Thesis

SIMULTANEOUS SOLUTION FOR DISTRIBUTION OF HEAD IN A TWO AQUIFER SYSTEM

Submitted by

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WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR SUPERVISION BY JOHN R. ECKHARDT ENTITLED Simultaneous Solution For Distribution of Head in a Two Aquifer System BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE.

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ABSTRACT

SIMULTANEOUS SOLUTION FOR DISTRIBUTION OF HEAD IN A TWO AQUIFER SYSTEM

A general computer program, WTSHED4, was developed to simulate the simultaneous areal distribution of head in a two aquifer system as a result of natural and artificial influences to each aquifer. WTSHED4 was constructed from a program developed by the professional groundwater staff at Colorado State University. The basic mathematic principal is to solve, by Gauss elimination technique, a two-dimensional form of the Boussinesq equation using an implicit central finite difference scheme. WTSHED4 is designed to simulate nonsteady-state conditions in a two-dimensional horizontal space including nonlinear flow conditions caused by a varying transmissivity. Physiographical influences such as impermeable, permeable and hydraulic boundaries are simulated without undue idealization. The hydrologic and geologic parameters which define a particular study area are incorporated into the model and can vary in both space and time.

To check the validity of WTSHED4, five problems were modeled and results compared with analytic solutions. Agreement was very good, the only differences being a result of the time and space dimensions used in the finite difference approximation. Although WTSHED4 has been verified with several analytic solutions, it should be carefully used until additional laboratory or field verification is conducted.

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I. INTRODUCTION

Groundwater evaluation often includes the analysis of vertical leakage from layers above or below the main aquifer. An aquifer that loses or receives significant quantities of water to or from these layers is termed a leaky aquifer and should be considered as part of a multiple aquifer system. In this system, shown in Figure 1, the main aquifer is generally overlain or underlain by a stratum of relatively low permeability, termed an aquitard or aquiclude. When these confining layers are of sufficiently low permeability to prevent flow or effectively act as no flow boundaries, the analysis of the confined aquifer is comparatively simple, depending upon boundary conditions. However, when one or more of the confining layers has a permeability large enough to allow significant flow to create a leaky aquifer condition, the problem becomes analytically intractable even when restrictive assumptions and idealizations are made.

The use of numerical methods in conjunction with high speed digital computers facilitates solutions to such complex problems. A general computer model which would treat confined and unconfined aquifer problems, with or without leakage present, by altering only data input would be of great value to water administrators. In developing such a management tool, hydrologic factors of precipitation, irrigation, pumping, evapotranspiration and influences from lakes and streams must be included. Also non-homogeneous aquifer conditions and physiographical influences such as impermeable, constant head and gradient boundaries should be included. One such general model has been developed by the professional groundwater staff at Colorado State University for unconfined aquifer analysis. The computer model uses a central finite difference approximation and in its various forms has been applied to several problems by

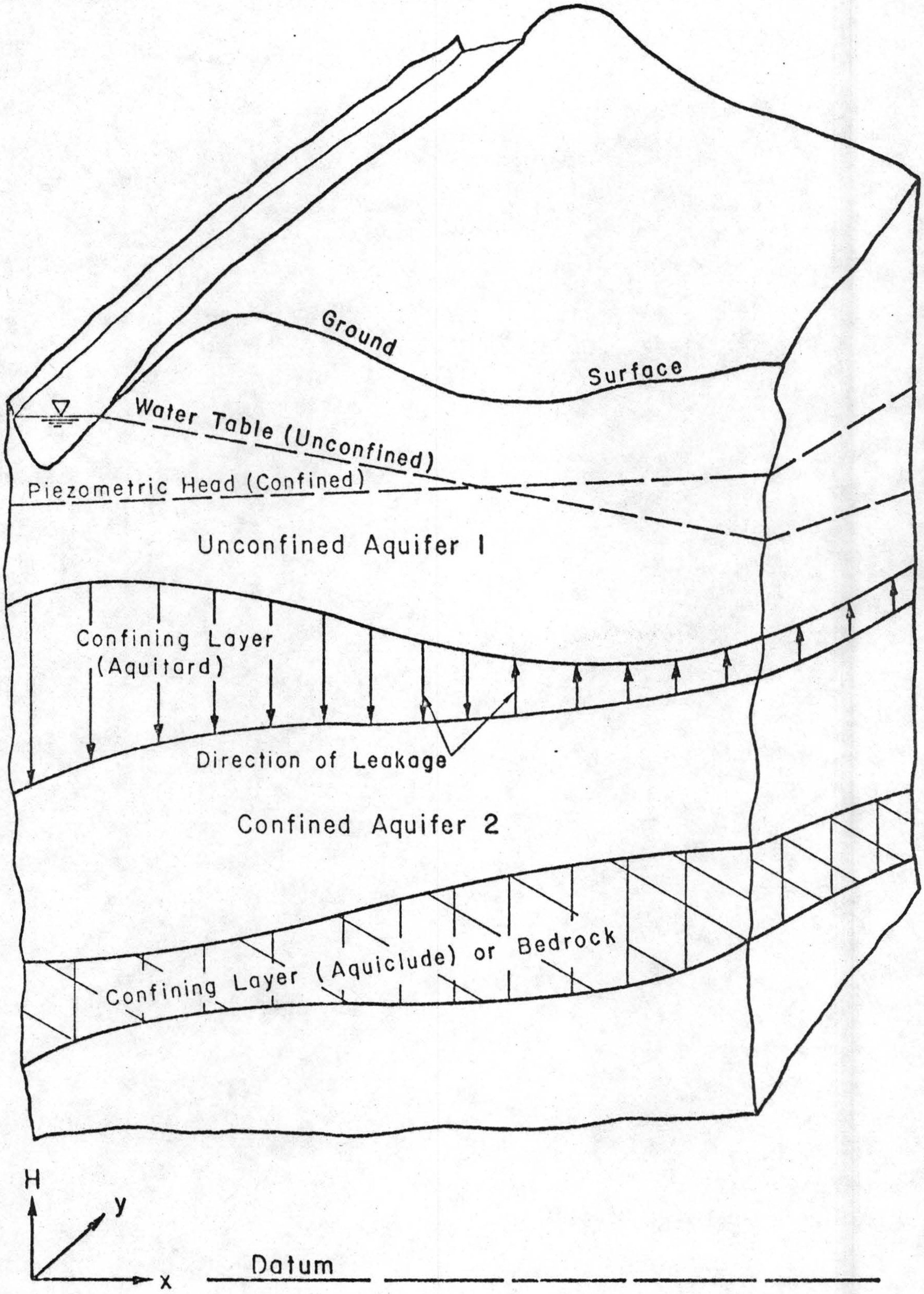


Figure 1. Two Leaky aquifer system.

Longenbaugh (37), Bibby (2), Olson (46), and Reddell and Sunada (49) and verified with a physical model by Stettner (55). Since this particular model includes all the hydrologic and geologic parameters required to model an unconfined aquifer study area, the inclusion of confined aquifer and leaky aquifer analysis would result in a general groundwater management tool.

II. OBJECTIVES

The computer program developed at Colorado State University for unconfined aquifer analysis will be modified to include confined aquifer analysis and simultaneous analysis of the interaction of a two aquifer system. After this is accomplished, the program, referred to as WTSHED4, will be verified with boundary value problems that have analytic solutions available in the literature.

III. REVIEW OF LITERATURE

SELECTED ANALYTICAL SOLUTIONS TO GROUNDWATER PROBLEMS

The utilization of groundwater dates from ancient times as witnessed by the references to wells and springs in the Old Testament. However, the occurrence and movement of groundwater was not fully understood until the latter half of the nineteenth century when Darcy (4) experimentally derived a formula known today as Darcy's law. Darcy's law has served as the basis for many analytic solutions to groundwater problems, too numerous to mention here. However, one of the first to use Darcy's law was Dupuit (9) in 1863 when he developed a steady-state formula for the flow of of water to a well. Almost 75 years later Theis (56) introduced an equation for non-steady-state flow to a well. Although Theis was the first to use the analogy of Darcy's law to the law of the flow of heat by conduction it had been recognized earlier. Subsequently many investigators have used the mathematical theory of heat conduction developed by Fourier and others to solve groundwater problems. One author used in this study, R. E. Glover, developed nonsteady-state solutions to many types of problems. Much of his work has not been published in journals but appears in reports and memoranda.

A class of problems with particular interest is the flow in leaky aquifers. The question often arises as to how much flow comes from the aquifer in which a well is completed and how much comes from beds lying above or below. If all of the well production comes from one aquifer, the behavior should follow the Theis solution. Otherwise, flow must be considered in a complex multiple aquifer system where drawdown in each layer depends on the flow behavior of the entire system. The resulting complex boundary conditions make the problem analytically intractable.

A simplifying assumption used in the solution of this problem is that the flow in the aquifers is horizontal and the flow in the aquitards is vertical. The validity of this assumption has been investigated for the case of two aquifers separated by an aquitard by Neuman and Witherspoon (44,45). They found when the permeability of the main aquifers is greater than two orders of magnitude of the aquitard permeability, the errors introduced by the above assumption are less than 5 percent. These errors increase with time and decrease with radial distance from the well.

The assumption of vertical flow in the aquitards implies that the rate of leakage into the pumped aquifer is proportional to the potential drop across the aquitard. DeGlee (5) in 1930 was the first to introduce a steady-state approach to flow in leaky aquifers using this assumption. Later on Steggeventz and Van Ness (54) used this same approach and similar work was also performed by Glebov (15), Myatiev (40,41) and Girinsky (14), as summarized by Polubarinova-Kochina (48) and Aravin and Numerov (1). Jacob (33) in 1946 used the same assumption to derive a partial differential equation for nonsteady flow in a leaky aquifer. In addition he assumed that the head in the unpumped aquifer remained constant and storage in the aquitard could be neglected. Utilizing this approach, Hantush (17, 18, 19 and 20) and Hantush and Jacob (28, 29, 30, 31 and 32) developed a large number of solutions to various problems involving flow in aquifers with vertical leakage. One of these solutions, Hantush and Jacob (30), generally referred to as the r/B solution, is of particular interest because it has been extensively tabulated (18) and the resulting type curves widely used by groundwater hydrologists. The solution describes the nonsteady radial flow to a well that completely penetrates an infinite leaky aquifer and discharges at a constant rate.

Since the r/B solution neglected storage in the aquitard, which can be of major consequence at early times, Hantush (21) later published a modified solution referred to as the β solution which included the effects of storage in the aquitard. Again by assuming no drawdown in the unpumped aquifer, Hantush was able to develop asymptotic solutions for the pumped aquifer that apply to small and large values of time. Solutions for small values of time have been extensively tabulated by Hantush (22). Hantush's modified approach has not been used much in subsequent work on leaky aquifers. From 1961 to 1967, Hantush (23, 24, 25, 26, and 27) and DeWiest (6,7) analyzed various leaky aquifer problems but storage in the aquitard was always neglected. Neuman and Witherspoon (43,44) have investigated the assumption of neglecting storage in the aquitard and found that as time increased, the error decreased and was insignificant for values of time $\geq 80\beta^2/(r/B)^4$

where:

$$\beta = \frac{r}{4}\left(\frac{K'/b'}{T} \cdot \frac{S'}{S}\right)^{\frac{1}{2}}$$

$$B = \sqrt{\frac{Tb'}{K'}}$$

K' = vertical permeability of aquitard

b' = thickness of aquitard

S' = storage coefficient of aquitard

r = horizontal radial distance from center of well

T = transmissivity of main aquifer

S = storage coefficient of main aquifer

However as time increases, the assumption that the drawdown in the unpumped aquifer remains constant may cause serious errors.

Two of the first steady-state solutions in the literature considering the drawdown to vary in the unpumped aquifer were described in 1962 by Polubarinova-Kochina (48) and Spiegel (52). Later on, Hantush (27) considered the nonsteady-state case of the problem but neglected storage in the aquitard. The most recent development was by Neuman and Witherspoon in 1969 (43,44) who investigated the problem of flow in a system consisting of two aquifers separated by an aquitard. They considered storage in the aquitard and drawdown due to leakage in the unpumped aquifer to arrive at a complete solution. Although the resulting equations are very complex, their validity was checked using a numerical integration technique and comparing results with a finite element method described by Javandel and Witherspoon (34, 35, and 36).

IV. METHOD OF INVESTIGATION

A. WTSHED4 Development

The computer program WTSHED2 developed by the professional staff at Colorado State University will first be modified to consider confined aquifer analysis including the problem of changing from confined to unconfined or unconfined to confined. After this is accomplished, WTSHED2 will then be modified to consider leaky aquifer conditions including the simultaneous analysis of the interaction of a two aquifer system. The resulting computer program will be referred to as WTSHED4.

The basic principal of WTSHED2 is to solve by numerical techniques a two-dimensional form of the Boussinesq equation. This equation is a second order non-linear partial differential equation which represents transient two-dimensional flow in a saturated water table aquifer. No general analytic solution has been obtained for this equation. However, utilizing a central finite difference approximation, solution for water table aquifer problems using a digital computer was developed (see Appendix A).

The area to be studied is represented by a system of rectangular grids which may be oriented in the horizontal or vertical plane. A mass balance equation written for each grid results in the central finite difference form of the Boussinesq equation. WTSHED2 is developed such that the implicit central finite difference form of the Boussinesq equation is written for each grid as a function of flow across each of its four surfaces and the net vertical withdrawal. The resulting system of equations is then solved simultaneously, using Gauss-Elimination, for the water table elevation at the end of a selected time step. This predicted value will then be used as the initial value for the next time step and the entire process repeated. Successive solutions for the following time increments form the complete analysis.

WTSHED2 consists of a main controlling program and several subprograms. The main program's primary function is to control the execution of subprograms for all time steps at which calculations are desired. The basic sequence of events is shown in Figure 2. The subprograms are designed for specific tasks, such as physical parameter input, solving a set of simultaneous equations and mass balance computations. A description of each subprogram is contained in Appendix D.

With minor modifications WTSHED2 may be used to model any unconfined aquifer system including those hydraulicly connected to surface water. The program is designed to simulate nonsteady-state conditions in a two-dimensional horizontal space including nonlinear flow conditions caused by a varying transmissivity. Physiographical influences such as impermeable, permeable and hydraulic boundaries are simulated without undue idealization. Essentially a physical boundary is represented by a grid with permeability equal to zero, piezometric head equal to a constant, or hydraulic gradient equal to a constant. The hydrologic and geologic parameters which define a particular study area are incorporated into the model and can vary in both time and space. The following average or representative parameters must be determined for each grid for both aquifers in addition to space and time dimensions.

1. G Ground surface or top of confined aquifer elevation (feet).
2. Z Bedrock or bottom of aquifer elevation (feet).
3. PHI Specific yield or storage coefficient (decimal).
4. FK Permeability (feet/day).
5. H Initial water level or piezometric head (feet).

