THESIS

LOW-SENSITIVITY ACTIVE FILTER DESIGN

Submitted by

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ABSTRACT OF THESIS

LOW-SENSITIVITY ACTIVE FILTER DESIGN

At high frequencies, the performance of active filters is impaired by the frequency dependent nature of the operational amplifiers. Much research has been devoted to the design of active filters with reduced dependence on the active devices; however no general analysis scheme or method of comparing existing filters has emerged.

A general characterization of second-order active-RC filters employing one, two, and three operational amplifier(s) is given. A method of comparing active filters with respect to the active devices based on the active sensitivity function is introduced. Conditions necessary for zero pole, ω_o, Q, and transfer-function-magnitude active sensitivities are derived. Several novel circuits that possess these zero-active-sensitivity properties are presented. The markedly superior performance of these filters is demonstrated by a comparison of second-order zero-active-sensitivity filters with other popular existing realizations. Experimental results that agree favorably with the theoretical results of these filters are presented.

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CHAPTER I

INTRODUCTION

Active RC filters are filters constructed with resistors, capacitors, and some type of amplifying device. For low frequency applications they are often more economical to construct, smaller in size, and easier to tune than their passive counterparts and, as a result, much research has been devoted to active-RC filter design. Many active-RC filter designs have appeared in the literature since the landmark paper of Sallen and Key [1] in 1955, but no dominant general design procedure has emerged.

Cost, size, tunability, component spread, and performance are some of the factors that must be considered when comparing two active filters that ideally have the same transfer function. This thesis addresses the performance of active RC filters. In active RC filters, the critical transfer-function parameters, such as the ω_o and Q of the poles and zeros, are determined by resistors, capacitors and the characteristics of the amplifying device (alternately, active device) itself. Consequently, changes in these circuit components from the values specified in the design will cause a change in the critical transfer-function parameters. In practical designs, the component values will vary from those specified in the design due to manufacturing tolerances, aging, temperature, etc. Many circuit designs exist that will realize a given transfer function; the critical parameters of some are affected much less by small changes in circuit components than others.

The sensitivity function is useful for comparing filters with respect to the passive components as it relates the changes in transfer-function parameters to the changes in the passive components. Many circuit designs exist with acceptably low passive sensitivities. In many designs the change in transfer-function parameters due to small changes in the passive components is small compared to those that result from changes in the properties of the active device.

Most recent active filter designs use an operational amplifier (OP AMP) as the active device. An ideal OP AMP has infinite input impedance, zero output impedance, infinite common-mode rejection ratio, and an infinite voltage gain. OP AMPs will be the only active devices discussed in this thesis.

Prior to 1972, most active filter designers assumed the OP AMP to be ideal. As a result, the experimentally observed performance of many of these designs differed significantly from that predicted by the design. A model of the internally compensated OP AMP was presented by Budak and Petrela [2] and others in 1972 and has been incorporated in most designs in recent years. The use of this model has resulted in quite close agreement of theoretical and experimental results. A discussion of this model is included later in this chapter.

Since the parameters of this model vary considerably from device to device, and since even the parameters of a specific device are affected by temperature, supply voltages, aging, signal levels, etc., the critical transfer-function parameters of a good active filter design should be insensitive to changes in the characteristics of the active device.

There is not an agreement among active filter designers on a method for comparing, with respect to the active device, any two active RC filters that ideally have identical transfer functions. A host of literature [3] - [21] has appeared in recent years discussing and introducing circuits in which some parameters are less dependent on the characteristics of the active devices than the circuits of Sallen and Key. However, these circuits all have one or more critical parameter(s) whose dependence on the active devices is of the same order of magnitude as the circuits of Sallen and Key.

Contributions of This Thesis

The active sensitivity function is introduced and shown to be a figure of merit that can be used to compare with respect to the active devices any two active filter designs that have ideally identical transfer functions. Designs with low active sensitivity are less dependent on the active-device characteristics than those with high active sensitivities.

The ω_o and Q of the poles and zeros, as well as the transfer-function magnitude of a transfer function, are affected by the active devices. The degree of the dependence is a function of the actual pole-zero locations, which are in turn functions of the characteristics of the available OAs. In some designs, only the poles are affected by the characteristics of the active devices. In this thesis, conditions are established to force the active sensitivities of either the poles or zeros or both to vanish. Several design examples are included in which both the zero and pole sensitivities, with respect to the active elements, are zero. These designs are compared with popular existing designs on a basis of incremental as well as infinitesimal changes in the characteristics of the OP AMPs. It is shown that any realizable transfer function can be obtained from a circuit with zero active sensitivities. Prior to this investigation, no active RC filters possessing this property were given. These zero-active-sensitivity properties are obtained without requiring matched OP AMPs.

Most existing two-input amplifiers ideally have an infinite common-mode rejection ratio and thus, identical gain functions on each input. The usefulness in active filter design of a two-input amplifier with a different gain function for each input is demonstrated.

Development of Notation

In a linear single-input single-output system, the Laplace transform of the output, R(s), is related to the Laplace transform of the input, E(s), by

$$R(s) \quad = \quad T(s)\ E(s) \tag{1}$$

where T(s) is dependent only upon the system and hence is independent of the particular input E(s). T(s) is called the transfer function of the system. In the case that the system is composed of a finite number of lumped circuit components and amplifiers, the transfer function is the ratio of two polynomials with real coefficients,

$$T(s) \quad = \quad \frac{\sum_{i=0}^{m} a_i\ s^i}{\sum_{j=0}^{n} b_j\ s^j} \quad = \quad \frac{N(s)}{D(s)} \tag{2}$$

where $a_m \neq 0$ and $b_n = 1$.

The poles and zeros of the transfer function T(s) are defined as the roots of D(s) and N(s), respectively. The transfer function, T(s), may be expressed in terms of the poles and zeros by

$$T(s) \quad = \quad \frac{a_m \prod_{i=1}^{m} (s-z_i)}{\prod_{j=1}^{n} (s-p_j)} \ . \tag{3}$$

The order of the transfer function is defined to be the maximum of m and n.

In the case that the order of the transfer function is two, the transfer function is said to be biquadratic. It is immediate that T(s) may be written as a product of biquadratic transfer functions with real coefficients and, at most, one first-order transfer function. In general, this decomposition is not unique.

Many higher-order transfer functions are realized by cascading biquadratic and first-order sections. Some advantages of cascaded biquadratic designs are the use of a common biquadratic building block and ease of tuning. Some direct designs may be less sensitive to changes in component values and require fewer components than the cascaded biquad designs.

Since the degree of the biquadratic transfer function is two, it may be expressed in the form

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} . \tag{4}$$

Two biquadratic-transfer-function parameters of interest are obtained when T(s) is written in the form

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s\omega_o/Q + \omega_0^2} . \tag{5}$$

ω_0 is called the pole frequency and Q is called the pole Q.

If a sinusoidal input A sin ωt is applied to a system with transfer function T(s), the steady state response is given by

$$r(t) = |T(j\omega)| \cdot A \cdot \sin(\omega t + \angle T(j\omega)). \tag{6}$$

The frequency-dependent amplitude-scaling factor $|T(j\omega)|$ is called the transfer function magnitude and $\angle T(j\omega)$ the phase of the system.

Sensitivity

If f is a function and a is a variable, then the sensitivity of f with respect to a is defined as

$$S_a^f = \frac{\partial f}{\partial a} \, . \tag{7}$$

Many authors define the sensitivity function by

$$S_a^f = \frac{a}{f} \frac{\partial f}{\partial a} \, . \tag{8}$$

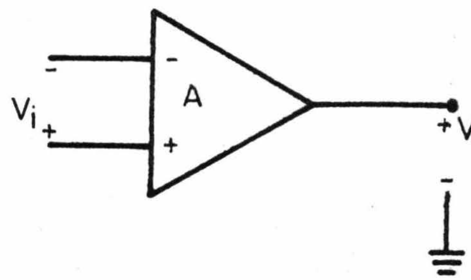
Some advantages of the normalization in the latter definition are that the sensitivity expressions are often less involved and that the sensitivity directly approximates the ratio of the percentage change in f due to a given percentage change in a. The former definition, however, has been adopted since it is useful when a or f vanish; a situation that is later shown to exist when comparing active filters.

Operational Amplifier Model

The active device in all filters discussed here is an internally compensated operational amplifier with infinite input impedance, zero output impedence, infinite common-mode rejection ratio, and is characterized by the transfer function [2].

$$A(s) = \frac{V_o}{V_i} = \frac{1}{\tau s} \, . \tag{9}$$

The symbol for the operational amplifier is shown in Fig. 1. The time constant, τ, is related to the gain-bandwidth product, GB, of the OP AMP



$$A = \frac{V_o}{V_i} = \frac{1}{\tau s}$$

Fig. 1. Operational Amplifier

(typical values of which are usually included in manufacturer specification sheets) by the expression

$$\tau \;=\; 1/GB. \tag{10}$$

The OP AMP thus is characterized by a single parameter, τ. In the case that $\tau = 0$ the OP AMP as said to be <u>ideal</u>.

CHAPTER II

SENSITIVITY CONSIDERATIONS

Active Sensitivity--A Figure of Merit

Let f be a transfer function parameter of an active RC filter consisting of r resistors, c capacitors and m OP AMPs. Let $\vec{R}$, $\vec{C}$, and $\vec{T}$ be rx1, cx1, and mx1 column vectors, respectively, of the component values specified in the active-RC filter design. Also, let $\vec{R} + \vec{R}_p$, $\vec{C} + \vec{C}_p$, and $\vec{T} + \vec{T}_p$ be the actual component values used in the design. Define the (r + c + m) x 1 component vectors $\vec{P}$ and $\vec{P}_p$ by the expressions

$$\vec{P} = \begin{pmatrix} \vec{R} \\ \hline \vec{C} \\ \hline \vec{T} \end{pmatrix} \quad \text{and} \quad \vec{P}_p = \begin{pmatrix} \vec{R}_p \\ \hline \vec{C}_p \\ \hline \vec{T}_p \end{pmatrix} . \tag{11}$$

The function, f, is a function of $\vec{P} + \vec{P}_p$ where $\vec{P}_p$ is ideally the (r + c + m) x 1 zero vector. If f is expanded in a Maclaurin series in terms of the r + c + m independent variables, one obtains

$$\begin{aligned} f(\vec{P}+\vec{P}_p) &= f(\vec{P}) + \sum_{i=1}^{r+c+m} \left(\frac{\partial f}{\partial p_i}\right) \Bigg|_{\vec{P}_p=0} p_{p_i} + \sum_{i=1}^{r+c+m} \sum_{j=1}^{r+c+m} \frac{\partial^2 f}{\partial p_i \, \partial p_j} \Bigg|_{\vec{P}_p=0} p_{p_i} p_{p_j} \\ &+ \sum_{i,\, j,\, k=1}^{r+c+m} \sum \sum \frac{\partial^3 f}{\partial p_i \, \partial p_j \, \partial p_k} \Bigg|_{\vec{P}_p=0} p_{p_i} p_{p_j} p_{p_k} + \cdots \end{aligned} \tag{12}$$

where p_{p_i} is the ith element of $\vec{P}_p$.

A first-order approximation of f is obtained by neglecting all second-order and higher terms. A first-order approximation is thus

$$f(\vec{P} + \vec{P}_p) \simeq f(\vec{P}) + \sum_{i=1}^{r+c+m} \left(\frac{\partial f}{\partial p_i}\right) \Bigg|_{\vec{P}_p=0} p_{p_i} . \tag{13}$$

This may be expressed in terms of the sensitivity function defined in (7) by the expression

$$f(\vec{P} + \vec{P}_p) \simeq f(\vec{P}) + \sum_{i=1}^{(r+c+m)} S^f_{p_i} \Big|_{\vec{P}_p=0} p_{p_i} \; . \tag{14}$$

The right hand side of (14) is equal to the desired part $f(\vec{P})$ plus the parasitic part

$$\sum_{i=1}^{(r+c+m)} S^f_{p_i} \Big|_{\vec{P}_p=0} p_{p_i} \; . \tag{15}$$

Although no assumption on the form of the statistical distribution from which the p_p's comes is made, if $\vec{P}_p$ is assumed to be a random variable, the variance of the first-order approximation of $f(\vec{P} + \vec{P}_p)$ is given by

$$\mathrm{var}(f(\vec{P} + \vec{P}_p)) = \sum_{i=1}^{(r+c+m)} \left(\left| S^f_{p_i} \Big|_{\vec{P}_p=0} \right| \mathrm{var}\; p_{p_i} \right) \tag{16}$$

where it is safely assumed that the random variables p_{p_i} and p_{p_j} are independent for $i \neq j$. To avoid cluttering the notation, no distinction is made between the random variable $\vec{P}_p$ and the sample point $\vec{P}_p$; the distinction should be clear from the context. The term $\mathrm{var}(p_{p_i})$ is dependent only on the available circuit components and is always nonnegative. Thus, the term $\mathrm{var}(f(\vec{P} + \vec{P}_p))$ is an increasing function of the configuration dependent quantity $\left| S^f_{p_i} \Big|_{\vec{P}_p=0} \right|$ for $i=1,\ldots r+c+m$.

The merit of the function $S^f_{p_i}$, $i=1,\ldots r+c+m$ for comparing active filter sections is immediate. In particular, the <u>active sensitivity</u> defined by

$$S^f_{t_i} = \frac{\partial f}{\partial t_i} \tag{17}$$

is a figure of merit for comparing active-RC filter sections with respect to the active devices. Unless stated otherwise, the term sensitivity is to be interpreted as active sensitivity throughout the remainder of this thesis.

Two schools of thought exist on the design of the active RC filter itself. One assumes that the OP AMPs are ideal for the determination of the resistor and capacitor values in the circuit. With this assumption the $\vec{T}$ vector defined above is the zero vector and the vector $\vec{T}_p$ is composed of the actual OP AMP time constants, that is, $t_i=0$ and $t_{p_i}=\tau_i$, $i = 1, \ldots m$.

The other assumes that the parameters of the OP AMP are equal to their expected (or in some cases measured) value when the resistor and capacitor values are determined. This is actually a form of predistortion discussed by several authors [20, 22]. Since the variance of the OP AMP time constant is large and dependent upon the environment, little is gained from such designs; the design itself is, in general, more involved, often requiring the use of a digital computer. Since $\vec{R}_p$ and $\vec{C}_p$ are usually assumed to have a zero mean, it follows from (12) that for the latter case the expected value of $f(\vec{P} + \vec{P}_p)$ is equal to $f(\vec{P})$. The estimate of $f(\vec{P} + \vec{P}_p)$ is, in general, not unbiased in the former case; however, in most good designs the bias is quite small. Nonetheless, the variance is the same in both cases. Unless stated otherwise, it is assumed that $\vec{T}$ is the zero vector in this thesis and hence, $\vec{T}_p$ is the vector of OP AMP time constants. This assumption is by far the most popular and does keep the analysis somewhat tractable.

Although a comparison of active RC filters should be made on the basis of changes in critical parameters due to both active and passive components, this thesis only discusses comparisons with respect to the active devices; comparisons with respect to the passive components have been quite successfully made by many others in the past twenty years. The assumption will thus be made that the resistors and capacitors are ideal. To see the effect of the nonideal OP AMPs on the active-filter transfer-function parameter, f, (12) thus reduces to

$$f(\vec{T}_p) = f(\vec{T}_p=0) + \sum_{i=1}^{m} \frac{\partial f}{\partial \tau_i}\Bigg|_{\vec{T}_p=0} \tau_i + \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial^2 f}{\partial \tau_i \partial \tau_j}\Bigg|_{\vec{T}_p=0} \tau_i \tau_j$$

$$+ \sum_{i,}\sum_{j,}^{m}\sum_{k=1} \frac{\partial^3 f}{\partial \tau_i \partial \tau_j \partial \tau_k}\Bigg|_{\vec{T}_p=0} \tau_i \tau_j \tau_k + \cdots \tag{18}$$

and the first-order approximation of (14) reduces to

$$f(\vec{T}_p) \simeq f(\vec{T}_p=0) + \sum_{i=1}^{m} S_{\tau_i}^{f}\Bigg|_{\vec{T}_p=0} \tau_i \quad . \tag{19}$$

Sensitivity Relations

For any circuit component, q, the transfer function may be written in the form

$$T(s) = \frac{N_o(s) + qN_1(s)}{D_o(s) + qD_1(s)} = \frac{N(s)}{D(s)} \tag{20}$$

where N_o, N_1, D_o, and D_1 are polynomials in s independent of q. The pole sensitivity of the pole p_i is defined by (7) as

$$S_q^{p_i} = \frac{\partial p_i}{\partial q} \; . \tag{21}$$

Since $D_o(p_i) + qD_1(p_i) = 0$, an implicit differentiation of this equation with respect to q yields

$$\frac{\partial D_o(p_i)}{\partial p_i} \frac{\partial p_i}{\partial q} + q \frac{\partial D_1(p_i)}{\partial p_i} \frac{\partial p_i}{\partial q} + D_1(p_i) = 0. \tag{22}$$

Solving for $\frac{\partial p_i}{\partial q}$, the following expression is obtained

$$S_q^{p_i} = \frac{-D_1(p_i)}{\frac{d}{dp_i} D(p_i)} \tag{23}$$

where it has been assumed that p_i is a simple pole to enable division by $\frac{d}{dp_i} D(p_i)$ in (22). An analogous expression for the sensitivities of the zeros can also be written.

The ω_o and Q sensitivities will now be derived. From (5), the pole p_i may be expressed in terms of ω_o and Q by the expression

$$p_i = \frac{1}{2} \omega_o \left[-\frac{1}{Q} \pm i\sqrt{4 - \frac{1}{Q^2}}\right]. \tag{24}$$

Upon differentiating this equation by q, it follows that

$$S_q^{\omega_o} = \omega_o \operatorname{Re}\left[\frac{1}{r_i} S_q^{r_i}\right] \tag{25}$$

and

$$S_q^Q = -Q^2\sqrt{4 - \frac{1}{Q^2}} \operatorname{Im}\left[\frac{1}{r_i} S_q^{r_i}\right]. \tag{26}$$

The transfer-function-magnitude sensitivity will now be computed two ways. The first, which will be stated in the form of a theorem, relates the transfer-function-magnitude sensitivity to the root sensitivities. The second is much easier to derive and apply. Before the theorem is stated, some notation must be developed. T(s) as given in (3) can be decomposed into the product of the desired and parasitic transfer functions defined by the expression

$$T(s) = T_D(s) \cdot T_P(s) = \left[\frac{a_{m_1} \prod_{i=1}^{m_1} (s-z_i)}{\prod_{j=1}^{n_1} (s-p_j)}\right] \cdot \left[\frac{a_{m_2} \prod_{i=m_1+1}^{m} (s-z_i)}{\prod_{j=n_1+1}^{n} (s-p_j)}\right] \tag{27}$$

where

$$T_P(s)\Big|_{\vec{T}_p=0} = \left[\frac{a_{m_2} \prod_{i=m_1+1}^{m} (s-z_i)}{\prod_{i=n_1+1}^{n} (s-p_i)}\right]\Bigg|_{\vec{T}_p=0} = 1, \tag{28}$$

a_{m_1} is independent of $\vec{T}_p$, and where

$$z_i\Big|_{\vec{T}_p=0} = p_j\Big|_{\vec{T}_p=0} = \infty \quad \text{for} \quad m \geq i > m_1, \text{ and } n \geq j > n_1. \tag{29}$$

Theorem: If τ is an OP AMP time constant, then

$$S_\tau^{|T(j\omega)|}\Bigg|_{\vec{T}_p=0} = |T(j\omega)| \; \mathrm{Re}\left[\sum_{i=1}^{n_1} \frac{S_\tau^{p_i}}{s-p_i} - \sum_{j=1}^{m_1} \frac{S_\tau^{z_i}}{s-z_j}\right]\Bigg|_{\vec{T}_p=0} \tag{30}$$

Proof: For any s and any $\vec{T}_p$, the poles and zeros of the parasitic factor, $T_p(s)$, may be decomposed into the four disjoint sets:

$G_{cz} = \{z_i | z_i$ is a zero of $T_p(s)$ with nonzero immaginary part$\}$

$G_{rz} = \{z_i | z_i$ is a zero of $T_p(s)$ with zero immaginary part$\}$

$G_{cp} = \{p_i | p_i$ is a pole of $T_p(s)$ with nonzero immaginary part$\}$

$G_{rp} = \{p_i | p_i$ is a pole of $T_p(s)$ with zero immaginary part$\}$.

