An effective choice, for this purpose, is to plot the quantity
\[
\frac{q_1}{Q_1} = \left[ 1 - p \left( \frac{\chi}{4\chi (1-\xi)} \right) \right]
\]
as ordinate, against \( \frac{dQ_1}{d\xi} \) as abscissa. The coupling variable is \( \xi \).

A chart constructed in this manner is shown in figure. The five curves apply to the five \( x_1 \) distances of table. These curves are plotted directly from the values in tables and . The results of the computation are shown in the following table.

Table

<table>
<thead>
<tr>
<th>Distance ( x_1 ) (Feet)</th>
<th>Area in(^2)</th>
<th>Factor ((0.1)(100))</th>
<th>( R_s ) Depletion Rate ((\text{ft}^3/\text{sec}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1320</td>
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<td>10</td>
<td>396.2</td>
</tr>
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<td>10</td>
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<td>10</td>
<td>164.4</td>
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<tr>
<td>21120</td>
<td>4.69</td>
<td>10</td>
<td>46.9</td>
</tr>
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</table>

The depletion rates computed in this way would be appropriate if all the pumping were done at the distance from the river, \( x_1 \), shown in the left hand column. It will be necessary to apply a factor which accounts for the distribution of the pumps. The distribution of pumps is as indicated on figure. This chart shows the distribution in terms of the decimal fraction of the total pumps which lie within the distance \( x_1 \) from the river. If \( p \) represents this fraction then the total stream depletion \( D_s \) would be given by the integral

\[
D_s = \int R_s \frac{dp}{d\chi_1} d\chi_1
\]
where $x_m$ represents the maximum value of $x_1$.

A good choice for graphical evaluation of this integral would be to plot $R_s$ against $\int \frac{dp}{dx_1} dx_1 = p$. The coupling variable is $x_1$.

This chart is shown in figure. When the area under the curve is measured with a planimeter and the appropriate factor is applied it is found that the stream flow is depleted at the end of the year 1957 by 251 cubic feet per second. Then at this time the effect of all of the pumping which has been done between 1930 and 1957 inclusive is estimated to reduce the flow of the stream leaving this section of the river by 251 cubic feet per second. This is the estimated amount by which the flow of the stream is reduced below what it would have been if the pumping had not been done.

Some further explanation of the nature of these charts is provided by the shaded areas. On figure the shaded area represents the stream depletion at the end of the year 1957 of the pumping which originated in the 18th year of pumping. A reference to table will show that this increment is $170-107 = 63$ cubic feet per second. This will be found to be, to scale, the width of the shaded increment. The ordinate of the shaded increment represents the river depletion at the end of 1957 resulting from the pumping of 63 second feet during the interval between 1948 and 1957. The evaluation is appropriate for pumps at a distance of 5280 feet or one mile from the river. The area under the entire curve represents the river depletion at the end of 1957 if all the pumping were done at the distance from the river shown on the curve.

The shaded element of figure shows the stream depletion due to the approximately 2 percent of the total pumping between 0.63 and 0.65. The coupling variable $x_1$, as plotted on this chart, indicates that the increment is located, roughly, at $1\frac{3}{4}$ miles from the river. For the
entire pumping, if concentrated at this distance, the depletion rate would be about 200 cubic feet per second.
References

The Effect of a Well on the Flow of a Nearby Stream, by C.V. Theis-Trans Amer Geophysical Union -Part 3, Page 734-738, 1941.


Bessel Functions for Engineers, by N.W. McLachlan- Oxford-1934.


Tables of the Bessel Functions $Y_0(x), Y_1(x), K_0(x), K_1(x)$ $0 \leq x \leq 1.$, National Bureau of Standards. Applied Mathematics Series 25-1952.


Tables of Functions-Jahnke and Emde-Dover-1945.

Mathematical Tables-Dwight-Dover-1958.


Heat Conduction, by L.R. Ingersoll, O.J. Zobel and A.C. Ingersoll. App F Tables of the Integral $\int_{x}^{\infty} \frac{e^{-y^2}}{y} dy$ Page 253-


A Short Table of $I(0,1;x)$ by J.C. Jaeger and Martha Clarke-Proc. Royal Soc. Edinburgh-Section a, Part III-Page 229-230., 1942.

