APPLICATION OF MODEL TECHNIQUES TO MASS TRANSFER AND EVAPORATION STUDIES

By

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Hydrologists and irrigation engineers very often find that they must answer either one or both of the following questions:

1. How much evaporation will take place from a given lake or reservoir during a specified time interval?

2. How much evaporation is to be expected from a reservoir created behind a dam which is still in the planning stage?

Attempts have been made to answer these questions for specific cases by making use of data obtained from evaporation pans placed at the site, by application of empirical formulae synthesized from evaporation pan data, or by extension of formulae arrived at by application of various mass transfer theories. Because of the lack of certainty in any of the available methods and because application of two or more of the methods, or different equations for one of the methods, to a given problem often gives widely varying results; a cooperative effort on the part of several Governmental agencies (2) was made to actually measure the evaporation from Lake Hefner and evaluate the various equations and methods which have been proposed for the estimation of evaporation.

Prior to the Lake Hefner studies much thought had been given to the feasibility of constructing a scale model of the lake or reservoir in question (together with a portion of the surrounding terrain), placing it in a low velocity wind tunnel, and measuring the evaporation. However, because of the great difference between

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a characteristic Reynolds number for the model and for the prototype which must exist when scaling a lake of any appreciable size, no attempts have been made along these lines. With the Lake Hefner prototype data for evaporation and wind structure available, and with more knowledge of boundary layer theory, an excellent opportunity was at hand to develop model techniques by which accurate estimates of evaporation from existing reservoirs or proposed reservoirs could be made. Accordingly, a contract was awarded the Civil Engineering Department of Colorado A & M College to construct a model of Lake Hefner and conduct evaporation measurements under controlled conditions in a wind tunnel in cooperation with the U. S. Geological Survey.

The purpose of this paper is to show how evaporation data obtained from a model lake may possibly be used to predict the amount of evaporation from its prototype. Dimensional analysis is used to group significant variables into parameters which may be measured both in the model and the prototype. Use is then made of von Kármán's extension of the Reynold's analogy and appropriate drag coefficient formulae for flat plates to form a basis for the comparison of evaporation from the model and the prototype.

Dimensional Analysis

The variables of major importance which affect the rate of evaporation E from a lake may be placed in the following equation:

$$E = \phi \left(\sqrt{\tau_0/\rho},\ \Delta C,\ V_e,\ V_f,\ k_l,\ k_w,\ S,\ D,\ A\right). \tag{1}$$

The following table lists the meaning of each symbol and the fundamental units used for each throughout the paper.

SYMBOL	DEFINITION	UNITS
E	Average rate of evaporation	$FL^{-2}T$ *
$\sqrt{\tau_0/\rho} = V_*$	Shear velocity measured at an upwind station or some other designated point	LT^{-1}
ΔC	Difference in water vapor concentration between an upwind station and the saturation concentration at lake surface temperature	FL^{-3}
ν_e	Molecular diffusion coefficient of water vapor	L^2T
ν_f	Molecular diffusion coefficient of momentum	L^2T
k_1	Roughness of upwind terrain	L
k_w	Roughness of lake surface	L
S	Shape of lake	
A	Area of lake	L^2
D	Wind direction	
ρ	Air density	$FL^{-4}T^2$
τ_0	Surface shear at upwind station	FL^{-2}

By dimensional analysis the variables of Eq 1 may be grouped into dimensionless parameters to form the following equation:

$$\frac{\sqrt{A}\,E}{\nu_e\,\Delta C} = \phi_1\left(\frac{\sqrt{A}\,V_*}{\nu_e}, \frac{\nu_f}{\nu_e}, \frac{k_w}{k_1}, \frac{\sqrt{A}}{k_w}, D\right) \qquad (2)$$

