SIMILITUDE FOR NON-STEADY DRAINAGE OF PARTIALLY SATURATED SOILS

by

G. L. Corey, A. T. Corey and R. H. Brooks

August 1965

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
Fort Collins, Colorado

9

SIMILITUDE FOR NON-STEADY DRAINAGE
OF PARTIALLY SATURATED SOILS

by

G. L. Corey, A. T. Corey and R. H. Brooks

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO

August 1965 No. 9

ABSTRACT

The similarity condition was applied to non-steady flow of liquids in partially saturated porous media. Inspectional analysis was applied to the differential equation describing two-phase flow where the gas phase is everywhere at atmospheric pressure. Criteria for similitude were developed to facilitate studying complicated field processes with small laboratory models.

Standard scaling units were so chosen that the capillary fringe depth can be precisely accounted for in a model and the time scale permits modeling a lengthy flow process in a relatively short period of time. The standard scaling units are readily measurable media and liquid properties. Some of these properties, however, are characteristic of the drainage cycle only. Theoretically, therefore, the scaling criteria are not applicable when hysteresis is important and the imbibition cycle is involved.

The validity of the criteria were confirmed experimentally by applying a non-steady drainage process to two columns of media which, according to the theory were similar. Even though they were similar, the media properties were sufficiently different so that one column was a small model of the other. It was also discovered that, at least during steady flow, processes involving hysteresis can, in fact, be accurately scaled with these criteria.

PREFACE

Colorado State University's contribution to W-51 Regional Research Project entitled "Factors Influencing the Flow of Subsoil Water in the Immediate Proximity of and into Drainage Facilities" includes a study of the possibility of using physical models of field drainage systems. Results are presented herein which indicate that the scaling theory is valid and future work will be directed toward modeling field situations. A More detailed report on this study is included in the senior author's dissertation entitled "Similitude for Non-Steady Drainage of Partially Saturated Soils" presented at Colorado State University.

The authors are grateful to Dr. D. E. Elrick, Associate Professor, Department of Soil Science, Guelph University, Ontario, Canada, and Dr. G. L. Bloomsburg, Associate Professor, Department of Agricultural Engineering, University of Idaho, for their critical review of this paper.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES (TEXT)

LIST OF TABLES (APPENDIX III)

Table Page

LIST OF SYMBOLS

Symbol	Definition	Dimension
A	Area	L^2
a	Mobility ratio	none
c	Constant of integration	
d	Microscopic characteristic length	L
e	Subscript meaning "effective"	none
f	Body force per unit volume	FL^{-3}
g	Acceleration due to gravity	LT^{-2}
H	Hydraulic head	L
h_o	Capillary rise in a capillary tube	L
K	Maximum permeability to a particular fluid phase when only one phase occupies the medium	L^2
K_e	Effective permeability - the permeability when the medium is occupied by more than one fluid phase	L^2
K_r	Relative permeability - K_e/K	none
L	Length	L
m	General subscript	none
n	General subscript	none
nw	Subscript meaning "non-wetting phase"	none
P	Pressure	FL^{-2}
P_b	Bubbling pressure - approximately the minimum capillary pressure on the drainage cycle at which a continuous non-wetting phase exists in a porous medium	FL^{-2}
P_C	Capillary pressure - the pressure difference $(P_{nw} - P_w)$	FL^{-2}
Q	Volume flow per unit area	L
Q_t	Volume flow rate	L^3T^{-1}
q	Liquid volume flux	LT^{-1}
R	Radius	L
r	Position vector	L
S	Saturation - the ratio of the volume of wetting fluid to the volume of the voids	none
S_e	Effective saturation $(S - S_r) / (1 - S_r)$	none
S_r	Residual saturation - the saturation at which K_{ew} 0	none
t	Time	T
w	Subscript meaning "wetting phase"	none
X	Distance measured parallel to the flow	L

LIST OF SYMBOLS (CONTINUED)

Y	Distance measured perpendicular to the flow	L
Z	Vertical coordinate direction	L
α	Contact angle of wetting - non-wetting fluid interface with medium grains	Radians
$f(\alpha)$	Dimensionless representation for contact angle	none
η	Pore size distribution index $-d(\ln K_e)/d(\ln P_c)$	none
θ	Moisture content of medium - Ratio of the volume of wetting fluid to the total bulk volume of medium	none
μ	Dynamic viscosity	$FL^{-2}T$
ρ	Fluid density	$FT^{2}L^{-4}$
ρ_b	Bulk density of the porous medium	FL^{-3}
ρ_s	Particle density of the medium	FL^{-3}
σ	Interfacial tension	FL^{-1}
ϕ	Porosity - the volume of pore space expressed as a fraction of bulk volume of the medium	none
ϕ_e	Effective porosity $(1 - S_r)\phi$	none
Δ	Denotes a difference	
∇	Gradient operator	L^{-1}
div	Divergence operator	L^{-1}

SIMILITUDE FOR NON-STEADY DRAINAGE OF PARTIALLY SATURATED SOILS

G. L. Corey, A. T. Corey and R. H. Brooks[1]

INTRODUCTION

Many physical phenomena are simple enough and sufficiently understood to allow complete mathematical analysis without recourse to auxiliary experimentation. Others, however, are so complicated as to render complete mathematical analysis difficult if not impossible. One need not await development of mathematical techniques, however, in order to learn something of the phenomena. Information can be obtained empirically by direct experimentation.

It is often impractical to perform laboratory investigations under prototype conditions and one must resort to model studies. In order that model results be applicable to prototype conditions, the model must be adequately scaled. Similitude for a given phenomenon requires that the model be scaled dynamically as well as geometrically. In fluid dynamics there are two generally accepted methods of defining the scaling criteria necessary for a given process.

Dimensional analysis consists of grouping the pertinent variables into dimensionless parameters. These are then studied empirically to define their interrelationships. Theoretically, since the parameters are dimensionless, their functional relationships apply to any size of the physical process providing the parameters are numerically equal. One need not know the mathematical relationships among variables in order to use this method.

The other method, called "inspectional analysis" by Ruark (18), requires that the fundamental equations relating variables be known. These equations are "scaled" or made dimensionless by applying suitable standard variables. This process results in dimensionless parameters which must be held equal in the model and prototype; thus the scaling criteria become apparent. This method is also referred to as "change of variable" or "dynamic similitude." The success of either of these methods depends, to a large degree, on the user's prior knowledge of the process. It is difficult without this to fully appreciate the validity of general conclusions drawn from small-scale observations.

Isothermal movement of liquids through porous materials has been a phenomenon of interest for many years. Since Darcy (8) first described the equation of steady flow, researchers have been concerned with developing a more thorough understanding of this complicated process. In 1931, Richards (16) proposed a general equation describing non-steady flow of liquids in either saturated or unsaturated materials. This equation, in various forms, has been the basis for much of the subsequent work in partially saturated media.

This non-linear, second order differential equation has thus far not been solved for anything other than simple one-dimensional flow. Since most physical problems involve either two or three-dimensional flow it would be advantageous to be able to model this type of phenomenon. The purpose of this paper is to describe a theory of similitude based on Richard's equation and applicable to drainage of soils. The validity of the theory was checked by experimentally testing similar media during a non-steady drainage process.

[1] Professor of Agricultural Engineering, University of Idaho, formerly graduate student at Colorado State University; Professor of Agricultural Engineering, Colorado State University, Agricultural Research Engineer, USDA, respectively.

LITERATURE REVIEW

The use of models to study flow through porous media is not new. However, in many laboratory investigations reported in the literature, scale relations between model and prototype have not been recognized. Conclusions developed do not apply to prototype systems because experimental design did not adequately account for scaling factors.

Scientists in the petroleum industry are interested in fluid flow through porous media since raw petroleum most often occurs in the porous mantle of the earth. They are, in general, interested in two-phase liquid flow or the displacement of oil by water or gas. Modeling techniques have been applied to this problem and although the process involves the flow of two liquids and attention is focused on modeling the displacement of oil by water or gas, their selection of pertinent variables and discussions of model design apply equally well to media partially saturated with a single liquid.

The author has taken the liberty of converting all the scaling criteria developed by the various investigators, to similar units for ease of comparison.

Scaling Theories

Leverett (12) was probably the first to use dimensional analysis to arrive at a scaling theory for flow through porous media. He developed the theory in 1937 and discussed it along with experimental techniques in 1942. He was interested in the displacement of oil by water. He fixed the model-prototype length ratio by a desired ratio. This fixed the time scale since the acceleration due to gravity, a variable he considered pertinent, would have the same value in both model and prototype. He fixed the mass scale by noting that the density difference between oil and water would be approximately the same for model and prototype. Of course, fixing ratios of length, time, and mass also fixed all other model-prototype ratios since no other fundamental dimensions were needed. His scaling criteria are summarized below.

$$K_{ro} = K'_{ro} \qquad (1)$$

$$K_{rw} = K'_{rw} \qquad (2)$$

$$\frac{P_c}{P'_c} = \frac{\sigma}{\sigma'} \sqrt{\frac{K'}{K}} \qquad (3)$$

The primed quantities refer to the model and the unprimed to the prototype and the symbols are defined as follows:

K is the maximum permeability of the material and is not a function of the liquid.

K_{ro} is the relative permeability to oil = $\frac{K_{eo}}{K}$.

K_{rw} is the relative permeability to water = $\frac{K_{ew}}{K}$.

K_{eo} is the effective permeability to oil and is a function of capillary pressure.

K_{ew} is the effective permeability to water and is a function of capillary pressure.

μ_o is the dynamic viscosity of the oil phase.

μ_w is the dynamic viscosity of the water phase.

P_c is the capillary pressure = pressure of oil - pressure of water.

σ is the interfacial tension between the two liquids.

According to Leverett's theory, equation 1, 2, and 3 must be satisfied. However, the quantity represented in equation 3 as well as $\left[\frac{K\mu_o'}{K'\mu_o}\right]$ and $\left[\frac{K\mu_w'}{K'\mu_w}\right]$ must all be equal to some specific value depending on the choice of the length, time, and mass scales. He indicated that since one may choose the porous material as well as the model fluid sufficient flexibility is available to meet these criteria.

Rapoport (15) was quite critical of the dimensional analysis approach and developed scaling laws for water-oil flows utilizing the inspectional analysis technique. His criteria are summarized below.

1. The model must be constructed in such a manner as to scale the prototype geometry.
2. The initial fluid distribution must be the same for model and prototype.
3. The relative permeability functions and the oil-water viscosity ratio must be the same for model and prototype. (Relative permeability functions refer to the relationship between relative permeability and saturation.)
4. The capillary pressure functions applying to model and prototype must be related to each other either by direct proportionality or by a general linear transformation.

5. The design and operation of the model must be conducted in accordance with the following expressions:

$$\left[Q_t \mu_w / L^2 K g \Delta\rho\right]_{model} = \left[Q_t \mu_w / L^2 K g \Delta\rho\right]_{prototype} \tag{5}$$

$$\left[Q_t \mu_w / LKP_c\right]_{model} = \left[Q_t \mu_w / LKP_c\right]_{prototype} \tag{6}$$

$$\left[K P_c^2/\phi \sigma^2\right]_{model} = \left[K P_c^2/\phi \sigma^2\right]_{prototype} \tag{7}$$

where Q_t is total water flow, ϕ is the porosity, and $\Delta\rho$ is the density difference between water and oil.

Croes and Schwarz (7) used model experiments to study the process of water driving oil from a porous material. They reported experimental results in dimensionless form. The dimensionless groups were

$$a, \frac{L}{Y}, \frac{\mu_o}{\mu_w}, \frac{\Delta\rho\ Yg\sqrt{K}}{\sigma \cos \alpha}, \frac{\Delta\rho g K}{q\mu_w}$$

where a, the mobility ratio, $= \dfrac{K'_{ro}\mu_w}{K'_{rw}\mu_o}$. The relative permeabilities refer to the unflooded region and L and Y are the lengths parallel to and perpendicular to the direction of flow respectively, g is the acceleration due to gravity, and α is the contact angle of the oil-water interface with the sand grains.

Craig, et.al. (6) performed a model study of oil recovery from a water flooded formation. They considered homogeneous material and neglected the gravity effect. They scaled the model geometrically and used the flow scaling criteria

$$\left[\frac{q_w \mu_o X}{\sigma \cos \alpha}\right]_{model} = \left[\frac{q_w \mu_o X}{\sigma \cos \alpha}\right]_{field} \tag{8}$$

where q_w was the injection rate per foot and X was the distance between wells. They used model materials that had relative permeability characteristics typical of many oil bearing formations. The tests covered a wide enough range of rock and fluid characteristics and injection rates to simulate a wide variety of field flooding conditions.

Scheidegger (19) scaled the general equation for non-steady flow through partially saturated porous media and arrived at the following scaling criteria:

$$\left[\frac{P_c}{L\ \Delta\rho g}\right]_{model} = \left[\frac{P_c}{L\ \Delta\rho g}\right]_{prototype} \tag{9}$$

and

$$\left[\frac{Q_t\ \mu}{L^2 K\ \Delta\rho g}\right]_{model} = \left[\frac{Q_t \mu}{L^2 K\ \Delta\rho g}\right]_{prototype} \tag{10}$$

which results in the scaled time being

$$\left[\frac{t\ Q_t}{L^3\ \phi}\right]_{model} = \left[\frac{t\ Q_t}{L^3\ \phi}\right]_{prototype} \tag{11}$$

where Q_t is the volume flow rate and t is time. These criteria are sufficient for macroscopic scaling only and in order to achieve full scaling one must assume that the relative permeability-saturation relationships are the same for model and prototype. Schiedegger suggested using the prototype material in the model.

Richardson (17) used inspectional analysis to scale the differential equation describing the displacement of oil by gas or water. He proposed the following scaling criteria:

1. The model and prototype are geometrically similar.

2. The model and prototype have similar initial and boundary conditions.

3. The model and prototype have the same relative permeability and dimensionless capillary-pressure-saturation relations.

4. The model and prototype have the same values for the parameters

   $$\frac{\sigma\ f(\alpha)}{q\mu_o\ L}\sqrt{\frac{K}{\phi}},\ \frac{K\Delta\rho g}{q\mu_o},\ \frac{\mu_w}{\mu_o},$$

   with time given by $\phi L/qt$, where $f(\alpha)$ is merely a dimensionless representation for the contact angle and q is the volume flux.

The previously discussed scaling laws were developed by petroleum engineers interested in the displacement of oil by water or gas. In this process two liquids are flowing side by side. Several investigators have proposed scaling laws for modeling unsteady flow of one liquid in a partially saturated material. Stallman (20), in 1964, used dimensional analysis to determine scaling criteria for unsteady flow. According to his results, correct dimensional scaling of a prototype flow system is attained only if the model and prototype characteristics satisfy the following criteria:

1. The model and prototype are geometrically similar and have similar boundary conditions.

2. The curve of the relative permeability versus saturation of the porous medium in the model must be identical with the same curve of the porous medium in the prototype.

3. $$[\phi]_{model} = [\phi]_{prototype} \tag{12}$$

4. $$\left[\frac{P_c}{\sigma}\sqrt{\frac{K}{\phi}}\right]_{model} = \left[\frac{P_c}{\sigma}\sqrt{\frac{K}{\phi}}\right]_{prototype} \tag{13}$$

5. $$\left[\sigma L^{2/3}\sqrt{\frac{t}{\mu\rho g}}\right]_{model} = \left[\sigma L^{2/3}\sqrt{\frac{t}{\mu\rho g}}\right]_{prototype} \tag{14}$$

6. $$\left[\frac{K\rho g t}{\mu L}\right]_{model} = \left[\frac{K\rho g t}{\mu L}\right]_{prototype} \tag{15}$$

where μ and ρ are the dynamic viscosity and density of the liquid respectively. Stallman's fourth criterion is the relationship discussed by Leverett, represented in equation 4.