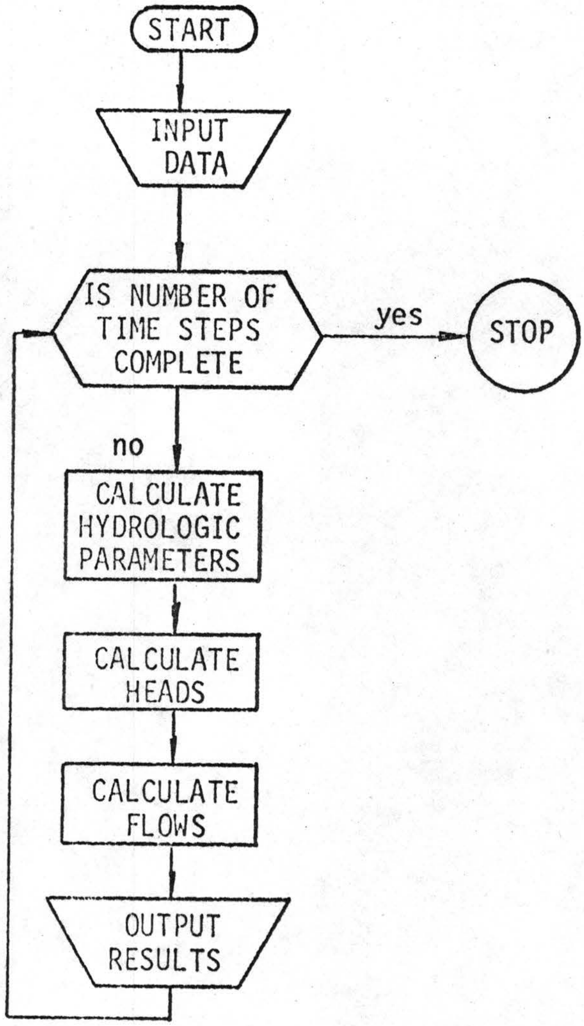


Figure 2. WTSHED2 - WTSHED4 sequence of events.

6. CA Ratio of irrigated area to total area of each grid (decimal) (unconfined aquifer only).

7. PHR Phreatophyte use (acre-feet/year) or phreatophytes present for each grid - use to be calculated based on depth to water table (unconfined aquifer only).

8. IWELL Well number if grid represents a well (integer) - may vary per year.

9. IPIT Recharge pit or line number if grid represents a pit or line (integer) - may vary per year (unconfined aquifer only).

In addition the following hydrologic parameters must be determined for every year of analysis:

1. PPT Precipitation (inches/year) - assumed uniform over the entire study area (unconfined aquifer only).

2. CPT Ratio of the amount of precipitation that reaches groundwater divided by total annual precipitation of the entire model (decimal)(unconfined aquifer only).

3. YPT Distribution of precipitation for each DT of one year (decimal) - sum of YPT=1.00 (unconfined aquifer only).

4. APW Applied water as irrigation (feet/year) - assumed uniform over study area (unconfined aquifer only).

5. CAW Ratio of the amount of applied water that reaches groundwater divided by the total annual applied water for the entire model (decimal)(unconfined aquifer only).

6. YAW Distribution of applied water for each DT of one year (decimal) - sum of YAW=1.00 (unconfined aquifer only).

7. NW Number of wells in study area (integer) - may vary per year.

8. RPUM Amount pumped by a well or wells from a grid, per year, one net well per grid (acre-feet/year).

9. CPM Ratio of the amount of water removed from groundwater due to pumping for each well divided by the total annual amount of water pumped for each well (decimal).

10. YPM Distribution of pumping for each well for each DT of one year (decimal) - sum of YPM-1.00.

11. NP Number of recharge pits or lines in study area (integer)- may vary per year (unconfined aquifer only).

12. RCHR Amount each pit or line recharges per year (feet).

13. YRC Distribution of recharge for each pit or line for each DT of one year (decimal) - sum of YRC=1.00 (unconfined aquifer only).

Two items had to be considered when WTSHED2 was adapted to handle confined aquifer conditions: (1) the Boussinesq equation in central finite difference form must be linearized or altered to utilize a time constant saturated thickness, and (2) the conditions when a confined aquifer becomes unconfined or when a confined aquifer is unconfined and becomes confined. The saturated thickness, $h^t_{i,j-1/2}$ between grids (i,j-1) and (i,j) is computed by the following method, (see Figure 3).

$$U_{i,j} = \min(G_{i,j}, H_{i,j})$$

$$U_{i,j-1} = \min(G_{i,j-1}, H_{i,j-1})$$

$$h^t_{i,j-\frac{1}{2}} = \max(U_{i,j}, U_{i,j-1}) - \max(Z_{i,j}, Z_{i,j-1})$$

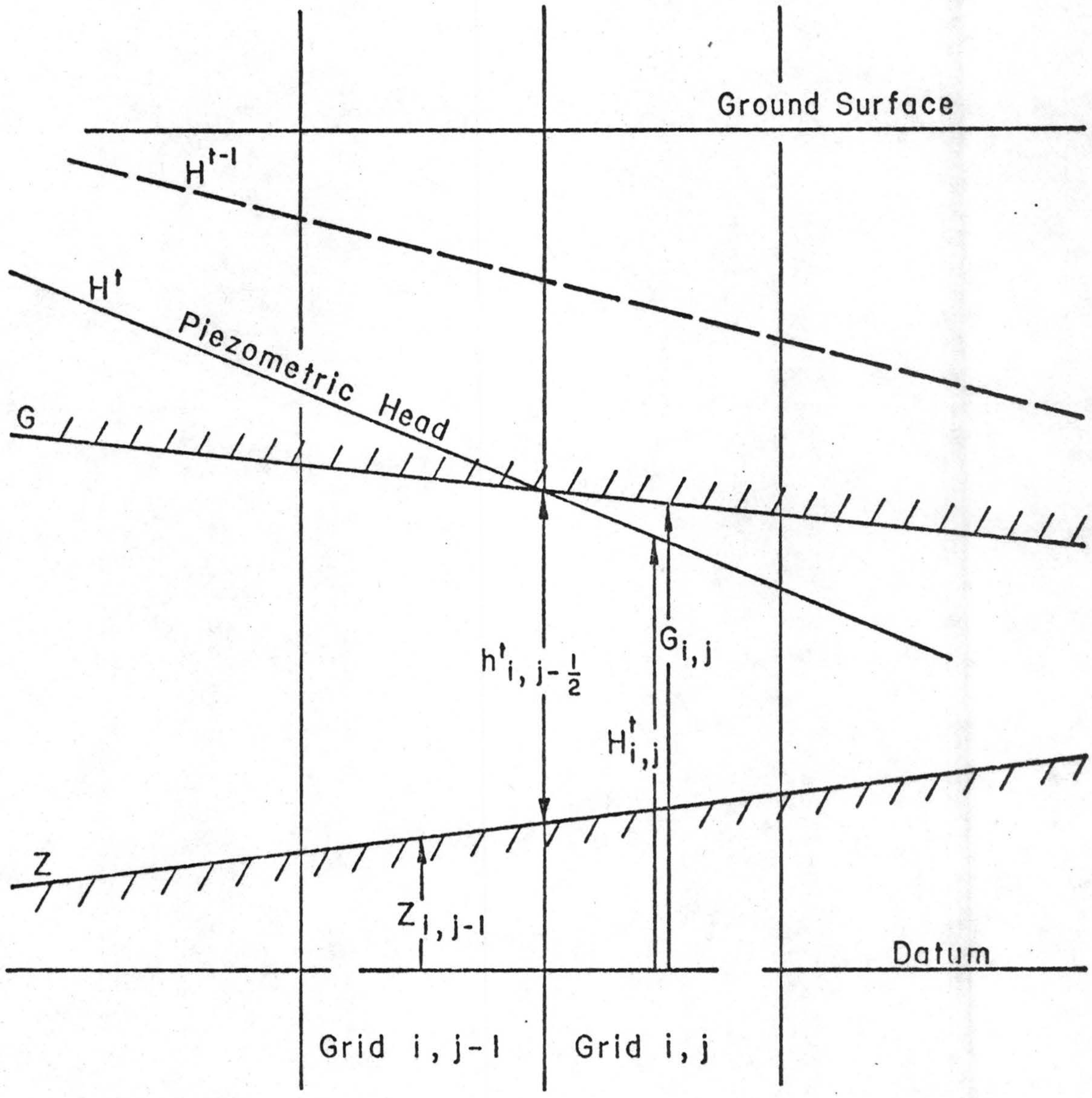


Figure 3. Finite difference grids for confined aquifer

where:

$G_{i,j}$ = top of confined aquifer elevation

$H_{i,j}$ = piezometric head

$Z_{i,j}$ = bottom of confined aquifer elevation.

These equations will ensure that the flux out of a dry grid will be zero. Also the same equations can be used for unconfined aquifers where G is replaced by the ground surface elevation. If a confined aquifer never becomes unconfined, the saturated thickness is constant in time and the coefficients A, B, C, and D in the central finite difference equation are constants for a particular grid (see Appendix A).

The condition when a confined aquifer becomes unconfined or when a confined aquifer is unconfined and becomes confined can be solved by considering the change in storage in a particular grid per time step. Referring to Figure 3,

S_u = unconfined aquifer specific yield

S_c = confined aquifer storage coefficient.

For the unconfined case:

$$\frac{\Delta\ \text{Storage}}{\Delta x \Delta y} = (H^{t+1} - H^{t})S_u \tag{1}$$

For the confined case:

$$\frac{\Delta\ \text{Storage}}{\Delta x \Delta y} = (H^{t+1} - H^{t})S_c \tag{2}$$

The case where the aquifer changes from confined to unconfined is:

$$\begin{aligned}\frac{\Delta\ \text{Storage}}{\Delta x \Delta y} &= (H^{t+1}-Z)S_u-[(G-Z)S_u+(H^{t}-G)S_c] \\ &= (H^{t+1}S_u-H^{t}S_c)-G(S_u-S_c) \qquad (3)\end{aligned}$$

The case where the aquifer changes from unconfined to confined is:

$$\frac{\Delta \text{ Storage}}{\Delta x \Delta y} = [(G-Z)S_u+(H^{t+1}-G)S_c] - (H^t-Z)S_u$$
$$= (H^{t+1}S_c - H^tS_u) + G(S_u - S_c) \qquad .(4)$$

For the confined and unconfined case, the change in storage per time step can be handled by using the appropriate storage coefficient but for the confined to unconfined or unconfined to confined case the following term must be considered:

$$(H^{t+1} - G)(S_u - S_c) \quad .$$

For example, when a confined aquifer becomes unconfined, equation 3 is used. If no change occurs, that is the aquifer remains confined, equation 2 is used. The difference between these two equations is:

$$H^{t+1}(S_u - S_c) - G(S_u - S_c)$$

or

$$(H^{t+1} - G)(S_u - S_c) \quad .$$

WTSHED2 was altered, hereafter referred to as WTSHED4, for confined aquifer analysis by first using the appropriate storage coefficient and proceeding through the normal calculations. Next the computed H values for each grid at the previous and present time steps are checked to see if the aquifer became unconfined or confined. If no grids change from confined to unconfined or unconfined to confined, WTSHED4 proceeds on with the next calculations. If there is a change, the term $(H^{t+1} - G)$ $(S_u - S_c)$ is added to equation 2 for a change from confined to unconfined and subtracted from equation 1 for a change from unconfined to confined. With these changes made, the equations are solved a second time and the program proceeds neglecting the first solutions. In addition, if a change from confined to unconfined or unconfined to confined takes place, a message such as "GRID 7 9 CONFINED TO UNCONFINED" is printed.

When WTSHED4 predicts a grid change from confined to unconfined or unconfined to confined, if a large space or time dimension is being used, the possibility of an oscillation between either equations 2 and 3 or 1 and 4 exists. This can be detected by comparing the computed piezometric head values with the confined to unconfined or unconfined to confined message referred to above at the time step of occurance. For example, the message: "GRID 5 10 CONFINED TO UNCONFINED" is printed at time step 6. The results indicate for this time step the piezometric head for grid 5, 10 is 166 feet. The top of the confined aquifer elevation for grid 5,10 is 165 feet, therefore, indicating grid 5,10 is still confined. This oscillation problem can be corrected by decreasing the space and/or time dimensions.

All of the changes referred to above made in development of WTSHED4 to consider confined aquifer problems were made in subprograms MATSOL and PARAM. Various input and output statements along with many other minor changes were made to input and output all required data.

In addition to the inclusion of confined aquifer analysis, leaky aquifer conditions are also incorporated in WTSHED4. A two aquifer system as shown in Figure 1 is considered. Several problems had to be solved before the leaky aquifer condition could be considered:

(1) Interaction between aquifers.

(2) Simultaneous analysis of both aquifers.

(3) Amount of computer storage.

(4) Input and output of data.

Two assumptions were made when formulating the leaky aquifer problem. (1) Since most problems considered by WTSHED4 will involve long period analysis, the storage in the aquitard is neglected, and (2) the flow between the aquifers is expressed by the product of the vertical permeability of the leaky aquitard and the difference between the water table elevation and the piezometric head. Using these assumptions and holding either the water table elevation or piezometric head constant, results in a simple confined or unconfined aquifer analysis with the leakage expressed as a vertical input per grid per time step as a function of the previous changing water table or piezometric head. This is actually a three-dimensional problem for which no known analytic solution exists. Since WTSHED4 is to be used for water resources management, the areal distribution of water table and piezometric head is of importance rather than a one- or two-dimensional vertical problem. This three-dimensional problem can be changed to two areal dimensions by assuming that the vertical flow through the aquitard is reflected to perfectly horizontal flow in the aquifers. As mentioned in the literature review, this assumption results in errors of less than 5% when the permeabilities of the aquifers are greater than two orders of magnitude of the aquitard.

The remaining problem in development of the leaky aquifer conditions was to simultaneously analyze both aquifers at one time. No equations exist for this condition except to treat the leakage as a boundary condition. Using the same concept to develop WTSHED4, first the unconfined aquifer is analyzed for time step one using the initial heads in both aquifers to compute the leakage term. Then the confined aquifer is analyzed using the same leakage from time step one of the unconfined aquifer analysis to yield a new head. The entire process is repeated for the required number of time steps to complete the solution.

Using this approach for simultaneous aquifer analysis requires essentially twice the computer space, therefore, generally limiting the size of problem that can be analyzed. A scheme was developed to conserve computer storage. It consisted of storing data and results for the aquifer not under analysis on a file rather than direct access storage and interchanging data and results back and forth as needed. For example, if the unconfined aquifer is being analyzed, the confined aquifer data and results would be stored on a file. Then when the confined aquifer is to be analyzed, the confined aquifer data and results would be transferred to direct access storage and the unconfined aquifer data and results stored on a file. Subprogram DATRANS was developed to handle the data transfer when called at the appropriate times. Subprogram QFIX was altered to calculate the leakage terms and add or subtract the values where necessary.

