Protections 2016 Ft. Collins, Colorado, USA, 7-9 September 2016
2nd International Seminar on
Dam Protection Against Overtopping
ISBN: 978-1-1889143-27-9 DOI:

Embankment Overtopping Protection by Riprap Considering Interstitial Flow

Tony L. Wahl
Bureau of Reclamation
P.O. Box 25007
Denver, CO 80225
USA
E-mail: twahl@usbr.gov

ABSTRACT

Riprap is commonly provided to protect embankment slopes from localized erosion caused by surface runoff during rainfall events, but it can also provide erosion protection during small overtopping events. When flow rates are small, most water may flow interstitially through the riprap layer, then gradually above the riprap layer as flow rates increase. On relatively steep slopes, interstitial flow may still be a significant proportion of the total flow—even 100% of the flow—when the failure discharge is reached. In the late 1990s the Bureau of Reclamation sponsored tests on 2H:1V (50%) embankment slopes to determine allowable rates of overtopping flow for large riprap with median stone sizes up to 26-inch diameter. These tests included measurements of interstitial flow, and Mishra (1998) and Frizell et al. (1998) used the collected data to develop a design procedure for riprap on steep slopes accounting for interstitial flow. Unlike other riprap design equations that relate allowable discharge or stone size to independent variables through simple closed-form equations, the method incorporates interstitial flow in a mechanistic way through iterative calculations that add complexity to the solution process.

This paper illustrates the Mishra/Frizell design method through example calculations and highlights key aspects of the solution process. Important conclusions are drawn related to the use of riprap for overtopping flow protection on typical embankment dam slopes.

Keywords: *Riprap design, interstitial flow, critical shear stress.*

1. INTRODUCTION

Riprap is a common material used to protect embankment slopes against erosion associated with waves on a reservoir or surface runoff during rainfall events. Riprap can also provide some protection against erosion of the downstream slope when flow overtops an embankment dam. Variables affecting the failure discharge of riprap installed on an embankment slope include the slope angle, stone size, riprap layer thickness, and various stone properties (e.g., gradation, angularity, uniformity, and unit weight).

Efforts to determine the discharge limits for embankment overtopping flow have focused over time on gradually steeper slopes and larger sized material. Although it has long been recognized that water can flow both through and over a riprap layer, there was not great interest in quantifying interstitial flow when testing was focused on relatively flat slopes. On such slopes, small stone sizes can protect against significantly large flow rates, and riprap failure tends not to occur until the depth of flow is well above the top of the riprap layer and interstitial flow is a small fraction of the total. Thus, early riprap design equations made no distinction between interstitial flow and flow above the riprap surface. Examples include the works of Maynord (1988) on slopes of 2% or less, Abt & Johnson (1991) on slopes of 1 to 20%, and Robinson et al. (1998), testing rock chutes on slopes of 2 to 40 percent (2.5H:1V). As the utility of riprap was demonstrated for relatively flat slopes, interest grew in the possibility of using riprap to protect steeper slopes. As slopes were increased, failure tended to occur with less flow over the riprap surface and more interstitial flow—up to 100% interstitial flow in some cases.

1.1. Steep Slope Tests at Colorado State University

During the 1990s the Bureau of Reclamation commissioned flume tests performed at Colorado State University to evaluate discharge limits of riprap on 50% slopes (2:1). Mishra (1998) and Frizell et al. (1998) reported on overtopping tests with median rock sizes from 10.5 to 26 inches at unit discharges up to 10 $ft^3/s/ft$. At such steep slopes, flow takes place primarily through the rock rather than over the rock, and a variety of multiphase flows can occur as noted by Peirson and Cameron (2006), including aerated water, water through rock, and potentially air and water through rock. Flow conditions were observed through clear acrylic windows in the side of the test flume. Flow depths were measured with piezometers embedded in the rock layer, and salt injectors and conductivity probes were used to measure interstitial velocities. Failures of the riprap slope (exposure of bedding material) occurred when measured flow depths were still below the top of the rock layer. (Most recent investigators have been consistent in defining riprap failure to have occurred when the bedding material is exposed.) Highly aerated flow was observed above the rock surface, but did not register at the piezometers and was only a small fraction of the total flow.

