Active Learning in the Secondary Mathematics Classroom: The Effect on Student Learning

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Abstract

Across the United States, secondary mathematics has been struggling to be at the standard in which government leaders in education think it should be. Active learning has been a strategy that many educators have been looking into as a way to improve this lack of proficiency for many decades. In this study, a summer academy which used active learning strategies with students around the Pueblo, Colorado area for two weeks is used to show active learning results. Pre and post assessment scores of the academy show an increase in student learning within a six day time period. There are many factors that play into the academy which could have had a negative effect on the academy’s results. If students were exposed to the active learning environment in a regular public education setting, the results could have been significantly better. Secondary mathematics teachers are called upon to integrate active learning regularly in their curriculum to see improved student understanding and learning.
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“Tell me and I forget. Teach me and I remember. Involve me and I learn” –Benjamin Franklin.

Introduction

“During the 1980s, the teaching of mathematics in secondary schools experienced a number of major changes, which can be characterized as a move away from expository teaching towards the use of a greater diversity of learning activities, and a greater emphasis on problem-solving and investigational approaches to tasks” (Kyriacou, 1992). As a college student studying to be a secondary mathematics teacher, effective strategies to improve student learning and understanding have been a concern of mine. This study will address the question: “Is active learning an effective strategy to improve student learning and understanding in the secondary mathematics classroom?”

I had the opportunity to be a junior mentor in the Noyce Summer Academy where students explored algebraic thinking skills, using active learning strategies. The academy seemed successful and the consensus among those who participated in it was that students came out with a better understanding of and enriched skills in algebra concepts. However, no one had computed the results or determined if the active learning strategies really resulted in improved knowledge among the students. In this analysis of the academy, the results do indeed show improvement. Active learning activities in secondary mathematics classrooms should be integrated consistently throughout any curriculum to improve student learning and understanding of mathematical concepts.
Background

What is Active Learning?

“In essence, active learning may be described as the use of learning activities where pupils are given a marked degree of ownership and control over the learning activities used, where the learning experience is open-ended rather than tightly pre-determined, and where the pupil is able to actively participate in and shape the learning experience” (Kyriacou, 1992). Active learning is also very closely related to project (or problem) based learning which can be described as “a student-driven, teacher-facilitated approach to learning” (Bell 2010). Active learning can look like many things. Sometimes it may look like students working in collaborative groups to problem solve. Other times students may be gathering information as a class and analyzing their results. However, active learning does not always have to be something that is strenuous, takes a lot of time, or requires a lot of materials. A small scale active learning activity could be rolling dice individually to find probabilities. Active learning has been integrated into primary and secondary classrooms more in the last several decades. Lecturing has been prominent throughout many classroom environments for centuries and is still common now.

How Do Students Learn?

In order to come to a conclusion about activity based learning in the classroom being effective for student learning, we must first talk about how students learn. George Brown states that “Students learn, with varying degrees of success, through reading, memorizing, thinking, writing, note-taking in lectures, observing, listening to and talking with others and by doing things.” Brown also goes on to say that while this is how students learn in varying contexts, it
does not explain how they learn. Diving into this topic can get messy because a behaviorist and constructivist will have different perspectives as do other belief systems. However, for our purposes in dealing with education, behaviorism and its beliefs seems to be the most related to how the majority of those in the education field believe.

Behaviorism is described as “a movement in psychology that advocates the use of strict experimental procedures to study observable behavior (or responses) in relation to the environment (or stimuli)” (Behaviorism). This branch in psychology fits closely with our topic of study, education. For decades it has been an ongoing effort between teachers, administrators, parents, and education experts to determine what stimuli in the classroom (material, delivery of information, classroom environment, etc) results in the best responses (effective learning) from students in the classroom. Since education has gone through so many phases with the last century or so, it has been tough to gauge what the best methods of teaching are to maximize student learning. To go even farther, in this research students’ comprehension of the material is the main focus, not just learning it. Determining whether active learning increases student understanding in the classroom takes a behaviorist approach of analyzing the aspects of a classroom.
Speaking of behaviors, “In 1956, Benjamin Bloom headed a group of educational psychologists who developed a classification of levels of intellectual behavior important in learning” also known as Bloom’s Taxonomy (Overbaugh). The original version was modified in the 1990’s to represent the tier shown below.

Remembering material that is presented is the lowest tier in the taxonomy. For anyone that has been in a classroom, attended a conference, or was in any kind of setting where material was being presented knows that often times remembering is hard enough. If it is achieved, comprehension is sometimes out of sight. In a classroom remembering might look something like definition recognition, memorizing, and students being able to repeat but not explain. These are just every day experiences that every person in these kinds of settings can relate to. It is my assumption that this could be a cause for why some students need to be re-taught material every new school year. They remember it when they need it but as soon as they don’t need to recall the information over and over; students cannot find it in their memory again. According to the taxonomy, the highest level of learning is creating. However, in order to attain the ability to create with the knowledge students learn, they must get past remembering and move on to truly understanding the material and be able to manipulate it. According to George Brown in “How Students Learn” in order to achieve effective learning for students the follow must be in place: “Ensure that your students are engaged in active learning in your classes and in their study time. Set some tasks that involve interaction with others. Provide some choices for students so they gain a sense of ownership of their learning.”
Research

Robert Noyce Teacher Scholarship Program

Researching how effective any type of activity is in the classroom is difficult. There are many factors that play into a classroom and the learning that takes place. The factors include, but are not limited to, the teacher, how qualified the teacher is, how experienced the teacher is, the students, the community, classroom management, classroom environment, grade level, parent involvement, subject, ability level of students (advanced, general education, or remedial), and the structure of the school. Studying the effect of a specific activity in a classroom somewhere in the Pueblo community would be challenging and would take an enormous amount of time considering that schools are on a time schedule with testing and set curriculums. However, we can at least get an idea of how effective active learning is in a classroom setting with a situation similar to a public school classroom.

Colorado State University-Pueblo’s Physics and Math Department received a grant from the Robert Noyce Teacher Scholarship Program of the National Science Foundation in 2011. The program is in place to “address a critical shortage of K-12 mathematics teachers by encouraging talented science, technology, engineering, and mathematics (STEM) majors and professionals to enter the teaching profession” (CSU-Pueblo). The grant money supports those who are exploring (or pursuing) the secondary math field in several ways. One of the two most prominent ways is providing college juniors and seniors who are getting a bachelor’s of mathematics degree with secondary certification and those who already have their degree but are back in school to get their secondary certification with financial support. The students do not have to pay any of the money back as long as they complete two years of work in a high needs school for every year of support granted.
The second way Noyce is helping draw secondary mathematics teachers into the southern Colorado community is by providing the university with a unique opportunity to have a summer internship program. The program’s goal is to recruit college freshmen and sophomores who are pursuing careers in science, technology, engineering, or math (STEM) into the secondary mathematics education field. The program offers a two week math enrichment experience for middle school and high school freshmen students in the Pueblo County area. Students explore math in an active and hands-on way from a variety of people with teaching experience. The program has been offered twice, the summers of 2013 and 2014. The research we will be taking a look into is from the program in the summer of 2014.

