T H E S I S

INFLUENCE OF SHAPE
ON THE FALL VELOCITY
OF SAND GRAINS

Submitted by
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In partial fulfillment of the requirements
for the Degree of Master of Science
in Irrigation Engineering
Colorado
Agricultural and Mechanical College
Fort Collins, Colorado
December, 1949

COLORADO AGRICULTURAL AND MECHANICAL COLLEGE

December 1949

WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR SUPERVISION BY ARTHUR THOMAS COREY

ENTITLED INFLUENCE OF SHAPE ON THE FALL VELOCITY OF SAND GRAINS

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE.

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ACKNOWLEDGEMENTS

The writer is indebted to Dr. M. L. Albertson, Associate Professor of Civil Engineering, Colorado A & M College, for his assistance in the preparation of this thesis.

Special acknowledgment is due to Mr. Bruno Klinger, Associate Professor of Botany, Colorado A & M College, for his valuable suggestions in regard to the photographic procedure, and to Dr. L. W. Durrell, Dean of the Division of Science and Arts, for the use of his personal equipment in the measurement of the dimensions of sand grains.

Acknowledgments also are due Miss Duvoid Burris and Mr. N. T. Mahoney, photographers, Fort Collins, Colorado, for their assistance in the photographic work connected with this investigation.

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Chapter I
INTRODUCTION

Sedimentation problems are becoming increasingly important in the field of irrigation. The annual cost of removing sediment from channels and the loss of storage space in reservoirs is very great. Unfortunately, the science of sedimentation engineering is in its infancy, and there is a considerable lack of fundamental knowledge concerning the movement of sedimentary particles in water.

For purposes of design, engineers would like to know in advance the probable rate of sedimentation in a given reservoir or canal. In the past, estimations have been made on the basis of experience with other projects, but often these estimations bear little relationship to actual conditions. Recently a number of theoretical methods for predicting the rate of sedimentation have been devised. All of these methods require a knowledge of the average hydraulic properties of the particular sediment under consideration.

The hydraulic properties of sediment particles include the rate at which they will settle out of suspension or in the case of the larger particles, the velocity of flow required to move them along the bed of a channel. It has been established by a number of workers in this field that the fall velocity of particles in still water serves as an ideal index for both of these hydraulic properties.

Fall velocities have been measured in the past by dropping particles in water and measuring their rate of fall with a stop watch. This method is satisfactory when the particles are small and the velocities are relatively slow, but the accuracy is decreased when the greater velocities of larger particles are measured.

It is the larger particles, corresponding to Reynolds numbers above 100, about which information is most needed. In the case of small particles, shape is less important because the flow pattern follows Stokes law more closely. As the particles become larger, shape becomes increasingly important and some means of correlating shape with fall velocity would be helpful. Practical methods of measuring both fall velocity and particle shape need to be developed, and for that reason this investigation is organized as shown.

The problem

How does the shape of rock particles influence the movement of the particles in water?

Problem analysis.--The following questions concerning this problem need to be answered:

1. Can a practical method be found for measuring the movement of particles in water?
2. Do different deposits of sand have different average shape characteristics which can be expressed mathematically?
3. Can a practical method be found for evaluating the shape of the particles?
4. What is the effect of the difference in shape upon the movement of the particles in water?

Delimitations.--In this investigation consideration is given only to those methods of evaluating shape that are sufficiently simple to be of practical value.

The study will be limited to sands of sieve diameters corresponding to sieves 4 to 14 meshes to the inch. Four sands will be considered--two from river sediment deposits, one wind-blown sand, and one sample of fragments from a rock crusher.

Chapter II

REVIEW OF LITERATURE

The great increase in interest in problems of sedimentation brought about by the silting of reservoirs and the clogging of irrigation and navigation channels has led to an extensive search for fundamental knowledge concerning the movement of sediments in suspension and along the beds of channels.

It has been recognized for some time that the distance which a given particle will be carried in suspension and the rate at which it moves is a function of its fall velocity. More recently it has been found that the velocity of flow required to move larger particles along the bed of a stream is also a function of their fall velocities, and in the case of larger particles, their velocity is a function of shape. For this reason a number of workers have gathered information on the fall velocities of sand particles and have attempted to correlate velocity with shape.

Among the first to gather information on the fall velocity of rock particles were metallurgical engineers concerned with ore-dressing problems. They investigated the problem from the standpoint of relating sieve size and fall velocity of mineral particles in water since that was the kind of information required in their industry. Richards (4), in 1908 made a study of the settling velocities of crushed quartz and galena grains. He found that for particles larger than 1.55 mm in diameter the coefficient of drag no longer was a function of Reynolds number only, but a distinct variation existed tetween the quartz and galena grains. This variation was apparently attributed to the difference in specific gravities of the minerals.

In 1933 Rubey (6) used Richards' data and much of his own to develop a formula which he thought could be used in place of Stokes law for calculating fall velocities of particles larger than about 0.14 mm in diameter. He said that the fall velocities of particles vary with their diameters according to two distinct laws. The first law is the law of Stokes which applies to the purely viscous range. The second law which he calls the impact law applies to the range in which resistance is practically independent of viscous forces. Rubey derived an expression for impact resistance by considering that the forces holding a spherical particle in suspension at a constant height in a rising current of some fluid is the impact of the fluid. By combining Stokes law and the impact law he derived a general expression for fall velocity as follows:

$$4/3 \pi r^3 (\rho_s - \rho_f) g = 6 \pi r \mu v + \pi r^2 v^2 \rho_f$$

In this expression r refers to the sieve radius, ρ_s is the particle density, ρ_f is the fluid density, μ is the viscosity, and v is the terminal velocity of the particle. Rubey added experimental coefficients to make the results fit the data for quartz particles, but he could not make it fit the data for both quartz and galena. He considered this to be due to the difference in average shape of the particles, galena being more nearly cubical whereas crushed quartz tended to be more flaky in average shape. He noted that particles generally fall so that resistance is a maximum. He also observed that the current required to move a particle along the bed of a channel is approximately the same as its settling velocity in still water.

Rubey considered that his expression for fall velocity was based on sound theory, but as illustrated by Rouse (5:245) in 1945, the relationship between the drag coefficient and Reynolds number beyond Stokes range cannot be expressed by an equation of the same form as Stokes law and the point of departure from Stokes law depends on the shape of the particle.

A Russian investigator, Zegrzda (10), 1934, was among the first to emphasize the importance of shape. He pointed out that the discrepancy between the drag coefficients of quartz and galena as found by Richards could not possibly result from a difference in specific gravity, and he attributed the discrepancy to the difference in average shape of these minerals.

Zegrzda plotted the data of Richards along with his own on a graph of drag coefficient as a function of Reynolds number. He pointed out that such a graph could be valid for only a specific shape and that to be theoretically accurate, graphs should be made for every conceivable shape of particles. However, he said that such graphs would not represent average conditions and could be of no practical use. Therefore he plotted the data, using the sieve diameters in the expressions for the drag coefficient and Reynolds number. He also compared the sieve diameters with the sedimentation diameter which is the diameter of a hypothetical sphere having the same density and fall velocity.

Zegrzda used petroleum products and water as a medium in which he measured the fall velocity of sand grains and particles of controlled shape made of steel. He noted that with the steel shapes there was no break in the drag coefficient as a function of Reynolds number curve regardless of the medium, whereas for irregular sand grains the curves obtained by using water as a medium did not fit the curves obtained with other mediums. This result puzzled Zegrzda, but he suggested that since sand grains contained minute pits and projections not found on the steel shapes, that flow around the sand particles might be influenced by fluid properties other than density and viscosity. Therefore he recommended that water alone be used as a medium in sedimentation studies.

In 1935 Wadell (9) defined true sphericity as the ratio:

$$\psi = \frac{s}{S}$$

where s is the surface area of a sphere of the same volume as the solid, S the actual surface area of the solid, and ψ denotes the degree of true sphericity. The maximum value obtained for ψ is 1, which is the numerical shape value of a sphere. Wadell showed that the coefficient of resistance, a function of Reynolds number for solids of various shapes is also a function of sphericity.

Since the surface areas of individual sand particles are very difficult to obtain, true sphericity is impractical to use for studies of the fall velocity of individual grains. For this reason, Wadell suggested the use of the following expression for shape:

$$\phi = \frac{d_c}{D_c}$$

where d_c is the diameter of a circle equal in area to the projected area obtained when the grain rests on one of its larger faces, more or less parallel to the plane of the longest and intermediate diameters, and D_c is the diameter of the smallest circle circumscribing the projected area.

Wadell said that this ratio was an indication of true sphericity for average particles, but he pointed out that for extreme cases, this definition gives entirely erroneous results, for example, the sphericity of a thin disk as defined by this ratio is 1.0, the same as for spheres.

The measurement of the projected area was accomplished by magnifying the grains to a standard size by a camera lucida mirror and the area was reproduced by drawing it on paper after which it was measured with a planimeter. A circle scale consisting of a number of circles drawn on the ground surface of a 20 cm by 20 cm sheet of transparent celluloid was used with a microscope to measure the circumscribed circle.

Wadell analyzed the experimental data on extremely thin disks (sphericity values of 0.12 and 0.22) and showed graphically the wide spread between the resistance curves for these disks and that for spheres even in Stokes range. He was the first to point out that elimination of the nominal diameter between Reynolds number and the drag coefficient,

$$R = \frac{\rho_f v_o d_n}{\mu}$$

$$C_D = \frac{4/3(\rho_s - \rho_f) g d_n}{v_o^2 \rho_f}$$

yields a series of lines,

$$\log C_D = \log R + \log \frac{4/3(\rho_s - \rho_f) g \mu}{\rho_f^2 v_o^3}$$

on the logarithmic C_D versus R graph, each of which represents a particular terminal settling velocity in the same fluid. In this expression ρ_s and ρ_f are the densities of the particle and fluid, μ is dynamic viscosity, and v_o is the terminal velocity.

In 1938 Heywood (1) suggested the use of a volume constant k for use in measuring shape. He defined k as follows:

$$k = \frac{\text{volume of particle}}{d^3}$$

where d is defined as the diameter of a circle of area equal to the projected area of the particle when placed in its most stable position. The volume constant k varies from $\pi/6$ for a sphere to values of less than 0.1. Heywood has prepared a nomograph such that fall velocity can be determined if an approximate value of k can be estimated which tends to be more nearly the true value than if the particle was assumed to be spherical. Heywood tabulated values of k for particles having equal length, breadth, and thickness, but of various angularities.

In 1942 W. C. Krumbein (2) published the results of an investigation correlating fall velocity and flume behavior for non-spherical particles. He found that the relative settling velocity furnishes an adequate index of flume behavior. Flume behavior refers to both velocity of water necessary to move a particle and the rate at which it moves once it starts.

By using particles of equal density and volume made from molded cement, the nominal diameter was held constant. The shapes studied were classified as cubes, disks, rollers, bricks, and fragments.

Krumbein suggested the use of an over-all "form-coefficient" which could be derived from settling velocities by introduction of the nominal diameter into the coefficient of resistance. He also suggested the following expression for sphericity:

$$\psi = \sqrt[3]{(b/a)^2\ (c/b)}$$

where a, b, and c are the long, intermediate, and short axes of the particle. The results of his experiments indicated that this ratio gives values comparable to true sphericity as defined by Wadell, and he made graphs on which the ratio b/a is the ordinate and c/b is the abscissa. Lines of equal sphericity based on the foregoing equation were also plotted on the graph.

In 1945 Serr (7) investigated the fall velocity of various sands of sizes corresponding to sieve diameters

10 to 100 of the Tyler standard screen scale. He compared the sieve diameters to the sedimentation diameters of sands of different shapes and proposed that the ratio of sedimentation diameter to the sieve diameter be used as a practical criteria for expressing shape.

The sieve diameter was used for computing Reynolds number and the drag coefficient in plotting the drag coefficient as a function of Reynolds number. Serr assumed that the sieve diameter was roughly comparable to the nominal diameter and found that this was more nearly true as the size of the particles increased. He also found that the sedimentation diameter was always smaller than the sieve diameter, and that for his most angular sand grains, this caused a reduction in fall velocity of about 20% to 30% in the larger sizes. Particles corresponding to a screen size of 100 showed little effect of shape. In other words, the ratio of sedimentation to sieve diameters was about unity. The drag coefficient as a function of Reynolds number curves were found to flatten at a nominal diameter of about 1.5 mm indicating to Serr that this is near the size where viscous effects become secondary and the drag coefficient becomes independent of Reynolds number for natural particles. For spheres, this does not occur until the diameter is about 5 mm. The data obtained by Serr did not extend beyond the Reynolds number at which the curves barely begin to flatten.

All of the investigators already mentioned measured fall velocity of particles in a fluid by timing the fall through a measured distance with a stop watch. This procedure is sufficiently accurate for particles of small size and slow rate of fall, but it becomes inaccurate when larger particles are used which is the reason that these investigators generally used particles of a size that are influenced primarily by viscous forces.

More recently, as reported by Malaika (3), McPherson adopted a photographic method for velocity measurement in order to utilize larger spheres, but he did not apply this method to irregular shaped particles. In 1949 Malaika (3) used the equipment developed by McPherson to study the resistance of steel particles of various shapes. For particles symmetrical about three mutually perpendicular axes Malaika found that the shape factor could be satisfactorily expressed as the ratio of the length to the largest diameter perpendicular to the long axis. In this investigation, the particles were carefully dropped by a magnetic release in the position that they normally assume while settling in a fluid.

From this investigation Malaika concluded the following:

1. The positional stability of the particle differs markedly for each of three regions of drag studied.
   In the deformation-drag zone, the particles are stable in any settling position; in the

surface-drag zone, they orient themselves with the largest section, in the planes of symmetry, normal to the direction of motion; in the zone of eddy formation, the particles start to oscillate and the motion becomes wholly unsteady.

2. Although an accurate prediction of settling velocity from geometry is very difficult, the normal and tangential areas of a particle related to similar values for the equivalent sphere are found to be significant. These factors are combined with circularity to yield the settling velocity as a semi-empirical function of shape characteristics.

3. All particles settling in a fluid in the deformation-drag zone follow a resistance law of a form similar to Stokes law for spheres with a correlation coefficient depending on the shape and orientation of the particle.

4. An analytical solution was found for the velocity of spheroids settling along a principal axis in the deformation-drag zone. To a close approximation, the effect of shape on settling velocity is found to be a function only of the l/s ratio of the particle, so that the analytical results can be used to predict settling velocity of a wide variety of shapes. (3:46)

Summary of literature.--The fact that shape has a considerable influence on fall velocity has been established. It has also been shown that fall velocity serves as an index of the behavior of particles in water. A number of schemes for expressing shape have been devised which seem to correlate with fall velocity under average conditions. These expressions for the shape factor require measurements which are tedious and time-consuming to make. A practical expression for the average shape factor of particles, the ratio of sedimentation diameter to sieve diameter, has been proposed by Serr. This ratio is not entirely satisfactory from the research standpoint, however, since it shows nothing of the fundamental factors involved in particle drag.

Most of the investigations have been made with small particles that are primarily influenced by viscous forces and little data has been accumulated with larger particles that are influenced primarily by inertia forces.

A method for determining the velocity of larger particles has recently been employed by Malaika (3) with which fundamental knowledge has been accumulated concerning the fall velocity of non-spherical particles. Before the present investigation, this method had not been adapted to the study of irregular sand grains.

