

RESEARCH REVIEW  
ON  
HYPERCONCENTRATED FLOWS



by

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## LIST OF SYMBOLS

A	constant in the velocity distribution
a,b,c	coefficients
B	channel width
$C_v$	volumetric sediment concentration
$C_{vf}$	volumetric concentration of fine particles
$C_{vo}$	critical concentration separating Newtonian from non-Newtonian fluids
$C_v^*$	maximum possible static volume concentration
$C_1$	coefficient in the quadratic rheological model
$\bar{C}_\omega$	depth averaged concentration of particles with fall velocity $\omega$
$\bar{C}_1$	average concentration near the bed from $y=y_1$ to $y=y_2$
D	grain diameter
$D_{av}$	average grain diameter
$D_{max}$	maximum particle diameter at which discrete coarse particles turn into neutral buoyant load
$D_{50}$	median grain diameter
E	parameter
$f, f_m$	Fanning's resistance coefficient in clear water and sediment-water suspension respectively
g	gravitational acceleration
h	flow depth
k	Von Karman constant
K	constant
$\lambda$	Prandtl's mixing length
$\lambda_m$	modified Prandtl's mixing length of suspension
m	exponent or constant
P	plastic number
Q	dynamic parameter
Ri	Richardson number
r	coefficient of correlation
s	distance between two neighboring particles in suspension
$S_e$	energy gradient
u	point flow velocity
$U_*$	friction velocity
$u_m$	maximum velocity in pipe flow
$U_o$	velocity of "plug flow"
V	cross-sectional averaged velocity
y	upward distance from the streambed
$\omega$	gross settling velocity
$\omega_o$	settling velocity of a single particle in an infinite mass of fluid
$\omega'$	settling velocity of particle in the mixture
$\alpha$	dynamic internal friction angle
$\lambda$	linear concentration defined by Bagnold
$\gamma$	specific weight of water or carrier fluid
$\gamma_m$	specific weight of suspension

$\gamma_s$	specific weight of sediment particles
$\xi = y/h$	relative flow depth
$\mu$	dynamic viscosity of clear water
$\mu_c$	consistency index in yield-pseudoplastic model
$\mu_p$	dynamic viscosity of the mixture
$\mu_r$	relative viscosity of suspension
$\nu$	kinematic viscosity of clear water
$\rho, \rho_s, \rho_m$	densities of clear water, sediment particle, and sediment-water mixture, respectively
$\tau$	shear stress
$\tau_y$	yield shear stress
$\tau_o$	bed shear stress
$\omega(\frac{y}{\delta})$	Cole's wake flow function
$\delta$	boundary sublayer thickness
$\bar{\delta}$	thickness of bonded water on the particle surface
$\Pi$	Coles' wake strength coefficient

## I. INTRODUCTION

The presence of suspended sediments affects the flow of natural streams. When the concentration of sediment is low, the effects are negligible. At higher concentrations of sediment, however, both the physical and dynamic characteristics of the flow differ from that of clear water flows. Many unusual phenomena, such as clogging of river, periodic flow, tearing off of the bed sediments, are observed in hyperconcentrated flows.

Although high concentrations are somewhat unusual, they are not rare, and at some locations they are common. For example, some of the rivers in the Yellow River Basin in China are known for their extremely high concentration of sediments. The mean annual sediment concentration in some tributaries can be as high as  $500 \text{ kg/m}^3$ , and the maximum can reach  $1600 \text{ kg/m}^3$ . Owing to the presence of the hyperconcentrated flows, a variety of problems in agriculture and industry might arise. Urgent research on the effects of high concentration on stream transport processes and flow phenomena is needed.

Back in the late sixties and early seventies, China began research on hyperconcentrated flow. In more recent years, studies on hyperconcentrated flow have gained the attention of many scientific experts all over the world. In the United States, several recent occurrences of hyperconcentrated flows caused extensive property damages and loss of life in the San Francisco Bay area, in the Wasatch Mountains of Utah, and near the volcanic Mt. St-Helens. The situation urged the ASCE to set up a Task Committee on the effects of High Sediment Concentration on flow and sediment transport to describe and report on these flow phenomena.

Studies on hyperconcentrated flows are generally classified according to: (1) rheological and kinematic behaviors; (2) settling velocities of sediment particles; (3) flow mechanisms including velocity distribution, flow resistance, unstable flow phenomena and (4) sediment transport mechanisms including bedload, suspended load and neutral buoyant load. Since the studies of flow with high concentration of sediments are still at an early stage, many conflicting opinions and experimental results have yet to be explained. For instance, Bagnold (1954), Zhang (1961), and Bruhl and Kazanskij (1976) concluded that the existence of sediment in flow reduces the turbulence intensity while Elata and Ippen (1961), Muller (1973), Bohlen (1969) reported from their experiments that the existence of sediment in flow increases the intensity of turbulence. Similarly, the resistance of hyperconcentrated flows has been reported to be: 1) smaller than (Vanoni, 1960; Zhang, 1961), 2) equal to (Einstein and Chien, 1955), and 3) greater than (Montes, 1973) that of a clear water.

This report presents a general review and comprehensive comparison including controversies of the research results published on various aspects of hyperconcentrated flows. The report includes four parts: 1) the classification of sediment-laden flows and forms of grain movement, 2) physical properties of hyperconcentrated flows, 3) velocity distribution in hyperconcentrated flows and 4) hyperconcentrated flow resistance.



## II. CLASSIFICATION OF SEDIMENT LADEN FLOWS AND TYPES OF GRAIN MOVEMENT

### 2.1 Classification of Hyperconcentrated Flows

The classification of flows with high sediment concentration differs from engineers, geologists, geomorphologists and researchers in different parts of the world. Generally such flows have been classified (Bradley and McCutcheon, 1985) according to: (1) triggering mechanism, (2) sediment composition and (3) rheological and kinematic behaviors.

Classification by flow triggering mechanism is preferred by geologists and includes grouping for lahars, till flows, semiarid mountain mud flows and alpine mud flows. This classification has a descriptive value in spite of some overlaps and lacks of coverage, most notably for flows generated on lower gradient slopes.

Classification by sediment content has advantage in quantitative description over schemes which are based on such qualitative methodology. The most common classification schemes based on sediment content are given in TABLE I and some disagreements among researchers are illustrated. One of the disagreements is the division between hyperconcentrated flows and mud or debris flows. The disagreement is mainly caused by the various composition of sediment samples used for study. Since the transition from a mud flow to a hyperconcentrated flow may be defined as the point at which there is a change from a uniform to a nonuniform concentration profile sediment distribution, the composition of sediment suspension will affect the determination of the transition. With the same sediment concentration, flows with larger content of fine sediments tend to have more uniform concentration profiles. On the other hand, rheology of sediment laden flows is affected considerably by the content of fine sediment. With less fine sediment flows may be classified as Newtonian fluid even if the total sediment concentration is extremely high. According to Fei's experiments, the critical concentration in volume at which a sediment-water mixture turns to a non-Newtonian fluid closely related to the content of fine particles, as shown in Fig. (1). Therefore, only sediment concentration is not sufficient to separate different flow patterns. Most hyperconcentrated flows in Japan are debris flows or begin as debris flows, while most flows in China tend to be fine-grained hyperconcentrated flows that grade into mud flows.

Pierson and Costa (1984) classified flows according to concentration and kinematic behaviors to avoid problems of using concentration as a single variable. The division between stream flow and slurry is assumed to be the transition from Newtonian to non-Newtonian behaviors. The break between slurry flow and granular flow is primarily a function of particle size and gradation. As of yet neither class can be closely associated with specific concentration. Sediment size distribution, shape, cohesion or composition all seem to be important factors in such classification schemes.

Qian and Wang (1984) made a distinction between two-phase flows (flow carrying the bed load and/or suspended load) and pseudo-one-phase flow (flow in which the neutral buoyant particles and water mix together to form a homogeneous fluid and move in its entirety). When the content of fine particles in the suspension is low, the flow belongs to two-phase flow. With increasing concentration of fine sediment particles and strengthening of the flocculated structure, more and more coarse particles are held or supported by the flocculated structure and turn into neutral buoyant load. A two-phase flow will be transferred to a pseudo-one-phase flow. When discrete coarse particles turn into neutral buoyant load, this critical condition can be deduced by balancing the submerged weight with the drag force,

TABLE I. CLASSIFICATION OF FLOW WITH HIGH CONCENTRATION  
(After Bradley and McCutcheon, 1985)

Sources	Sediment concentration in volume (S.G.=2.65)									
	10	20	30	40	50	60	70	80	90	100
Beverage, and Culbertson (1966)	high	extremely high	hyperconcentrated flow			mud flow				
Costa (1984)	water flood		hyperconcentrated flow		debris flow					
NRC from O'Brien and Julien (1984)	water flood		mud flood	mud flow	landslides					
Takahashi (1981)			debris grain flow				fall, landslides, creep sturzstorm, pyroclastic flow			
Chinese investigators (Fan, Dou, 1980)										
Pierson and Costa (1984)	<u>stream flow</u> hyperconcentrated		<u>slurry flow</u> (debris current) debris and mud flows solifunction				<u>granular flow</u> sturz-storm, debris avalanche, earth flow soil creep			

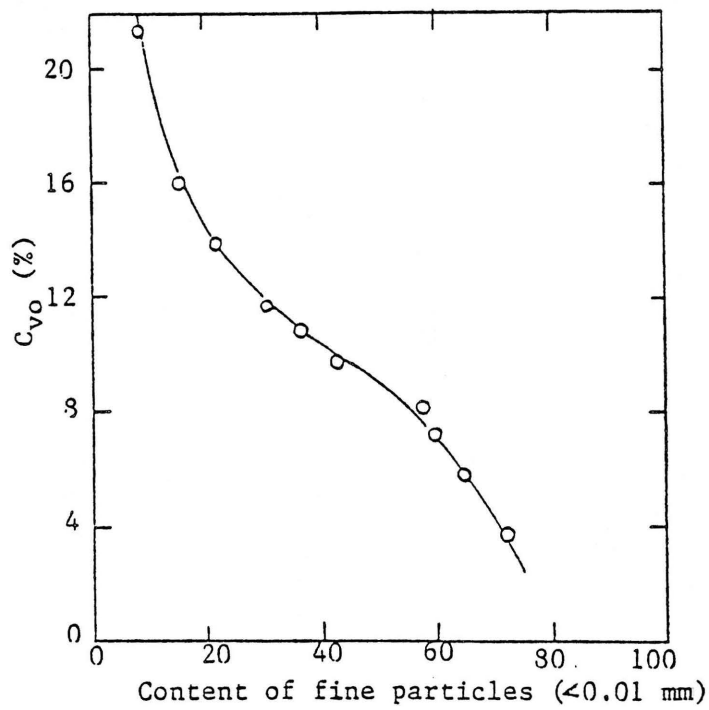


Fig.1. The relationship between the content of fine particles and the critical concentration turning to non-Newtonian fluid (after Fei)

$$D_{\max} = K \frac{\tau_y}{\gamma_s - \gamma} \quad (1)$$

where  $K$  is a constant,  $\tau_y$  is the Bingham plastic yield shear stress,  $\gamma_s$ ,  $\gamma$  are specific weights of particles and carrier fluid, respectively. The flow behaves as a pseudo-one-phase flow if the coarsest particles are finer than  $D_{\max}$  as given by Eq. (1).

TABLE II represents a summary from the above analysis. The classification of sediment-laden flows is given in detail. In the table,  $C_{vf}$  is the volumetric concentration of fine particles.

## 2.2 Different Types of Grain Movement

Grain movement in flows with high sediment concentration follows somewhat different laws from that of clear water flows. In general, moving particles can be classified into: (1) contact load, (2) suspended load and (3) neutral buoyant load.

Contact load is described as the transport of bed material in contact with the bed surface either as rolling, sliding or jumping. It's submerged weight is counterbalanced by the impact caused by mutual collisions with bed particles. When the flow intensity (or flow velocity) is low, contact load may have frequent contact with the bed surface. When the flow intensity increases, the movement of the grains will prevail in the layers below the surface, provided that the drag is large enough to overcome the frictional resistance of the surface layer. The particles moving or rolling in terms of a thin layer are thus called laminated load. In an ordinary sediment-laden flow, laminated load usually does not exist. Even if it does exist, it is relatively unimportant unless the slope of the river channel is larger than about 1 percent.

Suspended load relates to sediment particles held in suspension by the interaction of vortices in turbulent flow with the particles. In this case, potential energy from the flow is required to maintain the movement of suspended particles originated from turbulent energy.

Neutral buoyant load exists in non-Newtonian flow because of the yield stress. The settling velocity of fine grains is zero and no relative movement will occur unless the shear stress of the flow is larger than the yield stress. Consequently, particles with a certain submerged weight can be kept stationary without segregation with water owing to the existence of the yield stress. Qian and Wan (1986) gave a criterion to distinguish the suspended load and the neutral buoyant load according to the value of  $D_{\max}$  in Eq. (1).