**Noyce Summer Internship**

There were 9 interns total in the program for 2014. The interns had little teaching experience but were pursuing STEM areas such as mathematics, engineering, and physics. Joining them for the program were two professors of mathematics from CSU-Pueblo, two mentors who are experienced secondary math teachers from the Pueblo community, and two junior mentors who were in the process of getting their Bachelor’s of Science in Mathematics with Secondary Certification at CSU-Pueblo. The interns, professors, mentors, and junior mentors were split up into four classrooms. Each classroom had at least two interns and either one professor or one mentor. The junior mentors visited each classroom but were not consistently in any one classroom.

**Students**

The students attending the academy were incoming seventh graders for the upcoming fall, incoming eighth graders, or incoming ninth graders (freshmen). All students were from the
Pueblo County area. Most of the students were not from the same school and did not know each other. They were separated by their general age group. This way the students in each classroom could work on material that everyone in class had the same amount of knowledge about for the most part. The older students were in the same class together so they were able to extend some of the activities (explained in detail below) and explore some deeper concepts. Students were notified about the summer academy by the administrators or math teachers at their school at the end of the spring 2014 semester. It was free to all students who wished to attend and their parent or guardian had to sign them up. The academy was intended for those students wishing to extend their learning and further explore mathematics. However, many students were sent to the academy because they struggled with math in their school and were seeking some sort of extra help or tutoring. By the entrance forms and first day activities it became known that some students were there because their parents or guardians wanted them to improve their skills and some willingly signed up themselves. These are some important factors to take into consideration when looking into the research and results of the academy.

**Daily Routine**

The classrooms were set up with procedures and expectations closely related to any classroom in public education. The first day was spent on a pre-assessment of algebraic thinking skills in mathematics (which would become the post-assessment on the last day of the program) and the professors, interns, and students getting to know each other. Small games and activities to learn names and interests were played and students filled out a card stating their favorite subject(s), interests, reason for attending the program, and what they hoped to get from attending the program. Students were seated in groups of three or four for the duration of the program but were moved around occasionally. Every day following had a procedure as follows:
1. Students came into the classroom and if they were early had a challenging puzzle that required some sort of math to work on if they chose to.

2. The professors, mentors, and interns explained to their students what the objective was for the day and the group norm they should focus on. For example “Listen to every group members’ ideas” and “Stay on task” were two of the skills provided.

3. A quick recap of the work done the day before and any questions students had, were addressed.

4. The activity and directions were given and students began to work with their groups according to the activity. Professors and interns monitored, collaborated, and guided students and groups.

5. As students were working, periodically the interns, mentors, and professors would stop to have a dialogue about what students were finding, struggles they were having, questions, and anything the students found intriguing or interesting.

6. Snacks were distributed and students took a break.

7. The activity resumed with more dialogue and discussion in between.

8. A closing activity or discussion took place to provide purpose and meaning to the activity for the students.

9. A post-activity sheet was given to students that included a couple questions or problems related to the work they did that day and two questions “What was your favorite part of today and why?” and “What was your least favorite part of the day and why?”.

The last day of the two week program was spent on students taking the post-assessment (the same test given the first day of the program) and an ending ceremony. The students’ parents, guardians, and family members were invited to attend the ceremony where students were
recognized for hard work during the program. They demonstrated a problem solving activity they worked on in the academy for their families and professors, mentors, interns, and junior mentors were available for students and families to talk to.

**Before and After Students**

Two days before the program, hours before students arrived each day, hour after students left each day, and two days following the conclusion of the program the professors, mentors, inters, and junior mentors collaborated, planned, and organized. The daily activities were decided prior to the program but how to implement them was the majority of the collaboration. The adults in each classroom had the freedom to present the activities any way that they chose, but for the most part the pre-collaboration ideas were used in all the classrooms. As a group, the “group norms” were decided upon. Group norms can be compared to classroom rules in public classrooms and are a set of collaborative skills that every group member should improve on in order to have an efficient and effective group learning experience. Every day after the students left the professors, interns, and mentors read the post-activity sheets (discussed above) from their students. This provided feedback and sometimes adjusted how the next day’s work would look. If the student work showed that the majority of students did not grasp the concepts fully or the students did not like the way the material was presented (from the “What was your least favorite part and why?” question) each classroom of adults made changes for the next day’s plan. The same applied for when students showed that the material was too easy. Then, the material was revamped to be more challenging and meaningful.

This process of evaluating a day’s work in the classroom and assessing student learning and understanding is representative to what teachers do on a daily basis. Adjustments and small changes from personal experience can sometimes make the difference between students learning
and wasting their time. Even behind the scenes the program was designed to provide the most accurate and representative learning environment as that of a public classroom.

**Program Curriculum**

The topic for the summer program was algebraic thinking. The activities and lessons were built upon the book “Fostering Algebraic Thinking” by Mark Driscoll. He describes “three habits that seem to be critical to developing power in algebraic thinking”. All three habits were used in deciding the material and activities that the students in the academy would be exposed to. The first habit is “Doing-Undoing”. Driscoll describes this first habit as “the capacity not only to use a process to get to a goal, but also to understand the process well enough to work backward from the answer to the starting point”. This habit also plays an important role in the next two habits. The second habit of algebraic thinking is “Building Rules to Represent Functions”. He describes this as “the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined functional rules”. The third is “Abstracting from Computation” which he explains as “the capacity to think about computations independently of particular numbers that are used”.

Driscoll also focuses throughout his text on how teachers can provide the best guidance to foster successful algebraic habits of mind from their students. He suggests three general ideas that teachers can do: “Consistent modeling of algebraic thinking. Giving well-timed pointers to students that help them shift or expand their thinking, or that help them pay attention to what is important. Making it a habit to ask a variety of questions aimed at helping students organize their thinking and respond to algebraic prompts.” In preparation for students, the professors and mentors spent a great deal of time training and improving the interns’ and junior mentors’ skills of the questioning portion of what Driscoll describes.
Active Learning Activities

Hop to It

This activity was the first activity done at the academy and was revisited the next week. The first time the students saw this activity they were just exploring with it and problem solving. They were told that there was an even number of frogs that were split in half. All of the frogs were in a row of lily pads but there was one empty lily pad between the divided amount of frogs (for example 6 frogs on lily pads with 3 on right and 3 on the left with one empty lily pad in the middle). The goal was to get the frogs on the right to the left side of the lily pads, the frogs on the left to the right side of the lily pads, and still have one lily pad in the middle empty at the end.