In solving large areal problems not all boundaries of a study area can usually be considered constant head, constant gradient or impermeable. Therefore, a gradient or underflow boundary was developed by projecting a computed gradient interior to a study area to the outside or boundary grids. The procedure is to hold the water level or piezometric head at the gradient boundary grids constant at odd time steps and adjust the heads at even time steps. The adjustment is made by calculating the gradient between two and three grids inward of the boundary and projecting this gradient back to the boundary to adjust the water table or piezometric head. Using this procedure avoids the problem of a constant gradient boundary, but when used, results should be checked. All gradient boundary calculations are performed in subprogram BJUST.

Finally a method was devised to handle the great quantity of variable input and output. Utilizing several CDC system subprograms, a method developed at Colorado State University referred to as "Dynamic Core Allocation" was used so that no change in dimension statements with the program would have to be made for any size of problem considered. This allows the user to compile the program and store it on a permanent file requiring only data input for use. Data card input was changed to allow a small amount for simple problems and as much as desired for complex detailed problems (depending on grid density). The amount of computer storage is adjusted by the size of the problem considered. Also an input code was devised to distinguish confined, unconfined and leaky aquifer analysis. Output consists of a repeat of all input data and results at the desired time steps. Error messages such as, "WITHDRAWAL RESTRICTED IN GRID 6 8" are printed when encountered. A list of important variables and a detailed flow chart are included in the appendices. A complete listing of program WTSHED4 is available from the Groundwater Section at Colorado State University, upon request.

B. Analytical Verification.

In order to check the validity of program WTSHED4, four analytical solutions to boundary value problems were selected from the literature to be compared with solutions to WTSHED4 (see Table 1). One difficulty with this method, as with any numerical-analytical comparison, is to decide how detailed the numerical approximation should be to achieve a desired accuracy. For the case of WTSHED4, a finite difference model, this means that the proper Δx and Δy spacing in conjunction with the time increment must be determined. In order to conserve computer time, the space and

Table 1

Analytic Solutions

Description	Equation	Reference
Parallel Drains	$h = \frac{4H}{\pi} \sum_{n=1,3,5}^{\infty} \frac{e^{-n^{2\pi 2}(\frac{\alpha t}{L^2})}}{n} \sin(\frac{n\pi x}{L})$	Glover (16)
Well in infinite aquifer	$s = \frac{Q}{4\pi T} W(u)$	Theis (56)
Two leaky aquifer systems, (confined and unconfined) head remains constant in unconfined aquifer	$s = \frac{Q}{4\pi T} W(u,r/B)$	Hantush & Jacob (r/B) (30)
Two leaky aquifer systems, $(T_1/S_1 = T_2/S_2)$(confined and unconfined) head varies in both aquifers	$s_1 = \frac{Q_2}{4\pi(T_1+T_2)} [W(u) - W(u,\beta)]$ $s_2 = \frac{Q_2}{4\pi(T_1+T_2)} [W(u) + \delta_1 W(u,\beta)]$	Hantush 1967b (27)

time dimensions were selected to achieve 95% agreement with analytical results in a minimum number of time steps. Rushton (51) suggests that for a central finite difference model, detailed results in the minimum number of time steps can be obtained for:

$$\frac{\Delta tT}{\Delta x^2 S} < 0.1$$

where:

Δt = time increment (t)

T = transmissivity (L^2/t)

Δx = space dimension (L)

S = storage coefficient.

The first test to be considered was Glover's solution (16) to the parallel drain problem (see Figure 4). If drawdown is small, this solution can also be applied to unconfined aquifer problems. Space dimensions were held constant and the time increment varied in order to achieve different values of $\Delta tT/\Delta x^2S$ to obtain 95% accuracy in the least amount of time steps. The unconfined aquifer problem was checked at three values of $\Delta tT/\Delta x^2S$, 1.125, 0.375 and 0.075 (see Figure 5). The maximum error gradient at the end of 12 time steps for the three tests was approximately 8%. Test three, where $\Delta tT/\Delta x^2S < 0.1$, ran for 60 time steps and the maximum gradient error reduced to approximately 5% (see Figure 6).

The confined aquifer problem was also checked at three values of $\Delta tT/\Delta x^2S$, 2.25, 0.75 and 0.15 (see Figure 7). The maximum error in gradient for all three time increments at the end of 12 time steps was approximately 3%. Test three ran for 24 time steps and the error in gradient reduced from 10% at 4 time steps to 2% at 24 time steps (see Figure 8). The actual piezometric head differed from the theoretical values a maximum of 0.5 feet.

