CHARACTERISTICS OF BED FORMS AND REGIMES OF FLOW IN ALLUVIAL CHANNELS

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ABSTRACT

The characteristics of the bed and the water surface in an alluvial channel are changed by changing the characteristics of the flow, the fluid, and/or the sediment. This paper first reviews the existing literature on the characteristics of bed forms and regimes of flow over an alluvial bed and then it reports the studies made by the writers about the scale of ripples and dunes and about the criteria for the prediction of different regimes of flow in an alluvial channel.

INTRODUCTION

When the characteristics of the flow, the fluid and/or the sediment are changed, the nature of the bed and/or water surface changes and takes different forms. For known conditions of the flow, the fluid, and the sediment characteristics, predicting the nature of the bed and the water surface is of great importance in the study of river engineering. For example, a sudden shift in the rating curve for an alluvial stream, when the discharge exceeds a certain limiting value, can be explained sometimes by the change in the bed condition. Similarly, knowledge of the nature of the flow, for the given conditions, is quite essential in the design of alluvial irrigation channels - - as it is in the case of model studies with movable beds. In order to approach complete similarity, one would expect to have the same conditions for the bed and the water surface in the model and the prototype. Also, in order to understand the laws of velocity distribution, suspended sediment distribution, and sediment transport, it is essential for one to know the characteristics of bed forms and regimes of flow in an alluvial channel (2)*.

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* Numbers in the parenthesis refer to the number of reference in the bibliography.

REGIMES OF FLOW

As the sediment characteristics, the flow characteristics, and/or the fluid characteristics are changed in alluvial channels, the nature of the bed surface and the water surface changes accordingly. These types of bed and water surfaces are classified according to their characteristics and are called Regimes of Flow.

Simons, Richardson and Albertson (1, 10) 1958, have given a complete description of the different regimes of flow that are observed in an alluvial channel. For convenience these regimes can be divided into four stages as follows:

1. Plane-bed (without motion of sediment particles)
2. Ripples and dunes
3. Transition
4. Antidunes

The characteristics of these four regimes will now be discussed in brief.

Plane-bed (without motion of sediment particles): In this regime the depth and the velocity of the flow, for a given size of the bed material, are such that the average shear stress on the bed τ_0 is not large enough to move the sediment particles. Therefore, the boundary acts like a rigid boundary. This regime of flow is similar to subcritical flow over a plane rigid bed. The water surface in this regime is fairly smooth. In this discussion the term average shear is used. However, it should be realized that from the point of view of mechanics of motion of the sediment particle, it is the magnitude and the duration of instantaneous shear that is important, sometimes the instantaneous value of shear is much greater than the average shear, but since such high values of instantaneous shear are of only small duration, the inertia of the particle prevents its movement.

In summary, the movement of a particle depends upon:

1. The magnitude of the shear at a given time.
2. The duration of time this magnitude is equalled, or exceeded, during a single surge of the flow due to turbulence.
3. The inertia, orientation, stability, and shape of the particle.

Ripples and Dunes: When the average shear stress on the bed τ_o is equal to the critical shear stress for the given size of bed material, the individual particles on the bed start moving. According to Liu (8) there is a range of shear stress in which there is a general motion of the sediment particles and yet the bed is geometrically plane. With further increase in the shear, however, instability develops and ripples are formed on the bed which move slowly downstream. If the shear on the bed is increased further, the ripples grow into dunes. When the dunes are present on the bed, the water surface is characterized by boils; these boils are formed downstream from the troughs of the dunes in which a large roller is moving in an unsteady fashion so that boils periodically break off and spread to the water surface. In the latter part of this paper it is shown that the scale of these ripples and dunes is a function of the variables pertaining to the flow, the fluid and the sediment. The ripple and dune regime is the most common regime found in natural alluvial channels.

Transition : With further increase in shear stress τ_o , the bed changes from washed-out dunes or sand-bars to the plane bed and then to the symmetrical bed-waves. Similarly, the water surface also takes different forms - - boils, plane surface, and standing waves. This regime is unstable compared to the ripple and dune regime. Consequently, slight changes in the slope or the discharge change the flow pattern radically.

Antidunes : If the τ_o and Fr values ($Fr = \frac{V}{\sqrt{gD}}$ in which V is the mean velocity of flow and D is the depth of flow) are further increased, the standing waves which are symmetrical sand and water waves in phase, move upstream and break intermittently. For a given size of bed material, the frequency of the breaking of the waves is a function of the flow characteristics. This regime is called the antidune regime and has been observed in the flume experiments by different investigators (4 and 10). This regime has been observed also in natural streams, such as San Juan River in Utah (6). In this regime Fr is usually greater than approximately 0.60.

CRITERION FOR BEGINNING OF MOTION

Shields (11) 1936, was the first to suggest a criterion for the beginning of motion of a sediment particle in an alluvial channel, which was later proved to be functionally correct. The analysis presented by Shields is based on equating the resistance of the grain to the motion to the force exerted by the fluid on the grain. Because of the inability to take into consideration several factors - - such as porosity of the bed material, resistance coefficient of the grain, and effective area of the grain - - Shields reduced this equality to a functional relationship.

This functional relationship can be expressed as

$$\frac{\tau_c}{(\gamma_s - \gamma_f)d} = f\left(\frac{V_{*c}\, d}{\nu}\right) \qquad (1)$$

in which

τ_c is the average critical shear stress on the bed for the sediment particle of diameter d ,

V_{*c} is the corresponding critical shear velocity , $\left(= \sqrt{\frac{\tau_c}{\rho_f}} \right)$

ρ_f is the density of water,

γ_s, γ_f are the specific weights of sediment and water respectively.

