Civil Engineering Department
Colorado A. and M. College
Fort Collins, Colorado

A COMPARATIVE STUDY OF MOMENTUM TRANSFER, HEAT TRANSFER, AND VAPOR TRANSFER

PART III

FREE CONVECTION

by

C. S. Yih
Associate Professor

Prepared for the
Office of Naval Research
Navy Department
Washington, D. C.

Under ONR Contract No. N9Onr-82401
NR 063-071/1-19-49

February, 1951 Report 3

Foreword

This report is Part III of a preliminary study in connection with the Wind-Tunnel Project under contract with the ONR to be carried out by the Fluid Mechanics Laboratory of the Colorado Agricultural and Mechanical College, Fort Collins, Colorado. Besides being a review of existing literature, it also contains one solution and several proposed solutions by the writer.

The writer wants to express his thanks to Dr. D. F. Peterson, Head of the Civil Engineering Department of the College and Chief of the Civil Engineering Section of the Experiment Station, for critical reading of the manuscript and many valuable suggestions. Professor T. H. Evans is Dean of Engineering of the College and chairman of the Engineering Division of the Experimental Station.

The present work has been done under the supervision of Dr. M. L. Albertson, Head of Fluid Mechanics Research, to whom the writer owes many valuable discussions and suggestions, and much assistance in preparing this report.

To Mr. Fred Repper, Graduate Assistant, who has rendered indispensable assistance with his fine draftsmanship, and to the Multigraph Office of the College, which has kindly lent its able service, the writer also wants to show his appreciation.

CONTENTS

Part III. Free Convection

1. Table of Notations

a = radius of a cylinder or of a sphere

b = half-width as defined in the text

B : as defined in the text

C = a numerical constant

Δc = difference in concentrations of vapor at two significant points, corresponding to $\Delta \gamma$

c_p = specific heat at constant pressure

E = rate of mass-transfer of vapor per unit area

f, f_2, f_3,--- = functions as defined in the text

g = gravitational acceleration

G = strength of point source of heat, as defined in the text.

Gr = Grashof number for heat transfer = $gL^3 \Delta T/\nu^2 T_0$

Gr' = Grashof number for evaporation = $L^3 \gamma \Delta \gamma / g \mu^2$

h = coefficient of heat transfer

h' = coefficient of vapor transfer

h_m = mean value of h

h'_m = mean value of h'

k = thermal conductivity

K = vapor diffusivity

L = a length

n = a constant

Nu = Nusselt number for heat transfer = $h_m L/k$

Nu' = Nusselt number for vapor transfer = $h'_m L/K$

p = pressure

q = rate of heat transfer per unit area

q_m = mean value of q

$q_{h.u}$, $q_{h.d}$, $q_{v.u}$, $q_{v.d}$: as defined in the text

r = radial distance in two or three dimensions

T = absolute temperature

T_o = ambient temperature, absolute scale

T_s = absolute temperature of the surface

ΔT = $T_s - T_o$

U = longitudinal velocity-component

v = radial or transverse velocity - component

x = longitudinal distance

y = transverse distance

α = thermal diffusivity = $k/\rho c_p$

γ = specific weight

$\Delta\gamma$ = difference of specific weights at two significant points

ζ = a function of dimensionless parameters

η, ξ = dimensionless parameters

θ = a function of dimensionless parameters or an ordinate

θ_0, θ_1--- = functions of dimensionless parameters

μ = dynamic viscosity

ν = kinematic viscosity = μ/ρ

π = 3.14159---

ρ = density

σ = Prandtl number for heat transfer = ν/α

σ' = Prandtle number for evaporation = ν/K

φ = the third spherical coordinate

ψ = stream-function or Stokes' stream-function

χ = a function of dimensionless parameters

2. Introduction

When a convectional current is caused by differences in the specific weight of a substance or a mixture of substances, the phenomenon is called free convection. Since in free convection the distribution of velocity and that of temperature or moisture are interdependent, the answer to a particular problem can only be found by solving the equations of motion and of diffusion simultaneously in the case of laminar flow, and by systematic experimentation in the case of turbulent flow. Due to the difficulties introduced by the non-linearity and simultaneity of the equations in question, theoretical results in laminar free convection are very few. In the next section, two theoretical solutions in laminar free convection will be given, to be followed by experimental results presented in Section 5. Proposed theoretical solutions are presented in Section 4.

3. Theoretical Results in Laminar Free Convection

Because of the lack of a conclusive theory of turbulence, theoretical solutions for turbulent free convection are non-existent. In the laminar case, there exist only two theoretical solutions: one for the vertical plate by Pohlhausen (230, 1930) and the other for a point source by C. S. Yih (234, 1948).

(a) Vertical plate

In 1881, L. Lorenz (221) dealt with heat transfer from a vertical surface at a uniform and constant temperature T_s to a colder gas in contact with it when gravity is the only force acting on that gas. He correctly recognized that the gas close to the surface streams straight upwards, that the horizontal velocities were negligible and that consequently the pressure distribution is hydrostatic. This, obviously, is the case of streamline flow on a vertical wall. He further assumed that the ambient temperature T_o at an infinite distance from the wall was constant, that the physical constants of the gas are independent of the temperature, and that the temperature distribution is a function of the distance from the plate alone. This last assumption has since been proved to be wrong.

Setting up a heat valance for a differential section of the fluid and integrating, Lorenz obtained (denoting $T_s - T_o$ by ΔT)

$$h_m = C\sqrt[4]{\frac{g\rho^2 C_p k^3 \Delta T}{\mu L T_o}} \qquad (1)$$

where h_m is the mean coefficient of heat transfer by convection, L is the height of the surface, C is a constant for which Lorenz found the value 0.548, g the gravitational acceleration, and the other symbols have the same meanings as in Part I. This formula, in spite of Lorenz's wrong assumption about the temperature distribution, is in a form which has been proved valid with good approximation through more than half a century.

One consequence of Eq 1 is that the rate of heat exchange, q, is proportional to $(\Delta T)^{5/4}$. This has been proved by numerous experiments, beginning with those of Dulong and Petit (211, 1817), who arrived at an exponent 1.23, instead of 1.25. Nusselt (225, 1909) directed attention to the fact that, according to experiments with air, the equation becomes less exact at temperature excesses below 20°F and fails entirely when ΔT approaches zero. In 1928, Nusselt and Juerges (228) succeeded in essentially improving the theory, but the conclusive result is due to E. Schmidt and Beckmann (230) in 1930, in cooperation with Pohlhausen, who found a way to integrate the differential equations set up by these authors. In the following, their solution will be presented. Measuring x from the lower edge of the plate in a vertical direction and y in a direction normal to the plate, and denoting the velocity components in the x- and y-directions by u and v , and the absolute temperature by T, one has for the equations of motion and of diffusion:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\frac{T-T_0}{T_0} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where ν is the kinematic viscosity, α the thermal diffusivity, and T_0 the ambient temperature in the absolute scale. Eq 1 and 2 are to be solved simultaneously with the equation of continuity,

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0 \tag{4}$$

and the boundary conditions

$$u = v = 0 \quad \text{and} \quad T = T_s \quad \text{at} \quad y = 0 \tag{5}$$

$$u = 0 \quad \text{and} \quad T = T_0 \quad \text{at} \quad y = \infty \tag{6}$$

Eq 4 permits the use of the stream-function ψ in terms of which u and v can be expressed as follows:

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x} \tag{7}$$

Pohlhausen made the substitutions

$$\eta = B\frac{y}{x^{1/4}} \tag{8}$$

$$\Theta = \frac{T-T_0}{\Delta T} = \Theta(\eta) \tag{9}$$

$$\psi = 4\nu B x^{3/4} f(\eta) \tag{10}$$

where
$$B = \left[\frac{g\Delta T}{4\nu^2 T_0}\right]^{1/4}$$

Substituting Eqs 8 and 10 in Eq 7, he obtains

$$u = 4\nu B^2 x^{1/2} f'(\eta) \tag{11}$$

$$v = \nu B x^{-1/4} (\eta f' - 3f) \tag{12}$$

Substitution of Eqs 8, 9, 11 and 12 into Eqs 2 and 3 yields the ordinary differential equations

$$f''' + f f'' - 2(f')^2 + \theta = 0 \tag{13}$$

$$\theta'' + 3\sigma f \theta' = 0 \tag{14}$$

where $\sigma = \frac{\nu}{\alpha}$ is the Prandtl number. These equations are to be solved simultaneously, with the boundary conditions

$$f(0) = f'(0) = 0, \qquad \theta(0) = 1 \tag{15}$$

$$f'(\infty) = 0, \qquad \theta(\infty) = 0 \tag{16}$$

The system consisting of Eqs 13 to 16 was solved by Pohlhausen by numerical integration, after expanding f and θ into slowly convergent series in η and starting from the values of $f''(0)$ and $\theta'(0)$ experimentally determined by E. Schmidt and Beckmann (230, 1930) who used an unheated leading section. The results are shown in Fig. 1.