## 2.3 Patterns of Motion of Hyperconcentrated Flows

As shown in TABLE II, the motion of flow with high sediment concentration can be classified into two major types of flows: two-phase flow and one-phase flow. Generally, in two phase flows there exist bedload and suspended load while in pseudo-one-phase flow, bedload and neutrally buoyant load are present. As sediment concentration of fine particles increases, more and more particles will turn into neutral buoyant load. Finally, grain movement will approach one of these extreme cases (Wang and Qian, 1985a):

- (1) all the particles turn into neutral buoyant load;
- (2) turbulence dies out, suspended load no longer exists and all the particles move as laminated load; or
- (3) turbulence dies out, suspended load no longer exists, coarse

TABLE II. CLASSIFICATION OF SEDIMENT-LADEN FLOW

Phases	one-phase	Two phase flow			pseudo-one phase flow
Concentration in volume	zero	increasing →			increasing →
Rheology	Newtonian Clear water flow	Newtonian	Non-Newtonian		Non-Newtonian flow
Percent of fine sediment	Zero	Relative low. Particles settle selectively. $C_{vf}$ is less than 10%			$C_{vf}$ is relatively high to as high as 20-30%. Or cohesive material is dominant.
Percent of Neutral buoyant Sediment	Zero	less than 70% in general increasing →			Larger than 80% in general
Characteristics		No yield stress, content of fine material is low or zero. Concentration is not high.	No fine sediment exists. The concentration may be high. Can be dilatant fluid.	Fine particles play an important role in determine the behaviors of the flow. Can be Bingham plastic, pseudoplastic or dilatant model	Will not flow or move very slowly.
flow patterns		Hyperconcentrated		Debris flow	Landslides

particles move as laminated load with fine particles and water forming homogeneous liquid phase.

The pattern of motion of flow with high sediment concentration is always controlled by the following factors: (1) grain constituent and concentration; (2) flow density; and (3) flow intensity. These factors have been explained by Qian and Wan (1986). It is obvious that at low flow intensity, laminated load may not exist, whereas at high intensity laminated load turns out to be important.

None of the classification schemes found in the literature are appropriate to the analysis of hyperconcentrated flows. It seems imperative to understand the rheology of hyperconcentrated flows in order to classify flows based on physical processes rather than arbitrary criteria.

### III. RHEOLOGICAL PROPERTIES OF HYPERCONCENTRATED FLOWS

#### 3.1 Rheological Properties

Newtonian fluids are described by a linear relationship between shear stress  $\tau$  and rate of deformation  $du/dy$ . The Newtonian fluid model is applicable to laminar flow. In turbulent flows the relationship between shear stress  $\tau$  and rate of deformation  $du/dy$  is nonlinear:

$$\tau = u \frac{du}{dy} + \rho \lambda^2 \left( \frac{du}{dy} \right)^2 \quad (2)$$

where  $\lambda$  is Prandtl's mixing length and  $\rho$  is the mass density of the fluid. When the mixing length is small, the second term on the right hand side of Eq. (2) is negligible and the flow is laminar. However when the mixing length and the velocity gradient are large, the second term on the right hand side of Eq. (2) is dominant and the flow is turbulent.

When the sediment concentration in the flow exceeds a certain value, neither the Newtonian model nor the model described by Eq. (2) is valid. In hyperconcentrated flow, the shear stress is determined by the following factors: (1) effects of cohesion between particles (including chemical effects); (2) internal friction between particles; 3) dispersive pressure by collision between particles; and (4) turbulence. To consider these factors a number of models have been proposed.

The Bingham plastic model shown below incorporated a yield stress to describe non-Newtonian laminar motion:

$$\tau = \tau_y + u_p \frac{du}{dy} \quad (3)$$

where  $\tau$  = shear stress,  $u_p$  = plastic viscosity or rigidity coefficient and  $du/dy$  = the velocity gradient or rate of shear. Many researchers, such as Cao, et al. (1983), Fan and Dou (1980), Hou and Yang (1983), Higgins, et al. (1983), etc., have applied the Bingham plastic model. In most cases these were studies of slurry composed of clay or fine sediments.

It is generally accepted that the Bingham plastic model can be used for hyperconcentrated sediment flow when examined at low rates of shear (O'Brien and Julien (1986), Dai and Wan (1980) as shown in Fig. (2). Two empirical expressions for the calculation of yield stress and viscosity are

$$\begin{aligned} \tau_y &= a e^{bc v} \\ u_p &= c e^{dc v} \end{aligned} \quad (4)$$

where  $a, b, c, d$  are coefficients depending upon physical properties of particles and sediment composition. An evaluation of these coefficients for mud flow deposits is given in TABLE III and TABLE IV provides a summary of values  $\tau_y$  and  $u_p$  for various clay mixtures.

In open channel flow, shear rates for hyperconcentrated flow beyond the sublayer are on the order of 5 to 50  $s^{-1}$ , while in experiments, shear rates can reach as high as 1000  $s^{-1}$ . Fig. (2) shows that flow can not be modelled by Bingham plastic over a large range of shear rate (for instance, from 5 to

TABLE III. Mud flow Sample Viscosity and Yield Stress as a Function of Concentration by Volume (After O'Brien, 1986).

	$\mu_p = c e^{dC_v}$				$\tau_y = a e^{bC_v}$			
	Points	c	d	r <sup>2</sup>	Points	a	b	r <sup>2</sup>
1. Aspen Pit 1	10	3.60x10 <sup>-2</sup>	22.1	.99	10	1.81x10 <sup>-1</sup>	25.7	.92
2. Aspen Pit 4	10	5.38x10 <sup>-2</sup>	14.5	.95	10	2.72x10 <sup>-0</sup>	10.4	.93
3. Aspen Natural								
Soil	9	1.36x10 <sup>-3</sup>	28.4	.96	10	1.52x10 <sup>-1</sup>	18.7	.83
4. Aspen Mine Fill	9	1.28x10 <sup>-1</sup>	12.0	.87	9	4.73x10 <sup>-2</sup>	21.1	.91
5. Aspen Natural								
Soil Source	16	4.95x10 <sup>-4</sup>	27.1	.83	14	3.83x10 <sup>-2</sup>	19.6	.92
6. Aspen Mine								
Fill Source	10	2.01x10 <sup>-4</sup>	33.1	.85	10	2.91x10 <sup>-2</sup>	14.3	.84
7. Glenwood #1	14	2.83x10 <sup>-3</sup>	23.0	.93	14	3.45x10 <sup>-2</sup>	20.1	.96
8. Glenwood #2	8	6.48x10 <sup>-1</sup>	6.2	.94	8	7.65x10 <sup>-2</sup>	16.9	.97
9. Glenwood #3	10	6.32x10 <sup>-3</sup>	19.9	.95	8	7.07x10 <sup>-4</sup>	29.8	.91
10. Glenwood #4	9	6.02x10 <sup>-4</sup>	33.1	.96	9	1.72x10 <sup>-3</sup>	29.5	.75
11. Iida (1938)	8	3.73x10 <sup>-3</sup>	36.6	.99				

Notice: "r" in this table represents the correlation coefficient.



TABLE IV. Fluid Properties of Clay-Water Mixtures (after O'Brien, 1986).

Investigator	Clay Type	$C_v$	Range of Shear Rate $s^{-1}$	$\mu_p$ Poises	$\tau_y$ Dynes/cm <sup>2</sup>
Mills, 1983	Kaolin	.162	5 - 1021	.289	19
		.188	5 - 1021	.430	55
		.203	5 - 1021	.591	147
Valentik and Whitmore, 1965	China	.091	170 - 930	.040	78
		.112	170 - 930	.054	166
		.126	170 - 930	.067	250
		.138	170 - 930	.081	330
		.155	170 - 930	.095	435
Wan, 1982	Kaolin	.074	8 - 380	.093	53.2
		.087	8 - 930	.068	27.4
		.105	8 - 380	.051	14.2
		.126	8 - 380	.047	7.4
		.149	8 - 380	.039	4.7
	Bentonite	.0106	8 - 380	.042	6.3
		.0134	8 - 380	.054	10.5
		.0159	8 - 380	.054	20.0
		.0185	8 - 380	.049	31.0
		.0217	8 - 380	.062	~45
Pazwash and Robertson, 1971	Kaolin	.10 pH 10.1	0 - 59	.044	0.0
		.10 pH 6.0	0 - 69	.061	3.0
		.20 pH 8.0	0 - 67	.089	7.3
		.20 pH 7.0	0 - 80	.122	18.7
		.20 pH 5.0	0 - 61	.378	30.0
du Plessis and Ansley, 1967	Unknown	.08	6 - 1060	.027	9.6
		.115	6 - 1080	.039	16.8
		.145	6 - 1150	.053	33.5
Thomas, 1963	Kaolin	.25	40 - 1733	.890	400

1000 s<sup>-1</sup>). But it also shows that Bingham model is applicable for flow which has a shear rate less than about 20 s<sup>-1</sup>. Therefore in open channel, flow can be modelled approximately by Bingham plastic.

Viscosity changes rapidly as sediment concentration increases, especially in non-Newtonian fluid when concentration exceeds a certain value. Investigators have attempted to relate viscosity to sediment concentration and sediment composition, but most of the methods are dependent upon the rheological model chosen to describe the flow properties. Since these methods still require further experimental verification, estimates of viscosity are somewhat difficult to confirm.

Einstein, A. (1905, 1906, 1911) theoretically derived the dynamic viscosity  $\mu_p$  of dilute suspension of solid spherical particles;

$$\mu_p = \mu(1 + 2.5C_v) \quad (5)$$

where  $\mu_p$  is the dynamic viscosity of the suspension and  $\mu$  is the viscosity of the fluid. This equation has been shown to be valid for concentration less than 1 to 2 percent.

Thomas (1963) first proposed the following empirical equation:

$$\mu_p = \mu \left[ 1 + 2.5C_v + 10.05C_v^2 + 0.00273 e^{16.6C_v} \right] \quad (6)$$

which he later modified to

$$\mu_p = \mu \left[ 1 + 2.5C_v + 10.05C_v^2 + 0.062 \exp\left(\frac{1.875C_v}{1-1.595C_v}\right) \right] \quad (7)$$

and indicated that it fitted the available data reasonably well over the entire range of concentration.

The most comprehensive theoretical analysis of highly concentrated suspension is that of Frankel and Acrivos (1967) who showed that for concentration exceeding about 0.80 times the maximum possible concentration the viscosity depends upon the ratio between the concentration and the maximum possible concentration  $C_v^*$  rather than only the concentration itself. This is a reflection on the effects of maximum packing density which could be partially significant with mixed size particles. The Frankel-Acrivos equation is:

$$\mu_p = 1.125 \mu \left[ \frac{(C_v/C_v^*)^{1/3}}{1 - (C_v/C_v^*)^{1/3}} \right] \quad (8)$$

where  $C_v^*$  is the maximum possible static concentration in volume.

It was demonstrated by Landel, Moser and Bauman (1963) on water suspension of glass beads, aluminum powder, copper powder and ammonium perchlorate with mean particle diameters ranging from less than 10 to more than 100 microns (1

micron = 10<sup>-6</sup>m ) that  $\frac{\mu_p}{\mu}$  can be expressed as:

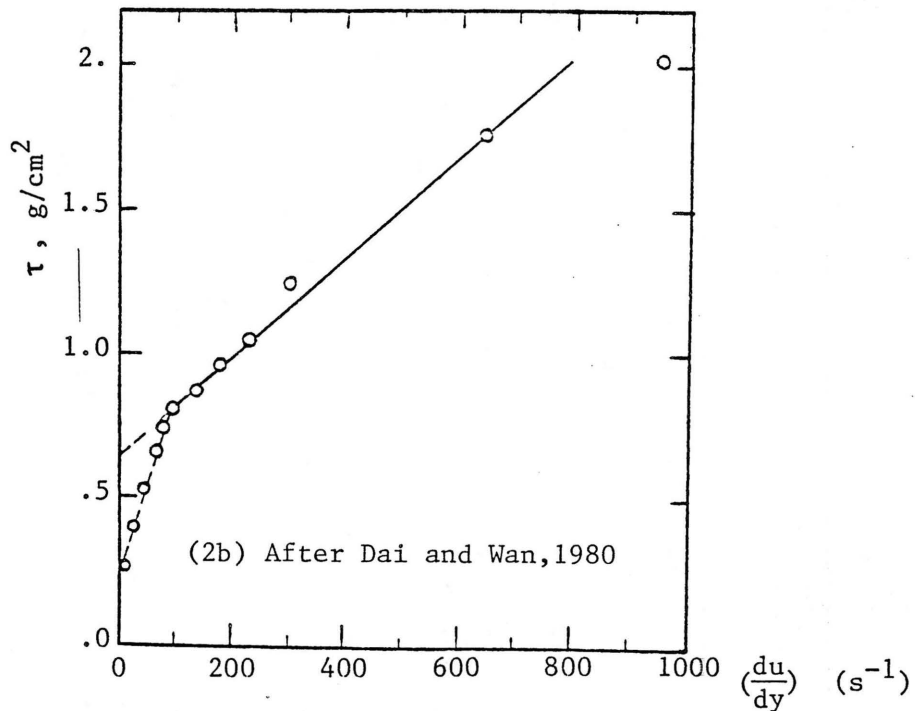
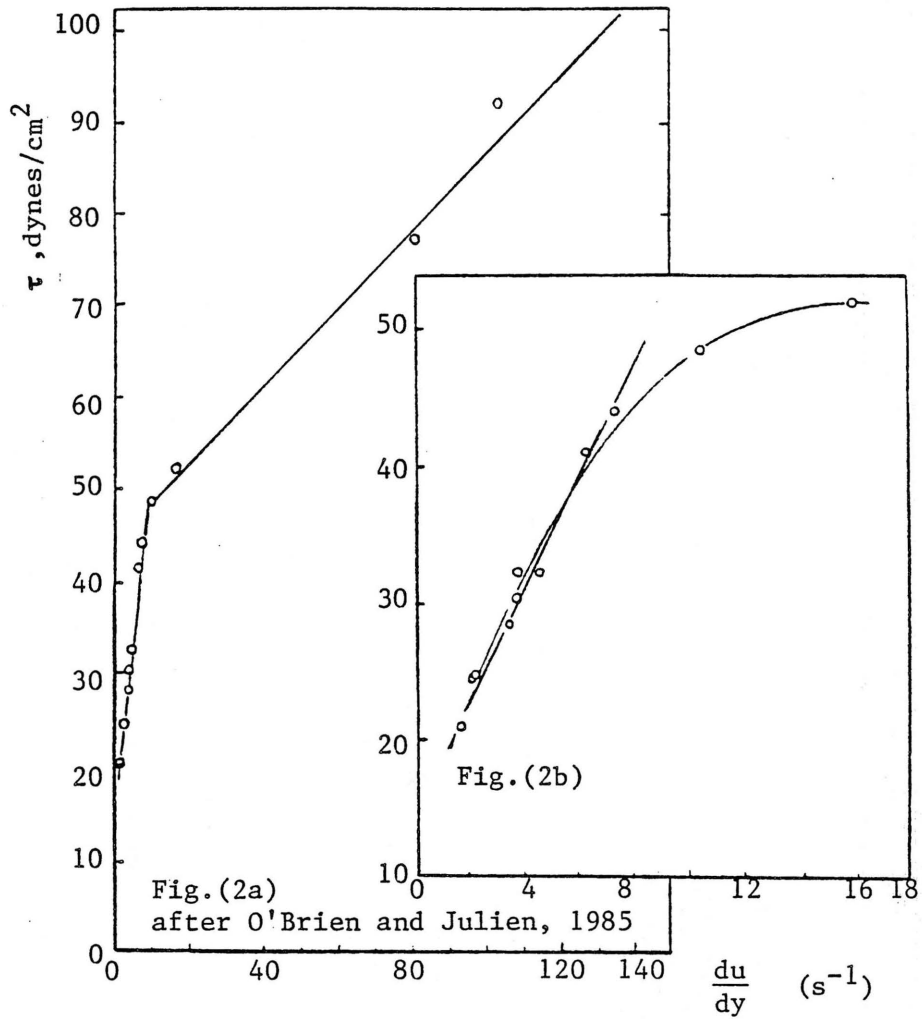


