EFFECT OF POROSITY AND PARTICLE SHAPE AND DIAMETER ON HYDRAULIC CONDUCTIVITY OF SANDS AND GRAVELS

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LIST OF SYMBOLS

a	A numerical constant.
C	Coefficient in the general Hagen-Poiseuille equation.
C_b	Hagen-Poiseuille coefficient based on the gross cross-section of flow.
C_s	Hagen-Poiseuille coefficient based on spatial average flow through the pores.
c_o	Coefficient in Hagen-Poiseuille equation written in a particular form.
d	Diameter, Mean diameter of particle, (L)[1].
d_e	Effective hydraulic diameter of particle computed by sieve analysis, (L).
d_p	Hydraulic length characteristic of pore, (L).
f	Coefficient in Darcy-Weisbach pipe flow equation.
g	Acceleration of gravity (FT^2L^{-1}).
h	Hydraulic head difference, (L).
K	Fluid conductivity, (L/T).
L	Mean, straight-line length of flow path, (L).
L_p	Actual total length of flow path, (L).
	Volumetric porosity of sand or gravel.
R	Reynolds number.
R_b	Reynolds number based on the gross cross section of flow.
R_p	Reynolds number based on spatial average flow through the pores.
R_H	Hydraulic Radius, (L).

[1] Dimensions in parenthesis are indicated unless the quantity is dimensionless.

S_s Ratio of hydraulic radius to particle diameter.

V Velocity based on gross cross section of flow, (LT^{-1}).

V_p Spatial average velocity through the pores, (LT^{-1}).

W Weight of a grain-size fraction of sand or gravel specimen, (F).

δ Effective hydraulic diameter of particle based on hydraulic measurements, (L).

ψ Resistance coefficient.

ψ_b Resistance coefficient based on the gross cross section of flow.

ψ_p Resistance coefficient based on the spatial average flow through the pores.

ρ Fluid density, $(FT^2 L^{-4})$.

μ Viscosity, $(FT L^{-2})$.

Effect of Porosity and Particle Shape and Diameter on Hydraulic Conductivity of Sands and Gravels.

Synopsis

Darcy's equation is commonly used to solve problems involving viscous flow of fluids through sands and gravels. The hydraulic conductivity term in Darcy's equation, however, includes at least the fundamental effects of porosity, particle diameter, particle shape and fluid density and viscosity. Equations in which the more fundamental factors are explicitly expressed would be more useful. Owing to the similarity of factors involved, such equations may take the form of the Hagen-Poiseuille and Darcy-Weisbach equations for pipe flow. The greatest handicap to the application of these equations to the problem of flow through sands and gravels has been the lack of understanding regarding the effects of porosity, particle shape and particle diameter. If these could be properly evaluated, then viscous flow through sands and gravels could be fully described by $\psi = CR$, where ψ is the resistance coefficient similar to the friction factor used in pipe flow, C is the constant, and R is Reynolds number.

In addition to conducting further experimentation, the writers have reviewed the work of Carman, Fair and Hatch, Bakhmeteff and Feodoroff, Franzini, and others and propose methods whereby the factors of porosity and particle diameter may be included in ψ and R. If this is done, C, depends

upon particle shape. Data for flow through sands and gravels may then be reduced to a series of universal curves or plots of ψ and R where each curve represents a particular shape. Such a series of curves is presented. Knowing the approximate particle shape, the sieve analysis, and the porosity, one should be able to predict with a high degree of accuracy the flow characteristics of sands and gravels. The relationship or porosity, particle shape and diameter with the conventional hydraulic conductivity term, K, of Darcy's law is also shown.

Development of Theory

The analogy between pipe flow and flow through porous materials is demonstrated by the similar factors involved in each case. These factors describing the fluid are density ρ, viscosity μ ; and the flow, piezometric head loss h , length of flow path L , and velocity V_p . The acceleration of gravity is represented by g . Several factors are necessary to describe the medium, and must include at least a characteristic length or diameter d_p , the porosity of the medium n , and the average shape of the particles. Other factors which may require consideration are roughness of the particles, and the statistical distribution of the length and shape characteristics of the particles.

If all the factors describing the medium, with the exception of d_p , are included in a constant; Fair and Hatch (7), and Bakhmeteff and Feodoroff (2) have shown by dimensional analysis that

$$h/_L = \frac{\text{constant}}{2g} (V_p)^a \quad (d_p)^{a-3} \quad (\mu/\rho)^{2-a} \quad . \qquad (1)$$

Following established practice, an arbitrary constant of 2 is included in the denomonator of Eq 1. Letting a = 1 , the Hagen-Pouiseuille equation for laminar flow may be obtained,

$$h/_L = \frac{C \mu V_p}{2g \rho d^2} \quad . \qquad (2)$$

Letting $a = 2$, the Darcy-Weisbach equation of pipe flow for both laminar and turbulent flow results,

$$h/_L = f \frac{V_p^2}{d_p \, 2g} \, , \qquad 2g\,d_p \tag{3}$$

where f is the friction factor. These are pipe flow equations. For laminar flow in pipes the constants in Eqs 1 and 2 are related by Reynolds number R.

$$C = f\,R \tag{4}$$

The constant C is dependent only upon the shape of the pipe. The constant f of Eq 3 for laminar flow is dependent upon shape and Reynolds number. With increasing Reynolds number the importance of viscosity decreases and f becomes also a function of roughness. An analogous situation can be expected for flow through porous media.

Eqs 2 and 3 may be adapted to flow through porous media. Since the velocity in pipe flow is actually a pore velocity, then a similar term for porous media should be used. The pore velocity V_p is then

$$V_p = \frac{V}{n} \tag{5}$$

where V is the bulk velocity found by dividing the discharge by the gross cross-sectional area, and n is the porosity.