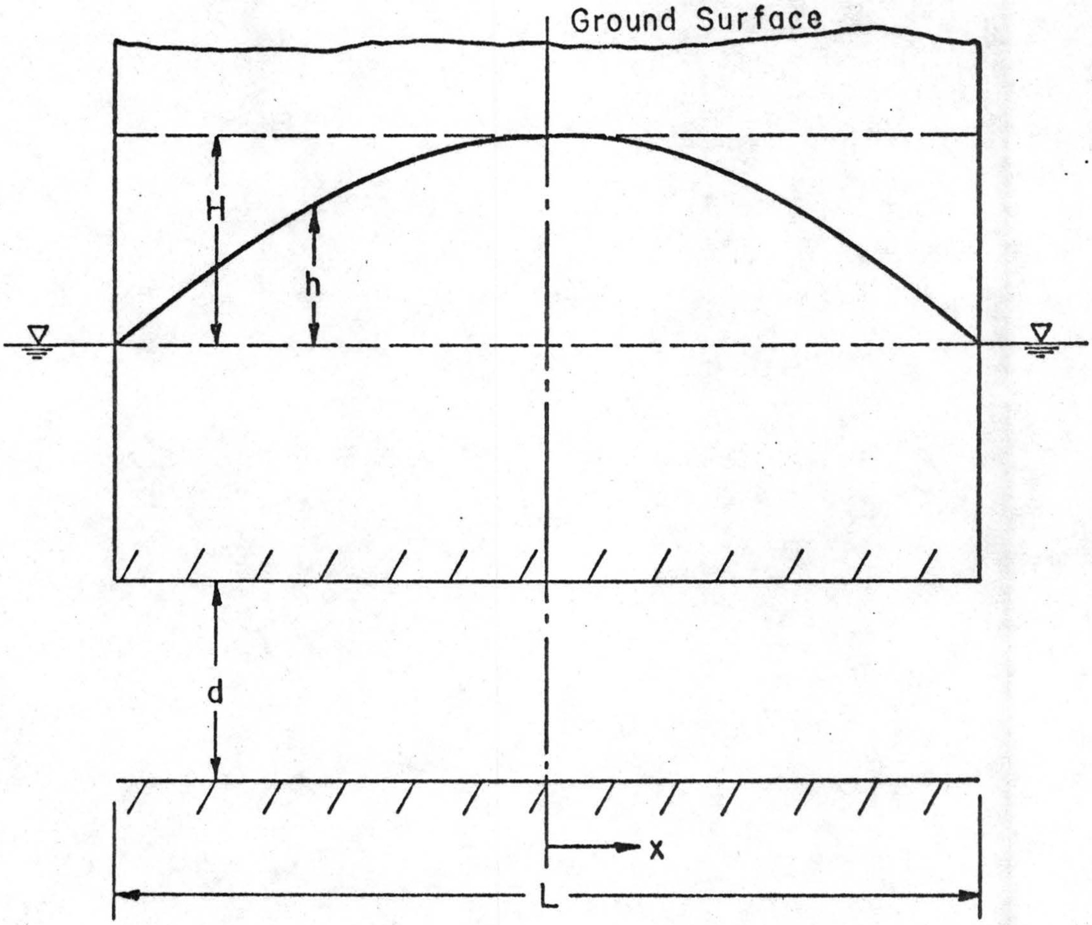


Figure 4. Parallel drain problem

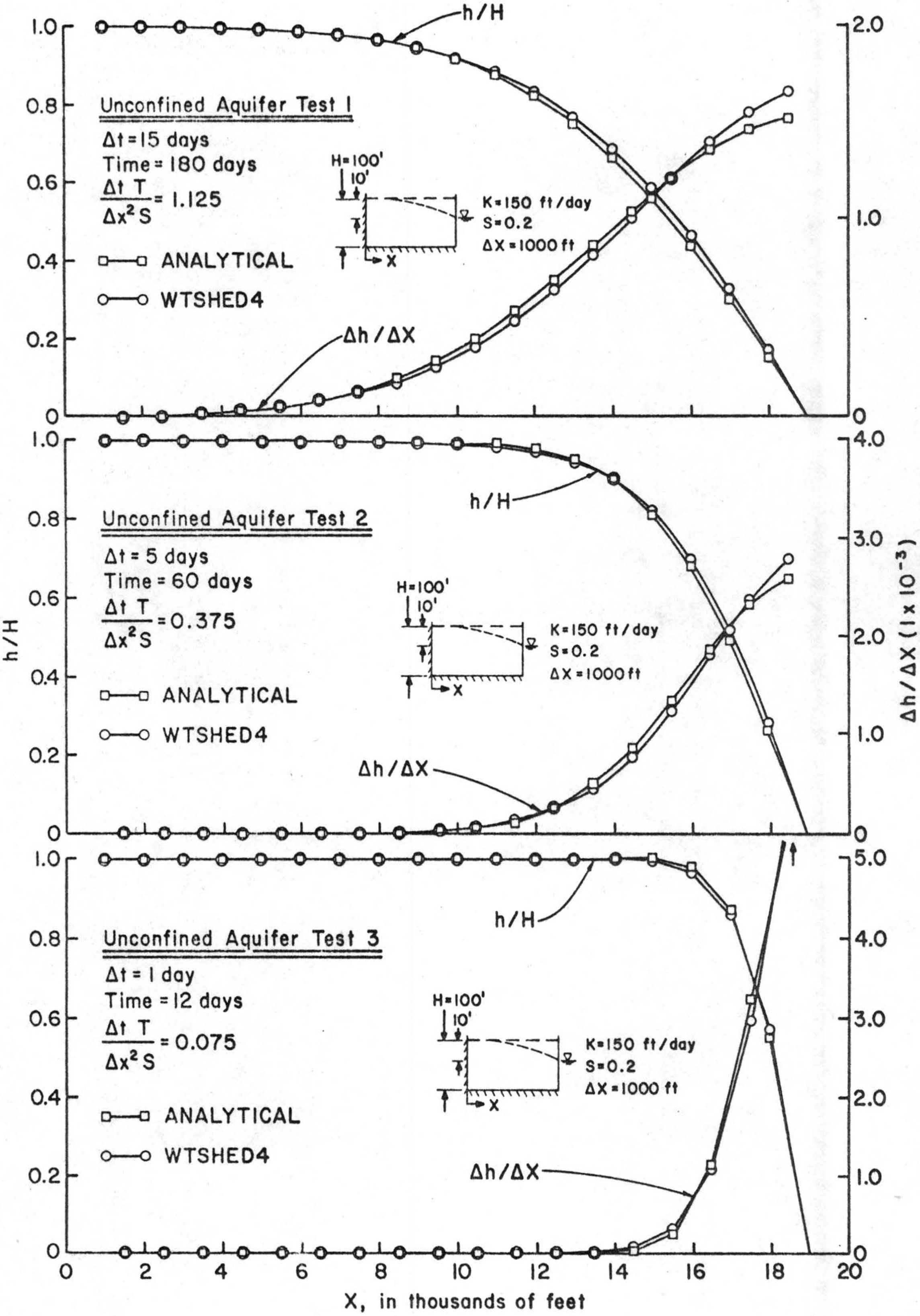


Figure 5. Unconfined aquifer tests 1, 2, 3

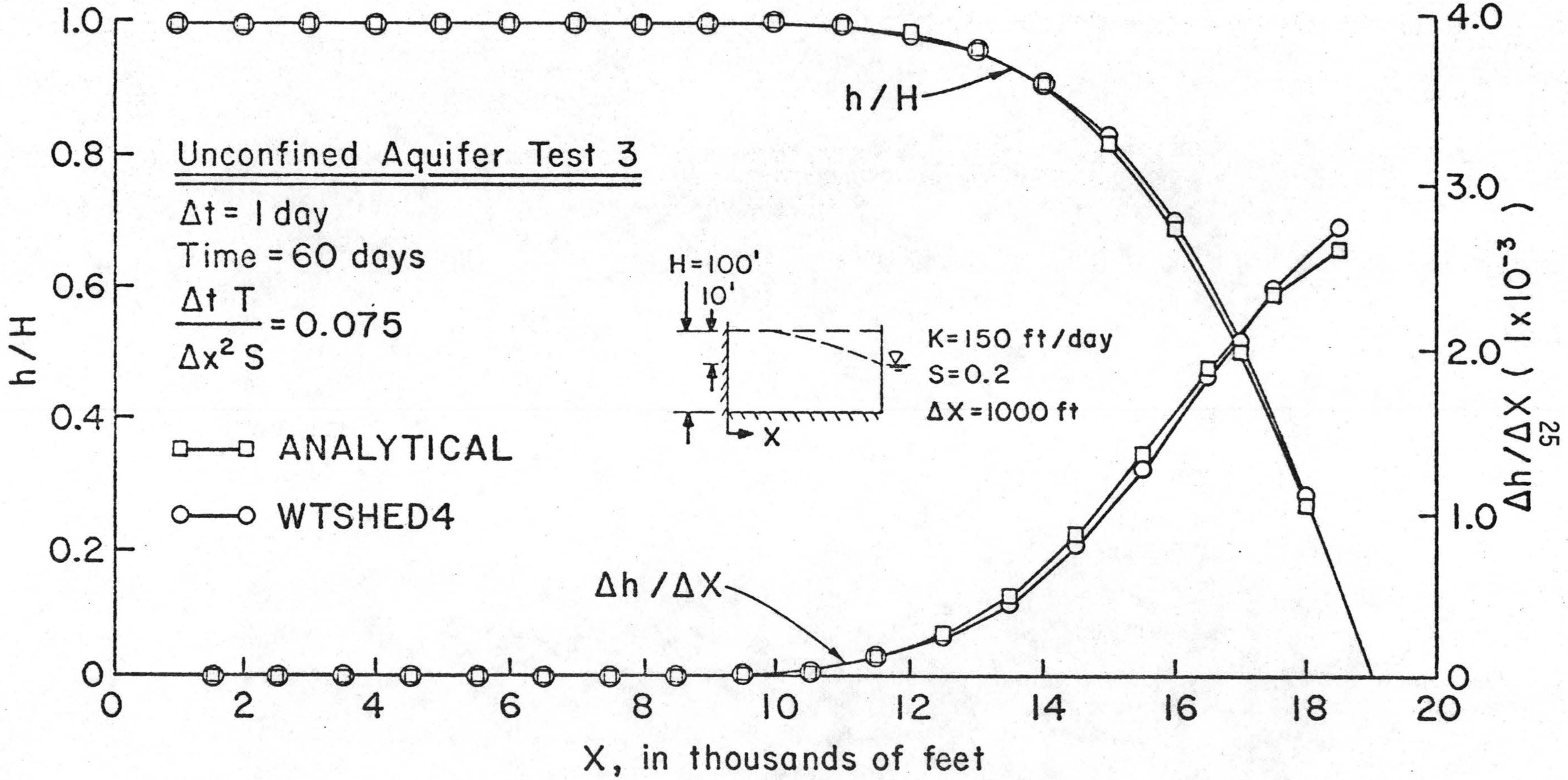


Figure 6. Unconfined aquifer test 3 - 60 days

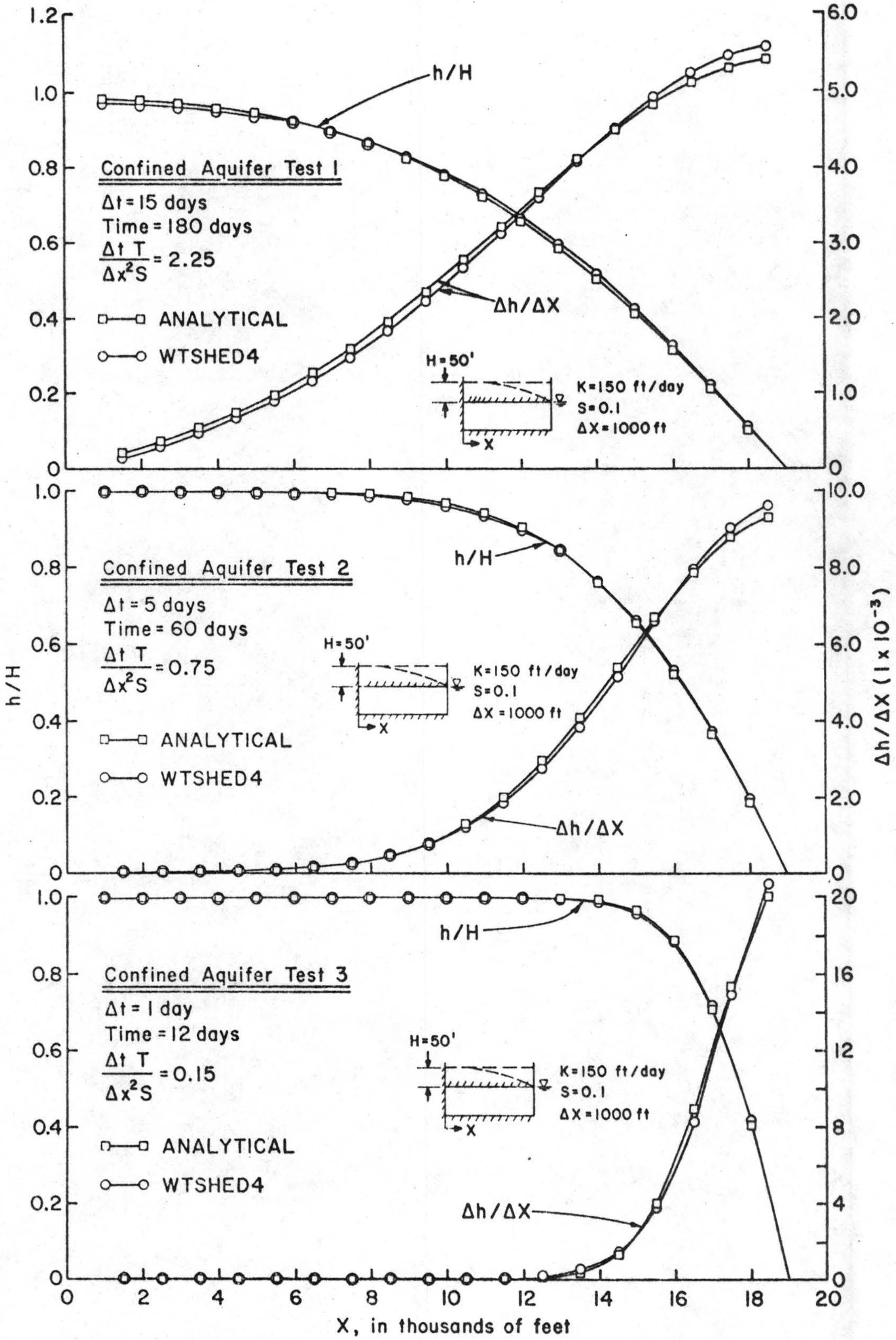


Figure 7. Confined aquifer tests 1, 2, 3

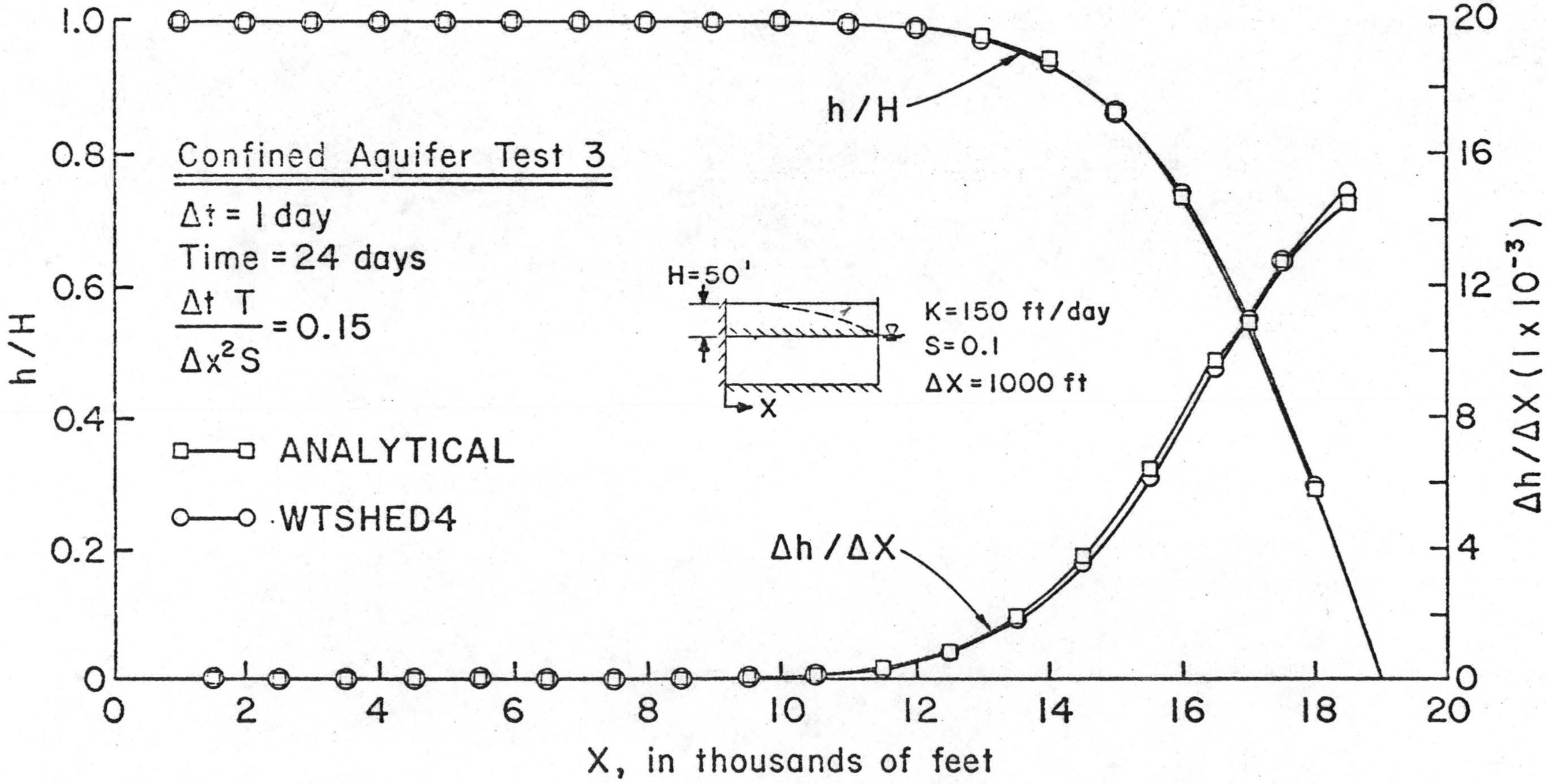


Figure 8. Confined aquifer test 3 - 24 days

The next set of tests performed were well problems as shown in Figure 9, where the Theis solution (56) is valid. A time increment of one day was selected for convenience and space dimensions were varied to achieve accuracy close to the well and yet approximate infinity or zero drawdown at the boundaries of the model. Several tests were performed with varying space dimensions and boundary conditions to obtain a good model. Confined aquifer test 8 with 21 rows and columns of grids and constant head boundaries was the most accurate. Time steps 9 and 12 (see Figure 10) were checked for accuracy and the WTSHED4 gradient was only 2.5% in error for 9 time steps and 2% in error for 12 time steps from the theoretical solution. The maximum difference in piezometric head was less than 0.4 feet except close to the well.

The third set of tests performed was the leaky aquifer tests. Using the same geometry and time increment as the confined aquifer test 8, two types of problems were considered (see Figure 11). The first problem was the case where the drawdown remains constant in the unconfined aquifer. The theoretical solution used for this case was the Hantush and Jacob r/B solution (30). The results are shown in Figure 12. Gradient errors between the WTSHED4 solution and the theoretical solution were less than 2% at 12 time steps. The maximum difference in piezometric head was less than 0.2 feet except at radial distances close to the well.

The second problem considered was the case where drawdown varies in both aquifers. The theoretical solution used for this case was the Hantush 1967b solution (27). The case where diffusivities of both the unconfined and confined aquifers are equal was used to allow easy analytic solution. The results are shown in Figure 13. The Theis solution for

