Financial Risk Management

MSBA in Financial Risk Management

The Model Building Approach

The Model-Building Approach

- The main alternative to historical simulation is to make assumptions about the probability distributions of the return on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically

- This is known as the model building approach or the variance-covariance approach

Daily Volatilities

- In VaR calculations we measure volatility "per day"

$$\sigma_{\text{day}} = \frac{\sigma_{\text{year}}}{\sqrt{252}}$$

- Theoretically, s_{day} is the standard deviation of the continuously compounded return in one day
- In practice we assume that it is the standard deviation of the percentage change in one day

Microsoft

- We have a position worth $10 million in Microsoft shares
- The volatility of Microsoft is 2% per day (about 32% per year)
- We use N=10 and X=99

Microsoft

- The standard deviation of the change in the portfolio in 1 day is \$200,000

- The standard deviation of the change in 10 days is

$$200{,}000\sqrt{10} = \$632{,}456$$

Microsoft

- We assume that the expected change in the value of the portfolio is zero (This is OK for short time periods)
- We assume that the change in the value of the portfolio is normally distributed
- Since $N(-2.33)=0.01$, the VaR is

$$2.33 \times 632{,}456 = \$1{,}473{,}621$$

AT&T

- Consider a position of $5 million in AT&T
- The daily volatility of AT&T is 1% (approx 16% per year)

- The S.D per 10 days is

$$50{,}000\sqrt{10} = \$158{,}144$$

- The VaR is

$$158{,}114 \times 2.33 = \$368{,}405$$

Portfolio

- Now consider a portfolio consisting of both Microsoft and AT&T
- Assume that the returns of AT&T and Microsoft are bivariate normal
- Suppose that the correlation between the returns

S.D. of Portfolio

- A standard result in statistics states that

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

- In this case s$_X$ = 200,000 and s$_Y$ = 50,000 and r = 0.3. The standard deviation of the change in the portfolio value in one day is therefore 220,227

VaR for Portfolio

- The 10-day 99% VaR for the portfolio is

$$220{,}227 \times \sqrt{10} \times 2.33 = \$1{,}622{,}657$$

- The benefits of diversification are (1,473,621+368,405)–1,622,657=$219,369

- What is the incremental effect of the AT&T holding on VaR?

Build The Model

This assumes

- The daily change in the value of a portfolio is linearly related to the daily returns from market variables

- The returns from the market variables are normally distributed

Markowitz Result for Variance of Return on Portfolio

$$\text{Variance of Portfolio Return} = \sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij} w_i w_j \sigma_i \sigma_j$$

w_i is weight of ith instrument in portfolio

σ_i^2 is variance of return on ith instrument in portfolio

ρ_{ij} is correlation between returns of ith and jth instruments

Variance of Return on Portfolio

VaR Result for Variance of Portfolio Value ($a_i = w_i P$)

$$\Delta P = \sum_{i=1}^{n} \alpha_i \Delta x_i$$

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \alpha_i \alpha_j \sigma_i \sigma_j$$

$$\sigma_P^2 = \sum_{i=1}^{n} \alpha_i^2 \sigma_i^2 + 2 \sum_{i<j} \rho_{ij} \alpha_i \alpha_j \sigma_i \sigma_j$$

σ_i is the daily volatility of ith instrument (i.e., SD of daily return)

σ_P is the SD of the change in the portfolio value per day

Covariance Matrix ($\text{var}_i = \text{cov}_{ii}$)

$$C = \begin{pmatrix} \text{var}_1 & \text{cov}_{12} & \text{cov}_{13} & \cdots & \text{cov}_{1n} \\ \text{cov}_{21} & \text{var}_2 & \text{cov}_{23} & \cdots & \text{cov}_{2n} \\ \text{cov}_{31} & \text{cov}_{32} & \text{var}_3 & \ldots & \text{cov}_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{cov}_{n1} & \text{cov}_{n2} & \text{cov}_{n3} & \ldots & \text{var}_n \end{pmatrix}$$

$\text{cov}_{ij} = \rho_{ij}\,\sigma_i\,\sigma_j$ where σ_i and σ_j are the SDs of the daily returns of variables i and j, and ρ_{ij} is the correlation between them

Alternative Expressions for σ_P^2

$$\sigma_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{cov}_{ij}\,\alpha_i\alpha_j$$

$$\sigma_P^2 = \boldsymbol{\alpha}^{\mathrm{T}} C \boldsymbol{\alpha}$$

where $\boldsymbol{\alpha}$ is the column vector whose ith element is α_i and $\boldsymbol{\alpha}^{\mathrm{T}}$ is its transpose

Build The Model Example

- Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

	A	B	C	D	E	F	G	H	I	J	K	L
1		DATA after Adjusting for Exchange Rates								RETURNS		
2	Day	Date	DJIA	FTSE 100	CAC 40	Nikkei 225						
3	0	8/7/2006	11219.38	6026.333	4345.084	14023.44			DJIA	FTSE 100	CAC 40	nikkei 225
4	1	8/8/2006	11173.59	6007.081	4347.993	14300.91			-0.00408	-0.00319	0.00067	0.019787
5	2	8/9/2006	11076.18	6055.3	4413.353	14467.09			-0.00872	0.008027	0.015032	0.01162
6	3	8/10/2006	11124.37	5964.004	4333.898	14413.32			0.004351	-0.01508	-0.018	-0.00372
7	4	8/11/2006	11088.02	5977.008	4338.86	14270.95			-0.00327	0.00218	0.001145	-0.00988
8	5	8/14/2006	11097.87	6014.24	4384.468	14491.32			0.000888	0.006229	0.010512	0.015441
9	6	8/15/2006	11230.26	6052.116	4458.963	14507.49			0.011929	0.006298	0.016991	0.001116