Since G_{cp} and G_{cz} contain only complex conjugate pairs, $T_p(s)$ may be written in the form

$$T_p(s) = a_{m_3} \cdot \prod_{\{p_i,p_i^*\}\varepsilon G_{cp}} \left[\frac{p_i p_i^*}{(s-p_i)(s-p_i^*)}\right] \cdot \prod_{\{z_i,z_i^*\}\varepsilon G_{cz}} \left[\frac{(s-z_i)(s-z_i^*)}{z_i z_i^*}\right]$$

$$\cdot \prod_{p_i \varepsilon G_{rp}} \left(\frac{p_i}{s-p_i}\right) \cdot \prod_{z_i \varepsilon G_{rz}} \left(\frac{s-z_i}{z_i}\right) \tag{31}$$

where

$$a_{m_3} = a_{m_2} \frac{\pi z_i z_j^*}{\pi p_i p_j^*} . \tag{32}$$

It thus follows that

$$\mathrm{Log}\ T_p(s) = \sum \mathrm{Log}\left(\frac{p_i p_i^*}{(s-p_i)(s-p_i^*)}\right) + \sum \mathrm{Log}\left(\frac{(s-z_i)(s-z_i^*)}{z_i z_i^*}\right)$$

$$+ \sum \mathrm{Log}\left(\frac{p_i}{s-p_i}\right) + \sum \mathrm{Log}\left(\frac{s-z_i}{z_i}\right) + \mathrm{Log}\ a_{m_3} + 2\pi ik \tag{33}$$

where the function Log(x) is the principle value of the logarithmic function and the integer k is the constant necessary to make (33) valid. The ranges on the summations in (33) are obvious from (31). Differentiation of (33) with respect to a parameter q results in the expression

$$\frac{\partial T_p(s)}{\partial q} = T_p(s) \cdot \left\{ -s\left[\sum\left(\frac{\frac{\partial p_i}{\partial q}}{p_i(s-p_i)} + \frac{\frac{\partial p_i^*}{\partial q}}{p_i^*(s-p_i^*)}\right) - \sum\left(\frac{\frac{\partial z_i}{\partial q}}{z_i(s-z_i)} + \frac{\frac{\partial z_i^*}{\partial q}}{z_i^*(s-z_i^*)}\right)\right.\right.$$

$$\left.\left. + \sum \frac{\frac{\partial p_i}{\partial q}}{p_i(s-p_i)} - \sum \frac{\frac{\partial z_i}{\partial q}}{z_i(s-z_i)} \right] + \frac{1}{a_{m_3}} \frac{\partial a_{m_3}}{\partial q} \right\} . \tag{34}$$

But

$$\left.\frac{\frac{\partial p_i}{\partial q}}{p_i(s-p_i)} + \frac{\frac{\partial p_i^*}{\partial q}}{p_i^*(s-p_i^*)}\right|_{\vec{T}_p=0} = -\lim_{p_i \to \infty} \left(\frac{\frac{\partial p_i}{\partial q}}{p_i^2} + \frac{\frac{\partial p_i^*}{\partial q}}{p_i^{*2}}\right). \tag{35}$$

The complex root p_i may be expressed as

$$p_i = u + iv \tag{36}$$

where the real valued functions u and v are, in general, functions of q. It thus follows from (36) that

$$\frac{\frac{\partial p_i}{\partial q}}{p_i^2} + \frac{\frac{\partial p_i^*}{\partial q}}{p_i^{*2}} = \frac{2\frac{\partial v}{\partial q}(u^2 - v^2) + 4uv\frac{\partial v}{\partial q}}{|p_i|^4}. \tag{37}$$

From (35) and (37), it is concluded that

$$\mathrm{Re}\left[i\omega \left\{ \frac{\frac{\partial p_i}{\partial q}}{(s-p_i)p_i} + \frac{\frac{\partial p_i^*}{\partial q}}{(s-p_i^*)p_i^*} \right\} \Bigg|_{\vec{T}_p=0} \right] = 0. \tag{38}$$

A similar argument establishes the same result for the complex zeros, real poles, and real zeros in (33).

It is a well known and easily verified fact that

$$S_q^{|T(j\omega)|} = |T(j\omega)| \ \mathrm{Re}\left[\frac{1}{T(j\omega)} S_q^{T(j\omega)}\right]. \tag{39}$$

Upon taking the logarithm of (27) and differentiating with respect to q, one obtains

$$S_q^{T(s)} = T(s)\left[\frac{1}{a_{m_1}} S_q^{a_{m_1}} + \sum_{i=1}^{m_1} \frac{S_q^{p_i}}{s-p_i} - \sum_{i=1}^{n_1} \frac{S_q^{z_i}}{s-z_i} + \frac{1}{T_p(s)} \frac{\partial T_p(s)}{\partial q}\right]. \tag{40}$$

In the case that q is an OP AMP time constant, τ (an element of the $\vec{T}_p$ vector), it follows from the decomposition in (27) that $S_\tau^{a_{m_1}} = 0$. Furthermore, since the OP AMP gain function is $(\tau s)^{-1}$, it follows from (28) and (32) upon evaluating at s = 0 that $a_{m_3} = 1$ and is independent of τ. It may be thus concluded from (38), (39) and (40) that

$$S_{\tau}^{|T(j\omega)|}\Big|_{\vec{T}_p=0} = |T(j\omega)| \text{ Re} \left\{ \sum \frac{S_{\tau}^{p_i}}{s-p_i} - \sum \frac{S_{\tau}^{z_i}}{s-z_i} \right\} \Bigg|_{\vec{T}_p=0} . \tag{41}$$

The proof is now complete.

The second derivation of the transfer-function-magnitude sensitivity is obtained from (20) and (39). Equation (20), repeated for convenience, states that T(s) may be written as

$$T(s) = \frac{N_o(s) + q\, N_1(s)}{D_o(s) + q\, D_1(s)} . \tag{42}$$

Differentiating with respect to q, it follows that

$$S_q^{T(s)} = \frac{[D_o(s) + q\, D_1(s)]\, N_1(s) - [N_o(s) + q\, N_1(s)] D_1(s)}{[D_o(s) + q\, D_1(s)]^2} . \tag{43}$$

Consequently, it follows from (39) that

$$S_q^{|T(j\omega)|} = |T(j\omega)| \text{ Re} \left\{ \frac{N_1(j\omega)}{N_o(j\omega) + q\, N_1(j\omega)} - \frac{D_1(j\omega)}{D_o(j\omega) + q\, D_1(j\omega)} \right\} . \tag{44}$$

In the case that q is an OP AMP time constant, this equation reduces to

$$S_{\tau}^{|T(j\omega)|}\Big|_{\vec{T}_p=0} = |T(j\omega)| \text{ Re} \left\{ \frac{N_1(j\omega)}{N_o(j\omega)} - \frac{D_1(j\omega)}{D_o(j\omega)} \right\} . \tag{45}$$

Merits of the Root-Sensitivity Function

From (25), (26), and (30), it follows that the ω_o, Q, and transfer-function-magnitude active sensitivities may be written in terms of the root sensitivities, thus emphasizing the importance of the root-sensitivity function. In particular, it should be observed from these equations that if the root sensitivities all vanish, then so do the ω_o, Q, and transfer-function-magnitude sensitivities with respect to the time constant of the OP AMP. The conditions necessary for zero root sensitivities in active filter design are developed in the next chapter.

CHAPTER III

ZERO-SENSITIVITY CRITERION

General Active Transfer Functions

The most general three-OP AMP active filter with output taken from the output of one of the OP AMPs is shown in Fig. 2. Applying superposition to this configuration, the following set of equations is obtained:

$$\left.\begin{aligned} V_4 &= T_{40}V_i + T_{41}V_1 + T_{42}V_2 + T_{43}V_3 \\ V_5 &= T_{50}V_i + T_{51}V_1 + T_{52}V_2 + T_{53}V_3 \\ V_6 &= T_{60}V_i + T_{61}V_1 + T_{62}V_2 + T_{63}V_3 \end{aligned}\right\} \tag{46}$$

where

$$T_{ij} = \left.\frac{V_i}{V_j}\right|_{V_k=0} \qquad \text{for } k \in \{0, 1, 2, 3\} - \{j\} \tag{47}$$

is the transfer function of the passive RC network from terminal j to terminal i. From (9), it follows that

$$\left.\begin{aligned} V_1\tau_1 s &= -V_4 \\ V_2\tau_2 s &= -V_5 \\ \text{and} \qquad V_3\tau_3 s &= -V_6. \end{aligned}\right\} \tag{48}$$

It follows from the two systems of equations (46) and (48) that the overall active-RC transfer function of the circuit of Fig. 2 is given by

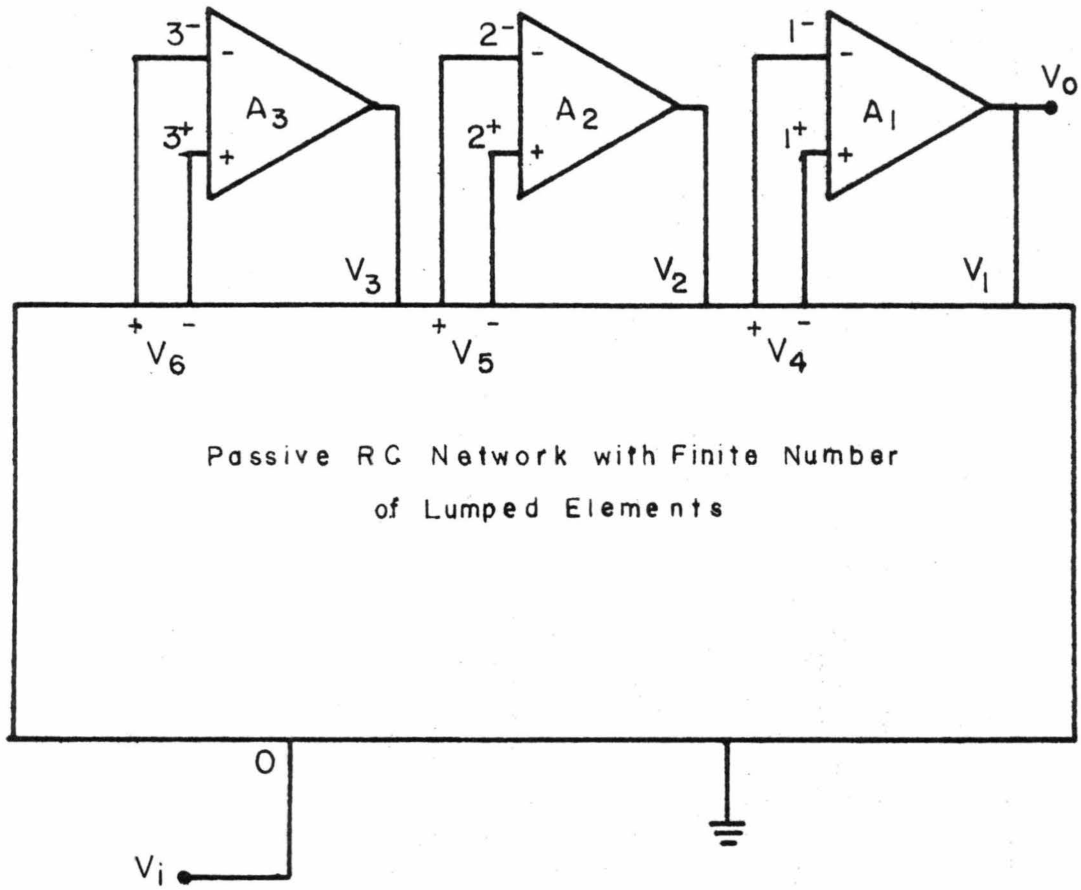


Fig. 2. General Three-OP AMP Active RC Filter

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{\begin{vmatrix} -T_{40} & T_{42} & T_{43} \\ -T_{50} & T_{52}+\tau_2 s & T_{53} \\ -T_{60} & T_{62} & T_{63}+\tau_3 s \end{vmatrix}}{\begin{vmatrix} T_{41}+\tau_1 s & T_{42} & T_{43} \\ T_{51} & T_{52}+\tau_2 s & T_{53} \\ T_{61} & T_{62} & T_{63}+\tau_3 s \end{vmatrix}} . \qquad (49)$$

From (49) it follows that for three, two, and one OP AMP(s), the transfer functions are given by:

THREE OP AMPS

$$T_3(s) = \frac{\begin{array}{c} T_{43}T_{52}T_{60}+T_{40}T_{53}T_{62}+T_{42}T_{50}T_{63}-T_{40}T_{52}T_{63}-T_{42}T_{53}T_{60} \\ -T_{43}T_{50}T_{60}+(T_{43}T_{60}-T_{40}T_{63})\tau_2 s+(T_{42}T_{50}-T_{40}T_{52})\tau_3 s-\tau_3\tau_3 s^2 T_{40} \end{array}}{\begin{array}{c} T_{41}T_{52}T_{63}+T_{42}T_{53}T_{61}+T_{43}T_{51}T_{62}-T_{43}T_{52}T_{61}-T_{41}T_{53}T_{62}-T_{42}T_{51}T_{63} \\ +\tau_1 s(T_{52}T_{63}-T_{53}T_{62})+\tau_2 s(T_{41}T_{63}-T_{43}T_{61})+\tau_3 s(T_{41}T_{52}-T_{51}T_{42})+\tau_1\tau_2 s^2 T_{63} \\ +\tau_1\tau_3 s^2 T_{52}+\tau_2\tau_3 s^2 T_{41}+\tau_1\tau_2\tau_3 s^3 \end{array}} \qquad (50)$$

TWO OP AMPS

$$T_2(s) = \frac{T_{42}T_{50}-T_{40}T_{52}-\tau_2 sT_{40}}{T_{41}T_{52}-T_{42}T_{51}+\tau_1 sT_{52}+\tau_2 sT_{41}+\tau_1\tau_2 s^2} \qquad (51)$$

SINGLE OP AMP

$$T_1(s) = \frac{-T_{40}}{T_{41}+\tau_1 s} . \qquad (52)$$

Since all transfer functions of passive RC networks with a finite number of lumped components are ratios of polynomials, multiplication by the least common multiple of all denominator polynomials in any of the three active-RC transfer functions just given enables these transfer functions to be written in the following form:

$$T_3(s) = \frac{N_{30}+N_{32}\tau_2+N_{33}\tau_3+N_{323}\tau_2\tau_3}{D_3+D_{31}\tau_1+D_{32}\tau_2+D_{33}\tau_3+D_{312}\tau_1\tau_2+D_{313}\tau_1\tau_3+D_{323}\tau_2\tau_3+D_{3123}\tau_1\tau_2\tau_3} \tag{53}$$

$$T_2(s) = \frac{N_{20}+N_{22}\tau_2}{D_2+D_{21}\tau_1+D_{22}\tau_2+D_{212}\tau_1\tau_2} \tag{54}$$

$$T_1(s) = \frac{N_{10}}{D_1+D_{11}\tau_1} \tag{55}$$

where all Ns and Ds are polynomials in s independent of the time constants of the OP AMPs.

Conditions for Zero Pole-Sensitivity

Conditions for zero active pole-sensitivity will be derived for the three-OP AMP active RC filter. The results for the single- and two-OP AMP cases will be subsequently stated.

From (23) and (53), the pole sensitivity of the kth pole, p_k (a simple root of $D_3(s)$) with respect to τ_1, the time constant of the first OP AMP, may be expressed as

$$S_{\tau_1}^{p_k} = -\frac{D_{31}(p_k) + D_{312}(p_k)\tau_2 + D_{313}(p_k)\tau_3 + D_{3123}(p_k)\tau_2\tau_3}{\frac{d}{dp_k}\left[D_3(p_k)+\tau_1 D_{31}(p_k)+\tau_2 D_{32}(p_k)+\tau_3 D_{33}(p_k)+\tau_1\tau_2 D_{312}(p_k) +\tau_1\tau_3 D_{313}(p_k)+\tau_2\tau_3 D_{323}(p_k)+\tau_1\tau_2\tau_3 D_{3123}(p_k)\right]} \tag{56}$$

Hence

$$S_{\tau_1}^{p_k}\Bigg|_{\vec{T}_p=0} = -\frac{D_{31}(p_k)}{\frac{d}{dp_k}[D_3(p_k)]} \tag{57}$$

where $\vec{T}_p$ is as defined on page 9. It follows from (57) that if the pole sensitivity of each simple pole of $D_3(s)$ with respect to the first OP AMP is to be identically zero, it is necessary and sufficient that

$$D_{31}(p_k) = 0 \tag{58}$$

where p_k is any simple root of $D_s(s)$.

By similar arguments, the conditions for zero pole-sensitivity of each simple pole of $D_3(s)$ with respect to each OP AMP time constant are obtained. The conditions for zero pole-sensitivity with respect to each OP AMP time constant for the three- and two-OP AMP active filters are, respectively:

CONDITION 1

$$D_{31}(p_k) = D_{32}(p_k) = D_{33}(p_k) = 0 \tag{59}$$

where p_k is any simple root of $D_3(s)$

CONDITION 2

$$D_{21}(p_k) = D_{22}(p_k) = 0 \tag{60}$$

where p_k is any simple root of $D_2(s)$.

The conditions for zero pole-sensitivity in the single-OP AMP case are likewise; $D_{11}(p_k) = 0$ where p_k is any simple root of $D_1(s)$. It will now be shown that no single-OP AMP active-RC filter with at least one pair of complex conjugate poles has zero pole-sensitivity. From (52) and (53), it follows that $D_{11}(s)/s$ is the characteristic polynomial of the passive RC network of Fig. 2, and as such, has only negative-real-axis roots. The claim now follows since the polynomial $D_1(s)$, which by assumption has at least one root off of the real axis, cannot be a divisor of $D_{11}(s)$, a necessary condition for zero sensitivity with respect to each root of $D_1(s)$.

From (18), it can be seen that for zero pole-sensitivity, the Maclaurin series expansion of the actual pole position is expressible as the sum of the desired pole position and only second-order and higher terms involving the OP AMP time constants. The actual pole position in zero-sensitivity designs is thus

$$p_k = p_k\Big|_{\vec{T}_p=0} + \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial^2 f}{\partial\tau_i\partial\tau_j}\Bigg|_{\vec{T}_p=0} \tau_i\tau_j + \text{higher order terms.} \qquad (61)$$

From (25) and (26), it follows that the Maclaurin series expansion of ω_o and Q also contain only second-order and higher OP AMP time constant terms when the pole sensitivities vanish.

Zero Transfer-Function-Magnitude Sensitivity

Conditions 1 and 2 can be strengthened to obtain zero transfer-function-magnitude sensitivity for all ω. From (30) it is sufficient that both the active pole and active zero sensitivities vanish to obtain zero transfer-function-magnitude sensitivity. The conditions for zero transfer-function-magnitude sensitivity with respect to each OP AMP time constant for the three- and two-OP AMP active filters are, respectively:

CONDITION 3

$$D_{31}(p_k) = D_{32}(p_k) = D_{33}(p_k) = 0$$

where p_k is any simple root of $D_3(s)$, and (62)

$$N_{32}(n_k) = N_{33}(n_k) = 0$$

where n_k is any simple root of $N_{30}(s)$.

CONDITION 4

$$D_{21}(p_k) = D_{22}(p_k) = 0$$

where p_k is any simple root of $D_2(s)$, and (63)

$N_{22}(n_k) = 0$ where n_k is any simple root of $N_{20}(s)$.

No claim has been made about the physical realizability of active RC filters with zero pole-sensitivities or zero transfer-function-magnitude sensitivities. In fact, although the sensitivity of the desired poles (the poles of $D_2(s)$ or $D_3(s)$) may be zero and hence the pole perturbations extremely small when actual OP AMPs with sufficiently small time constants are used, the parasitic poles introduced by the OP AMPs may be either quite close to the immaginary axis or even in the right half plane resulting in an unstable filter. Several realizations are given later that are stable and that do have zero pole-sensitivity.

Zero Second Derivatives

The second derivative is related to the sensitivity function defined in (7) by

$$\frac{\partial^2 f}{\partial_j \partial_i} = S_j^{S_i^f} . \tag{64}$$

The second derivative may thus be thought of as the sensitivity of the sensitivity function. It follows from (18) that zero active sensitivities and zero second derivatives with respect to all OP AMP time constants eliminate both the first- and second-order terms in the Maclaurin expansion of any critical transfer-function parameter. Unless stated otherwise, all second derivatives discussed in the remainder of this thesis are with respect to the OP AMP time constants.

The conditions for zero pole second derivatives will now be developed for the three-OP AMP active RC filter. It follows from (22) that

$$\left.\frac{\partial^2 p_k}{\partial \tau_1^2}\right|_{\vec{T}_p=0} = - \frac{\left[\frac{dD_3(p_k)}{dp_k}\left[\frac{\partial D_{31}(p_k)}{\partial p_k}\ \frac{\partial p_k}{\partial \tau_1}\right] - D_{31}(p_k)\left(\frac{d^2 D_3(p_k)}{dp_k^2}\ \frac{\partial p_k}{\partial \tau_1} + \frac{\partial D_{31}(p_k)}{\partial p_k}\right)\right]}{\left[\frac{dD_3(p_k)}{dp_k}\right]^2} \tag{65}$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_2^2}\right|_{\vec{T}_p=0} = -\frac{\left[\frac{dD_3(p_k)}{dp_k}\left(\frac{\partial D_{32}(p_k)}{\partial p_k}\,\frac{\partial p_k}{\partial \tau_2}\right) - D_{32}(p_k)\left(\frac{d^2D_3(p_k)}{dp_k^2}\,\frac{\partial p_k}{\partial \tau_2} + \frac{\partial D_{32}(p_k)}{\partial p_k}\right)\right]}{\left[\frac{dD_3(p_k)}{dp_k}\right]^2} \qquad (66)$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_3^2}\right|_{\vec{T}_p=0} = -\frac{\left[\frac{dD_3(p_k)}{dp_k}\left(\frac{\partial D_{33}(p_k)}{\partial p_k}\,\frac{\partial p_k}{\partial \tau_3}\right) - D_{33}(p_k)\left(\frac{d^2D_3(p_k)}{dp_k^2}\,\frac{\partial p_k}{\partial \tau_3} + \frac{\partial D_{33}(p_k)}{\partial p_k}\right)\right]}{\left[\frac{dD_3(p_k)}{dp_k}\right]^2} \qquad (67)$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_2 \partial \tau_1}\right|_{\vec{T}_p=0} = -\frac{\left[\frac{dD_3(p_k)}{dp_k}\left(\frac{\partial D_{31}(p_k)}{\partial p_k}\,\frac{\partial p_k}{\partial \tau_2} + D_{312}(p_k)\right) - D_{31}(p_k)\left(\frac{d^2D_3(p_k)}{dp_k^2}\,\frac{\partial p_k}{\partial \tau_2} + \frac{\partial D_{32}(p_k)}{\partial p_k}\right)\right]}{\left[\frac{dD_3(p_k)}{dp_k}\right]^2} \qquad (68)$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_3 \partial \tau_1}\right|_{\vec{T}_p=0} = -\frac{\left[\frac{dD_3(p_k)}{dp_k}\left(\frac{\partial D_{31}(p_k)}{\partial p_k}\,\frac{\partial p_k}{\partial \tau_3} + D_{313}(p_k)\right) - D_{31}(p_k)\left(\frac{d^2D_3(p_k)}{dp_k^2}\,\frac{\partial p_k}{\partial \tau_3} + \frac{\partial D_{33}(p_k)}{\partial p_k}\right)\right]}{\left[\frac{dD_3(p_k)}{dp_k}\right]^2} \qquad (69)$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_2 \partial \tau_3}\right|_{\vec{T}_p=0} = -\frac{\left[\frac{dD_3(p_k)}{dp_k}\left(\frac{\partial D_{32}(p_k)}{\partial p_k}\,\frac{\partial p_k}{\partial \tau_3} + D_{323}(p_k)\right) - D_{32}(p_k)\left(\frac{d^2D_3(p_k)}{dp_k^2}\,\frac{\partial p_k}{\partial \tau_3} + \frac{\partial D_{33}(p_k)}{\partial p_k}\right)\right]}{\left[\frac{dD_3(p_k)}{dp_k}\right]^2} \qquad (70)$$

Assuming zero pole-sensitivites, these equations reduce to:

$$\left.\frac{\partial^2 p_k}{\partial \tau_1^2}\right|_{\vec{T}_p=0} = 0 \qquad (71)$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_2^2}\right|_{\vec{T}_p=0} = 0 \tag{72}$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_3^2}\right|_{\vec{T}_p=0} = 0 \tag{73}$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_1 \partial \tau_2}\right|_{\vec{T}_p=0} = -\frac{D_{312}(p_k)}{\frac{dD_3(p_k)}{dp_k}} \tag{74}$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_3 \partial \tau_1}\right|_{\vec{T}_p=0} = -\frac{D_{313}(p_k)}{\frac{dD_3(p_k)}{dp_k}} \tag{75}$$

$$\left.\frac{\partial^2 p_k}{\partial \tau_2 \partial \tau_3}\right|_{\vec{T}_p=0} = -\frac{D_{323}(p_k)}{\frac{dD_3(p_k)}{dp_k}} \quad . \tag{76}$$

The condition for zero pole-sensitivity and zero pole second derivatives for the three-OP AMP active RC filter now follows from (71) - (76). This condition is:

CONDITION 5

Condition 1 must be satisfied and

$$D_{312}(p_k) = D_{313}(p_k) = D_{323}(p_k) = 0 \tag{77}$$

where p_k is any simple root of $D_3(s)$.

