

DISSERTATION

A CONCEPTS FOR CALCULUS INTERVENTION: MEASURING STUDENT  
ATTITUDES TOWARD MATHEMATICS AND ACHIEVEMENT IN CALCULUS

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WE HEREBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER OUR SUPERVISION BY MARY E. PILGRIM ENTITLED A CONCEPTS FOR CALCULUS INTERVENTION: MEASURING STUDENT ATTITUDES TOWARD MATHEMATICS AND ACHIEVEMENT IN CALCULUS BE ACCEPTED AS FULFILLING IN PART REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

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## ABSTRACT OF DISSERTATION

### A CONCEPTS FOR CALCULUS INTERVENTION: MEASURING STUDENT ATTITUDES TOWARD MATHEMATICS AND ACHIEVEMENT IN CALCULUS

Data indicate that about 40 percent of students initially enrolled in MATH 160: Calculus for Physical Scientists I finish the course with a grade of D or F, dropped, or withdrew from the course (Reinholz, 2009). The high failure rate led to an intervention course (MATH 180) for students at risk of failing MATH 160.

At-risk students were identified based on their calculus exam one scores. This dissertation reports on the effect of MATH 180 during the fall 2009 semester on both student achievement in MATH 160 and math attitude. Students identified as being at-risk of failing MATH 160 were invited to drop MATH 160 and enroll in MATH 180. Not all students that were invited accepted the invitation. After completing MATH 180 during the fall 2009 semester, students then had the option to enroll in MATH 160 for the spring 2010 semester.

MATH 180 students exhibited improvement in exam one scores. From the fall 2009 semester to the spring 2010 semester students raised their exam one scores by one-half of a standard deviation. Although MATH 180 students showed improvement in MATH 160 during the spring 2010 semester, there were no overall significant differences in achievement between students that took MATH 180 and those that did not.

Qualitative analysis indicated that MATH 180 students came to understand that calculus problems could be solved using multiple strategies, but they did not always know what those strategies were.

In class it was hard at first to understand the direction it was going but it was helpful to try to think at math differently than I have been taught all my life.

Math attitude was measured using the Modified Indiana Mathematics Belief Scales (MIMBS). MIMBS scores improved for students that took MATH 180, but there were no significant differences between MATH 180 students and non-MATH 180 students. There were significant correlations between constructs measured by the MIMBS and final course grade in MATH 160.

Despite there being no significant differences in academic performance, trends in the data indicate higher final exam scores and course grades for students in the intervention group.

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## CHAPTER I: INTRODUCTION

The purpose of this study was to test an intervention meant to increase student achievement in Calculus I at Colorado State University. In order to provide an introduction to the study presented in this dissertation, chapter one will follow the conceptual map provided in Figure 1.1.

### Conceptual Map of Chapter One



*Figure 1.1.* A conceptual map of the major topics discussed in chapter one.

### A Brief History of Calculus Reform

The 1986-87 academic year was not a successful one for undergraduate students enrolled in calculus for natural science and engineering majors at national four-year post-secondary institutions. According to a preliminary study conducted by the

Mathematical Association of America (MAA) with support from the National Science Foundation (NSF), about 47 percent of students initially enrolled in calculus for natural science and engineering majors finished the academic year with a grade of D or better (Anderson & Loftsgaarden, 1987). Whether failing the course, dropping the course, or withdrawing from the course, this 53 percent “failure” rate raised concern among mathematicians and educators.

As an incentive for change, the NSF announced that grant awards would be given for endeavors that addressed undergraduate calculus curriculum (Ganter, 2001). This announcement spurred a nation-wide calculus reform effort. The calculus reform called for new ways of instructing and learning. Although reform methods varied across institutions, all shared common themes.

### **Active Learning and Constructivism**

Creating an environment that was student centered and facilitated active learning was of great importance (Ganter, 2001; Smith, 1994). Reformists believed that students should have opportunities to explore calculus concepts in various forms. Student should be able to experience calculus concepts symbolically, numerically, and graphically (Ganter, 2001; Ross, 1996; Smith, 1994). In addition, students should have the ability to communicate such concepts both orally and in writing (Ganter, 2001; Ross, 1996; Smith, 1994; Stehney, 1992).

Such experiences require interaction with the course material as opposed to passive observation of a lecturing professor. As Douglas (1986) points out, “constant interaction with questions asked and answered” (p. 19) fosters an active learning environment necessary for calculus instruction and learning.

This was a paradigm shift in pedagogy. The guiding philosophy behind the reform was constructivism (Frid, 1994; Smith, 1996; Snook, 2002). The philosophy was that knowledge is created not discovered. “In constructivist principles, learning is a process in which individuals construct knowledge” (Figueira-Sampaio, dos Santos, & Carrijo, 2009, p. 484). Thus, in a calculus class, meaning is constructed by students as they interact with the course material in an attempt to discover mathematical concepts and truths. Frid (1994) describes a constructivist classroom as one in which learning is active, enabling students to “build knowledge” (p. 70).

The constructivism philosophy brought with it new calculus classroom designs in which new instructional techniques were implemented. Just as proponents of change believed calculus classrooms should foster a discovery frame of mind, educators adopted this same exploration mindset in order to determine what methods would be effective in their classrooms. Once the transition to a reform-type classroom had been made, empirical research on these methods needed to be conducted in order to identify what was actually facilitating student success.

### **Empirical Studies and Literature**

Since the reform movement began, there have been a multitude of studies to assess the various techniques for calculus instruction. Studies show that effective methods of teaching calculus incorporate active learning in some way. The methods in which active learning is implemented, however, has shown by the studies to be varied.

The common effective instructional methods incorporate one or more of the following: (a) multiple representations of calculus concepts, (b) exploration with the aid

of technology, (c) writing, and (d) cooperative learning. Multiple representations and exploration with technology are often employed together.

**Multiple representations and hands-on exploration of calculus concepts.**

Calculus is not simply a set of procedures to follow and formulas in which to plug values. Calculus is an area of mathematics that provides building blocks for natural science and engineering, fields where students must have a strong conceptual understanding of calculus. A student cannot understand calculus concepts by merely memorizing formulas and working template problems from the textbook.

The heart of calculus is conceptual, not procedural. Smith (1994) asserts, and other authors agree, that “students must experience calculus concepts in a rich interplay of symbolic, numerical, and graphical forms” (p. 4). Students learn to think more conceptually through interaction with the material and being exposed to multiple approaches (Douglas, 1986; Gehrke & Pengelley, 1996; Girard, 2002; Goerdt, 2007; Ross, 1996; Smith, 1994, 1996; Tiwari, 1999; Tucker, 1996).

A considerable number of studies have been conducted which illustrate methods using these three approaches (symbolic or algebraic, numerical, and graphical) in the college calculus classroom setting. Such studies have done so with the aid of technology via the hand-held graphing calculator or mathematical computer software. Technology was used to provide students with both numerical and graphical representations. Studies have found that utilizing multiple presentation techniques proved effective in teaching college students calculus concepts.

**Writing about calculus concepts.** Mathematics is a language of symbols that can be translated into words. Words are necessary in order to accurately communicate

mathematical concepts. Students often do not realize the importance words have in mathematics. Words describe a problem, explain a solution, and are used in mathematical proofs, which are all common to a calculus course.

Writing as a requirement in a calculus course enables students to construct mathematical knowledge and better understand calculus concepts (Beidleman, Jones, & Wells, 1995; Contreras, 2002; Cooley, 2002; Hackett, 1998; Wahlberg, 1998). It gives students a chance to reflect on what they have learned and to “clarify and organize” (Cooley, 2002, p. 263) their thoughts. The goal with writing is to have students grow in both their knowledge and understanding of calculus concepts. Beidleman et al. (1995) assert that writing gives students an opportunity to “internalize” (p. 299) calculus concepts in order to communicate them to others.

Studies have been conducted which analyzed the impact of writing in a college level calculus course. Positive findings with regard to students’ understanding of calculus concepts resulted. Through writing, students gained a deeper understanding of calculus concepts, allowing them to give meaning to and connect concepts. In addition, multiple studies showed significant improvement with regards to academic achievement.

**Cooperative learning.** Cooperative learning embodies the spirit of constructivism. Cooperative learning is learning that takes place within groups. Davidson and Kroll (1991, p. 362) define cooperative learning to be “learning that takes place in an environment where students in small groups share ideas and work collaboratively to complete academic tasks.” Similarly, Slavin (1980, p. 315) states that cooperative learning “refers to classroom techniques in which students work on learning activities in small groups and receive rewards or recognition based on their groups

performance.” Although definitions differ somewhat in the literature, all cooperative learning strategies incorporate small group learning. Cooperative learning is typically employed by educators that believe mathematical knowledge is best learned and understood through self-discovery rather than by watching a lecture. It is a constructivist teaching method based on the idea that mathematics is not knowledge awaiting discovery (Dubinsky & Schwingendorf, 1997).

Learning mathematics is done through interaction with the material, thus learning can be seen as a “social activity” (Davidson, Reynolds, & Rogers, 2001, p. 2).

Cooperative learning gives students the opportunity to interact and have a conversation about mathematical concepts. It also facilitates discussions and teamwork. Through cooperative learning students can learn how to correctly communicate mathematics and, in turn, gain a deeper understanding of the concept (Davidson, et al., 2001).

Cooperative learning can be structured in different ways and can take place in various environments. Cooperative learning has been used in conjunction with instruction, discovery learning, peer tutoring, group testing, and topic review, just to name a few (Davidson, 1997). Studies have shown that student achievement and mathematical understanding can be increased through the implementation of cooperative learning.

The literature indicates that by creating an appropriate “mix” of writing, multiple representations, technology, and cooperative learning, the calculus classroom can become an active learning environment in which students can explore calculus concepts. The constructive process establishes meaning and fosters knowledge development. Studies

show that if students deepen their conceptual knowledge by understanding and connecting concepts, academic achievement will improve.

The calculus course taught in a constructivist manner, using problem-based learning, is less restrictive than the traditional calculus course, which typically involves students sitting quietly while an instructor lectures. Incorporating writing, multiple representations of concepts, technology and cooperative learning creates a classroom that is more accessible to various learning styles. Mathematics is not learned through watching and listening; it is learned by doing.

### **The Research Problem**

The spring 2009 semester proved to be difficult for undergraduate students enrolled in MATH 160: Calculus for Physical Scientists I at Colorado State University (CSU). Only 50 percent of the 266 students initially enrolled finished the course with a grade of C or better, while the remaining 50 percent either received a grade of D or F, dropped the course, or withdrew from the course (K. Klopfenstein, personal communication, May 27, 2009).

CSU is not an anomaly, nor was this an unusual semester. In fact, according to the University of Colorado at Boulder (2009), the national average failure rate for such a calculus course is 40 percent. With deeper investigation, however, it was discovered that this statistic is based on data from the 1980s. The data do not clarify what percentage of students actually received a grade of F, dropped the course or withdrew from the course. Anecdotal evidence from instructors at both Colorado State University and Front Range Community College – Larimer Campus, supports that the percentage of students that complete the first semester of calculus with a grade of C or better has not improved much

since the 1980s. It lingers around 60 percent, leaving 40 percent of students receiving a grade of D or F, dropping the course, or withdrawing from the course. The data continue to raise concern among mathematics educators.

It is important to note that the majority students at Colorado State University who are enrolled in degree programs that require MATH 160 must receive a grade of C or better in MATH 160 in order to continue in their degree programs. Thus, on average about 40 percent of the students initially enrolled will most likely have to retake the course (Klopfenstein, 2009a). MATH 160 is a required course for students majoring in natural sciences and engineering. Without successfully completing MATH 160, students cannot continue to take courses in their respective degree programs as it is a prerequisite for natural science and engineering courses. A high failure rate in MATH 160 has the potential to lead to attrition in fields of study for which calculus is a required component.

### **Purpose of This Study**

The purpose of this study was to test an intervention meant to increase student achievement in MATH 160. This study reports on an intervention which focused on students who were at risk of failing MATH 160. These poor performing students were given the option to enroll in MATH 180, which was a "Concepts for Calculus" intervention. This intervention was composed of a 12 week re-examination of elementary mathematical functions from a more advanced standpoint to prepare students to understand the concepts and processes in beginning calculus. MATH 180 incorporated several methods of instruction that gained popularity during the calculus reform of the 1980s including multiple representations, writing, and cooperative learning. After completing MATH 180, students then had the option to re-enroll in Calculus I.

The goal of the intervention was to address two primary issues: (a) pre-calculus skills and, more importantly, (b) students' perceptions of mathematics and the ability to think mathematically (Klopfenstein, 2009c). Students enrolled in MATH 160 frequently have weak pre-calculus skills. In addition, they believe mathematics is a set of formulas and procedures that need to be memorized. Students need to have the ability to understand and connect mathematical concepts while working through multiple representations of functions (Douglas, 1986; Ganter, 2001; Klopfenstein, 2009a).

MATH 180 focused on helping students understand the conceptual basis of pre-calculus and did so through the use of writing, multiple representations, technology, and group activities. It is important to note that the focus of the intervention was not to re-teach prerequisite mechanics. The emphasis was on understanding and connecting concepts, that is, to help students gain a deeper understanding of mathematical functions and the importance of those functions as calculus tools. In addition, the study used the Modified Indiana Mathematics Belief Scales (MIMBS) which combines a modified version of the Indiana Mathematics Belief Scales (IMBS) and Mathematics Usefulness Scale (MUS) to measure students' perceptions of mathematics. The purpose was to measure students' beliefs about mathematics both before and after the intervention.

### **Overarching Research Questions and Data to Be Collected**

The purpose of this study was to investigate the effect of an intervention (MATH 180) on both academic achievement and beliefs about mathematics. Therefore, the following overarching research questions were investigated:

1. Is there a difference in academic performance and beliefs about mathematics between students who participate in the MATH 180 intervention and those who do not?
2. Does MATH 180 improve academic performance in MATH 160 and beliefs about mathematics?
3. Is there a relationship between belief about mathematics and academic performance in MATH 160?
4. What insights do students identify as the defining moments of the intervention and what suggestions do they have to improve the intervention?

### **Definition of Terms**

#### **Calculus for Physical Scientists I**

Calculus for Physical Scientists I is a semester long (16 weeks) course offered through the Colorado State University Department of Mathematics. The course number is MATH 160. MATH 160 is a four credit course required for undergraduate students majoring in certain natural science or engineering fields.

#### **Concepts for Calculus**

Concepts for Calculus is an experimental course offered through the Colorado State University Department of Mathematics. The course number is MATH 180. It is a 12 week long course designed to guide students in learning how to think and communicate mathematically. The purpose of MATH 180 is not to re-teach prerequisite mechanics. The emphasis is on understanding and connecting mathematical concepts. Specifically, MATH 180 is meant to help students gain a deeper understanding of the mathematical concept of a function and the importance of the function concept as a calculus tool.

Methods of instruction include multiple representations, writing, and cooperative learning. A more in-depth description and material used for the course (including the syllabus) can be found in Appendix A.

### **Failure in MATH 160**

Failure in MATH 160 is completing the course with a final grade of D or F, dropping the course, or withdrawing from the course. The reason for this is due to the majority of degree programs accepting only a grade of C or better for MATH 160.

### **Indiana Mathematics Belief Scales**

The Indiana Mathematics Belief Scales (IMBS) is a 30 item questionnaire designed to measure five constructs about mathematics (Kloosterman & Stage, 1992). Each item on the questionnaire is answered using a Likert-type scale. The five constructs measured are:

1. "I can solve time-consuming mathematics problems" (Kloosterman & Stage, 1992).
2. "There are word problems that cannot be solved with simple step-by-step procedures" (Kloosterman & Stage, 1992).
3. "Understanding concepts is important in mathematics" (Kloosterman & Stage, 1992).
4. "Word problems are important in mathematics" (Kloosterman & Stage, 1992).
5. "Effort can increase mathematical ability" (Kloosterman & Stage, 1992).

Six questions are associated with each construct, resulting in 30 questions. The IMBS is available with permission from Kloosterman and Stage (1992). See Appendix B for a copy of the IMBS.

### **Mathematics Usefulness Scale**

The Mathematics Usefulness Scale (MUS) was created by Fennema and Sherman (1976) as means for measuring “students’ beliefs about the usefulness of mathematics currently, and in relationship to their future education, vocation, or other activities” (p. 326). See Appendix B for a copy of the MUS, which is available with permission from Fennema and Sherman (1976).

### **Modified Indiana Mathematics Belief Scales**

The Modified Indiana Mathematics Belief Scales (MIMBS) is a modified version of the IMBS combined with a modified version of the MUS. The MIMBS is a 36 item questionnaire designed to measure six constructs about mathematics. Each item on the questionnaire is answered using a Likert-type scale

Modification and use of the IMBS were done so with permission from the authors. Constructs one, three, and five from the IMBS were kept in-tact. Modifications were done on constructs two and four. Construct two was changed to: “Mathematics problems are solved by identifying and applying the correct procedure”. Construct four was changed to: “Mathematics problems have a single, correct answer”. Modifications to the IMBS were completed by K. Klopfenstein, Mary Pilgrim, and Daniel Reinholz. See Appendix C for a copy of the MIMBS.

The sixth construct of the MIMBS is “Mathematics is useful in daily life.” This is a modified version of the MUS. Modifications were done by Kloosterman and Stage (1992) with permission from Fennema and Sherman (1976). Use of the modified version was done so with permission from all authors.

## **Natural Science**

According to the CSU course catalog (2009), an undergraduate student majoring in a natural science includes any of the following majors:

1. Applied Computing Technology
2. Biochemistry
3. Biological Science
4. Chemistry
5. Computer Science
6. Mathematics
7. Natural Sciences
8. Physics
9. Psychology
10. Zoology

However, not all require MATH 160. Majors requiring MATH 160 includes any of the following:

1. Applied Computing Technology
2. Computer Science
3. Mathematics
4. Physics

## **Success in MATH 160**

Success in MATH 160 is completing the course with a final grade of C or better.

The reason for this is due to the majority degree programs accepting only a grade of C or better for MATH 160.

## **Study Delimitations and Limitations**

### **Delimitations**

By choice, the study was delimited to students enrolled in MATH 160 at CSU during the fall 2009 semester as well as students that repeat MATH 160 during the spring 2010 semester. Although there are other calculus courses offered, such as a business calculus course and a biology calculus course, this study focused on students enrolled in

MATH 160. The business and biology calculus courses are different in content and applications, and the MATH 180 intervention course is designed for students that are majoring in natural sciences and engineering. Including other calculus courses could possibly require three different MATH 180 interventions.

### **Limitations**

There was no random assignment of students into the intervention. The intervention was optional for students. Therefore it cannot be assumed that the group of students in the intervention is representative of the population of students enrolled in a first semester calculus course or even MATH 160. In addition, it cannot be assumed that the students in the intervention are similar to students not in the intervention beyond their fields of study, as students enrolled in MATH 160 are natural science or engineering majors. There are also differences in students between the fall and spring semesters. Spring semesters can have a higher percentage of students retaking the course than fall semesters. These are threats to both internal and external validity (Gliner, Morgan, & Leech, 2009, pp. 104, 127).

Other threats to internal validity include attrition and carryover effects (Gliner, et al., 2009, p. 104). It is possible that students enrolled in MATH 160 have taken the course before and, thus, have seen the material before. However, ecological validity is “high” (Gliner, et al., 2009, p. 127), as the settings for both the intervention and non-intervention groups was a natural classroom setting.

Other limitations fall under instruction and grading. I was not an instructor or grader for either MATH 160 or MATH 180. Therefore, instruction and exam administration and grading for these courses are not things that I had control over.

However, the individuals that instructed, administered exams, and graded exams for MATH 160 have previous experience in these areas with this course. In addition, although the course coordinator wrote initial drafts of MATH 160 exams, exam critiquing and grading was a combined effort of all MATH 160 instructors and course coordinator.

MATH 180, on the other hand, was a new course at CSU. No one had experience in instruction or exam administration and grading for this course. The instructor for MATH 180 had over 40 years of experience teaching on the college level as well as over five years of experience teaching MATH 160. Therefore, such duties for MATH 180 were not out of his realm.

### **The Researcher's Perspective**

Objectivism is the epistemological stance that meaning is found in reality (Crotty, 1998, p. 8). The presence of human beings does not give meaning. Meaning and truth exist, waiting to be discovered. As a mathematician, I believe there are universal truths that exist regardless of human interaction and therefore maintain objectivism as my epistemological stance in studying the natural world.

