ROUTING DEMAND CHANGES WITH VOLUME COMPENSATION: AN UPDATE

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ABSTRACT

Using the gate-stroking method, this paper shows that a complex open-channel flow feedforward control problem can be treated as a series of linearly additive single flow-change control problems. A key element of this approach is determining the initial conditions for each single flow-change problem. An inadequate choice of initial conditions will result in under or overestimation of the canal storage volume change needed for the new steady-state conditions. These findings provide support to a simple feedforward control scheme based on volume compensation and time delay. An example is used to demonstrate that the simple scheduling approach is nearly as effective in controlling water levels as the complex gate-stroking approach.

INTRODUCTION

Bautista and Clemmens (1998) proposed a simple method for routing known demand changes through an open-channel water delivery system (the feedforward control problem) using the concept of volume compensation. Volume compensation refers to the volume of water that needs to be added or removed from a canal pool in going from an assumed initial steady-state to a desired new steady-state condition. That volume is delivered through a small number of step changes in inflow rate. The magnitude of those changes depends on estimates of the time needed for the flow changes to travel the length of the channel (the travel delay time \( \tau \)). A key problem of volume compensation is determining this delay, and thus, the timing of the inflow changes.

Simulation studies have demonstrated the application of the volume-compensating feedforward control method to specific water delivery systems (Bautista and Clemmens, 1998; Bautista and Clemmens, 1999a). Additional research is needed to generalize those results and to identify limitations of the method. A recent study used gate-stroking (Wylie, 1969) and volume compensation to examine the characteristics of feedforward control solutions for single-pool canals of uniform geometry (Bautista et al, 2002). The gate-stroking

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method solves the governing equations of unsteady open-channel flow inversely in space. The study considered a wide range of canal geometries and flow configurations. The gate-stroking method can fail to find a solution or can produce a solution requiring discharges exceeding the canal capacity or flow reversal under conditions where the time needed to supply the canal volume change is small relative to the disturbance wave travel time. Volume compensation offers a solution under those conditions and the resulting water level control is satisfactory. There are also conditions under which upstream flow changes travel with little attenuation and, therefore, the inflow hydrograph computed by gate-stroking nearly matches the desired outflow hydrograph. Under those conditions, a volume-compensating schedule can be easily identified and will produce water level control comparable to that obtained with gate-stroking.

Bautista and Clemmens (1998) outlined a volume-compensation strategy for multi-pool canal systems subject to multiple changes, but provided no justification for the approach. Recent tests, not reported here, with canal systems subject to multiple flow changes have resulted in adequate control for some demand changes but less adequate for others, suggesting problems with the original approach. The purpose of this paper therefore is to reexamine the basic concept used and to refine the method.

**MULTI-POOL SYSTEMS: ADDITIVITY OF SOLUTIONS**

The volume-compensating feedforward control method for multi-pool systems suggested by Bautista and Clemmens (1998) treats the multiple flow change problem as a series of linearly additive single flow change problems. Because the governing equations of unsteady open-channel flow are nonlinear, one can not expect this assumption to hold in general. This section analyzes the linearity of feedforward control solutions, using the full Saint Venant equations (the gate-stroking method) under a specific set of flow conditions. Determining conditions under which gate-stroking solutions are additive should suggest conditions under which the feedforward control problem can be treated as a linear problem.

This analysis uses one of the test cases proposed by the ASCE Task Committee on Canal Control Algorithms (Clemmens et al, 1998), ASCE Test Canal 2, Scenario 2. Canal characteristics and test details are given in Table 1. The canal is 28 km long and relatively flat. The canal's geometry, together with the specified flow conditions, results in a low Froude number for all pools. All pools are entirely in backwater for the initial flow conditions. This means that disturbances can travel up and down the canal for a long time and, thus, flow levels can oscillate for a long time. In a previous study, a finite-difference gate-stroking model for multiple pools (Bautista et al. 1997) was used to compute a feedforward flow schedule for this test case and was shown to produce satisfactory water level control (Bautista and Clemmens, 1999b). In this paper,
rather than processing all demand changes simultaneously as was done in that reference, each flow change was processed individually, as is described next.