ν. is the kinematic viscosity of water.

Shields' plot of Eq. 1 is shown in Fig. 1.

Recently Iwagaki (5) 1956, has treated this problem of beginning of motion extensively and from the point of view of the hydrodynamics involved. Considering the equilibrium of a spherical sediment particle resting on a sand bed, Iwagaki evaluated a dimensionless function of the cirtical shear stress; this function is similar to that of Shields. While studying the equilibrium of sediment particles, in addition to other forces, the resistance resulting from the pressure gradient was also considered and the velocity fluctuations resulting from the turbulence were taken into consideration. In order that this theoretical functional relationship would agree with the experimental data collected by Iwagaki, an experimental constant, called a "sheltering coefficient" was introduced. Iwagaki's function is shown in Fig. 1.

CRITERION FOR BEGINNING OF RIPPLE FORMATION

Liu, (8) 1957, has established a criterion for the beginning of ripple formation. According to Liu, the conditions at the moment sediment ripples appear on the bed must be:

a. The flow is capable of transporting the sediment, and

b. The interface between the flow and the movable bed becomes unstable.

Liu proposed that sediment ripples are caused by the instability of the laminar boundary layer. The index of this instability was obtained by Liu from the dimensional analysis point of view. Thus, based on the combination of the instability theory of the interface and the dimensional analysis, he showed that at the beginning of ripple formation the following relationship should hold

$$\frac{V_*}{\omega} = f\left(\frac{V_* d}{\nu}\right) \qquad (2)$$

in which W is the fall velocity of sediment.

Experimental data were used to establish this functional relationship, see Fig. 2. Recently Plate (9) 1957, has investigated Liu's criterion for ripple formation by analytical, as well as, experimental means. It was found that the beginning of ripples was well determined by Liu's curve.

CHARACTERISTICS OF RIPPLES AND DUNES

As mentioned in the discussion of the ripple and dune regime, for a given size of bed material, the size and shape of the ripples and dunes changes as the average shear stress τ_0 acting on the bed is increased. Observations in the natural channels and the flume studies indicate that these ripples and dunes take various forms - - some times they are perpendicular to the flow direction and other times they are oblique; in some cases the dunes run across the whole width of the flume and in other cases there are breaks in them, therefore, the ripples and the dunes can be described by their average length λ and average height h only as a first approximation.

From the point of view of dimensional analysis it can be shown that (12) these characteristics of the ripples and dunes can be described by the following functional relationships,

$$\frac{h}{D} = f\left(\frac{\tau_o}{(\gamma_s-\gamma_f)d}, \frac{V}{\sqrt{g\,d}}\,{}_{D}, \frac{V_* d}{\nu}, \frac{d}{D}, \sigma\right) \qquad (3)$$

and

$$\frac{\lambda}{D} = f\left(\frac{\tau_o}{(\gamma_s-\gamma_f)d}, \frac{V}{\sqrt{gD}}, \frac{V_* d}{\nu}, \frac{d}{D}, \sigma\right) \qquad (4)$$

in which

h is the average height of ripples or dunes,
λ is the average length of ripples or dunes,
τ_o is the average shear stress on the bed,
σ is the standard deviation of the bed material

combining the two equations, one gets

$$\frac{h}{\lambda} = f\left(\frac{\tau_o}{(\gamma_s-\gamma_f)d}, \frac{V}{\sqrt{gD}}, \frac{V_* d}{\nu}, \frac{d}{D}, \sigma\right) \qquad (5)$$

Some consideration must now be given to the relative importance of these parameters which govern the magnitude of $\frac{h}{\lambda}$. The standard deviation σ of the bed material is important in certain problems concerning the sediment and is of secondary importance in other cases. Unfortunately, very little information exists to define the significance of σ. Consequently, it is omitted here as a first approximation. Also, since $\frac{d}{D}$ can be expressed as

$$\frac{d}{D} = \frac{(\gamma_s - \gamma_f)d}{\tau_o} \cdot \frac{\gamma_f S}{(\gamma_s - \gamma_f)} \tag{6}$$

It can be assumed that the influence of $\frac{d}{D}$ is indirectly taken into consideration in the parameter $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$. For these reasons Eq. 5 can be reduced to a simpler form.

$$\frac{h}{\lambda} = f\left(\frac{\tau_o}{(\gamma_s - \gamma_f)d}, F_r, \frac{V_* d}{\nu}\right) \tag{7}$$

It can be recalled that $\frac{v_* d}{\nu}$ is directly proportional to the ratio $\frac{d}{\delta}$ in which δ is the thickness of the laminar sublayer. When the bed is geometrically plane or when the ripples have just formed it seems logical that the viscosity should play an important role by the existence of a laminar sublayer and therefore, in the beginning stages of ripple formation $\frac{v_* d}{\nu}$ should be an important parameter. When the ratio of the ripple height to the depth of water is very small, and Fr is small, the influence of Froude number is quite negligible. Therefore, in the presence of ripples, Eq. 7 can be simplified to the form

$$\frac{h}{\lambda} = f\left(\frac{\tau_o}{(\gamma_s - \gamma_f)d}, \frac{V_* d}{\nu}\right) \tag{8}$$