Fig. (2). Rheological Properties of Hyperconcentrated Flows

$$\frac{\mu_p}{\mu} = \left(1 - \frac{C_v}{C_v^*}\right)^{-2.5} \quad (9)$$

This formula is widely accepted now.

Recently Fei (1983) considered that both the particles and the bonded water layer, which is strongly attached to the surface of fine grains with a thickness  $\bar{\delta}$ , contribute to the viscosity of the suspension. In the case that the flocculated structure is developed by fine particles, some of the water is entrapped within the structure, which contributes also to the viscosity. The entrapped water cannot move freely and its amount varies with the concentration of suspension. By taking all these into consideration the following equation is deduced:

$$\mu_r = \frac{\mu_p}{\mu} = \left[ 1 - \left(1 + 6\alpha\bar{\delta} \sum \frac{\Delta P_i}{D_i}\right) C_v \right]^{-2.5} \quad (10)$$

where  $\alpha$  is a coefficient and  $\Delta P_i$  is the percentage of particles with diameter  $D_i$ ,  $\bar{\delta}$  is the thickness of bonded water layer. Taking  $C_v^* = (1 + 6\alpha\bar{\delta} \sum \frac{\Delta P_i}{D_i})^{-1}$  as the maximum concentration, Eq. (10) becomes

$$\mu_r = \left(1 - \frac{C_v}{C_v^*}\right)^{-2.5} \quad (11)$$

which is exactly the same as that obtained by Landel, et al. When there are no fine particles in suspension, the relative viscosity is much smaller than that given by Eq. (11) and follows

$$\mu_r = \left(1 - \frac{C_v}{C_v^*}\right)^{-2} \quad (12)$$

Since Eq. (11) and (12) consider the whole size distribution of particles in the suspension, the experimental data for different sediment and coal slurries can be unified. As long as the sediment concentration is not so high as to reach neutral buoyant load, Eq. (11) and (12) are recommended in usage. Also Fei observed the exponential relationships between yield stress and volumetric concentration  $C_v$  as almost the same as that given by Eq. (4).

. A pseudo-plastic model or power law model, and the Eyring model (Thomas, 1963) have been used to describe slurries of fine sediments, but they do not seem to show enough improvement over the Bingham plastic model to justify the added complexity.

. A dilatant model is used by Takahashi (1981) based on Bagnold's finding that the shear stress is proportional to the rate of deformation squared. This model is

$$\tau = (\alpha\sigma_s) \left[ \left(\frac{C_v}{C_v^*}\right)^{1/3} - 1 \right]^{-2} D^2 \sin\alpha \left(\frac{du}{dy}\right)^2 \quad (13)$$

where  $\tau$  = shear stress,  $a$  = empirical constant,  $\rho_s$  = mass density of grains,  $C_v^*$  = grain concentration by volume in the bed sediments,  $C_v$  = volumetric concentration of suspended sediments,  $D$  = grain diameter, the coefficient "a" observed by Bagnold and Takahashi remains so variable that the model described by Eq. (13) may not be adequate.

. Quadratic model. O'Brien and Julien (1984) proposed a quadratic model to describe the relationship between the shear stress and the rate of deformation,

$$\tau = \tau_y + \mu_p \frac{du}{dy} + C_1 \left(\frac{du}{dy}\right)^2 \quad (14)$$

On the right hand side of the equation the first term describes the yield stress due to cohesion between particles, the second term describes the viscous stress due to the fluid and friction between particles and the third term includes in the variable  $C_1$  both the effects of dispersive stress due to inertial particle collisions and turbulent stresses. Eq. (14) is very similar to Eq. (2) which is suitable to clear water flow, except that the yield stress term has been added. The coefficient  $C_1$  in the equation is given by

$$C_1 = \rho_m \lambda_m^2 + a \rho_s \left[ \left(\frac{C_v^*}{C_v}\right)^{1/3} - 1 \right]^{-2} D_{av}^2 \quad (15)$$

according to Bagnold and the conventional expression for the turbulent stress in flow with concentration  $C_v$ . In Eq. (15)  $\rho_m$ ,  $\lambda_m$  are the density and mixing length of the mixture whereas "a" is constant and  $D_{av}$  is the average diameter of suspended particles in the mixture.

The sediment particles can reduce the turbulent intensities in a suspension. According to Bagnold smaller eddies are suppressed more easily than large ones when grain flow is at high concentration. As the sediment concentration increases more and more eddies are damped. Finally the additional shear stress due to residual turbulence must be negligible at high grain concentration (Bagnold, 1954). On the other hand, the dispersive shear stress will increase with the increase of particle concentration. Consequently, the total shear stress caused by turbulence and dispersive shear is dependent on the relative and opposite changes in turbulent shear stress and dispersive stress. Generally, this stress will increase with the increase of sediment concentration.

Bagnold's data have been used to verify Eq. (14). Bagnold's data are replotted in Fig. (3) showing the relationship between  $\tau$  and  $\left(\frac{du}{dy}\right)^2$ . In logarithmic coordinates we can see that  $\tau$  and  $\left(\frac{du}{dy}\right)^2$  become linear when  $du/dy$  is large. This implies that the shear stress is strongly determined by turbulent shear stress or dispersive stress when the rate of deformation is high. In this analysis least-square binomial regression is used and the results are listed in TABLE V and shown in Fig. (4). From Fig. (4) we can find that Bagnold's data fit the rheological model described by Eq. (14) reasonably well although no data is available for the rate of shear less than  $10 \text{ s}^{-1}$ . Fig. (5) shows that as the sediment concentration increases both coefficients  $C_1$  and  $\mu_p$  increase exponentially with  $C_v^m$ , expressed as

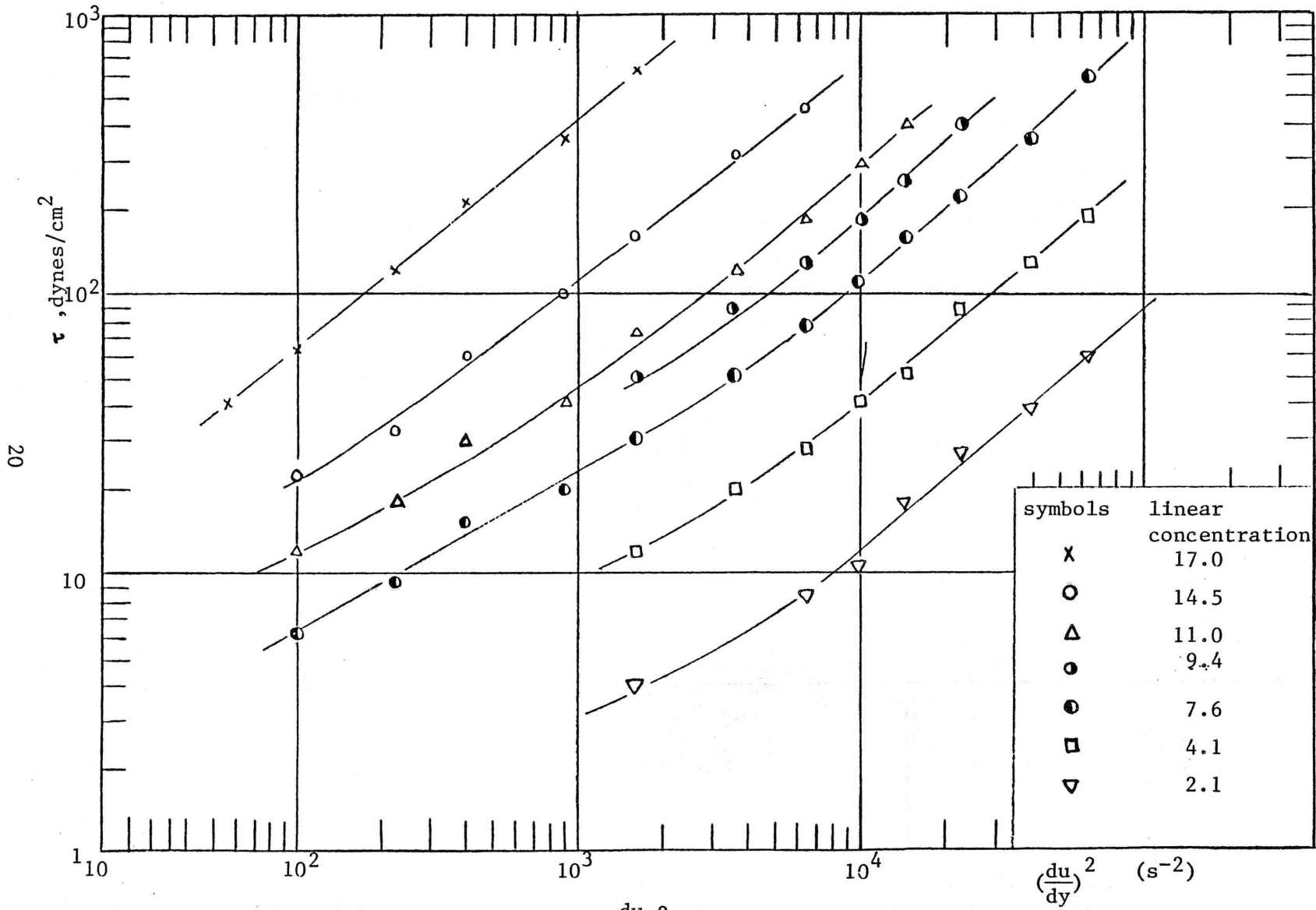


Fig.(3) Relation Between  $\tau$  and  $\left(\frac{du}{dy}\right)^2$  (After Bagnold's data, 1954)