Postpositivism goes hand in hand with objectivism. Postpositivists take a scientific approach to research, studying possible causes that “influence outcomes” (Creswell, 2008, pp. 6-7). It is my nature to follow a scientific procedure and quantify things. Therefore that was the primary manner in which this study was completed.

However, as an educator, I am student-centered. My main concern is to facilitate student success. I am looking for a solution to the problem of high failure rates in first semester calculus. I want to know “what works,” which adds pragmatism to my worldview (Creswell, 2007, p. 22). Therefore the methods utilized in this study will be

influenced by the postpositive and pragmatic perspectives. I will employ scientific methods to try to determine how to solve the problem.

### **Summary and Conclusion**

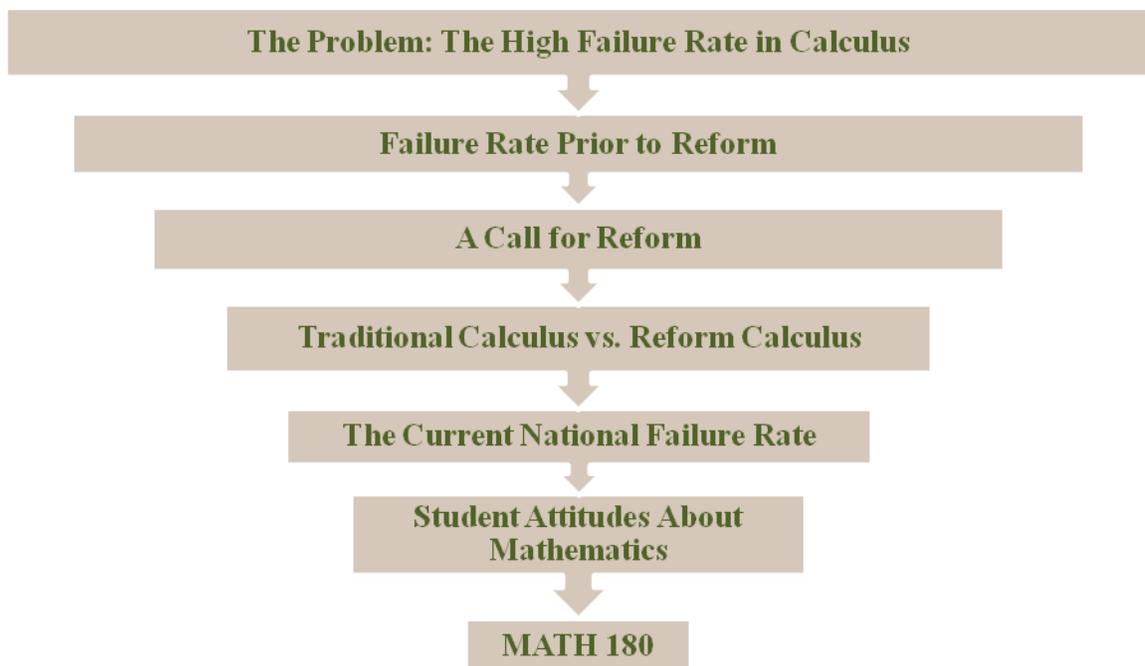
Despite these limitations, there is much to be gained from this study. The intervention addresses a national concern. It has the potential to increase student success in MATH 160 and thereby decrease attrition in fields such as natural sciences and engineering, to which MATH 160 is a gateway.

Colleges and universities each have their own approach to presenting the first semester of calculus for natural science and engineering majors. Some have stuck with the traditional methods of teaching calculus, while others have chosen to reform their calculus courses. Those that have chosen to reform usually use only one or two of the instructional techniques (writing, multiple representations, technology, and cooperative learning). The MATH 180 intervention incorporated all four of the instructional techniques in some form. Thus, in addition to addressing a national concern, this study will add a unique perspective to the body of literature on mathematics, specifically calculus, instruction.

## CHAPTER II: A REVIEW OF THE LITERATURE

In order to address the research problem, the review of the literature will follow the conceptual map provided in Figure 2.1.

### A Conceptual Map of the Review of Literature



*Figure 2.1.* A conceptual Map of the major topics discussed in chapter two for the review of literature.

As stated in chapter one, on average, 40 percent of all students initially enrolled in MATH 160: Calculus for Physical Scientists I at Colorado State University (CSU) finish the course with a grade of D or F, drop the course, or withdraw from the course (Klopfenstein, 2009a). This is clearly a problem, as MATH 160 is a gateway course for students majoring in natural sciences or engineering. Students that do not receive a grade

of C or better in MATH 160 typically have to take the course again and cannot continue in their respective degree programs.

There are a number of reasons why this failure rate should be of concern. Most importantly, if only a small proportion of students are successful in MATH 160, then only a small proportion of students continue on-track in natural sciences and engineering programs. A small proportion of students in such programs results in a small proportion of students receiving bachelor's degrees in natural sciences and engineering. Fewer bachelor's degrees awarded in natural sciences and engineering fields, in turn, contributes to attrition in natural sciences and engineering fields.

It is excellence in natural sciences and engineering fields that has enabled the United States to be a world leader both technologically and economically (Greenspan, 2000). The National Science Board (2004, p. 1) reported that over 2.8 million "first university" degrees were awarded worldwide in "science and engineering fields" in 2000. Data from the National Science Board (2004) also indicates that only about 14 percent of these degrees were awarded to students in the United States. Although it is important to note that not all degrees awarded in these fields required the same level of student effort, other countries are increasing the number of individuals trained these fields at a greater rate than the United States. And, as the National Science Board (2004, p. 1) points out, "A workforce trained in [natural science and engineering] is indispensable to a modern economy." The United States' once unchallenged role as technological and economical leader is diminishing. Thus the high failure rate in calculus has larger implications.

### **National Failure Rate in Calculus: Prior to Reform**

Any student aspiring to major in natural sciences or engineering is required to take a first semester calculus course. This is a requirement of any accredited university, since knowledge of calculus is required to understand primary aspects of natural science and engineering fields. Students are denied access to other courses required by their respective degree programs unless they pass the first semester of calculus, often with a grade of C or better. Calculus is a gatekeeper course to natural science and engineering fields.

Therefore it was seen as a problem when Anderson and Loftsgaarden (1987) reported that 53 percent of undergraduate students were failing the first year of calculus. This data does not include students who had received a grade of D. High failure rates in calculus as well as continued advances in technology forced mathematicians to rethink how a calculus course should be taught.

### **The Reform Movement**

Although the National Science Foundation stimulated a nation-wide calculus reform in the 1980s, the idea of changing how calculus is taught was nothing new. Calculus reform has been discussed among mathematicians for over a century. Teaching calculus in a manner different from the traditional methods was addressed by Durell (1894a, 1894b) who suggested introducing graphical representations of concepts in order to give students a better understanding. Rather than starting with abstract concepts, Durell (1894a) suggested introducing students to something more tangible, such as graphs, so that students could more easily make connections to theory.

Reform calculus has evolved a great deal since 1894, but the same basic ideas are present. Constructivism is the guiding philosophy behind reform calculus (Frid, 1994; Smith, 1996; Snook, 2002). The belief is that the process of learning mathematics is active and not passive. Supporters of reform calculus believe that exploration and interaction with mathematics is the only way in which students can truly learn and retain knowledge (Douglas, 1986; Frid, 1994; Smith, 1996; Snook, 2002). However, before discussing all the facets of reform calculus, clear distinctions should be made between the terms “traditional calculus” and “reform calculus.”

### **Traditional Calculus Instruction**

A traditional calculus course is typically lecture-based with homework and exams making up the bulk of a student’s grade (Frid, 1994; Matthews, 1998; Miller, 1999; Roddick, 1997). Traditional calculus emphasizes procedures, rote skills and symbolic manipulation (Frid, 1994). Students are shown problems in lecture that provide a template for homework exercises. Memorization combined with predetermined steps is requisite for the traditional calculus course.

The traditional calculus classroom is instructor centered. Students passively observe while the instructor lectures. Lectures involve symbolic explanations of concepts with little or no emphasis on multiple representations of concepts (Frid, 1994; Miller, 1999). Typically the traditional calculus classroom does not incorporate learning strategies that foster student interaction and exploration.

### **Reform Calculus Instruction**

Traditional calculus instruction, however, did not seem to be getting the job done. Research conducted in the 1980s showed a high national failure rate for undergraduate

students enrolled in calculus for physical science majors (Anderson & Loftsgaarden, 1987).

Shortly after Anderson and Loftsgaarden (1987) released their preliminary results, the National Science Foundation (NSF) announced that grant awards would be given for endeavors that address undergraduate calculus curriculum (Ganter, 2001). This announcement sparked a nation-wide calculus reform effort. The NSF announcement, coupled with the advent of the hand-held graphing calculator called for, as the Mathematical Association of America (MAA) aptly titled their 1987 colloquium,

### **Calculus for a New Century: A Pump, Not a Filter**

The calculus reform movement of the 1980s called for a new way of instructing and a new way of learning. Despite the focus of utilizing hand-held graphing calculators in university classrooms in order to reduce time-consuming calculations (Ganter, 2001), there was a greater emphasis on changing the way in which calculus was taught and learned. Although reform methods varied across institutions, all shared common themes. Calculus reform has multiple facets, but constructivism and pragmatism are at the foundation. These are the two common threads among reform calculus courses.

A constructivist finds meaning through interaction (Creswell, 2009). Therefore, a constructivist calculus classroom would foster an environment in which students would engage with mathematics. This changes the classroom from instructor centered to student centered.

Although constructivist calculus classrooms are student centered, they do not all look the same. Every instructor has his or her own unique approach. The goal, however, is not to establish a common pedagogy. The goal is to increase mathematical

understanding of calculus concepts as well as student achievement. Increasing student achievement may be accomplished in a variety of ways. Each instructor must find his or her own way and identify successful instructional techniques for their own classrooms and students (Ross, 1996; Slavin, 2008).

**Reform instructional techniques.** Creating an environment that was student centered and facilitated active learning was of great importance during the calculus reform movement (Douglas, 1986; Ganter, 2001; Smith, 1994). It was believed by mathematics instructors that students should have the opportunity to explore calculus concepts in various forms. Students should experience calculus concepts symbolically, numerically, and graphically (Ganter, 2001; Ross, 1996; Smith, 1994). In addition, Ganter (2001), Ross (1996), Smith (1994), Stehney (1992) and others agreed that students should have the ability to communicate such concepts both orally and in writing.

Such experiences require interaction with the course material as opposed to quiet observation of a lecturing professor. Instead, “constant interaction with questions asked and answered” (Douglas, 1986, p. 19) fosters an active learning environment necessary for calculus instruction and learning. The calculus reform movement produced a diverse collection instructional techniques that have been successfully employed in the classroom. The techniques that were introduced typically incorporated one or more of the following: (a) multiple representations of calculus concepts, (b) exploration with the aide of technology, (c) writing and (d) cooperative learning. Reform instructors believe that if such techniques are employed in the calculus classroom, then students will have the opportunity to interact with the mathematical material and therefore derive meaning.

**Multiple representations.** Until the national reform of the 1980s, much of calculus was taught by lecture using symbolic representations of calculus concepts. Such teaching methods emphasized the procedures for calculus, leaving the question of “why” for the student to ponder on their own. Calculus, however, is not simply a set of procedures to follow and formulas in which to plug values. Calculus is an area of mathematics that provides building blocks for natural sciences and engineering fields where students must have a strong conceptual understanding of calculus. The heart of calculus is conceptual, not procedural.

Understanding calculus concepts does not result from memorizing formulas and working template problems from the textbook. Through interaction with the material and being exposed to multiple approaches students can gain a deeper understanding of calculus concepts (Douglas, 1986; Gehrke & Pengelley, 1996; Girard, 2002; Goerd, 2007; Ross, 1996; Smith, 1994, 1996; Tiwari, 1999; Tucker, 1996).

Smith (1994) asserts, and other authors agree, that “students must experience calculus concepts in a rich interplay of symbolic, numerical, and graphical forms” (p. 4). This is not to say that the procedures of calculus are any less important. However, multiple representations give students the opportunity to explore calculus concepts symbolically, numerically, graphically, and through words or writing. It is through these experiences that students can develop problem solving skills and conceptual understanding (Douglas, 1986; Gehrke & Pengelley, 1996; Girard, 2002; Goerd, 2007; Ross, 1996; Smith, 1994, 1996; Tiwari, 1999; Tucker, 1996).

The use of multiple representations in a calculus course is exploring concepts in more than one form. The multiple forms in which mathematical concepts may be

presented are in symbolic, numerical, graphical, and written forms. Writing in calculus, however, is often treated in the literature as a stand-alone instructional technique.

Therefore, it will be discussed in a separate section in this review and the discussion of multiple representations will be inclusive of symbolic, numeric, and graphical forms.

**Multiple representations with technology.** Technology fits in easily when teaching calculus concepts using multiple representations. While symbolic representations are typically presented on a chalkboard, numeric and graphical representations are often illustrated on a computer or calculator screen. Creating tables containing numeric data or sketching an accurate graph by hand, although not difficult, can be time consuming. Hand-held graphing calculators and computer algebra systems (CAS) have the capability to provide students with accurate tables and graphs in a much shorter period of time, leaving more time for exploration and explanation rather than computation. In addition, the use of technology in the calculus classroom gives students the opportunity to explore concepts in various forms.

Studies addressing technology in the classroom have frequently had positive results. Bell (2001), Ellington (2003), Goerdts (2007), Heid (1988), Nasari (2008), Palmiter (1991), Schrock (1989) and Tiwari (1999, 2007) are a few of the studies that found the incorporation of technology in the classroom as an instructional tool helped improve students' understanding of calculus concepts. The data found in some studies also indicate that students provided with multiple presentations of calculus topics can have statistically significant higher academic achievement (Ellington, 2003; Goerdts, 2007; Heid, 1988; Nasari, 2008; Palmiter, 1991; Tiwari, 1999, 2007). Though not all studies had statistically significant results in academic achievement, groups provided

with multiple representations with technology often had higher exam means with smaller standard deviation (Bell, 2001; Schrock, 1989). Studies revealed that students taught with symbolic, numerical, and graphical forms demonstrate more flexibility in working with calculus concepts.

The literature illustrates the need for students to experience mathematical concepts in various forms. Instruction that incorporates multiple representations caters to more learning styles as opposed to the traditionally taught calculus course. There is potential for their mathematical thinking and understanding to grow and become more diverse when students are provided with multiple representations.

There are caveats with technology, however. Although studies indicate positive results regarding the use of technology as a tool for multiple representations, Judson (1991), Meagher (2005), and Tiwari (2007) recommend caution when implementing technology in the classroom. Admittedly, technology has the potential to uncover “gaps” (Judson, 1991, p. 40) in student knowledge, but it can also waste time and take away from the mathematical conversation (Maldonado, 1998). Instructors must be proficient in the technology that they plan to implement in their classroom and have the ability to tie it in with mathematical concepts. It is also important not to encourage dependence upon technology. Technology should be a tool and not a “crutch.”

**Writing.** Writing is another form in which mathematical concepts can be explored. Mathematics is a language of symbols that can be translated into words. Words are necessary in order to accurately communicate mathematical concepts. Students often do not realize the importance words have in mathematics. Words describe problems,

characterize mathematical relationships, explain solutions and reasoning, and are used in mathematical proofs, which are all common to a calculus course.

Writing as a requirement in a mathematics course enables students to construct mathematical knowledge and better understand mathematical concepts (Beidleman, Jones, & Wells, 1995; Contreras, 2002; Cooley, 2002; Hackett, 1998; Wahlberg, 1998). Writing gives students a chance to reflect on what they have learned and to “clarify and organize” (Cooley, 2002, p. 263) their thoughts. The goal with writing is to have students grow in both their knowledge and understanding of calculus concepts. Beidleman et al. (1995) assert that writing gives students an opportunity to “internalize” (p. 299) concepts in order to communicate them to others.

Beidleman et al. (1995), Contreras (2002), Cooley (2002), Hackett (1998), Porter and Masingila (2000), Taylor (2007), Wahlberg (1998) and Waywood (1992, 1994) had positive results in academic achievement as a result of incorporating writing in various forms into their mathematics classrooms. In fact, Waywood (1992, 1994) used daily journal writing as a way to identify when students were struggling with mathematical material. Studies illustrate that, through writing, students gained a deeper understanding of mathematical concepts, allowing them to give meaning to and connect concepts.

Though the majority writing studies are case studies or anecdotal in nature, Hackett (1998), Wahlberg (1998), Porter and Masingila (2000), and Taylor and McDonald (2007) conducted quasi-experimental studies involving writing. The studies showed that students who had writing components in their calculus courses scored higher on exams than students who were not required to write. The results in Hackett’s (1998) and Wahlberg’s (1998) studies were statistically significant, whereas the results in the

studies by Porter and Masingila (2000) and Taylor and McDonald (2007) were not. These studies were conducted in response to students lacking a deep understanding of calculus concepts and the ability to communicate mathematics. Additionally, it was found that writing enabled students to think through mathematical ideas in order to gain a conceptual understanding. Understanding of mathematical concepts in turn increased students' ability to communicate mathematics and then improve academic performance.

A major problem that can arise when incorporating writing in the mathematics classroom is students lacking the ability to write. The literature suggests that a writing component in a mathematics course can be beneficial to students. However, as Beidleman et al. (1995) learned, requiring writing can become difficult when students have to be trained to write. Although time should be spent helping students develop skills in writing about mathematics, it is not realistic to take the time to teach students how to write in general. Beidleman et al. suggest that giving students enough time to write and incorporating peer review may help reduce classroom time spent on teaching students to write, both in general and using mathematical terminology.

The literature shows that writing about calculus concepts can enhance students' conceptual knowledge and, in turn, improve understanding of mathematical concepts. Writing helps students work through their mathematical reasoning in order to gain a better understanding of their rationale. Writing can be used as a form of mathematical expression. It is not enough for students to simply write down an answer. They should be able to explain how they arrive to their conclusions and discuss mathematical meaning. The hope is that, through writing, students gain insight on work they do both correctly and incorrectly in order to better understand the mathematical concepts.

**Cooperative learning.** Cooperative learning is an active learning strategy that became popular during the late 1970s and early 1980s. Cooperative learning is defined differently among mathematics educators; however, all cooperative learning strategies have a common thread – students working together in small groups (usually two to six students) in a structured manner (Davidson, et al., 2001; Slavin, 1980). It is important to note that cooperative learning is not simply “group work.” The idea behind cooperative learning is to get students actively engaged with the material.

Students working together in small groups sharing and exchanging knowledge is the basic structure of cooperative learning. However, how that is done differs among researchers. For example, much of the research by Slavin (1980, 1981, 1987, 1988) addresses cooperative learning on the elementary and secondary education levels. Treisman (Fullilove & Treisman, 1990; Treisman, 1985, 1992), on the other hand, has implemented cooperative learning on the postsecondary level. Students in elementary and secondary schools are different from college students. Therefore, the cooperative learning strategies are structured differently.

**Slavin and cooperative learning.** Slavin (1980, p. 315) states that cooperative learning “refers to classroom techniques in which students work on learning activities in small groups and receive rewards or recognition based on their group’s performance.” Slavin (1987, 1988) argues that there are various forms of cooperative learning and not all are effective. Slavin (1987, 1988; 1996) also suggests that, in order for a cooperative learning model to be effective, groups must have a common goal toward which they are all willing to work and success toward the common goal must depend upon on all group

members learning the material. The common goal to which Slavin (1980, 1987, 1988) refers is some sort of reward, recognition, or special certificate or notoriety.

This reward-type model of cooperative learning has had a great deal of success on the elementary and secondary education levels. Slavin (1981, 1988, 1999; Slavin, Leavey, & Madden, 1984; Stevens & Slavin, 1995) has conducted multiple studies and research syntheses on cooperative learning for the elementary and secondary education levels. All show that cooperative learning, when designed as Slavin defines it, results in higher academic achievement. However, simply putting students in groups is not necessarily enough to foster learning. There must be both a “group goal” and “individual accountability” (Slavin, 1987, p. 9, 1988, p. 31, 1999, p. 74) in order to utilize cooperative learning effectively in the classroom.

The post-secondary level of education is different from the elementary and secondary levels. Rewards, recognitions or special notoriety are not a part of the classroom specifically. In college courses, the reward is succeeding in the course. Therefore, Slavin’s model for cooperative learning, although successful on the elementary and secondary levels, does not carry over to the post-secondary classroom. Modifications would be needed. However, Johnson, Johnson, and Smith (2007, p. 27) assert that “[c]ooperative learning is the instructional procedure of choice to maximize student learning (especially of highly complex or difficult material) and long-term retention”. In fact, Treisman (Fullilove & Treisman, 1990; 1985, 1992) has had success with a cooperative learning model designed for post-secondary calculus courses.