For a circular pipe the hydraulic radius R_H is

$$R_H = \frac{\text{Area}}{\text{wetted perimeter}} = \frac{\text{diameter}}{4}$$

By analogy the diameter of the pore can be expressed linearly proportional to the hydraulic radius. Following the development of Fair and Hatch (7), if the effective size of the pore d_p is made equal to its hydraulic radius,

$$d_p = R_H = \frac{\text{Area of flow}}{\text{Wetted perimeter}} = \frac{\text{Area of flow x Length}}{\text{Wetted perimeter x length}} =$$

$$\frac{\text{Volume of flow}}{\text{Surface area of solids}} = \frac{\text{Void Volume}}{\text{Surface area solids}} .$$

If n is porosity

$$\text{Void volume} = \text{Solid volume} \left(\frac{n}{1-n}\right) ;$$

substituting,

$$d_p = \frac{\text{Volume Solids}}{\text{Surface Area solids}} \frac{n}{(1-n)} .$$

For uniform-diameter spheres,

$$d_p = \frac{d}{6} \left(\frac{n}{1-n}\right) ,$$

where d is the diameter of a sphere.

For other shapes in general,

$$d_p = \frac{d}{S_s} \left(\frac{n}{1-n}\right). \qquad (6)$$

Where S_s is a shape factor varying from 6 to 7.7.

Substituting these values of V_p and d_p in Eqs 2 and 3,

$$h/_L = \frac{C_s \mu V}{2g \rho d^2} \frac{(1-n)^2}{n^3} , \qquad (2a)$$

where C_s equals $C(S_s)^2$, and

$$h/_L = \psi_p \frac{V^2}{2 g d} \frac{(1-n)}{n^3} . \qquad (3a)$$

ψ_p is the coefficient of resistance based on the pore velocity and pore diameter. A comparison of Eqs 2a and 3a shows the effect of porosity is different for turbulent and laminar flow.

From Darcy's law,

$$h/_L = V/_K , \tag{7}$$

where K is the conductivity[1] having the dimensions LT^{-1}. Combining Eqs 2a and 7 and solving for K,

$$K = \frac{2 g \rho d^2}{\mu C_s} \frac{n^3}{(1-n)^2} . \tag{8}$$

Eq 8 describes K in more fundamental terms than does Eq 7

As is C in pipe flow, C_s is primarily dependent upon shape although other factors such as arrangement of particles, statistical distribution of particle shape and direction of flow through the packing may have some effect.

Porosity Function

Eq 8 shows that the conductivity of a material varies with its porosity in accordance with the expression $n^3/(1-n)^2$. This relationship was first presented by two German workers, Kruger (1918) and Zunker (1920) who arrived at their conclusions empirically. Theoretical support was provided in 1927 by Kozeny, another German worker. In the United States, Fair and Hatch

[1] Conductivity is frequently termed "permeability" in engineering. The writers use "conductivity" to describe the transmissibility of a specific fluid (dimensionally LT^{-1}) and "permeability" to describe the general flow transmission property of the medium which is independent of the fluid (dimensionally L^2).

independently presented a similar theoretical development, which forms the basis, in general, for the presentation leading to the deduction of Eq 8.

Of interest are the various expressions which have been suggested by investigators to describe the variation of conductivity with porosity. Most of these expressions have been cited by Dalla Valle (6), Fransini (8), or Loudon (10).

Schlicter	$K \propto$	$n^{3.31}$
Terzaghi	$K \propto$	$\left(\frac{n-0.13}{\sqrt[3]{1-n}}\right)^2$
Hulbert & Feben	$K \propto$	$\left(\frac{1}{69.43-n}\right)$
Bakmeteff & Feodoroff	$K \propto$	$n^{4/3}$
Mavis & Wilsey	$K \propto$	n^6 or n^5
Rose	$K \propto$	n^a where $a = \phi(n)$
Buckingham, Edgar	$K \propto$	n^7
Smith, W. O.	$K \propto$	$(1-n)^{2/3}\left[\frac{1}{(1-n)^{2/3}} - 1\right]$
Rapier and Duffield	$K \propto$	$\frac{n^{3/2}}{1.115(1-n)\left[(1-n)^2 - 0.018\right]}$
Kozeny, Fair and Hatch	$K \propto$	$\frac{n^3}{(1-n)^2}$

The recent work of Alghita (1), Fransini (8), and Loudon (10) lends conclusive support to the validity of expression, $n^3/(1-n)^2$.

When resistance coefficient computed using bulk velocity and uncorrected for effect of porosity on pore size is plotted on a logarithmic scale as a function of Reynolds number similarly computed, with the data in the laminar range will fall on a straight line with a slope of minus one. The equation of this line is $R_b \psi_b = C_b$, where C_b corresponds to bulk flow. C_b is equal to ψ_b at $R = 1$ and is a function of porosity as well as pore shape. By proper correction for porosity, however, and plotting ψ_p as a function of R_p where ψ_p is as previously defined and R_p is the Reynolds number characteristic of pore flow, C_s may be determined. C_s is independent of porosity and depends principally on particle shape and possibly other minor factors. Substituting from Eqs 5 and 6, R_p and ψ_p may be defined in terms of ψ_b and R_b as follows:

$$R_p = \frac{V_p d_p \rho}{\mu} = \frac{V d \quad \rho}{\mu (1-n)} = {}^{R_b}/_{1-n} \text{, and} \tag{9}$$

$$\psi_p = \frac{2 \; g \; h \; d_p}{L \; V_p^2} = \frac{2 \; g \; h \; d \; n^3}{L \; V^2 \; (1-n)} = \frac{\psi_b n^3}{1-n} . \tag{10}$$

In the laminar range,

$$C_s = R_p \;\; \psi_p = R_b \; \psi_b \frac{n^3}{(1-n)^2} = C_b \frac{n^3}{(1-n)^2} \tag{11}$$

At a porosity of 57 percent the porosity function $\frac{n^3}{(1-n)^2}$ equals one. Therefore at 57 percent porosity C_s is numerically equal to C_b.