Table 1. ASCE Canal Control Test Case 2-2: geometric and flow data

<table>
<thead>
<tr>
<th>Pool</th>
<th>Pool Length (km)</th>
<th>Pool Bottom Width (m)</th>
<th>Pool Target Downstream Depth (m)</th>
<th>Initial Pool Inflow (m$^3$/s)</th>
<th>Initial Offtake Flow Change (m$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.0</td>
<td>7.0</td>
<td>2.1</td>
<td>2.7</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
<td>7.0</td>
<td>2.1</td>
<td>2.5</td>
<td>0.3</td>
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<td>3.0</td>
<td>7.0</td>
<td>2.1</td>
<td>2.2</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>4.0</td>
<td>6.0</td>
<td>1.9</td>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>4.0</td>
<td>6.0</td>
<td>1.9</td>
<td>1.7</td>
<td>0.2</td>
</tr>
<tr>
<td>6</td>
<td>3.0</td>
<td>5.0</td>
<td>1.7</td>
<td>1.5</td>
<td>0.3</td>
</tr>
<tr>
<td>7</td>
<td>2.0</td>
<td>5.0</td>
<td>1.7</td>
<td>1.2</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>2.0</td>
<td>0.6</td>
<td>1.7</td>
<td>1.0$^+$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

$^8$ For all pools, bottom slope = 0.0001, side-slope = 1.5, and Manning n = 0.02
$^1$ Flow past the canal's tail end is 0.7 m$^3$/s.

In the example, flows change at six of the eight turnouts three hours after the beginning of the test$^2$. Since all demand changes take place at the same time, it is clear that the change in the most-downstream pool has to be routed first (i.e., requires the earliest change in inflow at the head of the canal). Initial conditions for that sub-problem are, simply, the time-zero initial conditions (discharges and levels). The second demand change to be routed is that originating in the penultimate pool, 7. Assuming a new steady-state as a result of the demand change in pool 8, initial flows for this second sub-problem are the sum of the initial flows and the demand change for the first sub-problem (a flow increase of 4.0 m$^3$/s in all pools). Initial water levels depend on these flows and the prescribed downstream target level. The same logic can be applied to determine the initial conditions of all remaining flow changes.

Solutions were combined for each check structure by adding all flow increment hydrographs for that particular check structure to its time-zero initial discharge. As an example, for the head gate, the time-zero initial discharge is 2.7 m$^3$/s (table 1). Since six individual offtake flow changes need to be processed, six different hydrographs are computed for the head gate. The flow increment hydrograph

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$^2$ The Test Case originally requires changes to occur two hours after the beginning of the test (Clemmens et al., 1998). This time was modified to allow the initial flow changes at the head gate to occur at a time greater than time zero.
resulting from each demand change is the difference between the gate-stroking solution and the initial conditions for that particular sub-problem. Since demand changes at a location do not affect check flows downstream from that location (once unsteadiness caused by the change has dissipated), the number of flow increment hydrographs that needs to be combined decreases as the check is located farther downstream. For example, for the check structure between pools 6 and 7, the combined hydrograph is simply the solution to the individual demand change in pool 8 plus the flow increment hydrograph due to the change in 7.

Figure 1 compares the linearly combined and nonlinear simultaneous solutions obtained for the head gate. The solutions are nearly in agreement for most of the hydrograph. The mismatch in the initial part of the hydrograph suggests that the difference is related to the demand change or changes at downstream pools, since those changes would require the earliest flow changes at the head gate.

To understand the above mismatch, gate-stroking solutions were developed for a simpler problem, consisting of the demand changes in pools 7 and 8 only. Two different combination solutions (CS1, CS2) for the head-gate are shown in Figure 2, along with the simultaneous solution (SS). Solution CS1 is based on the same assumption used in the preceding analysis, namely that in processing the demand change in pool 7, prior changes (i.e., the change from pool 8) have reached steady-state conditions. In contrast, solution CS2 assumes that the prior change in pool 8 has not taken place. That change is larger than the initial canal flow so it is likely that the resulting steady state will not be reached until after the change in
pool 7 takes place. Because initial conditions are difficult to identify, the same initial conditions used to process the change in pool 8 were applied to process the demand change in pool 7. In comparison with the hydrograph from the simultaneous solution (SS), the CS1 hydrograph shows a large flow rate increase and then a large decrease. Those oscillations are not present in the CS2 hydrograph and, the hydrograph’s shape is closer to the simultaneous solution. Notice however that the volume of water delivered to the canal with CS2 is less than that delivered by the simultaneous solution (the volume can be calculated by integration of the hydrograph with respect to time). This volume mismatch should cause water levels to temporarily deviate from their target value. Clearly, the steady conditions assumed by the original approach, CS1, result in an incorrect estimation of the transient response, however they do account more accurately for the needed volume change (the resulting volume is in close agreement with the volume delivered by the simultaneous solution hydrograph).

