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WIND EFFECTS ON BUILDINGS AND STRUCTURES

FLUIDELASTIC FEATURES OF FLOW AROUND CYLINDERS

by Gerrit H. Toebes*

I - INTRODUCTION

Vortex induced vibrations of structural elements are non-linear and stochastic phenomena. Past studies often involved, either tacitly or explicitly, notions that were essentially linear and deterministic. As a consequence much work has been insufficiently systematic yielding results that preclude the generalizations necessary in structural optimization procedures.

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* School of Civil Engineering, Purdue University, Lafayette, Indiana, U.S.A.

Only recently reports on experimentally determined lift forces for circular cylinders have become available[1, 2, 3, 4] that are more systematic in the sense that the statistical and fluidelastic characteristics of the total lift force were studied using cylinders of reasonable length to diameter ratio. The term fluidelastic connotates that lifts were measured on cylinders oscillating at carefully controlled amplitudes.

Despite the progress they signify the reports also show that complete experimental delineation of lift is out of the question because of the abundance of geometric and kinematic variables that are involved. Therefore complementary studies of the flow field around oscillating cylinders are of significance. Without them it is not possible to arrive at a coherent picture that admits judicious extrapolation. The present report pertains to such a study and gives some initial results that bear on the vortex excitation aspects of flexible circular cylinders. Findings of more exclusive fluid mechanics interest will be reported elsewhere.

In selecting a framework for this discussion a number of theses came to mind advanced many years ago[5] in connection

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1. Bishop, R.E.D. and Hassan, A.Y.: "The Lift and Drag Forces on a Circular Cylinder Oscillating in a Flowing Fluid," Proc. Royal Soc. Series A, Vol. 277 (1964)

2. Cincotta, J.J.; Jones Jr., G.W. and Walker, R.W.: "Experimental Investigation of Wind Induced Oscillation Effects on Cylinders in Two-Dimensional Flow at High Reynolds Number," Proc. Meeting on Ground Wind Load Problems in Relation to Launch Vehicles, NASA Langley Research Center (June 1966)

3. Schmidt, L.V.: "Fluctuating Force Measurements upon a Circular Cylinder," Proc. Meeting on Ground Wind Load Problems in Relation to Launch Vehicles, NASA Langley Research Center (June 1966)

4. Toebes, G. H. and Ramamurthy, A.S.: "A Study of Hydroelastic Forces on Circular Cylinders," Proc. ASCE, Journ. of Eng. Mech. Vol. 93 (1967)

5. Toebes, G. H.: "The Hydroelastic Vibrations of Flat Plates related to Trailing Edge Geometry," Ph.D. Thesis, Mass. Inst. of Technology (1959)

with the vortex induced vibrations of flat plates. Similar ideas have been advanced independently by several investigators subsequent and possibly prior to formulations in Ref. 5; no attempt is made here to provide their chronology. The theses intended to make plausible the nature of observed flow induced vibration of flat plates that were restrained elastically around the leading edge and mounted in a water tunnel. They were only inferred from the plate response characteristics and stated that severe vibrations of the plates were hydroelastic in nature. The thus implied feedback feature would arise because

1. plate motions increased the circulatory strength of shed vortices;
2. the dynamic lift contained higher harmonics of the fundamental Strouhal frequency resulting in equations of motion that were non-linear;
3. plate motions caused phasing of separated shear layers and thus enhanced two-dimensionality of the early wake flow;
4. the Strouhal frequency was controlled by the frequency of plate motion over a substantial velocity range;
5. trailing edge geometry affected substantially the early wake flow.

Each of these notions will be considered below using an initial interpretation of actual measurements in the flow field of a vibrating body.

II - EXPERIMENTAL EQUIPMENT

A 6" diameter and 72" long test cylinder was mounted horizontally in the 6' x 6' test section of a wind tunnel. The cylinder was supported by streamlined struts that were connected to a 3" cylinder supported by bushings. The 3" cylinder was in turn connected to a floor mounted flywheel via a rod and a variable excentric. The flywheel was driven by a D.C. motor. Harmonic cylinder oscillations, transverse to the flow, at frequencies up to 25 cps and up to a double amplitude $2\epsilon = 2.0$ inches could thus be effected. The cylinder carried 1' x 1' end plates; the distance between end plates and tunnel walls was 0.07 inch.

Cylinder motion records were obtained from the output of a capacitance type transducer. Flow field measurements were recorded on magnetic tape or by means of a multi-channel oscillograph. Mean velocity measurements were made using a 1/8" diameter Prandtl tube attached to a pressure transducer readout device.

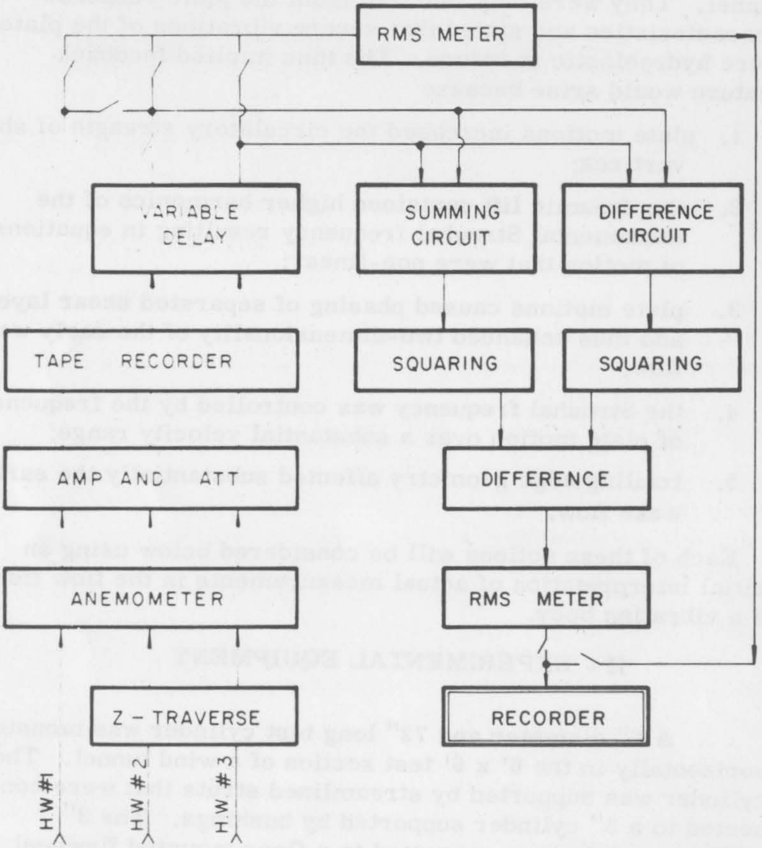

Figure 1 - Instrumentation For Correlation Measurements

Correlation measurements were made as indicated in Figure 1, showing how the product of two hot-wire outputs $e_1 \cdot e_2$ or $e_1 \cdot e_3$ were obtained by means of the familiar sum and difference method. A limited number of correlations involved the introduction of time delays in order to obtain autocorrelation functions. The movable wires were positioned by means of an electronically operated traversing mechanism. A number of measurements were made by recording on X-Y plotters the Prandtl tube or anemometer outputs, averaged by means of RMS meters, while the traversing mechanism was moving; the resulting graphs were subsequently averaged by eye.

