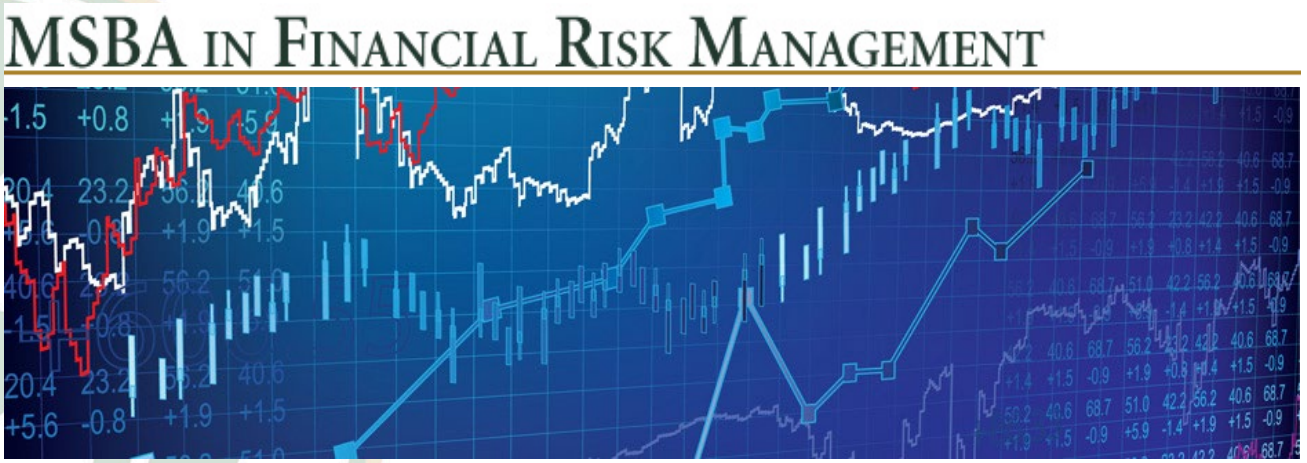
Financial Risk Management



Extreme Value Theory

Motivations for EVT

- "From the point of view of the risk manager, inappropriate use of the normal distribution can lead to an understatement of risk, which must be balanced against the significant advantage of simplification.
- From the central bank's corner, the consequences are even more serious because we often need to concentrate on the left tail of the distribution in formulating lender-of-last-resort policies. Improving the characterization of the distribution of extreme values is of paramount importance"
  – (Alan Greenspan, 1995)

Some statements on extremes and correlation

- "Extreme, synchronized rises and falls in financial markets occur infrequently but they do occur. The problem with the models is that they did not assign a high enough chance of occurrence to the scenario in which many things go wrong at the same time - the "perfect storm scenario" (Business Week, Sep 1998)

- "Regulators have criticized LTCM and banks for not "stress-testing" risk models against extreme market movements. . . The markets have been through the financial equivalent of several Hurricane Andrews hitting Florida all at once. Is the appropriate response to accept that it was mere bad luck to run into such a rare event - or to get new forecasting models that assume more storms in the future?" (The Economist, October 1998, after the LTCM rescue)

Some statements on extremes and correlation

- ". . . The trading floor is quiet. But this masks their attempt at picking up the pieces with a new fund, JWM Partners. Now, Mr. Meriwether is preaching new gospel: World financial markets are bound to hit **extreme turbulences** again. . . Mr. Meriwether's crew, once bitten, also is betting on more liquid securities: "With globalization increasing, you'll see more crises," he says. "Our whole focus is on the extremes now - **what's the worst that can happen to you in any situation** - because we never want to go through that again." (John Meriwether, The Wall Street Journal, 21/8/2000)
  "Over the last number of years, regulators have encouraged financial entities to use portfolio theory to produce dynamic measures of risk. VaR, the product of portfolio theory, is used for short-run day-to-day profit and loss exposures. Now is the time to encourage the BIS and other regulatory bodies to support studies on stress test and concentration methodologies. Planning for crises is more important than VaR analysis. And such new methodologies are the correct response to recent crises in the financial industry"
  - (Myron Scholes, American Economic Review, May 2000)