The coefficient of heat transfer is

$$h = -\frac{k}{\Delta t}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -k\sqrt[4]{\frac{g\Delta T}{4\nu^2 T_0 x}}\,\theta'(0) \tag{17}$$

where k is the thermal conductivity. Assuming $\sigma = 0.733$ for air, Pohlhausen's solution gives $\theta'(0) = -0.508$. Substituting this value in Eq 17 and integrating over the height L, one obtains the mean coefficient of heat transfer

$$h_m = 0.479\,\frac{k}{L}\sqrt[4]{\frac{gL^3\Delta T}{\nu^2 T_0}} \tag{18}$$

This may be compared with Lorenz's result by converting Eq. 1 into the form

$$h_m = 0.548 \frac{k}{L} \sqrt[4]{\frac{gL^3 \Delta T}{\nu^2 T_0}} \sqrt[4]{\frac{\nu}{\alpha}} = 0.513 \frac{k}{L} \sqrt[4]{\frac{gL^3 \Delta T}{\nu^2 T_0}} \tag{19}$$

on taking $\sigma = \frac{\nu}{\alpha} = 0.733$. The difference is only 7%.

Defining the Grashof's number as

$$G_r = \frac{gL^3 \Delta T}{\nu^2 T_0}$$

and the Nusselt number as

$$Nu = \frac{hL}{k}$$

Eq 18 can be written as

$$Nu = 0.479 (G_r)^{\frac{1}{4}} \tag{18a}$$

(b) Point source of heat

The problem of laminar free convection due to a point source of heat was solved by Yih (234) in 1948. A point source of heat is considered to be situated in an infinite plane above which the atmosphere was originally isothermal and at rest, and the resulting steady temperature and velocity distributions are sought. Taking the point source as the origin and the vertical line through it as the x-axis, from which r is measured radially, denoting the velocity components in the x- and r-directions by u and v, and remembering the equation of state, the equations of motion and of diffusion can be written;

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{\nu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) + g\left(\frac{T - T_0}{T_0}\right) \tag{20}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) \tag{21}$$

where ν, T_0, and α have the same meanings as in Eqs 2 and 3. The above equations are to be solved simultaneously, with the boundary conditions:

u, v, and $T-T_0$ vanish at $r = \infty$

u and v vanish at $x = 0$ except at the origin

v, $\frac{\partial T}{\partial r}$, and $\frac{\partial u}{\partial r}$ vanish at $r = 0$

The equation of continuity

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0$$

offers the use of the Stokes' stream-function ψ , from which the velocity components can be obtained as follows:

$$u = \frac{1}{r}\frac{\partial \psi}{\partial r} \quad , \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x} \tag{23}$$

It should be noted that, the flow being steady, the quantity

$$G = \int_0^\infty 2\pi r u (T - T_0) \frac{\gamma_0}{T_0} dr \tag{24}$$

where γ_0 is the specific weight of air at T_0, must be constant and is indeed a measure of the strength of the heat source.

Making the substitutions

$$\eta = \left(\frac{\rho^2 G}{\mu^3}\right)^{1/4} \frac{r}{x^{1/2}} \tag{25}$$

$$\frac{\gamma_0 (T - T_0)}{T_0} = \frac{G\rho}{x\mu}\,\theta(\eta) \tag{26}$$

$$\psi = 4\nu x f(\eta) \tag{27}$$

one obtains, in place of Eq 23,

$$u = 4\left(\frac{G}{\mu}\right)^{1/2} \frac{f'}{\eta} \tag{28}$$

$$v = 2\left(\frac{\mu G}{\rho^2}\right)^{1/4} x^{1/2}\left(f' - \frac{2f}{\eta}\right) \tag{29}$$

where the primes denote differentiation with respect to η .

Substitution of Eqs 25 to 29 into 20 and 21 yields the simultaneous ordinary differential equations

$$(1 - 4f)\frac{d}{d\eta}\left(\frac{f'}{\eta}\right) = f''' + \eta\theta \tag{30}$$

$$f = -\frac{1}{4\sigma}\frac{\theta'}{\theta}\eta \tag{31}$$

where $\sigma = \frac{\nu}{\alpha}$ is the Prandtl number. The boundary conditions, except the one that $v = 0$ when $x = 0$, which will be discussed later, are now replaced by the following

$$f(0) = f'(0) = \theta'(0) = 0 \tag{32}$$

$$\theta(\infty) = 0 \quad , \quad f(\infty) = a \text{ finite number} \tag{33}$$

Still another boundary condition is furnished by Eq 24, which can be put in the following dimensionless form:

$$\int_0^\infty f'\theta d\eta = \frac{1}{8\pi} \tag{34}$$

The differential system consisting of Eqs 30 to 34 is difficult to solve for arbitrary values of σ. But when σ has the values 1 and 2 solutions in closed forms can be obtained

When $\sigma = 1$, it may be assumed

$$f(\eta) = B\left(1 - \frac{1}{1+A\eta^2}\right) = \frac{AB\eta^2}{1+A\eta^2} \tag{35}$$

Substitution of Eq 35 into 31 and integration give:

$$\theta(\eta) = \frac{C}{(1+A\eta^2)^{2B}} \tag{36}$$

With these functional forms for f and θ, Eqs 30 to 33 are satisfied if $B = 3/2$ and $C = 24A^2$. Eq 34 can then be integrated to yield $C = \frac{1}{3\pi}$ from which $A = \frac{1}{6\sqrt{2\pi}}$. With these values for A, B, and C, Eqs 26 and 27 become, by virtue of Eqs 35 and 36:

$$\frac{\gamma_0 \mu x (T - T_0)}{T_0 \rho G} = \frac{1}{3\pi\left(1 + \frac{\eta^2}{6\sqrt{2\pi}}\right)^3} \tag{37}$$

and

$$\psi = 6\nu x \frac{\eta^2}{6\sqrt{2\pi} + \eta^2} \tag{38}$$

from which

$$\left(\frac{\mu}{G}\right)^{1/2} u = \frac{\sqrt{2}}{\sqrt{\pi}\left(1 + \frac{\eta^2}{6\sqrt{2\pi}}\right)^2} \tag{39}$$

and

$$\left(\frac{\rho^2 x^2}{\mu G}\right)^{1/4} v = -\frac{6\eta^3}{(6\sqrt{2\pi}+\eta^2)^2} \tag{40}$$

When $\sigma = 2$, it can be similarly shown that

$$f(\eta) = 1 - \frac{1}{1 + \frac{\sqrt{5}}{8\sqrt{2\pi}}\eta^2} \tag{41}$$

$$\Theta(\eta) = \frac{5}{8\pi} \frac{1}{\left(1 + \frac{\sqrt{5}}{8\sqrt{2\pi}}\eta^2\right)^3} \tag{42}$$

and results similar to Eqs 37 to 40 can be readily obtained.