If the pore values of Reynolds number and resistance coefficient are plotted for a particular medium, all points should fall on a single line regardless of porosity. Using this procedure, available data for various sands were plotted.

Effective diameter

For uniform-size material the diameter d in the foregoing analysis was considered to be the grain size. It was found by taking the mean size of the two adjacent-sized sieves used to separate the material. The question naturally arises as to proper diameter to use for a material of non-uniform particle size. Several definitions of such an effective diameter d_e have been suggested by other investigators. Hazen's definition of the effective diameter as the 10-percent size is probably the most widely known. Others have suggested the 50-percent size. Mavis and Wilsey found on the average that the 34-percent size was the effective diameter for the sands tested by them. Allen suggested the equation,

$$d_e = \sqrt[3]{\frac{\Sigma W}{\Sigma (W/d^3)}} \;,$$

where W is the weight of an approximately uniform material of mean diameter d.

The effective diameter of characteristic length of a graded material suggested by Fair and Hatch, Kozeny, and Blake is

$$d_e = \frac{\Sigma P}{\Sigma [P/d]} \; . \qquad (12)$$

This definition of effective diameter may be described as the grain diameter of a uniform-sized material which has the same area-volume ratio as the graded material and thus follows the concept of an equivalent hydraulic radius used to develop Eq 2a.

The writers propose that the effective diameter be measured by measuring the conductivity of the medium. Solving Eq 8 for d and using the symbol δ for effective diameter thus determined,

$$\delta = \sqrt{\frac{K \mu C_s}{2 g \rho} \frac{(1-n)^2}{n^3}} \quad . \tag{13}$$

The procedure for computing δ was to determine the shape factor C_s for a uniform-sized sample of sand. Then, applying this value of C_s to a graded sand, δ was computed from Eq 14 by utilizing the measured conductivity of the graded sand.

Under the definition of Eq 13 and if C_s has been defined so that it depends only on shape, roughness, and perhaps arrangement of the particles, the effective diameter δ must take into account the mean and statistical distribution of particle sizes. The effective diameter δ is the particle diameter of a uniform-grained sand which would have the same permeability as graded sand of the same shaped particles and having the same porosity.

Shape constant

As shown by Carman (5), the magnitude of the shape constant may be derived by comparison with the pipe flow equations. If the Hagen-Poiseuille equation is written in terms of the hydraulic radius of a circular pipe rather than the diameter and the arbitrary constant of 2 in Eq 2 omitted, the value of 2 for the other constant results. For other cross-sectional shapes this constant varies from approximately 1.2 to 3. Bartell and Osterhoff, and Carman assumed 2 and 2.5 respectively as good average values for the channel shapes found in sand. These investigators corrected for the length of flow path by applying $\frac{L_p}{L} = \frac{\pi}{2}$ and $\frac{L}{L_p} = \sqrt{2}$ respectively, where L_p is the actual pore length and L is the straight-line length of flow. Applying the length correction to velocity and straight-line length, the Hagen-Poiseuille equation, Eq 2, for sand becomes in the form used by these investigators,

$$h/_L = \frac{c_o \mu V}{\rho g d^2} ,$$

where,

$$c_o = 2 \left(^{L}p/_{L}\right)^2 = (\pi/2)^2 = 4.9, \text{ (Bartell and Osterhoff)}$$

or,

$$c_o = 2.5 \left(^{L}p/_{L}\right)^2 = 5 .$$

The equivalent hydraulic radius concept, Eq 6, (Carman) introduces the shape factor S_s, which is 6 for spheres and greater than 6 for other shapes. Since $C_s = C(S_s)^2$ this introduces a constant of 36 for spheres. Assuming the value of $c_o = 5$ to be approximately correct for flow through granular media and taking into account the arbitrary constant 2, of Eq 2 gives the value of the shape constant C_s for spheres as,

$$C_s = (5)(36)(2) = 360 .$$

Experimental Results

Apparatus

Although the data of others where sufficient information was available, have been drawn on freely, a large share of the data used were taken by the writers, therefore their apparatus will be described. A constant-head permeameter with vertical downward flow was used. The permeameter tube was of 2-in. OD plastic, 18-in. long. Two grooved wooden end blocks were fitted with 3/4-in. pipe for inlet and outlet. These and the tube assembly were held together by means of four 3/8-in. logitudinal rods. The pipe at the lower end of the tube was fitted with a valve to control the rate of flow. The sand was supported on a screen 3-in. from the bottom of the tube. The head loss across the sand was measured by means of piezometer taps connected to a manometer. The constant head tank was located approximately 15 ft above elevation of the permeameter. Water from the city supply lines entered this tank after passing through a gravel filter which removed some of the dissolved air. By means of the valve, positive pressure was maintained on the discharge end during tests to further reduce the possibility of air dissolving from the water. Special care was taken to assure that conditions of uniform temperature prevailed throughout the system before observations were begun. Fig 1 shows a 3-in.

drawing
photograph of the apparatus. The discharge was measured by timing flow into containers of known volume.

Drawing
Fig 1 - Photograph of Apparatus

The porosity was computed from the specific gravity of the sand, the dry weight and the total volume occupied by the sand. To measure the total volume of sand, the permeameter tube was pre-calibrated by adding known volumes of water. Air was removed from the uniform sands by reversing flow through the permeameter and agitating the sand vigorously for several minutes. In the case of graded sands agitation was
is not practical because of size separation. The procedure for graded sand was to evacuate the air from the sample using a vacuum pump and then draw up deaerated water through the bottom.

