DESCRIPTION AND MEASUREMENT OF UNCERTAINTY FOR STATE-SPACE MODEL OF LARGE CASCADE CANAL SYSTEM

Guan Guanghua[1*]
Wang Changde[1]
Feng Xiaobo[1]

ABSTRACT

Automatic control of canal network is a key solution for modern water-saving irrigation, and it is also a vital technique puzzle of the Middle Routine of the South-to-North water transfer project of China. The design of the controller is based on a liner mathematical model deduced from Saint-Venant Equation system, but the S-V Equations are a system of first order partial differential equations. A state-space model of a cascaded canal system is used in this paper to analyze the uncertainty including the uncertainty of the system itself and the uncertainty introduced in the procedure of mode-building. This uncertainty will be a precondition for the design of a robust controller. Using the liner model as the nominal case, the uncertainty is measured by the largest singular value of the distance matrix of the models. At last a simulation case of six canals is given together with quantificational describe of uncertainty.

INTRODUCTION

A canal system is a complicated system built with water flow, controlling gates and corresponding measuring equipments. The signal is mainly carried by water wave. The main equation system that describes the dynamic procedure of open canal flow is the S-V Equation system, which is a first order partial differential equation system. Ordinarily this equation system could not be solved directly. The water wave in open canal is a gravity wave, and the wave velocity is proportional to the square root proportion of the equivalent water depth. So it is usually not large. So the system has a large time-lag and is highly nonlinear. Additionally, there are strong coupling effects among the reaches, and adding with the uncertainty disturbance such as wind wave and unscheduled water withdraw, this system may be very complicated.

The design of the controller is based on the state-space linear mathematical model. One key limit of modern control theory is that the mathematical model must

[1]State Key Laboratory of Water Resources and Hydropower Engineering Science,Wuhan University, Wuhan 430072, China

*ggh@whu.edu.cn

describe the real system accurately. In another words, the design of the controller relies on the accuracy of the mathematical model. The linear state-space model of the canal system is built by truncating the Taylor series and taking the first order term. The ignored higher order items will bring uncertainty. How to account for this uncertainty and introduce it into the controller design procedure so that the system can tolerant this difference is the main task of robust control. The final robust controller can limit the influence of this uncertainty within limits, so the model uncertainty measurement is very important.

THEORY BACKGROUND

An effective canal automatic control system must choose an appropriate operation method according to hydraulic characteristic and function of its own. The method of operation directly affects the canal regulation volume of water and has a great influence to the canal operation, stability. The basic methods of operation are constant downstream depth, constant upstream depth, constant volume and controlled volume operation method. The main idea of the constant volume method is to keep the water volume in the canal pool constant by keeping the middle point of the water surface constant. In this way, the water volume of neighboring canal reaches can feed each other. The canal system can meet the water volume demand by itself, so it has a faster respone. The following model is based on this idea.

Much work has been carried out for the canal network modeling and many models have been developed. But these models may not make good balance on the accuracy and the simplicity. Usually the S-V equations are solved by characteristic method or explicit difference method. According to our experience, the characteristic method has larger error when discharge is large; its calculation will lead to a flow imbalance. The explicit difference method is easier to program but inaccurate. In this paper, the Preismann implicit scheme is used because it has high accuracy and can reach unconditioned convergence.

MATHEMATICAL MODEL

The continuity equation and the momentum equation of S-V Equations can be discretized by Preissmann implicit scheme, and expanded at the target operating point with Taylor series into the following two matrixes:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \end{bmatrix} \cdot \begin{bmatrix} \delta Q^{+}{}_{i} \\ \delta z^{+}{}_{i} \\ \delta Q^{+}{}_{j} \\ \delta z^{+}{}_{j} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \end{bmatrix} \cdot \begin{bmatrix} \delta Q_{i} \\ \delta z_{i} \\ \delta Q_{j} \\ \delta z_{j} \end{bmatrix} \tag{1}$$

The coefficients and the calculation procedure can be found in reference [1].