(Note: $\frac{G(\frac{4\sqrt{t}}{a^2})}{\pi} = \frac{4}{\pi} I(0,1;x)$.)


Non Steady Radial Flow in/Infinite Leaky Aquifer, by M.S. Hantush and C.E. Jacob—Trans Amer Geophysical Union—36 Pages 95-100-1955.


Bureau of Reclamation Technical Memorandum No.
Canal seepage.

The drawdown is

\[ y = \frac{Q_1 x}{2\pi K D} \int_{0}^{\infty} e^{-\frac{u^2}{4\pi (\frac{x}{4\pi c})}} du = \frac{Q_1}{2} \Psi(\frac{x}{4\pi c}) \]

The flow is

\[ F = \frac{Q_1}{2} \left[ 1 - \frac{2}{\pi} \int_{0}^{\infty} e^{-\frac{u^2}{4\pi c}} du \right] = \frac{Q_1}{2} \Psi(\frac{x}{4\pi c}) \]

For a canal at the boundary of the alluvial sediments in a river valley, there is no flow in the direction of the river and no increase in \( y \) at \( x=W \). This condition can be met by the series

\[ F = 2Q_1 \left[ S\left(\frac{W}{4\pi c}\right) - S\left(\frac{3W}{4\pi c}\right) + S\left(\frac{5W}{4\pi c}\right) - \ldots \right] \]

The rise of the water table is

\[ y = \frac{Q_1}{2} \left[ 2\Psi(x, t) - 2\Psi(2W-x, t) + 2\Psi(W+x, t) - 2\Psi(3W-x, t) - \ldots \right] \]

RGO Nov 30 1959
Computation of \[ \int \frac{e^{-u^2}}{u} \, du \]

\[ \sqrt{\frac{x}{4x^4}} \]

\[ \sqrt{\pi} = 1.77245 \]

Refer to table obtained from graphical integration

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \int \frac{e^{-u^2}}{u} , du )</th>
<th>( \sqrt{\pi} \int \frac{e^{-u^2}}{u} , du )</th>
<th>Values from Bättinges Table</th>
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</table>

Check is satisfactory

R.G. Mar 20 1959
Evaluation of \[ \prod \int_{0}^{\infty} \frac{e^{-u^2}}{u} \, du \]

Let \[ u_1 = e^{-u^2}, \quad \frac{du_1}{u} = -2ue^{-u^2} \, du \]

Then,

\[ \prod \int_{0}^{\infty} \frac{e^{-u^2}}{u} \, du = \prod \left[ \int_{0}^{\infty} \frac{e^{-u^2}}{u} \, du - \frac{1}{2} \int_{0}^{\infty} 2e^{-u^2} \, du \right] \]

\[ = \prod \left[ \int_{0}^{\infty} \frac{e^{-u^2}}{u} \, du \right] - \frac{1}{2} \int_{0}^{\infty} 2e^{-u^2} \, du \]

\[ = \int_{0}^{\infty} \frac{e^{-u^2}}{u} \, du - \frac{1}{2} \int_{0}^{\infty} e^{-u^2} \, du \]

Values

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<th>e^{-x^2}</th>
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<th>\frac{1}{2} \int_{0}^{\infty} e^{-u^2} , du</th>
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</table>

\[ \pi = 1.77245 \]

\[ \prod \int_{0}^{\infty} \frac{e^{-u^2}}{u} \, du = \prod \left[ \frac{e^{-x^2}}{x} - 1 + \frac{2}{\pi} \int_{0}^{\infty} e^{-u^2} \, du \right] \]
From Jaeger, "Heat Flow in the Region bounded Internally by a circular cylinder." - The heat flow per unit of area per unit of time is

\[ F = \frac{4 \pi K \nu}{\alpha t^2} I_0(0, 1, x) \]

Where

\[ \alpha = \frac{K t}{\alpha^2} \]

Here \( K \) represents the conductivity, \( \nu \) the diffuseness constant, \( \alpha \) the radius of the hole, \( I_0 \) the initial temperature difference.