Analysis of Data

Data from several other sources as well as that of the authors' were analyzed in an effort to check the validity of the porosity function $\frac{n^3}{(1-n)^2}$, establish the shape factor C_s for various sands, and determine the effective diameter of graded sands. The authors tested four natural sands, designated Ottawa Sand, Sand Luis Blow Sand, White Sand and Poudre River Sand, respectively, and a medium consisting of glass beads. With the exception of the Ottawa Sand both a uniform and a graded sample of each type of material was tested. The porosity was varied as much as possible for each material. A total of test runs were conducted. Table 1 summarizes the experimental program conducted by the authors.

Table 1 - Summary of Testing Program of Kiefer and Peterson

All of the data were summarized on plots of resistance coefficient ψ_b as a function of Reynolds number R_b. Examples of these are shown by the plots for the Poudre River Sand, Figs 2 and 3. For brevity plots of the results using the

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Fig 2 - Resistance Coefficient as a Function of Reynolds Number for 30-35 Poudre River Sand

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Fig 3 - Resistance Coefficient as a Function of Reynolds Number for Graded Poudre River Sand

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other materials are omitted from this paper[1]. As shown by Figs 2 and 3, data in the laminar flow region plot along a single line for each porosity. The validity of porosity function $\frac{n^3}{(1-n)^2}$ is demonstrated when the pore value of Reynolds number R_p and resistance coefficient ψ_p are plotted. When this is done, all data reduce to a single line regardless of porosity. This line is marked "pore values" in Figs 2 and 3.

A summary of shape factors from all the sources checked is presented in Table 2. In Fig 4 microphotographs are shown

[1] Complete data are included in a thesis, "Reynolds Number for Flow through Porous Media", submitted in partial fulfillment of the requirements for the degree of Master of Science in Irrigation Engineering by Fred W. Kiefer, Jr., Colorado A & M College, Fort Collins, Colorado, May, 1953

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Table 1 - Values of Shape Constant C_s for Various Materials

Fig 4 - Microphotographs of the Materials Tested

of four materials tested by the authors.

The effective diameter for each graded material tested by the authors was computed using Eq 13 and compared with the diameter as computed using Eq 12. The results are summarized in Table 3, and show that the effective diameter of a graded sand determined by sieve analysis and using Eq 12 almost exactly agrees with that determined hydraulically.

Table 3 - Summary of Experimental Data of The Writers

Fig 5 summarizes the information for sands of the shapes ordinarily found in nature. For spheres the shape constant

Fig 5 - Pore Resistance Coefficient as a Function of Pore Reynolds Number for Flow through Porous Media.

is approximately 350. An average sand would have a shape constant of around 500. Conceivably the shape constant could be much higher for other shaped materials which have a high surface are per unit volume.

Critical Reynolds Number

The value of Reynolds number at which flow becomes turbulent has been reported by various investigators to be between one and ten. Data on Ottawa sand and Poudre River sand extended into the turbulent region. For these two cases the critical value of Reynolds number R_b was approximately 3 to 6. The critical pore Reynolds number was slightly higher, from 6 to 10. Assuming that the critical Reynolds number should properly be expressed in terms of pore flow, the theoretical variation of the bulk critical Reynolds number is also shown in Fig 5 for assumed critical values of $R_p = 6$ and ψ_p corresponding to a shape factor C_s of 350. The corresponding values of bulk Reynolds number and bulk resistance coefficient were computed for porosities varying from 10 percent to 90 percent. Over the practical range of porosity, 30 percent to 50 percent, the critical bulk Reynolds number varied only from 3 to 4. The curve of bulk critical Reynolds numbers for various values of C_s will be slightly displaced for any constant value of R_p.

The shape of particles might have some effect on the critical Reynolds number. However, the tests which extended to the turbulent region were for sands of widely different shape, Ottawa sand and Poudre River sand. The critical pore Reynolds number was approximately the same in each case, indicating that shape has no major effect on the critical Reynolds number.

Application

The curves of Fig 5 may be used to estimate the hydraulic characteristics of sands and gravels. The shape constant for a sand can be estimated by comparison with the microphotographs of Fig 4. The effective diameter of the sand can be determined by sieve analysis using Eq 12. The effect of porosity is very important and great care must be exercised in its determination. For a change in porosity of from 35 percent to 41 percent the conductivity will be doubled. The presence of air in the sand can make the porosity very difficult to determine. If the amount of air present can be found, the porosity should be computed considering the air as part of the solids. Under these circumstances a change of temperature will have a pronounced effect on the conductivity, -more owing to the expansion or contraction of the air than to the change in viscosity of the fluid. The Hagen-Poiseuille equation for laminar flow, Eq 2a, in the form,

$$V = \frac{2\, g \rho\, d_e^2}{C_s \mu} \quad \frac{n^3}{(1-n)^2} \quad \left(\frac{h}{L}\right) ; \qquad (2a)$$

may be employed, or knowing the values of shape factor, effective diameter and porosity, the value of conductivity K may be computed using Eq 8. Experimental observations agreed very closely with these equations.

If flow is turbulent, R_p greater than approximately 6, the Darcy-Weisbach type equation,

$$\frac{h}{L} = \psi_p \frac{V^2}{2\, g} \left(\frac{1}{d_e}\right) \frac{(1-n)}{n^3} , \qquad (3a)$$

would be expected to apply. Very little experimental evidence is available to support this equation, however.

Most problems in civil engineering involving flow through sands and gravels are in the laminar flow range. For other uses, the investigations need to be extended into the turbulent region.

