Wind Forecasting Techniques for Input into an Automatic Air Traffic Control (ATC) System

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Research Division of the Systems Research and Development Service
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WIND FORECASTING TECHNIQUES FOR INPUT INTO AN AUTOMATIC AIR TRAFFIC CONTROL (ATC) SYSTEM

Final Report
Contract ARDS-450

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ABSTRACT

This report describes the progress of work accomplished during the first phase of the project as of 31 July, 1962.

The first part (A) outlines purpose and scope of the whole project, also including portions thereof, which have not yet been treated in the present study.

The second part (B) describes an error checking program, which is to be applied to the input data. This program package will detect inconsistencies and gross errors in the punch card data received from the National Weather Records Center, Asheville. It will at the same time perform a harmonic analysis of the vertical wind profiles, thus smoothing out irrelevant fluctuations in the wind speed measurements, and it will extract the Layer of Maximum Wind Parameters which are needed for further analyses.

In the third part (C) of the report a brief description is given of a horizontal analysis routine which may be applied to LMW data.

The fourth part (D) deals with experiments conducted with a kinematic prognostic equation. Extensive studies have been made of truncation errors resulting from the application of this prognostic equation to known functions. Tests with data extracted from facsimile charts revealed that the kinematic method works well if one considers these truncation errors properly.

The fifth and last section (E) of the report presents an outlook for future work. Main improvements are anticipated from the use of objectively analyzed punch-card data, and from the suppression of truncation errors.
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A. INTRODUCTION

Purpose and Scope of Project

by

Elmar R. Reiter
PURPOSE AND SCOPE OF PROJECT

During the years since World War II aviation weather forecasters have been confronted with an ever-increasing number of air-space users. The informations required in modern air traffic range all the way from present and forecast terminal conditions to normal and hazardous in-flight weather over a range that spans the whole troposphere (Travelers Res. Cent. 1960); they have to satisfy the needs of a great variety of air-borne vehicles.

It will be realized, that the demands made to weather-input into an ATC system can be met properly only, if future needs are anticipated and carefully taken into account (Borg-Warner Controls 1961). Most of the decisions of such a system will have to be based upon electronic computer techniques in view of:

a) the large number of air-space users requesting the services of the system, and

b) the large amounts of data that will have to be handled in order to satisfy the needs of the system. These data will have to be collected, processed, and disseminated in a minimum amount of time.

This report deals with the phase of such an automatic ATC system that is concerned with the development and testing of wind forecasting techniques suited for handling by an electronic computer. The altitude range for which these forecasts should provide information shall meet the requirements of present and future sub- and supersonic jet air traffic. It is anticipated that the layer between 20,000 and 50,000 feet will meet most of these specifications.

The report presented here describes a series of experiments -- preliminary in nature-- which have been designed to test certain automatic wind forecasting techniques.

The state of atmospheric modelling, at this time, does not allow for direct wind forecasts other than geostrophic or gradient-wind calculations from prognostic topographies. It is felt that such calculations
would introduce an undue amount of smoothing, especially when applied to levels close to the jet stream. Furthermore, due consideration of vertical wind shears in the jet stream region would require the use of a rather elaborate baroclinic model of the atmosphere, if forecasts were to be based upon dynamic considerations. It, therefore, was decided to approach the problem of upper-wind forecasting with strictly kinematic tools.

Several years ago, the introduction of the Layer of Maximum Wind (LMW) concept into jet-stream analysis had been proven useful (Reiter 1958), because the representation of the three-dimensional wind field near the jet-stream level could be reduced to the treatment of four parameters only:

1) mean speed
2) mean direction of the LMW.
3) thickness
4) height

Integrating the properties of flow over a layer rather than treating them at a surface will introduce a certain amount of smoothing. In view of the irregularities in vertical wind profiles, caused by errors in measurement (Reiter 1961) and by meso-meteorological disturbances of a local character (Reiter 1962), such a smoothing is considered beneficial to the representativeness of the LMW analyses.

The LMW usually does not extend from 20,000 to 50,000 feet, but is confined to a somewhat shallower layer. It, therefore, will eventually become necessary to modify the LMW concept in order to meet the full requirements of an automatic ATC system. In order not to make the forecasting problem too unwieldy from the beginning, it was decided to start from the original LMW definition, and to devise forecasting techniques applicable to this layer. Possible improvements and extensions of these techniques were left to a later phase of the project.

To make an automatic forecasting technique truly effective, it will have to be based upon automatically processed and analyzed data. The first part of this project, therefore, deals with error checks in the data input and with modifications of a smoothing routine of vertical wind profiles,
that has already been developed earlier (Riehl 1961). Secondly, a horizontal analysis technique is described. It will become the concern of this project to incorporate these routines into a uniform package which will allow an operational application of the whole procedure ranging from data processing to forecasting.

The design of verification procedures of any forecast poses a problem in itself. Strictly speaking, the skill of upper-wind forecasts should be evaluated in terms of improvement over geostrophic or gradient winds obtained from forecast topographies. The latter, however, are not readily available. The quality of LMW predictions, therefore, will have to be checked against actual observations.

Furthermore, since the forecasting procedures tested during this project will have to be applied to air-traffic control, the main emphasis in verification should be placed in route winds rather than spot winds. Any such verification procedures will have to be left to a later phase of the project, when objectively analyzed LMW charts will be available.

To test the intrinsic quality of the forecasting routine independently from errors introduced by poor data, artificial flow patterns have been simulated by geometric functions. The treatment of such functions gives an indication of the truncation errors introduced by using finite differences instead of differentials.

ACKNOWLEDGEMENT

The authors of this report wish to express their gratefulness to Messrs. Dr. Herbert Riehl, Cdr. J. W. Hinkelman Jr., and Mr. W. Eggert for helpful suggestions and discussions. The co-operation received from the National Weather Records Center, Asheville, North Carolina, is greatly appreciated.