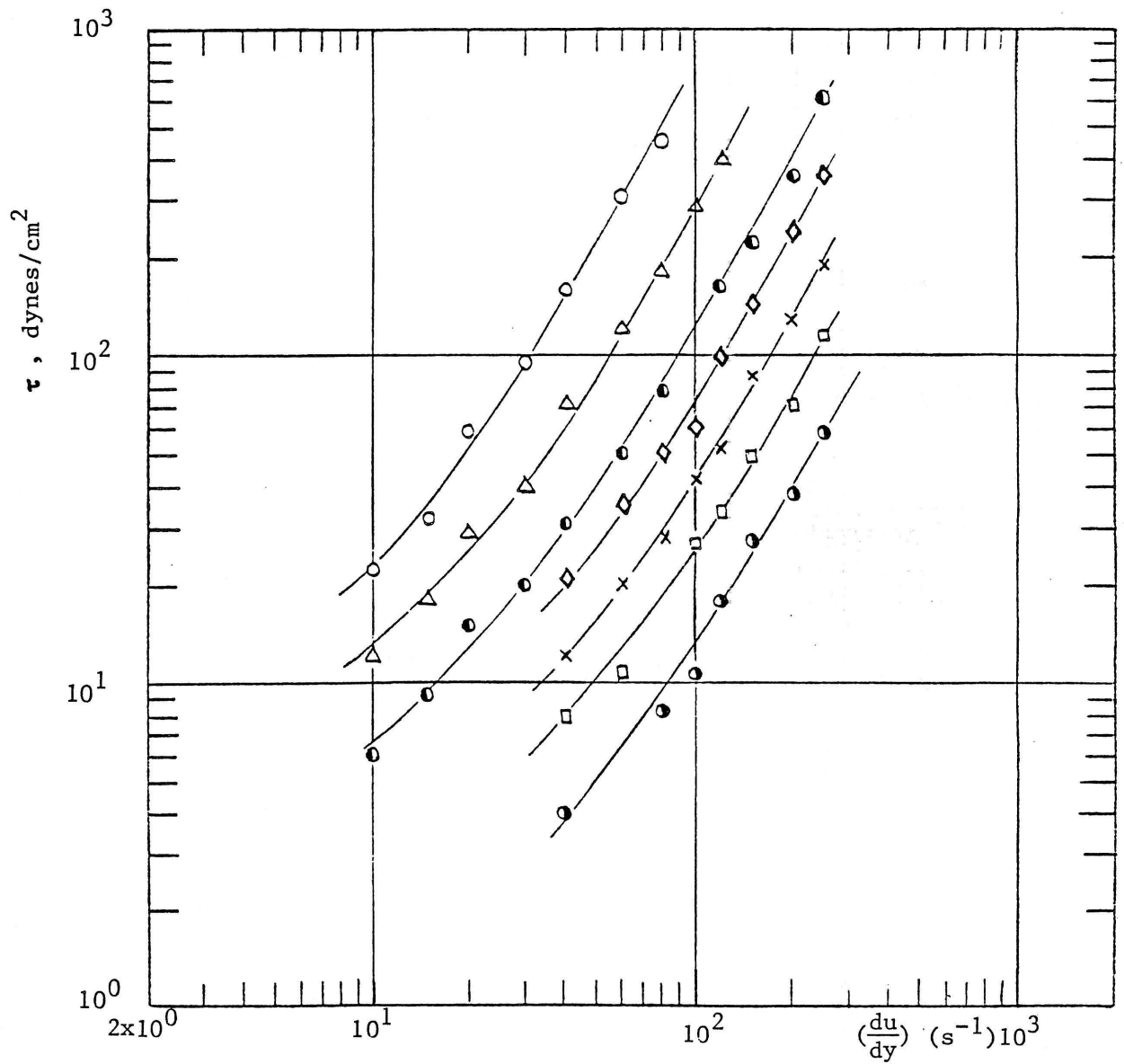


Fig.(4). Comparison Between Bagnold's Data and the Quadratic Model

○	=14.5;	×	=4.1
△	=11.0;	□	=3.1
●	= 7.6;	●	=2.1
◇	= 5.7;		

TABLE V. Coefficients in Eq.(14) (Bagnold's data, 1954)

Equation	$\tau = \tau_y + \mu_p \frac{du}{dy} + C_1 \left(\frac{du}{dy}\right)^2$						
Linear concentration $\lambda_{**}$	14.5	11	7.6	5.7	4.1	3.1	2.1
$C_v$ (%)	60.6	57.0	51.07	45.6	38.5	32.0	23.0
$\tau_y^*$	8.15	6.72	3.0	4.2	2.92	2.18	0.0
$\mu_p^*$	0.75	0.485	0.30	0.185	0.126	0.083	0.067
$C_1^{!*$	0.0342	.02236	.0088	.0048	.0025	.001435	.00064

\* Unit of  $\tau_y = \text{dynes/cm}^2$ , unit of  $\mu_p = \text{poises}$ ,

Unit of  $C_1 = \text{g/cm} = \text{s}^2 \cdot \text{dynes/cm}^2$

$$** \lambda = \left[ \left( \frac{C_v^*}{C_v} \right)^{1/3} - 1 \right]^{-1}$$

TABLE VI. Coefficients in Eq.(14) (Govier's data, 1957)

Volumetric concentration %	39.7	34.14	30.3	24.92	21.80	16.8
$\tau_y$	78.4	20.7	9.84	5.0	3.2	2.61
$\mu_p$	.351	.29	.137	.093	.067	.0315
$C_1$	$3.15 \times 10^{-3}$	$2.40 \times 10^{-4}$	$1.28 \times 10^{-4}$	$3.10 \times 10^{-5}$	$3.80 \times 10^{-5}$	$6.34 \times 10^{-5}$

\* Unit of  $\tau_y = \text{dynes/cm}^2$

unit of  $\mu_p = \text{poises}$

unit of  $C_1 = \text{s}^2 \cdot \text{dynes/cm}^2 = \text{g/cm}$



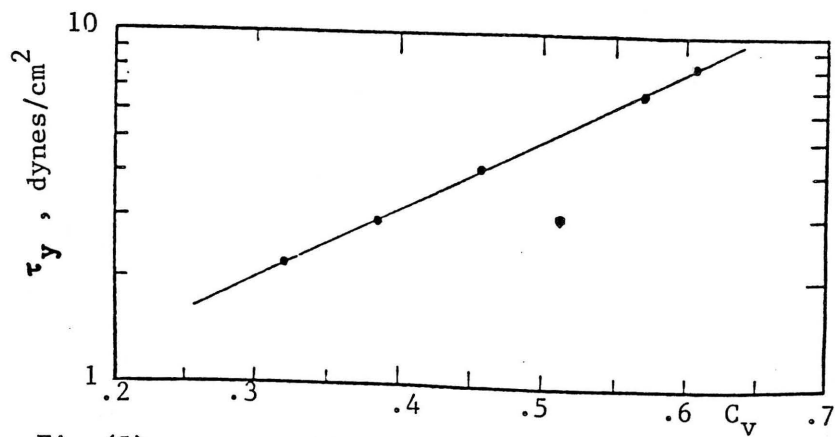
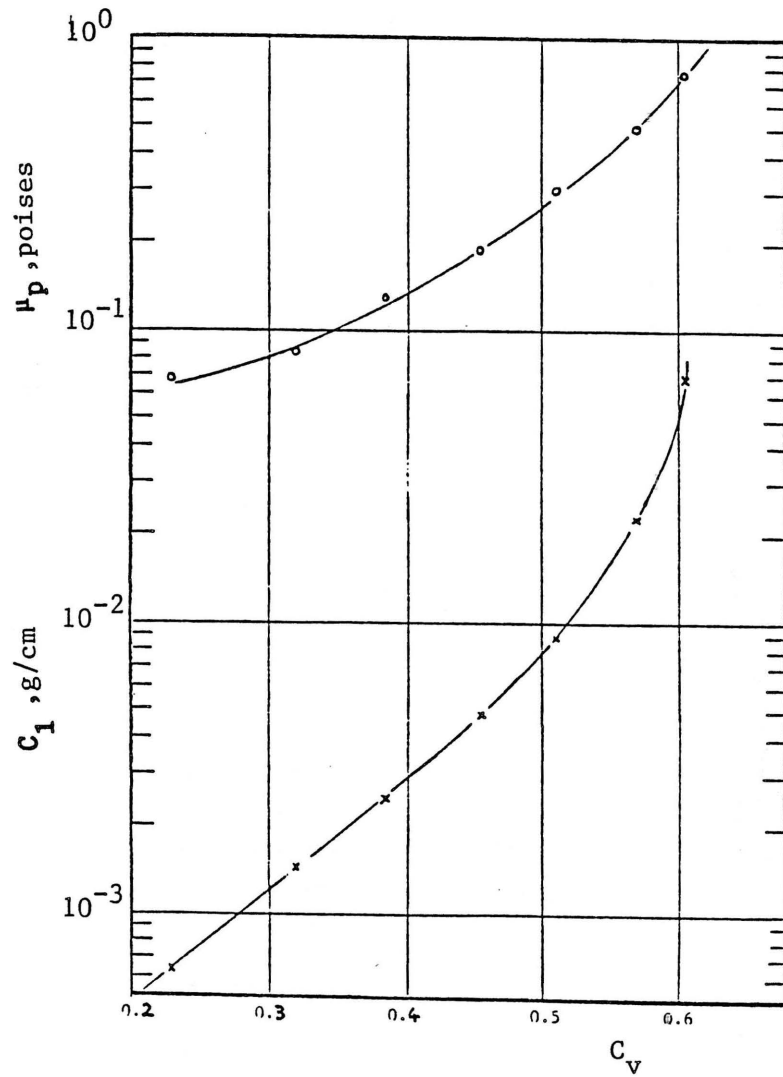


Fig. (5). Relations between  $C_V$  and  $C_1$ ,  $C_V$  and  $\mu_p$ , and  $C_V$  and  $\tau_y$  (Bagnold's data).

$$\begin{aligned}\tau_y &= \tau_{y_0} e^{K_1 C_v^{m_1}} \\ u_p &= u_{p_0} e^{K_2 C_v^{m_2}} \\ C_1 &= C_{1_0} e^{K_3 C_v^{m_3}}\end{aligned}\tag{16}$$

where  $\tau_{y_0}$ ,  $u_{p_0}$ ,  $C_{1_0}$ ,  $K_1$ ,  $K_2$ ,  $K_3$ ,  $m_1$ ,  $m_2$ , and  $m_3$  are parameters (or constants) to be determined. They are functions of density, size and composition of particles. A summary of the results from the flow samples in Bagnold's and Govier, et al.'s experiments is given in TABLE VII. Comparing these data with those in TABLE III it can be found that the only difference is that  $m_1$ ,  $m_2$  and  $m_3$  in Eq. (16) are not necessarily unity.

As seen from TABLE V, although the yield shear stress  $\tau_y$  has the tendency to increase with increasing sediment concentration, it is very small compared with the grain shear stress or the stress caused by particle collision or turbulence when the shear rate exceeds a certain value, for instance  $20 \text{ s}^{-1}$ .

This phenomenon is also observed by Savage and McKeown (1983). A small value of  $\tau_y$  in a fluid with gravity free dispersion of large solid spheres confirms that the content of fine particles, the density and the size of particles are very important variables in rheology of hyperconcentration. With neutrally buoyant large grains in the fluid, the yield shear stress is very small even if the solid concentration is high.

Fig. (6) shows the experimental data with galena obtained by Govier, et al (1957). The volumetric concentrations are between 17% and 40%. The curves in the figure are obtained by fitting Eq. (14) using least-square regression analysis. The analytical results of coefficients  $\tau_y$ ,  $u_p$  and  $C_1$  are given in TABLE VI. Again these coefficients are functions of particle concentration, following relationships in Eq. (16).

. Yield-pseudo-plastic model. It was first proposed by Herschel and Bulkley (1926) and expressed as

$$\tau = \tau_y + u_c \left(\frac{du}{dy}\right)^n\tag{17}$$

where  $\tau_y$ ,  $u_c$  and  $n$  are the characterizing coefficients. Chen (1983) reported that Eq. (17) is a generalized model that can cover the spectrum of Newtonian, Bingham, pseudo-plastic, dilatant and power law models depending on how the yield stress  $\tau_y$ , the consistency index  $u_c$  and flow behavior index  $n$  are chosen. This model is empirically acceptable, however, the difficulties in predicting the values of  $u_c$  and  $n$  which vary with the rate of shear make it impractical.

TABLE VII. Sample Viscosity Index, Quadratic Index, and Yield Stress as a Function of Concentration

1.	$\tau_y = \tau_{y_0} e^{K_1 C_v^{m_1}}$			
Sources	Points	$\tau_{y_0}$	$K_1$	$m_1$
Govier (1957)	6	2.0	79.8	3.28
Bagnold(1954)	5	0.5	4.61	1.00
2.	$\mu_p = \mu_{p_0} e^{K_2 C_v^{m_2}}$			
Sources	Points	$\mu_{p_0}$	$K_2$	$m_2$
Govier (1957)	6	$5.41 \times 10^{-3}$	11.11	1.00
Bagnold(1954)	7	$6.0 \times 10^{-2}$	10.76	2.36
3.	$C_1 = C_{1_0} e^{K_3 C_v^{m_3}}$			
Sources	Points	$C_{1_0}$	$K_3$	$m_3$
Govier	* 6	$6.63 \times 10^{-9}$	32.7	1.00
Govier	** 6	$2.32 \times 10^{-4}$	-8.31	1.00
Bagnold	7	$2.35 \times 10^{-4}$	11.65	1.67

\* For case  $C_v > 25\%$  (approximately)