Build The Model Example

- Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

CORRELATIONS

	DJIA	FTSE 100	CAC40	Nikkei 225
DJIA	1			
FTSE 100	0.489105943	1		
CAC40	0.495709627	0.918108253	1	
Nikkei 225	-0.061899208	0.200942213	0.21095096	1

COVARIANCES

	DJIA	FTSE 100	CAC40	Nikkei 225
DJIA	0.000122707			
FTSE 100	7.6812E-05	0.000200995		
CAC40	7.66715E-05	0.000181743	0.00019496	
Nikkei 225	-9.4745E-06	3.93641E-05	4.07E-05	0.00019093

Std Devs	0.011077298	0.014177255	0.01396281	0.01381775	
alpha's	4000	3000	1000	2000	alpha's
Variance-Covariance	0.0001227	0.0000768	0.0000767	-0.0000095	4000
	0.0000768	0.0002010	0.0001817	0.0000394	3000
	0.0000767	0.0001817	0.0001950	0.0000407	1000
	-0.0000095	0.0000394	0.0000407	0.0001909	2000

Build The Model Example

- Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

Portfolio Variance	8761.832891
Portfolio SD	93.60466277
1% Z Score	2.326347874
One-Day 99% VaR	217.7570082

The Monte Carlo Simulation Approach

Monte Carlo Simulation

Structured Monte Carlo

To calculate VaR using MC simulation we

- Value portfolio today
- Sample once from the multivariate distributions of the Dx_i
- Use the Dx_i to determine market variables at end of one day
- Revalue the portfolio at the end of day

Monte Carlo Simulation

- Calculate D*P*
- Repeat many times to build up a probability distribution for D*P*
- VaR is the appropriate fractile of the distribution times square root of *N*
- For example, with 1,000 trial the 1 percentile is the 10th worst case.

Example : Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

- Equal Weight Model

Simulation Approach				
	DJIA	*FTSE 100*	*CAC 40*	*nikkei 225*
DJIA	1			
FTSE 100	0.489106	1		
CAC 40	0.49571	0.918108	1	
nikkei 225	-0.061899	0.200942	0.210951	1

	DJIA	FTSE 100	CAC40	Nikkei 225
Return	0	0	0	0
Gross Return	1	1	1	1
Portfolio Loss	0			

Forecast Name	Portfolio
Standard Deviation	94.41
Variance	8,913.06
1%	-219.65
One-Day 99% VaR	219.65

Example : Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

- EWMA Model

Simulation Approach

	DJIA	FTSE 100	CAC 40	nikkei 225
DJIA	1			
FTSE 100	0.611	1		
CAC 40	0.629	0.971	1	
nikkei 225	-0.113	0.409	0.342	1

	DJIA	FTSE 100	CAC40	Nikkei 225
Return	0	0	0	0
Gross Return	1	1	1	1
Portfolio Loss	0			

Forecast Name	Portfolio Loss
Standard Deviation	206.29
Variance	42,555.04
1%	-482.95
One-Day 99% VaR	482.95

	Ret	FT	C	Nil
Return (EWMA)	1.000			
FTSE 100 (EWMA)	0.611	1.000		
CAC40 (EWMA)	0.629	0.971	1.000	
Nikkei 225 (EWMA)	-0.113	0.409	0.342	1.000

Monte Carlo Simulation

- Overcomes some of the shortcomings of the normal distribution approach
- Overview
  - Make assumptions about distributions for frequency and severity of individual losses
  - Randomly draw from each distribution and calculate the firm's total losses under alternative risk management strategies
  - Redo step two many times to obtain a distribution for total losses under each of the alternative strategies
  - Compare strategies (distributions)

Comparison of Approaches

- Historical simulation lets historical data determine distributions, but is computationally slower
- Model building approach assumes normal distributions for market variables. It tends to give poor results for low delta portfolios
- Monte Carlo approach overcomes some of the shortcomings of the normal distribution approach

Stress Testing

- This involves testing how well a portfolio performs under extreme but plausible market moves

- Scenarios can be generated using
  - Historical data
  - Analyses carried out by economics group
  - Senior management

Back-Testing

- Tests how well VaR estimates would have performed in the past
- We could ask the question: How often was the actual 1-day loss greater than the 99%/1- day VaR?