**The Treisman model.** The Treisman form of cooperative learning was the result of a dissertation study that began in 1975 at the University of California at Berkeley.

Initially, Treisman (1985) was concerned about the high failure rate in first semester calculus among African American students. The high failure rate caused such students to change majors and led to underrepresentation of African American students in mathematics and engineering fields.

Calculus teaching assistants (TAs) were questioned about their strong and weak students. Further investigation revealed that, while African American students were overrepresented among weak students, Chinese American students were overrepresented among strong students (Treisman, 1985). This raised the question: What were the “reasons for this apparent difference in [academic] performance?” (Treisman, 1985, p. 4).

The African American students studied primarily alone and for about eight hours a week, while the Chinese American students studied in small groups for about fourteen hours a week. The groups in which the Chinese American students studied created an environment in which knowledge was exchanged and difficult problems were discussed. In the end, the Chinese American students succeeded in the course, while nearly all of the African American students failed (Treisman, 1985).

These findings led to the creation of the Mathematics Workshop Program (MWP) in which the best features of the Chinese American study groups were adapted for African American and Latino/Latina students (Fullilove & Treisman, 1990). The MWP was an honors program that encouraged students to “achieve the highest levels of academic success possible” (Fullilove & Treisman, 1990, p. 472).

Workshops were open to freshman calculus students and consisted of two hour sessions that ran three to four days a week. Students met in small groups within the workshop to work on difficult mathematics problems while a “workshop leader” walked

around the room (Treisman, 1985, p. 41). The students had primary control, but the workshop leader was there to provide help. The workshops were not mandatory, and students involved were discouraged from studying alone. A cooperative learning environment was promoted. At the end of the semester, only one of the 42 workshop participants failed calculus and over half received a grade of B- or better.

An evaluation of the MWP from academic year 1978 through academic year 1984 revealed positive results for the program. The evaluation found that MWP students were two to three times more likely than non-MWP students to earn a grade of B- or better (Fullilove & Treisman, 1990). All analyses were significant with  $p$ -value less than 0.01 with MWP students significantly outperforming non-MWP students regardless of time period (Fullilove & Treisman, 1990).

The Treisman workshop model of cooperative learning has also had successful replications. For example Murphy, Stafford, and McCreary (1998) implemented a similar workshop at the University of Illinois at Urbana-Champaign. It was found students that had participated in the workshop had significantly higher achievement (with  $p$ -value of 0.002) in first semester calculus than students who had not participated in the workshop.

Advocates for cooperative learning believe that learning is a “social activity” (Davidson, et al., 2001, p. 2). The literature supports that students engaged in cooperative learning have a better chance of success in mathematics than students who work alone. Cooperative learning gives students the opportunity to actively engage with their peers and interact with the material. This process enables students to learn the importance of teamwork and fosters meaningful conversations about mathematics.

## **The Current National Failure Rate in Calculus**

With so much energy and funding going into calculus reform projects, one would anticipate that the national failure rate in calculus would be a great deal lower than it was in the 1980s. Such data, in fact, is difficult to find. However, data from CSU as well as personal experience, indicates that about 40 percent of students initially enrolled in Calculus I finish the course with a grade of D or F, dropped the course, or withdrew from the course (Reinholz, 2009).

This seems to be an improvement. However, it should be pointed out that the data from the 1980s discusses failure rate at the end of the academic year, not semester. The data for CSU is based on a single, semester-long course. A comparison between the data cannot really be made. Nevertheless, this issue can be resolved by looking at how success or failure in calculus has affected degree attainment over the years.

### **The Bigger Picture: The Impact of a High Failure Rate in Calculus**

The exact value for the national failure rate in calculus remains in question, but the general consensus is that it is around 40 percent. Regardless, the impact of the failure rate is clear. Failing first semester calculus is a barrier to any student majoring in natural sciences or engineering. As stated before, if such students do not pass the first semester of calculus (usually with a grade of C or better), then they cannot continue in their respective degree programs. This impacts the persistence of natural science and engineering degrees, which, in turn, can contribute to attrition in such fields.

Data compiled by the NSF does not indicate that the percentage of natural sciences and engineering bachelor's degrees are on the upward climb. The percentages of such bachelor's degrees awarded were actually at a peak in the mid 1980s. Additionally,

the percentages of bachelor's degrees in such fields attained by women and minorities continue to be lower than those awarded to white males.

### **Persistence in Natural Science and Engineering Undergraduate Majors**

Data collected by the National Center for Education Statistics (NCES) from colleges and universities across the United States does not indicate that the percentage of science and engineering degrees being awarded is on the upswing. The NSF has been keeping record of national data released from the NCES, which details the number of bachelor's, master's and doctorate degrees granted from 1966 to 2006. It should be noted, however, that data was not released during 1999.

**Bachelor's degrees.** According to the data compiled by the NSF (2008a), 32.1 percent of all bachelor's degrees awarded in 2006 were in science and engineering fields. The NSF (2008a) classifies a large number of degrees under the category of "science and engineering" – biological and agricultural sciences; earth, atmospheric, and ocean sciences; mathematics and computer sciences; physical sciences; psychology; social sciences; and engineering. Thus, it is worthwhile to look at the percentage of bachelor's degrees awarded in math and computer sciences, physical sciences, and engineering.

In 2006, of all bachelor's degrees awarded, only 9.7 percent were in mathematics and computer science, physical sciences, and engineering (National Science Foundation, 2008a). Interestingly, the percentage of bachelor's degrees awarded in these fields were at an all time high in 1986 with 15.2 percent, and the lowest was in 1976 with 8.3 percent awarded (National Science Foundation, 2008a). Recent (i.e. 2006) data does not indicate an upward trend of degrees awarded in mathematics and computer science, physical sciences, or engineering fields. In fact, the NSF (2008a) data indicates a downward trend

in percentage of degrees awarded in these fields, with percentages declining each year since 2003.

**Bachelor's degrees for women.** Despite the recent decline in the number of bachelor's degrees awarded, the percentage of such degrees awarded to women in science and engineering fields has been on a general upward trend. However, if the data is restricted to include only mathematics and computer science, physical sciences, and engineering fields, this is not the case. As of 2006, of all bachelor's degrees awarded to women, only 4.2 percent were in these fields (National Science Foundation, 2008a). The highest percent was 7.6 percent in 1986 and the lowest was 2.9 percent, which occurred in both 1975 and 1976 (National Science Foundation, 2008a). Of all bachelor's degrees awarded to women, social sciences maintain the highest percentages within science and engineering fields. However, degrees from non science and engineering fields have continued to dominate over 70 percent of bachelor's degrees awarded to women (National Science Foundation, 2008a).

As of 2006, of all bachelor's degrees awarded in mathematics and computer science, physical sciences, and engineering, women received 26.8 percent, 42.4 percent, and 19.5 percent, respectively (National Science Foundation, 2008a). In fact, according to the NSF (2008a) data, these percentages have been decreasing in the past three to four years.

**Bachelor's degrees for minorities.** The NSF has also compiled data detailing bachelor's degrees awarded to minorities. The data provided is from 1997 to 2006. The NSF (2008b) classifies minority groups as Asian/Pacific Islander, Black, Hispanic, American Indian/Alaskan Native, and other or unknown ethnicity. In addition, the data

reported breaks up areas of study into more specific fields. There is detailed data on bachelor's degrees awarded in computer sciences, mathematics and statistics, physics, and engineering fields.

Of all bachelor's degrees awarded in computer sciences, mathematics and statistics, physics, and engineering from 1997 to 2006, it is of no surprise that each minority groups had lowest percentages in 1997. Percentages Asian/Pacific Islander, Black, Hispanic, American Indian/Alaskan Native, and other or unknown ethnicity were 10.5 percent, 6.4 percent, 5.9 percent, 0.4 percent, and 2.5 percent respectively (National Science Foundation, 2008b). As of 2006, bachelor's degrees in said areas of study awarded to each group were 11.2 percent, 6.8 percent, 6.8 percent, 0.5 percent, and 5.8 percent respectively (National Science Foundation, 2008b). This is a total of 31.1 percent of all bachelor's degrees in computer sciences, mathematics and statistics, and engineering being awarded to minority students, leaving 68.9 percent awarded to white students.

**Master's and doctorate degrees.** Data compiled from the NSF (2008a) from 1966 to 2006 also includes data on master's and doctorate degrees awarded in mathematics and computer science, physical sciences, and engineering. The highest percentage of master's degrees awarded was in 1966 with 16.3 percent and the lowest was in 1977 with 8.1 percent. For doctorate degrees, the highest was in 1967 with 32 percent and the lowest was in 1980 with 19.2 percent (National Science Foundation, 2008a). As of 2006, the percentage of master's and doctorate degrees awarded was 10 percent and 30.5 percent respectively (National Science Foundation, 2008a).

The percentages of master's and doctorate degrees in mathematics and computer science, physical sciences, and engineering programs, although low, are not nearly as low as the percentages of bachelor's degrees awarded. Surprisingly, the highest percentages of bachelor's degrees awarded in such fields were in 1986, prior to the national calculus reform. Additionally, although the percentages for women and minorities have risen since the NSF began compiling data in 1966, white males still receive the majority of bachelor's degrees awarded in mathematics and computer science, physical sciences, and engineering. The reform does not seem to have impacted prevalence of bachelor's degrees in a positive manner, and equity in awarding such degrees is yet to be achieved.

### **Students' Attitudes Toward Mathematics**

How useful is mathematics? Is there a fixed way in which any mathematics problem can be solved? Is there always a single solution to a mathematics problem? How a student responds to such questions, as well as what their perceived beliefs about mathematics are, can either help or impede the learning of mathematics (Berkaliev & Kloosterman, 2009; Bookman & Friedman, 1998; Kloosterman & Cougan, 1994; Kloosterman, Raymond, & Emenaker, 1996; Kloosterman & Stage, 1992). Students' beliefs and attitudes toward mathematics influence how they approach mathematics. If a student does not believe that mathematics is useful or that it is too difficult, then the motivation to spend time working on mathematics will decline. Additionally, beliefs about mathematics can influence confidence, which, in turn, can affect performance. Therefore, a common assumption held by those that research students' attitudes toward mathematics is that there is a relationship between attitude and academic achievement (Kloosterman & Cougan, 1994; Kloosterman & Stage, 1992).

In response to the lack of instruments about the nature and learning of mathematics, Kloosterman and Stage (1992, p. 109) developed the Indiana Mathematics Beliefs Scales (IMBS), which measured students' beliefs "related to motivation and thus achievement on mathematical problems solving." Kloosterman and Stage (1992) combined the IMBS with a modified version of the Mathematics Usefulness Scale (MUS), created by Fennema & Sherman (1976), into a set of scales that measures six constructs and each construct is measured by answers to six questions. Therefore the combination of the IMBS and modified MUS form a questionnaire that is comprised of 36 questions, each measured on a five-point Likert-type scale. The six constructs measured are:

1. "I can solve time-consuming mathematics problems" (Kloosterman & Stage, 1992).
2. "There are word problems that cannot be solved with simple step-by-step procedures" (Kloosterman & Stage, 1992).
3. "Understanding concepts is important in mathematics" (Kloosterman & Stage, 1992).
4. "Word problems are important in mathematics" (Kloosterman & Stage, 1992).
5. "Effort can increase mathematical ability" (Kloosterman & Stage, 1992).
6. "Mathematics is useful in daily life" (Fennema & Sherman, 1976).

The scales are intended for secondary and postsecondary students. A copy of the IMBS and MUS, can be found in Appendix B.

Studies in which the IMBS and modified MUS were used found that beliefs and attitudes about mathematics are influential on academic achievement (Berkaliev &

Kloosterman, 2009; Kloosterman, 1998; Kloosterman & Cougan, 1994). These scales have been used with students in mathematics courses of varying levels. One would expect that an undergraduate student enrolled in a remedial mathematics course would have a lower IMBS and modified MUS scores (i.e. have a more negative attitude regarding the nature and learning of mathematics) than an undergraduate student majoring in engineering who is taking higher level mathematics courses. However, Berkaliev and Kloosterman (2009) found that this was not the case.

Berkaliev and Kloosterman (2009) used the IMBS and modified MUS to compare the perceived beliefs and attitudes of undergraduate engineering students to undergraduates enrolled in remedial or elementary mathematics. The mean scores for each constructs for the engineering students were 22.4, 16.4, 23.9, 18.7, 23.4 and 24.5 respectively. The mean scores for each construct for the remedial/elementary students were 20.5, 16.5, 25.3, 18.8, 22.4, and 23.2 respectively. Interestingly, the scores on each construct are not all that different. Both groups had similar attitudes regarding constructs two and four. These constructs involve ideas about the steps in solving mathematics problems and the importance of word problems.

Students' perceived beliefs about both the nature of the process and solutions of mathematics problems is of great concern. Students need to recognize mathematics is not about following predetermined steps and getting a single answer. If students, however, believe that mathematics is simply about following a sequence of steps to get to the solution, then they do not have the ability to think critically about mathematics. If students are in a fixed mindset, then they will not be able to successfully work through

complicated problems and situations frequently encountered by natural scientists and engineers practicing in the field.

### **The Status of Calculus at Colorado State University**

MATH 160 is the first semester calculus course required for students majoring in natural sciences or engineering at CSU. MATH 160 is not a reform calculus course, but it is also not a traditional calculus course. The course is a lecture-based course and a traditional calculus textbook is used. However, some reform techniques are integrated into the course. There is a strong emphasis on multiple representations in MATH 160 with both technology and writing being used, but neither group activities nor cooperative learning are employed in MATH 160.

On average, about 60 percent finish MATH 160 with a grade of C or better (Reinholz, 2009). In response to the large number of students failing MATH 160, MATH 180: Concepts for Calculus was created.

### **MATH 180: A Concepts for Calculus Intervention**

Reinholz (2009) found that the first exam score is a good predictor of student success in MATH 160. Thus, students that were identified as being at risk of failing MATH 160 during the fall 2009 semester (based on exam one scores) were invited to enroll in MATH 180: Concepts for Calculus. MATH 180 was designed to “help students transition from a mechanical to a conceptual understanding of mathematics” (Klopfenstein, 2009a, p. 1). MATH 180 incorporated both multiple representations as well as collaborate learning.

The course incorporated both in-class and online instruction. The in-class component was partially comprised of exploratory group activities as well as outside

homework assignments. Homework assignments involved both reading and writing about mathematical topics covered in class. Responses on both group activities and homework assignments were meant to reflect both “understanding and thoughtful analysis” (Klopfenstein, 2009a, p. 2). Group activities were structured, with both individual and group grades. The online component required students to complete problems and assessments on the web-based system ALEKS<sup>®</sup> (Assessment and LEarning in Knowledge Spaces). The ALEKS<sup>®</sup> component enabled students to practice and maintain computational skills necessary for calculus. Since the emphasis in MATH 180 was not on computational skills, the online component comprised only 25 percent of the course grade. Additionally, both a midterm exam and final exam were part of the course grade.

The purpose of MATH 180 was not to re-teach prerequisite mechanics. The emphasis was on understanding and connecting mathematical concepts, that is, to help students gain a deeper understanding of mathematical functions and the importance of those functions as calculus tools. The hope for students enrolled in MATH 180 was that they would succeed in MATH 160, a course in which they may initially have failed. A syllabus, course materials, and exams for MATH 180 can be found in Appendix A.

## **CHAPTER III: RESEARCH METHODOLOGY**

### **Research Design**

#### **Research Paradigm**

This study was a mixed methods design. The overall purpose of the study was to test the effectiveness of an intervention on both academic achievement and beliefs about mathematics of undergraduate students who were required to take MATH 160. This study involved both an experiment as well as a survey, which provided numeric data. Such designs are classified in the quantitative paradigm (Creswell, 2009). All data, with the exception of interviews, were described as numbers not words, which, as Creswell (2009, p. 3) states is often the “distinction between quantitative and qualitative research”.

#### **Data Sources and Collection**

All undergraduate students enrolled in MATH 160 during the fall 2009 semester filled out a questionnaire regarding their beliefs about mathematics by use of the MIMBS (modified IMBS). Reliability statistics of each construct for both the IMBS and MIMBS can be found in Table 3.1.

Table 3.1

*Reliability Statistics for the IMBS and MIMBS*

| Construct   | Cronbach's Alphas<br>from Kloosterman and<br>Stage (1992) for the<br>IMBS + MUS<br>(N = 251) | Cronbach's<br>Alphas for the<br>MIMBS<br>(N = 476) |
|---|--|--|
| "I can solve time consuming<br>mathematics problems"  | 0.80   | 0.76   |
| IMBS: "There are word problems that<br>cannot be solved with simple, step-by-<br>step procedures" | 0.50   | 0.55   |
| MIMBS: "Mathematics problems are<br>solved by identifying and applying the<br>correct procedure"  | 0.81   | 0.81   |
| "Understanding concepts is important in<br>mathematics"   | 0.52   | 0.47   |
| IMBS: "Word problems are important in<br>mathematics"   | 0.89   | 0.87   |
| MIMBS: "Mathematics problems have a<br>single correct answer"                                     | 0.87   | 0.77   |
| "Effort can increase mathematical ability"  |  |  |
| MUS: "Mathematics is useful in daily<br>life"   |  |  |

Section instructors administered the MIMBS. Consent forms were also signed by willing participants at this time. No data was collected from students that did not sign consent forms. This was done during the first and second weeks of the fall 2009 semester. All consent forms and MIMBS data were given to the researcher by the section instructors.

After the first exam in MATH 160, students who had receive a grade of D or F were encouraged to drop MATH 160 and enroll in the MATH 180 intervention class. Previous data from multiple semesters of MATH 160 indicate that the exam one scores predicted final grades (Reinholz, 2009). It is important to note that since assignment to MATH 180 was not random, the design is quasi-experimental.

Information about MATH 180 was presented to MATH 160 students by the researcher. The researcher spoke to the two 2:00 p.m. sections about the MATH 180 course offering. MATH 180 ran in the same time slot, which enabled students in the 2:00 p.m. MATH 160 sections to make the switch to MATH 180 easily. All students eligible to take MATH 180 (regardless of time slot) received an email invitation that described the course as well as instructions for dropping MATH 160 and enrolling in MATH 180. The first MATH 160 exam was on a Wednesday, and MATH 180 began the following Monday.

Near the end of the semester both MATH 160 and MATH 180 students retook the MIMBS, which was compared to their initial MIMBS scores. MIMBS scores and exam one scores were also compared across different academic performing groups:

1. MATH 160 students who received a final grade of D or F, dropped the course, or withdrew (W) from the course during the fall 2009 semester
2. Students who received a final grade of A, B, or C in MATH 160 during the fall 2009 semester.
3. Students who participated in MATH 180 during the fall 2009 semester.

Performance in MATH 160 during the spring 2010 was also compared between the group of students that took MATH 180 and the group of students that did not take MATH 180 but had to repeat MATH 160 due to receiving a grade of D, F, or W.

In addition, interviews were conducted with students from the intervention group. The purpose of the interviews was to identify “aha” moments during the MATH 180 course as well as suggestions for the course.

In summary, the data that were collected are:

1. The data collected from MATH 160 students during the fall 2009 semester were:
  - a. Exam one scores
  - b. Final exam scores
  - c. Final course grades
  - d. Beginning and end of semester MIMBS scores
2. The data collected from MATH 180 students during the fall 2009 semester were:
  - a. Fall 2009 MATH 160 exam one scores
  - b. MATH 180 final course grades
  - c. Beginning and end of semester MIMBS scores
  - d. Spring 2010 MATH 160 exam one scores
  - e. Spring 2010 MATH 160 final exam scores
  - f. Spring 2010 MATH 160 final course grades

3. The data collected from students that repeated MATH 160 during the spring 2010 semester due to not successfully completing MATH 160 during the fall 2009 semester (these students did not take MATH 180) were:
  - a. Spring 2010 MATH 160 exam one scores
  - b. Spring 2010 MATH 160 final exam scores
  - c. Spring 2010 MATH 160 final course grades
4. Interviews with students enrolled in the MATH 180 intervention during the fall 2009 semester.

The quantitative data in the form of exam scores, MIMBS scores, and course grades were collected section instructors and given to the researcher. Qualitative data in the form of interviews were conducted by the researcher.

