relatively long periods of time, is reasonably valid. The control section, usually in rock or gravel, is very stable; there is a minimum amount of channel vegetation; the natural channel slope is sufficiently steep that the hysteresis effect due to changing surface slope is virtually negligible; and the absence of alluvium in the channel bypasses the effect of changing friction factor. Therefore, in the subsequent analysis, it has been assumed that the major variation of points from a fitted stage-discharge relationship is caused by errors in single discharge measurements, and not to any great extent to conditions causing the natural relationship to be unstable.

## 2.4 The Use of a Stage-Discharge Relationship

### 2.4.1 The record of stage

Once a stage-discharge relationship has been established for a gaging station, a record of stage at the station can be translated into one of discharge. This phase of the development of a discharge record may introduce further sources of potential error. These sources include the procedure used for sampling the stage in time and the nature of the physical installation and its inherent limitations.

(a) Stage sampling in time

There are many methods for obtaining a river-stage measurement. However, each method yields either a record of discrete stage values, further or less apart in time, or a continuous time series of stage. The error introduced by the use of discrete values to approximate the continuous record is a function of the potential range in stage, and of the rate of change of stage in relation to the time interval between readings. For example, once or twice daily readings of stage on a large river may better describe the continuous record than hourly readings on a flashy mountain stream. The trend in recent years has been to obtain a series of discrete stage values with a very short time interval between successive values, as such a series can be readily recorded on a digital recording unit.

The errors arising from the use of a series of discrete values can be evaluated only by a comparison of this series with the continuous record for a sample period for each stream. If the time interval between readings is sufficiently short, the errors may be considered to be random and tend to cancel. For the purpose of this study, it has been assumed that a continuous record is available such that this sampling error may be neglected.

(b) Installation errors

Installations for the purpose of continuously recording water-level fluctatuions can introduce errors into the flow records. Such errors, dependent on the type and condition of the particular installation, will be considered with regard to both float- and pressure-operated instruments. More detailed accounts of the various types of instrumentation and of their inherent errors have been presented by Stevens (1919-20), Liddell (1927), and Learmonth (1964).

The errors involved in the records of properly designed float-operated recorders are generally too small to be of great importance. However, as equipment either poorly designed or improperly maintained may introduce significant errors, it is well to recognize the sources of errors. The sources have been given the following names in the literature: (i) float lag, (ii) line shift, (iii) submergence of the counterpoise, (iv) temperature and saturation, (v) humidity. Each source is defined below.

(i) Float lag: If a float performs any mechanical work such as turning gears, moving an index hand or a recorder pen, or operating a totalizer or an electric switch, there is always a lag of the index behind the true water level. The force required to move the mechanism must be supplied by the float and can be supplied only by the pressure of the water on the float. The lag varies directly with the force required to move the mechanism and inversely as the area of the float. Although, it cannot be entirely eliminated, this float lag can be made very small.

(ii) Line shift: With every change of stage, a portion of the float line passes from one side of the float pulley to the other. This change of weight alters the depth of floatation of the float, causing the stylus to deviate from the true water height by an amount dependent on the change in stage since the last correct setting, and on the weight of the line.

(iii) Submergence of counterpoise: When the counterpoise and any portion of the line becomes submerged, the tension in the float line is reduced and the depth of floatation increased. This error is always positive and tends to compensate for the error of line shift.

(iv) Temperature and saturation: Differences in thermal expansion from temperature changes of the stilling well and float line may cause significant errors if the well is very large. However, for normal stream-gaging installations, this error is negligible. If a wooden well is used, saturation of the wood will lift the recording instrument. Only in very deep wells need this source of error be considered.

(v) Humidity: All paper is affected to some extent by humidity changes. In extreme cases, Stevens (1919-20) has noted an expansion of two percent. Although the paper might expand two percent, it does not follow that the stage record is in error by that amount. The actual error depends upon the stage and the position of the neutral axis of expansion. When extreme accuracy is required, strip charts are available which are equipped with check points.

Since a float-operated recorder is used in conjunction with a stilling well and an intake pipe, errors inherent with these installations will be considered with those contributed by the recorder itself. Sources of errors in such an installation include: (i) a lag of the water rise in the well behind the actual stream use, (ii) silting effects, and (iii) a drawdown in the well due to high velocities past the intake pipe.

(i) Lag of the water rise in the well: The purpose of a float well is to avoid oscillations and surging of the float that would otherwise occur and result in an obscured record. With a similar purpose, the intake is kept relatively small. Although the above purpose is justified, the use of a stilling well and intake pipe may, in fact, cause a water rise or fall in the stilling well to lag sharp stream fluctuations. This source of error has neither received much mention in the literature nor been evaluated for various sizes of intakes and stilling wells.

(ii) Silting effects: If the stream carries considerable sediment, silting of the intake may reduce its capacity and cause an even more serious time lag of the record. Most float-operated installations are designed to either avoid severe silting or allow desilting maintenance.

(iii) Drawdown in the well: It was observed at recording installations during the 1930's that the passage of high flow velocities by the inlet to the intake pipe caused a drawdown effect on the well; that is, the level present in the stilling well was lower than the corresponding stream level. The U. S. Geological Survey (1937) presented their findings from a study of this drawdown effect in flume experiments. The maximum difference in the well of -1.75 feet occurred when the intake was at right angles to the current. This difference was exactly equal to the velocity head of the flow past the inlet. The minimum difference was + 0.25 feet when the leg was turned upstream at $30^0$. If a deflector was used on the end of the intake pipe to eliminate velocity and velocity head at the intake, the differences were minimized. The U. S. Geological Survey (1937) also referred to a study undertaken at the National Hydraulic Laboratory which revealed that fins protruding one and one-half times the pipe diameter were most effective in reducing drawdown in the well.

The effect of the drawdown is a function of the position and condition of the inlet. If both factors remain constant in time, then the drawdown is constant for the same flow conditions, and no error is introduced. The discharge is merely referenced to a set of drawdown-stage values.

The bubble-type pressure system commonly called the bubble-gage, eliminates the need for a stilling well and intake pipe. It usually consists of a specially designed servo-manometer, a transistor control, and a gas-purge system. The mercury manometers used by the U. S. Geological Survey, Buchanan (1966), have a sensitivity of ±0.005 feet and can be built to record ranges in stage in excess of 120 feet.

Learmonth (1964) considered the sources of error in pressure systems in great detail under the headings: (i) gas compression in closed systems, (ii) leakage of gas from closed systems, (iii) temperature errors in closed systems, (iv) insufficient flow of gas in open systems, (v) friction loss in feed line in open systems, (vi) temperature and regulator errors in open systems, (vii) blockages, (viii) leaks in open systems, (ix) temperature errors in transducers, (x) meniscus errors, (xi) friction and backlash. As in the case of float-operated instrumentation, properly adjusted equipment gives rise to negligible errors.

2.4.2 Computation of mean daily flow

Streamflow records are usually presented in tables of daily flow values. Each daily flow value is obtained by applying the mean daily gage height, usually estimated by eye, to the rating curve. The error arising from this procedure is dependent upon the curvature of the rating curve and the daily range in stage.

It was suggested by Grover and Hoyt (1916) that a maximum allowable error for translating mean daily stage to mean daily flow should be one percent. The U. S. Geological Survey continues to use this criterion. The amount of daily range in stage allowable for a given mean daily stage, in order that errors due to curvature of the rating curve shall not exceed one percent, can be found graphically. A chord is constructed to the rating curve such that the horizontal distance, measured by the discharge scale, from the midpoint of the chord to the curve equals one percent of the discharge at the corresponding stage. The difference in stage values at the ends of the chord is the allowable daily range.

The errors resulting from the application of mean stage values to the rating table are generally cumulative, and therefore may contribute to a systematically positive error in the estimated discharge values. This error can be controlled by the type of technique mentioned above.

### 2.5 Additional Sources of Error

#### 2.5.1 Winter records

Ice conditions in the stream during the winter present major difficulties to obtaining accurate flow records for this period of the year. The normal open-water stage-discharge relationship is altered by the presence of ice in its many forms, and the relationship continues to change as the ice conditions alter. Further, it is virtually impossible to maintain a continuous stage record unless very elaborate and expensive equipment is installed at the station. As a result, the accuracy of winter records are highly dependent upon the number and distribution of the discharge measurements made during that period.

Individual discharge measurements made under ice conditions are, in general, less accurate than those made in open water. The vertical velocity curve under ice cover is drawn back further in its upper position, and the shape of the curve is largely dependent on the roughness of the under-surface of the ice. Buchanan (1966) has recommended that the 0.2 and 0.8 depth method be used for effective depths (total depth of water minus the distance from water surface to bottom of ice) of 2.5 feet or greater, and the 0.6 depth method was recommended for effective depths less than 2.5 feet. By considering 352 vertical velocity curves under ice, Barrows and Horton (1907) found the average coefficient for obtaining mean velocity from 0.2 and 0.8 depths was 1.002, varying from 0.98 to 1.04. A coefficient of about 0.92 must be applied to the 0.6 depth velocity, or one of about 0.88 to a mid-depth velocity. In addition, the presence of ice crystals or floating ice may clog the meter; the meter parts themselves may become frozen when the meter is moved out of the water between velocity readings; it is difficult if not impossible to properly sample the section; and the hydrographer's physical discomfort does nothing to enhance an accurate measurement.

Rating curves may be constructed according to gage heights to the surface of the water or to the bottom of the ice. The curve as constructed with gage heights to the bottom of the ice, in general lies to the left of the open-water curve, but tends to approach it in its lower portion. This observation has led to the derivation and use of so-called winter backwater curves. With gage heights to the surface of the ice, the rating curve is approximately parallel to the curve determined by stages to the bottom of the ice.

It is not uncommon for ice to form in or completely block the intake pipe or stilling well at recorder installations. Such events render the record completely unrepresentative.

In the final analysis, available flow measurements, a special rating curve, and data of general climatic conditions are considered, and a largely subjective estimate of flow is made. As a result, individual daily flows during winter conditions may be grossly in error. However, it may be assumed that such flows are so small relative to the remainder of the flow during the year, that the annual discharge value is affected only slightly. This assumption may not be entirely justified in spring break-up conditions when accurate estimates of flow are difficult to obtain and the flow volumes may be large. In mountain watersheds, where major runoff volumes usually occur during open water at the gaging stations, the assumption is shown to be reasonably valid in Chapter IV.

#### 2.5.2 Human subjective errors

Subjective errors are those made by the hydrographer in reading the instruments, counting meter revolutions, making computations, and in making biased observations by consistently reading high or low. Factors such as weather conditions, traffic, mental attitude, training, and morale contribute to such errors. These errors cannot be controlled but they can be minimized. Except for gross errors which are usually self evident, human subjective errors cannot be evaluated. They are considered to be random in nature and small.

### 2.6 Classification of Errors

As an aid to summarizing the foregoing discussion, a classification of sources of errors has been prepared. The type of error resulting, its possible functional relationship, and some relative magnitudes are presented. The classification is given in Table 7. Besides its usefulness as a summary, it is hoped that this classification can serve as a focal point for attention and discussion regarding the topic of errors in streamflow records.

TABLE 7. A CLASSIFICATION OF ERRORS IN DISCHARGE DETERMINATIONS ON MOUNTAIN STREAMS

| Source of Error | Type | The Error Is A Function Of: | Relative Magnitude |
|---|---|---|---|
| A. A Single Stream-Gaging Measurement<br>1. A single discharge measurement<br>a) Velocity considerations<br>i) A single velocity determination | | | |
| (1) The current-meter calibration curve | Random | Precision of the calibration procedure; meter characteristics | For velocity > 1fps: $\pm 0.5\%$<br>For velocity < 1fps: $\gtrsim 0.5\%$ |

TABLE 7 - continued

| Source of Error | Type | The Error Is A Function Of: | Relative Magnitude |
|---|---|---|---|
| (2) Properties of the fluid (eg. viscosity, density) | Systematic | Climate or region; sediment load; meter characteristics | For normal conditions, the error is negligible. |
| (3) Boundary effects | Systematic | Meter characteristics; depth and relative roughness of section; velocity distribution | For depths > 1 ft: negligible |
| (4) Oblique currents | Systematic | Meter characteristics; obliquity of flow | For cup meter: positive<br>For propeller meter: negative<br>For screw meter: ± |
| (5) Micro-turbulence | Systematic | Degree of turbulence and its variability; particular meter characteristics | For cup meter: positive<br>For propeller meter: negative<br>For screw meter: f(meter) |
| (6) Time measurement | Random | Precision of time-piece and methodology used | ± negligible |
| ii) Sampling in space | | | |
| (1) Velocity distribution in vertical | Systematic | Difference between true and assumed distributions. | ± 1.5 to 5.0% |
| (2) Robustness of vertical sampling technique | Random | The error in placing the meter at the correct depths | Effect on total discharge |
| (3) Velocity distribution in the horizontal | Random | Difference between true and assumed distributions; the number of verticals used | ± 5% effect on total discharge |
| iii) Sampling in time | | | |
| (1) Pulsations in the flow regime | Random | The distribution of point velocities in time; the effect of position and roughness | ≥ ± 2% effect on total discharge |
| b) Area considerations<br>i) A single measurement | | | |
| (1) Measurement of depth | Random | Measuring instrument and technique | ± negligible |
| (2) Measurement of width | Random | Measuring technique | ± negligible |
| ii) Sampling in space | | | |
| (1) Depth distribution in the horizontal | Random | Difference between true and assumed bed configurations | ± negligible |
| iii) Sampling in time | | | |
| (1) Depth distribution at a point in time | Random | The distribution of depth at a point in time | ± negligible |
| 2. A single stage measurement | | | |
| a) Surface tension effects | Systematic | Measuring instrument and technique | - negligible |
| b) Sampling stage in time | Random | Stage fluctuations in time | ± negligible |

TABLE 7 - continued

| Source of Error | Type | The Error Is A Function Of: | Relative Magnitude |
|---|---|---|---|
| B. Establishment and Use of a Stage-Discharge Relationship<br>1. The natural stage-discharge relationship | | | |
| a) Instability of control | Random or Systematic | The control | ± a small random error for mountain watersheds |
| b) Conditions counteracting the effect of control | Systematic | Reach of river | |
| c) Hysteresis effects of flood waves | Random or Systematic | Water slope and flashiness of the stream | |
| d) Hysteresis effects of bed characteristics | Systematic | Bed material and range of bed forms | |
| 2. The record of stage in time<br>a) Sampling stage in time<br>i) Point measurements | | | |
| (1) Measurement technique | Random | Measurement device and method | Omitted since only recording installations are being considered |
| (2) Sampling stage in time | Random or Systematic | Deterministic and stochastic variability of stage during measurement intervals | |
| ii) Recording installations | | | |
| (1) Lag of water rise in well | Random | Fluctuations of river stage | ± negligible |
| (2) Drawdown effects on the well | Systematic | The installation and velocity past intake pipe | - negligible |
| (3) Temperature and humidity effects | Random | Variability of climate and the installation | ± negligible |
| (4) Stage-recording mechanism | Random | Instrumentation | ± negligible |
| 3. Fitting of a rating curve | Random | Fitting procedure and scatter of points | |
| 4. Mean daily discharge determination | | | |
| a) Mean daily stage | Random | Variation of daily stage | ± negligible |
| b) Mean daily discharge | Systematic | Daily range of stage and linearity of curve in that range | ≤ 1% |
| C. Computational Procedures | | | |
| 1. Monthly discharge | Random | Dependence among daily values | |
| 2. Annual discharge | Random | Dependence among daily values | |
| D. Other Sources | | | |
| 1. Human subjective errors | Random | Personnel involved | |
| 2. Winter records | Random | The climate, site, and flow characteristics | |

## CHAPTER III

## AN ERROR MODEL FOR A SINGLE DISCHARGE MEASUREMENT

### 3.1 Method of Approach

In order to analyze the manner of combination of the component errors discussed in the previous chapter, a hypothetical error model has been established for a single discharge measurement at a gaging station. Whereas error models have been prepared by Prochazka (1960), and by Carter and Anderson (1963) to be practical working expressions, this model has been designed as an analytical tool for a better understanding of the factors involved. At such time when sufficient data has been studied for a meaningful evaluation of each term in the model, its practical application could be considered.

The approach employed in this study involves the derivation of a general expression for the error in a discharge measurement in terms of the component error functions. Then the mean and variance of the basic model are considered in terms of hypothetical means and variances of factors involved in the components. Further studies involving the possible nature of these components are included. There has been an attempt to define the order of magnitude of some of the terms, where possible, but no attempt to evaluate all terms.

### 3.2 A General Model

The discharge past a measuring section in a river may be defined as the integral over the cross-sectional area and over the measurement time of the velocity vector perpendicular to the section. That is,

$$Q = \frac{1}{T}\iint_{(x,z)}\int_{t=0}^{T} V(x, z, t)\,dx\,dz\,dt$$

where

- $Q$ represents the mean discharge over the time interval $T$,
- $(x, z)$ are the coordinates defining the measuring section,
- $V(x,z,t)$ represents the velocity vector perpendicular to the section.

If the cross section is divided into n vertical sections, the above relationship may be expressed as,

$$Q = \sum_{i=1}^{n} A_i \overline{V}_i$$

where

- $A_i$ is the time-average area of the ith vertical,
- $\overline{V}_i$ is the time- and area-average velocity of the ith vertical.

In practice, the discharge is estimated by,

$$\widehat{Q} = \sum_{i=1}^{n} \widehat{A}_i \widehat{\overline{V}}_i$$

where $\widehat{\ }$ denotes the quantities estimated by measurement. Therefore, the error in a single discharge measurement may be expressed as

$$q = \widehat{Q} - Q = \sum_{i=1}^{n} (\widehat{A}_i \widehat{\overline{V}}_i - A_i \overline{V}_i)$$

If $\widehat{A}_i$ and $\widehat{\overline{V}}_i$ are written in the forms,

$$\widehat{A}_i = A_i + a_i ,$$

$$\widehat{\overline{V}}_i = \overline{V}_i + v_i ,$$

then

$$q = \sum_{i=1}^{n} (A_i v_i + \overline{V}_i a_i + v_i a_i)$$

where $a_i$, $v_i$, $q$ are errors in the determination of $A_i$, $\overline{V}_i$, and $Q$, respectively.

Since the last term within the summation expression above may be considered to be much smaller than the other two terms, it and terms similar to it have been omitted from subsequent equations, with the assumption that no accuracy of representation has been lost. That is,

$$q \approx \sum_{i=1}^{n} (A_i v_i + \overline{V}_i a_i)$$

This is the basic equation for the error in a single discharge measurement.

The error in a vertical sectional area, $a_i$, is considered first. The area in the ith vertical is given by,

$$A_i = \int_{W=0}^{W_i} D\,dW = W_i \overline{D}_i$$

and an estimate of $A_i$ is

$$\widehat{A}_i = \widehat{W}_i \widehat{\overline{D}}_i$$

where

- $\widehat{W}_i$ is an estimate of the width associated with the ith vertical, $W_i$,
- $\widehat{\overline{D}}_i$ is an estimate of the mean depth of the ith vertical, $\overline{D}_i$.

Therefore,

$$a_i = \widehat{W}_i \widehat{\overline{D}}_i - W_i \overline{D}_i$$

As before, the estimated values may be written as,

$$\widehat{W}_i = W_i + w_i ,$$

$$\widehat{\overline{D}}_i = \overline{D}_i + d_i ,$$

and the error in the ith vertical area expressed as,

$$a_i \approx W_i d_i + \overline{D}_i w_i .$$

Further, the error in the mean velocity estimate for the ith vertical can be considered as,

$$v_i = \widehat{\overline{V}}_i - \overline{V}_i .$$

Following the general approach used above,

$$\overline{V}_i = \sum_{j=1}^{m_i} (R_{ij} \overline{V}_{ij})$$

and

$$\widehat{\overline{V}}_i = \sum_{j=1}^{m_i} (R_{ij} \widehat{\overline{V}}_{ij} + v_p)$$

or

$$v_i = \sum_{j=1}^{m_i} (R_{ij} \widehat{\overline{V}}_{ij} + v_p - R_{ij} \overline{V}_{ij}) = \sum_{j=1}^{m_i} (R_{ij} v_{ij} + v_p)$$

where $R_{ij}$ is the appropriate weighting factor attributable to each time average velocity, $\overline{V}_{ij}$; there being $m_i$ points selected in the ith vertical for sampling the vertical velocity profile,

$v_p$ is the error in sampling the vertical profile of velocity,

$v_{ij}$ is the error in measuring the time-average velocity at a point.

The error in measuring $\overline{V}_{ij}$ may be expressed generally as,

$$V_{ij} = \widehat{\overline{V}}_{ij} - V_{ij} = v_c + v_o + v_s + v_t$$

where

$v_c$ is the error introduced by the current-meter calibration curve,

$v_o$ is the error caused by oblique currents,

$v_s$ is the error in sampling velocity in the horizontal direction,

$v_t$ is the error in sampling velocity in time.

Substitution of the various error relationships into the basic equation for the error in a single discharge measurement yields,

$$q \approx \sum_{i=1}^{n} \left\{ A_i \sum_{j=1}^{m_i} \left[ R_{ij} (v_c + v_o + v_s + v_t) + v_p \right] + \overline{V}_i \left[ W_i d_i + \overline{D}_i w_i \right] \right\}$$

This expression is a general error model, including the error components only in general form. The next sections afford a discussion of possible characteristics of these components.

### 3.3 The Component Errors

#### 3.3.1 The error in the area of a vertical section

This error, $a_i$, can be subdivided into components involving the linear measurements of width and depth. The error in measuring the depth may be considered as,

$$d_i = d_t + d_s + d_m$$

where

$d_t$ denotes the error due to sampling the mean depth, $\overline{D}_i$, in time,

$d_s$ denotes the error due to sampling in the horizontal direction,

$d_m$ denotes error due to measurement technique.

It seems reasonable to assume that the three component errors of $d_i$ are (i) independent among themselves, (ii) can each be expressed as a proportion of the true mean depth, and (iii) are normally distributed. That is:

$$d_t = \delta_t \overline{D}_i$$

where

$\delta_t$ = N $(0, \sigma^2_{dt})$ ; i.e., a normal distribution with mean 0 and variance $\sigma^2_{dt}$;

$$d_s = \delta_s \overline{D}_i$$

where

$\delta_s = N(0, \sigma^2_{ds})$

$\sigma^2_{ds}$ is a function of the bed shape and the number of verticals, n;

$$d_m = \delta_m \overline{D}_i$$

where

$\delta_m = N(\mu_{dm}, \sigma^2_{dm})$

$\mu_{dm}$ is a mean bias due to measurement technique.

Therefore,

$$d_i = (\delta_t + \delta_s + \delta_m)\overline{D}_i$$

or

$$d_i = \delta_i \overline{D}_i$$

where

$$\delta_i = N(\mu_d, \sigma^2_d)$$

$$\mu_d = \mu_{dm}$$

$$\sigma^2_d = \sigma^2_{dt} + \sigma^2_{ds} + \sigma^2_{dm}$$

Careful measurement technique can remove the bias from this error, and can reduce the variance, due to technique, to a very small value. The remaining two components of the variance may be functions of depth, velocity, the bed shape, and the number of vertical sections selected. These terms have not received any experimental attention, and no quantitative estimates of them are available. It seems reasonable to assume that for a well-selected gaging site, the standard deviation, $\sigma_d$, would be less than one percent of the mean depth.

Then

$$\delta_i = N(0, \sigma_d^2)$$

and

$$\sigma_d < 0.01 \overline{D}_i$$

The error in measuring the width of the ith vertical can likewise be subdivided into errors due to sampling in time, sampling in area, and measurement technique. The error due to sampling in area is a function of the width of the measuring section and the number of verticals. However, for the sake of this analysis, the error in measuring $W_i$ is considered to be negligible, as shown by Prochazka (1960).

From the above discussion, the error in the area of a vertical section may be expressed as,

$$a_i \approx \delta_i W_i \overline{D}_i = \delta_i A_i \ .$$

Further,

$$E\left[a_i\right] = 0$$

$$\mathrm{Var}\left[a_i\right] \approx \mathrm{Var}\left[\delta_i\right] A_i^2 \approx \sigma_d^2 A_i^2$$

where $\sigma_d^2$ approaches zero as the number of vertical sections increases and as the depth increases for well-chosen gaging stations.

3.3.2 The error in the mean velocity of a vertical section

This error term involves the error in sampling the vertical profile of velocity in addition to the errors attributable to sampling the velocity at a point. Each of the individual components is discussed below.