WIND FORECASTING TECHNIQUES FOR INPUT INTO AN AUTOMATIC AIR TRAFFIC CONTROL (ATC) SYSTEM

B. CHECKING AND PREPARING OF INPUT DATA

by

Ben Duran, Genevieve S. Garst and Elmar R. Reiter
Table B.1. Layout of data winds, containing minute-by-minute wind reports

<table>
<thead>
<tr>
<th>COLUMNS</th>
<th>ITEMS</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 5</td>
<td>Station No.</td>
<td>Schedules time 00 or 12 plus or minus 1-1/2 hours. If more than 1-1/2 hours off scheduled time punch actual time.</td>
</tr>
<tr>
<td>6 - 7</td>
<td>Year</td>
<td>Sfc is punched 000 minute (1st card for each flight). Cols. 14-16 will be punched in other cards indicating the first minute for which data is punched inCols. 17-22.</td>
</tr>
<tr>
<td>8 - 9</td>
<td>Month</td>
<td>Wind Direction 360°</td>
</tr>
<tr>
<td>10 - 11</td>
<td>Day</td>
<td>Wind direction and speed at consecutive 1 minute or 2 minute intervals for the entire flight. Winds reported in mps and tenths. Winds reported in knots will be punched to whole knots - rounding off tenths position if any. For wind speeds of 100 or more, 500 will be added to the wind direction. (Cols. 17-19, 23-25, etc.)</td>
</tr>
<tr>
<td>12 - 13</td>
<td>Hour</td>
<td>Height of last minute punched in card, prefixing '0' where necessary, when entered at the same minute wind direction and speed is reported. If height is not entered on the same minute as wind direction and speed is reported, punch next lowest height prefixing '0' where necessary.</td>
</tr>
<tr>
<td>14 - 16</td>
<td>Beginning Minute</td>
<td>Punch 1 when card contains wind direction and speeds at 1 minute intervals.</td>
</tr>
<tr>
<td>17 - 64</td>
<td>Wind Direction and Wind Speed</td>
<td>Punch 2 when card contains wind direction and wind speeds at 2 minute intervals.</td>
</tr>
<tr>
<td>65 - 69</td>
<td>Altitude of Last Minute Punched in Card</td>
<td>Wind speeds in mps - punch 0.</td>
</tr>
<tr>
<td>70</td>
<td>Minute Identifier</td>
<td>Wind speeds in knots - punch 1.</td>
</tr>
</tbody>
</table>
Table B.1. (cont)

<table>
<thead>
<tr>
<th>COLUMNS</th>
<th>ITEMS</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>Height Identifier</td>
<td>Punch X if height is reported for one minute earlier than for the last wind direction and speed punched in the card. Leave blank if the height is reported for the same minute as the wind direction and speed for the last minute punched in card.</td>
</tr>
<tr>
<td>73 - 79</td>
<td>Leave blank</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>Last card of flight</td>
<td>Punch 1 for the last card of the flight. Leave blank on all other cards.</td>
</tr>
</tbody>
</table>

FOR MISSING MINUTES:

When minutes are missing between values punched on the same card, punch XXXXXX for missing data.
When minutes are missing and continue to be missing beyond card on which reported wind directions and speeds end, punch XXXXXX in columns for remainder of card through column 64. Punch height of wind direction and speed at last minute reported and begin the next card with the minute for which wind direction and speeds are reported again.
CHECKING AND PREPARING OF INPUT DATA

1. INTRODUCTION

In an earlier paper (Reiter 1958) it has been pointed out that the present shortcomings of measurement and coding techniques may at times make the vertical wind profiles as obtained from teletype reports unrepresentative of actual conditions of flow in the atmosphere. It has to be expected that erroneous input data will have an adverse effect upon the quality of forecasts. It was decided, therefore, to use the best available data for this project. These were the minute-by-minute records of wind speed and wind direction, which are obtained directly from the angle and slant range readings of the radar instruments.

The data were supplied by the U. S. Weather Bureau National Weather Records Center, Asheville, North Carolina, in punch card form. Since the cost of double-punching or even checking these data cards would have been prohibitive a machine program has been devised which would detect any gross mistakes in the input data.

Table B.1. contains the lay-out of the data cards. The machine program for which these cards serve as an input has been written in SPS (Symbolic Programming System) in order to save storage locations in the machine which otherwise would be taken up by FORTRAN (Formula Translation). The whole package consists of three parts:

(a) error checking program
(b) harmonic analysis of vertical wind profiles
(c) computation of LMW parameters

2. ERROR CHECKING PROGRAM:

A flow chart of the machine operations involved in this part of the program is presented in Fig. B.1.

For each successive minute-by-minute pair of wind speed and wind direction data a special location is reserved in the memory of the computer.
This procedure is of particular advantage when missing data points have
to be interpolated, and at higher levels, where wind data are available only
every two minutes. These gaps in the data sequence are indicated by 000.
The harmonic analysis routine may now be set up to utilize minute-by-minute
data, because all gaps are bridged by interpolation.

Before a new wind profile for a particular station and map time
is calculated, the memory of the machine is cleared (Step No. 1 in Fig. B.1.).
Cards are then read into the machine and wind data stored in their proper
minute location, allowing at the same time for any missing data (No. 3 to 9).
The first check is aimed to detect any inconsistencies in station
number and ascent time notation (No. 11). Minute-by-minute wind data will be
re-arranged in the computer if they should prove to be out of sequence
(No. 13 and 14). If the ascent does not go beyond 9000 m it will be discarded
(No. 16).

Next the consistency between speed and direction reports for "calm"
situations is checked (No. 18 to 26). Missing data are interpolated (No. 27)
by assuming a linear wind profile across the gap:

\[
V_{last + 1} = V_{last} + \frac{V_{first} - V_{last}}{\text{COUNTER} - 1} \cdot 1
\]  

\text{B(1)}

\(V_{last}\) is the last complete wind report of the sounding before the onset of
the data gap, \(V_{first}\) is the first wind report after the gap of missing
data. (\text{COUNTER} - 1) indicates the number of missing data points across which
the interpolation has to be performed.

Wind speeds in excess of 99.9 m/sec are coded on the data cards by
adding 500 to the value of the direction. In steps No. 28 to 31 these values
are reduced again to their original meaning.

Excessive maxima and minima of wind speed, which do not show up in
any surrounding extremes, are probably caused by mis-punched data. They will
be detected and eliminated by step No. 35. (Fig. B.2.).

Fig. B.3. shows an example of data as they appear in the computer
at the end of this part of program. The "flags" over certain digits
indicate the start of the individual wind speed and wind direction groups.
According to this, the surface wind (at minute No. 0) is from 330° at 5.7 m/sec. The next pair of values (minute No. 1) is given by 297° and 7.5 m/sec etc.

3. HARMONIC ANALYSIS PROGRAM:

Fig. B.4. contains an abbreviated flow diagram of the operations involved in the harmonic analysis of vertical wind profiles. Procedures for these computations have been devised by Riehl (1961); they had to be adapted to SPS, however.