### **Research Questions and Data Analysis**

The purpose of this study was to address the effect of an intervention on both academic achievement and beliefs about mathematics. The overarching research questions posed in chapter one were:

1. Is there a difference in academic performance and beliefs about mathematics between students who participate in the MATH 180 intervention and those who do not?
2. Does MATH 180 improve academic performance in MATH 160 and beliefs about mathematics?
3. Is there a relationship between belief about mathematics and academic performance in MATH 160?

4. What insights do students identify as the defining moments of the intervention and what suggestions do they have to improve the intervention?

In order to address these overarching questions the following research questions and sub-questions were investigated. The data analysis used for each question is stated with each question:

1. Is there a difference between the scores on the first MATH 160 exam during the fall 2009 semester and the spring 2010 semester for students that complete the MATH 180 intervention?

To answer this question, two sub-questions were addressed. Each sub-question is a difference question involving repeated measures, therefore paired *t*-tests were used, as recommended by Gliner et al.(2009).

Sub-questions:

- a. Is there a difference between the scores on the first MATH 160 exam during the fall 2009 semester and the spring 2010 semester for students that complete the MATH 180 intervention?
  - b. Is there a difference between the scores on the first MATH 160 exam during the fall 2009 semester and spring 2010 semester for the group of students that complete the MATH 180 intervention with a grade of A, B, or C?
2. Is there a difference between the scores on the first MATH 160 exam during the spring 2010 semester between students that participated in MATH 180 and students that did not participate in MATH 180, but had to repeat MATH 160

due to receiving a grade of D, F, or W in the course during the fall 2009 semester?

To answer this question, two sub-questions were addressed. Each sub-question is a difference question involving two independent groups and required an independent t-test, as recommended by Gliner, et al. (2009).

Sub-questions:

- a. Is there a difference in scores on the first MATH 160 exam during the spring 2010 semester between students that participated in MATH 180 and students that did not participate in MATH 180, but had to repeat MATH 160 due to receiving a grade of D, F, or W?
  - b. Is there a difference in scores on the first MATH 160 exam during the spring 2010 semester between the group of students that participated in MATH 180 and completed the course with a grade of A, B, or C and the group of students that did not participate in MATH 180, but had to repeat MATH 160 due to receiving a grade of D or F?
3. Is there a difference in performance between students enrolled in MATH 160 during the spring 2010 semester that took MATH 180 and students enrolled in MATH 160 during the spring 2010 semester that did not take MATH 180 but are having to repeat MATH 160 due to receiving a grade of D, F, or W?

To answer this question, six sub-questions were addressed. Each sub-question is a difference question involving two independent groups, thus independent *t*-tests were used.

Sub-questions:

- a. Is there a difference in performance on the spring 2010 MATH 160 final exam between students that participated in MATH 180 and students repeating MATH 160 that did not take MATH 180?
- b. Is there a difference in performance on the spring 2010 MATH 160 final exam between the group of students that completed MATH 180 with a grade of A, B, or C and the group of students repeating MATH 160 that did not take MATH 180?
- c. Is there a difference in performance on the spring 2010 MATH 160 final exam between the group of students that completed MATH 180 with a grade of A, B, or C and the group of students repeating MATH 160 that did not take MATH 180 but received a grade of D or F on the first MATH 160 exam during the fall 2009 semester?
- d. Is there a difference in final grades for spring 2010 MATH 160 between students that participated in MATH 180 and students repeating MATH 160 that did not take MATH 180?
- e. Is there a difference in final grades for spring 2010 MATH 160 between the group of students that completed MATH 180 with a grade of A, B, or C and the group of students repeating MATH 160 that did not take MATH 180?
- f. Is there a difference in final grades for spring 2010 MATH 160 between the group of students that completed MATH 180 with a grade of A, B, or C and the group of students repeating MATH 160 that did

not take MATH 180 but received a grade of D or F on the first MATH 160 exam during the fall 2009 semester?

4. Is there a correlation between MIMBS score and performance in MATH 160 during the fall 2009 semester?

To answer this question, four sub-questions were addressed. Each sub-question is a correlation question involving approximately normally distributed variables, therefore bivariate correlation with Pearson's  $r$  was used.

Sub-questions:

- a. Is there a correlation between MIMBS score at the beginning of the semester and exam one score?
  - b. Is there a correlation between scores on the six constructs of the MIMBS at the beginning of the semester and exam one score?
  - c. Is there a correlation between end of the semester MIMBS score and final exam grade in MATH 160?
  - d. Is there a correlation between end of the semester MIMBS score and final course grade in MATH 160?
5. Is there a difference in end of the semester MIMBS scores between students enrolled in the MATH 180 intervention during the fall 2009 semester and students that remained in MATH 160 at end of the fall 2009 semester?

This question was addressed with two sub-questions, each of which are difference questions involving two independent groups, thus independent  $t$ -tests were used.

Sub-questions:

- a. Is there a difference in end of the semester MIMBS scores between the group of students who received a grade of A, B, or C in MATH 180 during the fall 2009 semester and students that remained in MATH 160 at end of the fall 2009 semester?
  - b. Is there a difference in end of the semester MIMBS scores between the group of students who received a grade of A, B, or C in MATH 180 during the fall 2009 semester and students that remained in MATH 160 at end of the fall 2009 semester and received a grade of D or F on exam one?
6. Is there a difference in MIMBS scores at the beginning of the fall 2009 semester versus the end of the fall 2009 semester for students that enroll in MATH 180?

This is a difference question involving repeated measures, therefore a paired  $t$ -test was used.

7. A one-way analysis of variance (ANOVA) was used to conduct a comparison in end of the semester MIMBS scores between: (a) students that received a grade of D or F in MATH 160 during the fall 2009 semester, (b) the groups of students that received a grade of A, B, or C in MATH 160 during the fall 2009 semester, and (c) students that enrolled in the MATH 180 during the fall 2009 semester.

An ANOVA will be chosen since the independent variable of MIMBS scores has more than two levels and is being compared between groups (Gliner, et

al., 2009, p. 290). The hypothesis was that there would be a significant difference between the D/F students and the students that go through the MATH 180 intervention.

8. What insights do students provide for the defining moments of the intervention and what suggestions do they have to improve the intervention? Interviews were conducted by the researcher to identify common threads that address this question. In addition a focus group session was conducted with the participants of MATH 180.

## **CHAPTER IV: RESULTS**

The purpose of this study was to assess the effects of a *Concepts for Calculus* intervention on students at risk of failing the first semester of calculus. This chapter presents the results of the eight research questions posed in chapter three. The findings to the research questions are both quantitative and qualitative in nature. Results are presented in the order in which the research questions were asked. Therefore all quantitative results are presented first followed by qualitative results.

### **Quantitative Results**

To answer the quantitative research questions, the statistical package SPSS Statistics 18.0 was used. Each research question with results is listed under its own heading.

#### **Research Question One**

The first research question asked whether or not there would be a difference between the scores on the first MATH 160 exam during the fall 2009 semester and the spring 2010 semester for students that complete the MATH 180 intervention. To answer this question, two sub-questions were addressed. Each sub-question is a difference question involving repeated measures, therefore paired *t*-tests were used, as recommended by Gliner et al.(2009).

Sub-questions:

- 1a. Is there a difference between the scores on the first MATH 160 exam during the fall 2009 semester and the spring 2010 semester for students that complete the MATH 180 intervention?

Of the 22 students that completed MATH 180, only 12 chose to enroll in MATH 160 during the spring 2010 semester. A comparison between their fall 2009 and spring 2010 MATH 160 exam one scores was conducted. The mean score for fall was 48.15 and the mean score for spring was 51.57. The paired  $t$  statistic that resulted was -0.669 with statistical significance  $p = 0.517$ . Thus there was no statistically significant difference between the two groups.

- 1b. Is there a difference between the scores on the first MATH 160 exam during the fall 2009 semester and spring 2010 semester for the group of students that complete the MATH 180 intervention with a grade of A, B, or C?

Of the 12 students that both took MATH 180 during the fall 2009 semester and then enrolled in MATH 160 for the spring 2010 semester, only nine of the students had received a grade of A, B, or C in MATH 180. A comparison of MATH 160 exam one scores was made with this group of students. The mean score for fall was 48.03 and the mean score for spring was 54.94. The paired  $t$  statistic that resulted was -1.107 with statistical significance  $p = 0.301$ . Thus, although the students gained almost seven points on the mean, it was not statistically significant.

The range for a grade of C in MATH 180 during the fall 2009 semester was larger than the ranges for grades of A and B. Both the A and B grade ranges were ten points while the C grade range was 20 points. Thus a comparison in exam ones was made for students that received a grade of A or B in MATH 180. Only four students fell into this category. The fall 2009 exam one mean score for these students was 51.67. The spring 2010 exam one mean score for these students was 74.44. The paired  $t$  statistic was -4.651 with significance  $p = 0.019$ . This indicates a statistically significant increase in mean score for exam one for these few students.

Due to the large difference in mean scores for the fall 2009 and spring 2010 semesters, exam one scores were converted to  $z$ -scores. Paired  $t$ -tests were then conducted. Results from the paired  $t$ -tests were statistically significant when comparing exam one means for the group of MATH 180 students that received a grade of A, B, or C. The mean  $z$ -score for fall exam one was -1.2697 and the mean  $z$ -score for the spring exam one was -0.5523. The paired  $t$  statistic was -2.35 with significance  $p = 0.047$ .

### **Research Question Two**

The second research question asked if there was a difference in scores on the first MATH 160 exam during the spring 2010 semester between students that participated in MATH 180 and students that did not participate in MATH 180, but had to repeat MATH 160 due to receiving a grade of D, F, or W in the course during the fall 2009 semester. To answer this question, two sub-questions were addressed. Each sub-question is a

difference question involving two independent groups and required an independent t-test, as recommended by Gliner, et al. (2009).

Sub-questions:

2a. Is there a difference in scores on the first MATH 160 exam during the spring 2010 semester between students that participated in MATH 180 and students that did not participate in MATH 180, but had to repeat MATH 160 due to receiving a grade of D, F, or W?

Twelve students continued from MATH 180 into MATH 160 and 59 students repeated MATH 160 due to receiving a grade of D, F, or W during the fall semester. The mean exam score for the students that took MATH 180 was 51.57. The mean exam score for the students that did not take MATH 180 was 69.02. The resulting independent  $t$  statistic (with equal variances assumed) was 2.871 with statistical significance  $p = 0.005$ , thus indicating that the students that did not take MATH 180 had a statistically significant higher mean exam one score. The confidence interval was (5.324, 29.572) and the effect size was  $|d| = 0.909$ , which is larger than typical.

2b. Is there a difference in scores on the first MATH 160 exam during the spring 2010 semester between the group of students that participated in MATH 180 and completed the course with a grade of A, B, or C and the group of students that did not participate in MATH 180, but had to repeat MATH 160 due to receiving a grade of D or F?

The sample sizes in this question were nine students from MATH 180 that received a grade of A, B, or C and 58 that did not take MATH 180, but

had to repeat MATH 160 due to receiving a grade of D or F. The mean exam score for those that took MATH 180 was 54.94 and the mean exam score for those that did not take MATH 180 was 69.60. The independent  $t$  statistic (with equal variances assumed) was 2.166 with statistical significance  $p = 0.034$ , indicating a statistically significant higher exam one score mean for students that did not participate in MATH 180. The confidence interval was (1.144, 28.178) and the effect size was  $|d| = 0.776$ , which is large.

As a follow-up to research question two, an independent  $t$ -test was run to see if there was a difference in exam one scores (spring semester) between the 12 students from MATH 180 and the 16 students that declined the MATH 180 invitation but were repeating MATH 160. That is, there were 16 students that were invited to MATH 180 during the fall 2009 semester. These students opted to remain in MATH 160 but had to repeat MATH 160 in the spring due to receiving a grade of D or F in the fall.

The mean score for the group of MATH 160 repeaters was 53.4, which was higher than the mean score for the MATH 180 students (51.57). But the difference was not statistically significant. The independent  $t$  statistic was 0.239 with significance  $p = 0.813$ .

### **Research Question Three**

Is there a difference in performance between students enrolled in MATH 160 during the spring 2010 semester that took MATH 180 and students enrolled in MATH 160 during the spring 2010 semester that did not take MATH 180 but are having to repeat MATH 160 due to receiving a grade of D, F, or W? To answer this question, six sub-

questions were addressed. Each sub-question is a difference question but compares independent groups of students. Therefore independent  $t$ -tests were used.

Sub-questions:

3a. Is there a difference in performance on the spring 2010 MATH 160 final exam between students that participated in MATH 180 and students repeating MATH 160 that did not take MATH 180?

Six students that had taken MATH 180 in the fall 2009 semester took the spring 2010 MATH 160 final exam, and 51 students that were repeating MATH 160 (and not taken MATH 180) took the spring 2010 MATH 160 final exam. The exam was worth 200 points. The mean score for the 51 MATH 160 repeaters was 95.0392, and the mean score for the six MATH 180 students was 92.6667. There was no statistically significant difference between the mean exam scores. The independent  $t$  statistic was 0.195 with significance  $p = 0.846$ .

3b. Is there a difference in performance on the spring 2010 MATH 160 final exam between the group of students that completed MATH 180 with a grade of A, B, or C and the group of students repeating MATH 160 that did not take MATH 180?

Five students that had received a grade A, B, or C in MATH 180 in the fall 2009 semester took the spring 2010 MATH 160 final exam, and 51 students that were repeating MATH 160 (and not taken MATH 180) took the spring 2010 MATH 160 final exam. The mean score for the 51 MATH 160 repeaters was 95.0392, and the mean score for the five MATH 180 students

was 95.2. There was no statistically significant difference between the mean exam scores. The independent  $t$  statistic was -0.012 with significance  $p = 0.990$ .

- 3c. Is there a difference in performance on the spring 2010 MATH 160 final exam between the group of students that completed MATH 180 with a grade of A, B, or C and the group of students repeating MATH 160 that did not take MATH 180 but received a grade of D or F on the first MATH 160 exam during the fall 2009 semester?

The five MATH 180 A/B/C students were compared with ten MATH 160 repeaters. The students repeating MATH 160 declined the invitation to take MATH 180 in the fall. The mean score for the 10 MATH 160 repeaters was 89.1, and the mean score for the five MATH 180 students was 95.2. There was no statistically significant difference between the mean exam scores. The independent  $t$  statistic was -0.396 with significance  $p = 0.669$ .

- 3d. Is there a difference in final grades for spring 2010 MATH 160 between students that participated in MATH 180 and students repeating MATH 160 that did not take MATH 180?

Six students that took MATH 180 in the fall 2009 semester completed MATH 160 in the spring, and 51 students repeated MATH 160 but did not take MATH 180. To compare final letter grades, numeric values were assigned to each letter grade (A = 4, B = 3, C = 2, D = 1, and F = 0). The MATH 180 students had a mean letter grade value of 1.883, and the students repeating MATH 160 had a mean letter grade value of 1.902. The difference

was not statistically significant. The independent  $t$  statistic was 0.146 with significance  $p = 0.885$ .

- 3e. Is there a difference in final grades for spring 2010 MATH 160 between the group of students that completed MATH 180 with a grade of A, B, or C and the group of students repeating MATH 160 that did not take MATH 180?

Five students that received grades of A, B, or C in MATH 180 completed MATH 160 in the spring, and 51 students repeated MATH 160 but did not take MATH 180. The MATH 180 students had a mean letter grade value of 1.8, and the students repeating MATH 160 had a mean letter grade value of 1.902. The difference was not statistically significant. The independent  $t$  statistic was 0.198 with significance  $p = 0.844$ .

- 3f. Is there a difference in final grades for spring 2010 MATH 160 between the group of students that completed MATH 180 with a grade of A, B, or C and the group of students repeating MATH 160 that did not take MATH 180 but received a grade of D or F on the first MATH 160 exam during the fall 2009 semester?

Five students that received grades of A, B, or C in MATH 180 completed MATH 160 in the spring, and ten students repeated MATH 160 but did not take MATH 180 although they had been invited into the course. The MATH 180 students had a higher mean letter grade value (1.8) than the students repeating MATH 160 (1.5). The difference was not statistically significant. The independent  $t$  statistic was -0.587 with significance  $p = 0.567$ .

## Research Question Four

Question four asked if there is a correlation between MIMBS score and performance in MATH 160 during the fall 2009 semester. To answer this question, four sub-questions were addressed. To answer this question, four sub-questions were addressed. Each sub-question is a correlation question involving approximately normally distributed variables, therefore bivariate correlation with Pearson's  $r$  was used.

Sub-questions:

4a. Is there a correlation between MIMBS score at the beginning of the semester (MIMBS-A) and exam one score?

There were 453 students that had a MIMBS-A score as well as an exam one score. The Pearson correlation was  $r = 0.187$ , indicating a positive relationship between MIMBS-A score and exam one. The coefficient of determination was 0.035. The significance was  $p < 0.001$ , which is statistically significant, however, the size of the effect was small.

4b. Is there a correlation between scores on the six constructs of MIMBS-A and exam one score?

The Pearson's  $r$  values, coefficients of determination, significances, and effect sizes for each construct can be found in Table 4.1.

Table 4.1

*Pearson's  $r$ , Coefficients of Determination, Significances, and Effect Sizes for MIMBS-A Constructs*

|                        | MATH 160 Exam One Score<br>(Fall 2009 for $N = 453$ ) |
|------------------------|---|
| Confidence             |   |
|                        | Pearson's $r$ : $r = 0.201$                           |
|                        | Coefficient of Determination: $R^2 = 0.04$            |
|                        | Significance: $p < 0.001$                             |
|                        | Effect Size: Small to medium                          |
| Nature of Mathematics  |   |
|                        | Pearson's $r$ : $r = 0.155$                           |
|                        | Coefficient of Determination: $R^2 = 0.024$           |
|                        | Significance: $p = 0.001$                             |
|                        | Effect Size: Small to medium                          |
| Understanding Concepts |   |
|                        | Pearson's $r$ : $r = 0.089$                           |
|                        | Coefficient of Determination: $R^2 = 0.008$           |
|                        | Significance: $p = 0.060$                             |
|                        | Effect Size: Smaller than typical                     |
| Single Correct Answer  |   |
|                        | Pearson's $r$ : $r = 0.081$                           |
|                        | Coefficient of Determination: $R^2 = 0.007$           |
|                        | Significance: $p = 0.084$                             |
|                        | Effect Size: Smaller than typical                     |
| Effort                 |   |
|                        | Pearson's $r$ : $r = 0.088$                           |
|                        | Coefficient of Determination: $R^2 = 0.008$           |
|                        | Significance: $p = 0.060$                             |
|                        | Effect Size: Smaller than typical                     |
| Usefulness             |   |
|                        | Pearson's $r$ : $r = 0.032$                           |
|                        | Coefficient of Determination: $R^2 = 0.001$           |
|                        | Significance: $p = 0.495$                             |
|                        | Effect Size: Smaller than typical                     |

The Pearson correlation values all indicated a positive relationship between score on MIMBS-A construct and exam one. The constructs dealing with confidence and the nature of mathematics were statistically significant with  $p < 0.001$  and  $p = 0.001$  respectively.

The correlation matrix containing the Pearson correlation coefficients for the constructs matrix is in Table 4.2.

Table 4.2

*Correlation Matrix for MIMBS-A Constructs*

|                        | Confidence | Nature of Math | Understanding Concepts | Single Answer | Effort | Usefulness |
|------------------------|------------|----------------|------------------------|---------------|--------|------------|
| Confidence             | 1          | 0.004          | 0.327                  | 0.190         | 0.229  | 0.374      |
| Nature of Math         | 0.004      | 1              | 0.118                  | 0.268         | -0.048 | 0.011      |
| Understanding Concepts | 0.327      | 0.118          | 1                      | 0.276         | 0.295  | 0.415      |
| Single Answer          | 0.190      | 0.268          | 0.276                  | 1             | 0.044  | 0.170      |
| Effort                 | 0.229      | -0.048         | 0.295                  | 0.044         | 1      | 0.372      |
| Usefulness             | 0.374      | 0.011          | 0.415                  | 0.170         | 0.372  | 1          |

4c. Is there a correlation between end of the semester MIMBS score (MIMBS-B) and final exam grade in MATH 160?

There were 331 students that had both a MIMBS-B and final exam score that were used to answer this research question. The Pearson correlation was  $r = 0.253$  with significance  $p < 0.001$  and coefficient of determination  $R^2 = 0.064$ . This indicated a statistically significant positive relationship between MIMBS-B score and final exam score, and the size of the effect was small to medium

As a follow-up to this question, bivariate correlation was run to see if there was a correlation between the six constructs of the MIMBS-B and final

exam score. The Pearson's  $r$  values, coefficients of determination, significances, and effect sizes for each construct can be found in Table 4.3.