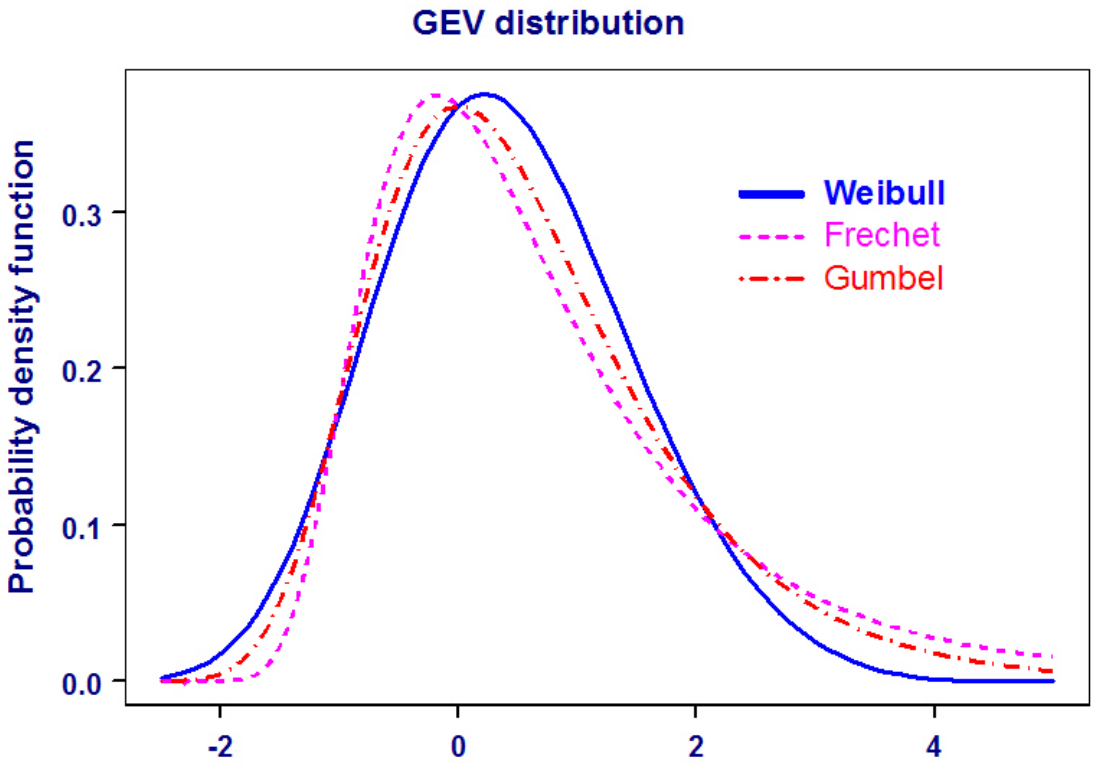
Extreme Value Theory: Block Maxima

- Worst Case Scenario (WCS): One important theorem states that the maximum of a sequence of observations, under very general conditions, is approximately distributed as the generalized extreme value (GEV) distribution. This distributio

(i) *Gumbel*
A distribution with a light upper tail and positively skewed.

(ii) *Frechet*
A distribution with a heavy upper tail and infinite higher order moments.

(iii) *Weibull*
A distribution with a bounded upper tail.



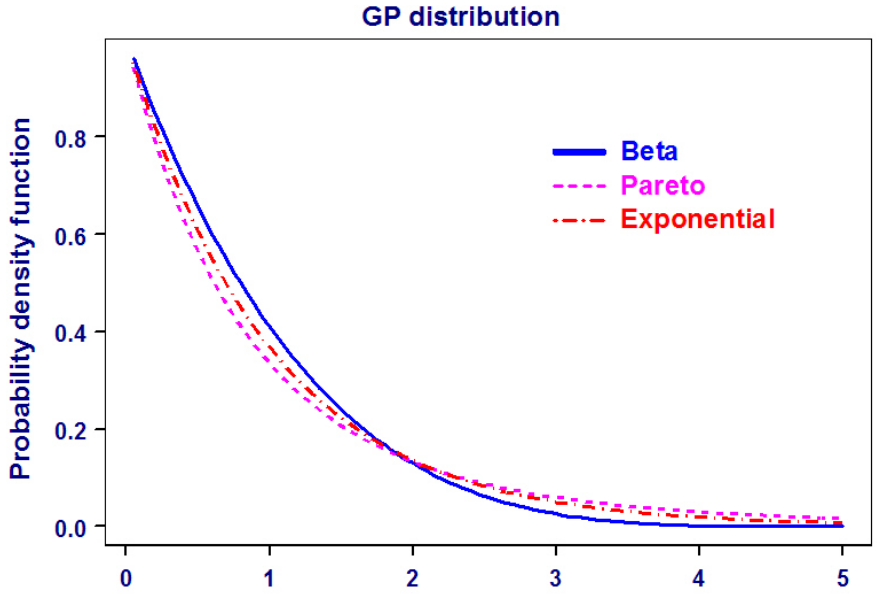
Extreme Value Theory: Peaks Over Threshold