However, when the ripples grow into dunes, the large percent of the energy loss in the alluvial channel is due to form drag on the dunes and other irregularities and a relatively small percent is due to grain roughness. Under such circumstances, it is questionable whether $\frac{v_* d}{\nu}$ should be a significant parameter. Also, when the height of dunes is several hundred times larger than the grain size, or the thickness of laminar sublayer, the concept of the laminar sublayer becomes quite ambiguous. On the other hand, with the increase in the average shear stress τ_0 on the bed, and thus an increase in the $\frac{h}{D}$ ratio, the Froude number becomes important and the water surface begins to affect the bed configuration. For these reasons, in the presence of dunes, Eq. 7 reduces to

$$\frac{h}{\lambda} = f\left(\frac{\tau_0}{(\gamma_s - \gamma_f)d}, F_r\right) \tag{9}$$

Tsubaki, Kawasumi, and Yasutomi (12), because of the limited amount of data, simplified Eq. 5 to the form

$$\frac{h}{\lambda} = f\left(\frac{\tau_0}{(\gamma_s - \gamma_f)d}\right) \tag{10}$$

The discussion in the foregoing paragraph will show that Eq. 10 is only a rough approximation, since it does not take into consideration the influence of either Fr or $\frac{v_* d}{\nu}$ on the $\frac{h}{\lambda}$ ratio.

The functional relationships given by Eq. 8 and Eq. 9 can now be verified by using the experimental data. Below is given a summary of the data used for the verification:

Investigator	Ref. No.	Sediment used	Flume
Simons and Richardson	10	0.45 mm	8'x2'x150' long.
Barton and Lin	2	0.18 mm	4'x2'x65' long.
Laursen	7	0.10 mm	3'x1.5'x105' long.
Tsubaki, Kawasumi and Yasutomi	12	1.16 mm and 1.46 mm	Concrete channels 6.56' and 2.62' wide respectively, with alluvial bed.
Plate	9	0.545 mm	8"x14"x10' long.
Liu	8	0.69 mm	1'x2'x40' long.

Figure 3 shows the variation of $\frac{h}{\lambda}$ with $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ in which $\frac{v_* d}{\nu}$ is the third variable for the case when the ripples were present on the bed. This figure shows the validity of Eq. 8 and also justifies the reasoning in arriving at this equation. In order to ascertain that $\frac{d}{D}$ or Fr are not as important as $\frac{v_* d}{\nu}$, the parameters $\frac{d}{D}$ and Fr were used as the third variables in the place of $\frac{v_* d}{\nu}$ on Fig. 3. This did not give any systematic correlation. Therefore, it is concluded that in the ripple regime $\frac{v_* d}{\nu}$ and $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ are more important than Fr in defining $\frac{h}{\lambda}$. The lines on Fig. 3 can be brought together by using standard techniques; use of these techniques gave a single curve when $\frac{h}{\lambda}$ was plotted against $\frac{\tau_o}{(\gamma_s - \gamma_f)d} \cdot \frac{\nu}{v_* d}$, See Fig. 4. Thus, it is possible to predict the scale of ripples, if one knows τ_o, γ_s, γ_f, d and ν. Note that for large values of the shear parameter the length ratio appears to increase less rapidly - - as would be expected by logic alone.

Similarly, Eq. 9 can be verified by using the data collected by Simons and Richardson, Barton and Lin, Laursen, and Tsubaki and others. See Fig. 5 in which $\frac{h}{\lambda}$ is plotted against $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ with Fr as the third variable. In spite of the scatter, the figure clearly indicates that Fr is quite important in determining the characteristics of the dunes. When the ripples grow into dunes, Fr becomes important and the water surface begins to affect the development of the dunes. As Fr increases the water surface influences the bed more and more and therefore, Fr becomes more and more important. During this process the bed configuration also influences the water surface. Thus the water surface and the bed affect each other. For given value of Fr, there is a certain value of $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ beyond which a further increase in $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ abruptly decreases the value of $\frac{h}{\lambda}$ and bed becomes plane.

It is believed that this transition is within a fairly narrow range of $\frac{\tau_0}{(\gamma_s - \gamma_f)d}$. Some of scatter on Fig. 5 is due to the fact that some points plotted on Fig. 5 are in this transition region.

In the case of model studies with alluvial beds, the objective is generally to attain complete similarity and hence the bed configuration in the model and the prototype should be geometrically similar for given flow conditions. In other cases, in which a particular roughness of an alluvial bed is desired, this procedure provides a means for determining in advance how to obtain the desired value of roughness. The foregoing discussion and the plots which relate $\frac{h}{\lambda}$ to the flow, the fluid, and the sediment characteristics can serve as a guide in tackling the problem of model studies with movable alluvial beds.

Even though it is recognized, by those who are engaged in sediment research, that characteristics of ripples are different from those of dunes, there is no way of drawing the line of demarcation between the two. The writers feel that the foregoing analysis provides a means for drawing this line of demarcation. As shown above, in the ripple regime the viscous forces are important while once the ripples grow into dunes the viscous forces become of secondary importance while gravitational forces become important.

CRITERIA FOR REGIMES OF FLOW

In spite of the fact that the knowledge of the techniques for predicting the regime of flow for given conditions is quite important, comparatively few attempts have been made to develop such techniques.

Gilbert's pioneer work (4) 1914, described various regimes of flow over an alluvial bed. He also stressed the importance of slope and the "capacity" VD in determining the regimes of flow in alluvial channels.