For air which has a Prandtl number σ of 0.73 under normal conditions, the solution corresponding to $\sigma = 1$ can be used to give a close approximation. For $\sigma = 1$, the value of v at $x = 0$ can be shown to be $-\frac{6\nu}{r}$. This has a value of only 0.03 fps. at $r = 0.05$ ft (ν being taken to be 0.00025 sq. ft. per sec for air) and varies inversely with r, so that the boundary condition $v = 0$ at $x = 0$ is approximately satisfied. This approximation will not introduce large errors if x is not extremely small. That the boundary-layer equations are valid can also be verified a posteriori from Eqs 37 and 39.

The patterns of streamlines and isotherms are shown in Fig. 2 and Fig. 3, respectively. The parameters used are dimensionless.

4. Proposals for Further Theoretical Investigations on Laminar Free Convection

In the following specific problems in laminar free convection and their proposed solutions are presented:

(a) Finite Source--two-dimensional case

α Statement of the problem. A heated plate of width 2b, serving as the finite source, is considered to be situated in an infinite plane above which the atmosphere was originally isothermal and at rest, and the resulting steady temperature and velocity distributions as well as the heat transfer from the source are sought. In the vertical plane perpendicular to the longitudinal direction of the heated plate, take the y-axis along the trace of the heated plate, the origin at its mid-point, and the x-axis in the vertical direction. The equations of motion and of diffusion can be written

$$u u_x + v u_y = \nu (u_{xx} + u_{yy}) + g \frac{T - T_0}{T_0} \tag{43}$$

$$u T_x + v T_y = \alpha (T_{xx} + T_{yy}) \tag{44}$$

where

u = velocity component in the x-direction

v = velocity component in the y-direction

ν = μ/ρ = kinematic viscosity

μ = dynamic viscosity

ρ = density

g = gravitational acceleration

α = $k/\rho c_p$ = thermal diffusivity

k = thermal conductivity

c_p = specific heat at constant pressure

T_0 = ambient temperature (abs. scale)

T = temperature at any point (abs. scale)

The equation of continuity is

$$u_x + v_y = 0 \tag{45}$$

which permits the use of the stream function ψ such that

$$u = \psi_y , \qquad v = -\psi_x \tag{46}$$

Eqs 43, 44 and 45 are to be solved with the boundary conditions

(i) When $x = 0$, $T = T_s$ for $|y|<b$ where T_s is the temperature of the plate and 2b is the width of the plate, and $T = T_o$ for $|y|>b$. This condition may be approximated by the continuous distribution

$$T_{x=0} - T_o = \frac{T_s - T_o}{1 + \left(\frac{y}{b}\right)^{2n}} = \frac{\Delta T}{1 + \left(\frac{y}{b}\right)^{2n}}$$

where $\Delta T = T_s - T_o$, and n is taken to be a very large positive integer.

(ii) When $x = 0$, u, v, and ψ are all equal to zero.

(iii) When $x = \infty$, $T = T_o$

(iv) When $y = 0$, $\psi = 0$, $v = 0$, $u_y = 0$, $T_y = 0$

(v) When $y = \pm\infty$, $T = T_o$

β. Dimensional Analysis. A dimensional analysis of the variables b, x, y, μ, ρ, k, c_p, T_o, $T - T_o$, ΔT, g, ψ with $T - T_o$ and ψ as the dependent variables and b, μ, ρ, c_p, and T_o as the repeating variables yields

$$\frac{T - T_o}{T_o} = F_1\left(\frac{x}{b}, \frac{y}{b}, \frac{\mu c_p}{k}, \frac{\Delta T}{T_o}, \frac{\rho^2 g b^3}{\mu^2}\right)$$

$$\frac{\psi}{\nu} = F_2\left(\frac{x}{b}, \frac{y}{b}, \frac{\mu c_p}{k}, \frac{\Delta T}{T_o}, \frac{\rho^2 g b^3}{\mu^2}\right)$$

Writing ξ for x/b, η for y/b, and σ for the Prandtl number $\mu c_p/k$, the above relationships can be written

$$\frac{T - T_o}{T_o} = \frac{\Delta T}{T_o}\, \theta\left(\xi, \eta, \sigma, \frac{\Delta T}{T_o}, \frac{g b^3}{\nu^2}\right) \tag{47}$$

$$\frac{\psi}{\nu} = \zeta\left(\xi, \eta, \sigma, \frac{\Delta T}{T_0}, \frac{g b^3}{\nu^2}\right) \tag{48}$$

Using Eq 46, Eq 45 is automatically satisfied. In terms of the dimensionless paraemters Eqs. 43 and 44 become

$$\zeta_\eta \zeta_{\xi\eta} - \zeta_\xi \zeta_{\eta\eta} = \zeta_{\eta\xi\xi} + \zeta_{\eta\eta\eta} + Gr\,\Theta \tag{49}$$

$$\zeta_\eta \Theta_\xi - \zeta_\xi \Theta_\eta = \frac{1}{\sigma}\left(\Theta_{\xi\xi} + \Theta_{\eta\eta}\right) \tag{50}$$

where $Gr = gb^3 \Delta T / \nu^2 T_0$ is the Grashof's number. The dependence of ζ and Θ on the Grashof's number and the Prandtl number σ is then obvious. The boundary conditions become

(i) When $\xi = 0$: $\zeta = \zeta_\xi = \zeta_\eta = 0$

(ii) When $\eta = 0$: $\zeta = \zeta_\xi = \zeta_{\eta\eta} = 0$

(iii) When $\xi = 0$: $\Theta = (1 + \eta^{2n})^{-1}$

(iv) When $\eta = 0$: $\Theta_\eta = 0$

(v) When $\xi = \infty$: $\Theta = 0$

(vi) When $\eta = \pm\infty$: $\Theta = 0$

γ. Proposed method of solution. Assume

$$\zeta = \xi f_2(\eta) + \xi^3 f_3(\eta) + \xi^4 f_4(\eta) + \cdots \tag{51}$$

the coefficients of ξ^0 and ξ being zero as required by B. C. (boundary condition) (i). The functions f should obviously be odd functions by symmetry. Assuming the odd functions f to be entire functions, it is obvious that B. C. (ii) is automatically satisfied. To satisfy B. C. (iii) and (v), assume

$$\Theta = e^{-\xi^2}\left[\Theta_0(\eta) + \xi\,\Theta_1(\eta) + \xi^2\,\Theta_2(\eta) + \cdots\right] \tag{52}$$

where

$$\Theta_0 = (1+\eta^{2n})^{-1} \tag{53}$$

The functions $\Theta_0, \Theta_1, ---$ are obviously even functions by symmetry, so that B. C. (iv) is automatically satisfied. In solving for the functions $\Theta_0, \Theta_1, ---$, only B. C. (vi) remains to be satisfied. It is of course assumed that the series in Eq 52 is convergent for all real values of η and all real finite values of ξ. If the series diverges at $\xi = \infty$, B. C. (v) will be satisfied if the degree of divergence is less than that of $\exp(\xi^2)$ as $\xi \to \infty$. This will be tenatively assumed.

Substituting Eqs. 51 and 52 into Eqs 49 and 50, and equating coefficients of like powers of ξ, two series of ordinary differential equations in η are obtained. The first series is furnished by Eq. 49:

$$\xi^0: \quad 2f_2' + Gr.\Theta_0 = 0 \tag{54}$$

$$\xi: \quad 6f_3' + Gr.\Theta_1 = 0 \tag{55}$$

$$\xi^2: \quad 12f_4' + f_2''' + Gr(\Theta_2 - \Theta_0) = 0 \tag{56}$$

$$\xi^3: \quad 2f_2'f_2' - 2f_2f_2'' = 20\, f_5' + f_3''' + Gr(\Theta_3 - \Theta_1) \tag{57}$$

$$\xi^4: 5f_2'f_3' - 3f_3f_2'' - 2f_2f_3'' = 30f_6' + f_4''' + Gr\left(\Theta_4 - \Theta_2 + \frac{\Theta_0}{2}\right) \tag{58}$$

etc.