(a) The error in sampling the vertical profile of velocity, $v_p$

Once again, this term may be best considered as involving two components. One is the error incurred in the selection of a theoretical vertical profile and suitable sampling points with their respective weights. A second error occurs when the streamgager fails to place the current meter at the correct sampling positions.

In order to gain an appreciation for the relative magnitude of these errors, the robustness of the standard one- and two-point sampling techniques was studied with regard to several theoretical vertical profiles of velocity. A word of definition regarding the term robustness in this context seems warranted. Since the current meter cannot be placed precisely at the selected sampling points in a vertical, it assumes positions on one side or the other of the points desired. A question arises with regard to how critical it is to the sampling method to place the meter as precisely as possible. In other words, how robust is the sampling technique? A very robust method could tolerate large deviations of meter placement with relatively minor effect on the estimate of mean velocity in a vertical. On the other hand, a method lacking robustness would require very precise placement of the meter in order to avoid introducing large errors into the estimate of mean velocity.

Two expressions have been evaluated to express the robustness of each sampling method as applied to each of twelve theoretical velocity profiles. The relative bias of the sampling scheme is represented by $v_d/\overline{V}$, where $v_d$ is the bias in the estimate of the mean velocity in the vertical, $\overline{V}$, due to misplacement of the meter. This expression is in terms of the variance of the placement error about the correct sampling point, the mean error having been assumed to be zero. If the variance is assumed to approach zero, the constant term remaining represents the bias due to the failure of the sampling scheme to yield the true mean of the velocity distributuion. The second term used to denote the robustness is the variance of the error, $\mathrm{Var}[v_d]/\overline{V}^2$, in relation to the square of the true mean velocity. Again, this expression is in terms of the variance of the placement error. When the placement variances at the 0.2 and 0.8 depths are equivalent, the expressions for the bias and variance introduced by the sampling scheme may be written in terms of the common variance, $\sigma^2_{2/8}$, and the correlation coefficient between the errors at the two points, $\rho_{2/8}$.

The results of the study of robustness are presented in Table 8. In order to illustrate the method used to arrive at the expressions, and to clarify the terminology, the relationships for the parabolic velocity distributuion, $V = \gamma Z^{1/2}$, are derived below. The velocity is given in terms of the proportionate or relative depth, Z, measured from the streambed. The average or mean velocity in the distribution may be evaluated to be,

$$\overline{V} = \int_0^1 \gamma Z^{1/2}\, dZ = 0.667\,\gamma$$

Further, the relative depth at which the mean velocity occurs in the profile is,

$$Z_{\overline{V}} = \left(\frac{\overline{V}}{\gamma}\right)^2 = 0.445 \text{ from the bottom,}$$

or 0.555 from the surface.

For the one-point sampling technique, the error in the depth measurement for sampling at the 0.60 depth may be characterized as

$$\Delta_{0.6} = N(0, \sigma^2_{0.6}) \ ;$$

that is, with a normal distribution of mean zero and variance $\sigma^2_{0.6}$. Then,

$$\hat{\overline{V}} = \gamma(0.40 + \Delta_{0.6})^{1/2} \ .$$

Employing a Taylor Series expansion,

$$\hat{\overline{V}} \approx \gamma(0.6325 + 0.7906\,\Delta_{0.6} - 0.4941\,\Delta_{0.6}^2 + 0.6176\,\Delta_{0.6}^3 + \ldots)$$

TABLE 8. ROBUSTNESS OF SAMPLING SCHEMES USED FOR VERTICAL VELOCITY DISTRIBUTUIONS

| Velocity Distribution | $\overline{V} = f(\gamma)$ | $Z_{\overline{V}}$ | 0.6 Point Sampling | | 0.2 And 0.8 Point Sampling | |
|---|---|---|---|---|---|---|
| | | | $v_d/\overline{V}$ | $Var(v_d)/\overline{V}^2$ | $v_d/\overline{V}$ | $Var(v_d)/\overline{V}^2$ |
| Parabolic<br>$V = \gamma Z^{1/2}$ | $0.667\gamma$ | 0.555 | $-(0.051 + 0.741\sigma^2_{0.6})$ | $1.406\sigma^2_{0.6}$ | $0.006 - (0.131\sigma^2_{0.2} + 1.048\sigma^2_{0.8})$<br>or<br>$-0.006 - 1.179\sigma^2_{2/8}$ | $(0.173\sigma^2_{0.2} + 0.677\sigma^2_{0.8}) + 0.703\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(0.849 + 0.703\rho_{2/8})$ |
| Reversed Parabolic<br>$V = \gamma(2Z - Z^2)$<br>(axis at the surface) | $0.667\gamma$ | 0.577 | $-(0.040 + 1.500\sigma^2_{0.6})$ | $3.240\sigma^2_{0.6}$ | $-0.010 - (0.750\sigma^2_{0.2} + 0.750\sigma^2_{0.8})$<br>or<br>$-0.010 - 1.500\sigma^2_{2/8}$ | $(0.090\sigma^2_{0.2} + 1.440\sigma^2_{0.8}) + 0.720\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(1.530 + 0.720\rho_{2/8})$ |
| Reversed Parabolic<br>$V = \frac{\gamma}{0.81}(1.8Z - Z^2)$<br>(axis 0.10 below surface) | $0.700\gamma$ | 0.593 | $-(0.010 + 1.500\sigma^2_{0.6})$ | $2.250\sigma^2_{0.6}$ | $-0.012 - (0.882\sigma^2_{0.2} + 0.882\sigma^2_{0.8})$<br>or<br>$-0.012 - 1.764\sigma^2_{2/8}$ | $(0.031\sigma^2_{0.2} + 1.524\sigma^2_{0.8}) + 0.436\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(1.555 + 0.436\rho_{2/8})$ |
| Reversed Parabolic<br>$V = \frac{\gamma}{0.64}(1.6Z - Z^2)$<br>(axis 0.20 below surface) | $0.729\gamma$ | 0.616 | $0.029 - 2.143\sigma^2_{0.6}$ | $2.352\sigma^2_{0.6}$ | $-0.014 - (1.071\sigma^2_{0.2} + 1.071\sigma^2_{0.8})$<br>or<br>$-0.014 - 2.142\sigma^2_{2/8}$ | $2.251\sigma^2_{0.8}$<br>or<br>$\sigma^2_{2/8}(2.251)$ |
| Elliptic<br>$V = \gamma(2Z - Z^2)^{1/2}$<br>(axis at the surface) | $0.786\gamma$ | 0.619 | $0.018 - 0.950\sigma^2_{0.6}$ | $0.910\sigma^2_{0.6}$ | $0.005 - (0.338\sigma^2_{0.2} + 1.473\sigma^2_{0.8})$<br>or<br>$0.005 - 1.811\sigma^2_{2/8}$ | $(0.017\sigma^2_{0.2} + 0.720\sigma^2_{0.8}) + 0.220\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(0.737 + 0.220\rho_{2/8})$ |
| Elliptic<br>$V = \frac{\gamma}{0.9}(1.8Z - Z^2)^{1/2}$<br>(axis 0.10 below surface) | $0.807\gamma$ | 0.631 | $0.030 - 1.327\sigma^2_{0.6}$ | $0.845\sigma^2_{0.6}$ | $0.005 - (0.701\sigma^2_{0.2} + 2.772\sigma^2_{0.8})$<br>or<br>$0.005 - 3.473\sigma^2_{2/8}$ | $(0.006\sigma^2_{0.2} + 0.725\sigma^2_{0.8}) + 0.131\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(0.731 + 0.131\rho_{2/8})$ |
| Logarithmic $Z_o = 0.01$<br>$V = C \ln \frac{Z}{Z_o}$ | | 0.632 | $0.023 - 0.867\sigma^2_{0.6}$ | $0.481\sigma^2_{0.6}$ | $0.023 - (0.108\sigma^2_{0.2} + 1.734\sigma^2_{0.8})$<br>or<br>$0.023 - 1.842\sigma^2_{2/8}$ | $(0.030\sigma^2_{0.2} + 0.481\sigma^2_{0.8}) + 0.240\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(0.511 + 0.240\rho_{2/8})$ |
| $Z_o = 0.001$ | | 0.632 | $0.014 - 0.529\sigma^2_{0.6}$ | $0.179\sigma^2_{0.6}$ | $0.014 - (0.066\sigma^2_{0.2} + 1.058\sigma^2_{0.8})$<br>or<br>$0.014 - 1.124\sigma^2_{2/8}$ | $(0.011\sigma^2_{0.2} + 0.179\sigma^2_{0.8}) + 0.090\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(0.190 + 0.090\rho_{2/8})$ |
| Hyperbolic $\epsilon = 0$<br>$V = \overline{V}[\gamma - \beta \cosh \epsilon(1-Z)]$ | | | 0 | 0 | 0 | 0 |
| $\epsilon = 1$ | | 0.584 | $-(0.028 + 1.611\sigma^2_{0.6})$ | $2.995\sigma^2_{0.6}$ | $-0.010 - (0.693\sigma^2_{0.2} + 0.909\sigma^2_{0.8})$<br>or<br>$-0.010 - 1.602\sigma^2_{2/8}$ | $(0.075\sigma^2_{0.2} + 1.457\sigma^2_{0.8}) + 0.660\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(1.532 + 0.660\rho_{2/8})$ |
| $\epsilon = 2$ | | 0.601 | $0.001 - 1.858\sigma^2_{0.6}$ | $2.399\sigma^2_{0.6}$ | $-0.008 - (0.555\sigma^2_{0.2} + 1.323\sigma^2_{0.8})$<br>or<br>$-0.008 - 1.877\sigma^2_{2/8}$ | $(0.044\sigma^2_{0.2} + 1.486\sigma^2_{0.8}) + 0.514\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(1.530 + 0.514\rho_{2/8})$ |
| $\epsilon = 3$ | | 0.625 | $0.031 - 2.078\sigma^2_{0.6}$ | $1.721\sigma^2_{0.6}$ | $-0.005 - (0.396\sigma^2_{0.2} + 1.858\sigma^2_{0.8})$<br>or<br>$-0.005 - 2.254\sigma^2_{2/8}$ | $(0.020\sigma^2_{0.2} + 1.485\sigma^2_{0.8}) + 0.346\rho\sigma_{0.2}\sigma_{0.8}$<br>or<br>$\sigma^2_{2/8}(1.505 + 0.346\rho_{2/8})$ |

Therefore,

$$\hat{\hat{V}} - \overline{V} \approx -\gamma(0.0342 - 0.7906\,\Delta_{0.6} + 0.4941\,\Delta_{0.6}^{2} - 0.6176\,\Delta_{0.6}^{3} + \ldots)$$

The bias of the sampling scheme is

$$E\left[\hat{\hat{V}} - \overline{V}\right] = v_d \approx -\gamma(0.034 + 0.494\,\sigma^2_{0.6})$$

$$\text{or } -\overline{V}\,(0.051 + 0.741\,\sigma^2_{0.6})$$

The variance of the sampling error is,

$$\text{Var}\left[v_d\right] = E\left[\hat{\hat{V}} - \overline{V}\right]^2 - E^2\left[\hat{\hat{V}} - \overline{V}\right]$$

$$\approx E\left[\gamma(0.0012 - 0.0541\,\Delta_{0.6} + 0.6588\,\Delta^2_{0.6})\right]$$

$$- \gamma^2\left[0.0012 + 0.0338\,\sigma^2_{0.6}\right]$$

$$\approx 0.625\,\gamma^2\sigma^2_{0.6}$$

$$\text{or } 1.406\,\sigma^2_{0.6}\,\overline{V}^2 \;.$$

For the two-point sampling technique, at 0.2 and 0.8 relative depths, the measurement errors in $\hat{Z}_{0.2}$ and $\hat{Z}_{0.8}$ may be characterized by $\Delta_{0.2} = N(0, \sigma^2_{0.2})$ and $\Delta_{0.8} = N(0, \sigma^2_{0.8})$, respectively. Further, it is assumed that the two errors are bivariate-normally distributed with a correlation coefficient $\rho_{2/8}$. Then,

$$\hat{\hat{V}} = \frac{\hat{V}_{0.2} + \hat{V}_{0.8}}{2} = \frac{\gamma(0.8 + \Delta_{0.2})^{1/2} + \gamma(0.2 + \Delta_{0.8})^{1/2}}{2}$$

$$\approx \frac{\gamma}{2}\left[(0.8944 + 0.5590\,\Delta_{0.2} - 0.1747\,\Delta^2_{0.2} + \ldots)\right.$$

$$\left. + (0.4472 + 1.1180\,\Delta_{0.8} - 1.3975\,\Delta^2_{0.8} + \ldots)\right]$$

$$\approx \gamma\;\left[0.6708 + (0.2795\,\Delta_{0.2} + 0.5590\,\Delta_{0.8})\right.$$

$$\left. - (0.0873\,\Delta^2_{0.2} + 0.6988\,\Delta^2_{0.8})\right] \;.$$

Therefore,

$$\hat{\hat{V}} - \overline{V} \approx \gamma\left[0.0042 + (0.2795\,\Delta_{0.2} + 0.5590\,\Delta_{0.8})\right.$$

$$\left. - (0.0873\,\Delta^2_{0.2} + 0.6988\,\Delta^2_{0.8})\right]$$

The bias of the sampling scheme is,

$$v_d \approx \gamma\left[0.004 - (0.087\,\sigma^2_{0.2} + 0.699\,\sigma^2_{0.8})\right]$$

$$\text{or } \overline{V}\left[0.006 - (0.131\,\sigma^2_{0.2} + 1.048\,\sigma^2_{0.8})\right] \;.$$

If $\sigma^2_{0.2} = \sigma^2_{0.8} = \sigma^2_{2/8}$,

$$v_d = \gamma\,(0.004 - 0.786\,\sigma^2_{2/8})$$

$$\text{or } \overline{V}(0.006 - 1.179\,\sigma^2_{2/8})$$

the variance of the sampling scheme is,

$$\text{Var}\left[v_d\right] \approx E\left[\gamma(0. + 0.0023\,\Delta_{0.2} + 0.0046\,\Delta_{0.8}\right.$$

$$\left. + 0.0774\,\Delta^2_{0.2} + 0.3067\,\Delta^2_{0.8} + 0.3125\,\Delta_{0.2}\Delta_{0.8})\right]$$

$$- \gamma^2(0. - 0.0007\,\sigma^2_{0.2} - 0.0058\,\sigma^2_{0.8})$$

$$\approx \gamma^2\left[(0.077\,\sigma^2_{0.2} + 0.301\,\sigma^2_{0.8}) + 0.313\,\rho_{2/8}\sigma_{0.2}\sigma_{0.8}\right]$$

$$\text{or } \overline{V}^2\left[(0.173\,\sigma^2_{0.2} + 0.677\,\sigma^2_{0.8}) + 0.703\,\rho_{2/8}\sigma_{0.2}\sigma_{0.8}\right]$$

If $\sigma^2_{0.2} = \sigma^2_{0.8} = \sigma^2_{2/8}$

$$\text{Var}\left[v_d\right] \approx \gamma^2\,(0.378\,\sigma^2_{2/8} + 0.313\,\rho_{2/8}\sigma^2_{2/8})$$

$$\approx \gamma^2\sigma^2_{2/8}(0.378 + 0.313\,\rho_{2/8})$$

$$\text{or } \overline{V}^2\sigma^2_{2/8}(0.849 + 0.703\,\rho_{2/8})$$

The variance of the placement error should approach zero. However, it has been included in order to recognize the fact that the meter is virtually never placed at exactly the correct sample point. Rather, it takes positions scattered about the correct point in a somewhat random manner.

This analysis of robustness may be considered from two viewpoints to estimate the relative magnitude of $v_p$. If the vertical profile of velocity at a given section is known, from detailed analysis, to correspond well to one of the theoretical profiles considered, the mean error and variance can be determined for a reasonable range of placement error variances. If such information is not available, but it is reasonable to assume that the vertical profiles lie somewhere within the family of theoretical profiles, a general indication of the mean and variance of $v_p$ may be obtained.

For example, considering the second viewpoint above, it might be reasonable to assume that the mean error in sampling the vertical profile is zero.

This could be reasoned from some of the distributions yielding slightly positive biases, and others slightly negative ones. Further, consideration of the variance terms for placement error standard deviations, in the order of 0.01 to 0.05, reveals coefficients of variation for the 0.6 depth sampling of from 0.004 to 0.09, and for the two-point sampling scheme of from 0.001 to 0.07, assuming $\rho$ to be zero. The lowest values were obtained by considering the logarithmic distribution, for which the sampling schemes are very suitable; and the upper values were determined from the reversed parabolic distributuions.

(b) The error in sampling velocity at a point, $v_{ij}$

As has been discussed in the previous chapter, and represented in the general error model at the beginning of the chapter, there are several possible sources of error in sampling velocity at a point. Error introduced by the use of a current-meter rating curve, error caused by oblique currents, error in sampling the horizontal area, and error in sampling velocity in time are considered.

(i) The error of the current-meter calibration, $v_c$: From a consideration of fig. 2 in Chapter II, it would appear reasonable to express this error as an exponential function of velocity, decreasing quite rapidly and levelling off at approximately one foot per second. In other words, the percentage error of the calibration can be very large at velocities below one foot per second, but tends to be in the order of $\pm 0.5$ to 1.0 percent at higher velocities. For the purpose of studying mountain streams, where the mean velocity in a vertical is rarely below one foot per second during summer flows, the lower range of velocities and the inherent errors have been omitted from the model. Considering only velocities greater than one foot per second, the error of the current-meter calibration can be represented as,

$$v_c = \nu_c(\overline{V}_{ij} + v_o + v_s + v_t)$$

where

$$\nu_c = N(0, \sigma^2_{vc})$$

$$\sigma_{vc} \approx 0.007 .$$

Recognizing that $v_o$, $v_s$, $v_t$ are one or two orders of magnitude smaller than $\overline{V}_{ij}$,

$$E\left[v_c\right] = 0$$

$$\text{Var}\left[v_c\right] \approx 0.00005\,\overline{V}^2_{ij}$$

(ii) The error caused by oblique currents, $v_o$: This is perhaps the most difficult error component to describe adequately. It is caused by the failure of the current meter to record the correct cosine component of the mean velocity vector and its angular fluctuations in the direction of flow perpendicular to the stream cross section. As the true nature of the effect of oblique angles and their fluctuations on current meters is not yet fully understood, the functional relationships hypothesized here are meant only to yield approximate estimates of the error term and not to fully describe the process.

In the case of the cup-type meter, it has been suggested in the literature that the major effect of obliquity is reflected in the failure of the meter to yield the correct cosine component of the velocity vector. Rather, the maximum velocity is registered. Under these assumptions, the error in an instantaneous velocity measurement due to obliquity could be represented as,

$$v'_o\,(\text{cup}) = (1 - \cos\alpha_{ij})\,V_{ij}$$

or the error in a time-averaged velocity measurement as,

$$v_o\,(\text{cup}) = \frac{1}{T}\int_{t=0}^{T} (1 - \cos\alpha_{ij})\,V_{ij}\;dt$$

Considering the suggestions of Kolupaila (1957), the error caused by using a propeller or screw meter in oblique currents may be represented by either,

$$v'_o\,(\text{prop. }1) = \cos\alpha_{ij}\,(\cos\alpha_{ij} - 1)\,V_{ij}\ ,$$

or $$v'_o\,(\text{prop. }2) = \cos\alpha_{ij}\,(\cos^2\alpha_{ij} - 1)\,V_{ij};$$

or $$v'_o\,(\text{screw}) = \nu_o\,V_{ij}$$

for the auto-component or screw-type meters, where

$$\nu_o = N(0, \sigma^2_{vo})\ .$$

In other words, for cup- and propeller-type meters, the error in velocity registration due to oblique currents has been assumed to be a function of the instantaneous oblique angle and its variability. For the propeller- and screw-type meters, it is doubtful that the above functions adequately describe the situation. Rather, each meter should be studied as an individual. However, it is felt that the relationship for the cup meters is reasonably representative.

Before the nature of such error functions can be further studied, certain assumptions must be made regarding the nature of variation of both $\alpha_{ij}$ and $V_{ij}$. Both variables have been assumed to be Gaussian processes, with stationary time series, and the variables are assumed to be independent. Under these assumptions, the mean and variance of $v_o$ have been computed below for the cases where $v_o$ is a function of both $\alpha_{ij}$ and $V_{ij}$.

The mean error caused by obliquity may be expressed as

$$E\left[v_o\right] = E\left\{\frac{1}{T}\int_0^T f\left[\alpha_{ij}(t)\right] V_{ij}(t)\,dt\right\}$$

$$= E\left\{f\left[\alpha_{ij}(t)\right] V_{ij}(t)\right\}$$

$$= E\left\{f\left[\alpha_{ij}(t)\right]\right\} E\left[V_{ij}(t)\right]$$

or, in words, the mean error is equal to the mean value of the function of $\alpha_{ij}$ multiplied by the mean velocity value. Now it is of interest to consider the mean values of the three functions of $\alpha_{ij}$ given above. They are

$$E\left[1 - \cos\alpha_{ij}\right] = 1 - e^{-\frac{1}{2}\sigma_\alpha^2} \cos\overline{\alpha}_{ij} \quad ;$$

$$E\left[\cos\alpha_{ij}(\cos\alpha_{ij} - 1)\right] = \frac{1}{2}\left[e^{-2\sigma_\alpha^2}\cos 2\overline{\alpha}_{ij} + 1\right] - e^{-\frac{1}{2}\sigma_\alpha^2}\cos\overline{\alpha}_{ij};$$

$$E\left[\cos\alpha_{ij}(\cos^2\alpha_{ij} - 1)\right] = \frac{1}{4}\; e^{-4.5\sigma_\alpha^2}\cos 3\overline{\alpha}_{ij} - e^{-\frac{1}{2}\sigma_\alpha^2}\cos\overline{\alpha}_{ij}$$

in terms of the mean angle $\overline{\alpha}_{ij}$, and the variance of the angle. The particular aspect that should be noted is that the mean of each function does not go to zero as $\overline{\alpha}_{ij}$ goes to zero. Rather, the value remains a function of $\sigma_\alpha^2$. The above expressions have been evaluated for values of $\overline{\alpha}_{ij}$ and $\sigma_\alpha$ ranging from zero to twenty degrees, and the results are presented in Table 9. For example, if a cup-type meter is employed at a point where the mean obliquity is zero, and the standard deviation is four degrees, then a bias would result in the velocity measurement amounting to 0.0024 $\overline{V}_{ij}$. As the mean angle and/or the variance of the angle increase, the bias increases at an increasing rate.

The variance of the error,

$$\mathrm{Var}\left[v_o\right] = E\left\{\frac{1}{T}\int_0^T f\left[\alpha_{ij}(t)\right] V_{ij}(t)dt\right\}^2 - \left\{\left[\overline{f(\alpha_{ij})}\right]^2 \overline{V}_{ij}^2\right\}$$

may be evaluated to be,

$$\mathrm{Var}\left[v_o\right] \approx \frac{2\overline{V}_{ij}^2\ \mathrm{Var}[f(\alpha_{ij})]}{\lambda_1^2}\left(1 + e^{-\lambda_1 T}\right) + \frac{2\, f(\alpha_{ij})^2\ \mathrm{Var}\left[V_{ij}\right]}{\lambda_2^2 T}\left(1 + e^{-\lambda_2 T}\right)$$

where $\rho_\alpha(\tau)$, the autocorrelation coefficient of the function $f\left[\alpha_{ij}(t)\right]$, has been expressed as $\rho_\alpha(\tau) = e^{-\lambda_1\tau}$; and $\rho_v(\tau)$, the autocorrelation coefficient of $V_{ij}$ has been expressed as $\rho_v(\tau) = e^{-\lambda_2\tau}$.

It seems reasonable to expect that the first term is approximately two or three orders of magnitude larger than the second term. Then

$$\mathrm{Var}\left[v_o\right] \approx \frac{2\overline{V}_{ij}^2\ \mathrm{Var}[f(\alpha_{ij})]}{\lambda_1^2 T}\left(1 + e^{-\lambda_1 T}\right)$$

The order of magnitude of the variance of the angle functions is revealed in Table 10, and that of the variance of the velocity magnitude in a subseqent section.

It is again interesting to note the parameters involved in the expression of the variance. As for the mean error, the mean and variance of the angle, and the mean of the velocity are involved. Also the variance of the velocity magnitude, the rate of decay of the autocorrelation coefficient, and the time interval of measurement are factors. As the time interval lengthens, and the time series becomes independent, the variance becomes a function of the variance of the angle function and of the velocity magnitude.

