A GRAPHICAL PROCEDURE TO ESTIMATE POTENTIAL EVAPOTRANSPIRATION BY THE PENMAN METHOD

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INTRODUCTION

The engineer or the hydrologist is often asked to estimate the amount of water required by a growing crop under conditions of good management. Such estimates are needed:

1. For new project investigations;
2. To determine inadequacies of existing water supplies;
3. To determine possible water salvage from phreatophyte eradication.

These estimates are made by deducting any precipitation which might occur from the potential evapotranspiration. A number of theoretical and empirical methods of computing evapotranspiration are available. They are divided into three groups depending upon the theory or procedure underlying the method: 1) Energy Budget Theory, 2) Mass Transfer Method, and 3) Empirical Methods.

ENERGY BUDGET THEORY

Any theoretical method which proposes to explain evapotranspiration or lake evaporation from a consideration of the energy available to change the water from liquid to vapor state belongs to a number of methods discussed broadly as the energy budget theory. Sources of energy available to produce evaporation are:

1. Incident solar radiation.
2. Advective energy carried into the region from the surrounding area.

Energy may be lost to the evaporation process by:

1. Solar radiation reflected from the water or leaf surface;
2. Long-wave radiation back to the atmosphere;
3. Sensible heat transfer by conduction to the atmosphere;
4. Energy absorbed by the liquid water in being converted to the gaseous state (±590 calories per gram of water evaporated).

The last form of energy lost from the water is directly proportional to the mass of water evaporated from either the plant surface, soil surface or lake surface. If careful measurements of all the other sources and losses of energy are obtained, then the amount of evaporation could be easily obtained from a simple accounting of the gains and losses of energy. Hence the term "Energy Budget".

Limitations of the Energy Budget Theory

There are practical limitations to the application of the energy budget. Firstly, the measurements of the various elements are not commonly available; therefore, the application of the energy budget methods might prove to be impractical for the present because of lack of basic data.

A second and more permanently disabling deficiency arises from the fact that the water vapor in the atmosphere surrounding the transpiring plant or over the evaporating lake serves to "throttle down" or control the further production of water vapor from the liquid surface. When the vapor pressure in the air becomes equal to the vapor pressure of the liquid, any additional water vapor produced is just balanced by a like amount of water vapor condensed on the liquid surface: hence there is no net evaporation. The formation of this "vapor blanket" limits further evaporation. There are a number of natural processes and forces which tend to carry away any water vapor formed at a surface. These include:

1. Molecular diffusion of the water vapor into a less saturated region;
2. Turbulent transportation by wind away and into higher levels of the atmosphere; and
3. Dynamic instability of the atmosphere.

The energy budget theory does not measure or explain the presence or condition of the "vapor blanket".

A third deficiency of the energy budget theory lies in the fact that it does not describe the nature of the evaporating surface. A lake surface, soil and plants all evaporate water; but for a given set of energy conditions, each would produce a different quantity of water vapor. Indeed, a bare rock surface would produce no water vapor even though it might have been exactly the same surface texture, color and aspect as an adjacent soil surface.

MASS TRANSFER THEORY

The amount of water evaporated may be explained by evaluating the forces which tend to carry away and disperse the water vapor from the evaporating surface. To determine these parameters, these measurements are required: 1) temperature of the air, 2) wind velocity, 3) type of air flow as measured from wind profile, and 4) type of vapor diffusion as measured from humidity profile.

There are many different equations used to determine the evaporation based on the previous listed measurements. The relatively large number of equations results from the type of assumptions made to make the resulting equation reasonably practical and upon experimental results obtained in many different locations. Most of these equations are based on the assumptions of:

1. Logarithmic velocity profile;
2. Logarithmic humidity profile;
3. Stable atmosphere.

Limitations of the Mass Transfer Theory

The equations used in the mass transfer theory are based on the type of air flow usually found over a lake surface of large extent. The equations fail if the lake is located in unusual topography such as in a deep canyon or in rugged topography. These equations also fail to predict evapotranspiration because the nature of plants, plant size, and the distribution of plants and bare ground surface is so complex that it almost defies analysis. Recent studies (6)(7) show that sizeable differences in evapotranspiration were observed depending upon the plant size and distribution of plants and bare soil.