Regulatory Capital

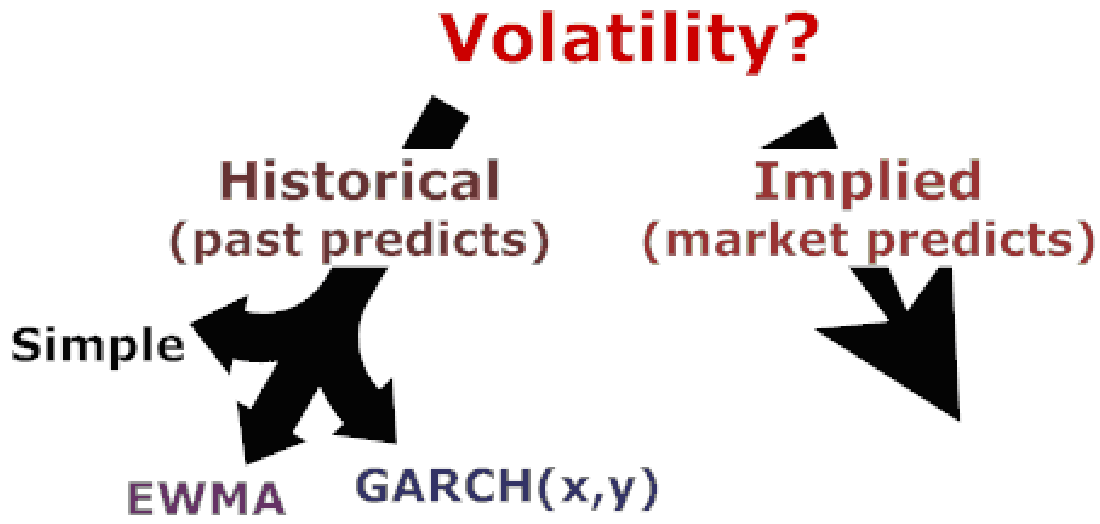
- Basel II
- Internal markets
- Cushion/Buffer for unexpected losses
- Traditional: Rigid, based on asset classifications
- Modern: Based on internal risk models such as VaR

- Externalities/systemic risk
- Deposit insurance
- Moral hazard problems

Volatility Term Structures

Return as Random Variable

- Think of return as a *random variable*



Estimating Volatility

- Define σ_n as the volatility per day between day n-1 and day n, as estimated at end of day n-1
- Define S_i as the value of market variable at end of day i
- Define $u_i = \ln(S_i/S_{i-1})$

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}(u_{n-i} - \bar{u})^2$$

$$\bar{u} = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}$$

Simplifications Usually Made

- Define u_i as $(S_i - S_{i-1})/S_{i-1}$
- Assume that the mean value of u_i is zero
- Replace m-1 by m

This gives

$$\sigma_n^2 = \frac{1}{m} \sum_{i=1}^{m} u_{n-i}^2$$

Weighting Scheme

Instead of assigning equal weights to the observations we can set

$$\sigma_n^2 = \sum_{i=1}^{m} \alpha_i u_{n-i}^2$$

where

$$\sum_{i=1}^{m} \alpha_i = 1$$

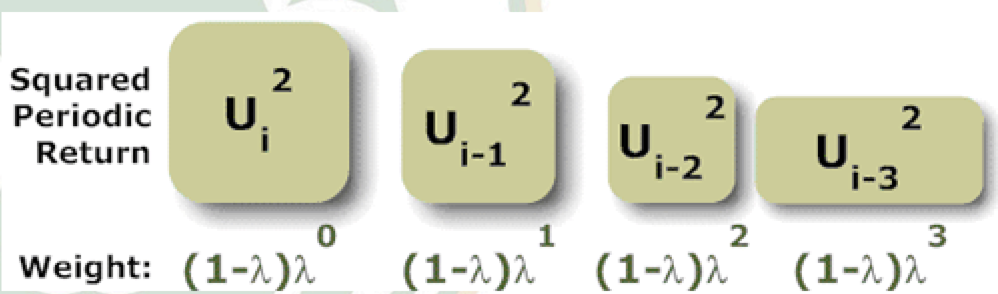
Squared Periodic Return	U_i^2	U_{i-1}^2	U_{i-2}^2	U_{i-3}^2
Weight:	α	α	α	α

$$\sigma_n^2 = \frac{1}{m} \sum_{i=1}^{m} u_{n-i}^2$$

EWMA Model

- In an **Exponentially Weighted Moving Average (EWMA)** model, the weights assigned to the u^2 decline exponentially as we move back through time
- This leads to

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$



Thomas *Bayes*
(1702–1761)

Attractions of EWMA

- Relatively little data needs to be stored
- We need only remember the current estimate of the variance rate and the most recent observation on the market variable
- Tracks volatility changes
- RiskMetrics uses $\lambda = 0.94$ for daily volatility forecasting

ARCH Model

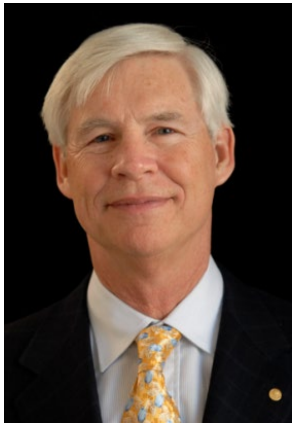
ARCH: AutoRegressive Conditional Heteroskedasticity

In an ARCH(q) model (Engle 1982) we also assign some weight to the long-run variance rate, V_L:

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{q} \alpha_i u_{n-i}^2$$

where

$$\gamma + \sum_{i=1}^{q} \alpha_i = 1$$



Robert Engle (1942-)

ARCH Model

- An ARCH(q) model can be estimated using ordinary least squares.
- A methodology to test for the lag length of ARCH errors using the Lagrange multiplier test was proposed by Engle (1982). This procedure is as follows:
  - Estimate the best fitting AutoRegressive model AR(q) .