The heat flow per unit of length is

\[ 2\pi a F = \frac{2\pi a R \nu}{\alpha t^2} I_0(0, 1, x) \]

\[ 2\pi a F = \frac{8 \pi K \nu}{\alpha t^2} I_0(0, 1, x) \]

For the Neuman function the total flow into the well is

\[ Q = 2\pi R \nu h_0 G(\frac{4\pi a t}{\alpha}) \]

The Jaeger & Clarke function translated into the hydraulic case would be

\[ Q = \frac{8 \pi R \nu h_0}{\alpha t^2} I_0(0, 1, x) \]

Then

\[ 2\pi R h_0 G(\frac{14\pi a t}{\alpha}) = \frac{8 \pi R \nu h_0}{\alpha t^2} I_0(0, 1, x) \]

\[ \frac{G(\frac{14\pi a t}{\alpha})}{\alpha t^2} = \frac{8 \pi R \nu h_0 I_0(0, 1, x)}{2\pi R h_0} = \frac{4}{\alpha t^2} I_0(0, 1, x) \]
<table>
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<th>$x^2$</th>
<th>$I_0(x, y)$</th>
<th>$\beta_0(0, 0)$</th>
<th>$\frac{4\beta_0}{x^2}$</th>
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\[
\frac{4}{\pi^2} = 0.405,284,734,57
\]
PREFACE

The science of groundwater hydraulics has advanced considerably since 1935 due to the development of mathematics of transient states. These mathematical procedures have been adapted, to a great extent, from the field of heat conduction in solids. Since steady state situations rarely occur in nature, these developments have provided the groundwater hydrologist more adequate tools to handle field problems.

Several good references are available to the student and the field worker dealing with steady state problems in groundwater flow. However, the mathematical theory and its application to transient groundwater situations is quite scattered through 25 years of technical and scientific journals. Because of its importance to the field, the authors have attempted to present all available information on the theory of transient groundwater situations, and its application to field problems. Workers and students in the fields of groundwater, irrigation and drainage, in particular, should find this text of value.

The book is divided into two parts. Chapters I through VI contain the mathematical development and proofs of first approximation equations for one-dimensional and radial flow under various boundary conditions. The development of these first approximation equations necessarily requires the assumption of many rather ideal conditions, for the sake of simplicity. Therefore, the last Chapters of the text are devoted to methods of adapting the first approximations to field conditions. Numerous worked examples are included to illustrate field applications.
A wide range of symbols, notations and units are in use throughout the World by engineers, physicists, geologists and agronomists working on groundwater problems. All equations presented herein may be used with any consistent set of units. A list of the symbols and notations used, along with their respective dimensions, is given just previous to the first Chapter. For use in converting from one system to another, a table of useful equivalents is given in Appendix I.

Acknowledgement is also made to members of the Civil Engineering Staff at Colorado State University and of the USDI Bureau of Reclamation. ---
Computation of

$$\sqrt{\frac{1}{\pi}} \int_0^\infty \frac{e^{-u^2}}{u^2} \, du = \pi \left[ \frac{e^{-w^2}}{w^{1/2}} - 1 + \frac{2}{\pi} \int_0^w e^{-u^2} \, du \right]$$

$$w = 0.001$$

\[
e^{-w^2} = e^{-0.000001} = 1 - w^2 + \frac{w^4}{4} - \ldots = 0.999,999,0000
\]

\[
\frac{e^{-w^2}}{w} = 999.9990000 \quad \frac{e^{-w^2}}{w^{1/2}} = 564.1890193
\]

\[
\frac{2}{\pi} \int_0^w e^{-u^2} \, du = 0.00112, 83788
\]

\[
\int_0^w \frac{e^{-u^2}}{u^2} \, du = 1769.314030
\]

D.G. Feb 24, 1961
Draws on the barrier.

Compute area under the curve $\frac{xL}{2}$ from Fig. 4 of Dunn's paper.

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<th>$y$</th>
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\[
\frac{(22.860)(0.2)}{6} = 0.762
\]

\[
\frac{1}{0.762} = 1.312
\]

\[
y = \frac{1.312}{\frac{9(xL^2)}{2} + 1}
\]

\[
(\frac{xL}{L^2})
\]

\[
y
\]

\[
4\frac{xL}{L^2}
\]

\[
\sqrt{\frac{4xL}{L^2}}
\]

Note. This is a recomputation for a curve with ordinates increased enough to represent the volume HL at time zero.