Acknowledgments

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LIST OF TABLES

Table 1 - SUMMARY OF TESTING PROGRAM OF KIEFER AND PETERSON

Material	Percent Porosity	Number of Test Runs
Ottawa sand 20-30 Mesh	34.0	10
	37.2	16
	39.2	14
	33.4	7
	32.7	14
	41.4	7
	41.1	14
	41.2	7
	41.6	9
	37.1	4
San Luis Blow sand 50-60 Mesh	46.2	13
	43.9	9
	39.7	10
San Luis Blow sand Graded	40.4	8
	33.9	7
	37.0	7
	34.1	9
	32.3	10
White sand 50-60 Mesh	42.7	10
	37.0	11
	43.6	12
	41.0	9
White sand Graded	37.0	11
	38.0	7
	43.1	8
	39.3	11
Poudre River sand 16-18 Mesh	47.3	30
	42.6	14
	42.2	14
	46.4	9
Poudre River sand 30-35 Mesh	45.0	8
	47.2	7
	45.5	7
	47.9	14
	45.7	9
	43.3	8

Table 1 - (Continued) SUMMARY OF TESTING PROGRAM OF KIEFER AND PETERSON

Material	Percent Porosity	Number of Test Runs
Poudre River Sand Graded	45.5	7
	47.2	7
	37.5	14
	44.4	14
	39.7	14
Glass Beads 70-80 Mesh	40.8	10
	37.5	8
	35.6	7
Glass Beads Graded	39.8	12
	37.0	8
	35.4	11

Table 2 - VALUES OF SHAPE CONSTANT C_s FOR VARIOUS MATERIALS

Material	Nature of Grain Shape	C_s	Experimentors
Glass spheres	approximately spherical, some irregular particles	340	Kiefer
Ottawa sand	almost spherical	330-360	Kiefer
Ottawa sand	almost spherical	330	Franzini
Lead shot	almost spherical	320	Franzini
Lead shot		384[a]	Bakhmetoff and Feodoroff
Red sand	semi-angular	400	Franzini
San Luis blow sand	irregular, rounded corners	490	Kiefer
White sand	irregular, rounded corners	480	Kiefer
Poudre River sand	angular - some mica flakes	680	Kiefer
1/16 to 5/16 spheres		346-360	Coulson

[a] Based on information partially estimated by the writers.

Table 3 - SUMMARY OF EXPERIMENTAL DATA OF THE WRITERS

Material	Sieve Size	C_s	Mean Sieve Diameter ft.	Dr by Permea- bility ft.	d_e by Sieve Analysis ft.
Poudre River sand	16-18	670	0.00353	-	-
Poudre River sand	30-35	680	0.00177	-	-
Poudre River (A) sand	Graded	680	-	0.00241	0.00239
Poudre River (B) sand	Graded	680	-	0.00169	0.00170
San Luis blow sand	50-60	490	0.000884	-	-
San Luis blow sand	Graded	490	-	0.00093	0.00090
White sand	50-60	480	0.000884	-	-
White sand	Graded	480	-	0.000815	0.00084
Glass beads	70-80	340	0.000626	-	-

