Structure of Vertical Wind Profiles

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An exponential relationship between the vertical wind shear and the layer thickness over which this shear is measured has been derived. From an analysis of the Richardson number it appears that shallow layers reaching maximum shearing conditions under the established power law should be turbulent, irrespective of the prevailing thermal stability conditions.

TURBULENCE INFORMATION FROM VERTICAL WIND PROFILES

It has been shown by *Essenwanger* [1963, 1967] that vertical wind shears measured by detailed wind soundings, such as the FPS-16 tracks of 'Jimspheres,' yield exponential relationships of the form

$$\langle \Delta v \rangle = a_0 (\Delta h)^{a_1} \qquad (1)$$

where $\langle \Delta v \rangle$ denotes the mean total shear in meters per second (measured as the velocity vector difference over the height increment Δh), a_0 is a constant, and a_1 is the exponent of the relationship in question. (For further details, see *Essenwanger and Reiter* [1969]). The following exponents a_1 have been found from vertical wind soundings:

Mean wind shears	$a_1 = 1/2$
Mean extreme shears	$a_1 = 1/3$
Transient (nonstationary) fluctuations	$a_1 = 0$
Persistent mesoscale	$a_1 = 4/5$

A transverse structure function may be defined as

$$D_{tt}(\mathbf{r}) = \overline{[v_t(\mathbf{r} + \mathbf{r}_1) - v_t(\mathbf{r}_1)]^2} \qquad (2)$$

where v_t is the velocity component transverse to the distance vector $\mathbf{r}_1$ [*Tatarski,* 1961]. For $\mathbf{r}_1 \equiv h$ and $\mathbf{r} \equiv \Delta h$, it is easily seen that (1) relates to (2) through the expression

$$\sigma_{\Delta h}^2 = -\langle \Delta v \rangle^2 + D_{tt}(\mathbf{r})$$

where $\sigma_{\Delta h}$ is the standard deviation of the shears Δv measured over the height incremental Δh.

It has been shown by *Tatarski* [1961] that for the structure function

$$D(r) = c^2 r^p \qquad \text{for} \qquad 0 < p < 2 \qquad (3)$$

the one-dimensional spectral density function

$$E(k) = \frac{\Gamma(p+1)}{2\pi} \sin \frac{\pi p}{2} c^2 k^b$$

holds, where

$$b = -(p + 1)$$

From the foregoing it may be seen that $a_1 = p$,

From this it appears that the exponential rel[illegible] ship for mean extreme shears leads to a '$-5/3$' [illegible]trum slope.

According to *Kolmogorov* [1941] and *Obu*[illegible] [1941], conditions in the inertial subrange are [illegible]acterized by

$$D_{tt}(\mathbf{r}) = \tfrac{4}{3} C \cdot (\epsilon r)^{2/3} \quad \text{for} \quad l_0 \ll r \ll L$$

where l_0 and L characterize the inner and the [illegible] scales of turbulence, respectively, and where C [illegible] constant of the order of unity.

$$a_0 = (\tfrac{4}{3} C)^{1/2} \cdot \epsilon^{1/3}$$

Figure 1 contains the data sample of mean [illegible]treme shears obtained by *Essenwanger* [1963] [illegible]tween the ground level and 30 km. Taking $a_1 =$ [illegible] we may evaluate the coefficient as $a_0 = 1.2$[illegible] units of $m^{2/3}\,sec^{-1}$.

As a matter of interest, it should be pointed [illegible] that *Vinnichenko and Dutton* [1969] found th[illegible] '$-5/3$' slope prevails in the horizontal compo[illegible] of perturbation motion from the inertial subr[illegible] through the synoptic scale. The vertical structu[illegible] the atmosphere, according to the preceding dis[illegible]sion, seems to align itself in the same '$-5/3$' ex[illegible]ential relationship of horizontal motions, if one [illegible]siders the *extreme* velocity variations only. Howe[illegible] the vertical structuring shows considerably [illegible]

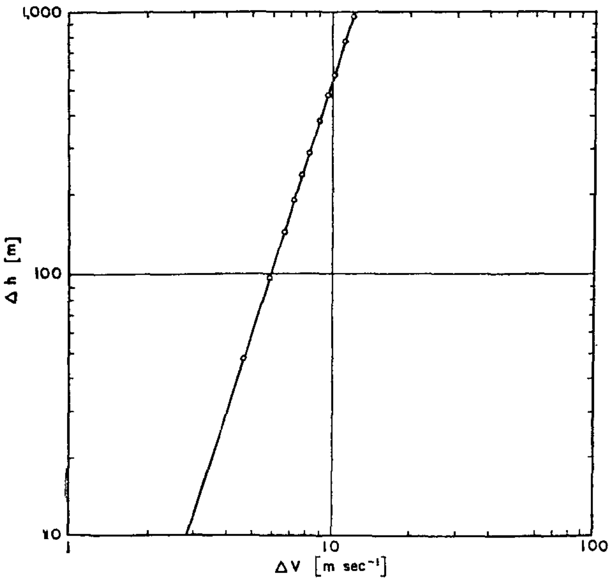

Fig. 1. Wind vector difference ΔV (m sec^{-1}) measured over height increment $\Delta h(m)$ on logarithmic scale. [From *Essenwanger,* 1963.]