Table 4.3

*Pearson's  $r$ , Coefficients of Determination, Significances, and Effect Sizes for MIMBS-B Construct*

|                        | MATH 160 Final Exam Score<br>(Fall 2009 for $N = 331$ )  |
|------------------------|--|
| Confidence             | Pearson's $r$ : $r = 0.400$<br>Coefficient of Determination: $R^2 = 0.16$<br>Significance: $p < 0.001$<br>Effect Size: Medium to large       |
| Nature of Mathematics  | Pearson's $r$ : $r = 0.075$<br>Coefficient of Determination: $R^2 = 0.006$<br>Significance: $p = 0.175$<br>Effect Size: Smaller than typical |
| Understanding Concepts | Pearson's $r$ : $r = 0.122$<br>Coefficient of Determination: $R^2 = 0.015$<br>Significance: $p = 0.026$<br>Effect Size: Small                |
| Single Correct Answer  | Pearson's $r$ : $r = 0.015$<br>Coefficient of Determination: $R^2 < 0.001$<br>Significance: $p = 0.783$<br>Effect Size: Smaller than typical |
| Effort                 | Pearson's $r$ : $r = 0.076$<br>Coefficient of Determination: $R^2 = 0.006$<br>Significance: $p = 0.167$<br>Effect Size: Smaller than typical |
| Usefulness             | Pearson's $r$ : $r = 0.162$<br>Coefficient of Determination: $R^2 = 0.026$<br>Significance: $p = 0.003$<br>Effect Size: Small to medium      |

All constructs had a positive correlation with final exam score. The confidence, understanding concepts, and usefulness constructs were statistically significant. The confidence construct had the strongest relationship with final exam score.

4d. Is there a correlation between MIMBS-B score and final course grade in MATH 160?

There were 334 MIMBS-B scores and final course grades used to answer this research question. The difference in sample sizes between this question and question 4c is because not all students took the final exam. A student that did not take the final exam may still have a final course grade. The Pearson correlation for this research question was  $r = 0.270$  with  $p < 0.001$ . This indicated a positive, statistically significant relationship between MIMBS-B score and final course grade with a small to medium effect size. The coefficient of determination was  $R^2 = 0.073$ .

As a follow-up to this question, multiple linear regression was run to see if there was a combination of constructs one and two from MIMBS-B (confidence and nature of mathematics) that could be used to predict final course grade. The Pearson correlations, coefficients of determination and significances can be found in Table 4.4.

Table 4.4

*Pearson Correlations and Significances for Confidence and Nature of Mathematics of MIMBS-B as Predictors for MATH 160 Final Course Grade*

| MATH 160 Final Course Grade<br>(Fall 2009 for $N = 334$ ) |  |
|---|--|
| Confidence  |  |
| Pearson Correlation:                                      | $r = 0.440$                            |
| Coefficient of Determination:                             | $R^2 = 0.1936$                         |
| Significance:   | $p < 0.001$                            |
| Effect Size:  | Medium to large                        |
| Nature of Mathematics                                     |  |
| Pearson Correlation:                                      | $r = 0.045$                            |
| Coefficient of Determination:                             | $R^2 = 0.002$                          |
| Significance:   | $p = 0.414$                            |
| Effect Size:  | Smaller than typical                   |
| Model Summary with Constructs<br>Combined                 |  |
| Pearson Correlation:                                      | $r = 0.448$                            |
| Coefficient of Determination:                             | $R^2 = 0.196$ , adjusted $R^2 = 0.191$ |
| Significance:   | $p < 0.001$                            |

Both the confidence and nature of mathematics constructs had positive correlation with MATH 160 final course grade. Only confidence was significant. The  $R^2$  value indicates that 19.4 percent of the variance in MATH 160 final course grade can be predicted from confidence.

The question then arose as to whether the MIMBS given at the beginning of the semester, specifically scores on the confidence and nature of mathematics constructs, could be used to predict performance on the MATH 160 final exam as well as course grade. Multiple regression and bivariate correlation were run on the data. Results were statistically significant, indicating that about seven percent of the variance in final course grade in

MATH 160 could be predicted by scores on the confidence and nature of mathematics constructs. Results from the data analysis can be found in Table 4.5.

Table 4.5

*Pearson Correlations and Significances for Confidence and Nature of Mathematics of MIMBS-A as Predictors for MATH 160 Final Course Grade*

| MATH 160 Final Course Grade<br>(Fall 2009 for $N = 334$ ) |  |
|---|--|
| Confidence  |  |
| Pearson Correlation:                                      | $r = 0.237$                            |
| Coefficient of Determination:                             | $R^2 = 0.0562$                         |
| Significance:   | $p < 0.001$                            |
| Nature of Mathematics                                     |  |
| Pearson Correlation:                                      | $r = 0.133$                            |
| Coefficient of Determination:                             | $R^2 = 0.0177$                         |
| Significance:   | $p = 0.009$                            |
| Model Summary with Constructs<br>Combined                 |  |
| Pearson Correlation:                                      | $r = 0.269$                            |
| Coefficient of Determination:                             | $R^2 = 0.072$ , adjusted $R^2 = 0.068$ |
| Significance:   | $p < 0.001$                            |

### Research Question Five

Question five asked whether or not MIMBS-B scores differed between MATH 180 students and MATH 160 students. This question was addressed with two sub-questions, each of which are difference questions that do not necessarily involve samples of the same size, thus independent  $t$ -tests were used.

Sub-questions:

5a. Is there a difference in MIMBS-B scores between the group of students who received a grade of A, B, or C in MATH 180 during the fall 2009 semester and students that remained in MATH 160 at end of the fall 2009 semester?

The MIMBS-B scores were compared between 16 of the MATH 180 students and 342 of the MATH 160 students. The mean MIMBS-B score for the MATH 180 students was 132.56 and the mean MIMBS-B score for the MATH 160 students was 131.60. The independent  $t$  statistic (with equal variances assumed) was -0.325 with significance 0.745. Thus there was no statistically significant difference between the MIMBS scores of the two groups.

5b. Is there a difference in MIMBS-B scores between the group of students who received a grade of A, B, or C in MATH 180 during the fall 2009 semester and the group of students that remained in MATH 160 at end of the fall 2009 semester and received a grade of D or F on exam one?

The MIMBS-B scores were compared between 16 of the MATH 180 students and 34 of the MATH 160 students. The mean MIMBS-B score for the MATH 180 students was 132.56 and the mean MIMBS-B score for the MATH 160 students was 128.62. The independent  $t$  statistic (with equal variances not assumed) was -1.507 with significance 0.138. The mean MIMBS score for MATH 180 students was not significantly higher. The effect size was  $|d| = 0.359$ , indicating a small to medium effect.

The grade range of C (20 points) for MATH 180 is larger than the grade ranges for A and B (ten points for each). Thus an independent  $t$ -test was run to see if there was a difference in MIMBS-B scores between the group of students who received a grade of A or B in MATH 180 during the fall 2009 semester and the group of students that remained in MATH 160 at end of the fall 2009 semester and received a grade of D or F on exam one. The mean MIMBS-B scores were 131.43 for the MATH 180 students and 130.12 for the MATH 160 students. The independent  $t$  statistic was -0.276 with  $p = 0.784$ , which does not indicate a significant difference in MIMBS-B scores between the groups.

### **Research Question Six**

Question six asked whether or not MIMBS-A and MIMBS-B scores differed for students that participated in MATH 180. This is a difference question involving repeated measures; therefore a paired  $t$ -test was used.

There were 22 students that participated in MATH 180 and had beginning and end of the semester MIMBS scores. The mean MIMBS-A score for these students was 131.86. The mean MIMBS-B score for these students was 132.09. The paired  $t$  statistic was -0.121 with significance 0.905, which does not indicate a statistically significant difference in mean MIMBS scores.

A follow-up paired  $t$ -test was run to see if there were any differences in construct scores between MIMBS-A and MIMBS-B for MATH 180 students. Table 4.6 contains the paired  $t$  statistics and significances for each construct.

Table 4.6

*Statistics and Significances for Differences Between MIMBS-A and MIMBS-B Constructs for MATH 180 Students*

| MATH 180 MIMBS Differences<br>(Beginning – End; $N = 22$ ) |  |
|--|--|
| Confidence   | $t$ statistic: $t = -0.126$<br>Significance: $p = 0.901$ |
| Nature of Mathematics <sup>1</sup>                         | $t$ statistic: $t = -1.976$<br>Significance: $p = 0.061$ |
| Understanding Concepts                                     | $t$ statistic: $t = 0.745$<br>Significance: $p = 0.465$  |
| Single Correct Answer                                      | $t$ statistic: $t = -0.893$<br>Significance: $p = 0.382$ |
| Effort   | $t$ statistic: $t = 0.631$<br>Significance: $p = 0.535$  |
| Usefulness   | $t$ statistic: $t = 1.517$<br>Significance: $p = 0.144$  |

Differences in scores on the constructs were not statistically significant. The biggest difference in the scores on the constructs was on the nature of mathematics construct. Students scored higher on this construct at the end of the semester than they did at the beginning of the semester. The 95% confidence interval for this construct was [-1.589, 1.407].

<sup>1</sup> The nature of mathematics construct had an effect size of  $|d| = 0.421$ , which indicated a small to medium effect.

A paired *t*-test was also run to see if there were any differences between the MIMBS-A and MIMBS-B scores as well as scores on specific constructs for the group of students who received a grade of A, B, or C in MATH 180. Table 4.7 contains the *t* statistics and significances for each construct.

Table 4.7

*Statistics and Significances for Differences in Overall MIMBS-A and MIMBS-B Scores and Construct Scores for MATH 180 A/B/C Students*

| MATH 180 MIMBS Differences<br>(Beginning – End; <i>N</i> = 16) |   |
|--|---|
| MIMBS Score  | <i>t</i> statistic: <i>t</i> = -0.205<br>Significance: <i>p</i> = 0.841 |
| Confidence   | <i>t</i> statistic: <i>t</i> = -0.156<br>Significance: <i>p</i> = 0.878 |
| Nature of Mathematics <sup>2</sup>                             | <i>t</i> statistic: <i>t</i> = -3.212<br>Significance: <i>p</i> = 0.006 |
| Understanding Concepts   | <i>t</i> statistic: <i>t</i> = 0.619<br>Significance: <i>p</i> = 0.545  |
| Single Correct Answer  | <i>t</i> statistic: <i>t</i> = -0.410<br>Significance: <i>p</i> = 0.688 |
| Effort   | <i>t</i> statistic: <i>t</i> = 1.046<br>Significance: <i>p</i> = 0.312  |
| Usefulness   | <i>t</i> statistic: <i>t</i> = 0.783<br>Significance: <i>p</i> = 0.446  |

<sup>2</sup> The nature of mathematics construct had an effect size of  $|d| = 0.803$ , which indicated a large effect.

Only the construct relating to the nature of mathematics was statistically significant. The mean score on this construct increased significantly for the MATH 180 students that received a grade of A, B, or C. Increase in score on this constructs indicates that students did not believe as strongly as the did at the beginning of the semester that mathematics is about following procedures.

Lastly a paired *t*-test run to see if there were any differences between the beginning and end of the fall 2009 semester in overall MIMBS scores as well as scores on specific constructs of the MIMBS for the group of students who received a grade of A or B in MATH 180. The choice to exclude the students that received a grade of C was due to the numerical range for C being larger than the grade ranges for both A and B. Table 4.8 contains the *t* statistics and significances for each construct.

The nature of mathematics construct was nearly significant with  $p = 0.059$  and effect size of  $|d| = 0.75$ , which is larger than typical. These students had an increase in score on this construct from beginning to end of semester. Understanding concepts was statistically significant with an effect size of  $|d| > 1.00$ , which is much larger than typical. However, the mean scores on this construct went down from the beginning of the semester. This indicates that students did not believe as strongly as they did at the beginning of the semester about the importance of understanding concepts in mathematics.

Table 4.8

*Statistics and Significances for Differences in Overall MIMBS-A and MIMBS-B Scores and Construct Scores for MATH 180 A/B Students*

| MATH 180 MIMBS Differences<br>(Beginning – End; $N = 7$ ) |  |
|---|--|
| MIMBS Score   | $t$ statistic: $t = 1.804$<br>Significance: $p = 0.121$  |
| Confidence  | $t$ statistic: $t = 0.315$<br>Significance: $p = 0.763$  |
| Nature of Mathematics                                     | $t$ statistic: $t = -2.328$<br>Significance: $p = 0.059$ |
| Understanding Concepts                                    | $t$ statistic: $t = 2.680$<br>Significance: $p = 0.037$  |
| Single Correct Answer                                     | $t$ statistic: $t = 0.420$<br>Significance: $p = 0.689$  |
| Effort  | $t$ statistic: $t = 1.247$<br>Significance: $p = 0.259$  |
| Usefulness  | $t$ statistic: $t = -0.367$<br>Significance: $p = 0.726$ |

### **Research Question Seven**

A one-way analysis of variance (ANOVA) was used to conduct a comparison in MIMBS-B scores between: (a) students that received a grade of D or F in MATH 160 during the fall 2009 semester, (b) students that received a grade of A, B, or C in MATH

160 during the fall 2009 semester, and (c) students that enrolled in the MATH 180 during the fall 2009 semester.

An ANOVA was chosen since the independent variable of end of semester MIMBS scores had more than two levels and was being compared between groups (Gliner, et al., 2009, p. 290). The hypothesis was that there would be a significant difference between the D/F students and the students that go through the MATH 180 intervention.

The mean MIMBS-B score for the D/F MATH 160 students ( $N = 97$ ) was 127.05. The mean MIMBS-B score for the A/B/C MATH 160 ( $N = 237$ ) students was 133.397. The mean MIMBS-B score for the A/B/C MATH 180 ( $N = 16$ ) students was 132.56. The one-way ANOVA showed that difference in means between the two MATH 160 groups was statistically significant with a value of  $p < 0.001$ . There was no statistical difference between any other two groups. Note also that the MATH 180 A/B/C students had a higher mean than MATH 160 D/F students.

### **Qualitative Results: Research Question Eight**

What insights do students provide for the defining moments of the intervention and what suggestions do they have to improve the intervention? Interviews were conducted by the researcher to identify common themes that address this question. In addition a focus group session was conducted with the participants of MATH 180. Qualitative results from the interviews are presented first, and the results from the focus groups will be presented second.

## Interviews

Nine MATH 180 students were contacted for interviews. Only four students completed interviews with the researcher either in person or via. Students were chosen based on the instructor's recommendation and academic status in the class. The intent was to interview students at various grade levels in the class. However, the four students that completed interviews received either a final grade of A or B in MATH 180. Each interviewee was asked the same eight questions. These questions were the same for each interview:

1. How well did the small group collaboration work for you?
2. Describe your reaction to the activities you worked on in groups.
3. How was MATH 180 different from previous math classes you have taken?
4. What was the least effective part of MATH 180?
5. Do you think about mathematics differently now than you did prior to taking MATH 180?

Sub-question:

If given a mathematics problem, would you approach it differently now than you did prior to taking MATH 180?

6. As a result of your experiences in MATH 180, are you more or less likely to take another mathematics course?
7. How do you feel about ALEKS<sup>®</sup>?

Sub-question:

Do you feel ALEKS<sup>®</sup> was useful or valuable?

Two interviews were done in person and two through email as these students were unable to meet with the researcher in person. However, one of the email responses did not provide an answer to each question; rather, the student just wrote a brief paragraph of reflection about MATH 180. Useable responses to each question were analyzed for common threads. The prevailing themes revolved around the class format – working in groups and content of group activities, feedback and communication with the instructor, ALEKS<sup>®</sup>, and future mathematics classes. Each theme will be discussed under separate headings in the following five sections, with class format discussed under two headings.

**MATH 180 class format: working in groups.** The MATH 180 class format was not lecture-based, as were nearly all of the previous mathematics courses that the students had taken. Students spent the majority of class time working in small groups collaborating on problems posed by the instructor. The students enjoyed working in small groups on problems posed during class. One student commented that working in groups was beneficial as it provided “very different perspectives” when working through problems. The students liked being able to see how their peers thought about problems as it was often helpful in understanding the concepts presented in the group activities.

On the other hand, another student acknowledged a concern of dependence on others for the correct answer. The student was accustomed to the regular group collaboration so she found it difficult to take exams individually, commenting:

I really liked that we worked in groups. It just didn't help me for the test because I was used to working and relying on other people if I couldn't get the right answer. And then on the test I was like “oh wait I'm not used to just doing it on my own.

Though MATH 180 exams did reflect concepts explored in the group activities and during class, the student found the transition to working alone difficult. On the other hand, another student mentioned that you can succeed in MATH 180 as long as “you pay attention in class” and “work diligently.” Essentially, in order to do well on a MATH 180 exam, students must have a thorough understanding of the concepts explored in group activities.

**MATH 180 class format: group activities.** Students agreed that the group activities in MATH 180 pushed for the “why” behind mathematics problems and were more conceptual than previous mathematics courses they had taken. In addition, students had not been exposed to the different perspectives of mathematical concepts.

The purpose of the different perspectives was to get students to think differently about mathematics. Two of the students acknowledged that they would think differently about mathematics, saying that knowing what the question is asking is “half the battle.” In addition, these two students expressed feeling comfortable working with any mathematics problem even if the problem had parameters rather than numbers. They felt that as long as they understood the concepts, they could work through any problem. One student explained:

I really liked that it wasn't number based. Like it was more conceptual. Because that taught us a lot more than just using numbers. Because like, if you teach somebody how to use numbers and like one method it can only be applied to that method.

However, one student commented she would not think differently about mathematics in the future and would not approach problems differently than she did prior to taking

MATH 180. It should be pointed out that this is the same student that found it difficult to take the exams individually.

Though students' opinions about the group activities were varied, all mentioned having difficulty at times in interpreting the instruction of the group activities. One student stated that she never knew how to begin the activities and had trouble identifying the purpose of the activities. A student who liked the activities expressed similar feelings stating:

I thought they were good. Um. I think one of the biggest problems we all had was kind of the wording on things. They were really vague. Sometimes we didn't even know what we were trying to find, and that's really difficult.

Though all students agreed that the activities encouraged them to think "outside the box," there was some resistance to the content of the activities. One student said:

I thought he was going to just like reinforce the values of calculus, but like go slower like – like a pre-calculus class. But like really look in depth of why that, if it's a limit, like why it happens.

**Feedback.** Although one student had nothing negative to say about MATH 180 and felt that all aspects of the course were effective, two of the students agreed that the lack of feedback regarding academic progress in the course was frustrating. Work was not graded and returned to the students in a timely fashion. They felt this made it difficult to study and expressed concern about being able to prepare for class exams. In fact, one student commented that:

All my notes are on my worksheets. I guess he like gave us our worksheets back, but then he didn't like grade them obviously because he didn't have time. But I could be studying them and it could be wrong.

Though the students expressed the need for feedback, only one was worried about poor exam performance due to not having received feedback on worksheets. The other two felt that the in-class experience was the best way to study for exams.

**ALEKS®.** Every student agreed that ALEKS® was helpful. They appreciated the instant feedback and felt ALEKS® provided good practice. Though students were unsure as to whether or not MATH 180 would prepare them for MATH 160 the following semester, they did feel that ALEKS® was a useful component for the course that provided techniques they could apply in calculus.

**Future mathematics classes.** Students were divided regarding taking mathematics classes in the future. Two students agreed that their MATH 180 experience made them feel more comfortable taking additional mathematics courses, commenting:

I definitely feel that it's helped me communicate like the math ideas behind it as well.

It was more conceptual than any other math class I have every [*sic*] taken which I think helps a lot because instead of using only numbers to describe why something is right [*sic*] Dr. K really opened my mind to the concepts behind the numbers [*sic*] I feel much more confident with any math problem I am presented with now.

The other two students decided they would only take more mathematics courses if required by their majors. At the end of MATH 180 two of the four interviewees had decided to change their majors to ones not requiring MATH 160.

### **Focus Group**

The focus group was conducted by the researcher and held during one of the MATH 180 class sessions. The MATH 180 instructor was not present during the focus group session. There were eighteen MATH 180 students that attended the focus group.

Before the discussion students were asked to take five minutes to write and reflect on the following three questions:

1. What did you like best about MATH 180?
2. What did you like least about MATH 180?
3. What did you find interesting about MATH 180?

**The following section and subsections discuss the qualitative data resulting from the written responses.**

**Written responses.** The responses to the written questions varied. The 18 responses were read multiple times by the researcher to find common themes. In addition, the researcher met with the MATH 180 instructor to review the written responses after the course had ended and final grades had been assigned. Common themes were also echoed during the focus group discussion.