EMPIRICAL METHODS

Any review of literature reveals a large number of empirical equations for computing evaporation and evapotranspiration. These equations have been developed from simultaneous observations of evaporation (or evapotranspiration) and a number of climatological factors.

The early evaporation equations were developed from Dalton's Law. Many experiments followed and each set of experiments added or changed the empirical coefficients. Probably the most popular equation of this type was the Meyer formula (8). The advance of the energy budget and the mass transfer theories have replaced or altered many of these equations.

Estimating evapotranspiration adds the complexity of plant type, plant size, bare soil which might be exposed to evaporation and soil texture to the evaporation process. There are four methods of computing potential evapotranspiration. They are:

1. Lowry-Johnson Method;
2. Thornthwaite Method;
3. Blaney Criddle Method;
4. Penman Method.

The Lowry-Johnson Method (9) utilizes the maximum temperature above freezing during the growing season as an indicator of the amount of water evaporated and transpired by the crop. The Thornthwaite method (2) uses the mean daily temperature, the latitude of the place, and the month of the year to compute the evapotranspiration. It assumes that a high degree of correlation exists between the mean temperature and other variables such as radiation, atmospheric moisture and wind. A graphical procedure (2) for computing the evapotranspiration is available.

The Blaney-Criddle Method (10) estimates consumptive use (considered to be the same as potential evapotranspiration) using the mean temperature, type of crop grown, the latitude of the place and the season of the year. Recent studies (11) have added a refinement to the crop use coefficients to account for the change in plant size and plant maturing characteristics.

The Penman method (3) is a method of computing the potential using principles of both the energy budget and the mass transfer theories. For this reason it is more sophisticated than any of the other methods. In its present form it does lack the ability to account for differences in crop type, plant sizes and plant maturity. The evapotranspiration is given by

$$E_o = f \frac{(\Delta R_N + \gamma E_a)}{\Delta + \gamma} \qquad \text{----(1)}$$

where

E_T is the daily evapotranspiration in inches,

Δ is the slope of the saturation vapor pressure curve at the mean air temperature,

γ is the psychrometer constant,

R_N is the net solar radiation received converted to the amount of water this amount of energy would evaporate,

E_a is the "drying power" of the air and is highly correlated with the evaporation obtained from the mass transfer method.

f is a seasonal coefficient correcting for crop development,

$$f = \frac{E_T}{E_o}$$

E_o is the daily evaporation of a free water surface in inches

Probably the weakest factor in Eq. (1) is the factor, f, which Penman has defined as the ratio of the potential evapotranspiration to the free surface evaporation, E_T/E_o. Penman found that this ratio changed seasonly. Penman's findings were based on experiments with short grass in Rothamsted in southeast England. Pruitt and Angus (4) in reporting lysimeter experiments in rye grass at Davis, California found somewhat different values of f.

By using some of the data reported by Blaney, Haise and Jensen (11), typical values of f can be estimated for Colorado. These values are compared in the next table.

TABLE I.

Values of $f = E_T/E_o$

Month	Penman (Reference 3) for SE England	Penman (Reference 3) for Near Equator	Pruitt and Angus for California (Ref. 4)	Estimates for Colorado
January	0.6	0.7	0.6	0.6
February	.6	.7	0.6	0.6
March	.7	.7	0.7	0.7
April	.7	.7	0.7	0.7
May	.8	.7	0.8	0.8
June	.8	.7	0.9	0.9
July	.8	.7	0.9	0.9
August	.8	.7	0.9	0.9
September	.7	.7	0.9	0.8
October	.6	.7	0.9	0.7
November	.6	.7	0.9	0.6
December	.6	.7	0.9	0.6

This f factor is probably related to type of plant, size of plant and cropping practices and varies somewhat like the crop use coefficient, k, in the Blaney-Criddle Method. Pruitt and Angus found that on days with high winds and dry conditions the factor, f, deviated widely from that computed from the equation.

A second handicap in the use of the Penman equation is the presence of the net solar radiation term, R_N. Weekly average direct and diffuse solar radiation are measured and published by the Weather Bureau for about 75 stations in the United States. These data are published in the National Summary, Climatological Data. The data are not now available at a sufficient number of points to make Eq. (1) generally applicable. Penman has published a number of empirical equations which relate the net solar radiation received, R_N, and the drying power of the air, E_a, to certain more commonly published climatological measurements.