Hysteresis plays an important role in unsteady flow in partially saturated media if the flow process involves both drainage and imbibition. The capillary pressure-permeability-saturation relationships are not identical for both systems since during imbibition air becomes entrapped in the medium thus reducing the effective flow area. Miller and Miller (13) were the first to consider scaling laws which would be valid for any flow system whether it be drainage or imbibition. They used inspectional analysis and gave attention to microscopic as well as macroscopic scaling. Microscopic refers to the medium itself and macroscopic to the over-all flow system. Their criteria are based on the model and prototype media being "similar" and in "similar states." To satisfy this, the two media must have equal values of $\frac{P_c d}{\sigma}$, α, $\frac{K_e}{d^2}$; where d is a characteristic length of the media.

Using these relationships the following scaled variables were developed:

Microscopic geometry

$$P_{c.} = \frac{P_c d}{\sigma} \tag{16}$$

$$\alpha. = \alpha \tag{17}$$

Flow properties

$$S. = S \tag{18}$$

$$K_{e.} = \frac{K_e}{d^2} \tag{19}$$

Macroscopic geometry

$r. = \frac{r}{L}$ r is a position vector and L a macroscopic characteristics length

$f. = \frac{Ld}{\sigma}f$ f is a body force per unit volume = ρg for gravity (21)

$$q. = \frac{\mu L}{\sigma d} q \tag{22}$$

$$t. = \frac{\sigma d}{\mu L^2} t \tag{23}$$

where the dots designate scaled variables.

Several experiments have been made to test the validity of the above theory. Klute and Wilkinson (11) used "similar" media consisting of washed natural sands bracketed between sieve sizes. Permeability - capillary pressure relationships were investigated for each of five media. It is not clear whether the data were taken on drainage on imbibition cycles but it is clear that both were not included. The scaled capillary pressure $\left(\frac{P_c d}{\sigma}\right)$ and permeability $\left(\frac{K_e}{d^2}\right)$ curves coalesced into one single curve fairly well, at least within experimental error. However, observation of individual curves indicates that the experimental error was quite large.

Elrick, et.al. (9) ran hysteresis loops of capillary pressure-saturation. They used one material and "modeled" by using two separate liquids, i.e. water and butyl alcohol. The capillary pressure was scaled according to the Miller theory. The microscopic length d, however, was not applicable since the same medium was used in all tests. These curves showed good agreement for granular alundum but poor agreement when a mineral soil containing clay was used. Scaled capillary pressure versus scaled permeability was also measured with excellent results.

Elrick, et. al. also measured distance of an imbibing front and total imbibition against time. This experiment was conducted with two similar media using the same liquid in each. These data show excellent agreement when plotted in reduced coordinates. Another test involved downward flow into initially saturated samples, driven by gravity and a scaled excess of pressure at the top. The scaled volume discharged per unit of scaled time showed good agreement.

Wilkinson and Klute (21) drained initially saturated columns containing "similar" media and plotted discharge versus scaled time. Their results were only fair. Better results were obtained when liquid was allowed to imbibe into initially dry columns.

The above-mentioned experiments by Klute, Elrick, and Wilkinson were all conducted using the Miller scaling theory in order to test its validity. Some of the results were impressive, others not.

Summary

Sand models have been used for many years to study flow processes through porous media. However, only recently have scaling criteria been given due consideration.

Several investigators, interested in the replacement of oil by water, have developed criteria for modeling such phenomenon. Recently, theories have been proposed concerning flow of a single liquid in a porous material. The latter theories are of interest in this study. The process of a single liquid flowing under partially saturated conditions is essentially the same as when two immiscible liquids are flowing since actually two fluids are involved in either case. However, during the drainage process the gas flow is generally neglected.

Some of the scaling laws reviewed here are theoretically valid only in limited cases. The theories of Leverett and Stallman consider acceleration to be an important variable. Inertial effects are ordinarily neglected in flow through porous media; in fact, Darcy's law applies only when inertial forces can be neglected. The validity of this law has been thoroughly demonstrated, therefore scaling theories based on equal accelerations between model and prototype cannot be considered relevant to most practical flow cases.

There is a great deal of similarity among several of the theories. For example, most of the authors agree that the functional relationships between capillary pressure, saturation, and permeability must be similar for the model and prototype. On the other hand, there are some important differences; for example, there are three separate methods proposed for scaling capillary pressure.

Many of the theories presented are quite involved and so restrictive that it is doubtful if one could succeed in meeting all criteria in order to model a specific prototype flow system. As an example, Richardson gave a hypothetical example of model properties as related to prototype properties and found that the surface tension of the model fluid had to be adjusted to 3.86 dynes per centimeter. It is very doubtful that a stable liquid could be produced with so low a surface tension. It should be pointed out that of the theories presented here the only one that has been experimentally verified is the Miller-Miller theory.

Several of the theories allow for an arbitrary choice of the macroscopic length scale. If gravity flow is important in the particular problem, an arbitrary choice of the length scale will not properly model the capillary fringe. The Miller-Miller theory, however, requires that Ld be the same for gravity flow with the same liquid in the model and prototype.

THEORETICAL ANALYSIS

Calculations of movement of fluids through porous media are extremely difficult if the flow is unsteady and the media not completely saturated with fluid. A scaled replica of the prototype system affords a convenient substitution for calculation. Systems of comparatively large size can be reproduced on a small scale where fluid movement can be observed with relative ease. The aim of model design is to reduce the prototype size to some size convenient for laboratory use and to reduce the time scale so the model observations can be completed in a comparatively short time.

Assumptions

Several basic assumptions are necessary in order to simplify the analysis and make it more meaningful. Some are highly important and others may not be. Only through future research can the importance of each be defined. The analysis presented here is based on the following basic assumptions:

1. Temperature and associated thermodynamic effects and chemical interactions are excluded from the liquid flow regime under consideration.
2. The medium is homogeneous, isotropic, and physically stable.
3. The medium contains interconnected internal spaces sufficiently small so that the shape of interfaces between non-wetting and wetting fluids is not influenced by gravity.
4. The liquid is uniform in its surface tension, contact angle, viscosity, and density throughout every portion of the flow system.
5. The magnitude of the dynamic viscosity and density of the non-wetting fluid is negligibly small compared to the wetting fluid.
6. The pressure of the non-wetting fluid is constant throughout the given flow system.
7. The compressive forces of the fluid are negligible.
8. The inertial effects are negligible, i.e. the flow system obeys Darcy's equation.

Basic Flow Equations

Darcy's equation which describes steady flow of an incompressible fluid through a homogeneous and isotropic porous medium can be written in the form:

$$q = -\frac{K_e}{\mu} \nabla (P + \rho g z) \tag{24}$$

where q = volume flux of the fluid --- LT^{-1}

K_e = effective permeability --- L^2

P = fluid pressure --- FL^{-2}

μ = fluid viscosity --- FTL^{-2}

ρ = fluid density --- $FT^2 L^{-4}$

g = gravitational acceleration --- LT^{-2}

z = vertical space coordinate --- L .

The continuity equation for fluid flow through a porous material can be written as

$$\text{div } \rho q = -\frac{\partial(\phi \rho S)}{\partial t} \tag{25}$$

where ϕ is the porosity of the medium and S is saturation; both quantities being dimensionless. Since, for incompressible fluids, ρ is not a function of time nor space and for a stable material the porosity is constant, equation 25 can be written as

$$\text{div } q = -\phi \frac{\partial S}{\partial t} \tag{26}$$

Substituting equation 24 into equation 26 results in

$$\text{div}\left[\frac{K_e}{\mu} \nabla (P_w + \rho g z)\right] = \phi \frac{\partial S}{\partial t} \tag{27}$$

which is the general relationship for non-steady flow of a fluid through a homogeneous isotropic porous medium.

The saturation, S , is expressed as the ratio of the volume of liquid occupying the pores to the total pore volume. Another means of expressing the liquid content of a material is referred to as moisture content, θ , which is the ratio of the volume of liquid occupying the pores to the bulk volume of the medium. Then, moisture content and degree of saturation are related by $\theta = \phi S$. The substitution of $\frac{\theta}{\phi}$ for S in equation 27 is often used by soil physicists and is pointed out here to indicate the agreement of equation 27 with other forms found in the literature.

Capillary pressure, P_c , is defined as the pressure difference between the non-wetting and wetting fluid phases or

$$P_c = P_{nw} - P_w . \tag{28}$$

Since the non-wetting pressure is considered to be constant throughout the system and equal to the atmospheric pressure, one may write

$$P_c = -P_w \tag{29}$$

if atmospheric pressure is selected as the pressure datum. Equation 29 can be substituted into equation 27 if the equation in terms of capillary pressure is desired.

Similitude for Homogeneous Media

Brooks and Corey (2) scaled a form of equation 27 in order to determine modeling criteria for unsteady flow of liquids through partially saturated porous media. They substituted $\phi_e \frac{\partial S_e}{\partial t}$ for the term on the right side of the equation; thus equation 27 becomes

$$\text{div}\left[\frac{K_e}{\mu}\nabla(P_w + \rho g z)\right] = \phi_e \frac{\partial S_e}{\partial t} \qquad (30)$$

where ϕ_e is the effective porosity and S_e is the effective saturation. This does not alter the equation in any way since the substituted quantity is identical to the original one, as the following discussion will show.

Burdine (3) developed an approximate equation for determining permeability. The term $(S-S_r)/(1-S_r)$ appeared in his analysis.

$$S_e = \frac{S - S_r}{1 - S_r} \ . \qquad (31)$$

The residual saturation, S_r, was defined as the saturation at which the effective permeability is assumed to approach zero. Corey (5) called this quality effective saturation or Collins (4), in 1961, introduced a term similar to effective saturation. His relationship is general in that it applies also to the case where two liquids are flowing in the medium.

The effective saturation actually refers to the fraction of effective pores that are occupied by the liquid. Those pores so small as to exhibit virtually zero permeability under hydraulic gradients ordinarily found in nature are excluded from the analysis. In a similar manner, these pores are excluded from the porosity of the material and effective porosity, ϕ_e, is defined as

$$\phi_e = (1 - S_r)\phi \quad . \qquad (32)$$

This quantity is equivalent to the "drainable porosity," a term used by drainage engineers. Since $(1 - S_r)$ is a constant for a given material and fluid and not a function of time, the expression $\phi_e \frac{\partial S_e}{\partial t}$ is identical to $\phi \frac{\partial S}{\partial t}$.

Brooks and Corey wrote equation 30 in dimensionless form by scaling the variables appearing in the equation. To accomplish this, standard units of permeability, length, pressure, and time were chosen. The standard units were designated as K_o, L_o, P_o, and t_o respectively, and were selected such that they would be significant characteristics of the system.

Scaling equation 30 with the standard units results in

$$\left[\frac{t_o K_o P_o}{L_o^{\,2}\mu\phi_e}\right] L_o \,\text{div}\left\{\frac{K_e}{K_o}(L_o\nabla)\left[\frac{P_w}{P_o} + \left(\frac{\rho g L_o}{P_o}\right)\frac{Z}{L_o}\right]\right\} = \frac{\partial S_e}{\partial t/t_o} \qquad (33)$$

which is a dimensionless equation in which the constants $\frac{t_o K_o P_o}{L_o^{\,2}\mu\phi_e}$ and $\frac{\rho g L_o}{P_o}$ appear.

The standard units were chosen as follows: K, the saturated permeability, was selected as the standard unit of permeability. It is a constant for a given medium. The bubbling pressure, P_b, was selected as the standard unit of pressure. Bubbling pressure is the minimum capillary pressure on the drainage cycle at which a continuous non-wetting fluid phase exists in a porous medium. It is a constant for a given wetting liquid and medium. The manner in which the bubbling pressure was obtained is explained on Page (Appendix I). Since P_b was chosen for the standard pressure, the standard length was necessarily chosen as $P_b/\rho g$ in order that the second constant, mentioned above, would be equal to unity. The standard unit of time was such as to cause the first constant, above, to be equal to unity.

Summarizing, the selected standard units were:

1. $K_o = K$, the permeability of the medium at complete saturation.

2. $P_o = P_b$, the bubbling pressure for the given medium and liquid.

3. $L_o = P_b/\rho g$.

4. $t_o = P_b\,\mu\,\phi_e/K\,(\rho g)^2$.

Substituting the standard units into equation 33 gives

$$\text{Div.}\,[K.\nabla.(P. + Z.)] = \partial S./\partial t \ . \qquad (34)$$

where the dots designate scaled variables or operators with respect to scaled variables. It will be noted that the scaled permeability is merely the relative permeability and the scaled saturation is effective saturation.

Equation 34 will yield identical particular solutions in terms of scaled variables provided that:

1. Geometric similitude exists. This condition is satisfied only if corresponding lengths which are characteristic of the macroscopic size of the system are identical multiples of the length, $P_b/\rho g$. It is necessary to choose only one characteristic length as the size requirement

is satisfied if the ratio $P_b/\rho g L$ is identical for any two systems under consideration.

Microscopic geometric similitude is not explicitly detailed in this theory. However, the following condition of similarity does, in fact, imply that microscopic similarity is obtained.

2. The functional relationships between $K.$, $P.$, and $S.$ are identical for both systems. The relationship between K_e and P_c, for any given material, has been found to be a power function for $P_c > P_b$. For capillary pressures less than bubbling pressure K_e does not vary with pressure and is therefore the constant K or maximum permeability. Then, if $K.$ is plotted as a function of P_c for a given material the relationship would be represented by a straight line on a log-log plot for $P_c > P_b$. The slope of this line is referred to as η. Now, if $K.$ is plotted a function of $P.$ the curves would coalesce for all materials having equal values of η.

This criterion, then, demands that two similar media must have the same value for η, where:

$$\eta = \frac{d(\ln K.)}{d(\ln P_c)} \text{ for } P_c > P_b \quad . \qquad (35)$$

Brooks and Corey (2) have found that if two materials have similar scaled pressure-permeability relationships, they will also have similar scaled capillary pressure-effective saturation relationships.

3. The initial and boundary conditions in terms of scaled variables are similar for systems, the scaled time being $\dfrac{tK(\rho g)^2}{P_b \phi_e}$.

It is interesting to note that, for non-gravity systems, the first requirement can be eliminated since it originated from the gravity term of equation 33. The length scale could then be chosen arbitrarily and the only requirement for similarity would be equal η's and equal scaled time values. Therefore, for a non-gravity system a prototype material could be used in the model. However, there are few such systems of practical significance. Of course, the scaled time requirement can be eliminated for flow systems involving only steady state.

It is convenient to tabulate the scaled variables resulting from this theory.

