CIVIL ENGINEERING

THE MOST EFFICIENT STABLE CHANNEL FOR COMPARATIVELY CLEAR WATER IN NON-COHESIVE MATERIALS

by

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FOREWARD

The research reported herein was initiated by E. W. Lane. The preliminary draft had been reviewed by P. N. Lin before he left the United States several years ago. Both Professors Lane and Lin have not reviewed the manuscript in its final form.

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H. K. Liu

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INTRODUCTION

SCOPE

A stable channel has been defined by E. W. Lane (7) as an unlined earth earth channel for carrying water, in which objectionable scour and deposits of sediment do not occur. When water is withdrawn from a storage reservoir, it will be normally free of sediment; and the basic problem in designing a stable channel is then to prevent scour or movement of the material forming the perimeter of the channel. Many canals carry so little sediment that for design purposes they may be considered to be clear water canals. The present paper is limited to the discussion of a stable channel in the case where the water is comparatively clear. It is also limited to canals constructed in coarse, non-cohesive material.

SHAPE OF STABLE CHANNEL

Since the prevention of scour is now the principal problem, the severest allowable condition to which the channel may be subject is that in which the motion of the bed and the banks is at all places incipient. Thus, for straight canal having a mild slope, the following five conditions are believed to determine for practical purposes, the shape of the most efficient stable channel for comparatively clear water: (1) at and above the water surface, the side slope is at the angle of repose of the material; (2) at points between the center and the edge of the channel, the particles are in a state of incipient motion, under the action of gravity component of their own submerged weight acting down the side slope and also the tractive force of the flowing water; (3) where the side slope is zero the tractive force alone is sufficient to cause incipient motion; (4) the particle is held against the bed by the component of the submerged weight of the particle acting normal to the bed; and (5) the tractive force on any area is equal to the component of the weight of the water above the area in the direction of flow. On the basis of these assumptions, R. E. Glover (5) derived the following equation for the cross-sectional shape of a stable channel.

$$y = y_o \cos\left(f \frac{x}{y_o}\right) \tag{1}$$

A derivation of Eq. 1 will be described in detail elsewhere in this paper.

In addition to Eq. 1, equations for the shape of a stable channel in clear water have also been proposed by Forchheimer (4), Niezery and Braudeau (11), Koechlin (13), and Fan (3).[*] The present paper will be based on Eq. 1.

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[*] After the studies by the U. S. Bureau of Reclamation were carried out, it was discovered that studies along similar lines had previously been carried on. However, the advantageous characteristics of the profile of a stable channel have not been recognized fully by these investigators. For example, Forchheimer recognized that this form of the channel was one of a minimum water cross-section, but he did not recognize that it also produced a minimum channel width, and one of a minimum excavation. All these characteristics are also for stable channels constructed on the side of a hill. The principal reason why Forchheimer's solution has not been extensively used in canal design is that for a given discharge and material size, only one canal slope can be used. This gives so little flexibility of design that it rarely fits the requirements of the case.

THE PROBLEM

The cross-sectional shape of a stable channel according to Eq. 1.will be a segment of a cosine curve symmetrical with respect to the center line, the maximum depth in the center being y_o and the top width being $2x_o = \frac{\pi}{f} y_o$. The capacity of a channel having such dimensions and characteristics is designated as Q_o. In practice, several problems may arise with regard to the use of such a cross section:

(1) Strict adherence to the theoretical shape may prove undesirable as far as actual construction is concerned. In this case, trapezoidal or any other more convenient forms approximating the theoretical cross section may be considered.

(2) For a given longitudinal slope and a given material composing the channel, if Q_o on the basis of Eq. 1 is greater than that required, the cross section of the designed channel may be reduced by joining the two side portions after an appropriate portion in the center of the theoretical section is removed.

(3) If Q_o is less than that required, the cross section of the designed channel may be increased by adding a level bottom in the center of the section computed on the basis of Eq. 1, so that the designed section will now consist of a horizontal bottom in the center and a curve dictated by Eq. 1 joining each end of this horizontal bottom.

In all the computations mentioned above, it is desirable to have a set of curves available for the speedy evaluation of any of the variables y, A, P, R, Q, and Q_c for any given value of $\frac{x}{x_o}$. In the present paper, expressions for the evaluation of the foregoing variables will be deduced. Diagrams prepared on the basis of these expressions will also be presented.

CROSS SECTION OF STABLE CHANNEL

TRACTIVE FORCE

As stated previously (see Introduction), R. E. Glover (5) proceeded from the five assumptions proposed by E. W. Lane (1) and discovered Eq. 1 for the cross-sectional shape of a stable channel carrying clear water. According to Lane's assumption there are essentially two forces tending to move the material on the bed and banks of an inclined canal. The two forces are (1) the drag of the flow on the bed and (2) the component of the submerged weight of the grain down the slope. If a particle remains at rest, the resultant of the foregoing forces must be balanced by the frictional force opposing the motion of the particle.

In the case of uniform flow in a channel, the mean flow is free from acceleration and consequently, the forces acting on a free body (Fig. 1) of the flow must be in equilibrium. Therefore, the resistance exerted by the bed and banks of the channel to the flow must be balanced by the longitudinal component of the weight of water in the free body. The total drag of the flow on the bed and banks, being equal to the resistance to flow, is therefore simply the longitudinal component of the weight of water in the free body.

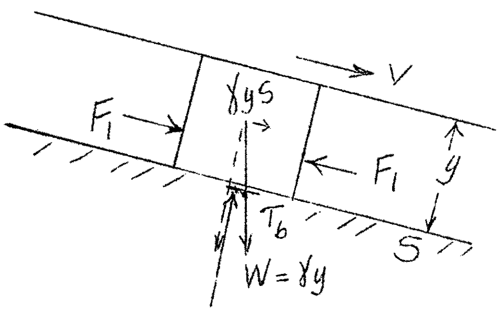

Fig. 1 - Free Body Diagram for a segment of uniform flow.

For wide channels, the flow per unit width of the channel may be considered. The drag of flow on a unit width of the bed is then equal to the longitudinal component of the wéight of water in the free body divided by the width of the channel. The tractive force T_b , being the drag per unit area of the bed, is therefore, equal to the drag per unit width divided by the length of the free body. Thus the tractive force is equal to the longitudinal component of a column of water having a unit cross-sectional area, and a height equal to the depth of flow.

Let γ be the weight of water per unit volume and y the depth of flow, then the weight of a water column of a height y and a base area of unit is γy. The component of this weight in the direction parallel to the channel bottom is $\gamma y S$ which is also the tractive force, i.e.,

$$T_b = \gamma y S \tag{2}$$

DIFFERENTIAL EQUATION OF A STABLE CHANNEL

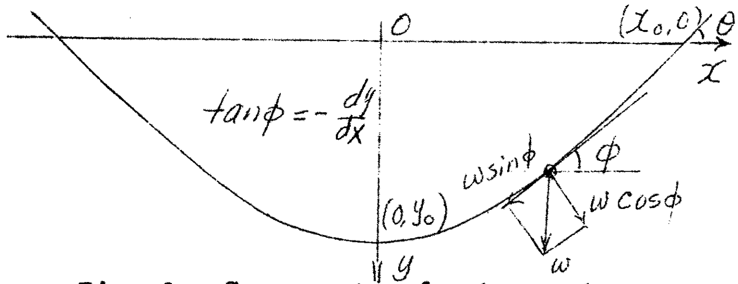

Fig. 2 - Components of submerged weight of a particle.

Let a particle be located on a segment of the perimeter where the local transverse inclination of the bed is ϕ. Also let the submerged weight of this particle be w.

Then the component of the submerged weight tending to move the particle downward along the slope is

$$w \sin \phi$$

And the component normal to the bed is

$$w \cos \phi$$

Because of the latter component, a frictional force is induced, of which the value is

$$f\, w \cos\phi = \tan\theta\, w \cos\phi$$

for cohesiveless material, where θ being the angle of repose.

As explained in the previous section, the tractive force per unit area acting on the channel bottom, where the depth of flow is y , is given by

$$\tau_b = \gamma y S \quad \text{lb. per sq. ft.} \tag{2}$$

On an area sloping at an angle ϕ , the intensity of force, that is, the force per unit area is reduced to

$$\gamma y S \cos\phi \quad \text{lb. per sq. ft.}$$

because now the same force is acting over a larger area $\frac{1}{\cos\phi}$. If the number of particles actually subjected to the action of the tractive force be N per unit area, then the drag acting on a particle located at a point of the bed where the transverse inclination is ϕ is

$$\frac{1}{N} \gamma y S \cos\phi$$

The resultant of forces tending to move the particles is then

$$\sqrt{w^2 \sin^2\phi + \left(\frac{1}{N}\gamma y S \cos\phi\right)^2}$$

Under the condition of incipient motion, the resultant must be equal and opposite to the frictional force, so that

$$\sqrt{w^2 \sin^2\phi + \left(\frac{1}{N}\gamma y S \cos\phi\right)^2} = w \tan\theta \cos\phi \tag{3}$$

At the center of the cross section, $y = y_o$, the bottom is transversely horizontal, i.e., $\phi = 0$. Inserting this value of ϕ in Eq. 3 leads to

$$\frac{1}{N} \gamma y_o S = \omega \tan \theta$$

i.e.,

$$\frac{\gamma S}{N} = \frac{\omega \tan \theta}{y_o} \tag{4}$$

Substituting from (4) in (3),

$$\omega^2 \sin^2 \phi + \frac{\omega^2 \tan^2 \theta}{y_o^2} y^2 \cos^2 \phi = \omega^2 \tan^2 \theta \cos^2 \phi$$

so that, on dividing throughout by $\omega^2 \cos^2 \phi$

$$\tan^2 \phi + \tan^2 \theta \left(\frac{y}{y_o}\right)^2 = \tan^2 \theta \tag{5}$$

Now $-\frac{dy}{dx} = \tan \phi$. Therefore, Eq. 5 may be written as

$$\left(\frac{dy}{dx}\right)^2 + \frac{\tan^2 \theta}{y_o^2} y^2 = \tan^2 \theta \tag{6}$$

Which is the differential equation of a stable channel carrying clear water.

By transposing terms and taking the square roots of both sides of Eq 6.

$$\frac{dy}{dx} = \tan \theta \sqrt{1 - \left(\frac{y}{y_o}\right)^2} = f \sqrt{1 - \left(\frac{y}{y_o}\right)^2} \tag{7}$$

where $f = \tan \theta$

So that

$$\frac{dy}{\sqrt{1 - \left(\frac{y}{y_o}\right)^2}} = f\, dx \tag{8}$$

Integrating (8) $$-\cos^{-1}\left(\frac{y}{y_o}\right) = \frac{f}{y_o}x + C \tag{9}$$

When $x = 0$, $y = y_o$; therefore, $C = 0$

and the equation of the shape of a stable channel becomes

$$y = y_o \cos\left(f \frac{x}{y_o}\right) \tag{1}$$

or when $x = x_o$, $y = 0$, therefore $f \frac{x_o}{y_o} = \frac{\pi}{2}$ (10)

i.e.,

$$x_o = \frac{\pi}{2} \frac{y_o}{f} \tag{11}$$

Eq. 1 can be expressed also as

$$\frac{y}{y_o} = \cos\left(\frac{\pi}{2}\frac{x}{x_o}\right) \tag{12}$$

The value of y_o in Eq. 1 may be determined from working values of tractive force (7), thus,

$$T_w = \gamma y_o S \tag{2a}$$

so that

$$y_o = \frac{T_w}{\gamma S} \tag{13}$$

Eqs. 1 and 9 are given in tabulation form shown as Table II in the Appendix and are plotted in Plate 1 shown by solid line. Equation 11 is tabulated as Table I in the Appendix.