2. STEEP-SLOPE RIPRAP DESIGN PROCEDURE

Results of the Colorado State tests were used to develop a design procedure that could determine the appropriate rock size and layer thickness to safely protect against a given unit discharge of overtopping flow (Mishra 1988; Frizell et al. 1998). The relations developed included the effects of varying the coefficient of uniformity, $C_u=D_{60}/D_{10}$, and the porosity of the rock mix, recognizing that in large-size rock mixes porosity can vary significantly depending on placement methods and other factors. The procedure allows for riprap layer thicknesses of 2 to 4 times D_{50}, the former being the minimum necessary to protect the bedding material and the latter being a practical upper limit for effective placement and economic protection of the slope. A very significant requirement of the procedure is that for slopes steeper than 4:1 (S = 25%), the entire computed flow must be conveyed interstitially. For flatter slopes, a portion of the flow can be conveyed above the rock surface, with a check that the surface flow will not cause shear stresses to exceed a critical shear stress limit for the rock. It should be noted that the testing performed at Colorado State also explored the question of toe stability, with a variety of measures employed in attempts to produce a test in which failure occurred at the toe of the slope, but no toe failure ever occurred. Neither this work nor that of Robinson et al. (1998) looked at the issue of increased protection for groin areas where embankment slopes meet converging abutments. The increased unit discharge in these areas may require locally larger stone sizes and an increased riprap layer thickness.

The dual consideration of performance limits related to both interstitial flow and surface-flow shear stress causes the Mishra/Frizell riprap design procedure to be more complex than many of the equations that have preceded it. Considering questions received at Reclamation from individuals attempting to use the procedures since they were first published, there appears to be some confusion about how the intersecting limiting conditions should be applied. This paper attempts to clarify the procedure through examples and presentation of some generalized guidance in graphical form. The procedure's results are compared to the equations developed by Abt & Johnson (1991) and Robinson et al. (1998), which do not consider interstitial flow. Some important insights are noted regarding the use of riprap to protect steep embankment slopes.

2.1. Mishra/Frizell Procedure

The Mishra/Frizell riprap design procedure consists of ten steps, performed iteratively until an acceptable design is obtained. As presented here, the design factor of safety is 1.0, indicating that the riprap placement would be expected to fail at the allowable discharge value used in the computations. For comparison, the Abt & Johnson (1991) and Robinson et al. (1998) equations will also be presented. The original development of the Abt & Johnson equation included a 1.35 factor of safety which has been removed here so that it also computes the rock size corresponding to the threshold of failure. Definitions for terms in the equations that follow are:

q = allowable discharge, above which riprap failure is expected (ft^3/s or m^3/s),
S = the embankment slope expressed as the tangent of the slope angle (i.e., for 2:1 slope, S=0.5),
D_{50} = the riprap diameter for which 50% by weight of the material is finer (ft or m),

n_p = the porosity of the riprap, (ratio of voids to total volume)
C_u = the coefficient of uniformity, $C_u = D_{60} / D_{10}$,
g = the acceleration due to gravity, (32.2 ft/s^2 or 9.81 m/s^2),
α= slope angle from horizontal, $\alpha = \tan^{-1}(S)$,
φ= angle of repose, typically about 42° for angular riprap,
G_s= specific gravity of stones, typically near 2.65
t = thickness of riprap layer (ft or m).

The Mishra/Frizell procedure consists of the following steps:

STEP 1: Calculate the overtopping head from a basic weir equation using an appropriate discharge coefficient, C. (This step is optional, but it is often desirable to know the overtopping head.)

$$H = (q/C)^{2/3} \tag{1}$$

STEP 2: Calculate a starting median rock diameter, D_{50}, from the universal design equation

$$D_{50} = \frac{K_M q^{0.52}}{{C_u}^{0.25} S^{0.75}} \left(\frac{\sin\alpha}{(G_s \cos\alpha - 1)(\cos\alpha \tan\varphi - \sin\alpha)} \right) \tag{2}$$

where K_M is a coefficient having a value of 0.55 s$^{0.52}$m$^{-0.04}$ or 0.5245 s$^{0.52}$ft$^{-0.04}$.
(Note that "s", "m" and "ft" are dimensions: seconds, meters, and feet. The decimal exponents are needed for dimensional consistency.)

STEP 3: Calculate the interstitial velocity, V_i, and the average velocity, V_{avg}. The interstitial velocity represents flow occurring only through the void areas within the riprap layer, whereas V_{avg} is a "virtual velocity" calculated by spreading the flow over the entire thickness of the riprap layer.

$$V_i = 2.48 S^{0.58} {C_u}^{-2.22} \sqrt{g D_{50}} \tag{3}$$

$$V_{avg} = V_i n_p \tag{4}$$

STEP 4: Compute the flow depth within the riprap layer corresponding to V_{avg}.

$$y = \frac{q}{V_{avg}} \tag{5}$$

If the calculated depth is less than or equal to $2D_{50}$, the design is acceptable and the depth of the riprap layer should be $2D_{50}$. If the calculated depth is greater than $2D_{50}$, two options are available. If the slope is less than or equal to 0.25, proceed with step 5 to determine if some flow can be conveyed above the surface of the riprap. If the slope is greater than 0.25, increase the D_{50} value (perhaps by 10%) and repeat steps 3 and 4.