LIST OF FIGURES

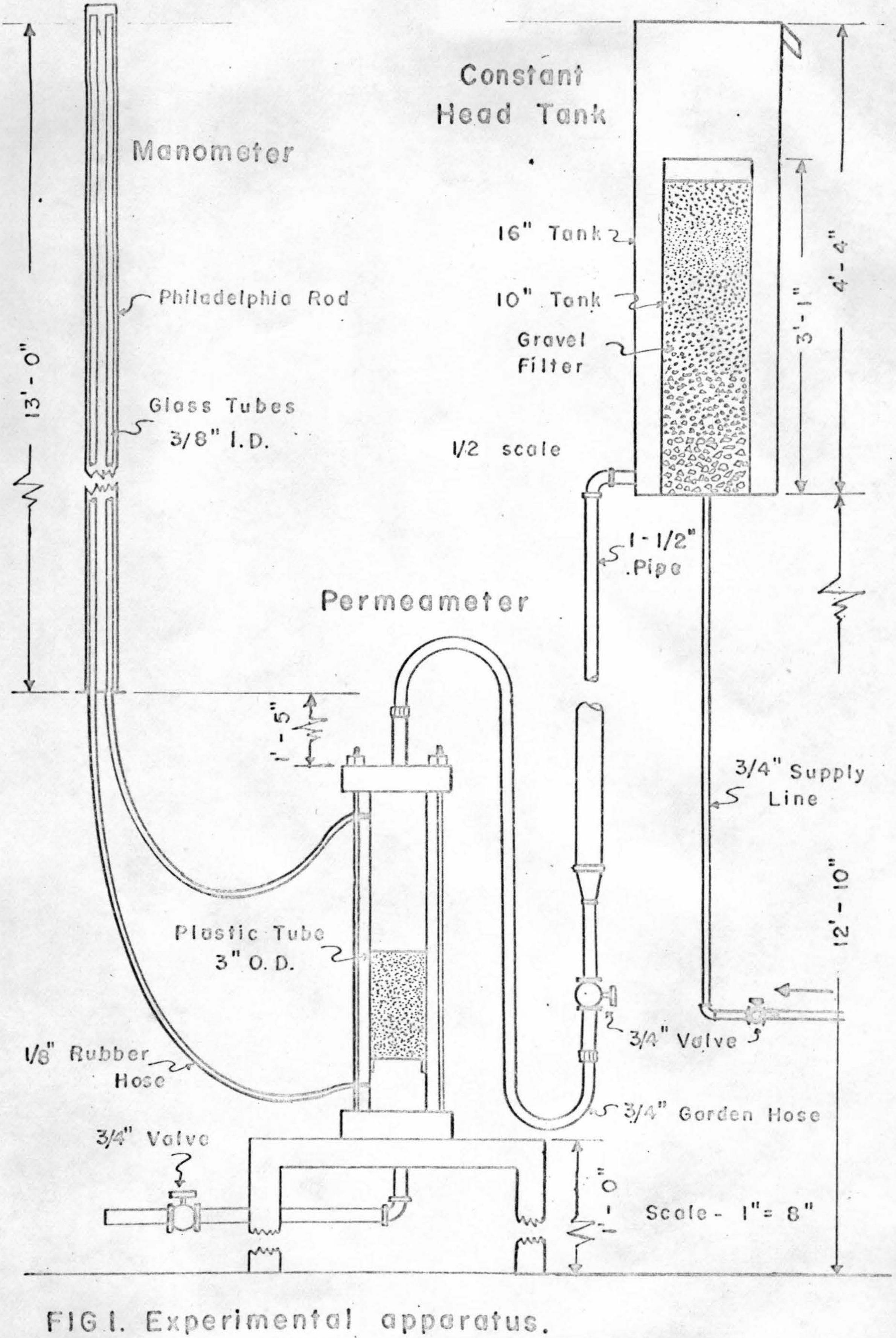


FIG.I. Experimental apparatus.

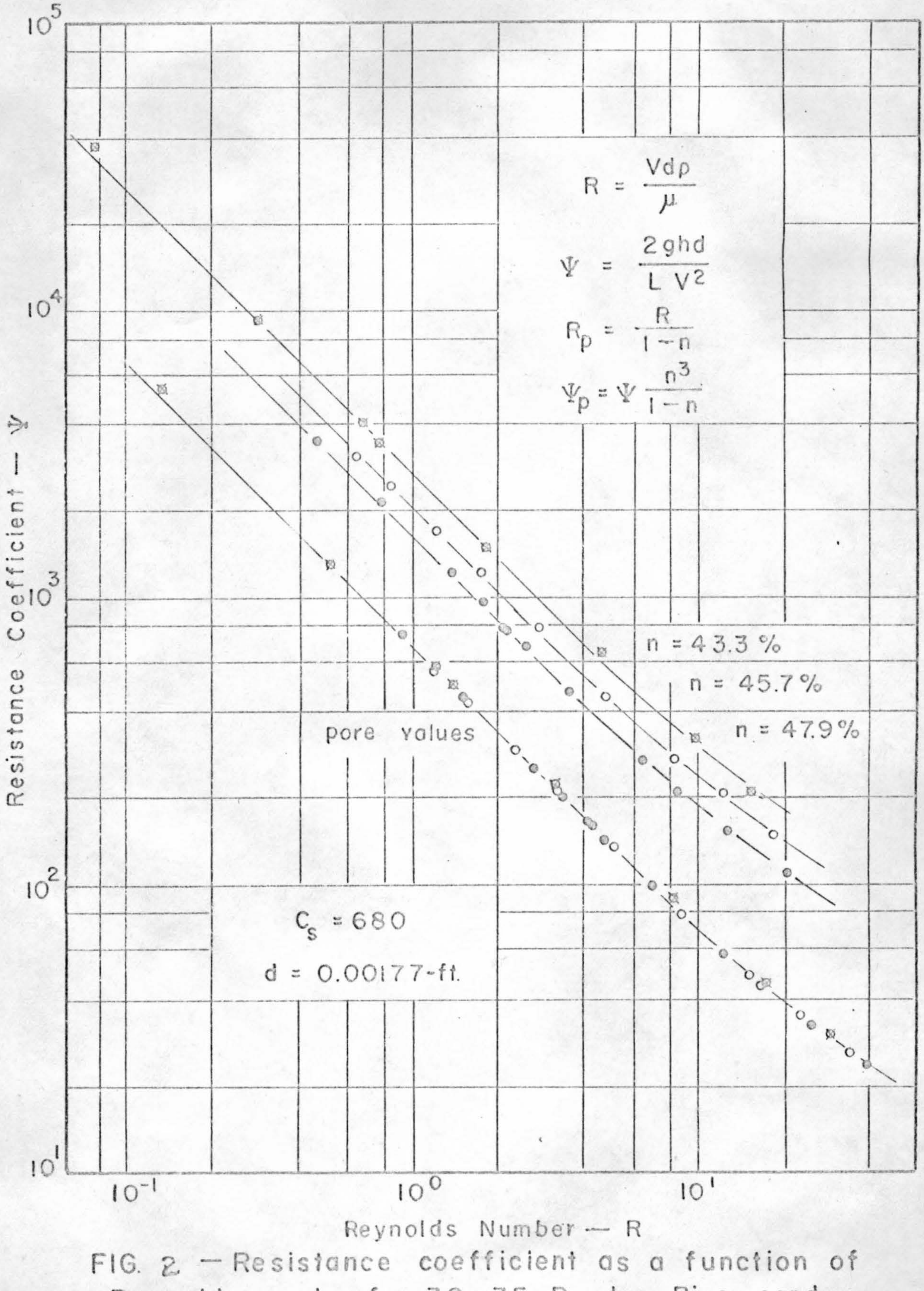


FIG. 2 — Resistance coefficient as a function of Reynolds number for 30-35 Poudre River sand.