THE MOST EFFICIENT SECTION

The most efficient section in hydraulic problems is one that has, for a given area, the maximum hydraulic radius or the minimum wetted perimeter, and the minimum top width.

In the case of a symmetrical cross section, if the cross-sectional area is to be maintained constant, then only two types of geometric shapes are possible (Fig. 3a and 3b)

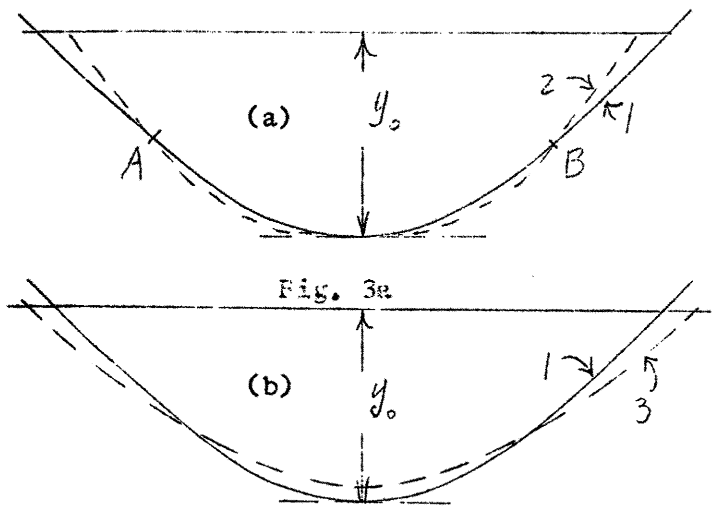

Fig. 3 - Variation of Channel Cross-Section having constant area.

In the case of Fig. 3a, a part of the alternate cross section is deeper than that based on Eq 1. In no case, however, does the maximum depth for a given material and a given longitudinal slope exceed y_o. Since the side of the dotted cross section at A and B is greater than that of the stable section, the dotted cross section in Fig. 3a is unstable, and therefore should be ruled out.

The dotted cross section in Fig. 3b may be stable, provided that, for the same value of y, the ϕ - value for the dotted cross section never exceeds the value of ϕ dictated by Eq. 1. The dotted cross-section is always shallower and wider than that given by Eq. 1, consequently, the hydraulic radius of the cross section of Fig. 3b is smaller than that of the cross section given by Eq. 1 and the top width is larger than that given by Equation 1. Thus, if the cross-section is to be stable, the cross-section dictated by Equation 1 is the most efficient. A most efficient cross-section also implies that for a given cross-sectional area, the discharge is the maximum and the top width is the minimum.

COMPUTATIONS OF AREA, WETTED PERIMETER, HYDRAULIC RADIUS AND DISCHARGE

AREA

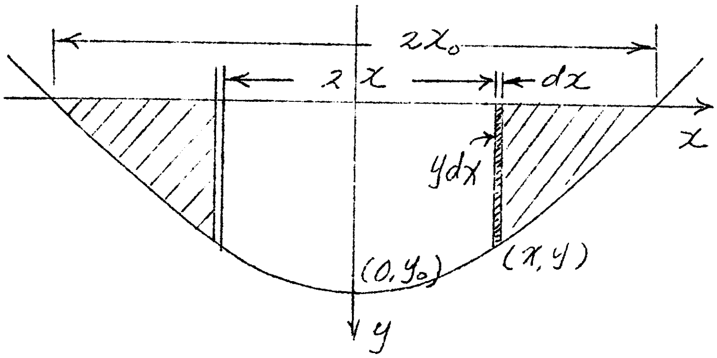

Fig. 4 - Definition Sketch for Stable Channel Formulas

Since the equation of the cross section of a stable channel is

$$y = y_0 \cos\left(\frac{f}{y_0} x\right) \tag{1}$$

The total area of the cross section is given by

$$\begin{aligned} A_0 &= 2\int_0^{x_0} y\,dx \\ &= 2y_0 \int_0^{x_0} \cos\left(\frac{f}{y_0}x\right) dx \\ &= \frac{2y_0^2}{f}\left[\sin\left(\frac{f}{y_0}x\right)\right]_0^{x_0} = \frac{2y_0^2}{f} \end{aligned}$$

namely

$$\frac{A_0}{y_0^2} = \frac{2}{f} \tag{14}$$

Therefore

$$\begin{aligned} A &= A_0 - 2\int_0^{x} y\,dx \\ &= \frac{2y_0^2}{f}\left\{1 - \left[\sin\left(\frac{f}{y_0}x\right)\right]_0^{x}\right\} \\ &= \frac{2y_0^2}{f}\left\{1 - \sin\left(\frac{f}{y_0}x\right)\right\} \end{aligned}$$

i.e.,

$$\frac{A}{y_o^2} = \frac{2}{f}\left(1 - \sin\left(\frac{f}{y_o}x\right)\right) \tag{15}$$

Eq. 15 is shown as Plate 2 and is tabulated in Table III.

WETTED PERIMETER AND HYDRAULIC RADIUS

By definition, the wetted perimeter P is

$$P = 2\int_{x}^{x_o} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \tag{16}$$

Since

$$\frac{dy}{dx} = f\sqrt{1 - \left(\frac{y}{y_o}\right)^2} \tag{7}$$

and

$$\frac{y}{y_o} = \cos\left(f\,\frac{x}{y_o}\right) \tag{10}$$

therefore

$$\frac{dy}{dx} = f\,\sin\left(\frac{f}{y_o}x\right) \tag{17}$$

and

$$P = 2\int_{x}^{x_o} \sqrt{1 + f^2\sin^2\left(\frac{f}{y_o}x\right)}\, dx \tag{18}$$

Let

$$\epsilon = \frac{\pi}{2} - f\,\frac{x}{y_o} \tag{19}$$

$$d\epsilon = -\frac{f}{y_o}\, dx \tag{20}$$

When $$x = x_o = \frac{\pi}{2} \frac{y_o}{f}, \qquad \epsilon = 0$$

Substitute Eq. 19 and 20 in Eq. 14 and change limits so that

$$P = -2\frac{y_o}{f}\int_{\epsilon}^{0}\sqrt{1+f^2\cos^2\epsilon}\; d\epsilon$$

$$= -2\frac{y_o}{f}\int_{\epsilon}^{0}\sqrt{1+f^2(1-\sin^2\epsilon)}\; d\epsilon$$

$$= -\frac{2y_o}{f}\sqrt{1+f^2}\int_{\epsilon}^{0}\sqrt{1-\frac{f^2}{1+f^2}\sin^2\epsilon}\; d\epsilon$$

$$= 2y_o\frac{1}{\sin\theta}\int_{0}^{\epsilon}\sqrt{1-\sin^2\theta\sin^2\epsilon}\; d\epsilon$$

$$= \frac{2y_o}{\sin\theta}\, E(\sin\theta, \epsilon)$$

$$= \frac{2y_o}{\sin\theta}\, E\left(\sin\theta, \left(1-\frac{x}{x_o}\right)\frac{\pi}{2}\right)$$

Therefore

$$\frac{P}{y_o} = \frac{2}{\sin\theta}\, E\left(\sin\theta, \alpha\frac{\pi}{2}\right) \tag{21}$$

Where

$$\alpha = 1 - \frac{x}{x_o} \tag{22}$$

and

$$k = \sin\theta \tag{23}$$

The quantity $E\left(k, \alpha\frac{\pi}{2}\right)$ is known as the elliptic integral of the second kind of the modulus k and the amplitude $\alpha\frac{\pi}{2}$, and has been tabulated in a number of mathematical tables (6).

The hydraulic radius is then given by

$$R = \frac{A}{P} \tag{24}$$

$$= y_0 \frac{\sin\theta}{f} \frac{1 - \sin(\frac{f}{y_0}x)}{E(k, \alpha\frac{\pi}{2})}$$

So that

$$\frac{R}{y_0} = \cos\theta \frac{1 - \sin(\frac{f}{y_0}x)}{E(k, \alpha\frac{\pi}{2})} \tag{25}$$

Eqs. 21 and 25 are plotted as Plate 3 and 4 respectively. They are also given in Table IV.

DISCHARGE

Applying the Manning formula one has

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{26}$$

$$= \frac{1.49}{n} S^{1/2} \left[y_0 \cos\theta \frac{1 - \sin\frac{f}{y_0}x}{E(k, \alpha\frac{\pi}{2})} \right]^{2/3} \tag{27}$$

Consequently, $Q = AV$

$$= \frac{2y_0^2}{f}\left(1 - \sin\frac{f}{y_0}x\right) \frac{1.49}{n} S^{1/2} \left[y_0 \cos\theta \frac{1 - \sin\frac{f}{y_0}x}{E(k\,\alpha\frac{\pi}{2})} \right]^{2/3}$$

$$= \frac{2.98}{n} \frac{y_0^{8/3} \sin^{2/3}\theta \; S^{1/2} \left(1 - \sin\frac{f}{y_0}x\right)^{5/3}}{f^{5/3} E^{2/3}(k, \alpha\frac{\pi}{2})} \tag{28}$$

from which

$$Q_0 = \frac{2.98}{n} \; \frac{S^{1/2} \, y_0^{8/3} \sin^{2/3}\theta}{f^{5/3} \, E^{2/3}(k, \frac{\pi}{2})} \tag{29}$$

and

$$\frac{Q_0}{\frac{1.49}{n} \, y_0^{8/3} \, S^{1/2}} = \frac{2}{f^{5/3}} \left[\frac{\sin\theta}{E(k, \frac{\pi}{2})}\right]^{2/3}$$

Hence

$$\frac{Q}{Q_0} = \left[1 - \sin\left(f \frac{x}{x_0}\right)\right]^{5/3} \left[\frac{E(k, \frac{\pi}{2})}{E(k, \alpha\frac{\pi}{2})}\right]^{2/3} \tag{30}$$

Applying the Chezy's formula, one has

$$V = C\sqrt{RS} \tag{31}$$

$$= C\sqrt{S}\left[y_0 \cos\theta \; \frac{1 - \sin\frac{f}{y_0}x}{E(k, \alpha\frac{\pi}{2})}\right]^{1/2} \tag{32}$$

$$Q = AV$$

$$= C S^{1/2} \frac{2}{f^{3/2}} \left[y_0 \sin\theta\right]^{1/2} \frac{\left[1 - \sin\frac{f}{y_0}x\right]^{3/2}}{E^{1/2}(k, \alpha\frac{\pi}{2})} \tag{33}$$

from which

$$Q_0 = \frac{2C_0 S^{1/2}}{f^{3/2}} \; \frac{(y_0 \sin\theta)^{1/2}}{E^{\frac{1}{2}}(k, \frac{\pi}{2})} \tag{34}$$

Hence

$$\frac{Q}{Q_0} = \frac{C}{C_0}\left[1 - \sin\frac{f}{y_0}x\right]^{3/2} \left[\frac{E(k, \frac{\pi}{2})}{E(k, \alpha\frac{\pi}{2})}\right]^{1/2} \tag{35}$$

The choice of using Chezy's or Manning's flow formula is a matter of personal preference. In the United States, the Manning formula is widely used. Such a formula was for fully developed turbulent flow. Manning's coefficient is usually taken as a constant for given channel surface roughness. Lane (9) expressed the Manning's n as proportional to $d_{25}^{1/6}$ - , where d_{25} is the size of the bed-material in inches of which 25 per cent is larger. The assumption that n is a constant for given bed-material enables Eq. 30 to be plotted as Plate 5 without knowing the value of n. The computation of Eq. 30 is shown in Table V.