Since the stratospheric portions of the soundings especially in the vicinity of a jet stream, may show large fluctuations of wind speed, the very last reported values of a sounding may not be considered representative for a harmonic analysis. Therefore, an artificial terminal point of the sounding is computed by taking the arithmetic mean over the last 9 reported wind values. Between the surface wind $A = V_1$ and this terminal point the harmonic "zero" is given by the straight-line wind profile:

$$B = \frac{V_{n-8} + V_{n-7} + \cdots + V_n}{9} - V_1$$  \hspace{1cm} \text{(B2)}

The harmonic analysis will only have to be performed with the residual wind values which exceed this straight-line profile, and which are given by:

$$G_j + 1 = V_j + 1 - A - B \cdot t_j \hspace{1cm} j = 0, \ldots, n-5$$  \hspace{1cm} \text{(B3)}

where $V_j$ is the original wind value reported at minute No. $j$, $G_j$ is the residual value after taking out the straight profile, and $t_j$ is the time in minutes.

Harmonic analysis is now performed, computing:

$$C_i = \frac{2}{n-5} \sum_{j=0}^{n-5} G_j + 1 \sin \left( \frac{n_i \cdot j}{n-5} \right) \hspace{1cm} i = 1, \ldots, 9$$
The smoothed, re-computed wind values are given by:

\[ V'_{j+1} = A + B \cdot j + \sum_{i=1}^{9} C_i \sin \left( \frac{i \pi j}{n-5} \right) \]

when nine harmonics are used in describing the wind profile.

4. COMPUTATION OF LMW PARAMETERS:

Fig. B.5. shows an outline of computational procedures involved in the determination of LMW parameters, whose definition may be taken from Fig. B.6.

Step No. 46 eliminates soundings which do not penetrate the level of maximum wind and for which, therefore, LMW parameters cannot be computed. A print-out of information on these soundings is provided by No. 47.

In No. 48 the maximum wind speed is compared with the speed measured at 6000 m. If the difference between these two values is less than 15 m/sec the sounding is considered to be barotropic, and it is marked as such (No. 49 to 51).

The upper and lower boundaries of the LMW are picked off by No. 53 and 54. In case no intersection can be obtained between the wind profile and the line \( V = 0.8 V_{\text{max}} \) a statistical relationship is used according to which on the average 45% of the thickness of the total LMW are found above the level of maximum wind, and 55% are found below this level (No. 55) (Reiter 1958).

If the thickness of the LMW, \( \Delta h \), is less than 5000 m (No. 57), the sounding, again, is considered barotropic and is indicated as such.

Sometimes one finds secondary wind maxima, even after the vertical profile has been subject to smoothing by harmonic analysis. A check for such maxima is made in step No. 58. Such secondary "humps" in the wind profiles are assumed to be real only if they are more than 4500 m apart (No. 62). In this case their values will be punched out together with the height at which they occur (No. 68).

If the vertical distance between successive maxima is less than 4500 m they will be eliminated by additional smoothing. This is accomplished by reducing the number of harmonics used for reconstructing the wind profiles.
successively by one (No. 63 to 65) until either condition No. 62 is met or the three last harmonics have been dropped. The wind profiles should not be reduced by more than 3 harmonics, however (No. 63).

Since a change in the number of harmonics used to describe the wind profiles may have a slight effect upon $V_{\text{max}}$ and $h_{\text{max}}$, these values are re-computed in No. 69 and 70, and any missing upper boundaries of the LMW are calculated from the statistical relationships mentioned earlier (No. 72 and 73).

The thickness of the LMW (No. 74), the mean height of the layer (No. 75), the difference between mean height and $h_{\text{max}}$ (No. 76) and the mean wind direction within the layer (No. 77) are finally computed and punched out.

The program, then, proceeds to the next sounding.
LITERATURE


FIGURES

D-1 through D-6
ERROR CHECKING PROGRAM

1. START
2. CLEAR MEMORY
3. READ A CARD
4. SET FLAGS OVER FIRST DIGITS OF SPEED AND DIRECTION VALUES
5. STORE SPEEDS & DIRECTIONS IN COMPUTER
6. LOCATIONS FOR MIN. BY MIN. VALUES RESERVED AT PROPER PLACE FOR ANY MISSG SPEEDS & DIRECTIONS
7. 1ST CARD
   YES
   8. SAVE STATION NO., YR., MO., DAY, HOUR, MIN.
   NO
   9. COL. 80
      = 0
      ≠ 0
10. COMPARE STATION NO., YR., MO., DAY, HOUR, MIN.
11. ARE STATION NO., YR., MO., DAY, HOUR SAME?
   NO
   12. PRINT ERROR & DATA FROM FIRST CARD & INCORRECT CARD
   YES
   13. MIN. IN CORRECT SEQUENCE
      YES
      14. REARRANGE IN THE COMPUTER
      NO
Fig. B.1.: Flow-diagram of error-checking program (See text).
Fig. B.2.: Elimination of excessive wind fluctuations due to erroneous data (See text). Maxima or minima which deviate by more than 15 m/sec from surrounding maxima or minima will be eliminated by smoothing.
Fig. B.3.: Example of minute-by-minute wind data as stored in the computer (Station No. 24225, Medford, Oregon, 1 Aug. 1961, 00 GMT); upper set of numbers: directions in degrees; lower set of numbers: speeds in tenths of meters per second.
HARMONIC ANALYSIS PROGRAM

37 HARMONIC ANALYSIS

38 FIND $V_i$ &
   COMPUTE:
   \[ B = \frac{\sum_{i=n-5}^n V_i}{n} - V_i \]

39 COMPUTE $G_{j+1}$

40 COMPUTE $C_i$

41 COMPUTE $V_{j+1}'$

42 REPLACE $V_{j+1}$ BY $V_{j+1}'$

43 FIND ALTITUDES BY
   LINEAR INTERPOLATION
   BETW. LAST ALTIT.
   AND STATION ELEV.

44 LMW PARAMETERS

Fig. B.4.: Flow-diagram of harmonic analysis program.
Fig. B.6.: Definition of various LMW parameters: $V_{\text{max}}$ = maximum wind speed; $h_{\text{max}}$ = height of maximum wind; $h_1$ and $h_2$ = lower and upper boundaries of LMW; $\Delta h$ = thickness of LMW; $\overline{h}$ = height of LMW.
C. CURRENT STATUS OF NUMERICAL ANALYSIS

by

Ferdinand Baer
CURRENT STATUS OF NUMERICAL ANALYSIS

An IBM-704 program has been written and checked out which fits a quadratic surface by the method of least squares to four parameters from the level of maximum wind (LMW) on a specified grid network for a selected number of stations. Two different but basically similar methods have been used to choose the data which are relevant for the fitting of any given grid point. If the data requirements, which will be specified below, are not satisfied at a grid point, no fitting will be made.