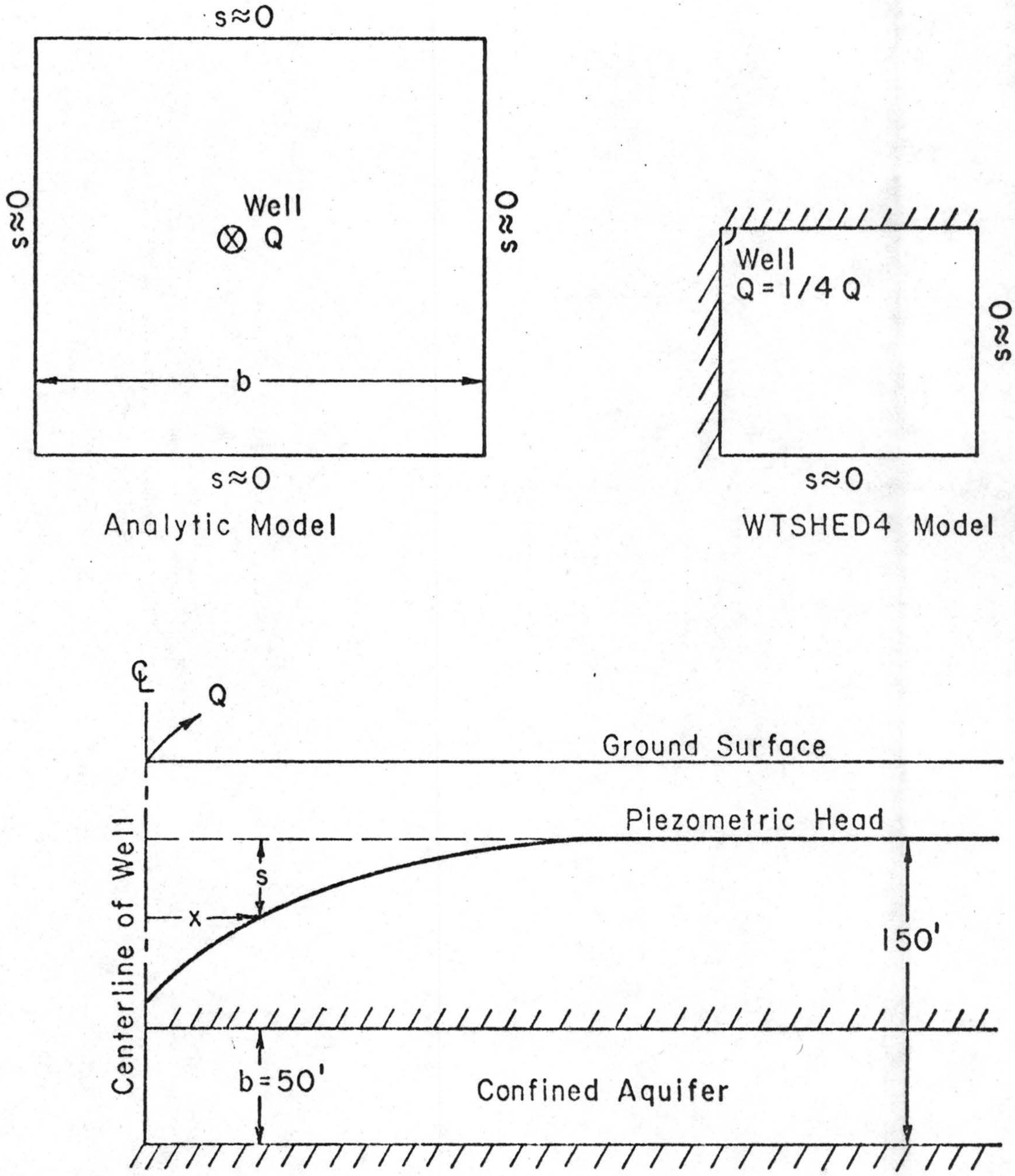


Figure 9. Confined aquifer well problem

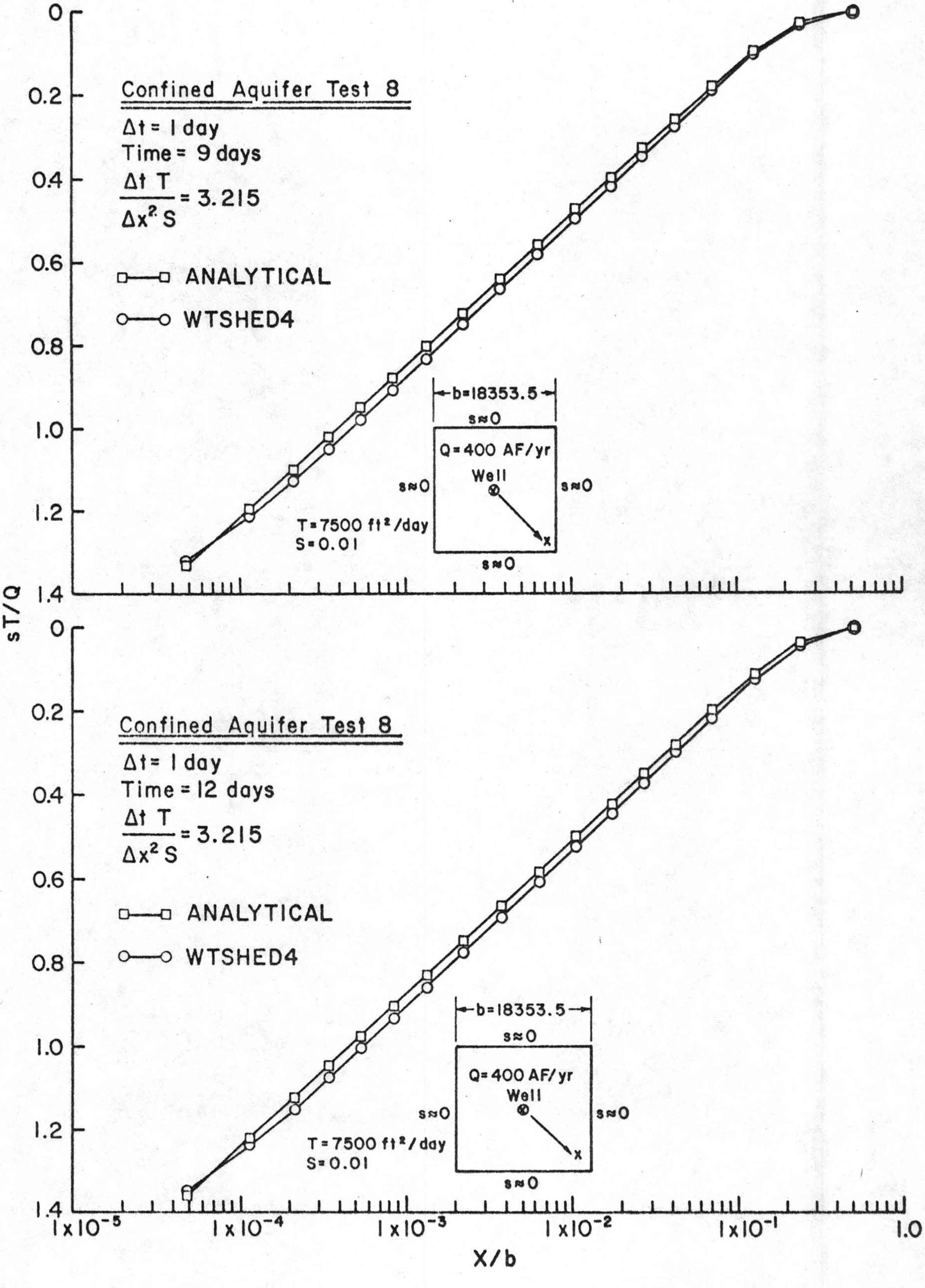
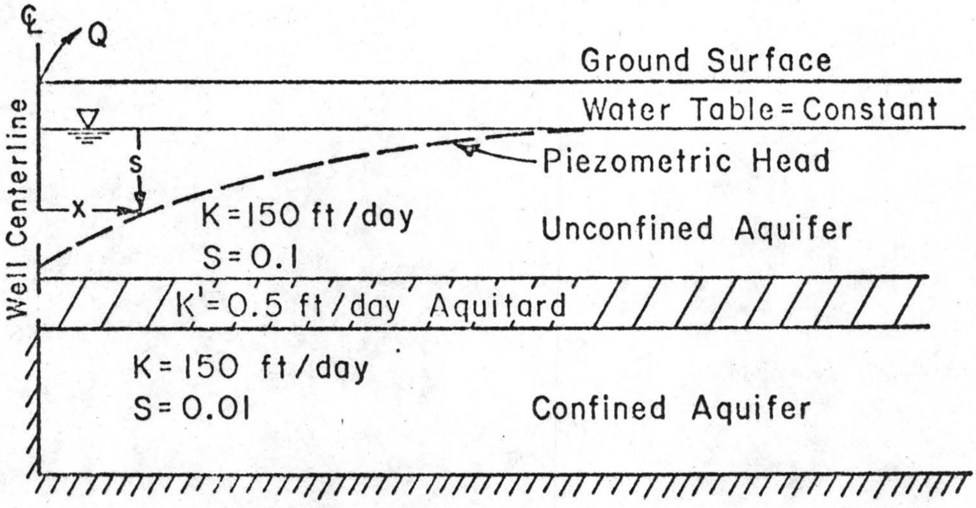
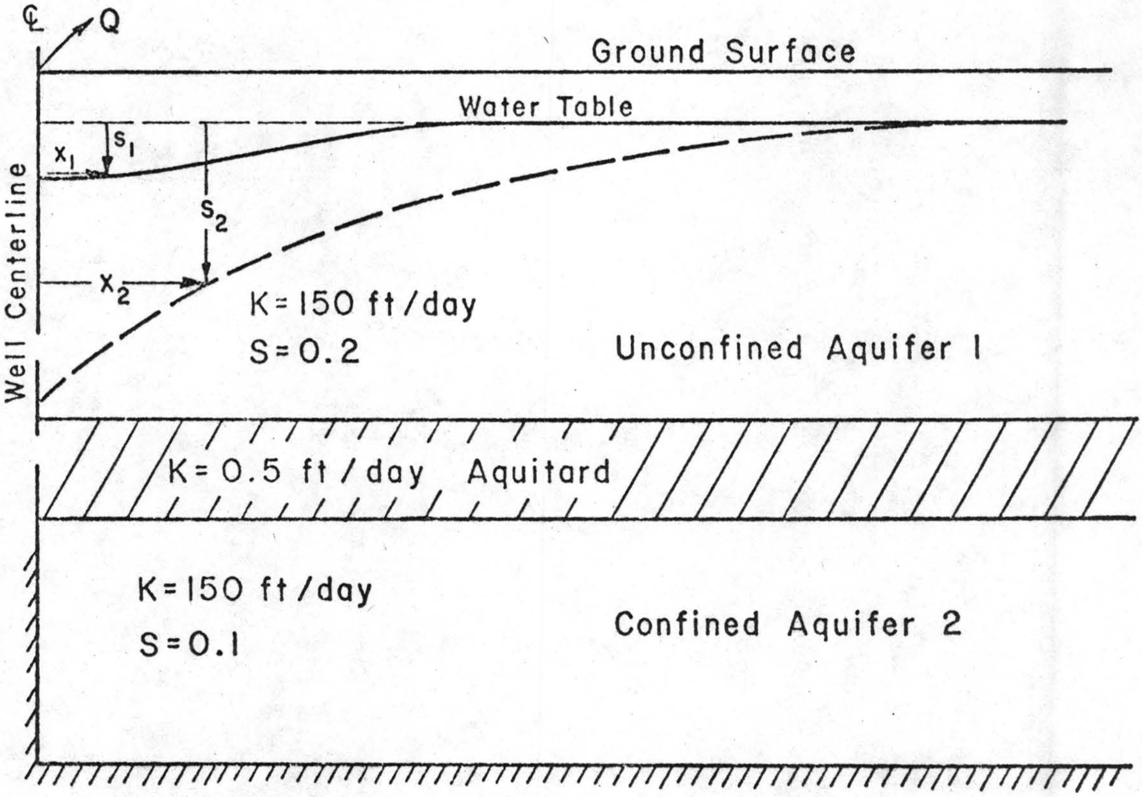


Figure 10. Confined aquifer test 8



Leaky Aquifer Problem #1



Leaky Aquifer Problem #2

Figure 11. Leaky aquifer problems 1 and 2

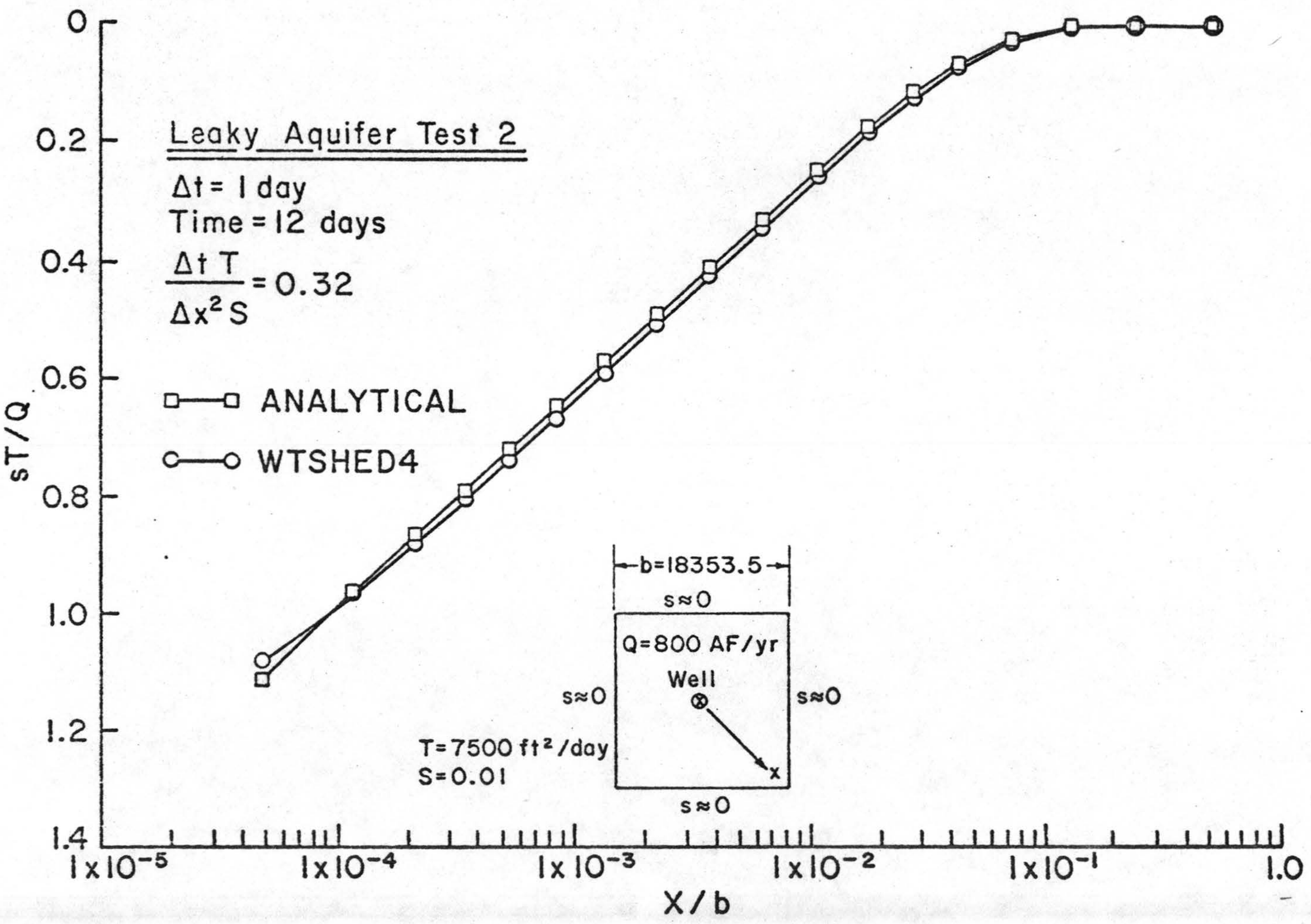


Figure 12. Leaky aquifer test 2

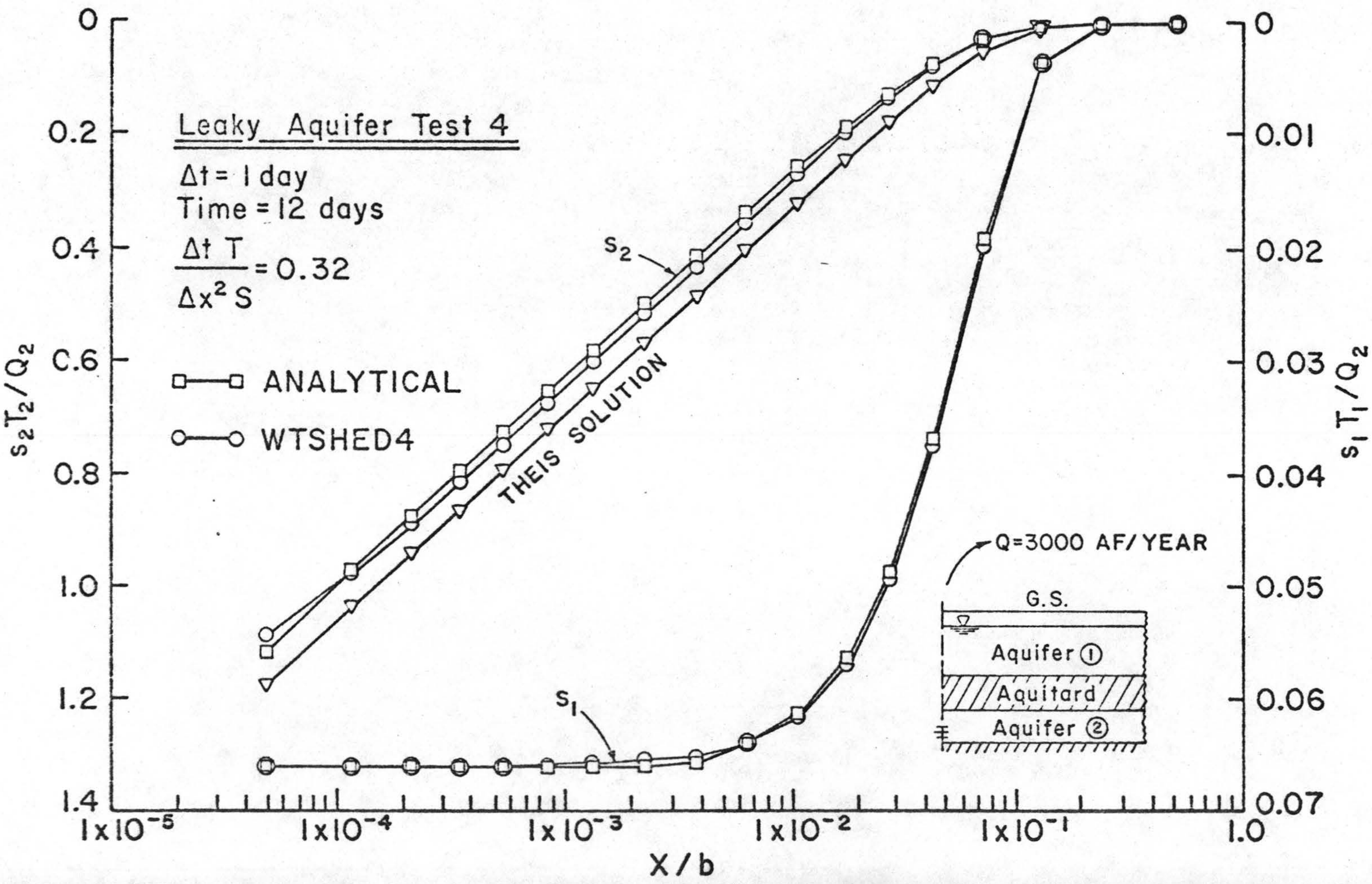


Figure 13. Leaky aquifer test 4

the confined aquifer is included for reference. Gradient results for the confined aquifer were less than 2% in error of the theoretical solution. The maximum difference in piezometric head was less than 0.2 feet except at radial distances close to the well. The same gradient accuracy is true for the unconfined aquifer except the water table elevation computed from WTSHED4 and the theoretical results agreed almost exactly.

