Financial Risk Management

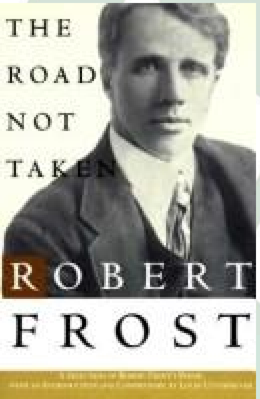
MSBA in Financial Risk Management

Colorado State University

the Right Solutions to Manage your Risk

Valuation and Risk Models

Value at Risk



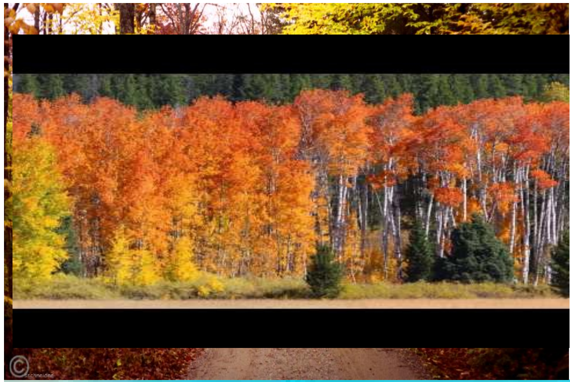
The Road Not Taken
By Robert Frost

TWO roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;

Then took the other, as just as fair,
And having perhaps the better claim,
Because it was grassy and wanted wear;
Though as for that the passing there
Had worn them really about the same,

And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.



Risk Management Options

- Avoidance (elimination)
- Reduction (mitigation)
- Acceptance (do nothing)
- Insurance
- Risk Reserve
- Risk Transfer
- Increase
- Get more information
- Contingency planning

Measures of Risk

- Value at Risk (VaR)
  - What is it?
    - Worst-case scenario dollar value loss (up to a specified probability level, for a given holding period) that could occur for a company exposed to a specific set of risks
      - Denoted in dollar (%) terms
      - Specify a probability level (confidence level)
      - Specify a time period
  - Also known as "Maximal Probable Loss"

The Question Being Asked in VaR

"What loss level is such that we are *X*% confident it will not be exceeded in *N* business days?"

- Probabilistic worst case
- Almost "perfect storm"
- 1/100 year flood level

VaR History

- Financial firms in the late 80's used it for their trading portfolios
- JP Morgan, CEO Dennis Weatherstone, 1990's
  - 4:15 and VaR → RiskMetrics, 1994
- 1997, the U.S. Securities and Exchange Commission ruled that public corporations must disclose quantitative information about their derivatives activity. Major banks and dealers chose to implement the rule by including VaR information in the notes to their financial statements.
- Now Basel II Accord, VaR is the preferred measure of market risk

VaR Uses

- Benchmark comparison
  - Interested in relative comparisons across units or trading desks
- Potential loss measure
  - Horizon related to liquidity and portfolio turnover
- Set capital cushion levels
  - Confidence level critical here