E.C. July 20 1960
\[ h = H \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{e^{-n^2 \pi^2 x^2}}{n^2} \sin \frac{n\pi}{L} x \]

Then, of the initial saturated volume \( HL \) the amount remaining is

\[ \int_0^L h \, dx = -H \frac{4L}{\pi} \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{e^{-n^2 \pi^2 x^2}}{n^2} \cos \frac{n\pi}{L} x \int_0^L \]

\[ = HL \frac{8}{\pi^2} \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{e^{-n^2 \pi^2 x^2}}{n^2} \]

The part of the original volume remaining is

\[ p = \frac{\int_0^L h \, dx}{HL} \]

\[ p = \frac{8}{\pi^2} \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{e^{-n^2 \pi^2 x^2}}{n^2} \]

Check

When \( t = 0 \) then \[ p = \frac{8}{\pi^2} \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{1}{n^2} \]

\[ \sum_{n=1,3,5}^{n=\infty} \frac{1}{n^2} = \frac{(2^2-1) \pi^2}{4} \cdot \frac{1}{6} = \frac{\pi^2}{8} \]

Hence when \( t = 0 \), \[ p = 1 \quad \text{OK}. \]

\[ \frac{8}{\pi^2} = 0.81056494691. \]
\[
\sum_{n=1,3,5 \text{ etc}}^{n=\infty} \frac{\frac{8}{\pi^2} \left( \frac{n^2 \pi^2 x^2}{L^2} \right)}{n^2} = p
\]

<table>
<thead>
<tr>
<th>( \frac{\sqrt{\frac{8}{\pi^2}}}{L^2} )</th>
<th>( \frac{\alpha t}{L^2} )</th>
<th>( \frac{\pi^2 x^2}{L^2} )</th>
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<td>.2500</td>
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<tr>
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(\( \text{H}^{\text{free}} \)) = 94284
\( \text{av.} \) 88675
\( \text{av.} \) 77433
\( \text{av.} \) 64205
\( \text{av.} \) 54875
\( \text{av.} \) 43973
\( \text{av.} \) 33345
\( \text{av.} \) 24195
\( \text{av.} \) 16711
\( \text{av.} \) 10985
\( \text{av.} \) 0.06805
\( \text{av.} \) 0.02321
\( \text{av.} \) 0.00643
\( \text{av.} \) 0.00217
\( \text{av.} \) 0.00026
\( \text{av.} \) 0.00004

\( \frac{8}{\pi^2} = 0.8105694691 \)
\( e = .20998 \)

Feb 28 1959

\[ \frac{8}{\pi^2} = 0.8105694691 \]
### Computation for the parallel stream chart

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<th>$X/L$</th>
<th>$P(X/L)$</th>
<th>$P(X/L)$</th>
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<tr>
<td>0.06</td>
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<td>0.1</td>
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### Additional Data

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</table>
\[
\frac{h}{H} = \frac{4}{\pi^2} \sum_{n=0}^{\infty} \frac{\varepsilon}{n^2} \sin \frac{n\pi x}{L}
\]

\[
\frac{\pi}{2} = 1.5707963 \quad \frac{h}{H} = 1.27323954
\]

\[
\sin \left( \frac{n\pi x}{L} \right)
\]

\[
\begin{array}{ccccccc}
N & 1 & 0 & 0.000 & 0.3090 & 0.5877 & 0.8091 & 0.9510 & 1.0000 \\
3 & 0 & 0.9425 & 1.88496 & 282744 & 3.76992 & 4.71237 & \\
5 & 0 & 1.57079 & 3.1416 & 4.7124 & 5.2832 & 6.2832 & 7.8540
\end{array}
\]

\[
\frac{4}{\pi^2} = 0.42441
\quad \frac{4}{5\pi^2} = 0.2546479
\quad \pi^2 = 9.8696
\]