The rules were as follows:

- The frogs must always be on a lily pad after each move.
- Frogs beginning on the left must only move to the right and vice versa.
- Frogs may “jump” over another frog if there is an empty lily pad on the other side.
- Frogs cannot jump more than one frog at a time.
- Frogs can step to an empty lily pad adjacent to them.
- Only one frog can move at a time.
- Only one frog is allowed on any lily pad at a time.

The students gathered in groups and were given large sticky notes to act as lily pads and they were acting as the frogs. Together they physically acted out the process and through trial and error problem solved the situation. If there were an uneven number of people in the group, one student would act as the leader and guided the moves the group was doing.
The students revisited this activity the next week of the program and did a little more with abstracting the process. They were given scenarios of different amounts of frogs (for example, 2 frogs and 3 lily pads or 12 frogs and 13 lily pads) and the challenge was to find the minimum number of moves (jumps or steps) that would get the frogs to the opposite sides while following the rules. Instead of using their bodies and large sticky notes, they were given small squares and pictures of frogs to manipulate. They had a table to record their data and were encouraged to find an algebraic rule to describe the number of steps, jumps, and total moves per amount of pairs of frogs. They then graphed all three data types (steps, jumps, and total moves) on the same coordinate plane. They compared the slopes, or steepness, of the three graphs. Some of the advanced or older students recognized the difference between the linear and quadratic graphs.

**Bowl-A-Fact**

Bowl-A-Fact focused on order of operations. This activity was broken up into three parts: A, B, and C. Students were given three dice per group. For Part A students were to roll all three dice. Using the three numbers on the dice, as a group they tried to come up with as many combinations of the three numbers using addition, subtraction, multiplication, and division to “knock down” bowling pins. The pins were labeled 1-10. They were allowed to use parentheses. For instance, if they rolled a 6, 5, and 3 they could use \((6/3)+5\) which has a value of 7 to knock down pin 7. They needed a combination that equaled every value from 1 to 10. If they could not get a strike (or knock down all ten pins) they had the opportunity to roll three dice again and use their new number to try to get a spare (knock down the remaining numbers). Part B and Part C were done the same way except students were given the numbers 2, 3, and 6 for Part B and 1, 2, and 4 for Part C for their combinations. They were not given the opportunity to
get a spare on these two parts but were asked if they got a spare, and if not what pins they were unable to knock down. If they had found all the possible combinations, they should have seen that with 2, 3, and 6 they had pins left over. With 1, 2, and 4 they were able to get a strike. Students were then encouraged to provide an explanation for why their person numbers that they rolled and the two other sets of numbers had different amounts of pins that were able to be knocked down.

Graph Stories

“Graph Stories” was an activity where the student groups were able to use their math and creative thinking skills. At this point in the academy students were exploring how slopes of lines differ when observing a person walking, running, jogging, or not moving on a graph with variables of distance versus time. In their groups the students were instructed to create a graph where multiple slopes were involved and write a story that explained the changes in the slope. Some students chose to write a skit and act out their stories, but it was a great way to give the students freedom in their learning and creativity. Each group had to present their story to those in their classroom similar to how students present projects in their normal classroom environments.

Tommy’s Toothpick Designs

This activity focuses on identifying patterns and abstracting the pattern into an algebraic rule. The “toothpicks” were used to create a pattern of growing stacks of squares. The students first focused on identifying a pattern with the small squares in the perimeter of each figure. They worked as a group to identify the next figure in each pattern, abstract a rule, create a table, create a graph, identify what figure number would have a given number of toothpicks (reversing the process), and doing some error analysis. Then, the students worked on similar aspects only with
counting the number of little squares and toothpicks in the figure rather than perimeter. Below are the first three figures that the students were given for this activity.

Baseball Jerseys
In this activity students were given two different t-shirt printing companies. Each company had different prices for the amount of t-shirts being ordered to print and had different flat fees to pay no matter the amount of t-shirts ordered. Students worked together in their groups to analyze these companies’ costs to order t-shirts. Several scenarios about baseball teams ordering different amounts of t-shirts were given and the students had to figure out which company would be the cheapest for each scenario. Also, there were situations where the baseball teams paid their amount but couldn’t remember which company they ordered from, so the students had to work backwards to determine the company in which the baseball team ordered from. This activity was an excellent way to bring real-world context into algebraic thinking. Students were observing how flat fees change the cost of ordering and how the cheapest company depends on the size of the order. The students then created their own t-shirt company with a different pricing plan that would always be the cheapest out of the three companies. They made a poster for their plan and had multiple representations of showing the cost the t-shirts would be when ordering from their
company. The students took turns presenting their plan to their class and also were able to show their work to their family and friends who came to the last day ceremony.

**Weekly Savings Plans**

This activity is another one that had the students looking at algebraic concepts through a real-world context. Students saw multiple representations (graphs, equations, tables, etc) of different types of savings account systems. They matched graphs to equations, tables to equations, graphs to tables, and so on that had to do with the amount of money a person was putting into their savings account on a routine basis. This not only improved their algebra skills, but also gave them a sense of purpose for the math and got them thinking about their futures.

**Function Machines**

The “Function Machine” idea was to get the students to really practice the idea of “undoing”.

There are many times in a math student’s life where they see a problem like this:

\[ 6(3) + 4 \div 2 =? \]

They must have a proficient understanding of order of operations and math-fact skills. However, these activities challenged students to problems that looked like:

\[ 6(\_\_) + \_\_\_\_ \div 2 = 20 \]

Students used their knowledge and skills to “undo” the process of order of operations. This is a skill that must be mastered in order to solve equations and other topics in more advanced math.
Results

As mentioned before, the pre-assessments were given to students on the first day of the academy. They were encouraged to do as much as they knew and to try all of the problems. The concepts on the assessment were those that the students should have been taught in school or at least had been exposed to. Since many of the students attending the academy had struggled grasping some ideas in their previous school year, it was not a surprise that the pre-assessment scores were not generally passing scores. Also, since there was a grade span of around three school years within the students attending, many of the lower scores could be accounted for by those younger students who had not reached these concepts yet within their normal academic math classes. Below is the average test score results of the pre and post assessments separated by classroom and by the total students attending the academy.
Pre/Post Assessment average test results measured by the amount of points correct out of 32.

Pre/Post Assessment average test results measured by the percentage correct.