Microscopic geometry

$$P. = \frac{1}{P_b} P_c \quad \text{pressure of wetting phase} \qquad (36)$$

$$\eta. = \eta \quad \text{representation of relationship between } K_e \text{ and } P_c \qquad (37)$$

Macroscopic variables

$$L. = \frac{\rho g}{P_b} L \quad \text{length} \qquad (38)$$

$$t. = \frac{(\rho g)^2 K}{P_b \mu \phi_e} t \quad \text{time} \qquad (39)$$

$$q. = \frac{\mu}{\rho g K} q \quad \text{flow volume flux} \qquad (40)$$

$$Q. = q.t. = \frac{\rho g}{P_b \phi_e} qt = \frac{\rho g}{P_b \phi_e} Q \quad \text{flow volume} \qquad (41)$$

Flow properties of the medium

$$K. = K_r = \frac{K_e}{K} \quad \text{permeability} \qquad (42)$$

$$S. = S_e = \frac{S - S_r}{1 - S_r} \quad \text{saturation} \qquad (43)$$

$$\phi. = \phi_e = (1 - S_r)\phi \quad \text{porosity} \qquad (44)$$

Summary

The approach presented here is similar to that of Miller's, however it is restricted to the drainage cycle. Brooks and Corey stated, however, that even when hysteresis is a factor there is a possibility that the theory is valid. Neither theory allows for an arbitrary choice of length scale if gravity flow is to be considered. This indicates that the capillary fringe should be adequately accounted for in either In fact, the length scale called for by the Brooks-Corey theory demands that the capillary fringe be precisely scaled since bubbling pressure head is analogous to the capillary fringe depth.

Brooks and Corey scaled capillary pressure by dividing it by the bubbling pressure, a property of the medium and the liquid. Miller and Miller scaled this quantity by dividing by the surface tension of the liquid and multiplying by some standard microscopic length. Thus, in both cases, pressure was scaled using properties of the medium and the liquid. Since,

$$P_c = \sigma \left[\frac{1}{R_1} + \frac{1}{R_2}\right] \qquad (45)$$

where R_1 and R_2 refer to the radii of the interface between the wetting and non-wetting fluids, it was logical to choose σ and a microscopic length, d, as parameters to scale capillary pressure. Bubbling pressure, being a specific capillary pressure, is also a function of σ and d.

Miller and Miller scaled permeability by dividing it by d^2. Brooks and Corey scaled it by dividing by the permeability at complete saturation. Here again the similarity of scaling is apparent. In fact, Elrick, in testing the Miller theory, defined d as the quantity which would cause $\frac{K_e}{d^2}$ at complete saturation to be equal to one. This, of course, is exactly what is called for in the Brooks-Corey theory.

According to Miller and Miller, the Hysteresis aspect of their theory is based on the conclusion that "as the capillary pressure is changed until the end point pressure for the existing interface shape is attained, the indicated cross over (hysteresis) will occur just as this pressure is reached, regardless of the time rate at which it was approached". If this, in fact, is the case, it is likely that the Brooks-Corey criteria will scale hysteresis as well as the Miller-Miller criteria.

EXPERIMENTAL PROCEDURE

The validity of the scaling laws was checked by testing non-steady state flow through materials scaled according to the theory. One-dimensional flow was used because of its experimental simplicity. The experiment provided a valid test of the theory, since the scaling criteria are not related to the number of dimensions used.

All tests were made with columns of porous material packed into lucite tubes. The columns had a cross-sectional area of 7.94 cm^2 and were assembled to the desired lengths by taping short sections of tubing together. This was done to allow air to enter the media at all joints between the short lengths of lucite tubing.

Media and Liquid

Sands were used as the porous media for most of the experiments. One experiment was performed with a fragmented clay material in order to represent properties unattainable with sands. All media were unconsolidated.

The liquid (wetting fluid) used was a hydrocarbon oil, which is a core test fluid referred to as Soltrol "C". This oil was used because its low (23 dynes/cm.) surface tension facilitates experimental work involving a mixture of liquid and air. Also, oil being non-polar does not create swelling of clay particles which might alter the physical properties of the media during experiments.

Packing Columns

The tubes were filled with media through a small (1/2 inch I.D.) lucite tremie which initially extended to the bottom of the columns. The tremie was filled with thoroughly mixed dry material through a funnel and was slowly withdrawn until the tubes were completely filled. The entire column was then vibrated until the desired bulk density was reached. The short columns were vibrated with a Jackson (10) soil column packer. Columns too long for this packer were packed by vibrating the entire column with a hand vibrator. One of the columns (146.5 cm. in length) required tamping as well as vibrating in order to obtain the desired bulk density.

The vibrations, of course, caused settling of material and initially full tubes were less than full after vibrating. To eliminate discontinuities in the media, the columns were made extra long, filled, vibrated, and then the extra length was removed before experimentation. The extra length was provided for by merely taping an extra 10 cm section of lucite tubing on the end of the column.

Determining Media Properties

The scaling criteria require that certain fluid and medium properties be known before adequate modeling studies can be made. These properties include permeability, K, bubbling pressure, P_b, η, porosity, ϕ, residual saturation, S_r, effective porosity, ϕ_e, and bulk density ρ_b.

The method used to determine the relationship between pressure and effective permeability for a given medium, required measuring the effective permeability at various capillary pressures. A short column of material was used for this. All of the above properties were obtained either from the capillary pressure-effective permeability data or from samples taken from the column used to obtain these data. A detailed discussion of the procedures for determining these properties is presented in Appendix I.

Testing the Theory

The adequacy of the scaling criteria was determined by first finding two media that behaved similarly on the drainage cycle. The two materials used were

1. Poudre sand - This material came from the Poudre River in Colorado and is mostly of granitic origin but also contains considerable mica. The natural sand was screened and size fractions were recombined in order to give a uniform particle size distribution.

2. Crab Creek sand - This sand came from wind-blown deposits along Crab Creek in Washington. It is of basaltic origin and undoubtedly exhibits some secondary porosity. This material was used in its natural state.

It was discovered that these two sands have a very similar scaled pressure - permeability relationship on the drainage cycle. The η values for each are approximately 10 depending somewhat on the density of the particular packing.

It was necessary to establish an unsteady state process in order to check the time scale. This was done by allowing an initially saturated column of material to drain under the force of gravity. The two materials proved quite suited to such a study since even though the η values were the same, the bubbling pressure head for the Poudre sand was 39.1 cm of oil, while that of Crab Creek sand was only 13.9 cm. This meant a length scale ratio of 2.81:1.

A column of Poudre sand was packed to a length of 146.5 cm and a column of Crab Creek sand to a length of 52.1 cm. The properties K, $P_b/\rho g$, and η were determined on these long columns. Actually, since the bubbling pressures were not accurately known beforehand, the columns were not cut to the above mentioned lengths until after these properties were determined. At this time a graded filter was constructed on the bottom of each column so that there would be a minimum of exit resistance during the drainage process.

The columns were then saturated and connected to the inflow barrier. The experimental apparatus was essentially the same as that used to measure media properties which is presented in Figure 9. The tensiometers were connected to their respective manometers. The outflow tube was adjusted so that the water table in the column, as measured with a tensiometer previously installed around the sand filter, was exactly at the bottom of the column of material. The inflow was lowered until the scaled capillary pressure, $P_{.} = P_c/P_b$, on the top of the column was approximately 0.4. At this point all tensiometer leads were pinched off and then the inflow lead was pinched off. The instant that the inflow was stopped, time was started and outflow measurements versus time were made. They were continued until the flow rate was essentially zero.

These data were then properly scaled and compared to determine if the two media having similar η values could be represented by the same scaled curve. The scaled discharge, $Q_{.} = \frac{\rho g}{P_b \phi_e} Q$ was plotted versus scaled time, $t_{.} = \frac{K(\rho g)^2}{P_b \mu \phi_e} t$.

The time scaling parameter was verified by allowing two columns, similar in all respects, to drain under gravity flow. The materials, Crab Creek and Poudre sand, were packed into long columns. The various properties were determined on these columns and the resulting parameters are presented in Table 1.

Table 1. Scaling parameters for two long columns of similar media.

	Poudre Sand	Crab Creek Sand
η	9.90	9.7
$P_b/\rho g$ (cm)	39.1	13.9
K (μ^2)	3.4	29.47
L (cm)	146.5	52.1
ρ_b gm/cc	1.47	1.54
ρ_s gm/cc	2.74	2.79
S_r	0.125	0.148
ϕ	0.464	0.448
ϕ_e	0.406	0.382

Combining parameters reported in Table 1 into scaled quantities indicated that

1. The length ratio was $(P_b)_p/(P_b)_c = 2.81$
2. The scaled length of each column was $\frac{L\rho g}{P_b} = 3.75$
3. The time ratio was approximately $\left(\frac{K\,(\rho g)^2}{P_b\,\mu\,\phi_e}\right)_c \Big/ \left(\frac{K\,(\rho g)^2}{P_b\,\mu\,\phi_e}\right)_p = 25.9$ where the subscripts p and c refer to Poudre and Crab Creek sands respectively.

This indicates that the Crab Creek column, which can be considered to be a model of the Poudre column was only 0.36 times as long as the Poudre column and the flow process would occur approximately 26 times faster.

When the columns had been resaturated and installed for the drainage test, the inflow height was adjusted to the initial condition. The initial condition obtained on each column was: $P_. = 0$ at the bottom of each column and the scaled pressure at the top was 0.49 for the Poudre sand and 0.42 for the Crab Creek sand. These top initial conditions were not precisely the same; however, immediately upon stopping the inflow, the pressure at the column tops increased to $P_. \geq 1.0$ and the minor difference was considered negligible. The reason that a small amount of capillary pressure was desired at the top initially was to ensure that there would be no leaks through the lucite section joints forming the column.

As the inflow was stopped, a stop watch was started and the unsteady drainage was begun. The outflow and time were recorded at sufficiently small intervals to allow complete descriptions of the discharge-time relationships.

The unscaled discharge-time relationships are shown graphically in Figure 1, while the coalescence of these data when scaled is demonstrated in Figure 2. Since $Q_. = \frac{\rho g}{P_b \phi_e} Q$ and $t_. = \frac{K\,(\rho g)^2}{P_b\,\mu\,\phi_e} t$, it was necessary to record the temperature in order to compute μ/ρ. Complete data obtained during the drainage process are presented in Appendix III.

The unscaled data together with the scaled relationship indicate the validity of the time scaling criterion. The coalescence of data into one relationship is excellent. The data for the longer column do not follow the model data exactly near the end of the experiment. However, in order to arrive at the scaled time indicated, the long column drained for a total of 101.2 hours while the data for the short (model) column were obtained in only 10.3 hours. Error due to evaporation at the outflow tube and diffusion of liquid from the column through the joints could account for the minor difference in discharge since such a long time was required to drain the long column.

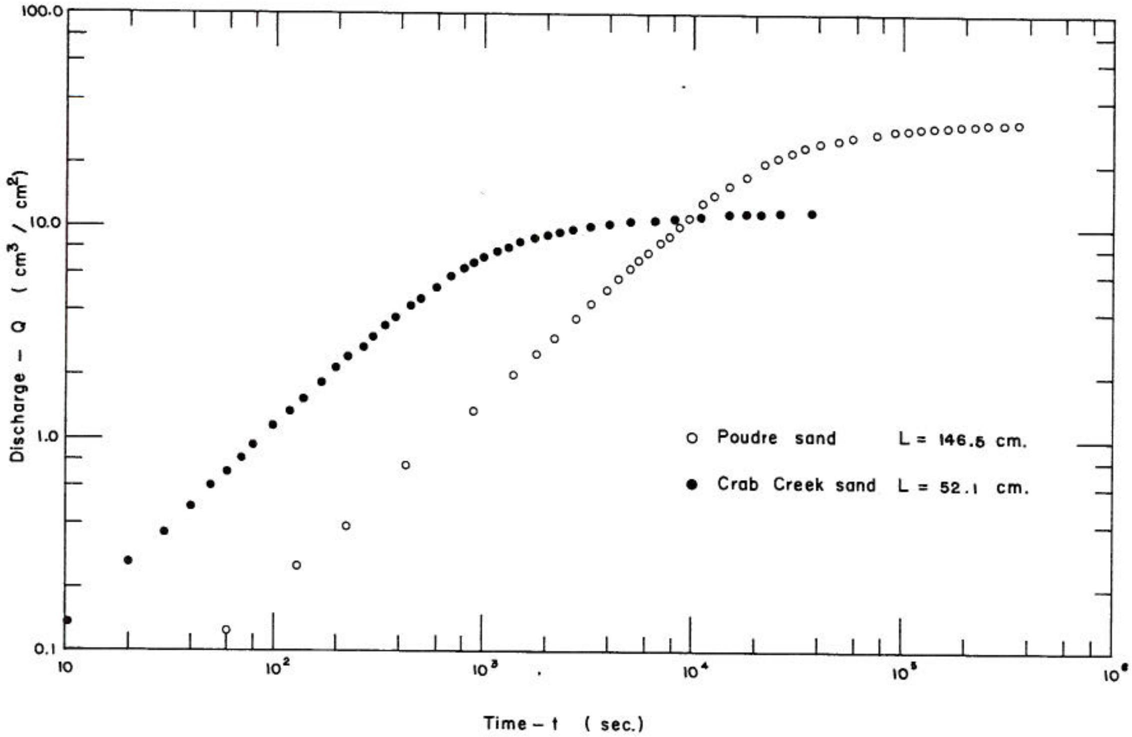

Figure 1. Discharge-time relationships of unsteady drainage from two columns of similar media. Scaled length, L. , is 3.75.

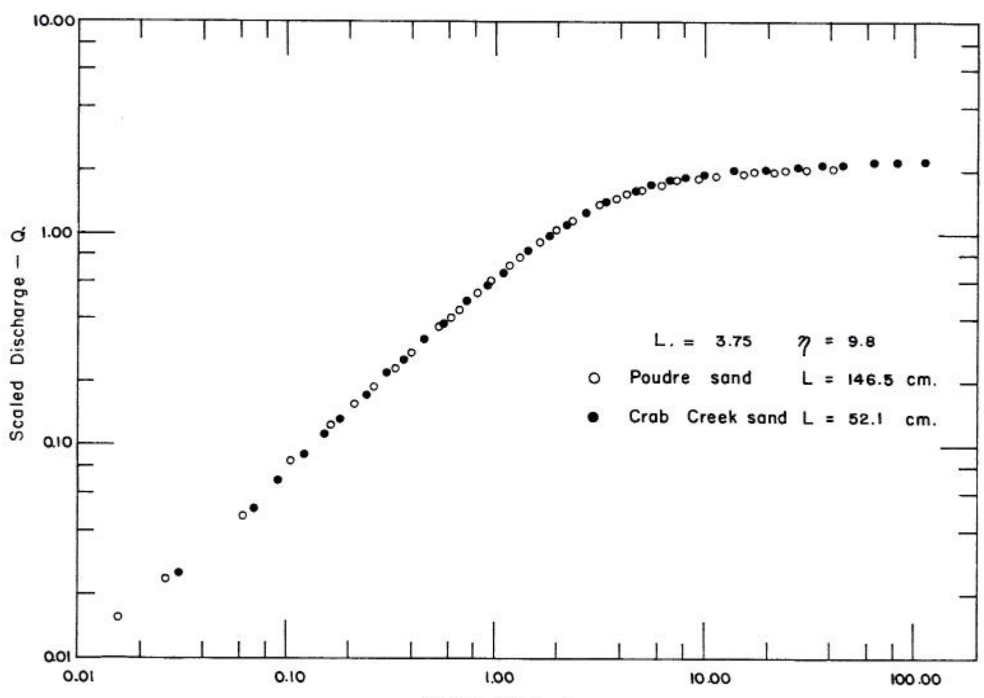

Figure 2. Scaled discharge-time relationship of unsteady drainage from two columns of similar media.