V. DISCUSSION OF RESULTS

After making all the changes to the original program WTSHED2 in developing WTSHED4, it was decided to check the original unconfined aquifer solution. Since no exact analytic solutions exist for unconfined aquifer problems, Glover's parallel drain solution (16) was used for the case of small drawdowns so that the saturated thickness could be assumed constant. When using WTSHED4 or any finite difference approximation, as the number of time steps increase the more closely the exact solution is approximated. However for the WTSHED4 unconfined aquifer tests, even after 60 time steps the solution was still 5% in error. Part of the reason for this discrepancy could be the size of space dimensions used, but probably the main reason for the error is the approximation of constant saturated thickness used in the analytic solution as mentioned above.

Two different types of problems with known closed form solutions were used to verify the confined aquifer solutions of WTSHED4. Glover's solution to the parallel drain problem (16) was selected since it involved boundary conditions of constant head and the Theis solution (56) was selected to consider a well problem. WTSHED4 results for the confined aquifer parallel drain problem agreed within 3% of the analytic solution for 12 time steps and converged as indicated by the 24 time steps used in confined aquifer test 3. The maximum gradient error always occurred at the point where drawdown was greatest, at a drain, for the times selected. The reason for this is the fact that the drain is a constant water level or piezometric head boundary and allowed to drop instantaneously causing steep gradients. When modeling steep gradients using a central finite difference method, the smaller the grids used the more closely the theoretical gradient is approximated. If larger grids are used, this gradient error can be significant causing large differences between the computed and theoretical solution. This is the reason for the larger gradient error next to the constant head boundary between WTSHED4 and the theoretical solution. This error should decrease as time increases since the water table or piezometric head slope will decrease.

Many model problems exist when trying to simulate a well for the Theis solution since the well radius is zero and the aquifer is infinite. Several models were considered, each having different boundary conditions and varying in number of grids and grid size. Confined aquifer test 8 was finally used where constant head boundaries were utilized and the number and size of outside grids increased to approximate infinity or zero drawdown at the boundaries. The computed and theoretical solutions were checked at 9 and 12 time steps and found to converge. The only remaining model error was the non-zero well radius. This was the cause of the difference in gradient, approximately 2%, between the WTSHED4 solution and the theoretical solution. If the well grid and grids near the well were decreased to approximate a well of zero radius, the WTSHED4 solution should exactly equal the Theis solution.

Finally leaky aquifer problems were verified using the Hantush and Jacob r/B solution (30) for a constant head in the unconfined aquifer and the Hantush 1967b solution (27) for drawdown in both aquifers. The same WTSHED4 model was used as the confined aquifer test 8 since it resulted in a solution very close to the Theis solution. The WTSHED4 solution agreed within 2% of the theoretical r/B solution. The reason for the small gradient error is the fact that the WTSHED4 model did not approximate the zero radius well as required by the r/B solution. The gradient error for the Hantush 1967b solution was very small, less than

2%, in both the confined and unconfined aquifer. Again the reason for the gradient error being the non-zero well radius in WTSHED4. The water table for the unconfined aquifer agreed within 0.1 foot.

VI. CONCLUSIONS AND RECOMMENDATIONS

Using WTSHED4, one can obtain a solution to a large majority of groundwater problems normally encountered. Input has been devised so that no special training is required to use WTSHED4 and with the increasing availability of large, high speed computers, solutions to complex problems are more easily accessible. Although WTSHED4 has been verified with several analytic solutions to relatively simple problems, it should be carefully used until additional laboratory or field verification is conducted.

Another study that should be performed is a determination of the accuracy of a solution as a result of space and time dimension variations. This would be very helpful when modeling a study area because space and time dimensions could be selected based on available data and desired accuracy of solution.

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VIII. APPENDICIES

APPENDIX A - Mathematical Formulation

APPENDIX B - List of Important Variables in WTSHED4

APPENDIX C - Flow Chart for WTSHED4

APPENDIX D - Description of Subprograms

APPENDIX A

Mathematical Formulation

The nonlinear partial differential equation for transient, two-dimensional flow in a saturated porous medium can be derived from the mass continuity equation and Darcy's law and may be written as:

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kh\frac{\partial H}{\partial y}\right)=S\frac{\partial H}{\partial t}+\frac{Q}{\Delta x\Delta y}$$

where

K = hydraulic conductivity (L/T),

h = saturated thickness of aquifer or confined aquifer thickness (L),

H = water table elevation or piezometric head, referred to an established datum (L),

S = specific yield (dimensionless),

Q = net groundwater withdrawal (L^3/T),

x,y = space dimensions (L),

t = time dimension (T).

Dividing the region of groundwater flow into a rectangular grid system and using an implicit central finite difference scheme, the above equation, written for one grid, becomes:

$$A^tH^{t+\Delta t}_{i,j-1}+B^tH^{t+\Delta t}_{i,j+1}+C^tH^{t+\Delta t}_{i-1,j}+D^tH^{t+\Delta t}_{i+1,j}-(A+B+C+D+E)^tH^{t+\Delta t}_{i,j}$$

$$=Q^{t+\Delta t/2}_{i,j}-EH^t_{i,j}$$

where

$$A^t=\frac{2K_{i,j}K_{i,j-1}\Delta y_{i,j}\Delta y_{i,j-1}h^t_{i,j-1/2}}{\Delta y_{i,j}K_{i,j}\Delta x_{i,j-1}+\Delta y_{i,j-1}K_{i,j-1}\Delta x_{i,j}}$$

$$B^t=\frac{2K_{i,j}K_{i,j+1}\Delta y_{i,j}\Delta y_{i,j+1}h^t_{i,j+1/2}}{\Delta y_{i,j}K_{i,j}\Delta x_{i,j+1}+\Delta y_{i,j+1}K_{i,j+1}\Delta x_{i,j}}$$

$$C^t = \frac{2K_{i,j}K_{i-1,j}\Delta x_{i,j}\Delta x_{i-1,j}h^t_{i-1/2,j}}{\Delta x_{i,j}K_{i,j}\Delta y_{i-1,j} + \Delta x_{i-1,j}K_{i-1,j}\Delta y_{i,j}}$$

$$D^t = \frac{2K_{i,j}K_{i+1,j}\Delta x_{i,j}\Delta x_{i+1,j}h^t_{i+1/2,j}}{\Delta x_{i,j}K_{i,j}\Delta y_{i+1,j} + \Delta x_{i+1,j}K_{i+1,j}\Delta y_{i,j}}$$

$$E = \frac{S_{i,j}\Delta x_{i,j}\Delta y_{i,j}}{\Delta t} .$$

The i,j notation (see Fig. A-1) refers to the grid for which a particular equation is written and the superscripts represent the time level of computation. The term $(h^t_{i,j-1/2})$ in the coefficient A^t and its counterpart in the other coefficients is the effective saturated thickness between the grids (i,j-1) and (i,j). $h^t_{i,j-1/2}$ is computed by the following equation.

$$h^t_{i,j-1/2} = \max(H_{i,j}, H^t_{i,j-1}) - \max(z_{i,j}, z_{i,j-1})$$

where

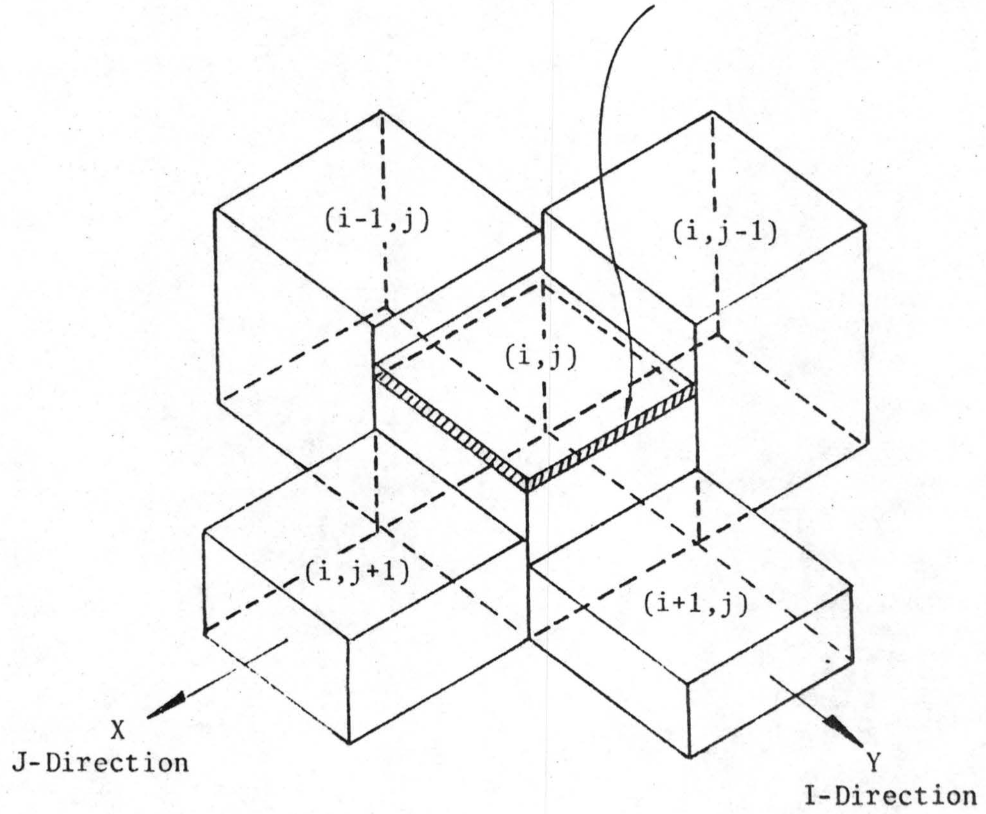
H = water table elevation above a datum (L),

z = bedrock elevation, referred to a datum (L).

This equation ensures that the flux out of a dry grid will be zero.

The coefficients A^t, B^t, C^t, D^t are computed at the beginning of each time increment and held constant throughout the time increment. This approximation effectively linearizes the difference equation for the unconfined case and makes solution possible. The coefficient E is held constant for each grid throughout the analysis. The net groundwater withdrawal, Q, which may be positive or negative, is the net volume of water added or withdrawn through the top or bottom of each grid in a given time interval. It is held constant throughout each time step at the average value it has at the middle or the increment. All other variables are defined above.

$$Q^{t+\Delta t/2} + S_{i,j}\Delta x_{i,j}\Delta y_{i,j}\ \frac{H^{t+\Delta t}-H^{t}}{\Delta t}$$



$$\frac{\partial}{\partial x}\ (K_x h\Delta y\ \frac{\partial H}{\partial x})\Delta x$$
$$= A^t(H_{i,j-1}-H_{i,j})^{t+\Delta t}$$
$$-B^t(H_{i,j}-H_{i,j+1})^{t+\Delta t}$$

$$\frac{\partial}{\partial y}\ (K_y h\Delta x\ \frac{\partial H}{\partial y})\Delta y$$
$$= C^t(H_{i-1,j}-H_{i,j})^{t+\Delta t}$$
$$-D^t(H_{i,j}-H_{i-1,j})^{t+\Delta t}$$

Fig. A-1. Finite difference grid and its physical significance.

The flow equations used in WTSHED4 are derived using the same grid system as in Fig. A-1. Flows are computed in the I and J-directions for each grid using Darcy's equation to and from the intersections of the grids. The hydraulic gradients from the middle of each grid to its intersections (or edges) are used. The resulting equations are:

<u>J-direction between grids J to J+1</u>

$$\text{Flow} = \frac{2K_{i,j}K_{i,j+1}\Delta y_{i,j}\Delta y_{i,j+1}h^t_{i,j+1/2}}{\Delta y_{i,j}K_{i,j}\Delta x_{i,j+1} + \Delta y_{i,j+1}K_{i,j+1}\Delta x_{i,j}}(h_{i,j+1}-h_{i,j})^{t+1}$$

$$= B^t(h_{i,j+1}-h_{i,j})^{t+1}$$

<u>I-direction between grids I to I+1</u>

$$\text{Flow} = \frac{2K_{i,j}K_{i+1,j}\Delta x_{i,j}\Delta x_{i+1,j}h^t_{i+1/2,j}}{\Delta x_{i,j}K_{i,j}\Delta y_{i+1,j} + \Delta x_{i+1,j}K_{i+1,j}\Delta y_{i,j}}(h_{i+1,j}-h_{i,j})^{t+1}$$

$$= D^t(h_{i+1,j}-h_{i,j})^{t+1}$$

APPENDIX B

List of Important Variables in WTSHED4

Symbol	Description
APW	Applied water as a result of surface irrigation (feet/year).
BBC	Bottom boundary code.
CA	Fraction of grid to which water is applied (decimal).
CAW	Coefficient of deep percolation of applied water (decimal).
CCA	Uniform fraction of each grid irrigated for all grids (decimal).
CMATRX	Coefficient matrix.
CPM	Coefficient of groundwater removed by pumping (decimal).
CPT	Coefficient of effective precipitation to groundwater (decimal).
CR	Right hand side matrix.
DLX	Uniform X-dimension of all grids (feet).
DLY	Uniform Y-dimension of all grids (feet).
DT	Time increment (days).
DTWT	Depth to water table from ground surface.
DX	X-dimension of grid (feet).
DY	Y-dimension of grid (feet).
ET	Evapotranspiration (feet) - calculated from ET program.
FACFTA	Amount flooded between buffer zone boundaries (acre-feet).
FACFTT	Total amount flooded (acre-feet).
FFK	Uniform permeability of all grids (feet/day).
FK	Permeability (feet/day).
FKL	Vertical Permeability of Leaky layer (feet/day).
FVA	Amount flooded between buffer zone boundaries (acre-feet).
FVT	Total amount flooded (acre-feet).
FWTOP	Desired time of output (multiple of DT).
G	Ground surface or top of confined aquifer elevation (feet).