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**Pre/Post Assessment Scores (Points)**

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<tr>
<th>Classroom</th>
<th>Average Test Scores (Out of 32)</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>Classroom 2</td>
<td>22</td>
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<tr>
<td>Classroom 4</td>
<td>16</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
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**Pre/Post Assessment Scores (Percentages)**

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<tr>
<th>Classroom</th>
<th>Average Test Scores (Percentage)</th>
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</thead>
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<td>Total</td>
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</tbody>
</table>
Discussion

While the increase in scores from the pre-assessment to the post-assessment among the students attending the academy does not seem dramatic, we must take a closer look into the factors that played into them. These students were mostly those who struggled with math at least within the past school year. They were already lacking in many mathematical concepts and some may have not seen much of the material before because of their age. One of the largest factors to consider within the academy and the scores that resulted are that the students were not held accountable for doing badly or doing well. In their regular classrooms grades are assigned. Their work, participation, and test scores are factored into their grade in some way and grades are used for students to pass that grade level and to be recommended for classes the next school year. Therefore, without grades being assigned and no accountability, students had to rely on their will to do well. Considering this, some students may not have taken the task seriously or tried with their full capability.

Another underlying factor we must look into is that usually when pre-assessments are given in a regular classroom, students have not learned or seen the material before. That way when the teacher or instructor gives the post-assessment at the end of the unit or concept they can see the increase from having no knowledge to being taught the material. So, a dramatic increase in scores would be expected. In the academy’s situation, many of the students had seen the material in some form. Thus, the pre-assessment was not given with a blank slate of knowledge. This could be another reason why a dramatic increase was not seen. Finally, the students attended the academy for eight days (if they attended every day). Two of these days were spent on pre and post assessments as well as getting to know the students and the end celebration. This leaves only six days that the students really explored the concepts. In a normal
school schedule, this is equivalent to one week and a day (in a five-day week schedule) or a week and two days (in a four-day week schedule). Units and concepts are usually between three to five weeks long in a normal classroom setting. These students learned and accomplished a lot of material in a short amount of time.

Since the students did not know each other for the most part nor the professors, interns, and mentors this could also play into their learning and assessment results. In an ideal classroom situation, teachers take the time to make students feel comfortable with each other and with their instructor. When students feel comfortable they could be more likely to ask questions and interact with each other more. It is hard to say whether or not the students in the academy felt comfortable, but it definitely was not a normal classroom setting.

Ultimately, whether the scores are affected or not, there is no doubt that active learning causes students to be more involved. When they have some ownership over what they are learning and have options, they are more likely to care and put more effort into the activity or material. Any time student motivation and self-drive can be improved, it can only help students with their learning. To demonstrate how students felt about the activities and what they feel they learned. Some student comments have been included.

The following are post-survey results from students who attended the academy.

Question: “What activity from the Summer Academy did you like best? Why did you like it?”

Responses:

- “Hop to It ‘cause we got to do math in a different way.”
- “Hop-to-It. I liked Hop to It because we went outside and had to communicate.”
- “Baseball Jerseys because it was fun and it really got you thinking.”
“Baseball Jerseys because I really like baseball, so I learned more because I liked it.”

“Hop to It. It challenged us to use our brains to switch the sides in the least amount of steps or jumps.”

Question: “What activity from Summer Academy did you learn the most from? What math did you learn from it?”

Responses:

“Weekly Savings Plans. I learned about the graph and how to use it. I also learned a lot of vocab.”

“Baseball Jerseys. I learned how to keep working and talking with a group.”

“Tommy’s Toothpicks. I learned that just because a box has 4 sides doesn’t mean you get 4 every time.”

“Hop to It. I learned to see what will happen before I do it.”

“Baseball Jerseys. I learned more about how math can tie into everyday problems.”

Other Sources

Dr. Pilgrim of Colorado State University in Fort Collins, CO and Ms. Bloemker of Boltz Middle School in Fort Collins were part of a similar academy done in the summer of 2013. The academy was held at CSU Fort Collins and was focused on low performing 8th grade students who would be entering high school that fall into geometry. The students attended the academy for free and were there for a total of 8 days. In a presentation at the Colorado Council of Teachers of Mathematics conference, Pilgrim and Bloemker described their academy to narrow in on algebra concepts that tie into geometry. The program’s curriculum was “influenced by problem-based learning and inquiry-based strategies” and encouraged the use of hands-on tools.
There were 19 participants with an average pre-test score of 39.1 percent and an average post-test score of 55.1 percent. So, a 16 percent increase was accomplished in a total of 8 days.

A study done on active learning strategies versus lecturing strategies and the effects on students retaining important concepts was done in Chicago, Illinois by three college students. The study included middle school and high school students from grades 7 through 12. The study did not focus solely on math, but within a 13 week intervention study of embedding active learning strategies into three different classrooms (in different schools) the results showed positive results. At the end of their study they came to a few conclusions about using active learning in the secondary classroom, “student engagement and motivation for learning significantly increased, increases in student retention of essential concepts, and active learning strategies were very cost effective” (Bachelor, 2012).

**Conclusion**

The Noyce Summer Academy focused on many rigorous concepts in algebraic thinking. The active learning strategies and activities that were used involved rich mathematical thinking and problem solving techniques. With a total of six days doing active learning, students showed a 2-7 point or a 9-20 percent increase in assessment results from the first day of the academy to the last. The students were not in an ideal classroom situation with many factors playing into their learning. Had the students been in a regular classroom environment, spent more time on the active learning material, held accountable for their results, and were in a known and comfortable environment, it is safe to say that the increase in scores would be expected to be greater. While active learning may not be suitable and realistic to do all the time in a regular secondary classroom setting, it is possible to integrate it into any curriculum. Many of the activities done in
the academy did not require many materials or a significant amount of space. However, they still had the students collaborating, in some cases moving, problem solving, and taking some sort of ownership of what work they were producing. The students themselves demonstrated that they enjoyed the activities for academic reasons and learned not only math skills, but problem solving skills. They commented on the activities getting them to think and “use their brains”. That is often times what all teachers strive for in their classrooms.

Other studies also indicate active learning being a successful tool to improve student learning and understanding. Not only at the middle school level are results showing growth using active learning, but also at the high school level. Any secondary mathematics teacher should consider integrating active learning into their curriculums. If these results can come from a setting where many factors were against the results, a public, secondary mathematics classroom should expect significant results in their students’ learning and understanding of mathematical concepts. All secondary mathematics educators should take active learning into serious consideration and see their students’ results grow.

**Acknowledgments**

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Resources


Overbaugh, Richard and Schultz, Lynn. “Bloom’s Taxonomy.” Old Dominion University.

http://ww2.odu.edu/educ/roverbau/Bloom/blooms_taxonomy.htm


Retrieved From


Google Image¹: http://ww2.odu.edu/educ/roverbau/Bloom/fx_Bloom_New.jpg

Google Image²: http://media.tumblr.com/tumblr_mb4pw9eNR41qzt83e.png

Pilgrim, M, and Bloemker, J. (2014). The Impact of an 8th Grade Algebra Summer Camp on 9th Grade Geometry Performance [PowerPoint slides].