III - VORTEX STRENGTH

The question whether the circulatory strength of vortices developing from the free shear layers and, as a consequence, the induced lift on the cylinder do increase as a result of cylinder motion may be judged from Figure 2. It shows one of the wake traverses that were made in the transverse or Y-direction, i.e., along verticals in planes parallel to the flow and perpendicular to the cylinder. Figure 2 presents the traverse that was made closest to the cylinder and thus bears most directly on the matter of lift force increase. The coordinate system adopted for this and subsequent figures is: X/d - downstream direction; Y/d - vertically upwards; Z/d - axial direction; d = cylinder diameter = 6"; the origin is at the mean position of the cylinder axis. A Prandtl tube and a hot wire were positioned at $X/d = 0.6$ and with a separation distance of $\Delta Z/d = 0.03$; the probes were thence traversed slowly over the interval $-1.0 < Y/d > 3.0$. The lower coordinate axis of the figure gives the traverse in proper geometric relation to the cylinder, a quarter of which has been shown.

The solid curves in Figure 2 pertain to a steady cylinder. The dashed curves are for a cylinder oscillating with a double amplitude of 1.2", i.e., a relative double amplitude $2\epsilon/d = 0.20$. The cylinder frequency, f_s, was equal to the Strouhal frequency, f_f, as determined during the corresponding $2\epsilon = 0$ condition. The frequency ratio, f_r, equaled

$$f_r = f_s/f_f = 1.0 \pm 0.03 \tag{1}$$

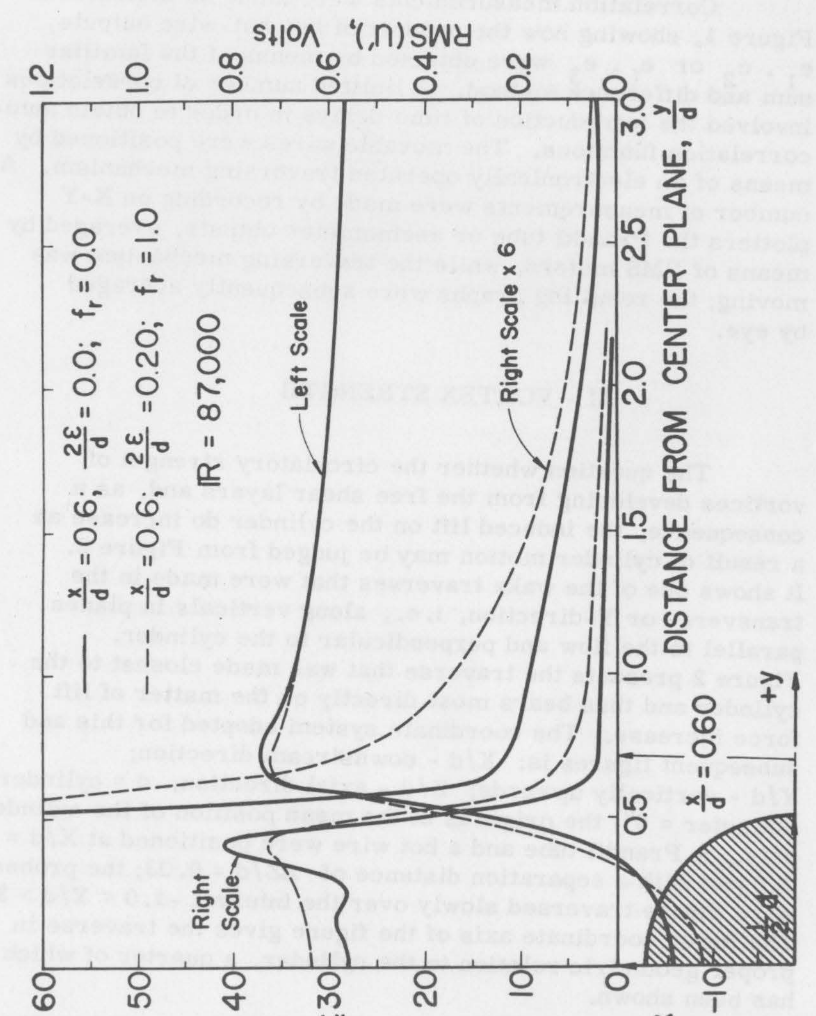

Figure 2 - Mean Velocity And Turbulency Intensity Variations In A Traverse Perpendicular To The Mean Flow And The Cylinder Axis. The Traverse Is Located 0.10 Diameter Downstream Of The Cylinder. The Solid And The Dashed Curves Pertain To A Steady And A Vibrating Cylinder, Respectively.

The indicated variation in Eq. 1 arises from the fact that in the average the Strouhal frequency wandered over a 6% interval. This was determined by continual counting of 20 cycle records; by means of analog averaging such variations can not be determined.

It is seen that the mean velocity distribution in the potential flow field beyond $Y/d = 0.7$ is affected little or not at all by cylinder oscillation. However, the intensity of flow field oscillation is increased in the order of 100%. This is significant since it means that pressure oscillations over the front half of the cylinder can be increased beyond the amount associated with the apparent mass stresses. Depending on the resultant's phase relation with $\epsilon(t)$ this may contribute to fluidelastic excitation of the cylinder. In addition the location of boundary layer separation is affected leading to some change in the early wake structure which in turn influences the bounding potential flow. The finding is thus indicative of at least a facet of the feedback mechanism that is the essence of a fluidelastic phenomenon.