Net Solar Radiation

The net solar radiation, R_N, is equal to the integrated sum of the solar flux less the radiation absorbed by the atmosphere less the reflected radiation less the long-wave radiation returned back to space.

$$R_N = R_s - R_a - R_r - R_b$$

where

R_s is the solar radiation received by the earth at the top of the atmosphere,

R_a is the solar radiation absorbed by the atmosphere,

R_r is the solar radiation reflected by the earth back to space,

R_b is the long-wave radiation radiated back to space.

The solar radiation incident to the earth at the top of the atmosphere (based on a constant solar flux of 1.94 calories per square centimeter per minute) varies with the latitude and the sun's declination. Values of the radiation received at the top of the atmosphere are published in the Smithsonian Meteorological Tables (12) for each month of the year and for various latitudes. As published, these figures are expressed in Langleys. One langley is equivalent to one calorie per square centimeter or 3.69 Btu per square foot. Since we are concerned with evaporation or evapotranspiration, it is more convenient to convert the solar energy to the depth of water this energy would evaporate; thus

1 langley/day = .0169 mm water/day

= .000667 inches water/day

The average monthly values of solar radiation at the top of the atmosphere are shown in part (a) of the enclosed graph. The units are evaporation equivalent in inches/day.

The amount of solar energy absorbed by the atmosphere depends upon the cloudiness of the atmosphere and the length of travel through the atmosphere.

$$R_a = 0.29 \cos\varphi + .55\, n/N$$

where

R_a is the solar radiation absorbed by the atmosphere

φ is the latitude of the place

n/N is the ratio of actual sunshine to possible sunshine. Average monthly values for this ratio expressed as a percent are published for about 200 stations in the United States in the National Summary of the Climatological Data.

Assuming the latitude to be 40°, this part of the computation is made on part (c) of the enclosed computing graph.

The amount of solar radiation reflected from the surface of the earth depends upon the nature of the surface and upon the angle of the sun relative to the surface of the earth. The ratio of the reflected radiation to the incident radiation is called albedo. Budyko (1) lists mean values of the albedo for different types of natural land surfaces. Mean values of the albedo, r , and some selected ranges of values published by Budyko are shown in the next table.

TABLE II.

Mean Values of Albedo from Natural Land Surfaces

Type of Surface	Albedo, r
Stable Snow Cover	
Latitude 60° and higher	0.80
Latitude below 60°	0.70
Unstable Snow Cover	0.45
Water	0.05
Coniferous Forest	0.14
Tundra, Steppe, Deciduous Forest,	
Savanna in moist season	0.18
Savanna in dry season and	
Semideserts	0.25
Deserts	
(The color of soil surfaces in deserts is variable, a range of albedo from bare soil is also included)	
Dark soils	.05 - .15
Moist gray soils	.10 - .20
Dry clay or gray soils	.20 - .35
Dry light sandy soils	.25 - .45

"The albedo of water surfaces depends greatly on the altitude of the sun and varies from a few per cent at high sun (noon) to almost 100% for the sun near the horizon.----A relatively great absorption of short-wave radiation in water reservoirs is explained by the fact that the sun rays penetrate the upper translucent water layers, where they are scattered and almost completely absorbed. This is why the albedo of muddy water reservoirs is considerably higher." This comment on the albedo of water surfaces from Budyko's work is also included.

The values of albedo are used in part (b) of the enclosed computing graph. They are included so that the user may select appropriate values of the albedo to use in the computation. After using parts (a), (b) and (c) of the computing graph, the user has an intermediate value of net solar radiation, R_c , which will be used later.

$$R_c = R_s(1-r)\left[0.22 + 0.55\ {}^{n}/_{N}\right].$$

The long wave radiation back to space, R_b , is related to the temperature of the air and its transparency. The transparency is related to the amount of water vapor present and the degree of cloudiness. Penman gives the next empirical equation for the back radiation:

$$R_b = \sigma T_a^4(0.56 - 0.09\sqrt{e_d})(0.1 + 0.9\ {}^{n}/_{N})$$

where

σ is the Stefan-Boltzman constant,

= 8.26×10^{-11} calories per square centimeter per minute,

or 2.01×10^{-9} millimeters of H_2O per day

T_a is the mean temperature of the air, °K

σT_a^4 is the black body radiation at the mean air temperature,

e_d is mean vapor pressure, mm Hg,

${}^{n}/_{N}$ is ratio of actual sunshine to the possible sunshine.