No two-OP AMP active RC filter with at least one pair of complex conjugate poles has both zero pole-sensitivity and zero pole second derivatives. This fact will now be verified. The conditions necessary for zero sensitivity and zero second derivatives in the two-OP AMP case can be shown to be

$$D_{21}(p_k) = D_{22}(p_k) = D_{212}(p_k) = 0 \tag{78}$$

where p_k is any simple root of $D_2(s)$. However, $D_{212}(s)$ is the characteristic polynomial (or product of characteristic polynomials) of a passive RC network. It must be concluded that $D_{212}(s)$ has no roots off of the real axis and hence that $D_{212}(p_k) = 0$ at all roots of $D_2(s)$.

Predistortion for Low Active Sensitivity

In some applications, it may be desirable to predistort the active filter design so that the desired poles are in precisely the desired position when the OP AMP time constants agree with those specified in the design. In the predistorted designs, the OP AMP time constant vector $\vec{T}$ of Chapter II is no longer the zero vector but rather the vector of either expected or measured OP AMP time constants. Several authors have discussed predistorted designs, among which are [20] and [22].

Predistorted designs may not be practical for large scale production at the present time since the properties of most commercially available OP AMPs vary considerably from device to device. Some precision applications may well benefit from such designs, however. A computer aided design procedure is often needed in the predistorted design itself. In precision applications that have used predistorted design techniques incorporating measured OP AMP time constants, it is particularly crucial that the active sensitivities be small since the OP AMP time constant is also affected by temperature, age, supply voltages, etc.

The active pole sensitivity for the predistorted designs will now be computed for the three-OP AMP case; the results for the two-OP AMP and single-OP AMP cases will be stated.

In the three-OP AMP predistorted design, the transfer function may be written as

$$T(s) = \frac{N_{30}+N_{32}(\tau_2+t_{p2})+N_{33}(\tau_3+t_{p3})+N_{323}(\tau_2+t_{p2})(\tau_3+t_{p3})}{\begin{array}{l} D_3+D_{31}(\tau_1+t_{p1})+D_{32}(\tau_2+t_{p2})+D_{33}(\tau_3+t_{p3})+D_{312}(\tau_2+t_{p2})(\tau_1+t_{p1}) \\ +D_{313}(\tau_3+t_{p3})(\tau_1+t_{p1})+D_{323}(\tau_2+t_{p2})(\tau_3+t_{p3})+D_{3123}(\tau_3+t_{p3})(\tau_2+t_{p2})(\tau_1+t_{p1}) \end{array}} \tag{79}$$

where τ_1, τ_2, and τ_3 are the OP AMP time constants used in the design and t_{p1}, t_{p2}, and t_{p3} are the differences in the actual OP AMP time constants and the values specified in the design. The position of the desired pole p_k is thus a function of $\vec{T}_p = (t_{p1}, t_{p2}, t_{p3})^T$; $\vec{T}$ is assumed to be constant. It follows from (23) and (53) that the pole sensitivities of the pole p_k with respect to the perturbation in the OP AMP time constants may be expressed as

$$S_{t_{p1}}^{p_k} = -\frac{D_{31}(p_k)+D_{312}(p_k)(\tau_2+t_{p2})+D_{313}(p_k)(\tau_3+t_{p3})+D_{3123}(p_k)(\tau_2+t_{p2})(\tau_3+t_{p3})}{\frac{d}{dp_k}D(p_k)} \tag{80}$$

$$S_{t_{p2}}^{p_k} = -\frac{D_{32}(p_k)+D_{312}(p_k)(\tau_1+t_{p1})+D_{323}(p_k)(\tau_3+t_{p3})+D_{3123}(p_k)(\tau_1+t_{p1})(\tau_3+t_{p3})}{\frac{d}{dp_k}D(p_k)} \tag{81}$$

$$S_{t_{p3}}^{p_k} = -\frac{D_{33}(p_k)+D_{313}(p_k)(\tau_1+t_{p1})+D_{323}(p_k)(\tau_2+t_{p2})+D_{3123}(p_k)(\tau_1+t_{p1})(\tau_2+t_{p2})}{\frac{d}{dp_k}D(p_k)} \tag{82}$$

where

$$D(p_k) = D_3(p_k)+D_{31}(p_k)(\tau_1+t_{p1})+D_{32}(p_k)(\tau_2+t_{p2})+D_{33}(p_k)(\tau_3+t_{p3})$$
$$+D_{312}(p_k)(\tau_2+t_{p2})(\tau_1+t_{p1})+D_{313}(p_k)(\tau_3+t_{p3})(\tau_1+t_{p1})$$
$$+D_{323}(p_k)(\tau_2+t_{p2})(\tau_3+t_{p3})+D_{3123}(\tau_3+t_{p3})(\tau_2+t_{p2})(\tau_1+t_{p1}). \quad (83)$$

In the two-OP AMP case, the sensitivity expressions are

$$S_{t_{p1}}^{p_k} = -\frac{D_{21}(p_k) + D_{212}(p_k) \cdot (\tau_2+t_{p2})}{\frac{d}{dp_k} D(p_k)} \quad (84)$$

$$S_{t_{p2}}^{p_k} = -\frac{D_{22}(p_k) + D_{212}(p_k) \cdot (\tau_1+t_{p1})}{\frac{d}{dp_k} D(p_k)} \quad (85)$$

where

$$D(p_k) = D_2(p_k) + D_{21}(p_k) \cdot (\tau_1+t_{p1}) + D_{22}(p_k) \cdot (\tau_2+t_{p2}) + D_{212}(p_k) \cdot (\tau_1+t_{p1}) \cdot (\tau_2+t_{p2}) \quad . \quad (86)$$

The sensitivity expression in the single-OP AMP case is

$$S_{t_{p1}}^{p_k} = \frac{D_{11}(p_k)}{\frac{d}{dp_k} [D_1(p_k) + D_{11}(p_k) \cdot (\tau_1+t_{p1})]} \quad . \quad (87)$$

It can be seen from (80) - (81) that zero pole-sensitivity is not attainable in the predistorted designs; conditions necessary to eliminate the first-order and second-order time constant terms in the Maclaurin series expansion of the pole position will be given, however. From (18) the Maclaurin series expansion of the desired pole p_k is in the three-OP AMP case

$$p_k = p_k\Big|_{\vec{T}_p=0} + \sum_{i=1}^{3} S_{t_{pi}}^{p_k}\Big|_{\vec{T}_p=0} \cdot \tau_{pi} + \text{higher order terms.} \quad (88)$$

From (80) - (83) this equation may be expressed as

$$p_k = p_k\Big|_{\vec{T}_p=0} - \frac{1}{\frac{d}{dp_k}D(p_k)\Big|_{\vec{T}_p=0}} \cdot \sum_{i=1}^{3} D_{3i}(p_k)\Big|_{\vec{T}_p=0} \cdot \tau_{pi} - \frac{1}{\frac{d}{dp_k}D(p_k)\Big|_{\vec{T}_p=0}} \cdot \Big\{ D_{312}(p_k)$$

$$[\tau_2 t_{p1} + \tau_1 t_{p3}] + D_{313}(p_k)[\tau_3 t_{p1} + \tau_1 t_{p3}] + D_{323}(p_k)[\tau_3 t_{p2} + \tau_2 t_{p3}]$$

$$+ D_{3123}(p_k)[\tau_2\tau_3 t_{p1} + \tau_1\tau_3 t_{p2} + \tau_1\tau_2 t_{p3}] \Big\} \Big|_{\vec{T}_p=0} + \text{higher order terms.} \tag{89}$$

Since the OP AMP time constants τ_1, τ_2, and τ_3 are small, the time constant-time constant perturbation product terms of the quantity in parenthesis may be considered as second- and third-order effects. With this interpretation, the conditions necessary to eliminate first-order time constant terms in the Maclaurin series expansion of the desired pole become in the three- and two-OP AMP cases, respectively:

CONDITION 6

$$D_{31}(p_k) = D_{32}(p_k) = D_{33}(p_k) = 0 \tag{90}$$

where p_k is the pole position specified in the design.

CONDITION 7

$$D_{21}(p_k) = D_{22}(p_k) = 0 \tag{91}$$

where p_k is the pole position specified in the design.

For the single-OP AMP case, it is not possible to eliminate the first-order time constant terms when p_k has a nonzero imaginary part. The proof of this fact is similar to the argument given on page .

It should be pointed out that it is possible to realize circuits with either Condition 6 or Condition 7 satisfied at each desired pole. Note also that in the predistorted designs, p_k is not, in general, a root of $D_3(s)$ in the three-OP AMP case or of $D_2(s)$ in the two-OP AMP case.

It can be shown by using (45) and (79) that the first-order time constant terms in the Maclaurin series expansion of the transfer-function magnitude vanish if the following conditions hold in the three- and two-OP AMP cases, respectively:

CONDITION 8

$$D_{31}(p_k) = D_{32}(p_k) = D_{33}(p_k) = 0$$

where p_k is any simple desired pole and (92)

$$N_{32}(n_k) = N_{33}(n_k) = 0$$

where n_k is any simple desired zero.

CONDITION 9

$$D_{21}(p_k) = D_{22}(p_k) = 0$$

where p_k is any simple desired pole and (93)

$$N_{22}(n_k) = 0$$

where n_k is any simple desired zero.

It can also be shown that if, in addition to Condition 8, the following condition holds, then both the first-order and second-order time constant terms are zero in the Maclaurin series expansion of the desired pole p_k.

CONDITION 10

$$D_{312}(p_k) = D_{313}(p_k) = D_{323}(p_k) = 0 \tag{94}$$

where p_k is the pole position specified in the design.

Although zero active sensitivity was not attainable for the predistorted designs, for sufficiently small changes in the OP AMP time constants the desired poles will move less when the corresponding conditions are satisfied than in the zero-active-sensitivity unpredistorted designs. This follows since, in general,

$$t_{pi} < \tau_j \qquad \text{for } i, j \in \{1, 2, 3\}. \tag{95}$$

The inequality is often quite strong.

In order to keep the analysis tractable, no further considerations of low-sensitivity predistorted designs will be made in this thesis. Slight modifications of the designs that follow can be made, however, to obtain low sensitivity predistorted designs.

CHAPTER IV

ZERO-SENSITIVITY DESIGNS

Many circuits can be designed whose transfer functions satisfy the conditions for zero pole, ω_o, Q, and transfer-function-magnitude sensitivity. The zero-sensitivity conditions derived in Chapter III do not, however, imply stability; it is possible in some realizations that the desired poles are quite insensitive to the OP AMP time constants but the parasitic poles introduced by the OP AMP lie in the right half plane rendering the filter unstable.

Several general configurations with zero active sensitivities are presented in this chapter. Active filters realizing specific transfer functions are subsequently introduced, stability discussed, and performance comparisons made with previous state of the art designs. The comparisons are made for both infinitesimal and incremental changes in the OP AMP time constants. Experimental performance of the zero-sensitivity realizations is compared with the theoretical.

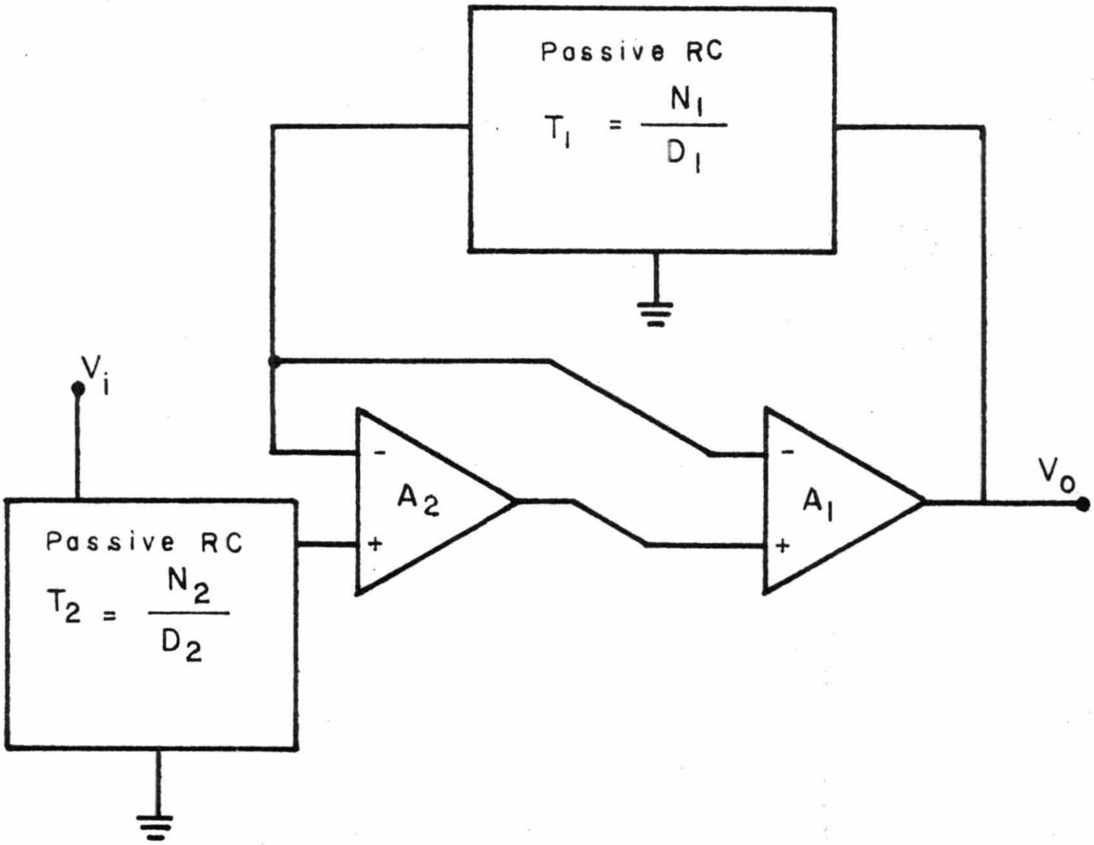
Functional Forms

Many functional forms exist that have zero active sensitivities. Three of these are presented and discussed in this section.

It is assumed that the passive RC networks shown in Fig. 3, Fig. 4, and Fig. 6 have identical characteristic polynomials and that the zeros of T_1 are coincident with the desired transfer-function poles and the zeros of T_2 are coincident with the desired active transfer-function zeros.

The transfer function of the circuit of Fig. 3 is given by

$$T(s) = \frac{V_o}{V_i} = \frac{N_2}{N_1 + \tau_2 s N_1 + \tau_1 \tau_2 s^2 D} . \tag{96}$$



$$D_1 = D_2 = D$$

Fig. 3. Two-OP AMP Zero Sensitivity Filter

It is immediate from (96) that T(s) satisfies both Condition 2 and Condition 4. The active filter of Fig. 3 thus has zero pole, ω_o, Q, and transfer-function-magnitude sensitivities.

Close matching of the two characteristic polynomials of the passive RC networks of Fig. 3 is not crucial. This can be explained as follows. With D_1 and D_2 as shown in Fig. 3, the transfer function of the active filter is

$$T(s) = \frac{N_2 D_1}{D_2 N_1 (1 + \tau_1 s) + \tau_1 \tau_2 s^2 D_1 D_2} . \tag{97}$$

When $\tau_1 = \tau_2 = 0$,

$$T(s) = \left(\frac{N_2}{N_1}\right) \left(\frac{D_1}{D_2}\right) . \tag{98}$$

Since D_1/D_2 should be approximately unity, the transfer function attains approximately its desired value when $\tau_1 = \tau_2 = 0$. Furthermore, the transfer-function-magnitude sensitivity expressions are from (45)

$$S_{\tau_1}^{|T(j\omega)|} \Bigg|_{\tau_1=\tau_2=0} = -|T(j\omega)| \ \mathrm{Re}\left(\frac{sD_2N_1}{D_2N_1}\right) = 0 \tag{99}$$

$$S_{\tau_2}^{|T(j\omega)|} \Bigg|_{\tau_1=\tau_2=0} = -|T(j\omega)| \ \mathrm{Re}\left(\frac{\tau_1 s^2 D_1 D_2}{D_2 N_1 (1 + \tau_1 s)}\right)\Bigg|_{\tau_1=0} = 0 . \tag{100}$$

The transfer-function-magnitude sensitivities thus vanish even if D_1 and D_2 are not matched.

With the appropriate choice of T_1 and T_2, any realizable transfer function can be obtained from the zero-sensitivity configuration of Fig. 3. Any realizable transfer function can also be obtained by a cascade of biquadratic zero-sensitivity sections and at most one first-order section. Since a cascade of zero-sensitivity sections still has zero sensitivities, the cascaded zero-sensitivity biquadratic design is also a zero-sensitivity configuration.

If the passive RC circuits of Fig. 4 have the same characteristic polynomials, then this active filter has transfer function given by

$$T(s) = \frac{V_o}{V_i} = \frac{N_2}{N_1 + \tau_3 s N_1 + \tau_2\tau_3 s N_1 + \tau_1\tau_2\tau_3 s^2 D} \; . \tag{101}$$

It can be seen that this transfer function satisfies Conditions 1, 3, and 5 so has zero desired pole, ω_o, Q, and transfer-function-magnitude sensitivities and also zero second derivatives of the desired poles.

The active devices in both Fig. 3 and Fig. 4 can be replaced by single two-input active devices that have different gain functions on each input. The equivalent active devices are shown in Fig. 5 along with the inverting and noninverting transfer functions. In terms of the inverting and noninverting active transfer functions given in Fig. 5, the transfer functions of the circuits of Fig. 3 and Fig. 4 may be expressed as

$$\frac{V_o}{V_i} = \frac{N_2 A^+}{D - N_1 A^-} \; . \tag{102}$$

The single-OP AMP RC filter with the OP AMP modeled by a single pole and an infinite common-mode-rejection ratio is shown in Chapter III to have nonzero active sensitivity. If, however, the model of the active device is changed, it is possible to obtain zero sensitivity with respect to each parameter of the active device. For example, a zero

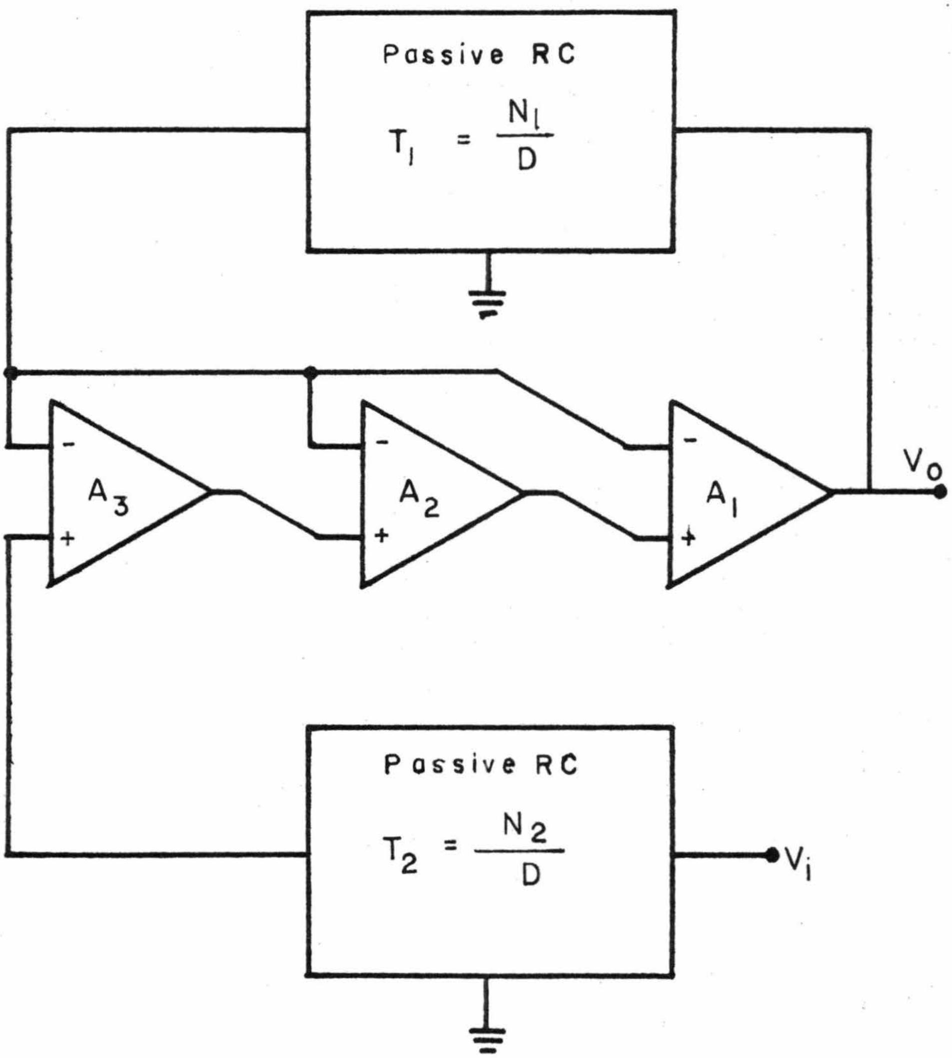


Fig. 4. Three-OP AMP Zero Second-Derivative Filter

active sensitivity filter results when the active devices in either Fig. 3 or Fig. 4 are replaced with the appropriate two-input equivalent device of Fig. 5. The equivalent active devices of Fig. 5 do not have an infinite common-mode-rejection ratio.