For $Y/d < 0.7$ turbulent fluid was traversed. As a result of cylinder vibration the free shear turbulence is diffused in particular towards the wake centerline. The velocity record shows on the average, a narrowing of the wake. Both facts indicate a decrease in pressure near the wake center. Such a pressure drop is a prerequisite for the formation of vortices of increased circulation.

Note finally that the free shear layer appears to be diffused less than the double amplitude of cylinder motion. This points to a shifting of separation points such that motion in the + Y-direction associates with shifts in the + X-direction. Again this would contribute to a fluidelastic mechanism.

All in all the Figure 2 confirms the notion that cylinder oscillation leads to increased circulatory strength of developing vortices. Indicating a periodic shifting of the separation zone it also points to the importance of body geometry in that zone as a fluidelastic factor.

IV - HARMONIC CONTENT OF THE LIFT FORCE

The Figure 3 presents a remarkable picture of 20 hot-wire traces taken in groups of five. The measurement locations are indicated by dots drawn to scale relative to the cylinder a quarter of which has been shown. The proper relative turbulence or velocity fluctuation intensity may be inferred from Figure 2 which pertains to the same traverse at X/d = 0.6. They are also indicated in the right hand side scale in which the lower trace has been given an arbitrary reference magnitude of 10.0.

The data show how the flow field well away from the cylinder participates of the oscillatory wake characteristics. In conjunction with Figure 2 it renders a self-sustained feedback between wake and separation zone oscillation more than plausible. The flow field shows substantial correlation in the transverse direction which is interpreted to signify its relative stability; this would contradict past reports that even the smallest blemishes on the cylinder surface would have a substantial effect in the flow.

The major observation afforded by Figure 3 is of course the decidedly non-harmonic nature of velocity variation when one approaches the free shear layer zone near the cylinder. The pulse or whip type nature of the flow provides a fair number of higher harmonics in the velocity traces. In spectral analysis results, not now presented, harmonics up to the 5th one were discernible. This is taken as a confirmation of the notion that the lift force itself will contain higher harmonics so that the equation of motion will be non-linear even though the structural system can be classified as a linear, one-degree of freedom system.

Other noteworthy features are the apparent relaxation towards harmonic velocity variations when moving away from the cylinder and the presence on the other side of the shear layer of double the Strouhal frequency that is discernible even to the eye. Also evident is the random nature of the flow even though its frequency content is confined to a narrow band.

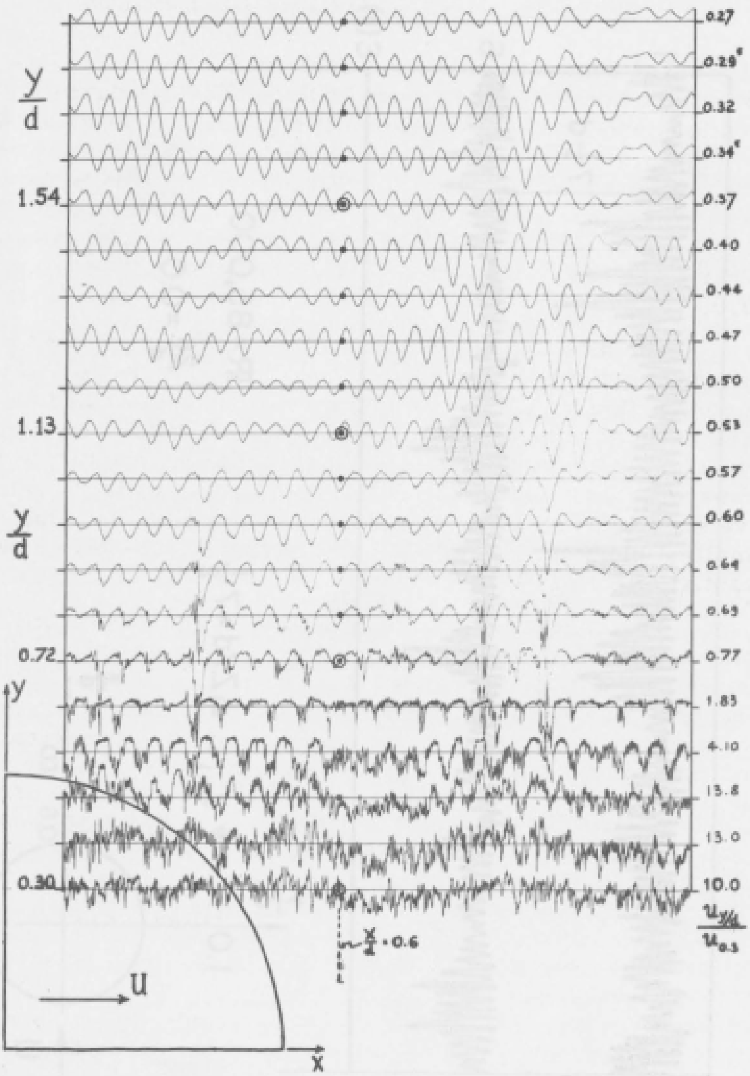

Figure 3 - A Sample Of 20 Hot-Wire Anemometer Traces, Taken In Groups Of Five At Measurements Locations Indicated By Dots. Steady Cylinder. Reynolds Number = 85,000