The back radiation, R_b , can be obtained from parts (d) and (e) of the computing graph. This value of the back radiation must be subtracted from the value of R_c found previously to obtain the value of the net solar radiation R_N.

$$R_N = R_c - R_b.$$

Drying Capacity of the Air

The drying capacity of the air is a measure of the air to carry away and scatter the water vapor from the surface. This term in the equation utilizes some of the principles of the mass transfer concept. Penman has given the following equation for the drying capacity of the air, E_a .

$$E_a = (0.175 + 0.0035U_2)(e_a - e_d)$$

where

E_a is the drying capacity of the air - mm per day,

U_2 is the wind velocity as measured at 2 meters-mpd,

e_a is the saturation vapor pressure at the mean air temperature - mm Hg,

e_d is the saturation vapor pressure at the mean dew point which is equivalent to the actual vapor pressure of the air - mm Hg.

The coefficients in this equation are converted to the following when the units are changed to the more commonly available form.

$$E_a = (.00683 + .003276\ U_2)(e_a - e_d)$$

where

E_a in inches per day

U_2 in miles per hour

e_a in mm Hg

e_d in mm Hg

The solution for the drying capacity of the air may be obtained from parts (f) and (g) of the computing graph using the wind velocity at the two-meter level (U_2), mean air temperature (t_a) and the mean dew point (t_d).

Adjustment of U_2

A comment is in order regarding the elevation at which the wind measurements are made. The coefficients in the Penman equation have been evaluated for the two-meter elevation which is equivalent to 6.56 feet. Usually the wind measurements are made at a higher elevation than this. The measurements made at a higher elevation may be reduced to a measurement made at 2 meters using this relationship:

$$\frac{U}{U_o} = \left(\frac{Z}{Z_o}\right)^{1/7} .$$

This equation reduces to this form for two-meter elevation:

$$U_2 = U_x \left(\frac{6.56}{Z_x}\right)^{.143} .$$

Figure 2 is an auxiliary graph giving the velocity measured at 2 meters when the measurements are made at some other elevation, x. Before using part (f) of the computing graph, the published wind velocity must be reduced to wind velocity at the two-meter elevation.

Final Computation of E_o

The different elements of the Penman Equation have now been obtained from the computing graph. The final steps may be computed next. Using the net radiation, R_N, which was computed from the difference between R_c (from part (c)) and R_b (from part (e)), enter part (h) of the computing graph and then proceed to part (i). The drying capacity of the air, E_a, (from part (g)) is needed in part (i). Finally, potential free surface evaporation, E_o, is obtained from part (j) of the computing graph. The potential evapotranspiration may be computed by multiplying E_o by an appropriate f factor. Table I may be consulted for selection of an f factor.

CONCLUDING REMARKS

A series of graphs have been prepared to facilitate the use of the Penman Equation to compute the potential evapotranspiration. The general form of the Penman Equation as shown in Eq. (1) is somewhat impractical because the basic data are not extensively published. On the basis of some empirical relationships, Eq. (1) has been modified to the following form:

$$E_o = \frac{\Delta\left\{R_s(1-r)\left[0.22 + 0.55\ n/N\right] - \sigma T_a^4(0.56 - 0.09\sqrt{e_d})(0.1 + 0.9\ n/N)\right\}}{\Delta + \gamma}$$

$$\frac{\gamma(0.175 + 0.0035\ U_2)(e_a - e_d)}{\Delta + \gamma} . \qquad \text{---- (2)}$$

Equation (2) is called the modified Penman Equation. As more data become available, it is likely that the use of Eq. (2) will be replaced by Eq. (1). The computing graphs given herein are similar to the graphs published by Palmer and Havens (2) for the Thornthwaite method and by Purvis (5) for the Penman method. The graphs given herein differ from the graphs published by Purvis in that Budyko's findings in connection with albedo have been included and Purvis' graphs are for one location - Columbia, S. C., whereas these graphs could be used anywhere in the northern hemisphere up to latitudes of 60°. All scales are shown in the units normally used in publishing the data in the United States.