EFFECT OF LIFTING FORCE UPON THE CROSS SECTION OF A STABLE CHANNEL

In actual conditions, the dynamic force exerted by the flow on a particle at the boundary is not always horizontal (10). Depending on the shape of particles, the dynamic force exerted may be inclined either upward or downward. If the particle is more or less spherical in shape, the lift F_L is away from the boundary (2), i.e., it is inward; if the sediment is somewhat flat, the "lift" may be toward the boundary (8), i.e., it is outward.

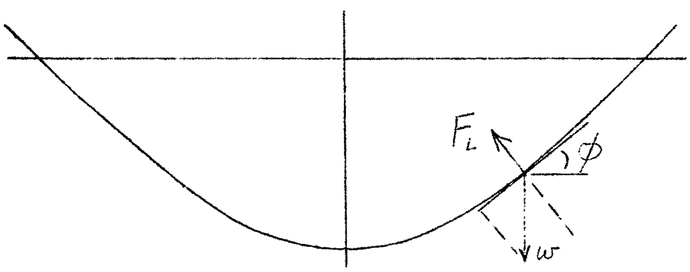

Fig. 5 - Hydrodynamic lift force acting on a particle resting on a side slope.

Let the ratio of the lift to the longitudinal drag be B, which is also the ratio of the lift coefficient and the drag coefficient. Then the lift acting on a particle is given by

$$\frac{\beta}{N} \gamma y S \cos \phi$$

Where β is positive when F_L is directed inward, and negative when F_L is directed outward. In this case the frictional force acting on a particle is given by

$$\left(w \cos\phi - \frac{\beta}{N} \gamma y S \cos\phi\right) \tan\theta$$

Eq. 3 is changed to

$$w^2 \sin^2\phi + \left(\frac{1}{N} \gamma y S \cos\phi\right)^2 = \left(w - \frac{\beta}{N} \gamma y S\right)^2 \cos^2\phi \tan^2\theta \qquad (36)$$

When $\phi = 0$, $y = y_0$ so that

$$\frac{1}{N} \gamma y_0 S = \left(w - \frac{\beta}{N} \gamma y_0 S\right) \tan\theta$$

$$\frac{\gamma y_0 S}{N} (1 + \beta \tan\theta) = w \tan\theta$$

$$\frac{\gamma S}{N} = \frac{w}{y_0} \frac{\tan\theta}{1 + \beta \tan\theta} = \frac{w}{y_0} \frac{f}{1 + \beta f} \qquad (37)$$

Eq. 6 is therefore changed to

$$\left(\frac{dy}{dx}\right)^2 + \left(\frac{f}{1+\beta f}\right)^2 \left(\frac{y}{y_0}\right)^2 = \left[1 - \frac{\beta f}{1+\beta f} \frac{y}{y_0}\right]^2 f^2$$

Rearrange terms

$$\left(\frac{dy}{dx}\right)^2 = f^2\left[1 - \frac{2\beta f}{1+\beta f}\left(\frac{y}{y_0}\right) - \frac{1-\beta^2 f^2}{(1+\beta f)^2}\left(\frac{y}{y_0}\right)^2\right] \qquad (38)$$

Substitute

$$\frac{y}{y_0} = z, \qquad \frac{dy}{dx} = y_0 \frac{dz}{dx}$$

Separate variables and take square root of both sides:

$$\frac{dz}{\sqrt{1 - \frac{2\beta f}{1+\beta f} z - \frac{1-\beta^2 f^2}{(1+\beta f)^2} z^2}} = \frac{f}{y_o} dx \qquad (39)$$

Integrate

$$\frac{1+\beta f}{\sqrt{1-\beta^2 f^2}} \sin^{-1}\left[\frac{1-\beta^2 f^2}{1+\beta f} z + \beta f\right] = f\frac{x}{y_o} + C_1 \qquad (40)$$

When $x = 0$, $\quad z = \frac{y_o}{y_o} = 1$

Therefore

$$C_1 = \frac{1+\beta f}{\sqrt{1-\beta^2 f^2}} \frac{\pi}{2}$$

Consequently

$$\frac{y}{y_o} = \frac{1+\beta f}{1-\beta^2 f^2}\left[-\beta f + \cos\left(\frac{f\sqrt{1-\beta^2 f^2}}{1+\beta f}\frac{x}{y_o}\right)\right] \qquad (41)$$

in which

$$\frac{x_o}{y_o} = \frac{1+\beta f}{f\sqrt{1-\beta^2 f^2}} \cos^{-1}\beta f \qquad (42)$$

Eq. 41 is a general equation for determining the shape of a stable channel. Eq. 41 expresses half of the channel width as a function of y_o, β and f. It should be noted from this development that the shape of a stable section is affected by any appreciable value of β, although in the determination of y_o the effect of lift is accounted for by using a working value of the tractive force. It should not be difficult to see that when β is positive, the cross section of a stable channel is flatter than that of $\beta = 0$.

In the case of a spherical particle placed on a plane surface, experiments conducted by Lossievsky and d'Abramov ([10]) indicated that the lift is about 25 per cent of the horizontal drag. In the case of closely placed hemispheres, Einstein and El - Samni ([2]) found that the lift coefficient is 0.178, namely that

$$F_L = 0.178\, a \rho \frac{V^2}{2}$$

Where V is to be measured at 0.15d above the top of the hemispheres of diameter d . According to Einstein and El - Samni, this coefficient also applies to natural gravel, provided that the diameter is taken as the sieve diameter of which 35 per cent by weight of the mixture is finer and that V is measured at 0.15d above the top of the grains.

For lack of better information, it will be tenatively suggested that the value of β determined by Einstein and El - Samni may be used for more or less rounded material; For coarser material of flat shape, β may be set equal to zero.

If β is assumed to be 0.18 , Eq. 41 becomes

$$\left(\frac{y}{y_0}\right)_\beta = 1.0507 \left[-0.04822 + \cos 87.2472^\circ \frac{x}{x_0})\right], \text{ for } \theta = 15^\circ \quad (43)$$

$$\left(\frac{y}{y_0}\right)_\beta = 1.0701 \left[-0.06552 + \cos \left(86.2431 \frac{x}{x_0}\right)\right], \text{ for } \theta = 20^\circ \quad (44)$$

$$\left(\frac{y}{y_0}\right)_\beta = 1.0878 \left[-0.08393 + \cos \left(85.1861^\circ \frac{x}{x_0}\right)\right], \text{ for } \theta = 25^\circ \quad (45)$$

$$\left(\frac{y}{y_0}\right)_\beta = 1.1159 \left[-0.10393 + \cos \left(84.2250^\circ \frac{x}{x_0}\right)\right], \text{ for } \theta = 30^\circ \quad (46)$$

$$\left(\frac{y}{y_o}\right)_\beta = 1.1442 \left[-01.2604 + \cos\left(82.7639^\circ \frac{x}{x_o}\right)\right] \quad , \text{ for } \theta = 35^\circ \quad (47)$$

$$\left(\frac{y}{y_o}\right)_\beta = 1.1779 \left[-0.15104 + \cos\left(81.3128 \frac{x}{x_o}\right)\right] \quad , \text{ for } \theta = 40^\circ \quad (48)$$

$$\left(\frac{y}{y_o}\right)_\beta = 1.2195 \left[-0.18000 + \cos\left(79.6306 \frac{x}{x_o}\right)\right] \quad , \text{ for } \theta = 45^\circ \quad (49)$$

Table VI has been computed by using Eqs. 43 - 49, and is shown by broken lines in Plate 1 (Appendix) . It is clear from Plate 1 that if uplift force is taken into consideration, the channel side slope is slightly reduced. The top width of the channel is slightly increased. The increase of the top width for $\beta \neq 0$ can be also found by comparison of Tables I and VII. The same result can be achieved by using a smaller value of θ . For the cases shown in plate I. a factor of safety defined as $\frac{\tan \theta}{\tan \theta'}$, varying from 1.049 to 1.022 , is needed. It is recommended, however, a factor of safety 1.25, which is used in obtaining the working value of the shear force, should be used in order to be consistant in choosing factor of safety.

Examples of Stable Channel Design

A certain irrigation system requires three canals carrying discharge 100 cfs, 30 cfs, and 20 cfs respectively. The slope of these canals is chosen to be 0.001. All these canals will be excavated through a region where the ground is composed of graded sand and gravel of slightly rounded shape. The size d_{25} is 1/4 inch. Design the most economical and stable cross sections for these canals.

Step 1 Determination of Q_0

As mentioned previously, the discharge Q_0 is the discharge through a canal whose two side slopes are symmetrically shaped according to Eq. 1,

$\frac{y}{y_0} = \cos \left(\frac{\tan \theta}{y_0} x \right)$ with $y = y_0$ at the center of the canal, where

y_0 is the maximum depth of flow for given earth material and canal slope, and is determined as follows:

According to Lane

$$T_c \ (\text{lbs}/\text{sq. ft}) = 0.4 \ d_{25} \ (\text{in}) \tag{50}$$

for

$$d_{25} = 0.25 \text{ in}$$

$$T_c = 0.4 \times 0.25 = 0.1 \ \text{lbs}/\text{sq ft.}$$

Since

$$T_c = 62.4 \ y_0 \ S \tag{2}$$

Therefore,

$$y_0 = \frac{Tc}{62.4 \quad S} = \frac{0.1}{62.4 \times 0.001} \doteq 1.6 \text{ ft.}$$

In order to apply Eq. 1 it is necessary to determine the angle of repose of the earth material. The angle of repose for slightly rounded, cohesionless earth at d_{25} = 0.25 in. has been estimated from Plate 6 (7) to be 25.4°. If a factor of safety 1.25 is used. A working value of the angle of repose can be found as follows:

$$\tan \quad \theta' = \frac{\tan \quad 25.4^\circ}{1.25}$$

therefore

$$\theta' = 21.5^\circ$$

The profile of the side slope can be determined according to Eq. (1) for y_0 = 1.6', and $\theta = \theta' = 21.5^\circ$. The coordinates of the side slope from Eq. 1 is given as

$$y = 1.6 \quad \cos \quad (0.25 \; x)$$

which is plotted as Fig. 6, such a canal has the following properties:

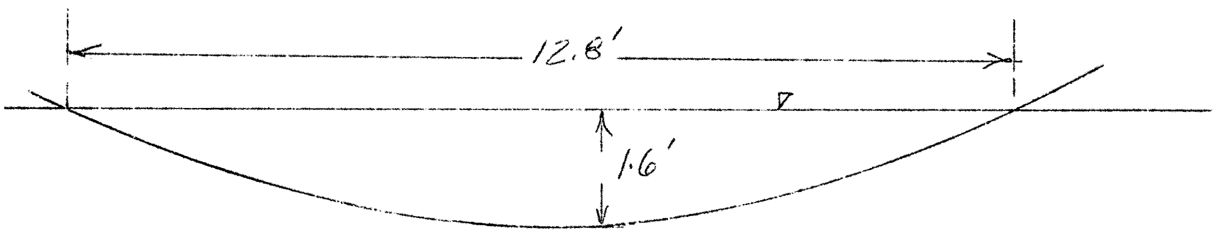

Fig. 6 Cross section for Q + 30 cfs, d_{25} = 0.25 inch and = 21.5°

(a) The top width $T = 2\chi$ is $\cdot \frac{\pi}{2} \frac{y_0}{\tan\theta} = \frac{\pi}{2} \frac{1.6}{\tan 21.5} = 12.8$ ft

(b) The total cross-sectional area A_0 can be determined from Eq. 14 for $\theta = 21.5^\circ$, and $y_0 = 1.60$ ft.