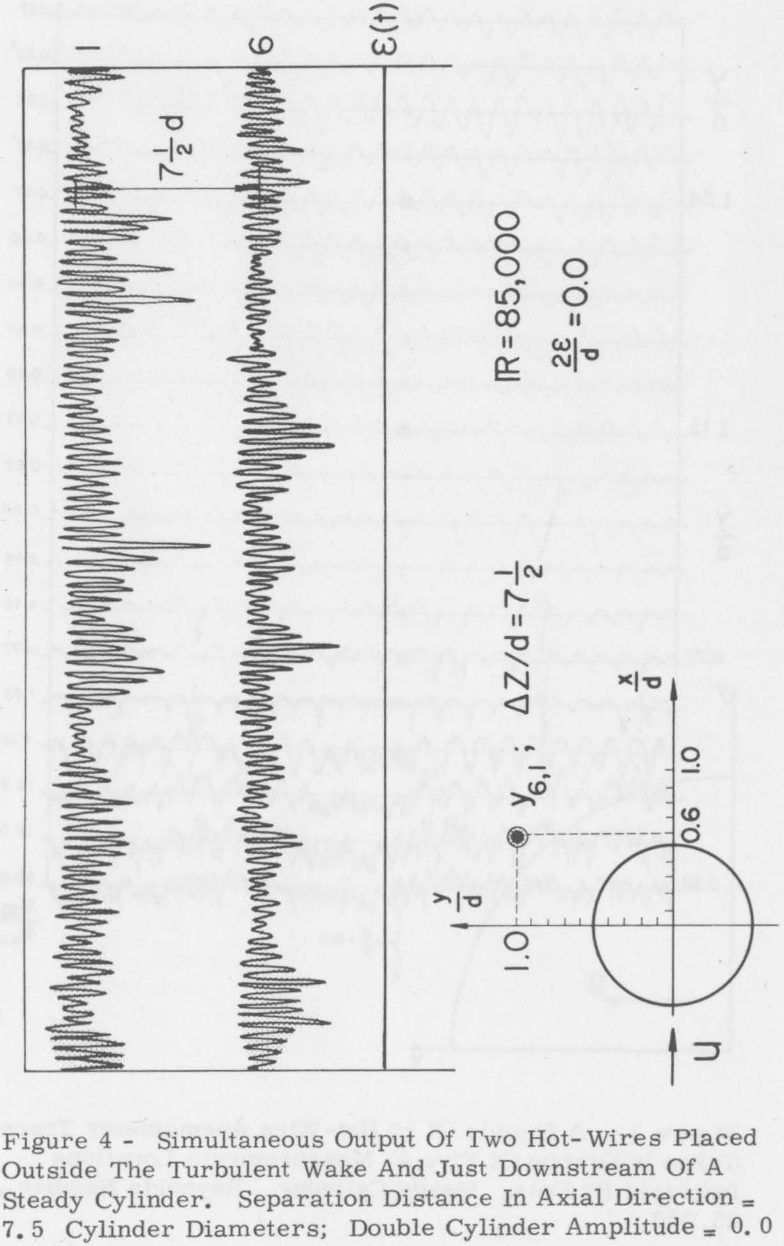

Figure 4 - Simultaneous Output Of Two Hot-Wires Placed Outside The Turbulent Wake And Just Downstream Of A Steady Cylinder. Separation Distance In Axial Direction = 7. 5 Cylinder Diameters; Double Cylinder Amplitude = 0. 0

V - PHASING OF THE NEAR WAKE

In regard to the question of fluidelastic excitation the flow structure or "two-dimensionality" in the axial or Z - direction was assumed to be of major importance. Therefore a large number of measurements were made with hot-wires placed in lines parallel to the cylinder axis. Figure 4 shows the simutaneous outputs of the two outer hot wires (#1 and #6) of a group of six that were placed parallel to the cylinder at 1.5 cylinder diameter intervals. The cylinder displacement trace, ϵ (t), is shown too; it indicates that the data pertain to a steady cylinder case. Evident are once more the asymmetry of the velocity traces and their narrow-band random noise nature giving rise to the stochastic character of cylinder excitation. There is little or no amplitude correlation that can be discerned by eye in the long records of which Figure 4 is a representative sample.

Figure 5 is similar to Figure 4. Now the hot-wire separation distance was 0.5 d so that the outer wires were placed 2.5 d apart. Also the line of measurement was moved upstream to $X/d = 0.0$ revealing far more symmetrical velocity variations. This could be expected since in front of the cylinder no asymmetry can be present. A greater amount of amplitude correlation may be discerned, certainly when viewing all six traces in the records of which Figure 5 is a partial sample. Nevertheless, the correlation is by no means perfect.

In addition to amplitude, the phase correlation of velocity fluctuations will determine the total flow and hence lift force correlation along the cylinder. Figure 6 presents a portion of a record similar to that shown in Figure 5, but taken with an expanded time base. It is apparent that substantial phase differences may exist in the flow structure over the relatively small distance of $2\frac{1}{2}$ cylinder diameters, also, that these differences occur randomly. Similar observations can be made about the amplitude. The time interval e-f-g and beyond shows substantial amplitude correlation. At the same time appreciable phase difference can occur. Conversely, around time b one notes low amplitude and high phase correlation.

Figure 7 presents a sample for the same conditions as in the Figures 5 and 6, except that now the cylinder was

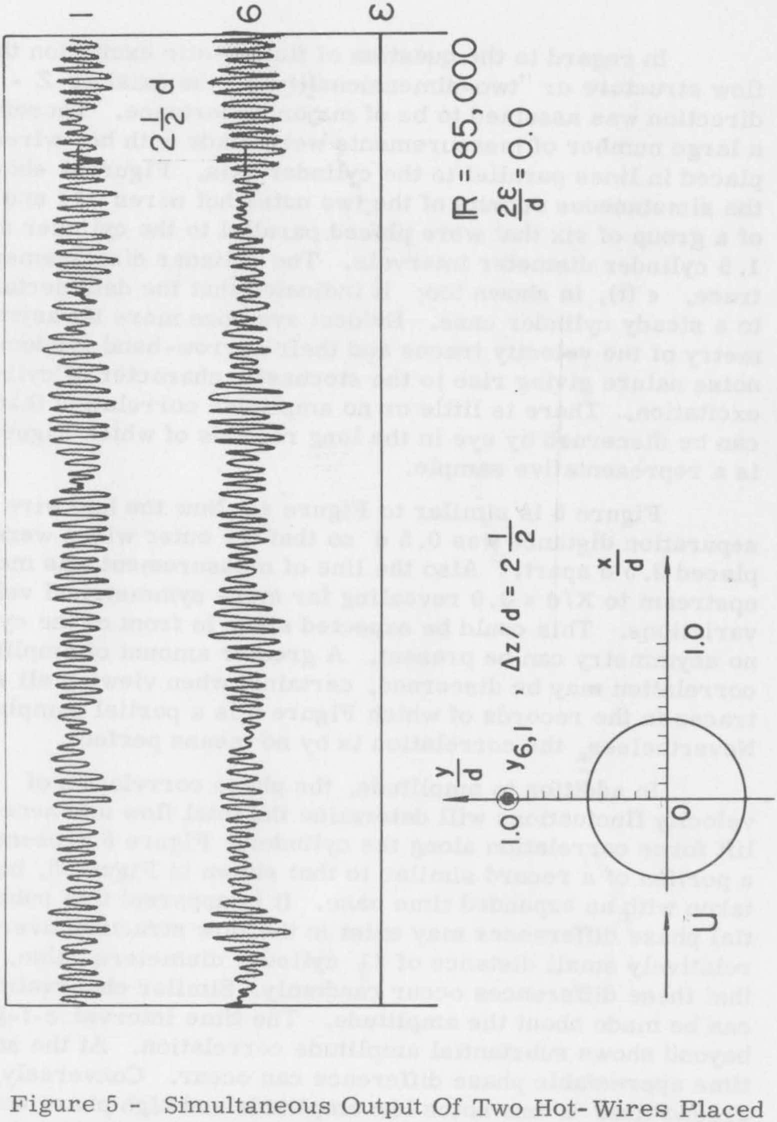

Figure 5 - Simultaneous Output Of Two Hot-Wires Placed In Potential Flow Near Cylinder. Separation Distance In Axial Direction = 2.5 Cylinder Diameters; Double Cylinder Amplitude = 0.0