Computations are performed at grid points on the rows $3 \leq i \leq 12$ and on the columns $2 \leq j \leq 15$, where one grid interval represents three degrees of latitude. There are 99 relevant stations which apply to this grid, although they do not all report at any given time. The four parameters of the LMW which are given as input data and for which grid points are to be fitted, are wind magnitude to three significant figures, wind direction, thickness, and height, the last three quantities given to two significant figures. The wind magnitude and direction are converted by the program to rectangular velocity components in the form:

$$u = VV \cos DD$$
$$v = VV \sin DD$$

and are fitted in this form. They are reconverted to $VV$ and $DD$ before machine output.

The quadratic surface to be fitted about any grid point as origin is of the form, if $F$ represents any of the four input parameters:

$$F = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2$$

from which we see that at least six data points are required to fit this function. Also, since $F$ will apply only to the grid point, we need compute only the value of $a_0$. Applying the method of least squares:

$$\delta \sum_{k=1}^{n} (F - F_k(\Delta i, \Delta j))^2 = 0$$
where \( k \) represents the relevant data points, we arrive at a matrix equation of the form:

\[
A \{a\} = \{D\}
\]

where \( A \) is a 6 x 6 matrix depending only on the relative coordinates of the data points \((\Delta i)_k, (\Delta j)_k\), \(\{a\}\) is the column vector whose components are the \( a_e \) coefficients of the expansion of \( F \) and \(\{D\}\) is a column vector which is a function of both the coordinates of the data points and their given values, \( F_k(\Delta i, \Delta j) \). This equation is formally solved for \( a_0 \) by finding the inverse of \( A \), \((A^{-1})\). This method has been used in our program by means of a prewritten inversion subroutine, and is a convenient method since \( A^{-1} \) is independent of the data (i.e., is merely a function of the coordinates) and may be used to get a solution for all four parameters.

To check the results of this method, the Crout (1941) method of solving a matrix equation has also been programmed and checked.

Since the data at the given stations are investigated before being made available, and are considered good, and also since the purpose of this analysis is not smoothing -- which would obliterate the jet features for which we are searching -- the minimum requirements for fitting were selected as the lower limit on required data at any given grid point. Hence, we have the first requirement for fitting; there must be at least six significant data points which apply at the grid point under consideration.

The influence region about the grid point for which data would apply was chosen empirically. By scrutinizing the coordinates of the stations with respect to the grid points and in relation to the minimum fitting requirements, a three grid-interval square was chosen as the minimum influence region. If this region contains insufficient data, it is expanded by units of 0.4 grid-intervals to a maximum of a five grid-interval square. If the data are still insufficient for this limit, no fitting is performed at that grid point.

In order that the relevant data points for a given grid point should not be too poorly distributed about the grid point, an empirically derived criterion was established which requires the average data point to lie within a circle 0.6 grid-intervals from the grid point. The test was set up such that:
\[
\sum_{k=1}^{n}(\Delta i)_k \leq 0.6n, \quad \sum_{k=1}^{n}(\Delta j)_k \leq 0.6n
\]

If this criterion is not satisfied, more data from an increased influence region are required. If the criterion is not satisfied for the maximum influence region, no fitting is made at the grid point.

The two models which were programmed are the following:

A. Grid points at which the above requirements were satisfied for the minimum influence region are fitted to the data points. The influence region is then expanded in the intervals specified above to the maximum size, and all grid points for which data points satisfy the requirements at each expansion are fitted.

B. Grid points for which data points in the minimum influence region satisfy the above requirements are fitted. Beginning again with the minimum influence region, now using precomputed grid points as additional data points, the uncomputed grid points are fitted, subject to the above restrictions. The influence region is then expanded as in method A, using here also the precomputed grid points.

In both methods A and B, no grid point is ever refitted. In other words, once a value has been fitted, it is not changed again during the computation. At the end of the computation, all grid point values and data points are lumped together as data points, and a new field of grid points is computed using all data for the minimum influence region, subject to the restraints described above. This field, called the summary field, is computed for both methods A and B. Thus four grid point fields for each parameter are made available as output from the program and take roughly eight minutes of IBM-704 machine time. A routine has also been written which will compare the computed values at grid points with input grid point values - \( F_{ij} \) (input) - chosen from a hand analysis. The output from this routine will give a histogram of the number of grid points in a given difference interval; i.e., \( F_{ij} \) (output) - \( F_{ij} \) (input).

Although the program described here has been checked out, certain points in the output field give extreme values for all parameters. This
difficulty may be explained as follows. If the coordinates of the relevant data for a given grid point lie on a quadratic curve in the coordinate \((x,y)\) plane, then one can show that the data points lie on a plane which cuts the quadratic surface which we desire to find. However, if these points lie on any plane, they cannot describe a unique quadratic surface. It would be extremely remote to find any selection of stations lying exactly on a quadratic curve. However, it is possible for them to approach such a curve closely, and the less the number of stations, the greater is this possibility. Since we allow a minimum of six data points, this possibility is in fact a reality.

A plot of the stations making up the data for those grid points which give extreme values in one experiment shows in most cases a close approximation to a quadratic curve. The significant fact to note here is that the approximation to such a curve and the resulting extreme value of the fitted quantity are relative. There is no precise value for which degeneracy occurs.

Several methods have been suggested to bypass this difficulty.

(a) Since the degeneracy condition arises with the minimal number of data points, for those grid points where there are no extra data values for smoothing, use a lower order fitting. Although this method is adequate when near degeneracy occurs, an actual computation, made by passing a plane through the six data points which apply to a grid point near the jet, shows that this lower order fitting obliterates the jet. The corresponding quadratic fit for these data points gives a satisfactory value.

(b) In order to test for the significance of a computed value, we may compare it to the station data which are used in fitting. Since the quadratic fit does not require the resulting value to remain bounded between the maximum and minimum of the input data, we may allow some excess of the computed value in either direction. Thus, we may require the computed value to be within some percentage of the maximum or minimum of the input data.