It will now be shown that the two-OP AMP filter of Fig. 6 has zero pole-sensitivities and zero second pole derivatives with respect to the OP AMP time constants. If T_1 and T_2 are assumed to have identical characteristic polynomials, the transfer function of this filter is

$$T(s) = \frac{V_o}{V_i} = \frac{N_2}{N_1+\tau_2 sN_1+s^2[\tau_1\tau_2-\tau_2R_3C_3+\tau_2R_3C_3N_1]+s^2\tau_1\tau_2R_3C_3D}. \quad (103)$$

If the R_3C_3 product is chosen so that

$$R_3C_3 = \tau_1 \quad (104)$$

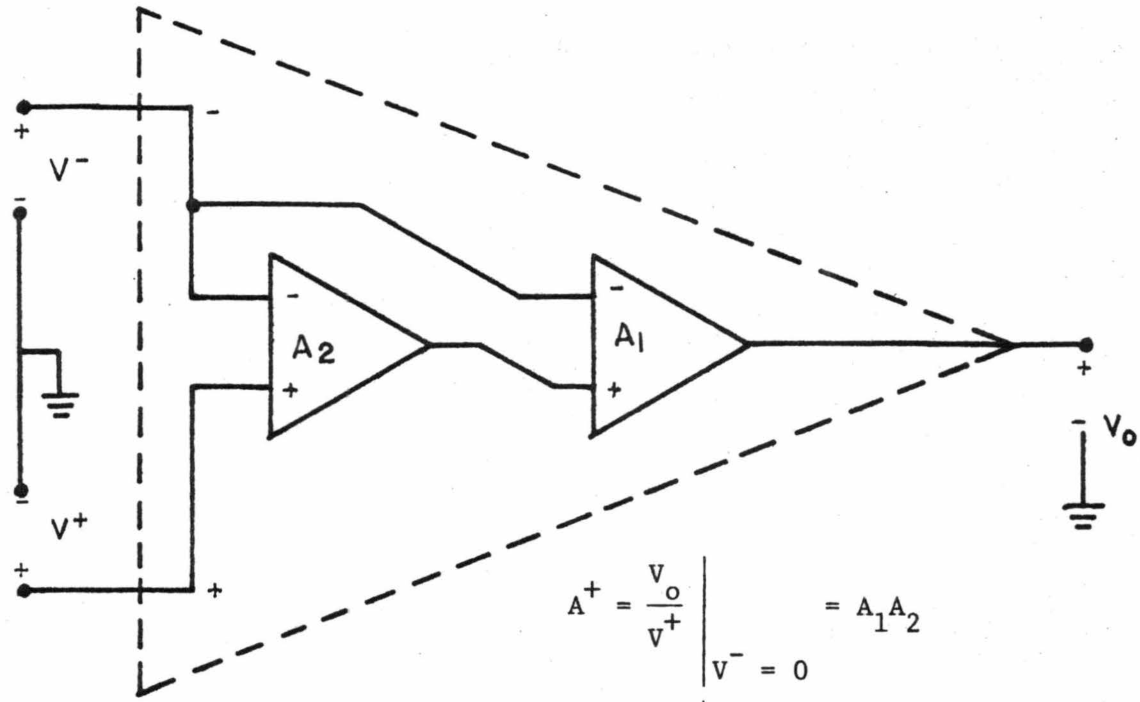
then the transfer function reduces to

$$T(s) = \frac{N_2}{N_1 + \tau_2 sN_1 + \tau_1\tau_2 s^2 N_1 + \tau_1^2\tau_2 s^3 D}. \quad (105)$$

Since this transfer function satisfies Conditions 1, 3, and 5, the pole, ω_o, Q, and transfer-function-magnitude sensitivities and second pole derivatives can be made zero. These zero-sensitivity properties are obtained with only two OP AMPs; however, matching a passive time constant and an active time constant is necessary to obtain these zero-sensitivity properties.

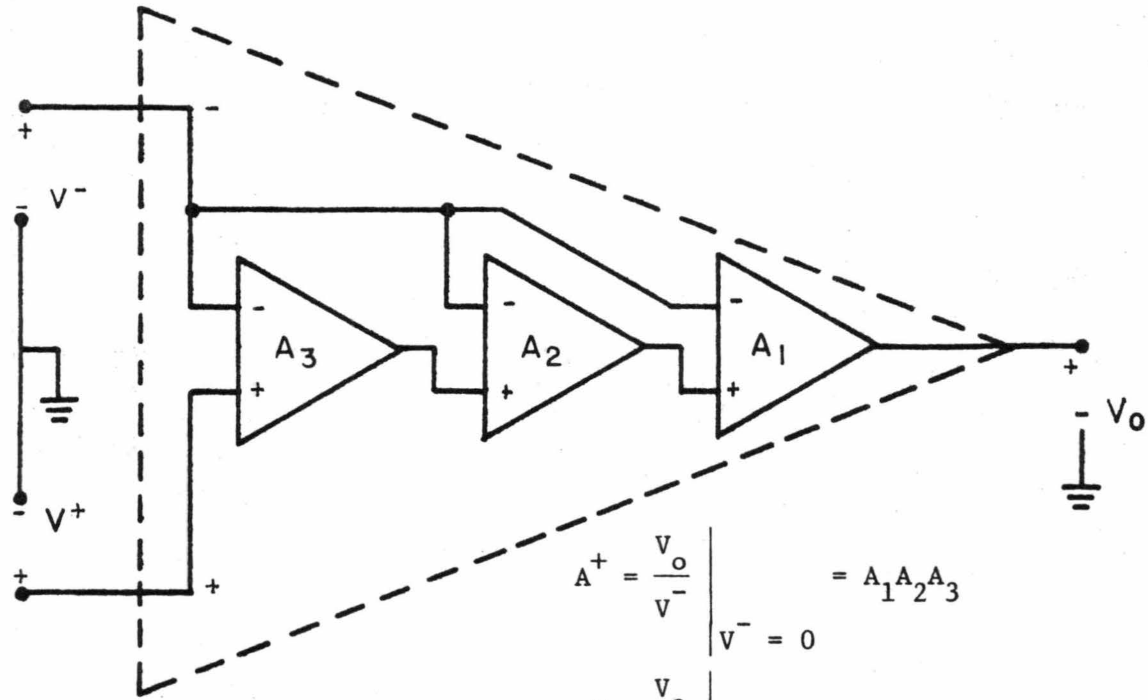
Cascaded Amplifier Designs

It will now be shown that the cascaded amplifier designs of Fig. 7 have zero-sensitivity transfer functions. If T_1 and T_2 have the same



$$A^{+} = \left.\frac{V_o}{V^{+}}\right|_{V^{-} = 0} = A_1A_2$$

(a) $$A^{-} = \left.\frac{V_o}{V^{-}}\right|_{V^{+} = 0} = -(A_1A_2+A_1)$$



$$A^{+} = \left.\frac{V_o}{V^{-}}\right|_{V^{-} = 0} = A_1A_2A_3$$

(b) $$A^{-} = \left.\frac{V_o}{V^{-}}\right|_{V^{+} = 0} = -(A_1+A_1A_2+A_1A_2A_3)$$

Fig. 5. Equivalent Active Devices for Circuits of Fig. 3 and Fig. 4

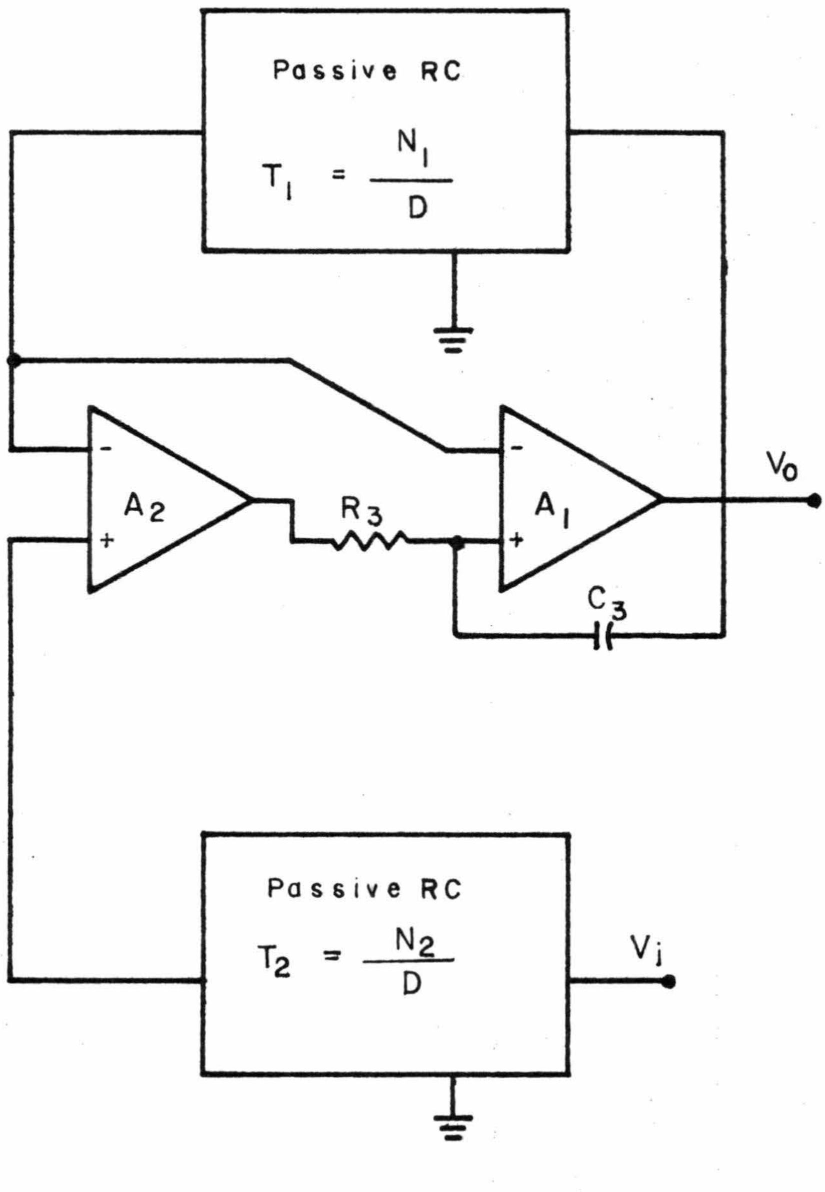


Fig. 6. Two-OP AMP Zero Second-Derivative Active Filter

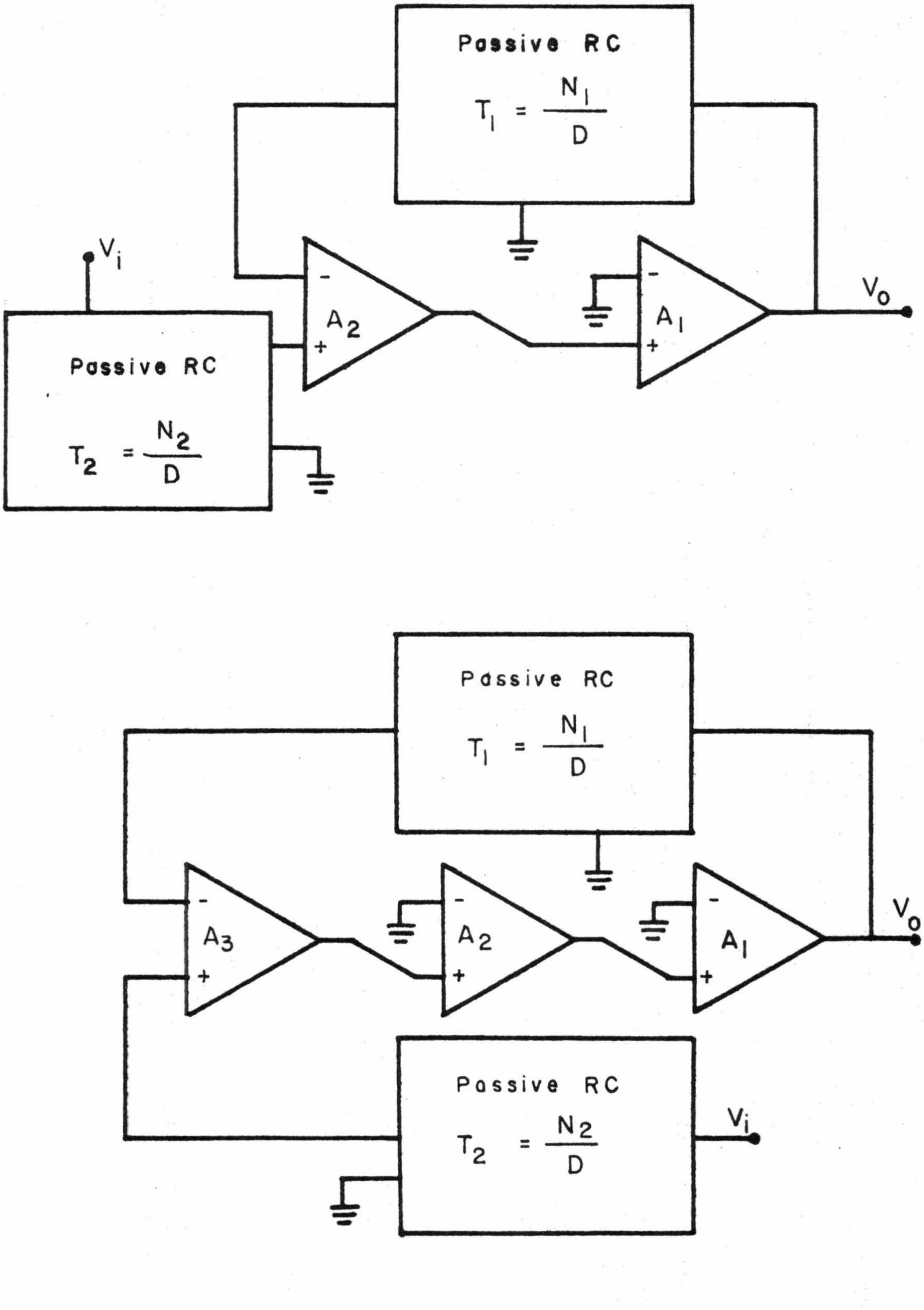


Fig. 7. Cascaded-OP AMP Zero-Sensitivity Designs

characteristic polynomial, then the transfer function of the cascaded amplifier filters (a) and (b) are, respectively:

$$T(s) = \frac{V_o}{V_i} = \frac{N_2}{N_1 + \tau_1\tau_2 s^2 D} \tag{106}$$

and

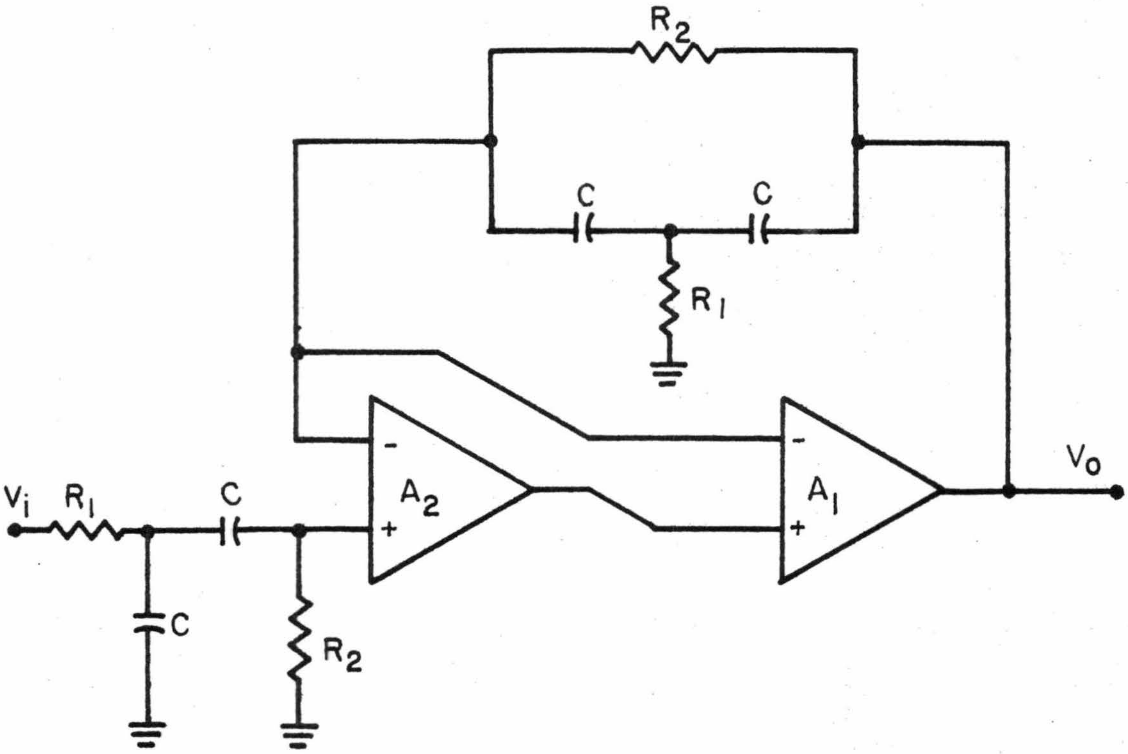
$$T(s) = \frac{V_o}{V_i} = \frac{N_2}{N_1 + \tau_1\tau_2\tau_3 s^3 D} . \tag{107}$$

These transfer functions respectively satisfy the zero-sensitivity and zero second derivative conditions of Chapter III. It will be shown in a following section that in some cases the cascaded amplifier designs become unstable. An investigation has not been made to determine if the cascaded amplifier designs are ever useful. They have been included, nonetheless, to emphasize the fact that the zero-sensitivity conditions do not guarantee stability.

Specific Zero-Sensitivity Biquadratic Bandpass Realizations

Any realizable transfer function can be realized with zero sensitivity. The only zero-sensitivity active filters discussed here will be of the second-order bandpass type. Even the treatment of the bandpass case is far from exhaustive. A similar discussion with similar results could be made for other classes of transfer functions but has not been included here to save space and avoid redundancy. A comment is, however, included at the end of this section discussing modifications that will yield arbitrary lowpass and highpass transfer functions.