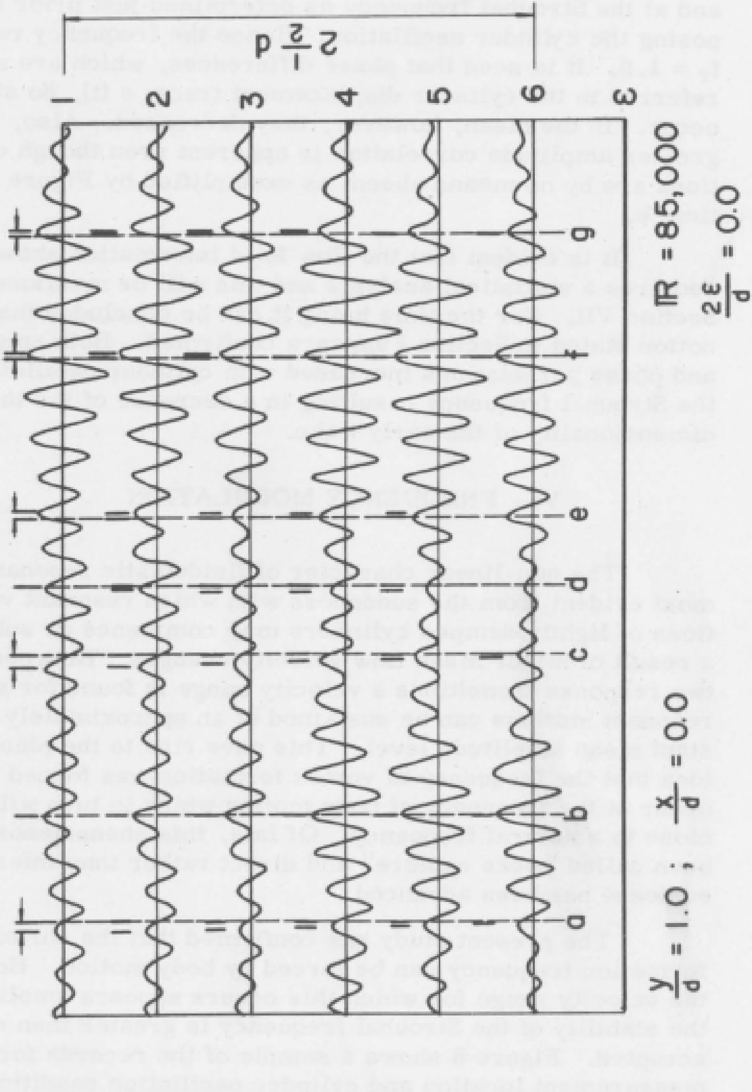

Figure 6 - Simultaneous Output Of Six Hot-Wires Placed In Potential Flow Near Cylinder. Separation Distances In Axial Direction = 0.5 Cylinder Diameter Each; Double Cylinder Amplitude = 0.0

oscillated at a double amplitude of 0.2 cylinder diameters, and at the Strouhal frequency as determined just prior to imposing the cylinder oscillation. Hence the frequency ratio was $f_r = 1.0$. It is seen that phase differences, which are now referred to the cylinder displacement trace, ϵ (t), do still occur. In the mean, however, they decreased. Also, a greater amplitude correlation is apparent even though deviations are by no means absent as exemplified by Figure 7 - time b.

It is evident that the flow field information shown requires a statistical analysis and this will be mentioned in Section VII. For the time being it can be concluded that the 3rd notion stated in Section I appears confirmed. Both amplitude and phase correlations increased with cylinder oscillation at the Strouhal frequency resulting in a decrease of the three-dimensionality of the early wake.

VI - FREQUENCY MODULATION

The non-linear character of fluidelastic resonance is most evident from the suddeness with which resonant vibrations of lightly damped cylinders may commence or subside as a result of minor mean flow velocity changes. Between these two response transitions a velocity range is found for which resonant motions can be sustained at an approximately constant mean amplitude level. This gave rise to the plausible idea that the frequency of vortex formation was forced to occur at the frequency of body motion which in turn will be close to a natural frequency. Of late, this phenomenon has been called "wake capture" and direct rather than inferential evidence has been advanced.

The present study has confirmed that the vortex formation frequency can be forced by body motion. However, the velocity range for which this occurs appears smaller and the stability of the Strouhal frequency is greater than commonly accepted. Figure 8 shows a sample of the records for the same measurement location and cylinder oscillation conditions (i.e., f_s = 10 cps; $2\epsilon/d = 0.2$) as presented in Figure 7. However, the velocity of flow was increased about 20% from 28 fps to 34 fps. The frequency ratio $f_r = 0.81$. This is yet within the range of resonant conditions of lightly damped elements. The velocity record of Figure 8 shows the presence of modulation.