The second series is furnished by Eq 50:

$$\xi^0: \quad \Theta_2 = \Theta_0 - \frac{\Theta_0''}{2} \tag{59}$$

$$\xi:\ 2f_2\Theta_0' + \frac{1}{\sigma}\left(6\Theta_3 - 6\Theta_1 + \Theta_1''\right) = 0 \qquad (60)$$

$$\xi^2:\ f_2'\Theta_1 - 3f_3\Theta_0' - 2f_2\Theta_1' = \frac{1}{\sigma}\left(12\Theta_4 - 12\Theta_2 + 6\Theta_0 + \Theta_2'' - \Theta_0''\right) \qquad (61)$$

$$\begin{aligned}\xi^3:\ & f_3'\Theta_1 + 2f_2'(\Theta_2 - \Theta_0) - 4f_4\Theta_0' - 3f_3\Theta_1' - 2f_2(\Theta_2' - \Theta_0') \\ & = \frac{1}{\sigma}\left(20\Theta_5 - 20\Theta_3 + 10\Theta_1 + \Theta_3'' - \Theta_1'\right)\end{aligned} \qquad (62)$$

$$\begin{aligned}\xi^4:\ & f_4'\Theta_1 + 2f_3'(\Theta_2 - \Theta_0) + 2f_2'(\Theta_3 - \Theta_1) - 5f_5\Theta_0' \\ & - 4f_4\Theta_1' - 3f_3(\Theta_2' - \Theta_0) - 2f_2(\Theta_3' - \Theta_1') \\ & = \frac{1}{\sigma}\left(30\Theta_6 - 30\Theta_4 + 12\Theta_2 - 5\Theta_0 + \Theta_4'' - \Theta_2'' + \frac{\Theta_0''}{2}\right)\end{aligned} \qquad (63)$$

A close inspection of Eqs 54 to 63 shows that f_2 , Θ_2 and f_4 are at once known from Eqs 54, 59 and 56 since Θ_0 is given, and further, that if one function is known in addition, all the other functions can be solved, at least theoretically. Since the power series in Eqs 51 and 52 must be convergent for finite values of ξ , it follows that

$$\lim_{n\to\infty} f_n = 0\ , \quad \lim_{n\to\infty} \Theta_n = 0$$

Consequently, for an approximate solution, it is possible to take either f_n or Θ_n to be zero, provided n is sufficiently large. Concerning actual computation, f_6 can be first taken to be zero. Then Eqs 54, 55, 56, 58, 59, 60, 61 furnish the solution of f_3 , and all functions are known. The differential equation one finally obtains after elimination among the above-mentioned equations is linear and of second order, thus presenting no difficulty for its solution. The degree of approximation can now be checked by computing $\Theta_6, \Theta_7, \ldots, \Theta_n$. The smallness and decreasing magnitudes of these functions serve as a check for the closeness of the approximation. If greater accuracy is required one can take f_7 or f_8, or in general $f_n\ (n>6)$ to be zero. If the resulting system furnishes after elimination a linear differential equation, it can be solved directly. Otherwise, the results previously obtained can be used to linearize the non-linear differential equation, and the perturbation can be iterated until the corrective term becomes sufficiently small. If the final results thus obtained do not differ appreciably from those previously obtained, the problem can be considered as solved. Otherwise, a larger n has to be taken until two consecutive sets of results do not differ appreciably. In the process of solution, B. C. (vi) must of course be taken into account.

(b) Finite source, axially symmetric case

α. Statement of the Problem. The problem is completely analogous to the one presented in (a) α, except that instead of a plate of width 2b the heat source is now realized by a heated circular disc of radius a. The center of the disc is taken to be the origin, from which the x-axis is erected in the vertical direction. Instead of y, r is used, which is measured radially from the x-axis. Thus, replacing b by a and y by r, one obtains the variables for the present problem from those in the last problem. Understanding v to be the radial component of velocity, the differential equations to be satisfied are

$$u u_x + v u_r = \nu\left[u_{xx} + \frac{1}{r}(r u_r)_r\right] + g\,\frac{T - T_0}{T_0} \tag{64}$$

$$u T_x + v T_r = \alpha\left[T_{xx} + \frac{1}{r}(r T_r)_r\right] \tag{65}$$

$$(ru)_x + (rv)_r = 0 \tag{66}$$

the last of which permits the relations

$$u = \frac{1}{r}\psi_r \quad , \quad v = -\frac{1}{r}\psi_x \tag{67}$$

where ψ is the Stokes' stream-function. The boundary conditions are similar to those in (a)$\cdot\alpha$.

β. Dimensional Analysis. Taking the variables from (a)$\cdot\beta$ and replacing b by a and y by r, the following relations can be obtained:

$$\frac{T-T_0}{T_0} = \frac{T_s-T_0}{T_0}\,\theta\left(\xi, \eta, \sigma, \frac{\Delta T}{T_0}, \frac{ga^3}{\nu^2}\right) \tag{68}$$

$$\frac{\psi}{a\nu} = f\left(\xi, \eta, \sigma, \frac{\Delta T}{T_0}, \frac{ga^3}{\nu^2}\right) \tag{69}$$

where $\xi = x/a$ and $\eta = r/a$. Substituting Eqs 67 to 69 in Eqs 64 and 65, the following dimensionless equations are obtained:

$$\eta f_\eta f_{\xi\eta} + f_\xi f_\eta - \eta f_\xi f_{\eta\eta} = \eta^2 f_{\eta\eta\eta} + \eta^2 f_{\xi\eta\eta} - \eta f_{\eta\eta} + f_\eta + Gr\,\eta^3\theta \tag{70}$$

$$f_\eta \theta_\xi - f_\xi \theta_\eta = \frac{1}{\sigma}\left(\eta\theta_{\xi\xi} + \eta\theta_{\eta\eta} + \theta_\eta\right) \tag{71}$$

where now

$$Gr = \frac{ga^3\Delta T}{\nu^2 T_0}$$

The boundary conditions for Eqs 70 and 71 are similar to those in (b).β.

γ. Proposed Method of Solution. The proposed method of solution is similar to that in (a) . γ .

(c) <u>Horizontal cylinder</u>

α. Statement of the Problem. A heated horizontal cylinder of radius a at a temperature T_s is placed in the atmosphere originally at rest and isothermal of temperature T_0, and the resulting temperature and velocity distributions as well as the heat transfer from the cylinder are sought. In a plane perpendicular to the axis of the cylinder, the trace of the cylinder is a circle, the center of which is chosen as the origin, from which r is measured radially. The line drawn from the origin vertically upward is taken as the polar axis, from which θ is measured counterclockwise. Denoting by u and v the velocity components in the r and θ directions respectively, one has the equations

$$u u_r + \frac{v}{r} u_\theta - \frac{v^2}{r} = \nu\left(u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} - \frac{2}{r^2} v_\theta - \frac{u}{r^2}\right) + g\frac{T - T_0}{T_0}\cos\theta \qquad (72)$$

$$u T_r + \frac{v}{r} T_\theta = \alpha\left(T_{rr} + \frac{1}{r} T_r + \frac{1}{r^2} T_{\theta\theta}\right) \qquad (73)$$

$$(ur)_r + v_\theta = 0 \qquad (74)$$

the last of which permits the use of the stream-function ψ such that

$$u = \frac{1}{r}\psi_\theta , \qquad v = -\psi_r \qquad (75)$$

The boundary conditions are

(i) When $\theta = 0$ or $\pm\pi$: $\psi = 0$, $v = 0$, $u_\theta = 0$

(ii) When r = a; $\psi = 0$, $u = 0$, $v = 0$

(iii) When $\theta = 0$ or $\pm\pi$: $T_\theta = 0$

(iv) When r = a : $T = T_s$

(v) When $r = \infty$ $T = T_0$

β Dimensional Analysis. Taking the variables from (a) .β and replacing x and y by r and θ and b by a, a dimensional analysis yields