STEP 5: Calculate the depth of water that can flow over the riprap surface at the critical shear stress threshold.

$$h = \frac{0.06(G_w - G_s) D_{50} \tan\varphi}{0.97(S)} \tag{6}$$

where: G_w = specific gravity of water (1.0)
G_s = specific gravity of rock (usually about 2.65)
h = depth of water flowing above riprap surface.

STEP 6: Calculate the Manning's roughness coefficient for the riprap surface, $n = 0.0414 {D_{50}}^{1/6}$. ($D_{50}$ must be given in meters.)

STEP 7: Calculate the unit discharge that can be carried above the riprap layer,

$$q_1 = \frac{1.0}{n} h^{5/3} \sqrt{S} \tag{7}$$

(If h is given in feet rather than meters, change the 1.0 coefficient to 1.486.)

STEP 8: Calculate the unit discharge that must be carried in the riprap layer,

$$q_2 = q - q_1 \tag{8}$$

STEP 9: Compute the required riprap layer thickness to convey the unit discharge, q_2.

$$h_2 = \frac{q_2}{V_{avg}} \tag{9}$$

If h_2 is less than or equal to $4D_{50}$, the design is acceptable and the riprap layer thickness should be $4D_{50}$. If h_2 is greater than $4D_{50}$, increase the D_{50} value and repeat the procedure starting at step 3.

Careful examination of the steps in this procedure reveals that only two riprap layer thicknesses are possible. If the slope is greater than 25%, the riprap layer can only have a thickness of $2D_{50}$ and all of the flow must be carried interstitially. If the slope is less than or equal to 25%, the layer thickness can be $2D_{50}$ if all of the flow can be carried interstitially within that thickness, or $4D_{50}$ with some of the flow computed to be above the riprap surface. On flat slopes, the interstitial flow capacity is small, so it is likely that the procedure will yield flow above the riprap surface and a layer thickness of $4D_{50}$. Regarding the flow depth being above or below the riprap surface, it should be noted that in the testing performed at Colorado State, even when piezometers indicated that the effective flow depth was below the riprap surface, water and spray were visible above the rock surface. It is crucial to notice that as step 9 is worded (Mishra 1998; Frizell et al. 1998), there is no provision for an intermediate riprap layer thickness between $2D_{50}$ and $4D_{50}$. It is also important to emphasize that computed flow of water above the riprap surface is only allowed when the slope is 25% or flatter; for slopes steeper than 25% all flow must be interstitial. These requirements of the procedure lead to some dramatic discontinuities in the rock sizes that are required on either side of the 25% slope boundary, which will be discussed in more detail below; it is not clear that all of these limitations were intended results of the procedure.

2.2. Other Procedures

Results from applying the Mishra/Frizell design procedure will be compared in this paper to the Abt & Johnson (1991) and Robinson et al. (1998) equations. The Abt & Johnson equation was developed from testing performed on relatively flat slopes ranging from 50:1 to 10:1 (S=2% to 10%). Equations for directly computing allowable discharge and the required D_{50} value at the threshold of failure are:

$$q = K_1 S^{-0.768} D_{50}^{1.786} \tag{10}$$

$$D_{50} = K_2 q^{0.56} S^{0.43} \tag{11}$$

Values of the K_1 and K_2 coefficients are listed in Table 1 for both U.S. Customary and S.I. units.

The Robinson equation was developed from tests carried out at slopes ranging from 6:1 to 2.5:1 (S = 17% to 40%). The allowable discharge and required D_{50} value at the threshold of failure are:

$$q = K_3 S^{-0.58} D_{50}^{1.89} \tag{12}$$

$$D_{50} = K_4 q^{0.53} S^{0.307} \tag{13}$$

with K_3 and K_4 also given in Table 1.

Table 1. Coefficients for riprap design equations.