(iii) The error in sampling the horizontal area, $v_s$: This error arises from the estimation of velocity at a given level for a width $W_i$, rather than of the velocity at a point, $\overline{V}_{ij}$. That is, the estimate of mean velocity at a point, $\hat{\overline{V}}_{ij}$, is meant to represent the mean velocity at that level over the width $W_i$. This term in the model may be considered to be proportional to $\overline{V}_{ij}$, as

$$v_s = \nu_s\, \overline{V}_{ij}$$

where

$$\nu_s = N(0, \sigma_{vs}^2)$$

$\sigma_{vs}^2$ is a function of n .

Although the true functional relationship for $\sigma_{vs}^2$ is not known, it may be hypothesized that is is inverse in nature, $\sigma_{vs}^2$ decreasing exponentially, or otherwise, as n increases. That is,

$$E\left[v_s\right] = 0$$

$$\mathrm{Var}\left[v_s\right] = \sigma_{vs}^2\, \overline{V}_{ij}^2 \quad .$$

(iv) The error in sampling velocity in time, $v_t$: Pulsations or variability of velocity at a point in time cause an error which is a function of the length of the measurement interval at the point. Although the current-meter registration does not reflect the natural frequency of the velocity magnitude fluctuations, it has been assumed that the registration is merely a dampening of the physical situation, with no bias introduced. This assumption is perhaps more valid for the cup-type meters. For propeller- and screw-type meters, there is reason to believe that the true fluctuations are not merely dampened when translated to the meter registration.

Suitable data was obtained from records on the Mississippi River, Mackenzie (1884), for an analysis of this error component. Time series of velocity at various relative depths at different points of the river was considered in an analysis of the

TABLE 9. MEAN VALUES OF ANGLE FUNCTIONS USED TO DESCRIBE ERROR DUE TO OBLIQUE CURRENTS

Mean of $(1 - \cos \alpha_{ij})$

| Std. Dev. $\sigma_\alpha$ (Degrees) | Mean Angle, $\overline{\alpha}_{ij}$, Degrees | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 0 | 0. | .0006 | .0024 | .0055 | .0097 | .0152 | .0219 | .0297 | .0387 | .0489 | .0603 |
| 2 | .0060 | .0012 | .0030 | .0061 | .0103 | .0158 | .0224 | .0303 | .0393 | .0495 | .0609 |
| 4 | .0024 | .0030 | .0049 | .0079 | .0121 | .0176 | .0242 | .0321 | .0411 | .0513 | .0626 |
| 6 | .0055 | .0061 | .0079 | .0109 | .0151 | .0206 | .0272 | .0350 | .0440 | .0541 | .0654 |
| 8 | .0097 | .0103 | .0121 | .0151 | .0193 | .0247 | .0313 | .0391 | .0481 | .0582 | .0694 |
| 10 | .0151 | .0157 | .0175 | .0205 | .0247 | .0301 | .0366 | .0444 | .0533 | .0633 | .0745 |
| 12 | .0217 | .0223 | .0241 | .0271 | .0312 | .0366 | .0431 | .0508 | .0596 | .0696 | .0807 |
| 14 | .0294 | .0300 | .0318 | .0347 | .0389 | .0442 | .0506 | .0582 | .0670 | .0769 | .0879 |
| 16 | .0382 | .0388 | .0406 | .0435 | .0476 | .0529 | .0593 | .0668 | .0755 | .0853 | .0962 |
| 18 | .0482 | .0487 | .0505 | .0534 | .0574 | .0626 | .0690 | .0764 | .0850 | .0947 | .1056 |
| 20 | .0591 | .0597 | .0614 | .0643 | .0683 | .0734 | .0797 | .0871 | .0956 | .1052 | .1158 |

Mean of $\left[\cos \alpha_{ij}(\cos \alpha_{ij} - 1)\right]$

| Std. Dev. $\sigma_\alpha$ (Degrees) | Mean Angle, $\overline{\alpha}_{ij}$, Degrees | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 0 | 0. | -.0006 | -.0024 | -.0054 | -.0096 | -.0150 | -.0214 | -.0288 | -.0372 | -.0465 | -.0567 |
| 2 | -.0006 | -.0012 | -.0030 | -.0060 | -.0102 | -.0155 | -.0219 | -.0293 | -.0377 | -.0470 | -.0570 |
| 4 | -.0024 | -.0030 | -.0048 | -.0078 | -.0119 | -.0171 | -.0234 | -.0307 | -.0390 | -.0482 | -.0581 |
| 6 | -.0054 | -.0060 | -.0077 | -.0106 | -.0146 | -.0198 | -.0259 | -.0331 | -.0412 | -.0501 | -.0598 |
| 8 | -.0094 | -.0100 | -.0117 | -.0145 | -.0184 | -.0234 | -.0294 | -.0363 | -.0441 | -.0528 | -.0622 |
| 10 | -.0144 | -.0150 | -.0166 | -.0193 | -.0231 | -.0278 | -.0336 | -.0402 | -.0478 | -.0561 | -.0651 |
| 12 | -.0203 | -.0208 | -.0224 | -.0250 | -.0285 | -.0331 | -.0385 | -.0449 | -.0520 | -.0599 | -.0685 |
| 14 | -.0269 | -.0274 | -.0288 | -.0312 | -.0346 | -.0389 | -.0440 | -.0500 | -.0567 | -.0641 | -.0721 |
| 16 | -.0340 | -.0344 | -.0358 | -.0380 | -.0412 | -.0452 | -.0499 | -.0555 | -.0617 | -.0686 | -.0760 |
| 18 | -.0414 | -.0418 | -.0431 | -.0452 | -.0481 | -.0517 | -.0561 | -.0612 | -.0669 | -.0732 | -.0800 |
| 20 | -.0490 | -.0494 | -.0506 | -.0524 | -.0551 | -.0584 | -.0623 | -.0670 | -.0721 | -.0778 | -.0840 |

Mean of $\left[\cos \alpha_{ij}(\cos^2 \alpha_{ij} - 1)\right]$

| Std. Dev. $\sigma_\alpha$ (Degrees) | Mean Angle, $\overline{\alpha}_{ij}$, Degrees | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 0 | 0. | -.0012 | -.0049 | -.0109 | -.0192 | -.0297 | -.0423 | -.0568 | -.0730 | -.0908 | -.1099 |
| 2 | -.0012 | -.0024 | -.0060 | -.0120 | -.0203 | -.0307 | -.0432 | -.0577 | -.0738 | -.0915 | -.1105 |
| 4 | -.0048 | -.0060 | -.0096 | -.0154 | -.0235 | -.0338 | -.0461 | -.0602 | -.0761 | -.0934 | -.1121 |
| 6 | -.0107 | -.0118 | -.0153 | -.0210 | -.0288 | -.0388 | -.0507 | -.0644 | -.0798 | -.0966 | -.1147 |
| 8 | -.0186 | -.0197 | -.0230 | -.0284 | -.0360 | -.0455 | -.0569 | -.0700 | -.0848 | -.1009 | -.1181 |
| 10 | -.0282 | -.0293 | -.0324 | -.0376 | -.0447 | -.0537 | -.0645 | -.0769 | -.0908 | -.1060 | -.1224 |
| 12 | -.0394 | -.0403 | -.0432 | -.0481 | -.0547 | -.0631 | -.0732 | -.0848 | -.0978 | -.1120 | -.1272 |
| 14 | -.0515 | -.0524 | -.0551 | -.0596 | -.0657 | -.0735 | -.0827 | -.0934 | -.1054 | -.1184 | -.1325 |
| 16 | -.0644 | -.0652 | -.0677 | -.0717 | -.0773 | -.0844 | -.0928 | -.1025 | -.1134 | -.1252 | -.1379 |
| 18 | -.0776 | -.0784 | -.0805 | -.0842 | -.0892 | -.0955 | -.1030 | -.1117 | -.1215 | -.1321 | -.1434 |
| 20 | -.0907 | -.0914 | -.0933 | -.0965 | -.1009 | -.1065 | -.1132 | -.1209 | -.1294 | -.1388 | -.1488 |

relationship between the coefficient of variation of velocity, relative depth, and meter exposure time. The results, presented in fig. 7, reveal that an equation which represented the scatter of points rather well is of the form,

$$C_v(V_{ij}) = (0.015 + 0.049\, Z^2)\, T^{-0.50}$$

where

$C_v(V_{ij})$ is the coefficient of variation of velocity,

Z is the relative depth measured from the water surface,

T is the measurement time or meter exposure time in minutes.

The function is parabolic with respect to relative depth, reveals that the velocity is independent of time, and that the variance is proportional to the square of velocity. Table 11 also summarizes the results.

TABLE 10. VARIANCE OF ANGLE FUNCTIONS USED TO DESCRIBE ERROR DUE TO OBLIQUE CURRENTS

Variance of $(1 - \cos \alpha_{ij})$

| Std. Dev. $\sigma_\alpha$ (Degrees) | Mean Angle, $\overline{\alpha}_{ij}$, Degrees | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 0 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 2 | 0. | 0. | 0. | 0. | 0. | 0. | .0001 | .0001 | .0001 | .0001 | .0001 |
| 4 | 0. | 0. | 0. | .0001 | .0001 | .0002 | .0002 | .0003 | .0004 | .0005 | .0006 |
| 6 | .0001 | .0001 | .0001 | .0002 | .0003 | .0004 | .0005 | .0007 | .0009 | .0011 | .0013 |
| 8 | .0002 | .0002 | .0003 | .0004 | .0006 | .0008 | .0010 | .0013 | .0016 | .0020 | .0024 |
| 10 | .0004 | .0005 | .0006 | .0008 | .0010 | .0013 | .0017 | .0022 | .0027 | .0032 | .0039 |
| 12 | .0009 | .0010 | .0011 | .0014 | .0017 | .0022 | .0027 | .0033 | .0040 | .0048 | .0057 |
| 14 | .0017 | .0017 | .0019 | .0023 | .0027 | .0033 | .0040 | .0049 | .0058 | .0069 | .0081 |
| 16 | .0028 | .0029 | .0032 | .0036 | .0042 | .0049 | .0058 | .0069 | .0081 | .0094 | .0109 |
| 18 | .0044 | .0045 | .0048 | .0053 | .0061 | .0070 | .0081 | .0094 | .0109 | .0125 | .0144 |
| 20 | .0066 | .0067 | .0071 | .0077 | .0085 | .0096 | .0110 | .0125 | .0143 | .0163 | .0185 |

Variance of $\left[\cos \alpha_{ij} (\cos \alpha_{ij} - 1)\right]$

| Std. Dev. $\sigma_\alpha$ (Degrees) | Mean Angle, $\overline{\alpha}_{ij}$, Degrees | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 0 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 2 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | .0001 | .0001 | .0001 | .0001 |
| 4 | 0. | 0. | 0. | .0001 | .0001 | .0001 | .0002 | .0003 | .0003 | .0004 | .0004 |
| 6 | .0001 | .0001 | .0001 | .0002 | .0002 | .0003 | .0004 | .0006 | .0007 | .0008 | .0009 |
| 8 | .0002 | .0002 | .0002 | .0003 | .0005 | .0006 | .0008 | .0010 | .0012 | .0014 | .0016 |
| 10 | .0004 | .0004 | .0005 | .0006 | .0008 | .0010 | .0012 | .0015 | .0018 | .0021 | .0023 |
| 12 | .0007 | .0007 | .0008 | .0010 | .0012 | .0015 | .0018 | .0021 | .0024 | .0028 | .0031 |
| 14 | .0012 | .0012 | .0013 | .0015 | .0017 | .0020 | .0024 | .0028 | .0031 | .0035 | .0039 |
| 16 | .0018 | .0018 | .0019 | .0021 | .0024 | .0027 | .0030 | .0034 | .0038 | .0042 | .0046 |
| 18 | .0024 | .0025 | .0026 | .0028 | .0030 | .0034 | .0037 | .0041 | .0045 | .0049 | .0053 |
| 20 | .0032 | .0032 | .0033 | .0035 | .0037 | .0040 | .0044 | .0047 | .0051 | .0055 | .0058 |

Variance of $\left[\cos \alpha_{ij} (\cos^2 \alpha_{ij} - 1)\right]$

| Std. Dev. $\sigma_\alpha$ (Degrees) | Mean Angle, $\overline{\alpha}_{ij}$, Degrees | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 0 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 2 | 0. | 0. | 0. | .0001 | .0001 | .0001 | .0002 | .0002 | .0003 | .0003 | .0004 |
| 4 | 0. | .0001 | .0001 | .0002 | .0004 | .0005 | .0007 | .0009 | .0011 | .0013 | .0015 |
| 6 | .0002 | .0003 | .0004 | .0006 | .0009 | .0012 | .0016 | .0020 | .0024 | .0028 | .0032 |
| 8 | .0006 | .0007 | .0009 | .0012 | .0017 | .0022 | .0028 | .0035 | .0041 | .0047 | .0052 |
| 10 | .0014 | .0015 | .0017 | .0022 | .0028 | .0035 | .0043 | .0051 | .0060 | .0067 | .0075 |
| 12 | .0025 | .0026 | .0029 | .0035 | .0042 | .0050 | .0059 | .0069 | .0079 | .0088 | .0097 |
| 14 | .0040 | .0041 | .0045 | .0050 | .0058 | .0067 | .0077 | .0087 | .0098 | .0108 | .0116 |
| 16 | .0058 | .0059 | .0063 | .0068 | .0076 | .0084 | .0094 | .0105 | .0115 | .0125 | .0133 |
| 18 | .0077 | .0078 | .0082 | .0087 | .0094 | .0102 | .0111 | .0121 | .0130 | .0139 | .0147 |
| 20 | .0097 | .0098 | .0101 | .0105 | .0111 | .0118 | .0126 | .0134 | .0143 | .0151 | .0158 |

Data for one vertical in a Russian mountain stream were extracted from the paper by Dement'ev (1962) and analyzed in like manner to that of the Mississippi River. These points plot in approximately the same area as the previous data but appear to reflect some dependence in time. However, considerably more data is required before any inference can be made regarding the dependence, if in fact such exists.

One cannot compare the results directly with those presented by Dement'ev (1962) since the statistics evaluated are entirely different. However, the relative order of magnitude of the results is the same, the above relationship tending to give a more optimistic view of the error.

For the purpose of the error model, this component of error will be considered as,

$$v_t = \nu_t \overline{V}_{ij}$$

where

$$\nu_t = N(0, \sigma_{vt}^2)$$

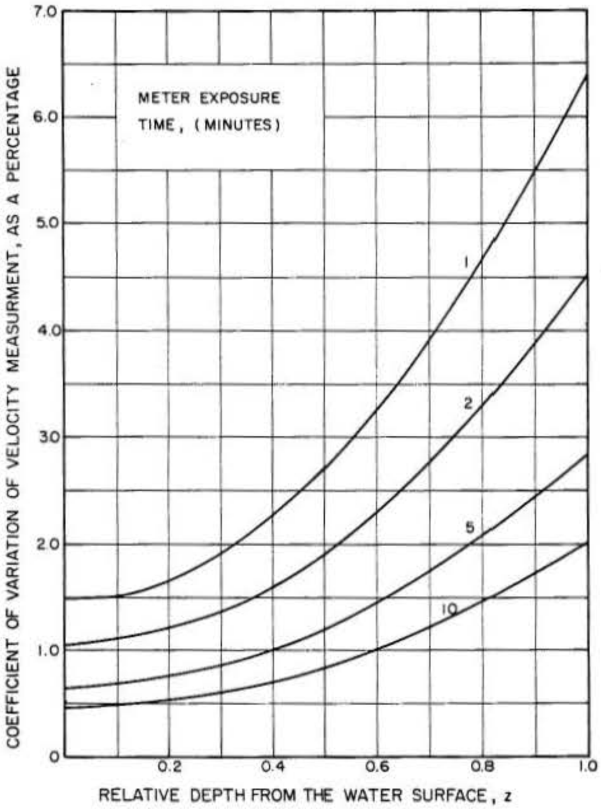


Fig. 7 The coefficient of variation of a velocity measurement as a function of relative depth and meter exposure time

TABLE 11. VARIABILITY OF VELOCITY AT A POINT AS A FUNCTION OF THE RELATIVE DEPTH AND THE METER EXPOSURE TIME

| Coefficient of Variation of Velocity, Percentage | | | | |
|---|---|---|---|---|
| Relative Depth (From Surface) | Time, Minutes | | | |
| | 1 | 2 | 5 | 10 |
| 0.1 | 1.53 | 1.08 | 0.68 | 0.48 |
| 0.2 | 1.68 | 1.19 | 0.75 | 0.53 |
| 0.3 | 1.93 | 1.36 | 0.86 | 0.61 |
| 0.4 | 2.27 | 1.60 | 1.01 | 0.72 |
| 0.5 | 2.72 | 1.92 | 1.22 | 0.86 |
| 0.6 | 3.26 | 2.30 | 1.46 | 1.03 |
| 0.7 | 3.91 | 2.76 | 1.75 | 1.24 |
| 0.8 | 4.65 | 3.29 | 2.08 | 1.47 |
| 0.9 | 5.49 | 3.88 | 2.45 | 1.73 |

and

$$\sigma_{vt}^2 \approx (0.015 + 0.049\, Z^2)^2\, T^{-1.0}$$

3.4 The Error Model - Its Mean and Variance

Substitutuion of the relationships hypothesized in the previous section into the general model developed in section two of this chapter, yields the following expression.

$$q \approx \sum_{i=1}^{n} \left\{ A_i \sum_{j=1}^{m_i} \left\{ R_{ij} \left[ \nu_c \overline{V}_{ij} + \frac{1}{T} \int_0^T f(\alpha_{ij})\, V_{ij}\, dt + \nu_s \overline{V}_{ij} \right.\right.\right.$$

$$\left.\left.\left. + \nu_t \overline{V}_{ij} \right] \right\} + \nu_p A_i \overline{V}_i + \overline{V}_i \left\{ W_i \delta_i \overline{D}_i \right\} \right\}$$

or

$$q \approx \sum_{i=1}^{n} \left\{ A_i \sum_{j=1}^{m_i} \left[ R_{ij} \overline{V}_{ij} (\nu_c + \nu_s + \nu_t) + \frac{R_{ij}}{T} \int_0^T f(\alpha_{ij}) V_{ij} dt \right] \right.$$

$$\left. + A_i V_i (\nu_p + \delta_i) \right\}$$

This equation represents a model of error in a single discharge measurement. The more meaningful terms of the mean and the variance are now considered.

The mean error in a single discharge measurement may be derived from the above equation as

$$\overline{q} = E\left[q\right] \approx \sum_{i=1}^{n} \left\{ A_i \sum_{j=1}^{m_i} \left[ R_{ij} \overline{V}_{ij} E\left[\nu_c + \nu_s + \nu_t\right] \right.\right.$$

$$\left.\left. + \frac{R_{ij}}{T} \int_0^T E\left[f(\alpha_{ij}) V_{ij}\right] dt \right] + A_i \overline{V}_i\, E\left[\nu_p + \delta_i\right] \right\}$$

Using the hypotheses advanced in the previous section regarding the expected values of the component errors,

$$\overline{q} \approx \sum_{i=1}^{n} \left\{ A_i \left[ \sum_{j=1}^{m_i} R_{ij}\, \overline{f(\alpha_{ij})}\, \overline{V}_{ij} \right] + A_i \overline{V}_i\, \nu_p \right\}$$

This expression defines the mean error or the bias in a single discharge measurement. As such bias can be estimated only by a detailed study of the role of the factors contained in this equation for each gaging site, or by the use of an independent, more accurate, method of determining discharge such as a weir, it is assumed to be zero in practice. However, the major potential contributors to bias should be recognized and steps taken to minimize this effect.

This mean error is primarily a function of the ability of the current meter to respond correctly to oblique and turbulent streamflow, and of the robustness of the technique for sampling the vertical distribution of velocity. If the function representing the oblique effect is relatively constant across the section, the bias caused by this effect may be considered to be proportional to the total discharge. Until the turbulence in natural channels has been adequately described and measured, and the function relating the effect on the meter is verified or replaced, the magnitude of the bias caused by this factor will be in question. The study of robustness presented in a previous section also revealed that the present sampling techniques do introduce bias when applied to the theoretical profiles considered. This bias may be positive or negative depending on the particular distributuion. There may be reason to believe that this bias is negligible, when different velocity distributions are present across the gaging section. However, proper sampling of those distributuions in the sections containing the major part of the discharge is essential.

The variance of the error in a total discharge measurement is the other statistic considered. It should be noted that, since the true discharge has been assumed to remain constant during the measurement interval, the variance of the error is equal to the variance of the discharge measurement itself. Beginning with the general model,

$$\mathrm{Var}\left[q\right] \approx \mathrm{Var}\left[\sum_{i=1}^{n} (A_i v_i + \overline{V}_i a_i)\right]$$

In order to put this expression in a tractable form, it is necessary to make a number of assumptions relative to the independence of the parameters involved. In most instances, the dependence is a function of the bias in the depth and velocity measurements. Therefore, it has been assumed that there is no bias in the measurements. This infers that the total discharge measurement is also unbiased.

If the errors in the discharge estimates in the n vertical sections are independent, then

$$\mathrm{Var}\left[q\right] \approx \sum_{i=1}^{n} \mathrm{Var}\left[A_i v_i + \overline{V}_i a_i\right]$$

$$\approx \sum_{i=1}^{n} \left\{\mathrm{Var}\left[A_i v_i\right] + \mathrm{Var}\left[\overline{V}_i a_i\right]\right\}$$

$$\approx \sum_{i=1}^{n} \left\{A_i^2 \mathrm{Var}\left[v_i\right] + \overline{V}_i^2 \mathrm{Var}\left[a_i\right]\right\}$$

In a previous section it was shown that,

$$\mathrm{Var}\left[a_i\right] \approx A_i^2 \sigma_d^2 .$$

Also

$$\mathrm{Var}\left[v_i\right] \approx \mathrm{Var}\left[\sum_{j=1}^{m_i} R_{ij}(v_c + v_o + v_s + v_t) + v_p\right] .$$

Assuming the errors in estimates of mean velocity at individual points are independent,

$$\mathrm{Var}\left[v_i\right] \approx \sum_{j=1}^{m_i} R_{ij}^2 \mathrm{Var}\left[v_c + v_o + v_s + v_t\right] + \mathrm{Var}\ v_p$$

$$\approx \sum_{j=1}^{m_i} R_{ij}^2 \overline{V}_{ij}^2 (\sigma_{vc}^2 + \sigma_{vo}^2 + \sigma_{vs}^2 + \sigma_{vt}^2) + \overline{V}_i^2 \sigma_{vp}^2 .$$

Then,

$$\mathrm{Var}\left[q\right] \approx \sum_{i=1}^{n} \left\{A_i^2\left[\sum_{j=1}^{m_i} R_{ij}^2 \overline{V}_{ij}^2 (\sigma_{vc}^2 + \sigma_{vo}^2 + \sigma_{vs}^2 + \sigma_{vt}^2) + \overline{V}_i^2 \sigma_{vp}^2\right] + A_i^2 \overline{V}_i^2 \sigma_d^2\right\}$$

where

$\sigma_{vc}^2 \approx 0.00005$ for velocities greater than one foot per second,

$$\sigma_{vo}^2 \approx \frac{2\left(1 - e^{-\lambda_1 T}\right)}{\lambda_1^2 T} \mathrm{Var}\left[f(\alpha_{ij})\right],$$

$\sigma_{vs}^2$ is inversely proportional to n ,

$$\sigma_{vt}^2 \approx \frac{(0.015 + 0.049\ Z^2)^2}{T} ,$$

$\sigma_{vp}^2$ may be estimated from Table 8 ,

$\sigma_d^2$ is inversely proportional to n .