Chapter III

THEORETICAL ANALYSIS

The theory involved in a study of the fall velocity of a particle is largely a matter of analyzing the gravitational forces tending to move the particle downward and the resistance forces developed by the fluid as the particle moves. When the actuating and resisting forces are equal, the particle no longer accelerates and is said to have reached terminal velocity. In this discussion, fall velocity is considered to be synonomous with terminal velocity.

Balance of forces

Every fluid exerts a buoyant force on a submerged solid equal to the weight of fluid displaced so that the effective gravitational force acting on the solid is normally considered to be equal to the difference in unit weights of the fluid and solid times the volume of the solid or:

$$F = (\text{volume})\ (\rho_s - \rho_f)g \qquad (1)$$

At terminal velocity the net gravitational force F is equal to the fluid resistance which is sometimes called frictional resistance although this definition is not concise. The resistance developed by the fluid is actually a combination of two entirely different phenomena.

The first phenomenon and the one which is most important in the case of small particles moving relatively slowly is viscous resistance. Under these conditions, fluid moving past the particles is deformed in a manner not ?
wholly unlike that which would be indicated by a flow net. The velocity gradient created by these conditions results in a resistance to the motion of a small sphere which can be expressed by the well known law of Stokes,

$$F = 3\pi d v_o \mu \qquad (2)$$

in which F is the net gravitational force, d is the diameter, v_o is the relative velocity of the particle when the forces are in balance, and μ is the coefficient of dynamic viscosity. This kind of resistance is called deformation drag.

As the weight of the particle is increased, other factors remaining constant, the relative velocity is increased thereby causing inertia forces to become a factor in the drag on the particle. The inertia forces of the fluid are a function of both fluid velocity relative to the solid particle and the radius of curvature of the path it must take in passing the particle. The inertia forces, therefore, depend upon the velocity v_o of the fluid relative to the particle, the dimension d

of the particle perpendicular to the major axis of fluid motion, the shape of the particle surface, and the density ρ of the fluid.

The ratio of inertia forces to fluid viscosity is called Reynolds number which is expressed as

$$\frac{\rho v_o d}{\mu}$$

As this ratio increases, the viscous forces become less important, and the velocity and pressure distribution within the fluid surrounding the particle are changed. The value of Reynolds number at which viscous forces are no longer important depends upon the shape of the particle. When Reynolds number reaches a critical value (about 10^5 in the case of spheres) the pattern of flow around the particle reaches a constant for a given shape, and resistance to the fall of a particle of a given weight is a function of shape and velocity only. The exact value of Reynolds number at which this occurs is also a function of shape.

When inertia forces are dominant over viscous forces, a zone of separation is created on the downstream side of the particle. This separation takes place at a point where the velocity is increased because of the resistance of the surrounding field to a change in direction. The stream lines are forced closer together by fluid inertia, and tangential acceleration takes place.

As the velocity increases, the pressure is reduced according to Bernoullli's principle.

The area of low pressure is maintained throughout the region of discontinuity downstream from the particle. On the upstream face there is a region of stagnation in which tangential velocity is reduced to a minimum and pressure is a maximum. The combination of high pressure on the upstream face and low pressure on the downstream face produces a resistance totally unrelated to viscous shear and is known as form drag.

<u>Consideration of variables</u>

From the above discussion it can be seen that the variables involved in the fall velocity of a particle are as follows:

1. The net gravitational force F which can be expressed as:

   $F = (\text{volume})\ (\rho_s - \rho_f)\ g$

   where ρ_s, and ρ_f are the densities of the particle and the fluid respectively.
2. The fluid viscosity μ.
3. The terminal velocity v_o.
4. The projected dimension or dimensions of the particle perpendicular to the direction of flow.

5. The curvature of the surface of the particle which is called shape.

It is obvious that, for a particle of given weight, the last two variables listed are directly related. A particle of non-spherical shape will have either a greater or lesser projected area perpendicular to the flow than a sphere of the same weight depending on the direction of flow.

This is a point of major significance in the fall velocity of particles of random shape because, as pointed out in Chapter II, the particles generally fall so that their resistance is a maximum in the size range of surface and form drag. The exact position in which the particles fall is influenced by their eccentricity, however, because for stability the center of gravity and the center of resistance must be in a line with these forces. Therefore, the projected dimension of a particle of given volume is also a function of shape.

In the case of true spheres the shape is a constant and the remaining variables can be summarized in the following expression:

$$v_o = f(F, d, \rho, \mu) \qquad (3)$$

Dimensional analysis of this expression yields the parameters, $\frac{\rho v d}{\mu}$ and $\frac{F/d^2}{\rho {v_o}^2}$. The first of these is the familiar Reynolds number R , and the second is related to Eulers number $\frac{\Delta P}{\rho v^2/2}$, in which ΔP is the increment in pressure. By replacing d^2 with A , the projected area, and v by $v_o/2$, an expression is obtained that is even more closely related to Eulers number. This parameter is called the drag coefficient and is expressed as follows:

$$\frac{F/A}{\rho v_o^2/2}$$

A graphical representation of C_D as a function of R for spheres and disks appears in the textbook, Elementary Mechanics of Fluids, by Rouse (5:245). An inspection of this graph shows that the value of C_D for disks is considerably greater than that for spheres beyond the range of Stokes law. This effect is caused by the increased inertial resistance to flow as already explained.

In order to obtain parameters for use with irregular particles that are comparable to those used for symmetrical shapes, the tentative assumption may be made that the form of the particle is described by the lengths of three mutually perpendicular axes, the major axis a , the intermediate axis b , and the minor axis c . The variables influencing the problem now may be expressed as:

$$f(v_o, F, \mu, \rho_f, \rho_s, a, b, c) = 0 \qquad (4)$$

By applying the Buckingham π-Theorem to these variables the following dimensionless equation can be written:

$$f_1\left(\frac{v_o b \rho}{\mu}, \frac{F/b^2}{\rho v_o^2}, \frac{\rho_s}{\rho_f}, \frac{b}{a}, \frac{c}{b}\right) = 0 \qquad (5)$$

Since the ratio $\frac{\rho_s}{\rho_f}$ is held constant in this investigation, it may be eliminated as one of the variables.

In order to obtain parameters with more significance, the remaining four variables may be recombined. Since the product ab is roughly related to the projected area perpendicular to the motion of falling particles, the parameter $\frac{F/b^2}{\rho v_o^2}$ is multiplied by the ratio b/a and it becomes $\frac{F/ab}{\rho v_o^2}$. It is probable that the dimension b has as much influence on Reynolds number as any other dimension therefore Reynolds number may be left as it is expressed in Eq. (5).

Consideration of shape factor

Since a number of investigators (3, 6, 9) have found that particles generally fall with their largest projected area perpendicular to the line of motion, it may be reasoned that flatness is the shape factor having the most significance. To obtain a parameter expressing flatness the ratios b/a and c/b can be recombined by squaring the ratio c/b and multiplying this ratio by b/a . The square root of this product $c/\sqrt{ab}$ may be used to express flatness. The dimensionless equation that results from the foregoing operations is as follows:

$$f_2\left(\frac{v_o b}{\mu}, \frac{F/ab}{\rho v_o^2/2}, \frac{c}{\sqrt{ab}}, \frac{c}{b}\right) = 0 \tag{6}$$

In order to reduce the number of variables so that they can be correlated with a single graph, the assumption may be made that the ratio c/b is relatively insignificant so that the following dimensionless equation results:

$$f_3\left(\frac{v_o b\rho}{\mu}, \frac{F/ab}{\rho v_o^2/2}, \frac{c}{\sqrt{ab}}\right) = 0 \tag{7}$$

The shape factor $c/\sqrt{ab}$ is probably not as complete from the theoretical standpoint as the volume constant of Heywood because it does not take roundness into account. Furthermore, the value for both spheres and cubes is 1.0, therefore this shape factor cannot accurately be called sphericity. However, it has a decided advantage in that it does not involve the tedious and time consuming process of measuring surface or sectional areas of irregular particles and yet it does describe the flatness. For this reason the ratio $c/\sqrt{ab}$ was used to express shape in this study.

Other ratios were considered for use in this investigation but were eliminated because of theoretical and practical considerations. One of these is the ratio $\frac{b/a + c/a + c/b}{3}$ which seemed to have no advantage over the ratio $c/\sqrt{ab}$ and is less convenient. Another was the ratio $d_n/\sqrt{ab}$ which was eliminated because it gives a value for spheres less than the value for cubes. The ratio c/d_n was not used because an angular particle can give a high value to a non-spherical particle.

A shape factor analogous to the volume constant of Heywood (1:27) was considered. Heywood's volume constant is expressed by the ratio:

$$k = \frac{\text{volume}}{d^3}$$

where d is the diameter of a circle having an area equal to the largest projected area of the particle. The possibility of replacing d^3 with $(ab)^{3/2}$ was studied since $(ab)^{\frac{1}{2}}$ is roughly related to d . However, $(ab)^{\frac{1}{2}}$ is normally somewhat greater than d and the discrepancy is considerable in the case of round particles which should have higher values of the ratio if it is to correlate with fall velocity.

It is not probable that a simple ratio can be found that adequately expresses shape from the theoretical point of view. However, it is believed that the shape factor used in this study is superior to the simplified ratio suggested by Wadell (9:264):

$$\frac{d_c}{D_c}$$

where d_c is the diameter of a circle equal in area to the largest projected area and D_c is the diameter of the smallest circle circumscribing this area. This ratio not only gives values of shape for disks and spheres that are identical, but it also is difficult to measure.

Coefficient of drag based on nominal dimensions

Sedimentation engineers cannot conveniently measure the dimensions of individual sand particles in the field. Therefore, they are interested in relating the influence of shape to the behavior of a given volume or weight of sediment. For a given density, the weight of a particle is described by its nominal diameter. For this reason another group of parameters was studied using the same shape factor and the nominal diameter in Reynolds number and the coefficient of drag as follows:

$$f\left(\frac{F/d_n^{\,2}}{\rho v_o^{\,2}/2}, \frac{v_o d_n}{\mu}, \frac{c}{\sqrt{ab}}\right) = 0 \qquad (8)$$

The dimensional analysis resulting in this equation can be accomplished in the same way as the analysis of true spheres, Eq. (3), except that the nominal diameter d_n is substituted for the actual diameter and the shape parameter is added.

It should be thoroughly understood that the nominal coefficient is by no means comparable to the coefficient of drag expressed in terms of projected areas perpendicular to movement. However, since it is desirable in most cases to compare directly the fall velocity of particles of a given volume, the nominal coefficient is probably more significant. It should be remembered that for a given volume, Reynolds numbers and drag coefficients based on projected areas are functions of shape. Therefore, when these variables are used, shape is already largely taken into account.

The nominal coefficient of drag may also be thought of as a ratio of two drag forces

$$C_D = \frac{F}{\rho A_n v_o^2/2} = \frac{\text{drag on the particle}}{\left(\begin{array}{l}\text{drag on sphere having same}\\ \text{weight and velocity}\end{array}\right)\phi} \qquad (9)$$

where ϕ is a variable depending on Reynolds number. At high Reynolds number ϕ is a constant.

From this relationship it is obvious that the nominal coefficient of drag may become extremely large as the particles become very thin. On the other hand, the plot illustrated by Rouse (5:245) shows that the conventional coefficient of drag based on projected areas hardly exceeds 1 beyond a Reynolds number of 10^3 even with extremely thin disks.

The pertinent projected area is impossible to measure in the case of irregular particles because there is no sure way of determining the position in which a given irregular particle will fall.

If the settling position of particles was purely a matter of chance, average results obtained by using a very large number of fall velocity measurements should show no difference whether nominal or true areas are used. The settling position is not entirely a matter of chance, however, as it has been shown by many investigators that the particles normally fall so that the resistance is a maximum.

Since sieve diameters are roughly comparable to nominal diameters as was shown by Serr (7:27), the same principle applies to the use of sieve diameters in calculating the coefficient of drag.

Chapter IV

MATERIALS AND METHODS

The principal object of this investigation was to evaluate the effect of shape on the movement of sand grains in water. It was known beforehand that the effect of shape is greatest in the case of particles that move too fast to be accurately timed with a stop watch. Because information concerning the effect of shape on particles of this size was needed, a satisfactory method of measuring fall velocity of this magnitude was sought.

Equipment.--The basic principle used for measuring fall velocity in this investigation was adapted from the method used by Malaika (3) which was developed by McPherson at the Iowa Institute of Hydraulic Research. Using a light from a strobolux, the particles were photographed as they fell through the water. The period of the flashes was controlled by a strobotac, and because this period was known, the position of the particle at the time of successive flashes could be detected on the photographic film. By photographing a scale under the same conditions, a reference is obtained with which the distance traveled by a particle between flashes can be measured. From these data, the fall velocity was

Fig. 1 -- Equipment for photographing
falling particles.

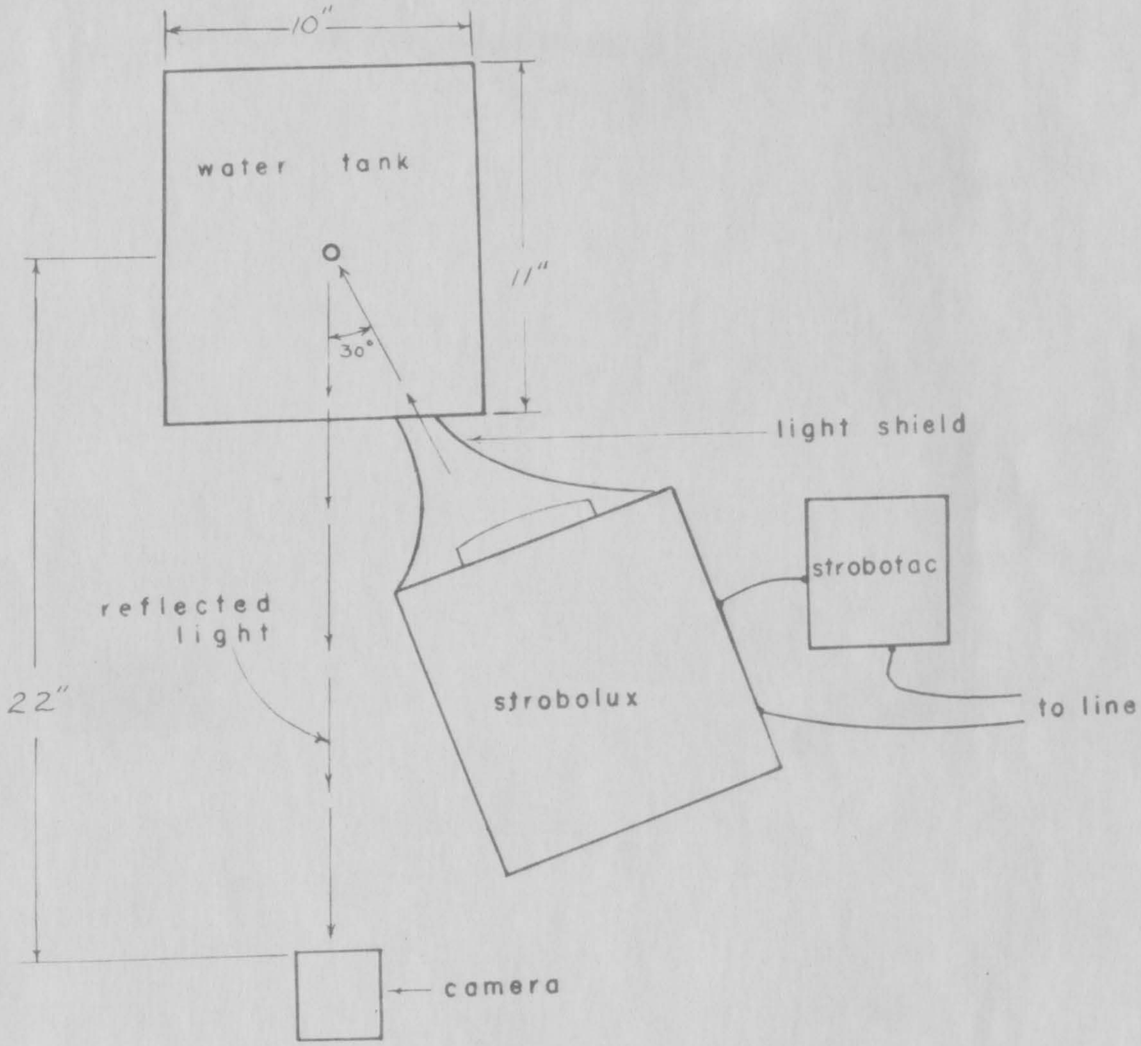


Fig. 2--Schematic layout of equipment for photographing falling particles.

calculated.