To demonstrate the effect of the value of the parameter η on nonsteady drainage of a column, two columns having vastly different values of this parameter were packed and drained. The materials used were Pullman clay and the 246-295 micron fraction of Poudre sand referred to herein as Poudre sand (uniform). The columns were cut to the same scaled length as those used in testing the theory. They could not, however, be considered similar because of their different η values. The scaled capillary pressure-effective permeability data for each are presented in Figure 3 along with that of Crab Creek sand for comparison. The pertinent scaling parameters are given in the following table.

Table 2. Scaling parameters for three dissimilar media.

	Pullman Clay	Crab Creek Sand	Poudre Sand (uniform)
η	3.8	9.7	18.4
$P_b/\rho g$ (cm)	7.6	13.9	11.6
K (μ^2)	14.7	29.47	56.67
L (cm)	28.5	52.1	43.5
ρ_b (gm/cc)	1.25	1.54	1.53
ρ_s (gm/cc)	2.60	2.79	2.67
S_r	0.000	0.148	0.191
ϕ	0.519	0.448	0.429
ϕ_e	0.519	0.382	0.347

These columns were drained starting with the initial condition of $P_\cdot = 0$ at the bottom, while the top pressure was $0.40\ P_\cdot$ and $0.44\ P_\cdot$ for the clay and sand respectively. The scaled results appear in Figure 4. Complete data are presented in Appendix III.

Due to the complexity of the differential equations describing flow, there are two assumptions often used in approximating ground-water flow. The saturated flow theory ignores the flow in the capillary fringe and assumes that all flow takes place below the water table. The capillary tube theory treats the porous media as if it were a series of capillary tubes.

These assumptions were used to determine the scaled discharge - time relationships resulting from each in order to compare them with the data obtained from actual media and are also presented in Figure 4. The theoretical analysis resulting from each assumption is presented in Appendix II.

The information shown in Figure 4 indicates the influence of the value of the parameter η on the discharge - time relationship. It should be pointed out that these data represent discharge from columns having a scaled length of 3.75. The curves would be different for various other scaled lengths. In fact, it appears reasonable that the greater the scaled length, the less influence that η would have on the relationships. Since the scaled length is the actual length divided by the bubbling pressure head, the greater the scaled length becomes, the less influence the capillary fringe depth (represented by bubbling pressure) has on the total amount drained from a particular column. In other words, if the columns were long enough, most of the drainable liquid would be removed and the total scaled amount removed would be almost the same for all columns regardless of the value of η for the particular media.

These data also indicate that the influence of η becomes less pronounced as η increases. The difference in the relationship for $\eta = 3.8$ and $\eta = 9.7$ is greater than the difference between $\eta = 9.7$ and $\eta \rightarrow \infty$. This is to be expected since, for high η values, the effective permeability reduces tremendously with small increases in capillary pressure and whether η is 15 or 25 makes very little difference since these values are both very high. The saturated pores of media with high η's empty rapidly until the capillary pressure reaches a value greater than bubbling pressure after which the drainage process is greatly slowed due to the sudden drop in effective permeability associated with small capillary pressure increases above bubbling pressure. After the initial surge of outflow there is also very little drainable liquid left since a medium with a high η value must have rather uniform pore sizes. In fact, as the pore sizes all approach one particular size $\eta \rightarrow \infty$. This is the reason for indicating that a capillary tube is analogous to a medium having infinite η.

The theoretical systems represented in Figure 4 indicate the extent of error involved in making such assumptions. The saturated flow theory which ignores the capillary fringe does not represent the flow process well and the amount drained in a given time was approximately 40 percent in error early in the flow process and this error became greater as the column drains further. The capillary tube theory follows the true relationships quite well for media with high η values. This should be true since media with uniform pore sizes are, in fact, analogous to capillary tubes.

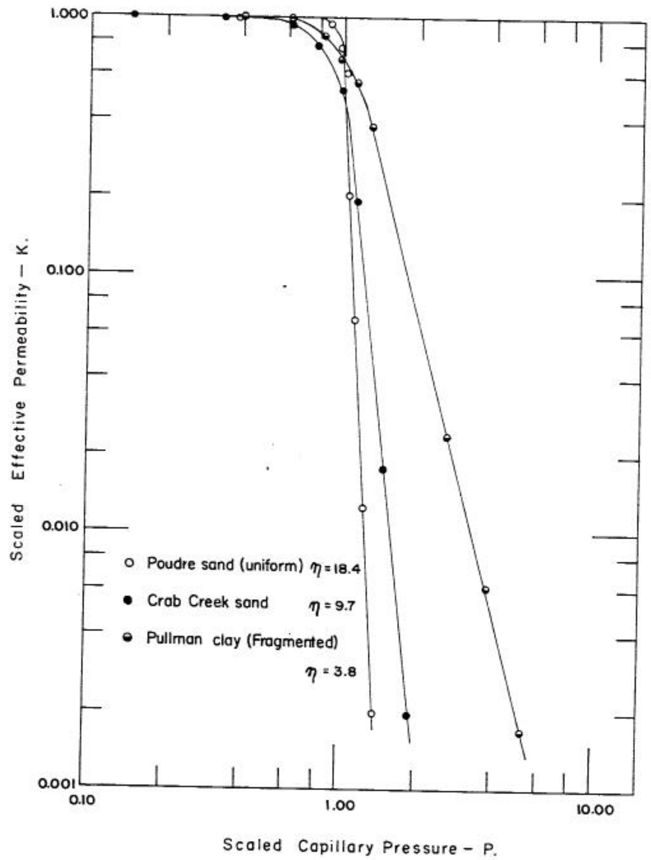

Figure 3. Scaled capillary pressure-effective permeability relationships for three dissimilar media. (obtained on the drainage cycle)

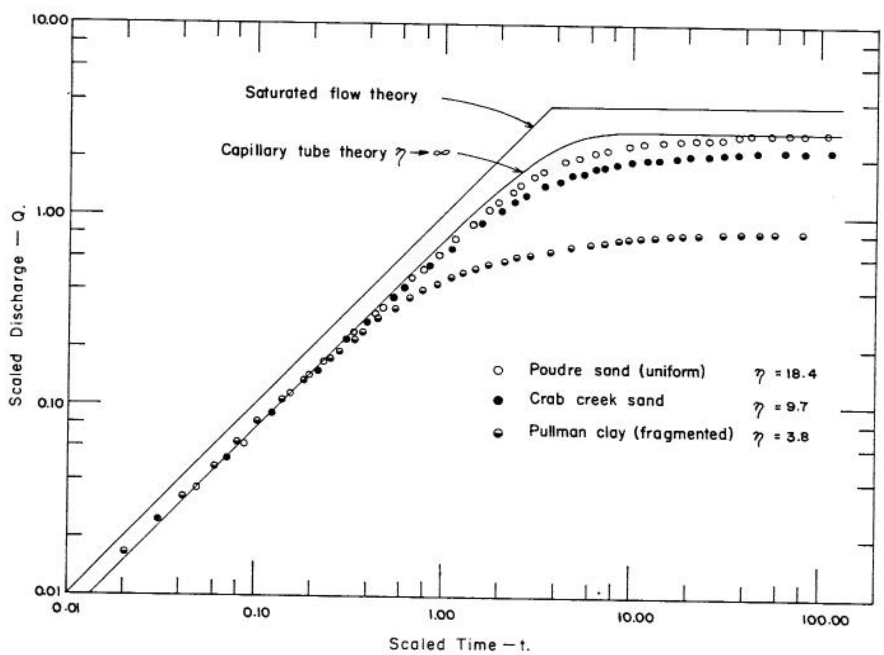

Figure 4. Scaled discharge-time relationships of unsteady drainage from three columns of dissimilar media and two theoretical systems. Scaled length, L. , is 3.75.

SELECTING MEDIA FOR MODELS

Actual model studies could be made for prototype systems which contained media of either high η values or low η values. There are many problems in engineering which would involve filter materials constructed of rather uniform sand sizes. In this case, the model would of necessity have to be constructed with material having a high η value. On the other hand, agricultural soils generally possess a wide range in pore sizes and therefore have low η values. There is need, therefore, for model materials with a wide range of the parameter η.

The model medium should also have a much lower bubbling pressure than the prototype material. Since bubbling pressure is the standard unit used to compute the length scales, if the model media did not have a sufficiently low bubbling pressure, the size of the model would approach that of the prototype. The amount of model size reduction, of course, depends on the particular problem being studied but in general one would want a size reduction factor of at least two.

Model media, then, should be available with a wide range of η values and all should possess rather low bubbling pressures. It appears that this will present little problem since in this study such materials were found or synthesized and as future work progresses other media will become available.

Finding high η media presents little difficulty since any material of uniform particle size will possess a high η value because the pore spaces will also be rather uniform. The bubbling pressure of such media can be controlled by selection of the particular size of uniform material used.

Intermediate η values (8-12) can be obtained from sands which themselves exhibit secondary porosities or the grains themselves are porous. Sands of basaltic origin possess this property and also possess rather low bubbling pressures. It would appear that one should be able to mix sands in such a manner as to create media of intermediate η values also.

Media possessing low η values and low bubbling pressures can be artificially developed by starting with tightly cemented clay aggregates and breaking them up into fine aggregates until the desired aggregate size is obtained. Such a medium would possess a very wide range of pore sizes yet have large pores due to the large aggregates present. Such a material would have to be modeled with a nonpolar liquid, however, so that the aggregates would remain stable throughout the flow process.

The question of how close the model η value must be to the prototype value before the media are sensibly similar will be important in model studies. Of course, to be truly similar the values must be exactly the same. This question was not studied in this research; however, some comments can be made.

Prototype materials possessing high η values (above 15) could undoubtedly be modeled with materials having η values within $\pm$ 20 percent of the prototype media. However, media with low values (3-8) should be modeled with media having η values within $\pm$ 5 percent of the true values. The above statement is based only on the experience gained in this study and the data shown in Figure 9, therefore it should be evaluated as such. Further studies are needed to answer this important question more precisely.

The scaling criteria were developed utilizing the various parameters (K, $P_b/\rho g$, η) determined from the drainage cycle. The values of these three parameters are different for imbibition than they are for drainage. It was not known, therefore, whether or not the laws would be valid if pressure reversals were encountered and hysteresis became important in a particular problem.

The adequacy of the scaling criteria for imbibition, given that two media behaved similarly on the drainage cycle, was determined by first finding media that behaved similarly on the drainage cycle. The two materials, Crab Creek sand and Poudre sand, used for the non-steady drainage test were used to run hysteresis loops of the capillary pressure-effective permeability relationships. After the drain-cycle relationships were determined for these materials, the imbibition cycle relationships were determined by measuring the capillary pressures and effective permeabilities at various points on this cycle. The scaled data were then plotted using the parameters from the drainage cycle.

The hysteresis loops were determined on short columns of media. The two materials, Poudre sand and Crab Creek sand, which behaved similarly on the drainage cycle, also reacted similarly on the imbibition cycle. The pertinent parameters of each are presented in Table 3.

Table 3. Scaling parameters for two similar media.

	Poudre Sand	Crab Creek Sand
η_d	9.60	9.70
K_d (μ^2)*	4.84	27.35
$P_b/\rho g_d$ (cm)	32.20	13.00
η_i	6.20	7.00
K_i (μ^2)	2.53	15.40
$P_b/\rho g_i$ (cm)	18.00	7.70

* μ^2 is the symbol for square microns = $cm^2 \times 10^{-8}$

The subscripts (i) and (d) refer to the quanities as measured on the imbibition and drainage cycles respectively. The graphical representation of these relationships is shown in Figure 5.

The scaling theory requires that effective permeability, K_e, be scaled by dividing it by the permeability, K, and that capillary pressure be scaled by dividing it by bubbling pressure or $K. = K_e/K$ and $P. = P_c/P_b$. When the information presented in Figure 5 was scaled according to the above, the result was a coalescence of data as indicated in Figure 6.

Klute and Wilkinson (11), in testing the Miller scaling theory, found that individual size fractions of a given sand reacted quite well as similar media. Poudre sand was sieved into various size fractions and hysteresis loops were run on each particular size fraction. The measured parameters of each are represented in Table 4.

Table 4. Scaling parameters for various size fractions of Poudre sand

Sand Size Microns	η_d	K_d (μ^2)	$P_b/\rho g_d$ (cm)	η_i	K_i (μ^2)	$P_b/\rho g_i$ (cm)
74-104	16.6	11.46	27.0	9.6	7.61	15.0
104-147	16.7	16.68	21.2	11.7	9.93	12.9
147-175	17.5	26.31	16.8	11.4	11.21	9.8
175-246	16.7	44.36	12.8	10.5	14.87	7.6
246-295	18.4	56.67	11.6	10.0	29.55	6.7
295-417	15.7	79.91	9.3	10.8	40.11	5.4

The drainage cycle data for the various size fractions are shown in Figure 7. For clarity, the data from the imbibition cycle were not included. Upon scaling these relationships, it was found that the fractions do, in fact, behave as similar media. Figure 8 contains these scaled data. The data from all of the various hysteresis loops are contained in Appendix III.

The hysteresis loops of the capillary pressure - effective permeability relationships for similar media indicate that the criteria is, in fact, valid for imbibition as well as drainage. At least for steady state flow, two media which are found to react similarly to drainage will perform similarly on the imbibition cycle. Since non-steady imbibition was not studied, one cannot state that the scaled time factor is valid for this cycle. However, scaled time includes those parameters used in scaling the hysteresis loop and it is quite likely that this scaled parameter is also valid for the imbibition cycle.

Evidently there is some relationship between the parameters as determined by drainage and those found by imbibition. The hysteresis loops were run on similar media having only two different values of η. Therefore insufficient experimental evidence was obtained to deduce these relationships. These data indicate, however, the possibility of such relationships. For example,

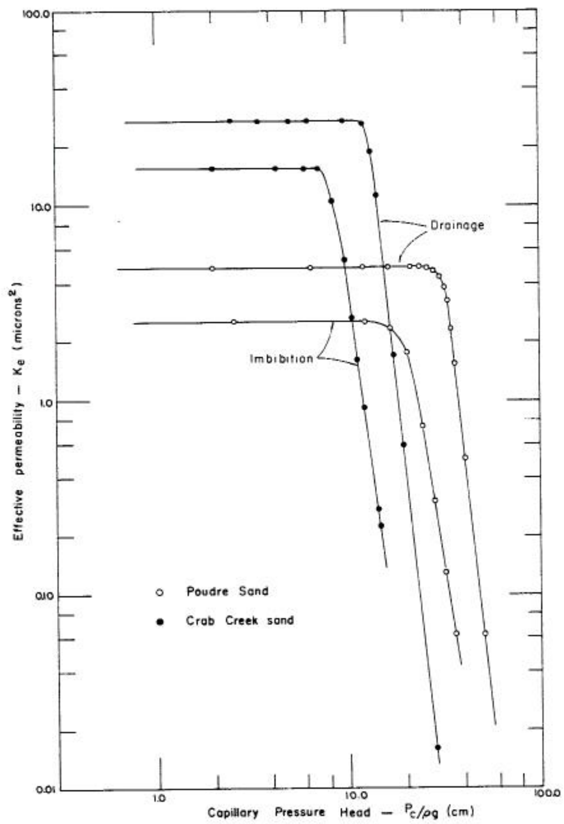

Figure 5. Capillary pressure-effective permeability relationships for Poudre and Crab Creek sands.