therefore

$$A_0 = \frac{2}{\tan 21.5^\circ} \times 1.6^2 = 13 \text{ sq. ft.}$$

(c) The total length of the wetted perimeter P_0 can be determined from Plate 3. At

$$\theta = 21.5^\circ , \quad \frac{P_0}{y_0} = 8.25$$

therefore

$$P_0 = 8.25 \; y_0 = 8.25 \times 1.60 = 13.2 \text{ ft.}$$

(d) The corresponding hydraulic radius, R_0, can be determined from Plate 4, at

$$\theta = 21.5^\circ , \quad \frac{R_0}{y_0} = 0.615 ,$$

therefore

$$R_0 = 0.615 \; y_0 = 0.615 \times 1.60 = 0.985 \text{ ft.}$$

(e) The Manning n, which is assumed to depend upon the size of material only, can be computed by using Eq. 51

$$n = \frac{(d_{25})^{1/6}}{39} \tag{51}$$

$$= \frac{(0.25)^{1/6}}{39} = 0.02$$

(f) The average velocity through such a section is therefore

$$V_0 = \frac{1.49}{0.02} \times 0.985^{2/3} \times 0.001^{1/2} = 2.32 \text{ fps}$$

(g) The capacity of such a section, Q_0, is

$$Q_0 = A_0 V_0 = 13 \times 2.32 = 30.2 \text{ cfs}$$

Step II Design of Canal Cross Sections

The design of a canal cross section depends upon the design discharge Q relative to Q_0 found from step I, there are three cases namely: (1) $Q = Q_0$, (II) $Q > Q_0$ and (III) $Q < Q_0 = 30.2$ cfs, a discharge of 100 cfs should be considered as case II, that of 20 cfs as case III, and that of 30 cfs approximately as case I. Each of these cases is discussed as following:

Case I $Q = Q_0$

Since $Q = 30$ cfs is approximately equal to Q_0, a cross section such as shown in Fig. 6 can be used, and a certain free board is needed for wave action and other requirements. The shape of the cross section may be modified slightly for construction purpose. It should be noted that case I is not very common. In other words, Q is frequently not equal to Q_0.

Case II $Q > Q_0$

In this particular case Q is 100 cfs, and Q_0 is 30.2 cfs. An increase of the canal depth is prohibited because of erosion problems. The canal should be widened to accomodate the additional discharge which is 100 - 30.2 = 69.8 cfs.

If the mean flow velocity is assumed to be the same as $V_0 = 2.32$ fps, the total cross sectional area, A, for Q = 100 cfs should be

$$A = \frac{100}{2.32} = 43 \text{ Sq. ft.}$$

The additional width, b at a uniform depth $y_0 = 1.6$ is

$$b = \frac{43 - 13}{1.60} = \frac{30}{1.6} = 18.75 \text{ ft.}$$

the total wetted perimeter P is 18.75 + 13.2 = 31.95 ft. and the total hydraulic radius R is $\frac{43}{31.95} = 1.345$ ft.

therefore, the mean velocity is

$$V = \frac{1.49}{0.02} \times 0.001^{1/2} \times 1.345^{2/3} = 2.36 \times 1.218 = 2.87 \text{ fps}$$

and the discharge Q = 43 x 2.87 = 123.5 cfs is greater than required. In order to determine the correct width of the canal, several values of b have been assumed and the corresponding capacities of the canal are obtained. A graph showing Q Vs b has been prepared as shown in Fig. 7. The correct b - value for Q = 100 obtained from Fig. 7 is 14.4 ft. Therefore, the total width of the canal carrying Q = 100 cfs is 14.4 + 12.8 = 27.2 ft.

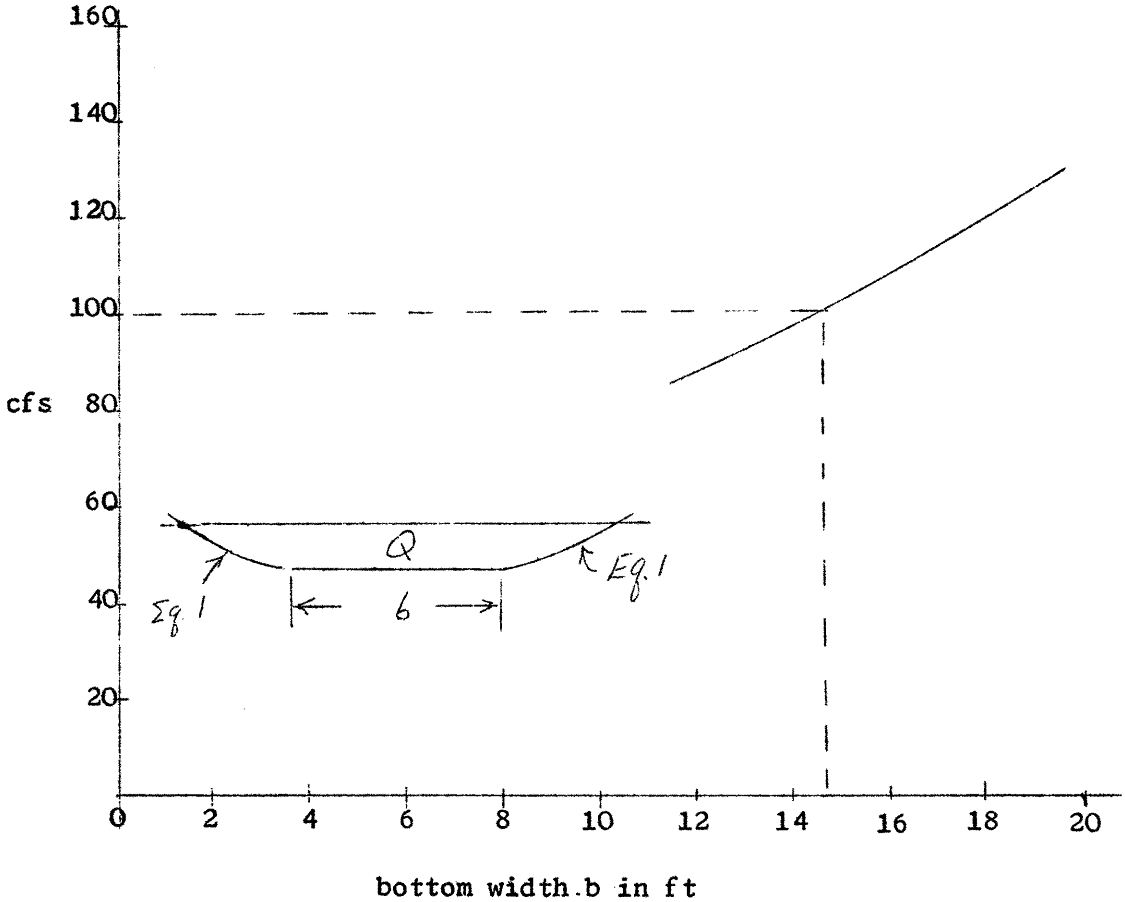

Fig. 7 Graph showing discharge against additional canal width

The cross section of such a canal is shown as Fig. 8.

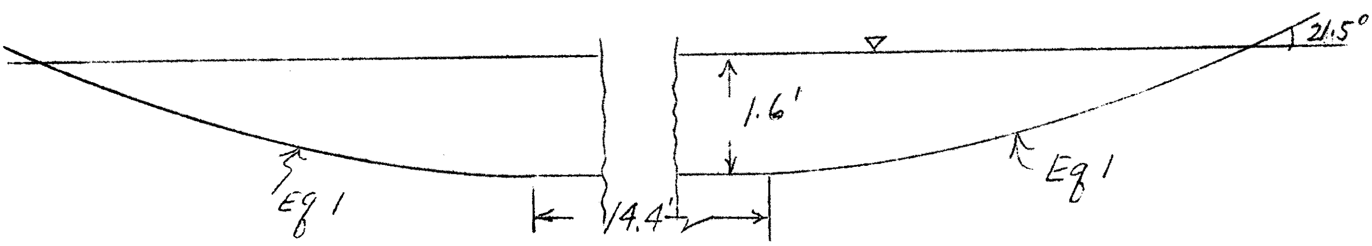

Fig. 8 Canal cross section Q = 100 cfs
d_{25} . 0.25 inch.
= 21.5° , S = 0.001

Case III $Q < Q_0$

In this particular case Q is 20 cfs, and Q_0 is 30.2 cfs. The cross section shown in Fig. 6 is larger than is required. Therefore, for economical reasons the central portion of the section should be removed and the remaining section is shown as Fig. 9. The extent of the central portion which is to be removed, depends upon the capacity of the shaded area shown in Fig. 4. This can be done easily as follows: The r elative capacity of the shaded area is taken as

$$\frac{Q}{Q_0} = \frac{20}{30.2} = 0.663 ,$$

According to Plate 5, it corresponds to $\frac{x}{x_0} = 0.178$ for $\theta = 21.5^\circ$, or $x = 6.4 \times 0.1778 = 1.13$ ft. The top width of the canal is then

$$12.8 - 2 \times 1.13 = 10.54 \text{ ft.}$$

The cross section of the designed canal is shown as Fig. 9.

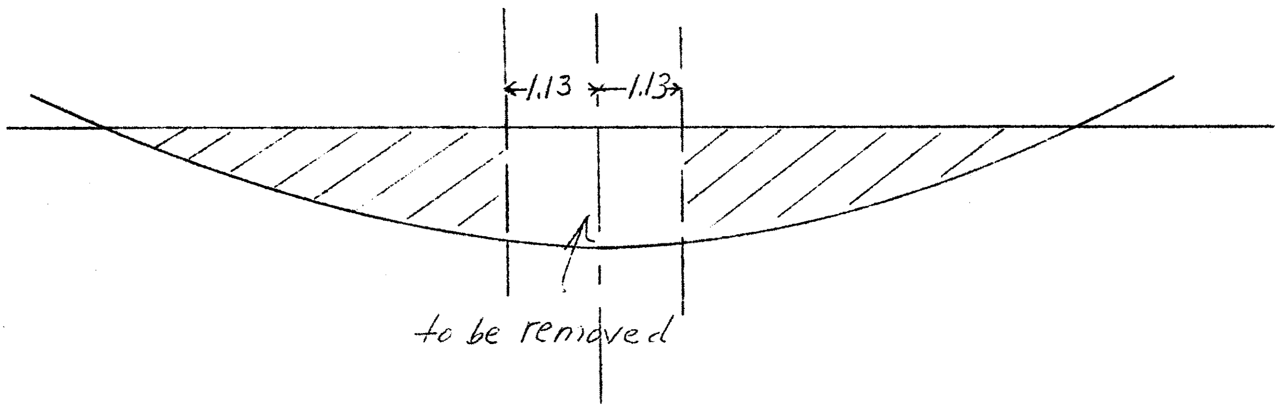

Fig. 9 Cross section for $Q = 20$ cfs. $d_{25} = 0.25$ inch and
$= 21.5^\circ$

Discussion

The design of a canal depends upon the type of the boundary and the sediment load in the canal.

The various boundary materials and sediment loads can be summarized as following:

Type of boundary:
- (a) rigid boundary
- (b) erodible boundary composed of cohesionless material
- (c) erodible condition composed of cohesive material
- (d) alluvial boundaries
- (e) movable but not alluvial boundary

Type of flow:
- (a) clear water
- (b) water with wash load
- (c) water with bed load
- (d) water with both bed load and suspended load

The present engineering knowledge of canal design is mostly for the case of clear water with or without wash load flowing in rigid channels. The method of designing stable channels discussed in this report is for clear water flowing in a channel composed of granular cohesionless material. Much less is known about the stable channel profile for clear water flowing in a channel composed of cohesive material. The regime channel theory (16, 17, and 18) which originated from India is intended for alluvial channels carrying sediment laden water. The problem for clear water in movable but not alluvial channels such as ditches covered with heavy vegetation, is not one concerning stable channel profiles but is one concerning the discharge coefficient.