Besides the strobolux and strobotac, the equipment used for this measurement consists of a water tank with two sides of glass in which the sand grains were dropped, a camera, equipment for developing pictures, micrometer, scale, and tweezers for dropping the particles.

The strobolux is a General Radia Type 648-A, and the strobotac is a Type 631-B. The strobolux was mounted beside the camera, and the light was directed toward the center of the water tank in a vertical beam about 2 inches in width and about 12 inches in depth. The beam was directed as nearly parallel to the camera as possible without interfering with the pictures. This was done in order to get as much reflection as possible from the face of the particles toward the camera. A pasteboard hood painted white on the inside was placed over the front of the strobolux to direct the light in a narrow vertical beam.

The tank, 10 in. by 11 in. by 21 in., was made of 1/4 in. plywood and polished glass with a welded angle-iron frame. The glass and plywood were sealed to the frame with rubberized cement. Aquarium cement was used on the inside corners as an additional precaution against leakage. Plywood sides were painted flat black to serve as a background for the particles. The tank was built with the idea of using it as a constant temperature bath in which a glass cylinder would be placed according to the procedure of McPherson. However, it was found that a constant temperature bath was not needed. All of the photography was done in the furnace room of the Colorado A & M Hydraulic Laboratory which is below ground level so that the temperature is uniform over reasonably long periods of time. During the two weeks in which pictures were taken, the temperature varied less than 1° F. In addition, there is a dark room equipped for developing film adjacent to the furnace room. Since neither a water bath nor a glass cylinder were required, the particles were dropped directly into the center of the tank.

This greatly improved the photography because the use of a glass cylinder within the tank gives an objectionable reflection especially when the light is directed practically parallel to the camera.

In taking the pictures of the falling sand grains, several cameras were tried. It was found that only a camera with a very good lens would give satisfactory results. A Speedographic press-type camera with a 3.5 lens and ground glass focusing was rented for the purpose. The camera was mounted so that the lens was 22 inches from the line of fall of the particle and so that the center of the camera field was at the center of the tank. The depth of the camera field at this distance is only about 20 cm. Consequently the particles fell through about 12 cm of water before they came within the camera field.

A reference scale was made which consisted of a thin steel rod painted flat black. Narrow marks were painted on the scale 5 cm apart with aluminum paint. The scale stood on a metal base in the center of the water tank at the exact position of the line of fall of the particles.

Pictures were taken with the fastest film obtainable, Dupont High Speed Pan Type 428, which has an emulsion speed of 200. Slower film was found to be unsatisfactory since the reflection from rough sand grains is faint and is necessarily of very short duration. Cut film 2¼ in. by 3¼ in. was used, and it was developed with an Eastman Kodak D-11 developing solution.

The distance between images on the film was measured with a micrometer graduated in millimeters which can be read to the nearest 0.1 mm.

Particles were dropped just at the surface by means of tweezers which were clamped rigidly in position over the center of the tank. Another pair of tweezers was used to place the particles into the fixed tweezers. A magnetic dropping device such as was employed by Malaika cannot be used except on metal particles. The tank was filled with distilled water which was frequently changed. It was found necessary to use very clean water for best results.

A Chainomatic analytical balance was used to weigh the particles to the nearest 0.1 mg. Axes of particles smaller than a seive size of 8 meshes to the inch were measured with an ocular micrometer. Larger particles were measured with a micrometer to the nearest 0.1 mm.

Procedure.--The photography was done in complete darkness. Pictures were taken in batches of ten since that was the number of cut-film holders available and was a convenient number to develop at one time.

The strobolux and strobotac were allowed to warm up twenty minutes before each run to insure a uniform flash period. The strobotac was adjusted to the line frequency by turning an adjusting screw until the adjusting reed appeared to stand still at speeds of 900 rpm and 3600 rpm. The strobolux was then set at 600 rpm which is the slowest speed possible. This was found to be a convenient speed for the range of particle sizes used for both photographic results and convenience in calculating velocity. Since 600 rpm equal 10 flashes per second, it is only necessary to multiply the distance traveled between flashes by 10 to get the fall velocity.

Each particle was placed in the tweezers in the position at which particles normally fall, with their largest cross section in a horizontal position. They were usually placed just at the water surface. After the particle was placed and held by the dropping tweezers, the release could be accomplished with one hand while the camera shutter was operated with the other.

The particles were allowed to drop a few inches before the shutter was opened. The shutter was closed as soon as the particle fell beyond the camera field. It was found that the shorter the interval of time the shutter was open the more distinct were the images on the film. Because it is impossible to prevent some diffusion of light by the water or to entirely eliminate reflections from the background, any unnecessary flashes tend to mask the images made by reflections from the particle.

The scale was photographed in the same way as the particles and in exactly the same position as the normal line of fall of the particle in order to prevent inaccuracies in the distance relationships.

Because many of the larger particles varied considerably from this line in their fall, the error resulting from this was determined by photographing the scale in positions 8 cm forward and to the rear of the dropping position. It was found that this caused a 10% change in the distance ratio of the scale points to the images on the film. A few large particles fell at a distance greater than 8 cm from the normal line. When this happened the images on the film were either invisible or too blurred to measure as the focal range of the camera at this distance is very short.

Before each run, the camera was carefully focused on a string in the line of fall by means of ground glass. The string, which was weighted to make it hang vertically, was then removed from the tank and the water was allowed to become still before particles were dropped. It was impossible to prevent minute particles of dust from getting into the water and these were visible when the strobolux flashed in the darkness. The dust particles were utilized to determine when the water became still since these could be seen moving when the water was unsettled.

Although the water temperature was measured before and after each run, it varied not more than 1° F. during the entire investigation. The water temperature seemed to increase gradually with the flashing of the strobolux until after several hours of continuous operation, it reached a temperature about 1° higher than the air temperature. Transverse temperature gradients were not detectable. The air temperature varied even less than the water temperature, being around 69° F. for all runs.

After several runs, the water became cloudy because at the end of each run the particles were removed from the tank by hand. In addition, the paint used on the inside of the tank contained carbon black, some of which went into suspension in the water.

In dropping the smaller particles, it was discovered that unless they were pre-wetted they would remain at the surface when released. For very flat particles, pre-wetting was not sufficient to prevent the particles from being buoyed up by surface tension in which case it was found necessary to release the particles just below the surface. The very slight disturbance of the surface caused by this is not believed to have affected the terminal velocity.

The velocity was obtained by measuring the distance between several images near the center of the negative, the exact number depending on the velocity of fall. The average of these was taken for calculating the fall velocity. Irregularity of fall velocity was recorded where this was significant which was chiefly in the case of the larger particles. The deviation from a vertical line was also measured. Measurements were made from the edge of one image to the same edge of another.

The long, intermediate, and short axes of each particle were measured. In the case of the smaller particles, this was accomplished with an ocular micrometer. The particle was placed on a glass slide under the lens of the microscope with its long axis parallel to the long axis of the slide. The long axis was measured first, and the slide was turned 90° without moving the particle. The intermediate axis was then measured. In the case of particles of roughly rectangular shape, the axes were measured parallel to the sides and not from corner to corner. This was done so that the product ab would be more closely related to the maximum cross-sectional area. The short axis was measured last. This was accomplished by sliding the particle to the edge of the glass slide where it could be grasped and turned on edge with a pair of tweezers that had been ground to a fine point. An attempt was made to measure the short axis at 90° to the dimensions a and b. The particles too large to measure with the ocular micrometer were measured directly with an ordinary micrometer to the nearest 0.1 mm.

From the dimensions a, b, and c the shape factor $c/\sqrt{ab}$ was computed for each particle and the average shape factor for each type of sand was determined.

Altogether 190 particles were measured. There were fifty from each of the samples except the wind-blown sample. The largest fraction was missing from this sample, so that only 40 particles were selected. The particles varied in weight from 4 mgs to 500 mgs.

Calculations of the coefficient of drag and Reynolds number were based on the assumption that all particles selected have the average specific gravity of quartz, 2.65. Particles of quartz and feldspar only were selected for measurement.

<u>Errors inherent in technique</u>.--The most serious inaccuracy of the method of measuring fall velocity used in this investigation results from the fact that the larger particles do not fall in a vertical line. When the particles move out of focus the results are in error by as much as 10%. Measurements could seldom be made on particles that were out of focus to a greater degree since their image on the negative was blurred.

The assumption of all particles having the average specific gravity of quartz may lead to errors of 5% in individual cases even though only quartz and feldspar particles were selected. Both of these minerals have an average specific gravity close to 2.65. A great majority of the particles were quartz so that the average of all particles dropped should be very close to 2.65.

The measurements of the shape factor of particles are reproducible only within about ± 5% owing to the fact that a slight change in angle of the particle may make a difference in the measurement of an axis. In the case of the more compact shapes, the longest axis is not always obvious, and unless care is taken, the longest axis will not be found. Slight differences in measurement make considerable difference in the shape factor especially as this ratio approaches 1.0.

The settling position of irregular particles is controlled to a considerable extent by their shape, but it is also affected somewhat by the position in which they are dropped. While all particles were dropped in what was thought to be their most stable position, this was probably not always accomplished. The extent of error resulting from this factor alone is not known. However, in one case a rather irregular particle was dropped five times in different positions and was found to have a deviation of fall velocity of 5%. The extent of error resulting from this particle being out of focus is not known. A relatively flat particle with a deviation from the normal fall line of about 8 cm was dropped five times in different positions, and it was found that the particle fell in essentially the same position regardless of how it was dropped, but the fall velocity deviation was 10% which can be attributed entirely to the particle getting out of focus. Small particles that fall in a reasonably straight line such as most of those passing a screen with 10 meshes to the inch have an insignificant deviation of fall velocity regardless of how many times they are dropped if they are dropped with their largest cross section horizontal.

A minor source of error results from the fact that the period of the strobolux flash is accurate only within $\pm$ 1%.

Recommendations for improving technique.--It is recommended that if this method is used for future studies of irregular particles that a camera telephoto lens be obtained. It might be possible with such a lens to place the camera at a considerably greater distance from the line of fall than was possible with the lens used. This would materially reduce the error resulting from particles being out of focus.

It is also recommended that the camera be placed so that pictures are obtained of particles near the beginning of their fall, where their deviation from the normal fall line is a minimum. Because the small particles were dropped first in this investigation, the desirability of doing this was not recognized until a great amount of data was collected. It was thought best not to change the conditions when the larger particles were dropped so that results would be comparable.

Chapter V

PRESENTATION AND DISCUSSION OF DATA

The principal aims of this analysis are to determine the usefulness of the photographic method for measuring fall velocity of sand grains, to develop a practical method of expressing shape, and to determine the correlation between fall velocity and shape.

Measurement of fall velocity

It can be seen from the photographs of falling sand grains, Figs. 3, 4, and 5, that the photographic method of measuring fall velocity permits a more complete analysis of the motion of particles in water than does the timing of their fall with a stop watch. The pattern of the paths of particles and the position of the particles while falling are shown by the photographs. By comparing the spacing between images, the deviation from a constant velocity also can be studied.

From the standpoint of cost the photographic method has the disadvantage of requiring film which has a very high emulsion speed and is quite expensive. Furthermore, the time involved in making the measurements is greater then the time required to use a stop watch because of the necessity of developing the film and making

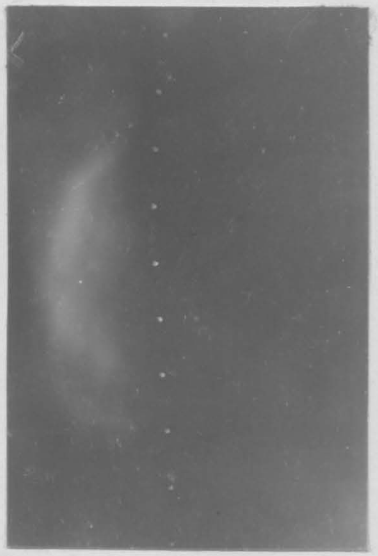


Fig. 3 -- Falling particle.
Axes dimensions:
a = 2.6
b = 2.5
c = 2.0
sf = 0.78
Weight = 16.8 mgs
Fall velocity = 220 mm/sec
C_{Dn} = 0.86

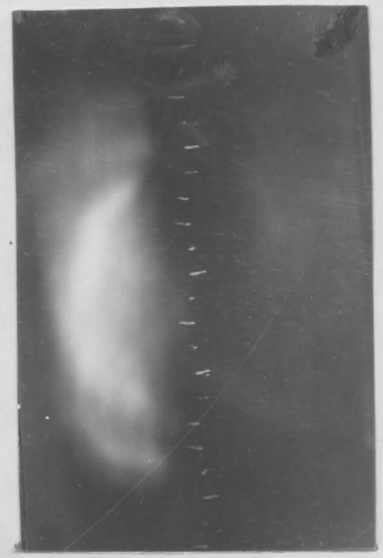


Fig. 4 -- Falling particle.
Axes dimensions:
a = 7.5
b = 2.1
c = 1.0
sf = 0.25
Weight = 16.2 mgs
Fall velocity = 96.6 mm/sec
C_{Dn} = 4.1

Fig. 5 -- Falling particle
Axes dimensions:
a = 4.0
b = 2.4
c = 1.9
sf = 0.61
Weight = 14.9 mgs
Fall velocity = 132 mm/sec
C_{Dn} = 2.15

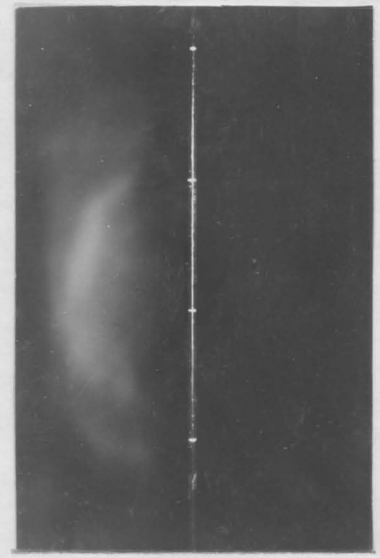


Fig. 6 -- Photograph of scale with marks 5 cm apart.