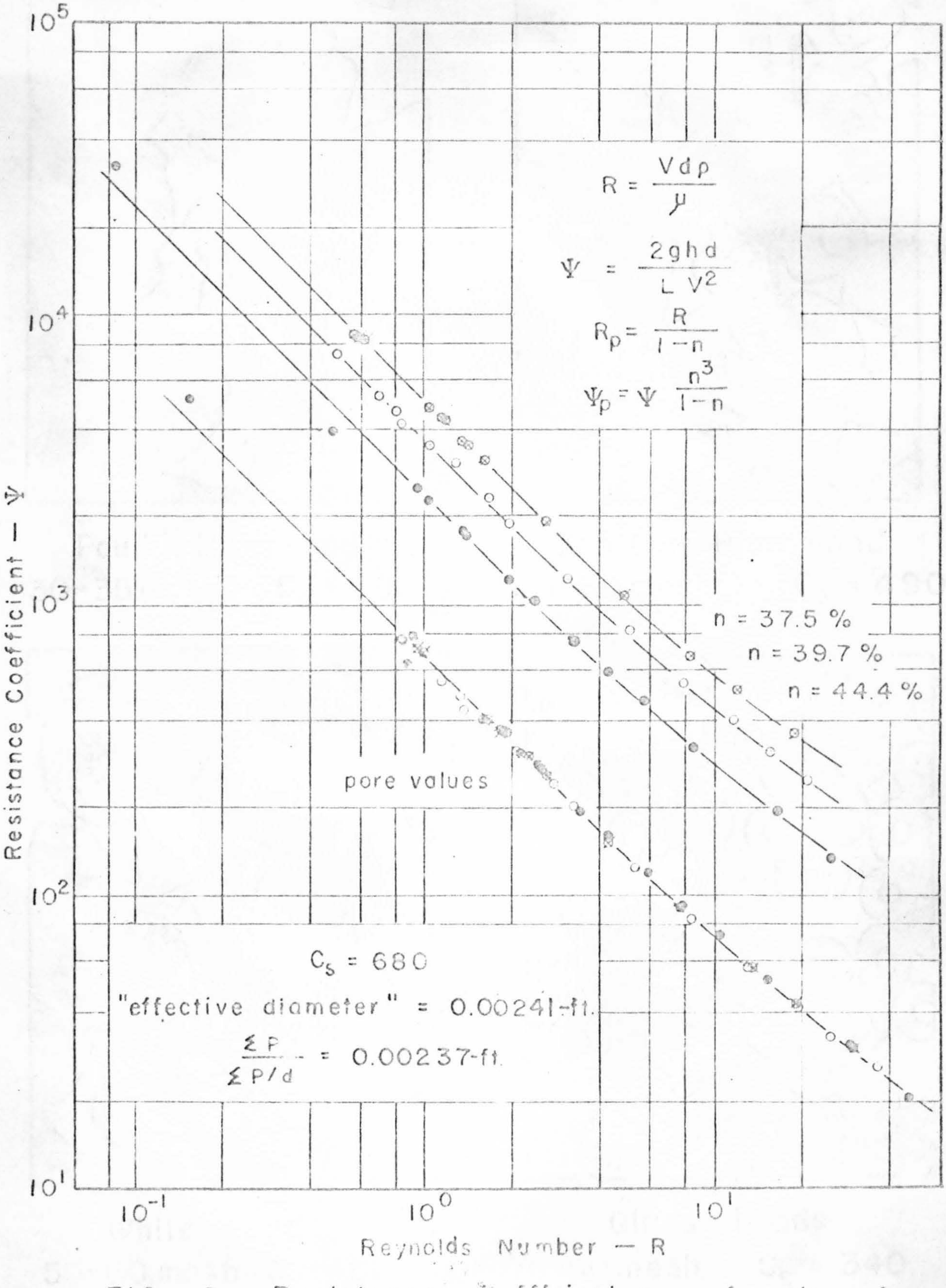
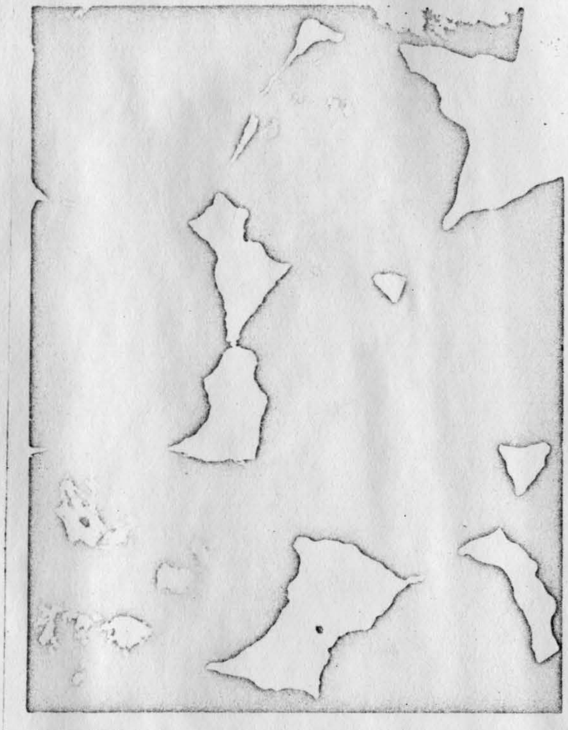
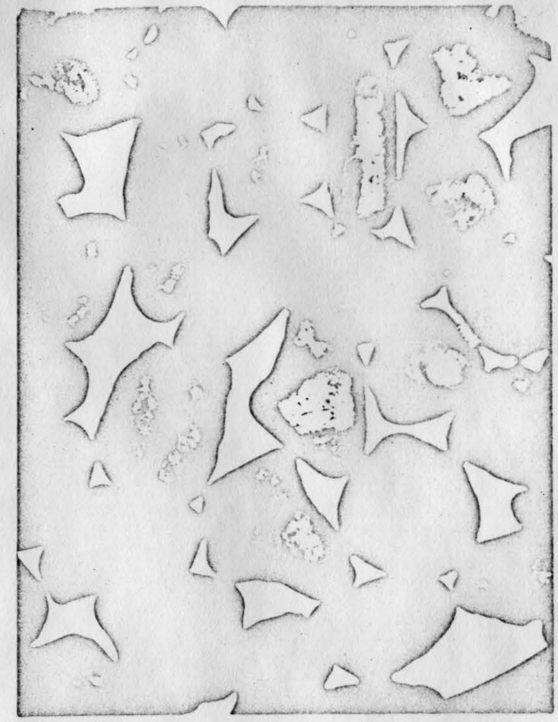


FIG. 3 - Resistance coefficient as a function of Reynolds number for graded Poudre River sand.