Equation	Parameter	U.S Customary units (ft, s, ft^3/s)	S.I. units (m, s, m^3/s)
Abt & Johnson (1991)	K_1	3.26	2.528
	K_2	0.516	0.595
Robinson et al. (1998)	K_3	4.3	3.773
	K_4	0.462	0.496

3. APPLYING THE MISHRA/FRIZELL PROCEDURE

To demonstrate the Mishra/Frizell procedure and compare it to results from the other procedures, a set of spreadsheets was developed to perform the calculations and apply the rules described in the steps above. The calculations were first carried out in the form of a design problem, setting the allowable unit discharge to values of 1, 2, 5, and 10 ft^3/s/ft. For each unit discharge, the procedures were applied for slopes varying from 2 to 50%. Rock properties for all cases were assumed to be:

- Porosity of riprap placement = 0.45
- Coefficient of uniformity, $C_u = D_{60}/D_{10} = 2.1$
- Angle of repose, $\varphi = 42°$ (angular riprap)
- Specific gravity of riprap stones = $G_s = 2.65$

Figure 1 shows the results for the 1 ft^3/s/ft design unit discharge. At the lowest slopes (2 to 4%), the Mishra/Frizell design equation yields a starting rock size that conveys only a small fraction of the flow interstitially in a $4D_{50}$–thick layer but is able to convey all the remaining flow above the riprap surface. For slopes of 5 to 25%, the interstitial flow is still small and the design equation's stone size is too small to resist the shear stress associated with the necessary surface flow, so the stone size must be increased. The riprap layer is still $4D_{50}$ thick, since all flow cannot be carried interstitially. For this unit discharge, the percentage of the flow carried above the riprap varies from 99% at 2% slope to 65% at 25% slope. For slopes greater than 25% the starting rock size given by the design equation is still too small to convey all of the flow interstitially, so the procedure requires the rock size to be increased until all flow can be conveyed interstitially in a $2D_{50}$ layer. This leads to a dramatic near-tripling of the required rock size once the slope exceeds 25%. A surprising result for steep slopes is that the required rock size actually decreases as slope increases above 25%, since the interstitial flow capacity increases as slope increases.

Figure 1 also illustrates the results that would be obtained if some limitations of the procedure were relaxed.

- If a $4D_{50}$ riprap layer thickness was allowed for slopes steeper than 25% the required rock size in the steep slope zone would drop by about one third (see "Mishra/Frizell interstitial flow limit for $t = 4D_{50}$").
- If flow of water above the riprap surface were allowed for slopes steeper than 25% the required rock size would drop down to one of the blue dotted lines, depending on whether the thickness of the riprap layer were $2D_{50}$ or $4D_{50}$ (if a $4D_{50}$ layer was allowed for slopes steeper than 25%).
- For slopes less than 25%, if flow above the riprap surface was allowed in combination with a $2D_{50}$-thick layer, the rock size could be increased by about 15%, but the total volume of rock required would be reduced, since the layer thickness would be cut in half.