Such a requirement which may filter out very extreme values, is not able to isolate those values which are not too excessive but yet are obviously wrong. A simple example of this is given by a situation in which the data points very near the grid point have small values, but one data
point on the edge of the influence region has a large value. An obviously erroneous computed value having the same magnitude as the large peripheral input value would not be excluded by this technique.

(c) A more precise way to determine the degeneracy of the system is to investigate the geometry of the relevant input data points in detail. The degree of degeneracy will be determined by the size of the determinant of the system which describes these input points as a quadratic curve. Since this degeneracy is relative, as discussed above, the criterion which determines when a computed point is satisfactory must be arrived at statistically. The determinant of the system must be correlated with some estimate of the goodness of fit of the computed grid point value. This has not yet been attempted.

(d) We may use another method of fitting (smoothing) which does not depend on a particular functional form. One such technique is to use a weight function for the data points in the influence region, where the weight function is dependent on the radial distance of the data point to the grid point. This method has been used in meteorological literature, see Bergthorsson and Döös (1955):
LITERATURE


Crout, P., 1941: A short method etc. Trans. AIEE, 60, 1235-1241.
WIND FORECASTING TECHNIQUES FOR INPUT INTO AN AUTOMATIC AIR TRAFFIC CONTROL (ATC) SYSTEM

D. FORECASTING EXPERIMENTS WITH A KINEMATIC EXTRAPOLATION TECHNIQUE

by

Elmar R. Reiter and Patricia White
FORECASTING EXPERIMENTS WITH A KINEMATIC EXTRAPOLATION TECHNIQUE

1. INTRODUCTION:

In a Cartesian coordinate system fixed to the earth the differential operator describing the rate of change of a certain characteristic property of an air particle has the form (Godske et al. 1957):

\[ \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \]

In a moving coordinate system we obtain:

\[ \frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{V} - \mathbf{c}) \cdot \nabla \]

where \( \frac{\partial}{\partial t} \) gives the "local" change of the respective property in the moving system, and \( \mathbf{c} \) is the rate of displacement of this system with respect to the earth's surface. Subtraction of (2) from (1) renders

\[ \frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \mathbf{c} \cdot \nabla \]

If we wish to study the movement of an iso-line of the characteristic property, \( \frac{\partial}{\partial t} \) naturally will be zero, and we arrive at

\[ \frac{\partial}{\partial t} = -\mathbf{c} \cdot \nabla \]

Specifically, let \( u \) and \( v \), the two components of the wind velocity characterizing the layer of maximum wind, be the property whose motion shall be predicted. This gives us the set of two equations

\[ \frac{\partial u}{\partial t} = -c_x \frac{\partial u}{\partial x} -c_y \frac{\partial u}{\partial y} \]

\[ \frac{\partial v}{\partial t} = -c_x \frac{\partial v}{\partial x} -c_y \frac{\partial v}{\partial y} \]
We may solve these two equations with respect to \( c_x \) and \( c_y \), the two components of the displacement of the system, and we obtain

\[
c_x = \frac{\frac{\partial u}{\partial y} \frac{\partial v}{\partial t} - \frac{\partial v}{\partial y} \frac{\partial u}{\partial t}}{\partial x} \quad \text{D(6)}
\]

\[
c_y = \frac{-\frac{\partial u}{\partial x} \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \frac{\partial u}{\partial t}}{\partial y}
\]

where \( D = \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \).

\( \frac{\partial u}{\partial t} \) and \( \frac{\partial v}{\partial t} \) may be evaluated from two successive maps; the differentials \( \frac{\partial}{\partial x} \) and \( \frac{\partial}{\partial y} \), when converted into differences, may be computed from grid points on the latest map. One will have to bear in mind in this case, that a non-centered time-interval has been used.

One of the most serious restrictions in the application of equations D(5) and D(6) is based on the assumption of \( c_x \) and \( c_y \) being constant. Even if a positive or negative acceleration of the system were taken into account by using three previous maps instead of two, improvements are anticipated to be small, because only average accelerations over 24 hours can be obtained with this method, while the actual accelerations may be larger and may be active over periods of time less than 12 hours. Thus, it might be hazardous to extrapolate a trend \( \frac{\partial c}{\partial t} \) linearly.

The local "accelerations" of the flow pattern may be obtained from equations D(5) and D(6) by partially differentiating with respect to time, and by considering, that when following a given isotach, \( \frac{\partial c}{\partial t} \) will be zero:

\[
\frac{\partial^2 u}{\partial t^2} = -\frac{\partial c_x}{\partial t} \frac{\partial u}{\partial x} - \frac{\partial c_y}{\partial t} \frac{\partial u}{\partial y} - c_x \frac{\partial^2 u}{\partial x^2} - c_y \frac{\partial^2 u}{\partial y^2} \quad \text{D(7)}
\]

A similar equation holds for the \( v \)-component. Furthermore,

\[
\frac{\partial c_x}{\partial t} = \frac{\partial}{\partial t} \left[ \frac{\frac{\partial u}{\partial y} \frac{\partial v}{\partial t} - \frac{\partial v}{\partial y} \frac{\partial u}{\partial t}}{D} \right] \quad \text{D(8)}
\]
An analogous equation for \( \frac{\partial c}{\partial t} \) may be derived from D(6).

The computational procedure would be the following:

1. \( c_x \) and \( c_y \) are determined from the map pairs \( t = 0 \), \( t = 12 \), and \( t = 12 \), \( t = 24 \).

2. The differences between these two sets of \( c_x \) and \( c_y \) values render \( \frac{\partial c_x}{\partial t} \) and \( \frac{\partial c_y}{\partial t} \), according to D(8).

3. \( \frac{\partial^2 u}{\partial t^2} \) and \( \frac{\partial^2 v}{\partial t^2} \) are computed from D(7), where \( \frac{\partial u}{\partial x} \), \( \frac{\partial v}{\partial x} \), \( \frac{\partial u}{\partial y} \), and \( \frac{\partial v}{\partial y} \) should be taken from the map \( t = 12 \).

4. The forecast chart is obtained by computing \( u_{24} + \frac{\partial u}{\partial t} \Delta t + \frac{\partial^2 u}{\partial t^2} (\Delta t)^2 \), and an analogous expression for \( v \). The index 24 refers to the values taken at grid points of the map \( t = 24 \).