The change of discharge is reflected by the change of water level, so accuracy of the discharge calculation is vital to the accuracy of discharge adjustment. Actually, the gates are vary in type, shapes and dimension, so there may be no unique discharge calculation formula. The most popular one is the discharge formula of large orifice outflow. But in this formula the discharge coefficient varies with the change of flow rate so it is hard to fix. In experimental study, Henry's (1950) method for the calculation of discharge coefficient for free outflow and submerged outflow is widely accepted. So in this paper, Henry's formula is used to calculate the gate discharge. The control matrix for gate node is:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & \\ A_{21} & A_{22} & A_{23} & A_{24} & \\ & -(\frac{\partial f}{\partial z_j})_e & 1 & -(\frac{\partial f}{\partial z_k})_e & \end{bmatrix} . \begin{bmatrix} \delta Q^+{}_i \\ \delta z^+{}_i \\ \delta z^+{}_j \\ \delta Q^+{}_k \\ \delta z^+{}_k \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} & \\ B_{21} & B_{22} & B_{23} & B_{24} & \\ & -(\frac{\partial f}{\partial z_j})_e & 1 & -(\frac{\partial f}{\partial z_k})_e & \end{bmatrix} . \begin{bmatrix} \delta Q_i \\ \delta z_i \\ \delta z_j \\ \delta Q_k \\ \delta z_k \end{bmatrix} . + \begin{bmatrix} 0 \\ 0 \\ (\frac{\partial f}{\partial u})_e \end{bmatrix} . \Delta \delta u$$

(2)

Here the function f(x) is the conducted form Henry's equations [1].

Generally, water intake in canal network is in the vertical direction to flow. With the scheduled increase or decrease of discharge, the water level at the intake point will rise or drop. This is called lateral flow. For canals at the size of the South-to-North water transfer project (China), while the water intake flow is 20 or 50m^3/s, the intake point will have a drop of 1mm and 3mm separately. Because this drop is relatively small, we ignore it in modeling. So the water level relationship at intake node is continuous.

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \end{bmatrix} . \begin{bmatrix} \delta Q^+{}_j \\ \delta z^+{}_j \\ \delta Q^+{}_l \\ \delta z^+{}_l \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \end{bmatrix} . \begin{bmatrix} \delta Q_j \\ \delta z_j \\ \delta Q_l \\ \delta z_l \end{bmatrix} . + \begin{bmatrix} A_{11} & -B_{11} \\ A_{21} & -B_{21} \end{bmatrix} . \begin{bmatrix} \delta Q_p \\ \delta Q^+{}_p \end{bmatrix}$$

(3)

To simplify the model, in the control of multiple-canals, we assume that upstream and downstream ends of the canal system are large reservoirs which insure that the first and last node can remain constant. Because the state variable that we take is the deviations value of water depth and flow rate from the operating point, the water depth variables at the up and down border point equal zero.

Putting the coefficients about the δz and δQ at ordinary node, the gate node and the water intake point node together into the corresponding position at the $\boldsymbol{A_L}$ *and* $\boldsymbol{A_R}$ matrixes, the coefficient of δu into the matrix B and coefficient of δq into matrix C, we can get:

$$A_L \delta x(t+1) = A_R \delta x(t+1) + B\delta u(t) + C\Delta \delta q(t) \tag{4}$$

Multiply both sides with $(A_L)^{-1}$, then

$$\delta x(t+1) = \Phi \delta x(t) + \Gamma \delta u(t) + \Psi \delta q(t) \tag{5}$$

Here $\Phi = (A_L)^{-1} * A_R$, it is the feedback matrix of the system, $\Gamma = (A_L)^{-1} * B$, the control matrix, $\Psi = (A_L)^{-1} * C$, the system disturbance matrix. $\delta x(t)$ is the state vector, $\delta u(t)$ is control vector and $\delta q(t)$ is disturbance vector.

We define the output equation as $\delta y(t) = H\delta x(t)$, where H is the output matrix.

MODEL UNCERTAINTY

Because the nonlinear model is "waving" around the linear nominal model, we can use corresponding "amplitude" to describe this range. If we can calculate the maximum value of difference inform the target, it can serve as a measurement of this uncertainty. Robust controller designed from this maximum uncertainty can stabilize the system in this series of uncertainty.