$$y_t = a_0 + a_1 y_{t-1} + \cdots + a_q y_{t-q} + \epsilon_t = a_0 + \sum_{i=1}^{q} a_i y_{t-i} + \epsilon_t.$$

  - Obtain the squares of the error and regress them on a constant and q lagged values: where q is the length of ARCH lags.

$$\hat{\epsilon}_t^2 = \hat{\alpha}_0 + \sum_{i=1}^{q} \hat{\alpha}_i \hat{\epsilon}_{t-i}^2$$

  - The null hypothesis is that, in the absence of ARCH components, we have αi = 0 for all . The alternative hypothesis is that, in the presence of ARCH components, at least one of the estimated αi coefficients must be significant.

GARCH Model

- If an **AutoRegressive Moving Average model (ARMA model)** is assumed for the error variance, the model is a **Generalized AutoRegressive Conditional Heteroskedasticity (GARCH**, Bollerslev(1986)) model.

- In GARCH (1,1) model we assign some weight to the long-run average variance rate

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

Since weights must sum to 1

$$\gamma + \alpha + \beta = 1$$

Tim Bollerslev
(1958-)

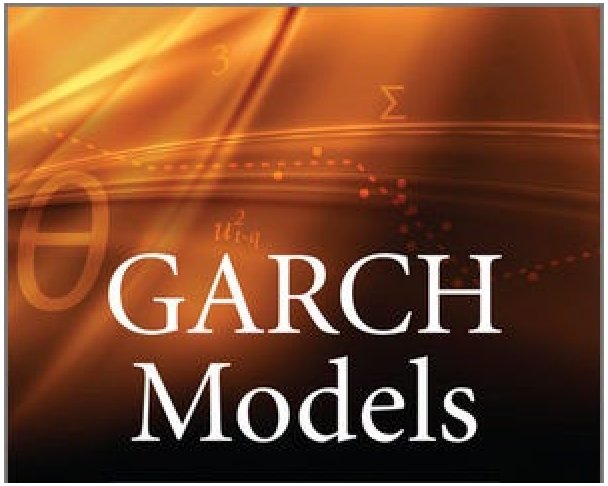
GARCH Model

Setting $\omega = \gamma V$ the GARCH (1,1) model is

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

and

$$V_L = \frac{\omega}{1 - \alpha - \beta}$$



Example

- Suppose

$$\sigma_n^2 = 0.000002 + 0.13u_{n-1}^2 + 0.86\sigma_{n-1}^2$$

- ω=0.000002, α=0.13, β =0.86, γ=1−α−β=0.01

$$V_L = \frac{\omega}{1-\alpha-\beta} = \frac{0.000002}{0.01} = 0.0002$$

- The long-run variance rate is 0.0002 so that the long-run volatility per day is $1.4\% = \sqrt{0.0002}$

Example continued

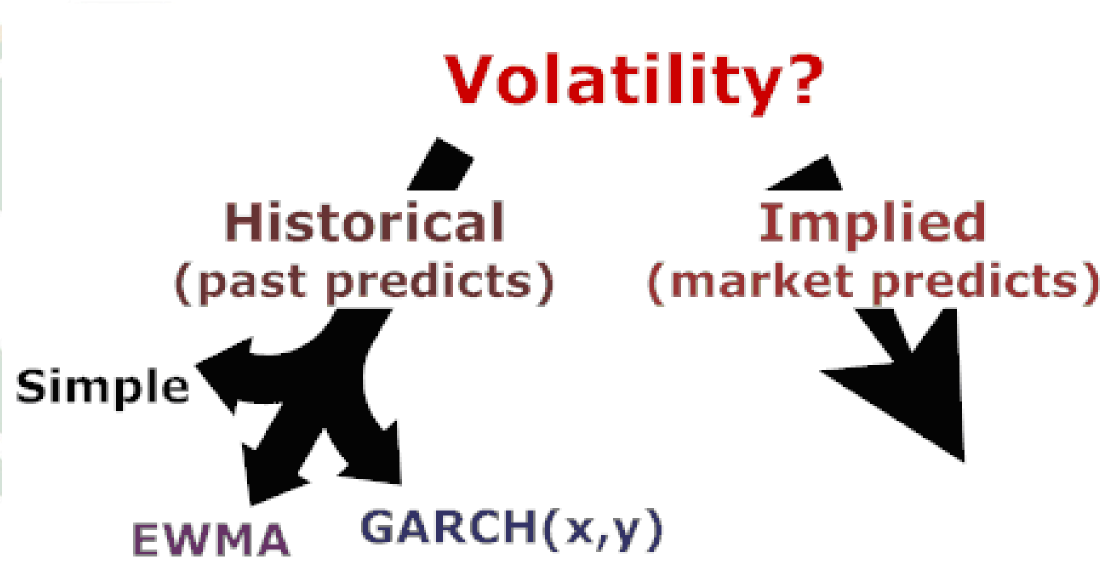
- Suppose that the current estimate of the volatility is 1.6% per day and the most recent percentage change in the market variable is 1%.
- The new variance rate is

$$0.000002 + 0.13 \times 0.0001 + 0.86 \times 0.000256 = 0.00023336$$

The new volatility is 1.53% per day

GARCH (p,q)

$$\sigma_n^2 = \omega + \sum_{i=1}^{p} \alpha_i u_{n-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{n-j}^2$$



