Uncertainty in flood routing: Diffuse wave models by *fuzzy* set theory approach

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Abstract. Impacts caused by flood routing affect families, economic activities, public and private systems. Impacts and economic losses has increased considerably, because these areas, in general, there is a growing concentration of people and economic activities present in the region. Flood routing is a problem, both developed and developing countries. Many hydrodynamic problems that involve of the propagation flood waves along the length in natural channels are solved by Saint – Venant equations. In most practical applications of flow routing in open channels, inertia terms are negligible, thus the system of Saint – Venant equations is reduced to a parabolic equation, known as the diffusive wave equation. This research aims to apply the *fuzzy* set theory in diffusive wave models at natural channels, in order to verify the uncertainties related to the hydrodynamic parameters present in these models. Through the implicit finite difference schemes was solved partial differential equations present in Saint - Venant equations. Simulations were carried out for different scenarios in the water body. It was developed a computer program, coded in Fortran. Results allowed establishing some interesting analysis with regard to the behavior of diffusive wave flood routing, it is strong influence of hydraulic parameters, the slope feature and Manning roughness coefficient. Results allowed concluding that the application of the *fuzzy* set theory in the hydrodynamic systems, it is a viable alternative for determining the uncertainty in flooding and thus be more a support tool in water resources management programs

Keywords: Flood Routing, *Fuzzy* Theory, Diffusive Wave Models.

1. Introduction

Flood routing is very danger to the environment, both developed and developing countries, according BAUTZER (2011) in USA, the last flood routing Mississippi River occurred in 2011 in which about 1.45 million hectares were submerged bringing a loss of around nine million dollars, according to economist Michael Hicks, director of Economic Research Center of Ball State University, Tennessee.

Hydrologic and hydraulic models to determine spatially distributed in flood area are an important tool that allows opportunity to good planning and risk analysis management. The representation of the ground surface is a critical factor in hydrologic and hydraulic modeling of flood, because as model input data, determines the inflow and the extent of flooding. (HORRIT and BATES, 2001).

Fuzzy Logic currently stimulates interest of researchers, engineers and industrial, and in general, all those who need the formalization of empirical methods, generalization of natural reasoning, automation decisions, artificial systems carrying out the enterprises that human are proposing it. (TANSCHEIT and SHARF, 1990).

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The Saint-Venant equations are used to describe the waves of the rivers. System of the Saint-Venant equations is reduced to the diffusive wave equation that can be solved using finite difference algorithm. The choice of the numerical method, stages of time and space to be accumulated depends essentially on the form of hydrographs and the hydraulic properties of the river. (CHAGAS, 2005)

For this research a methodology was developed that combined the model hydrodynamic analysis with the *Fuzzy* theory for the propagation of the diffuse wave in order to study the uncertainties and sensitivity in relation to the variation of the hydrodynamic parameters such as slope and roughness coefficient. For this, a FORTRAN language program was developed that allowed several simulations to evaluate the diffusion wave propagation behavior for different scenarios proposed

2. Methods

In this work a methodology was developed, based in a combination among the formulations that describe propagation of diffuse wave and the *Fuzzy* Set Theory, to determine uncertain during the flood routing. The proposed methodology allows to modify those indexes in membership functions and, with that, an evaluation of the risk of flood can be accomplished.

2.1. Diffuse wave equation

The Saint-Venant equations, first developed by Barre de Saint-Venant in 1871 (YEVJEVICH and MAHMOUD, 1975). These equations are derived from the application of the principles of continuity (Equation 1) and momentum equation (Equation 2), known as equations of Saint-Venant (CUNGE et al., 1980), in honor of its formulator.

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + g \left(\frac{\partial y}{\partial x} - S_0 + S_f \right) = 0 \tag{2}$$

As shown in Equation 2 is the dynamic wave, when neglects the inertia terms (local and convective acceleration terms) have diffusion wave routing:

$$\frac{\partial Q}{\partial t} + \frac{dQ}{dA} \frac{\partial Q}{\partial x} = \frac{Q}{2S_0 B} \frac{\partial^2 Q}{\partial x^2} \tag{3}$$

with
- Initial conditions

$$Q(x, 0) = Q_0$$
$$A(x, 0) = A_0$$

where Q_0 is the steady state flow of the channel, and the A_0 is the cross-section area for the steady state conditions.

- Boundary conditions

$$Q(0, t) = f(t)$$

where f(t) is hydrograph.

2.2.   *Fuzzy* theory

Many human experiences need an including manipulation than the simple treatment of false or true, yes or no, certain or wrong. It is in this context that the logic *fuzzy* if it turns an appropriate tool to treat vague and uncertain information, in general described in a natural language (LIMA, 2002).

An eminent factor of that theory is its capacity to capture intuitive concepts, besides considering psychological aspects used by the human beings in its usual reasoning, avoiding that its representation is plastered by traditional models (OLIVEIRA, 1999).