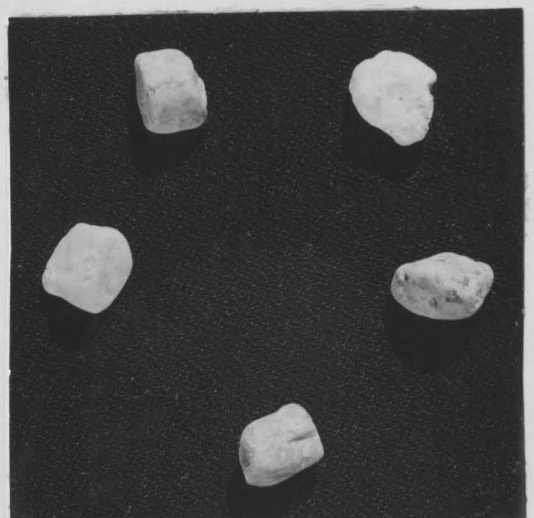
measurements from the negative.

An inspection of the photographs of falling grains shows that the particles do not always fall in a straight line. As shown in Table 2, when a particle with an irregular path of fall was photographed five times, the maximum deviation from the average of the five trials was 10%. Another particle with a somewhat less irregular path had a maximum deviation of about 5%, and still another particle had an insignificant deviation. There is a tendency for the paths of larger particles to deviate more from a vertical line than is the case for smaller particles as can be seen from the tabulation of these data in Table 3 in the Appendix. In most cases the images on the photographs substantiated the findings of previous investigations in that they showed the particles to be falling with the largest projected area horizontal. The photograph, Fig. 4, is an example of this tendency.

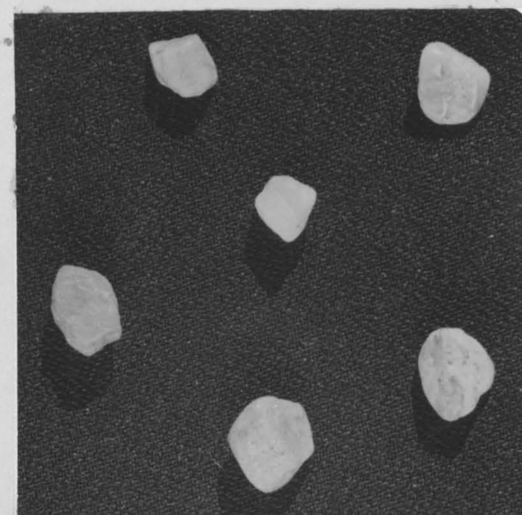
Measurement of shape

It was found that the necessary measurements for calculating the shape-factor ratio $c/\sqrt{ab}$ for about 25 particles could be made in an hour.

Measurements of individual particles from four different sediment deposits were made in order to determine whether or not these sediment deposits have different average shape characteristics which can be expressed by



a) Sample of sand grains from Cache Le Poudre River near Bellvue, Colorado.
Ave. S. F. - 0.59 to 0.76.



b) Sample of sand grains from the Middle Loup River at Dunning, Nebraska.
Ave. S. F. - 0.67 to 0.71.

c) Sample of fragments from rock crusher at Bellvue, Colorado.
Ave. S. F. - 0.45 to 0.53

d) Sample of wind -blown sand grains from Laporte, Colorado.
Ave. S. F. - 0.53 to 0.64.

(Note: Enlarged 1 1/3 times natural size.)

PHOTOGRAPHS OF SAMPLES STUDIED BY COREY.

Fig. 26.

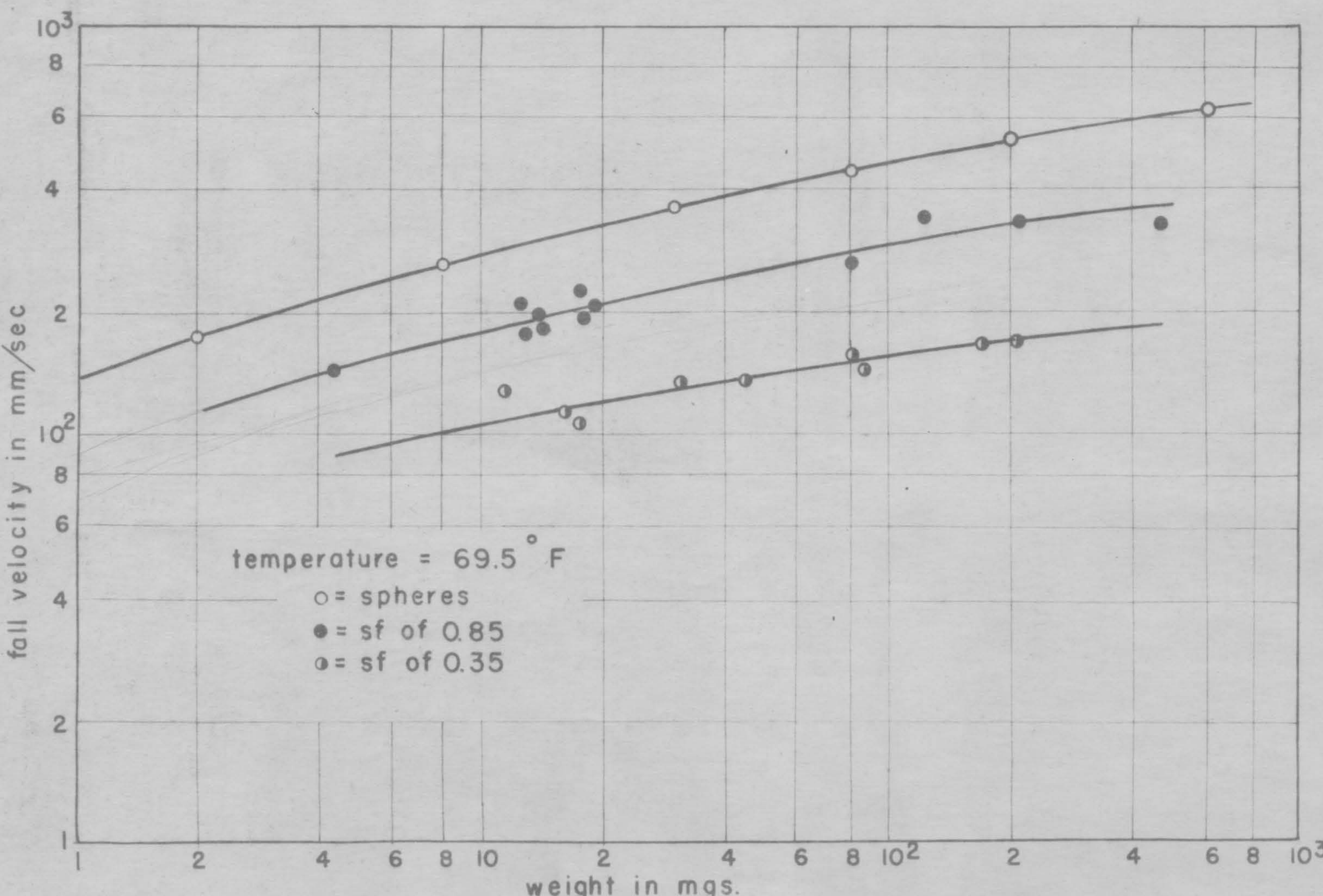


Fig. 11-- Fall velocity as a function of weight

the ratio $c/\sqrt{ab}$. The values of the shape-factor ratio for each particle are shown in Table 1.

The average value of this ratio is lowest in the case of crushed rock -- being about 0.45. The fact that crushed rock tended to be somewhat flatter and more angular than the others is evident in the photographs of the sand grains, Figs. 7, 8, 9, and 10.

For both the Loup River sample and the Poudre River sample, the average shape factor is about 0.7. Although the Loup River particles have the appearance of being somewhat more highly polished and worn than the Poudre River particles, this apparent difference in shape is not reflected in the measured ratios. Intermediate between shape-factor ratios of the crushed rock and the river deposits is the wind-blown sand which has an average of about 0.6. The appearance of these particles also indicates that they are intermediate in shape between the other two samples.

Correlation of fall velocity with shape

The correlation between fall velocity and the degree of flatness can easily be visualized by examining the plot, Fig. 11, of fall velocity as a function of weight, keeping the same fluid at constant temperature. It can be seen from this plot that particles with a shape-factor ratio of 0.85 show an average fall velocity nearly

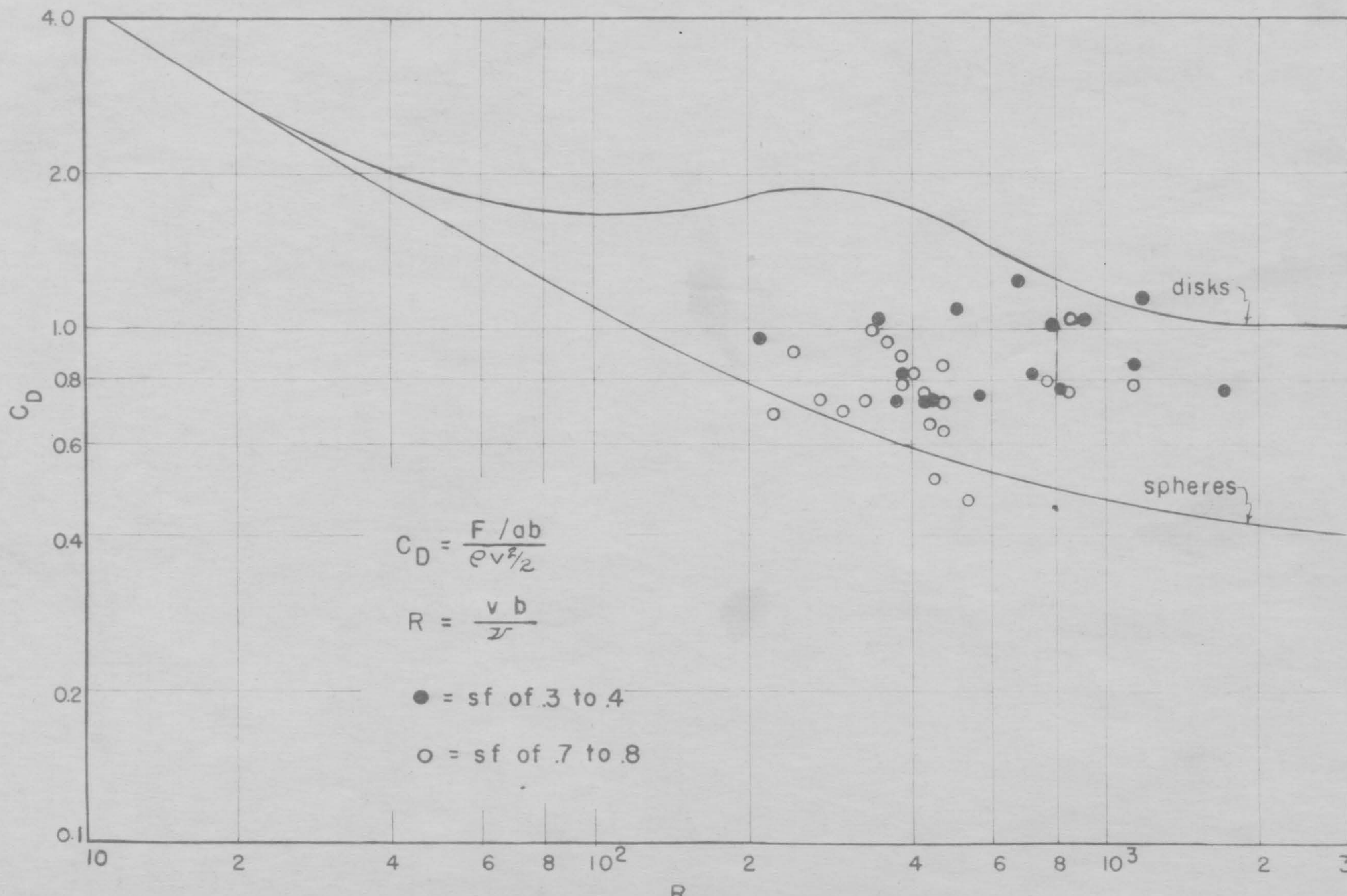


Fig. 12--Coefficient of drag as a function of Reynolds number

twice as great as particles with a shape-factor ratio of 0.35, and perfect spheres show a fall velocity more than three times as great in the size range studied.

In the plot of the coefficient of drag as a function of Reynolds number using nominal dimensions, Fig. 13, the data obtained by Serr (7:24) are represented by broken lines. Serr used sieve diameters in the drag coefficient and Reynolds number in his original plot whereas this plot utilizes nominal diameters. Consequently, the data are not entirely comparable.

However, as pointed out by Wadell (9:258), the intermediate axis b is the dimension which usually determines whether or not a particle will pass through a given sieve opening. As can be seen by comparing the data compiled in Tables 1 and 4, the value of b does not differ widely from d_n for most of the particles measured in this investigation. Therefore, the curves plotted by Serr using sieve diameters have been transferred directly so that a comparison of the data of both studies can be made.

In order to make this comparison as direct as possible, a range of the shape-factor ratios corresponding to the average of the two river samples was plotted. The curve which fits these data best is a continuation of the curve drawn by Serr for the same kind of material. Serr's curve for talus debris passes through points corresponding

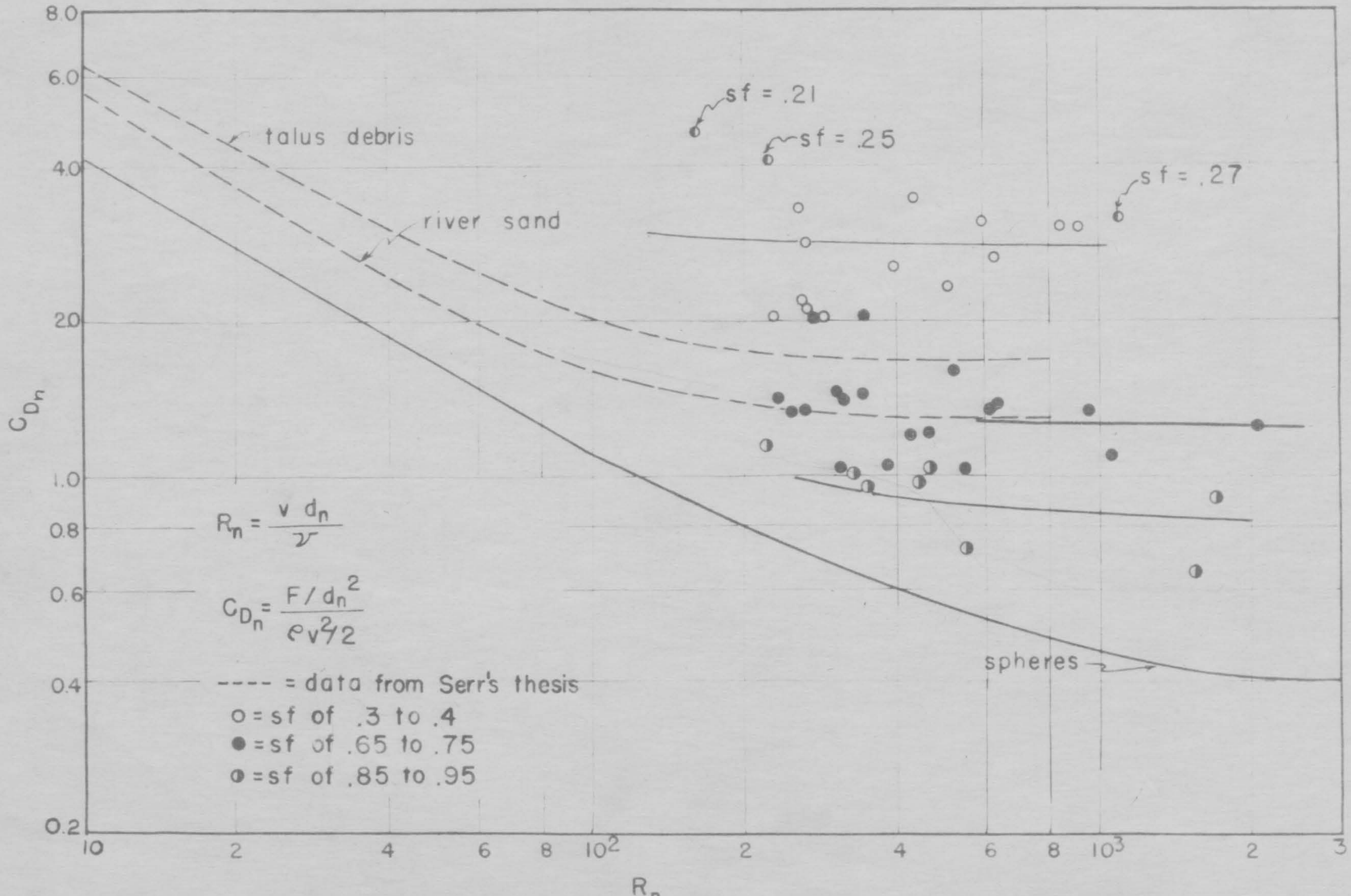


Fig. 13 -- Coefficient of drag as a function of Reynolds number

to about the average shape-factor ratio of the crushed rock used in this study. Individual particles of the crushed rock, however, have even smaller shape-factor ratios and much larger values of C_{Dn} as shown by the curve drawn for a shape factor of 0.3 to 0.4.