Application to EWMA

Estimate the variance of observations from a normal distribution with mean zero: $u_i \sim N(0, v_i)$

$$f(u_i) = \frac{1}{\sqrt{2\pi v_i}} \exp\left(\frac{-u_i^2}{2v_i}\right)$$

$$v_i = \lambda v_{i-1} + (1-\lambda) u_{i-1}^2$$

We choose parameters that maximize

$$\prod_{i=1}^{m} \frac{1}{\sqrt{2\pi v_i}} \exp\left(-\frac{u_i^2}{2v_i}\right)$$

or

$$\sum_{i=1}^{m} \left[-\ln(v_i) - \frac{u_i^2}{v_i}\right]$$

Correlations and Covariances

- Define $x_i=(X_i-X_{i-1})/X_{i-1}$ and $y_i=(Y_i-Y_{i-1})/Y_{i-1}$

Also

- $\sigma_{x,n}$: daily vol of X calculated on day n-1
- $\sigma_{y,n}$: daily vol of Y calculated on day n-1
- cov_n: covariance calculated on day n-1
- The correlation is $\text{cov}_n/(\sigma_{u,n}\,\sigma_{v,n})$

Updating Correlations

- We can use similar models to those for volatilities

- Under EWMA

$$\text{cov}_n = \lambda \, \text{cov}_{n-1} + (1-\lambda) x_{n-1} y_{n-1}$$

Volatilities and Correlations for Four-Index on Sept 25, 2008 with Equal Weights

	DJIA	FTSE	CAC 40	Nikkei 225
DJIA	1			
FTSE	0.489	1		
CAC 40	0.496	0.918	1	
Nikkei 225	−0.062	0.201	0.211	1

	DJIA	FTSE	CAC 40	Nikkei 225
Vol. per day (%)	1.11	1.42	1.40	1.38

Volatilities and Correlations for Four-Index on Sept 25, 2008 for EWMA and λ=0.94

	DJIA	FTSE	CAC 40	Nikkei 225
DJIA	1			
FTSE	0.611	1		
CAC 40	0.629	0.971	1	
Nikkei 225	−0.113	0.409	0.342	1

	DJIA	FTSE	CAC 40	Nikkei 225
Vol. per day (%)	2.19	3.21	3.09	1.59

Example : Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
			VARIANCEs						COVARIANCE			
lambda		DJIA	FTSE 100	CAC 40	Nikkei 225		DJIA/FTSE	DJIA/CAC	DJIA/Nikkei	FTSE/CAC	FTSE/Nikkei	CAC/Nikkei
0.94												
		0.0001227	0.000201	0.000195	0.0001909		7.6812E-05	7.66715E-05	-9.4745E-06	0.0001817	3.93641E-05	4.06997E-05
		0.0001163	0.0001895	0.0001833	0.000203		7.29856E-05	7.19072E-05	-1.37514E-05	0.0001707	3.32096E-05	3.90526E-05
		0.0001139	0.000182	0.0001859	0.0001989		6.44077E-05	5.97298E-05	-1.90043E-05	0.0001677	3.68134E-05	4.71898E-05
		0.0001082	0.0001848	0.0001941	0.0001878		5.66074E-05	5.14463E-05	-1.88342E-05	0.0001739	3.79664E-05	4.83728E-05
		0.0001024	0.000174	0.0001826	0.0001824		5.27835E-05	4.81351E-05	-1.57675E-05	0.0001636	3.43962E-05	4.47919E-05
		9.628E-05	0.0001658	0.0001783	0.0001857		4.99485E-05	4.58072E-05	-1.39985E-05	0.0001578	3.81036E-05	5.18431E-05
		9.904E-05	0.0001583	0.0001849	0.0001747		5.14593E-05	5.522E-05	-1.23597E-05	0.0001547	3.62391E-05	4.98704E-05
		9.756E-05	0.0001491	0.0001792	0.0001841		4.9486E-05	5.68342E-05	-2.17907E-06	0.0001467	3.64211E-05	5.72984E-05
		9.173E-05	0.0001407	0.0001687	0.0001732		4.63926E-05	5.35117E-05	-2.08929E-06	0.0001375	3.44128E-05	5.37357E-05
		8.724E-05	0.0001348	0.0001601	0.0001635		4.19777E-05	4.90755E-05	-1.10465E-06	0.0001312	3.09603E-05	4.94691E-05

Example : Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

alpha's			4000	3000	1000	2000		alpha's
Variance-Covariance			0.0004801	0.0004303	0.0004257	-0.0000396		4000
			0.0004303	0.0010314	0.0009630	0.0002095		3000
			0.0004257	0.0009630	0.0009535	0.0001681		1000
			-0.0000396	0.0002095	0.0001681	0.0002541		2000
Standard Deviations			0.0219107	0.0321151	0.0308795	0.0159408		
Portfolio Variance			40995.765					
Portfolio SD			202.47411					
1% Z Score			2.3263479					
One-Day 99% VaR			471.02521					
			0.0219107	0.0321151	0.0308795	0.0159408		
Correl		0.0219107	1.000	0.611	0.629	-0.113		
Matrix		0.0321151	0.611	1.000	0.971	0.409		
		0.0308795	0.629	0.971	1.000	0.342		
		0.0159408	-0.113	0.409	0.342	1.000		