$$\frac{T-T_0}{T_0}=\frac{T_s-T_0}{T_0}\chi\left(\xi,\theta,\sigma,\ \frac{\Delta T}{T_0},\ \frac{ga^3}{\nu^2}\right) \tag{76}$$

$$\frac{\psi}{\nu}=f\left(\xi,\theta,\sigma,\ \frac{\Delta T}{T_0},\ \frac{ga^3}{\nu^2}\right) \tag{77}$$

where ξ = r/a. Substituting Eqs 76 and 77 into Eqs 72 and 73, one obtains

$$\xi f_\theta f_{\theta\xi}-f_\theta f_\theta-\xi f_\xi f_{\theta\theta}-\xi^2 f_\xi f_\xi=\xi f_{\xi\xi\theta}+\xi f_{\xi\theta}+f_{\theta\theta\theta} + \xi^3 Gr.\,\chi\cdot\cos\theta \tag{78}$$

$$\xi(f_\theta\chi_\xi-f_\xi\chi_\theta)=\frac{1}{\sigma}(\xi^2\chi_{\xi\xi}+\xi\chi_\xi+\chi_{\theta\theta}) \tag{79}$$

The boundary conditions can be readily written in dimensionless form.

γ. Proposed Method of Solution. The proposed method of solution is similar to that in (a) .γ , with a few modifications.

(d)Sphere

α. Statement of the Problem. A heated sphere of radius a at temperature T_s is placed in the atmosphere originally at rest and isothermal at temperature T_o , and the resulting temperature and velocity distributions as well as the heat transfer are sought. Spherical coordinates (r,θ,φ) will be used. By symmetry all unknown quantities are not functions of φ . Using u and v to denote the velocity components in the r- and θ-directions, respectively, one has the equations

$$uu_r+\frac{v}{r}u_\theta-\frac{v^2}{r}=\nu\left(u_{rr}+\frac{2}{r}u_r+\frac{\cot\theta}{r^2}u_\theta+\frac{1}{r^2}u_{\theta\theta}-\frac{2u}{r^2}-\frac{2\cot\theta}{r^2}v-\frac{2}{r^2}v_\theta\right)+g\cdot\cos\theta\cdot\frac{T-T_0}{T_0} \tag{80}$$

$$uT_r + \frac{v}{r}T_\theta = \nu\left(T_{rr} + \frac{2}{r}T_r + \frac{\cot\theta}{r^2}T_\theta + \frac{1}{r^2}T_{\theta\theta}\right) \tag{81}$$

$$\sin\theta(ur^2)_r + (rv\sin\theta)_\theta = 0 \tag{82}$$

the last of which permits the use of the stream-function ψ such that

$$u = \frac{1}{r^2\sin\theta}\psi_\theta \;, \qquad v = -\frac{1}{r\sin\theta}\psi_r \tag{83}$$

The boundary conditions are

(i) When $\theta = 0$, or $\pm\pi$: $\psi = 0$, $v = 0$, $u_\theta = 0$

(ii) When $r = a$: $\psi = 0$, $u = 0$, $v = 0$

(iii) When $\theta = 0$ or $\pm\pi$; $T_\theta = 0$

(iv) When $r = a$: $T = T_s$.

(v) When $r = \infty$; $T = T_o$

β. Dimensional Analysis. Understanding r now as in three dimensions, the variables in (c) . β can be used here and the resulting relationships are

$$\frac{T - T_o}{T_o} = \frac{T_s - T_o}{T_o}\,\chi\left(\xi, \theta, \sigma, \frac{\Delta T}{T_o}, \frac{ga^3}{\nu^2}\right) \tag{84}$$

$$\frac{\psi}{a\nu} = f\left(\xi, \theta, \sigma, \frac{\Delta T}{T_o}, \frac{ga^3}{\nu^2}\right) \tag{85}$$

Substitution of Eqs 83 to 85 into Eqs 80 and 81 yields two dimensionless equations comparable to Eqs 78 and 79, the unknowns being χ and f . The boundary conditions can be readily put in dimensionless form.

γ. Proposed method of solution. The proposed method of solution is similar to that in (a) . γ , with a few modifications.

5. Experimental results in free convection

In the following, experimental results in free convection from vertical and horizontal plates and cylinders and from a point source (turbulent case) will be presented.

(a) Vertical plates

The theory pertaining to free convection from a vertical plate has been presented in 3 (a). Experiments by Lorenz (221) as well as more recent (1935) ones of R. Weise (232) led to values about 25% higher than that given by Eq 18. This may be due to the influence of the edges and to motions of the ambient air. Schmidt and Beckmann (230) used vertical plates about 5 and 20 inches in height, with sharpened lower edges and vertical bounding plates at the edges to prevent lateral convection. For the small plates their experimental coefficients are in perfect agreement with Eq 18, while for large plates they are 4% higher than the theoretical values, ν being taken to be that of the ambient air. The temperatures were measured with a thermocouple with wires of 0.015 mm. in diameter, 10 to 20 mm. long. The deflections were read by a microscope, the calibration showing that the deflection is not proportional to the velocity.

The use of the theory developed in 3 (a) is restricted to certain heights depending on the fluid and on the temperature difference. For air and the temperature differences used in (28), the theory is not valid beyond h = 2 ft. Above this height turbulence plays an essential role. Experiments on turbulent heat transfer from a vertical plate were done by Griffiths and A. H. Davis (213, 1922). Their results showed that the mean heat transfer coefficient h_m is proportional to $(\Delta T)^{1/3}$, or that the mean rate of heat transfer per unit area, q_m, is proportional to $(\Delta T)^{4/3}$, both h_m and q_m being independent of the height of the plate L. Experiments by the authors on vertical cylinders undergoing turbulent heat transfer showed approximately the same coefficient 1/3 for ΔT in the expression for h_m. The form of the formula

$$Nu = C\,(Gr \cdot \sigma)^{1/3} \tag{86}$$

where Gr and σ were defined before, has been verified by a correlation of King (218, 1932) and by Jakob and Linke (217, 1933) to be valid for not only vertical plates and cylinders, but also horizontal plates and cylinders, when the flow is turbulent.

In the laminar range, King recommended the following formula in British technical units for vertical plates of less than 2 or 3 feet in height:

$$h = 0.275\sqrt[4]{\frac{\Delta T}{L}} \tag{87}$$

The height-limits correspond to the ΔT -range of the data which King correlated.

Jacob and Linke (217) suggested that when $4 < \log_{10}(Gr \cdot \sigma) < 8$, the formula

$$Nu = 0.555\,(Gr \cdot \sigma)^{\frac{1}{4}} \tag{88}$$

be used, and when $8 < \log_{10}(Gr \cdot \sigma) < 12$, the formula

$$Nu = 0.129\,(Gr \cdot \sigma)^{1/3} \tag{89}$$

be used.

According to the most recent measurements of Touloukian, Hawkins and Jakob (231, 1948) on vertical cylinders in water and ethylene glycol, with σ ranging from 2.4 to 117.8 and Gr from 22 (10^6) to 326 (10^9), the results may be represented as follows:

$$Nu = 0.726 \ (Gr \ . \sigma \ )^{\frac{1}{4}} \quad (0.2(10^9) < Gr \ \sigma < 40(10^9)\ ) \tag{90}$$

$$Nu = 0.0674 \ (Gr \ . \sigma^{1.29})^{1/3} \quad (40\ (10^9) < Gr \ . \sigma < 900(10^9)) \tag{91}$$

both with a maximum deviation of about $\pm$ 10%. Correlation with

$$Nu = 0.086 \ (\ Gr \ . \sigma \ )^{1/3} \tag{92}$$

was possible with about the same maximum deviation, but with a considerable systematic deviation. The authors claimed that since the cylinder diameter was 2.75 inches the influence of curvature was probably so small that the results will not differ appreciably from those for vertical plates.