\*\* For case  $C_v < 25\%$

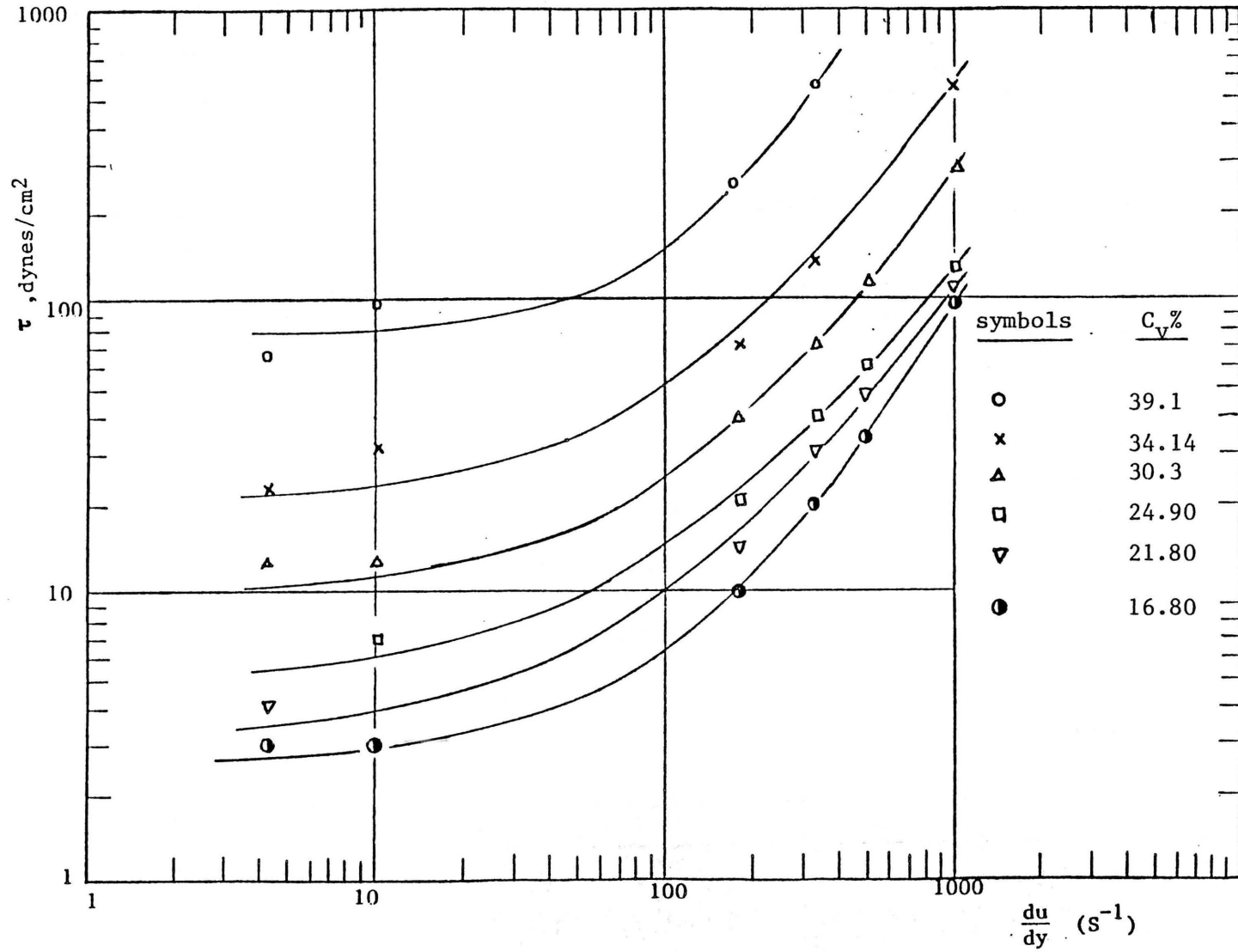


Fig.(6) Verification of Eq.(6) from Govier's data

### 3.2 Settling Velocity of Sediment Particles

#### 3.2.1 The collective settling of particles in hyperconcentrated flow of coarse particles.

The gross settling of uniform coarse grains generally obeys the formula

$$\frac{\omega'}{\omega_0} = (1 - C_V)^m \quad (18)$$

in which  $\omega'$  is the gross settling velocity,  $\omega_0$  the settling velocity of a single particle in an infinite mass of fluid and the exponent  $m$  is a function of the Reynolds number  $Re = \frac{\omega_0 D}{\nu}$ , in which  $D$  is the diameter of sediment particle and  $\nu$  is the kinematic viscosity of clear water. According to Richardson and Zaki's studies (1954),  $m$  approaches a maximum constant value of 4.6 for Reynolds number smaller than 0.4 and approaches a minimum of 2.3 when the Reynolds number is greater than  $10^3 \sim 10^4$ , as shown in Fig. (7). The exponent  $m$  obtained by Wang and Qian (1985a) and by Xie and Wang (1982) are all larger than the values given by Fig. (7) and close to those proposed by the Beijing College of Mining. This implies that the exponent  $m$  is not only a function of grain Reynolds number but also a function of other factors. Yue (1983) contended that  $m$  should decrease with concentration.

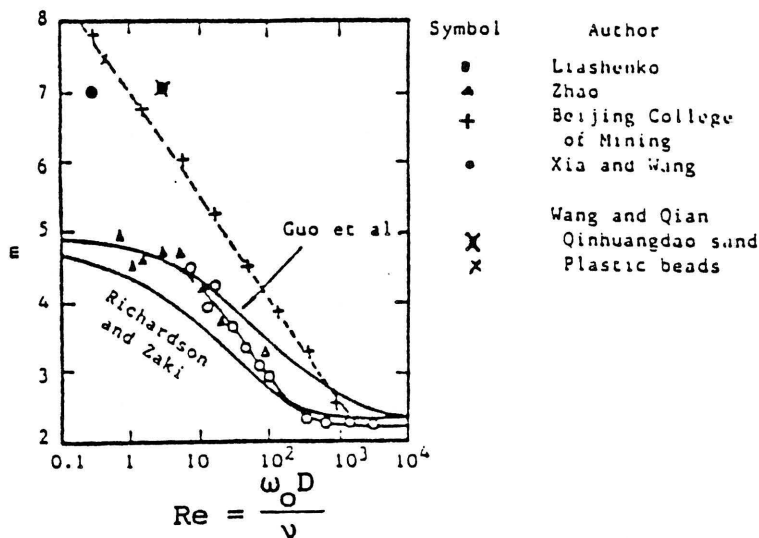


Fig.(7) The relationship between grain Reynolds number and the exponent  $m$  in Eq.(18) (After Qian and Wan, 1986)

### 3.2.2 Settling velocity in a suspension of both fine and coarse particles.

With both fine and coarse particles in suspension the settling properties of particles become much more complicated. Collisions and flocculation might occur after the sediment concentration reaches a certain value and the change of fluid properties also affects the hydrodynamic interaction between particles.

For the simplest case where a single particle falls in a clay suspension, yield stress is of significance in determining the drag forces of the particles. Plessis and Ansley (1967), Ansley and Smith (1967) thus combined the effects from yield stress and viscous stress to come out with the following equation

$$C_D = f(Re_B, He) \quad (19)$$

where  $C_D$  is the drag coefficient,  $Re_B = \rho \omega' D / u_p$  and  $He = \frac{\rho \tau_y D^2}{u_p^2}$  are called Bingham Reynolds number and the Hedstrom number, respectively. More specifically, Plessis and Ansley (1967) defined

$$C_D = f\left(\frac{He + Re_B}{Re_B^2}\right) = f(P) = kP^{0.5} \quad (20)$$

while Ansley and Smith (1967) defined

$$C_D = f(Q) = f\left(\frac{Re_B^2}{Re_B + \frac{7\pi}{24} He}\right) \quad (21)$$

in which  $Q$  was called dynamic parameter,  $P$  the plastic number and  $\omega'$  is the fall velocity in the mixture.

As a matter of fact, Eq. (20) and (21) are essentially the same. It is discussed by Woo (1985) that these two methods could provide results in good agreement with experimental data within a certain range of accuracy. But Woo's analysis also indicated that the scatter of data in the  $C_D \sim P$  (or  $Q$ ) diagram was excessive when compared with data sets from other sources.

The influence of fine particles on the particle settling velocity is significant. It can be described by (Qian and Wan, 1986)

$$\frac{\omega'}{\omega_0} = (1 - C_v)^m (1 - KC_{vf})^{2.5}$$

$$C_{vf} = \frac{rC_v}{1 - (1 - r)C_v} \quad (22)$$

where  $\omega'$  is the particle fall velocity in the mixture,  $C_{vf}$  is the concentration of fine particles in the suspension which has concentration  $C_v$ ,  $r$  is the volumetric ratio of fine particles to all particles in the suspension. Obviously, fine particles in suspension reduce the particle fall velocity.

### 3.2.3 The settling of sand particles in hyperconcentrated flow.

The settling of mixed particles in hyperconcentrated flow can be divided into three stages: (1) hindered settling or restricted settling of discrete particles and discrete flocs. In this stage the initial concentration in the fluid is low and there is no mixing interface occurring when experiments of sedimentation are done; (2) selective settling when the settling of coarse and fine particles is selective; and (3) slow settling of flocculant structure as a whole. Following an increase in concentration the settling of sediment will gradually transform from stage 1 to stage 3. The coarser the sediment constituent, the higher the concentration at which the transition takes place.

Chu (1983) put the effect of bonded water on the particle surface into consideration, gave the gross settling velocity of a swarm of non-flocculated particles for high concentration as

$$\frac{\omega}{\omega_0} = [1 - \epsilon KC_V]^{3.5} \quad (23)$$

where  $K$  is the ratio of the volume of bonded particles to that of unbonded particles and  $\epsilon$  is the coefficient of the pores caused by collision particles ( $\epsilon$  is taken to be 1.4) and  $K$  is expressed as

$$K = 1 + 6 \int_0^1 \left(\frac{\bar{\delta}}{D}\right) dP \quad (24)$$

in which  $\bar{\delta}$  is the thickness of the bonded water on the particle surface,  $\bar{\delta} = 1$  mm according to Woodruff's experiments and  $dP$  is the percentage of the volume of a certain particle diameter to the total particle volume. Then Eq. (23) becomes

$$\frac{\omega}{\omega_0} = \{1 - 1.4 [1 + 6 \int_0^1 \left(\frac{\bar{\delta}}{D}\right) dP] C_V\}^{3.5} \quad (25)$$

or for convenience of application

$$\frac{\omega}{\omega_0} = \{1 - 1.4 [1 + 6 \sum_{i=1}^n \left(\frac{\bar{\delta}}{D}\right) \Delta P_i] C_V\}^{3.5} \quad (26)$$

After comparison with many data obtained by various researchers, Chu claimed that Eq. (26) is not only in good agreement with the experimental data (Fig. 8) but also more reasonable.

From Eq. (26) we can see the influence of fine particles on the settling velocity of the swarm of non-flocculated particles by evaluating the term

$$\sum_{i=1}^n \left(\frac{\bar{\delta}}{D}\right) \Delta P_i. \quad \text{When there are no fine particles in the suspension, the term}$$

$$\sum_{i=1}^n \left(\frac{\bar{\delta}}{D}\right) \Delta P_i \rightarrow 0.$$

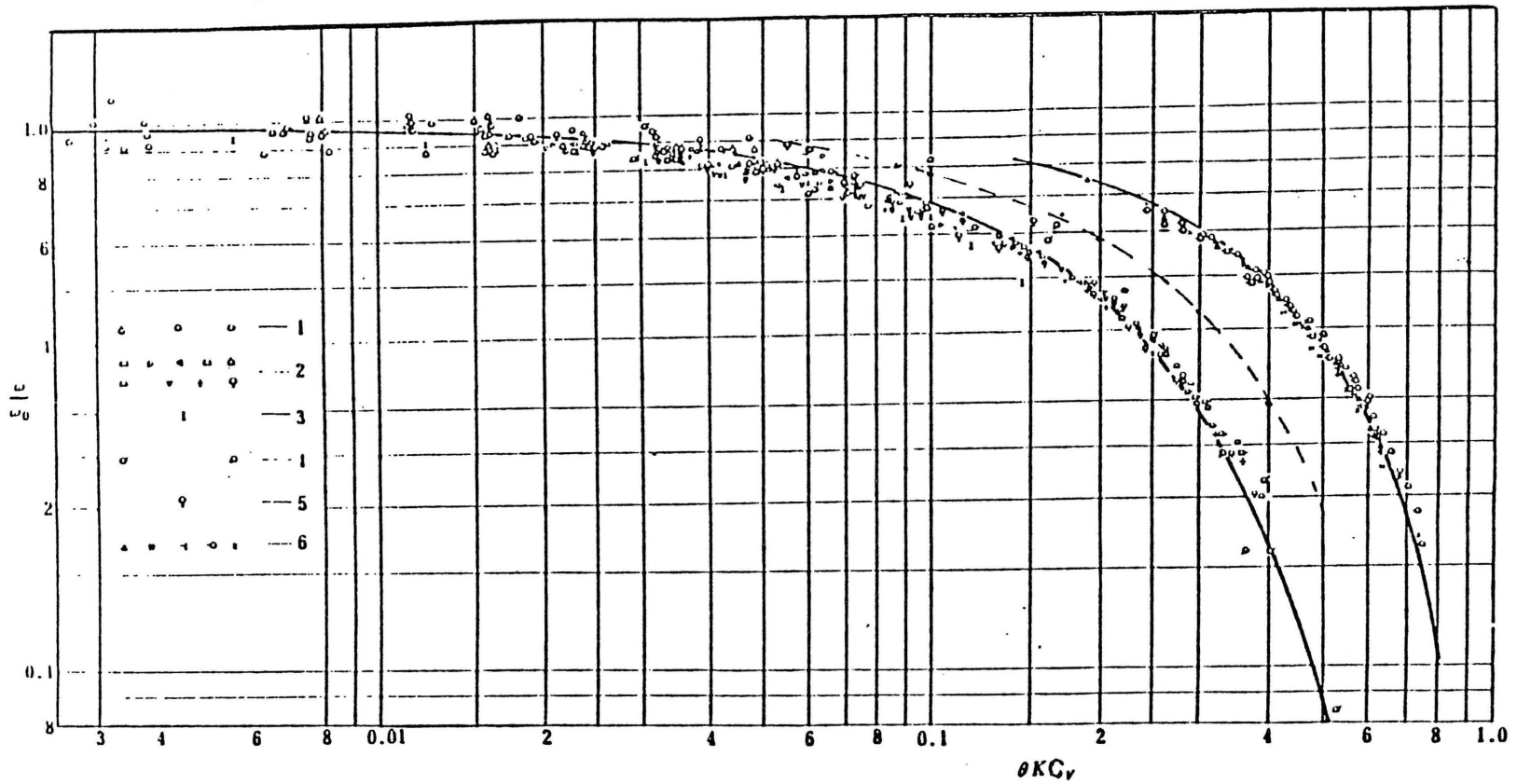


Fig. (8). Comparison between experimental data and Equation (23) for fall velocity in hyper-concentrated flows



#### IV. VELOCITY DISTRIBUTION OF HYPERCONCENTRATED FLOWS

As long as sediment particles exist in the flow the velocity distribution is changed due to the interaction between sediment particles and the fluid. Several factors should be considered: (1) the change of viscosity; (2) the damping effects of particles on turbulent eddies; and (3) two-phase flow.