Langbein (6) 1942, reanalysed Gilbert's data and showed that VD and Fr are the parameters which govern the regimes of flow. His plot for Gilbert's data (0.50 mm size bed material) is shown in Fig. 6. This plot is quite significant in that it includes Fr as one of the factors which govern the regimes of flow. This plot, however, does not include any parameter involving size of the bed material and the parameter VD is not dimensionless.

Albertson, Simons, and Richardson (1) 1958, were the first to present a generalized criterion for the determination of regimes of flow in an alluvial channel. Plotting $\frac{v_*}{w}$ against $\frac{v_* d}{\nu}$ for a wide range of flume data, they could draw empirical lines showing the limits of different regimes of flow. Hence if one knows v_*, d, ν, and w, for a given flow then the regime of flow could be predicted. In the foregoing discussion ν is the kinematic viscosity of water and w is the fall velocity of the sediment of median diameter d.

In order to see how this criterion describes the regime of flow for natural river data, a few data based on the observations from Niobrara River near Cody (Nebraska, U.S.A.) and from the Mississippi River are plotted in Fig. 7. In the actual case for both the Niobrara and the Mississippi Rivers, the flow is in the ripple or dune regime; the $\frac{v_* d}{\nu}$ --- $\frac{v_*}{w}$ criterion, however, predicts transition or the antidune regime. Hence based on the use of river data it seems that the variation of $\frac{v_* d}{\nu}$ with $\frac{v_*}{w}$ may need some modification so that it can be used to predict the regime of flow for natural river data.

The Albertson-Simons-Richardson criterion for regimes of flow was developed as an extension of the work done by Liu (8) on ripple formation. In fact, the parameters on the abscissa and the

Ordinate scales are the same as those proposed by Liu. As discussed in the foregoing analysis, $\frac{v_* d}{\nu}$ should be a significant parameter just after the motion of sediment particles starts, and the flow is in the ripple regime. However, when the ripples grow into dunes the viscosity effects (as represented in the parameter $\frac{v_* d}{\nu}$) become of secondary importance and because of the large ratio of the dune height to the depth of flow, Fr becomes significant. Therefore, it is felt that the inability of $\frac{v_* d}{\nu}$ -- $\frac{v_*}{w}$ criterion to predict the correct regime for natural river data stems from the fact that Fr effects are not considered.

In the study of velocity distribution in alluvial channels, it was found by Garde (3) that in the ripple and dune regime, as well as, in the transition regime $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ is an important parameter. Furthermore, in the transition regime Fr also becomes significant. That is verified by Langbein (6), who also shows that Fr is important in determining the regimes of flow. Therefore, it seems logical to expect that the variation of $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ with Fr should be significant in determining the regimes of flow. Garde (3) has presented this plot. See Fig. 8.

When the motion of sediment particles just begins, $\frac{\tau_o}{(\gamma_s - \gamma_f)d} = 0.05$ for the coarse material; therefore for $\frac{\tau_o}{(\gamma_s - \gamma_f)d} < 0.05$ the coarse material will not move. For the finer material (smaller than approximately 4.0 mm dia.) $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ has different magnitude at incipient motion -- it being a function also of $\frac{v_* d}{\nu}$.

As $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ increases the bed develops into ripples and with further increase in $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ the ripples grow into dunes. In a relatively narrow range of $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ in which the bed changes from plane bed with motion to development of full scale ripples, $\frac{v_* d}{\nu}$ is important. But once the dunes start forming, Fr number becomes more important than $\frac{v_* d}{\nu}$. In the dune and transition regime $\frac{\tau_o}{(\gamma_s - \gamma_f) d}$ and Fr are both important, the importance of Fr increasing with $\frac{\tau_o}{(\gamma_s - \gamma_f) d}$. When the Fr is in the vicinity of unity it is extremely important. For this reason the line separating the antidune regime from the transition regime could be almost vertical for larger values of $\frac{\tau_o}{(\gamma_s - \gamma_f) d}$.

Figure 9 can be used to predict the regime of flow if one knows the quantities V, D, S, and d. In natural streams the discharge is measured at the gaging station; measurements of the slope S and bed-material size d at the gaging station are also usually available. Therefore, if the cross-sectional area of the stream at the gaging station is determined, then the data necessary for determining the regime of flow are available and Fig. 9 can now be used for the prediction of flow regime.

SUMMARY

This study has revealed that in the analysis of the characteristics of flow over an alluvial bed, the parameters of primary importance are $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ and Fr and $\frac{v_* d}{\nu}$. When the motion of sediment has just begun or when the flow is in the initial stage of ripple formation, the parameters $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ and $\frac{v_* d}{\nu}$ are important, while in the dune, transition and the antidune regimes $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ and Fr are important parameters. Beyond the range of ripple regime the variation of $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ with Fr is adequate to define the limits of various flow regimes in an alluvial channel as is done in Fig. 9.