The bandpass circuits of Figs. 8 - Fig. 11 are all zero-active-sensitivity configurations. The frequency normalized transfer functions



$$R_2 = 4Q^2R_1$$

$$\omega_o = \frac{1}{2QRC}$$

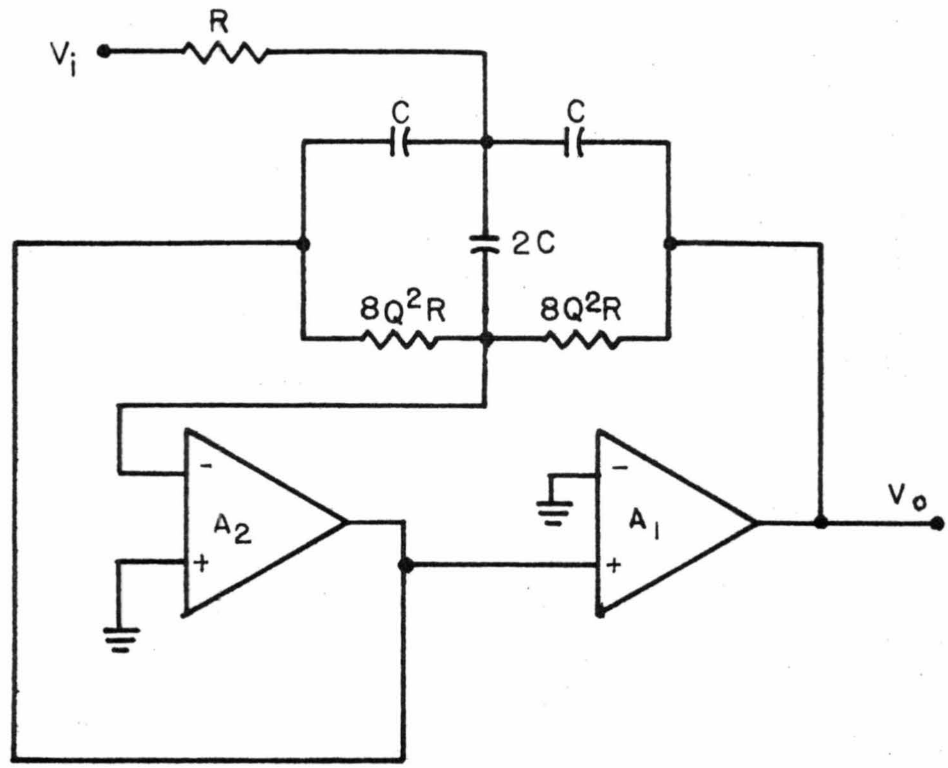
$$s_n = s/\omega_o$$

$$\tau_{1n} = \tau_1\omega_o$$

$$\tau_{2n} = \tau_2\omega_o$$

$$\frac{V_o}{V_i} = \frac{2Qs_n}{(s_n^{\ 2} + s_n/Q + 1)(1 + \tau_{2n}s_n) + \tau_{1n}\tau_{2n}s_n^{\ 2}(s_n^{\ 2} + s_n\ [2Q + 1/Q] + 1)}$$

Fig. 8. Zero-Active-Sensitivity Four-Capacitor Bandpass Filter



$$\omega_o = \frac{1}{4QRC}$$

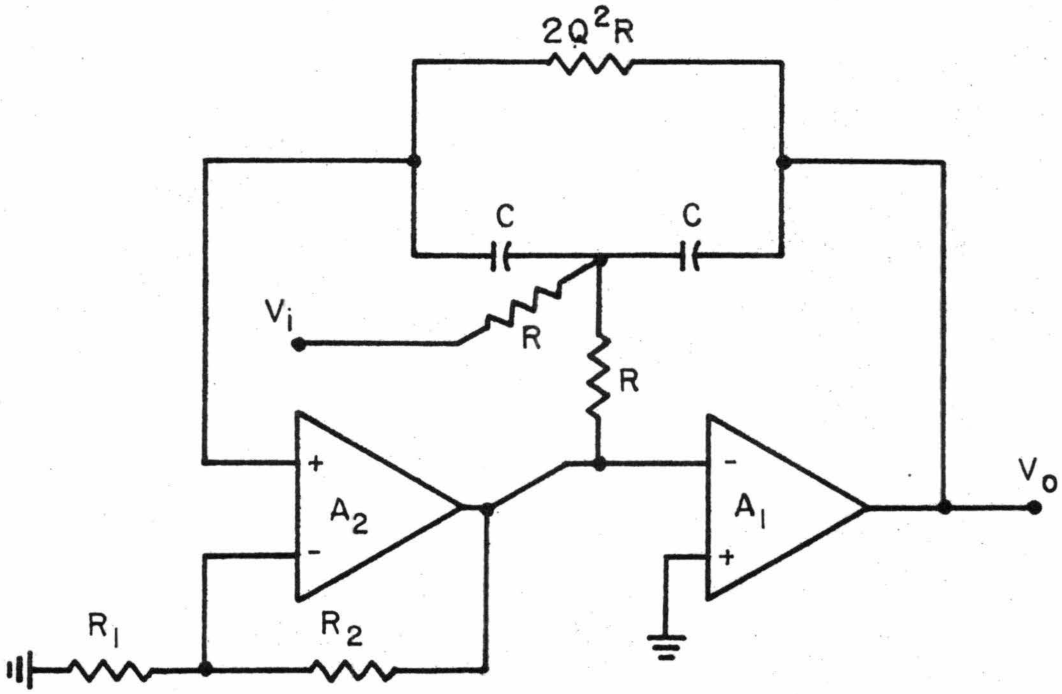
$$s_n = s/\omega_o$$

$$\tau_{1n} = \tau_1\omega_o$$

$$\tau_{2n} = \tau_2\omega_o$$

$$\frac{V_o}{V_i} = \frac{-4Qs_n}{(s_n^2 + s_n/Q + 1)(1 + \tau_{1n}s_n) + \tau_{1n}\tau_{2n}s_n^2(s_n^2 + s_n[2Q + 1/Q] + 1)}$$

Fig. 9. Zero-Active-Sensitivity Three-Capacitor Bandpass Filter



$$R_1 = R_2$$

$$\omega_o = \frac{1}{QRC}$$

$$s_n = s/Q$$

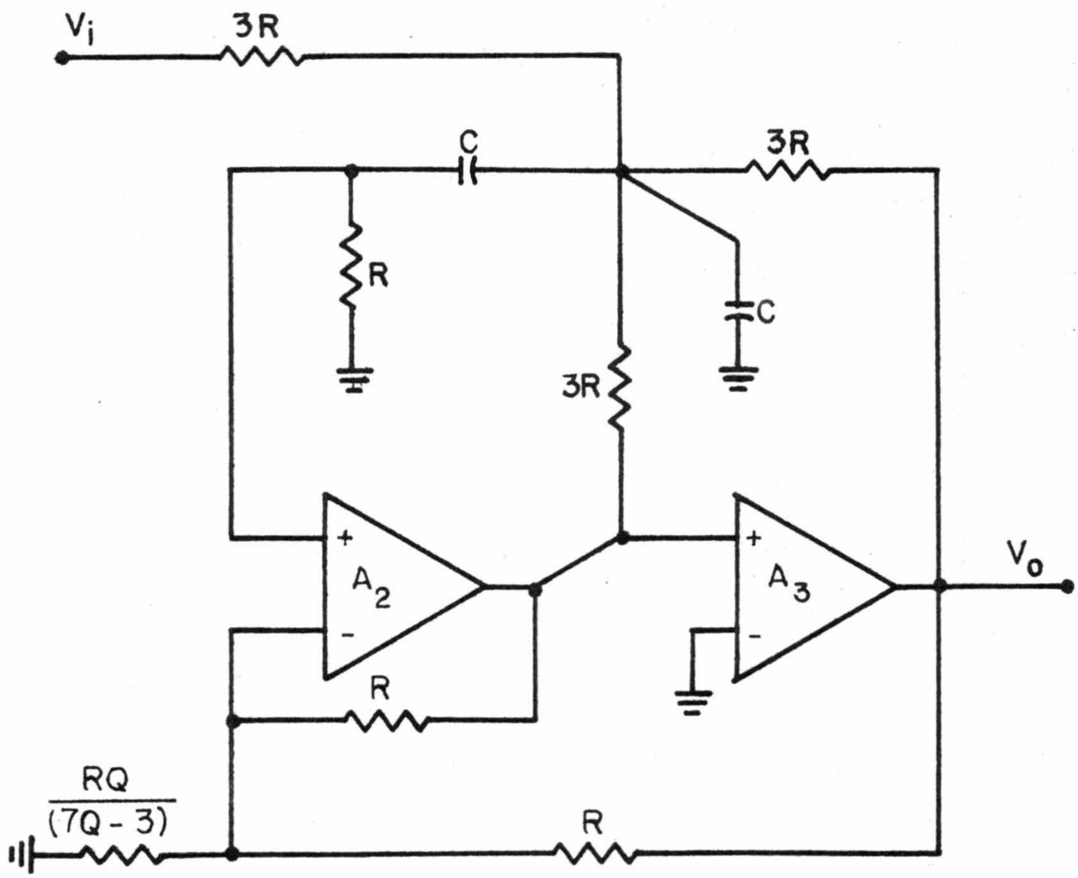
$$\tau_{1n} = \tau_1\omega_o$$

$$\tau_{2n} = \tau_2\omega_o$$

$$\frac{V_o}{V_i} = \frac{-Qs_n}{s_n^{\ 2} + s_n/Q + 1 + \tau_{1n}s_n(s_n^{\ 2} + s_n/Q + 1)}$$

$$+ \tau_{1n}\tau_{2n}s_n^{\ 2}(s_n^{\ 2} + s_n[2Q + 1/Q] + 1)$$

Fig. 10. Zero-Active-Sensitivity Two-Capacitor Bandpass Filter



$$\omega_o = \frac{1}{RC}$$

$$s_n = s/\omega_o$$

$$\tau_{1n} = \tau_1\omega_o$$

$$\tau_{2n} = \tau_2\omega_o$$

$$\frac{V_o}{V_i} = \frac{s_n(3 - 1/Q)}{s_n^2 + s_n/Q + 1 + \tau_{1n}s_n(s_n^2 + s_n/Q + 1) + \tau_{1n}\tau_{2n}(9 - 3/Q)(s_n^2 + 3s_n + 1)}$$

Fig. 11. Zero-Active-Sensitivity Low Component Spread Active Filter

are included in the figures. A brief discussion about each of these four circuits follows:

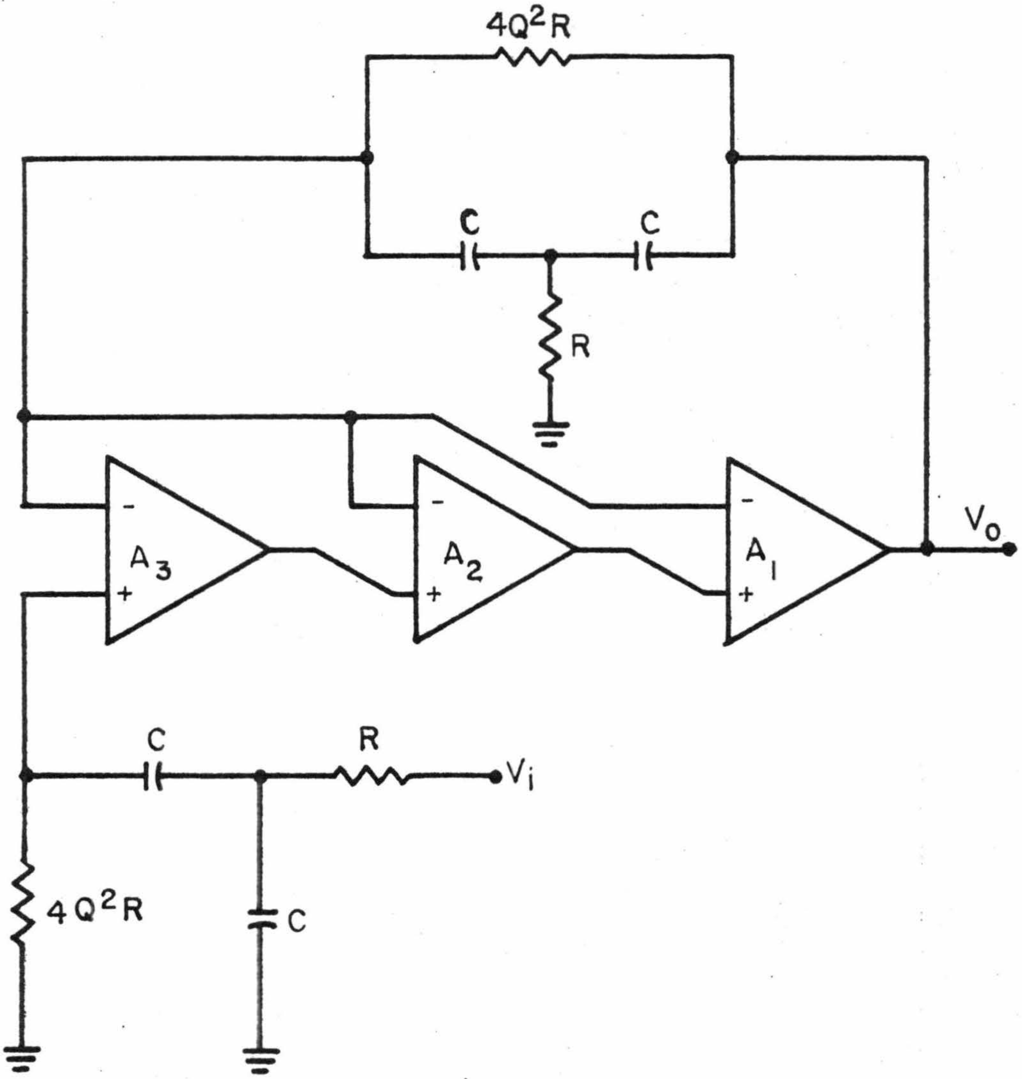
a) The circuit of Fig. 8 is obtained from the general form shown in Fig. 3. The passive RC network that realizes T_1 of Fig. 3 is chosen to be the popular complex-zero-producing bridged-T structure while the network that realizes T_2 has been chosen to have the same characteristic polynomial but a bandpass transfer function.

b) The bandpass filter of Fig. 9 uses the bridged-T structure but uses one less capacitor than the circuit of Fig. 8.

c) The circuit of Fig. 10, introduced by A. Budak [23], uses only two capacitors. The desired poles of the active transfer function are, as in the previous two designs, determined by the zeros of the bridged-T feedback network. Whereas in the previous two structures the zero-sensitivity properties were dependent only upon the circuit topology, it can easily be shown that the zero-sensitivity properties of this configuration are also dependent upon R_1 and R_2.

d) The zero-sensitivity circuit of Fig. 11 uses only two capacitors and has a much smaller component spread than the previous structures. The component spread of the bridged-T structure is proportional to Q^2; the component spread of the circuit of Fig. 11 is bounded by 21. The circuit of Fig. 11 does have, in general, higher passive sensitivities than the previous configurations and may be more difficult to tune.

The transfer function of the circuits of Fig. 12 and Fig. 13 have zero sensitivities and zero second derivatives. The circuit of Fig. 12



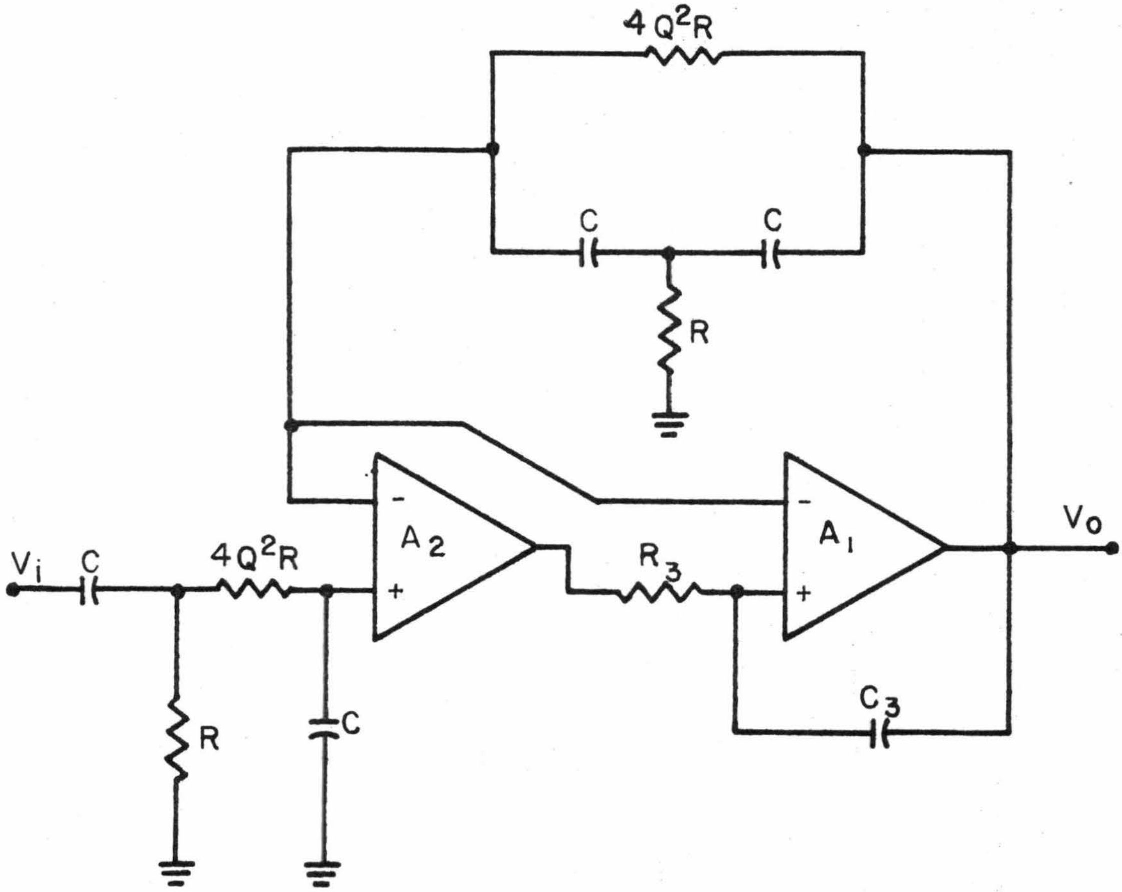
$$\omega_o = \frac{1}{2QRC}$$

$$s_n = s/\omega_o$$

$$\tau_{in} = \tau_i \omega_o$$

$$\frac{V_o}{V_i} = \frac{2Qs_n}{(s_n^2 + s_n/Q + 1)(1 + \tau_{3n}s_n + \tau_{2n}\tau_{3n}s_n^2) + \tau_{1n}\tau_{2n}\tau_{3n}s_n^3(s_n^2 + s_n[2Q + 1/Q] + 1)}$$

Fig. 12. Four-Capacitor Three-OP AMP Zero Second-Derivative Filter



$$\omega_o = \frac{1}{2QRC}$$

$$s_n = s/\omega_o$$

$$R_3C_3 = \tau_1$$

$$\tau_{in} = \tau_i\omega_o$$

$$\frac{V_o}{V_i} = \frac{s_n/2Q}{(s_n^2 + s_n/Q + 1)(1 + \tau_{2n}s_n + \tau_{1n}\tau_{2n}s_n^2) + \tau_{1n}^2\tau_{2n}s_n^3(s_n^2 + s_n[2Q + 1/Q] + 1)}$$

Fig. 13. Five-Capacitor Two-OP AMP Zero Second-Derivative Active Filter

is of the same form as the general circuit of Fig. 4 where the passive network that realizes T_1 has been chosen to be the bridged-T structure. The circuit of Fig. 13 which is of the general class of Fig. 6 also uses the bridged-T feedback structure but uses only two OP AMPs at the expense of an additional resistor and capacitor. The latter circuit has zero second derivatives only as long as $R_3C_3 = \tau_1$.

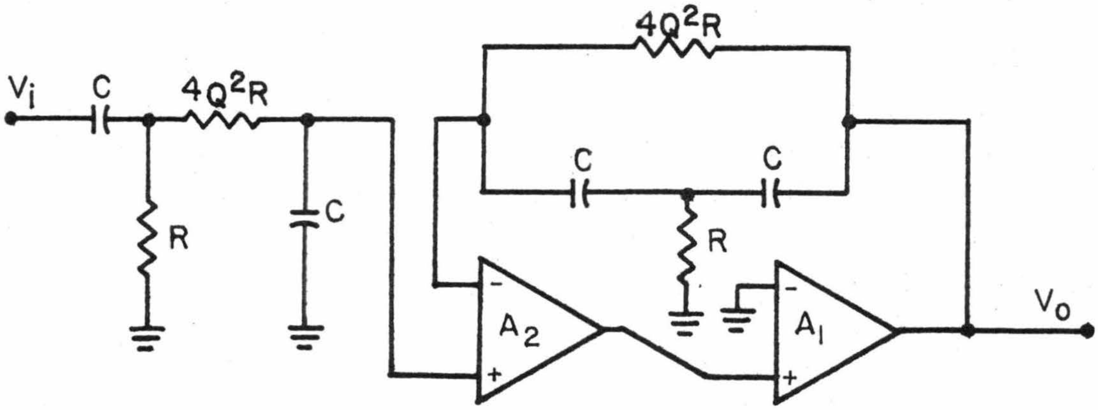
If the passive RC structures that determine T_1 and T_2 in Fig. 7 are chosen to be the bridged-T and the bandpass networks shown in Fig. 14, the zero-sensitivity and zero-second-derivative cascaded-amplifier active filters are obtained. It will be shown later in this chapter that both circuits of Fig. 14 can become unstable.

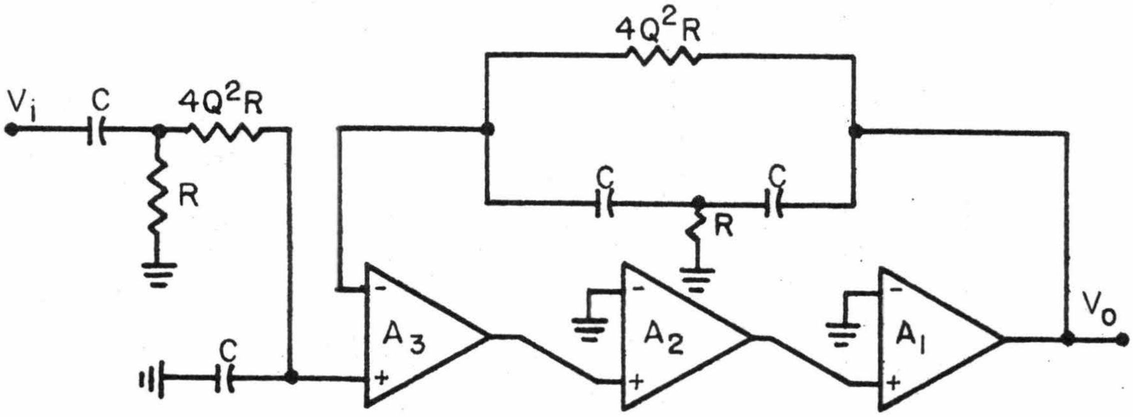
If the feedback network of Fig. 8, Fig. 12, and Fig. 13 is changed to the twin-T structure, the resulting component spread is proportional to Q instead of Q^2 of the bridged-T structure. However, one additional capacitor is used in the twin-T structure.

The networks of Fig. 8, Fig. 12 and Fig. 13 can be made lowpass and highpass, respectively, if the input passive RC network is altered as shown in Fig. 15.