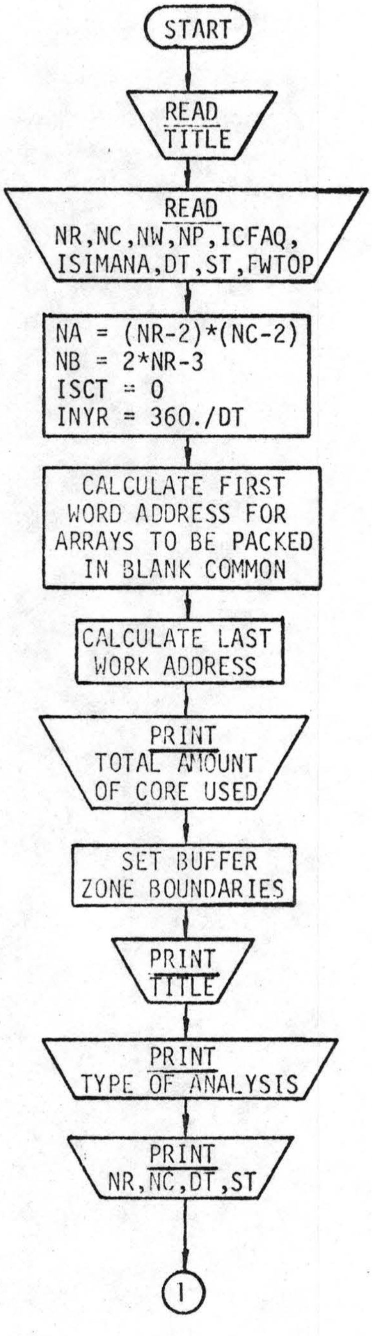
Symbol	Description
GG	Uniform ground surface or top of confined aquifer elevation of all grids (feet).
H	Initial water table elevation or piezometric head (feet).
HL	Head value causing leaky aquifer (feet).
HP	Water table elevation or piezometric head at previous time level (feet).
HT	Present water table elevation or piezometric head (feet).
HW	Uniform water table elevation or piezometric head (feet).
ICFAQ	Equals 1 for confined aquifer analysis, otherwise blank.
INYR	Number of time increments per year (360./DT).
ISIMANA	Equals 1 for simultaneous analysis of a two aquifer leaky system.
LBC	Left boundary code.
LCIE	Right (J) buffer zone.
LCIW	Left (J) buffer zone.
LCJE	Bottom (I) buffer zone.
LCJW	Top (I) buffer zone.
NA	Number of rows in reduced band matrx.
NB	Number of columns in reduced band matrx.
NC	Number of columns (integer).
NGPU	Number of grids with phreatophyte use (integer).
NP	Number of recharge pits or lines (integer).
NR	Number of rows (integer) (NR $\leq$ NC).
NW	Number of wells (integer).
OACFTA	Overdraw between buffer zone boundaries (acre-feet).
OACFTT	Total overdraw (acre-feet).
OVA	Overdraw between buffer zone boundaries (acre-feet).
OVT	Total overdraw (acre-feet).

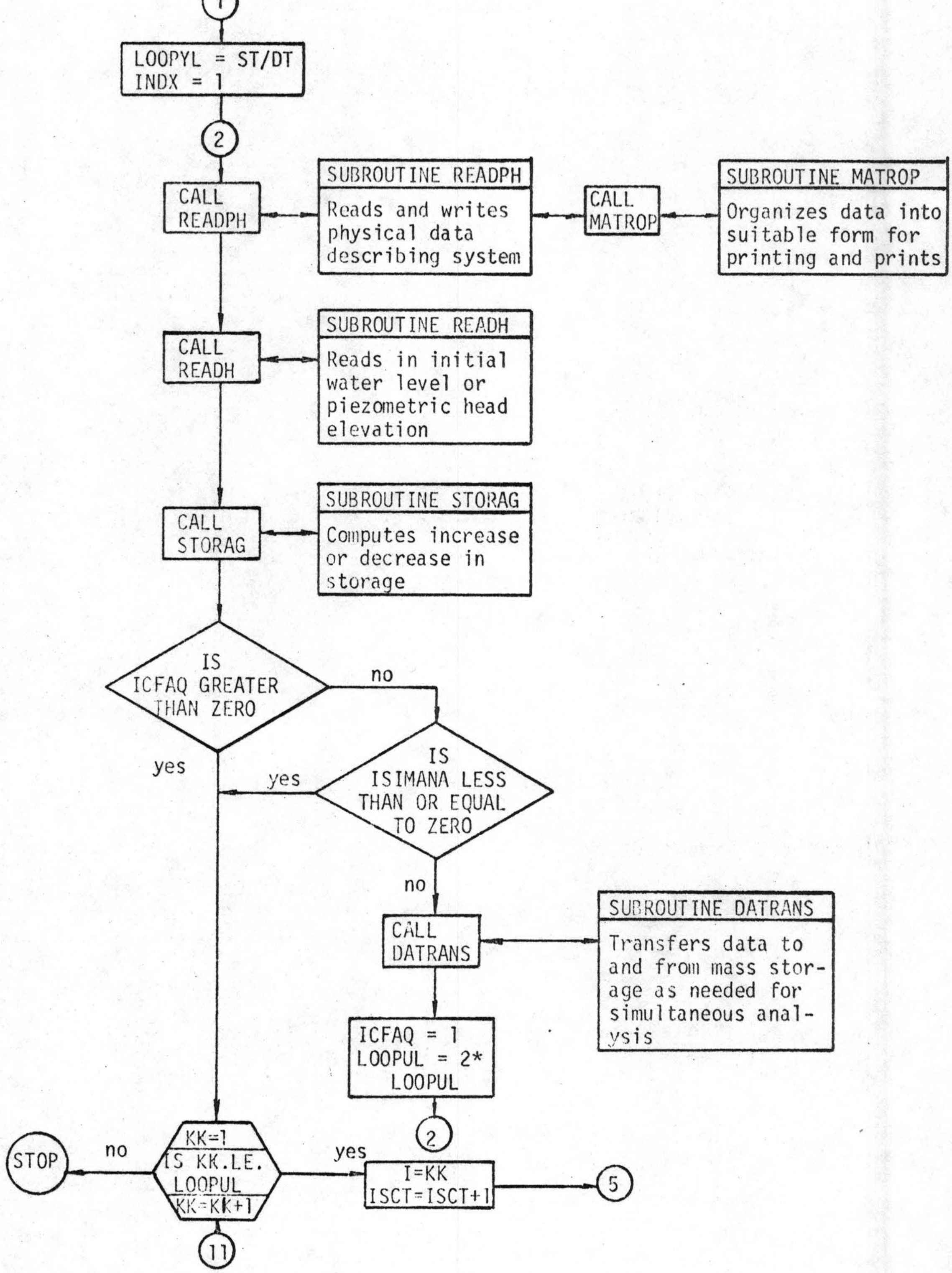
Symbol	Description
PHI	Specific yield or storage coefficient (decimal).
PHIC	Storage coefficient of confined aquifer (decimal).
PHR	Phreatophyte use (acre-feet/year) or phreatophytes present - use calculated in program.
PIT	Recharge pit or line number code.
PPHI	Uniform specific yield or storage coefficient of all grids (decimal).
PPHIC	Uniform storage coefficient of confined aquifer for all grids (decimal).
PPT	Precipitation (inches/year).
Q	Net value of hydrologic and artificial input per grid (acre-feet/day).
RBC	Right boundary code.
RCHR	Amount each pit or line recharges per year (feet/year).
REPEAT	Data input code for multiple year analysis (blank indicates to read in data).
RPUM	Amount each well pumps per year (acre-feet/year).
SQA	Total Q per DT between stations (acre-feet).
SQBA	Total inflow through buffer zone boundaries (acre-feet).
SQBT	Total inflow through boundaries (acre-feet).
SQGGI	Flow between grids in I-direction (acre-feet).
SQGGJ	Flow between grids in J-direction (acre-feet).
SQR	Inflow from constant head grids (acre-feet).
SQRA	Inflow from constant head grids within buffer zone boundaries (acre-feet).
SQT	Total Q per DT (acre-feet).
SQRT	Total inflow from constant head grids (acre-feet).
ST	Total time of analysis (days).
STA	Between stations storage (acre-feet).

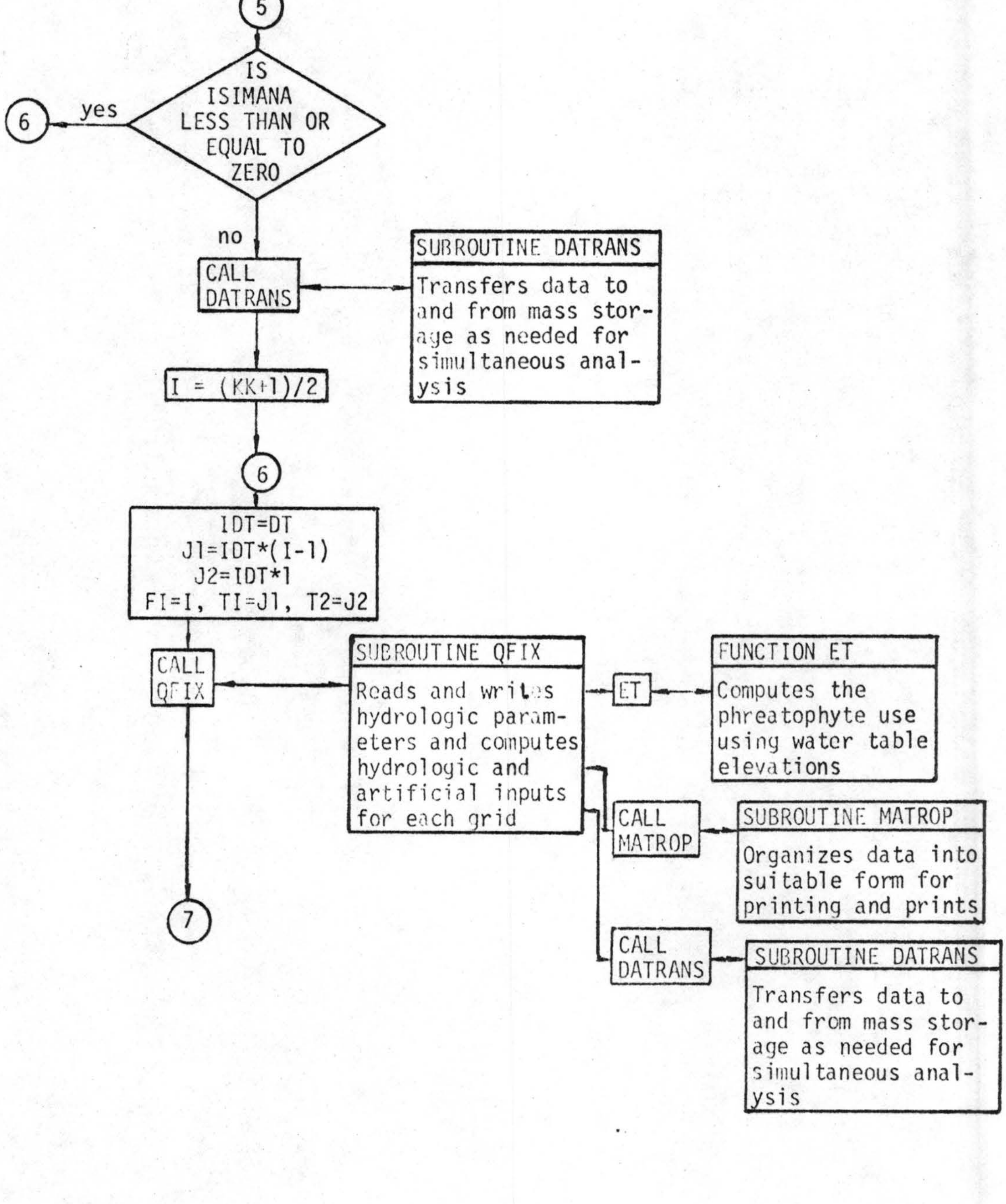
Symbol	Description
STATEM	Decrease of storage between stations (acre-feet).
STOL	Overlap area storage (acre-feet).
STT	Total area storage (acre-feet).
STTTEM	Total decrease of storage (acre-feet).
TBC	Top boundary code.
TL	Thickness of leaky layer (feet).
WELL	Well number code.
YAW	Distribution of applied water for each DT for one year (decimal).
YPM	Distribution of pumping for each DT for one year (decimal).
YPR	Distribution of phreatophyte use for each DT for one year (decimal).
YPT	Distribution of precipitation for each DT for one year (decimal).
YRC	Distribution of pit recharge for each DT for one year (decimal).
Z	Bedrock elevation (feet).
ZZ	Uniform bedrock elevation of all grids (feet).