The shape parameter S has been omitted from Eq 2 since the shape for a particular lake will be practically constant and will of course be the same in the model as in the prototype. For convenience the terms in Eq 2 may be renamed such that

$$N = \phi_1(R_*, \sigma, r, r', D) \qquad (3)$$

where N represents $\frac{\sqrt{A}\,E}{\nu_e \Delta C}$ and is similar to Nusselts number

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* The letters F, L, and T represent force, length, and time respectively.

in heat transfer, R_* replaces $\frac{\sqrt{A}\, V_*}{\nu_e}$ and represents a type of Reynolds number, σ is $\frac{\nu_f}{\nu_e}$ or the Prandtl number, r is the roughness ratio $\frac{k_w}{k_l}$, and r' is equal to $\frac{\sqrt{A}}{k_w}$.

In order to obtain complete geometrical and dynamical similarity between the model and the prototype or in other words the same function ϕ_1 for the model as occurs for the prototype, the model should be tested and designed such that the five parameters in Eq 3 R_*, σ, r, r' and D have values comparable to those of the prototype. However, as will be illustrated equality for the five parameters cannot be obtained. For example, a practical scale for the Lake Hefner model is 1:2000. Typical values of the various variables for the prototype are as follows:

$\sqrt{A}$	--	10,000 ft
k_w	--	4 in
k_l	--	4 in

The Prandtl number is the same for model and prototype as is the range of values for ν_f. The value of r' for the prototype has a typical value of about 30,000 which can be equaled in the model provided the value of k_w is in the neighborhood of 0.002 in. By casting the lake surface upon a glass surface using plaster of Paris, roughnesses in the order of magnitude of 0.001 to 0.003 in. may be attained. The value of r for the prototype is approximately one and may be duplicated in the model provided the terrain is constructed with a roughness of about 0.002 in. The parameter R_* varies from about 10^7 to 10^8 for the prototype and inasmuch as V_* for the model is of the same order of magnitude as for the prototype, R_* for the model is in the range from 5×10^3 to 5×10^4. The parameter

D may be varied for the model by turning the model in the wind tunnel.

The immediate problem is to find some sound basis to extrapolate the model data obtained at a value of R_* 2000 times smaller than the value of R_* for the prototype. A possible method of attack is to obtain a theoretical relationship between the parameters of Eq 3 and then proceed by making laboratory and field measurements to verify the results. In the following section use is made of the von Kármán extension of Reynold's analogy to form a basis for extrapolation. The effect of r, and D upon N is not predicted theoretically and must be determined by experiment.

Evaporation Equations for Flat Surfaces with Zero Pressure Gradient

In the case of zero longitudinal pressure gradient -- see Yih (7:55)* -- Kármán expresses the analogy between momentum transfer and mass transfer by

$$\frac{1}{C_e} = \frac{2}{C_f} + 5\left(\frac{2}{C_f}\right)^{\frac{1}{2}} \left\{\sigma - 1 + \ln\left[1 + \frac{5}{6}(\sigma - 1)\right]\right\} \tag{4}$$

where $C_e = \frac{q}{\rho \Delta C' U_o}$ in which q is the mass of water vapor transferred for each unit of area and unit time, $\Delta C'$ is the same as ΔC except that $\Delta C'$ is expressed in weight of water vapor per unit weight of dry air, and U_o is the ambient velocity of the air stream approaching the evaporation surface. The Prandtl number σ has the value of 0.6. The drag coefficient C_f for smooth and rough plates is expressed as a function of the Reynolds number $R = \frac{U_o L}{\nu_f}$ and $\frac{L}{K_w}$ -- L is the plate length which is comparable to $\sqrt{A}$ in R_*. Before proceeding further, C_e should be expressed in terms of N, and R in terms of R_*.

* The first number in parenthesis is the bibliographical entry number and the number following a colon is the page number.