There are many ways to describe uncertainty: multiplication uncertainty, addition uncertainty, and coprime factor uncertainty, among these the first two ones are more popular.

(1) multiplication uncertainty

$$\tilde{p}(s) = p(s)(1 + W(s)\Delta p), \; \|\Delta p\|_\infty < 1 \tag{6}$$

Here p(s) is the nominal model, Δp is the unknown disturbance, W(s) is the border function of Δp, it is also called weight function.

(2) addition uncertainty

$$\tilde{p}(s) = p(s) + W(s)\Delta p, \quad \|\Delta p\|_\infty < 1 \tag{7}$$

Ordinarily, most uncertainty measurement of robust control uses multiplication uncertainty, because it is easier to deal in math. Using multiplication uncertainty, the relations between nominal model and original model can be described as:

$$G(j\omega) = [I + L(j\omega)]G_A(j\omega) \tag{8}$$

The matrix $L(j\omega)$ bounds G around G_A, it describes the uncertainty in the form of multiplication.
When G and G_A are known, we can get $L(j\omega)$ from equation (8), usually largest

singular value $\bar{\sigma}[L(j\omega)]$ is taken as the norm of $L(j\omega)$.

UNCERTAINTY SOURCES

In the modeling procedure of this paper, the non-structural uncertainty comes from two aspects. First the physical model is a dynamic model, the geometry parameter has errors and operating point varies with time. The second kind of uncertainty comes from the model-building procedure, the discrete and liberalizing all can introduce uncertainty.

The uncertainty of the physical model itself

(1) Uncertainty introduced in construction

Geometry parameters include length of canal, bottom width, side slope, vertical slope, canal depth and so on. Ordinarily, when a project is designed and constructed, the canal geometry parameters fit some Chinese design and construction codes or standards. According to these codes and standards, the construction dimension error must not exceed 10mm. In such condition, the construction error is limited in 0.5%. For the canal length, we always get this data by measuring after construction, and the measuring error is also within 0.5%. So for large canal system like in the South-to North water transfer project of China, the geometry parameter error is small.

(2) Outside disturbance in operating

Real canal systems operate in natural conditions, so there are many natural and man-made disturbances in its operation process.

First we discuss the natural disturbance. One main natural disturbance is wind waves. Here we take a wave as a kind of stochastic disturbance like white noise. It is described by a sine function of time (t):

$$\Delta h_{dis} = A \sin Kt \tag{9}$$

Here A is the amplitude, K is the wave frequency. We will do simulations to demonstrate its influence later.

Man-made disturbance is mainly caused by unscheduled water inflows, and unexpected offtakes downstream. It generally leads to the change of flow rate in canals, and departure from designed operating point. So the next section focuses on the uncertainty caused by the unscheduled water intake and the change of operating point.

<u>The uncertainty introduced in the modeling procedure</u>

The gap between the nonlinear model and the linear model is the truncation error of Taylor series. It is an extreme-value problem and in real operating practices the independent variables alter in a very limited zone. If we seek for the extreme value without any precondition, the result model uncertainty will be much larger than necessary and it is almost impossible to design a controller which can 'endure' such a large uncertainty. So we linearize the nonlinear system at many operating points, measure the difference between these models and the original one, and take this as a measure of the truncation error of Taylor series. Of course, this method is quite conservative, but it can reach acceptable result for the desired limited changes. So the uncertainty problem becomes:

$$\overline{\sigma}[L(j\omega)] = \max_i \overline{\sigma}\left[G_i(j\omega)G_A^{-1}(j\omega) - I\right] \tag{10}$$

CASE STUDY

In order to study the uncertainty of the canal system model in quantity, we are to build model of a given case. It is a multi-reach canal system made of six reaches, the geometric parameters come from the design material of South-to North water transfer project, as shown in Figure 1 and the canal dimensions can be found in Table1.