(b) Horizontal plates

Jakob and Linke (217) showed that the form of Eq 89 can also be used for the turbulent convection on the upper side of horizontal plate, according to their experiments with water boiling on such surfaces. Their results can be represented by the formula

$$Nu = 0.273 \ (Gr \ . \sigma \ )^{1/3} \tag{93}$$

which gives a Nu more than twice as large as that given by Eq 89. This difference is probably due to the better mechanism of heat transfer from horizontal plates, and due to the boiling of the liquid.

From the experiments of Griffiths and Davis (213) it can be shown that the rate of heat transfer for a horizontal plate, facing upward in air, is, in British technical units.

$$q_{h.u} = 0.275 \ (\Delta \ T)^{4/3} \tag{94}$$

Compared with the same authors' values of q_v for vertical plates 2 and 8 2/3 feet high, one has

$$q_{h.u} \approx 1.28 \ q_v \tag{95}$$

the difference being only 28% instead of the more than 100% found for boiling water.

For horizontal plates facing downward, the same authors found

$$q_{h.d} = 0.50 \ q_{h.u} \tag{96}$$

The factor 0.50 might be even smaller but for the secondary influences at the edge of the plate. Taking mean,

$$\frac{q_{h.u} + q_{h.d}}{2} = 0.96\, q_v \approx q_v \tag{97}$$

which is true of course only when the flow is turbulent.

Experiments to determine the temperature distribution in the air surrounding a heated horizontal plate were done by R. Weise (232, 1935), who used square aluminum plates, about 16 and 24 cm in side length, 1.0 and 1.5 cm. thick, respectively, which were hung up in a wide room and heated electrically. The boundary layer thickness is about 1.5 cm in the center below the plate and 1 cm close to the edge. Detailed data are presented in the form of isotherms. These data are not useful for the case of free evaporation from the surface of a lake, since the heated plate has two surfaces in contact with the atmosphere.

In October, 1944, G. H. Hichox published his University of California Dissertation (106), a part of which dealt with evaporation in still air. He used an evaporation pan 1 ft in diameter and an interferometer to measure the fall of the water surface. From his experimental data he suggested the formula

$$Nu' = 0.645\,(Gr'\sigma')^{1/4} \tag{98}$$

where

Nu' = the nusselt number for evaporation = $\dfrac{h'_m L}{K} = \dfrac{E\,L}{K\,\Delta C}$

Gr' = the Grashof number for evaporation = $\dfrac{L^3\,\gamma\,\Delta\gamma}{g\,\mu^2}$

σ' = the Prandtl number for evaporation = $\dfrac{\nu}{K}$

h'_m being the mean coefficient of vapor transfer, E being the mass transferred per unit time per unit area, L being a representative length which can be taken to be the diameter d in this case, K being the vapor diffusivity, μ and ν having the usual meanings,

Δc being equal to the difference of the saturation concentration c_1 at the prevailing temperature and the concentration c_0 of the ambient air, γ being the specific weight of the ambient air and $\Delta\gamma$ being corresponding to ΔC. It must be noted that Eq 98 is of the same form as Eqs 88 and 90, and that the coefficient 0.645 in Eq 98 is almost the mean of the coefficients 0.555 and 0.726 occurring in Eqs 88 and 90, respectively, in spite of the difference in orientations of the evaporation surface.

Since σ and g are practically constant, it can be concluded from Eq 98 that

$$E \sim \Delta c \left(\frac{\mu^2 \Delta c}{\gamma^3 L} \right)^{0.25} \tag{99}$$

Since $\gamma \sim \frac{p}{T}$, and $\mu \sim \sqrt{T}$ approximately, the above proportionality further gives

$$E \sim (\Delta c)^{1.25} \frac{T}{L^{0.25} p^{0.75}} \tag{100}$$

Eq 98 apparently corresponds to laminar flow only. When the flow is turbulent, the exponent 1/4 in Eq 98 can be expected to change to 1/3, and the rate of evaporation E can be expected to be independent of L, as in the case of heat transfer.

The effect of the position of the water surface in the evaporation pan can be systematically treated by allowing the pertinent dimensionless parameter to vary (See Part IV). In this way more conclusive results for pan evaporation can be obtained.

Transpiration rate from a leaf-shaped surface as a function of temperature and relative humidity has been measured by Emmett Martin (222,1943).

(c) Horizontal and vertical cylinders

Correlation by Nusselt (226,1915) shows that for $(Gr.\sigma) < 10^{-5}$

$$h \approx 0.4 \frac{k}{D} \tag{101}$$

and for $(Gr.\sigma) > 10^{5}$

$$Nu = 0.52 \, (Gr.\sigma)^{\frac{1}{4}} \tag{102}$$

In the range between 10^{-5} and 10^{5}, the trend gradually varies from Eq 101 to 102. The Nu and Gr are of course based upon the diameter D.

The correlation by Jakob and Linke (217) shows that free convection from horizontal cylinders also obeys Eqs 88 or 89, according as the flow is laminar or turbulent.

When the diameters are large enough for the curvatures to be neglected, free convection from vertical cylinders obeys the same equations as free convection from vertical plates. Correlation by Jakob and Linke (217) shows that turbulent free convection from vertical cylinders obeys Eq 89. As has been mentioned in 5 a, more recent results obey Eqs 90 and 91.

The heat transfer from vertical wires, 15.5 to 12.3 inches long, to air by free convection has been investigated by Meuller (223, 1942) in the range of temperature differences from 1 to 100° C. No effect of wire length was noticed, while the effect of the diameter was marked. Based on the diameter, the pertinent parameters obey the relation

$$Nu = (Gr.\sigma)^{0.1} \tag{103}$$

in the range $10^{-7} \leq Gr.\sigma \leq 10^{-2}$ and possibly up to $Gr.\sigma = 1$.

For free convection in enclosed plane and cylindrical gas layers, see (32,1949), pp. 534-542.

(d) Point source of heat

The laminar case of the free convection due to a point source of heat has been treated in 2(b). When the flow is turbulent, Yih's experiments (234,1948) give the following results (the symbols that appeared in 2 (b) retaining their meanings):

(i) For temperature distribution

$$\frac{\gamma_0}{T_0}\left(\frac{x^5}{\rho G^2}\right)^{\frac{1}{3}}(T-T_0) = 11.0\, e^{-\frac{1}{2}\left(\frac{r}{0.084x}\right)^2} \tag{104}$$

(ii) For the distribution of the longitudinal velocity

$$\left(\frac{\rho x}{G}\right)^{1/3} u = 4.7\, e^{-\frac{1}{2}\left(\frac{r}{0.072x}\right)^2} \tag{105}$$

(iii) For the Stokes' streamfunction

$$\psi = 0.0244\left(\frac{Gx^5}{\rho}\right)^{\frac{1}{3}}\left[1 - e^{-\frac{1}{2}\left(\frac{r}{0.072x}\right)^2}\right] \tag{106}$$

(iv) For discharge through a horizontal section at an elevation x:

$$Q = 0.153\left(\frac{Gx^5}{\rho}\right)^{\frac{1}{3}} \tag{107}$$

(v) For the momentum flux through a horizontal section at an elevation x:

$$M = 0.36\,(G^2\rho)^{\frac{1}{3}}\, x^{4/3} \tag{108}$$

(vi) For the height h at which transition from laminar to turbulent flow occurs:

$$\frac{\rho^2 h^2 G}{\mu^3} = 9\times 10^9 \tag{109}$$

It may be remarked that in Yih's experiments, low flames were used for the heat source. The mass-source and momentum-source characters of the flames are believed to be negligible at sufficiently high elevations.