Comparisons with Existing Designs

The merit of any new circuit is best established by comparing it with accepted existing designs. The zero-sensitivity circuits of Fig. 8 and Fig. 12 have transfer functions that are representative of the zero-sensitivity and zero second derivative circuits, respectively, of Fig. 8 through Fig. 13. For this reason only the zero-sensitivity circuits of Fig. 8 and Fig. 12 will be compared on a theoretical basis with accepted existing bandpass filter designs.





$$\omega_o = \frac{1}{2QRC}$$

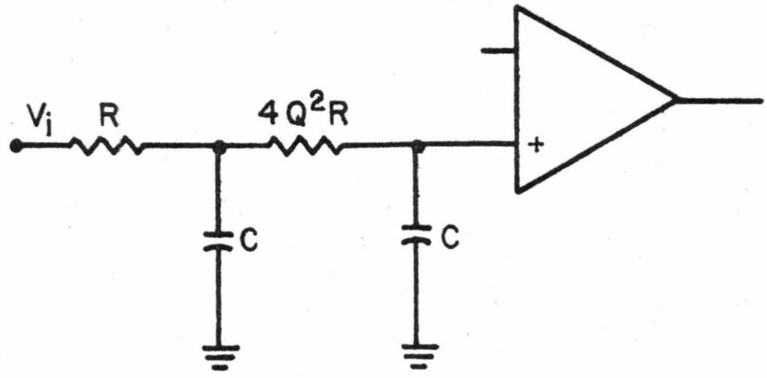
$$s_n = s/\omega_o$$

$$\tau_{in} = \tau_i\omega_o$$

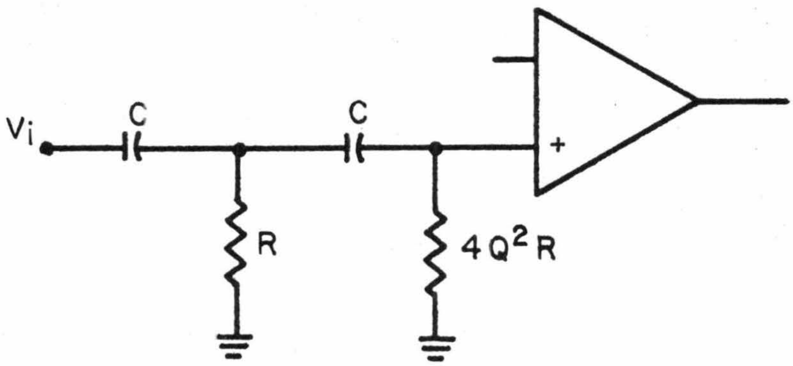
$$\frac{V_{o1}}{V_i} = \frac{s_n/2Q}{s_n^{\ 2} + s_n/Q + 1 + \tau_{1n}\tau_{2n}s_n^{\ 2}(s_n^{\ 2} + s_n[2Q + 1/Q] + 1)}$$

$$\frac{V_{o1}}{V_i} = \frac{s_n/2Q}{s_n^{\ 2} + s_n/Q + 1 + \tau_{1n}\tau_{2n}\tau_{3n}s_n^{\ 3}(s_n^{\ 2} + s_n[2Q + 1/Q] + 1)}$$

Fig. 14. Cascaded-Amplifiers Zero-Sensitivity Filters



(a) Lowpass Modification



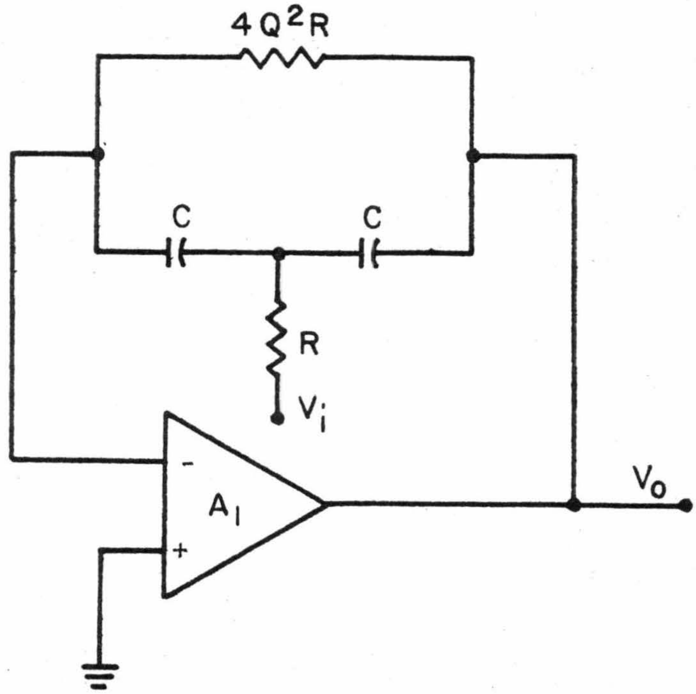
(b) Highpass Modification

Fig. 15. Modifications for Lowpass and Highpass Transfer Functions

The three existing circuits against which the zero-sensitivity designs are compared are shown in Fig. 16 - Fig. 18. The circuit of Fig. 16, discussed by several authors including [17], [19], and [20] uses only a single OP AMP. The circuit of Fig. 17 was recently claimed by Sedra [11] to be the "state of the art" two-OP AMP bandpass filter. The commonly used state-variable filter, a form of which is shown in Fig. 18, uses three OP AMPs; it appears in many texts and papers including [21] and [24]. The transfer functions of these filters are also included in the corresponding figures.

Comparison of the performance of these filters due to infinitesimal changes in the OP AMP time constants can be obtained from a study of the sensitivity functions and Maclaurin expansions discussed previously. The zero-sensitivity designs are obviously much superior to the existing designs in such comparisons. This follows from (19) since zero sensitivity was a design constraint in the zero sensitivity designs.

In this section the effect of incremental changes in the OP AMP time constants are studied on a theoretical basis. The comparisons of the zero-sensitivity and existing designs of Fig. 16 - Fig. 18 are made for values of $Q = 10$ and 25 on the basis of the position of the desired poles and on the transfer-function-magnitude response using the time constants of the OP AMPs as parameters. The comparisons show the actual pole positions and magnitude characteristics for a given OP AMP time constant. The positions of the parasitic poles, those poles introduced by the OP AMPs, are also discussed. Although the assumption of equal OP AMP time constants is not necessary to obtain the zero-sensitivity conditions for the circuits of Fig. 8 and Fig. 12, it will be assumed for convenience in these comparisons that all OP AMPs are identical.



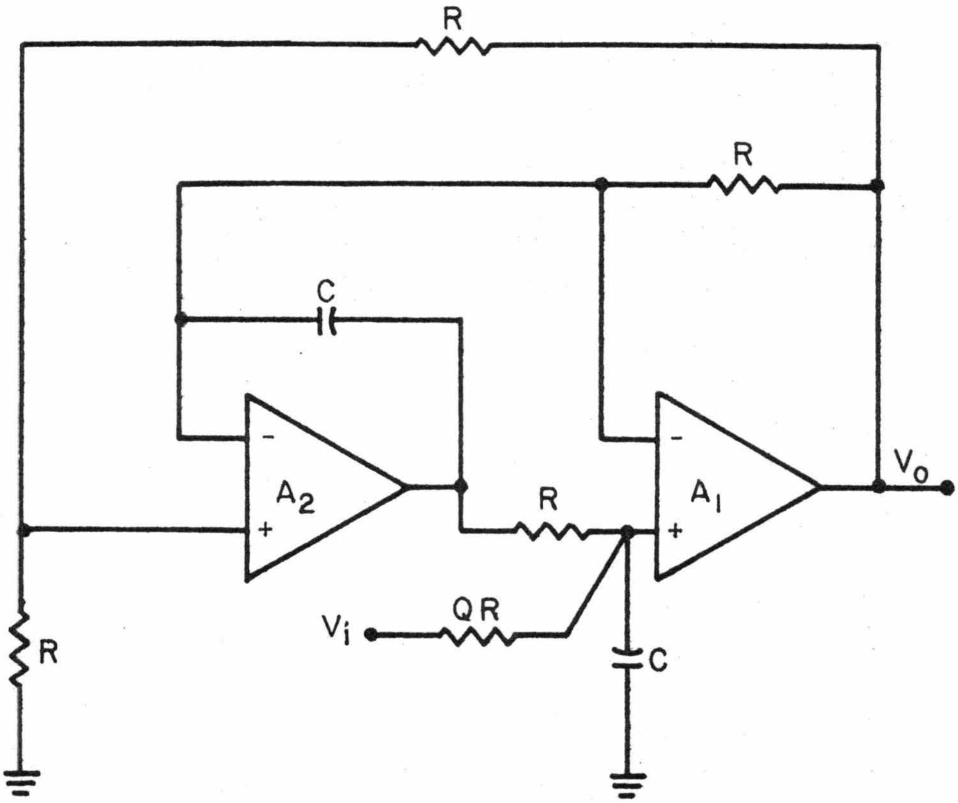
$$\omega_o = \frac{1}{2QRC}$$

$$s_n = s/\omega_o$$

$$\tau_n = \tau_1\omega_o$$

$$\frac{V_o}{V_i} = \frac{2Qs_n}{s_n^2 + s_n/Q + 1 + \tau_n s_n(s_n^2 + s_n[2Q + 1/Q] + 1)}$$

Fig. 16. Popular Single-OP AMP Bandpass Filter



$$\omega_o = \frac{1}{RC}$$

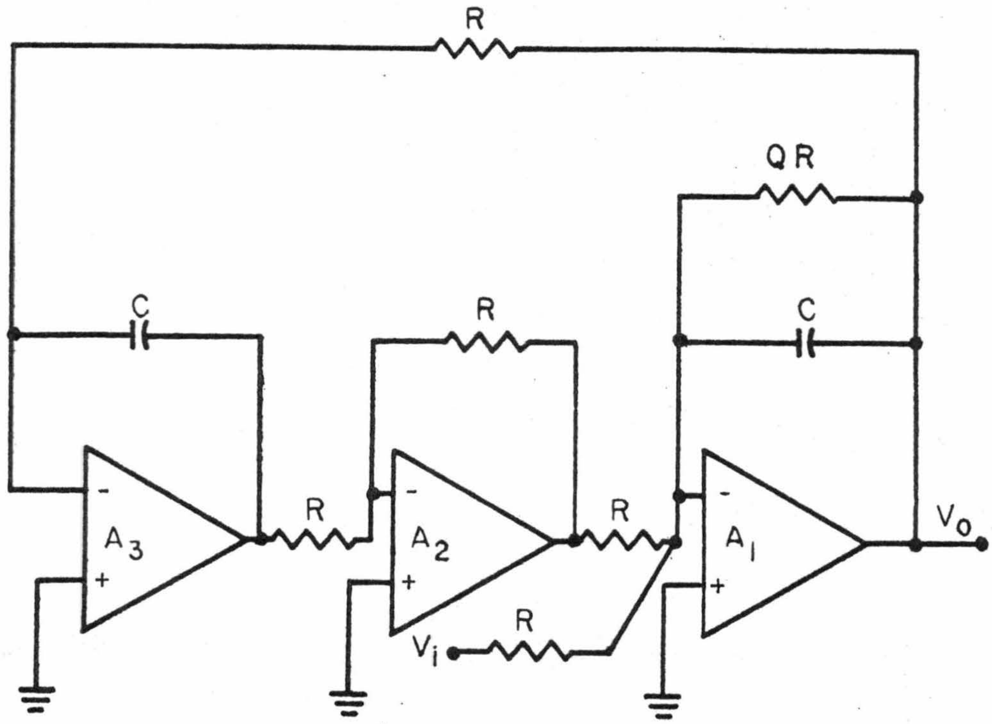
$$s_n = s/\omega_o$$

$$\tau_{1n} = \tau_1\omega_o$$

$$\tau_{2n} = \tau_2\omega_o$$

$$\frac{V_o}{V_i} = \frac{2s_n/Q + \tau_{2n}2s_n(s_n + 1)/Q}{s_n^{\,2} + s_n/Q + 1 + \tau_{1n}2s_n^{\,2}(s_n + 1 + 1/Q) + \tau_{2n}2s_n(s_n + 1 + 1/Q) + \tau_{1n}\tau_{2n}2s_n^{\,2}(s_n^{\,2} + s_n[2 + 1/Q] + 1 + 1/Q)}$$

Fig. 17. Popular Two-OP AMP Bandpass Filter



$$\omega_o = \frac{1}{RC}$$

$$s_n = s/\omega_o$$

$$\tau_{in} = \tau_i\omega_o$$

$$\frac{V_o}{V_i} = \frac{s_n + \tau_{2n}2s_n^{\ 2} + \tau_{3n}(s_n^{\ 2} + s_n) + \tau_{2n}\tau_{3n}s_n^{\ 2}2(s_n + 1)}{s_n^{\ 2} + s_n/Q + 1 + \tau_{1n}s_n^{\ 2}(s_n + 2 + 1/Q) + \tau_{2n}2s_n^{\ 2}(s_n + 1/Q)}$$

$$+ \tau_{3n}s_n(s_n^{\ 2} + s_n[1 + 1/Q] + 1/Q) + \tau_{1n}\tau_{2n}2s_n^{\ 3}(s_n + 2 + 1/Q)$$

$$+ \tau_{1n}\tau_{3n}s_n^{\ 2}(s_n^{\ 2} + s_n[3 + 1/Q] + 2 + 1/Q)$$

$$+ \tau_{2n}\tau_{3n}s_n^{\ 2}2(s_n^{\ 2} + s_n[1 + 1/Q] + 1/Q)$$

$$+ \tau_{1n}\tau_{2n}\tau_{3n}2s_n^{\ 3}(s_n^{\ 2} + s_n[3 + 1/Q] + 2 + 1/Q)$$

Fig. 18. State Variable Filter

The desired pole loci for the circuits of Fig. 16 - Fig. 18 and the zero-sensitivity circuits are shown in Fig. 19 and Fig. 20 for Q = 10 and Q = 25, respectively. It can be seen that for the two-OP AMP circuit of Fig. 8, the actual pole location is practically identical to the desired pole location for $\tau_n = \tau\omega_o < .01$ and that in the three-OP AMP circuit of Fig. 12, the pole location moves very little for $\tau_n < .1$. It should be noted that one vertical unit on the graph represents about five horizontal units in Fig. 19 and ten horizontal units in Fig. 20. Therefore, the magnitude of the pole movement for Qs of 10 and 25 for either of the zero-sensitivity circuits is about an order of magnitude or more smaller than that for any of the other circuits compared for $\tau_n = .01$. As τ_n becomes smaller, the differences in the pole movement become even more pronounced. If the GB of an OP AMP is $2\pi 10^6$ rad./sec, a typical value for the 741 type OP AMP, then the two-OP AMP zero-sensitivity circuit of Fig. 8 should perform nearly as if the OP AMP is ideal for a Q of 10 or 25 for center frequencies up to 10 KHz. The three-OP AMP zero-sensitivity design of Fig. 12 should correspondingly be useful for center frequencies up to about 100 KHz.

The complex-conjugate parasitic poles of the five circuits under comparison are shown in Fig. 21 and Fig. 22. The parasitic poles for the circuits of Fig. 16 and Fig. 18 are not shown since they are all on the negative real axis. Although the parasitic poles for the zero-sensitivity circuits are closer to the imaginary axis than those of the existing designs, they are, nonetheless, in the left half plane. It is seen in the transfer-function-magnitude comparisons that the parasitic poles do not have any significant effect on the frequency response.