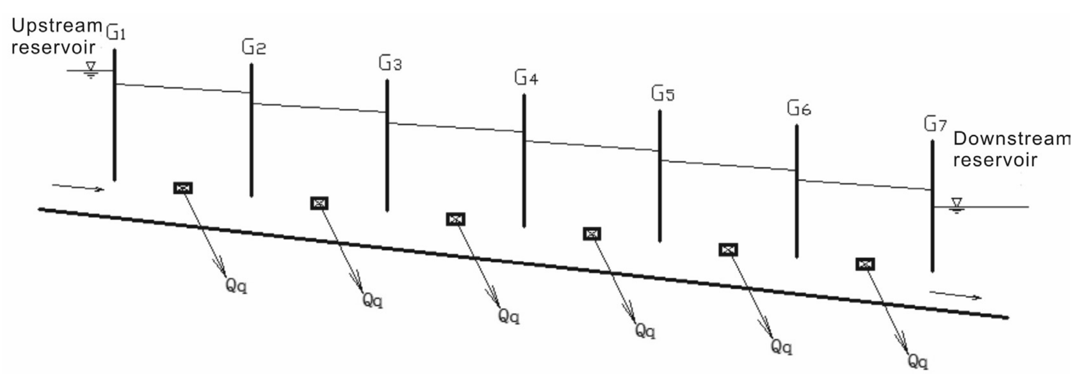

Figure 1. Profile of test case

Table 1.Canal parameters

parameter \ Canal number		1	2	3	4	5	6
Bottom height(m)	upstream	4.8	4.0	3.2	2.4	1.6	0.8
	downstream	4.0	3.2	2.4	1.6	0.8	0.0
length (m)		2000	2000	2000	2000	2000	2000
Initial downstream water level (m)		4.0	3.8	3.6	3.4	3.2	3.0
Roughness		0.015	0.015	0.015	0.015	0.015	0.015
Bottom width (m)		15	15	15	15	15	15
Side slope		2.0	2.0	2.0	2.0	2.0	2.0
Bottom slope i		0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
Width of control gate(m)		15	15	15	15	15	15
Initial discharge(m^3/s)		60	55	50	45	40	35

The whole system is made up of six reaches, and 32 nodes. The size of the state-space equation: A_L is a matrix of 55×55, A_R is 55×55, B is 55×7, C is 55×12. The parameter matrix Φ is 55×55, Γ is 55×4, disturbance matrix Ψ is 55×12.

(1) Influence of geometric parameter uncertainty

Construction error is within 10mm, the length error of reaches is within 5%, so here we give the variability of canal geometric parameters in Table 2.

Table 2. Variability in canal geometric parameters

	Length of reach (m)	Bottom width(m)	Width of control gates(m)
Nominal system	2000	15	15
Case1	2010	15.01	15.01
Case2	1990	14.99	14.99

Now we get three groups of linear system models. If we measure the uncertainty of case1 and case2 from the nominal one by method above, we can get the result of Table 3.

Table 3. Measurement for uncertainty form turbulence in canal geometric parameter

	Uncertainty for matrix Φ (%)	Uncertainty for matrix Γ (%)
Case1	0.088	0.28
Case2	0.09	0.27

From the above results we can see that the influence of variability in canal geometric parameters is making Φ matrix has ±0.1% uncertainty, and Γ matrix has ±0.3% uncertainty.

(2) Wave disturbance analysis

We simulate two kinds of operating condition. First we assume A=0.15 and K=$\pi/40$ (it is inverse solution of a cycle of 80s), the gate discharge line is in Fig. 2.