This model may be further simplified if the variances of the error components are assumed to be constant for a single discharge measurement, and if $R_{ij}$ is assumed to be equal to $1/m$, where m is the number of velocity measurements made in each vertical. Then,

$$\mathrm{Var}\left[q\right] \approx \sum_{i=1}^{n} \left\{A_i^2\left[(\sigma_{vc}^2 + \sigma_{vo}^2 + \sigma_{vs}^2 + \sigma_{vt}^2)\sum_{j=1}^{m} \frac{\overline{V}_{ij}^2}{m^2} + \overline{V}_i^2 \sigma_{vp}^2 + \overline{V}_i^2 \sigma_d^2\right]\right\}.$$

But

$$\sum_{j=1}^{m} \frac{\overline{V}_{ij}^2}{m^2} = \frac{\mathrm{Var}\left[\overline{V}_{ij}\right]}{m} + \frac{\overline{V}_i^2}{m} = \frac{\overline{V}_i^2}{m} \left\{1 + \left[C_v^2 (\overline{V}_{ij})\right]\right\}$$

where $C_v(\overline{V}_{ij})$ is the coefficient of variation of the m, $\overline{V}_{ij}$ values. In the above expression, it has further been assumed that

$$\sum_{j=1}^{m} \frac{\overline{V}_{ij}}{m} \approx \overline{V}_i \ .$$

If the weights were unequal, rather than all equal to $1/m$, such an assumption could also be made of the weighted mean in order to arrive at the above expression. Then,

$$\mathrm{Var}\left[q\right] \approx \left\{(\sigma^2_{vc} + \sigma^2_{vo} + \sigma^2_{vs} + \sigma^2_{vt}) \left\{\frac{1 + \left[C^2_v (\overline{V}_{ij})\right]}{m}\right\} + \sigma^2_{vp} + \sigma^2_d\right\} \sum_{i=1}^{n} Q^2_i$$

where

$$Q^2_i = A^2_i \, \overline{V}^2_i$$

For the standard two-point velocity sampling technique, the term involving the variability of the two velocity values may be expressed as

$$\frac{1 + [C^2_v (\overline{V}_{ij})]}{m} = \frac{(\overline{V}_{0.2} / \overline{V}_{0.8})^2 + 1}{[\overline{V}_{0.2} / \overline{V}_{0.8}) + 1]^2}$$

As the ratio of $\overline{V}_{0.2} / \overline{V}_{0.8}$ varies from 1.5 to 2.5, the above expression varies from 0.52 to 0.59. In other words, the effect of average variance in a point velocity measurement is magnified from four to eighteen percent due to the variability of the velocities sampled in each vertical.

Further,

$$\sum_{i=1}^{n} Q^2_i = n \ \mathrm{Var}\left[Q_i\right] + \frac{\left(\sum_{i=1}^{n} Q_i\right)^2}{n} = n\mathrm{Var}\left[Q_i\right] + \frac{Q^2}{n}$$

$$= \frac{Q^2}{n} \left\{1 + \left[C^2_v (Q_i)\right]\right\}$$

Therefore,

$$\mathrm{Var}\left[q\right] = \left\{\frac{\left[\sigma^2_{vc} + \sigma^2_{vo} + \sigma^2_{vs} + \sigma^2_{vt}\right]\left[1 + C^2_v(\overline{V}_{ij})\right]}{m} + \sigma^2_{vp} + \sigma^2_d\right\} \left\{\frac{\left[1 + C^2_v(Q_i)\right]}{n}\right\} Q^2$$

A number of important inferences can be made from the variance model.

(i) The standard deviation of the error in a discharge measurement, and of the measured discharge itself, is proportional to the discharge. This proportionality may not be truly constant over the range of discharge values, due to the variation of the component errors. However, it is reasonable to assume that there is a representative coefficient of proportionality for discharges exceeding the low flow conditions.

(ii) Of the component errors considered, those arising from sampling both the vertical velocity profile and the depth have potentially the greatest effect on the total measurement. As correct sampling of the velocity profile is also important with regard to reducing the chance of bias, this phase of discharge measurements requires close consideration.

(iii) The variability of the n sectional discharge values can have a major effect on the error in the discharge measurement. This variability can be readily reduced by a subdivision of the stream cross section into verticals containing approximately equal discharges. Although such a subdivision has been suggested in stream-gaging manuals, proof of its importance for minimizing errors has not been given before. As a result, such methodology has not been adopted in practice. The variance model clearly reveals that the variability of the partial discharges should be considered.

(iv) The minimum number of sampling points required in each vertical to achieve a prescribed level of accuracy is a function of the total variance of a point velocity measurement, the robustness of the technique for sampling the velocity distributuion, and the variability of the sampled velocities in the distribution. The inference here is that sampling the vertical distributuion of velocity at points considerably distant in space and velocity magnitude, such as in the 0.2 and 0.8 point scheme, may give rise to a larger variance in the discharge measurement due to the greatly increased coefficient of variation term. The question arises whether this effect is balanced by a significant decrease in $\sigma^2_{vp}$, and by a reduction in the chance of bias. Improved sampling schemes centered more around the position of the mean velocity vector might be preferable in order to obtain a minimum variance unbiased estimate of the mean velocity in a vertical.

(v) The best measurement technique for a particular stream site, involving the minimum number of verticals and sampling points in each vertical for a prescribed level of accuracy, can be determined only after the relative role of each term of the variance model has been estimated from a preliminary study. For accurate research measurements, such as those required for the evaluation of weather modification attainments, there seems little question that such studies should be undertaken. For common mountain stream measurements, it would be difficult to justify such extensive investigations. However, the stream sites could be classified according to their flow and geometric characteristics. Then streams exhibiting similar characteristics could be sampled in one manner; and groups of gaging sites varying widely in properties could be sampled in different ways.

## CHAPTER IV

## THE EVALUATION OF DISCHARGE ESTIMATES

### 4.1 General Approach

At the outset, it was noted that there is a need for objective methodology for evaluating the accuracy of daily, monthly, and annual discharge estimates. Whereas, a hypothetical error model for a single discharge measurement has been developed in the previous chapter, a method is presented here for estimating the accuracy of a single discharge estimate from use of the appropriate stage-discharge relationship. A mathematical representation is made of the rating curve; sample curves are fitted; and both confidence and tolerance limits are prepared and interpreted. On the basis of this representation and of a consideration of the correlation between daily errors, daily, monthly and annual discharge estimates are evaluated.

### 4.2 Mathematical Representation of the Rating Curve

#### 4.2.1 Basic hypotheses

Before proceeding, it is necessary to establish a number of hypotheses. Firstly, it is hypothesized that, for a given stream-gaging station and a particular period of time during which the river control may be considered to be stable, there exists a true stage-discharge relationship, or at least a mean relationship about which the true one varies somewhat randomly. Secondly, it is hypothesized that for many mountain-gaging stations, the period of stability mentioned above is sufficiently long so that a number of discharge measurements may be obtained, and the true stage-discharge relationship may be estimated by a simple mathematical expression. Finally, it is hypothesized that the majority of scatter of plotted measurement points about a mountain stream rating curve estimated for a stable period, arises from measurement error, and not to any great extent to minor incidents of instability or hysteresis effects.

#### 4.2.2 Selection of sample data

A sample of nine stream-gaging stations was selected for analysis from those situated in the regions of the mountains of Colorado which have been deemed to be suitable for weather modification research and practice. Descriptive material regarding these stations is given in Table 12, and they are located on the map in fig. 8. This information has been obtained from U. S. Geological Survey Water Supply Papers, and the Station Analyses maintained in the Denver Office of the Surface Water Branch, Water Resources Division, U. S. Geological Survey.

TABLE 12. DESCRIPTIVE INFORMATION REGARDING SAMPLE GAGING STATIONS

| Basin and Station Name | Station Number | Drainage Area (Square Miles) | Average Discharge (cfs) | Specific Yield (cfs/Sq. Mi.) | Nature of Control |
|---|---|---|---|---|---|
| Arkansas River | | | | | |
| Arkansas River at Granite | 07- 860 | 427 | 353 | 0.83 | Rock and small boulder riffle. |
| Gunnison River | | | | | |
| Taylor River below Taylor Park Res. | 09-1090 | 245 | 187 | 0.76 | Boulder and gravel bar. |
| East River at Almont | 09-1125 | 295 | 347 | 1.17 | Coarse aggregates, cobble, and rock. |
| Quartz Creek near Ohio City | 09-1180 | 106 | 54.8 | 0.52 | Rock and cobble bar. |
| Curecanti Creek near Sapinero | 09-1250 | 31.8 | 33.7 | 1.06 | Rock and gravel bar. |
| Dolores River | | | | | |
| San Miguel River near Placerville | 09-1725 | 308 | 232 | 0.75 | Rock and gravel riffle. |
| San Juan River | | | | | |
| Animas River at Howardsville | 09-3575 | 55.9 | 106 | 1.90 | Rock and gravel riffle. |
| Hermosa Creek near Hermosa | 09-3610 | 172 | 143 | 0.83 | Gravel and cobble bar. |
| Animas River at Durango | 09-3615 | 692 | 862 | 1.24 | Low boulder dam. |

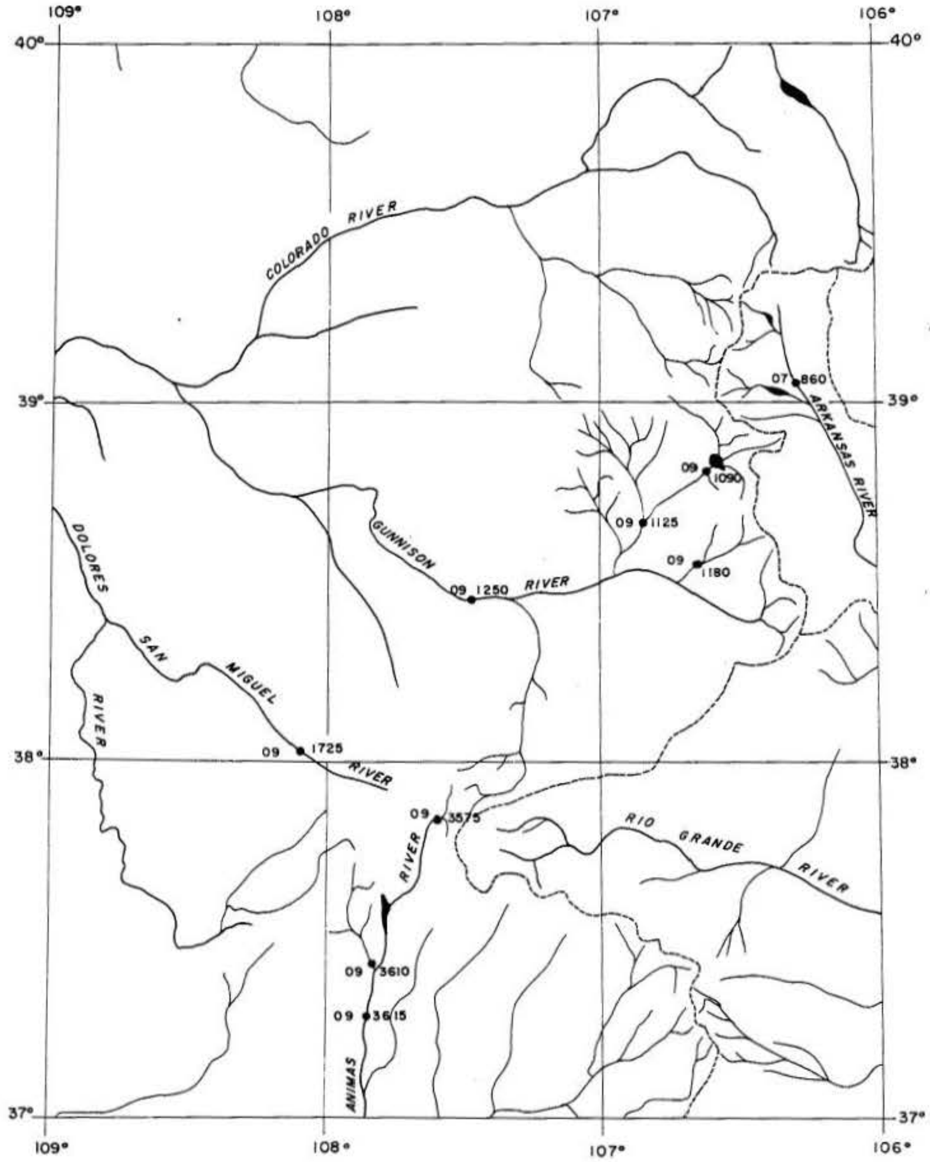


Fig. 8 Stream-gaging stations in the Colorado mountains selected for error evaluation

In order to ascertain the relative stability of the control sections of the above-mentioned stations, and hence the stability of the rating curves, copies of the data of all discharge measurements performed at each site were obtained. From this information, the values of gage height, or stage, and discharge were plotted sequentially in time for each station. This procedure proved to be a simple manner in which to note major shifts in the rating curves.

It was observed that in the majority of cases, a significant shift of a curve would occur only after the values at high discharges had been plotted. That is to say, a period of high flow appeared to be required before the rating curve would shift noticeably. Therefore, the shifts could be attributed to either a change in the cross-sectional shape of the channel, or a change in the relative roughness of the bed. In the first case, scour action could have altered the gaging site by degradation and/or aggradation. Secondly, the high velocities could have caused the removal of fine material from the bed, leaving a coarser armour plate and an increased relative roughness.

A minimum of a dozen points, and more usually at least thirty to fifty points, were available between shifts. The dates where the shifts were observed were noted, and the total period of station record was thus divided into periods of stability. These dates tended to correspond to notations made on the data sheets or the station analyses regarding shifts in the channel or a complete relocation of the station.

Although some subjectivity was involved in the above procedure, it proved to be an effective way to subdivide the period of record at a station into lengths when there appeared to be a relatively stable stage-discharge relationship. Further, the original hypothesis regarding the existence of a stable relationship for a relatively long period of time was proven valid for the mountain stations considered.

4.2.3 A rating-curve equation

In order to represent the stage-discharge relationship mathematically, it is desirable to choose a regression equation that adequately describes the relationship with a minimum number of parameters. The following equation was selected.

$$\overset{*}{Q} = \overset{*}{b}_1 (H - \overset{*}{Ho})^{\overset{*}{b}_2}$$

where

| | |
|---|---|
| $\overset{*}{Q}$ | is the estimated discharge, |
| $H$ | is the observed stage, |
| $\overset{*}{b}_1, \overset{*}{b}_2, \overset{*}{Ho}$ | are the estimated parameters of the equation. |

This equation has been used by a number of persons for the extension of station rating curves. Usually, $\overset{*}{Ho}$ has been assumed to be zero. However, the equation has seldom been employed to fit available data, an "eye fit" being deemed to be more suitable. Carter and Davidian (1965) have recently suggested the use of this equation, with a graphical procedure for estimating Ho.

Before a method of fitting the above equation is presented, it is worthwhile to note the significance of the estimated parameters. The first parameter, $b_1$, reflects the scales being used for stage and discharge; $b_2$ denotes the degree of curvature or slope of the estimated relationship; and Ho may be defined as the virtual stage at zero discharge. In the physical situation, Ho represents the lowest bed elevation at the stream station, or the pond elevation at zero flow, translated to the stage scale. This parameter could be estimated from a careful study of the station. However, it is more readily obtained by considering it as the third estimated parameter in the equation.

### 4.2.4 Fitting the rating curve

The method of least squares is chosen for estimating the parameters of the selected regression equation. Further, the method is employed on the logarithms of discharge, treating the equation in a linear form. Justification for this logarithmic transformation is given in a subsequent section. The equation considered for the least squares analysis becomes,

$$\ln \overset{*}{Q} = \ln \overset{*}{b}_1 + \overset{*}{b}_2 \ln (H - \overset{*}{Ho})$$

For ease of representation, the term $(H - \overset{*}{Ho})$ is denoted as Hr, or relative stage. That is,

$$\ln \overset{*}{Q} = \ln \overset{*}{b}_1 + \overset{*}{b}_2 \ln Hr \ .$$

Considering the least squares technique as applied to the logarithms, the objective is to minimize the squared deviations between the logarithms of the estimated and observed discharge values, or to minimize

$$\sum_1^p (\ln \widehat{Q}_i - \ln \overset{*}{b}_1 - \overset{*}{b}_2 \ln Hr_i)^2$$

where p represents the number of observed stage-discharge values. The three estimated parameters required to meet the above objective may be determined by setting the partial derivative of the summation with respect to each of the parameters equal to zero. The resulting equations are:

$$\sum_1^p \ln \widehat{Q}_i - p \ln \overset{*}{b}_1 - \overset{*}{b}_2 \sum_1^p \ln Hr_i = 0 \ ;$$

$$\sum_1^p \ln \widehat{Q}_i \ln Hr_i - \ln \overset{*}{b}_1 \sum_1^p \ln Hr_i - \overset{*}{b}_2 \sum_1^p (\ln Hr_i)^2 = 0 \ ;$$

$$\sum_1^p \frac{\ln \widehat{Q}_i}{Hr_i} - \ln \overset{*}{b}_1 \sum_1^p \frac{1}{Hr_i} - \overset{*}{b}_2 \sum_1^p \frac{\ln Hr_i}{Hr_i} = 0 \ .$$

Solving the above equations simultaneously for $\overset{*}{b}_1$, $\overset{*}{b}_2$, and $\overset{*}{Ho}$, the following equations are obtained. The equation for estimating $\overset{*}{Ho}$ is,

$$\frac{\sum_1^p \ln\widehat{Q}_i \sum_1^p \ln Hr_i - p \sum_1^p \ln\widehat{Q}_i \ln Hr_i}{\left[\sum_1^p \ln Hr_i\right]^2 - p \sum_1^p \left[\ln Hr_i\right]^2} - \frac{\sum_1^p \ln\widehat{Q}_i \sum_1^p 1/Hr_i - p \sum_1^p \ln\widehat{Q}_i / Hr_i}{\sum_1^p \ln Hr_i \sum_1^p 1/Hr_i - p \sum_1^p \ln Hr_i / Hr_i} = 0 \ .$$

Once Ho has been estimated, $\overset{*}{b}_1$ and $\overset{*}{b}_2$ may be determined explicitly from,

$$\overset{*}{b}_1 = \exp \left[ \frac{\sum_1^p \ln \widehat{Q}_i - b \sum_1^p \ln Hr_i}{p} \right]$$

and

$$\overset{*}{b}_2 = \frac{\sum_1^p \ln \widehat{Q}_i \sum_1^p \ln Hr_i - \sum_1^p \ln \widehat{Q}_i \ln Hr_i}{\left[\sum_1^p \ln Hr_i\right]^2 - \sum_1^p (\ln Hr_i)^2} \ .$$

The equation for estimating $\overset{*}{Ho}$ must be solved by an iterative procedure. Manually, this procedure would be monumental; by computer, it is readily accomplished. The equation is of the form, $f(\overset{*}{Ho}) = 0$, and the object of the iteration is to solve for the real root of the function. The Bolzano or bisection method is a simple procedure for this determination and can be easily programmed for the computer. Starting at a point $\overset{*}{Ho}$ that is virtually certain to be larger than the root, $\overset{*}{Ho}$ is decreased by an increment ho until an interval of width ho is found, such that $f(\overset{*}{Ho})$ and

$f(\overset{*}{H}o + ho)$ have opposite signs. A root is then known to fall between $\overset{*}{H}o$ and $(\overset{*}{H}o + ho)$. The increment ho is then halved at every step, with the new interval $(\overset{*}{H}o, \overset{*}{H}o + ho/4)$ used when $f(\overset{*}{H}o)$ and $f(\overset{*}{H}o + ho/2)$ are of opposite signs, and the new interval $(\overset{*}{H}o + ho/2, \overset{*}{H}o + 3ho/4)$ used when $f(\overset{*}{H}o + ho/2)$ has the same sign as $f(\overset{*}{H}o)$. The root is thus always approached from the left.

In applying the Bolzano method to the estimation of $\overset{*}{H}o$, the initial $\overset{*}{H}o$ selected was the lowest stage value of the data minus 0.01. For data from perennial streams, the minimum stage is inevitably more than 0.01 above the root; but it is important to select a point below the minimum value to insure that all relative stage values are positive. The iterative procedure was performed until the interval ho had been reduced to less than 0.0005.

4.2.5 Justification of the logarithmic transformation

A basic premise of the method of least squares is that the variance of the estimated variable be a constant over the range of applicability of the estimated relationship. From the previously-estimated model for error in a single discharge measurement, it was learned that the variance of the error, and of the measured discharge, is not a constant. Rather, the variance may be expected to be proportional to the square of the discharge. The logarithmic transformation tends to stabilize the variance when it is proportional to the square of the discharge, and the least squares approach is applicable on the logarithms.

In order to further verify the proportionality of the variance and the square of the discharge, the most recent rating-curve data for the mountain stream-gaging stations was studied. Rating curves were fitted by the above-described procedure, and the squared deviations, $(\overset{*}{Q}_i - Q_i)^2$, were determined and plotted with the respective estimated discharge values, as shown in fig. 9. On this plot, there is a tendency for the most dense grouping of points to follow a straight line of slope two. No single line on this plot would have much significance, since data from the nine sample stations were pooled together. However, the tendency suggested above reflects the fact that the variance of a discharge value is approximately proportional to the square of that value. Hence, the model presented in the previous chapter is strengthened, and the logarithmic transformation is further justified.

4.2.6 The divisive discharge value

In order to develop the hypothetical error model for a single discharge measurement, it was necessary to restrict consideration to discharges which did not involve low velocities and/or shallow depths. At velocities less than one foot per second, it was noted that most current meters failed to register accurately. Also, at depths less than one foot, the vertical distributuion of velocity, and hence the mean velocity in a vertical, became very difficult to estimate properly. Therefore, the error in a single relatively small discharge measurement occurring under the above conditions may be expected to be relatively high, and not abide by the conditions of the model.

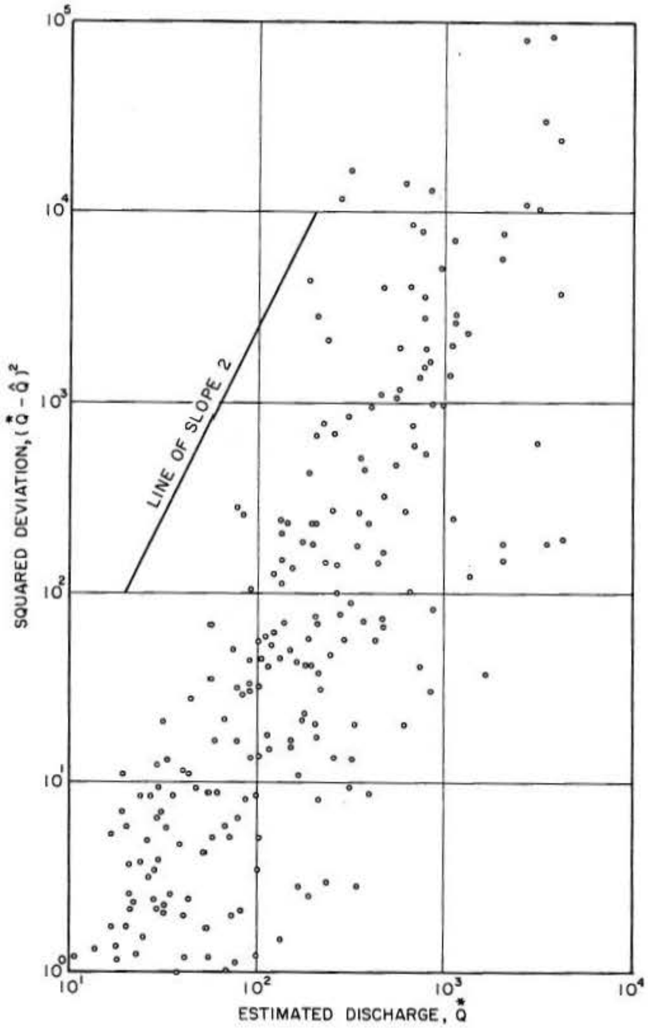


Fig. 9 Squared deviations from estimated rating curves versus estimated discharges

As a result, a "divisive discharge value" was established for each stream-gaging site. By definition, this value is that discharge which can be expected to exceed all discharges arising from velocities less than one foot per second and/or depths less than one foot. In other words, the divisive discharge value, determined on the basis of the above physical criteria, divides the discharge measurement and the rating curve for a gaging station into two portions. The upper, and longer portion of the curve, may be expected to have errors of estimate whose distributions are represented by the error model. That is, along this portion, the standard deviation of a single discharge is proportional to that discharge. On the short, lower portion of the curve, relative errors may become quite large, although in absolute value they will be small. The divisive discharge values for the nine sample stations are given in Table 13.

It was further observed that the divisive discharge values selected essentially divided the mean daily discharge estimates for the mountain stations considered into winter and summer flows. For more

TABLE 13. DIVISIVE DISCHARGE VALUES FOR THE SAMPLE STATIONS

| Station Name | Divisive Discharge Value, $Q_D$ (cfs) |
|---|---|
| Arkansas River at Granite | 120 |
| Taylor River below Taylor Park Reservoir | 50 |
| East River at Almont | 150 |
| Quartz Creek near Ohio City | 30 |
| Curecanti Creek near Sapinero | 20 |
| San Miguel River near Placerville | 95 |
| Animas River at Howardsville | 40 |
| Hermosa Creek near Hermosa | 35 |
| Animas River at Durango | 300 |

explicit information on this point, a duration curve was prepared for each sample station, and percent volume plots were developed from graphical integration of the duration curves. Both duration and volume curves are presented in fig. 10.