- In terms of the tail of a distribution, the corresponding theorem states that the observations exceeding a high threshold, under very general conditions, are approximately distributed as the generalized Pareto (GP) distribution. This distribution has three forms

(i) *Exponential*
A light-tailed distribution with a "memoryless" property.

(ii) *Pareto*
A heavy-tailed distribution (sometimes called "power law").

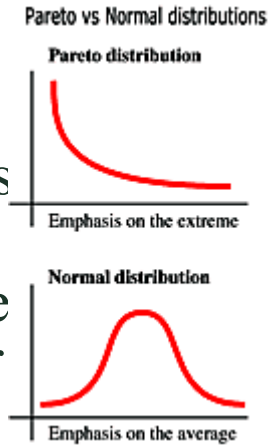
(iii) *Beta*
A bounded distribution.



Vilfredo Pareto

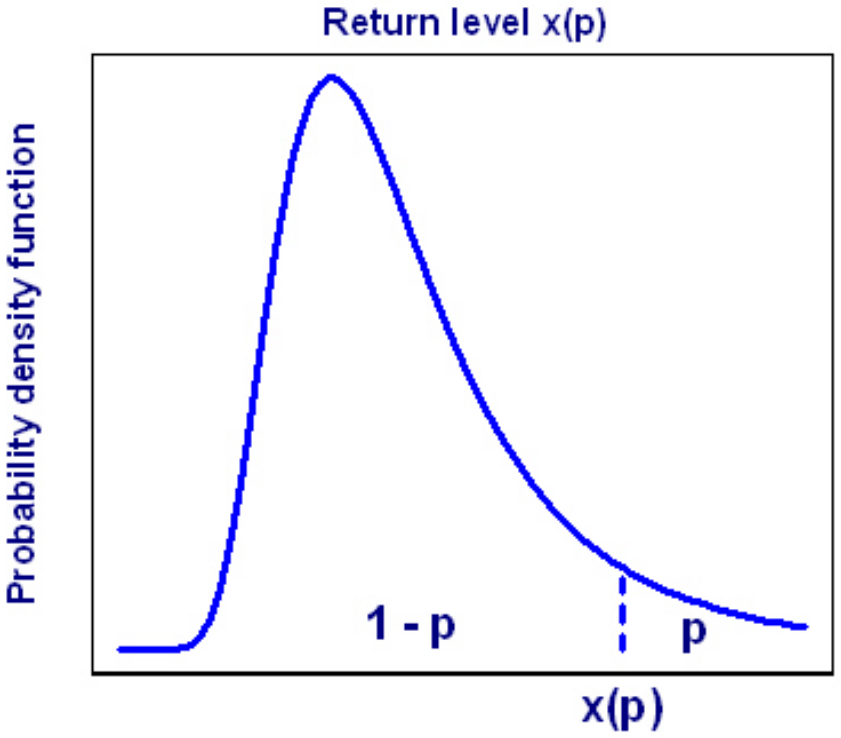
The Pareto Principle

- The Pareto principle is a mathematical formulation which states that the distribution of incomes and wealth in society is not random, but exhibits a consistent pattern. This relationship follows a regular logarithmic pattern and can be charted in a similar shape, regardless of the time period or country studied. The formula is: $\mathbf{Log}\ N = \mathbf{log}\ A + m\ \mathbf{log}\ x$
where N is the number of income earners who receive incomes higher than x, and A and m are constants.
- In simplified terms, 80% of the wealth is owned by 20% of the population. In its generalized form, the principle states that for many phenomena 80% of consequences stem from 20% of the causes.
- Employment of the Pareto principle improves everyday problem-solving efficiency greatly. Rather than wasting time, energies and money on efforts to correct everything, it is more profitable to focus the attention only on those few variables, which are shown to account for most of the problem