According to the measurements of Klebanoff and Diehl (3:12), velocity distributions in the boundary layer on a flat plate with zero pressure gradient are better expressed by the 1/7th power law than by the logarithmic laws. Taking the 1/7th power expression

$$\frac{u}{V_*} = 8.16\left(\frac{yV_*}{\nu_f}\right)^{\frac{1}{7}} \tag{5}$$

in which u is the local mean velocity at a distance of y above the boundary and remembering that when y is equal to δ -- δ being the thickness of the boundary layer -- u is equal to U_o, an expression for R

$$R = 11.85\ R_*^{10/9} \tag{6}$$

results when the formula (see Rouse (5:188))

$$\delta = \frac{0.377\ L}{R^{1/5}} \tag{7}$$

is substituted into Eq 5. Furthermore,

$$N = \sigma C_e R \tag{8}$$

which upon substitution for R by Eq 6 becomes

$$N = 7.11\ C_e R_*^{10/9}. \tag{9}$$

<u>Smooth Boundaries</u> -- For the approximate range $10^3 \leq R_* \leq 10^5$, the drag coefficient may be expressed by

$$C_f = 0.074\ R^{-1/5} \tag{10}$$

as may be seen from examination of the work of Schlichting (6:117). Upon substitution of Eq 10 into Eq 4 and making use of Eqs 6 and 9, an equation for N

$$N^{-1} = 6.23R_*^{-8/9} - 3.79R_*^{-1} \tag{11}$$

results which is valid for the range of R_* indicated.

For values of R_* greater than 10^5 up to about 10^8, C_f may be expressed in the form

$$C_f = \frac{0.427}{(-0.407 + \log R)^{2.64}} \tag{12}$$

which is commonly known as the [illegible] equation. Eq 12 may be [illegible] with Eq 6 and 9 to obtain

$$N^{-1} = R_*^{-10/9}\left[0.659\left(0.667 + \frac{10}{9}\log R_*\right)^{2.64} - 1.228\left(0.667 + \frac{10}{9}\log R_*\right)^{1.32}\right]. \tag{13}$$

Rough Boundaries -- From the work of Schlichting (6:41), the drag coefficient C_f for rough boundaries -- where $V_* k_w/\nu_f$ exceeds about 70 -- may be expressed as a function of only L/k_w

$$C_f = (1.89 + 1.62 \log L/k_w)^{-2.5} \tag{14}$$

in which L/k_w is analogous to r'. When Eq 14 is substituted into Eq 4 with Eq 6 and Eq 9, N may be expressed as

$$N^{-1} = R_*^{-10/9}\left[0.281(1.89 + 1.62 \log L/k_w)^{2.5} - 0.801(1.89 + 1.62 \log L/k_w)^{1.25}\right]. \tag{15}$$

Comparison of Available Data with Evaporation Equations

Fig. 1 is a plot of Eqs 11, 13, and 15. Eq 15 is plotted for different values of L/k_w and the transition region between $V_* k_w/\nu_f = 70$ and the curves for smooth boundaries is taken according to Fig. 89 of Schlichting (6:118). Also Eq 20 of Albertson (1:250) is plotted along with some of the actual points which were obtained by measurement of evaporation from a smooth, wetted porous-porcelain boundary placed in a wind tunnel.

Of particular importance is the excellent agreement of the data of Albertson with conversion Eq 11. For values of R_* less than 5×10^2 the agreement becomes poor as is to be expected since Eq 10 is no longer valid. The data of Albertson includes a range of x'/x -- x' is the length of the evaporation boundary and x is the distance measured from the leading edge of the plate to the corresponding point determined on the evaporation boundary by

x' -- from 0.0204 to 0.40. A close examination of the points indicates a very slight influence of x'/x upon the relationship between N and R_* which is important because the value of x'/x for a natural lake is a difficult quantity to define.