The plot of C_D as a function of R, Fig. 12, in which C_D is defined as $\frac{F/ab}{\rho v_o^2/2}$ and R as $\frac{v_o b}{\nu}$, shows almost no correlation between the curve for true spheres and the curve for disks taken from the plot illustrated by Rouse (5:245). All values are less than 2 for Reynolds numbers greater than 10^2 and less than 1 for Reynolds numbers greater than 10^3.

Significance of results

In spite of the many difficulties encountered with the photographic method, there is probably no simpler method of equal accuracy and latitude of use. No direct comparison was made in this investigation, but it is probable that in the range of sand grains corresponding to sieve sizes of the Tyler series 10 to 14 meshes to the inch, the photographic method of measuring fall velocity is considerably more accurate than the method of using a stop watch. Although measurements on smaller particles were not made in this study, it is likely that such measurements could have been made without difficulty.

The procedure as used in this investigation was

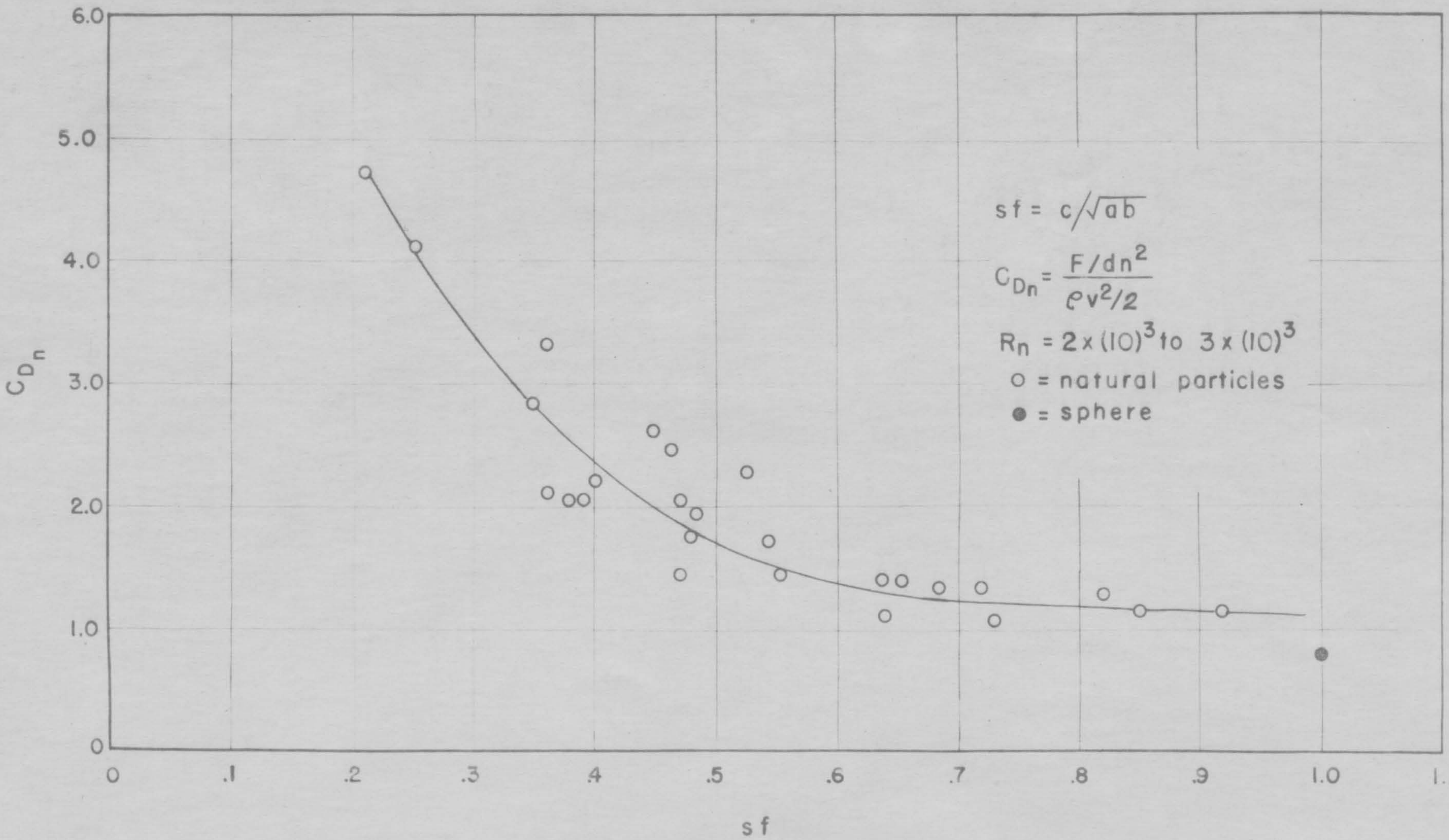


Fig. 14 --Coefficient of drag as a function of shape factor

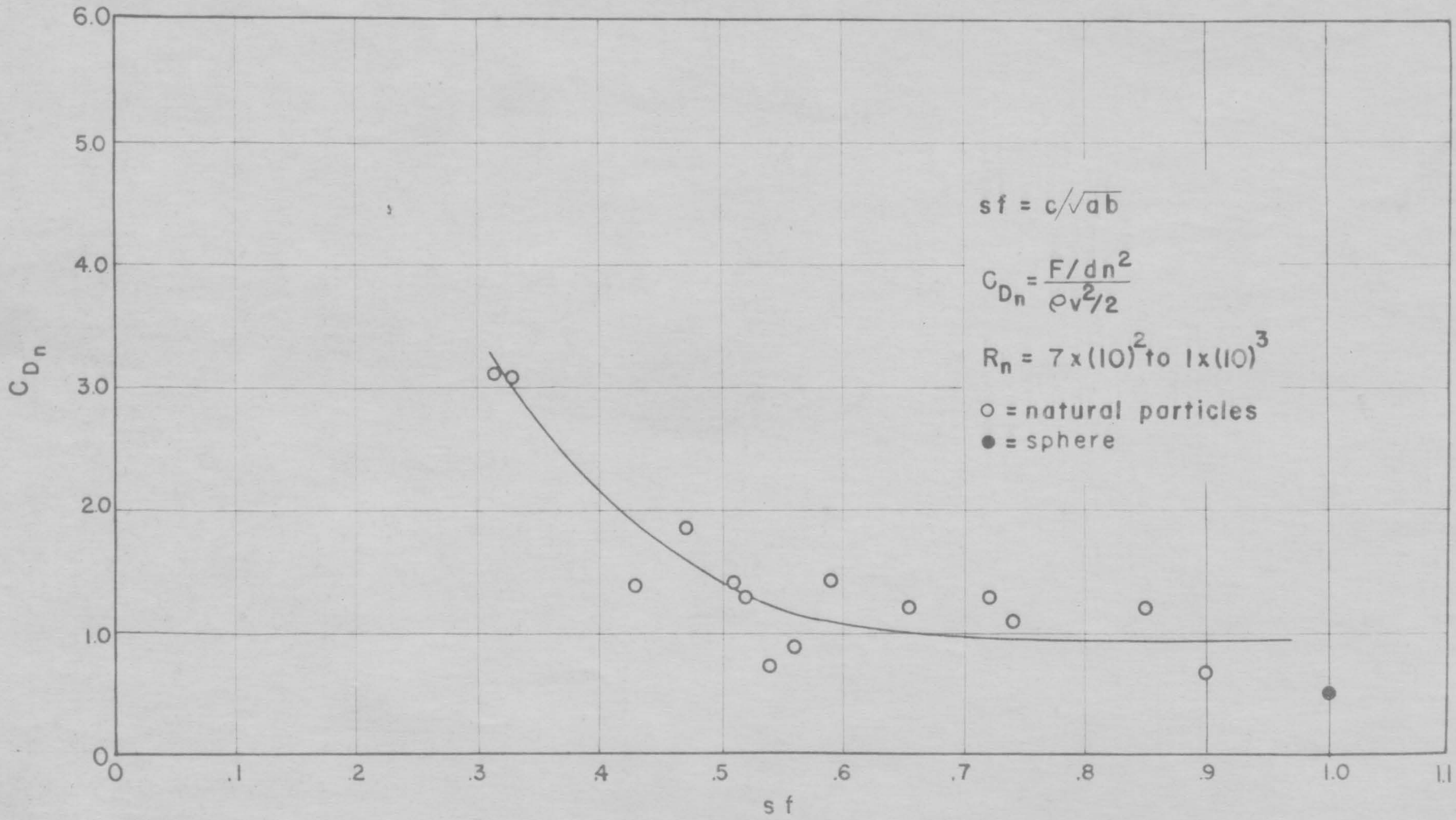


Fig. 15--Coefficient of drag as a function of shape factor

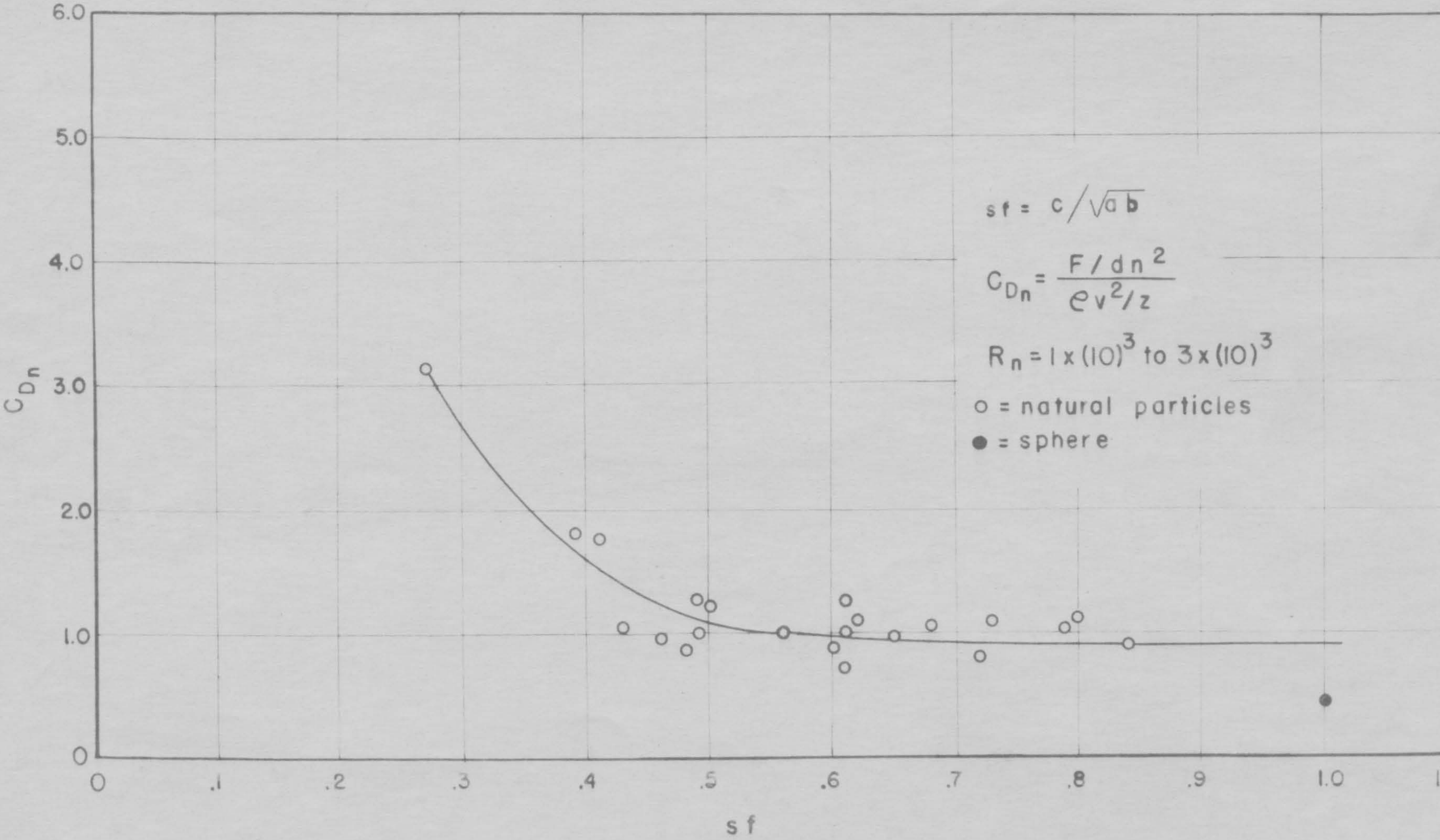


Fig. 16 -- Coefficient of drag as a function of shape factor

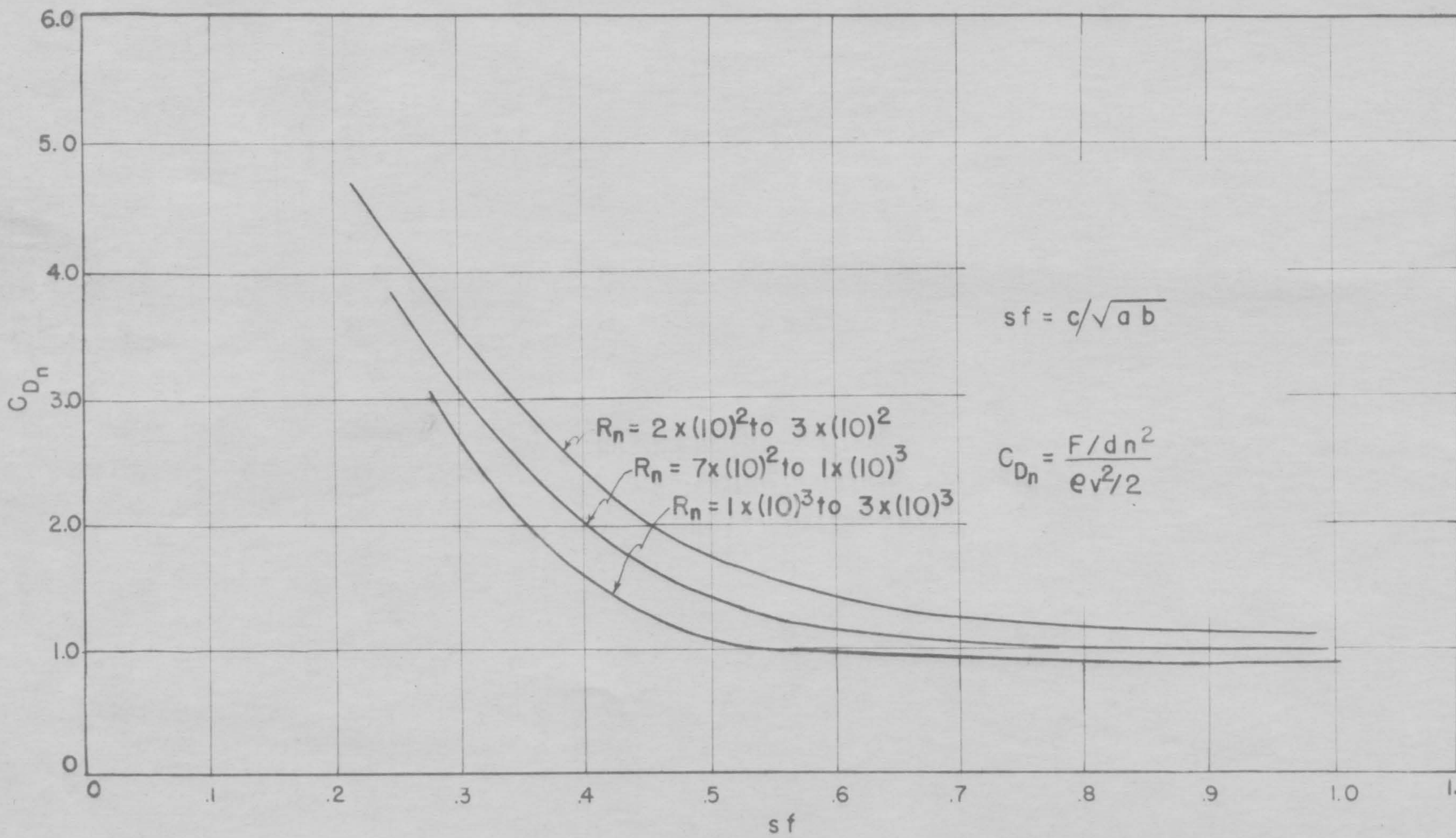


Fig. 17--Coefficient of drag as a function of shape factor

not satisfactory for measuring the fall velocity of the larger particles. Because of the instability of eddies formed at higher Reynolds numbers, the larger particles may drift out of the plane of focus resulting in considerable error. Since data on the larger particles are needed, a method of overcoming this difficulty would be helpful. There is a possibility that most of this inaccuracy could be overcome by using a telephoto lens and placing the camera at a much greater distance from the falling particles.

For particles small enough to fall within the range of Stokes law, the stop watch is probably as accurate as the photographic method and is faster and cheaper.

By photographing the particles in the water at various points during their fall, much information of fundamental nature concerning the movement of particles in water can be obtained such as the variation in orientation, velocity, and direction.

As pointed out in the review of literature there are many different types of shape factors. The ratio $c/\sqrt{ab}$ was used to express shape in this experiment because it was believed that flatness is the shape characteristic which has the greatest influence on fall velocity and because of the convenience of determining this ratio. The time involved in determining the ratio $c/\sqrt{ab}$ is much less than the time required to make measurements for shape factors involving surface or projected areas of particles.

The data indicate that deposits of totally different origin have average shape differences that can be measured by the shape-factor ratio. It is possible, however, that this method of measuring shape may not be sufficiently sensitive to distinguish between samples with minor shape differences such as those which exist between sand deposits from different rivers. The ratio does not take into account roundness or eccentricity.

When C_D is expressed in terms of nominal dimensions there is a definite correlation between C_D and the shape-factor ratio. Fall velocities of sand particles are inversely related to their flatness which is a measure of their projected area per unit volume. Flatness may reduce the average fall velocity of crushed rock over that of river-worn sand grains by more than 100% for Reynolds numbers greater than 100. The effect of flatness on fall velocity can be visualized by comparing the photographs, Figs. 3 and 4, of falling particles with different degrees of flatness. As was pointed out in Chapter IV, there is no theoretical limit to the extent to which flatness can increase the coefficient of drag when this coefficient is expressed in terms of nominal diameters. It is probable, for example, that particles such as mica flakes have values of C_{Dn} far greater than anything found in this investigation.

Because the coefficient of drag expressed in terms of projected areas shows virtually no correlation between C_D and shape factor, it appears that the shape-factor ratio $c/\sqrt{ab}$ is not related to the angularity of the particle edges. In other words, this shape factor apparently does not describe the particle sufficiently to show a correlation between the shape factor and the coefficient of drag based on projected areas -- the numerator of this coefficient F/ab being in itself an expression for flatness. Possibly, a correlation could be found if the ratio c/b had not been eliminated from Eq. (6) or if an improved method of measuring fall velocity could be devised.

Even when C_D is expressed in terms of nominal dimensions, there is a pronounced difference between the C_D of true spheres and the C_D of irregular particles with shape-factor ratios approaching 1. The most probable reason for this is that the shape-factor ratio does not express the degree of roundness -- the value for cubes being the same as that for spheres. While the shape-factor ratio shows the effect of increased inertial resistance due to flatness and to the flow, it cannot satisfactorily show the effect of an increased inertial resistance due to sharp edges.

Chapter VI

SUMMARY

Recently, a number of theoretical methods for predicting the rate of sedimentation have been devised. All of these methods require a knowledge of the average hydraulic properties of the particular sediment under consideration. It also has been established that the fall velocity of particles in still water serves as an index of the velocity of flow required to move them along the bed of a channel and the rate at which they will move for a given velocity of flow.

Fall velocities have been measured in the past by dropping particles in water and measuring their rate of fall with a stop watch. The accuracy of this method is not satisfactory when the greater velocities of relatively large particles are measured.

In the case of particles of a size which are normally carried in channels as bed load, shape is a factor having considerable influence on the fall velocity.

A photographic method for measuring fall velocities has been adopted recently at the Iowa Institute of Hydraulic Research. This method had not been used in the study of fall velocities of sand grains before the present investigation.

The purpose of this investigation has been to study the usefulness of the photographic method for determining fall velocities of sand grains, to develop a practical method of expressing shape, and to determine the relationship between shape and fall velocity.

Samples of sand from the Cache la Poudre River and the Middle Loup River together with samples from a wind-blown deposit were studied. Fragments from a rock crusher also were studied to obtain data on particles with an angular shape. Altogether data were obtained on 190 particles varyimg in weight from 4 mgs to 500 mgs.