ACKNOWLEDGMENTS

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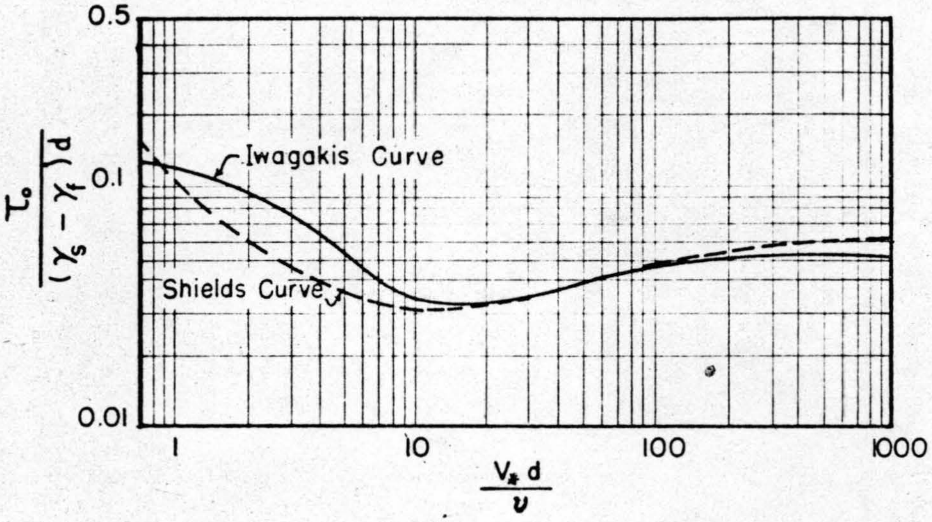

FIG. 1 CRITERIA FOR BEGINNING OF MOTION

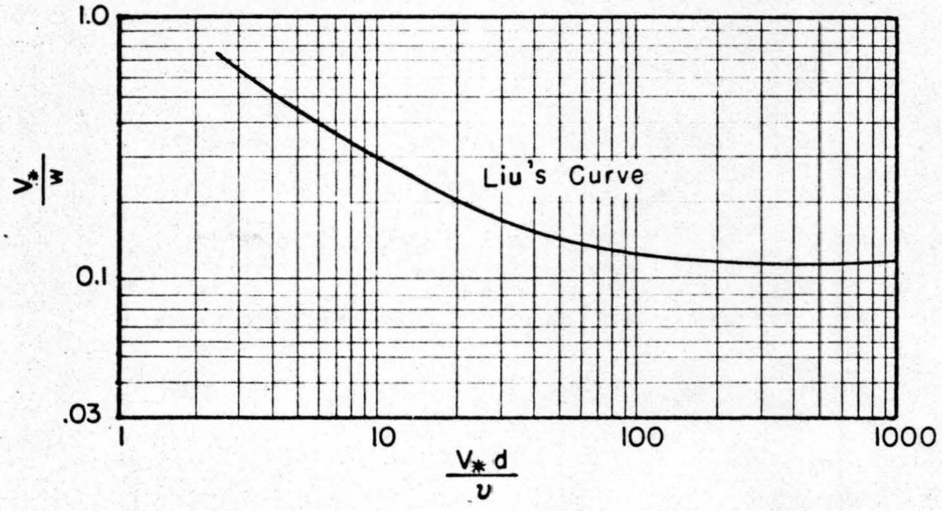

FIG. 2 CRITERIA FOR BEGINNING OF RIPPLE FORMATION

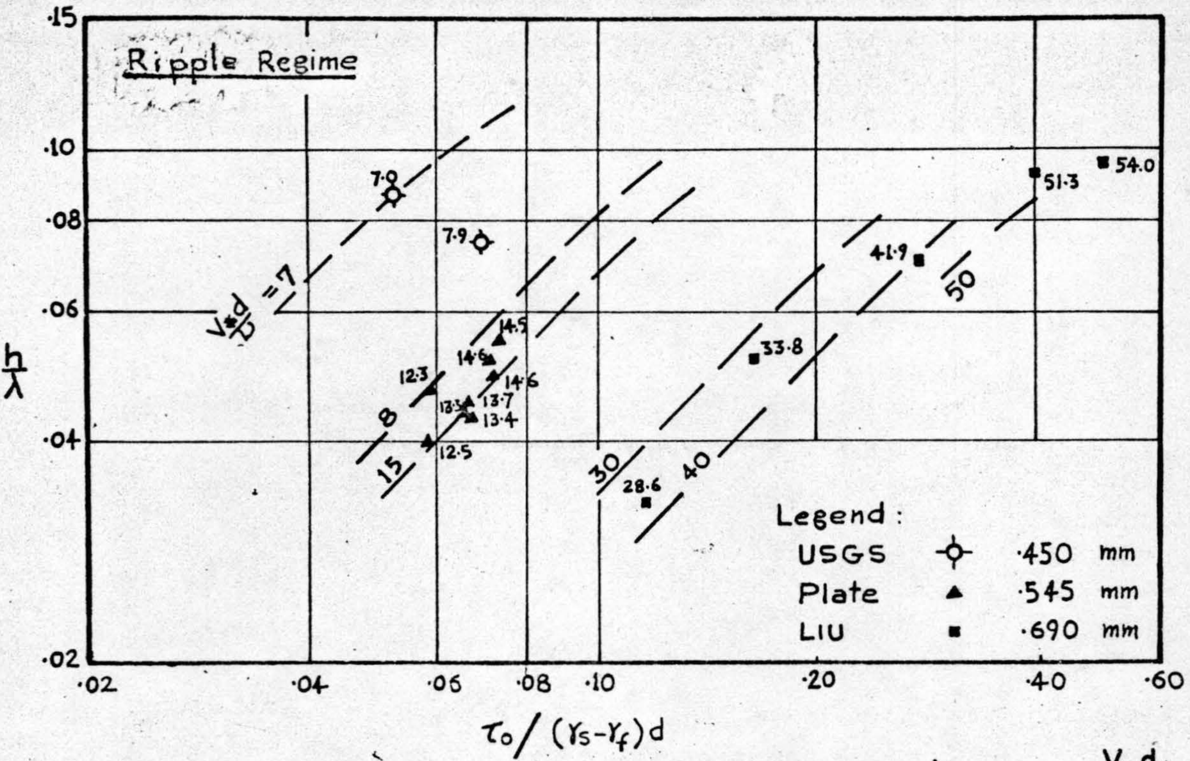

FIG. 3 VARIATION OF $\tau_o / (\gamma_s - \gamma_f) d$ WITH $\frac{h}{\lambda}$ AND $\frac{V_* d}{\upsilon}$