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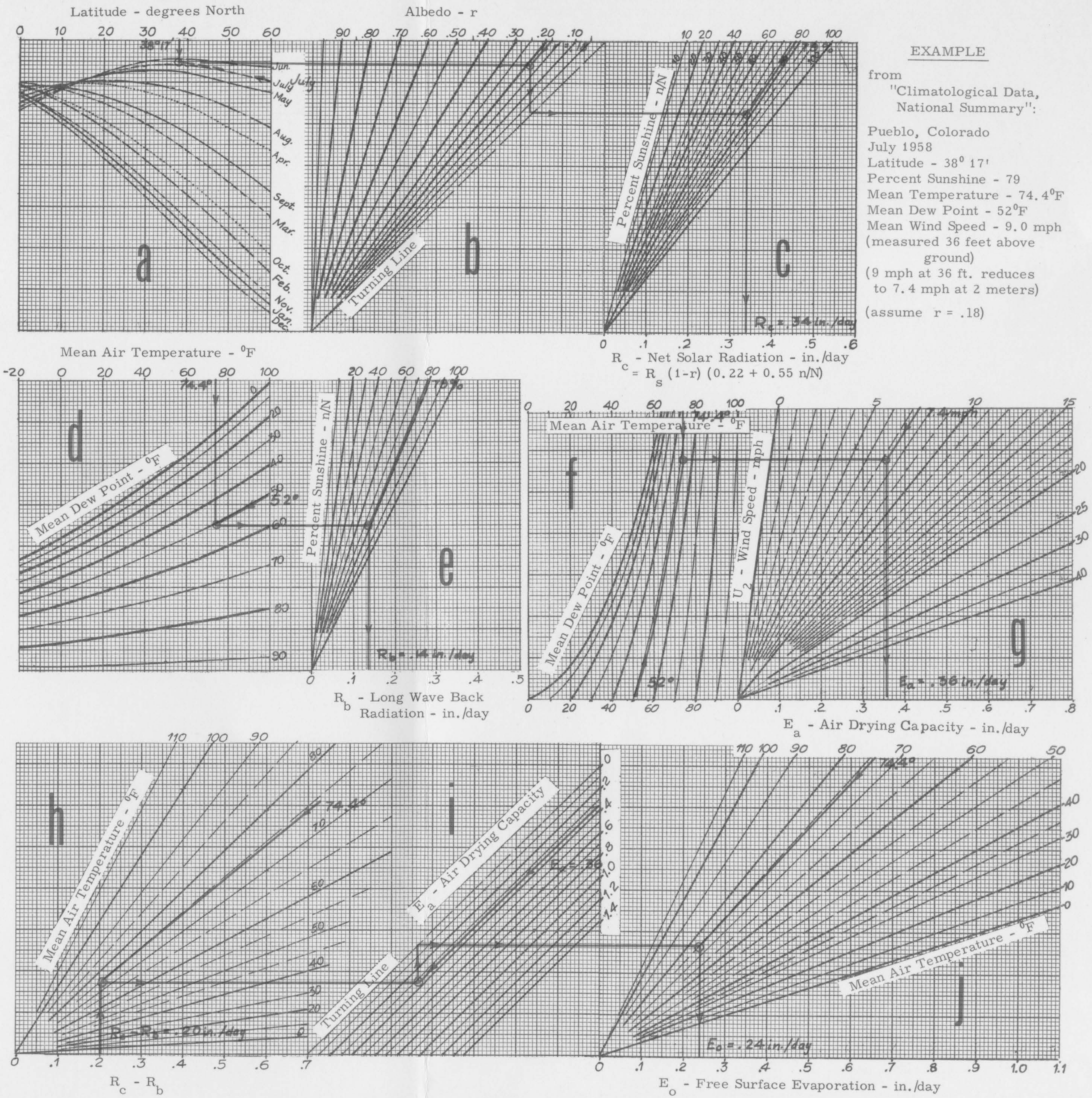

EXAMPLE

from
"Climatological Data,
National Summary":

Pueblo, Colorado
July 1958
Latitude - 38° 17'
Percent Sunshine - 79
Mean Temperature - 74.4°F
Mean Dew Point - 52°F
Mean Wind Speed - 9.0 mph
(measured 36 feet above ground)
(9 mph at 36 ft. reduces to 7.4 mph at 2 meters)
(assume r = .18)