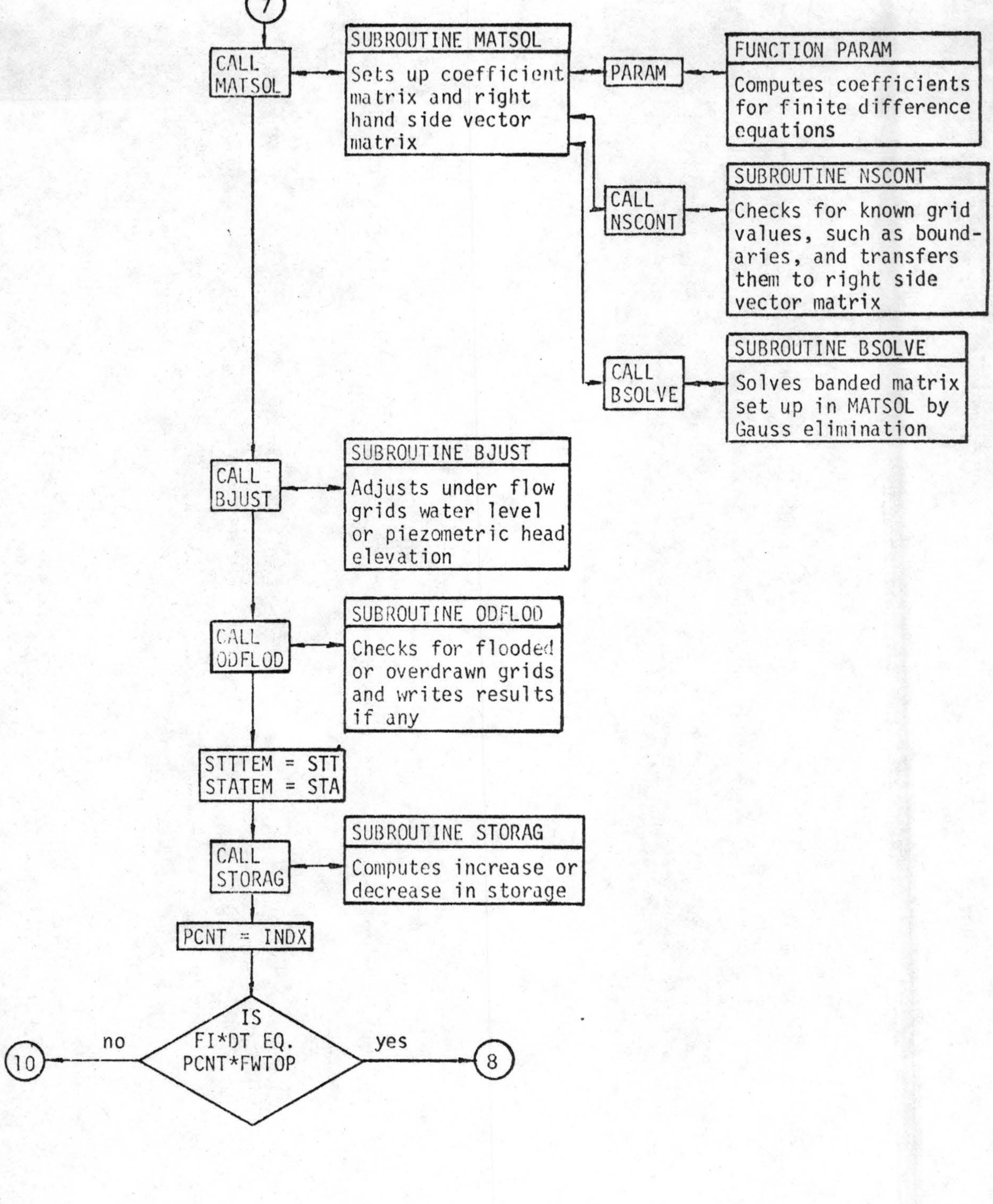
APPENDIX C

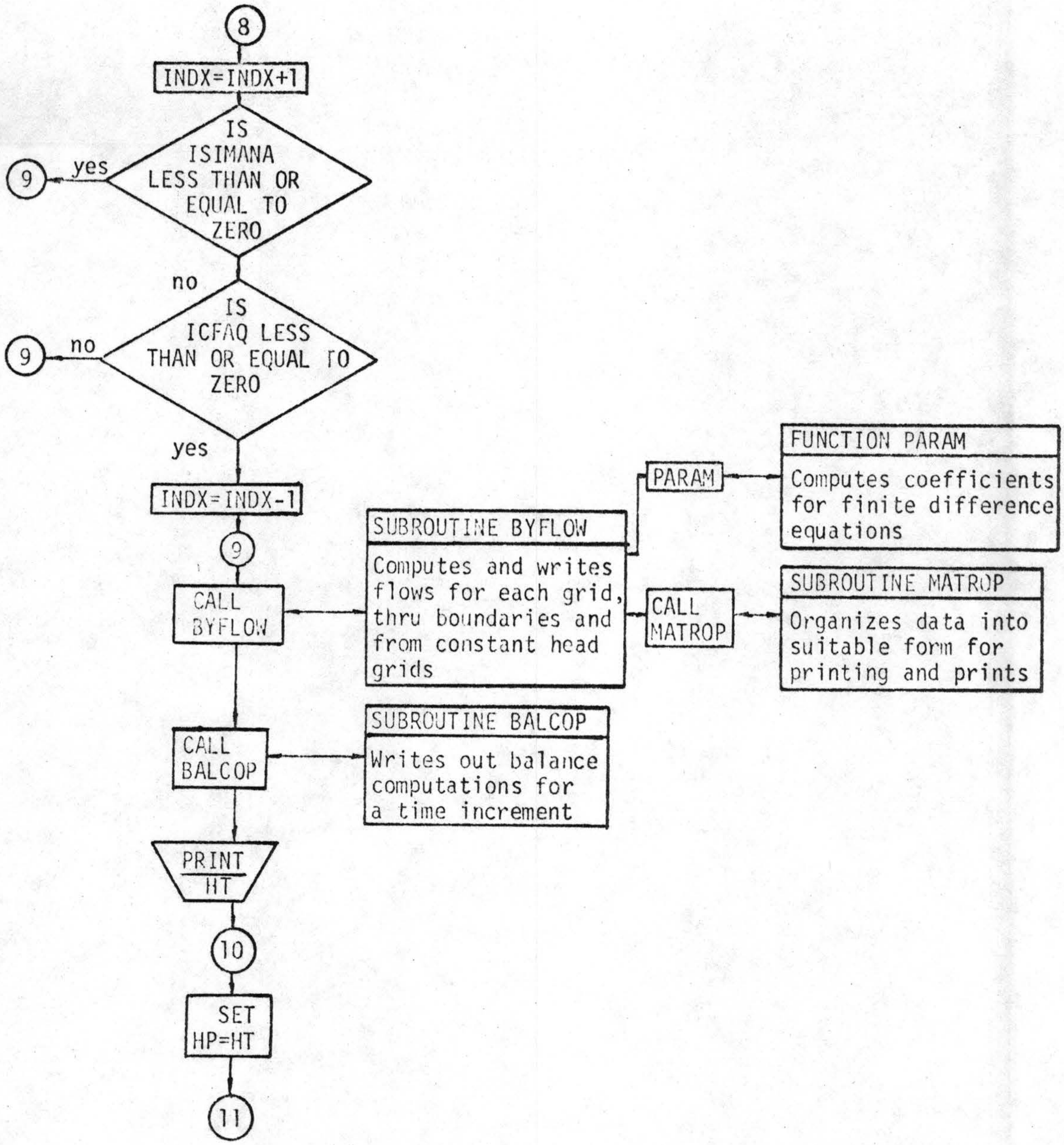
Flow Chart for WTSHED4


START
READ
TITLE
READ
NR,NC,NW,NP,ICFAQ,
ISIMANA,DT,ST,FWTOP
NA = (NR-2)*(NC-2)
NB = 2*NR-3
ISCT = 0
INYR = 360./DT
CALCULATE FIRST
WORD ADDRESS FOR
ARRAYS TO BE PACKED
IN BLANK COMMON
CALCULATE LAST
WORK ADDRESS
PRINT
TOTAL AMOUNT
OF CORE USED
SET BUFFER
ZONE BOUNDARIES
PRINT
TITLE
PRINT
TYPE OF ANALYSIS
PRINT
NR,NC,DT,ST
1


1
LOOPYL = ST/DT
INDX = 1
2
CALL READPH
SUBROUTINE READPH
Reads and writes physical data describing system
CALL MATROP
SUBROUTINE MATROP
Organizes data into suitable form for printing and prints
CALL READH
SUBROUTINE READH
Reads in initial water level or piezometric head elevation
CALL STORAG
SUBROUTINE STORAG
Computes increase or decrease in storage
IS ICFAQ GREATER THAN ZERO
no
yes
IS ISIMANA LESS THAN OR EQUAL TO ZERO
yes
no
CALL DATRANS
SUBROUTINE DATRANS
Transfers data to and from mass storage as needed for simultaneous analysis
ICFAQ = 1
LOOPUL = 2* LOOPUL
2
KK=1
IS KK.LE. LOOPUL
KK=KK+1
STOP
no
yes
I=KK
ISCT=ISCT+1
5
11


5
IS ISIMANA LESS THAN OR EQUAL TO ZERO
yes
6
no
CALL DATRANS
SUBROUTINE DATRANS
Transfers data to and from mass storage as needed for simultaneous analysis
I = (KK+1)/2
6
IDT=DT
J1=IDT*(I-1)
J2=IDT*1
FI=I, TI=J1, T2=J2
CALL QFIX
SUBROUTINE QFIX
Reads and writes hydrologic parameters and computes hydrologic and artificial inputs for each grid
ET
FUNCTION ET
Computes the phreatophyte use using water table elevations
CALL MATROP
SUBROUTINE MATROP
Organizes data into suitable form for printing and prints
CALL DATRANS
SUBROUTINE DATRANS
Transfers data to and from mass storage as needed for simultaneous analysis
7


7
CALL MATSOL
SUBROUTINE MATSOL
Sets up coefficient matrix and right hand side vector matrix
PARAM
FUNCTION PARAM
Computes coefficients for finite difference equations
CALL NSCONT
SUBROUTINE NSCONT
Checks for known grid values, such as boundaries, and transfers them to right side vector matrix
CALL BSOLVE
SUBROUTINE BSOLVE
Solves banded matrix set up in MATSOL by Gauss elimination
CALL BJUST
SUBROUTINE BJUST
Adjusts under flow grids water level or piezometric head elevation
CALL ODFLOD
SUBROUTINE ODFLOD
Checks for flooded or overdrawn grids and writes results if any
STTTEM = STT
STATEM = STA
CALL STORAG
SUBROUTINE STORAG
Computes increase or decrease in storage
PCNT = INDX
IS FI*DT EQ. PCNT*FWTOP
no
10
yes
8


8
INDX=INDX+1
IS ISIMANA LESS THAN OR EQUAL TO ZERO
yes
9
no
IS ICFAQ LESS THAN OR EQUAL TO ZERO
no
9
yes
INDX=INDX-1
9
CALL BYFLOW
SUBROUTINE BYFLOW
Computes and writes flows for each grid, thru boundaries and from constant head grids
PARAM
FUNCTION PARAM
Computes coefficients for finite difference equations
CALL MATROP
SUBROUTINE MATROP
Organizes data into suitable form for printing and prints
CALL BALCOP
SUBROUTINE BALCOP
Writes out balance computations for a time increment
PRINT HT
10
SET HP=HT
11

APPENDIX D

Description of Subprograms

Subroutine READPH

This subroutine reads and writes the physical data describing the study area. The following variables are read and printed: DX,DY,FK,Z, G,PHI and PHIC. CA is also read but printed later. Only one data card is required if all variables are uniform for each grid, otherwise each parameter that is variable must be read in matrix form. Variables DX and DY require only NC and NR values respectively.

Called From: Main Program

Subprograms Used: MATROP

Important Variables: DX DY FK Z G PHI PHIC CA

Subroutine READH

This subroutine reads the initial coded water level or piezometric head elevations. H is decoded and set equal to HT and HP. One data card is required if the initial water level is horizontal, otherwise the entire H-matrix must be read.

Called From: Main Program

Subprograms Used: None

Important Variables: H HT HP

Subroutine DATRANS

This subroutine transfers data to and from mass storage for simultaneous leaky aquifer analysis. Confined aquifer data is stored on unit 9 during unconfined analysis and unconfined data is stored on unit 8 during confined analysis. In addition the HL matrix of head causing the leaky aquifer conditions is set.

Called From: Main Program QFIX

Subprograms Used: None

Important Variables: FK G Z H HP HT HF WELL RPUM CPM YPM HL

Subroutine STORAG

This subroutine computes the initial storage and increase or decrease of storage. Total area and between station (between buffer zone boundaries), storage is calculated. Also storage of overlap areas is computed.

Called From: Main Program

Subprograms Used: None

Important Variables: STA STT STOL H HT Z

Subroutine QFIX

This subroutine reads and writes the hydrologic parameters. The hydrologic and artificial inputs are then calculated for each grid. A value of zero on the input card indicates a particular parameter is not used (see listing). The exception to this is the number of grids with phreatophyte use, NGPU. If NGPU is blank, the entire PHR matrix must be read, otherwise the number of grids specified is read, NGPU equal to zero indicates no phreatophyte use.

Coding PHR less than one indicates that phreatophyte use should be calculated every time increment from the previous time step water level elevation. The ET subprogram is used for this.

The factors considered in QFIX are (1) precipitation, (2) applied water as irrigation, (3) phreatophyte use, (4) wells, (5) recharge areas or lines, and (6) leaky aquifer conditions.

Called From: Main Program

Subprograms Used: DATRANS ET MATROP

Important Variables: PPT CPT YPT APW CAW YAW NGPU PHR YPR WELL RPUM CPM YPM PIT RCHR YRC Q SQT SQA REPEAT

Function ET

This subprogram computes the phreatophyte use for each grid using the water level elevations from the previous time step. If the depth to water table, DTWT, is negative, an error message is printed. It is anticipated this program, if used, will change with each study area.

Called From: QFIX

Subprograms Used: None

Important Variables: ET DTWT

Subroutine MATSOL

This subroutine sets up the coefficient matrix, CMATRX, and the right hand side vector matrix, CR. CMATRX is a reduced matrix containing only the band of known values in the left side of the difference equations and is written vertically rather than diagonally. Its dimensions are (NR-2)*(NC-2) by 2*NR-3. The coefficients are computed using Function PARAM and checked for adjacent boundary values of H, being treated accordingly, in subroutine NSCONT. MATSOL treats known grid values of H. BSOLVE is used to solve the matrix equation set up.

Called From: Main Program

Subprograms Used: PARAM NSCONT BSOLVE

Important Variables: CMATRX CR

Function PARAM

This subprogram computes the coefficients in the left side of the finite difference equation. For confined aquifer analysis, saturated thickness is compared to aquifer thickness and the smallest of the two is used to calculate the coefficient.

Called From: MATSOL BYFLOW

Subprograms Used: None

Important Variables: PARAM

Subroutine NSCONT

This subroutine transfers the coefficients, in CMATRX, multiplied by their respective H-value, to the right hand side vector matrix in case of adjacent head or known boundary conditions. It also sets coefficients equal to zero in case of adjacent impermeable grids.

Called From: MATSOL

Subprograms Used: None

Important Variables: None

Subroutine BSOLVE

This subroutine solves the matrix equation set up in MATSOL by Gauss Elimination. BSOLVE is designed specifically for a diagonal matrix that results from analysis of groundwater systems.

Called From: MATSOL

Subprograms Used: None

Important Variables: None

Subroutine BJUST

This subroutine adjusts the underflow boundary water level elevations. Gradients are calculated three grids in from the exterior boundary grids and the gradients are projected back to the exterior boundary grids to obtain new water level elevations. This calculation is performed at even time steps. At odd time steps the water level elevations are held constant and the exterior boundary grids are treated as constant head grids.

Called From: Main Program

Subprograms Used: None

Important Variables: H HT

Subroutine ODFLOD

This subroutine checks for overdrawn or flooded grids. If either should occur, a message is printed indicating such. For confined aquifer analysis the flooded grid computations are bypassed. Total flooded and overdraw amounts are computed for the total area and between stations.

Called From: Main Program

Subprograms Used: None

Important Variables: OACFTT=OVT OACFTA=OVA FACFTT=FVT FACFTA=FVA

Subroutine BYFLOW

This subroutine computes flows for each grid. Total flow through model boundaries and buffer zone boundaries is calculated as well as flow into the system from constant head grids. The flow equation used is developed from the finite difference equations and uses particular values of the CMATRX. These values are transferred from MATSOL except for boundary values which are calculated in BYFLOW using Function PARAM. Flow is not allowed to or from an impermeable grid and between any two adjacent underflow grids. I-direction and J-direction flows are printed and flows from constant head grids are interpreted and printed as flow from river grids.

Called From: Main Program

Subprograms Used: PARAM MATROP

Important Variables: SQGGI SQGGJ SQBT SQBA SQR SQRT SQRA

Subroutine BALCOP

This subroutine writes the balance computations at the desired time steps specified by FWTOP. Mass balance for the entire area cannot always be obtained, due to accounting procedures used to compute mass flow at exterior boundary grids. However, for between stations, which refers to the area between the buffer zone boundaries, mass balance must always be satisfied except for the case when a confined grid becomes unconfined. This error should be small and is indicated by the "TOTALS" in the mass balance output being different than zero. To reduce this error, decrease the value of ΔT. For confined aquifer analysis, a message is printed indicating if a grid becomes unconfined.

Called From: Main Program

Subprograms Used: None

Important Variables: SQA SQT SQRA SQRT SQBA SQBT STT STA STTTEM STATEM STOL OVA OVT

Subroutine MATROP

This subroutine organizes data or results into a suitable form for printing and then prints.

Called From: READPH QFIX BYFLOW

Subprograms Uses: None

Important Variables: NR=NOROW NC=NOCOL