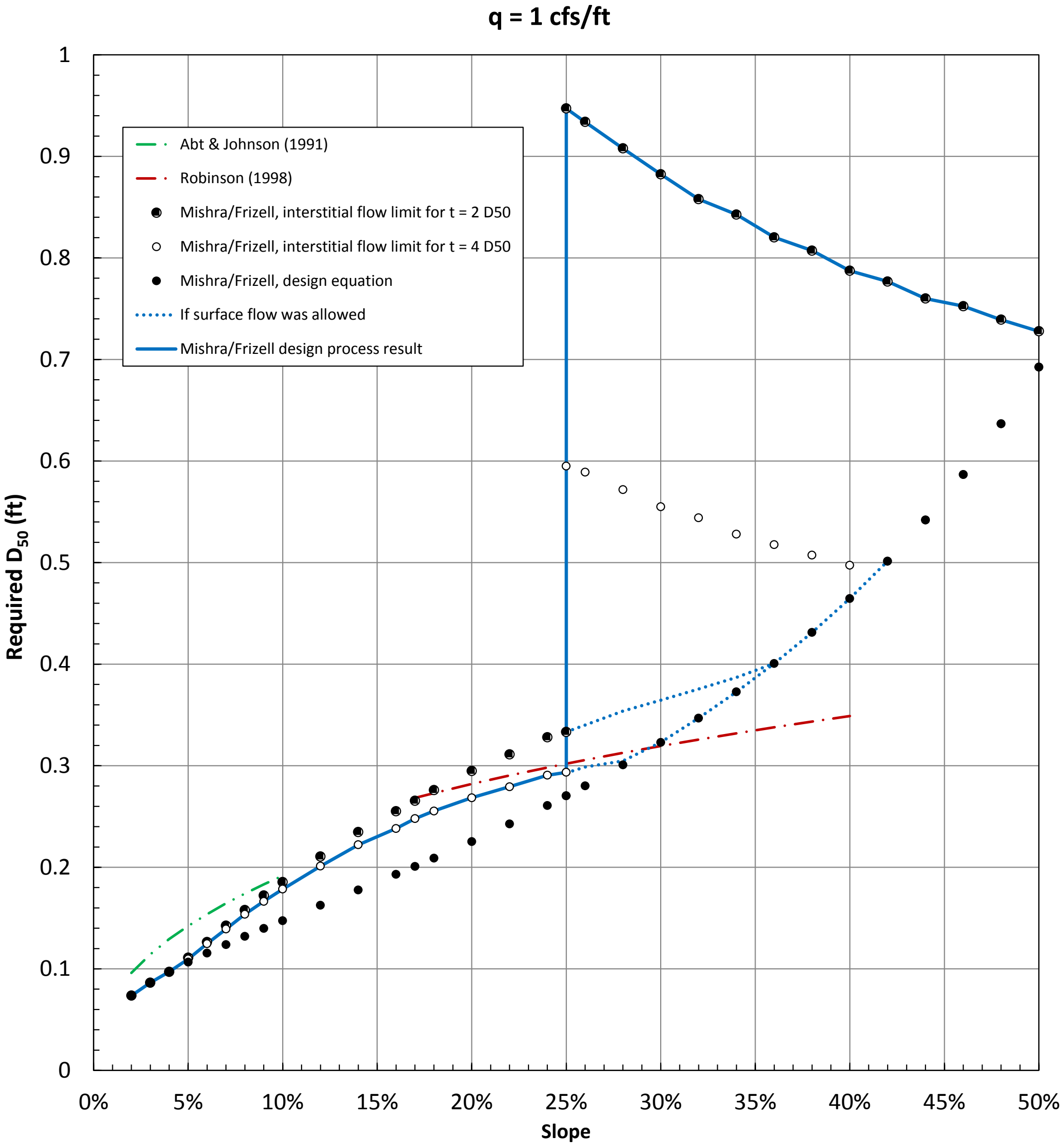

Figure 1. Required median stone sizes (D_{50}) as a function of embankment slope, using the Mishra/Frizell riprap design procedure and the Abt & Johnson and Robinson equations.

Comparing the Mishra/Frizell results to the rock sizes computed from the Abt & Johnson and Robinson equations, the Mishra/Frizell procedure produces similar results for slopes below 25%. For steep slopes, the Mishra/Frizell equation requires much larger rock sizes, although relaxing limitations of the procedure as outlined above would yield results that were more comparable in the 25-40% slope range.

For increasing values of the allowable unit discharge, Figure 2 shows curves similar to those developed in Figure 1. The general character of the charts is the same, with dramatic increases in D_{50} needed when the slope exceeds 25%. These increases again arise because the Mishra/Frizell procedure requires all flow to be interstitial in a $2D_{50}$ layer for slopes steeper than 25%. The figure also shows that as the allowable unit discharge is increased, the Mishra/Frizell procedure becomes more conservative in comparison to the Abt & Johnson and Robinson equations, even for slopes flatter than 25% where a majority of the flow can be carried above the riprap surface and interstitial flow is small.

When analyzing an existing riprap placement it is often desirable to determine the allowable unit discharge at the threshold for failure. Figure 3 shows how the allowable unit discharge varies as a function of embankment slope when the median size of rock is fixed at D_{50} = 0.5 ft.