2. THE INFLUENCE OF INPUT ERRORS ON THE QUALITY OF FORECASTS:

Since the space and time derivatives in equation D(6) will have to be estimated from actual data, certain "errors" will have to be anticipated, which may be due to transient micro- or meso-structural features in the upper-wind measurements, to errors in measurement and analysis, and most seriously, to the truncation involved by replacing differentials by differences. Therefore, the values computed for \( c_x \) and \( c_y \) will be slightly in error too, and they will in turn affect the computed changes \( \frac{\partial u}{\partial t} \) and \( \frac{\partial v}{\partial t} \) of the wind field.

We may express the effect of erroneous measurements in the form of percent corrections necessitated by errors in the gradients:

\[
P_{c_x} = \frac{P_{uy} P_{vt}}{P_{ux} P_{vy}} \left[ \frac{\partial u}{\partial y} \frac{\partial v}{\partial t} - P_{vy} P_{ut} \left( \frac{\partial v}{\partial y} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \right) \right]
\]

D(9)

\( P_{ux} \), \( P_{uy} \), etc. are the percent corrections in the gradients \( \frac{\partial u}{\partial x} \), \( \frac{\partial u}{\partial y} \), etc. From D(9) it can be seen that a maximum percent error \( P_{c_x} \) in the estimate of \( c_x \) will have to be expected, if \( P_{vt} \) and \( P_{ut} \) are both
< l (> l), and simultaneously \( P_{vx} \) and \( P_{ux} > l (< l) \). Otherwise the errors tend to cancel each other.

For \( c_y \) the corresponding terms are \( P_{vt} \) and \( P_{ut} \) as compared with \( P_{vy} \) and \( P_{uy} \).

Example: \( P_{uy} = 0.9 \quad P_{vt} = 0.9 \)
\( P_{vy} = 0.9 \quad P_{ut} = 0.9 \)
\( P_{ux} = 1.1 \quad P_{vx} = 1.1 \)

From this we obtain: \( P_{c_x} = 0.8 c_x \), which means that there will be a -20% error in the values of \( c_x \) computed from actual data. In \( c_y \) the corresponding error would be nil, since \( P_{c_y} = c_y \). Thus it appears as though extreme errors are likely to occur in one component of \( c \) only. They tend to cancel each other in the other component.

Sizable errors in the measurements of wind speed gradients are most likely to occur near the jet axis, where the wind shear reverses its sign. Let us consider a grid point who lies less than one grid distance away from the jet axis. From a reliable analysis the errors made in computing \( \frac{\partial u}{\partial t} \) and \( \frac{\partial v}{\partial t} \) at this grid point will be rather small. The horizontal gradients of speed normal to the direction of flow will, however, be underestimated—the more, the closer the grid point under consideration lies to the jet axis. For zonal flow the derivatives \( \frac{\partial u}{\partial y} \) and \( \frac{\partial v}{\partial y} \) will be affected strongest. From what has been said above, this will influence the computation of \( c_y \) most severely. Since both derivatives will be underestimated, according to equation D(6) the computed component \( c_y \) will be too large.

For meridional flow analogous considerations hold for \( c_x \). It, therefore, appears that the isotachs in the vicinity of the jet axis will move too fast, depending on the location of the jet axis with respect to the grid points. This, of course, will produce sizable errors in the forecast changes \( \frac{\partial u}{\partial t} \) and \( \frac{\partial v}{\partial t} \), and therefore, also in the prognostic charts.
3. **THE INFLUENCE OF TRUNCATION ERRORS ON KINEMATIC EXTRAPOLATION:**

In order to estimate the truncation errors introduced by using finite differences instead of differentials, tests have been made with artificial flow patterns which could be expressed analytically (see also Wurtele 1961).

**Example I:**

The flow is represented by a non-shearing sinusoidal pattern:

\[
\begin{align*}
u &= A \cdot \sin \frac{2\pi}{L} (x - ct) + B \\
v &= A \cdot \cos \frac{2\pi}{L} (x - ct)
\end{align*}
\]

In essence, this flow pattern is one-dimensional. Thus, it will suffice to forecast the u-component only.

\[
\frac{\partial u}{\partial t} = -c \frac{\partial u}{\partial x}
\]

and

\[
c = -\frac{\partial u}{\partial x}
\]

The computations carried out on an IBM 1620 electronic computer were set up in the following way: First, from equation D(10), the true values of \(u\) were computed for each grid point, and for one-hour time intervals. Subsequently, \(\frac{\Delta u}{\Delta t}\) was computed from these values. (All lengths were expressed in grid units, time was given in hours.) \(\frac{\Delta u}{\Delta t}\) was obtained from \(\frac{(u_{12} - u_0)}{12}\), where the indices refer to the time at which these values were "observed." Values for \(c\) were computed from equation D(12), and then, again substituted into D(11) in order to obtain new values for \(\frac{\Delta u}{\Delta t}\). Addition of these to the previous values of \(u_t\) render \(u_{t+\Delta t}\). This process was repeated iteratively.
In order to make the results obtained from those artificial flow patterns comparable with atmospheric conditions, all lengths in above equations have been expressed in units of grid distance, time in hours, and speeds in grid distances per hour. Assuming a wave-length of \( L = 20 \) grid units, a speed of \( c = 0.1 \) to \( 0.2 \) grid units per hour would correspond to short waves and cyclone waves. A speed of approximately \( c = 0.5 \) would be equivalent to group velocity.

In equation (10) the amplitude \( A \) has been assumed constant. Any changes in this amplitude factor in the course of extrapolation with time, therefore, will be due to truncation errors. Another effect of truncation will have to be expected in the phase angles of the disturbances.

Fig. D.1. shows the distribution of wind speeds \( u \) along the co-ordinate \( x \) for the case of a wave travelling at a speed \( c = 0.1 \) grid units per hour. The original amplitude has been assumed with \( 0.5 \) grid units per hour, the wave length \( L \) with \( 20 \) grid units.

The hourly values of \( u \) between \( T = 0 \) and \( T = 12 \) reveal a very slight damping of the amplitudes. This computational stability obviously is produced by the interpolation process, by which \( \frac{Au}{Ax} \) was obtained. Values of \( u \) beyond \( T = 12 \) are the result of actual extrapolation. \( T = 24 \) constitutes a 12-hour forecast from the latest map available (which, in our case, was for \( T = 12 \)). In this example of a slow moving wave the truncation error in the amplitudes of \( u \) (shaded area) amounts to about \( 15\% \) for a 12-hour extrapolation. There is hardly any truncation error in the phase angle of the extrapolated wave.