The *fuzzy* set theory (ZADEH, 1965; ZIMMERMANN, 1985) is a mathematical method used to characterize and to quantify the uncertainty and imprecision in the data and functional relationships. Diffuse sets are especially useful when the number of data is not enough to characterize the uncertainty through measures pattern statistical involving the estimate of frequencies.

According Ganoulis (1994), the central concept of the *fuzzy* theory is the membership function that represents numerically the degree in that an element belongs to a group. If an element belongs to a group *fuzzy* in some degree, the value of the membership function can be any number between 0 and 1. When the membership function of an element can only have the values 0 or 1, the set theory reduces to the classical theory of sets.

2.2.1 Diffuse wave equation with *fuzzy* characteristics:

Practical application, like industries, measuring voltage, current, temperature, etc., there may be a negligible error. This causes data inaccuracy. This imprecision can be represented by membership functions.

A membership function is one that represents the level of pertinence of the parameters, in a certain much defined physical process. This way, as bigger is the degree of pertinence of this variable in the context, the bigger will be the value of this function. Thus the "fuzzification" is carried out.

Parameters of the deterministic model of propagation Diffuse wave can be transformed into a *fuzzy* model simulation by "fuzzification". The deterministic equations take the following form as *fuzzy* equations:

$$\frac{\partial \tilde{Q}}{\partial t} + \frac{d\tilde{Q}}{d\tilde{A}}\frac{\partial \tilde{Q}}{\partial x} = \frac{\tilde{Q}}{2\tilde{S}_0 B}\frac{\partial^2 \tilde{Q}}{\partial x^2} \qquad (4)$$

where:
$\tilde{A}$ membership function for transversal area of the river;
$\tilde{Q}$ membership function for flow in the river;
$\tilde{S}_0$ membership function for bed slope;
$\tilde{S}_f$ membership function for friction slope;

3. Numerical Solution

With respect the numeric solution of the *fuzzy* differential equations of diffuse wave (Equation 4), the implicit discretization formulation will be used and defined through the relationship (GOMES, 2006):

$$\frac{\partial \tilde{Q}}{\partial t} \approx \frac{\tilde{Q}[i, j+1] - \tilde{Q}[i, j+1]}{\Delta t} \tag{5}$$

$$\frac{\partial \tilde{Q}}{\partial x} \approx \frac{1}{2}\left[\frac{\tilde{Q}[i+1, j+1] - \tilde{Q}[i-1, j+1]}{2\Delta x} + \frac{\tilde{Q}[i+1, j] - \tilde{Q}[i-1, j]}{2\Delta x}\right] \tag{6}$$

$$\frac{\partial^2 \tilde{Q}}{\partial x^2} \approx \frac{1}{2}\left[\frac{\tilde{Q}[i+1, j+1] - 2\tilde{Q}[i, j+1] + \tilde{Q}[i-1, j+1]}{\Delta x^2}\right.$$
$$\left. + \frac{\tilde{Q}[i+1, j] - 2\tilde{Q}[i, j] + \tilde{Q}[i-1, j]}{\Delta x^2}\right] \tag{7}$$

In order to solve the differential equations involved in the mathematical model of propagation of the diffusive wave, a computational program was developed in language FORTRAN 90. Thus, it is possible to calculate the variables pertinent to the control of flow in a river.

4. Results

The hydrodynamic *fuzzy* model developed in this paper was used to accomplish   several simulations, where the bed slope and roughness coefficient were values varied, for the same initial and boundary conditions. In order to evaluate the uncertainties of the *fuzzy* flow routing along the open channel. The software developed have a *fuzzy* routine which considers changes in hydrodynamic parameters. these are transformed in membership functions input for the *fuzzy* model. Parameters such as Manning's number, bed slope and inflow where software returns as a result the following "fuzzification" parameters both over time as space:

- Flow;
- Depth

The first simulations study the flow field, Q, for each membership function α-cut =0, α-cut=0,5 and α-cut=1, as the section vary.