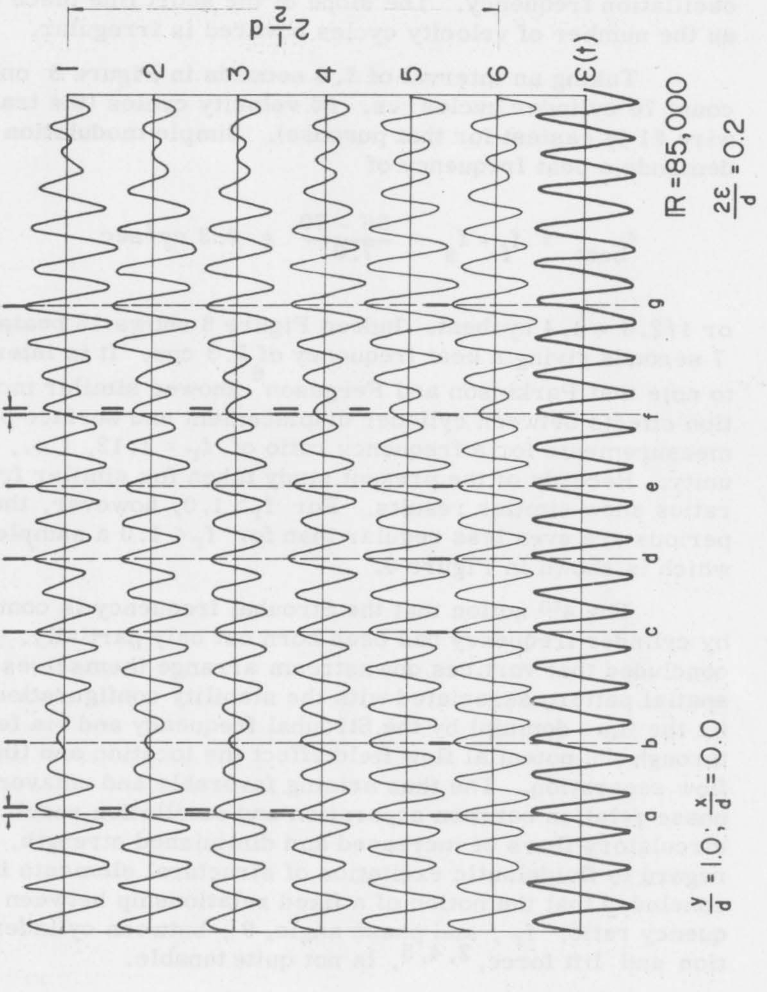

Figure 7 - Simultaneous Output Of Six Hot-Wires Placed In Potential Flow Near Cylinder. Separation Distance In Axial Direction = 0.5 Cylinder Diameter Each; Double Cylinder Amplitude = 0.2 Diameters. Frequency Ratio = 1.0

This modulation has a substantial random element as may be seen from the attempt to match beat frequency with cylinder oscillation frequency. The slope of the short line piece as well as the number of velocity cycles covered is irregular.

Taking an interval of 7.0 seconds in Figure 8 one may count 70 cylinder cycles vs. 86 velocity cycles (the trace for wire #1 is easiest for that purpose). Simple modulation demands a beat frequency of

$$f_{beat} = f_f - f_s = \frac{86 - 70}{7.0} = 2.3 \text{ cy/sec} \qquad (2)$$

or 1/2.3 = 4.4 cy/beat. Indeed Figure 8 shows 16 beats per 7 seconds giving a beat frequency of 2.3 cps. It is interesting to note that Parkinson and Ferguson[6] showed similar modulation effects between cylinder displacement and surface pressure measurements for a frequency ratio of $f_r = 1.12$, i.e., above unity. Records of the present study taken for similar frequency ratios show similar results. For $f_r > 1.0$, however, the beat periods are even less regular than for $f_r < 1.0$ a sample of which is shown in Figure 8.

The 4th notion that the Strouhal frequency is controlled by cylinder frequency has been born out only partially. It is concluded that vortices downstream arrange themselves in the spatial pattern associated with the stability configuration given (in the time domain) by the Strouhal frequency and via feedback through the potential flow field effect the location and time of flow separation. The thus arising favorable and unfavorable phase relation between separation and oscillation result in circulatory flows of increased and diminished strength. In regard to fluidelastic excitation of structural elements it is concluded that the notion of a fixed relationship between frequency ratio, f_r, and phase angle, θ, between cylinder motion and lift force,[2, 4, 5] is not quite tenable.

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6. Parkinson, G. V. and Ferguson, N.: Amplitude and Surface Pressure Measurements for a Circular Cylinder in Vortex-Excited Oscillation at Sub-Critical Reynolds Numbers", Proc. Meeting on Ground Wind Load Problems in Relation to Launch Vehicles, NASA Langley Research Center (June 1966)

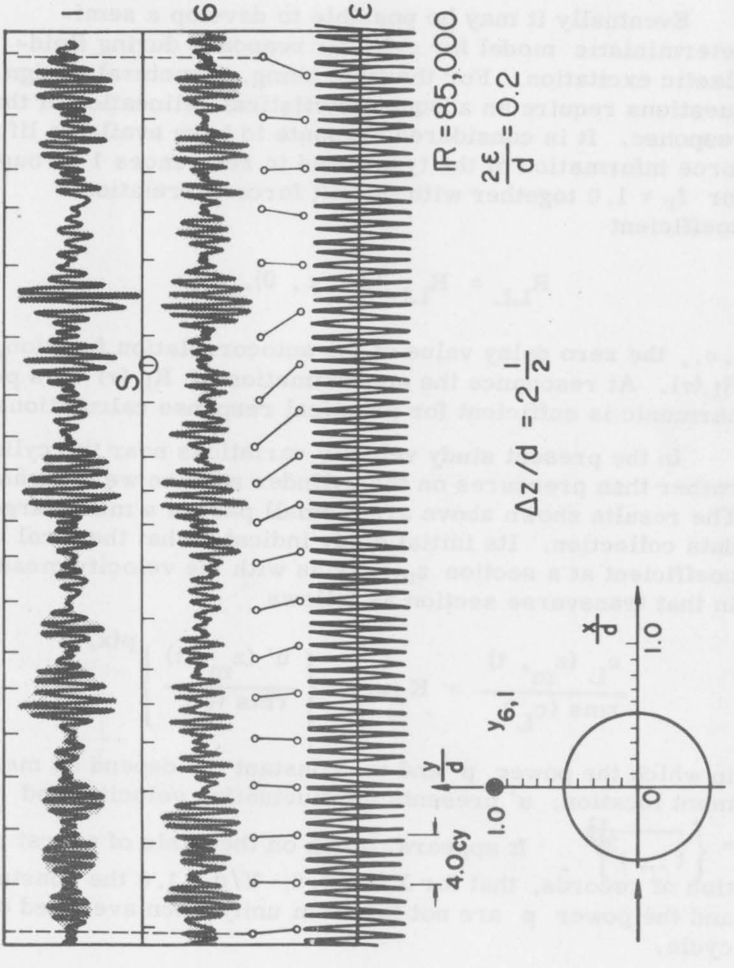

Figure 8 - Simultaneous Output Of Two Hot-Wires Placed In Potential Flow Near Cylinder. Separation Distance In Axial Direction = 2.5 Cylinder Diameters; Double Cylinder Amplitude = 0.2 Diameters; Frequency Ratio = 0.81; Reynolds Number = 102,000

VII - FLUIDELASTIC CYLINDER RESPONSE

Eventually it may be possible to develop a semi-deterministic model for cylinder response during fluid-elastic excitation. For the time being, structural design questions require an adequate statistical delineation of that response. It is considered adequate to have available lift force information of the type found in references 1 through 4 for $f_r = 1.0$ together with the lift force correlation coefficient

$$R_{LL} = R_{LL}\ (\Delta z, \epsilon,\ 0), \tag{3}$$

i.e., the zero delay value of the autocorrelation function, $R_L(\tau)$. At resonance the approximation of $R_L(\tau)$ by a pure harmonic is sufficient for practical response calculations.