The dimensions a, b, and c, (the major, intermediate, and minor axes) of each particle were measured, and shape was expressed by the ratio $c/\sqrt{ab}$. Each particle was weighed to the nearest 0.1 mg, and the fall velocities were determined by photographing the particles as they fell through water by means of periodic flashes of light from a strobolux.

Fall velocity and shape were correlated graphically by plotting the drag coefficient as a function of Reynolds number with the shape-factor ratio $c/\sqrt{ab}$ as a third variable. Graphs were made in which the drag coefficient and Reynolds number were expressed in terms of both projected dimensions perpendicular to movement and nominal dimensions.

When projected dimensions were used, little correlation was found between the drag coefficient and the shape factor. Evidently, the failure of this shape factor to take into account the angularity of edges is the principal reason for the lack of correlation -- the value of the shape-factor ratio for cubes being the same as for spheres.

When nominal dimensions were substituted for projected dimensions there was a definite correlation between shape and fall velocity -- particles with low ratios having much larger values for the drag coefficient than particles with higher ratios. In general, the values of the drag coefficient when nominal dimensions are used is much higher than when projected dimensions are employed.

The drag coefficient based on projected dimensions is an expression for resistance in terms of unit area. Theoretically, when this expression is used, the only influence that shape can have results from the effect upon inertial resistance of the angularity or roundness of particle edges. Apparently, the ratio $c/\sqrt{ab}$ does not express this shape characteristic satisfactorily.

The drag coefficient based on nominal dimensions is an expression for resistance in terms of unit volume. Flatness of natural particles may increase the projected area per unit volume over that of true spheres and thereby greatly increase the drag coefficient when this is expressed in terms of nominal dimensions. There is no theoretical limit to the extent to which flatness may increase the drag coefficient, and evidently the ratio $c/\sqrt{ab}$ expresses the degree of flatness fairly well.

A P P E N D I X

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DEFINITIONS

Drag coefficient.-- The coefficient in the drag force equation ($F = C_D A \rho_f v_s^2/2$) expressing the relative resistance of bodies of the same projected area under the same flow conditions.

Nominal dimension.-- The dimensions of a sphere of the same volume as the given particle.

Nominal drag coefficient.-- A coefficient of the same form as the drag coefficient, but with the projected area replaced by the nominal diameter squared.

Nominal Reynolds number.-- A parameter of the same form as Reynolds number in which the nominal diameter is used as the length dimension in the numerator.

Projected dimensions.-- The dimensions of the cross-sectional area of particles.

Reynolds number.-- The ratio of the inertia forces to the viscous forces acting on a body which has motion relative to a fluid surrounding the body.

Sediment.-- Fragmental rock material transported by, suspended in, or deposited by water or air.

Sedimentation diameter.-- The diameter of a sphere of the same terminal fall velocity as the given particle in the same fluid and under the same conditions.

Sieve diameter.-- The size of opening through which the given particle will just pass.

Sphericity.-- The ratio of a sphere of the same volume as the particle to the actual surface area of the particle.

Terminal fall velocity.-- The vertical component of the uniform velocity of a particle falling under gravitation forces when the forces of fluid resistance equal the gravitational force acting on the particle.

NOTATION

A_n = nominal projected area of the particle

a = major axis of the particle

b = intermediate axis of the particle

c = minor axis of particle

C_D = drag coefficient = $\dfrac{F/A}{\rho_f v_o^2/2}$

C_{Dn} = drag coefficient = $\dfrac{F/d_n^2}{\rho_f v_o^2/2}$

d = diameter of particle

d_n = nominal diameter

F = net gravitational force on particle

= $\frac{\pi}{6} d_n^3 (\rho_s - \rho_f) g$

g = acceleration of gravity

μ = dynamic viscosity of the fluid

ν = kinematic viscosity of the fluid

R = $\dfrac{\rho_f v_o d}{\mu}$ = Reynolds number

ρ_s = density of particles

ρ_f = density of fluid

sf = shape-factor ratio = $c/\sqrt{ab}$

v_o = terminal fall velocity

Table 1.--CALCULATION OF SHAPE FACTOR RATIO $c/\sqrt{ab}$

Sample of sand from Cache la Poudre River
at Bellvue, Colorado

Group	Number	Axis in mm a	b	c	sf ratio $c/\sqrt{ab}$
Retained on 4-mesh sieve	1	9.1	7.8	5.3	0.63
	2	6.4	5.0	4.5	0.80
	3	8.3	7.4	5.7	0.73
	4	9.9	8.8	4.0	0.43
	5	5.5	5.0	4.3	~~0.80~~ .82
	6	10.1	6.8	6.3	0.76
	7	8.2	6.8	5.7	0.76
	8	6.4	5.3	4.9	0.84
	9	8.3	6.4	4.4	0.60
	10	9.6	6.4	3.1	0.40
				Average	0.68
Retained on 8-mesh sieve	1	7.8	5.3	3.0	0.47
	2	5.9	4.0	2.9	0 9.59
	3	5.9	3.6	3.4	0.74
	4	5.4	3.8	3.0	0.66
	5	6.5	4.8	2.4	0.43
	6	5.4	3.7	2.5	0.56
	7	6.4	4.7	3.4	0.62
	8	5.8	3.7	3.3	0.72
	9	5.9	4.8	3.4	0.64
	10	7.3	4.8	2.9	0.49
				Average	0.59
Retained in 9-mesh sieve	1	4.3	3.0	2.4	0.66
	2	4.0	2.8	2.3	0.68
	3	3.1	2.9	2.5	0.83
	4	5.3	2.5	2.5	0.69
	5	4.3	3.1	2.5	0.69
	6	4.1	2.6	2.6	0.80
	7	3.5	2.5	2.3	0.76
	8	3.1	3.0	2.1	0.68
	9	4.9	2.6	1.8	0.49
	10	4.1	2.4	2.4	0.76
				Average	0.70

Table 1.--CALCULATION OF SHAPE FACTOR RATIO $c/\sqrt{ab}$ -- Continued

Sample of sand from Cache la Poudre River at Bellvue, Colorado

Group	Number	Axis in mm a	Axis in mm b	Axis in mm c	sf ratio $c/\sqrt{ab}$
	1	2.4	2.5	1.9	~~0.65~~ .78
	2	3.8	2.1	1.8	0.62 .64
	3	3.0	2.4	1.9	0.75 .71
	4	3.3	2.4	1.9	0.68
Retained on	5	4.3	2.5	2.0	0.62
10-mesh sieve	6	3.0	2.8	2.5	0.87
	7	3.4	2.1	2.1	0.80
	8	2.5	2.4	2.4	0.98
	9	3.1	2.6	2.3	~~0.79~~ .81
	10	3.1	2.4	2.3	0.83
				Average	0.76
	1	3.4	2.5	1.3	0.44
	2	2.1	1.8	1.6	0.82
	3	2.4	1.9	1.3	0.63
	4	2.5	2.2	1.9	~~0.59~~ .81
Retained on	5	2.6	2.2	1.9	0.78
14-mesh sieve	6	2.7	2.0	1.6	0.68
	7	2.3	2.1	1.6	0.73
	8	2.2	2.1	1.6	~~0.68~~ .74
	9	2.2	2.1	2.1	0.97
	10	2.3	1.9	1.5	0.73
				Average	0.70

Table 1.--CALCULATION OF SHAPE FACTOR RATIO $c/\sqrt{ab}$ -- Continued

Sample of sand from Middle Loup River at Dunning, Nebraska

Group	Number	Axis in mm a	Axis in mm b	Axis in mm c		sf ratio $c/\sqrt{ab}$
Retained on 4-mesh sieve	1	7.3	5.9	4.0		0.61
	2	8.0	7.5	4.7		0.61
	3	7.9	5.6	4.5		0.68
	4	6.5	5.8	3.4		0.55
	5	8.4	5.5	3.8		0.56
	6	7.7	6.4	3.2		0.46
	7	9.4	7.1	4.0		0.49
	8	8.5	6.0	5.0		0.70
	9	9.9	7.3	6.8		0.80
	10	8.0	6.6	3.5		0.48
					Average	0.60
Retained on 8-mesh sieve	1	5.2	4.4	3.5		0.73
	2	4.6	3.0	2.7		0.72
	3	5.7	3.6	2.8		0.61
	4	5.0	4.0	2.9		0.65
	5	5.3	4.4	3.8		0.79
	6	4.0	3.5	2.0		0.54
	7	3.7	3.0	3.0		0.90
	8	5.4	4.3	4.1		0.86
	9	6.9	5.2	2.5		0.41
	10	5.8	5.0	2.7		0.50
					Average	0.67
Retained on 9-mesh sieve	1	3.6	2.5	2.5		0.83
	2	3.5	3.1	1.5		0.45
	3	3.9	3.4	2.0		0.55
	4	2.9	2.6	2.5		0.91
	5	3.8	2.8	1.9		0.58
	6	3.4	2.6	2.3		0.75
	7	4.1	2.5	1.9		0.58
	8	3.6	2.6	2.6		0.85
	9	2.6	2.5	2.0		0.78
	10	4.6	2.5	2.1		0.62
					Average	0.69