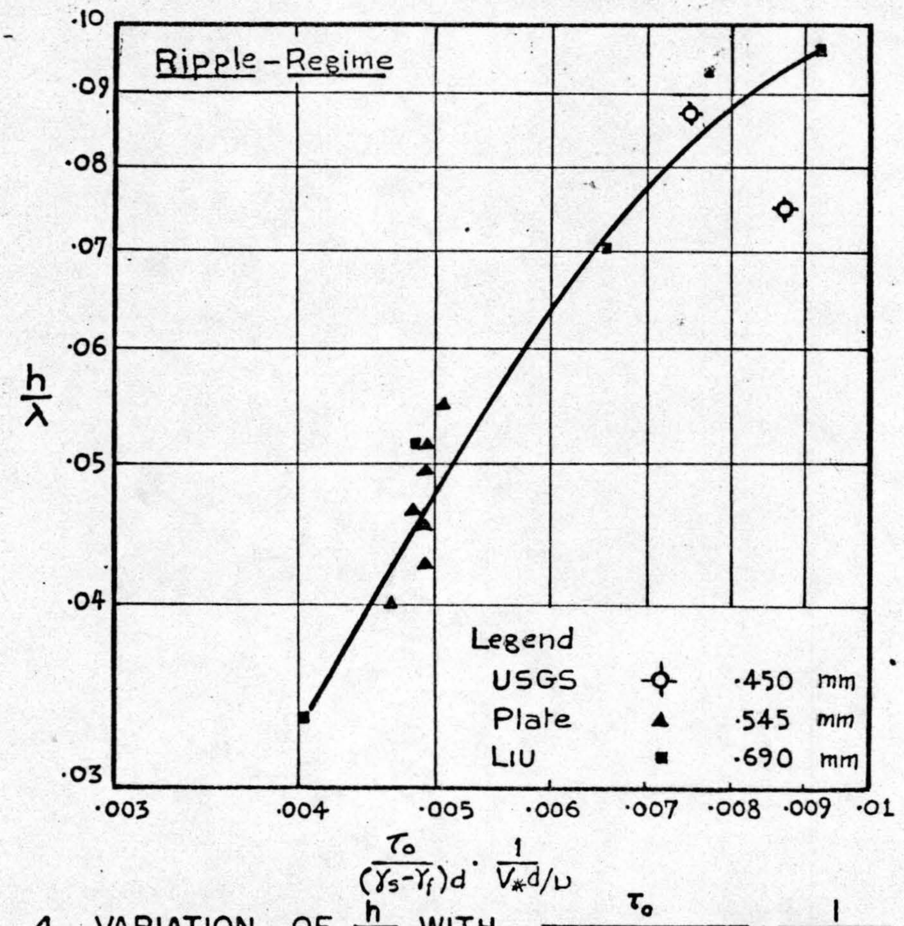

FIG. 4 VARIATION OF $\frac{h}{\lambda}$ WITH $\frac{\tau_o}{(\gamma_s - \gamma_f)d} \cdot \frac{1}{V_* d/\upsilon}$