GRAPH FOR COMPUTING EVAPORATION BY THE MODIFIED PENMAN EQUATION

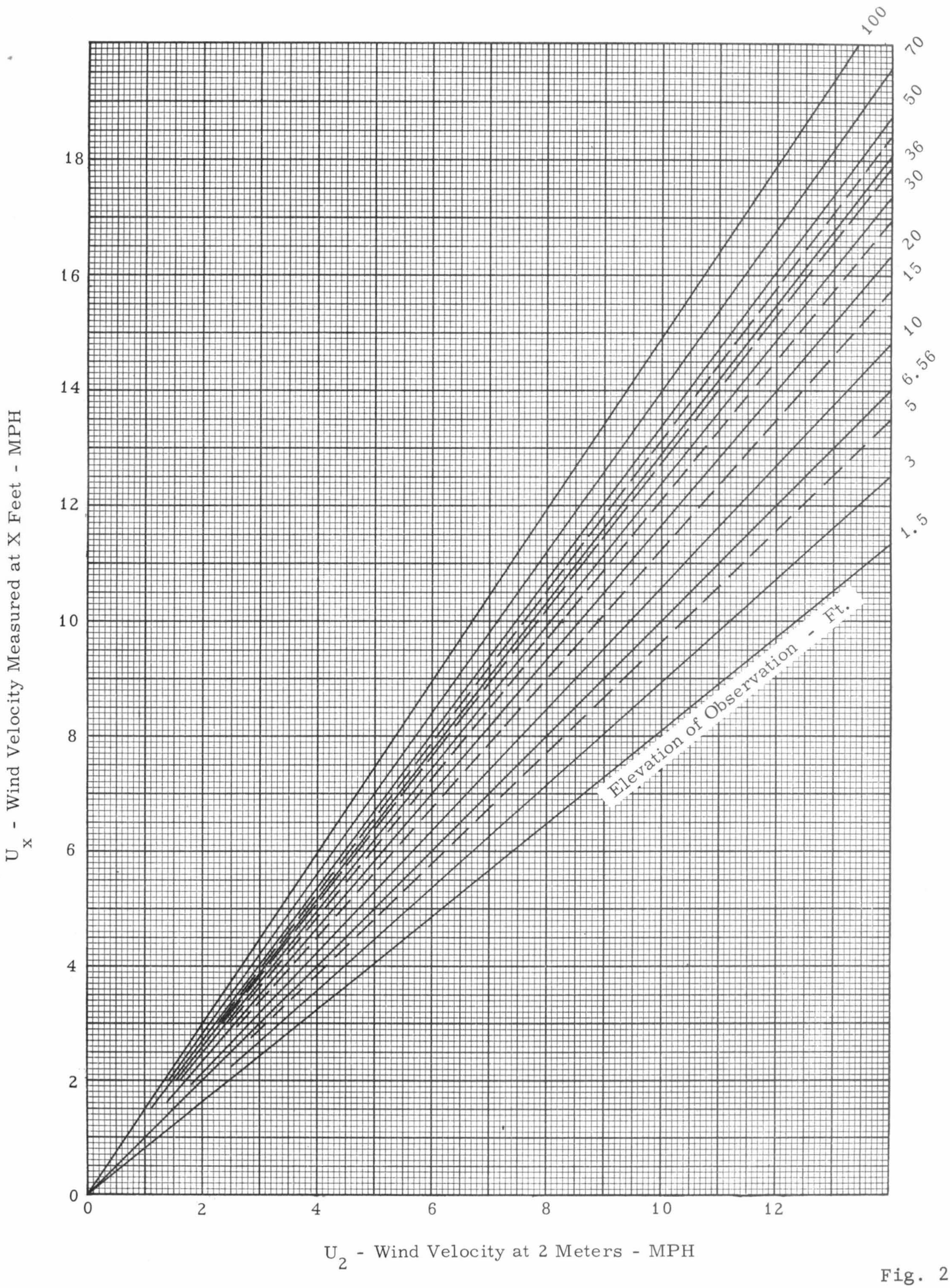
100
70
50
36
30
20
15
10
6.56
5
3
1.5
Elevation of Observation - Ft.
U_x - Wind Velocity Measured at X Feet - MPH
0
2
4
6
8
10
12
14
16
18
U_2 - Wind Velocity at 2 Meters - MPH
0
2
4
6
8
10
12

Fig. 2