Table 1.--CALCULATION OF SHAPE FACTOR RATIO $c/\sqrt{ab}$ -- Continued

Sample of sand from Middle Loup River at Dunning, Nebraska

Group	Number	Axis in mm a	b	c	sf ratio $c/\sqrt{ab}$
Retained on 10-mesh sieve	1	3.0	2.9	1.8	0.60
	2	3.9	2.8	2.0	0.61
	3	2.8	2.5	1.9	0.71
	4	3.1	2.5	1.9	0.67
	5	2.8	2.4	2.1	0.83
	6	3.1	2.6	2.0	0.70
	7	3.5	2.5	1.9	0.63
	8	3.1	2.4	2.0	0.74
	9	2.9	2.8	2.4	0.85
	10	3.1	2.8	1.9	0.64
				Average	0.70
Retained on 14-mesh sieve	1	3.1	2.2	2.0	0.77
	2	2.7	1.9	1.4	0.63
	3	2.3	1.9	1.6	0.73
	4	2.3	1.8	1.8	0.89
	5	2.4	1.6	1.3	0.64
	6	3.0	2.3	1.6	0.60
	7	3.1	2.1	1.7	0.67
	8	3.4	1.8	1.6	0.65
	9	3.8	1.8	1.5	0.59
	10	1.9	1.6	1.6	0.92
				Average	0.71

Table 1.--CALCULATION OF SHAPE FACTOR RATIO $c/\sqrt{ab}$ -- Continued

Sample of fragments from rock crusher at Bellvue, Colorado

Group	Number	Axis in mm a	b	c	sf ratio $c/\sqrt{ab}$
Retained on 4-mesh sieve	1	13.5	5.3	2.7	0.32
	2	8.1	7.1	1.5	0.27
	3	13.9	7.9	2.8	0.27
	4	11.4	7.2	3.5	0.39
	5	6.3	4.5	2.2	0.41
	6	14.2	6.4	4.0	0.42
	7	10.6	7.4	1.8	0.20
	8	11.0	6.8	2.8	0.32
	9	9.0	7.1	2.4	0.30
	10	10.0	6.1	1.9	0.24
				Average	0.31
Retained on 8-mesh sieve	1	8.5	4.6	2.1	0.34
	2	6.7	4.1	1.9	0.37
	3	6.4	4.2	2.7	0.52
	4	7.4	3.7	1.8	0.35
	5	8.1	6.3	2.3	0.32
	6	6.8	3.8	2.8	0.55
	7	6.5	4.6	2.1	0.38
	8	7.8	5.5	3.0	0.46
	9	4.5	4.0	3.0	0.71
	10	5.9	4.7	4.5	0.85
				Average	0.48
Retained on 9-mesh sieve	1	7.5	2.1	1.0	0.25
	2	3.9	2.9	1.5	0.45
	3	3.9	3.1	1.4	0.40
	4	5.8	2.5	1.9	0.50
	5	4.0	2.4	1.9	0.61
	6	4.5	2.5	1.5	0.45
	7	6.8	2.3	1.4	0.35
	8	3.6	2.5	1.9	0.62
	9	3.9	3.0	1.6	0.48
	10	4.0	3.0	1.3	0.36
				Average	0.45

Table 1.--CALCULATION OF SHAPE FACTOR RATIO $c/\sqrt{ab}$ -- Continued

Sample of fragments from rock crusher at Bellvue, Colorado

Group	Number	Axis in mm a	b	c	sf ratio $c/\sqrt{ab}$
Retained on 10-mesh sieve	1	7.8	2.8	1.6	0.35
	2	4.0	2.8	2.1	0.64
	3	5.3	2.8	2.5	0.66
	4	5.5	3.1	2.4	0.58
	5	4.9	3.0	2.5	0.66
	6	4.6	3.6	0.9	0.21
	7	5.3	3.3	1.5	0.36
	8	3.4	2.9	2.0	0.64
	9	5.9	2.0	2.0	0.59
	10	4.4	3.0	2.3	0.62
				Average	0.53
Retained on 14-mesh sieve	1	3.3	3.1	1.5	0.47
	2	4.3	2.1	1.4	0.46
	3	4.3	2.3	1.5	0.49
	4	4.8	2.1	1.5	0.47
	5	3.8	2.8	1.5	0.47
	6	4.4	1.9	1.5	0.52
	7	4.0	2.8	1.8	0.53
	8	5.8	1.9	1.3	0.38
	9	4.8	2.0	1.1	0.37
	10	5.0	2.4	1.9	0.55
				Average	0.47

Table 1.--CALCULATION OF SHAPE FACTOR RATIO $c/\sqrt{ab}$--Continued

Sample of wind-blown sand from Laporte, Colorado

Group	Number	Axis in mm a	b	c	sf ratio $c/\sqrt{ab}$
Retained on 8-mesh sieve	1	6.5	4.5	2.8	0.52
	2	6.9	5.8	3.2	0.51
	3	5.9	4.0	2.5	0.51
	4	6.5	3.7	1.4	0.29
	5	4.7	2.4	2.3	0.69
	6	5.5	4.7	3.0	0.59
	7	5.0	3.1	2.0	0.50
	8	3.3	3.0	1.9	0.60
	9	4.0	3.3	2.0	0.56
	10	5.7	3.7	2.3	0.50
				Average	0.53
Retained on 9-mesh sieve	1	3.9	3.0	2.1	0.63
	2	3.6	3.4	1.4	0.39
	3	3.8	3.3	2.1	0.61
	4	4.0	2.6	2.3	0.70
	5	4.1	3.1	1.6	0.45
	6	4.5	2.9	1.5	0.42
	7	4.0	3.0	1.9	0.54
	8	5.3	2.9	2.0	0.51
	9	4.3	2.6	2.5	0.75
	10	4.6	2.6	2.1	0.61
				Average	0.56
Retained on 10-mesh sieve	1	3.6	2.6	2.0	0.65
	2	3.6	2.8	1.9	0.59
	3	3.4	2.5	2.1	0.73
	4	3.4	2.5	1.9	0.65
	5	4.0	2.6	1.8	0.54
	6	3.9	2.5	2.0	0.64
	7	3.3	2.5	1.9	0.66
	8	3.3	2.4	1.5	0.54
	9	3.8	2.6	1.9	0.60
	10	2.9	2.5	1.8	0.65
				Average	0.63

Table 1.--CALCULATION OF SHAPE FACTOR RATIO $c/\sqrt{ab}$ -- Continued

Sample of wind-blown sand from LaPorte, Colorado

Group	Number	Axis in mm a	b	c	sf ratio $c/\sqrt{ab}$
	1	3.1	1.8	1.0	0.72
	2	2.4	1.6	1.3	0.69
	3	2.0	1.6	1.5	0.85
	4	2.6	1.6	1.3	0.66
Retained on 14-mesh sieve	5	2.5	1.4	1.1	0.56
	6	2.1	1.4	1.1	0.60
	7	2.4	1.7	0.9	0.47
	8	2.9	1.3	1.3	0.64
	9	2.3	1.9	1.3	0.64
	10	2.3	1.4	1.0	0.56
				Average	0.64

Table 2.--CALCULATION OF DEVIATION OF FALL VELOCITY MEASUREMENTS

Source of particle	Wt. of particle in mgs	sf of particle	Fall velocity measurements in mm/sec					Maximum deviation from mean in percent
			1	2	3	4	5	
Wind-blown	81.5	0.59	233	210	210	212	212	10
Poudre River	12.5	0.73	203	202	193	202	193	5
Loup River	13.3	0.64	176	175	175	175	176	0.6

Table 3.--MAXIMUM DEVIATION OF FALL PATHS OF PARTICLES FROM A VERTICAL LINE

Sample of sand from Cache la Poudre River at Bellvue, Colorado

Group	Number	Weight in mg	sf ratio	Maximum deviation in mm
	1	446	0.63	13
	2	190	0.80	7
	3	471	0.73	5
	4	399	0.43	2
Retained on 4-mesh sieve	5	175	0.80	6
	6	493	0.76	5
	7	444	0.76	8
	8	209	0.84	4
	9	388	0.60	0
	10	---	0.40	4
			Average	5.4
	1	147.0	0.47	11
	2	91.8	0.59	2
	3	77.6	0.74	0
	4	64.1	0.66	3
Retained on 8-mesh sieve	5	92.2	0.43	5
	6	84.1	0.56	5
	7	143.0	0.62	9
	8	96.2	0.72	12
	9	121.0	0.64	3
	10	118.0	0.49	1
			Average	5.1
	1	24.2	0.66	2
	2	22.4	0.68	0
	3	17.4	0.83	2
	4	39.3	0.69	3
Retained on 9-mesh sieve	5	20.6	0.69	2
	6	22.1	0.80	-
	7	15.9	0.76	10
	8	17.6	0.68	8
	9	23.7	0.49	6
	10	18.6	0.76	12
			Average	5.0

Table 3.--MAXIMUM DEVIATION OF FALL PATHS OF PARTICLES FROM A VERTICAL LINE--Continued

Sample of sand from Cache la Poudre River at Bellvue, Colorado

Group	Number	Weight in mg	sf ratio	Maximum deviation in mm
	1	13.9	0.65	2
	2	12.7	0.62	2
	3	13.8	0.75	-
	4	12.1	0.68	6
Retained on 10-mesh sieve	5	13.3	0.62	3
	6	12.9	0.87	2
	7	16.6	0.80	0
	8	8.7	0.98	0
	9	14.7	0.70	2
	10	14.1	0.83	6
		13	Average	2.6
	1	12.6	0.44	4
	2	8.4	0.82	4
	3	5.7	0.63	0
	4	9.6	0.59	11
Retained on 14-mesh sieve	5	11.7	0.78	2
	6	9.7	0.68	9
	7	12.5	0.73	2
	8	10.0	0.68	4
	9	8.6	0.97	9
	10	6.2	0.73	2
		9.5	Average	4.7

Table 3.--MAXIMUM DEVIATION OF FALL PATHS OF PARTICLES FROM A VERTICAL LINE--Continued

Sample of sand from Middle Loup River at Dunning, Nebraska

Group	Number	Weight in mg	sf ratio	Maximum deviation in mm
Retained on 4-mesh sieve	1	184	0.61	7
	2	305	0.61	5
	3	304	0.68	-
	4	152	0.55	-
	5	198	0.56	10
	6	197	0.46	9
	7	326	0.49	12
	8	303	0.70	-
	9	466	0.80	5
	10	272	0.48	2
			Average	8.6
Retained on 8-mesh sieve	1	95.3	0.73	14
	2	68.0	0.72	2
	3	88.5	0.61	0
	4	90.0	0.65	0
	5	108.0	0.79	-
	6	42.0	0.54	3
	7	52.2	0.90	0
	8	122.0	0.86	0
	9	151.0	0.41	7
	10	114.0	0.50	11
			Average	4.2
Retained on 9-mesh sieve	1	18.7	0.83	2
	2	16.5	0.45	5
	3	25.0	0.55	2
	4	16.5	0.91	2
	5	18.7	0.58	0
	6	19.7	0.75	9
	7	19.8	0.58	3
	8	17.9	0.85	0
	9	6.8	0.78	0
	10	25.5	0.62	2
			Average	2.5

Table 3.--MAXIMUM DEVIATION OF FALL PATHS OF PARTICLES FROM A VERTICAL LINE--Continued

Sample of sand from Middle Loup River at Dunning, Nebraska

Group	Number	Weight in mg	sf ratio	Maximum deviation in mm
Retained on 10-mesh sieve	1	12.9	0.60	5
	2	15.7	0.61	5
	3	11.7	0.71	6
	4	12.3	0.67	0
	5	12.6	0.83	11
	6	12.2	0.70	0
	7	14.5	0.63	12
	8	13.8	0.74	10
	9	13.9	0.85	2
	10	13.3	0.64	0
			Average	5.1
Retained on 14-mesh sieve	1	15.1	0.77	7
	2	10.4	0.63	6
	3	7.8	0.73	18
	4	8.9	0.89	6
	5	6.0	0.64	8
	6	13.0	0.60	4
	7	12.1	0.67	4
	8	10.8	0.65	2
	9	12.4	0.59	2
	10	4.4	0.92	4
			Average	6.1

Table 3.--MAXIMUM DEVIATION OF FALL PATHS OF PARTICLES FROM A VERTICAL LINE--Continued

Sample of fragments from rock crusher at Bellvue, Colorado

Group	Number	Weight in mg	sf ratio	Maximum deviation in mm
Retained on 4-mesh sieve	1	169	0.32	8
	2	103	0.27	-
	3	292	0.27	-
	4	274	0.39	5
	5	---	0.41	2
	6	234	0.42	-
	7	126	0.20	-
	8	206	0.32	-
	9	123	0.30	-
	10	83	0.24	14
			Average	7.3
Retained on 8-mesh sieve	1	85.5	0.34	11
	2	31.2	0.37	3
	3	45.2	0.52	5
	4	44.2	0.35	5
	5	81.6	0.32	3
	6	79.4	0.55	6
	7	46.9	0.38	6
	8	106.0	0.46	5
	9	43.5	0.71	6
	10	80.2	0.85	24
			Average	7.4
Retained on 9-mesh sieve	1	16.2	0.25	0
	2	13.2	0.45	8
	3	11.4	0.40	0
	4	21.5	0.50	5
	5	14.9	0.61	0
	6	19.0	0.45	0
	7	16.0	0.35	2
	8	17.8	0.62	4
	9	11.9	0.48	4
	10	11.7	0.36	13
			Average	3.6

Table 3.---MAXIMUM DEVIATION OF FALL PATHS OF PARTICLES FROM A VERTICAL LINE--Continued

Sample of fragments from rock crusher at Bellvue, Colorado

Group	Number	Weight in mg	sf ratio	Maximum deviation in mm
Retained on 10-mesh sieve	1	23.4	0.35	-
	2	11.4	0.64	6
	3	20.2	0.66	-
	4	19.6	0.58	5
	5	20.0	0.66	0
	6	9.7	0.21	7
	7	17.4	0.36	7
	8	17.6	0.64	7
	9	18.0	0.59	4
	10	22.9	0.62	8
			Average	5.5
Retained on 14-mesh sieve	1	---	0.47	6
	2	6.1	0.46	5
	3	11.6	0.49	0
	4	9.1	0.47	4
	5	8.8	0.47	2
	6	---	0.52	0
	7	9.7	0.53	5
	8	8.5	0.38	5
	9	6.4	0.37	0
	10	12.3	0.55	3
			Average	3.0