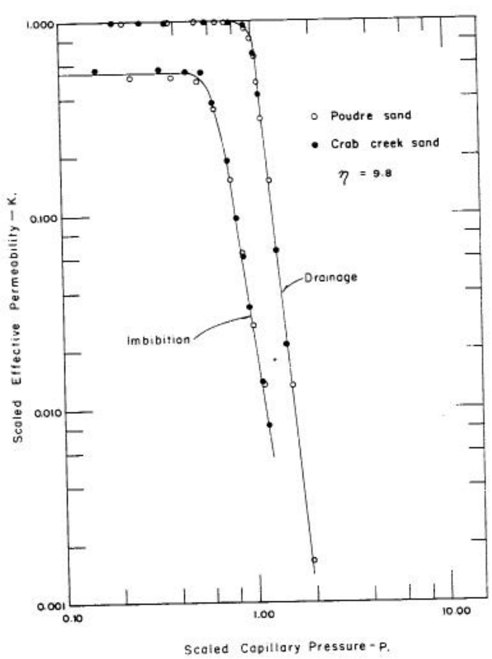

Figure 6. Scaled capillary pressure-effective permeability relationship for Poudre and Crab Creek sands.

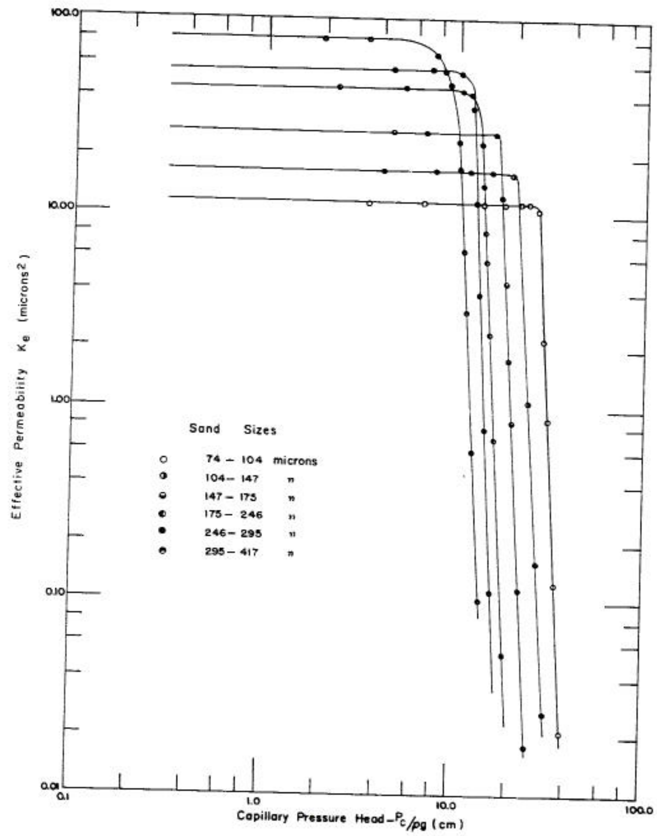

Figure 7. Capillary pressure-effective permeability relationships for various size fractions of Poudre sand. (obtained on the drainage cycle)

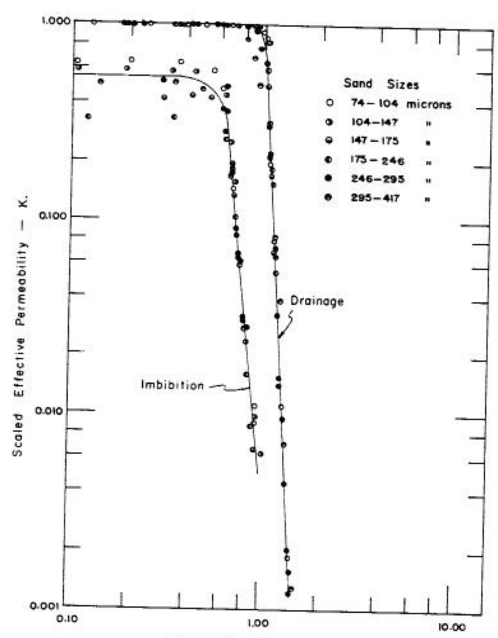

Figure 8. Scaled capillary pressure-effective permeability relationship for various size fractions of Poudre sand.

$$(P_b)_i = 0.6(P_b)_d \qquad (46)$$

is valid for each of the materials tested in this study, where the subscripts refer to the imbibition and drainage cycles.

There was no obvious constant relationship between the η values nor the permeabilities at zero capillary pressure as determined from the two cycles. This was to be expected with permeability since the value of the maximum permeability obtained by imbibition is a function of the time the column is allowed to flow at the low values of capillary pressure. Bloomsburg and Corey (1) found, that, if a column is allowed to flow sufficiently long, the imbibition permeability will eventually increase to a value equal to the drainage permeability. The reason for the lower value initially is due to the air that is entrapped and if allowed to operate long enough the air leaves the media by diffusion, thus the permeability increases to its drainage value. In this study no attempt was made to allow for diffusion of air, therefore the permeability values varied somewhat from column to column and undoubtedly were a function of the rapidity with which the data were obtained. The rate at which air diffuses out is a function of pore size and flow rate according to Bloomsburg and Corey. For static conditions a fine grained material becomes completely saturated faster than a coarse grained material. However, when flow occurs dissolved air is carried out with the liquid and coarse media, being more permeable than fine media, might reach complete saturation first. Therefore whether or not this range of pressures can be scaled is somewhat dependent on the flow rate.

SUMMARY

Scaling criteria were developed by scaling the differential equation describing non-steady flow in a partially saturated porous medium. The equation

$$\mathrm{div}\left[\frac{K_e}{\mu}\nabla\ (P_w + \rho g Z)\right] = \phi_e \frac{\partial S_e}{\partial t} \qquad (30)$$

is the result of combining Darcy's law with the continuity equation as it applies to flow in a porous material in which the gas phase is everywhere at atmospheric pressure.

The equation was scaled by selecting standard units of length, pressure, permeability and time. The bubbling pressure head was selected as the standard unit of length, the bubbling pressure as the unit of pressure, and the saturated permeability as the standard unit of permeability. Scaling equation (30) with these units resulted in two dimensionless parameters $L\rho g/P_b$ and $tK(\rho g)^2/P_b\mu\phi_e$.

The first of these parameters establishes the length scale ratios between model and prototype and the second establishes the time scale ratio. Neither of these ratios can be arbitrarily selected since they depend on the particular properties of the media.

The properties $P_b/\rho g$ and K are properties defined from the drainage cycle of a particular material. Therefore, theoretically, the scaling laws apply to drainage of media. The criteria for similar media are summarized below.

Two flow systems are similar if:

1. Geometric similarity exists.
2. The functional relationships between scaled values of K_e, P_c, and S are identical for both media. These relationships will be identical if both media have the same value of η where $\eta = -d(\ln K_e)/d(\ln P_c)$.
3. The initial and boundary conditions in terms of scaled variables are the same for both systems.

This theory was experimentally tested by measuring media properties and selecting those media with equal η values as model and prototype materials for further testing.

It was discovered that two media reacting similarly on the drainage cycle would also react similarly on the imbibition cycle even though the standard scaling units applied only to the drainage cycle. At least, this was true for steady state imbibition flow. A non-steady experiment involving imbibition was not performed.

Two columns of similar media having quite different values of permeability and bubbling pressure were subjected to an unsteady drainage process. Since the similar media had a wide variation in bubbling pressure one column was considerably shorter than the other and since the permeabilities were also quite different the time scales between the two were tremendously different. The short column, considered here as the model, was only 0.36 times as long as the long column (prototype). The process occurred approximately 26 times faster in the model than in the prototype.

The columns were initially completely saturated and then allowed to drain under gravity flow until the flow rate became negligible. The volume of liquid removed was measured at increments of time. These quantities were scaled according to the theory and plotted to determine if each column followed the same scaled discharge - time relationship. The scaled quantities were $Q_{.} = \frac{\rho g}{P_b \phi_e} Q$ and $t_{.} = \frac{(\rho g)^2 K}{P_b \mu \phi_e} t$ where $Q_{.}$ is the scaled volume removed per unit area and $t_{.}$ is scaled time. These data coalesced into a single scaled curve exceptionally well indicating that the scaled parameters were well chosen and the theory valid.

To demonstrate the effect of using dissimilar media, two columns of media with quite different η values were also drained. This allowed a comparison of the drainage relationship between three values of η for one scaled column length. These values of η (3.8, 9.7, and 18.4) represented a wide range since, according to Brooks and Corey (2), theoretically the smallest η value possible is 2 and the value of 18.4 is quite high for a natural medium. These drainage data indicated that the value of this parameter does significantly affect the drainage process. Differences, however, are more pronounced with low values than with high values of η.

Modeling media must possess the same value of η as the prototype media. Therefore, it is essential to have available model materials with various values of this parameter. The modeling materials should also possess low bubbling pressures in order that the model - prototype length ratio will be reasonable. The variations of η listed above were readily obtained by proper selection or synthesis of media, therefore these criteria should place little restriction on future model studies.

BIBLIOGRAPHY

1. Bloomsburg, G. L., and Corey, A. T. Diffusion of entrapped air from porous media. Colorado State University, Fort Collins, Colorado, Hydrology Paper No. 5, August 1964.

2. Brooks, R. H., and Corey, A. T. Hydraulic properties of porous media. Colorado State University, Fort Collins, Colorado, Hydrology Paper No. 3, March 1964.

3. Burdine, N. T. Relative permeability calculations from pore-size distribution data. Petroleum Transactions, AIME, 198:71-77, 1952.

4. Collins, R. E. Flow of Fluids Through Porous Materials. Reinhold Publishing Corp., New York, 1961, 270 pp.

5. Corey, A. T. The interrelation between gas and oil relative permeabilities. Producer's Monthly, 19(1):38-41, November 1954.

6. Craig, F. F., Geffen, T. M., and Morse, R. A. Oil recovery performance of pattern gas or water injection operations from model tests. Petroleum Transactions, AIME, 204:7-15, 1955.

7. Croes, G. A., and Schwarz, N. Dimensionally scaled experiments and the theories on the water-drive process. Petroleum Transactions, AIME, 204:35-48, 1955.

8. Darcy, H. Les Fontaines Publique de la Ville de Dijon. Dalmont, Paris, 1856, 570 pp.

9. Elrick, D. E., Scandrett, J. H., and Miller, E. E. Tests of capillary flow scaling. Soil Science Society of America Proceedings, 23(5):329-332, September - October 1959.

10. Jackson, R. D., Reginato, R. J., and Reeves, W. E. A mechanized device for packing soil columns, U. S. D. A., ARS 41-52, April 1962.

11. Klute, A., and Wilkinson, G. E. Some tests of the similar media concept of capillary flow: I. Reduced capillary conductivity and moisture characteristics data. Soil Science Society of America Proceedings, 22:278-281, July - August 1958.

12. Leverett, M. C., Lewis, W. B., and True, M. E. Dimensional-model studies of oil-field behaviour. Petroleum Technology, T. P. 1413:175-193, January 1942.

13. Miller, E. E., and Miller, R. D. Physical theory for capillary flow phenomena. Journal of Applied Physics, 27(4):324-332, April 1956.

14. Muskat, M. The Flow of Homogeneous Fluids Through Porous Media. J. W. Edwards Inc., Ann Arbor, Michigan, 1946, 763 pp.

15. Rapoport, L. A. Scaling laws for use in design and operation of water-oil flow models. Petroleum Transactions, AIME, 204:143-150, 1955.

16. Richards, L. A. Capillary conduction of liquids through porous mediums. Physics, 1:318-333, November 1931.

17. Richardson, J. G. Flow through porous media. Section 16, Handbook of Fluid Dynamics edited by V. L. Streeter, McGraw-Hill Book Company Inc., New York, 1961.

18. Ruark, R. E. Inspectional analysis: A method which supplements dimensional analysis. Journal of Elisha Mitchell Scientific Society, 51:127-133, 1935.

19. Scheidegger, A. E. The physics of flow through porous media. University of Toronto Press, Toronto, Canada, 1960, 313 pp.

20. Stallman, Robert W. Multiphase fluids in porous media - A review of theories pertinent to hydrologic studies. Geological Survey Professional Paper 411-E, U. S. Government Printing Office, Washington, D. C. 1964.

21. Wilkinson, G. E., and Klute, A. Some tests of the similar media concept of capillary flow: II. Flow systems data. Soil Science Society of America Proceedings 23(6): 434-437, November - December 1959.

APPENDIX I. MEASUREMENT OF MEDIA PROPERTIES

The scaling criteria required that certain fluid and medium properties be known before adequate modeling studies could be made. These properties included permeability, K, bubbling pressure, P_b, η, porosity, ϕ, residual saturation, S_r, and bulk density, ρ_b. A discussion of the methods used to determine these properties is presented here. The technique may be considered general in that they apply whenever one wants to measure any of the above regardless of the type of study.

Capillary Pressure - Effective Permeability Relationship

A sketch of the apparatus used for these determinations is shown in Figure 9. The 14 cm column of material is contained in 5 short sections of lucite tubing. The bottom 4 cm section is provided with a porous barrier to provide for drainage of the column. Two of the 2 cm sections contain tensiometers completely encircling the column. An inflow porous barrier is provided at the top. The material used for the porous inflow and outflow barriers and the tensiometers can be any porous material which has a higher bubbling pressure than the greatest capillary pressure anticipated in the particular experiment. Of course, if it did not have the high bubbling pressure, air would become entrapped in the system and erroneous results would be obtained. The material used in this study was a porous plastic, called "Porvic", which has a bubbling pressure head of approximately 180 cm of Soltrol oil, or 360 cm of water.

The connections between sections of lucite tubing are provided with small openings to facilitate air entry into the media at capillary pressures greater than bubbling pressure. These openings are not continuous from outside to inside the column. A groove is cut in the butt end of each section of tubing so that an air space is provided within the tubing wall at this point. This eliminates the problem of liquid being withdrawn from the media through the fine space which would be present at the joint if the sections were butted together without the annular air space.

The column is completely saturated before attaching it to the apparatus since drainage cycle properties are desired. This is accomplished by immersing the entire column in liquid in a chamber connected to a vacuum pump. Air is expelled from the chamber until a nearly full vacuum is reached. This pressure is held for approximately 30 minutes and upon release of the vacuum the material is completely saturated with liquid.

The column is then connected to the apparatus as shown in Figure 9. Care must be taken so that air does not enter the material or the outflow, inflow, and tensiometer leads during this operation. The inflow supply and outflow tubes are arranged beforehand such that the initial capillary pressure is near zero.

The inflow and outflow tubes are then carefully lowered until a small capillary pressure exists in the material. After the flow becomes steady, the discharge rate is measured and the tensiometer manometers read to determine the capillary pressure head. These readings should be corrected for the amount of capillary rise exhibited by the manometer tubes.

These data then are used to calculate the effective permeability from Darcy's law written in the form.

$$K_e = \frac{\mu Q \, \Delta L}{\rho \, g \, t \, \Delta H} \, 10^8 \tag{47}$$

where K_e is the effective permeability in square microns, Q is the volume of flow (cm^3/cm^2) through the column in time, t(sec), and $\Delta H/\Delta L$ is the pressure gradient (see Figure 9). Care should be taken to keep ΔH equal to ΔL so that the capillary pressure is constant throughout the column. This is accomplished by adjusting inflow and outflow heights to maintain ΔH at the desired amount.

After computing K_e for the particular $P_c/\rho g$, the inflow and outflow tubes are again lowered to a new value of capillary pressure. When the flow becomes steady the effective permeability is measured as before. This process is repeated until the entire relationship between capillary pressure and effective permeability is obtained.

The effective permeability does not vary greatly until the capillary pressure has reached bubbling pressure. This is to be expected, since, until bubbling pressure is reached, the pores are completely filled with liquid and the effective permeability is a constant in this region. This constant permeability is the maximum for the medium and is referred to merely as permeability.