**What students liked about MATH 180.** Students liked the format of the class as well as the ALEKS<sup>®</sup> component. They felt that the group collaboration and non-lecture format of the class helped them think about mathematics problems differently. Students felt that looking at problems from different points of view enabled them to better understand mathematical concepts. Additionally, a majority of students liked the extra practice they could get with the ALEKS<sup>®</sup> program. ALEKS<sup>®</sup> gave students a chance to maintain their mechanical skills as well as provide instant feedback about their performance.

Students liked that the instructor emphasized exploring problems from various perspectives. This idea paired well when working in groups as each group member often had a unique way in which they would interpret the problem. In addition, students were

pushed to think carefully about problems that were posed, and some students recognized that this process was new for them. The following writing excerpts reflect this.

I liked how it forced us to look at problems and think differently about them. I enjoyed the group aspect as well because it brought into view different views to the problem and different ways to answer it.

In class it was hard at first to understand the direction it was going but it was helpful to try to think at math differently than I have been taught all my life.

Mr. K is a fun teacher to have. I really liked how he doesn't just tell us the answer, we had to think about things.

I enjoyed looking at problems in a different manner.

Dr. K has really opened up my mind and I am so thankful for this course.

Students also enjoyed the ALEKS<sup>®</sup> component of the course. The purpose of ALEKS<sup>®</sup> was to “firm up the pre-calculus skills” needed in MATH 160 (Klopfenstein, 2009a, p. 1). Students were in agreement with this, stating:

ALEKS<sup>®</sup> was one of the most practical parts of this class that will prepare me most for calculus.

ALEKS<sup>®</sup> was a good refresher.

I liked the review with Alex, it helped me the most with me in getting ready for Math 160.

**What students did not like about MATH 180.** Students expressed their concerns about MATH 180 when asked to write about what they did not like. Common themes in the written responses revolved around grading, calculus the following semester and ALEKS<sup>®</sup>.

Students were apprehensive about the grading of the course. They did not know the grading structure of the written homework and had little or no feedback from the

instructor on which to base this component of their grade. Students did not know where they stood in the class academically. They were concerned about this and expressed a desire for more feedback from the instructor, writing:

I did not like the grading style. It was very unclear to me throughout the whole course what my grade was and how final grades would be processed. I did not like that I rarely got feedback on assignments I was getting low scores on. I had no idea what I was getting wrong.

I have no clue what grade I'm going to receive.

In addition to the lack of feedback, students were concerned about being prepared for MATH 160 the following semester. MATH 180 was so different from what they had been exposed to in MATH 160 at the beginning of the semester that they were unsure if they were prepared to think about calculus. Students did not feel that the MATH 180 experience would be beneficial for future mathematics classes. This concern is clear in the following writing excerpts:

I did not feel like I learned anything to help prepare me for Calc 160 or if I even learned anything.

I feel like there is too much "English" (class) involved. This is a math class not a writing class.

He wanted the "perfect" answer but not anything close to it.

I don't like how we have to think about our work. It seems like this is not the best style of learning for me.

These excerpts indicate that students had a specific idea about the nature of mathematics. MATH 180 encouraged students to think differently about mathematics problems and to write about mathematical concepts in order to prepare them for MATH 160. MATH 160 requires non-patterned mathematical thinking as well as writing about

calculus concepts. Several students felt that mathematics is about working with formulas and following procedures. They did not feel that writing should be involved, only numbers.

Although the majority of students liked ALEKS<sup>®</sup>, about one third of the students did not. The students that did not like ALEKS<sup>®</sup> expressed a dislike for the time constraints and self-paced structure of the program. A couple of students also mentioned a disconnect between the in-class content of MATH 180 and ALEKS<sup>®</sup>.

The worst part of 180 was the online homework ALEKS<sup>®</sup> because we have to teach ourselves.

We never talked about ALEKS<sup>®</sup> so it was never really a factor to the class.

ALEKS<sup>®</sup> time constraints: dislike

This illustrates a misconception among students that the purpose of the in-class component of MATH 180 is to learn and maintain procedural pre-calculus skills.

**What students found interesting about MATH 180.** Students found several things interesting about MATH 180. Most commonly, students found the problems and course material, multiple points of view, and applications to the real world to be interesting. Students liked that the problems presented involved real-world scenarios. In addition, they found it interesting when the instructor showed them that the problems could be approached from various perspectives. Students were intrigued by the connection of concepts to real-world and were even more fascinated when these problems could be approached in multiple ways. The following statements reflect this:

The interesting part of math 180 was the way he had us look at problems It took my level of thinking to a place it usually doesn't go.

The most interesting thing about Math 180 is how it makes you think about problems, and how to come up with a plan to solve it.

The most interesting part of this class was that I really don't feel like this was much of a math based class.

Since being in this class I have found myself thinking of real-life problems as functions and figuring it out that way.

Also teaching how there are so many different ways to solve a problem was a great way to work our minds.

**Focus group discussion.** After students spent five minutes writing and reflecting on the three questions posed by the researcher, a discussion about MATH 180 began. The researcher recorded the discussion which was fueled by reactions to the three writing questions as well as the five following questions posed by the researcher:

1. How well did small group collaboration work?
2. How did you feel about the actual group activities?
3. How was MATH180 different from past math classes you have taken?
4. Do you think about math differently now after going through MATH 180?
5. Do you have anything additional to add with regard to MATH 180?

All of the five questions listed were asked during the discussion. These questions were not modified in any way. However, students were allowed to respond freely. The researcher allowed students to continue their discourse for as long as someone had something to add to the conversation. A new question was posed once the conversation revolving around a particular question became quiet. Common themes that arose in the written responses were also echoed during the discussion when reflecting on additional questions posed by the researcher. The discussion revolved around three major themes: the non-lecture based format of the class, their future in MATH 160, and feedback. The

MATH 180 class format and the connection of the course with MATH 160 will be discussed together under one heading, with feedback following under a separate heading.

**MATH 180 class format and continuing to MATH 160.** Group work was a regular classroom activity, and the students responded positively to it. They recognized that “different people think in different ways” and found group collaboration helpful in approaching the assigned problems. They liked being able to work together to solve problems, stating:

For me personally it worked out really well ah just ‘cause um I thought, I thought it kind of like when we were working on problems it brought into light different ways to solve them, and it also kind of showed that different people work in different ways.

Like how you could use other people’s work in different ways to do the problems.

I definitely liked the groups in class. They’re definitely like a huge like advantage I feel like you know compared to the disadvantages of them.

However, because the students had grown accustomed to working with groups for nearly every class session, they expressed anxiety regarding individual exams. They were unsure of how to study and did not feel confident testing on their own. For example, one student explained:

I liked the groups, um but I felt that I wasn’t prepared for the tests because I was used to thinking with other people.

The only way to study for the exams was to be regularly engaged and attentive in class.

One student mentioned that the tests were a good indication of what “you, not necessarily your group, learned from class.”

The students were divided on their feelings about the problems that were assigned for the group activities. Students liked the activities and enjoyed working on applications of concepts. Students agreed that the activities made them think about mathematics differently. They recognized that problems could be approached in more than one way and there was not always “one right answer.”

There were students, however, that wanted more structure to the activities with instructions about how to begin the assignments. They felt the current instructions were “vague” and “abstract.” In addition, some students wanted problems they felt were more closely related to calculus. The following comments reflect the diverse opinions regarding the activities:

I liked that they weren't just sheets full of problems. They were word problems, and I tend to work better when there's an application of a mathematical concept rather than the concept itself.

I think if the problems he gave us were a little more structured and like he gave us an objective, what our goal was. Sometimes like I just didn't know what direction to go or how to start the problem. What am I supposed to look for in how to solve these problems?

It would have helped if there were more equations, actual math problems, not word problems.

Nearly all the students expressed a concern regarding adequate preparation for calculus. The students had spent the first few weeks enrolled in MATH 160 and were expecting MATH 180 to have similar assignments and activities. They did not see a connection between MATH 180 and MATH 160. In fact, a few of the students wondered if it would have been better to stay in MATH 160 and fail rather than take MATH 180.

We're trying to get prepared for calc one, why are we doing stuff that's not in calc one?

I don't really see how it's going to prepare me for calculus.

At least in calc you get your feet wet.

**Feedback.** The students reiterated the need for feedback from the instructor. They were frustrated about not knowing how they were doing on assignments. They did not know what the instructor's expectations were for solutions and stated they were "flying blind." The students also added that the instructor had no formal office hours and they had no place to go when they needed help. They suggested the class have a teaching assistant in order to help with such issues. The following are statements reflecting students' frustrations with feedback:

Explain why my reasoning is wrong or why I can't use it.

He could have gone over what we did for homework.

We haven't gotten any papers back just saying where we're at.

### **Qualitative Summary**

At the end of the fall 2009 semester, the researcher interviewed four MATH 180 students and conducted a focus group with eighteen MATH 180 students. Their responses were analyzed for common themes.

Overall, students enjoyed the group collaboration and found the course content interesting. They liked the idea of connecting concepts to real world situations. Students also found the multiple approaches to problems interesting and helpful. Seeing the various ways in which a problem could be solved was beneficial to their understanding of the concept being studied.

Students were frustrated by the lack of feedback from the instructor. They said that they knew the instructor was busy and felt that MATH 180 was not a priority. They expressed a need for homework problems to be graded and returned more consistently as well as regular office hours to be scheduled. The students said that they often felt discouraged with homework but did not know where to go for help.

Finally, students were unsure as to whether or not MATH 180 had prepared them for MATH 160. MATH 180 content was different from what they had briefly seen in MATH 160. One student wondered whether or not it would have been better to have been exposed to MATH 160 and failed rather than to have taken MATH 180.

A recurring theme that emerged in both the interviews and focus group involved the purpose of MATH 180. Students did not see the connection between MATH 180 and MATH 160. Some expected MATH 180 to be a pre-calculus course, while others thought it was a calculus course that progressed more slowly, delving into beginning calculus concepts such as limits. This indicates a misunderstanding of the course content of MATH 180 and how it relates to MATH 160.

As stated previously, MATH 160 involves studying functions, and MATH 180 content involved reexamining “elementary functions from a more advanced standpoint” (Klopfenstein, 2009b, p. 1). For some reason the connection between the two courses was unclear to students. It is uncertain whether this is a result of miscommunication between the instructor and students or a result of the students simply not understanding the syllabus.

## **CHAPTER V: DISCUSSION, CONCLUSIONS, AND RECOMMENDATIONS**

The overall purpose of this study was to test the effectiveness of an intervention on both academic achievement and beliefs about mathematics of undergraduate students who were required to take MATH 160 at Colorado State University. This chapter discusses the results of the eight research questions posed in chapter three, draws conclusions, and makes recommendations for future research. The discussion for the research questions is presented in the order in which the research questions were asked and listed under separate headings.

### **Research Question One**

Research question one involved comparing MATH 160 exam one scores between the fall 2009 and spring 2010 semesters for the MATH 180 students. Although the mean was higher for these students in the spring, there was no statistically significant difference in mean exam scores. When the means of only the MATH 180 A/B students were compared, the spring exam one mean was significantly higher than the fall exam one mean.

It is of no surprise students had a higher mean on exam one in the spring. Although MATH 160 exams change each semester, the content remains the same. Students had previously been exposed to MATH 160 exam one, so the experience was not foreign to them. However, a significant increase in mean exam score was expected.

There are various reasons for this outcome. For one, only 12 students continued on to MATH 160 in the spring, and of these students, five received a grade of C in MATH 180 and four received an A or B in MATH 180. These numbers are below the recommended guideline of 30, recommended by Gliner et al. (2009).

The difference in perceived difficulty between the two exams may have been significant. The students may have found the spring 2010 exam to be harder than the first exam the previous fall semester. This would not be surprising, as the mean score for exam one in fall 2009 was 64.48, and in spring 2010 the mean score on MATH 160 exam one was 52.45, which could be indicative of a harder exam.

For these reasons, the exam scores were converted to standardized  $z$ -scores. The paired  $t$ -test did show a significant improvement in exam scores for the spring semester (with  $p = 0.47$ ). Student scores for the fall were about 1.3 standard deviations below the mean, while scores in the spring were only about 0.6 standard deviations below the mean.

Although students still scored below the mean, improvement was made. This improvement coincides with qualitative results. Students agreed that MATH 180 helped them think about and approach problems in multiple ways. Perhaps these students were trying to think about calculus problems more critically, and, therefore, exhibited improvement in finding solutions.

### **Research Question Two**

Research question two compared exam one means from the spring 2010 semester between MATH 180 students and students that were repeating MATH 160 due to receiving a grade of D, F, or W in the course during the fall 2009 semester. Regardless of

the groups being compared, the non-MATH 180 students scored higher on exam one during the spring 2010 semester.

The argument could be that remaining in MATH 160 (and not succeeding in the course) is better preparation for MATH 160 than MATH 180. However, it is important to note that this study is based on the first course offering of MATH 180. The course has been changed in various ways in hopes to better meet the needs of the students. In addition, the initial offering of MATH 180 had 22 students enrolled and only 12 of those continued on to MATH 160 the following semester. The second offering of MATH 180 currently has about 40 students.

Hackett (1998) and Wahlberg (1998) had statistically significant results in student achievement after incorporating writing into the calculus classroom. Ellington (2003), Goerdt (2007), Heid (1988), Nasari (2008), Palmiter (1991), and Tiwari (1999, 2007) found that multiple representations with technology significantly increased performance on calculus exams. The use of multiple representations in the mathematics classroom is also encouraged by Douglas (1986), Ross (1996), and Smith (1994, 1996). All authors assert that experiencing concepts in multiple forms, whether through writing or technology, enable students to develop problem solving skills and conceptual understanding.

MATH 180 used multiple representations with writing and technology. In addition, MATH 180 utilized group collaboration as a means for instruction and exploration of concepts. Although it was not a form of cooperative learning, the hope was that such interaction would produce positive results, as Slavin (1980, 1988, 1999) and Treisman (1985, 1992) have had with cooperative learning.

The results of this study, though, conflict with the results produced by these authors. In fact, the results were the opposite with the MATH 180 students performing more poorly than students that had not taken MATH 180 and been exposed to reform instruction techniques. MATH 180 was conducted in such a way to facilitate active learning. Students were encouraged to interact with the course material.

However, as the instructor commented on more than one occasion, not all students were completely and regularly engaged. In addition, not all students attended class on a regular basis. This behavior was also noticeable when attempting to set up interviews for this study. About ten students were contacted, but only three students attended one-on-one interviews. This reflects the lack of engagement students had with the course. This is an issue. Irregular attendance and passive involvement in the course is not sufficient for students to gain an understanding of mathematical concepts.

Although others have been successful in incorporating multiple representations and cooperative learning as individual techniques, a combination of all may be too difficult to implement in MATH 180. The students that enroll in the course are those that are at a high risk of failing MATH 160. MATH 180 does not have the traditional structure that they were accustomed to in both MATH 160 and previous mathematics courses.

These non-lecture techniques are not as familiar to the students, and, as the discussion with the focus group revealed, several students were resistive to these instructional methods, which can make teaching and learning in such an environment difficult. The qualitative results reinforce the possibility of passive student involvement as well as resistance to the instructional techniques. The student comments indicated a

resistance to both reading and writing, as well as frustration with not knowing how to approach problems.

Although students liked thinking about problems in different ways, they often had to be told the different ways in which to think about the problems. It seems apparent that they knew multiple approaches to problems existed, but they struggled with understanding what those approaches were as well as how to use them.

### **Research Question Three**

The third research question addressed the performance in the spring 2010 semester of MATH 160 of both MATH 180 students and MATH 160 repeaters. The MATH 160 repeaters had both a higher mean final exam grade and mean course letter grade (except when comparing final exam grades with MATH 180 A/B/C students). However, when MATH 180 students were compared with repeaters that had declined the invitation to MATH 180, the results changed.

Ten of the 51 MATH 160 repeaters had been identified in the fall 2009 semester as being at-risk of failing MATH 160 (based on exam one scores). These students declined the invitation to take MATH 180, and chose to remain in MATH 160, which they failed. When retaking the course in the spring, these students did not perform as well as the MATH 180 students.

Although the results were not statistically significant, the at-risk students that participated in MATH 180 performed better on the final exam and had a higher mean letter grade than the at-risk students that chose not to take MATH 180. This would indicate that if a student is identified as being at-risk (based on Math 160 exam one score), it would be to their benefit to take MATH 180.

#### **Research Question Four**

The fourth research question asked if there was an association between MIMBS scores and performance in MATH 160. The confidence construct of the MIMBS correlated most strongly with performance in MATH 160.

This raised the question: Can scores on the confidence and nature of mathematics constructs of MIMBS-A be used to predict success in MATH 160? The results of the statistical analysis for this question, which was a follow-up question to research question 4(d), was worthwhile. Statistical analysis showed that about seven percent of variance in MATH 160 final course grade could be predicted with scores on both the confidence and nature of mathematics construct of the MIMBS-A. However, these results should be interpreted carefully and with caution. As stated in chapter three, the reliability of the nature of mathematics construct was low with Cronbach's alpha value of 0.55. In addition,

However, the confidence construct was reliable, with Cronbach's alpha 0.76. With such a high percentage of students not succeeding in calculus, it could be beneficial to utilize the confidence construct of the MIMBS with a placement exam at the beginning of the semester, rather than one month after the start, in order to have an idea of the students that are at risk of failing MATH 160. If at-risk students could be identified before entering calculus, they could be given the opportunity to develop the critical thinking skills needed for success in calculus as well as other future mathematics classes.

#### **Research Question Five**

At the end of the fall 2009 semester, the scores on the MIMBS for MATH 180 A/B/C students were compared with all MATH 160 students as well as the group of

MATH 160 students that received a grade of D or F on exam one. Although the mean score was higher for the MATH 180 students, differences were not statistically significant. It should be noted that the sample sizes being compared were markedly different.

There were 16 MATH 180 students being compared with all 342 MATH 160 students as well as the group of 34 MATH 160 students that received a grade of D or F on exam one. Regardless of the differences in sample size, MATH 180 students exhibited a more positive attitude (i.e. higher mean MIMBS-B score) towards mathematics at the end of the semester than the MATH 160 students.

The qualitative data reinforce these results. Comments from some of the MATH 180 students indicated that perceived beliefs and attitudes towards mathematics changed, although the change was not significant. For example, several students mentioned that mathematics problems be approached in different ways. Although not all of the MATH 180 students developed the ability to follow through with multiple approaches to problems, students did recognize the possibility to solve mathematics problems in various ways.

### **Research Question Six**

MATH 180 students had a higher mean MIMBS-B score than MATH 160 students. However, this does not indicate the overall impact that MATH 180 had on students. Thus a comparison between MIMBS-A and MIMBS-B scores was necessary. The overall MIMBS scores for MATH 180 students went up. The differences in overall MIMBS scores as well as individual construct scores were not statistically significant for the MATH 180 students. However, when only MATH 180 A/B/C or A/B students were

included, statistical significance arose. Both MATH 180 A/B/C and A/B student had significantly higher scores on the nature of mathematics construct at the end of the semester.

One of the goals of MATH 180 was to get students to think differently about mathematics. It is not uncommon for students to believe that mathematics is procedural and all mathematics problems can be solved by following a pattern or formula. This gets students into trouble when they take calculus. They try to apply templates to problems in order to solve them, but this does not always work. The majority of problems on MATH 160 exams are not patterned from textbook problems. Students have to be able to think critically and have the ability to apply conceptual knowledge.

Attitudes are a function of beliefs (Ajzen, 2001; Fishbein & Ajzen, 1972; Fishbein & Middlestadt, 1995; Fishbein & Middlestadt, 1997). The MIMBS measures students perceived attitudes about mathematics, which, in turn, gives insight about their beliefs about mathematics. Ajzen (2001) asserts that in order to influence beliefs, attitudes must be changed. If MATH 180 encourages students to change their attitudes toward mathematics changes, then their beliefs about mathematics can be influenced.

As previously stated, it was clear from the discussion group that students did believe that mathematics problems could be approached in various ways. However, these students were still rigid in their thinking. This is clear from comments such as:

It would have helped if there were more equations, actual math problems, not word problems.

I feel like there is too much “English” (class) involved. This is a math class not a writing class.

I don’t really see how it’s going to prepare me for calculus.

Students had preconceived notions of the content of MATH 180 and the connection it should have with MATH 160. Thus it is of no surprise that their MIMBS scores changed little.