Figure 1 shows membership *fuzzy* se for diffuse wave in different section, for a time of 3 hours. The goal is to see the flow field can be.
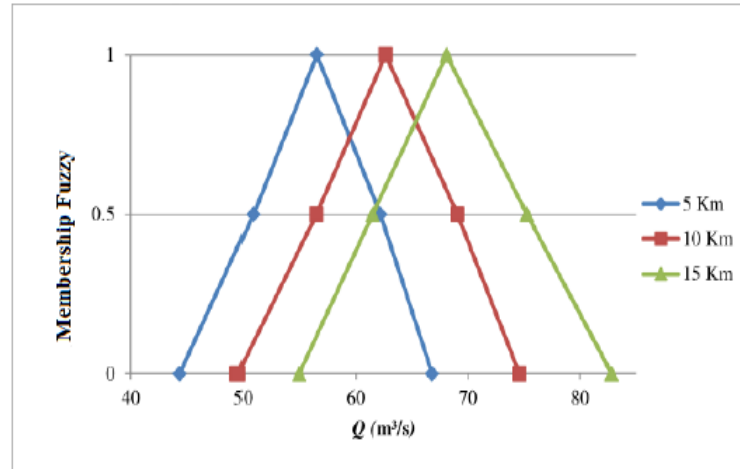
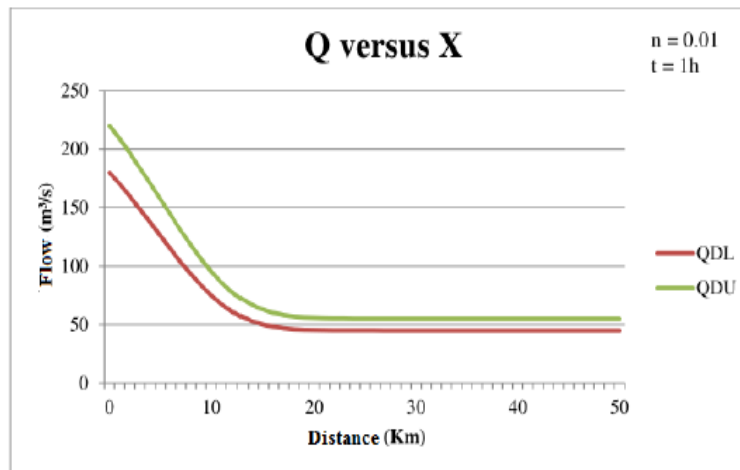**Figure 1.** Membership for *fuzzy* variable flow for sections: 5km, 10km and 15km, t = 3h.

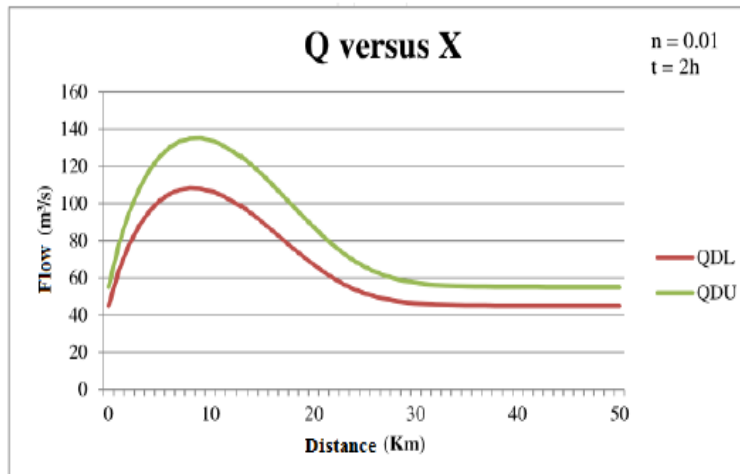**Figure 2**: Membership functions for *fuzzy* variable flow at α-cut=0.5, S_0=0.00005, n=0.01 and t=1h

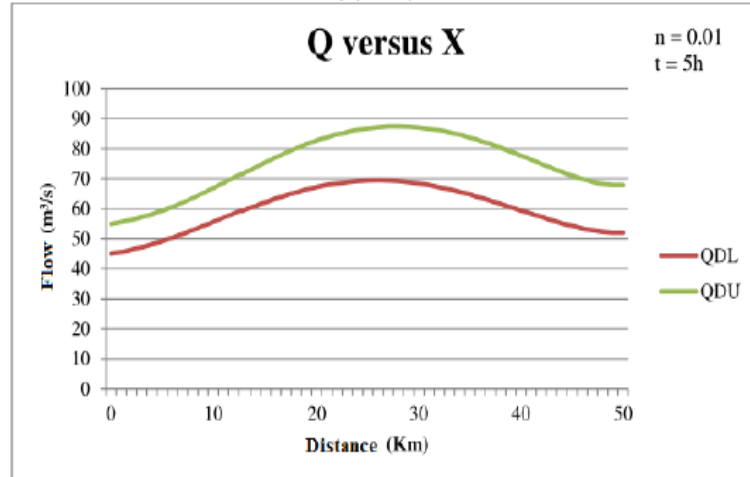**Figure 3**: Membership functions for *fuzzy* variable flow at α-cut=0.5, S_0=0.00005, n=0.01 and t=2h

Figure 4: Membership functions for *fuzzy* variable flow at α-cut=0.5, S_0=0.00005, n=0.01 and t=5h