The parameter η is defined as the negative of $d(\ln K_e)/d(\ln P_c)$ for $P_c > P_b$. The value of η, then, can be determined from the graph of the capillary pressure - effective permeability data. Brooks and Corey (2) defined bubbling pressure as the extrapolation of the straight line relationship between P_c and K_e to the ordinate representing $K_e = K$. This particular value of capillary pressure is called bubbling pressure because it has been found experimentally that, for homogeneous and isotropic media, it is very close to the capillary pressure at which the first non-wetting phase flow can be observed. The values of η and P_b can be determined by a least squares analysis of the data, since this is undoubtedly a more objective method than determining these parameters graphically.

The technique of using a short column to define media properties has several advantages over using longer columns. The column can be packed more easily and undoubtedly more uniformly. It is easier to control the capillary pressure within the column, since the entire column responds to changes in outflow and inflow heights quite rapidly. Perhaps the

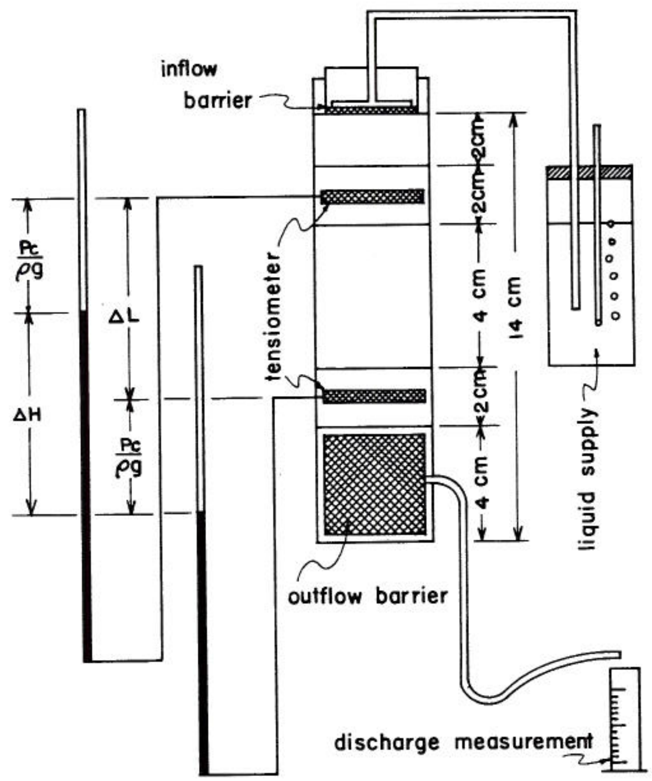

Figure 9. Sketch of apparatus used to determine capillary pressure-effective permeability relationships.

greatest advantage is the time required in determining the complete capillary pressure - effective permeability relationship. Each time the inflow and outflow is lowered to produce a higher capillary pressure in the column, one must wait until the flow has reached steady state before a determination of permeability can be made. With a short column this may be a matter of hours while a long column can take days to reach such a condition.

This is not to imply that one should always use a short column to measure the properties of a material. For model studies, one should construct the model and then measure the properties of the medium in the model itself since with present techniques it is difficult to pack materials with the degree of reproducibility that is necessary to insure knowing the properties from a sample. This is, perhaps, the most objectionable feature of using the modeling criteria presented herein to scale a given prototype problem. In other words, one does not know if the model contains a medium similar to the prototype until it is constructed and only then after these relationships have been measured in place in the model. This problem is not insurmountable, however, and with improved packing techniques, perhaps eventually one will be able to construct a model and know beforehand what the properties are by measuring them in a column and packing them precisely the same in the model.

A great many materials were tested in this study since a search for similar media was made. In making these many tests, several significant effects concerning experimental techniques were noted and are worthy of mention.

Packing Columns

As was previously mentioned, the permeability is constant at capillary pressures below bubbling pressure and varies as a power function of capillary pressure above bubbling pressure. The transition zone between these two relationships does not appear to be unique and depends on experimental technique to a certain extent.

It was discovered that this transition can be governed by the boundary effect between the column wall and the medium itself. The first columns studied were packed with the Jackson (10) packer which vibrated the column during the filling operation. However, some columns were packed by filling the tubes with material and vibrating only after the entire length was filled. It was discovered that the transition of the K_e - P_c curve from the constant K_e value to the power relationship was much sharper in those colums which were not vibrated during the filling operation. In fact, some of the columns which were vibrated while filling had such a large transition zone that there were apparently two η values for the material.

Since η is a function of pore-size distribution, it seemed unlikely that a given material packed uniformly could possess such a characteristic. Upon close inspection of a vibrating column, it was observed that at the top of the sand violent movement was taking place. The material seemed to circulate throughout the top inch of the column. This action caused a sorting of the particles with the finer particles boiling up in the center and the larger ones being shifted to the column boundary. Then, if one vibrates during the complete filling operation, this effect of sorting grain sizes is present throughout the entire column length. When the K_e - P_c relationship is run on such a column, the material near the wall desaturates first due to the large pores there and the permeability lowers prematurely since the material in the center may still be completely saturated. Thus, the K_e - P_c curve may exhibit an inflection as air finally enters the central section, after which the η for the finer material is obtained.

For the above reasons all columns used for the verification of the scaling theory were vibrated only after being filled with material. Since vibration (even then) caused sorting of materials on top, the top 10 cm of each newly packed column was removed before testing.

Technique for Raising Capillary Pressure

The capillary pressure was raised to a new value simply by lowering the inflow and outflow tubes of the apparatus. It was discovered that the manner in which this was performed could also affect the transition zone between saturated permeability and the variable permeability.

If, at capillary pressures just below bubbling pressure, the tubes were lowered a large amount, an unusually low permeability would result. This capillary pressure is very near the pressure at which air enters the material, therefore, a sudden jump to a pressure larger than bubbling pressure caused air to enter quite suddenly. The new pressure was not instantaneously experienced throughout the entire column and, at first, only affected the material near the inflow and outflow barriers. Air entered these areas and acted as a dam for the liquid remaining above. This sudden reduction in permeability is accompanied by a much lower discharge through the column which reduces the head loss through the capillary inflow and outflow barriers. If the reduction is sufficiently large, the capillary pressure within the column will actually be decreased even though the inflow and outflow tubes have been lowered. This lowering of capillary pressure was observed many times before the technique of using very small increments of pressure, near bubbling pressure, was employed.

The decrease in capillary pressure is not associated with an increase in permeability since the system is approaching the imbibition process rather than retreating on the drainage cycle. Inspection of any of the figures representing capillary pressure - effective permeability relationships show that reductions of capillary pressure caused no change in permeability until the curve representing the imbibition process is reached.

Capillary pressure reversals during the drainage process, then, should not be tolerated since the flow system immediately approaches the imbibition cycle and erroneous results will be obtained. Should imbibition occur, one need not terminate the experiment and start anew. He need only increase the capillary pressure sufficiently so that the process will again revert to the drainage cycle. Of course, no data will be available in the area of the pressure reversal.

This analysis emphasizes another advantage of a short column. It is always desirable to keep the hydraulic gradient within the column equal to unity during these tests so that the capillary pressure will be constant throughout. Then, since pressure reversals should not be allowed, if one lowers the inflow an excessive amount, he cannot correct this by raising it but must obtain the correct gradient by lowering the outflow. In a long column, this could mean no data would be taken over a large range of capillary pressure. Excessive lowering of either inflow or outflow is not uncommon since one cannot predict the change in capillary pressure which will result from lowering the inflow and outflow a given amount. The reason is that the capillary barriers used at the inlet and outlet provide a head loss and this loss is dependent on the discharge through them. Since one does not know what the flow rate will be at some higher capillary pressure, he does not know what this head loss will be. Since a great deal of time is required to reach steady flow in a long column, one is too late in knowing that he has lowered either the inflow or outflow excessively.

Porosity, Residual Saturation, and Bulk Density

The porosity, ϕ , of a material can be determined by utilizing the bulk density, ρ_b , of the medium and the particle density, ρ_s , through the relationship

$$\phi = \frac{\rho_s - \rho_b}{\rho_s} \qquad (48)$$

The bulk density can be measured by taking the entire amount of material in a section of the column, oven drying it, and weighing the dry material from the known volume of the column. The particle density, ρ_s is measured with a standard procedure using pycnometer bottles.

Since effective porosity, ϕ_e , is defined as $(1 - S_r)\,\phi$ it is necessary to measure residual saturation, S_r, beforehand. For sand materials this can be done by measuring the saturation at a very high capillary pressure but for fragmented clay material a complete curve of saturation versus capillary pressure should be drawn and the method discussed by Brooks and Corey (2) used to calculate S_r. The reason that it is necessary to run the entire curve for clay material is that it has very fine pores and one cannot conveniently subject it to a sufficiently high capillary pressure to ensure the medium being at residual saturation.

APPENDIX II. THEORETICAL ANALYSIS OF TWO ASSUMED DRAINAGE SYSTEMS

Two theoretical drainage systems were presented in Figure 4. The analysis leading to these results is presented below.

1. Saturated flow theory:

Assume that drainage is complete as the front progresses down through the column and that it continues until the front reaches the bottom after which there is no flow. Darcy's law for this case can be written in the form

$$q = \frac{K}{\mu} \rho g \frac{dH}{dZ} \tag{49}$$

where H is the hydraulic head and Z is the elevation, both measured upward from the bottom of the column. Since q is the volume flux of fluid it can be replaced by dQ/dt. The gradient, dH/dZ, is always 1 since this was the original assumption. Substituting these in equation 49 gives

$$dQ/dt = \frac{K}{\mu} \rho g \ . \tag{50}$$

Integrating we obtain

$$Q = \frac{K\rho g}{\mu} t + c \ .$$

But $c = 0$, since $Q = 0$ when $t = 0$. Then

$$Q = \frac{K}{\mu} \rho g \, t \ . \tag{51}$$

Now, from the scaling theory

$$Q = \frac{P_b}{\rho g} \phi_e Q.$$

and

$$t = \frac{P_b \phi_e \mu}{(\rho g)^2 K} t.$$

Substituting these scaled variables into equation 51 gives

$$Q. = t. \tag{52}$$

This indicates that Q. increases exactly as t. until all fluid is drained or until Q. = L. = 3.75 in this case.

2. Capillary tube theory:

Here, it is assumed that the tube drains similarly to that in the above theory; however, in this case the tube cannot drain completely to the bottom since there will be an amount equal to the capillary rise which will never drain from the tube. This assumption is equivalent to assuming a soil with an η value approaching infinity. The Hagen - Poiseuille equation describes such flow. It is written as

$$q = \frac{R^2 \rho g}{8 \mu} \frac{dH}{dZ} \tag{53}$$

where R is the radius of the capillary tube. This expression is analogous to Darcy's law where $K = R^2/8$. For this case

$$dH/dZ = \frac{Z - h_o}{Z} \tag{54}$$

where h_o is the capillary rise in the tube. Since Q is the volume removed per unit area,

$$Q = L - Z \tag{55}$$

where L is the column length. As in the saturated flow theory,

$$q = dQ/dt$$

or from equation 55

$$q = - dZ/dt \ . \tag{56}$$

Substituting equations 54 and 56 into 53 gives

$$-dZ/dt = \frac{R^2 \rho g}{8\mu} \left(\frac{Z - h_o}{Z}\right)$$

or

$$\frac{Z}{h_o - Z} dZ = \frac{R^2 \rho g}{8\mu} dt. \tag{57}$$

Integration of 57 results in

$$ho - Z - h_o \ln (h_o - Z) = \frac{R^2 \rho g}{8\mu} t + c.$$

When $t = 0$, $Z = L$ then $c = h_o - L - h_o \ln(h_o - L)$

and

$$L - Z - h_o \ln\left(\frac{h_o - Z}{h_o - L}\right) = \frac{R^2 \rho g}{8\mu} t \ . \tag{58}$$

Substituting $Z = L - Q$ from equation 55 gives

$$Q - h_o \ln \left(1 - \frac{Q}{L - h_o}\right) = \frac{R^2 \rho g}{8\mu} t \ . \tag{59}$$

For a capillary tube $K = R^2/8$, $P_b/\rho g = h_o$, and $\phi_e = 1$; therefore

$$Q = h_o Q. \quad \text{and} \quad t = \frac{8\mu h_o}{R^2 \rho g} t.$$

Substitution of these relationships into equation 59 gives

$$Q. - \ln \left(1 - \frac{h_o Q}{L - h_o}\right) = t.$$

or

$$Q. - \ln \left(1 - \frac{Q.}{L. - 1}\right) = t. \tag{60}$$

since $L_* = \frac{L}{h_o}$ for a capillary tube. Equation 60 describes the scaled discharge - time relationship for a capillary tube. For the particular case studied $L_* = 3.75$. This equation was used to determine the relationship shown in Figure 4.

APPENDIX III

Media Properties and Drainage Data

TABLE 1. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR POUDRE SAND*, SHORT COLUMN

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	$\Delta H/\Delta L$ $\Delta L = 6.0$	$q \times 10^4$ cm/sec	$P_c/\rho g$ cm	K_e μ^2	K_*	P_*
			Drainage				
25.0	1.868	0.983	25.61	6.8	4.86	1.01	0.21
"	"	0.983	25.37	12.1	4.81	1.00	0.38
"	"	0.992	25.54	16.2	4.81	1.00	0.50
"	"	0.983	25.39	21.2	4.82	1.00	0.66
"	"	0.983	25.61	23.5	4.87	1.01	0.73
"	"	1.000	25.49	26.1	4.76	0.99	0.81
"	"	1.000	25.22	28.0	4.71	0.98	0.87
"	"	1.000	23.49	30.4	4.39	0.91	0.94
"	"	1.000	20.56	32.2	3.84	0.79	1.00
"	"	1.016	17.77	33.5	3.26	0.67	1.04
"	"	1.000	12.69	34.8	2.37	0.49	1.08
"	"	1.083	8.96	36.4	1.55	0.32	1.11
"	"	1.058	2.89	40.2	0.51	0.11	1.25
"	"	1.124	0.38	50.7	0.064	0.013	1.57
"	"	0.992	0.04	62.6	0.008	0.0016	1.94
			Imbibition				
25.0	1.868	0.917	0.17	41.0	0.034	0.007	1.37
"	"	1.066	0.37	36.3	0.064	0.013	1.13
"	"	0.959	0.68	32.2	0.13	0.027	1.00
"	"	1.000	1.65	28.3	0.31	0.064	0.88
"	"	1.032	4.05	24.6	0.73	0.15	0.76
"	"	0.950	9.00	20.4	1.77	0.37	0.63
"	"	0.983	12.83	17.0	2.44	0.50	0.53
"	"	1.000	13.60	12.7	2.54	0.53	0.39
"	"	0.983	13.32	2.6	2.53	0.52	0.24

*$\eta = 9.6$, $P_b/\rho g = 32.2$ cm, $K = 4.84\mu^2$, $\rho_b = 1.41$ gm/cc, $\rho_s = 2.74$ gm/cc, $\phi = 0.485$

TABLE 2. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR CRAB CREEK SAND*, SHORT COLUMN