This paper can be considered as an extension of Lane's previous work on design of stable channel. Sufficient number of plates and tables have been given so that the design of a stable channel for clear water in non-cohesive material can be followed easily. It should be pointed out that the presentation is based upon the assumption that the local tractive force on the boundary is proportional to the local depth of flow, and the effect of lateral momentum mixing is neglected. (See reference (14)). The analysis in this report is limited to the case of straight channels. The secondary current due to bend is not considered.

The Manning-Strickler formula is used in the report for computing the discharge. The formula is valid only for turbulent flow in channels with rough boundary. Liu and Hwang (15) found that the Manning-Strickler formula is applicable for plane bed if the bed material is larger than 4 mm. A revision of the computations should be made if the discharge formula proposed by Liu and Hwang is used. However, the outlined procedure can remain the same. The present analysis is based upon the concept of critical shear at the boundary. In order to improve the analysis it is necessary to study thoroughly the distribution of the boundary shear. The analysis does not consider the wave action which can be detrimental to the canal banks. On the other hand, the effect of draw-down on the canal bank seems to be unimportant, because the seepage pressure due to draw-down is negligible in cohesionless materials.

Summary

In this paper the shape of stable channels obtained earlier by Lane and Glover, was again derived in a clear way. It is intended for clear water flowing in a channel composed of granular cohesionless material. It is based on the concept of the critical tractive force. Everywhere along the bank, the tractive force plus the tangential weight-component of the particle should always be equal or less than the resistance of the particle.

The maximum allowable depth y_0 in a channel can be determined for a given slope and a given size of the bed material. The channel profile is a function of the size of the bed material, the angle of repose of the material and the channel slope. The profile of such a canal is a cosine curve with $y = y_0$ at $x = 0$ and $y = 0$ at $x = x_0$, where y is measured downward from the water surface. A channel of such a shape is stable and efficient for one discharge Q_0 only.

In case the design discharge is greater than Q_0, a wider channel should be used. The channel can be widened by increasing the central portion at a depth y_0 and still maintaining the stable profile along the banks. In case the design discharge is less than Q_0, a certain central portion of the stable profile can be eliminated.

Sufficient plates and tables have been prepared for design purposes.

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APPENDIX

NOTATION

A Area of a Cross Section Between $2X$ and $2X_0$.

A_0 Area of the entire Cross Section.

C Chezy's Coefficient pertaining to A.

C_0 Chezy's Coefficient pertaining to A_0.

E Elliptic function of the second kind.

F_L Lifting Force.

N Number of Particles per unit area of wetted surface.

P Wetted parameter pertaining to area A.

P_0 Wetted parameter pertaining to area A_0.

Q Discharge through A.

Q_0 Discharge through A_0.

R = Hydraulic Radius pertaining to A.

R_0 = Hydraulic Radius pertaining to A_0.

S Longitudinal Slope of Channel.

T_w Working Value of Tractive Force.

V = Q/A.

V_0 = Q_0/A_0.

a	Project Area of bed
d	Diameter of bed
f	Ten θ , Coefficient of friction.
n	The Manning coefficient.
v	Velocity of Flow near the bed.
x	Lateral distance from the center line of a cross section.
x_0	Half surface width of a cross section.
y	Depth of flow at x.
y_0	Depth of flow in the center of a cross section.
w	Submerged weight of bed material particle.
α	$1- \frac{x}{x_0}$
β	Ratio of lift and drag coefficient
γ	Unit weight of water
η	$\frac{E(\sin\theta, \frac{\pi}{2})}{E(\sin\theta, \alpha\frac{\pi}{2})}$
ϵ	$\frac{\pi}{2} \frac{f}{y} x$
ϕ	Lateral inclination of bed at point x.y.
θ	Angle of repose of material composing the bottom and sides of the channel.
θ'	Angle of repose of material when lift force is considered.