shear than does the horizontal mesoscale. This fact is apparent even from a comparison of the mean large-scale vertical and the horizontal shears around a jet stream.

II RICHARDSON NUMBER

It has been pointed out by *Reiter and Lester* [1967, 1968] that the Richardson numbers evaluated from measurements in the free atmosphere, by virtue of the presence of a pronounced mesoscale in vertical wind profiles, reveal a certain dependence on the scale length Δh over which wind shear and thermal stability are evaluated. From (1) and from the observational evidence supporting it, we have seen that mean extreme wind shears themselves are scale-dependent. We are not aware of any corresponding statistics for the vertical gradient of temperature or potential temperature. Recent developments in small-lag temperature sensors [*Caplan and Camp,* 1968] may, however, allow such an evaluation in the near future.

We will assume that the statistical relationship between mean maximum vertical shear and layer thickness (shown in Figure 1) is brought about by the onset of turbulence that prohibits the development of excessive shears. Using the scale dependence of these maximum shears expressed by (1) and by an exponent $a_1 = 1/3$, in the Richardson number, we obtain

$$Ri = \frac{(g/\theta)\Delta\theta/\Delta h}{(\Delta V/\Delta h)^2} \qquad (8)$$

and substituting for Δv from (1) (with $a_1 = 1/3$), we arrive at

$$Ri = g\cdot\Delta\theta\cdot\Delta h^{1/3}/\theta\cdot a_0^2 \qquad (9)$$

Businger [1968] argues that turbulence will develop out of shear flow when the Richardson number is below a critical level of 0.25. Let us assume that this value of Ri also characterizes the critical limit to which the mean maximum wind shears in expression (9) may develop. With this assumption, we arrive at

$$\Delta\theta = 11.77/(\Delta h)^{1/3} \qquad (10)$$

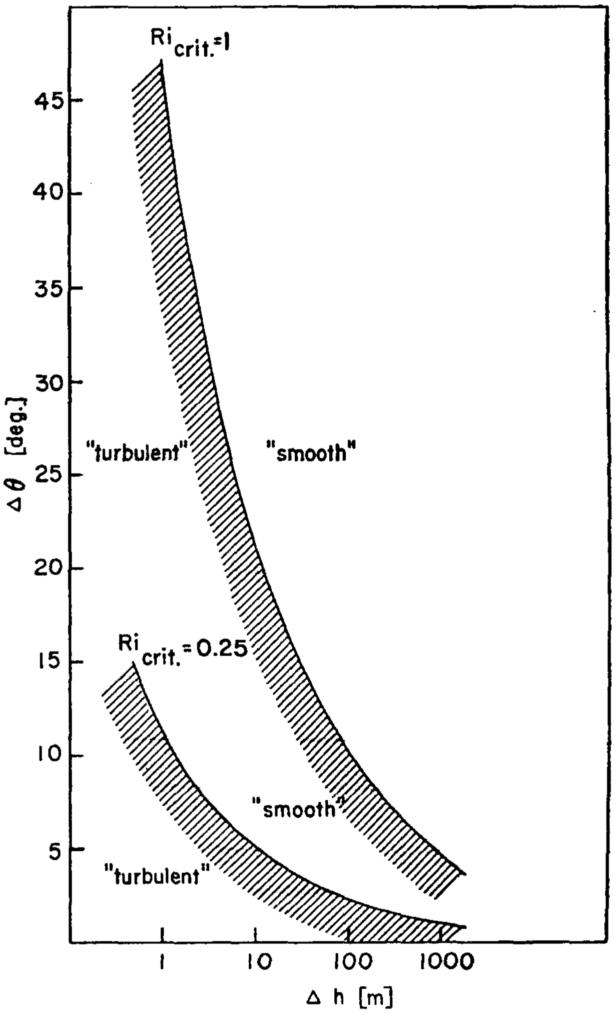

Fig. 2. Maximum vertical temperature differences $\Delta\theta$ in degrees under conditions of maximum vertical wind shear as given by equation 1 ($a_0 = 1.22$ m$^{2/3}$ sec^{-1}, $a_1 = 1/3$) for 'critical' Richardson numbers as indicated.

where $\theta = 310°$, $g = 9.8$ m sec^{-2}, and $a_0 = 1.22$ m$^{2/3}$ sec^{-1}. A critical Richardson number of 1 would yield, for the same parameter values as above,

$$\Delta\theta = 47.08/(\Delta h)^{1/3} \qquad (11)$$

where $\Delta\theta$ indicates the maximum vertical potential temperature gradient as a function of layer thickness Δh, which would be required to maintain a state of 'just no turbulence' under (mean) extreme shear conditions. The two functional relationships (10) and (11) are shown in Figure 2.