In general velocity distributions along the vertical depend on the range of sediment concentration. When the sediment concentration is less than 20 percent, many investigators find that the velocity profiles still obey the log law of the wall except for the near bed region where a large sediment concentration gradient exists. When the sediment concentration is high enough to form yield stress, particular phenomena, such as plug flow, have been observed. The effect of sediment concentration on the velocity distribution is determined for different ranges of sediment concentration.

##### 4.1 Velocity Distribution in Flows With Relatively Low Sediment Concentration

Experiments on the effects of concentration on velocity distribution may be traced back to Vanoni, Einstein and Qian. It was observed that the velocity distribution in the main flow zone can be also described by the conventional theory, the law of the wall, as shown in Fig. (9).

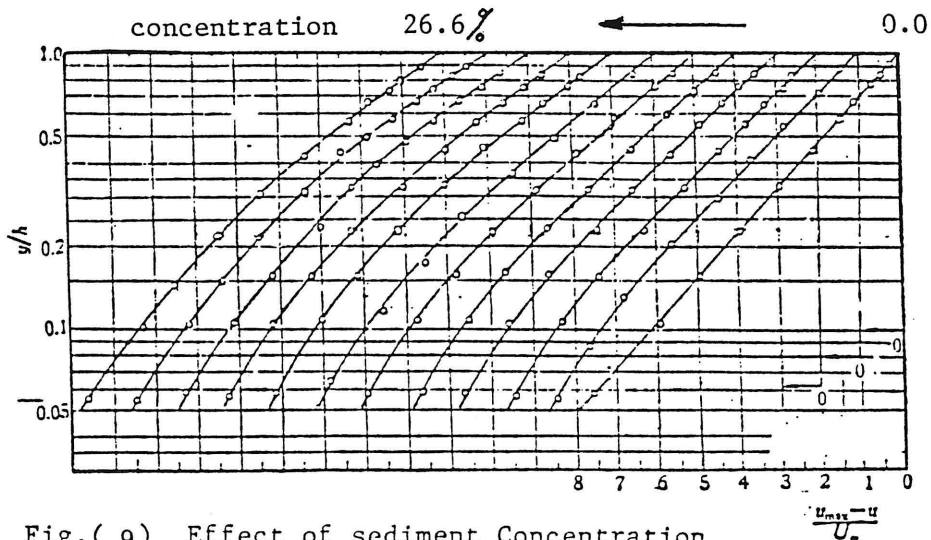


Fig.( 9) Effect of sediment Concentration  
On Velocity Distribution (after Einstein & Qian)

The velocity near the bed deviates sharply from the log law. Experimental data show that the velocity in the boundary layer starts to deviate from the logarithmic distribution at a dimensionless distance from the boundary,  $\frac{U_* y}{\nu}$ , of between 500 to 1000. Many investigators, such as Einstein and Qian, Vanoni, Barton and Lin (1955), considered it to be contributed by the decrease of the von Karman constant. Einstein and Qian (1955) established a

relationship between the von Karman constant and parameter  $E = \frac{\rho_s - \rho}{\rho_s} \frac{\sum \bar{C}_\omega \omega}{VS_e}$ ,

where  $\omega$  is the fall velocity of the particle,  $\bar{C}_\omega$  is the vertical average concentration belonging to those particles which have a fall velocity of  $\omega$ , and  $\rho_s$ ,  $\rho$  are the densities of particles and carrier fluid respectively.

Vanoni and Nomicos (1960) considered the significance of the high concentration near the bed surface and obtained the parameter E as

$$E = \frac{\rho_s - \rho}{\rho} \frac{\bar{C}_1 \omega}{VS_e} \frac{0.01h - 0.001h}{h}$$

where  $\bar{C}_1$  is the average concentration near the bed from  $y = 0.001h$  to  $y = 0.01h$ ,  $S_e$  is the energy gradient, and  $V$  is the mean velocity at the vertical.

Results obtained by Einstein and Qian, Vanoni and Nomicos are compared in Fig. (10a) and (10b). It shows that Vanoni and Nomicos' relation is much better than Einstein and Qian's. Fig. (10a) also includes the data from Ippen and Elata's experiments with neutrally buoyant particles, it shows that Elata and Ippen's data obviously are inconsistent with other data obtained from natural sediment. It demonstrated that the mechanism of the reduction of von Karman's constant is different with different types of testing materials.

It is generally accepted that the presence of sediment particles in open channel flow reduces the velocity gradient near the bed with an increase in the main flow region. Coleman (1981) re-examined the data from earlier experiments to show that the change in  $k$  was caused by the incorrect application of the logarithmic velocity distribution to the outer flow region where the log law is not really valid. Coleman's experiments were conducted in suspension over a smooth bed in an open channel under the condition that no bed form was present. He was able to quantify the effect of suspended sediment on the flow in terms of the wake strength coefficient  $\pi$  of the velocity profile, which was first used by Coles to describe the deviation of the velocity from the logarithmic distribution in turbulent flow. This velocity distribution is given by,

$$\frac{u}{U_*} = \frac{1}{k} \ln \frac{yU_*}{\nu} + A - \frac{\Delta u}{U_*} + \frac{\pi}{k} \omega\left(\frac{y}{\delta}\right)$$

where  $k$  denotes von Karman's constant,  $A$  is a constant traditionally set equal to about 5.5 for turbulent flow over a smooth boundary, and  $\pi$  is Coles' wake strength coefficient. The function  $\omega\left(\frac{y}{\delta}\right)$  denotes Coles' wake flow function, which is normalized to have the properties  $\omega(1) = 1$ , and  $\omega(0) = 0$  and  $\delta$  denotes the boundary layer thickness. The term  $\frac{\Delta u}{U_*}$  denotes the downshift in

the velocity distribution because of wall roughness.

Coleman used Eq. (27) to examine von Karman's constant  $k$  from straight line fitting to experimental velocity profiles in the lower 15% of the flow, in the region where the wake flow terms are negligible and not out in the more

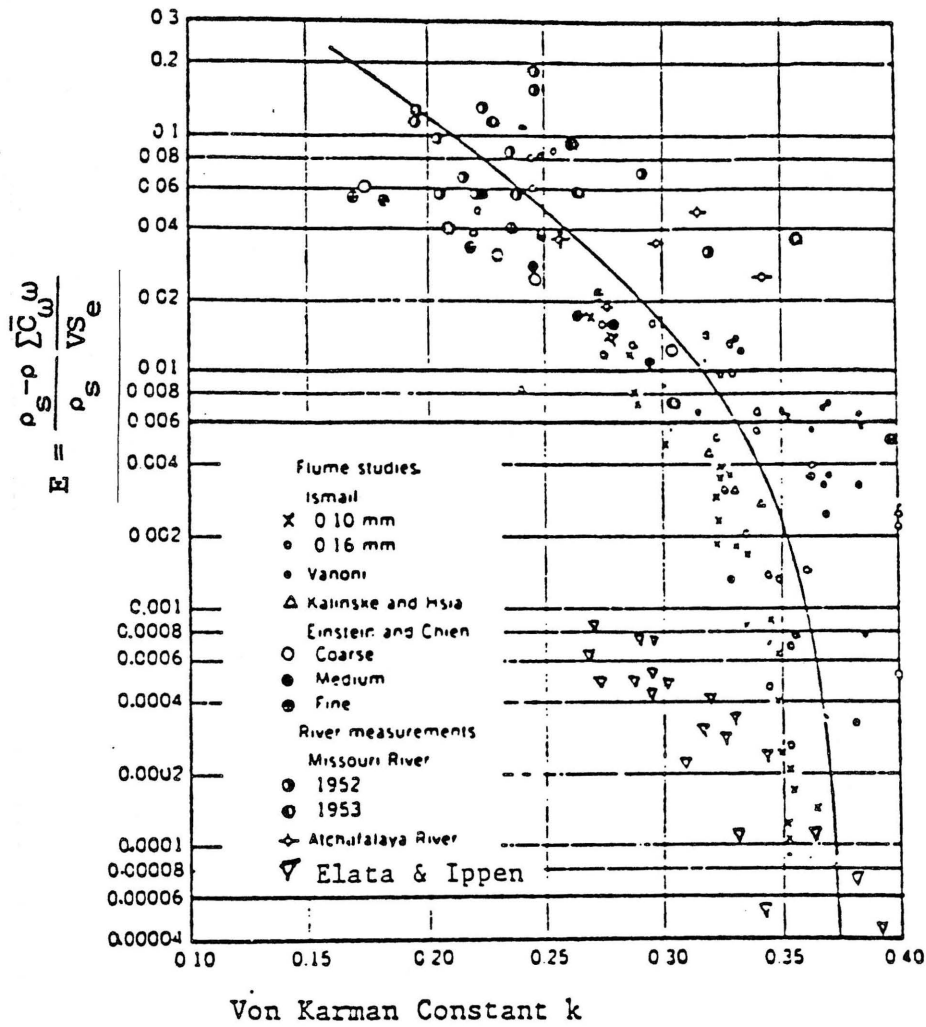


Fig.(10a). Relation between Von Karman constant and parameter E. (after Einstein & Qian)

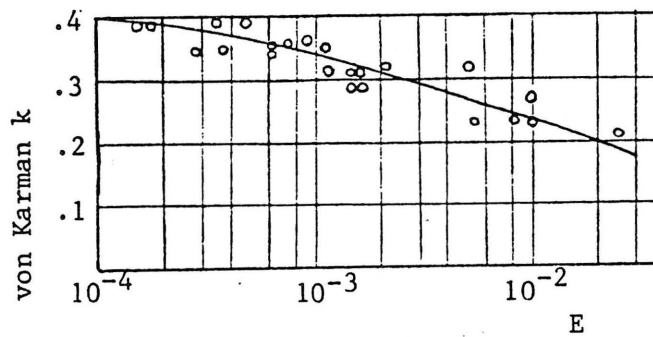


Fig.(10b). Relation Between  $k$  and  $E$   
(after Vanoni and Nomicos, 1960)

central regions of the flow. He observed, after re-evaluation of the early data together with his new data, that  $k$  is essentially constant over a range of conditions from flows with no sediment in suspension to flows carrying a near capacity load of suspended sediment. Also Coleman showed that the wake strength coefficient  $\pi$  for uniform nonseparating flows varies directly with the gross-flow Richardson number as shown in Fig. (4) in his paper (see Fig. 11 in this report).

The finding that  $k$  is not dependent on sediment concentration is far-reaching and important as claimed by Coleman. However, the invariance of  $k$  is accompanied with a wake flow function which requires further investigation. Julien and Lan (1986) argued that the presence of sediment is not synonymous to an increase in wake strength coefficient  $\pi$  as claimed by Parker and Coleman. With large concentrations of fines the wake strength coefficient remains equal to its clear water value as long as the concentration profile is uniform, because the Richardson number remains zero. Instead the effect of sediment should be concluded in the changes of kinematic viscosity and the coefficient  $A$  in the equation.

#### 4.2 Velocity Distribution in Hyperconcentrated Flow

With more and more sediment in suspension, even the wake-defect law of the velocity distribution cannot describe the influence of suspended sediments. Although the velocity distribution is much complicated in this case, some of the following phenomena are found to be true.

##### 4.2.1 Velocity distribution of hyperconcentrated flow of fine particles.

In hyperconcentrated flow containing fine particles or cohesive particles, Bingham yield stress (or yield stress in general) appears beyond a certain concentration. In the region where the shear stress is smaller than the yield stress, there is no deformation or relative motion between layers. That is, the flow in that region moves as a whole and a plug appears, as shown in Fig. (12). Assuming the Bingham plastic model to be valid, it is easy to derive that in a laminar flow the plug flow exists in the following region:

$$y > \frac{1}{\gamma_m S_e} (\tau_o - \tau_y) \quad (28)$$

The larger the bed shear stress  $\tau_o$  is, the smaller the plug will be, which indicates that as the flow turns into turbulent the plug will disappear. The velocity of plug flow is

$$U_o = \frac{1}{2u_p \gamma_m S_e} (\tau_o - \tau_y)^2 \quad (29)$$

In a laminar flow the velocity distribution beyond the plug region follows the parabolic formula; in turbulent flow it follows the logarithmic formula or even more accurately it follows the wake-defect law.