6. Concluding remarks

From the foregoing, the following remarks can be made:

(a) Equations for turbulent free convection have not been mathematically formulated, while the well formulated ones for laminar free convections are in general difficult to solve.

(b) Due to the many variables involved in free convection, dimensional analysis is particularly important in this branch of research.

(c) So far, the physical "constants" except γ and ρ have been considered as invarient. Future research should be concerned with the effect of the changes in these "constants." This is more true in the case of laminar flow where ν and the (heat or vapor) diffusivity play important roles, than in the case of turbulent flow.

7. Bibliography

19. Goldstein, S : Modern Developments in Fluid Dynamics, Vol. II, Oxford Clarenden Press, 1938. (Th. and Exp.)

32. Jakob, M.: Heat Transfer, Vol. I, Wiley, 1949. (Th. and Exp.)

38. McAdams, W. H.: Heat Transmission, 2nd Ed., McGraw-Hill Book Company, New York, p. 243, 1942. (Th. and Exp.)

105. Hickox, G. H.: Evaporation from a free water surface, Univ. of Calif. Thesis, 1939. (Exp. and correlation of data then existing)

166. Rohwer, C.: Evaporation from free water surfaces. Tech. Bulletin No. 271, U. S. Dept. of Agric., 1931. (Exp.)

208. Beckmann, W.: Forschung a. d. Geb. d. Ingenieurwes. 2, p.165, p. 213, p. 407, 1931. (Exp.)

209. Boussinesq, J.: Journ - de physique (4) 1, p. 65, 1902.

210. Davis, A. H.: Phil. Mag. (6) 44, 920, 1922.

211. Dulong and petit: Annales de chim, et physique 1, p.225 and p. 337, 1817. (Exp.)

212. Fishenden, Margaret, and Saunders, O. A.: Engg. 130, 177, 193, 1930. (Exp.)

213. Griffiths, E., and Davis, A. H.: The transmission of heat by radiation and convection, Department of Scientific and

Industrial Research, Food Investigation Board, Special Rep. No. 9, His Majesty's Stationery Office, London, 1922. (Exp.)

214. Hermann, R.: Physik, Zeitschr, 33, p.425, 1932.

215. Jakob, M.: Trans. A. S. M. E. 68, p. 189, 1946. (Exp.)

216. Jakob, M., and Fritz, W.: Forsch. a. d. Geb. d. Ing. 2, p. 435, 1931. (Exp.)

217. Jakob, M., and Linke, W.: Forsch, a. d. Geb. d. Ing. 4, p. 75, 1933. (Exp.)

218. King, W. J.: Mech. Engg. 54, p. 347, 1932. (Correlation of existing data).

219. Koch, W.: Gesundh. -Ing. Beihefte, Reihe 1, No. 22, 1927. (Exp.)

220. Kraussold, H.: Forsch. a. d. Geb. d. Ing. 5, p. 186, 1934. (Exp.)

221. Lorenz, L.: Wiedemanns Annaler 13, p. 581, 1881. (Th.)

222. Martin, E.: Studies of evaporation and transpiration under controlled conditions, Carnegie Institute of Washington Publication 550, Wash. D. C., 1943. (Exp.)