The selected divisive discharge values were equalled or exceeded, on the average, 47.3 percent of the time, and the range was from 26.5 to 56.0 percent. However, the same discharge values and flows below them accounted for only an average of 11.2 percent of the annual volume, the range being from 3.4 to 16.0 percent. In other words, although the flows in the lower portion of the rating curve occur over fifty percent of the time, they account for only about ten percent of the volume.

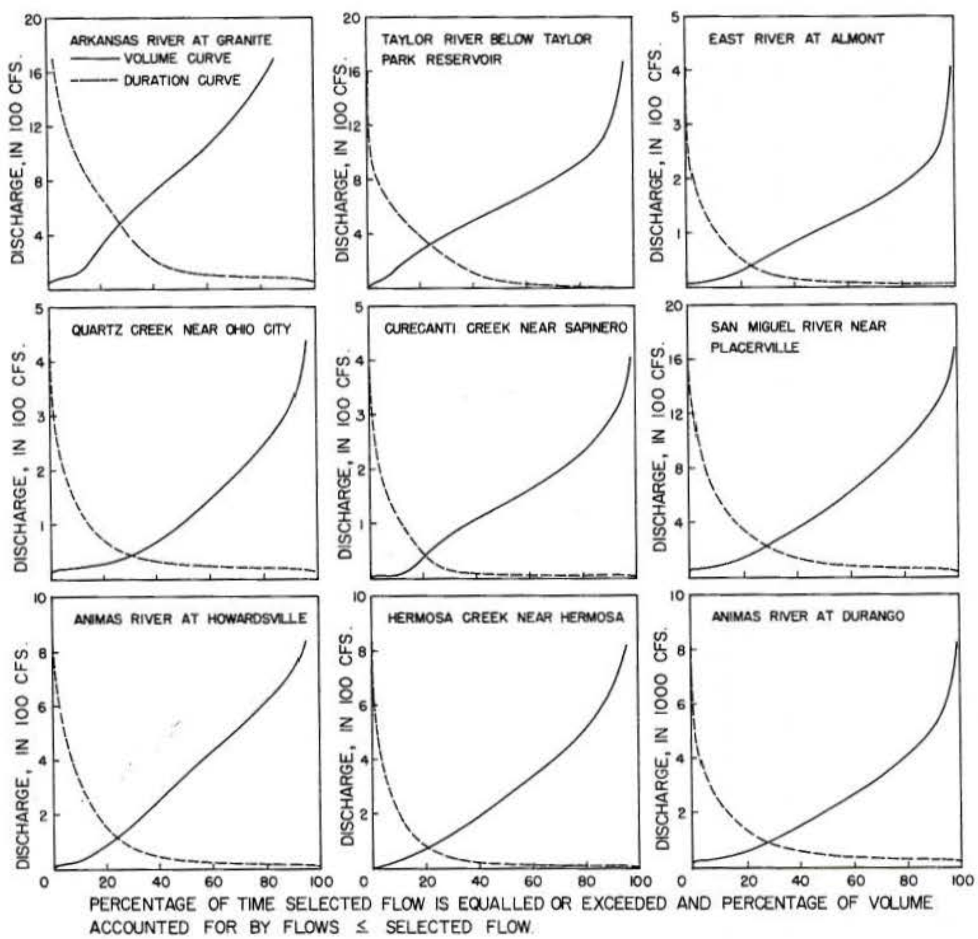


Fig. 10 Duration and volume curves for selected gaging stations

Therefore, the use of a divisive discharge value assumes several connotations. Not only are there physical justifications for considering the discharge measurements and the rating curve in two portions, but also statistically the values are consistent for the mountain streams with regard to both the duration and volume aspects. Since the low flows occur in the winter months during adverse weather and flow conditions, there is further physical justification for the errors in low-flow measurements to become relatively large.

### 4.2.7 Confidence and tolerance limits

On the basis of the foregoing discussion, different criteria are used for fitting confidence and tolerance limits to the two portions of a rating curve. For the upper portion of the curve, the limits reflect the fact that the variance is proportional to the square of the discharge; whereas on the lower portion, the limits are based on the assumption that the absolute error remains constant, with the result that the relative error increases rapidly for small discharge values. The limits for each portion are further discussed below.

For the upper portion of the fitted rating curve, the regular statistical confidence and tolerance limits are determined. Since the curve has been fitted by use of a logarithmic transformation, the limits must be evaluated on the same basis. In other words, the limits are determined by the variability of $\ln \overset{*}{b}_1$, $\overset{*}{b}_2$ and $\overset{*}{H}o$. Further, since the correlation between the estimated parameters is not known, the limits are determined with regard to a fixed value of $\overset{*}{H}o$. That is, once $\overset{*}{H}o$ has been estimated, it is assumed to be exact, and the confidence and tolerance limits for the regression curve are based on the variability of only $\ln \overset{*}{b}_1$ and $\overset{*}{b}_2$, which are uncorrelated parameters.

Then the confidence limits for the upper portion of the rating curve may be expressed as,

$$\ln \overset{*}{Q} - t_p s_{\overset{*}{Q}} \leq \ln Q \leq \ln \overset{*}{Q} + t_p s_{\overset{*}{Q}}$$

where $s_{\overset{*}{Q}}$ is the sample standard deviation of $\ln \overset{*}{Q}$, and $t_p$ is the appropriate "student" t value for the selected confidence level. In terms of the absolute rather than the logarithmic values,

$$\exp(-t_p s_{\overset{*}{Q}})\, \overset{*}{Q} \leq Q \leq \exp(t_p s_{\overset{*}{Q}})\, \overset{*}{Q}$$

Further, it is of interest to consider the distance between the limits and the estimated value as a percentage of the estimated value. The upper limit may be expressed as $100[\exp(t_p s_{\overset{*}{Q}}) - 1]\overset{*}{Q}$, and the lower limit as $100\,[\exp(-t_p s_{\overset{*}{Q}}) - 1]\, \overset{*}{Q}$; or the two limits as $100[\exp(\pm t_p s_{\overset{*}{Q}}) - 1]\, \overset{*}{Q}$. Then Q lies between a positive and a negative percentage of the estimated discharge.

Similarly, the tolerance limits for the upper portion of the fitted rating curve may be expressed as,

$$\ln \overset{*}{Q} - t_p s_Q \leq \ln Q \leq \ln \overset{*}{Q} + t_p s_Q$$

or

$$\exp(-t_p s_Q)\, \overset{*}{Q} \leq Q \leq \exp(t_p s_Q)\, \overset{*}{Q}$$

or

$$100\left[\exp(\pm t_p s_Q) - 1\right] \overset{*}{Q}\,.$$

where $s_Q$ is the sample standard deviation of $\ln Q$.

On the lower portion of the rating curve, the absolute error has been assumed to remain constant. The confidence and tolerance limits for this portion are, therefore, obtained by using the absolute deviations of the limits calculated for the upper portion at the divisive discharge value, and extending lines parallel to the rating curve from the divisive discharge value downwards to the minimum discharge. That is, the absolute error at the divisive discharge value is considered to be representative of the absolute errors to be expected at lower discharge values. As a result, the relative errors increase quite rapidly as the discharge decreases from the divisive value.

Although the relative errors, reflected by the limits, diverge slightly on the upper portion of the curve as the point considered departs from the geometric mean of the discharge measurements, they may be expected to remain fairly constant since the measurement range is relatively small. Then the relative errors begin to increase rapidly below the divisive value. This type of error distribution along the curve would appear to reflect the true situation rather well.

As the limits on the upper portion of the curve are fitted on the basis of logarithms, the errors are essentially assumed to be log-normally distributed. Hence, the relative upper bound is always larger than the relative lower one. The error is bounded on the lower side of a discharge value, whereas there is no bound on the positive side. Therefore, the log-normal error distributuion seems reasonable.

### 4.2.8 Sample stage-discharge rating curves

The data for each stable period of record considered for the sample stations were fitted by the procedure previously outlined with a regression equation of the form, $\overset{*}{Q} = \overset{*}{b}_1\, (H - \overset{*}{H}o)^{\overset{*}{b}2}$. The values obtained for the three estimated parameters, along with the sample coefficient of determination are given in Table 14 for each period. The standard errors of estimate were determined for each portion of each curve, and are listed in Table 15 in the dimensionless form of the estimated coefficient of variation from regression. The total number of data points, p, and the number of points on the lower portion, $p_L$, and upper portion $p_U$, of the curve are also given.

The question arises regarding how well the estimated regression equations fit the sample data. The best indicators of the nature of the fit are the coefficients of determination and the standard errors. Each is considered below, with regard to the results in Table 14 and 15.

The coefficient of determination represents the amount of variability in the discharge values accounted for by the estimated rating curve. The values obtained for the periods of record considered are all greater than 0.95, with the majority of them above 0.99. Such high values definitely infer that the

TABLE 14. ESTIMATED PARAMETERS AND SAMPLE COEFFICIENTS OF DETERMINATION FOR FITTED STAGE-DISCHARGE RELATIONSHIPS

| Station and Period of Record | $b_1^*$ | $b_2^*$ | $H_o^*$ | $r^2$ |
|---|---|---|---|---|
| Arkansas River at Granite | | | | |
| Mar. '38 - May '41 | 137.92 | 1.8940 | 0.8057 | 0.9932 |
| May '41 - June '43 | 18.410 | 2.9102 | 0.6129 | 0.9910 |
| June '43 - June '46 | 94.791 | 2.1087 | 1.3736 | 0.9922 |
| July '46 - Dec. '50 | 78.993 | 2.2748 | 1.2008 | 0.9950 |
| Feb. '51 - June '56 | 142.17 | 1.9697 | 1.6230 | 0.9958 |
| June '58 - Oct. '60 | 186.87 | 2.0582 | 2.5022 | 0.9849 |
| April '61 - Oct. '64 | 16.240 | 3.2796 | 0.1905 | 0.9950 |
| Taylor River below Taylor Park Reservoir | | | | |
| May '39 - July '43 | 78.499 | 2.0988 | 0.4691 | 0.9972 |
| Aug. '43 - Jan. '47 | 90.242 | 2.0067 | 0.5848 | 0.9974 |
| May '47 - Sept. '50 | 92.202 | 1.9437 | 0.5993 | 0.9970 |
| Sept. '50 - Nov. '52 | 73.661 | 2.0344 | 0.5183 | 0.9964 |
| Nov. '53 - Sept. '54 | 9.6592 | 2.8879 | 0.7161 | 0.9948 |
| Sept. '54 - Sept. '57 | 40.359 | 2.3480 | 1.7873 | 0.9968 |
| Sept. '57 - May '62 | 11.248 | 3.0928 | 1.2821 | 0.9972 |
| May '62 - Oct. '65 | 10.745 | 3.1252 | 1.2727 | 0.9968 |
| East River at Almont | | | | |
| Sept. '35 - June '39 | 31.012 | 2.8097 | -0.1067 | 0.9954 |
| July '39 - Jan. '44 | 36.329 | 2.7896 | -0.1362 | 0.9726 |
| Feb. '44 - Mar. '48 | 47.286 | 2.5356 | -0.2499 | 0.9942 |
| Mar. '48 - Sept. '50 | 35.941 | 2.6269 | -0.5087 | 0.9960 |
| Apr. '55 - May 58 | 14.397 | 2.9401 | 0.0238 | 0.9761 |
| May '58 - Aug. 61 | 86.845 | 2.1568 | 1.1260 | 0.9714 |
| Sept. '61 - Oct. '65 | 186.07 | 1.7469 | 1.4609 | 0.9942 |
| Quartz Creek near Ohio City | | | | |
| Apr. '38 - July '41 | 65.909 | 2.1968 | 0.0291 | 0.9775 |
| Aug. '41 - July '46 | 0.6876 | 5.3424 | -1.3969 | 0.9563 |
| July '46 - Oct. '50 | 14.164 | 3.0905 | 0.2315 | 0.9724 |
| Oct. '60 - May '62 | 6.1207 | 4.3352 | 0.4316 | 0.9785 |
| June '62 - June '65 | 40.621 | 2.9720 | 0.9398 | 0.9918 |
| Curecanti Creek near Sapinero | | | | |
| May '48 - Aug. '52 | 2.7988 | 4.0839 | 0.5110 | 0.9653 |
| Apr. '53 - June '57 | 8.2646 | 3.6291 | 1.2529 | 0.9952 |
| May '58 - Jan. '61 | 3.1275 | 4.6462 | 1.0963 | 0.9968 |
| Mar. '61 - Nov. '65 | 0.7263 | 5.4003 | 0.7777 | 0.9860 |
| San Miguel River near Placerville | | | | |
| Sept. '43 - Dec. '46 | 13.088 | 3.1663 | -1.3939 | 0.9837 |
| Jan. '47 - Apr. '48 | 5.2177 | 3.7253 | -1.5969 | 0.9942 |
| June '49 - May '52 | 39.224 | 2.6085 | -0.7028 | 0.9954 |
| June '52 - Oct. '55 | 21.439 | 2.9125 | -1.0454 | 0.9952 |
| Nov. '55 - Apr. '57 | 0.5799 | 4.9768 | -2.0859 | 0.9964 |
| Nov. '58 - Mar. '62 | 0.0188 | 6.0913 | -1.1756 | 0.9761 |
| Apr. '62 Nov. '64 | 13.513 | 3.1566 | 1.0193 | 0.9809 |
| Animas River at Howardsville | | | | |
| Apr. '40 - Apr. '42 | 1.6349 | 4.5126 | -0.9386 | 0.9960 |
| June '42 - May '48 | 8.9388 | 3.6290 | -0.3216 | 0.9954 |
| June '48 - Sept. '52 | 7.4809 | 3.6835 | -0.3921 | 0.9904 |
| Oct. '52 - Mar. '56 | 4.6810 | 3.8196 | -0.6330 | 0.9934 |
| Mar. '56 - June '59 | 14.649 | 3.2704 | -0.1863 | 0.9874 |
| June '59 - May '61 | 8.6052 | 3.6338 | -0.3121 | 0.9972 |
| June '61 - Nov. '64 | 14.549 | 3.3094 | -0.1014 | 0.9932 |
| Hermosa Creek near Hermosa | | | | |
| May '40 - July '43 | 15.852 | 3.3636 | -0.1344 | 0.9797 |
| Aug. '43 - Jan. '48 | 0.0009 | 8.5814 | -2.3477 | 0.9946 |
| Feb. '48 - May '52 | 23.230 | 3.5385 | -0.0023 | 0.9714 |
| June '52 - Apr. '54 | 36.505 | 3.4431 | 0.0107 | 0.9942 |
| Apr. '54 - June '57 | 70.300 | 2.6701 | 0.2547 | 0.9954 |
| Dec. '60 - Nov. '64 | 41.443 | 2.6908 | -0.1005 | 0.9880 |
| Animas River at Durango | | | | |
| Nov. '36 - Sept. '46 | 70.490 | 2.4842 | -0.3450 | 0.9932 |
| Sept. '46 - Oct. '53 | 67.517 | 2.5608 | 0.1239 | 0.9637 |
| Oct. '53 - Apr. '57 | 37.290 | 2.9156 | 0.4672 | 0.9759 |
| Apr. '57 - Oct. '64 | 232.73 | 1.9222 | 1.1732 | 0.9972 |

TABLE 15. COEFFICIENTS OF VARIATION FROM REGRESSION FOR SAMPLE RATING CURVES

| Station and Period of Record | P | $P_L$ | $P_U$ | Coefficients of Variation From Regression, as a Percentage | | |
|---|---|---|---|---|---|---|
| | | | | Entire Curve | Low Portion | High Portion |
| Arkansas River at Granite | | | | | | |
| Mar. '38 - May '41 | 39 | 10 | 29 | + 9.6 - 8.8 | +15.8 -13.3 | + 8.0 - 7.5 |
| May '41 - June '43 | 22 | 6 | 16 | +10.5 - 9.7 | +13.9 -12.2 | +10.8 - 9.8 |
| June '43 - June '46 | 28 | 10 | 18 | +10.5 - 9.5 | +15.9 -15.7 | + 5.7 - 5.4 |
| July '46 - Dec. '50 | 45 | 9 | 36 | + 7.8 - 7.3 | +15.4 -13.3 | + 6.0 - 5.7 |
| Feb. '51 - June '56 | 65 | 22 | 43 | + 6.6 - 6.2 | + 9.2 - 8.4 | + 5.3 - 5.0 |
| June '58 - Oct. '60 | 34 | 4 | 30 | +11.7 -10.5 | +12.6 -11.2 | +12.1 -10.8 |
| April '61 - Oct. '64 | 66 | 7 | 59 | + 6.5 - 6.1 | +11.3 -10.2 | + 7.2 - 5.8 |
| Taylor River below Taylor Park Res. | | | | | | |
| May '39 - July '43 | 42 | 4 | 38 | + 6.3 - 5.9 | +12.8 -11.3 | + 6.2 - 5.8 |
| Aug. '43 - Jan. '47 | 50 | 9 | 41 | + 6.0 - 5.7 | + 6.8 - 6.4 | + 6.0 - 5.7 |
| May '47 - Sept. '50 | 43 | 6 | 37 | + 6.1 - 5.8 | +13.1 -11.6 | + 5.4 - 5.1 |
| Sept. '50 - Nov. '52 | 28 | 5 | 23 | + 7.5 - 7.0 | +14.8 -12.9 | + 6.9 - 6.5 |
| Nov. '53 - Sept. '54 | 33 | 5 | 28 | + 8.7 - 8.0 | +14.7 -12.8 | + 8.4 - 7.8 |
| Sept. '54 - Sept. '57 | 50 | 9 | 41 | + 9.2 - 8.4 | +19.4 -16.2 | + 7.0 - 6.5 |
| Sept. '57 - May '62 | 50 | 24 | 26 | + 9.0 - 8.3 | +12.4 -11.1 | + 5.4 - 5.1 |
| May '62 - Oct. '65 | 40 | 14 | 26 | +10.5 - 9.5 | +17.4 -14.8 | + 6.5 - 6.1 |
| East River at Almont | | | | | | |
| Sept. '35 - June '39 | 43 | 21 | 22 | + 8.8 - 8.1 | +11.0 -10.0 | + 6.9 - 6.5 |
| July '39 - Jan. '44 | 40 | 23 | 17 | +21.9 -18.0 | +29.3 -22.7 | +11.0 -10.0 |
| Feb. '44 - Mar. '48 | 38 | 20 | 18 | + 9.4 - 8.6 | +13.1 -11.6 | + 4.6 - 4.4 |
| Mar. '48 - Sept. '50 | 29 | 15 | 14 | + 8.6 - 7.9 | + 9.2 - 8.5 | + 8.6 - 7.9 |
| April '55 - May '58 | 44 | 18 | 26 | +28.0 -21.9 | +38.4 -27.8 | +22.0 -18.1 |
| May '58 - Aug. '61 | 42 | 16 | 26 | +22.9 -18.6 | +38.4 -27.8 | +12.2 -10.9 |
| Sept. '61 - Oct. '65 | 47 | 20 | 27 | +10.3 - 9.3 | +14.0 -12.3 | + 7.5 - 7.0 |
| Quartz Creek near Ohio City | | | | | | |
| April '38 - July '41 | 41 | 12 | 29 | +16.8 -14.4 | +27.2 -21.4 | +12.9 -11.4 |
| Aug. '41 - July '46 | 34 | 11 | 23 | +19.5 -16.3 | +23.9 -19.3 | +18.8 -15.8 |
| July '46 - Oct. '50 | 44 | 23 | 21 | +16.5 -14.2 | +21.4 -17.7 | +11.4 -10.2 |
| Oct. '60 - May '62 | 31 | 13 | 18 | +13.9 -12.2 | +17.9 -15.2 | +11.0 - 9.9 |
| June '62 - June '65 | 37 | 11 | 26 | + 8.6 - 8.0 | + 7.9 - 7.3 | + 9.2 - 8.5 |
| Curecanti Creek near Sapinero | | | | | | |
| May '48 - Aug. '52 | 39 | 17 | 22 | +34.2 -25.5 | +47.8 -32.4 | +25.2 -20.2 |
| April '53 - June '57 | 46 | 24 | 22 | +11.5 -10.3 | + 9.1 - 8.4 | +14.0 -12.3 |
| May '58 - Jan. '61 | 33 | 16 | 17 | + 9.4 - 8.6 | + 9.7 - 8.9 | + 9.7 - 8.9 |
| Mar. '61 - Nov. '65 | 52 | 26 | 26 | +18.5 -15.7 | +26.2 -20.8 | + 9.6 - 8.8 |
| San Miguel River near Placerville | | | | | | |
| Sept. '43 - Dec. '46 | 36 | 14 | 22 | +12.9 -11.4 | +19.6 -16.4 | + 8.6 - 8.0 |
| Jan. '47 - April '48 | 18 | 8 | 10 | + 7.7 - 7.2 | +11.7 -10.5 | + 5.6 - 5.4 |
| June '49 - May '52 | 41 | 22 | 19 | + 6.5 - 6.1 | + 8.1 - 7.5 | + 4.7 - 4.5 |
| June '52 - Oct. '55 | 52 | 23 | 29 | + 7.1 - 6.6 | + 7.5 - 7.0 | + 7.0 - 6.6 |
| Nov. '55 - April '57 | 20 | 13 | 7 | + 6.4 - 6.1 | + 6.7 - 6.3 | + 7.0 - 6.5 |
| Nov. '58 - Mar. '62 | 46 | 11 | 35 | +15.8 -13.7 | +10.4 - 9.5 | +17.4 -14.8 |
| April '62 - Nov. '64 | 41 | 10 | 31 | +13.9 -12.2 | +33.6 -25.2 | + 5.1 - 4.9 |
| Animas River at Howardsville | | | | | | |
| April '40 - April '42 | 21 | 9 | 12 | + 7.7 - 7.2 | + 6.9 - 6.5 | + 8.8 - 8.1 |
| June '42 - May '48 | 62 | 32 | 30 | + 8.7 - 8.0 | +11.6 -10.4 | + 4.9 - 4.6 |
| June '48 - Sept. '52 | 56 | 27 | 29 | +14.2 -12.5 | +18.8 -15.8 | + 9.9 - 9.0 |
| Oct. '52 - Mar. '56 | 39 | 18 | 21 | + 8.4 - 7.8 | +11.7 -10.5 | + 5.5 - 5.2 |
| Mar. '56 - June '59 | 46 | 20 | 26 | +17.9 -15.2 | +28.4 -22.1 | + 7.6 - 7.1 |
| June '59 - May '61 | 28 | 12 | 16 | + 7.2 - 6.7 | + 7.9 - 7.4 | + 7.2 - 6.8 |
| June '61 - Nov. '64 | 42 | 15 | 27 | +11.2 -10.1 | +18.4 -15.6 | + 6.7 - 6.3 |
| Hermosa Creek near Hermosa | | | | | | |
| May '40 - July '43 | 39 | 16 | 23 | +23.1 -18.2 | +30.9 -23.6 | +18.9 -15.9 |
| Aug. '43 - Jan. '48 | 40 | 16 | 24 | + 8.8 - 8.1 | +10.6 - 9.6 | + 8.1 - 7.5 |
| Feb. '48 - May '52 | 59 | 28 | 31 | +29.7 -22.9 | +40.6 -28.9 | +19.6 -16.4 |
| June '52 - April '54 | 32 | 19 | 13 | + 9.8 - 8.9 | +12.0 -10.7 | + 7.2 - 5.7 |
| April '54 - June '57 | 56 | 24 | 32 | + 9.9 - 9.0 | +12.1 -10.8 | + 8.3 - 7.7 |
| Dec. '60 - Nov. '64 | 71 | 32 | 39 | +15.5 -13.4 | +23.6 -19.1 | + 5.9 - 5.6 |
| Animas River at Durango | | | | | | |
| Nov. '36 - Sept. '46 | 137 | 56 | 81 | + 9.9 - 9.0 | +13.7 -12.0 | + 6.6 - 6.2 |
| Sept. '46 - Oct. '53 | 134 | 55 | 79 | +24.6 -19.8 | +34.0 -25.4 | +16.9 -14.5 |
| Oct. '53 - April '57 | 68 | 30 | 38 | +13.5 -11.9 | +16.5 -14.2 | +11.3 -10.2 |
| April '57 - Oct. '64 | 150 | 51 | 99 | + 5.9 - 5.6 | + 7.3 - 6.8 | + 5.2 - 4.9 |
| Average Values | | | | +12.4 -10.8 | +17.3 -14.3 | + 9.37 - 8.40 |

selected mathematical expressions very adequately represents the stage-discharge relationships of the sample mountain-gaging stations. Further, it can be said that the logarithms of stage and discharge appear to be functionally linearly related.