##### 4.2.2 Velocity distribution of hyperconcentrated flow of only coarse particles.

###### A. Turbulent Flow.

With only coarse particles in suspension, the fluid behaves as dilatant or as that described by Eq. (14) with the omission of  $\tau_y$ , that is,

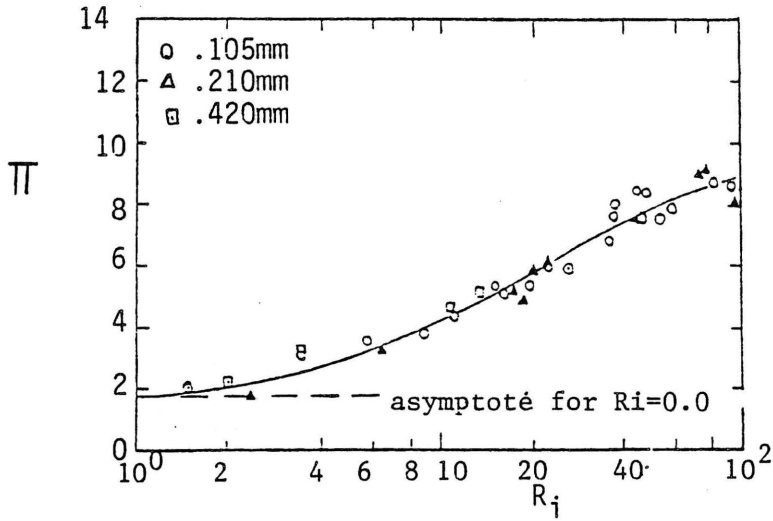


Fig.(11) Relation between Coles' strength coefficient and Richardson number (after Coleman,1981)

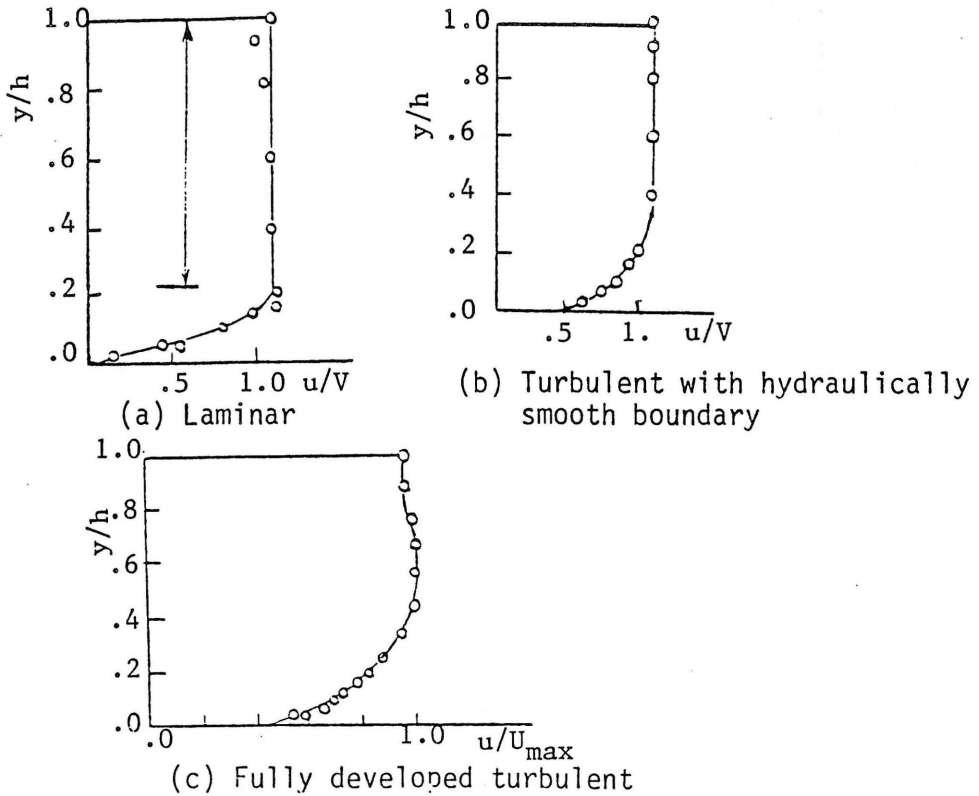


Fig.(12) "plug" appearing under conditions of different flow patterns ( after Qian and Wan, 1985)

$$\tau = \mu_p \frac{du}{dy} + C_1 \left( \frac{du}{dy} \right)^2$$

from which the velocity distribution is formed to obey the logarithmic law in the main flow region but to deviate from the logarithmic law in the lower region near the bed surface. Actually this case is the same as that discussed in the first part of this section where Coleman's equation was found to be valid.

#### B. Pseudo-laminar Flow at Hyperconcentration.

When the sediment concentration is extremely high turbulence disappears and all the particles move in laminated form, which looks like laminar flow although it is not really laminar. To distinguish it from the ordinary laminar flow, we call this kind of flow pseudo-laminar flow in this report. There is yet no available data for this kind of flow in open channels, experiments in pipes demonstrated that the velocity distribution is deduced as follows (Wang and Qian, 1986):

$$\frac{u}{U_m} = \left[ 1 - \left( 1 - \frac{y}{y_m} \right)^2 \right]^{1/2} \quad (30)$$

in which  $U_m$  is the maximum velocity at  $y = y_m$  and  $y_m$  is half of the pipe height.

## V. HYPERCONCENTRATED FLOW RESISTANCE

### 5.1 Resistance of Pseudo-One Phase Flow

The effects of high sediment concentration on the flow resistance in open channels as well as pipes have been of concern for several decades, but disagreement still exists especially for flow under turbulent conditions. Montes and Ippen's experiments on flow over smooth bed surfaces with large bottom slopes indicate that the Darcy-Weisbach resistance coefficient invariably increases if sediment is present, as shown in Fig. (13). Qian et al. (1980) also concluded that energy losses in smooth turbulent hyperconcentrated flow are larger than that in clear water flow under the condition of the same velocity. On the other hand, Zhang et al. (1980) demonstrated that the presence of sediment particles in suspension reduce the resistance or energy losses when examined in turbulent flow. There is still the third opinion which states that resistance in turbulent hyperconcentrated flow is somewhat less than that in clear water flow, while in laminar flow resistance of hyperconcentrated flow is much greater than that of clear water flow (Zhang, et al., 1980). These conflicts are generally caused by the following factors.

(1) Different usage of criterion for comparison. Some investigators compare the results from hyperconcentrated flow with that from clear water flow under the condition of the same Reynolds number, while others make the comparison under the condition of the same velocity. Since discharge reflects the transport capacity of the flow, it is recommended that comparison be given with the same discharge. Unfortunately, this kind of comparison has not yet been made.

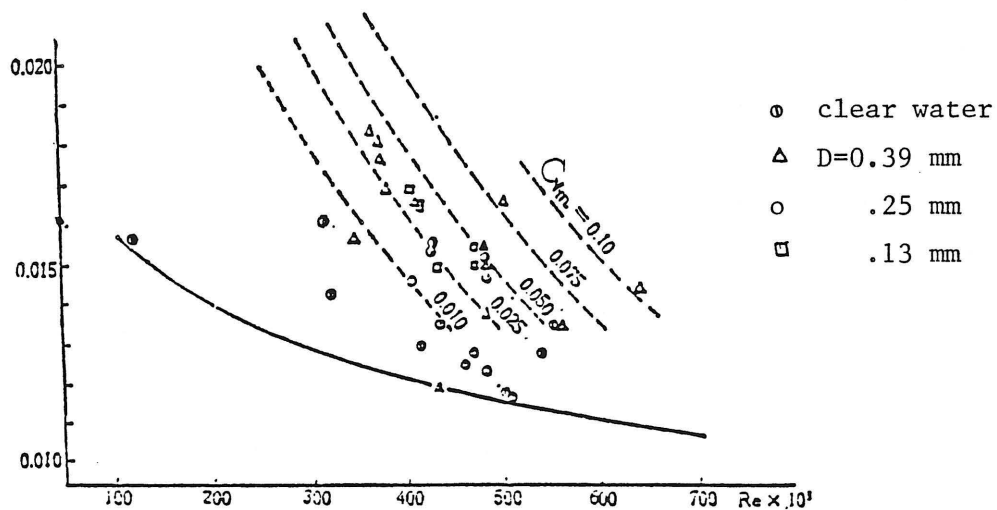


Fig. (13). Experimental data from Elata & Ippen showing  $f$  increases with the increase of sediment concentration

(2) Different boundary conditions make it difficult to compare. Strictly speaking experiments should be conducted under the conditions that no bedload exists; otherwise, the presence of bedload will radically increase the energy loss or resistance. It was also found that the resistance coefficient would be increased several fold if bedforms appear. In open channel flow, bedforms appear when flow intensity is increased to a certain value.

(3) The determination of flow regime. In different flow regimes (that is, with different Reynolds numbers), the resistance between sediment water mixture and clear water flow should be different. It has been noted that the viscosity of hyperconcentrated flow increases with increasing sediment concentration. However, in turbulent flow the effects of turbulence and dispersion of particles becomes dominant while the viscous term is negligible. Then the resistance is mainly dependent on boundary conditions. In laminar flow the viscous term in Eq. (14) is dominant, thus the resistance coefficient increases with increased viscosity or decreased effective Reynolds number. From experimental data in homogeneous slurry, Fei demonstrated that in laminar condition the effective resistance coefficient  $f_m$  and the effective Reynolds number  $Re_m$  have the relation

$$f_m = 16/Re_m \quad (31)$$

no matter how high the sediment concentration is. Therefore, under the same velocity the resistance coefficient in slurry is larger than that in clear water (Fei, 1985).

For turbulent hyperconcentrated flow in pipe, Fei integrated the logarithmic velocity distribution in turbulent pipe flow and carried out the expression for resistance coefficient

$$\frac{1}{\sqrt{f}} = \frac{0.353}{k} \left( \ln \frac{Re\sqrt{f}}{m} - 3.22 \right) \quad (32)$$

$$f = 8 \left( \frac{U_*}{V} \right)^2$$

where  $V$  is the cross-section average velocity,  $U_* =$  shear velocity,  $Re =$  Reynolds number and  $m$  denotes the factor reflecting the change in flow regime from turbulent to laminar. Approximately,  $m$  is taken equal to 0.10. Fig. (14) shows experimental data from his experiments with slurry. It indicates that the resistance coefficient of sediment-water mixture is less than that of clear water flow with the same Reynolds number. Now we change Eq. (32) into

$$\frac{1}{\sqrt{f}} = \frac{0.353}{k} \left( \ln \frac{V\sqrt{f}}{\nu} - \ln \frac{m}{D} - 3.22 \right) \quad (33)$$

We can see that  $f$  decreases with the decrease of  $k$ , but increases with the increase of kinematic viscosity  $\nu$  and coefficient  $m/D$ . Then with the same velocity, the resistance of the mixture may be less than, larger than or equal to that in clear water flow depending on the change in the value of  $k$ ,  $\nu$  and  $m/D$ .

In the previous chapter we stated that the wake-defect law of velocity distribution represents the most convincing argument to date on the effects of suspended sediments on the velocity distribution. Lau (1983) used Eq. (27) to consider that the change of resistance in hyperconcentrated flow is decreased. Parker and Coleman (1986) also obtained the same results based on their theoretical flow model. For further discussion we rewrite Eq. (27) as



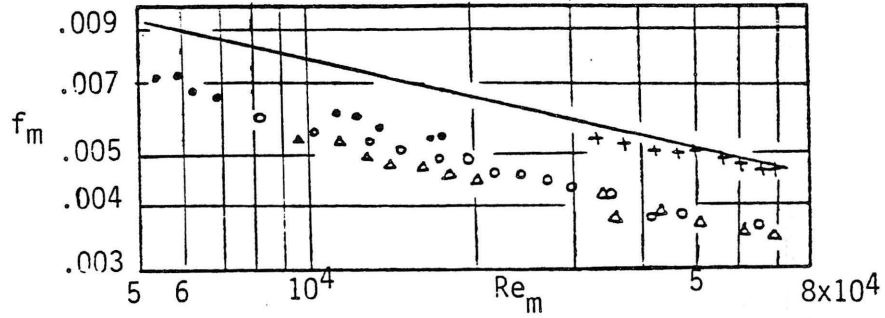


Fig.(14) Experimental data showing "resistance Reduction".(after Fei)

$$\frac{u}{U_*} = \frac{1}{k} \ln \left( \frac{U_* y}{\nu} \right) + A + \frac{2\pi}{k} \omega \left( \frac{y}{\delta} \right) \quad (34)$$

and apply Coles' empirical relationship for the wake function

$$\omega = \sin^2 \left( \frac{\pi}{2} \frac{y}{\delta} \right) \quad (35)$$

Integrate Eq. (34) over  $[0, h]$  to give the average velocity of the cross section,

$$\frac{V}{U_*} = \frac{1}{k} \left( \ln \frac{U_* h}{\nu} - 1 \right) + \frac{\pi}{k} + A \quad (36)$$

Here,  $\omega \left( \frac{y}{\delta} \right) = 1$  is used and  $\delta = h$ . If we still use the definition,

$$f = 8 \left( \frac{U_*}{\nu} \right)^2 \quad (37)$$

then we obtain

$$f = 8 \left[ \frac{1}{k} \left( \ln \frac{U_* h}{\nu} - 1 \right) + \frac{\pi}{k} + A \right]^{-2}$$

let  $\frac{U_* h}{\nu} = \frac{Vh}{\nu} \frac{U_*}{V} = \frac{Vh}{\nu} \frac{\sqrt{f}}{2\sqrt{2}}$ , then

$$f = 8 \left[ \frac{1}{k} \left( \ln \frac{Vh}{\nu} - 2.04 + \frac{1}{2} \ln f \right) + \frac{\pi}{k} + A \right]^{-2} \quad (38)$$

Although  $f$  appears on both sides of the preceding equation, the changes in term  $\left( \frac{1}{2} \ln f \right)$  are not significant. Then we can see  $f$  decreases as  $\pi$  increases when the discharge is kept unchanged. But it is noticed that the viscosity in Eq. (38) should be increasing with sediment concentration. Influence of viscosity on resistance coefficient  $f$  is significant especially if  $C_v$  is large and the concentration profile is nearly uniform. Then the opposite effects of  $\nu$  and  $\pi$  on the resistance remains unclear.