\[
\begin{array}{cccccccc}
\frac{4\pi^2}{L^2} & = 0.2 \quad \frac{4\pi^2}{L^2} & = 0.4 \quad \frac{8\pi^2}{L^2} & = 0.1 \quad \frac{16\pi^2}{L^2} & = 0.098696 \quad \frac{9\pi^2}{L^2} & = 0.8882 \\
\frac{25\pi^2}{L^2} & = 2.4674 \quad e \approx 0.9060 \quad e^{-2} \approx \frac{2.4674}{0.8882} \approx 2.75
\end{array}
\]

\[
\begin{array}{cccccccc}
N & 1 & 0 & 0.000 & 0.3564 & 0.6778 & 0.9333 & 1.0970 & 1.1534 \\
3 & 0 & 1.1412 & 1.660 & 2.0538 & -1.024 & -1.746 & \\
5 & 0 & 0.0216 & 0 & -0.0216 & 0 & 0.0216 & \\
\end{array}
\]

(Checks With The Bureau of Reclamation Graph.)
\[
\begin{align*}
\frac{\pi^2 x^2 t}{L^2} &= 0.3 \\
\frac{4x^2 t}{L^2} &= 0.09 \\
\frac{\alpha t}{L^2} &= 0.0225 \\
\frac{\pi^2 x^2 t}{L^2} &= 2.221 \\
\end{align*}
\]
\[
\begin{align*}
\frac{9 \pi^2 x^2 t}{L^2} &= 1.9986 \\
\frac{2.5 \pi^2 x^2 t}{L^2} &= 5.552 \\
\end{align*}
\]
\[
\begin{align*}
\eta &= .8008 \\
\epsilon &= .1355 \\
\zeta &= .00388 \\
\end{align*}
\]
\[
\begin{array}{ccccccc}
2 & 0.00 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 (X/L) \\
3 & 0.00 & 0.465 & 0.547 & 0.178 & -0.348 & -0.575 \\
5 & 0.00 & 0.010 & 0 & -0.010 & 0 & +0.010 \\
\end{array}
\]
\[
\begin{align*}
\frac{\pi^2 x^2 t}{L^2} &= 0.4 \\
\frac{4x^2 t}{L^2} &= 0.16 \\
\frac{\alpha t}{L^2} &= 0.04 \\
\frac{\pi^2 x^2 t}{L^2} &= 0.39478 \\
\end{align*}
\]
\[
\begin{align*}
\eta &= .6738 \\
\epsilon &= .0286 \\
\end{align*}
\]
\[
\begin{array}{ccccccc}
2 & 0.00 & 0.120 & 0.504 & 0.694 & 0.815 & 0.858 \\
3 & 0.00 & 0.010 & 0.012 & 0.004 & -0.007 & -0.012 \\
\end{array}
\]
\[
\begin{align*}
\frac{\pi^2 x^2 t}{L^2} &= 0.5 \\
\frac{4x^2 t}{L^2} &= 0.25 \\
\frac{\alpha t}{L^2} &= 0.0625 \\
\frac{\pi^2 x^2 t}{L^2} &= 0.6168 \\
\end{align*}
\]
\[
\begin{align*}
\eta &= .5397 \\
\epsilon &= .00388 \\
\end{align*}
\]
\[
\begin{array}{ccccccc}
2 & 0.00 & 0.212 & 0.404 & 0.556 & 0.654 & 0.687 \\
3 & 0.00 & 0.001 & 0.002 & 0.000 & -0.001 & -0.002 \\
\end{array}
\]
\[
\begin{align*}
\frac{\pi^2 x^2 t}{L^2} &= 0.6 \\
\frac{4x^2 t}{L^2} &= 0.36 \\
\frac{\alpha t}{L^2} &= 0.09 \\
\frac{\pi^2 x^2 t}{L^2} &= 0.8883 \\
\end{align*}
\]
\[
\begin{align*}
\eta &= 0.8883 \\
\epsilon &= 4.114 \\
\end{align*}
\]
\[
\begin{array}{ccccccc}
2 & 0.00 & 0.162 & 0.308 & 0.424 & 0.498 & 0.524 \\
3 & 0.00 & 0.001 & 0.002 & 0.000 & -0.001 & -0.002 \\
\end{array}
\]
\[
\begin{align*}
\frac{\pi^2 x^2 t}{L^2} &= 0.7 \\
\frac{4x^2 t}{L^2} &= 0.49 \\
\frac{\alpha t}{L^2} &= 0.1225 \\
\frac{\pi^2 x^2 t}{L^2} &= 1.2090 \\
\end{align*}
\]
\[
\begin{align*}
\eta &= .307 & 0.361 & 0.380 \\
\end{align*}
\]
\[
\begin{array}{ccccccc}
2 & 0.00 & 0.117 & 0.223 & 0.307 & 0.361 & 0.380 \\
3 & 0.00 & 0.001 & 0.002 & 0.000 & -0.001 & -0.002 \\
\end{array}
\]
\[
\frac{4x^2}{L^2} = 0.8 \quad \frac{4x^2}{L^2} = 6.4 \quad \frac{x^2}{L^2} = 0.16 \quad \frac{\pi^2x^2}{L^2} = 1.579 \quad \frac{\pi^2x^2}{L^2} = 1.579 = 0.2062
\]