Table 3.--MAXIMUM DEVIATION OF FALL PATHS OF PARTICLES FROM A VERTICAL LINE--Continued

Sample of wind-blown sand from LaPorte, Colorado

Group	Number	Weight in mg	sf ratio	Maximum deviation in mm
	1	72.7	0.32	8
	2	110.0	0.51	12
	3	64.0	0.51	8
	4	26.6	0.29	5
Retained on 8-mesh sieve	5	33.4	0.69	5
	6	81.5	0.59	9
	7	34.4	0.50	2
	8	20.5	0.60	4
	9	26.4	0.56	2
	10	50.4	0.50	6
		52.		Average 6.1
	1	22.0	0.63	0
	2	12.5	0.39	8
	3	16.8	0.61	9
	4	21.3	0.70	0
Retained on 9-mesh sieve	5	21.3	0.45	3
	6	15.2	0.42	7
	7	12.9	0.54	0
	8	27.6	0.51	4
	9	17.2	0.75	4
	10	20.1	0.61	-
		18.6		Average 3.9
	1	15.6	0.65	9
	2	16.7	0.59	5
	3	13.5	0.73	5
	4	15.5	0.65	6
Retained on 10-mesh sieve	5	15.1	0.54	12
	6	17.7	0.64	2
	7	15.7	0.66	7
	8	10.1	0.54	-
	9	12.4	0.60	9
	10	11.1	0.65	9
		14.3		Average 7.1

Table 3.--MAXIMUM DEVIATION OF FALL PATHS OF PARTICLES FROM A VERTICAL LINE--Continued

Sample of wind-blown sand from LaPorte, Colorado

Group	Number	Weight in mg	sf ratio	Maximum deviation in mm
	1	7.5	0.72	14
	2	6.7	0.69	2
	3	4.3	0.85	4
	4	6.1	0.66	0
Retained on 14-mesh sieve	5	4.0	0.56	8
	6	3.8	0.60	6
	7	4.8	0.47	6
	8	5.1	0.64	5
	9	6.1	0.64	-
	10	5.8	0.56	0
		5.4	.64	Average 5.0

Table 4.--TABULATION OF REYNOLDS NUMBER AND COEFFICIENT OF DRAG

Sample of sand from Cache la Poudre River at Bellvue, Colorado

Group	Number	Nominal diameter d_n in mm	With nominal dimensions $\frac{v_o d_n}{\nu}$	With nominal dimensions $\frac{F/d_n^2}{\rho v_o^2/2}$	With projected dimensions $\frac{v_o b}{\nu}$	With projected dimensions $\frac{F/ab}{\rho v_o^2/2}$
	1	6.85	1700	1.95	1930	1.28
	2	5.08	1430	1.17	1410	0.95
	3	7.00	2310	1.12	2440	0.90
	4	6.60	2220	1.03	2950	0.52
Retained on 4-mesh sieve	5	5.01	1260	1.40 1.56	1250	1.25
	6	7.07	1560	~~2.56~~ ?	3150	1.82
	7	6.83	2070	1.26	2100	1.05
	8	5.32	1700	0.91	1700	0.76
	9	6.54	2280	0.95	2220	0.76
	10	----	----	----	1900	----
	1	4.73	990	1.90	1110	1.05
	2	4.04	1060	0.84	1050	0.70
	3	3.82	925	1.14	865	0.79
	4	3.58	810	1.23	860	0.79
Retained on 8-mesh sieve	5	4.05	920	1.38	1090	0.73
	6	3.38	875	0.88	960	0.50
	7	4.68	1280	1.11	1280	0.81
	8	4.10	956	1.33	860	1.07
	9	4.43	1330	0.71	1440	0.60
	10	4.39	1090	1.25	1190	0.69
	1	2.59	547	1.02	635	0.53
	2	2.53	612	0.75	665	0.44
	3	2.32	534	0.77	662	0.46
	4	3.04	601	1.37	495	0.96
Retained on 9-mesh sieve	5	2.46	465	1.20	590	0.55
	6	2.51	467	1.28	484	0.75
	7	2.25	362	1.53	400	0.89
	8	2.33	452	1.08	581	0.60
	9	2.57	521	0.09 ? 1.19	535	0.57
	10	2.38	455	1.13	455	0.66

Table 4.--TABULATION OF REYNOLDS NUMBER AND COEFFICIENT OF DRAG--Continued

Sample of sand from Cache la Poudre River at Bellvue, Colorado

Group	Number	Nominal diameter d_n in mm	With nominal dimensions $\frac{v_0 d_n}{\nu}$	With nominal dimensions $\frac{F/d_n^2}{\rho v_0^2/2}$	With projected dimensions $\frac{v_0 b}{\nu}$	With projected dimensions $\frac{F/ab}{\rho v_0^2/2}$
	1	2.15	430	0.94	501	0.51
	2	2.09	360	1.23	366	0.67
	3	2.15	---	----	---	----
	4	2.06	275	2.03	317	1.12
Retained on 10-mesh sieve	5	2.12	320	1.63	380	0.69
	6	2.10	375	1.15	491	0.62
	7	2.29	430	1.13	400	0.82
	8	1.84	---	----	---	----
	9	2.20	390	1.24	461	0.73
	10	2.17	400	1.13	435	0.74
	1	2.09	420	0.91	500	0.48
	2	1.82	290	1.29	286	1.19
	3	1.60	---	----	---	----
	4	1.90	345	1.01	398	0.67
Retained on 14-mesh sieve	5	2.04	330	1.13	350	0.99
	6	1.91	313	1.03	326	0.85
	7	2.08	430	0.85	445	0.75
	8	1.93	306	1.35	340	0.70
	9	1.83	326	1.01	369	0.75
	10	1.64	270	1.08	310	0.70

Table 4.--TABULATION OF REYNOLDS NUMBER AND COEFFICIENT OF DRAG--Continued

Sample of sand from Middle Loup River at Dunning, Nebraska

Group	Number	Nominal diameter d_n in mm	With nominal dimensions		With projected dimensions	
			$\frac{v_o d_n}{\nu}$	$\frac{F/d_n^2}{\rho v_o^2/2}$	$\frac{v_o b}{\nu}$	$\frac{F/ab}{\rho v_o^2/2}$
Retained on 4-mesh sieve	1	5.10	1530	1.00	1770	0.60
	2	6.03	1750	1.25	2180	0.76
	3	6.03	1910	1.05	1770	0.86
	4	4.79	----	----	----	----
	5	5.20	1570	1.01	1660	0.59
	6	5.20	1600	0.97	1970	0.53
	7	6.16	2010	1.02	2320	0.57
	8	6.03	1810	1.17	1800	0.83
	9	6.94	2250	1.17	2360	0.78
	10	5.81	2010	0.85	2280	0.54
Retained on 8-mesh sieve	1	4.09	1060	1.09	1130	0.79
	2	3.66	1040	0.79	855	0.76
	3	3.99	1230	0.73	1110	0.55
	4	4.02	1080	0.97	1075	0.78
	5	4.27	1090	1.05	1170	0.86
	6	3.11	850	0.74	1350	0.51
	7	3.34	970	0.70	870	0.71
	8	4.44	1540	0.65	1490	0.56
	9	4.78	1040	1.75	1130	1.11
	10	4.34	1090	1.21	1260	0.78
Retained on 9-mesh sieve	1	2.38	490	0.99	514	0.62
	2	2.28	440	1.07	601	0.51
	3	2.62	490	1.31	631	0.69
	4	2.28	540	0.72	620	0.50
	5	2.38	430	1.30	491	0.72
	6	2.42	440	1.28	480	0.85
	7	2.42	480	1.10	494	0.62
	8	2.35	465	1.05	518	0.61
	9	2.29	510	0.86	557	0.64
	10	2.64	550	1.06	523	0.64

Table 4.--TABULATION OF REYNOLDS NUMBER AND COEFFICIENT OF DRAG--Continued

Sample of sand from Middle Loup River at Dunning, Nebraska

Group	Number	Nominal diameter d_n in mm	With nominal dimensions $\frac{v_o d_n}{\nu}$	With nominal dimensions $\frac{F/d_n^2}{\rho v_o^2/2}$	With projected dimensions $\frac{v_o b}{\nu}$	With projected dimensions $\frac{F/ab}{\rho v_o^2/2}$
	1	2.10	375	1.15	514	0.59
	2	2.24	410	1.19	500	0.56
	3	2.04	435	0.77	540	0.47
	4	2.07	---	----	---	----
Retained on 10-mesh sieve	5	2.08	445	0.81	505	0.54
	6	2.06	360	1.17	463	0.61
	7	2.13	415	1.05	476	0.57
	8	2.15	340	1.50	375	0.95
	9	2.15	420	0.97	540	0.57
	10	2.12	380	1.16	491	0.61
	1	2.22	403	1.16	400	0.85
	2	1.95	355	1.04	352	0.76
	3	1.79	310	1.03	340	0.73
	4	1.83	342	0.96	323	0.84
Retained on 14-mesh sieve	5	1.63	231	1.41	232	0.97
	6	2.10	330	1.50	354	0.97
	7	2.06	381	1.05	380	0.70
	8	1.98	305	1.45	280	0.92
	9	2.08	365	1.17	308	0.77
	10	1.47	220	1.15	233	0.85

Table 4.--TABULATION OF REYNOLDS NUMBER AND COEFFICIENT OF DRAG--Continued

Sample from rock crusher at Bellvue, Colorado

Group	Number	Nominal diameter d_n in mm	With nominal dimensions $\frac{v_o d_n}{\nu}$	With nominal dimensions $\frac{F/d_n}{\rho v_o^2/2}$	With projected dimensions $\frac{v_o b}{\nu}$	With projected dimensions $\frac{F/ab}{\rho v_o^2/2}$
Retained on 4-mesh sieve	1	4.96	828	3.12	885	1.07
	2	4.21	----	----	----	----
	3	5.94	1080	3.15	1440	1.01
	4	5.82	1370	1.81	1700	0.76
	5	----	----	----	750	----
	6	5.53	----	----	----	----
	7	4.50	----	----	----	----
	8	5.29	915	3.10	1180	1.15
	9	4.46	----	----	----	----
	10	3.92	630	2.65	980	0.67
Retained on 8-mesh sieve	1	3.95	590	3.14	670	1.25
	2	2.82	395	2.53	570	0.75
	3	3.19	630	1.43	830	0.54
	4	3.16	430	3.01	507	1.11
	5	3.88	620	2.64	1020	0.78
	6	3.65	770	1.67	760	0.96
	7	3.23	500	2.35	715	0.82
	8	4.25	850	1.51	1100	0.78
	9	3.15	630	1.38	865	0.76
	10	3.86	910	1.22	1110	0.65
Retained on 9-mesh sieve	1	2.27	223	4.10	208	1.32
	2	2.12	250	2.61	342	1.06
	3	2.02	260	2.18	396	0.74
	4	2.49	460	1.06	465	0.54
	5	2.20	295	2.15	318	1.10
	6	2.39	410	1.44	426	0.66
	7	2.26	266	2.84	212	0.96
	8	2.34	407	1.35	434	0.82
	9	2.05	290	1.77	425	0.64
	10	2.04	265	2.11	390	0.73

Table 4.--TABULATION OF REYNOLDS NUMBER AND COEFFICIENT OF DRAG--Continued

Sample from rock crusher at Bellvue, Colorado

Group	Number	Nominal diameter d_n in mm	With nominal dimensions		With projected dimensions	
			$\frac{v_o d_n}{\nu}$	$\frac{F/d_n^2}{\rho v_o^2/2}$	$\frac{v_o b}{\nu}$	$\frac{F/ab}{\rho v_o^2/2}$
	1	2.57	---	----	---	----
	2	2.02	360	1.12	485	0.41
	3	2.44	---	----	341	1.45
	4	2.41	380	1.42	487	0.59
Retained on 10-mesh sieve	5	2.43	340	2.20	417	0.89
	6	1.91	160	4.74	304	0.95
	7	2.32	260	3.31	360	1.05
	8	2.33	400	1.39	495	0.78
	9	2.35	325	2.11	278	1.00
	10	2.54	450	1.45	576	0.71
	1	----	---	----	358	----
	2	1.64	137	----	---	1.63
	3	2.01	274	1.95	306	0.83
	4	1.87	238	2.03	270	0.70
Retained on 14-mesh sieve	5	1.85	212	2.47	316	0.82
	6	----	---	----	260	----
	7	1.91	233	2.25	334	0.75
	8	1.91	230	2.04	225	0.69
	9	1.66	---	----	---	----
	10	2.07	300	1.73	344	0.63

Table 4.--TABULATION OF REYNOLDS NUMBER AND COEFFICIENT OF DRAG--Continued

Sample of wind-blown sand from Laporte, Colorado

Group	Number	Nominal diameter d_n in mm	With nominal dimensions $\frac{v_o d_n}{\nu}$	With nominal dimensions $\frac{F/d_n^2}{\rho v_o^2/2}$	With projected dimensions $\frac{v_o b}{\nu}$	With projected dimensions $\frac{F/ab}{\rho v_o^2/2}$
	1	3.74	840	1.31	1010	0.63
	2	4.20	850	1.93	1150	0.89
	3	3.58	760	1.41	845	0.75
	4	2.67	324	3.21	446	0.96
Retained on	5	2.88	514	1.58	430	1.20
8-mesh sieve	6	3.88	850	1.43	1030	0.83
	7	2.91	600	1.21	635	0.64
	8	2.45	425	1.44	518	0.86
	9	2.66	467	1.52	580	0.83
	10	3.30	627	1.32	702	0.84
	1	2.51	457	1.33	545	0.72
	2	2.08	278	2.05	450	0.73
	3	2.29	360	1.63	511	0.70
	4	2.49	---	----	---	----
Retained on	5	2.50	480	1.15	604	0.56
9-mesh sieve	6	2.22	523	1.84	418	0.70
	7	2.10	---	----	---	----
	8	2.71	435	1.83	461	0.89
	9	2.31	427	1.19	465	0.64
	10	2.44	---	----	---	----
	1	2.24	350	1.58	412	0.88
	2	2.29	410	1.25	494	0.66
	3	2.14	340	1.45	400	0.78
	4	2.23	365	1.47	409	0.86
Retained on	5	2.28	360	1.48	413	0.73
10-mesh sieve	6	2.33	430	1.20	461	0.67
	7	2.24	400	1.23	446	0.76
	8	1.94	300	1.42	368	0.69
	9	2.08	300	1.71	382	0.75
	10	2.00	315	1.41	392	0.78

Table 4.--TABULATION OF REYNOLDS NUMBER AND COEFFICIENT OF DRAG--Continued

Sample of wind-blown sand from Laporte, Colorado

Group	Number	Nominal diameter d_n in mm	With nominal dimensions		With projected dimensions	
			$\frac{v_o d_n}{\nu}$	$\frac{F/d_n^2}{\rho v_o^2/2}$	$\frac{v_o b}{\nu}$	$\frac{F/ab}{\rho v_o^2/2}$
Retained on 14-mesh sieve	1	1.75	265	1.34	275	0.72
	2	1.70	250	1.33	236	1.04
	3	1.46	217	1.15	231	0.79
	4	1.64	235	1.41	222	0.87
	5	1.42	173	1.70	175	0.96
	6	1.40	157	1.93	161	1.21
	7	1.51	205	1.43	230	0.82
	8	1.54	215	1.39	182	0.86
	9	1.64	263	1.10	302	0.71
	10	1.61	225	1.45	201	1.16

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