## 5.2 Resistance of Hyperconcentrated Flow Consisting of only Coarse Particles

### A. Turbulent Flow.

Hyperconcentrated flow consisting of only coarse particles usually behaves as a two-phase flow of a dilatant fluid. The resistance of this kind of flow depends strongly on the bed configuration and the intensity of bedload motion. If all particles move as suspended load, the resistance is close to that of clear water flow. In the case of hydrotransport in pipes a cut in flow velocity may result in a transformation of part of the suspended load into the bedload. If the velocity is further reduced, a critical point may be reached in which deposition on the bed begins to come into existence. Such a bed may assume a rippled form. Under such circumstances the frictional resistance will rise rapidly and is a function of the sediment concentration. In open channel flow such a kind of flow may not exist.

### B. Pseudo-laminar Flow.

When all the particles move as a laminated load turbulence no longer exists. The bed is usually kept in plane form under such high shear stress. In comparison with the dispersive shear stress caused by mutual collision between particles, the viscous resistance can also be neglected. In this case, the rheological model can be reduced to

$$\tau = C_1 \left( \frac{du}{dy} \right)^2 = \rho U_*^2 (1-\xi) \quad (39)$$

where  $\xi = y/h$  and the other parameters are as mentioned above. Integrating Eq. (39) yields

$$\frac{u}{U_*} = \frac{2h}{3} \sqrt{\frac{\rho}{C_1}} [ 1 - (1-\xi)^{3/2} ] \quad (40)$$

The average velocity at the vertical is given by

$$V = \frac{4h}{15} \sqrt{\frac{\rho}{C_1}} U_* \quad (41)$$

It is observed by many investigators (such as Wan, et al., 1979; Wang and Qian, etc., 1985a) that for laminar hyperconcentrated flow the relationship between the resistance and the corresponding effective Reynolds number

$$Re = \frac{\gamma_m V^2}{g \left[ u_p \frac{V}{4R} + \frac{1}{8} \tau_y \right]}$$

behaves as that in clear water flow, that is,

$$f_m = 96/Re \quad (42)$$

which is shown in Fig. (15).

Substituting Eq. (41) into Eq. (42) ( $\tau_y = 0$ ), and using Eq. (37) yields

$$f_m = \frac{96}{Re} = \frac{96v_m}{Vh} = \left[ \frac{720 \sqrt{2} v_m \sqrt{C_1}}{Vh^2 \sqrt{\rho}} \right]^{2/3} \quad (42a)$$

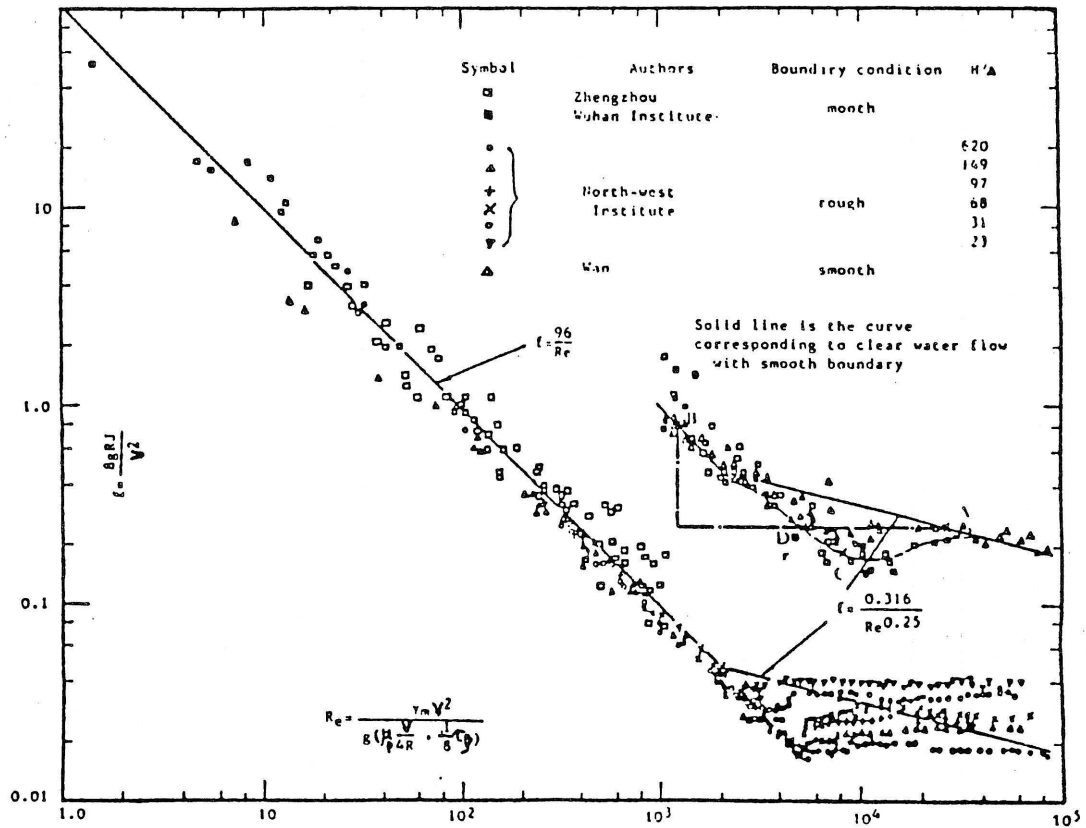


Fig.(15) Relationship Between Coefficient  $f$  and Reynolds number for Pseudo-one-phase Flow (after Qian and Wan, 1986)

which indicates that  $f_m$  increases with increasing  $C_1$ . That implies that the resistance of pseudo-laminar flow increases with the increase of sediment concentration.

Wang and Qian (1985b) also gave an expression for the resistance of such flow in terms of energy gradient,

$$S_e = 4.83 \frac{B+h}{B} \frac{\gamma_s D}{\gamma_m h} [ F_r^* f(\lambda) F_r ]^{1/2}$$

in which  $F_r^* = \omega_o^2 / gD$

$$F_r = V^2 / gh$$

$$f(\lambda) = \frac{2\pi}{12} \frac{\lambda^2}{1+\lambda} \left[ 1 - \frac{1}{12} \left( 1 + \frac{1}{\lambda} \right)^2 \right]$$

$$\lambda = \left[ \left( \frac{C_v^*}{C_v} \right)^{1/3} - 1 \right]^{-1}$$

$\lambda$  is the linear concentration defined by Bagnold,  $\omega_o$  is the settling velocity of a single particle in clear water. They suggested that, as the kinematic energy in maintaining the laminated load motion is directly taken from the potential energy of the flow, the resistance will be much larger than that of a clear water flow. The resistance of laminated load motion is proportional to the settling velocity of the particles. The coarser the size constituent of sediment the more the energy needed for maintaining the movement will be.

### 5.3 Effect of Fine Particles on Hyperconcentrated Flow Resistance and Movement of Coarse Particles.

Bruhl and Kazanskij (1976) observed, in pipe experiments with coarse particle suspension ( $D_{50} = 0.3$  mm,  $C_v = 24.5\%$ ), that head loss is larger than that in clear water pipe flow. As soon as fine particles of size less than 0.01 mm in diameter are added to the flow, energy head loss begins to decrease. The more fine particles are added to the suspension, the smaller the energy head loss is. However when the content of fine particles is greater than about 6.6% in volume, the energy head loss will no longer decrease. Also the energy loss is not affected by fine particles when the flow velocity is high, as shown in Fig. (16).

Kikkawa and Fukuoka (1969) also studied the effects of fine particles in suspension on resistance of hyperconcentrated flow and the movement of coarse particles. They found that the presence of fine particles in suspension not only makes the velocity gradient and concentration gradient become larger, but also increases the transport rate of coarse particles.

It has become a fact that the presence of bedload increases the bed resistance and the energy loss of the flow. In Bruhl and Kazanskij's experiments when the flow velocity is low it is reasonable to believe that a certain part of those coarse particles move in bedload form, then the resistance of the flow increases. After fine particles are added to the flow the viscosity of the flow is increased and the settling velocity of particles is decreased. Then some bedload turns into suspension which decreases the

resistance to flow. When the velocity is high almost every particle is in suspension. Then adding more fine particles has very limited effects on the resistance of the flow.

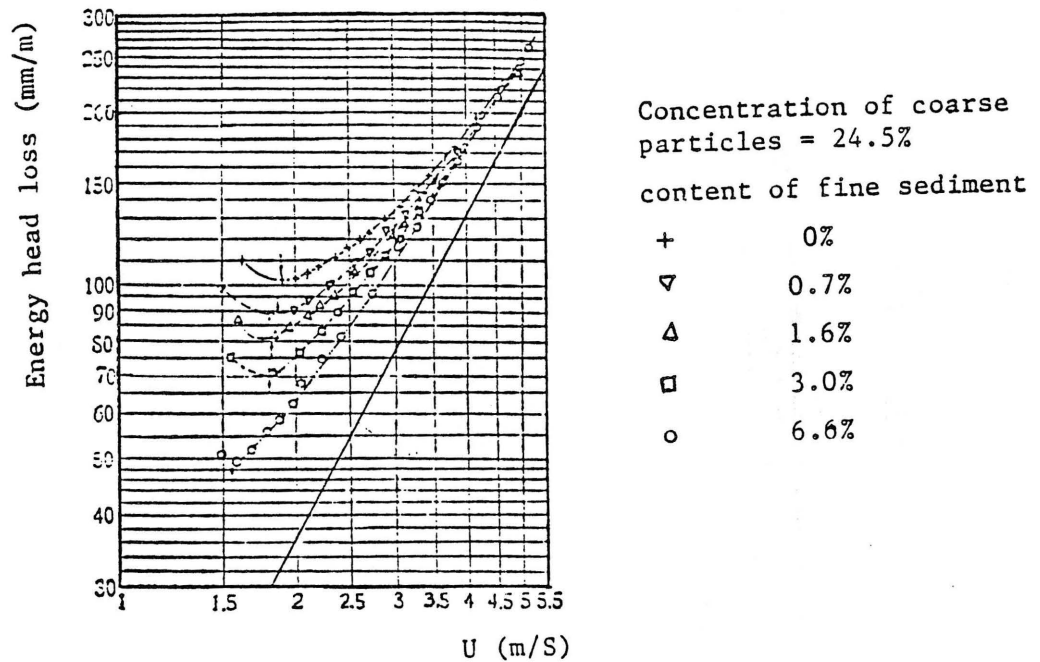


Fig.(16) Experimental data showing the effects of fine particles on flow resistance (after Bruhl and Kajanskij,1976)

## VI. CONCLUSION

Classification of sediment-laden flow, rheological properties of hyperconcentrated flows, velocity distribution and hyperconcentrated flow resistance have been reviewed in this report. The ultimate objective of the study is to summarize recent and past developments in hyperconcentrated flow research and to expand some points of view in this area; for instance, the quadratic rheological model, flow resistance of pseudo-laminar flows, etc. Although this review is not exhaustive, the following conclusions have been assessed:

(1) Sediment-laden flows have been classified according to triggering mechanisms, sediment compositions and rheological and kinematic behaviors. Although many methods are proposed, none of them are appropriate to the analysis of hyperconcentrated flow. As proposed in this report (TABLE II), it seems imperative to better understand the rheology of hyperconcentrated flows in order to classify flow based on physical process rather than more arbitrary criterion.

(2) Grain movement in hyperconcentrated flow generally can be classified into contact-load, suspended load and neutral buoyant load. The patterns of motion of hyperconcentrated flows are always controlled by grain constituent and concentration, flow density and flow intensity.

(3) Rheological approaches, including Newtonian, Bingham, pseudo-plastic, dilatant, power-law and quadratic models have been used to describe fluid flows ranging from low to high sediment concentration with fine and coarse particles. The report shows that the quadratic model proposed by O'Brien and Julien (Eq. 14) has a good deal of merit in describing hyperconcentrated flow of coarse particles, especially in the intermediate region of shear rate. When this model is used the coefficients in the model can all be expressed as function of volumetric concentration (Eq. 16).

(4) The settling velocity of sediment is affected by many factors of which few can be analytically considered. Up to now, Chu's (or Fei's) formula is found the best to predict the settling velocity of non-flocculated sediment particles in hyperconcentrations.

(5) The influence of sediment concentration on velocity distributions differs in flows with small concentrations from flows with large concentrations of sediment. When concentration is low, velocity distribution of defect-law with a wake function proposed by Coleman may be used (Eq. 27). But it does not have any advantage over the method used by Einstein and Chien, Vanoni and Nomicos. The invariance of the von Karman constant in Eq. (27) is maintained only by introducing the wake function in the velocity distribution and is still disputable. When concentration in the flow is high, plug flow appears and then velocity distribution needs to be treated specifically.

(6) Hyperconcentrated flow resistance is determined by so many factors, such as flow regimes, boundary conditions, sediment composition, etc., that any arbitrary judgment on resistance adjustment to sediment concentration is not recommended. For laminar and pseudo-laminar flow, the resistance of sediment-laden flow increases with the increase of sediment concentration. For turbulent flow, the effects of increase in sediment on the resistance remain unclear and can be determined under specific conditions.

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