In the present study velocity variations near the cylinder rather than pressures on the cylinder surface were measured. The results shown above are a small part of a much larger data collection. Its initial study indicates that the local lift coefficient at a section z_m varies with the velocity measured in that transverse section as follows

$$\frac{c_L\ (z_m,\ t)}{\text{rms}\ (c_L)} = K\ (x, y) \left\{ \frac{u'\ (z_m,\ t)}{\text{rms}\ (u')} \right\}^{p(x, y)} \tag{4}$$

in which the power p and the constant K depend on measurement location; u' presents the fluctuating velocity; and rms(...) $= \left\{ \overline{(\ \dots\ )^2} \right\}^{\frac{1}{2}}$. It appears, again on the basis of a first inspection of records, that for $X/d = 0.6$; $Y/d = 1.0$ the constant K and the power p are not far from unity when averaged over a cycle.

The instantaneous total lift coefficient for a cylinder of length Z is given by

$$C_L\ (t) = \int_0^Z c_L\ (t)\ dz \tag{5}$$

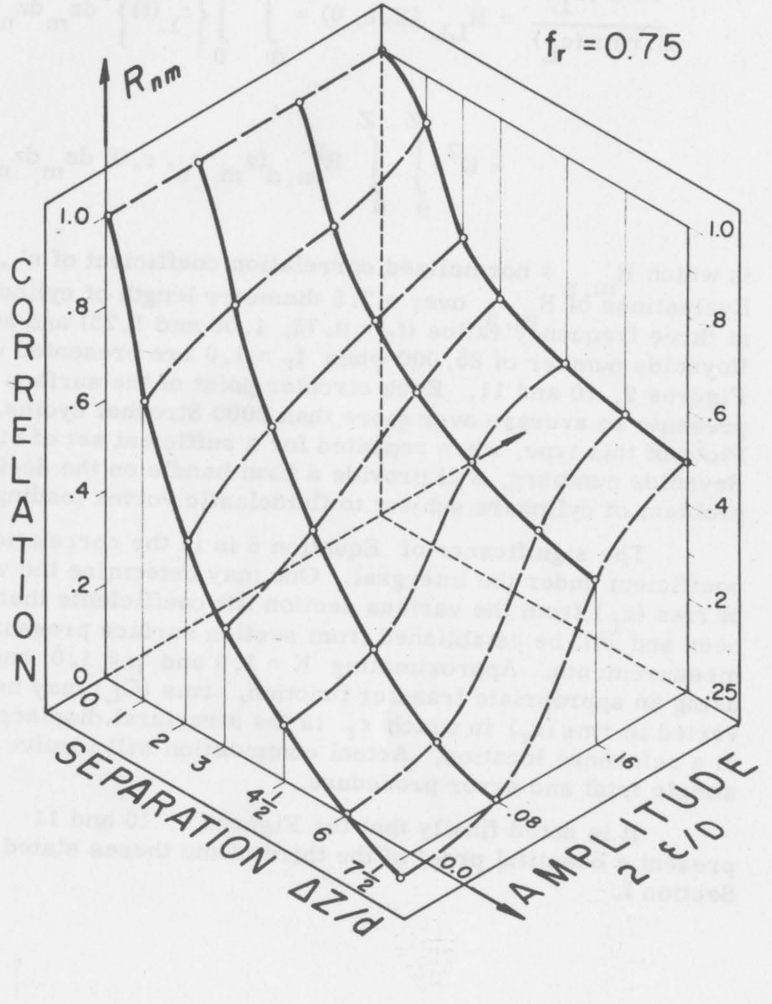

Figure 9 - Correlation Coefficient of Longitudinal Fluctuating Velocity Components At X/d = 0.6, Y/d = 1.0. Reynolds Number = 64,000. Cylinder Oscillated At f_s = 10.0 CPS.

hence

$$\frac{\mathrm{rms}\,(C_L)}{Z.\,\mathrm{rms}\,(c_L)} = R_{LL}\,(Z, \epsilon, 0) = \int_0^Z \int_0^Z \left\{ c_L(t) \right\}^2 dz_m\, dz_n =$$

$$= K^2 \int_0^Z \int_0^Z R^p_{m,n}(z_m\ z_n, \epsilon, 0)\, dz_m\, dz_n \qquad (6)$$

in which $R_{m,n}$ = normalized correlation coefficient of u'. Evaluations of $R_{m,n}$ over a 7.5 diameter length of cylinder, at three frequency ratios (f_r = 0.75, 1.00 and 1.25) and at a Reynolds number of 85,000 when f_r = 1.0 are presented in Figures 9, 10 and 11. Each circular point of the surface presents an average over more than 2000 Strouhal cycles. Plots of this type, when repeated for a sufficient set of other Reynolds numbers, will provide a firm handle on the design problem of cylinders subject to fluidelastic vortex loading.

The significance of Equation 6 is in the correlation coefficient under the intergral. One may determine the value of rms (c_L) from the various section lift coefficients that have been and will be established from section surface pressure measurements. Approximating K = 1.0 and p = 1.0 and using an appropriate transfer function, rms (C_L) may be converted in rms (ϵ_r) in which ϵ_r is the structural displacement at a reference location. Actual computation will involve a simple trial and error procedure.

It is noted finally that the Figures 9, 10 and 11 present a beautiful proof of the third of the theses stated in Section I.