Included with Fig. 1 is a group of points determined from the data obtained at Lake Hefner. These data are for the periods of January 6 to 20, April 1 to 15 and July 1 to 15, 1951. In the determination of N for these points, L was taken as $A^{\frac{1}{2}}$ and the values of ΔC and v_e were taken as an arithmetical average of the eight separate three-hour averages determined for each day and corresponding to a measured value of E. In the determination of R_*, v_e has the same average value as was used in N, and V_* was obtained by plotting the velocity profile at the upwind meteorological station for each three hour period, calculating V_* for the three hour period by using the equation

$$u/V_* = 5.75 \log y/k + 8.5 \tag{16}$$

and finally averaging the eight values obtained for each day. In using Eq 16 to calculate V_*, the roughness k was eliminated by solving simultaneously the two equations resulting from substitution of u at the 2-meter and at the 16-meter elevations along with the corresponding values for y. In this same manner the value of k was also determined. With only two exceptions was the value of $V_* k_w/v_f$ less than 70 which indicates that during a given day the average roughness k_w of the lake surface is large enough to be classified as rough. Munk (4) gives evidence that for wind speeds in excess of 6 to 8 meters per second at a 15 meter elevation a sea surface always becomes hydrodynamically rough, while for smaller wind velocities the roughness of the sea surface is uncertain. Upon examining the data for a stratification caused by L/k_w similar to that predicted by Eq 15 none was apparent. In part, this may be due to the inaccurate determination of k_w since the water surface does not in general present a surface of zero horizontal velocity and furthermore, an average value of k_w may obscure the anticipated trend. A point of major interest is that the major axis passing through the near elliptical swarm of data has very nearly a slope of 10/9 in accordance with Eq 15 for rough boundaries.

With an average value of r' in the neighborhood of 3×10^4, the center of gravity of the data for the Lake Hefner prototype instead of falling near R_* equal to 6×10^7 would be expected to be more nearly located at R_* equal to 1.5×10^7 if evaporation were to occur similarly to that from a square plate. One of the main reasons for the shifting of the data toward a value of R_* approximately four times larger than the one predicted by conversion Eq 15 is believed to be the dissimilarity in shape between the lake and the rectangular plates for which the conversion formulae are applicable. The length L of the rectangular evaporation boundary is replaced by $A^{\frac{1}{2}}$ in calculating N and R_* for the lake data therefore, any deviation of the lake shape from that of a square will cause a variation in the relationship between N and R_*. Other factors which tend to scatter the data are the fact that arithmetical averages were used in determining R_* when actually N is proportional to $R_*^{10/9}$, that during a given day the wind direction D is not constant, and that atmospheric lapse rates may affect the rate of evaporation; however, upon investigation of the relationship between lapse rates and velocity profiles using three-hour averages of the data, no correlations were found.

Extrapolation of Model Data

As has been pointed out under the section on the dimensional analysis; σ for model and prototype is 0.6, r' may be made the same in both model and prototype provided an average value of k_W is taken, and r may be made the same once an investigation of the site is made to determine a representative value for k_1. The values of R_* for the model will be approximately the values of R_* for the prototype multiplied by the scale factor since V_* will be nearly equal for the model and prototype and may be controlled to some extent by placing a roughened boundary upstream from the terrain leading to the lake surface -- Klebanoff and Diehl (3) -- or by varying the ambient velocity in the wind tunnel.

Of first importance is the fact that the model and prototype are similar in shape and accordingly the model data should be shifted toward larger values of R_* than is predicted by Eq 11 or 13 by an amount approximately the same as the prototype data. No experimental work has been done on measuring the evaporation from or the drag on smooth flat surfaces to determine the effect of shape which would serve as a basis for comparison.

In order to effect an extrapolation, the model must be tested at a value of N which may be obtained by extending the curve given by Eq 15 for the appropriate value or r' until it intersects the curve defined either by Eq 11 or Eq 13. Tests conducted at smaller values of N than that determined by the intersection, a value which will be called N', will be of little value because the surface will be hydrodynamically smooth and N will no longer be proportional to $R_*^{10/9}$. Tests conducted at N greater than N'

will be for a surface in a transition phase from hydrodynamically smooth to hydrodynamically rough; however, these data will have more utility because the transition follows fairly well the relationship $N \sim R_*^{10/9}$. Letting the subscript m be understood to mean model and the subscript p to mean prototype, extrapolation may possibly be carried out by use of the equation

$$N_p = N_m R_*^{-10/9} \quad (R_*)_p^{10/9} \quad . \tag{17}$$

The utility of Eq 17 will be determined when sufficient model data have been obtained.