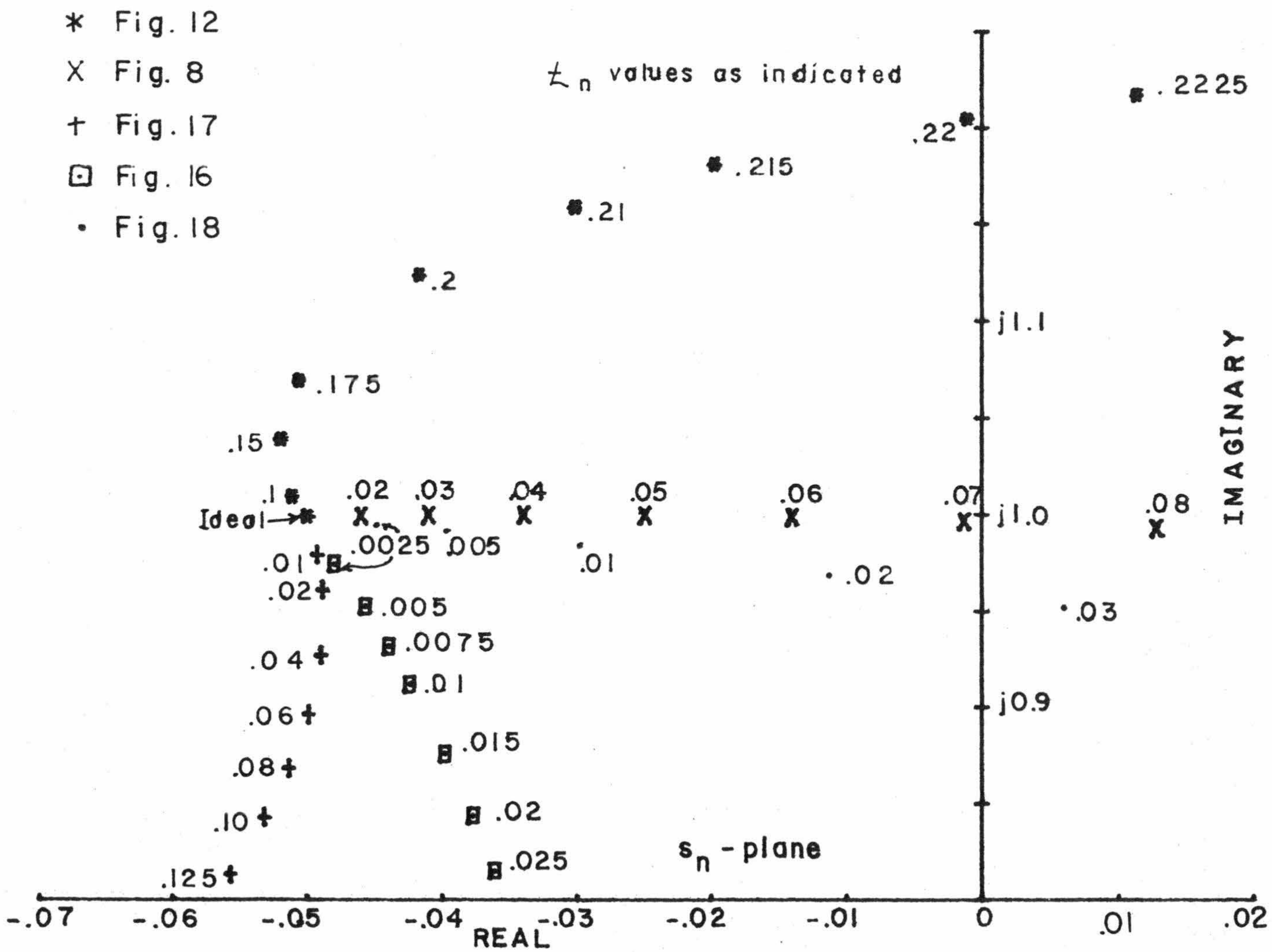


Fig. 19. Desired Pole Locus for Q = 10

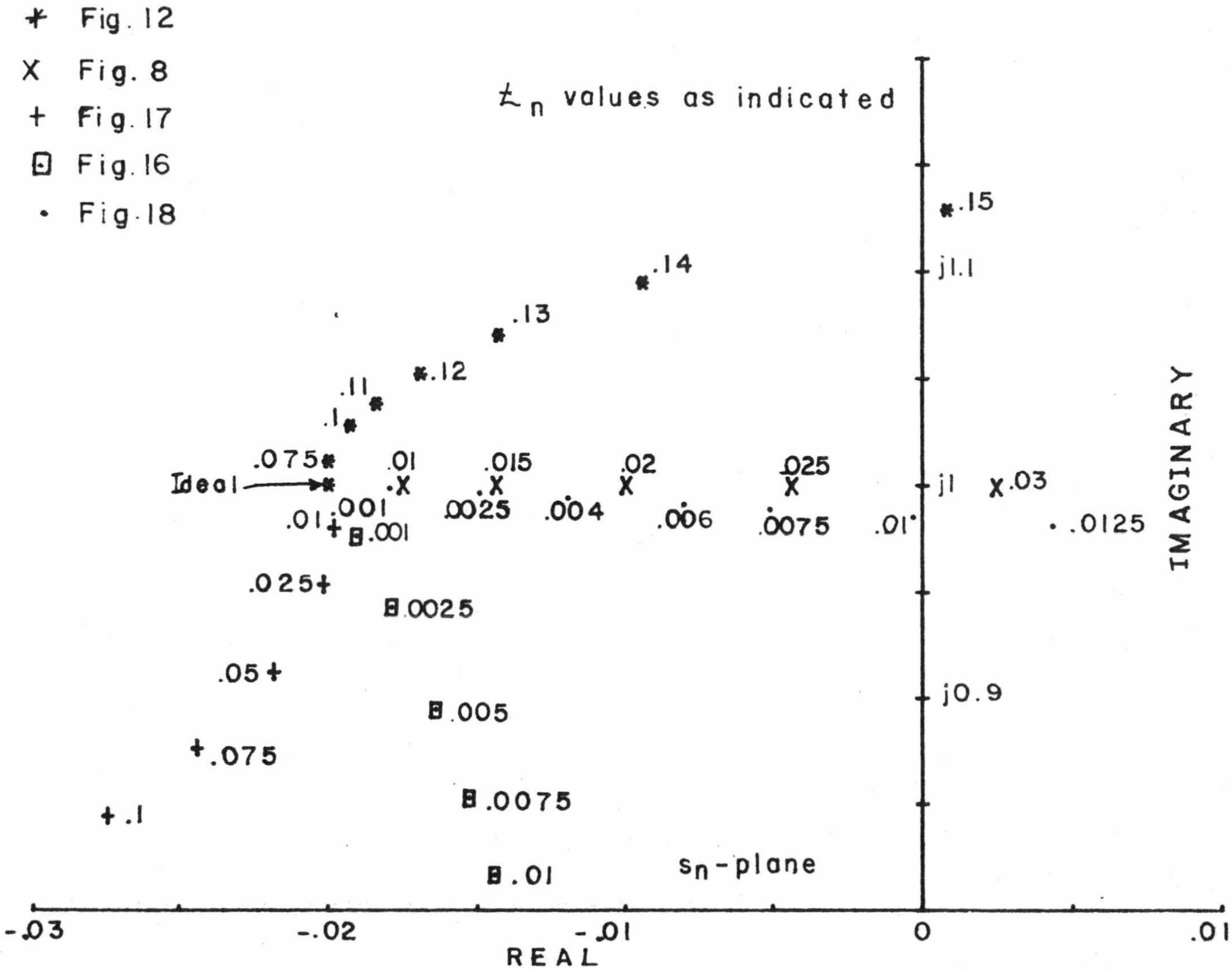


Fig. 20. Desired Pole Locus for Q = 25

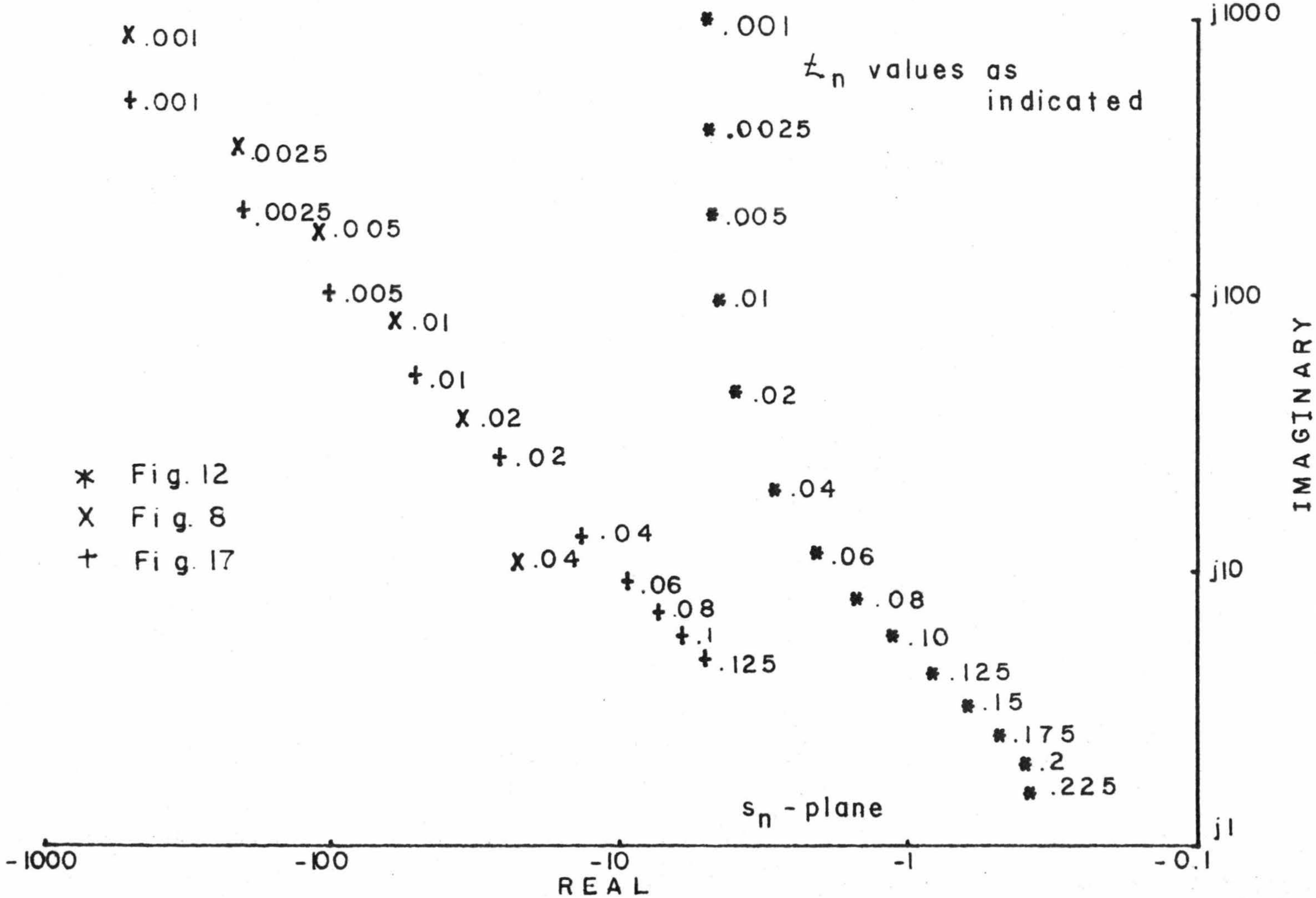


Fig. 21. Comparison of Parasitic Poles, Q = 10

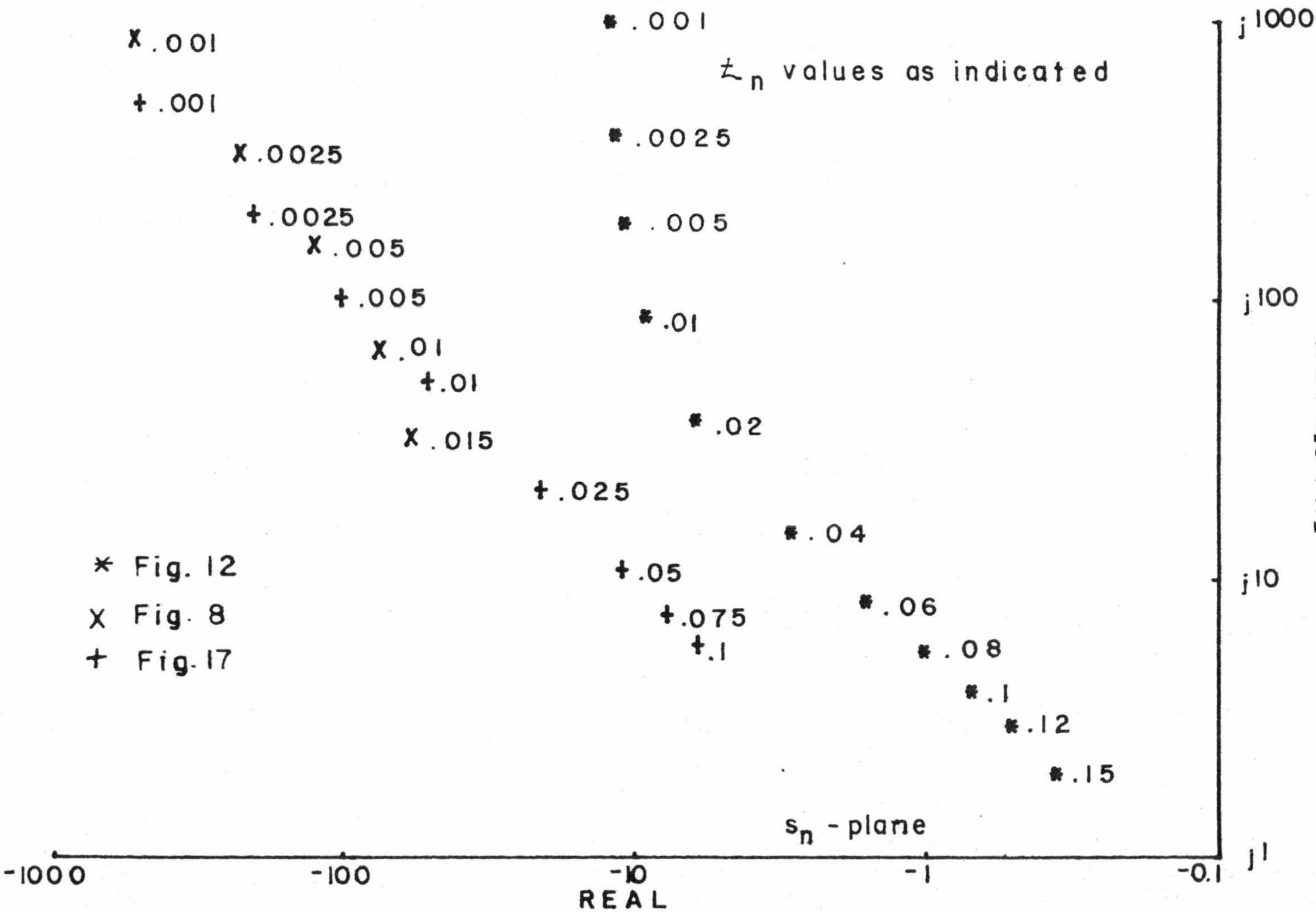


Fig. 22. Comparison of Parasitic Poles, Q = 25

The transfer-function magnitudes are compared in Fig. 23 - Fig. 25 for Q = 10 and in Fig. 26 and Fig. 27 for Q = 25. The magnitudes have been normalized so that when the OP AMPs are ideal, the peak amplitude is unity in all cases. For Q = 10, the normalized magnitude response is plotted for values of τ_n = .01, .025, and .05; and for Q = 25, the response is plotted for τ_n = .005 and .01. In all cases, the response of the zero-sensitivity three-OP AMP filter of Fig. 12 is practically indiscernable from the ideal response. The superior performance of the two-OP AMP zero-sensitivity circuit of Fig. 8 is also apparent. It is interesting to note that even when the two-OP AMP zero-sensitivity circuit does depart from the ideal, the center frequency remains nearly constant. This result could, of course, have been predicted from Fig. 19 and Fig. 20 since the desired pole movement is nearly horizontal for this circuit.

The effects of the parasitic poles on the transfer-function magnitude for the zero-sensitivity circuits of Fig. 8 and Fig. 12 are shown for a Q of 25 in Fig. 28 and Fig. 29, respectively. The frequency axis has been purposely extended from that in Fig. 23 - Fig. 27 to include frequencies that are comparable to the parasitic pole magnitudes. The effects of the parasitic poles on the frequency response are quite negligible.

Experimental Results

The two-OP AMP zero-sensitivity filter of Fig. 8 was tested using two 741 type OP AMPs with measured GBs of 857 KHz $\pm$ 1 percent. The bridged-T network was designed for a Q of 10 and center frequency of 10 KHz. The following measured component values were used in the design:

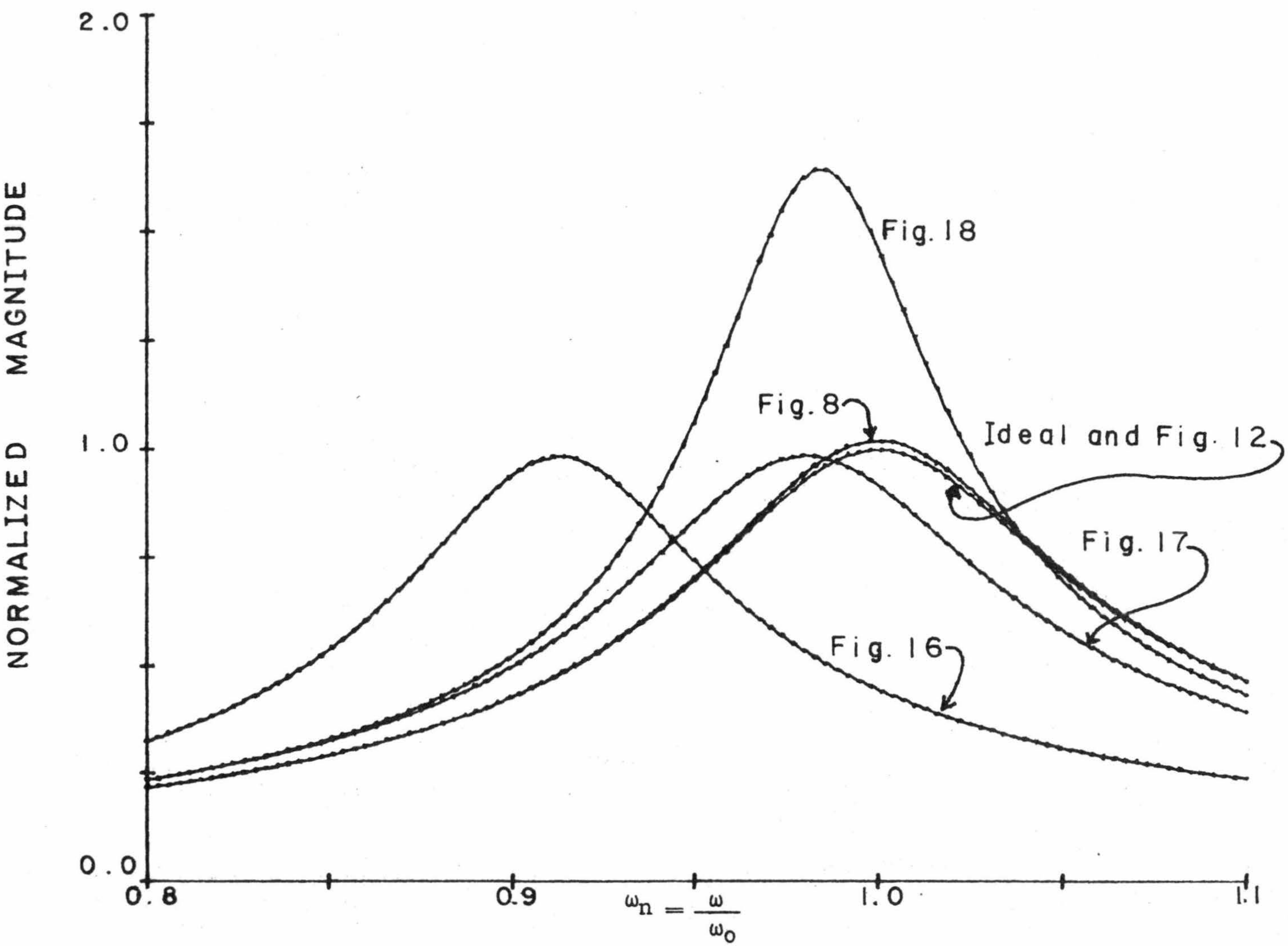


Fig. 23. Transfer Function Magnitude, Q = 10, τ_n = .01

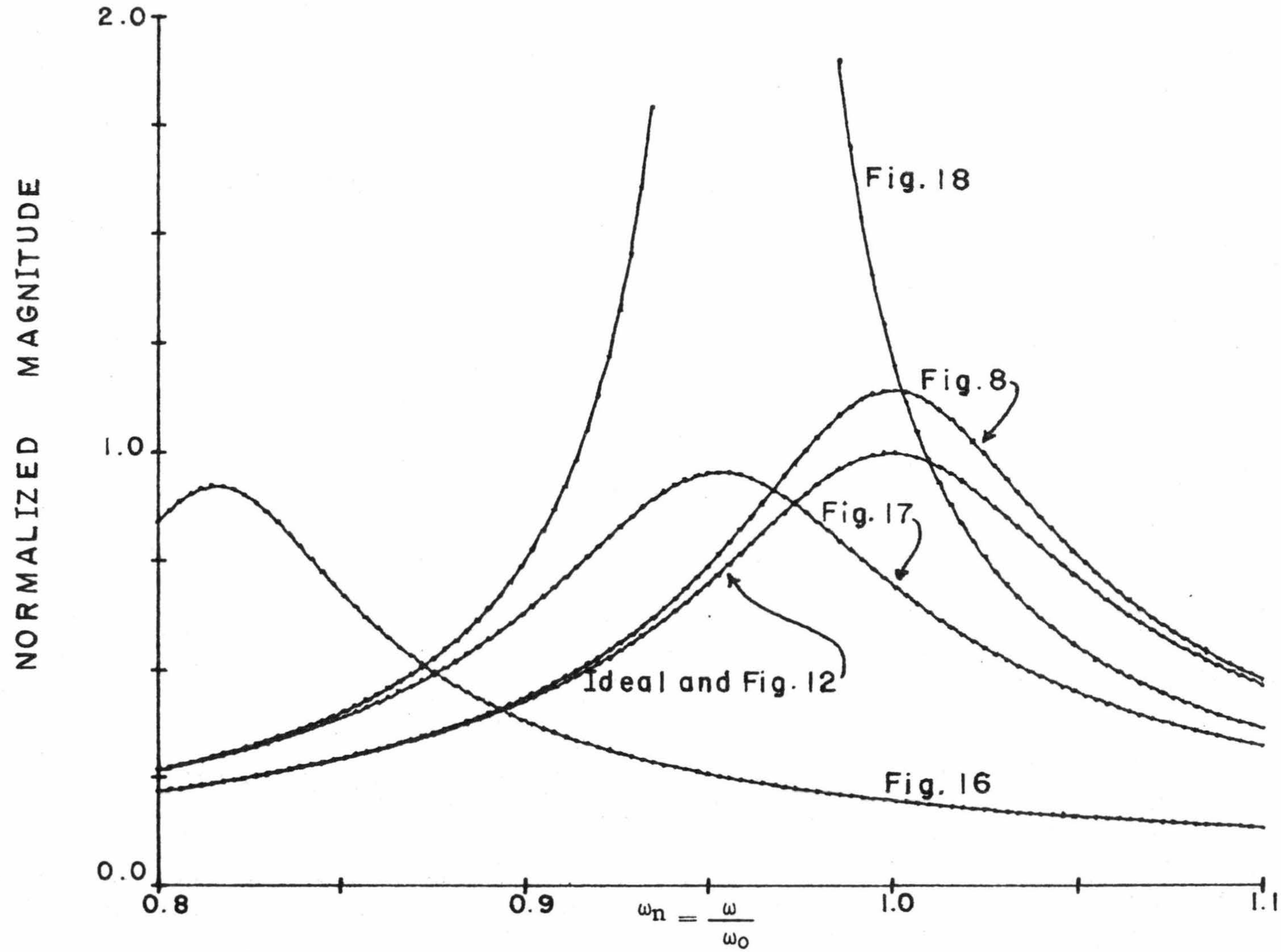


Fig. 24. Transfer Function Magnitude, Q = 10, τ_n = .025

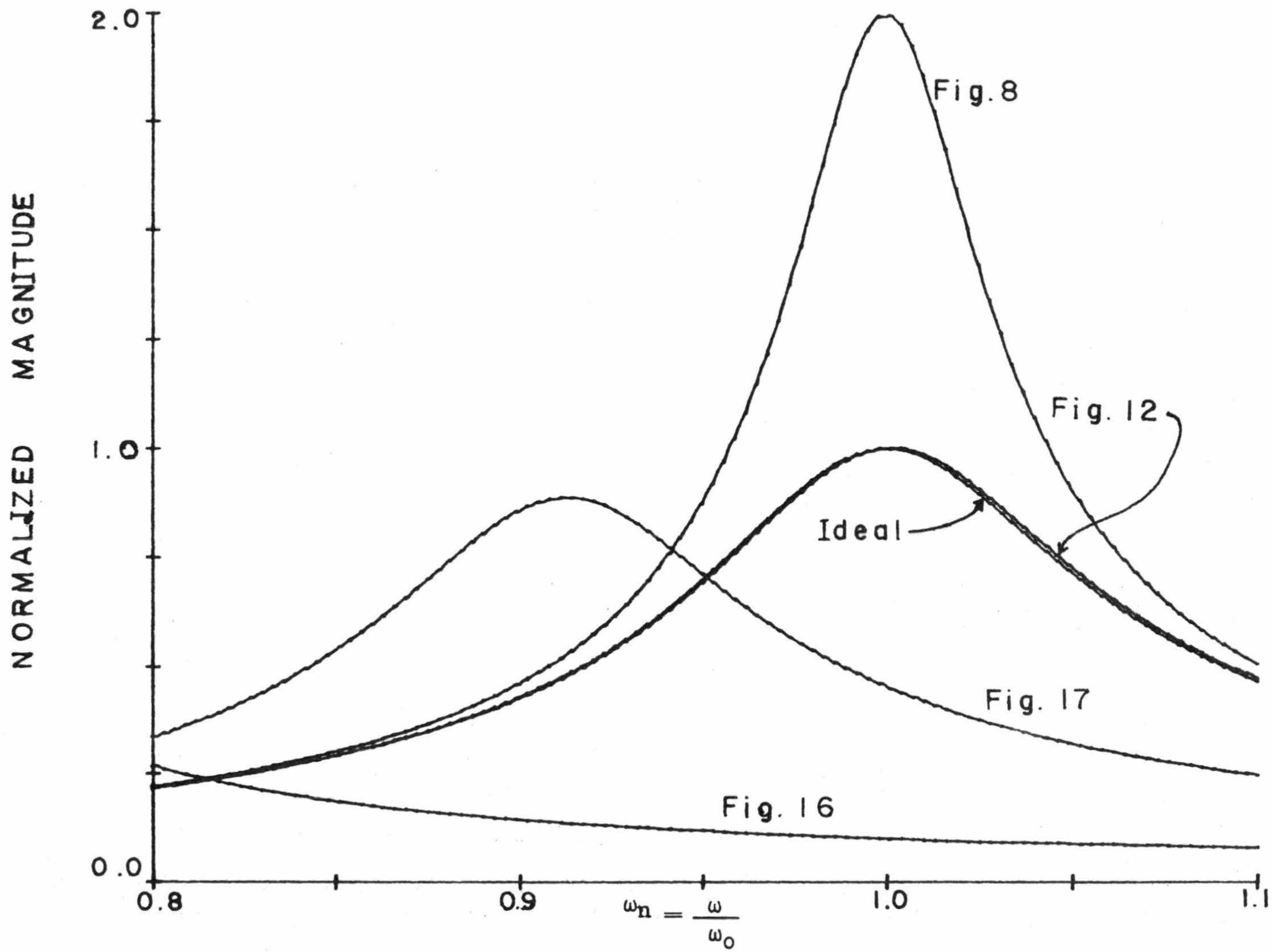


Fig. 25. Transfer Function Magnitude, Q = 10, τ_n = .05

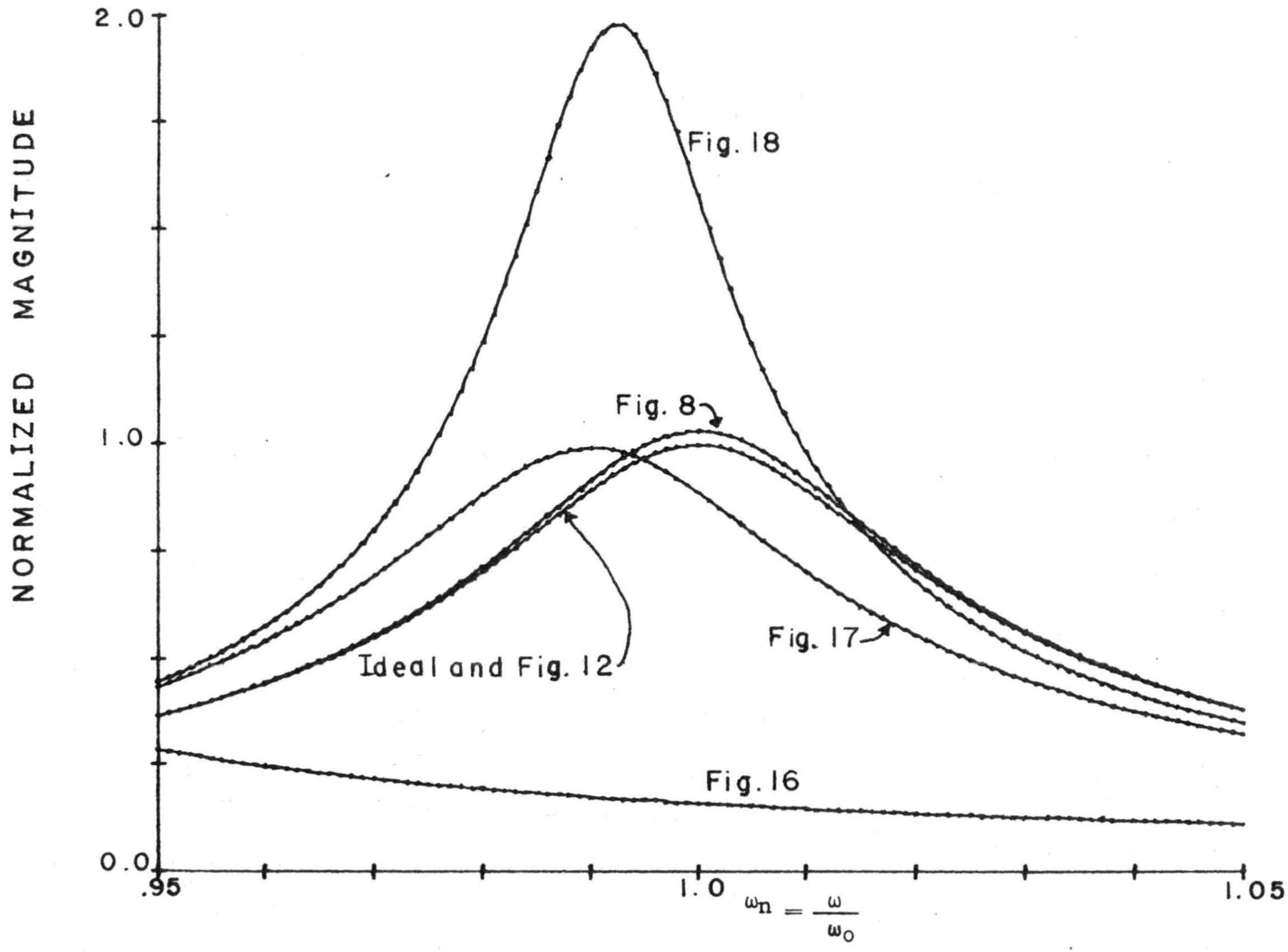


Fig. 26. Transfer Function Magnitude, Q = 25, τ_n = .005

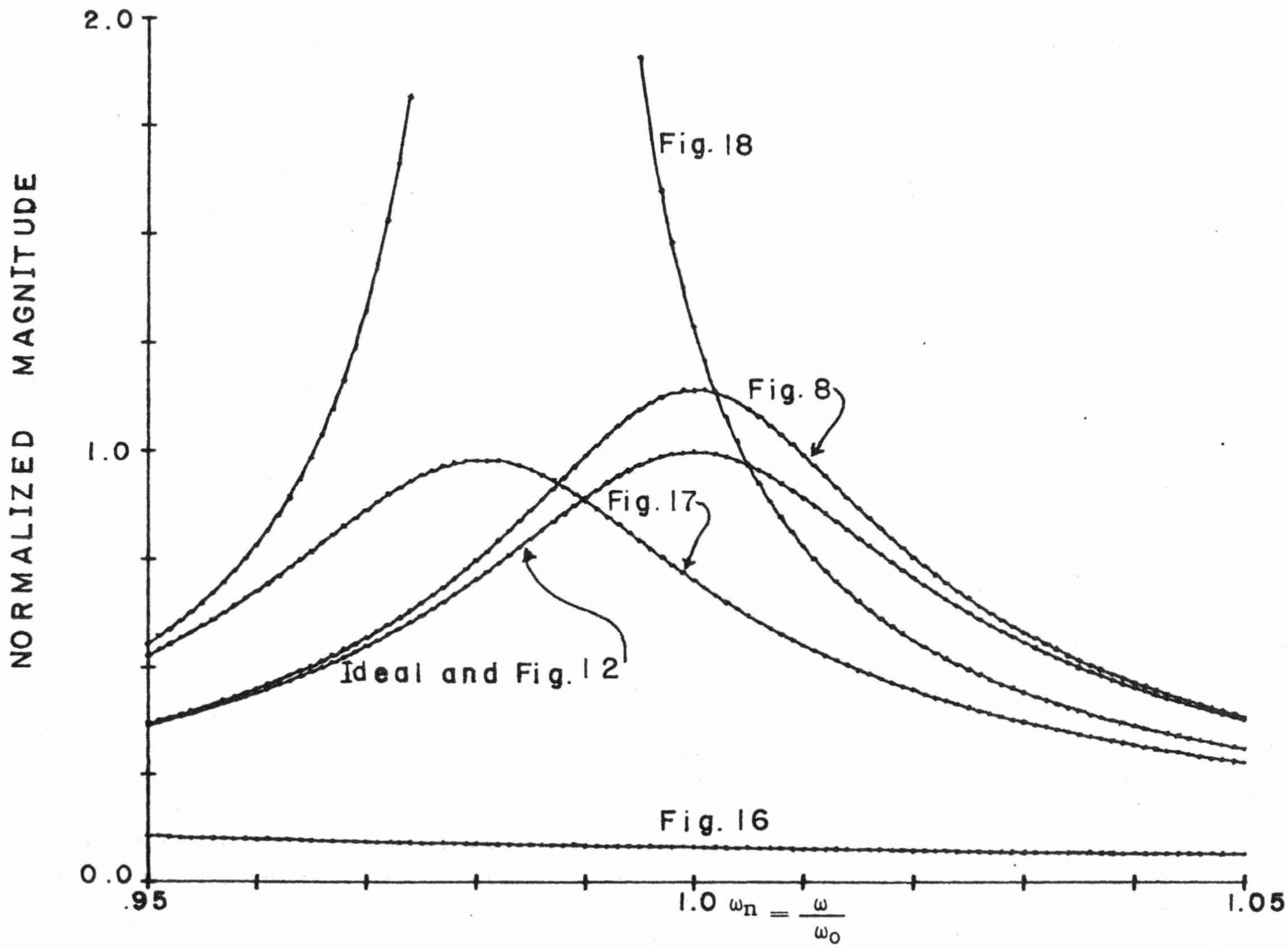


Fig. 27. Transfer Function Magnitude, Q = 25, τ_n = .01

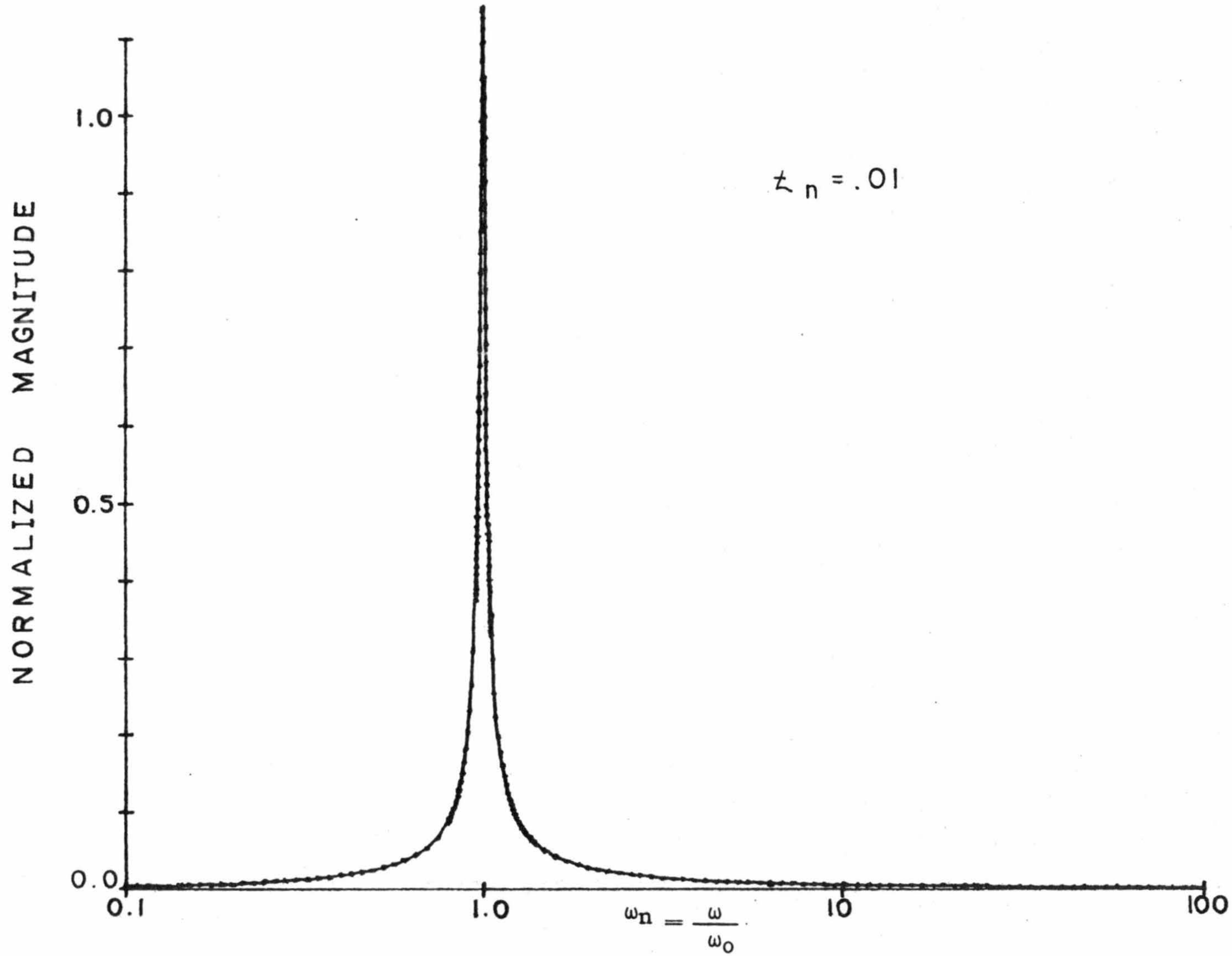


Fig. 28. Effect of Parasitic Poles on Transfer Function Magnitude for Zero-Sensitivity Circuit of Fig. 8

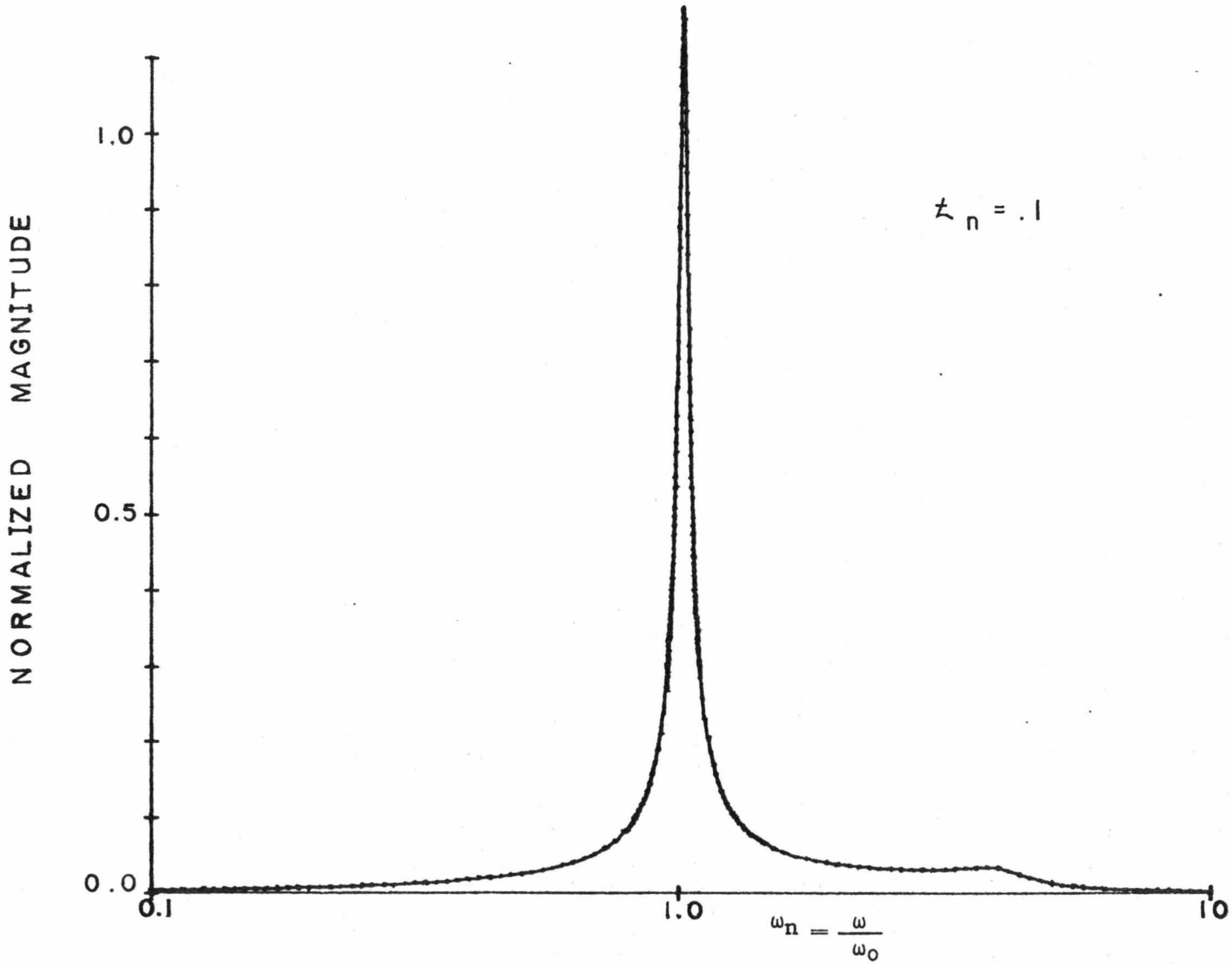


Fig. 29. Effect of Parasitic Poles on Transfer Function Magnitude for Zero-Sensitivity Circuit of Fig. 12

$$C = 315 \text{ pf}$$
$$R_2 = 949 \text{ K}\Omega \tag{108}$$
$$R_1 = 2.73 \text{ K}\Omega$$

With these component values, the normalized time constant, τ_n, is approximately .0117. The theoretical and measured values of f_0, Q, and peak amplitude are compared in Table 1.

TABLE 1

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

	Experimental	Theoretical	Percent Difference
f_o	9.991 KHz	9.932 KHz	0.6
Q	9.330 KHz	8.930 KHz	4.3
$Av(f_o)$	174.100 KHz	173.000 KHz	0.6

Designs at other center frequencies below 10 KHz gave similar results except that the gain at resonance was sometimes in the few percent range instead of the few tenths of a percent indicated in Table 1.

With three type 741 OP AMPs, the three-OP AMP zero-sensitivity filter of Fig. 12 proved to be unstable. With the single-pole model of the OP AMP used in the design, it follows from Fig. 19 and Fig. 21 that this filter possesses two pairs of left-half-plane complex-conjugate poles. Since the fifth pole is on the negative real axis, the filter is theoretically stable. However, the instability can be explained if the second pole of the 741 is included in the analysis. Thus, a more accurate model for the 741 is

$$A(s) = \frac{GB}{s}\left(\frac{\alpha GB}{s + \alpha GB}\right) \tag{109}$$

where α is typically around unity. With $\alpha = 1$, $\frac{\omega_o}{GB} = .01$, and $Q = 10$, the values used in the unstable design, the poles of the resulting eighth-order transfer function are located at:

$$\left.\begin{aligned} s &= \omega_o[-.0500021 \pm j(.998758)] \\ s &= \omega_o[-130 \pm j(64.9586)] \\ s &= \omega_o[-55.4558 \pm j(88.9402)] \\ s &= \omega_o[+26.3493 \pm j(59.7557)]. \end{aligned}\right\} \tag{110}$$

One of the three pairs of parasitic complex-conjugate poles is in the right half plane, thereby making the filter unstable. This right-half plane pole can, however, be moved back into the left half plane by decreasing α or by using different circuit configurations. The right-half-plane pole problem may not be present with other OP AMPs. No investigation was conducted in this respect.

Instabilities of Cascaded Amplifier Filters

For a Q of 10 and $\omega_o = .01GB$, the denominator polynomial of the three-OP AMP cascaded amplifier filter of Fig. 14 is

$$D(s_n) = s_n^2 + .1s_n + 1 + (.01)^3 s_n^3(s_n^2 + 20.1s_n + 1). \tag{111}$$

The roots of this polynomial are at

$$\left.\begin{aligned} s_n &= -.0500019 \pm (.998759)j \\ s_n &= 43.5686 \pm (86.2288)j \\ s_n &= -107.137 \quad . \end{aligned}\right\} \tag{112}$$

Because of the right-half-plane poles, this circuit is unstable when the single-pole model of (9) for the OP AMP is used.

For the same values of Q and ω_o, the two-OP AMP cascaded amplifier filter has denominator polynomial

$$D(s_n) = s_n^2 + .1s_n + 1 + (.01)^2 s_n^2 (s_n^2 + 20.1s_n + 1). \tag{113}$$

The roots of this polynomial are at

$$\left.\begin{aligned} s_n &= -.04900092 \pm .998895\, j \\ s_n &= -10.0009 \pm 99.4887\, j \;. \end{aligned}\right\} \tag{114}$$

This filter is thus stable if the single-pole model of the OP AMP is used. If, however, the two-pole model of the OP AMP given by (109) is used, the poles of the two-OP AMP cascaded amplifier zero-sensitivity filter are located at

$$\left.\begin{aligned} s_n &= -.0503863 \pm 1.00062\, j \\ s_n &= 3.61753 \pm 6.04157\, j \\ s_n &= -18.1888 \\ s_n &= -11.0454 \;. \end{aligned}\right\} \tag{115}$$

It can thus be concluded that for OP AMPs such as the 741 with a second pole as given by (109) either cascaded amplifier filter of Fig. 14 will be unstable for Q = 10 and ω_o = .01GB. The two-OP AMP cascaded amplifier filter may, however, be stable with other OP AMPs that have the second pole farther removed than that of the 741.

CHAPTER V

CONCLUSION

A consistent method for comparing active filters with respect to the active devices based on the active sensitivity function has been established.

Conditions for zero pole, ω_o, Q, and transfer-function-magnitude active sensitivities have been developed assuming a single-pole model of the active device. Conditions for zero second derivatives with respect to the OP AMP time constants have also been derived. Methods for obtaining low sensitivity predistorted active filters have been discussed.

Any realizable transfer function can be realized so that the transfer function is of the zero-active-sensitivity type. Several general zero-active-sensitivity filters have been presented. Some specific biquadratic bandpass zero-active-sensitivity filters were introduced.

The usefulness of a two-input active device with different gain functions on each input in zero-sensitivity active-filter design has been established.

Comparisons of the performance of the zero-active-sensitivity filters with popular existing bandpass filters have been made on a theoretical basis for both infinitesimal and incremental changes in the OP AMP time constants. The theoretical performance of the zero-sensitivity designs was shown to be significantly superior to that of the existing designs used in the comparison.

The experimental performance of the zero-sensitivity designs was discussed. With 741 type OP AMPs, the experimental and theoretical performance of the two-OP AMP zero-sensitivity filter tested agree quite closely. The three-OP AMP zero-second-derivative filter tested was unstable. This result was subsequently verified theoretically by using a more accurate two-pole model of the 741. For other OP AMPs of the present or future, this design may well be stable.

Suggestions for Future Work

Only a small number of zero-sensitivity designs have been investigated. Further investigation of zero-sensitivity designs with particular attention centered around the location of the parasitic poles introduced by both the single-pole and two-pole model of the OP AMP is necessary. The relation between component spread and passive sensitivity in zero-sensitivity designs remains to be studied. Finally, the use of two-input active devices with a different gain function on each input in low-sensitivity active filter designs warrants additional investigation.

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