\[
1 \quad 0.000 \quad 0.0812 \quad 1.1544 \quad 2.12 \quad 0.250 \quad 0.263
\]

\[
\frac{4x^2}{L^2} = 0.9 \quad \frac{4x^2}{L^2} = 8.1 \quad \frac{x^2}{L^2} = 0.2025 \quad \frac{\pi^2x^2}{L^2} = 1.9985 \quad \frac{\pi^2x^2}{L^2} = 1.9985 = 0.1355
\]

\[
1 \quad 0.000 \quad 0.0533 \quad 1.015 \quad 1.397 \quad 1.642 \quad 1.728
\]

\[
\frac{4x^2}{L^2} = 1.0 \quad \frac{4x^2}{L^2} = 1.0 \quad \frac{x^2}{L^2} = 0.25 \quad \frac{\pi^2x^2}{L^2} = 2.4674 \quad \frac{\pi^2x^2}{L^2} = 2.4674 = 0.08480
\]

\[
1 \quad 0.000 \quad 0.0333 \quad 0.0635 \quad 0.0872 \quad 0.1027 \quad 0.1080
\]

\[
1.2 \quad 3.56 \quad 0.02844
\]

\[
1.4 \quad 4.83 \quad 0.03625
\]

\[
1.6 \quad 6.32 \quad 0.01596
\]

\[
1.8 \quad 8.00 \quad 0.00229
\]

\[
2.0 \quad 9.869 \quad 0.00043
\]

\[
9.869 \quad 0.00052
\]

\[
.000066
\]
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</tr>
</tbody>
</table>

Note: The values are placeholders and should be replaced with actual numerical data.
| 0.05 | 0.00325 | 0.0625 | 0.0893 | 0.1410 | 0.4764 | 1.03516 | 0.35517 | 1.3779 | 0.2521 |
| 0.10 | 0.0025 | 0.09859 | 0.16553 | 0.29855 | 0.05351 | 0.01645 | 0.055165 | 0.00388 |
| 0.15 | 0.0100 | 0.79944 | 0.0034 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.20 | 0.0250 | 0.0067 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.25 | 0.0400 | 0.0625 | 0.0625 | 0.0625 | 0.0625 | 0.0625 | 0.0625 | 0.0625 |
| 0.30 | 0.0900 | 0.1225 | 0.1225 | 0.1225 | 0.1225 | 0.1225 | 0.1225 | 0.1225 |
| 0.35 | 0.1600 | 0.2025 | 0.2025 | 0.2025 | 0.2025 | 0.2025 | 0.2025 | 0.2025 |
| 0.40 | 0.2500 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
\[ \frac{\partial^2 h}{\partial \beta \partial \alpha} = 4 \sum_{n=1}^{\infty} \sum_{\alpha} \frac{n^2 \pi^2 + \pi^2}{L^2} e^{-\frac{n^2 \pi^2 + \pi^2}{L^2}} \]

<table>
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<th>( \frac{\alpha \pi}{L} )</th>
<th>( \frac{\pi}{L} )</th>
<th>( \frac{2}{\sqrt{\pi \frac{4 \alpha \pi^2}{L^2}}} )</th>
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</tbody>
</table>