Fig. D.2. shows the truncation errors for \( c = 1.0 \) under otherwise same conditions. Again the interpolation for \( T < 12 \) indicates a decrease in amplitudes. \( T = 24 \) (equivalent to a 12-hour forecast from the latest map) on the other hand reveals an increase in amplitudes of \( 185\% \), thus showing a very substantial computational instability. The phase angles, too, are subject to large truncation errors in this example. So, for instance, the interpolation for \( T = 6 \) is \( 180^\circ \) (or \( 10 \) grid units) out of phase as compared with the actual flow conditions.
Fig. D.3. summarizes the truncation errors for various wave speeds $c$ as functions of time. According to this diagram a 12-hour extrapolation forecast ($T = 24$) produces a maximum truncation error in the magnitude of wind speeds, for waves with a rate of displacement $c = 1$ grid distance per hour. For waves travelling faster than this the truncation error decreases again. Nevertheless, even for slower moving waves the truncation error for a 12-hour forecast may be appreciable.

The behavior of the phase angle (expressed in grid units $x$) is shown in Fig. D.4. For slow moving waves ($c = 0.1$ grid units per hour) the truncation error remains insignificant. For $c = 0.2$ the error for a 12-hour forecast ($T = 24$) amounts to about 0.8 grid units. The forecast waves move somewhat slower than is actually the case. For $c = 1.0$ truncation errors, again, become extremely large. The phase shift amounts to 20 grid units for $T = 12$, i.e. for the departure time of the forecast interval. That means, the forecast and the actual wave will be in phase; the interpolation as well as the extrapolation will, however, simulate a retrograde wave while in reality the waves are rapidly progressive. Similar conditions hold for $c = 1.5$. Even for $c = 2.0$, where both, forecast and actual conditions, show a progressive wave, in the interpolation for $T = 12$ the wave lags already behind by a full wave length.

From the foregoing it appears that the present kinematic extrapolation routine will produce reasonable results only for rather slow moving waves, with individual jet maxima showing a rather wide separation. It should be pointed out here, that the computations so far do not contain any damping devices which would suppress the computational instability.

The results outlined above apply to all one-dimensional fields of flow with time-constant amplitude factors. This also includes fields of the form

$$ u = A \sin \frac{2\pi}{L_x} (x - ct) \sin \frac{2\pi}{L_y} y $$

because the periodic function in $y$ may be considered as part of the amplitude factor of a wave which propagates in the $x$-direction.
It should be mentioned here, that a variety of time steps has been applied for arriving at a 12-hour forecast: $\Delta T = 1, 2, 3, 4, 6$ hours. The differences in the results at the appropriate time were, however, nil. In applying this kinematic forecasting method it apparently makes no difference whether one arrives at a 12-hour forecast by 12-hourly extrapolation steps or by four 3-hourly steps, or by any other combination. This is due to the circumstance, that the quantities to be extrapolated have been obtained by an interpolation between the maps $T = 0$ and $T = 12$. Since this interpolation was made linearly, the choice of the time interval $\Delta T$ becomes irrelevant.

**Example II:**

A set of one-dimensional velocity fields in which the amplitudes are time dependent has also been investigated by extrapolating the function:

$$u = A \cdot e^{bt} \sin \frac{2\pi}{L}(x - ct)$$

in the same manner as has been described above. Fig. D.5. contains the maximum amplitudes obtained for $A = 0.5$, $L = 20$, and $b = +0.1$, for the two different wave speeds $c = 0.1$ and $c = 1.0$. The differences between the curve showing actual conditions and the ones obtained by iterative extrapolation (heavier lines) constitute the truncation errors in the amplitudes. As is evident from this diagram, slow moving waves ($c = 0.1$) extrapolate rather poorly. As the dashed lines in Fig. 4 show, the truncation errors in the phase positions of the waves are by far smaller for slow waves than for faster ones.

The conditions for $b = -0.1$, i.e. for maximum wind speeds decreasing with time, are shown in Fig. D.6. In this case slow moving waves extrapolate by far better than fast moving waves. It appears, that the dissipating tendency of a jet maximum is not followed through the whole 12-hour period of extrapolation. Even with slow waves ($c = 0.1$) the last three hours of the forecasting interval simulate an increase in wind speeds, while waves with a speed of $c = 1.0$ do so from $T = 9$ on.

Again, as in the previous example the length of the time steps taken in the extrapolation process ($\Delta T = 1, 2, 3, 4, 6$ hours) does not influence results.
Experiment with LMW facsimile data

Figs. D.7. and D.8. contain the LMW isotachs (full lines) and isogons (thin dashed lines) taken from the facsimile charts of 9 January 1962, 00 and 12 GCT, which are transmitted by the U. S. Weather Bureau. Areas with wind speeds exceeding 100 knots are shaded. Jet axes are indicated by bold dashed lines.

Fig. D.9. contains the actual flow pattern of the LMW observed on 10 January 1962, 00 GCT. This figure is to be compared with Fig. D.10. which contains the wind speed forecast obtained by applying the kinematic forecasting routine to Figs. D.7. and D.8. Horizontal derivatives were evaluated from grid points in Fig. D.8., while time derivatives were computed from differences between charts D.7. and D.8. Thus, a non-centered time interval has been used in this forecast.

When comparing the position of the jet axes of Fig. D.10. with the ones in Fig. D.9. it appears, that the prediction is quite adequate. The somewhat poor performance of the forecast near the southern border of the map may be due to the sparsity of data in this area.

The wind speeds are grossly exaggerated by the forecast, as a comparison of the two charts reveals. Forecasting errors of more than 20 knots are indicated by wide slant hatching and of more than 60 knots by narrow slant hatching. The hatching is directed from the lower left to the upper right for too large forecast speeds and it points from the upper left to the lower right for too small speeds.

The main errors of the forecast seem to be concentrated along the jet axis, except for an area with too small wind speeds in the northeastern corner of the map, which is probably caused by poor input data. The maximum wind speeds forecast for the region south of the Great Lakes are too high by about 75 knots. While Fig. D.9. calls for maximum winds of about 150 knots, the forecast shows winds in excess of 225 knots. Since a kinematic routine cannot forecast the intensification of a system, this increase in jet-core wind speeds must be caused mainly by truncation errors.