LIST OF TABLES

LIST OF PLATES

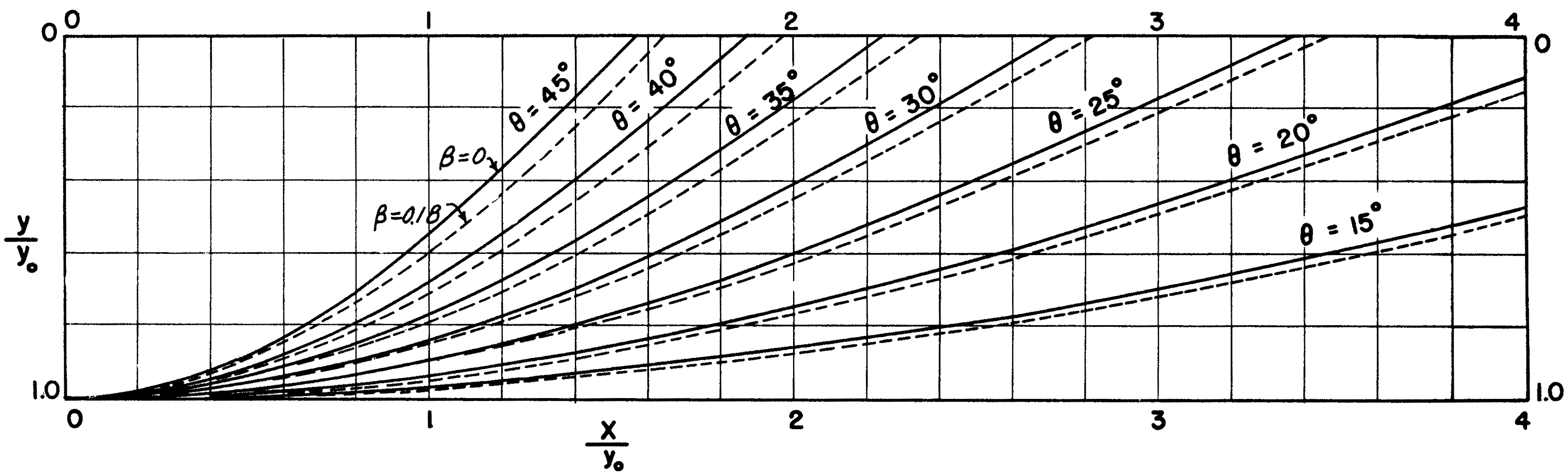


PLATE I CROSS SECTION OF STABLE CHANNELS

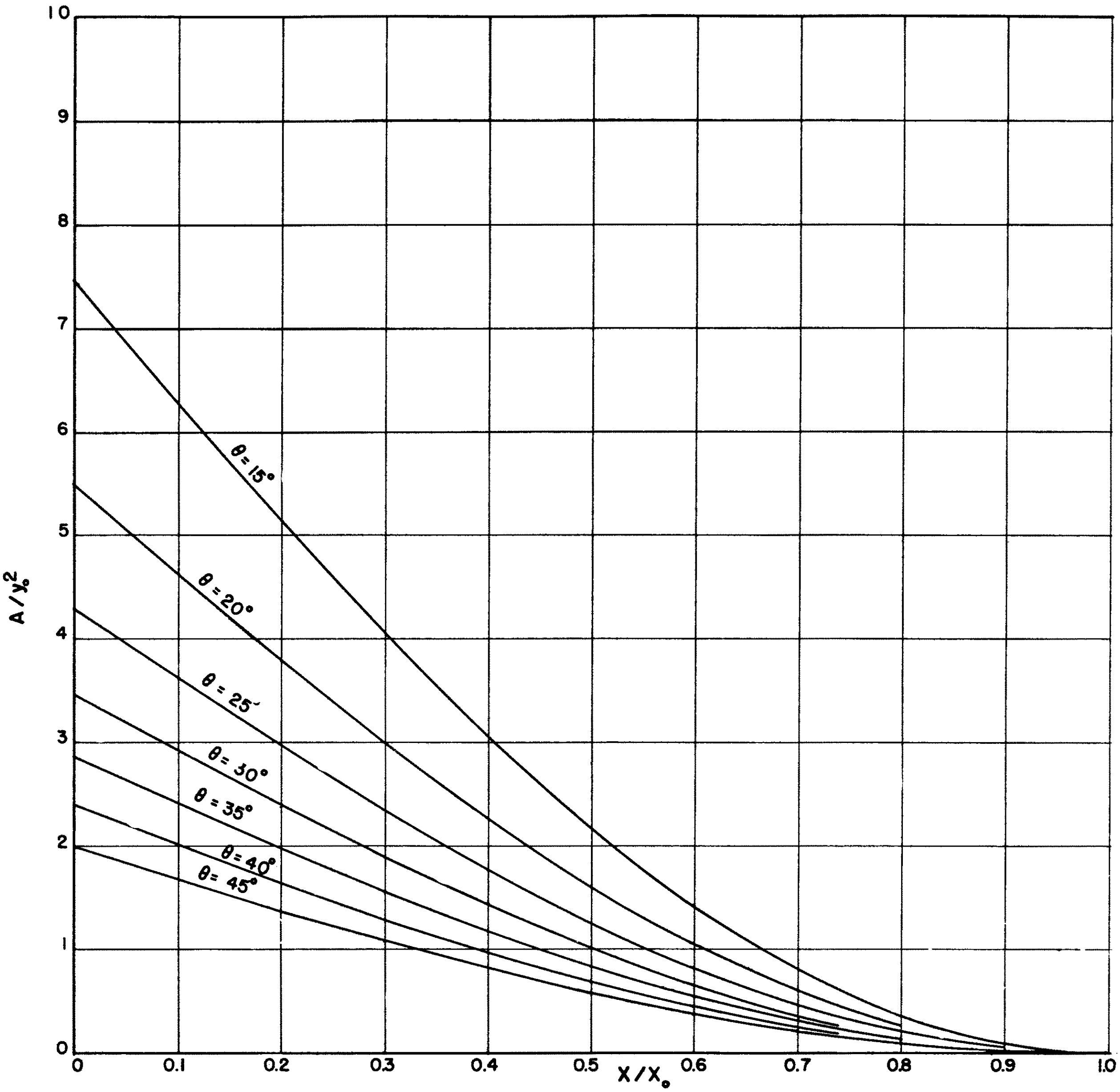


PLATE 2 CROSS SECTIONAL AREA OF STABLE CHANNELS

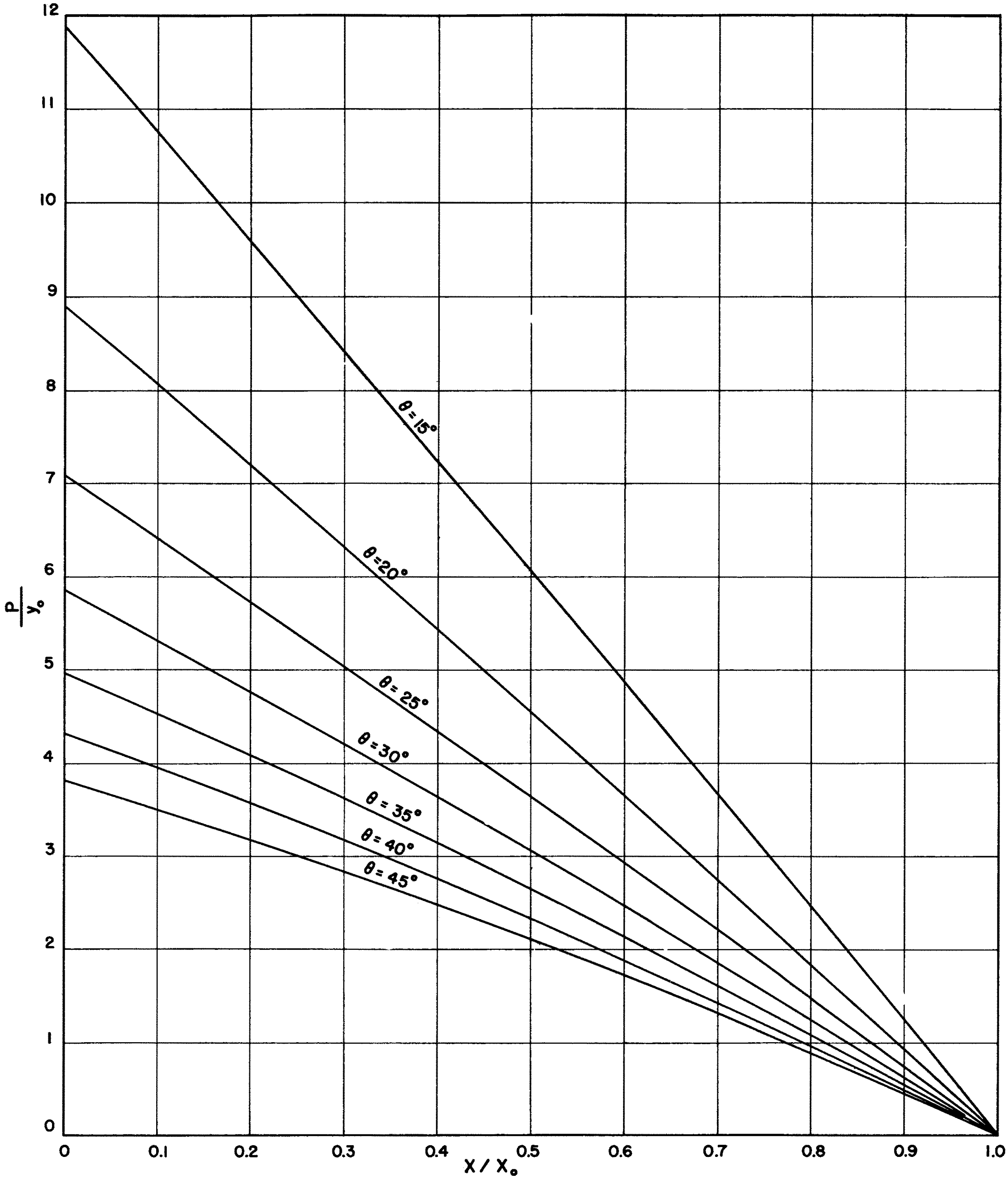


PLATE 3 WETTED PERIMETER OF STABLE CHANNELS

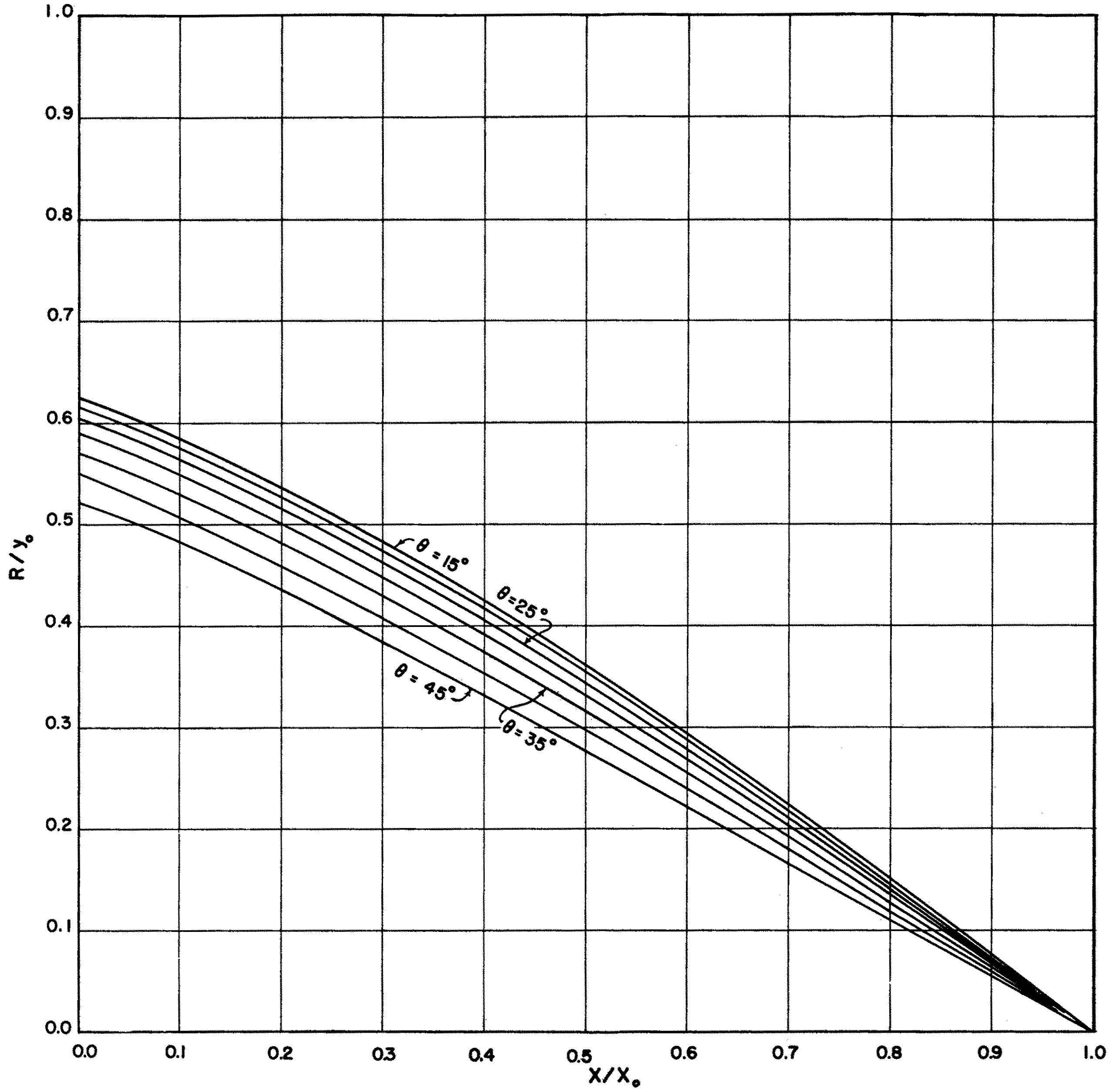


PLATE 4 HYDRAULIC RADIUS OF STABLE CHANNELS

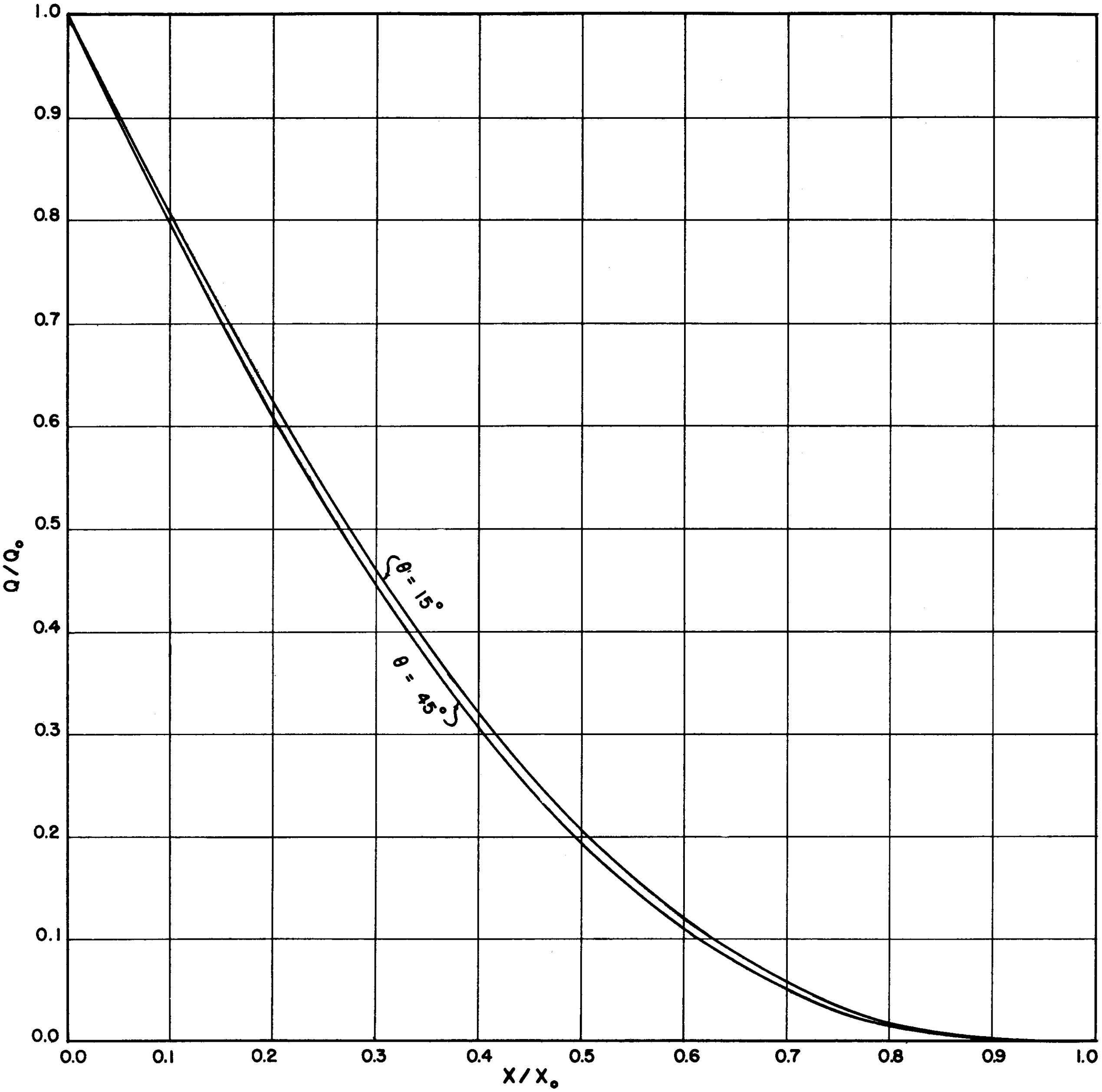


PLATE 5 CAPACITY OF STABLE CHANNELS

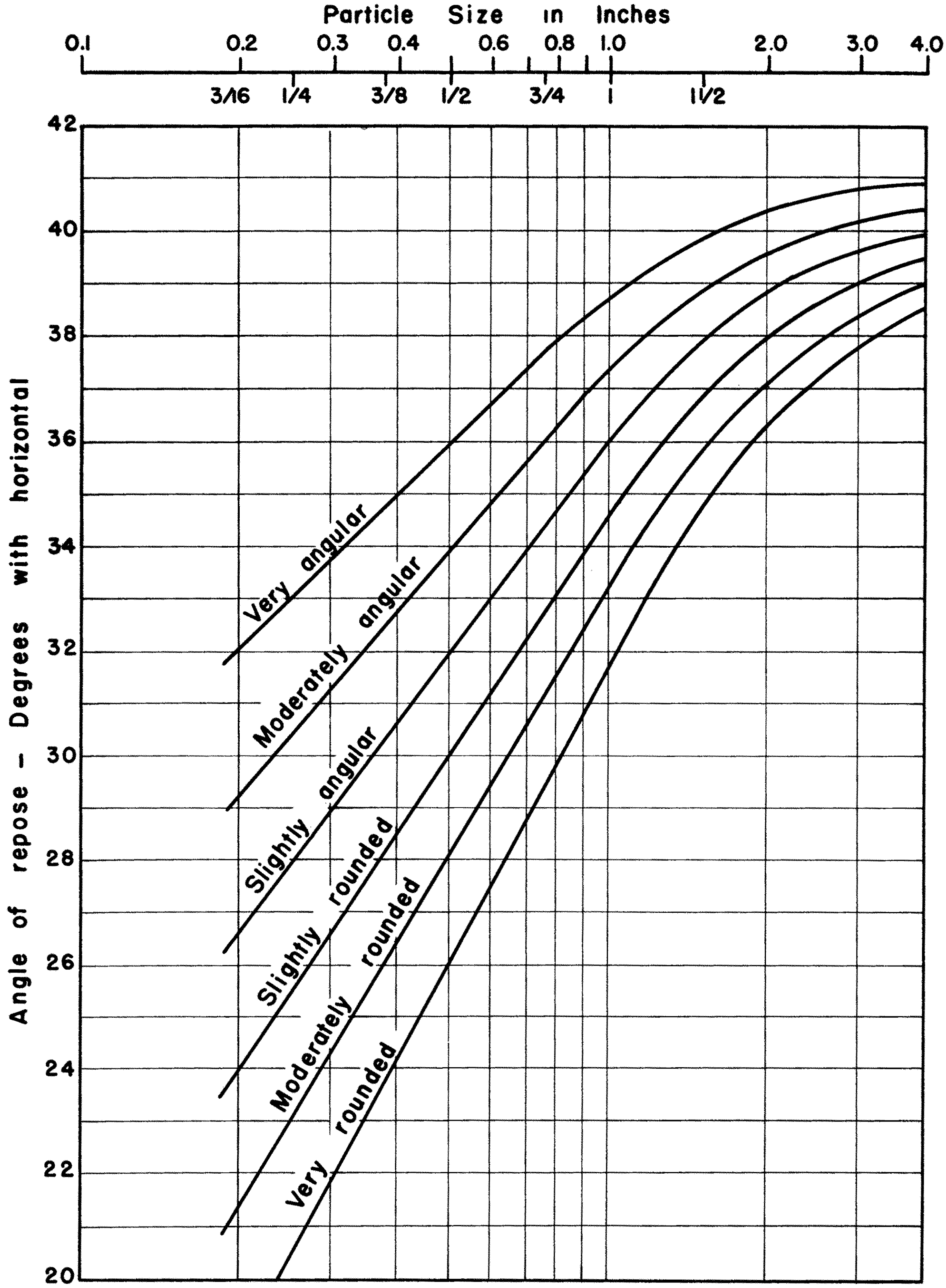


PLATE 6 ANGLE OF REPOSE OF NON-COHESIVE MATERIAL

TABLE I

Width-Depth Ratio of Stable Channels $\quad 2\,\dfrac{X_0}{Y_0} = \dfrac{\pi}{f}$

θ	15°	20°	25°	30°	35°	40°	45°
$2\,\dfrac{X_0}{Y_0}$	11.7248	8.6316	6.7372	5.4416	4.4866	3.7442	3.1416

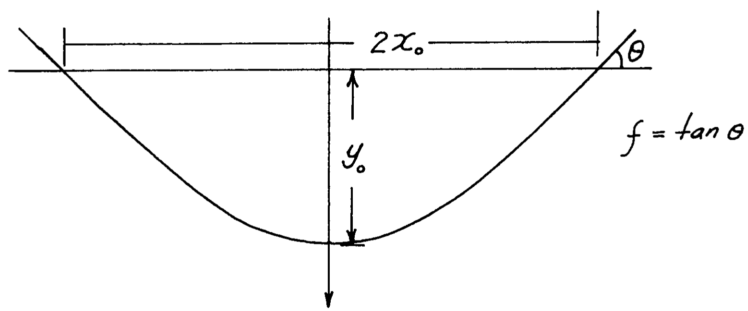


TABLE II

Cross Section of Stable Channels $$\frac{Y}{Y_0} = \cos\left[\tan\theta\ \frac{X}{Y_0}\right] = \cos\left(\frac{\pi}{2}\ \frac{X}{X_0}\right)$$

	$\frac{X}{X_0}$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.98	1.00
	$\frac{Y}{Y_0}$	1.0000	0.9877	0.9511	0.8910	0.8090	0.7071	0.5878	0.4540	0.3090	0.1564	0.0785	0.0314	0.0000
$\frac{X}{Y_0}$	$\theta = 15°$	0.0000	0.5862	1.1728	1.1758	2.3450	2.9312	3.5174	4.1037	4.6899	5.2762	5.5693	5.7452	5.8624
	$\theta = 20°$	0.0000	0.4316	0.8632	1.2947	1.7263	2.1579	2.5895	3.0211	3.4526	3.8842	4.1000	4.2295	4.3158
	$\theta = 25°$	0.0000	0.3369	0.6737	1.0106	1.3474	1.6843	2.0212	2.3580	2.6949	3.0317	3.2002	3.3012	3.3686
	$\theta = 30°$	0.0000	0.2721	0.5442	0.8162	1.0883	1.3604	1.6325	1.9046	2.1766	2.4487	2.5848	2.6664	2.7208
	$\theta = 35°$	0.0000	0.2243	0.4487	0.6730	0.8973	1.1217	1.3460	1.5703	1.7946	2.0190	2.1311	2.1984	2.2433
	$\theta = 40°$	0.0000	0.1872	0.3744	0.5616	0.7488	0.9361	1.1233	1.3105	1.4977	1.6849	1.7785	1.8347	1.8721
	$\theta = 45°$	0.0000	0.1571	0.3142	0.4712	0.6283	0.7854	0.9425	1.0996	1.2566	1.4137	1.4923	1.5394	1.5708