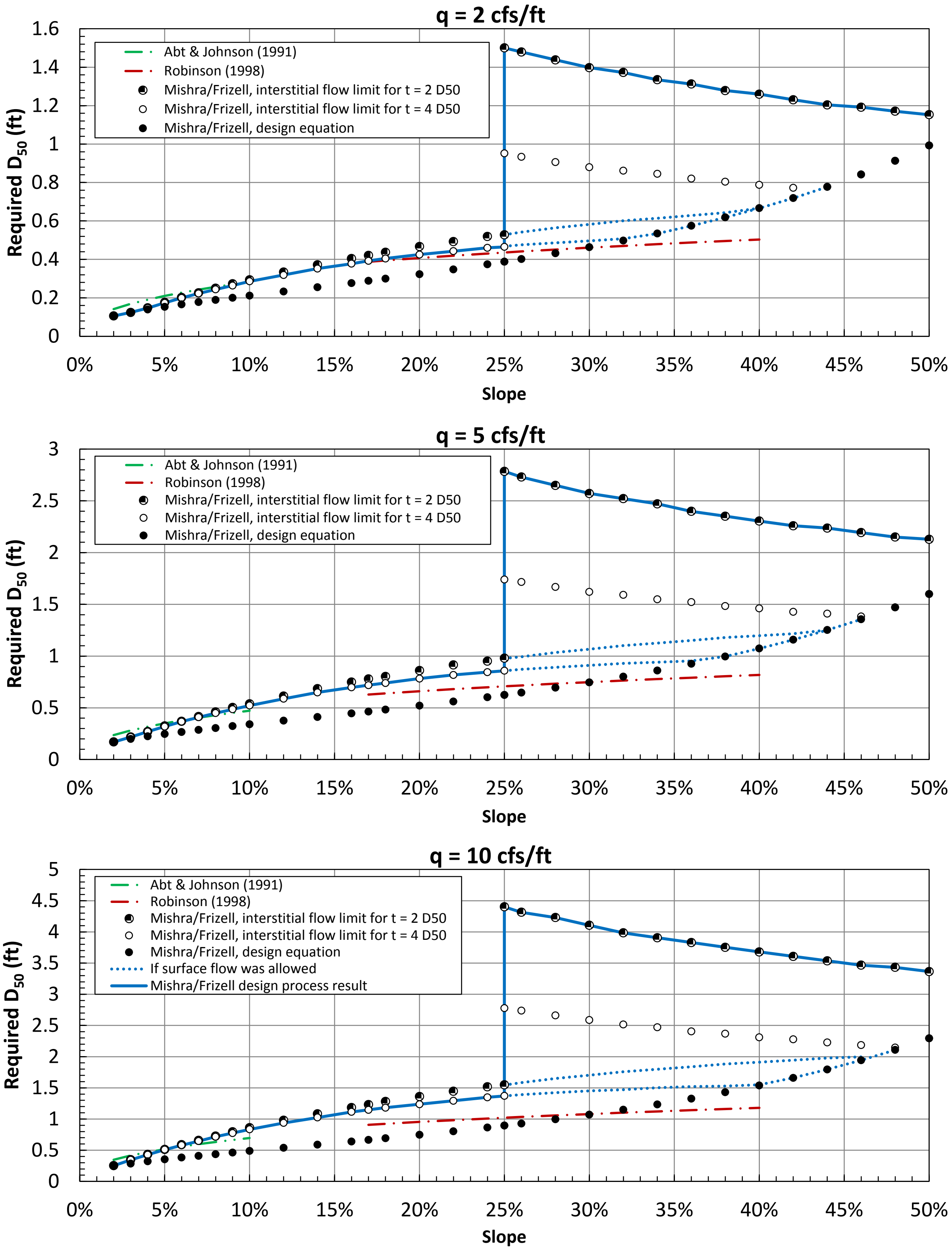

Figure 2. Required median stone sizes versus embankment slope for increasing values of the allowable unit discharge.