An appraisal of the table of coefficients of variation reveals that in virtually every case, the values for the lower portion are considerably larger than those for the upper portion. It was this result which was anticipated and led to the establishment of the divisive discharge value. Further, under the original hypothesis that the majority of scatter about the curve is due to measurement error, these values reflect the relative measurement error. Although there are no values with which to compare, the averages of +9.37 and -8.40 percent appear reasonable for mountain discharge measurements. Closer consideration of the individual values reveals that for some gaging sections, the errors of estimate can be as low as ±5.0 percent. However, they are seldom less than this. Such values might represent about the best accuracy that can be expected for current-meter methodology on mountain streams. On the other hand, the values can be quite large, as in the case of Curecanti Creek. These latter cases inevitably include considerable minor random instability of the basic stage-discharge relationship, which could not be detected by the stability check. The mean maximum values for the lower portion of the curve, +17.3 and -14.3 percent, are almost double those for the upper portion. The particularly large individual values are found in cases where the discharge measurements become very small. As a result, the relative errors are large on the lower portion, although the absolute errors are very small.

In summary, the regression equations of the form selected can be used very well as estimators of the true stage-discharge relationships on mountain streams. The relationships estimated appear to be very stable, and the errors of estimate reflect the measurement error to be expected in mountain flow measurements. This final statement can only be verified by global comparisons of field measurements or by evaluation of the components in the error model.

#### 4.2.9 Fitted error bounds

Considering the rating curves to be divided by the divisive discharge values, both confidence and tolerance limits were established. The limits existing at the lowest discharge point, the geometric mean point, and the lower or upper end of the upper portion, whichever yielded the maximum limits, are given for the 80 and 95 percent confidence levels in Tables 16 and 17, respectively. An illustration is given in fig. 11 as an aid to visualizing the appearance of the limits. This example was selected as being representative of the average values for the stations analyzed.

The error bounds on a single discharge estimate are given by the confidence limits for selected levels of confidence. The computed mean maximum confidence limits are +2.40 and -2.33 percent for the upper portion of the curve at the 80 percent confidence level, and +3.82 and -3.65 percent at the 95 percent level. Although the actual bounds for a particular curve should be used, these general values serve to indicate the general order of magnitude for the stations and periods of record sampled. Therefore, a single discharge estimate can be expected to be very good for these stations, particularly for the summer range of flows.

The tolerance limits yield information regarding the region in which a future discharge measurement may be expected to fall, assuming the river regime to have remained in the same stable condition. For the 55 periods of record studied, the mean percentage tolerance limits at the mean discharge are +12.9 and -11.2 percent at the 80 percent confidence level, and +21.3 and -16.9 percent at the 95 percent level. In general terms, a future measurement may be expected to be within about ±12 or ±19 percent of the corresponding estimated discharge on the upper portion of the curve. These limits may appear somewhat large to the field worker who expects a new measurement to fall within five percent of the existing curve. However, for the small number of data points usually available, such an optimistic viewpoint is not justified, particularly at the higher confidence levels.

### 4.3 Accuracy of Discharge Estimates

#### 4.3.1 Types of estimates

In practice, single discharge estimates are made for mean daily discharge values. A study of the error in monthly and annual discharge estimates requires further knowledge regarding the correlation between the errors in daily estimates. All three types of estimates, daily, monthly, and annual, are considered below with a discussion of the possible correlation between errors.

#### 4.3.2 A mean daily discharge estimate

The methodology for obtaining a mean daily discharge estimate involves the estimation of mean daily stage from stage-time records, and the application of this mean stage value to the rating curve to obtain the estimate of mean daily discharge. As has been pointed out in a previous chapter, the errors inherent in the determination of mean daily stage may be largely controlled and rendered minimal. Therefore, for the sake of this analysis, the error in a single mean daily discharge estimate is assumed to be equivalent to the error in any single discharge estimate for which the stage is given.

On the basis of the above-mentioned assumption, bounds on the error in an estimate of mean daily discharge may be determined from the confidence limits fitted to the rating curve. For the stations analyzed, the upper limit of discharges to be considered was taken to be the maximum discharge measurement used for the curve. Then the maximum confidence limits are those in Tables 16 and 17. Although it is not advisable to extend the curve upwards without more measurements, it may be done with the result of increased confidence limits and the risk of reaching a range of very high flows where more representative limits would be even larger than those reflected by the model.

#### 4.3.3 Correlation of daily errors

The correlation between errors in mean daily flow estimates determines the nature of the error in an estimate which is obtained by the summation of daily discharge estimates, such as a monthly or yearly value. The situations involving perfectly correlated and uncorrelated errors are considered, and the conditions for each to exist are discussed. Cases in discharge estimation where partial correlation may occur are also noted.

If the errors in two mean daily discharge estimates, $q_1$ and $q_2$, are perfectly correlated, then the

TABLE 16. 80 PERCENT CONFIDENCE AND TOLERANCE LIMITS FOR SAMPLE DATA

| Station And Period of Record | 80 Percent Limits (As A Percentage Of Estimated Discharge) | | | | | |
|---|---|---|---|---|---|---|
| | Confidence Limits | | | Tolerance Limits | | |
| | Maximum For Low Range | At Mean | Maximum For High Range | Maximum For Low Range | At Mean | Maximum For High Range |
| Arkansas River at Granite | | | | | | |
| Mar. '38 - May '41 | + 4.21 - 4.14 | +1.87 -1.84 | +2.04 -2.00 | + 21.2 - 20.9 | +10.9 - 9.80 | +10.9 - 9.83 |
| May '41 - June '43 | + 5.86 - 5.65 | +3.41 -3.30 | +3.45 -3.33 | + 26.0 - 22.5 | +15.3 -13.3 | +15.3 -13.3 |
| June '43 - June '46 | + 3.75 - 3.69 | +1.72 -1.68 | +1.79 -1.76 | + 16.6 - 15.4 | + 7.89 - 7.31 | + 7.91 - 7.33 |
| July '46 - Dec. '50 | + 2.36 - 2.33 | +1.26 -1.25 | +1.30 -1.29 | + 14.5 - 13.4 | + 8.04 - 7.44 | + 8.04 - 7.45 |
| Feb. '51 - June '56 | + 1.90 - 1.88 | +1.01 -1.00 | +1.08 -1.07 | + 12.7 - 11.9 | + 6.95 - 6.50 | + 6.96 - 6.51 |
| June '58 - Oct. '60 | + 3.52 - 3.42 | +2.73 -2.66 | +2.84 -2.76 | + 20.4 - 17.5 | +16.5 -14.1 | +16.5 -14.2 |
| April '61 - Oct. '64 | + 1.22 - 1.19 | +1.00 -0.99 | +1.01 -1.00 | + 9.80 - 9.07 | + 8.11 - 7.50 | + 8.11 - 7.50 |
| Taylor River below Taylor Park Res. | | | | | | |
| May '39 - July '43 | + 5.12 - 5.05 | +1.26 -1.25 | +1.38 -1.36 | + 30.8 - 28.4 | + 8.27 - 7.64 | + 8.29 - 7.66 |
| Aug. '43 - Jan. '47 | + 4.75 - 4.69 | +1.19 -1.17 | +1.27 -1.26 | + 30.1 - 27.8 | + 8.05 - 7.45 | + 8.06 - 7.46 |
| May '47 - Sept. '50 | + 2.34 - 2.32 | +1.12 -1.11 | +1.21 -1.20 | + 13.9 - 13.0 | + 7.20 - 6.72 | + 7.22 - 6.73 |
| Sept. '50 - Nov. '52 | + 5.09 - 5.00 | +1.82 -1.79 | +2.00 -1.96 | + 24.2 - 22.1 | + 9.44 - 8.63 | + 9.48 - 8.66 |
| Nov. '53 - Sept. '54 | + 9.01 - 8.86 | +2.00 -1.96 | +2.03 -1.99 | + 51.1 - 45.8 | +11.4 -10.3 | +11.5 -10.3 |
| Sept. '54 - Sept. '57 | +10.4 .-10.2 | +1.36 -1.35 | +1.52 -1.50 | + 65.2 - 58.4 | + 9.29 - 8.50 | + 9.31 - 8.52 |
| Sept. '57 - May '62 | +10.0 - 9.92 | +1.33 -1.31 | +1.35 -1.33 | + 54.2 - 5.06 | + 7.25 - 6.76 | + 7.26 - 6.76 |
| May '62 - Oct. '65 | +23.0 -22.6 | +1.62 -1.59 | +1.66 -1.63 | +123. -113. | + 8.85 - 8.13 | + 8.86 - 8.14 |
| East River at Almont | | | | | | |
| Sept. '35 - June '39 | + 6.85 - 6.73 | +1.86 -1.83 | +1.88 -1.85 | + 34.5 - 31.3 | + 9.45 - 8.64 | + 9.46 - 8.64 |
| July '39 - Jan. '44 | +20.2 -19.5 | +3.36 -3.25 | +3.39 -3.28 | + 92.5 - 80.0 | +15.5 -13.4 | +15.5 -13.4 |
| Feb. '44 - Mar. '48 | + 5.45 - 5.37 | +1.40 -1.38 | +1.43 -1.41 | + 24.6 - 23.0 | + 6.43 - 6.04 | + 6.43 - 6.04 |
| Mar. '48 - Sept. '50 | + 8.67 - 8.40 | +2.93 -2.84 | +2.98 -2.90 | + 35.6 - 31.7 | +12.2 -10.9 | +12.2 -10.9 |
| April '55 - May '58 | +23.6 -22.3 | +5.18 -4.93 | +5.30 -5.04 | +136. -114. | +30.6 -23.5 | +30.7 -23.5 |
| May '58 - Aug. '61 | +14.2 -13.8 | +2.96 -2.87 | +3.11 -3.02 | + 77.2 - 66.2 | +16.7 -14.3 | +16.7 -14.3 |
| Sept. '61 - Oct. '65 | + 8.75 - 8.57 | +1.83 -1.79 | +1.99 -1.95 | + 45.1 - 41.0 | +10.2 - 9.28 | +10.3 - 9.31 |
| Quartz Creek near Ohio City | | | | | | |
| April '38 - July '41 | + 8.69 - 8.44 | +2.95 -2.87 | +3.07 -2.98 | + 5.07 - 43.1 | +17.6 -14.9 | +17.6 -15.0 |
| Aug. '41 - July '46 | + 8.77 - 8.40 | +4.75 -4.53 | +4.75 -4.54 | + 48.3 - 38.3 | +26.1 -20.7 | +26.1 -20.7 |
| July '46 - Oct. '50 | + 7.82 - 7.57 | +3.11 -3.02 | +3.13 -3.03 | + 39.7 - 34.2 | +15.8 -13.7 | +15.8 -13.7 |
| Oct. '60 - May '62 | +10.8 -10.4 | +3.56 -3.44 | +3.57 -3.44 | + 51.1 - 43.6 | +16.9 -14.5 | +16.9 -14.5 |
| June '62 - June '65 | + 5.50 - 5.37 | +2.26 -2.21 | +2.28 -2.23 | + 30.4 - 27.0 | +12.6 -11.2 | +12.6 -11.2 |
| Curecanti Creek near Sapinero | | | | | | |
| May '48 - Aug. '52 | +52.5 -49.2 | +6.42 -6.03 | +6.45 -6.06 | +290. -214. | +35.6 -26.3 | +35.6 -26.3 |
| April '53 - June '57 | +20.5 -19.8 | +3.70 -3.56 | +3.73 -3.60 | +107. - 89.5 | +19.5 -16.3 | +19.5 -16.3 |
| May '58 - Jan. '61 | +17.1 -16.7 | +2.98 -2.90 | +2.99 -2.90 | + 70.9 - 69.1 | +13.7 -12.0 | +13.7 -12.0 |
| Mar. '61 - Nov. '65 | +13.7 -13.4 | +2.35 -2.29 | +2.35 -2.29 | + 76.1 - 67.4 | +13.1 -11.6 | +13.1 -11.6 |
| San Miguel River near Placerville | | | | | | |
| Sept. '43 - Dec. '46 | + 4.53 - 4.42 | +2.31 -2.26 | +2.33 -2.27 | + 23.3 - 21.6 | +11.8 -10.6 | +11.8 -10.6 |
| Jan. '47 - April '48 | + 4.76 - 4.65 | +2.37 -2.31 | +2.38 -2.32 | + 17.0 - 15.6 | + 8.44 - 7.79 | + 8.45 - 7.79 |
| June '49 - May '52 | + 2.60 - 2.56 | +1.38 -1.36 | +1.40 -1.38 | + 12.1 - 11.4 | + 6.46 - 6.07 | + 6.47 - 6.07 |
| June '52 - Oct. '55 | + 7.86 - 7.38 | +1.64 -1.62 | +1.66 -1.63 | + 20.3 - 18.5 | + 9.50 - 8.68 | + 9.51 - 8.68 |
| Nov. '55 - April '57 | + 8.46 - 8.17 | +3.59 -3.46 | +3.59 -3.47 | + 26.3 - 23.7 | +11.2 -10.0 | +11.2 -10.0 |
| Nov. '58 - Mar. '62 | + 9.02 - 8.73 | +3.57 -3.45 | +3.57 -3.45 | + 60.1 - 48.4 | +23.8 -19.2 | +23.8 -19.2 |
| April '62 - Nov. '64 | + 2.19 - 2.15 | +1.16 -1.15 | +1.16 -1.15 | + 12.8 - 12.0 | + 6.84 - 6.40 | + 6.84 - 6.41 |
| Animas River at Howardsville | | | | | | |
| April '40 - April '42 | + 9.35 - 9.07 | +3.27 -3.16 | +3.27 -3.17 | + 36.5 - 32.3 | +12.8 -11.3 | +12.8 -11.3 |
| June '42 - May '48 | + 3.46 - 3.42 | +1.13 -1.11 | +1.13 -1.12 | + 19.8 - 18.7 | + 6.55 - 6.14 | + 6.55 - 6.15 |
| June '48 - Sept. '52 | + 7.80 - 7.62 | +2.29 -2.24 | +2.30 -2.25 | + 45.5 - 40.0 | +13.4 -11.8 | +13.4 -11.8 |
| Oct. '52 - Mar. '56 | + 4.71 - 4.64 | +1.52 -1.50 | +1.52 -1.50 | + 23.2 - 21.6 | + 7.50 - 6.98 | + 7.50 - 6.98 |
| Mar. '56 - June '59 | + 6.93 - 6.80 | +1.89 -1.85 | +1.91 -1.87 | + 37.7 - 34.3 | +10.4 - 9.41 | +10.4 - 9.41 |
| June '59 - May '61 | + 6.87 - 6.72 | +2.30 -2.25 | +2.32 -2.27 | + 30.1 - 27.3 | +10.1 - 9.21 | +10.1 - 9.21 |
| June '61 - Nov. '64 | + 6.12 - 6.03 | +1.62 -1.60 | +1.64 -1.61 | + 33.9 - 31.1 | + 9.07 - 8.31 | + 9.07 - 8.31 |
| Hermosa Creek near Hermosa | | | | | | |
| May '40 - July '43 | +11.0 -10.4 | +4.78 -4.56 | +4.90 -4.67 | + 59.3 - 47.1 | +26.3 -20.8 | +26.3 -20.8 |
| Aug. '43 - Jan. '48 | + 4.29 - 4.20 | +2.07 -2.03 | +2.07 -2.03 | + 22.8 - 20.5 | +11.0 - 9.91 | +11.0 - 9.91 |
| Feb. '48 - May '52 | +12.8 -11.8 | +4.23 -4.06 | +4.32 -4.14 | + 77.2 - 60.8 | +26.9 -21.1 | +26.9 -21.2 |
| June '52 - April '54 | + 6.71 - 6.55 | +2.56 -2.50 | +2.61 -2.55 | + 26.7 - 24.2 | +10.3 - 9.34 | +10.3 - 9.35 |
| April '54 - June '57 | + 9.81 - 9.62 | +1.84 -1.81 | +1.94 -1.90 | + 58.6 - 52.6 | +11.2 -10.1 | +11.3 -10.1 |
| Dec. '60 - Nov. '64 | + 4.65 - 4.58 | +1.19 -1.17 | +1.22 -1.20 | + 30.1 - 27.9 | + 7.85 - 7.28 | + 7.86 - 7.28 |
| Animas River at Durango | | | | | | |
| Nov. '36 - Sept. '46 | + 2.68 - 2.66 | + .93 - .92 | + .966 - .957 | + 24.8 - 22.8 | + 8.72 - 8.02 | + 8.73 - 8.03 |
| Sept. '46 - Oct. '53 | + 5.42 - 5.31 | +2.28 -2.23 | +2.37 -2.31 | + 52.3 - 42.7 | +22.5 -18.3 | +22.5 -18.4 |
| Oct. '53 - April '57 | + 4.70 - 4.58 | +2.26 -2.21 | +2.29 -2.24 | + 31.3 - 27.1 | +15.2 -13.2 | +15.2 -13.2 |
| April '57 - Oct. '64 | + 1.49 - 1.48 | + .65 - .65 | + .740 - .734 | + 14.6 - 13.6 | + 6.77 - 6.34 | + 6.78 - 6.35 |
| Average Values | + 8.60 - 8.34 | +2.35 -2.28 | +2.40 -2.33 | + 46.6 - 39.6 | +12.9 -11.2 | +12.9 -11.2 |

TABLE 17. 95 PERCENT CONFIDENCE AND TOLERANCE LIMITS FOR SAMPLE DATA

| Station And Period of Record | 95 Percent Limits (As A Percentage Of Estimated Discharge) | | | | | |
|---|---|---|---|---|---|---|
| | Confidence Limits | | | Tolerance Limits | | |
| | Maximum For Low Range | At Mean | Maximum For High Range | Maximum For Low Range | At Mean | Maximum For High Range |
| **Arkansas River at Granite** | | | | | | |
| Mar. '38 - May '41 | + 6.62 - 6.43 | + 2.94 -2.85 | + 3.20 -3.10 | + 37.1 - 31.7 | +17.5 -14.9 | +17.5 -14.9 |
| May '41 - June '43 | + 9.43 - 8.94 | + 5.49 -5.21 | + 5.56 -5.27 | + 43.3 - 34.5 | +25.5 -20.3 | +25.5 -20.3 |
| June '43 - June '46 | + 5.99 - 5.83 | + 2.73 -2.66 | + 2.85 -2.77 | + 27.0 - 24.0 | +12.8 -11.3 | +12.8 -11.4 |
| July '46 - Dec. '50 | + 3.67 - 3.59 | + 1.97 -1.93 | + 2.04 -1.99 | + 23.1 - 20.5 | +12.8 -11.3 | +12.8 -11.3 |
| Feb. '51 - June '56 | + 2.93 - 2.90 | + 1.56 -1.54 | + 1.67 -1.64 | + 19.9 - 17.9 | +10.9 - 9.85 | +10.9 - 9.86 |
| June '58 - Oct. '60 | + 5.53 - 5.29 | + 4.29 -4.12 | + 4.47 -4.28 | + 33.2 - 26.2 | +26.8 -21.2 | +26.9 -21.2 |
| April '61 - Oct. '64 | + 2.17 - 2.13 | + 1.79 -1.76 | + 1.80 -1.77 | + 17.9 - 15.6 | +14.9 -12.9 | +14.9 -12.9 |
| **Taylor River below Taylor Park Res.** | | | | | | |
| May '39 - July '43 | + 8.00 - 7.85 | + 1.97 -1.93 | + 2.02 -1.98 | + 48.9 - 43.0 | +13.1 -11.6 | +13.2 -11.6 |
| Aug. '43 - Jan. '47 | + 7.41 - 7.26 | + 1.85 -1.82 | + 1.98 -1.95 | + 47.8 - 42.4 | +12.8 -11.3 | +12.8 -11.3 |
| May '47 - Sept. '50 | + 3.66 - 3.58 | + 1.74 -1.71 | + 1.89 -1.86 | + 22.1 - 19.8 | +11.4 -10.2 | +11.4 -10.3 |
| Sept. '50 - Nov. '52 | + 8.07 - 7.81 | + 2.88 -2.80 | + 3.16 -3.06 | + 39.0 - 33.8 | +15.2 -13.2 | +15.3 -13.3 |
| Nov. '53 - Sept. '54 | +14.2 -13.8 | + 3.14 -3.05 | + 3.19 -3.09 | + 82.5 - 69.5 | +18.5 -15.6 | +18.5 -15.6 |
| Sept. '54 - Sept. '57 | +16.1 -15.8 | + 2.12 -2.08 | + 2.37 -2.31 | +102. - 88.5 | +14.8 -12.9 | +14.8 -12.9 |
| Sept. '57 - May '62 | +15.8 -15.5 | + 2.09 -2.05 | + 2.12 -2.07 | + 86.4 - 77.5 | +11.6 -10.4 | +11.6 -10.4 |
| May '62 - Oct. '65 | +36.1 -35.3 | + 2.54 -2.48 | + 2.60 -2.54 | +197. -173. | +14.2 -12.4 | +14.2 -12.4 |
| **East River at Almont** | | | | | | |
| Sept. '35 - June '39 | +10.8 -10.5 | + 2.95 -2.86 | + 2.98 -2.89 | + 55.6 - 48.3 | +15.3 -13.3 | +15.3 -13.3 |
| July '39 - Jan. '44 | +32.5 -30.8 | + 5.39 -5.12 | + 5.43 -5.15 | +154. -122. | +25.7 -20.5 | +25.7 -20.5 |
| Feb. '44 - Mar. '48 | + 8.65 - 8.48 | + 2.23 -2.18 | + 2.27 -2.22 | + 43.5 - 35.9 | +10.4 - 9.41 | +10.4 - 9.41 |
| Mar. '48 - Sept. '50 | +14.0 -13.4 | + 4.75 -4.53 | + 4.83 -4.61 | + 59.5 - 49.2 | +20.4 -16.9 | +20.4 -16.9 |
| April '55 - May '58 | +37.3 -34.6 | + 8.23 -7.60 | + 8.43 -7.78 | +231. -152. | +52.0 -34.2 | +52.0 -34.2 |
| May '58 - Aug. '61 | +22.5 -21.4 | + 4.67 -4.47 | + 4.91 -4.68 | +126. - 99.1 | +27.3 -21.5 | +27.4 -21.5 |
| Sept. '61 - Oct. '65 | +13.8 -13.3 | + 2.87 -2.79 | + 3.13 -3.04 | +131. - 62.0 | +16.5 -14.1 | +16.5 -14.2 |
| **Quartz Creek near Ohio City** | | | | | | |
| April '38 - July '41 | +13.7 -13.1 | + 4.65 -4.44 | + 4.84 -4.61 | + 83.0 - 64.4 | +28.8 -22.3 | +28.8 -22.4 |
| Aug. '41 - July '46 | +14.0 -13.0 | + 7.57 -7.03 | + 7.57 -7.04 | + 81.5 - 56.5 | +44.0 -30.6 | +44.0 -30.6 |
| July '46 - Oct. '50 | +12.1 -11.8 | + 4.95 -4.72 | + 4.97 -4.74 | + 65.2 - 51.9 | +26.1 -20.7 | +26.1 -20.7 |
| Oct. '60 - May '62 | +17.2 -16.3 | + 5.71 -5.40 | + 5.71 -5.41 | + 85.0 - 66.2 | +28.2 -22.0 | +28.2 -22.0 |
| June '62 - June '65 | + 8.63 - 8.39 | + 3.56 -3.44 | + 3.59 -3.47 | + 49.2 - 41.0 | +20.3 -16.9 | +20.4 -16.9 |
| **Curecanti Creek near Sapinero** | | | | | | |
| May '48 - Aug. '52 | +84.1 -76.5 | +10.3 -9.33 | +10.3 -9.37 | +500. -310. | +61.6 -38.1 | +61.6 -38.1 |
| April '53 - June '57 | +32.8 -30.9 | + 5.88 -5.55 | + 5.94 -5.60 | +178. -135. | +32.3 -24.4 | +32.3 -24.4 |
| May '58 - Jan. '61 | +27.5 -26.3 | + 4.78 -4.57 | + 4.80 -4.58 | +130. - 10.6 | +22.6 -18.4 | +22.6 -18.4 |
| Mar. '61 - Nov. '65 | +21.6 -20.8 | + 3.70 -3.57 | + 3.70 -3.57 | +124. -102. | +21.2 -17.5 | +21.2 -17.5 |
| **San Miguel River near Placerville** | | | | | | |
| Sept. '43 - Dec. '46 | + 7.17 - 6.92 | + 3.66 -3.53 | + 3.69 -3.56 | + 37.7 - 31.5 | +19.3 -16.2 | +19.3 -16.2 |
| Jan. '47 - April '48 | + 8.05 - 7.75 | + 3.99 -3.84 | + 4.00 -3.85 | + 29.2 - 25.4 | +14.5 -12.7 | +14.5 -12.7 |
| June '49 - May '52 | + 4.12 - 4.02 | + 2.18 -2.14 | + 2.22 -2.17 | + 19.5 - 17.6 | +10.4 - 9.42 | +10.4 - 9.42 |
| June '52 - Oct. '55 | +13.5 -12.7 | + 2.58 -2.51 | + 2.60 -2.54 | + 32.5 - 28.3 | +15.2 -13.2 | +15.2 -13.2 |
| Nov. '55 - April '57 | +14.9 -14.1 | + 6.33 -5.95 | + 6.34 -5.96 | + 47.7 - 39.7 | +20.2 -16.8 | +20.2 -16.8 |
| Nov. '58 - Mar. '62 | +14.3 -13.5 | + 5.65 -5.35 | + 5.65 -5.35 | +100. - 71.9 | +39.7 -28.4 | +39.7 -28.4 |
| April '62 - Nov. '64 | + 3.41 - 3.35 | + 1.82 -1.79 | + 1.83 -1.79 | + 20.5 - 18.4 | +10.9 - 9.83 | +10.9 - 9.83 |
| **Animas River at Howardsville** | | | | | | |
| April '40 - April '42 | +15.4 -14.6 | + 5.36 -5.08 | + 5.37 -5.10 | + 61.5 - 50.8 | +21.6 -17.7 | +21.6 -17.7 |
| June '42 - May '48 | + 5.42 - 5.33 | + 1.76 -1.73 | + 1.77 -1.74 | + 31.7 - 28.8 | +10.4 - 9.42 | +10.4 - 9.42 |
| June '48 - Sept. '52 | +12.2 -11.9 | + 3.59 -3.47 | + 3.62 -3.49 | + 73.5 - 60.5 | +21.7 -17.8 | +21.7 -17.8 |
| Oct. '52 - Mar. '56 | + 7.45 - 7.28 | + 2.41 -2.35 | + 2.41 -2.36 | + 37.4 - 33.4 | +12.1 -10.8 | +12.1 -10.8 |
| Mar. '56 - June '59 | +10.9 -10.5 | + 2.97 -2.88 | + 3.00 -2.92 | + 60.9 - 52.1 | +16.7 -14.3 | +16.7 -14.3 |
| June '59 - May '61 | +11.0 -10.7 | + 3.70 -3.57 | + 3.73 -3.60 | + 49.2 - 42.2 | +16.7 -14.3 | +16.7 -14.3 |
| June '61 - Nov. '64 | + 9.63 - 9.35 | + 2.55 -2.49 | + 2.58 -2.51 | + 54.5 - 47.5 | +14.6 -12.7 | +14.6 -12.7 |
| **Hermosa Creek near Hermosa** | | | | | | |
| May '40 - July '43 | +17.5 -16.2 | + 7.62 - 7.08 | + 7.81 -7.25 | +100. - 69.6 | +44.3 -30.7 | +44.4 -30.7 |
| Aug. '43 - Jan. '48 | + 6.75 - 6.56 | + 3.27 -3.16 | + 3.27 -3.16 | + 37.0 - 31.4 | +17.8 -15.1 | +17.8 -15.1 |
| Feb. '48 - May '52 | +19.5 -18.3 | + 6.68 -6.26 | + 6.82 -6.38 | +129. - 89.4 | +44.9 -31.0 | +45.0 -31.0 |
| June '52 - April '54 | +10.9 -10.4 | + 4.17 -4.00 | + 4.25 -4.08 | + 44.4 - 38.0 | +17.2 -14.6 | +17.2 -14.7 |
| April '54 - June '57 | +15.3 -14.9 | + 2.89 -2.81 | + 3.04 -2.95 | + 94.0 - 79.6 | +18.1 -15.3 | +18.1 -15.3 |
| Dec. '60 - Nov. '64 | + 7.24 - 7.10 | + 1.85 -1.82 | + 1.89 -1.86 | + 47.5 - 42.3 | +12.4 -11.1 | +12.5 -11.0 |
| **Animas River at Durango** | | | | | | |
| Nov. '36 - Sept. '46 | + 4.14 - 4.08 | + 1.42 -1.40 | + 1.49 -1.47 | + 39.0 - 34.3 | +13.7 -12.1 | +13.7 -12.1 |
| Sept. '46 - Oct. '53 | + 9.14 - 8.13 | + 3.52 -3.41 | + 3.66 -3.53 | + 85.1 - 62.4 | +36.6 -26.8 | +36.6 -26.8 |
| Oct. '53 - April '57 | + 7.31 - 7.05 | + 3.54 -3.42 | + 3.58 -3.46 | + 50.4 - 40.5 | +24.6 -19.7 | +24.6 -19.7 |
| April '57 - Oct. '64 | + 2.29 - 2.27 | + 1.01 - .998 | + 1.14 -1.13 | + 22.9 - 20.6 | +10.6 - 9.58 | +10.6 - 9.60 |
| **Average Values** | +13.7 -13.1 | + 3.74 -3.58 | + 3.82 -3.65 | + 76.9 - 59.6 | +21.3 -16.9 | +21.5 -16.9 |