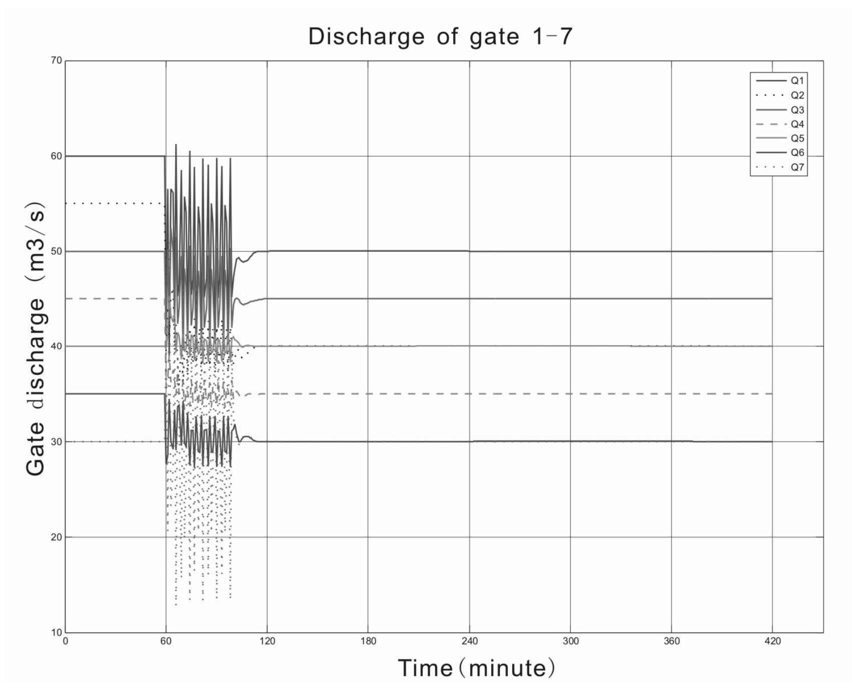

Figure 2. Gate discharge in waving disturbance (K = $\pi/40$)

The second condition, A=0.15 and K=$\pi/30$ (a cycle of 60s), the gate discharge line is in Fig 3.

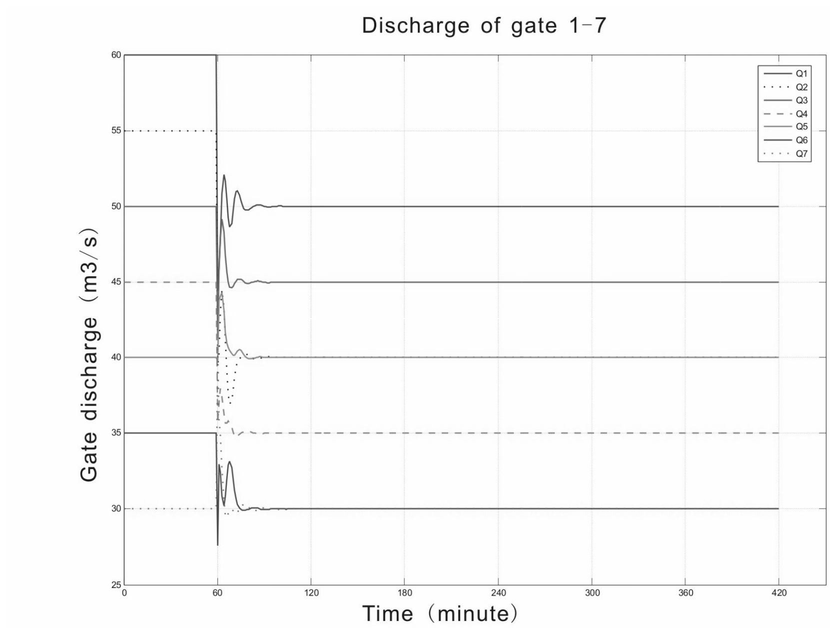

Figure 3. Gate discharge in waving disturbance (K = $\pi/30$)

Comparisons and analyzes of the two simulation results show that, $K = \pi/40$ is the worst condition. If the wave cycle is not an integer multiple of the system time step, the wave influence is quite limited. If $K = \pi/30$, the discharge can have very good dynamic performance, the wave only influences the water level error visibly. If amplitude was made 2 times of original, the system also has acceptable dynamic performance, which will not be listed here.

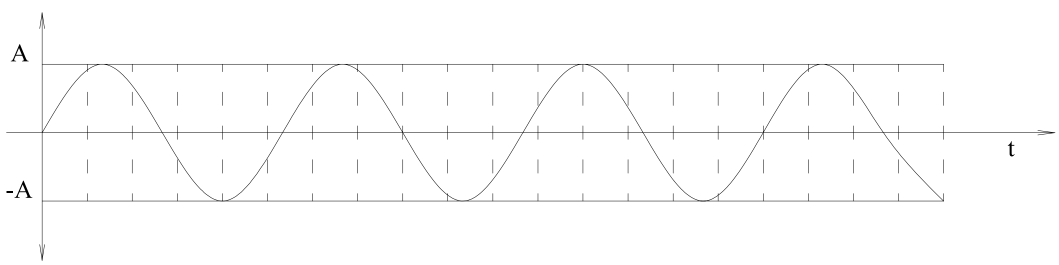

Figure 4. Illustration of wave cycle and system time step

Figure 4 illustrates the wave height when the wave cycle differs from the system time step, the real sinusoid line is the water level wave near the gate, and the dash line is the system time step. The cross point is the water level height at measure time. We can read from the plot that this height reaches only one max in one cycle. Mostly it is far smaller than amplitude A, so the influence of wave is cut down greatly.