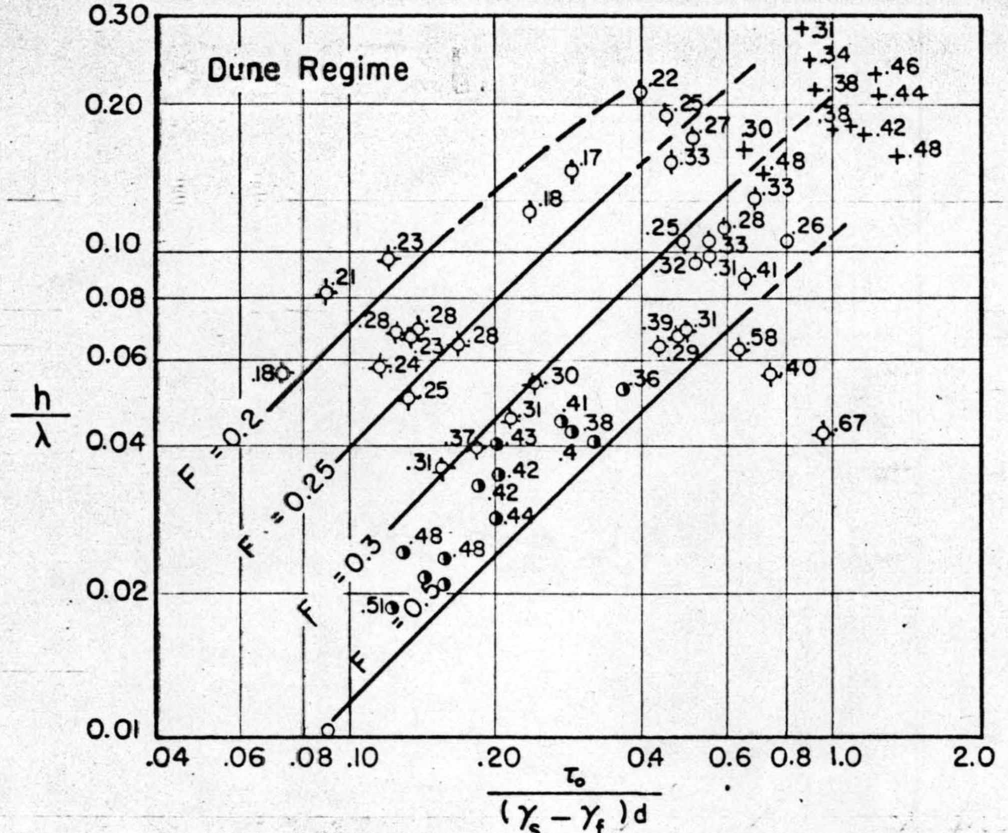

FIG. 5 VARIATION OF $\frac{h}{\lambda}$ WITH $\frac{\tau_o}{(\gamma_s - \gamma_f)d}$ AND F_r