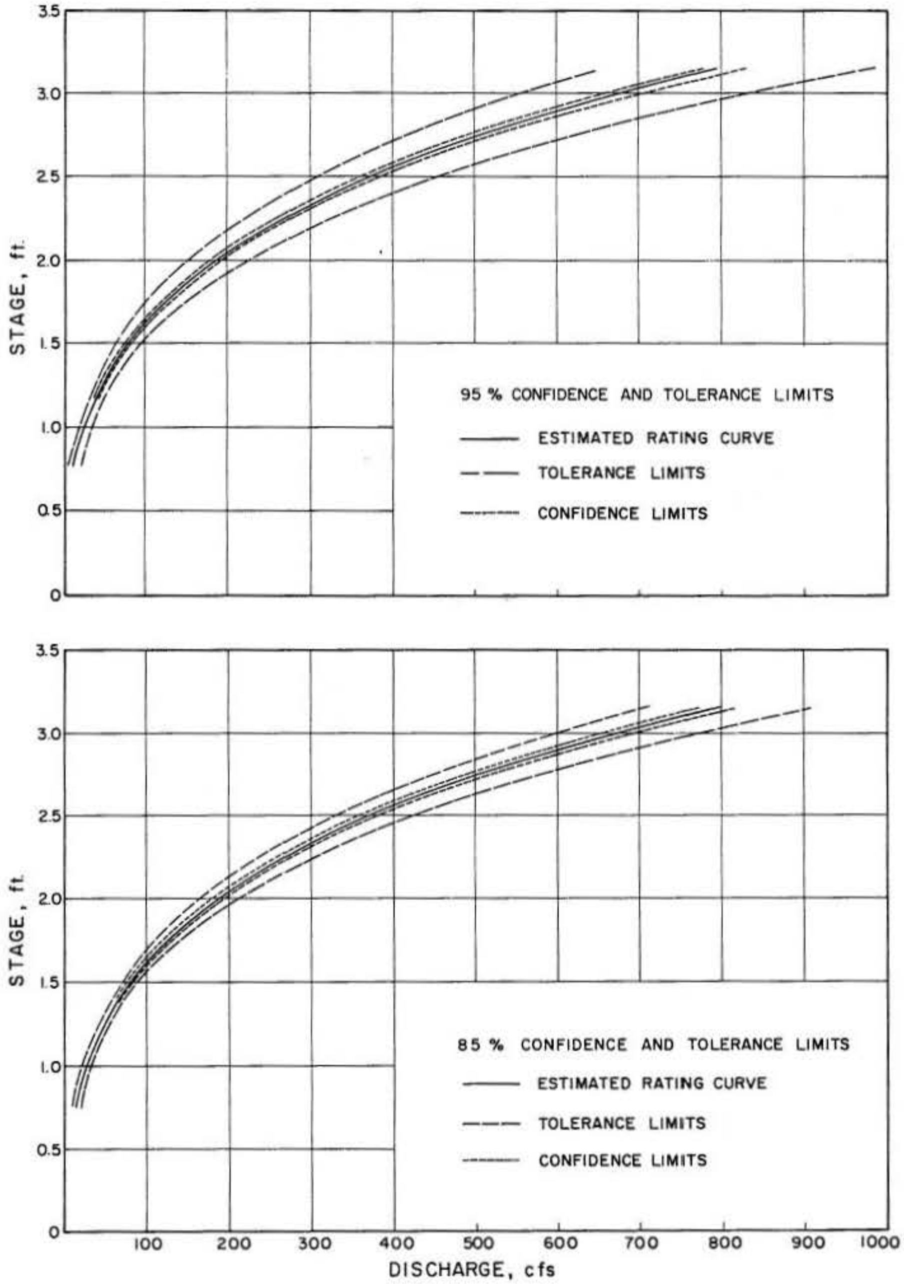


Fig. 11. Illustration of confidence and tolerance limits on a rating curve. (Animas River at Howardsville, June'48 to September '52).

variance of the error in the summation of the two estimates may be expressed as,

$$\mathrm{Var}\left[q_1 + q_2\right] = \mathrm{Var}\left[q_1\right] + \mathrm{Var}\left[q_2\right] + 2\sqrt{\mathrm{Var}\ q_1 \mathrm{Var}\ q_2}$$

$$= \sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2$$

$$= (\sigma_1 + \sigma_2)^2$$

where

$\sigma_1^2$ and $\sigma_2^2$ are the variances of $q_1$ and $q_2$, respectively.

That is,

$$\sigma_{(1+2)} = \sigma_1 + \sigma_2$$

where

$\sigma^2_{(1+2)}$ is the variance of the sum of $q_1$ and $q_2$.

When the variance of the errors, and of the discharge, is proportional to the square of the discharge, then the coefficient of variation of a summed total of estimated discharges is equal to that of an individual estimate. That is, when the errors are perfectly correlated, the percentage error bounds on a summed estimate are the same as those on an individual estimate.

If the errors in two estimates are uncorrelated, then

$$\mathrm{Var}\left[q_1 + q_2\right] = \mathrm{Var}\left[q_1\right] + \mathrm{Var}\left[q_2\right]$$

or

$$\sigma^2_{(1+2)} = \sigma^2_1 + \sigma^2_2 \ .$$

In words, for uncorrelated terms, the variances are additive rather than the standard deviations. Further, if the variance is proportional to the square of the discharge, the coefficient of variation of a summed total of estimated discharges may be expressed as,

$$C_v(\overset{*}{Q}_k) = \frac{K}{\sqrt{k}} \sqrt{1 + C_v^2(\overset{*}{Q}_i)}$$

where

$$\overset{*}{Q}_k = \sum_{i=1}^{k} \overset{*}{Q}_i \ ,$$

K reflects the coefficient of proportionality in $\sigma_i^2 = K^2 Q_i^2$,

$C_v(\overset{*}{Q}_i)$ is the coefficient of variation of the series of discharge values.

If $C_v(\overset{*}{Q}_i)$ is very small, then

$$C_v(\overset{*}{Q}_k) \approx \frac{K}{\sqrt{k}} \ .$$

Therefore, the percentage error bounds on a summed estimate are a function of the percentage error bounds on a single estimate, the number of estimates summed, and the variability of the estimates.

If the errors are uncorrelated, but the variance is constant, then the coefficient of variation of a summed total may be expressed as

$$C_v(\overset{*}{Q}_k) = \frac{\sigma\sqrt{k}}{\overset{*}{Q}_k}$$

where $\sigma$ is the constant standard deviation.

Now, if there is assumed to be a true stage-discharge relationship at a station for a sufficiently long period of time to establish a regression equation for the rating curve; and if this single rating curve is utilized to estimate a series of discharge values, then the errors of the individual estimates are perfectly correlated. As shown above, the error bounds for the summation of the series is identical to those for an individual estimate. This situation may be expected to exist for the estimation of summer flows at the mountain stream stations.

In the case of winter or low flow estimates, each daily estimate is often treated individually by use of the rating curve, flow measurements, temperature information, and inferences from nearby stations. As a result, the errors from day to day are virtually independent. Further, the absolute error bounds may be expected to be constant for the low flows. Therefore, the relative error bounds on a summed estimate are influenced by the number of estimates summed, their absolute values, and the constant absolute error bounds.

When the true stage-discharge relationship undergoes a process of random shifting about a mean relationship, the errors in individual discharge estimates may be expected to be highly correlated, although not perfectly. The correlation becomes a function of the nature of the shifting and the variability of the parameters of the relationship. When the variation of data about the rating curve is due almost entirely to measurement error, then the action of shifting can be neglected, and the errors of estimate considered to be perfectly correlated.

For the case when a shifting, rather than a single, rating curve is employed for estimating a series of values, the errors may be more or less correlated depending on the difference between the assumed and true shifting of the stage-discharge relationship. Such a curve-shifting procedure is often used in practice. Unfortunately, for this case, it is impossible to estimate either the correlation between errors in individual estimates or the error bounds on the summed estimate. They may be smaller or larger than the dependent situation would infer.

#### 4.3.4 Monthly discharge estimates

For stable mountain stream sections, it is assumed that for the summer months, there is a true stable stage-discharge relationship. Further, a single estimated rating curve is used to estimate all mean daily flows. As a result, the errors in the daily estimates are perfectly correlated. When a new measurement is added to the curve, and the entire curve is refitted, the correlation between the errors before and after the change will be slightly less. However, as the one point is only one in many, the errors are still virtually perfectly correlated. Therefore, the error bounds on a summer monthly discharge estimate are equivalent to those for a single mean daily estimate.

Winter or low flows which are treated as individuals, however, may be considered to be uncorrelated. Further, the absolute error bounds on these low flows have been assumed to be constant, and equal to $\pm \chi_s Q_D$, where $\chi_s$ represents the percentage error bounds on a single summer estimate, and $Q_D$ is the divisive discharge value. Therefore, the percentage error bounds on a winter monthly estimate may be represented by $\pm \chi_w$, where

$$\pm \chi_w = \frac{\pm \chi_s Q_D \sqrt{30}}{\sum_{i=1}^{30} \overset{*}{Q}_i}$$

It can be seen that although the relative error bounds on winter mean daily flow estimates may be considerably larger than those on summer daily estimates, a winter monthly estimate could be better than a summer estimate if the term $\sum_{i=1}^{30} \overset{*}{Q}_i / Q_D$ is greater than $\sqrt{30}$, and the errors are uncorrelated.

In practice, the summer monthly estimates might be expected to be somewhat better than the confidence limits would indicate, if less than perfect correlation existed between daily errors. On the other hand, winter monthly estimates may not be as good as the independent and constant variance assumptions would yield. However, since the actual correlation between daily errors cannot be determined, there is considerable value in studying the bounds and when they might be expected to occur.

4.3.5 Annual discharge estimates

From the previous discussion about daily and monthly discharge estimates, the assumptions are maintained that all summer flows are predicted from a single curve, but the winter flows are treated independently. Then the error bounds on a total summer discharge estimate are equal to those on a single summer daily estimate. The error bounds on an annual estimate are perhaps most clearly illustrated by a simple example.

Consider summer flows to (i) occur six months of the year (i. e., 50 percent of the time), (ii) exhibit errors in daily and monthly flow estimates that are perfectly correlated, and (iii) have error bounds for a single discharge estimate of $\pm\chi_s$ percent. For the winter flows, consider them to (i) occur the other six months of the year, (ii) be characterized by errors in daily and monthly estimates that are uncorrelated, and (iii) have absolute error bounds for a single flow estimate of $\pm\chi_s Q_D$, where $Q_D$ is the divisive discharge value. Then the following statements can be made:

The relative error bounds for a total summer flow estimate are $\pm\chi_s$.

The relative error bounds for a total winter flow estimate are

$$\pm \frac{\chi_s Q_D \sqrt{180}}{\sum_{i=1}^{180} \overset{*}{Q}_i}$$

The absolute error bounds for the total annual flow are

$$\pm \chi_s \sqrt{\overset{*}{Q}{}_s^2 + 180\, Q_D^2} \quad ,$$

where $\overset{*}{Q}_s$ is the estimate of total summer flow. If $\overset{*}{Q}_a$ represents the estimated annual flow; and $\overset{*}{Q}_w$ represents the estimated winter flow; and $R_{s/a}$, the ratio of $\overset{*}{Q}_s$ to $\overset{*}{Q}_a$, is of the order of 0.9, then the following statement can be made:

The relative error bounds for a total annual flow estimate are

$$\pm \chi_s \sqrt{0.81 + \frac{1.8\, Q_D^2}{\overset{*}{Q}{}_w^2}}$$

which is approximately $\pm 0.9\,\chi_s$.

It is evident that the errors in winter flow estimates play a negligible role in the error of estimate of total annual flow.

On the topic of annual discharge estimates, it is interesting to consider another possibility. If the annual estimate were based purely on the summer flow records and a predicted value of the ratio between annual and summer flows, $\overset{*}{R}_{a/s}$, then the equation could be written,

$$\overset{*}{Q}_a = \overset{*}{R}_{a/s}\, \overset{*}{Q}_s$$

Further,

$$\mathrm{Var}\left[\overset{*}{Q}_a\right] = \mathrm{Var}\left[R_{a/s}\, \overset{*}{Q}_s\right]$$

$$\simeq \overset{*}{R}{}^2_{a/s}\ \mathrm{Var}\left[\overset{*}{Q}_s\right] + \overset{*}{Q}{}_s^2\ \mathrm{Var}\left[\overset{*}{R}_{a/s}\right]$$

where $\overset{*}{R}_{a/s}$ and $\overset{*}{Q}_s$ have been assumed to be independent, and a product term involving the two variances has been considerd to be negligible in relation to the others. Then the relative error bounds on an annual estimate would be,

$$\pm \sqrt{\chi_s^2 + C_v^2\,(R_{a/s})}$$

It can be seen that if the ratio, $R_{a/s}$, could be predicted very accurately, either by use of a mean value or a correlation with similar ratios at downstream stations, then the relative error bounds would approach $\pm\chi_s$.

Although these latter bounds will be larger than those for the estimate on a full year of record, the increase is not very large for mountain streams where such a high percentage of flow occurs in the summer months. In fact, if only annual flow estimates are of importance at a station, the question arises whether costly winter measurements, maintenance of records, and analysis, are justified for such a small increase in accuracy of the estimate. In such cases, it might be more important to improve the estimation of summer records and forget about regular field work in the winter. At least the methodology given above affords an approach to a study of the possible increase in the accuracy of estimating annual flows by maintaining winter records versus the cost of such maintenance.

4.3.6 Flow estimates in general

The discussion presented on the accuracy of discharge estimates has been rather general in nature and based largely on the correlation between errors in single estimates. Whether or not the assumptions made are strictly applicable to a particular case, the approach is considered to give valuable estimates of the magnitude of the error bounds which could be expected. The one point that becomes very clear is that the use of a single estimated rating curve to make daily flow estimates maintains all estimates at a similar degree of accuracy. It infers that an annual or monthly estimate cannot be expected to be any better than a single estimate. If the curve has very narrow confidence limits, this result is not serious. However, if the limits tend to be wide, it is important to make many more field measurements to better estimate the stage-discharge relationship and to reduce the correlation between errors.

## CHAPTER V

## CONCLUSIONS AND RECOMMENDATIONS

### 5.1 Conclusions

From the results of this study of the accuracy of stream discharge determinations, a number of conclusions can be drawn. They are grouped below according to the type of discharge determinations for which they are applicable. It is important that each conclusion is considered in its particular grouping.

In general:

(i) The error in a discharge determination lends itself to a detailed analysis and classification of the component errors, their nature, and a consideration of their relative importance. Such an analysis and classification is an aid to the understanding of the problem of errors, and serves as an excellent basis for both theoretical and experimental investigations.

The development of the hypothetical mathematical error model for a single discharge measurement on a mountain stream reveals that:

(ii) Oblique currents, and their variability in time, can be a major contributor to bias in a point-velocity measurement.

An analysis of the effect reveals that there can be a bias in the measurement even when the mean oblique angle is zero. The bias increases as the angle, and its variance, increases. In general, cup-type meters may exhibit a positive bias due to obliquity and propeller-type meters exhibit a negative one. Screw-type meters must be studied individually.

(iii) The method of sampling the vertical distribution of velocity can also contribute to bias in a measurement. The common one- and two-point sampling techniques give biased estimates of the mean velocity for parabolic, elliptic, logarithmic, and hyperbolic velocity distributions. Further, a study of robustness of these techniques reveals that failure to place the meter at the prescribed sampling points may serve to increase or decrease the bias of the sampling technique, depending on the particular velocity distribution.

(iv) The variance in a point velocity measurement, due to sampling velocity in time, is proportional to the square of the expected velocity, varies parabolically with relative depth, and is inversely proportional to the square root of the measurement time.

(v) For other than low flow measurements, the standard deviation of the error in a single discharge measurement, and of the measured discharge itself, is proportional to the discharge.

(vi) Of the component errors considered, those arising from sampling both the vertical velocity distribution and the depth can potentially have the greatest effect on the variance of the measured discharge. Since the velocity sampling technique can also contribute to bias, as concluded in (iii) above, this phase of the measurement methodology requires critical appraisal.

(vii) Subdivision of a stream cross section into vertical sections containing approximately equal discharges is important for minimizing the variance of the measurement. The variability of the n partial discharge values is a significant term in the variance model for a single discharge measurement.

(viii) The variation between the velocities in a vertical section can be important in selecting a velocity sampling scheme for minimizing the error in the estimate of mean velocity in a vertical.

(ix) The best measurement technique for a particular stream can be determined only after the relative role of each term in the error model has been estimated from a detailed study of the stream site. For accurate research measurements, such studies may be justified and should be undertaken. For common stream-gaging practice, mountain streams exhibiting similar flow and geometric characteristics could be sampled in one manner; and groups of gaging sites varying widely in properties could be sampled in different ways.

The study of mountain stream-discharge relationships, including an analysis of data from nine representative gaging stations in the mountains of Colorado, indicates that:

(x) The stage-discharge relationship at a station may be expected to remain stable for a relatively long period of time. Major shifts of a relationship usually occur only after the passage of extremely high discharges.

(xi) Stage and discharge appear to be functionally related as

$$Q = b_1 (H - Ho)^{b_2} ,$$

where Q is discharge; H is stage; Ho is the stage at zero discharge; $b_1$, $b_2$ are parameters. Fitted rating curves of this form account for upwards of 99 percent of the variability in the sample data.

(xii) The least squares fitting procedure, applied to the logarithms of the stage and discharge values, is a very useful and convenient approach for estimating the rating curve for a station.

(xiii) The concept of a divisive discharge value, introduced to divide the rating curve into two portions at a point where the error changes in nature from a constant relative value to a constant absolute value, appears to have much merit. For mountain stations, the divisive discharge value also separates the mean daily flow estimates into two groups, each occurring about 50 percent of the time. The group including the summer flows accounts for approximately 90 percent of the total annual flow.

(xiv) The magnitude of measurement error in a single discharge measurement and the degree of minor random shifting of a rating curve are reflected in the coefficient of variation from regression for the sample data. The average coefficients for the upper portion of the curve are +9.37 and -8.40 percent; and the average coefficients are +17.3 and -10.9 percent on the lower portion.

(xv) Summer mean daily discharge values can be expected to be estimated within +2.40 and -2.33 percent error bounds, at the 80 percent confidence level; and within +3.82 and -3.65 percent, at the 95 percent level. Mean daily estimates in the winter months can be said to be within +8.60 and -8.34 percent error bounds, at the 80 percent confidence level; and +13.7 and -13.1 percent, at a 95 percent level.

(xvi) Future summer discharge measurements may be expected to lie within +12.9 and -11.2 percent of the rating curve values, 80 percent of the time; and within +21.5 and -16.9 percent bounds, 95 percent of the time. Single discharge measurements made in the winter months can be expected to fall within +47.6 and -39.6 percent of the curve, with 80 percent confidence; or within +76.9 and -59.6 percent, with 95 percent confidence.