Maximum Likelihood Methods

Maximum Likelihood Methods

- In maximum likelihood methods we choose parameters that maximize the likelihood of the observations occurring

Example 1

- We observe that a certain event happens one time in ten trials. What is our estimate of the proportion of the time, *p*, that it happens?
- The probability of the event happening on one particular trial and not on the others is
- We maximize this to obtain a maximum likelihood estimate. Result: $p=0.1$

$$\max p(1-p)^9 \qquad \text{s.t.} \quad p <= 1, \quad p >= 0$$

$$\xrightarrow{\ln(p(1-p)^9)}$$

$$\max \ln(p) + 9\ln(1-p) \qquad \text{s.t.} \quad p <= 1, \quad p >= 0$$

Example 2

Estimate the variance of observations from a normal distribution with mean zero: $u_i \sim N(0, v)$

$$f(u_i) = \frac{1}{\sqrt{2\pi v}} \exp\left(\frac{-u_i^2}{2v}\right)$$

$$\text{Maximize}: \quad \prod_{i=1}^{m} \left[\frac{1}{\sqrt{2\pi v}} \exp\left(\frac{-u_i^2}{2v}\right)\right]$$

Taking logarithms this is equivalent to maximizing:

$$\sum_{i=1}^{m} \left[-\ln(v) - \frac{u_i^2}{v}\right]$$

$$\text{Result}: \quad v = \frac{1}{m} \sum_{i=1}^{m} u_i^2$$

Application to EWMA

Estimate the variance of observations from a normal distribution with mean zero: $u_i \sim N(0, v_i)$

$$f(u_i) = \frac{1}{\sqrt{2\pi v_i}} \exp\left(\frac{-u_i^2}{2v_i}\right)$$

$$\boxed{v_i = \lambda v_{i-1} + (1-\lambda)u_{i-1}^2}$$

We choose parameters that maximize

$$\prod_{i=1}^{m} \frac{1}{\sqrt{2\pi v_i}} \exp\left(-\frac{u_i^2}{2v_i}\right)$$

or

$$\sum_{i=1}^{m}\left[-\ln(v_i) - \frac{u_i^2}{v_i}\right]$$

Estimate EWMA with S&P 500 Data

Date	Day	S_i	$u_i=(S_i-S_{i-1})/S_{i-1}$	$v_i=s_i^2$	$-\ln(v_i)-u_i^2/v_i$
18-Jul-2005	1	1221.13			
19-Jul-2005	2	1229.35	0.006731		
20-Jul-2005	3	1235.20	0.004759	0.00004531	9.5022
21-Jul-2005	4	1227.04	−0.006606	0.00004389	9.0395
…….	…..	…….	………..	………….	…………
13-Aug-2010	1279	1079.25	−0.004024	0.00016813	8.5945
Total					10192.5104

Estimate EWMA with S&P 500 Data

	A	B	C	D	E
1282					
1283		lambda	0.937443227	Obj	10192.5104
1284					
1285					
1286					
1287					
1288					
1289					
1290					
1291					

Solver finds value of lambda (cell C1283) that maximizes the likelihood function in cell E1283

The EWMA Volatility Chart

Application to GARCH

Estimate the variance of observations from a normal distribution with mean zero: $u_i \sim N(0, v_i)$

$$f(u_i) = \frac{1}{\sqrt{2\pi v_i}} \exp\left(\frac{-u_i^2}{2v_i}\right)$$

$$\boxed{v_i = \omega + \alpha u_{i-1}^2 + \beta v_{i-1}}$$

We choose parameters that maximize

$$\prod_{i=1}^{m} \frac{1}{\sqrt{2\pi v_i}} \exp\left(-\frac{u_i^2}{2v_i}\right)$$

or

$$\sum_{i=1}^{m} \left[-\ln(v_i) - \frac{u_i^2}{v_i}\right]$$

Estimate GARCH with S&P 500 Data

- Start with trial values of ω, α, and β
- Update variances
- Calculate $$\sum_{i=1}^{m}\left[-\ln(v_i)-\frac{u_i^2}{v_i}\right]$$
- Use solver to search for values of ω, α, and β that maximize this objective function
- Important note: set up spreadsheet so that you are searching for three numbers that are the same order of magnitude

Estimate GARCH with S&P 500 Data

Date	Day	S_i	$u_i=(S_i-S_{i-1})/S_{i-1}$	$v_i = s_i^2$	$-\ln(v_i) - u_i^2/v_i$
18-Jul-2005	1	1221.13			
19-Jul-2005	2	1229.35	0.006731		
20-Jul-2005	3	1235.20	0.004759	0.00004531	9.5022
21-Jul-2005	4	1227.04	−0.006606	0.00004447	9.0393
…….	…..	…….	………..	………….	…………
13-Aug-2010	1279	1079.25	−0.004024	0.00016327	8.6209
Total					10,228.2349