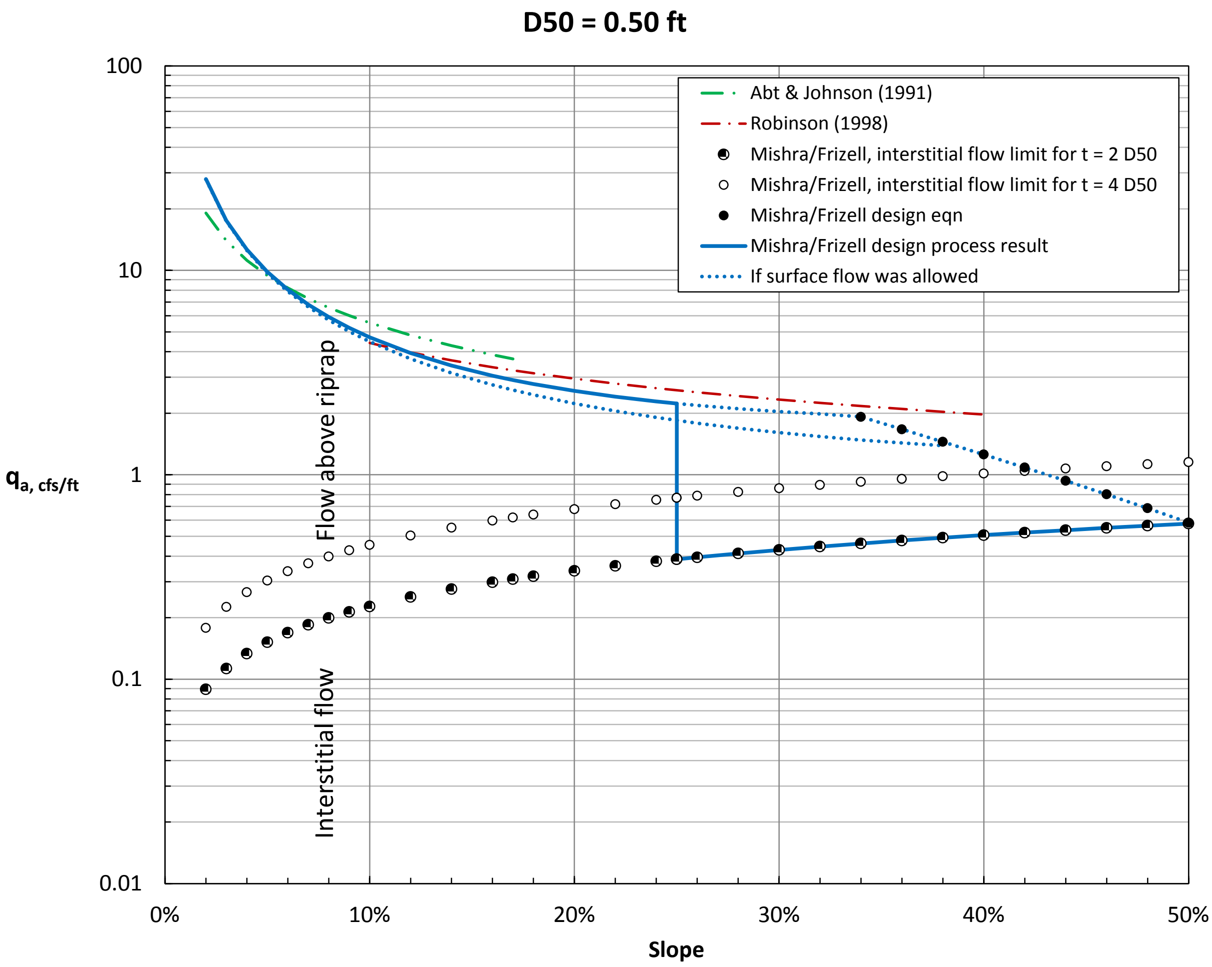

Figure 3. Allowable unit discharges as a function of slope for a fixed median rock size.

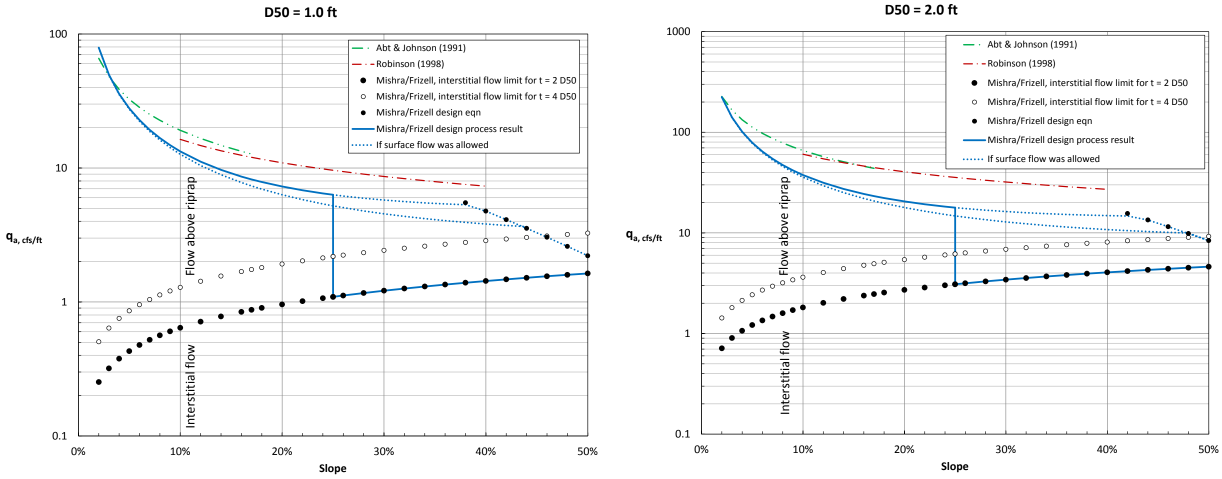

Figure 4. Allowable unit discharges for larger rock sizes of 1 and 2 ft.