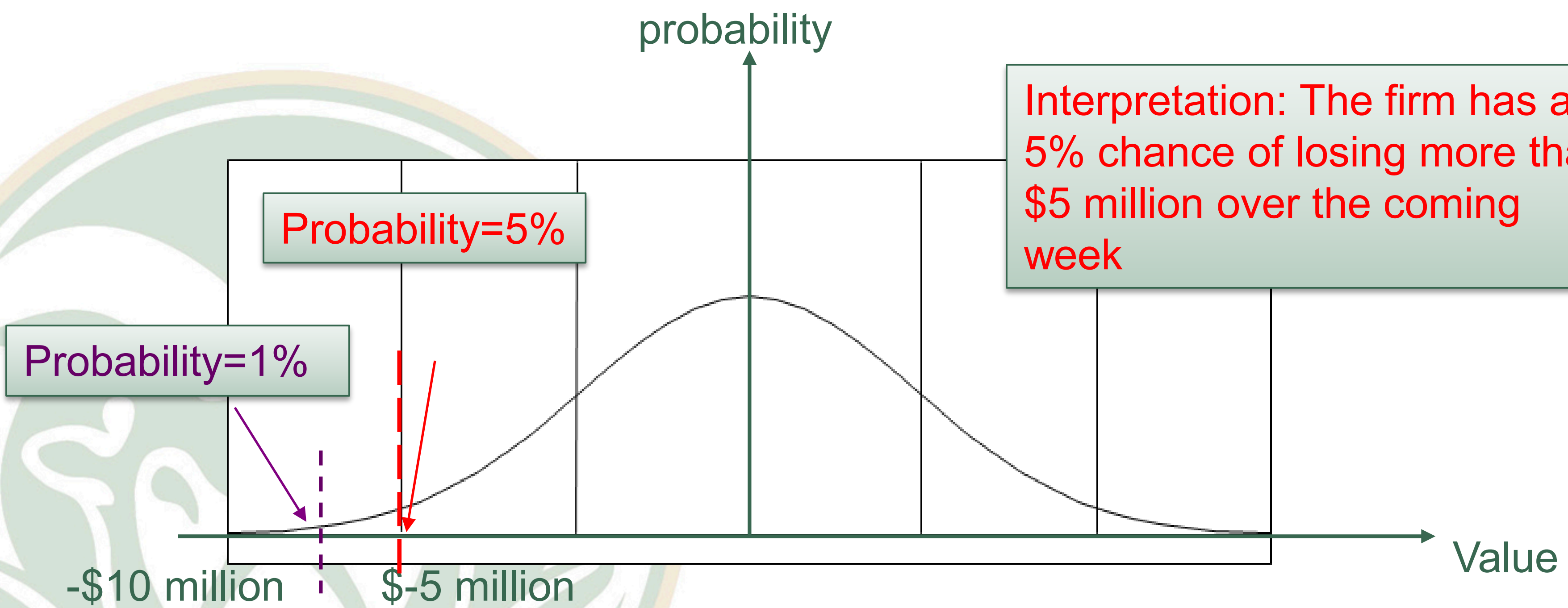
VaR Parameters

- Holding period: e.g. 10 day horizon
  - Risk environment
  - Portfolio constancy/liquidity
- Confidence level: e.g. 99 percent confidence level
  - How far into the tail?
  - VaR use
  - Data quantity
- At least one year of historical data

Measures of Risk

With a Normal Approximation: $VaR_\alpha = -(\mu - \sigma \times z_\alpha)$

With a lognormal Approximation $VaR_\alpha = P_{t-1} \times (1 - e^{\mu - \sigma \times z_\alpha})$



α-quantile of $F_X(x)$: $VaR_\alpha = F_X^{-1}(\alpha)$

Normal Approximation

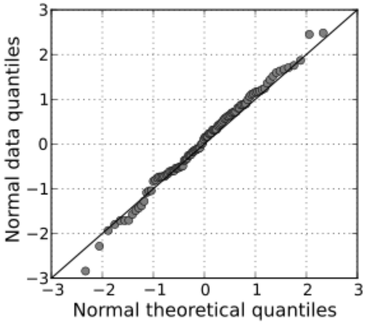
- Most loss distributions are not normal
- From the central limit theorem, using the normal distribution will nevertheless be appropriate when
  - Number of exposures is large
  - Losses across exposures are independent
- Examples where it might be appropriate
  - Worker injury losses for firms with a large number of employees
  - Automobile accident losses for firms with large fleets of cars
- Limitations of Normal distribution assumption
  - Independency
  - Applies only to aggregate losses, not individual losses
  - It cannot be used to analyze decisions about per occurrence deductibles and limits

Lognormal VAR

The lognormal distribution is right-skewed with positive outliers and bounded below by zero

Therefore, the lognormal distribution is commonly used to counter the possibility of negative asset prices

If we assume that geometric returns follow a normal distribution, then the natural logarithm of asset prices follows a normal distribution and asset prices themselves follow a lognormal distribution



Quantile-Quantile (QQ) Plot

- The QQ plot is a way to visually examine if empirical data fits the theoretical distribution (e.g., the normal distribution)
- The process graphs the quantiles at regular confidence intervals for the empirical distribution against the theoretical distribution
- As an example, if the middles of the QQ plot match up, but the tails do not, then the empirical distribution can be interpreted as symmetric with tails that differ from the theoretical distribution (either fatter or thinner)

Confidence Interval

- Estimators are only as useful as their precision

  Suppose that x is the qth quantile of the loss distribution when it is estimated from n observations. The standard error of x is

$$se(q) = \frac{1}{f(x)}\sqrt{\frac{q(1-q)}{n}}$$

  where $f(x)$ is an estimate of the probability density of the loss at the qth quantile calculated by assuming a probability distribution for the loss (probability mass in bin (width of interval).

- Confidence interval:
- $q - se(q) \times z_\alpha \leq VAR_\alpha \leq q + se(q) \times z_\alpha$

Non-Parametric VAR Estimation

- The Historical Simulation Approach
- The Model Building Approach
- The Monte Carlo Simulation Approach

The Historical Simulation Approach

Historical Simulation

- Create a database of the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day
- and so on

Historical Simulation

- Suppose we use 501 days of historical data (Day 0 to Day 500)
- Let v_i be the value of a variable on day i
- There are 500 simulation trials
- The ith trial assumes that the value of the market variable tomorrow is

$$v_{500} \frac{v_i}{v_{i-1}}$$

Historical Simulation Example

- Calculation of 1-day, 99% VaR for a Portfolio on Sept 25, 2008

Index	Value ($000s)
DJIA	4,000
FTSE 100	3,000
CAC 40	1,000
Nikkei 225	2,000

BEAR STEARNS

LEHMAN BROTHERS

Data After Adjusting for Exchange Rates

Day	Date	DJIA	FTSE 100	CAC 40	Nikkei 225
0	Aug 7, 2006	11,219.38	6,026.33	4,345.08	14,023.44
1	Aug 8, 2006	11,173.59	6,007.08	4,347.99	14,300.91
2	Aug 9, 2006	11,076.18	6,055.30	4,413.35	14,467.09
3	Aug 10, 2006	11,124.37	5,964.90	4,333.90	14,413.32
…	……	…..	…..	……	……
499	Sep 24, 2008	10,825.17	5,109.67	4,113.33	12,159.59
500	Sep 25, 2008	11,022.06	5,197.00	4,226.81	12,006.53