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	$\Delta H/\Delta L$ $\Delta L = 6.0$	$q \times 10^3$ cm/sec	$P_c/\rho g$ cm	K_e μ^2	K.	P.
			Drainage				
25.0	1.868	1.083	15.83	2.5	27.30	1.00	0.19
"	"	1.000	14.59	3.5	27.25	1.00	0.27
"	"	0.983	14.47	5.0	27.49	1.01	0.39
"	"	0.983	14.41	7.4	27.37	1.00	0.57
"	"	1.000	14.50	9.6	27.08	0.99	0.74
"	"	1.000	14.28	12.2	26.68	0.98	0.94
"	"	1.050	10.50	13.0	18.68	0.68	1.00
"	"	1.015	6.26	14.4	11.51	0.42	1.10
"	"	0.950	0.92	17.1	1.81	0.066	1.32
"	"	0.950	0.30	19.2	0.59	0.022	1.47
"	"	1.000	0.09	28.2	0.016	0.0006	2.17
			Imbibition				
25.0	1.868	0.983	0.12	14.8	0.23	0.008	1.14
"	"	0.933	0.19	14.3	0.38	0.014	1.10
"	"	1.032	0.51	12.3	0.93	0.034	0.95
"	"	1.015	0.89	11.4	1.63	0.060	0.88
"	"	1.032	1.40	10.8	2.53	0.093	0.83
"	"	1.025	1.46	10.6	2.65	0.097	0.82
"	"	1.025	2.89	9.7	5.27	0.19	0.75
"	"	0.942	5.30	8.3	10.50	0.38	0.64
"	"	0.975	8.05	7.2	15.28	0.56	0.55
"	"	1.000	8.22	6.0	15.35	0.56	0.46
"	"	1.000	8.26	4.3	15.43	0.56	0.33
"	"	1.000	8.37	2.0	15.64	0.57	0.15

*η = 9.7, $P_b/\rho g$ = 13.0cm, K = 27.35 μ^2, ρ_b = 1.54 gm/cc, ρ_s = 2.79 gm/cc, ϕ = 0.448

TABLE 3. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR THE 74-104 MICRON FRACTION OF POUDRE SAND**

			Drainage				
28.3	1.836	0.983	61.45	3.4	11.47	1.00	0.13
"	"	0.950	59.71	6.8	11.54	1.01	0.25
28.4	1.833	0.967	60.09	10.6	11.39	0.99	0.39
"	"	0.983	62.09	13.4	11.57	1.01	0.50
28.5	1.830	0.967	60.66	17.8	11.48	1.00	0.66
"	"	0.992	61.45	21.7	11.34	0.99	0.80
"	"	0.933	58.17	23.8	11.41	1.00	0.88
28.6	1.827	0.992	57.52	26.7	10.60	0.92	0.99
28.5	1.830	0.900	49.83	27.5	10.15	0.88	1.02
28.0	1.845	0.817	10.04	29.6	2.27	0.20	1.09
27.9	1.848	0.967	4.65	31.5	0.89	0.078	1.17
27.6	1.858	1.000	0.68	35.1	0.13	0.011	1.30
27.3	1.868	1.150	0.13	39.8	0.02	0.002	1.47
			Imbibition				
27.0	1.877	1.200	0.81	25.6	0.13	0.011	0.95
27.5	1.861	1.000	3.63	20.8	0.68	0.06	0.77
27.0	1.877	0.950	8.54	19.0	1.69	0.15	0.70
"	"	0.850	24.83	16.6	5.48	0.48	0.61
"	"	1.000	35.98	14.7	6.75	0.59	0.54
28.0	1.845	0.933	37.59	9.8	7.43	0.65	0.36
"	"	1.015	41.97	5.4	7.62	0.66	0.20
28.2	1.838	1.000	40.10	2.8	7.37	0.64	0.10

**η = 16.6, $P_b/\rho g$ = 27.0 cm, K = 11.46 μ^2, ρ_b = 1.38 gm/cc, ρ_s = 2.74 gm/cc, ϕ = 0.496

TABLE 4. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR THE 104-147 MICRON FRACTION OF POUDRE SAND*

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	$\Delta H/\Delta L$ $\Delta L = 6.5$	$q \times 10^4$ cm/sec	$P_c/\rho g$ cm	K_e μ^2	K.	P.
			Drainage				
29.1	1.811	1.000	92.52	4.1	16.76	1.00	0.19
"	"	0.938	88.57	7.3	17.09	1.02	0.37
"	"	1.000	92.58	11.9	16.79	1.01	0.56
"	"	1.015	92.23	15.2	16.45	0.99	0.71
28.7	1.823	0.970	89.53	19.5	16.32	0.98	0.92
28.0	1.845	0.862	5.16	24.7	1.09	0.065	1.16
28.7	1.823	1.031	0.91	28.0	0.16	0.010	1.32
28.2	1.839	1.045	0.15	31.3	0.027	0.0016	1.48
			Imbibition				
28.5	1.829	1.015	0.90	19.9	0.16	0.010	0.94
"	"	0.993	2.14	17.7	0.40	0.024	0.83
"	"	1.000	14.57	15.1	2.66	0.16	0.71
28.6	1.827	0.993	23.63	14.3	4.35	0.26	0.67
28.3	1.836	1.000	40.49	13.4	7.43	0.45	0.63
"	"	1.022	53.40	12.4	9.58	0.57	0.58
"	"	1.045	55.71	7.0	9.78	0.59	0.33
28.5	1.829	1.000	54.30	2.2	9.93	0.60	0.10

*$\eta = 15.4$, $P_b/\rho g = 20.7$ cm, $K = 16.67\ \mu^2$, $\rho_b = 1.40$ gm/cc, $\rho_s = 2.74$ gm/cc, $\phi = 0.489$

TABLE 5. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR 147-175 MICRON FRACTION OF POUDRE SAND**

			Drainage				
31.0	1.755	0.754	114.50	6.8	26.66	1.01	0.41
"	"	0.770	113.90	9.8	25.99	0.99	0.58
"	"	0.792	119.20	15.3	26.41	1.00	0.94
30.8	1.760	1.045	75.50	17.6	12.70	0.48	1.05
30.5	1.768	1.000	25.58	18.5	4.52	0.17	1.10
31.4	1.747	0.938	9.60	19.5	1.79	0.068	1.16
30.0	1.783	0.954	4.57	20.5	0.85	0.032	1.22
29.5	1.799	1.015	0.66	22.9	0.12	0.004	1.36
29.1	1.812	1.061	0.11	25.5	0.02	0.001	1.52
			Imbibition				
29.0	1.815	1.030	0.97	15.6	0.17	0.006	0.93
29.8	1.789	1.015	4.56	13.5	0.80	0.03	0.80
30.0	1.783	1.015	12.54	12.3	2.20	0.08	0.73
"	"	1.015	20.43	11.8	3.59	0.14	0.70
"	"	1.015	29.89	11.3	5.25	0.20	0.67
"	"	0.855	46.94	10.1	9.80	0.37	0.60
"	"	1.015	63.81	8.9	11.20	0.42	0.53
"	"	1.045	65.81	7.1	11.22	0.43	0.42
"	"	0.977	60.54	5.1	11.05	0.42	0.30

**$\eta = 17.5$, $P_b/\rho g = 16.8$ cm, $K = 26.31\ \mu^2$, $\rho_b = 1.43$ gm/cc, $\rho_s = 2.74$ gm/cc, $\phi = 0.478$

TABLE 6. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR THE 175-246 MICRON FRACTION OF POUDRE SAND*

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	$\Delta H/\Delta L$ $\Delta L = 6.5$	$q \times 10^3$ cm/sec	$P_c/\rho g$ cm	K_e μ^2	K.	P.
			Drainage				
29.5	1.799	0.570	14.04	2.4	44.38	1.00	0.18
29.0	1.815	0.570	14.09	5.2	44.92	1.01	0.41
"	"	0.585	14.10	10.2	43.78	0.99	0.80
"	"	0.985	14.00	13.3	25.80	0.58	1.04
"	"	0.985	12.18	13.5	22.45	0.51	1.06
29.2	1.808	0.985	7.68	13.8	14.09	0.32	1.08
29.1	1.811	0.969	4.35	14.1	8.12	0.18	1.10
29.0	1.815	0.985	3.65	14.3	6.73	0.15	1.12
28.3	1.836	0.955	1.25	15.3	2.41	0.054	1.19
28.5	1.829	1.006	0.38	16.3	0.69	0.015	1.28
28.0	1.845	1.022	0.03	19.1	0.05	0.001	1.50
			Imbibition				
28.5	1.829	1.015	0.22	11.5	0.39	0.009	0.90
29.0	1.815	1.000	0.40	10.8	0.72	0.016	0.85
"	"	1.000	0.73	10.2	1.33	0.03	0.80
"	"	1.006	1.54	9.6	2.77	0.06	0.75
28.0	1.845	1.006	2.47	9.2	4.53	0.10	0.72
28.5	1.829	1.006	3.62	8.9	6.57	0.15	0.69
28.8	1.821	1.006	4.74	8.6	8.56	0.19	0.67
29.0	1.815	0.925	5.96	8.1	11.72	0.26	0.63
"	"	1.170	9.58	5.6	14.87	0.34	0.44
"	"	1.030	8.45	1.6	14.87	0.34	0.13

*$\eta = 16.7$, $P_b/\rho g = 12.8$ cm, $K = 44.36\ \mu^2$, $\rho_b = 1.44$ gm/cc, $\rho_s = 2.67$ gm/cc, $\phi = 0.461$

TABLE 7. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR 246-295 MICRON FRACTION OF POUDRE SAND**

			Drainage				
29.0	1.815	0.858	26.37	4.5	55.72	0.98	0.39
"	"	0.879	27.45	7.1	56.67	1.00	0.61
29.1	1.812	0.899	27.15	10.2	54.71	0.97	0.88
"	"	1.032	24.82	11.2	45.57	0.77	0.97
"	"	1.016	20.32	11.9	36.23	0.64	1.02
"	"	1.004	6.52	12.7	11.77	0.21	1.09
29.0	1.815	1.000	2.16	13.4	3.93	0.07	1.15
28.0	1.845	1.000	0.43	14.5	0.80	0.014	1.25
"	"	1.008	0.06	16.3	0.11	0.002	1.41
			Imbibition				
27.8	1.852	1.004	0.86	9.8	1.59	0.028	0.85
28.0	1.845	1.004	2.06	8.7	3.79	0.07	0.75
28.5	1.829	1.004	5.55	8.0	10.11	0.18	0.69
28.9	1.818	1.004	11.27	7.5	20.42	0.36	0.65
29.0	1.815	1.008	15.18	7.1	27.33	0.48	0.61
"	"	1.052	17.13	3.3	29.55	0.52	0.29

**$\eta = 18.4$, $P_b/\rho g = 11.6$ cm, $K = 56.67\ \mu^2$, $\rho_b = 1.52$ gm/cc, $\rho_s = 2.67$ gm/cc, $\phi = 0.429$

TABLE 8. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR 295-417 MICRON FRACTIONS OF POUDRE SAND*

Temp. ^{o}C	$\mu/\rho g \times 10^5$ cm sec	$\Delta H/\Delta L$ ΔL = 6.0	q x 10^3 cm/sec	$P_c/\rho g$ cm	K_e μ^2	K.	P.
			Drainage				
28.8	1.821	0.700	30.71	1.9	79.89	1.00	0.20
"	"	0.723	32.19	3.3	79.94	1.00	0.35
"	"	0.866	32.10	7.5	67.45	0.84	0.81
28.7	1.824	1.032	31.05	8.3	54.80	0.69	0.89
"	"	1.000	26.04	8.8	47.49	0.59	0.94
28.5	1.829	1.024	13.46	10.0	24.02	0.30	1.08
"	"	0.992	9.29	10.2	17.13	0.21	1.10
"	"	0.992	3.53	11.0	6.51	0.08	1.18
28.6	1.827	0.992	1.71	11.5	3.14	0.039	1.23
28.7	1.824	1.032	0.34	12.6	0.60	0.008	1.36
28.4	1.830	1.065	0.06	14.2	0.10	0.001	1.53
			Imbibition				
28.8	1.821	1.073	0.18	10.8	0.30	0.004	1.16
28.5	1.829	1.065	0.43	8.8	0.73	0.009	0.94
"	"	1.040	1.30	7.7	2.28	0.029	0.82
28.8	1.821	1.015	2.67	7.2	4.78	0.06	0.77
28.9	1.817	1.015	4.12	6.8	7.37	0.09	0.73
"	"	1.015	7.75	6.3	13.85	0.17	0.67
28.5	1.829	1.032	12.85	5.7	22.75	0.28	0.61
28.7	1.824	1.032	21.09	4.4	37.23	0.47	0.47
28.5	1.829	1.006	22.12	3.2	40.12	0.50	0.34
"	"	1.015	22.20	1.3	39.94	0.50	0.13

*η = 15.7, $P_b/\rho g$ = 9.3 cm, K = 79.91 μ^2, ρ_b = 1.53 gm/cc, ρ_s = 2.67 gm/cc, ϕ = 0.427

TABLE 9. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR A LONG COLUMN OF CRAB CREEK SAND**

27.8	1.851	0.961	15.36	2.0	29.61	1.00	0.14
"	"	0.946	15.59	4.3	30.57	1.04	0.34
"	"	0.987	15.04	10.0	28.22	0.96	0.72
27.9	1.848	1.013	11.24	10.9	20.51	0.70	0.79
"	"	0.974	7.84	13.9	14.88	0.51	1.00
"	"	1.000	3.16	16.6	5.85	0.20	1.19
27.6	1.858	1.020	0.29	20.7	0.53	0.018	1.49
27.7	1.855	0.987	0.03	26.6	0.058	0.002	1.97
26.2	1.894	1.046	0.01	28.5	0.019	0.0006	2.05

**η = 9.7, $P_b/\rho g$ = 13.9 cm, K = 29.57 μ^2, ρ_b = 1.54 gm/cc, ρ_s = 2.79 gm/cc, ϕ = 0.448

TABLE 10. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR A LONG COLUMN OF POUDRE SAND*

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	$\Delta H/\Delta L$ $\Delta L = 26.8$	$q \times 10^4$ cm/sec	$P_c/\rho g$ cm	K_e μ^2	K.	P.
30.0	1.782	0.930	16.80	4.3	3.23	0.95	0.11
32.2	1.777	0.885	16.57	7.0	3.33	0.98	0.18
28.4	1.833	0.903	17.39	13.7	3.53	1.03	0.35
28.8	1.820	0.888	16.06	21.6	3.30	0.97	0.55
26.0	1.910	0.884	14.28	31.6	3.09	0.91	0.81
26.2	1.904	1.004	8.61	40.2	1.63	0.48	1.03
24.7	1.955	1.002	2.36	48.0	0.46	0.13	1.23
26.0	1.910	1.002	1.09	52.3	0.20	0.06	1.34
27.9	1.848	1.010	0.19	62.3	0.035	0.01	1.59

*$\eta = 9.9$, $P_b/\rho g = 39.1$ cm, $K = 3.40\ \mu^2$, $\rho_b = 1.47$ gm/cc, $\rho_s = 2.74$ gm/cc, $\phi = 0.464$

TABLE 11. CAPILLARY PRESSURE--EFFECTIVE PERMEABILITY DATA FOR A LONG COLUMN OF PULLMAN CLAY**

28.2	1.839	0.940	75.45	2.0	14.77	1.00	0.26
31.0	1.755	0.981	70.00	3.6	12.53	0.85	0.47
"	"	1.041	66.20	6.3	10.64	0.72	0.83
"	"	1.075	53.50	7.3	8.73	0.59	0.96
"	"	1.067	43.60	8.4	7.17	0.49	1.11
"	"	1.007	27.40	9.9	4.78	0.32	1.30
31.2	1.751	0.978	1.75	20.2	0.306	0.021	2.55
29.5	1.800	0.769	0.33	29.2	0.077	0.0052	3.84
29.0	1.815	0.835	0.10	40.1	0.021	0.0014	5.27