Figures 2 through 4 indicate the distance between the lower and upper limits of each triangular *fuzzy* sets for variable diffusive flow, uncertainty region. By comparing figures to each other, it is checked for different times a damping process along the river. This is due to existing diffusive term in the differential equation for diffusion wave.
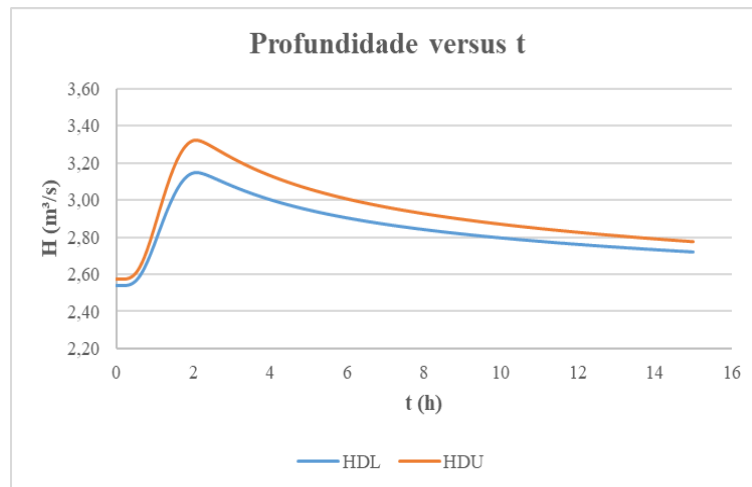

Figure 5 – Membership functions for *Fuzzy* depth variable para α-cut=0,5, S_0=0,00005, n=0,1 section 10 km
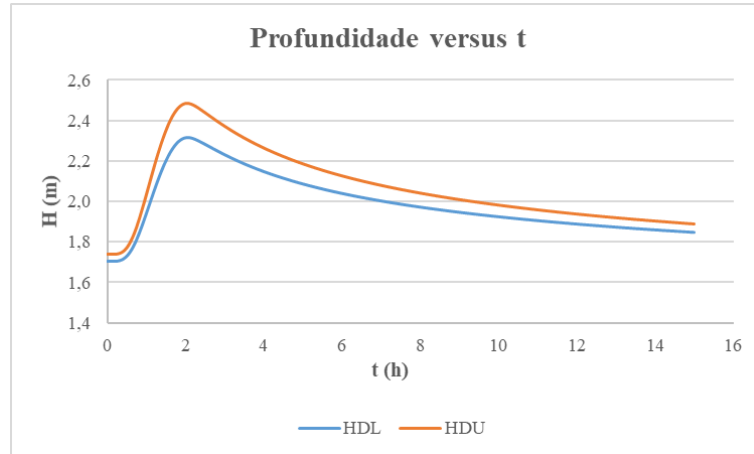
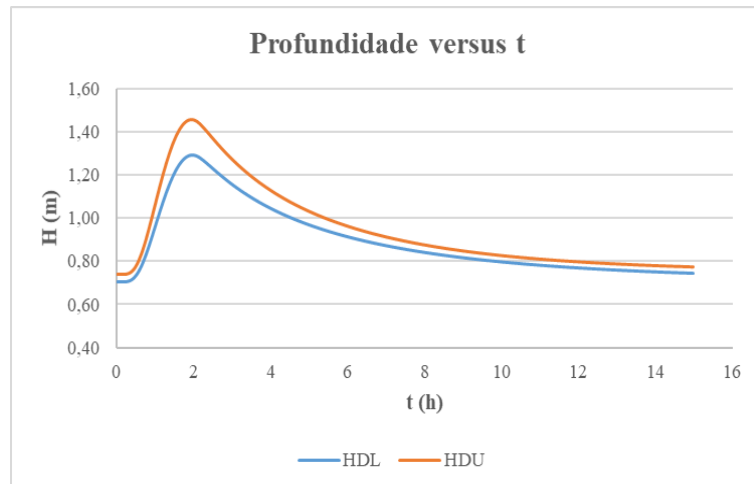**Figure 6** – Membership functions for *Fuzzy* depth variable para α-cut=0,5, S0=0,00005, n=0,05 section 10 km

Figure 7 – Membership functions for *Fuzzy* depth variable para α-cut=0,5, S0=0,00005, n=0,01 section 10 km

Figures 5, 6 and 7 shows that the higher the value of the Manning's number (n) the greater the depth value, increasing the risk of inundations.

4. Conclusions

Results allowed to establish that it is strongly influenced by the relation between hydraulic parameters and the Manning's number (n). The developed software allowed to evaluate the *Fuzzy* shape of the diffusion wave propagation behavior in aspects of flow, cross-sectional area, velocity and depth along the channel for different time intervals of observation and different sections, for this study the parameters were emphasized flow and depth.

In the reality, this study allows that subsidies could be obtained, in order to apply in the development of programs of water resources management, especially, in the prevention of inundations. However, it is important to point out that there are still many aspects to research and to implement in the model.

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