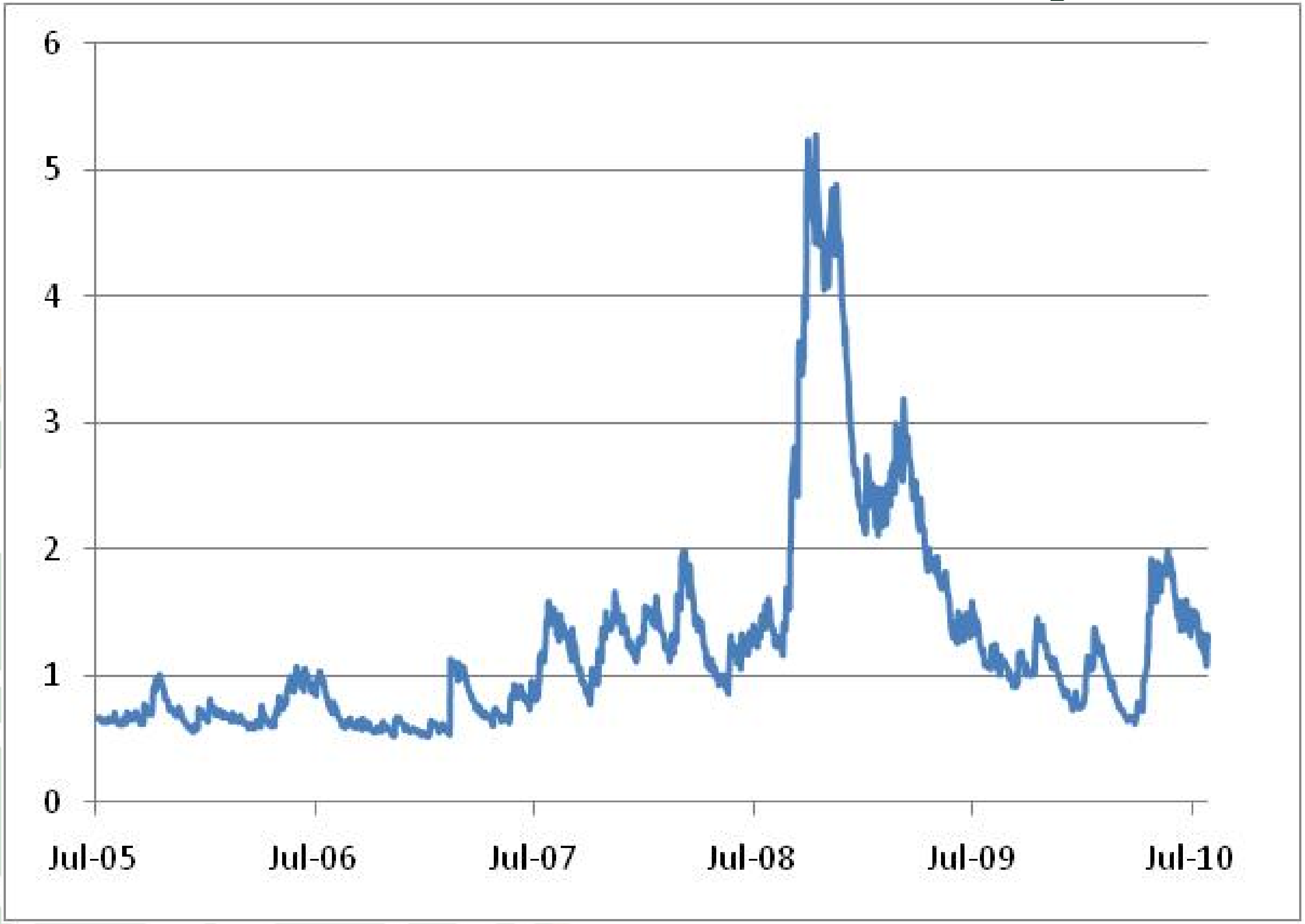
Estimate GARCH with S&P 500 Data

	A	B	C	D	E	F
1282						
1283	w	1.3465	0.0000013465	Obj	10228.2349	
1284	beta	0.910119357	0.910119			
1285	alpha	0.8339	0.083392			
1286						
1287	Long run variance per day V		0.000207524			
1288	Long run volatility per day		0.014406			
1289	Long run volatility per year		0.22868304			
1290						
1291						
1292						
1293						
1294						
1295						
1296						
1297						

Solver searches over
B1283 (which is ω*100000)
B1284 (which is β)
B1285: (which is α*10)
(scaled parameters so that they are on the same measurement)

The likelhood function (to be maximized) is in E1283
ω, β, and α are in C1283:C1285.

The GARCH Volatility Chart



Variance Targeting

- One way of implementing GARCH(1,1) that increases stability is by using variance targeting
- We set the long-run average volatility equal to the sample variance
- Only two other parameters then have to be estimated