223. Mueller, A. C.: Trans. Am. Inst. Chem. Engrs. 38, p. 613, 1942. (Exp.)

224. Mull, W., and Reiher, H.: Gesundh -Ing. Beihefte, Reihe 1, No. 28, 1930. (Exp.)

225. Nusselt, W.: Forsch, a. d. Geb. d. Ingenieurwes., No. 63, p. 81, 1909. (Exp.)

226. Nusselt, W.: Gesundh -Ing. 38, p.477 and p. 490, 1915. (Correlation of exp. data)

227. Nusselt, W.: Zeilsch, d. Ver. deutsch, Ing. 73, p.1475, 1929. (Exp.)

228. Nusselt, W. and Juerges, W.: Zeitschr. d. Ver. deutsch. Ing. 72, p. 597, 1928. (Th.)

229. Saunders, O. A.: Proc. Ray. Soc. London (A) 157, p. 278, 1936. (Exp.)

230. Schmidt, E., and Beckmann, W.: Techn. Mech. U. Thermodynamik 1, p. 341 and p. 391, 1930. (Exp. and Th.)

231. Touloukian, Y. S., Hawkins, G. A., and Jakob, M.: Trans. A. M. S. E. 70, p. 13, 1948. (Exp.)

232. Weise, R.: Forsch. a. d. Geb. d. Ing. 6, p. 281, 1935. (Exp.)

233. Wilkes, G. B., and Peterson, C. M. F.: Heating, Piping, Air Conditioning 9, p.505, 1937. (Exp.)

234. Yih, C. S.: Free convection due to a point source of heat, Univ. of Iowa Diss., 1948. (Th. and Exp.)

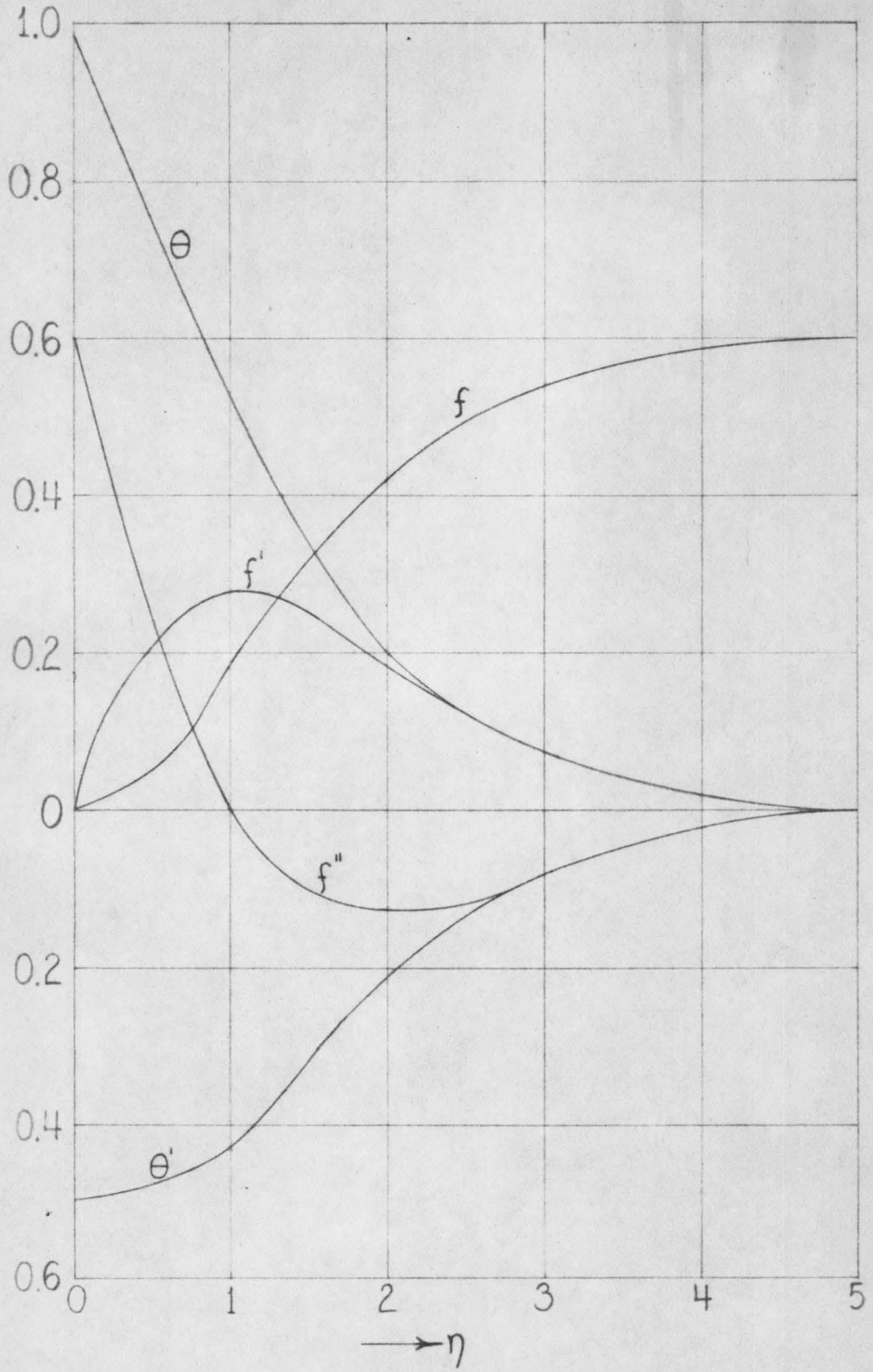

FIG. 1

RESULTS OF POHLHAUSEN'S SOLUTION

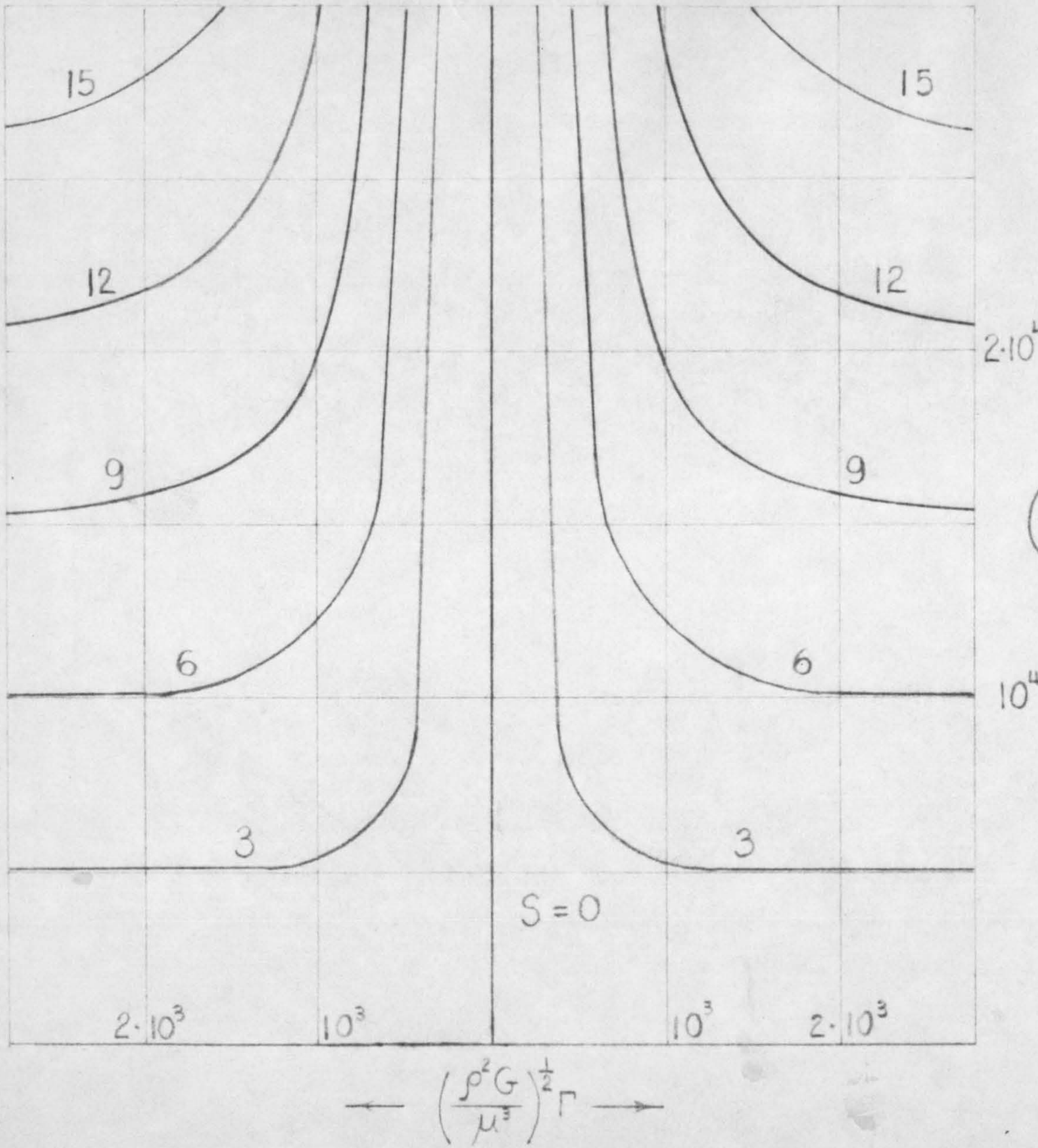

FIG. 2 LAMINAR FLOW PATTERN FOR POINT SOURCE OF HEAT

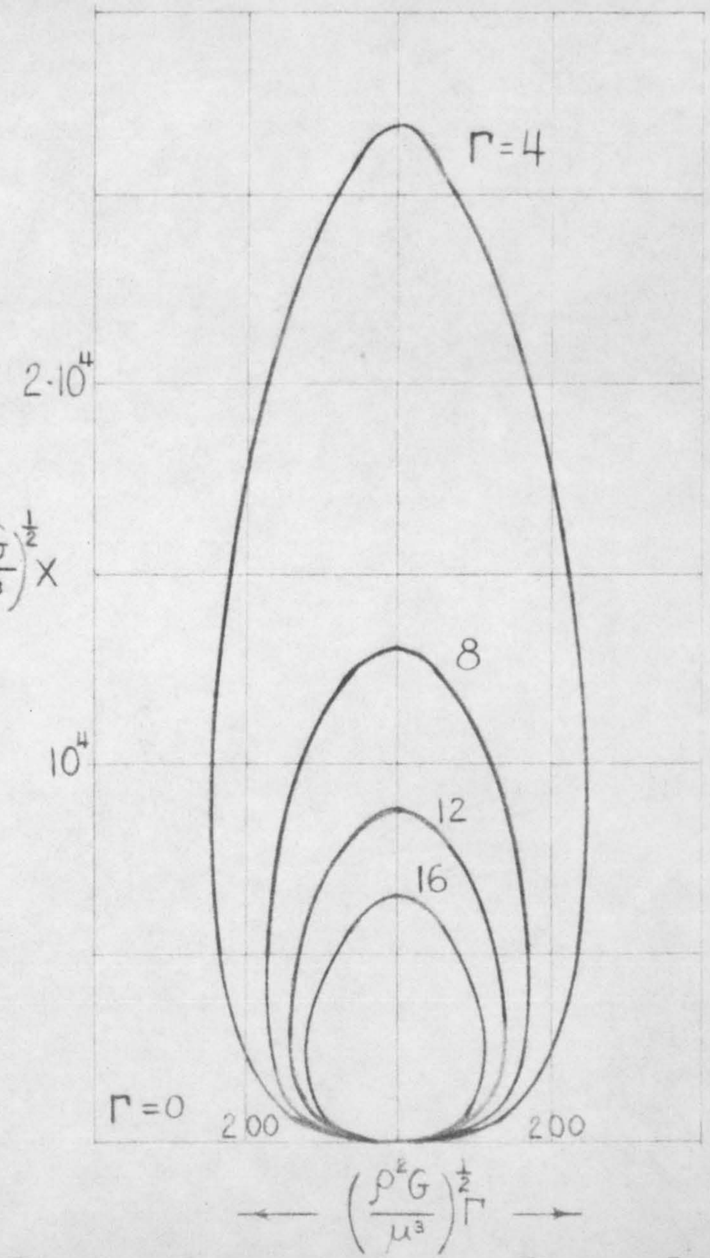

FIG 3 ISOTHERMS FOR POINT SOURCE OF HEAT