For slopes less than 25% the allowable discharges from the Mishra/Frizell procedure are comparable to the Abt & Johnson and Robinson equations—slightly larger for slopes less than 5% and slightly smaller for higher slopes. In this zone, the procedure allows flow above the riprap surface, but only if the placement thickness is $4D_{50}$. The figure also illustrates the slightly lower unit discharge that would be allowable if surface flow were allowed over a $2D_{50}$ layer. For slopes greater than 25%, only a $2D_{50}$ thickness is allowed and all of the flow must be interstitial, which greatly reduces the allowable discharge. The figure also illustrates the increase in allowable discharge that would occur if a $4D_{50}$ thickness were allowed with all flow still required to be interstitial, or if flow above the riprap surface were allowed with a $4D_{50}$ thickness.

Figure 4 shows allowable unit discharges for larger rock sizes, up to $D_{50} = 2.0$ ft. The Mishra/Frizell procedure again becomes more conservative than the Abt & Johnson and Robinson equations as the rock size increases.

4. DISCUSSION

The large discontinuity in all of the curves at the 25% slope threshold is a troubling feature of the design process, since it is extremely doubtful that it represents a true shift in riprap performance at that particular slope. The Colorado State tests showed that riprap failure would occur at a 50% slope with no flow above the riprap surface, but because tests were not run at other slopes, there is not strong justification for beginning to require all flow to be interstitial above a 25% slope, and it even seems unlikely that there is a sudden inability for riprap to resist surface flow above any certain slope. Instead, as slope increases, the failure flow condition probably exhibits a gradual shift from mostly surface flow and low interstitial flow to mostly interstitial flow and low surface flow.

The Mishra/Frizell method is predicated on the idea that there is an interstitial flow limit for the riprap layer that depends on the layer thickness, the slope, and the riprap properties. In addition, for slopes below 25%, surface flow can occur if it does not create stresses exceeding incipient motion criteria related to stone size. The shear stress calculation used to evaluate whether the point of incipient motion has been reached does not consider that the interstitial flow also will apply stresses to the riprap layer that will tend to destabilize the stones. A more realistic analysis would estimate the stresses applied by both interstitial flow and surface flow and consider their combined ability to destabilize the riprap layer. The curves shown in this paper that illustrate results when some of the key limitations of the method are relaxed give some idea of the results that might be obtained, but they do not represent an actual analysis of combined stresses. Unfortunately, the allowable flow capacities that are indicated for riprap on steep slopes—even if surface flow and thicker riprap layers are allowed—do not suggest that riprap will be an economical method for protecting against significant amounts of overtopping flow. On embankments with slopes in the 2:1 to 3:1 range, the allowable unit discharge for riprap with D_{50}=2 ft is less than 15 cfs/ft with surface flow allowed and less than 5 cfs/ft if the Mishra/Frizell method is applied as written. The relatively small level of overtopping flow protection provided by such large rock suggests that riprap would probably only be economically feasible for small-height dams with limited drainage area and minimal downstream hazards.

5. CONCLUSIONS

The Mishra/Frizell riprap design method considering interstitial flow was demonstrated to produce discontinuous predictions of required rock size or allowable unit discharge that probably do not accurately represent the behavior of real riprap across the full range of embankment slopes. The method's requirement that all flow be conveyed interstitially for any slope steeper than 25% leads to predictions of very low allowable unit discharge or very large rock size requirements. An analysis that yields more gradually varying results as a function of slope may be possible by considering the combined stresses created in the riprap layer by both interstitial flow and surface flow. That analysis might also help to refine the estimate of the slope threshold beyond which no surface flow is possible.

6. REFERENCES

Abt, S.R. and T.L. Johnson. 1991. Riprap design for overtopping flow. *Journal of Hydraulic Engineering*, 117(8):959 - 972.

Frizell, K.H., J.F. Ruff, and S. Mishra. 1998. Simplified design guidelines for riprap subjected to overtopping flow. In: *Dam Safety '98*, Proceedings of the Annual Conference of the Association of State Dam Safety Officials, Las Vegas, Nevada.

Maynord, S.T. 1988. Stable riprap size for open channel flows. Technical Report HL-88-4. U.S. Army Corps of Engineers, Waterways Experiment Station, Vicksburg, Mississippi.

Mishra, S.K. 1998. Riprap design for overtopped embankments. Ph. D. Thesis, Colorado State University, Department of Civil Engineering, Fort Collins, Colorado.

Peirson, W.L. and S. Cameron. 2006. Design of rock protection to prevent erosion by water flows down steep slopes. *Journal of Hydraulic Engineering*, 132(10):1110–1114.

Robinson, K.M., C.E. Rice, and K.C. Kadavy. 1998. Design of rock chutes. *Transactions of the ASAE*, 41(3):621-626.