**$\eta = 3.8$, $P_b/\rho g = 7.6$ cm, $K = 14.77\ \mu^2$, $\rho_b = 1.25$ gm/cc, $\rho_s = 2.60$ gm/cc, $\phi = 0.519$

TABLE 12. UNSTEADY DRAINAGE DATA FROM COLUMN OF CRAB CREEK SAND

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	t sec	Q cc/cm²	t.	Q.
29.0	1.815	10	0.134	0.031	0.026
		30	0.368	0.092	0.069
		40	0.485	0.122	0.091
		50	0.611	0.153	0.115
		60	0.720	0.183	0.135
		70	0.829	0.214	0.153
		80	0.946	0.244	0.178
		100	1.172	0.305	0.220
		110	1.247	0.336	0.235
		120	1.356	0.366	0.255
		130	1.456	0.397	0.274
		140	1.563	0.427	0.294
		150	1.664	0.458	0.313
		170	1.871	0.488	0.352
		180	1.967	0.549	0.370
		190	2.067	0.580	0.389
		200	2.176	0.610	0.409
		210	2.270	0.641	0.427
		220	2.369	0.672	0.446
		240	2.553	0.733	0.480
		260	2.745	0.794	0.516

TABLE 12. UNSTEADY DRAINAGE DATA FROM COLUMN OF CRAB CREEK SAND* (CONTINUED)

Temp. ^oC	$\mu/\rho g \times 10^5$ cm sec	t sec	Q cc/cm^2	t.	Q
		280	2.904	0.855	0.546
		300	3.080	0.916	0.579
		330	3.331	1.008	0.626
		360	3.582	1.099	0.673
		420	4.048	1.282	0.761
		480	4.478	1.465	0.842
		510	4.679	1.559	0.882
		540	4.879	1.649	0.918
		600	5.259	1.832	0.989
		660	5.616	2.013	1.056
		720	5.951	2.198	1.119
		780	6.260	2.381	1.177
		840	6.548	2.564	1.231
		900	6.815	2.748	1.282
		1020	7.288	3.114	1.372
		1140	7.693	3.480	1.447
		1260	8.040	3.847	1.512
		1380	8.336	4.213	1.568
		1500	8.597	4.580	1.617
29.0	1.815	1620	8.821	4.946	1.659
		1740	9.019	5.312	1.696
		1860	9.188	5.679	1.728
		1980	9.340	6.045	1.757
		2100	9.461	6.411	1.779
		2220	9.573	6.778	1.800
		2340	9.679	7.144	1.820
		2700	9.941	8.243	1.870
		3300	10.261	10.075	1.930
		4020	10.527	12.273	1.980
		4620	10.696	14.105	2.012
		5700	10.929	17.402	2.055
		7500	11.195	22.898	2.105
		8400	11.282	25.680	2.120
		9300	11.361	28.393	2.137
		10200	11.430	31.141	2.148
		12000	11.533	36.636	2.169
		15300	11.677	46.711	2.196
		21300	11.851	65.031	2.229
		27300	11.937	83.357	2.245
		37500	12.019	114.485	2.260

*η = 9.7, L. = 3.75, K = 29.47 μ^2, $P_b/\rho g$ = 13.9 cm, ϕ_e = 0.382

TABLE 13. UNSTEADY DRAINAGE DATA FROM COLUMN OF POUDRE SAND*

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	t sec	Q cc/cm^2	t.	Q.
29.5	1.799	60	0.126	0.007	0.008
"	"	130	0.252	0.016	0.016
"	"	225	0.378	0.027	0.024
"	"	530	0.755	0.063	0.047
"	"	900	1.385	0.107	0.087
"	"	1410	2.015	0.168	0.127
"	"	1800	2.518	0.214	0.159
"	"	2210	3.022	0.263	0.190
"	"	2850	3.777	0.339	0.238
30.0	1.783	3330	4.407	0.396	0.278
"	"	3960	5.162	0.472	0.325
"	"	4500	5.792	0.537	0.365
"	"	5190	6.547	0.620	0.412
29.9	1.786	5640	7.051	0.674	0.444
"	"	6360	7.807	0.760	0.492
"	"	7100	8.562	0.849	0.539
29.8	1.789	7830	9.317	0.936	0.587
30.0	1.783	8980	10.451	1.074	0.658
30.1	1.780	9780	11.206	1.170	0.706
"	"	10055	12.339	1.323	0.777
30.4	1.772	11375	13.347	1.481	0.841
"	"	12995	14.606	1.677	0.920
30.8	1.760	15110	16.117	1.934	1.015
"	"	16880	17.250	2.150	1.086
31.0	1.755	18530	17.880	2.351	1.126
30.8	1.760	22280	20.272	2.807	1.277
"	"	25820	21.720	3.348	1.368
"	"	29780	23.720	3.720	1.451
"	"	34520	24.238	4.296	1.526
31.0	1.755	41360	25.540	5.131	1.608
32.0	1.736	51320	26.925	6.359	1.696
31.8	1.740	59900	27.844	7.415	1.754
"	"	77720	28.436	9.608	1.791
31.0	1.755	93245	29.073	11.484	1.831
30.3	1.774	108260	29.625	13.296	1.866
31.0	1.755	122960	30.069	15.085	1.894
"	"	142220	30.490	17.435	1.920
30.5	1.768	168980	30.963	20.676	1.950
29.5	1.799	192380	31.222	23.476	1.966
29.2	1.808	217280	31.447	26.425	1.980
"	"	227540	31.554	27.640	1.986
29.0	1.815	257300	31.610	31.151	1.991
29.2	1.808	308420	32.011	37.205	2.016
"	"	364400	32.196	43.835	2.028

*η = 9.9, L. = 3.75, K = 3.40 μ^2, $P_b/\rho g$ = 39.1 cm,

ϕ_e = 0.406

TABLE 14. UNSTEADY DRAINAGE DATA FROM COLUMN OF THE 246-295 MICRON FRACTION OF POUDRE SAND*

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	t sec	Q cc/cm^2	t.	Q.
28.5	1.829	5	0.151	0.039	0.037
		10	0.250	0.077	0.062
		15	0.364	0.116	0.091
		20	0.469	0.154	0.116
		25	0.567	0.193	0.141
		30	0.669	0.231	0.166
		35	0.773	0.270	0.192
		40	0.878	0.308	0.218
		45	0.980	0.347	0.244
		50	1.084	0.385	0.270
		55	1.188	0.424	0.295
		60	1.288	0.462	0.320
		70	1.486	0.539	0.370
		80	1.690	0.616	0.420
		90	1.891	0.693	0.470
		100	2.092	0.770	0.520
		120	2.460	0.924	0.612
		130	2.640	1.001	0.657
		140	2.818	1.079	0.700
		150	3.002	1.156	0.747
		160	3.171	1.233	0.788
		180	3.510	1.387	0.873
		195	3.815	1.502	0.935
		225	4.307	1.733	0.949
		255	4.755	1.965	1.183
		300	5.388	2.311	1.340
		330	5.776	2.542	1.437
		390	6.435	3.005	1.601
		450	6.969	3.467	1.733
		570	7.767	4.392	1.932
		690	8.306	5.316	2.066
		810	8.686	6.241	2.161
		960	0.019	7.396	2.244
		1260	9.446	9.707	2.350
		1500	9.671	11.556	2.406
		1860	9.909	14.330	2.465
		2340	10.132	18.028	2.520
28.5	1.829	2940	10.326	22.651	2.569
		3300	10.413	25.424	2.590
		3840	10.525	29.585	2.618
		4740	10.681	36.519	2.657
		5340	10.754	41.141	2.675
		7140	10.914	55.010	2.715
		8940	11.020	68.876	2.741
		10740	11.094	82.745	2.759
		12540	11.152	96.613	2.774
		13740	11.184	105.585	2.782

*η = 18.4, L.= 3.75, K = 56.67 μ^2, $P_b/\rho g$ = 11.6 cm, ϕ_e = 0.347

TABLE 15. UNSTEADY DRAINAGE DATA FROM COLUMN OF PULLMAN CLAY*

Temp. °C	$\mu/\rho g \times 10^5$ cm sec	t sec	Q cc/cm²	t.	Q.
28.2	1.839	10	0.067	0.020	0.017
"	"	20	0.131	0.041	0.033
"	"	30	0.194	0.061	0.049
"	"	40	0.258	0.081	0.065
"	"	50	0.318	0.102	0.081
"	"	70	0.429	0.142	0.108
"	"	90	0.541	0.183	0.137
"	"	120	0.696	0.244	0.176
"	"	135	0.770	0.274	0.195
"	"	165	0.904	0.336	0.229
"	"	180	0.968	0.367	0.245
"	"	195	1.029	0.397	0.261
"	"	225	1.140	0.457	0.288
"	"	270	1.291	0.550	0.327
"	"	330	1.465	0.668	0.371
"	"	390	1.614	0.793	0.409
"	"	450	1.727	0.915	0.437
"	"	540	1.878	1.100	0.475
"	"	630	1.995	1.283	0.506
"	"	720	2.092	1.465	0.531
"	"	840	2.196	1.710	0.557
28.5	1.829	1020	2.318	2.076	0.587
"	"	1200	2.410	2.440	0.610
"	"	1440	2.506	2.932	0.634
"	"	1800	2.618	3.670	0.660
"	"	2400	2.745	4.900	0.696
"	"	3000	2.841	6.130	0.719
"	"	3600	2.903	7.350	0.735
28.8	1.820	4200	2.954	8.580	0.749
"	"	4800	2.991	9.822	0.757
"	"	5400	3.026	11.050	0.767
"	"	6600	3.072	13.531	0.778
29.0	1.815	7800	3.103	16.002	0.787
"	"	9000	3.127	18.477	0.791
"	"	10800	3.144	22.191	0.795
"	"	15060	3.225	30.980	0.817
"	"	18840	3.239	38.778	0.820
29.1	1.811	22680	3.244	46.714	0.821
"	"	28320	3.249	58.369	0.823
"	"	39660	3.252	81.384	0.825

*η = 3.8, L. = 3.75, K = 14.77 μ^2, $P_b/\rho g$ = 7.6 cm, ϕ_e = 0.519

Key Words: Similitude, Non-steady, Drainage, Partially saturated, Soils, Capillary fringe, Scaling criteria, Hysteresis, Imbibition, Media properties.

Abstract: The similarity condition was applied to non-steady flow of liquids in partially saturated porous media. Inspectional analysis was applied to the differential equation describing two-phase flow where the gas phase is everywhere at atmospheric pressure. Criteria for similitude were developed to facilitate studying complicated field processes with small laboratory models. Standard scaling units were so chosen that the capillary fringe depth can be precisely accounted for in a model and the time scale permits modeling a lengthy flow process in a relatively short period of time. The standard scaling units are readily measurable media and liquid properties. Some of these properties, however, are characteristic of the drainage cycle only. Theoretically, therefore, the scaling criteria are not applicable when hysteresis is important and the imbibition cycle is involved. The validity of the criteria were confirmed experimentally by applying a non-steady drainage process of two columns of media which, according to the theory were similar. Even though they were similar, the media properties were sufficiently different so that one column was a small model of the other. It was also discovered that, at least during steady flow, processes involving hysteresis can, in fact, be accurately scaled with these criteria.

Reference: Corey, G. L., Corey A. T. and Brooks, R. H., Colorado State University, Hydrology Paper No. 9 (August 1965) "Similitude for Non-Steady Drainage of Partially Saturated Soils"

Key Words: Similitude, Non-steady, Drainage, Partially saturated, Soils, Capillary fringe, Scaling criteria, Hysteresis, Imbibition, Media properties.

Abstract: The similarity condition was applied to non-steady flow of liquids in partially saturated porous media. Inspectional analysis was applied to the differential equation describing two-phase flow where the gas phase is everywhere at atmospheric pressure. Criteria for similitude were developed to facilitate studying complicated field processes with small laboratory models. Standard scaling units were so chosen that the capillary fringe depth can be precisely accounted for in a model and the time scale permits modeling a lengthy flow process in a relatively short period of time. The standard scaling units are readily measurable media and liquid properties. Some of these properties, however, are characteristic of the drainage cycle only. Theoretically, therefore, the scaling criteria are not applicable when hysteresis is important and the imbibition cycle is involved. The validity of the criteria were confirmed experimentally by applying a non-steady drainage process of two columns of media which, according to the theory were similar. Even though they were similar, the media properties were sufficiently different so that one column was a small model of the other. It was also discovered that, at least during steady flow, processes involving hysteresis can, in fact, be accurately scaled with these criteria.

Reference: Corey, G. L., Corey A. T. and Brooks, R. H., Colorado State University, Hydrology Paper No. 9 (August 1965) "Similitude for Non-Steady Drainage of Partially Saturated Soils"

Key Words: Similitude, Non-steady, Drainage, Partially saturated, Soils, Capillary fringe, Scaling criteria, Hysteresis, Imbibition, Media properties.

Abstract: The similarity condition was applied to non-steady flow of liquids in partially saturated porous media. Inspectional analysis was applied to the differential equation describing two-phase flow where the gas phase is everywhere at atmospheric pressure. Criteria for similitude were developed to facilitate studying complicated field processes with small laboratory models. Standard scaling units were so chosen that the capillary fringe depth can be precisely accounted for in a model and the time scale permits modeling a lengthy flow process in a relatively short period of time. The standard scaling units are readily measurable media and liquid properties. Some of these properties, however, are characteristic of the drainage cycle only. Theoretically, therefore, the scaling criteria are not applicable when hysteresis is important and the imbibition cycle is involved. The validity of the criteria were confirmed experimentally by applying a non-steady drainage process of two columns of media which, according to the theory were similar. Even though they were similar, the media properties were sufficiently different so that one column was a small model of the other. It was also discovered that, at least during steady flow, processes involving hysteresis can, in fact, be accurately scaled with these criteria.

Reference: Corey, G. L., Corey A. T. and Brooks, R. H., Colorado State University, Hydrology Paper No. 9 (August 1965) "Similitude for Non-Steady Drainage of Partially Saturated Soils"

Key Words: Similitude, Non-steady, Drainage, Partially saturated, Soils, Capillary fringe, Scaling criteria, Hysteresis, Imbibition, Media properties.

Abstract: The similarity condition was applied to non-steady flow of liquids in partially saturated porous media. Inspectional analysis was applied to the differential equation describing two-phase flow where the gas phase is everywhere at atmospheric pressure. Criteria for similitude were developed to facilitate studying complicated field processes with small laboratory models. Standard scaling units were so chosen that the capillary fringe depth can be precisely accounted for in a model and the time scale permits modeling a lengthy flow process in a relatively short period of time. The standard scaling units are readily measurable media and liquid properties. Some of these properties, however, are characteristic of the drainage cycle only. Theoretically, therefore, the scaling criteria are not applicable when hysteresis is important and the imbibition cycle is involved. The validity of the criteria were confirmed experimentally by applying a non-steady drainage process of two columns of media which, according to the theory were similar. Even though they were similar, the media properties were sufficiently different so that one column was a small model of the other. It was also discovered that, at least during steady flow, processes involving hysteresis can, in fact, be accurately scaled with these criteria.

Reference: Corey, G. L., Corey A. T. and Brooks, R. H., Colorado State University, Hydrology Paper No. 9 (August 1965) "Similitude for Non-Steady Drainage of Partially Saturated Soils"