Summary

The evaporation measurements at Lake Hefner have, for the first time, produced accurate data from a fairly large body of water which may be used to verify the results of evaporation studies made on a scale model. The Civil Engineering Department of Colorado A & M College under contract with the Department of the Navy, Bureau of Ships, and in cooperation with the U. S. Geological Survey has begun a model study of Lake Hefner with the aim of perfecting model techniques which may be generally usable.

By a dimensional analysis, similarity between evaporation from a model and its prototype is shown to depend primarily upon the equality of the parameters within the parenthesis of the following equation:

$$\frac{\sqrt{A}E}{\nu_e \Delta C} = \phi_1\left(\frac{\sqrt{A}\ V_*}{\nu_e},\ \frac{\nu_f}{\nu_e},\ \frac{k_w}{K_l},\ \frac{\sqrt{A}}{K_w},\ D\right).$$

Making use of preliminary prototype data, satisfactory evidence is available to demonstrate that all parameters in the parenthesis with the exception of $\frac{\sqrt{A}\ V_*}{\nu_e}$ or R_* may be made equal for model and prototype if average values are used for the prototype data.

Because V_* and V_e are of the same order of magnitude in the model and prototype, the ratio of R_* for the model to that for the prototype will be approximately the scale factor or 1:2000 for the Lake Hefner model. Furthermore, with $\frac{\sqrt{A}}{K_w}$ equal in magnitude for model and prototype, the model surface will be hydrodynamically smooth in the range of R_* available for testing.

In order to permit prediction of prototype evaporation from the model evaporation data obtained at a value of R_* approximately 2000 times smaller than that for the prototype, the technique is suggested of using the von Kármán extension of Reynolds analogy which allows calculation of the evaporation coefficient N for a plane rough or smooth rectangular boundary from expressions for the drag coefficient C_f. Verification of the conversion for a smooth boundary at values of R_* less than about 3×10^3 by experimental data is excellent. Tentative values of N for the Lake Hefner prototype correspond to values of R_* approximately four times larger than the values of R_* predicted by the conversion formula Eq 15 for rough surfaces. This variation is believed to be the result of a difference in shape between the rectangular plates and the lake surface and perhaps of the averaging procedure used to determine the elements comprising the parameters N, R_*, $\frac{\sqrt{A}}{K_w}$, and D. However, agreement between the conversion formula for rough surfaces Eq 15 and the prototype data rests in the observation that N is approximately proportional to $R_*^{10/9}$. Although no model data is yet available, these data when obtained are also expected to fall at values of R_* approximately four times greater than that predicted by the conversion formula because of shape similarity between model and prototype.

Since the prototype surface seems to be hydrodynamically rough when a daily average roughness is considered, a procedure for extrapolating the model data is as follows:

1. Select an average value of $\frac{\sqrt{A}}{k_w}$ for the prototype.
2. On Fig. 1, or an equivalent chart, draw a straight line for the value of $\frac{\sqrt{A}}{K_w}$ chosen by interpolating between the lines drawn by previous calculations from Eq 15 or calculate the coordinates directly from Eq 15.
3. Extend the line drawn in Step 2 until it intersects the curve defined by either Eq 11 or Eq 12 for smooth surfaces.
4. Test the model at a value of N -- N' -- defined by Step 3.
5. Extrapolate to values of R_* for the prototype by the following expression:

$$N_p = N_m' R_*'^{-10/9} (R_*)_p^{10/9}.$$

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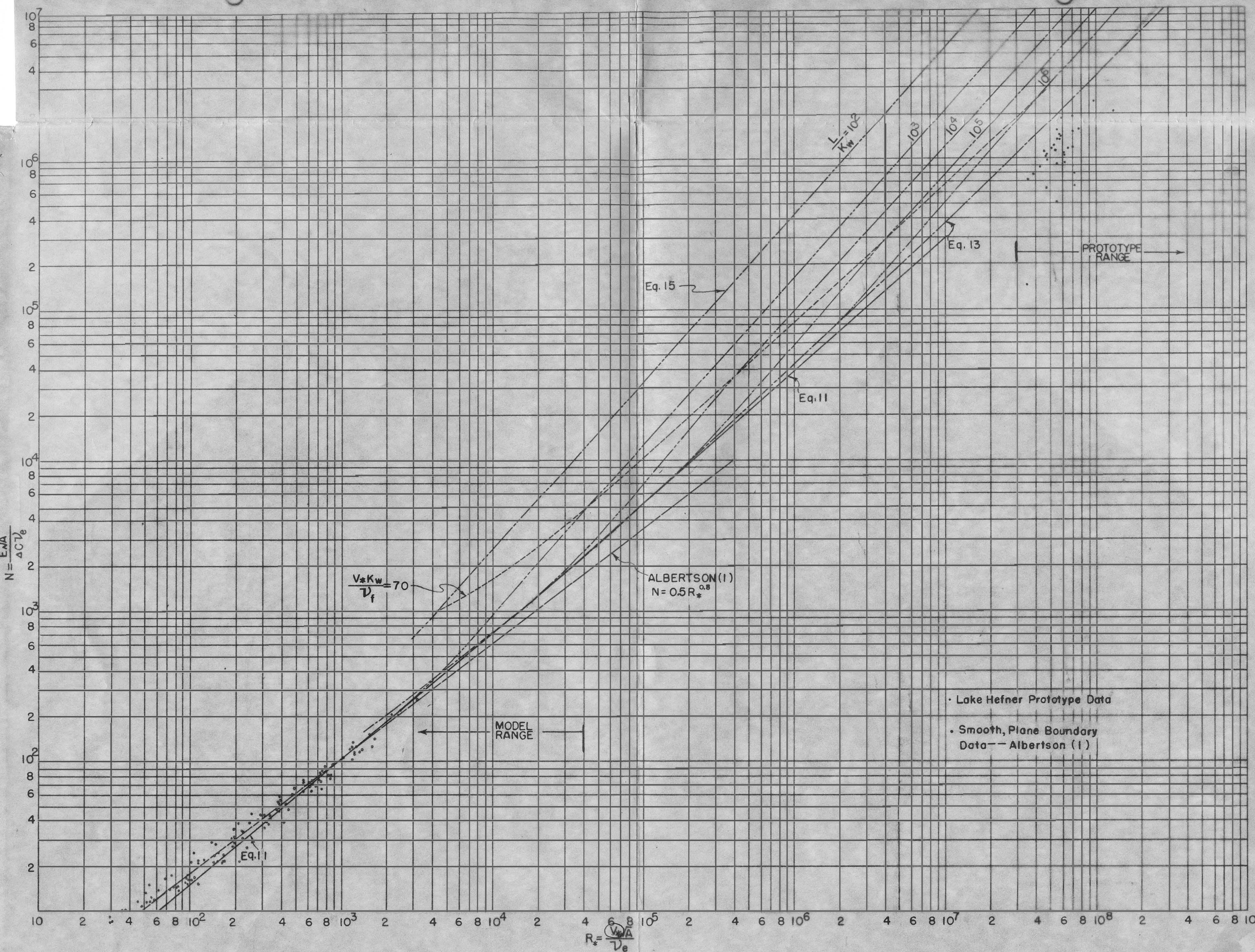

Fig 1 -- Comparison of conversion formulae with evaporation data