The distance between the jet maximum over the United States east coast and the trough over the Mountain States is about 8 grid units (Fig. D.8.). The next wind maximum upstream is only about two grid distances away in the x-direction. The full wave length, therefore would be \( L = 10 \). The speed
of the jet maximum cannot be estimated too accurately, due to some inconsistencies in the input analyses. A comparison of Fig. D.7. with D.8. seems to indicate a speed of 1 grid distance per 12 hours for the jet maximum as well as from the trough (evident from the crowding of isogons); this corresponds to about \( c = 0.1 \) grid units per hour. For a standard wave length of \( L = 20 \), on which the truncation error computations have been based, this speed would correspond to \( c = 0.2 \).

According to Fig. D.3. we would have to expect an exaggeration in maximum wind speeds by about 50 percent for a 12-hour forecast \( (T = 24) \), and for \( c = 0.2 \) (maximum amplitudes increase from 0.5 to about 0.75). Thus, the increase in maximum wind speeds from Fig. D.8. to Fig. D.10. can be explained adequately by the truncation which is involved in using differences rather than differentials. This means that the actual forecasting errors of wind speeds along the jet axis, which appear in Fig. D.10. are rather small and irrelevant, because the truncation errors which produce the gross exaggeration of maximum wind speeds, can be predicted too.

The forecast isogons (full lines) for 10 January 1962, 00 GCT are shown in Fig. D.11., together with the shaded area of forecast wind speeds in excess of 100 knots. The errors in degrees of wind direction are analyzed with dashed lines. As a whole, considering the inaccuracies and inconsistencies in the input data, the forecast wind directions verify quite well. There are only small areas in which the forecasting errors exceed 20 degrees.

From the foregoing one may conclude, that a kinematic forecasting routine should well be able to extrapolate the upper wind field to a sufficient degree of reliability, provided that the amplifications of maximum wind speeds produced by truncation can be eliminated. This should not be too difficult, however, since scanning techniques can easily be employed, which would reduce the exaggerated speed forecasts to their original values. Experiments of this kind are in progress.

FIGURES

D - 1 through D - 11
Fig. D.1.: Distribution of wind speed along the coordinate $x$ at various times ($T = 0, 6, 12$ and $24$ hours), for a wave with speed $c = 0.1$ grid units per hour, and a wave length of $20$ grid units. The shaded areas indicate truncation errors.
Fig. D.2.: Same as Fig. D.1., except for a wave with speed $c = 1.0$ grids per hour.
Fig. D.3: Maximum amplitudes as function of time, of waves of various speeds,
Wave length $L = 20$ grid units; original amplitude at $T = 0$
is 0.5 grid units per hour.

$u = A \sin \frac{2\pi}{L} (x - ct)$

$A = 0.5$
$L = 20$
Fig. D.4.: Displacement of wave crests in x-direction with time (ordinate). Actual conditions are indicated by thin lines and slant inscriptions; forecasts with the kinematic routine are given by heavier lines and upright inscriptions. The dashed lines represent the results from an exponentially amplifying ($b = 1$) or decreasing ($b = -1$) wave.
Fig. D.5.: Maximum amplitudes of waves with speed $c = 0.1$ and 1.0 and with exponentially increasing amplitudes. Thin line gives actual, heavier lines give forecast conditions.
**MAXIMUM AMPLITUDES**

\[ u = A e^{b t} \sin \frac{2\pi}{L} (x - ct) \]

- \(A = 0.5\)
- \(L = 20\)
- \(b = -0.1\)

Fig. D.6.: Same as Fig. D.5., except for exponentially decreasing amplitudes.
Fig. D.7.: LMW speeds (full lines, knots) and directions (arrows are grid points, dashed lines labelled in degrees) for 9 January 1962, 0000 GCT. Areas with wind speeds in excess of 100 knots are shaded. Jet axes are indicated by bold dashed lines.
Fig. D.8.: Same as Fig. D.7., except 9 January 1962, 1200 GCT.
Fig. D.9.: Same as Fig. D.7., except 10 January 1962, 0000 GCT.
Fig. D.10.: Forecast of LMI speeds (knots) obtained from numerical kinematic prediction, verifying 10 January 1962, 0000 GCT. Closely shaded areas mark wind speeds in excess of 100 knots. Shading slanting from lower left to upper right indicates wind speeds forecast too high, shadings slanting from upper left to lower right indicates wind speeds forecast too slow; light shading too high by 20 knots, dark shading too high by 50 knots. Jet axes in this diagram have been taken from Fig. D.9.
Fig. D.11.: Forecast of LMW directions (degrees) obtained from numerical kinematic prediction, verifying 10 January 1962, 0000 GCT. Jet stream indicated as in Fig. D.10. Thin dashed lines give errors of direction forecast (degrees). Slant shading stands for errors in excess of 20 degrees.
WIND FORECASTING TECHNIQUES FOR INPUT INTO AN
AUTOMATIC AIR TRAFFIC CONTROL (ATC) SYSTEM

E. OUTLOOK FOR FUTURE WORK

by

Elmar R. Reiter
The report presented here contains the preliminary results of investigations conducted to design a wind forecasting procedure which would be of operational use to the common aviation weather service. The outcome of these studies has been encouraging enough to make a continuation of the work desirable.

In essence there are three factors that control the quality of every forecast:

1. the accuracy and representativeness of input and verification data;
2. the adequacy of mechanical computation procedures, notably in the suppression of large truncation errors;
3. the adequacy of the atmospheric model used in the prediction.

Only if an estimate of factors (1) and (2) can be obtained, the performance of the forecasting model can be judged correctly.

The error checking and harmonic analysis program described in Section B of this report, as well as the objective horizontal analysis procedure of Section C are in accordance with the first of these three factors. Additional work will be necessary to adapt the programs outlined here to routine operations, making the best possible use of data, and controlling adverse effects of "shrinking" boundaries as the forecasting steps proceed.

The suppression of truncation errors will have the greatest effect upon the quality of kinematic forecasts. The example presented in Section D shows, that practically all errors in wind speed forecasts along the jet axis result from these truncation errors. Various damping devices will have to be tested, which will eliminate the computational instability introduced by using differences rather than differentials in the computations.

The kinematic forecasting procedure outlined in this report will certainly be open to improvements. It will be the task of further
investigations to test the effects of different grid sizes, and of accelerations in the rate of displacement of the flow patterns upon the quality of the forecasts.

Finally, the skill of various forecasting procedures will have to be tested objectively, as indicated in the Introduction. In this, again, the requirements of a common aviation weather service will have to be borne in mind.