### **Research Question Seven**

The one-way ANOVA conducted in order to compare MIMBS-B scores between (a) students that received a grade of D or F in MATH 160 during the fall 2009 semester, (b) students that received a grade of A, B, or C in MATH 160 during the fall 2009 semester, and (c) students that enrolled in the MATH 180 during the fall 2009 semester. Although the MATH 160 A/B/C students had the highest mean MIMBS-B score, the MATH 180 A/B/C students still had a higher mean MIMBS-B score than the MATH 160 D/F students. This indicates that students in MATH 180 course had a more positive attitude about mathematics than the MATH 160 D/F students.

It would seem that, for students at risk of failing MATH 160, MATH 180 has the potential to improve their attitudes, and thus possibly beliefs as well, about mathematics. Studies have been conducted that tie attitude about mathematics to performance (House, 1995; Pettersson & Scheja, 2008). House (1993, 1995) has found statistically significant data to support the relationship between attitude and academic achievement. MATH 180 has the potential to improve attitude toward mathematics for students at risk of failing MATH 160, which could then improve their chance for success in MATH 160.

### **Research Question Eight: Qualitative Results**

The statements that students made during the interviews and focus group discussion made several things clear. For one, they were missing the connection between MATH 180 and MATH 160 even though the purpose and content of MATH 180 was

stated in their syllabus. There was a common misunderstanding of the course content of MATH 180 and how it related to MATH 160. Broadly stated, MATH 160 involves studying functions, and MATH 180 content involved reexamining “elementary functions from a more advanced standpoint” (Klopfenstein, 2009b, p. 1). For some reason the connection between the two courses was unclear to the students.

Perhaps the students did not recognize that one must have a good understanding of functions in order to do well in calculus. On the other hand, they may have felt that understanding functions was not important in calculus. Regardless, students did not see how MATH 180 linked to MATH 160.

After all, this group of students had been exposed to nearly a month of MATH 160. This enabled students to develop assumptions of what they believed calculus to be as well as expectations of what MATH 180 should do for them. This may have caused some students to be resistant to MATH 180 and the opportunity it provided.

Second, it was clear that not all of the students felt comfortable working on their own. A majority of the students felt the need to rely on their peers in order to solve problems. A heavy reliance on peers for the correct answer can be detrimental in MATH 160, as students need to be able to think critically (on their own) about calculus concepts. MATH 160 exams do not consist of template problems and are completed individually. Students must have a good understanding of calculus concepts and have the ability to apply them to problems, often in an imaginative way.

### **Conclusion, Suggestions for Future Research, and Cautions**

Drawing conclusions about the success of MATH 180 must be done with great caution. Twenty-two students completed the course, and just over half ( $N = 12$ ) continued

to MATH 160. It is unknown what happened to the ten students that did not continue on to MATH 160. The sample size is well below the recommended 30 (Gliner, et al., 2009).

Although we cannot draw conclusions about the population of students that are at-risk of failing MATH 160, the qualitative results of this study as well as personal experiences of teaching MATH 160 indicate that, in general, students do not understand the purpose of calculus. The students that are at risk of failing the course have even less of an understanding of the nature of mathematics, let alone, calculus. The majority of students believe that mathematics is a series of formulas and procedures. This idea was clear from multiple statements made by MATH 180 students. One specific statement causes great concern:

I don't like how we have to think about our work. It seems like this is not the best style of learning for me.

Students that enroll in MATH 160 are students majoring in fields such as engineering, physics, and mathematics. These are not fields in which one can solve problems using templates. Buildings and bridges are not constructed in this manner. Advances in physics and mathematics do not occur through passive learning. To excel in these fields one must think creatively and critically. One must have an understanding of foundational concepts and have the ability to apply them. However, in general, the MATH 180 students do not understand the importance of understanding concepts. They have preconceived ideas of what MATH 160 should be and this does not involve having an understanding of mathematical concepts.

Perhaps more time is needed for students to let go of their misconceptions about MATH 160 and MATH 180. Ten weeks may not be enough time to significantly change

students' attitudes about mathematics. The exposure to MATH 160 for four weeks may be causing more problems with these at-risk students than if they had not been exposed to the course at all.

One suggestion for a future study is to use the confidence and nature of mathematics constructs of the MIMBS together with a mathematics placement exam as a way to identify students at risk of failing MATH 160 prior to them entering the class. Although students may meet the prerequisites for MATH 160, if they are identified as being at-risk of failing MATH 160, they could be given the recommended opportunity to take MATH 180. However, caution should be taken when using constructs of the MIMBS that have low reliability. Only the confidence construct had a high reliability and correlation with success in MATH 160.

In addition, future studies should look specifically at the population of students that are identified as being at-risk. Within this population, when MATH 180 students were compared with non-MATH 180 students that were repeating MATH 160, MATH 180 students performed better. Both mean final exam scores and final letter grades were higher for the MATH 180 students.

Although both qualitative and quantitative data indicate that the MATH 180 students still had misconceptions and were lacking in conceptual understanding, these students were still in a better position academically than those that chose not to take MATH 180.

### **Implications and Revisions of MATH 180**

MATH 180 ran as an experimental course for the first time during the fall 2009 semester. The enrollment during this initial offering of the course was 27 with 22

completing the course. MATH 180 was approved to run again during the spring 2010 semester. Based on the pilot run of the course, much of the content was changed with the hope to better address the needs of students at-risk of failing MATH 160.

The spring 2010 semester enrolled 40 students. During the spring 2010 semester, the course coordinator received news that the CSU Curriculum Committee wanted to see MATH 180 become a regular course. More changes to the course content are planned for the fall 2010 semester. The course is constantly evolving as new issues come to light.

Improving student achievement in calculus is a daunting task. High failure rates in college calculus have been a problem for decades. Clearly it is not an easy problem to solve, as no one has come up with a solution that can be applied or generalized to every post-secondary institution.

MATH 180 is a creative and unique approach to an old problem. The key is gaining an understanding of at-risk students and finding a way to get them to actively engage with mathematical material. Literature supports the idea that the constructivist classroom fosters learning. However, if the student refuses to engage, there is not much that can be done. Hopefully, as it evolves, MATH 180 will become a course in which students let go of their preconceived ideas about mathematics, become more active learners, and, in turn, succeed in calculus, a course they might otherwise had failed.

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## APPENDIX A: MATH 180 MATERIALS

### MATH 180 Syllabus

MATH 180 sec 1, 2 – 2:50 pm, MTWF, Engineering E103

Fall 2009

#### *MATH 180 Concepts for Calculus Course Policies and Procedures*

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**Why MATH 180 Concepts for Calculus?** Over the past several semester on average about 60% of the students who start MATH 160 finish the course with an A, B, or C. In other words, 40% of the students who start MATH 160 typically do not finish the course successfully. We hope *Concepts for Calculus* will change that.

Teachers often say that the main reasons students have difficulty with calculus are that their algebra skills are weak and they perceive mathematics as made up of rules to memorize and procedures to follow. The on-line component of MATH 180 will help students improve their algebra and pre-calculus skills. The classroom component is designed to help students transition from a mechanical to a conceptual understanding of mathematics. Students who successfully complete MATH 180 will be prepared to not just succeed, but to excel, in MATH 160.

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**Instructor:** Prof. Ken Klopfenstein    **Office:** Weber 116

**E-mail:** [kenk@math.colostate.edu](mailto:kenk@math.colostate.edu)    **Phone:** 491-6573

**Registration:** Written consent of the instructor is required to add or drop this course.

**Course description:** Intensive review of and practice with algebra and other precalculus skills (ALEKS).

Re-examination of selected topics in pre-calculus from a more conceptual point of view.

**Course Format and Expectations:** This 4-credit course is a blend of on-line and classroom instruction: 25% of the instruction is on-line; 75% is classroom based. To meet the requirements of the on-line portion of MATH 180, you must spend at least 75 minutes each week engaged with the on-line instructional program ALEKS. The part of your grade based on the on-line component will be determined from (i) the scores you earn on the on-line assessments and (ii) the amount of time you spend working on the on-line material.

To meet the requirements of the classroom component of MATH 180, you must attend all class meetings, participate actively and constructively in class discussion and activities, complete the outside reading and homework, and demonstrate competence on the written

midterm and final examinations. Expect to study at least 2 hours outside of class for each hour in class to meet these requirements.

MATH 180 will be a demanding course – at least as demanding as MATH 160. Expect to devote 13 – 16 hours to this course every week (4 hours in class, 8 – 10 hours on homework and reading, 1¼ hours on ALEKS).

**Textbook and Course Materials:**

On-line component: *Preparation for Calculus*, delivered on-line by ALEKS. Instructions for subscribing to ALEKS will be distributed.

Classroom component: None (Notes will be distributed in class and/or on line.)

**Course goals:** Concepts for Calculus is about *doing* mathematics. Doing mathematics is much more than solving textbook exercises. Doing mathematics is about asking questions, understanding concepts, coming up with ideas, and using the concepts and ideas to answer the questions and solve problems. The goals of this course are for you to

- become proficient and confident with fundamental pre-calculus skills;
- draw connections among different representations of functions and choose a representation appropriate to the situation;
- understand that there are reasons why mathematics “works” and appreciate the importance of understanding why;
- be able to clearly explain your understanding of mathematical concepts orally and in writing; and
- be able to read and interpret mathematical exposition.

**ALEKS:** The ALEKS component of MATH 180 will help you firm up the pre-calculus skills you must build on to learn calculus. ALEKS is a web-based, artificially intelligent assessment and learning system. ALEKS uses adaptive questioning to quickly and accurately determine what you know and don’t know. ALEKS then instructs you on the topics you are most ready to learn. ALEKS provides one-on-one instruction, 24/7, from virtually any web-based computer.

Subscribe to ALEKS directly from ALEKS Corporation at their web site. The information you will need to register will be issued in class. The subscription is expected to cost about \$30.

Your work on ALEKS will count 25% of your final grade. To meet the requirements of the ALEKS portion of MATH 180, you must spend at least 75 minutes each week working on ALEKS. The part of your grade based on ALEKS will be determined from (i) the scores you earn on the ALEKS assessments and (ii) the amount of time you spend working on ALEKS.

**Written Homework:** Homework requiring written responses that show understanding and thoughtful analysis will be assigned frequently and due on specified dates. Many of these assignments are to be done in collaboration with other members of the class. Some are to be done individually. Assignments will be accepted late *only in the case of absence because of participation in official university activities, documentable illness, or*

*other extenuating circumstances.* Class participation and written homework will count 25% of your final grade.

**Midterm Exam:** A midterm exam will be given Wednesday, October 28, 5:15 – 7:00 PM. Students who have an unavoidable, documentable time conflict with the evening mid-term exam will be allowed to take the exam at another mutually convenient time. Your score on the midterm exam will count 25% of your final grade.

**Final Exam:** The final exam will be given Tuesday, Dec 15, 9:10 AM. Attendance at the final exam is required. Don't expect to take the final early or late! If you have three or more final exams on the same day you may negotiate a time change with the instructors involved. If the parties involved cannot find a mutually agreeable time, the Registrar's Office indicates which exams must be rescheduled. If you have three exams on the same day, talk with instructors involved at least 4 weeks in advance.

Midterm and final exam questions will emphasize understanding concepts, thoughtful analysis, and clear writing. However, you may have to use algebra and other precalculus skills to analyze a problem and show your understanding. Your score on the final exam will count 25% of your final grade.

**Grading:** The 400 points possible in this course are calculated as follows:

$$\begin{aligned} \text{Point Total} = & \text{ALEKS (100 pts) + Class participation and homework (100 pts)} \\ & + \text{Mid-term exam (100 pts) + Final exam (100 pts)} \end{aligned}$$

Your final grade will be determined from your Point Total using a grading scale no more restrictive than the following:

|                 |           |   |               |           |   |
|-----------------|-----------|---|---------------|-----------|---|
| 90% – 100% .... | 360 – 400 | A | 55% – 60%     | 220 – 239 | D |
| 80% – 89% ..... | 320 – 359 | B | less than 55% | 0 – 219   | F |
| 60% – 79% ..... | 240 – 319 | C |               |           |   |

Plus/minus grades will be assigned only in exceptional situations. A grade of incomplete (I) will be assigned only in extenuating circumstances (beyond the student's control and could not reasonably have been anticipated or avoided) and with approval of the Mathematics Department Undergraduate Director.

**Academic Appeals:** Concerns about the course or any of your instructor's decisions that affect your success in the course should first be discussed with the instructor. Issues that cannot be resolved with the instructor should be discussed with Prof. Gerhard Dangelmayr, Undergraduate Director. To see Prof. Dangelmayr, make an appointment in the Math Dept. Office (Weber 101). The University Policy on Appeals of Academic Decisions, including grade appeals, is published under "Student Rights and Responsibilities" in the current CSU General Catalog.

**Policy on Academic Honesty:** The University Policy on Academic Integrity (see CSU General Catalog) is enforced in this course. Misrepresenting someone else's work as your own (plagiarism) and possessing unauthorized reference information in any form that could be helpful while taking an exam are examples of cheating. Submitting work

from a web site or other source as your own is an example of plagiarism. Students judged to have engaged in cheating may be assigned a reduced or failing grade for the assignment or the course and may be referred to the Office of Conflict Resolution & Student Conduct Services for additional disciplinary action.

### **MATH 180 Intended Weekly Schedule**

#### ***MATH 180 Tentative Topic Outline & Schedule***

##### ***Fall Semester, 2009***

|                     |  |
|---------------------|--|
| Week 19/21 – 9/25   | Multiple views of functions  |
| Week 29/28 – 10/02  | The function concept and equality of functions   |
| Week 310/05–10/09   | Mathematical models, formulating mathematical questions                                      |
| Week 410/12 – 10/16 | Writing polynomials in various forms for various purposes                                    |
| Week 510/19 – 10/23 | Zeros and factors of polynomials; solving polynomial equations algebraically and numerically |
| Week 610/26 – 10/30 | Numbers and sequences<br>Wednesday 10/28, 5:15 – 7:00 PM Midterm exam. Location tba          |
| Week 711/02– 11/06  | Power functions, exponential functions, and inverses   |
| Week 811/09 – 11/13 | The pavement art problem - posed   |
| Week 911/16 – 11/20 | Geometry of circles and angles   |
| 11/21 – 11/29       | <b>Thanksgiving Break</b>  |
| Week 101/30 – 12/04 | The pavement art problem – solved<br>Connecting with trigonometry                            |
| Week 11/07 – 12/11  | Tabulating the chord function  |
| Week 122/14 – 12/18 | Final Exam Week<br>Final Exam in MATH 180 is Tuesday, Dec 15, 9:10 AM.                       |

### *Classroom Expectations and Common Courtesies*

1. Come to every class. Arrive at class on time. Stay through the end of the class hour.
2. Be constructively involved and engaged in class. Listen actively. Be ready and willing to contribute to class discussion. Ask questions. Be ready to respond to questions – even if your response is “I don’t know; let me think about that a minute.”
3. Get to know your classmates in social conversations before class. Avoid social conversations during class.
4. **Turn off your cell phone.**
5. Don’t read the paper, do homework, solve SODOKU puzzles, play games on your calculator, surf the web, or listen to your i-pod in class.
6. Be a responsible, positive, contributing member of your collaborative group, both in and outside of class.
7. Always have pencil and paper ready in class – even if you don’t take notes.
8. Always bring your calculator to class – and have it available.

## MATH 180 Sample Group Activity

MATH 180 Concepts for Calculus

Fall Semester, 2009

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### Topic 3: Functions from Narratives; Mathematical Models

Functions are useful because they model the world as we experience it. But where do functions that model a situation come from? Sometimes they come from describing physical or geometric relationships symbolically/mathematically.

#### Advance Preparation/Homework (or in-class collaborative exercise):

Situation 0: A sector is cut from a disk.

1. The words “sector” and “disk” have very specific meanings in mathematics. What are meanings of the words “sector” and “disk” in the context of mathematics?  
(If you are not sure, it’s OK to look these words up. Try [www.dictionary.com](http://www.dictionary.com).)
2. Write three meaningful “non-mathematical” questions one could ask in the situation where a sector is cut from a disk.  
(Possible answers: What color is the disk? What is the disk made of? How is the disk being cut – scissors, knife, torch, or ... ? As the disk is cut, are OSHA requirements being met? Is something useful being made by cutting a sector from the disk?)
3. Write three questions one could ask about the result of cutting a sector from a disk that could be addressed using mathematics.
4. For each mathematical question, give/describe a function that could be used to address the question. In each case, what is the independent variable in the function? What is the dependent variable? Explain how the function might be used to address the question.
5. Sketch a qualitatively accurate graph of each function you describe in #4.

Goals: Critical reading, vocabulary, reading comprehension, translating narrative into more mathematical description using functions. Realizing the limitations of descriptions based on idealizing assumptions.

Situation 1: Helium is being pumped at a constant rate into a spherical weather balloon.

1. Write three meaningful mathematical questions one could ask about the situation presented by inflating the balloon.
2. For each of these mathematical questions, describe a function that could be used to address the question. Identify the independent and dependent variables.
3. Sketch a qualitatively accurate graph of each function.
4. If possible, find an expression/equation for each function.
5. Graph each function from its equation/expression.  
Are these graphs consistent with the graphs sketched in response to #3?

6. Use the expressions for the functions to answer the questions posed in #2.

Situation 2: You are watching a weather balloon rise at a constant rate straight up (no wind) from a point some distance across a level plain from the point where the balloon was launched.

1. Write three meaningful mathematical questions one could ask about the situation presented by the rising balloon.
2. For each of these mathematical questions, describe a function that could be used to address the question. Identify the independent and dependent variables.
3. Sketch a qualitatively accurate graph of each function.
4. If possible, find an expression/equation for each function.
5. Graph each function from its equation/expression.  
Are these graphs consistent with the graphs sketched in response to #3?
6. Use the expressions for the functions to answer the questions posed in #2.

Situation 3: Sand falls from a conveyor belt at a constant rate onto the top of a conical pile. The height of the pile is always the same fraction of the diameter of the pile.

1. Write several meaningful mathematical questions that involve the volume of the pile of sand.
2. Describe functions that could be used to address the questions you posed.
3. Without finding equations/expressions for these functions, sketch qualitatively accurate graphs of these functions.
4. Find equations/expressions for the functions you created in #3.
5. Use the functions you created to answer the questions you posed.

Situation 4: To be sent through the US mail, the sum of the length and girth of a package may be at most 108 inches.

1. Write several meaningful mathematical questions that involve mailing a package.
2. Describe functions that could be used to address the questions you posed.
3. Without finding equations/expressions for these functions, sketch qualitatively accurate graphs.
4. Find equations/expressions for the functions you created in #3.
5. Use the functions you created to answer the questions you posed.

Situation 5: A rectangular sheet of paper is placed on a flat surface. One corner is placed on the longer opposite edge and held there as the paper is folded flat.

1. Write three meaningful mathematical questions that involve folding a sheet of paper in this way.
2. Describe functions that could be used to address the questions you posed.
3. Without finding equations/expressions for these functions, sketch qualitatively accurate graphs.
4. Find equations/expressions for the functions you created in #3.
5. Use the functions you created to answer the questions you posed.

Situation 6: Jane is training for a multisport endurance race. She is in her kayak 2 miles off the shore of the lake. Her beach house is 5 miles down the straight coastline. She can paddle 6 miles per hour and run 10 miles per hour. It's late in the day, and Jane wants to get home as quickly as possible.

1. Describe (in words!) a function that could be used to address Jane's problem. What are the independent and dependent variables in the function you created? What is the domain of your function?
2. Without finding an equation, sketch qualitatively accurate graph of the function.
3. Find an equation or expression for the function you described in #1 and graphed in #2. Use the equation you found to graph this function. Is the graph consistent with the qualitative graph you sketched in #2?
4. Use the function you created in #3 to solve Jane's problem.
5. Write two other questions that could be answered using the function you created.
6. Use the function you created in #3 to answer the questions you posed in #5.

Situation 7: The fly and the train-wreck problem? (Omit or include only in text material?)

What is a model? Dictionary definitions. Types of models.

Molecular model

Atmospheric model; weather model

Architectural model

Engineering model (Discovermagazine.com, July/August 2009 issue, pages 6-7)

Artist's model

Working model

Computer model (aka simulation)

You constructed mathematical models of the situations described in 1 - 6 above. A mathematical model is a description given in mathematical terms (often, but not always, a function) that represents *elements of the situation that are important for a particular purpose*. Elements of the situation that are thought to be less important (or beyond our methods of analysis) are not accounted for in the model.