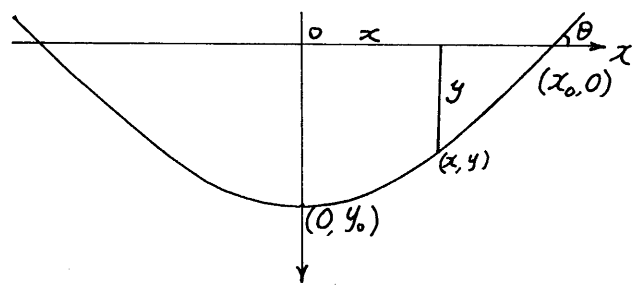


TABLE III

Cross-Sectional Area of Stable Channels $\frac{A}{Y_o^2} = \frac{2}{\tan\theta}\left[1 - \sin\left(\frac{X}{X_o}\frac{\pi}{2}\right)\right]$

$\frac{X}{X_o}$		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.98	1.00
$\frac{X}{X_o}\frac{\pi}{2}$		0.00	9°	18°	27°	36°	45°	54°	63°	72°	81°	85°30′	88°12′	90°00′
$\sin(\frac{X}{X_o}\frac{\pi}{2})$		0.0000	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511	0.9877	0.9969	0.9995	1.0000
$1 - \sin(\frac{X}{X_o}\frac{\pi}{2})$		1.0000	0.8436	0.6910	0.5460	0.4122	0.2929	0.1910	0.1090	0.0489	0.0123	0.0031	0.0005	0.0000
$\frac{A}{Y_o^2}$	$\theta = 15°$	7.4654	6.2979	5.1586	4.0761	3.0773	2.1866	1.4259	0.8137	0.3651	0.0918	0.0231	0.0037	0.0000
$\frac{A}{Y_o^2}$	$\theta = 20°$	5.4945	4.6352	3.7967	3.0000	2.2648	1.6093	1.0495	0.5989	0.2687	0.0676	0.0170	0.0027	0.0000
$\frac{A}{Y_o^2}$	$\theta = 25°$	4.2891	3.6183	2.9638	2.3418	1.7680	1.2563	0.8192	0.4675	0.2097	0.0528	0.0133	0.0021	0.0000
$\frac{A}{Y_o^2}$	$\theta = 30°$	3.4662	2.9240	2.3951	1.8925	1.4288	1.0153	0.6620	0.3778	0.1695	0.0426	0.0107	0.0017	0.0000
$\frac{A}{Y_o^2}$	$\theta = 35°$	2.8563	2.4096	1.9737	1.5596	1.1774	0.8366	0.5456	0.3113	0.1397	0.0351	0.0089	0.0014	0.0000
$\frac{A}{Y_o^2}$	$\theta = 40°$	2.3869	2.0107	1.6470	1.3014	0.9820	0.6981	0.4552	0.2598	0.1166	0.0293	0.0074	0.0012	0.0000
$\frac{A}{Y_o^2}$	$\theta = 45°$	2.0000	1.6872	1.3820	1.0920	0.8244	0.5858	0.3820	0.2180	0.0978	0.0246	0.0062	0.0010	0.0000

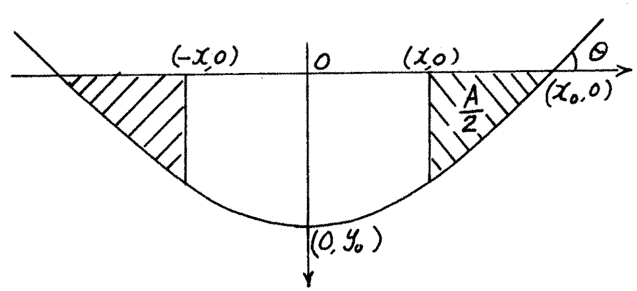
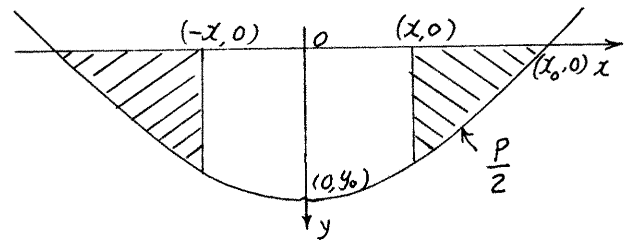


TABLE IV

Wetted Perimeter and Hydraulic Radius of Stable Channels

$$\frac{P}{Y_0} = \frac{2}{\sin\theta}\, B\left(\sin\theta \quad , (1 - \frac{X}{X_0})\frac{\pi}{2}\right) \qquad \frac{R}{Y_0} = \frac{A}{Y_0 P}$$