Check by use of:

\[ h = \frac{2}{\sqrt{\pi}} \int e^{-\alpha x} dx \]

\[ \frac{dh}{dx} = \frac{2}{\sqrt{\pi \frac{4 \alpha \pi^2}{L^2}}} \left( \frac{dh}{dx} \right) = \frac{2}{\sqrt{\pi \frac{4 \alpha \pi^2}{L^2}}} \]

\[ F_0 = KD_{a} \left( \frac{2H}{\pi \frac{4 \alpha \pi^2}{L^2}} \right) = \frac{KD_{a} 2H}{\pi \frac{4 \alpha \pi^2}{L^2}} = \frac{KD_{a} H}{L} \cdot \frac{2L}{\frac{1}{14 \alpha + \pi}} \cdot \frac{2}{\pi} = 1.12838 \]
\[ \frac{\sin \frac{n\pi}{L}}{n} \left( \sin \frac{n\pi x}{L} \right) \left( x = \frac{L}{2} \right) \]

| n | \(
\begin{array}{cccccccccc}
1000 & 99388 & 97553 & 90602 & 78056 & 67382 & 54204 & 41137 & 29850 & 20616 \\
3 & 0.33333 & 3.1545 & 2.6771 & 1.3712 & 0.3586 & 0.0954 & 0.0135 & 0.0011 & 0.0001 \\
5 & 0.20000 & 1.7161 & 1.0687 & 0.1696 & 0.0041 & 0.0001 \\
7 & -0.14286 & 1.0582 & 0.4243 & 0.00107 \\
9 & \begin{array}{c}
\frac{\sin \frac{n\pi}{L}}{n} \\
7.8479 & 7.7451 & 6.6429 & 5.4069 & 4.1124 & 2.9849 & 2.0616 \\
999.99 & 94.867 & 8.4576 & 0.68842 & 0.52362 & 0.38003 & 0.26249 \\
1.000 & 0.90 & 1.00 \\
13553.08396 \\
.13553 & 0.08396 \\
.1754.10688 & \text{Cement} \\
1.27324 & \text{Jan 23 1959}
\end{array}
\end{array} \right) \]

Checks from Cooling of
Concrete slabs.
For small values of $S$

\[
\frac{Y_2}{D} = 1 - \sqrt{1 - 2S \int e^{-u^2} \frac{1}{u} \, du}
\]

Suppose $S$ is very small so that $2S$ is small compared to unity.
Then approximately:

\[
\frac{Y_2}{D} = 1 - 1 + S \int e^{-u^2} \frac{1}{u} \, du
\]

\[
\frac{(Y_2' D)}{S} = \int e^{-u^2} \frac{1}{u} \, du
\]

or

\[
Y_2 = \frac{D}{\int e^{-u^2} \, du}
\]

\[
Y_2 = \frac{D}{\int e^{-u^2} \, du}
\]
Polubarinova-Kochina (in Russian)
(in English) - in Advances in Mechanics
V. Karman - V. Moses
( Harvard)

Aravin - Numerov
Chernii

Aug 1958 AGU

1951 Explanation of Darcy Law to Unaccompanied Media
January 31, 1959

Mr. Malcolm Johnson
Editor, College Department
McGraw-Hill Building
330 West 42nd Street
New York 36, New York

Dear Mr. Johnson:

As you suggested in your letter of August 5, 1958, I am transmitting herewith some extracts from the manuscript "Ground-Water Hydraulics" which Mr. R.E. Glover and myself are preparing.

The Preface and Table of Contents will give you an idea of the scope and content we have planned for this book. We intend to prepare it in such a way that it not only will serve as a text for a graduate course in the subject, but will also be useful as a reference and handbook for the field technician working in this field. Detailed worked examples are included to show typical applications of the theory, thus it is not necessary for the user to understand the mathematical developments. We feel that this text should be of value to all engineers and ground-water hydrologists who must deal with ground-water supplies for industries, municipalities and irrigation, as well as to students preparing for work in these fields.

Chapter I, "Well Pumped at a Constant Rate" is substantially complete in content. Chapters II and III contain only the development of formulas, but will be completed soon.

Chapters VII and VIII are complete enough to indicate how the usefulness of the formulas presented in the earlier chapters is to be extended.

We will appreciate having you review this material and having your comments and suggestions on it.

Sincerely yours,

[Signature]
Morton W. Bittinger
Assistant Civil Engineer