This means that while considering the influence of wave, the cycle is the key factor. To stabilize the canal system in some range of frequency, the discrete time step should be chosen carefully. Since, the S-V equation is solved in the mathematical model using the Preissmann implicit scheme, the choosing of time step can ignore the restriction of the so-called Courant-Friederichs-Levy condition. In reality, a natural wave is not just simple sinusoid, so the cycle is a series of cycles ranging in a certain field instead of a fixed one, so the choice of time step should go beyond this field.

(3) Uncertainty of operating point change

In the test cases of this paper, the change of operating point can be realized by changing the flow rate of each water intakes. Since there are six water intakes. If we increase and decrease each water intake's flow rate by 50% separately, and assume this to be the ranging filed of possible operating point change, we can get Table 4.

Table 4. Operating point change

Gate discharge		QG1	QG2	QG3	QG4	QG5	QG6	QG7
Nominal point		60	55	50	45	40	35	30
Flow rate increase 50%	case1	60	52.5	47.5	42.5	37.5	32.5	27.5
	case2	60	55	47.5	42.5	37.5	32.5	27.5
	case3	60	55	50	42.5	37.5	32.5	27.5
	case4	60	55	50	45	37.5	32.5	27.5
	case5	60	55	50	45	40	32.5	27.5
	case6	60	55	50	45	40	35	27.5
Flow rate decrease 50%	case7	60	57.5	52.5	47.5	42.5	37.5	32.5
	case8	60	55	52.5	47.5	42.5	37.5	32.5
	case9	60	55	50	47.5	42.5	37.5	32.5
	case10	60	55	50	45	42.5	37.5	32.5
	case11	60	55	50	45	40	37.5	32.5
	case12	60	55	50	45	40	35	32.5
No water intake	case13	60	60	60	60	60	60	60

From calculating we get 13 groups of linear model. Measuring the uncertainty of all cases by the method above separately, the result is in Table 5.

Table 5. Measurement for operating point change

		Uncertainty for matrix Φ (%)	Uncertainty for matrix Γ (%)
Flow rate increase 50%	case1	2.04	2.33
	case2	6.51	6.23
	case3	6.72	6.48
	case4	6.75	6.72
	case5	6.75	6.96
	case6	6.75	7.19
Flow rate decrease 50%	case7	1.94	2.22
	case8	5.91	5.64
	case9	6.07	5.82
	case10	6.08	5.98
	case11	6.08	5.90
	case12	6.08	6.26
No water intake	case13	298.45	327.49

From the above results we can see, the uncertainty brought by the changing of operating point is within 8%. We then calculate a extreme condition for shutting down all water intake, giving ±300% of uncertainty. So if the change of discharge is bounded in a certain range, it will introduce little uncertainty, larger flow rate change will lead to larger uncertainty. So if the change is in a small field, we can use a single mathematic model and the controller will stabilize the system. If the change is large, it should be broken down into n sub-procedures, modeling should be done in n steps, and controller should be solved separately to cut down the oscillation.

SUMMARY

The uncertainty of the system model impacts the design of a controller. The flaw of modern control theory is it relies too much on the accuracy of model. As a highly nonlinear system, the canal control system will loose high-order terms through linearization, leading to an inaccurate mathematic model. In this paper, we try to discuss this uncertainty introduced in modeling and the uncertainty of the physical model. The multiple uncertainty (2 norm) is used to describe this. If we consider this uncertainty in the design of a robust controller, we will be able to stabilize the system while it switches in a certain field. At last a test case is simulated and the results show that the ignored high-order items bring the main uncertainty.

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