Determining the initial of conditions of each sub-problem is easy for the Test Case and the order in which each demand change needs to be routed is evident. If the demand changes take place at different times, determining the order in which they need to be routed, and the resulting impact on initial conditions of subsequent flow changes, is less obvious. This problem was solved as follows: individual gate-stroking solutions were generated for a set of demand changes (with changes in the pools at different times) using the time-zero initial conditions for each individual sub-problem. The solution requiring the earliest flow change at the head gate was then assumed the first to be routed. The final conditions resulting from this first demand change were then used to define new initial conditions for the remaining set of demand changes, from which the next demand change to be routed was identified. The process was continued until all demand changes were processed. This approach was applied to modified versions of the Test Case, with demand changes taking place at different times. Results of these tests, which are not presented here, again showed reasonable agreement between the hydrographs computed by routing all changes simultaneously and those computed by routing the changes individually and then combining them.

These results show that the complex feedforward control problem, consisting of multiple pools and flow changes, is somewhat linear. Difficulties in applying this approach are likely to be encountered when dealing with very large flow rate changes, as such changes would result in long-lasting unsteady flow. In such cases, one could consider interpolation, to estimate a more representative set of initial conditions for a given flow change. While that approach may reflect better the dynamics of the transient, it will not satisfy its volume compensation requirements. The simpler and more consistent approach is to assume that each individually routed demand change completely defines the initial conditions for the next change.
SIMPLE VOLUME COMPENSATION SOLUTION

A volume-compensating feedforward control schedule for a single demand change in a single-pool canal can be obtained by dividing the pool's volume change $\Delta V$ by the travel delay $\tau$ (Bautista and Clemmens, 1998; Bautista et al. 2002):

$$\Delta Q_1 = \frac{\Delta V}{\tau}$$

(1)

$\Delta Q_1$ represents the flow rate change at the upstream check structure. The desired final steady-state check discharge, $Q_f$, is the sum of the initial steady-state check discharge, $Q_0$, and the demand change, $\Delta q_d$. Depending on the value of $\tau$, $Q_0 + \Delta Q_1$ may not match $Q_f$. Therefore, a second check-flow change, $\Delta Q_2$, will likely be needed to adjust the check discharge to $Q_f$.

$$\Delta Q_1 = \Delta q_d - \Delta Q_1$$

(2)

For the range of conditions examined in Bautista et al (2002), suggested bounds for $\tau$ are:

$$\tau_{DW} \leq \tau \leq \tau_{AV}$$

(3)

$\tau_{DW}$ is a delay estimate based on dynamic wave theory,

$$\tau_{DW} = \frac{L}{v_0 + c_0}$$

(4)

where $L$ is the canal length, $v_0$ the average flow velocity under the initial flow conditions, and $c_0$ average celerity under the initial flow conditions. $\tau_{AV}$ in (3) is a delay based on the time needed to supply $\cdot \cdot \cdot$ at a rate equal to the demand change:

$$\tau_{AV} = \frac{\Delta V}{\Delta q_d}$$

(5)

In cases where the wave introduced by upstream flow changes travels with little attenuation, $\tau_{AV}$ can also be interpreted as a kinematic shock travel time. With $\tau$ in (1) given by (5), $\Delta Q_2 = 0$.

Bautista and Clemmens (1988) computed $\tau$ using kinematic and dynamic wave theory. That approach requires estimates of the pool length affected by backwater for the given flow conditions. When applied to the Test Case, this approach proved inappropriate as it yielded discharge changes at the check structures.
greater than the canal capacity as a result of very small delay values. A simpler and more conservative approach was used here, by using (5) as the delay. As noted, this reduced the inflow schedule to a single change,

\[ \Delta Q = \Delta q_d \]  

(6)

and, more importantly, bounded the magnitude of the check-flow change.

If the canal has multiple pools and a single demand change occurs in pool \( J \), then a schedule of inflow changes needs to be computed for all check structures upstream from pool \( J \). The schedule of check \( J \) (pool \( J \)'s upstream check) is a function of pool \( J \) only. For pool \( J-1 \), the schedule is a function of the sum of volume changes and accumulated delays of pools \( J-1 \) and \( J \). For \( j \)-th check structure, the expression for the discharge change is (Bautista and Clemmens, 1998):

\[ \Delta Q_j = \frac{\sum_{k=J}^{j} \Delta V_k}{\sum_{k=J}^{j} \Delta \tau_k} \]  

(7)

This equation applies to the general case in which \( \tau \) in (1) is obtained by any reasonable procedure. In such case, the timing for \( \Delta Q_j \) for structure \( j \) is given by:

\[ t(\Delta Q_j) = t_a - \sum_{k=J}^{j} \Delta \tau_k \]  

(8)

while the timing for the second check-flow change, \( t(\Delta Q_2) \), is the demand change time, \( t_d \). If the delays are given by (5), then application of (7) yields simply \( \Delta q_d \) (Eq. 6) while \( \Delta Q_2 = 0 \). For a canal subject to multiple demand changes, each change has to be processed separately. The resulting time sequence of \( \Delta Q_j \)'s then defines the feedforward control schedule for check structure \( j \).