*Information and materials from Noyce Summer Internship attained from personal experience and inventory from being a part of the program in the summer of 2014.*
PART A: Your answers to these questions will help us understand how we can do a better job. There are no right or wrong answers. Simply indicate your opinion by circling your response.

<table>
<thead>
<tr>
<th>No</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. School has an important purpose in my life.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>2. The subjects taught in my classes are usually interesting.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>3. I like math.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>4. Classes are almost always about something boring.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>5. My teachers really care about me and want me to succeed.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>6. I will be able to use what I learn in math all my life.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>7. Good math skills help people get and hold good jobs.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>8. I am an excellent math student.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>9. I intend to graduate from high school.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>10. I plan to go to college.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>11. Sometimes I think that I would like to quit school.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>12. At school I am learning things that will help me succeed in life.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>13. Knowing a lot of math will help me succeed in life.</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>14. In general I like school.</td>
<td>1 2 3 4 5</td>
</tr>
</tbody>
</table>
Answer each math question as well as you can.

PART B: Write your answers for each of the following questions in the space provided.

1. The number of toothpicks needed to build "square trains" of different lengths are shown here:

<table>
<thead>
<tr>
<th>Number of Squares (x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Toothpicks (y)</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

a. How many **toothpicks** are needed in a train of 4 squares? ___________ toothpicks
   Draw a picture to show this:

b. How many **toothpicks** are needed in a train of 7 squares? ___________ toothpicks
   Show your thinking:

c. How many **squares** are in a train that uses exactly 34 toothpicks? ___________ squares
   Show your thinking:

d. You are told how many squares are in a train.
   In your own words, describe the pattern for finding the number of toothpicks:

e. Let $x$ be the number of squares in a train. Find a formula for computing the number of toothpicks ($y$).
2. Larry saved $800 by mowing lawns this summer. He spends exactly $20 each week.

Which graph shows how much money there is left in his savings account each week? Matching Graph ________

- Graph a
- Graph b
- Graph c
3. The first part of a pattern using the letters “A” and “B” are shown below.

Fill in the blanks to show what you think the next six letters will be.

A B B A A B B A A A

EXPLAIN YOUR THINKING:

4. The numbers “2”, “3” and “6” are used once each to write arithmetic sentences using +, - , ×, ÷ and parentheses.

a) Compute the value of each of the following arithmetic sentences:

\[ 3 + 6 - 2 = \quad 6 \times 3 - 2 = \]

\[ 6 - (3 + 2) = \quad 2 + 6 \div 3 = \]

b) Write an arithmetic sentence that equals “11”.
Use each of the numbers “2”, “3” and “6” just once. 

\[ \quad = 11 \]

c) Write an arithmetic sentence that equals “5”.
Use each of the numbers “2”, “3” and “6” just once.

\[ \quad - 5 \]
PART C: Circle the answer you think is correct for each question about Paul and Lisa.

Paul and Lisa are both saving money for a special class trip.
Paul is able to save $15 every week.
Lisa is only able to save $10 every week, but she started with $30 that she got on her birthday.

1. How much money will Paul have saved after 4 weeks?
   a) $40  b) $60  c) $70  d) $130  e) $160

2. How much money will Lisa have saved after 4 weeks?
   a) $40  b) $60  c) $70  d) $130  e) $160

3. How many weeks will it take Paul to save $150?
   a) 6 weeks  b) 8 weeks  c) 10 weeks  d) 12 weeks  e) 20 weeks

4. How many weeks will it take Lisa to save $150?
   a) 6 weeks  b) 8 weeks  c) 10 weeks  d) 12 weeks  e) 20 weeks

5. After how many weeks will Paul and Lisa have exactly the same amount saved?
   a) 6 weeks  b) 8 weeks  c) 10 weeks  d) 12 weeks  e) 20 weeks

6. If Paul and Lisa put their savings together, how many weeks will it take them to save $225?
   a) 6 weeks  b) 8 weeks  c) 10 weeks  d) 12 weeks  e) 20 weeks
PART D: Each table below is based on a different number rule. The Rule for Table 1 is given.

Table 1: Fill in each blank space in the table with the missing number.

<table>
<thead>
<tr>
<th>Starting Number (x)</th>
<th>Ending Number (y)</th>
<th>The rule for Table 1 in words is:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>Multiply starting number by 2. Then add 3.</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

The rule for Table 1 as a formula:

\[ y = 2x + 3 \]

Table 2: Fill in each blank space in the table with the missing number.
Then describe a rule in words and as a formula in the spaces provided.

<table>
<thead>
<tr>
<th>Starting Number (x)</th>
<th>Ending Number (y)</th>
<th>Describe the rule for Table 2 in words:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>52</td>
<td></td>
</tr>
</tbody>
</table>

Write the rule for Table 2 as a formula:

\[ y = \]
Table 3: Fill in each blank space in the table with the missing number. Then describe a rule in words and as a formula in the spaces provided.

<table>
<thead>
<tr>
<th>Starting Number (x)</th>
<th>Ending Number (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Describe the rule for Table 3 in words:

Write the rule for Table 3 as a formula:

\[ y = \]

PART E: Match each part of the graph to what Joe was doing along the way.

Joe likes to walk to school, but sometimes jogs or sprints part of the way to get there on time. The graph below shows his distance from home on Monday morning.

Joe was walking
Joe was jogging
Joe was sprinting
Joe stopped to talk to a friend
MATH ACADEMY GROUP WORK NORMS

Keep in your group.

Involve everyone in your group in the activity.

Show up on time.

Show respect for each other’s ideas.

Explain math ideas to others.

Stay on task.

&

Hand in your work at the end of class.

Use your group members for help before asking a teacher.

Give recognition and credit to others.

Share work materials.
Getting to Know You
NOYCE SUMMER MATH ACADEMY

Record the following information about yourself. Then compare with others to see what you may have in common.

1. What is your shoe size?
2. How tall are you (in feet and inches)?
3. What is your favorite color?
4. What is your favorite sport?

For #5 - #8, put an X on the line on the point that best describes you.

5. How athletic are you?
   NOT AT ALL  SO- SO  VERY!

6. How well do you sing?
   NOT AT ALL  SO- SO  GREAT!

7. How much do you like animals?
   NOT AT ALL  SO- SO  ALOT!

8. How much do you like playing video games?
   NOT AT ALL  SO- SO  ALOT!

9. What is the month and day of your birthday?

10. If math were a food, it would be a __________________, because ____________________

NAME ________________  MONDAY, JUNE 16, 2014
Hop To It!

Here's how the problem starts:

There are seven lily pads and six frogs.
On the three left-hand lily pads, facing the center, stand three of the frogs.
The other three frogs stand on the three right-hand lily pads, also facing the center.
The center lily pad is not occupied.