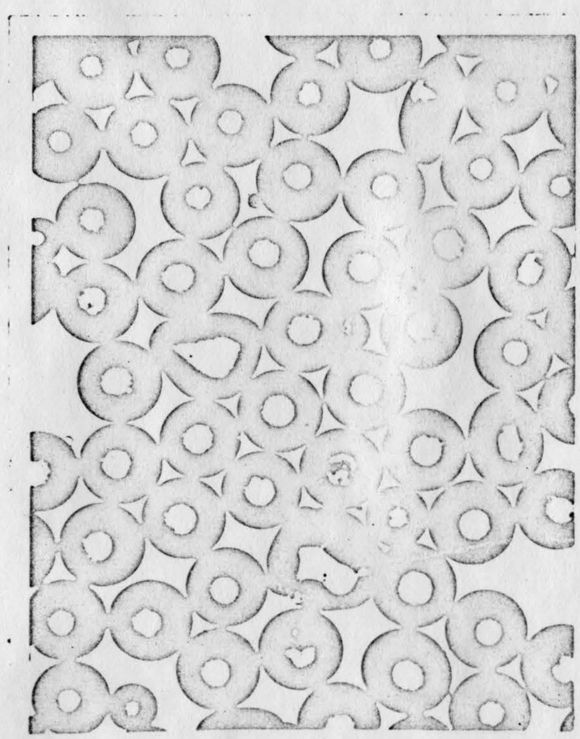


Poudre River sand
30-35 mesh $C_s = 680$



San Luis Blow sand
50-60 mesh $C_s = 490$

White sand
50-60 mesh $C_s = 480$



Glass beads
70-80 mesh $C_s = 340$

FIG 4.

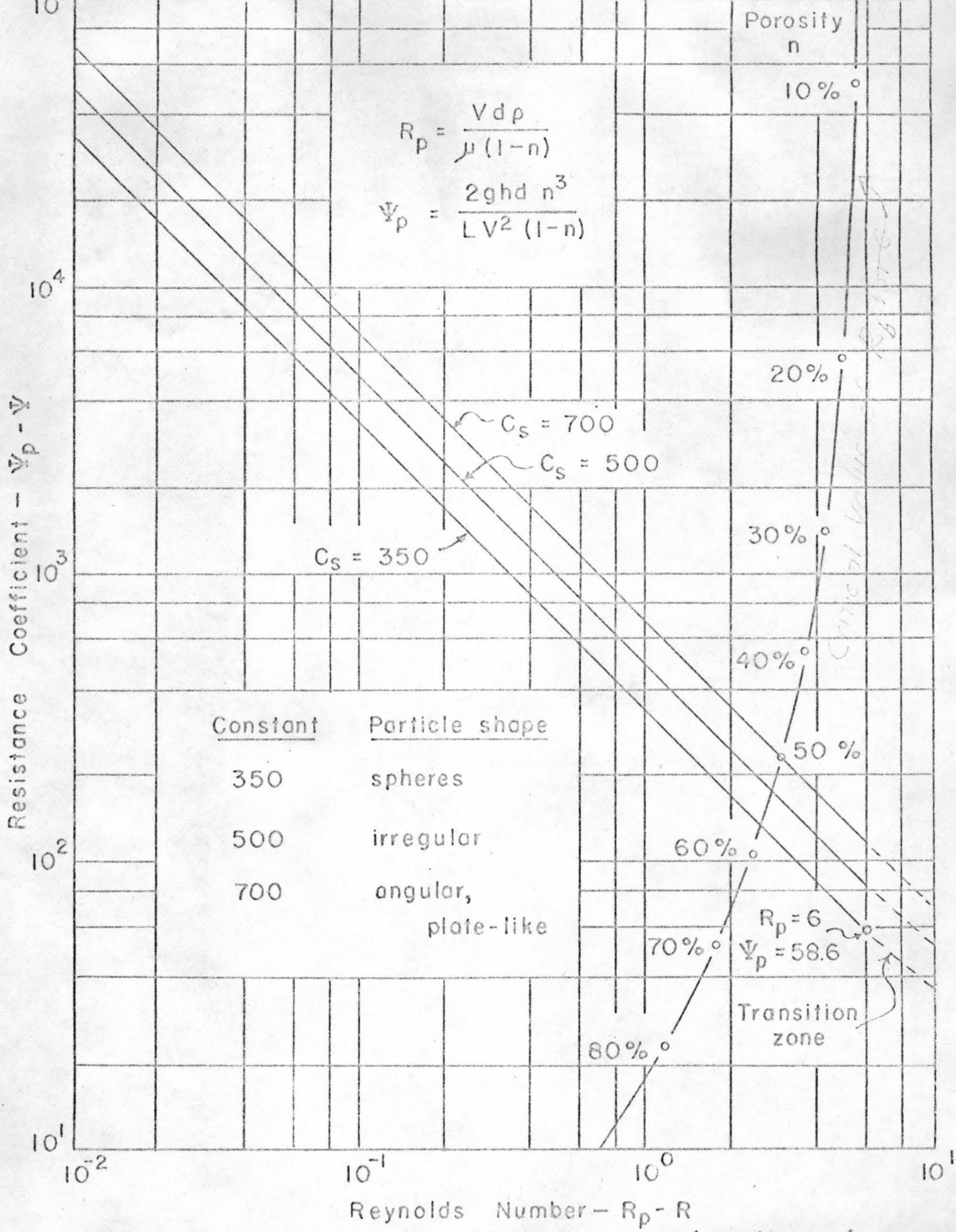


FIG. 5 - Pore Resistance Coefficient as a function of pore Reynolds number for flow through porous media. Theoretical variation of critical values of bulk R and Ψ with porosity for C_s = 350.

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