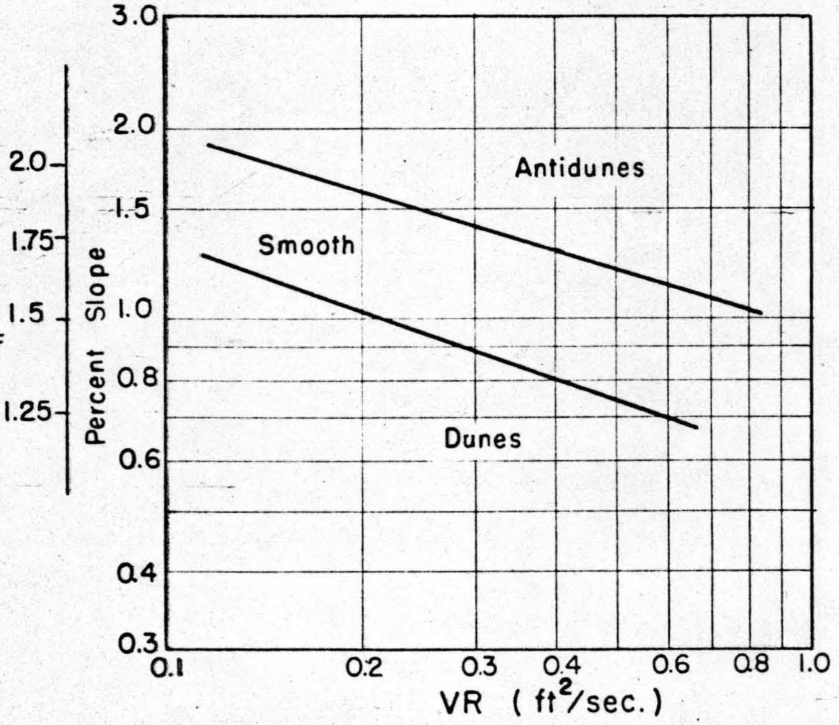

FIG. 6 LANGBEIN'S CRITERIA FOR REGIMES OF FLOW

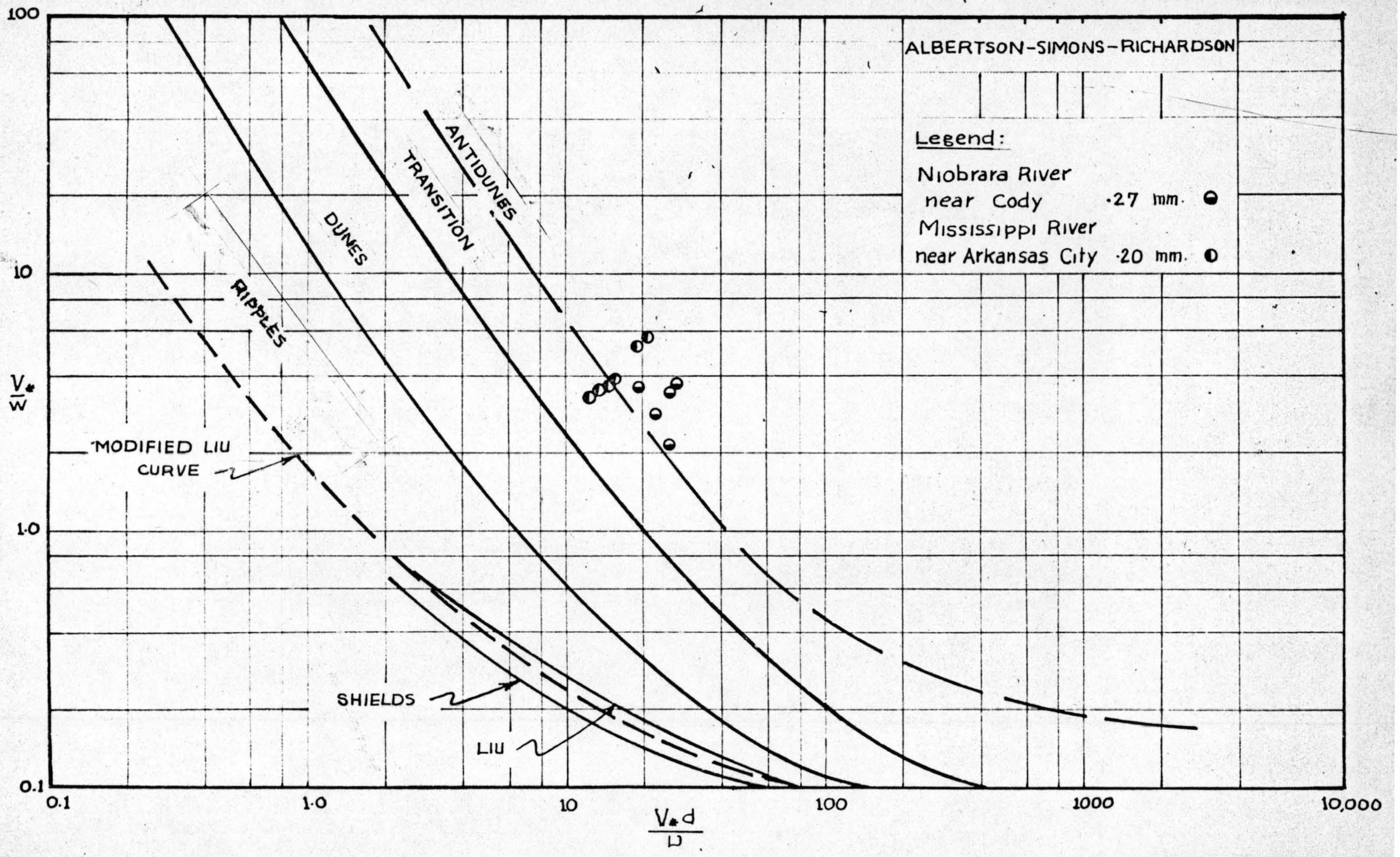

FIG. 7 CRITERIA FOR REGIMES OF FLOW (ALBERTSON - SIMONS - RICHARDSON)

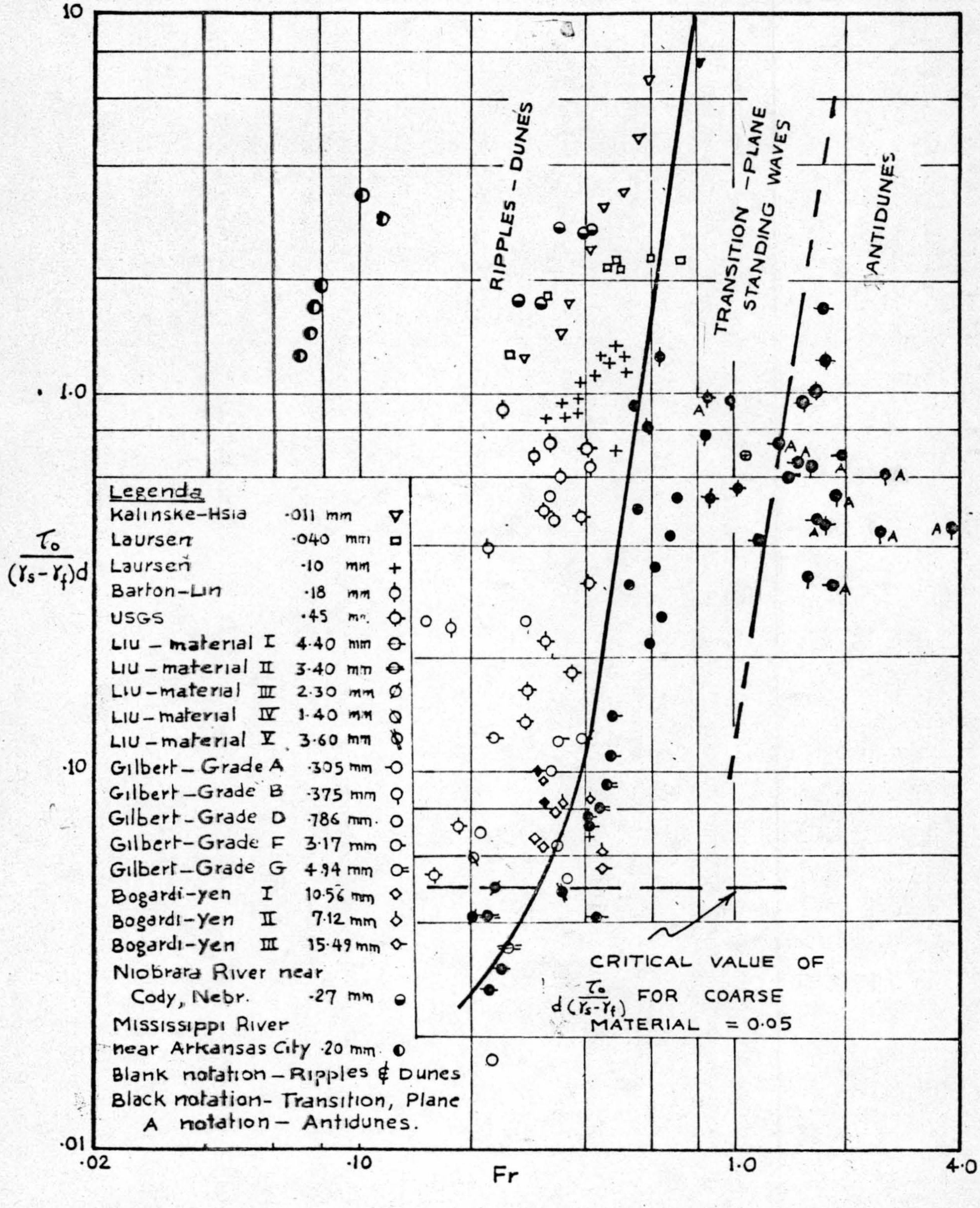

FIG. 8 CRITERIA FOR REGIMES OF FLOW