The challenge: exchanging places

Find a way for the frogs to move so that:

- the frogs originally standing on the right-hand lily pads end up on the left-hand lily pads;
- the originally standing on the left-hand lily pads end up on the right-hand lily pads; and
- the center lily pad is again unoccupied.

The order that they are standing in does not matter.

The rules:

- After each move, each frog must be standing on a lily pad.
- If a frog starts on the left, it may only move to the right.
- If a frog start on the right, it may only move to the left.
- A frog may "jump" over another frog if there is an empty lily pad on the other side.
- A frog may not "jump" more than one frog at a time.
- Only one frog can move at a time.
- Only one frog can step on a lily pad at a time.
Hop To It - Revisited

Remember the "Hop to It" rules:

- An equal number of frogs start off on two different ends of a row of lily pads, with the center lily pad empty. The frogs then move so that those that were originally on the right end of the row up on the left end of the row, and vice versa.
- After each move, each frog must be standing on a lily pad.
- If a frog starts on the left, it may only move to the right.
- If a frog starts on the right, it may only move to the left.
- A frog may "jump" over another frog if there is an empty lily pad on the other side.
- A frog may not "jump" more than one frog at a time.
- Only one frog can move at a time.
- Only one frog can step on a lily pad at a time.

Today's Challenge

Try to find the fewest number of moves necessary to complete the exchange of places for different numbers of frogs:

1. What if there are only 2 frogs (1 pair) and 3 lily pads?
   How many moves does it take for the two frogs to exchange positions?
   How many are jumps? How many are slides?

2. What if there are 4 frogs (2 pairs) and 5 lily pads?
   How many moves does it take for 4 frogs to exchange positions?
   How many are jumps? How many are slides?

3. What about 6 frogs (3 pairs)?

4. What about 8 frogs (4 pairs)?

5. What about 10 frogs (5 pairs)?

Use the materials provided to simulate the movement of the frogs.

As you work on each question, have someone in your group be the recorder.
The recorder should keep track of your work by listing the moves in the order they occur.

Then make a data table using the information you've gathered, and use your table to create a graph.

Can you find a pattern for any number of frogs? Look at steps, jumps, and total moves.
Bowl-A-Fact

Part A

1. Play a game of Bowl-A-Fact.
   Roll three dice, and write the three numbers you get here: _____, _____, _____
   (Roll again if you get 3, 4, 6 or 3, 3, 4.)
   Use your numbers to knock down as many pins as you can.
   (For this part and the remaining parts, record your figuring the spaces provide.)

   Did you get a strike? __________
   If not, what pins still need to be knocked down? ________________________

2. If you got a strike in #1, go on to Part B.
   If you did not get a strike, try for a spare.
   Roll the three dice again, and write the number you get here: _____, _____, _____
   Now use your new numbers to knock down as many of the remaining pins as you can.
Bowl-A-Fact
Part B

Suppose you play again, and the numbers you roll are 2, 3, 6.
Knock down as many pins as you can using these numbers.

Did you get a strike? 

If not, what pins still need to be knocked down? 

Name ________________________
Bowl-A-Fact

Part C

Suppose you play again, and the numbers you roll are \[1, 2, 4\]
Knock down as many pins as you can using these numbers.

\[
\begin{array}{ccc}
7 & 8 & 9 \\
4 & 5 & 6 \\
2 & 3 & 1 \\
\end{array}
\]

Name ____________________

Did you get a strike? _________
If not, what pins still need to be knocked down? __________________________
Tommy loves to make designs with toothpicks. He makes them in lots of different shapes, and usually mixes together different colors to make them more interesting to look at.

Tommy also likes to think about the different numbers of things in his design.

In this activity, we will think about some of these numbers for one of Tommy's favorite designs: Toothpick Squares.

Here are the first three sizes of Tommy’s Toothpick Squares:

<table>
<thead>
<tr>
<th>Toothpick Square #1</th>
<th>Toothpick Square #2</th>
<th>Toothpick Square #3</th>
</tr>
</thead>
</table>

1) Sketch a picture of Toothpick Square #4 below.

2) Write down 2-3 things that you notice about numbers of things in a Toothpick Square.
One thing that Tommy noticed about numbers in Toothpick Square is that the number of the square in the sequence is always equal to the number of toothpicks along each of its sides. For example, every side of Toothpick Square #3 has exactly three toothpicks.

3) Explain why this helps Tommy to figure out that he will need 12 toothpicks to build the outside, or perimeter, of Toothpick Square #3.

4) How many toothpicks are in the perimeter of Toothpick Square #4? Explain how you figured this number out.

5) Complete the following table for the number of toothpicks in the perimeter of the first five Toothpick Squares.

<table>
<thead>
<tr>
<th>Number of Toothpicks along one Side</th>
<th>Number of Toothpicks in Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

6) Write down 2 – 3 things you notice about the numbers in this table.
7) Draw a graph of the information in the table. 

*Use the horizontal (x) axis for the number of toothpicks along each side, and the vertical (y) axis for the number of toothpicks in the perimeter.*

---

8) How many toothpicks are in the perimeter of a Toothpick Square with 8 toothpicks on each side? Explain how you know.

9) How many toothpicks are on each side of a Toothpick Square that contains exactly 64 toothpicks in its perimeter? Explain how you know.
10) Tommy’s little sister Tina found the perimeter of four different square designs by counting all of the toothpicks around the outside, one by one. She’s worried she made a mistake.

Here are the numbers she got:

100 128 254 600

Which of these numbers must be wrong? Explain how you know.

11) Suppose Tina decides to count only the number of toothpicks on one side of a Toothpick Square. Write a rule that she can use to find the number of toothpicks in the perimeter of the design.

12) Let $x$ represent the number of toothpicks along one side of the design and let $y$ represent the number of toothpicks in its perimeter.

Re-write your rule from question #11 above as an equation of the form '$y = \ldots$'.
Another number Tommy likes to think about is the **number of small squares** contained in a Toothpick Square. For example, Toothpick Square #3 contains 9 small squares in it.

![Diagram of Toothpick Squares](image)

1) How many small squares are contained in Toothpick Square #4? ________

   Explain how you figured this number out.

2) Complete the following table for the number of small squares in the first five Toothpick Squares.

<table>
<thead>
<tr>
<th>Number of Toothpicks along one side</th>
<th>Number of Small Squares contained in design</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

3) Write down 2 – 3 things you notice about the numbers in this table.
4) Draw a graph of the information in the chart.

*Use the horizontal (x) axis for the number of toothpicks along each side, and the vertical (y) axis for the number of toothpicks in the perimeter.*

How is this like your graph from Part A, showing the number of toothpicks in the perimeter?