(xvii) If a single rating curve is employed to estimate all mean daily discharges during the summer, the percentage error bounds on the total summer flow, on each monthly flow, and on each mean daily value are equivalent.

(xviii) During the winter months, when daily flows are estimated independently, the percentage error bounds on a monthly flow estimate are a function of the percentage error bounds on a summer mean daily flow estimate, and the relative magnitude of the monthly estimate and the divisive discharge value.

(xix) The percentage error bounds on an annual discharge estimate are negligibly affected by the errors in winter flow estimates. They are approximately equal to the product of the ratio of summer to total annual flow and the percentage error bounds on a mean daily discharge estimate in the summer months.

## 5.2 Recommendations for Future Research

During this investigation, it has become evident that there are a number of areas in which continued research would be worthwhile. Topics are suggested below under the headings: determination of component errors, improvement of sampling techniques, global evaluation of errors, and other measurement techniques.

### 5.2.2 Determination of component errors

Because insufficient quantitative information is presently available for the determination of the various component error functions in the hypothetical error model, it remains a qualitative one. Further field research and analysis would allow the development of the components, and render the model a very useful tool for the estimation of error in a single measurement. The most important component errors which require investigation are those arising from the effects of turbulence in the flow, and improper sampling of the velocity distributuion. Suggestions are given below.

(a) Turbulence and its effect

The first requirement is the development of methodology to measure turbulence in streamflow. After such methods are developed, the effect of turbulence on current-meter registration may be properly evaluated. Also, the level of turbulence in mountain streams may be determined in relation to measurement positions and flow characteristics.

(b) Velocity distribution

Although considerable research has been undertaken in this area, sufficient detail is not available for developing improved measurement methodology. Field experiments involving continuous velocity recording for relatively long periods of time at several points in both the vertical and horizontal directions at stream sites are needed. Such investigations would yield excellent information regarding the distribution of velocity, and both its variability in time and area. Knowledge of these aspects would be invaluable for evaluating measurement techniques, and for developing new ones.

### 5.2.2 Improvement of sampling techniques

(a) Velocity distributuion

In conjunction with the field studies suggested above, a theoretical study is required to analyze the data with regard to optimizing sampling techniques. If a minimum variance,unbiased estimate is to be made of the mean velocity in a stream section, further consideration should be given to sampling of the distribution. A fresh look at discharge measuring from the sampling point of view would be most worthwhile.

(b) Discharge distribution

Useful research should be conducted regarding the distribution of discharge in various shaped cross sections, and with respect to the best ways of dividing the section for measurement purposes. Further, a study of the error induced by a changing stage could be undertaken by an analysis of the distribution of both the discharge and the percentage changes in partial discharge per unit change of stage.

### 5.2.3 Global evaluation of errors

In the introduction, the experimental approach was suggested as another method for evaluating the total or global error in discharge measurements. The detailed analysis presented in this investigation would serve well as a basis for extensive field studies, involing the comparison of various gaging techniques employed by different field teams at controlled gaging sites. In the light of the analytical approach, the experimental investigation could clarify and extend evaluation techniques. As considerable equipment and labor would be involved in such a study, it is suggested that it would be best handled by a large agency that is equipped and has had experience in the field.

### 5.2.4 Other measurement techniques

Other techniques, such as those involving dye dilution and the use of equal velocity contours, should be studied with regard to accuracy and cost. If, in comparison with present methodology, other measurement approaches prove to be considerably more accurate, they should be adopted for use, particularly where improved accuracy is demanded. Even if the cost of new techniques is considerable, it may well be justified by the increased accuracy for such studies as the evaluation of weather modification attainments.

## BIBLIOGRAPHY


1. Addison, H., Hydraulic Measurements, New York: John Wiley and Sons, Inc., 1949.

2. Anderson, I. E., and Reck, C. W., "Errors in Discharge Measurements," USGS Water Res. Bull., August 10, 1958.

3. Anderson, I. E., "Errors in Streamflow Measurement," USGS Professional Paper 424-C, 1961.

4. Anderson, I. E., Reck, C. W., and Hazelton, K., "Errors in Discharge Measurements," USGS Report on Research Activities, 1960-61.

5. Anonymous, "The Price Current Meter," Engineering News, Vol. 29: 196-197, 1893.

6. Barrows, H. K., "Work of the Hydrographic Branch of the USGS in New England and a Discussion of the Methods Used for Estimating Streamflow," Jour. Assoc. Eng. Soc., Vol. 35: 1-33, 1905.

7. Barrows, H. K., and Horton, R. E., "Determination of Streamflow During the Frozen Season," USGS Water Supply Paper 187, 1907.

8. Beckman, E. W., and Furness, L. W., "Flow Characteristics of Elkhorn River near Waterloo, Nebraska," USGS Water Supply Paper 1498-B, 1962.

9. Bitterli, S., "Making Water Measurements with Current Meters," Power, Vol, 75: 102-104, 1932.

10. Bonnafoux, R., "Influence de l'Inclinaison sur le Fonctionnement d'un Moulinet," International Current Meter Group, Report No. 21, Glasgow, 1966.

11. Boyer, M. C., "The Fantail Rating Curve and its Application," USGS Water Res. Bull., February 10, 1936.

12. Boyer, M. C., "Analysis of Methods of Adjusting Stage and Discharge," USGS Water Res. Bull., November 10, 1937.

13. Boyer, M. C., "Streamflow Measurement," Section 15 of Handbook of Applied Hydrology, Editor-in-Chief, V. T. Chow, New York: McGraw-Hill Bk. Co., Inc., 1964.

14. Brown, E. H., and Nagler, F., "Preliminary Report of Current Meter Investigations," Proc. Engrs. Soc. of West Penn., Vol. 30: 415-424, 1914-15.

15. Buchanan, T. J., Stream Gaging Manual, A Manual Being Prepared by the USGS for Field Use. 1966.

16. Carter, R. W., and Anderson, I. E., "Accuracy of Current Meter Measurements," Proc. ASCE, Vol. 89, HY4: 105-115, 1963.

17. Carter, R. W., and Davidian, J., "Discharge Ratings at Gaging Stations," USGS Surface Water Techniques, Hydraulic Measurements and Computations, Book 1, Chapter 12, 1965.

18. Chaix, B., "Essous en Campagne et en Laboratoire Relatifs a l'Influence de la Turbulence sur Differents Moulinets Hydrometriques," U. S. Army Eng. Waterways Expt. Sta. Translation No. 66-15, Corps of Engineers, Vicksburg, Mississippi, April 1966.

19. Chamberlain, A. R., and Ham, C. B., "Behavior of Experimental Current Meters in Still Water and Turbulent Flows," USGS Report, Prepared for Surface Water Branch, Water Resources Division, by Colorado State University, Fort Collins, Colorado, February 1958.

20. Colby, B. R., "Studies of Current Meters," USGS Water Res. Bull., April 10, 1936.

21. Colby, B. R., Discussion of "Accuracy of Current Meter Measurements," by R. W. Carter and I. E. Anderson, Proc. ASCE, Vol. 90, HY4: 349-351, 1964.

22. Corbett, D. M et al. "Stream-Gaging Procedure," USGS Water Supply Paper 888, 2nd Edition, 1962.

23. Craig, J. I., "On the Accuracy of Stream Measurement," Cairo Scientific Jour., Vol. 6: 279-284, 1912.

24. Dawdy, D. R., "Depth-Discharge Relations of Alluvial Streams-Discontinuous Rating Curves," USGS Water Supply Paper 1498-C, 1961.

25. Dement'ev V. V., "Investigations of Pulsations of Velocities of Flow of Mountain Streams and of its Effect on the Accuracy of Discharge Measurements," Soviet Hydrology: Selected Papers, No. 6, 1962.

26. Eisenhuth, H. and Odell, J., USGS Water Res. Bull., February 10, 1937.

27. Eisenlohr, W. S., "The Adjustment for the Amount the Stage Changes," USGS Water Res. Bull., May 10, 1937.

28. Eisenlohr, W. S., "Effect of Water Temperature on Flow of a Natural Stream," Trans. AGU, Vol. 29: 240-242, 1948.

29. Fischer, M., "Versuche uber den Einfluss der Turbulenten Stromung auf Hydrometrische Flugel durch Nachahmung der Turbulenz im Schlepptank," International Current Meter Group, Report No. 19, Glasgow, 1966.

30. Fortier, S., and Hoff, E. J., "Defects in Current Meters and a New Design," Eng. News Record, Vol. 85: 923-924, 1920.

31. Fteley, A., and Stearns, F. P., "Current Meter," Trans. ASCE, Vol. 12: 117-118, 1883.

32. Godfrey, R. G., "Evaluation of Current Meters," USGS Research Seminar, 1958.

33. Groat, B. F., "Characteristics of Cup and Screw Current Meters; Performance of These Meters in Tail-Races and Large Mountain Streams; Statistical Synthesis of Discharge Curves," Trans. ASCE, Vol. 76: 819-840, 1913.

34. Groat, B. F., "Pitot Tube Formulas - Facts and Fallacies," Proc. Engrs. Soc. of West Penn., Vol. 30: 323-383, 1914-15.

35. Groat, B. F., "Chemi-Hydrometry and its Application to the Precise Testing of Hydro-Electric Generators," Trans. ASCE, Vol. 80: 1231-1271, 1916.

36. Groat, B. F., "In Defense of Current Meters," Eng. News Record, Vol. 98: 370, 1927.

37. Grover, N. C., and Hoyt, J. C., "Accuracy of Streamflow Data," USGS Water Supply Paper 400-D, 1916.

38. Grover, N. C., "The Effect of Channel on Stream Flow," Jour. Boston Soc. Civ. Engrs., Vol. 3, No. 9: 465-476, 1916.

39. Grover, N. C., and Harrington, A. W., Stream Flow. New York: John Wiley and Sons, Inc. 1943.

40. Gutmann, I., "Glorifying the Current Meter," Eng. News Record, Vol. 97, Pt. 2: 840, 1926.

41. Hall, M. R., Hall, W. E., and Pierce, C. H., "A Method of Determining the Daily Discharge of Rivers of Variable Slope," USGS Water Supply Paper 345, 1914.

42. Harding, S. T., "Experiments in the Use of Current Meters in Irrigation Canals," Jour. of Agr. Res., Vol. 5: 217-232, 1915.

43. Henry, D. G., "On the Flow of Water in Rivers and Canals," Jour. Franklin Inst., Vol. 92: 167-389, 1871.

44. Hogan, M. A., Current Meters for Use in River Gauging. London: H. M. S. Printing Office, 1922.

45. Hogan, M. A., River Gauging. London: H. M. S. Printing Office, 1925.

46. Horton, T., "Flow in the Sewers of the North Metropolitan Sewerage System of Massachusetts," Trans. ASCE, Vol. 46: 78-92, 1901.

47. Hoyt, J. C., "A Universal Current Meter," Eng. News, Vol. 57: 263, 1907.

48. Hoyt, J. C., "Recent Changes of Methods and Equipment in the Water Resources Work of the USGS," Eng. News, Vol. 60: 15-16, 1908.

49. Hoyt, J. C., "The Use and Care of the Current Meter as Practiced by the USGS," Trans. ASCE, Vol. 66: 70-105, 1910.

50. Hoyt, J. C., "The Effects of Ice on Stream Flow," USGS Water Supply Paper 337, 1913.

51. Hoyt, J. C. and Grover, N. C., River Discharge. New York: John Wiley and Sons, Inc., 1914.

52. Hoyt, K. K., "Defense of the Price Current Meter," Eng. News Record, Vol. 97, Pt. 2: 1010, 1926.

53. Ionides, M. G., "River Stage-Discharge Curves," Engineering, Vol. 138: 396, 1934.

54. Jones, B. E., "A Method of Correcting River Discharge for a Changing Stage," USGS Water Supply Paper 375, 1915.

55. Kalinske, A. A., "Application of Statistical Theory to Measurements of Velocity and Suspended Sediment in Rivers," Trans. AGU, Vol. 26: 261-264, 1945.

56. Kerr, S. L., "Research Investigation of Current-Meter Behavior in Flowing Water," Trans. ASME, Vol. 57: 295-301, 1935.

57. Kinnison, H. B., "Stream-Flow Data - Its Collection and Use," Jour. Boston Soc. Civ. Engrs., Vol. 17: 171-248, 1930.

58. Kolupaila, S., "Recent Developments in Current-Meter Design," Trans AGU, Vol. 30: 916-918, 1949.

59. Kolupaila, S., "Use of Current Meters in Turbulent and Divergent Channels," Inst. Assoc. Sci. Hydrology, General Assembly of Toronto, Vol. 1, 1957.

60. Kolupaila, S., "Early History of Hydrometry in the U. S., "Proc. ASCE, Vol. 86, HY1: 1-51, 1960.

61. Kolupaila, S., Bibliography of Hydrometry. Notre Dame: University of Notre Dame Press, 1961.

62. Kolupaila, S., Discussion of "Accuracy of Current Meter Measurements," by R. W. Carter and I. E. Anderson, Proc. ASCE, Vol. 90, HY1: 352-355, 1964.

63. Learmonth, J. R., "The Collection and Analysis of Rainfall and Stream Level Data," Univ. of New South Wales, Water Res. Lab. Report No. 70, 1964.

64. Liddell, W. A., Stream Gaging. New York: McGraw-Hill Bk. Co., In., 1927.

65. Linford, A., Flow Measurement and Meters. London: E. and F. N. Spon, Ltd., 1949.

66. Linsley, R. K., Kohler, M. A., and Paulhus, J. L. H., Hydrology for Engineers. New York: McGraw-Hill Bk. Co., Inc., 1927.

67. Mackenzie, A., Report on Current Meter Observations in the Mississippi River Near Burlington, Iowa, Oct. 1879. Washington: U. S. Govt. Printing Office, 1884.

68. Markovic, R. M., "Evaluation of Weather Modification Attainments by Analysis of Streamflow Responses," Colorado State University Ph. D. Dissertation, Fort Collins, Colorado, May 1966.

69. Matalas, N. C., and Conover, W. J., "Derivation of the Velocity Profile from a Statistical Model of Turbulence," Water Res. Res., Vol. 1, No. 2: 235-261, 1965.

70. Matthes, G. H., "Price Current Meter: Serviceability vs Scientific Design," Eng. News Record, Vol. 98, Pt. 1: 126, 1927.

71. Mesnier, G. N., "Stream-Gaging Program," USGS Water Res. Bull., August 10, 1958

72. Moody, L. F., "Measurement of the Velocity of Flowing Water," Proc. Engrs. Soc. of West. Penn., Vol. 30: 279-323, 1914-15.

73. Morel, H., "Sur les Courbes de Reponse des Moulinets, aux Faibles Bitesses et en Canal d'Etalonnage," International Current Meter Group, Report No. 18, Glasgow, 1966.

74. Murphy, E. C., "Accuracy of Stream Measurements," USGS Water Supply and Irrigation Paper 64, 1902.

75. Murphy, E. C., "Current Meter and Weir Discharge Comparisons," Trans ASCE, Vol. 47: 370-378, 1902.

76. Murphy, E. C., Hoyt, J. C., and Hollister, G. B., "Hydrographic Manual of the USGS," USGS Water Supply and Irrigation Paper 94, 1904.

77. Murphy, E. C., "Accuracy of Stream Measurements," USGS Water Supply and Irrigation Paper 95, 1904.

78. Nagler, F. A., "Use of Current Meters for Precise Measurement of Flow," Trans. ASME, Vol. 57: 59-67, 1935.

79. Newell, F. H., "Operations at River Stations, 1900," USGS Water Supply and Irrigation Paper 47, 1901.

80. Newell, F. H., "Methods of Stream Measurement," USGS Water Supply and Irrigation Papers 56, 1901.

81. O'Brien, M. P., and Folsom, R. G., "Notes on the Design of Current Meters," Trans. AGU, Vol. 29: 243-250, 1948.

82. Oltman, R. E., "Performance of the Price Current Meter at Natural Stream Sites," USGS Preliminary Report, No. 24, 1954.

83. Orem, H. M., "Effect of Fluctuations in Velocity," USGS Water Res. Bull., August 10, 1936.

84. Ott, L. A., "Observations on the Use of Current Meters for Precise Flow Measurement," Trans. ASME, Vol. 57: 227-228, 1935.

85. Pardoe, W. S., "Methods of Stream Gaging," Engineering News, Vol. 75, Pt. 2: 889, 1916.

86. Pierce, C. H., "Conditions Requiring the Use of Automatic Gages in Obtaining Records of Stream Flow," USGS Water Supply Paper 375, 1915.

87. Pierce, C. H., "Surface Waters of Vermont," USGS Water Supply Paper 424, 1917.

88. Pierce, C. H., "Investigations of Methods and Equipment Used in Stream Gaging," USGS Water Supply Paper 868-A, 1941.

89. Pierce, C. H., "Methods of Stream Gaging," Hydrology. Editor-in Chief, V. T. Chow, New York: Dover Publ., 1942.

90. Pierce, C. H., "Current Meters, Their Performance, Accuracy and Reliability," USGS Water Res. Bull., May 10, 1946.

91. Price, W. G., "A New Current Meter and a New Method of Rating Current Meters," Engineering News, Vol. 33: 27, 1895.

92. Prochazka, J., "Notes on the Questions of Accuracy of Discharge Measurements With a Current Meter," Int. Assoc. Sci. Hydrology, General Assembly of Helsinki, Publ. 53, 1960.

93. Robson, A. D., "The Effect of Water Temperature Upon the Calibration of a Current Meter," Trans. AGU, Vol. 35: 647-648, 1954

94. Rohwer, C., "The Rating and Use of Current Meters," Colorado Agr. Coll., Tech. Bull. 3, 1933.

95. Rouse, H., "Research Institute for Hydraulic Engineering and Water Power," Trans. ASME, Vol. 54: HYD-54-3, 1932.

96. Rouse, H., Engineering Hydraulics. New York: John Wiley and Sons, Inc., 1949.

97. Rumpf, C. P., "An Investigation of the Use and Rating of the Current Meter," Engineering News, Vol. 71, Pt. 2: 1083-1084, 1914.

98. Schoof, R. R., "Stream Gaging Precision Related to Meter Suspension and Procedural Error," Oklahoma State Univ. M. S. Thesis, 1965.

99. Schubauer, G. B., and Mason, M. A., "Performance Characteristics of a Water Current Meter and in Air," Jour. Nat. Bur. Stds., Vol. 18: 351-360, 1937.

100. Scobey, F. C., "Behavior of Cup Current Meters Under Conditions Not Covered by Standard Ratings," Jour. Agr. Res., Vol. 11: 77-83, 1914.

101. Simons, D. B., and Richardson, E. V., "The Effect of Bed Roughness on Depth-Discharge Relations in Alluvial Channels," USGS Water Supply Paper 1498-A, 1961.

102. Stearns, F. P., "On the Current Meter," Trans. ASCE, Vol. 12: 301-338, 1883.

103. Stevens, J. C., "Comparison of Formulas for Computation of Stream Discharge," Engineering News, Vol. 59: 682-684, 1908.

104. Stevens, J. C. "Hydrometry as an Aid to the Successful Operation of an Irrigation System," Trans. ASCE, Vol. 71: 323, 1911.

105. Stevens, J. C., "The Accuracy of Water-Level Recorders and Indicators of the Float Type," Trans. ASCE, Vol. 83: 894-903, 1919-20.

106. Stevens, J. C., Hydrographic Data Book. Sixth Edition, Portland: Leopold and Stevens Instruments, Inc.

107. Tiffany, J. B., "Turbulence in the Mississippi River," Midwestern Conference on Fluid Dynamics, Univ. of Illinois, May 13, 1950.

108. Townsend, F. W., and Blust, F. A., "A Comparison of Stream Velocity Meters," Proc. ASCE, Vol. 86, HY4: 11-19, 1960.

109. Trestman, A. G., New Aspects of River Runoff Calculations. Jerusalem: Israel Program for Scientific Translations, 1964.

110. Troskolanski, A. T., Hydrometry. New York: Pergammon Press, Inc., 1960.

111. U. S. Bureau of Reclamation, Water Measurement Manual. Denver: U. S. Govt. Printing Office, 1953.

112. U. S. Geological Survey, "Ice Effects," USGS Water Res. Bull., Dec. 10, 1936.

113. U. S. Geological Survey, "Drawdown in the Stilling Well," USGS Water Res. Bull., August 10, 1937.

114. Unwin, W. C., "Current-Meter Observations in the Thames," Proc. Inst. Civ. Engrs., Vol. 71: 338-349, 1882-83.

115. Vahs, W. A., "The Influence of Contact Mechanism Torque on the Calibration Equation and Starting Velocity of Current Meters," International Current Meter Group, Report No. 17, Glasgow, 1966.

116. Vanoni, V. A., "Velocity Distributuion in Open Channels," Civil Engineering, Vol. 11, No. 6: 356-357, 1941.

117. Williams, G. S., Hubbell, C. W., and Fenkell, G. H., "Experiments on Flow on Water in Pipes," Trans. ASCE, Vol. 47: 337-341, 1902.

118. Williams, G. S., "Measurements of Water," Jour. West. Soc. Engrs., Vol. 17: 22-34, 1912.

119. Wisler, C. O., and Brater, E. F., Hydrology. New York: John Wiley and Sons, Inc., 1959.

120. Wood, G. K., "Accuracy of Stream-Flow Records," Trans. AGU, Vol. 25: 985-989, 1944.

121. Yarnell, D. L., and Nagler, F. A., "Effects of Turbulence on the Registration of Current Meters," Trans. ASCE, Vol. 95: 766-795, 1931.

122. Young, K. B., "A Comparative Study of Mean-Section and Mid-Section Methods for Computation of Discharge Measurements," USGS Water Res. Div., February 1950.

Key Words: Hydrology; Errors; Discharge determinations; Mountain streams; Classification; Mathematical model; Rating curve; Divisive discharge; Confidence limits; Correlation.

Abstract: The objective of this study was to analyze the errors that may be incurred in discharge determinations made on mountain streams. The possible sources of error were carefully considered and a classification of these sources, including notations on the nature of the resulting errors, was prepared. A mathematical error model for a single discharge measurement has been hypothesized, and methodology presented for the evaluation of daily, monthly, and annual discharge estimates.

An exhaustive literature review was undertaken regarding both the qualitative and quantitative aspects of the topic. This material was sorted in an attempt to divide the total error in a discharge determination into various component errors. Each component was analyzed separately, and with respect to the others, in order to yield information about the random or systematic nature of the error, and about possible functional relationships which might be involved. This information has been summarized in the form of a classification of errors.

Upon the foundation developed in the first phase, a hypothetical error model was developed for a single discharge measurement. No attempt has been made to render this model a practical working tool. Rather, it was essentially a qualitative undertaking to reveal the manner of combination of the various component errors, and to clarify the nature of some of the errors. The expected value and variance of the model were studied in order that inferences could be made regarding the significant error terms.

Finally, consideration was given to the errors arising from use of an estimated rating curve. A mathematical representation was given to the stage-discharge relationship and found to account for virtually all the variability in sample data for nine mountain stream-gaging stations in Colorado. The concept of a divisive discharge value was introduced to separate the rating curve into two portions: one along which the relative error was virtually constant; and the other along which the absolute error remained constant. Both confidence and tolerance limits were established for the estimated curves, and used for inferences regarding the error bounds on daily discharge estimates and future discharge measurements. After consideration was given to the correlation between errors in single discharge estimates, conclusions were drawn regarding the magnitude of the error bounds on monthly and annual discharge estimates.

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Upon the foundation developed in the first phase, a hypothetical error model was developed for a single discharge measurement. No attempt has been made to render this model a practical working tool. Rather, it was essentially a qualitative undertaking to reveal the manner of combination of the various component errors, and to clarify the nature of some of the errors. The expected value and variance of the model were studied in order that inferences could be made regarding the significant error terms.

Finally, consideration was given to the errors arising from use of an estimated rating curve. A mathematical representation was given to the stage-discharge relationship and found to account for virtually all the variability in sample data for nine mountain stream-gaging stations in Colorado. The concept of a divisive discharge value was introduced to separate the rating curve into two portions: one along which the relative error was virtually constant; and the other along which the absolute error remained constant. Both confidence and tolerance limits were established for the estimated curves, and used for inferences regarding the error bounds on daily discharge estimates and future discharge measurements. After consideration was given to the correlation between errors in single discharge estimates, conclusions were drawn regarding the magnitude of the error bounds on monthly and annual discharge estimates.

Key Words: Hydrology; Errors; Discharge determinations; Mountain streams; Classification; Mathematical model; Rating curve; Divisive discharge; Confidence limits; Correlation.

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