$\frac{X}{X_0}$		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.98	1.00
$\alpha = (1 - \frac{X}{X_0})\frac{\pi}{2}$		90°	81°	72°	63°	54°	45°	36°	27°	18°	9°	4°30′	1°48′	0°00′
$\theta = 15°$	$B(\sin\theta, \alpha\frac{\pi}{2})$	1.5442	1.3924	1.2403	1.0878	0.9346	0.7806	0.6258	0.4701	0.3138	0.1570	0.0782	0.0314	0.0000
	P/Y_0	11.9325	10.7595	9.5842	8.4058	7.2219	6.0319	4.8357	3.6326	2.4248	1.2139	0.6043	0.2426	0.0000
	R/Y_0	0.6256	0.5853	0.5382	0.4849	0.4261	0.3628	0.2949	0.2240	0.1506	0.0756	0.0382	0.0153	0.0000
$\theta = 20°$	$B(\sin\theta, \alpha\frac{\pi}{2})$	1.5238	1.3761	1.2280	1.0789	0.9287	0.7770	0.6238	0.4693	0.3136	0.1570	0.0782	0.0314	0.0000
	P/Y_0	8.9106	8.0469	7.1809	6.3090	5.4307	4.5436	3.6477	2.7443	1.8338	0.9181	0.4573	0.1836	0.0000
	R/Y_0	0.6166	0.5760	0.5287	0.4755	0.4170	0.3542	0.2877	0.2182	0.1465	0.0736	0.0372	0.0147	0.0000
$\theta = 25°$	$B(\sin\theta, \alpha\frac{\pi}{2})$	1.4981	1.3556	1.2124	1.0678	0.9212	0.7725	0.6214	0.4682	0.3133	0.1570	0.0782	0.0314	0.0000
	P/Y_0	7.0895	6.4151	5.7374	5.0532	4.3594	3.6557	2.9407	2.2157	1.4826	0.7430	0.3700	0.1486	0.0000
	R/Y_0	0.6050	0.5640	0.5166	0.4634	0.4056	0.3437	0.2786	0.2110	0.1414	0.0711	0.0360	0.0141	0.0000
$\theta = 30°$	$B(\sin\theta, \alpha\frac{\pi}{2})$	1.4675	1.3312	1.1939	1.0546	0.9125	0.7672	0.6186	0.4670	0.3129	0.1569	0.0782	0.0314	0.0000
	P/Y_0	5.8700	5.3248	4.7756	4.2184	3.6500	3.0688	2.4744	1.8680	1.2516	0.6276	0.3128	0.1256	0.0000
	R/Y_0	0.5905	0.5491	0.5015	0.4486	0.3915	0.3308	0.2675	0.2023	0.1354	0.0679	0.0342	0.0135	0.0000
$\theta = 35°$	$B(\sin\theta, \alpha\frac{\pi}{2})$	1.4323	1.3034	1.1729	1.0397	0.9026	0.7613	0.6155	0.4657	0.3125	0.1569	0.0782	0.0314	0.0000
	P/Y_0	4.9941	4.5447	4.0897	3.6252	3.1472	2.6545	2.1461	1.6238	1.0896	0.5471	0.2727	0.1095	0.0000
	R/Y_0	0.5719	0.5302	0.4826	0.4302	0.3741	0.3152	0.2542	0.1917	0.1282	0.0642	0.0326	0.0128	0.0000
$\theta = 40°$	$B(\sin\theta, \alpha\frac{\pi}{2})$	1.3931	1.2725	1.1498	1.0233	0.8919	0.7549	0.6122	0.4643	0.3121	0.1568	0.0782	0.0314	0.0000
	P/Y_0	4.3345	3.9593	3.5775	3.1839	2.7751	2.3488	1.9048	1.4446	0.9711	0.4879	0.2433	0.0977	0.0000
	R/Y_0	0.5507	0.5078	0.4604	0.4087	0.3539	0.2972	0.2390	0.1798	0.1201	0.0601	0.0304	0.0123	0.0000
$\theta = 45°$	$B(\sin\theta, \alpha\frac{\pi}{2})$	1.3506	1.2391	1.1250	1.0060	0.8806	0.7482	0.6087	0.4628	0.3116	0.1568	0.0782	0.0314	0.0000
	P/Y_0	3.8200	3.5047	3.1820	2.8454	2.4907	2.1162	1.7217	1.3090	0.8813	0.4435	0.2212	0.0888	0.0000
	R/Y_0	0.5236	0.4814	0.4343	0.3838	0.3310	0.2768	0.2219	0.1665	0.1110	0.0555	0.0280	0.0113	0.0000

TABLE V

Discharge Capacity of Stable Channels by Use of Manning's Formula

$$\frac{Q}{Q_o} = \left(1 - \sin \frac{X}{X_o}\,\frac{\pi}{2}\right)^{5/3} \eta^{2/3}, \quad \text{where} \quad \eta = \frac{B\left[\sin\theta\,,\ \frac{\pi}{2}\right]}{B\left[\sin\theta\,,\ \left(1 - \frac{X}{X_o}\right)\frac{\pi}{2}\right]}$$

$\frac{X}{X_o}$		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.98	1.00
$1 - \sin \frac{X}{X_o}\frac{\pi}{2}$		1.0000	0.8436	0.6910	0.5460	0.4122	0.2929	0.1910	0.1090	0.0489	0.0123	0.0031	0.0005	0.0000
$(1 - \sin \frac{X}{X_o}\frac{\pi}{2})^{5/3}$		1.0000	0.7532	0.5401	0.3647	0.2283	0.1292	0.0633	0.0249	0.0065	0.0007	0.0001	0.0000	0.0000
$(1 - \frac{X}{X_o})\frac{\pi}{2}$		90°	81°	72°	63°	54°	45°	36°	27°	18°	9°	4°30′	1°48′	0.0000
$\theta = 15°$	η	1.0000	1.1090	1.2450	1.4196	1.6523	1.9782	2.4676	3.2848	4.9210	9.8357	19.7468	49.1783	∞
	$\eta^{2/3}$	1.0000	1.0714	1.1573	1.2631	1.3912	1.5758	1.8261	2.2097	2.8931	4.5906	7.3058	13.423	∞
	Q/Q_o	1.0000	0.8070	0.6251	0.4607	0.3176	0.2036	0.1156	0.0550	0.0188	0.0032	0.0007	0.0000	0.0000
$\theta = 20°$	η	1.0000	1.1073	1.2409	1.4124	1.6408	1.9611	2.4428	3.2470	4.8591	9.7057	19.4859	48.5286	∞
	$\eta^{2/3}$	1.0000	1.0703	1.1548	1.2588	1.3911	1.5668	1.8138	2.1927	2.8688	4.5501	7.2413	13.305	∞
	Q/Q_o	1.0000	0.8061	0.6237	0.4591	0.3176	0.2024	0.1148	0.0546	0.0186	0.0032	0.0007	0.0000	0.0000
$\theta = 25°$	η	1.0000	1.1051	1.2357	1.4030	1.6262	1.9393	2.4108	3.1997	4.7817	9.5420	19.1572	47.4101	∞
	$\eta^{2/3}$	1.0000	1.0689	1.1515	1.2535	1.3829	1.5551	1.7978	2.1714	2.8383	4.4988	7.1595	13.154	∞
	Q/Q_o	1.0000	0.8051	0.6219	0.4572	0.3157	0.2009	0.1138	0.0541	0.0184	0.0031	0.0007	0.0000	0.0000
$\theta = 30°$	η	1.0000	1.1024	1.2292	1.3915	1.6082	1.9128	2.3723	3.1424	4.6900	9.3531	18.7659	46.7356	∞
	$\eta^{2/3}$	1.0000	1.0672	1.1475	1.2464	1.3726	1.5409	1.7787	2.1454	2.8019	4.4392	7.0618	12.975	∞
	Q/Q_o	1.0000	0.8038	0.6171	0.4546	0.3134	0.1991	0.1126	0.0534		0.0031	0.0007	0.0000	0.0000
$\theta = 35°$	η	1.0000	1.0990	1.2212	1.3776	1.5869	1.8814	2.3271	3.0756	4.5834	9.1287	18.3158	45.6146	∞
	$\eta^{2/3}$	1.0000	1.0650	1.1425	1.2381	1.3605	1.5240	1.7561	2.1149	2.7593	4.3679	6.9484	12.767	∞
	Q/Q_o	1.0000	0.8022	0.6171	0.4515	0.3106	0.1969	0.1112	0.0527	0.0182	0.0031	0.0007	0.0000	0.0000
$\theta = 40°$	η	1.0000	1.0948	1.2116	1.3614	1.5619	1.8454	2.2756	3.0004	4.4636	8.8846	17.8145	44.3662	∞
	$\eta^{2/3}$	1.0000	1.0622	1.1348	1.2284	1.3462	1.5045	1.7301	2.0803	2.7110	4.2897	6.8211	12.532	∞
	Q/Q_o	1.0000	0.8000	0.6129	0.4480	0.3073	0.1944	0.1095	0.0518	0.0176	0.0030	0.0007	0.0000	0.0000
$\theta = 45°$	η	1.0000	1.0900	1.2005	1.3425	1.5337	1.8051	2.2188	2.9183	4.3344	8.6135	17.2710	43.0127	∞
	$\eta^{2/3}$	1.0000	1.0591	1.1296	1.2110	1.3299	1.4825	1.7012	2.0421	2.6584	4.2020	6.6816	12.276	∞
	Q/Q_o	1.0000	0.7977	0.6101	0.4438	0.3036	0.1915	0.1077	0.0508	0.0173	0.0029	0.0007	0.0000	0.0000

TABLE VI

Modified Cross Section of Stable Channels

$$\left(\frac{Y}{Y_o}\right)_\beta = \frac{1 + \beta f}{1 - \beta^2 f^2}\left[-\beta f + \cos \frac{f\sqrt{1 - \beta^2 f^2}}{1 + \beta f}\frac{X}{Y_o}\right]$$

$$\beta = 0.18$$

	$\frac{X}{X_o}$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.98	1.00
$\theta = 15^\circ$	$\frac{X}{Y_o}$	0.0000	0.5967	1.1934	1.7900	2.3867	2.9834	3.5801	4.1768	4.7734	5.3701	5.6685	5.8475	5.9668
	$\frac{Y}{Y_o}$	1.0000	0.9879	0.9517	0.8923	0.8111	0.7099	0.5912	0.4576	0.3122	0.1584	0.0795	0.0317	0.0000
$\theta = 20^\circ$	$\frac{X}{Y_o}$	0.0000	0.4146	0.8831	1.3247	1.7662	2.2078	2.6494	3.0909	3.5325	3.9740	4.1948	4.3273	4.4156
	$\frac{Y}{Y_o}$	1.0000	0.9685	0.9519	0.8927	0.8118	0.7109	0.5924	0.4589	0.3135	0.1593	0.0801	0.0321	0.0000
$\theta = 25^\circ$	$\frac{X}{Y_o}$	0.0000	0.3468	0.6937	1.0405	1.3874	1.7342	2.0810	2.4279	2.7747	3.1216	3.2950	3.3990	3.4684
	$\frac{Y}{Y_o}$	1.0000	0.9845	0.9488	0.8901	0.8097	0.7095	0.5916	0.4587	0.3136	0.1595	0.0803	0.0322	0.0000
$\theta = 30^\circ$	$\frac{X}{Y_o}$	0.0000	0.2819	0.5637	0.8456	1.1275	1.4094	1.6912	1.9731	2.2550	2.5368	2.6778	2.7623	2.8187
	$\frac{Y}{Y_o}$	1.0000	0.9879	0.9523	0.8937	0.8134	0.7132	0.5950	0.4617	0.3160	0.1610	0.0810	0.0325	0.0000
$\theta = 35^\circ$	$\frac{X}{Y_o}$	0.0000	0.2342	0.4683	0.7025	0.9367	1.1709	1.4050	1.6392	1.8734	2.1075	2.2246	2.2949	2.3417
	$\frac{Y}{Y_o}$	1.0000	0.9881	0.9526	0.8942	0.8143	0.7143	0.5965	0.4513	0.3173	0.1618	0.0815	0.0326	0.0000
$\theta = 40^\circ$	$\frac{X}{Y_o}$	0.0000	0.1969	0.2689	0.5908	0.7878	0.9847	1.1816	1.3786	1.5755	1.7725	1.8709	1.9300	1.9694
	$\frac{Y}{Y_o}$	1.0000	0.9881	0.9529	0.8949	0.8153	0.7157	0.5982	0.4650	0.3190	0.1629	0.0821	0.0330	0.0000
$\theta = 45^\circ$	$\frac{X}{Y_o}$	0.0000	0.1646	0.3293	0.4939	0.6585	0.8232	0.9878	1.1524	1.3170	1.4817	1.5640	1.6134	1.6463
	$\frac{Y}{Y_o}$	1.0000	0.9822	0.9532	0.8955	0.8164	0.7172	0.6000	0.4670	0.3207	0.1640	0.0827	0.0332	0.0000

TABLE VII

Modified Width-Depth Ratio of Stable Channels on Account of Lift Force

$$\left(\frac{X_o}{Y_o}\right)_\beta = \frac{1 + \beta \tan\theta}{\tan\theta \sqrt{1 - (\beta \tan\theta)^2}} \cos^{-1} \beta \tan\theta \,, \quad \beta = 0.18 \,, \quad f = \tan\theta$$

θ	15°	20°	25°	30°	35°	40°	45°
$\left(\frac{X_o}{Y_o}\right)_\beta$	5.9668	4.4156	3.4684	2.8187	2.3417	1.9694	1.6463