Bautista and Clemmens (1998) applied this approach to situations with multiple demand changes by assuming that a pool’s flow was equal to the time zero discharge plus all demand changes ordered prior to the time of the requested \( \Delta q_d \). Only demand changes in the pool being processed or in pools downstream from it were included in this sum. That approach was modified to properly identify the initial conditions that need to be used to process each individual demand change, as discussed in the previous section. However, instead of using gate-stroking solutions, accumulated delays (the denominator of (7)) were used to determine the order in which individual demand changes needed to be routed.

The head-gate inflow hydrograph obtained with this method is shown in Figure 3 along with the hydrograph obtained via gate-stroking. It should be noted that the
final steady-state conditions of the test case are close to the canal’s maximum discharge capacity (Clemmens et al., 1998) and, therefore, the gate-stroking solution exceeds temporarily that maximum value.

Water level control produced with the gate-stroking and volume-compensation feedforward control schedules are shown in Figures 4. These results were computed with the unsteady flow simulation model CanalCAD (Holly and Parrish, 1995). The simulator used the control schedules to determine check flow rate setpoints as a function of time and internally computed a gate position for the new flow setpoint. Flow through the gravity offakes varied in response to water level fluctuations in the canal.

Three things are evident from Figure 4. First, water-level deviations were much larger with the simple approach (Figure 4b) than with gate-stroking (Figure 4a). Second, despite these large deviations, near-steady-state conditions were achieved shortly after the time at which the offtake flow changes occur. Lastly, in both cases the deviations were small relative to the target levels (Table 1).
Table 2. Maximum Absolute Error (MAE) and Integrated Average Error (IAE) for test case, from simulation with gate-stroking and volume compensating solutions

<table>
<thead>
<tr>
<th></th>
<th>Pool 1</th>
<th>Pool 2</th>
<th>Pool 3</th>
<th>Pool 4</th>
<th>Pool 5</th>
<th>Pool 6</th>
<th>Pool 7</th>
<th>Pool 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gate-Stroking</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>1.8%</td>
<td>0.8%</td>
<td>1.5%</td>
<td>0.4%</td>
<td>0.4%</td>
<td>0.7%</td>
<td>1.1%</td>
<td>4.5%</td>
</tr>
<tr>
<td>IAE</td>
<td>0.8%</td>
<td>0.2%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Volume-Compensation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>5.7%</td>
<td>4.0%</td>
<td>3.7%</td>
<td>4.4%</td>
<td>4.4%</td>
<td>5.2%</td>
<td>7.6%</td>
<td>7.2%</td>
</tr>
<tr>
<td>IAE</td>
<td>0.6%</td>
<td>0.1%</td>
<td>0.2%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.3%</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

Two performance measures recommended by the ASCE Task Committee on Canal Control Algorithms (Clemmens et al, 1998) were computed for these tests. The Maximum Absolute Error (MAE) is a measure of the maximum water level deviation relative to the target. The Integrated Average Error is a measure of the average absolute error relative to the target. Results are summarized in Table 2. The MAE for the simple feedforward control is as much as ten times greater than with gate-stroking, however these errors are short lived and have little impact on the average performance. The average error for all pools with both feedforward control methods is less than 1% of the target level.

**CONCLUSIONS**

For the example presented, similar gate-stroking results were obtained by processing all demand changes simultaneously and by treating the problem as a linear combination of single-flow change problems. The analysis assumed a succession of steady states and, thus, differences in results were due to unsteady flow effects not accounted for in defining initial conditions for individual flow change problems. Results show that even under conditions where strong unsteady effects would persist for long times, reasonable results can be obtained by assuming that each demand change creates a new set of steady initial conditions for the next flow change to be routed. Such an approach also assures volume compensation. It has been previously shown that a simple feedforward control method based on volume compensation can produce reasonable water level control in single-pool canals subject to a single demand change. A strategy was developed to apply the volume compensation method to multiple-pool canals subject to multiple flow changes. The resulting water level control over the test period was, on the average, comparable to that obtained with gate-stroking. This
suggests that the proposed volume compensation approach is both practical and effective.

REFERENCES