Scenarios Generated

Example of Calculation:

$$11{,}022.06 \times \frac{11{,}173.59}{11{,}219.38} = 10{,}977.08$$

Scenario	DJIA	FTSE 100	CAC 40	Nikkei 225	Portfolio Value ($000s)	Loss ($000s)
1	10,977.08	5,180.40	4,229.64	12,244.10	10,014.334	−14.334
2	10,925.97	5,238.72	4,290.35	12,146.04	10,027.481	−27.481
3	11,070.01	5,118.64	4,150.71	11,961.91	9,946.736	53.264
…	…….	…….	…….	……..	…….	……..
499	10,831.43	5,079.84	4,125.61	12,115.90	9,857.465	142.535
500	11,222.53	5,285.82	4,343.42	11,855.40	10,126.439	−126.439

Ranked Losses

Scenario Number	Loss ($000s)
494	477.841
339	345.435
349	282.204
329	277.041
487	253.385
227	217.974
131	205.256

99% one-day VaR

Accuracy

- We estimate the 0.01-quantile from 500 observations as \$25 million
- We estimate $f(x)$ by approximating the actual empirical distribution with a normal distribution mean zero and standard deviation \$10 million
- The 0.01 quantile of the approximating distribution is NORMINV(0.01,0,10) = -23.26 and the value of $f(x)$ is NORMDIST(-23.26,0,10,FALSE)=0.0027
- The estimate of the standard error is therefore

$$\frac{1}{f(x)}\sqrt{\frac{q(1-q)}{n}} = \frac{1}{0.0027} \times \sqrt{\frac{0.01 \times 0.99}{500}} = 1.67$$

The N-day VaR

- The N-day VaR for market risk is usually assumed to be $\sqrt{N}$ times the one-day VaR
- In our example the 10-day VaR would be calculated as $\sqrt{10} \times 253{,}385 = 801{,}274$
- Pick a horizon that is as short as feasible
- This assumption is in theory only perfectly correct if daily changes are normally distributed and independent

$$VAR(T\ \text{days}) = VAR(1\ \text{day}) \times \sqrt{T}$$

Historical Simulation Extension 1

- Age-weighted Historic Simulation
- Let weights assigned to observations decline exponentially as we go back in time
- Rank observations from worst to best
- Starting at worst observation sum weights until the required quantile is reached
- $W(i) = \lambda W(i-1) = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$
  - λ : decay parameter
  - Historical simulation is the special case of λ=1 (i.e. no decay).

Application to 4-Index Portfolio λ=0.995

Scenario Number	Loss ($000s)	Weight	Cumulative Weight
494	477.841	0.00528	0.00528
339	345.435	0.00243	0.00771
349	282.204	0.00255	0.01027
329	277.041	0.00231	0.01258
487	253.385	0.00510	0.01768
227	217.974	0.00139	0.01906
131	205.256	0.00086	0.01992

One-day 99% VaR=$282,204

Historical Simulation Extension 2

- Volatility-weighted Historic Simulation
- Use a volatility updating scheme and adjust the percentage change observed on day *i* for a market variable for the differences between volatility on day *i* and current volatility
- Value of market variable under *i*th scenario becomes

$$r^*_{t,i} = \frac{\sigma_{T,i}}{\sigma_{t,i}} r_{t,i}$$

- More, Correlation-weighted Historical Simulation
- Even more, Filtered Historical Simulation

Volatilities (% per Day) Estimated for Next Day in 4-Index Example

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	1.11	1.42	1.40	1.38
1	Aug 8, 2006	1.08	1.38	1.36	1.43
2	Aug 9, 2006	1.07	1.35	1.36	1.41
3	Aug 10, 2006	1.04	1.36	1.39	1.37
….	……	……	……	……	……
499	Sep 24, 2008	2.21	3.28	3.11	1.61
500	Sep 25, 2008	2.19	3.21	3.09	1.59

Volatility Adjusted Losses

Scenario Number	Loss ($000s)
131	1,082.969
494	715.512
227	687.720
98	661.221
329	602.968
339	546.540
74	492.764

Historical Simulation Extension 3

- Bootstrap Historical Simulation Method
- Suppose there are 500 daily changes
- Calculate a 95% confidence interval for VaR by sampling 500,000 times with replacement from daily changes to obtain 1000 sets of changes over 500 days
- Calculate VaR for each set and calculate a confidence interval

Computational Issues

- To avoid revaluing a complete portfolio 500 times a delta/gamma approximation is sometimes used
- When a derivative depend on only one underlying variable, S

$$\Delta P \approx \delta \Delta S + \frac{1}{2}\gamma(\Delta S)^2$$