A mathematical model is different from the types of models listed above, but serves much the same purpose. A mathematical model can help us understand a situation, make predictions about it, answer questions about it, and, sometimes, control or influence the situation.

There are several commonly used ways to present mathematical models:

- narratively, as a description in words
- tabularly, as a table of numbers
- geometrically, as a drawing
- graphically, in an x-y coordinate system
- symbolically, using mathematical expressions and functions

1. What kind of model was used in the Elastic Band Problem? What elements of the situation are left out of the model? What idealizing assumptions were made in the model?
2. What kind of model was used in the Sliding Ladder Problem? What idealizing assumptions were made in this model?
3. What kind of model did you construct in the situations assigned? Idealizing assumptions?

### **Dimensions and Units**

#### Dimension analysis

Use dimension analysis and estimation to recognize that some functions that MATH 160 students commonly construct to model situations 1 – 6 above are incorrect.

**MATH 180 Exams**

**MATH 180 Midterm Exam**

MATH 180 Concepts for Calculus

Name: \_\_\_\_\_

October 28, 2009 5:15 – 7:00 PM

**Midterm Exam**

Instructions:

Write a complete response to each problem in the space following the problem. Clearly label each part. If you need more space, ask for more paper. If you use additional pages for your response to a problem, clearly write the name of the problem and your name at the top of each page. Write the work for no more than one problem on each page.

Do not hesitate to ask questions during the exam.

When you have finished, be sure your name is on every page of your work, put all the pages in order, and staple the pages together.

**CONFIDENTIALITY PLEDGE**

I agree that I will not share any information, either specific or general, about the problems on this examination with any other student until authorized to do so.

\_\_\_\_\_  
(Signature)

\_\_\_\_\_  
(Date)

MATH 180 Concepts for Calculus

Name: \_\_\_\_\_

**1. The heated metal rod problem**

The composition of a certain straight metal rod differs along its length. As a result, different parts of the rod conduct heat at different rates and different parts expand or contract at different rates as the rod is heated or cooled. Imagine that the left end of the rod is cooled and the right end is heated.

- (a) Must there necessarily be a point somewhere along the rod where the temperature remains unchanged? Give a mathematical argument to support your conclusion. (Hint: Use a function.)
- (b) Parts of the rod that are cooled contract (and become shorter). Parts that are heated expand (and become longer). Must there necessarily be a point somewhere along the rod that remains in its original position? Give a mathematical argument to support your conclusion.

**2. A function defined geometrically**

Draw a square with sides of length 1. With each point P on the square associate the distance between P and the nearest corner of the square.

- (a) Explain why this association can be thought of as a function.  
What is the independent variable? What is the dependent variable?
- (b) Explain why it is **impossible** to sketch a graph in a standard x-y coordinate system that represents the function in (a).
- (c) Describe the same association between points on the square and distances to a corner in another way that **can be represented** by a graph in a standard x-y coordinate system. Explain why this association is also a function.
- (d) Sketch the graph of the association (or function) you described in (c).
- (e) Are the functions in (a) and (c) equal? Explain why or why not.

**3. The open-topped box problem**

Squares of the same size are cut out of the corners of a 11" by 14" rectangular sheet of heavy paper. The sides are folded up and edges taped together to form an open-topped box as illustrated in the drawings below.

- (a) What is a natural, reasonable, or best choice of independent variable in this situation? Explain why this is a good choice. (Yes, you have to write!)
- (b) What is a natural, reasonable, or best choice of dependent variable in this situation? Explain why this is a good choice. (What are boxes used for?)
- (c) Explain why the association between the independent and dependent variables you identified in (a) and (b) can be thought of as a function. What is the domain of this function?
- (d) Sketch a qualitatively accurate graph of the function that relates the independent and dependent variables you identified. Label the units on the axes.
- (e) What simplifying assumptions are given in the description of the situation or have you made to construct this mathematical model of open topped boxes? Explain why these assumptions are reasonable.
- (f) Find an explicit equation for the function that relates the independent and dependent variables you specified in (a) and (b) above.
- (g) Formulate at least two interesting questions you could answer by using the function that relates the independent and dependent variables you identified.
- (h) Use the function you created in (f) to answer one of the questions you posed in (g).

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**3. The open-topped box problem (page 2)**

**4. A cartoon fantasy problem**

Suppose two trains are 60 miles apart traveling toward each other on the same track. The westbound train is traveling 45 miles per hour. The eastbound train is traveling 75 miles per hour. The engineers are not aware that they are headed toward a disastrous head-on collision.

Mighty Mouse (the cartoon hero) tries to get the engineers' attention and prevent the collision by flying at 100 miles per hour back and forth between the two engines. Being a cartoon character, he doesn't have to slow down when he reaches one engine and turns around to fly back to the other, so he's always flying at exactly 100 miles per hour. In spite of Mighty Mouse's desperate efforts, the trains collide in a shower of sparks and hot metal. (Imagine the YouTube video!)

- (a) Draw the graph of a function that relates time and the distance between the trains. Clearly show the units on each axis.
- (b) Draw the graph of a function that relates time and the distance Mighty Mouse has flown. Clearly show the units on each axis.
- (c) Use the graphs you created in (a) and (b) to determine how far Mighty Mouse flew in his ill-fated effort to save the trains. Explain clearly how you used the graphs to determine how far Mighty Mouse flew.

# MATH 180 Final Exam

MATH 180 Concepts for Calculus

Name: \_\_\_\_\_

December 15, 2009 9:10 AM – 1:10 PM

## Final Exam

Instructions:

Write a complete response to each problem in the space following the problem. Clearly label each part. If you need more space, ask for more paper. If you use additional pages for your response to a problem, clearly write the number of the problem and your name at the top of each page. Write the work for no more than one problem on each page.

Do not hesitate to ask questions during the exam.

When you have finished, be sure your name is on every page of your work, put all the pages in order, and staple the pages together.

### CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any student who has not taken the exam before Friday, December 18.

\_\_\_\_\_  
(Signature)

\_\_\_\_\_  
(Date)

|                 |  |  |
|-----------------|--|--|
| 1. (15 pts)     |  |  |
| 2. (35 pts)     |  |  |
| 3. (20 pts)     |  |  |
| 4. (30 pts)     |  |  |
| 5. (45 pts)     |  |  |
| TOTAL (145 pts) |  |  |

(total will be scaled to 100 points)

1. Aloysius claims that the functions  $f(x) = \frac{2x+3}{(x+1)^2} - \frac{5}{(x+1)} + \frac{3}{(3x-2)}$  and  $g(x) =$

$$\frac{7+6x-6x^2}{(3x^3+4x^2-x-2)}$$

are equal.

(a) (5 points) What does Al mean when he says these two functions are equal?

(b) (10 points) Is Al right? Are these two functions equal?

Give convincing mathematical reasons for your answer.

2. (a) (5 points) Explain why one might want to write polynomials in different forms.
- (b) (7 points) Describe (and name) at least four different forms in which polynomials are written.
- (c) (10 points) Write the polynomial  $p(x) = x^3 + x^2 - 5x - 3$  in Taylor form around the point  $x = 1$ .
- (d) (8 points) Determine whether the part of the graph of  $p(x) = x^3 + x^2 - 5x - 3$  in some short interval around  $x = 1$  is exactly a parabola. Show/explain in detail how to see this from your work.
- (e) (5 points) What would you expect to see in the Taylor form for a fourth degree polynomial  $q(x)$  around a point where the function has its largest value? Why?

3. **Another Elastic Band Problem:** An elastic band of varying width (and, therefore, varying “stretchiness”) is stretched out flat in a straight line on a tabletop. The ends of the band are slowly released so there is no tension in the band and it lies unstretched in a straight line along the tabletop. Must there necessarily be some point of the unstretched band that is in the same position as it was in the stretched band?
- (a) (5 points) What mathematical ideas or tools do you have to think about the question?
  - (b) (5 points) Use one or more of these tools to formulate the question in mathematical terms.
  - (c) (10 points) Use your mathematical formulation of the question to find and justify an answer.

4. **The Canoe on the Lake Problem** The drawing shows a map of Lake Wanabedun. A canoe is at the point on the lake labeled C.
- (a) (5 points) With each direction  $d$  associate the point on the lake shore in direction  $d$  from the canoe.
    - (i) Explain why this association is a function.
    - (ii) Explain why this function *cannot* be graphed in a standard  $x$ - $y$  coordinate system.
  - (b) (5 points) With each direction  $d$  associate the distance in direction  $d$  from the canoe to the lake shore. (i) Explain why this association is also a function. (ii) This function models the same situation as the function in (a), and can be graphed in a standard  $x$ - $y$  coordinate system. Explain why it can be graphed in a standard  $x$ - $y$  coordinate system.
  - (c) (5 points) If a point on the lake shore is chosen randomly, must there *necessarily* be another point on the shore that is the same distance from the canoe? Give convincing mathematical reasons for your answer.
  - (d) (5 points) Must there necessarily be two (or more) points on the lake shore that are the same distance from the canoe? Give convincing mathematical reasons for your answer.
  - (e) (10 points) Must there necessarily be two points on the lake shore that are the same distance from the canoe and in directly opposite directions from the canoe? Give convincing mathematical reasons for your answer.

5. **The Two Towers Problem** Two radio towers are 100 feet apart. One of the towers is 60 feet high and the other is 180 feet high. The towers are to be stabilized by two cables (called guy-wires) attached at the tops of the towers and anchored to the ground at the same point between the towers as shown in the figure.
- (a) (5 points) What is a natural, reasonable, or best choice of an independent variable in this situation?  
Explain why this is a good choice.
  - (b) (5 points) What is a natural, reasonable, or best choice of a dependent variable in this situation?  
Explain why this is a good choice.
  - (c) (5 points) Explain why the association between the independent and dependent variables you identified in (a) and (b) can be thought of as a function. What is the domain of this function? (In other words, what values can the independent variable have?)
  - (d) (5 points) Sketch a qualitatively accurate graph of the function that relates the independent and dependent variables you identified in (a) and (b). Clearly indicate the domain of the function on one of the axes.
  - (e) (5 points) What simplifying assumptions are given in the description of the situation or have you made to construct the function that models this situation?
  - (f) (5 points) Find an explicit equation for the function you identified in (c) and graphed in (d).
  - (g) (7 points) Formulate at least two different interesting questions you could answer by using the function that you identified in (c) and graphed in (d).
  - (h) (8 points) Use this function you created in (f) to answer one of the questions you posed in (g). Show/explain clearly how you used the function to answer the question. (Maximum of 4 points for answering an uninteresting question correctly. Up to 8 points for answering a more interesting question correctly.)

## APPENDIX B: IMBS AND MUS

### Indiana Mathematics Belief Scales and Mathematics Usefulness Scale

Belief 1 (Student's Self Confidence): *I can solve time-consuming mathematics problems.*

- + Math problems that take a long time don't bother me.
- + I feel I can do math problems that take a long time to complete.
- + I find I can do hard math problems if I just hang in there.
- If I can't do a math problem in a few minutes, I probably can't do it at all.
- If I can't solve a math problem quickly, I quit trying.
- I'm not very good at solving math problems that take a while to figure out.

Belief 2 (The Nature of Mathematics): *There are word problems that cannot be solved with simple, step-by-step procedures.*

- + There are word problems that just can't be solved by following a predetermined sequence of steps.
- + Word problems can be solved without remembering formulas.
- + Memorizing steps is not that useful for learning to solve word problems.
- Any word problem can be solved if you know the right steps to follow.
- Most word problems can be solved by using the correct step-by-step procedure..
- Learning to do word problems is mostly a matter of memorizing the right steps to follow.

Belief 3: *Understanding concepts is important in mathematics.*

- + Time used to investigate why a solution to a math problem works is time well spent.
- + A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem.
- + In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.
- It's not important to understand why a mathematical procedure works as long as it gives the correct answer.
- Getting a right answer in math is more important than understanding why the answer works.
- It doesn't really matter if you understand a math problem, if you can get the right answer.

Belief 4: *Word problems are important in mathematics.*

- + A person who can't solve word problems can't really do math.
- + Computational skills are of little value if you can't use them to solve word

- problems.
- + Computational skills are useless if you can't apply them to real life situations.
  - Learning computational skills is more important than learning to solve word problems.
  - Math classes should not emphasize word problems.
  - Word problems are not a very important part of mathematics.

Belief 5: *Effort can increase mathematical ability.*

- + By trying hard, one can become smarter in math.
- + Working can improve one's ability in mathematics.
- + I can get smarter in math by trying hard.
- + Ability in math increases when one studies hard.
- + Hard work can increase one's ability to do math.
- + I can get smarter in math if I try hard.

This is the Mathematics Usefulness Scale as modified from the Fennema-Sherman (1976)

Mathematics usefulness scale by Kloosterman and Stage (1992):

Belief 6 (Relevance): *Mathematics is useful in daily life.*

- + I study mathematics because I know how useful it is.
- + Knowing mathematics will help me earn a living.
- + Mathematics is a worthwhile and necessary subject.
- Mathematics will not be important to me in my life's work.
- Mathematics is of no relevance to my life.
- Studying mathematics is a waste of time.

## APPENDIX C: MIMBS

### Modified Indiana Mathematics Belief Scales

Belief 1 (Student's Self Confidence): *I can solve time-consuming mathematics problems.*

- + Math problems that take a long time don't bother me.
- + I feel I can do math problems that take a long time to complete.
- + I find I can do hard math problems if I just hang in there.
- If I can't do a math problem in a few minutes, I probably can't do it at all.
- If I can't solve a math problem quickly, I quit trying.
- I'm not very good at solving math problems that take a while to figure out.

Belief 2 (The Nature of Mathematics): *Mathematics problems are solved by identifying and applying the correct procedure.*

- + Learning to solve math problems is mostly a matter of memorizing the right steps to follow.
- + Most math problems are easy to solve once you figure out what type of problem they are.
- + Any math problem can be solved if you know the right steps to follow.
- Many math problems cannot be solved by following a predetermined sequence of steps.
- Some math problems aren't like any of the common types of problems.
- There is no procedure to solve many math problems.

Belief 3: *Understanding concepts is important in mathematics.*

- + Time used to investigate why a solution to a math problem works is time well spent.
- + A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem.
- + In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.
- It's not important to understand why a mathematical procedure works as long as it gives the correct answer.
- Getting a right answer in math is more important than understanding why the answer works.

- It doesn't really matter if you understand a math problem, if you can get the right answer.

Belief 4: *Mathematics problems have a single, correct answer.*

- + Doing math is about finding the right answer to a problem.
- + Math problems always have one right answer.
- + A question that must be answered in writing is not a math problem.
- Math problems can have more than one right answer.
- A math problem can often be solved correctly in several different ways.
- Which answer to a math problem is correct (or best) depends on how the answer is going to be used.

Belief 5: *Effort can increase mathematical ability.*

- + By trying hard, one can become smarter in math.
- + Working can improve one's ability in mathematics.
- + I can get smarter in math by trying hard.
- + Ability in math increases when one studies hard.
- + Hard work can increase one's ability to do math.
- + I can get smarter in math if I try hard.

Belief 6 (Relevance): *Mathematics is useful in daily life.*

- + I study mathematics because I know how useful it is.
- + Knowing mathematics will help me earn a living.
- + Mathematics is a worthwhile and necessary subject.
- Mathematics will not be important to me in my life's work.
- Mathematics is of no relevance to my life.
- Studying mathematics is a waste of time.

## Administered MIMBS

Name: \_\_\_\_\_

This questionnaire is being used for a research study. The purpose of the study is to examine factors that influence student success in MATH160. This questionnaire is voluntary. You may skip any questions that you do not wish to answer.

If you hand in a completed questionnaire, it will count as one homework score.

Please indicate your level of disagreement or agreement with each of the following statements by circling the appropriate response. After you have responded to all the items, please record your response on the Scantron sheet provided. Write your name on both this sheet and the Scantron sheet and hand both in to your instructor.

1. I find I can do hard math problems if I just hang in there.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

2. A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

3. Studying mathematics is a waste of time.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

4. Ability in math increases when one studies hard.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

5. Math problems can have more than one right answer.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

6. I feel I can do math problems that take a long time to complete.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

7. Learning to solve math problems is mostly a matter of memorizing the right steps to follow.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

8. I can get smarter in math if I try hard.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

9. Knowing mathematics will help me earn a living.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

10. Doing math is about finding the right answer to a problem.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

11. I can get smarter in math by trying hard.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

12. Mathematics is of no relevance to my life.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

13. Many math problems cannot be solved by following a predetermined sequence of steps.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

14. Math problems that take a long time don't bother me.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

15. Most math problems are easy to solve once you figure out what type of problem they are.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

16. Hard work can increase one's ability to do math.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

17. If I can't solve a math problem quickly, I quit trying.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

18. Mathematics is a worthwhile and necessary subject.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

19. It doesn't really matter if you understand a math problem, if you can get the right answer.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

20. I'm not very good at solving math problems that take a while to figure out.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

21. Any math problem can be solved if you know the right steps to follow.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

22. Working can improve one's ability in mathematics.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

23. Mathematics will not be important to me in my life's work.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

24. If I can't do a math problem in a few minutes, I probably can't do it at all.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

25. Getting a right answer in math is more important than understanding why the answer works.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

26. I study mathematics because I know how useful it is.

|                   |          |           |       |                |
|-------------------|----------|-----------|-------|----------------|
| Strongly Disagree | Disagree | Uncertain | Agree | Strongly Agree |
| A                 | B        | C         | D     | E              |

27. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

28. By trying hard, one can become smarter in math.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

29. Some math problems aren't like any of the common types of problems.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

30. A question that must be answered in writing is not a math problem.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

31. Which answer to a math problem is correct (or best) depends on how the answer is going to be used.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

32. There is no procedure to solve many math problems.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

33. It's not important to understand why a mathematical procedure works as long as it gives the correct answer.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

34. A math problem can often be solved correctly in several different ways.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

35. Time used to investigate why a solution to a math problem works is time well spent.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

36. Math problems always have one right answer.

Strongly Disagree    Disagree    Uncertain    Agree    Strongly Agree  
A                            B                            C                            D                            E

37. What is your major?

Mathematics    Engineering    Physics    Chemistry    Other  
A                            B                            C                            D                            E

38. What is your gender?

Female    Male  
A                            B

39. Have you taken calculus before?

No                            Yes, in high school                            Yes, in college  
A                            B                            C

## APPENDIX D: INSTITUTIONAL REVIEW BOARD APPROVAL

Research Integrity & Compliance Review Office  
Office of the Vice President for Research  
321 General Services Building - Campus Delivery 2011  
Fort Collins, CO  
TEL: # (970) 491-1553  
FAX: # (970) 491-2293

### NOTICE OF APPROVAL FOR HUMAN RESEARCH

**DATE:** August 26, 2009  
**TO:** Gloeckner, Gene, Education  
DeVoe, Dale, Education, Klopfenstein, Kenneth, Mathematics, Pilgrim, Mary, Education  
**FROM:** Barker, Janell, CSU IRB 1  
**PROTOCOL TITLE:** Thinking About Mathematics Conceptually Rather Than Procedurally  
**FUNDING SOURCE:** NONE  
**PROTOCOL NUMBER:** 09-1285H  
**APPROVAL PERIOD:** Approval Date: August 26, 2009 Expiration Date: August 25, 2010

The CSU Institutional Review Board (IRB) for the protection of human subjects has reviewed the protocol entitled: Thinking About Mathematics Conceptually Rather Than Procedurally. The project has been approved for the procedures and subjects described in the protocol. This protocol must be reviewed for renewal on a yearly basis for as long as the research remains active. Should the protocol not be renewed before expiration, all activities must cease until the protocol has been re-reviewed.

If approval did not accompany a proposal when it was submitted to a sponsor, it is the PI's responsibility to provide the sponsor with the approval notice.

This approval is issued under Colorado State University's Federal Wide Assurance 00000647 with the Office for Human Research Protections (OHRP). If you have any questions regarding your obligations under CSU's Assurance, please do not hesitate to contact us.

Please direct any questions about the IRB's actions on this project to:

Janell Barker, Senior IRB Coordinator - (970) 491-1655 [Janell.Barker@Research.Colostate.edu](mailto:Janell.Barker@Research.Colostate.edu)  
Evelyn Swiss, IRB Coordinator - (970) 491-1381 [Evelyn.Swiss@Research.Colostate.edu](mailto:Evelyn.Swiss@Research.Colostate.edu)



Barker, Janell

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**Approval Period:** August 26, 2009 through August 25, 2010  
**Review Type:** EXPEDITED  
**IRB Number:** 00000202