Frequency and Severity

The concepts of **return level** and **return period** are commonly used to convey information about the likelihood of rare events such as floods. A **return level** with a **return period** of $T = 1/p$ years is a high threshold $x(p)$ (e.g., annual peak flow of a river) whose probability of exceedance is p. For example, if $p = 0.01$, then the return period is $T = 100$ years.



Extreme Value Theory: Peaks Over Threshold

- Extreme value theory can be used to investigate the properties of the right tail of the empirical distribution of a variable x. (If we interested in the left tail we consider the variable $-x$.)
- We first choose a level u somewhat in the right tail of the distribution
- We then use Gnedenko's result which shows that for a wide class of distributions as u increases the probability distribution that v lies between u and $u+y$ conditional that it is greater than u tends to a generalized Pareto distribution

Maximum Likelihood Estimator

- Generalized Pareto Distribution has two parameters ξ (the shape parameter) and β (the scale parameter)
- The cumulative distribution is $1-\left[1+\frac{\xi}{\beta}y\right]^{-1/\xi}$

- The observations, x_i, are sorted in descending order. Suppose that there are n_u observations greater than u
- We choose ξ and β to maximize $\sum_{i=1}^{n_u}\ln\left[\frac{1}{\beta}\left(1+\frac{\xi(v_i-u)}{\beta}\right)^{-1/\xi-1}\right]$

Using Maximum Likelihood for 4-Index Example, u=160

Scenario	Loss ($000s)	Rank	$\ln\left[\frac{1}{\beta}\left(1+\frac{\xi(v_i-u)}{\beta}\right)^{-1/\xi-1}\right]$
494	477.841	1	-8.97
339	345.435	2	-7.47
349	282.204	3	-6.51
329	277.041	4	-6.42
487	253.385	5	-5.99
304	160.778	22	-3.71
Total			-108.37

Tail Probabilities

Our estimator for the cumulative probability that the variable v is greater than x is

$$\frac{n_u}{n}\left[1+\xi\frac{x-u}{\beta}\right]^{-1/\xi}$$

Setting $u=\beta/\xi$ we see that this corresponds to the power law

$$\text{Prob}(v>x)=Kx^{-\alpha}$$

where

$$K=\frac{n_u}{n}\left(\frac{\xi}{\beta}\right)^{-1/\xi} \qquad \alpha=\frac{1}{\xi}$$

Extreme value theory therefore explains why the power law holds so widely

Estimating VaR Using Extreme Value Theory

The estimate of VaR when the confidence level is q is obtained by solving

$$q = 1 - \frac{n_u}{n}\left(1 + \xi\frac{\mathrm{VaR} - u}{\beta}\right)^{-1/\xi}$$

It is

$$\mathrm{VaR} = u + \frac{\beta}{\xi}\left\{\left[\frac{n}{n_u}(1-q)\right]^{-\xi} - 1\right\}$$

Implications

- Tail behavior of F, not the specific distribution, determines the limiting distribution of the (normalized) maximum (or minimum). The normalization factors (location and scale parameters may depend on F
- Implications to VaR (or other risk measure) estimation
- The tail index (shape parameter) is time invariant so that time aggregation is easier
- EVT is extended to when X's are not i.i.d.
  - Multivariate EVT

Conclusions

- Market risk measurement applies to large-scale portfolio and requires simplifications
- Among major design choices are
  (1) the choice and number of risk factors
  (2) the choice of a local versus full valuation method for the instruments
- These choices depend on the nature of the portfolio and reflect tradeoffs between speed and accuracy
- The ultimate goal of risk measurement is to understand risk better so as to manage it effectively
- Risk management should not only prevent losses, but add value to the decision process
- Tools such as marginal and component VAR are integral to portfolio management
- Proper risk management requires competent risk managers

Cartoon


McHUMOR.com by T. McCracken
Ben Franklin jump starts his car.


PERFECT STORM