Estimate GARCH with S&P 500 Data

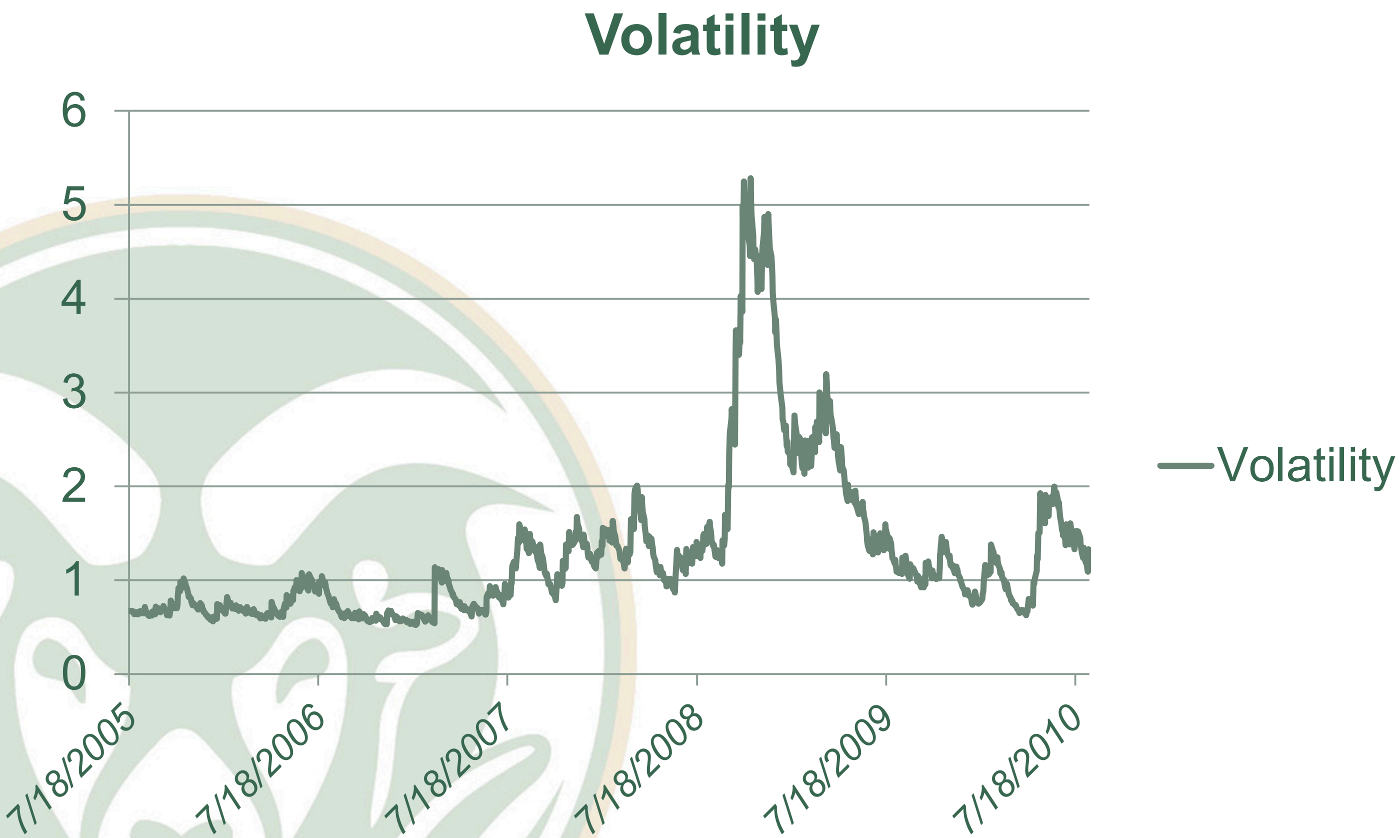
- Variance Targeting

	A	B	C	D	E	F	G
1282							
1283	w	1.3195	0.0000013195	Obj	10228.1941		
1284	beta	0.910105	0.910105				
1285	alpha		0.084425				
1286							
1287	Sample Variance		0.000241217				
1288	Sample Volatility		0.015531				
1289							
1290							
1291							
1292							
1293							
1294							

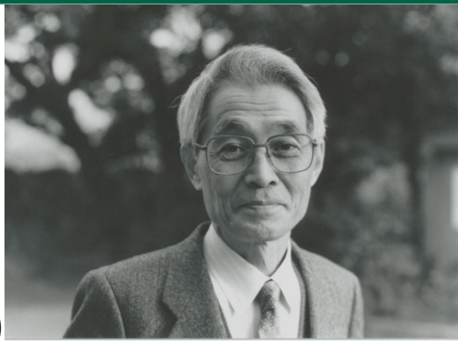
In the variance targeting approach, the long run average variance, V_L ,is set equal to the sample variance.
This means that w =V_L* (1-a-b).or
a=1-b-w/V_L
Solver searches for values of B1283 and B1284 that maximize the likelihood function in E1283.
w, b and a are in cells C1283:C1285

The GARCH Volatility Chart

- Variance Targeting



Model Selection Criteria



Hirotugu Akaike (1927-2009)

- Akaike Information Criterion (AIC)
  $$AIC = -2\ln(L) + 2k$$
- Akaike Information Criterion correction (AICc)

AICc is AIC with a correction for finite sample sizes:

$$AICc = AIC + \frac{2k(k+1)}{n-k-1}$$

where n denotes the sample size. Thus, AICc is AIC with a greater penalty for extra parameters.

Ideally, the AIC and AICc should be as small as possible

Model Selection Criteria

- Bayesian Information Criterion (BIC)/Schwartz Bayesian Criterion (SBC) Gideon E. Schwarz

$$SBC = -2\ \ln(L) + k\ (\ln(n) + \ln(2\pi))$$

Often Omitted for large n

L = likelihood function
k = number of parameters,
n = number of observations.