How is it different than that graph?
5) How small squares will there be in a design with 10 toothpicks on each side? Explain how you know.

6) How many toothpicks are on each side of a design that contains exactly 64 small squares? Explain how you know.

7) Is there a way to make a Toothpick Square that contains exactly 40 small squares? Explain how you know.

8) Is there a way to make a Toothpick Square that contains exactly 144 small squares? Explain how you know.

9) Suppose Tommy knows how many toothpicks he wants along each side of a design. Write a rule that he can use to find the number of small squares that it will contain.

10) Let x represent the number of toothpicks along one side of the design and let y represent the number of small squares it contains. Re-write your rule from question #9 above as an equation of the form 'y = ...'.
Another number that Tommy needs to know for his designs is the total number of toothpicks.

For example, Toothpick Square #1 uses a total of 4 toothpicks, while Toothpick Square #2 uses a total of 12 toothpicks.

1) How many toothpicks are needed to build Toothpick Square #3?  
How many toothpicks are needed to build Toothpick Square #4?  
Explain how you figured these numbers out.

2) Complete the following table for the total number of toothpicks in the first five designs.

<table>
<thead>
<tr>
<th>Number of Toothpicks along one side</th>
<th>Total Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
3) Use the paper provided to graph the information in the chart.
   Use the horizontal (x) axis for the number of toothpicks along each side,
   and the vertical (y) axis for the total number of toothpicks.

How is this graph like the graphs that you drew in Part A and Part B?

How is it different from the two earlier graphs?

Which of the two earlier graphs is it most like?
4) How many toothpicks will be needed for a design with 10 toothpicks on each side? Explain why you think this is correct.

5) Suppose Tommy knows how many toothpicks he wants along each side of a design. Write a rule that he can use to find the total number of toothpicks he will need.

6) Let \( x \) represent the number of toothpicks along one side of the design and let \( y \) represent the total number of toothpicks needed. Re-write your rule from question #5 above as an equation of the form \( y = \ldots \).
Bill needs to order new jerseys for the baseball team, the Bears. He has advertisements from two printing companies: "WE PRINT IT" and "A-ONE PRINTING".

**WE PRINT IT**

Get your baseball jerseys printed with your own team names here!
No set-up cost, and only $20 per jersey!

**A-ONE PRINTING**

We print your baseball jerseys – just supply us with your design. Pay a one-time set-up cost of $80; we then print each jersey for only $15!

Bill doesn’t know which company to choose.

He has asked you for help, and you have agreed to help him by studying the advantages of both pricing plans.

Work with your group to investigate the following questions:

- Under what circumstances would it be better to choose "WE PRINT IT"?
- Under what circumstances would it be better to choose "A-ONE PRINT"?

**NOTE:** You can use any of the materials in the room to help you study the two plans.
Bill thought it could be useful to talk with the managers of other teams in the league—but he needs your help to figure out if these managers made a good choice or not!

1. The manager of the Aces bought their jerseys from WE PRINT IT, and paid a total of $360.
   How many jerseys did the Aces’ manager buy? ______

   Would it have cost more money or less money with A-ONE? ______
   How much more or less? ______

2. The manager of the Cougars bought jerseys from A-ONE and paid a total of $290.
   How many jerseys did the Cougars’ manager buy? ______

   Would it have cost more money or less money with WE PRINT IT? ______
   How much more or less? ______
3. The manager of the Dynamites can’t remember which printing company they used, but remembers paying a total of $300.

Can we figure out which printing company the Dynamites bought from? 

- If so, say which company it was, and explain how you know this.
- If not, explain why we are not able to figure this out.

4. The manager of the Elites also can’t remember which printing company they used, but remembers paying a total of $380.

Can we figure out which printing company the Elites bought from? 

- If so, say which company it was, and explain how you know this.
- If not, explain why we are not able to figure this out.
5. The manager of the Frogs ordered jerseys at the start of the season from WE PRINT IT.

The parents of the team then decided they should have two different jerseys: one for the team to wear at home games, and the other for away games.

For the second set of jerseys, the manager decided to order from A-ONE.

In all, the Frogs spent a total of $500 on jerseys.

How many members were on the team? _________

Explain how you found this:
6. Bill plans to pay for all the jerseys that he needs for his Bears by holding fund raisers. But he also wants to buy as many jerseys as possible with this money they raise, so that he can add some more players to the team later on.

   a. Before the season begins, the Bears held several car washes and raised $260.

      Which company should Bill choose? 

      Explain why he should choose this company.

   b. Bill is also planning a spaghetti dinner fund raiser after the Bears' first game.

      He hopes to raise another $150.

      Should this make a difference in which company he chooses? 

      Explain why or why not:
BASEBALL JERSEYS
A New Plan

Bill just learned that there is a third company, IN-BETWEEN PRINTING, about to open up.

You contact Sue, the owner of IN-BETWEEN PRINTING, to find out about their pricing plan.

Sue tells you that she still is working out the details.

She wants to set her prices so that IN-BETWEEN PRINTING will never be the cheapest of the three printing companies in town, and also never be the most expensive of the three.

Since you’ve already studied the other two plans, she asks you for your help!

SUE’S IN-BETWEEN PRINTING

Let Sue print your baseball jerseys!
Pay a one-time set-up cost of $____.
Then each jersey will cost $____.

Use what you’ve learned about the other two printing companies to look for a pricing plan that Sue could use so that her prices are always in between the other two companies.

After you have tested out your ideas for Sue’s pricing plan, you will prepare a poster to present your findings to Bill and Sue.
BASEBALL JERSEYS
POSTER GUIDELINES

Your poster needs to include each of the following:

- A description of how the different pricing plans work
- A copy of your tables for the different plans
- A copy of your graphs for the different plans
- What you think the advantages of each pricing plan are

Your report to Bill:

- Under what circumstances would it be better to choose "WE PRINT IT", and why?
- Under what circumstances would it be better to choose "A-ONE PRINT", and why?

Your report to Sue:

- What ideas did you come up with for finding a plan with prices that will always be in between the other two companies?
- What did you do to test out your ideas?

You can use any of the materials in the room to get ready your poster ready.
# Baseball Jerseys: Comparison Sheet

As you read the other posters, use this chart to keep track of their conclusions.