If for a given height increment Δh, the observed temperature difference falls *below* the lines shown in Figure 2, turbulence should be expected, provided that the corresponding wind shear in this layer reached maximum values. From Figure 3 it appears that strong inversion conditions of the order of 12°/1 meter, or 5°/10 meters should rarely occur in the atmosphere. We may conclude, therefo *shallow* layers revealing maximum shear con should *always* ensue in turbulent flow. Deep however, may be turbulent or 'smooth,' even the wind shear conditions given by (1) with a_1 depending on the actual temperature structure der isothermal conditions in the stratospher $\Delta\theta \sim 10°$ for $\Delta h = 1000$ meter. Such a value still render 'smooth' conditions for Ri_{crit} = Turbulence may exist, however, for Ri_{crit} = 1

Reiter [1968], *Reed* [1968], and *Businger* | have speculated that the development of a turbulent region within a relatively thick sh layer should result in the development of an batic region with a nearly constant wind bo by two stable layers in which strong vertical shears are concentrated. The mixing pro within the turbulent layer also apply to passiv

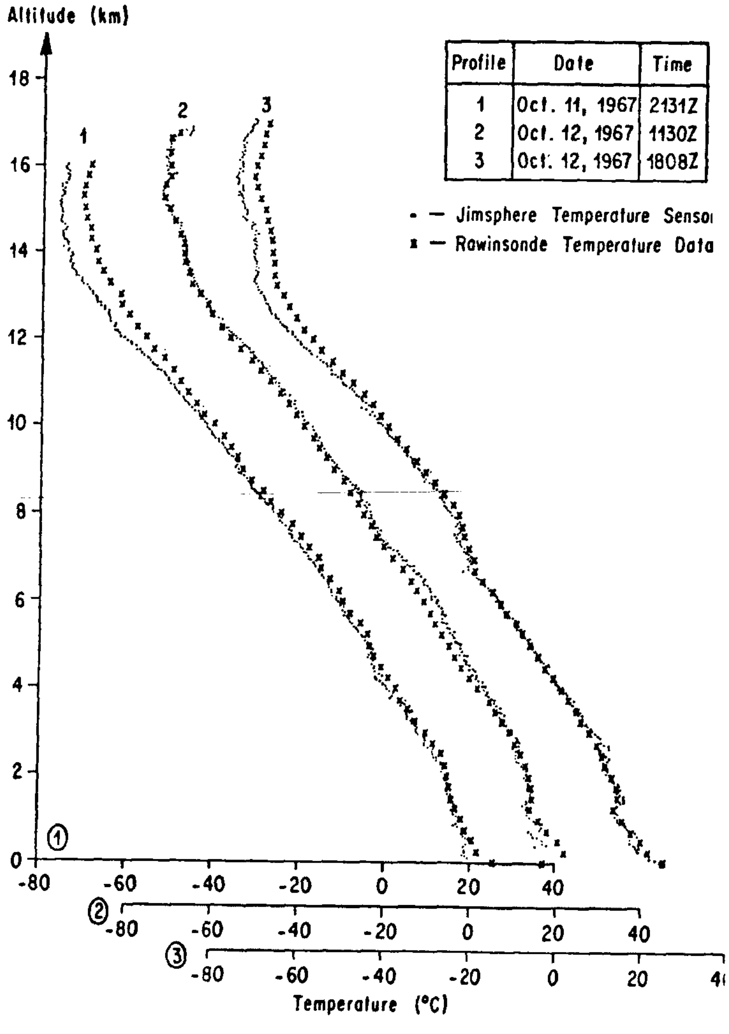

Fig. 3. Temperature measurements over Cape Kennedy with Jimsphere sensor (dots) and rawinsonde (crosses) for dates as indicated. [After *Caplan and Camp*, 1968.]

mixtures, such as water vapor. If a vertical gradient of this quantity existed prior to the onset of turbulence, the mixing process will 'concentrate' the gradient along the upper and lower boundaries of the turbulent layer. Backscattering of radio or radar signals will be enhanced by the resulting sharp gradients in the refractive index.

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