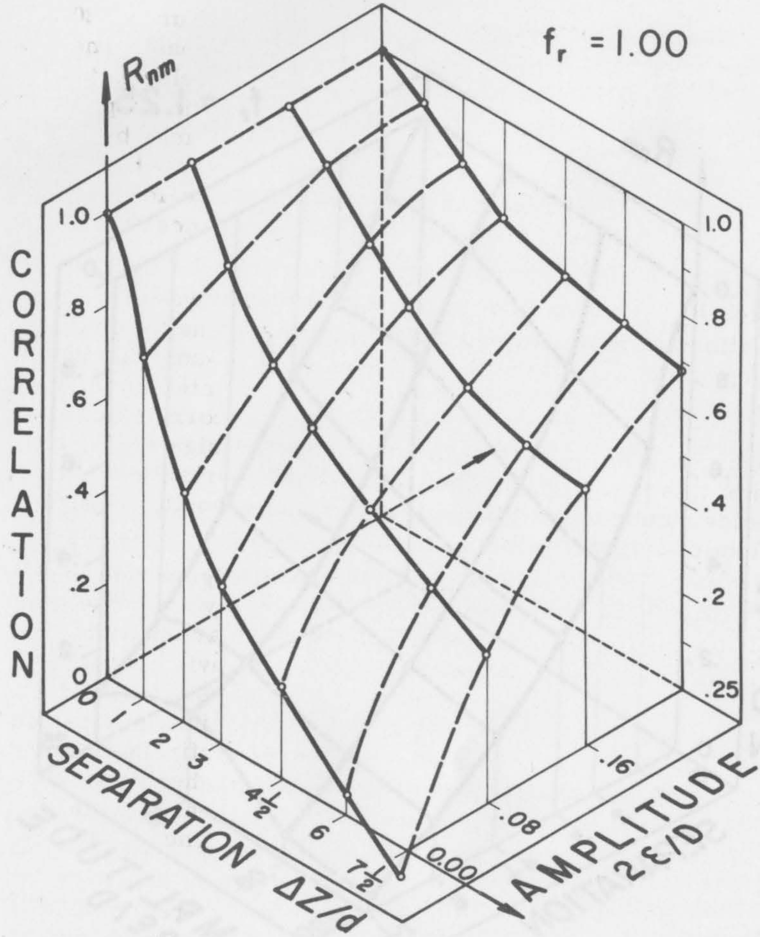

Figure 10 - Correlation Coefficient of Longitudinal Fluctuating Velocity Components At X/d = 0.6, Y/d = 1.0. Reynolds Number = 85,000. Cylinder Oscillated At f_s = 10.0 CPS.

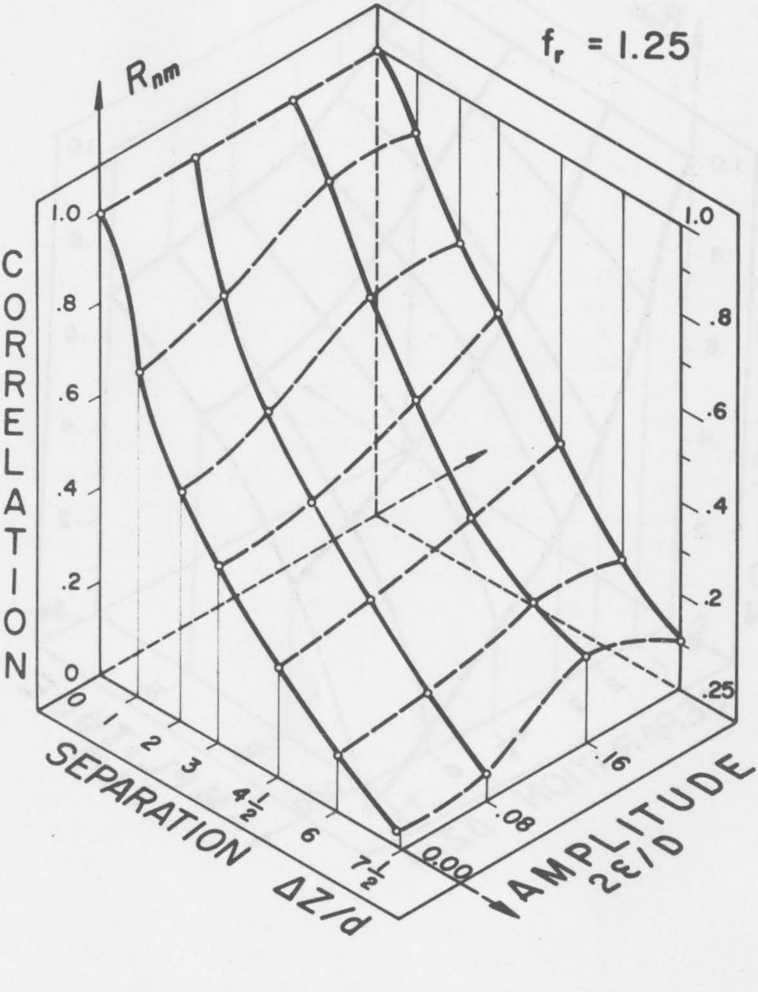

Figure 11 - Correlation Coefficient of Longitudinal Fluctuating Velocity Components at X/d = 0.6, Y/d = 1.0. Reynolds Number = 106,000. Cylinder Oscillated at f_s = 10.0 CPS.

VIII - SUMMARY AND CONCLUSIONS

This paper may be summarized as presenting initial results of a study of the fluidelastic flow field of a circular cylinder. Singled out are facets that bear on the question of resonant response of a lightly damped cylinder. It is concluded that

1. Cylinder motions increase the circulatory strength of developing vortices;
2. Cylinder motions cause increased "two-dimensionality" of the flow field;
3. The dynamic lift contains higher harmonics of the Strouhal frequency;
4. Striking flow field modulations occur when the ratio of cylinder motion to Strouhal frequencies is between 0.8 and 1.1 but not close to unity;
5. Correlation coefficient surfaces are presented that clearly show the fluidelastic character of the flow. For fluidelastic resonance at Reynolds numbers of the order of 10^5 they also permit an approximation of a cylinder's averaged fluidelastic response.

IX - ACKNOWLEDGMENTS

The reported data are partial results of a study on structuring of turbulent flow undertaken at the Fluid Dynamics and Diffusion Laboratory at Colorado State University, where the writer spent a sabbatical leave of absence from Purdue University as a National Science Foundation Postdoctoral Fellow. He wishes to acknowledge the hospitality and help received from the staff of the Fluid Dynamics and Diffusion Laboratory; the use of the Army Meteorological Wind Tunnel and associated equipment in that laboratory; and the financial support from the National Science Foundation under Fellowship No. 46061 as well as Grant GK-414.