<table>
<thead>
<tr>
<th>Group #</th>
<th>Group's idea about pricing plan for Sue's In-Between Printing</th>
<th>When is it better to choose &quot;WE PRINT IT&quot;?</th>
<th>When is it better to choose &quot;A-One Printing&quot;?</th>
<th>When is it better to choose &quot;Sue's In-Between Printing&quot;?</th>
<th>Other Notes about poster</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>One-time set-up cost = $_______</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cost per jersey = $_______</td>
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<tr>
<td>2</td>
<td>One-time set-up cost = $_______</td>
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<tr>
<td></td>
<td>Cost per jersey = $_______</td>
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<tr>
<td>3</td>
<td>One-time set-up cost = $_______</td>
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<tr>
<td></td>
<td>Cost per jersey = $_______</td>
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<tr>
<td>4</td>
<td>One-time set-up cost = $_______</td>
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<td></td>
<td>Cost per jersey = $_______</td>
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<tr>
<td>5</td>
<td>One-time set-up cost = $_______</td>
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<tr>
<td></td>
<td>Cost per jersey = $_______</td>
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<tr>
<td>6</td>
<td>One-time set-up cost = $_______</td>
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<tr>
<td></td>
<td>Cost per jersey = $_______</td>
<td></td>
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<td></td>
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</table>
A1. My Weekly Savings Plan

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td>Amount</td>
<td>$0</td>
<td>$17</td>
<td>$34</td>
<td>$51</td>
<td>$68</td>
<td>$85</td>
</tr>
</tbody>
</table>

A2. You and your friend, Diana, decide to combine your savings to buy an Xbox One (cost: $500). **How many weeks will it take to buy the Xbox together?**

Diana’s Weekly Savings Plan

The rule for Diana’s weekly savings plan can be described by $y = 60 + 3x$ where $y$ is the amount saved and $x$ is the number of weeks.

B1. My Weekly Savings Plan

I save $10 every week.

B2. You and your friend, Michael, decide to combine your savings to buy an Xbox One (cost: $500). **How many weeks will it take to buy the Xbox together?**

Michael’s Weekly Savings Plan

The rule for Michael’s weekly savings plan can be described by $y = 50 + 5x$ where $y$ is the amount saved and $x$ is the number of weeks.

C1. My Weekly Savings Plan

C2. You and your friend, Danny, decide to combine your savings to buy an Xbox One (cost: $500). **How many weeks will it take to buy the Xbox together?**

Danny’s Weekly Savings Plan

The rule for Danny’s weekly savings plan can be described by $y = -70 + 27x$ where $y$ is the amount saved and $x$ is the number of weeks.
F1. My Weekly Savings Plan

My Weekly Savings

<table>
<thead>
<tr>
<th>Week Number</th>
<th>Amount Saved ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8, -14</td>
</tr>
<tr>
<td>4</td>
<td>7, -21</td>
</tr>
<tr>
<td>6</td>
<td>6, -28</td>
</tr>
<tr>
<td>8</td>
<td>5, -35</td>
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<tr>
<td>10</td>
<td>4, -42</td>
</tr>
<tr>
<td>12</td>
<td>3, -49</td>
</tr>
<tr>
<td>14</td>
<td>2, -56</td>
</tr>
<tr>
<td>16</td>
<td>1, -63</td>
</tr>
<tr>
<td></td>
<td>0, 70</td>
</tr>
</tbody>
</table>

G1. My Weekly Savings Plan

I had $300 in my savings account. I spend $3 every week for a club membership card to Game Stop.

F2. You and your friend, Lizzie, decide to combine your savings to buy an Xbox One (cost: $500). How many weeks will it take to buy the Xbox together?

Lizzie’s Weekly Savings Plan

The rule for Lizzie’s weekly savings plan can be described by $y = -90 + 23x$ where $y$ is the amount saved and $x$ is the number of weeks.

G2. You and your friend, Brian, decide to combine your savings to buy an Xbox One (cost: $500). How many weeks will it take to buy the Xbox together?

Brian’s Weekly Savings Plan

The rule for Brian’s weekly savings plan can be described by $y = 40 + 10x$ where $y$ is the amount saved and $x$ is the number of weeks.
D1. My Weekly Savings Plan

I had $30 from my birthday. I also save $5 every week.

D2. You and your friend, Lisa, decide to combine your savings to buy an Xbox One (cost: $500). How many weeks will it take to buy the Xbox together?

Lisa’s Weekly Savings Plan

The rule for Lisa’s weekly savings plan can be described by \( y = -40 + 12x \) where \( y \) is the amount saved and \( x \) is the number of weeks.

E1. My Weekly Savings Plan

E2. You and your friend, Ben, decide to combine your savings to buy an Xbox One (cost: $500). How many weeks will it take to buy the Xbox together?

Ben’s Weekly Savings Plan

The rule for Ben’s weekly savings plan can be described by \( y = -120 + 12x \) where \( y \) is the amount saved and \( x \) is the number of weeks.
Question 2

Here is the equation of a weekly savings plan:

\[ y = 12x + 40 \]

Describe in words how this saving plan works.

Question 3

Here is the graph of a weekly savings plan:

Describe in words how this saving plan works.
\begin{align*}
y &= 7x \\
y &= 10x + 30 \\
y &= 30 \\
y &= 30 - 5x
\end{align*}

Carol started out with $30 in savings, and managed to save $10 each week.

Evelyn started out with $30 in savings, and never saved any more money. She never spent any of her savings either.

Bob started out with $30 in savings, and spent $5 from his savings each week.

Alice started out with $30 in savings, and managed to save $7 each week.

David started out with no savings, and managed to save $7 each week.

<table>
<thead>
<tr>
<th># of Weeks</th>
<th>Dollars Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>37</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
</tr>
<tr>
<td>3</td>
<td>51</td>
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<td>60</td>
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<tr>
<td>4</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
</tr>
<tr>
<td>$y = 7x$</td>
<td>$y = 7x + 30$</td>
</tr>
<tr>
<td>---------------</td>
<td>------------------</td>
</tr>
<tr>
<td>$y = 10x + 30$</td>
<td></td>
</tr>
<tr>
<td>$y = 30$</td>
<td>Carol started out with $30 in savings, and managed to save $10 each week.</td>
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<tr>
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<td>10</td>
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</table>
Machines at Work

Use the example below to complete the following machines.

**EXAMPLE:**

```
Input
Short

The Opposite

Tall

Output
```

1. 

```
3 sides

The shape name

The opposite

```

```
hexagon
```

2. 

```
Arizona

The first letter of the state

```

```
The first letter of the state

F
```

3. 

```
Meow

Animal that makes the noise

```

```
Animal that makes the noise

Pig
```
4. Texas
   USA capital city
   
5. 10
   Add 4 to the number
   
6. -2
   Multiply the number by 4
   
7. Make 2 different machines of your own below for a friend to complete.
Input-Output Machines

1) IN 2 x 3 → + 1 → OUT

2) IN 16 x 3 → + 1 → OUT

3) IN -5 x 2 → + 4 → OUT

4) IN 36 x 2 → + 4 → OUT

5) IN 32 ÷ 8 → + 5 → OUT

6) IN 13 ÷ 8 → + 5 → OUT

7) IN 48 ÷ 5 → - 2 → OUT