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ANALYSIS OF HYDRAULIC GEOMETRY RELATIONSHIPS IN ALLUVIAL CHANNELS

by

Pierre Y. Julien and Daryl B. Simons

Civil Engineering Department
Engineering Research Center
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ACKNOWLEDGMENTS

This study has been completed at the Engineering Research Center at Colorado State University during the post-doctoral studies of the first author. A NATO post-doctoral fellowship made this study possible, and this support provided by the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. The writers wish to extend their appreciation to all those who reviewed this report, and express special thanks to Dr. C. R. Thorne, Dr. R. M. Li, G. O. Brown, C. Mendoza and J. S. O'Brien. Their suggestions and comments were very helpful in the final preparation of this document.

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LIST OF VARIABLES

a	exponent of the resistance equation
b	coefficient of the resistance equation
c	distance between F_c and F_p
C	Chézy coefficient
d	distance between F_p and F_s
d_b	sediment size of bank material
d_s	sediment size of bed material
e	exponent of geometry relationships
f	Darcy-Weisbach friction factor
F_c	centrifugal force
F_p	pressure force
F_s	shear force
g	gravitational acceleration
G	density of grains
h	average depth
i,j,m	exponents of geometry relationships
k_l	longitudinal Shields number
k_t	transversal Shields number
Q	discharge
r	radius of curvature
R	ratio of bank to bed stability
R_d	ratio of bank to bed sediment size
S	slope
S_t	transverse slope
$\bar{u}$	local longitudinal velocity
$\bar{U}$	average longitudinal velocity

w	channel width
z	vertical coordinate
γ	specific weight of water
ρ	mass density of water
τ_o	longitudinal bed shear stress
τ_t	bed shear stress in the transverse direction

I. INTRODUCTION

Equilibrium of alluvial streams has been thoroughly studied in the past century. Many investigators have extended analysis to explain meandering (or braiding) of streams, and attempted to describe the hydraulic geometry of alluvial streams. This study points at the derivation of the characteristics of alluvial streams from fundamental principles. More precisely, this research aims to determine the downstream geometry of alluvial streams (channel width, depth, velocity, slope and radius of curvature), as a function of sediment size and water discharge. In this report, a brief review of literature is presented, then the concept of a new approach is detailed including the analysis of variables and fundamental equations. The theoretically derived hydraulic geometry relationships are then compared with existing empirical equations, followed by similar derivations for smooth channels and few notes on channel adjustment.

II. LITERATURE REVIEW

Excellent reviews of previous studies were presented by Graf (1971), Chitale (1973), Engelund and Skovgaard (1973), Callander (1978), and Engelund and Fredsøe (1982).

Many studies in the past have considered the case of meandering starting from a straight channel condition. Callander (1969) pointed out that straight bank channels with loose boundaries are unstable with the possible exception of channels just beyond the threshold of grain movement. Langbein and Leopold (1966) stated that meandering is the most probable form of channel. Its geometry is more stable than one of non-meandering alignment. Chang (1979a) concluded that a meandering river is more stable than a straight one as it expends less stream power per unit channel length for the system. He also stated that a stable alluvial channel represent the best hydraulic efficiency under the given condition. Onishi et al. (1976) also suggest that meandering channels can be more efficient than a straight one as for a given water discharge it can transport a larger sediment load and can require a smaller energy gradient.

Most of the research found in the literature can be classified under one of the following categories, namely: a) regime approach, b) minimum stream power, c) statistical theory and spectral analysis, d) secondary currents and e) stability analysis.

2.1 Regime Approach

The regime approach was developed by Kennedy (1895), Lindley (1919), Lacey (1929), Lane (1937), and Blench (1969, 1972) after replacing the word "equilibrium" with "regime". With the purpose to define the geometry of alluvial channels, several empirical relationships supported by field observations were derived. Simons and Albertson (1963) differentiated several channel conditions and their graphical relationships were supported analytically by Henderson (1966). From dimensional analysis and physical reasoning, several authors, Chien (1957), Henderson (1961) Stebbins (1963), Gill (1968) and White et al. (1982) have presented some physical support to the regime equations.

2.2 Minimum Stream Power

The theory of minimum variance was first stated by Langbein and Leopold (1966). Though it does not explain the processes, the method describes the net behavior of a river. The minimization involves the adjustment of the planimetric geometry and the hydraulic factors of depth, velocity and local slope. Yang (1971a, 1976) stated that the time rate of energy expenditure explains the formation of meandering streams. He also describes alluvial processes in terms of minimum stream power. Other studies by Maddock (1970) and Chang and Hill (1977) and Chang (1979b, 1980) use the principle of minimum stream power. As summarized by Cherkauer (1973), streams adjust their flow so as to minimize total power expenditure, and to minimize the sums of variances of power and of the dependent variables.

2.3 Statistical Theory and Spectral Analysis

Thakur and Scheidegger (1968) analyzed the probability for a stream to deviate by an angle $d\Phi$ in progressing an elemental distance d_s along its course. Their statistical study confirm the probabilistic view of meander development suggested by Langbein and Leopold (1966). Further developments were provided by Surkan and Van Kan (1969) showing that neither the directions, curvatures, nor their changes in natural meanders are Gaussian independent. Spectral analysis of meanders by Speight (1965), Ferguson (1975) and Dozier (1976) indicate that the characteristic meander wavelength is a poor indicator of the dominant frequencies of oscillation. As pointed out by Thakur and Scheidegger (1970) there seems to be more than one characteristic wavelength in a meander system.

2.4 Secondary Currents

According to Quick (1974), the meander mechanism is basically a fluid mechanics problem in which vorticity plays a leading role. Flow in a meander bend has been studied in detail by Rozovskii (1957), Yen (1967, 1970, 1972), Muramoto (1967), Chiu et al. (1978, 1981) and others. The problem is extremely complex and the Navier-Stokes Equation must be simplified to obtain a theoretical approximation. Rouse (1965) and Odgaard (1982) recognize that the energy gradient of flow in a meandering channel is Froude number dependent. Einstein and Li (1958) made a theoretical investigation of secondary currents under laminar and turbulent conditions. Einstein and Shen (1964) defined two types of meander patterns of straight alluvial channels with nonerodible banks: 1) those when the flow is nearly critical; and 2) those flows with alternating scour holes between rough banks. These studies were extended by Shen and Komura (1968) and Shen and Vedula (1969).

2.5 Stability Analysis

Several attempts have been made to explain the origin of meandering. Local disturbances, earth rotation, excessive energy and hydrodynamic stability figure among the best hypothesis so far. What causes meanders is still a question without a complete answer, although the case for dynamic stability is strong. This statement by Callander (1969) appears to be still valid. The stability of the sediment-water interface was presented by Exner (1925). Einstein (1926) described the effect of earth rotation and Coriolis forces to induce circulation. An analytical approach to local disturbances was presented by Werner (1951). A similar relationship for meander length was also derived from the concept of transverse oscillations by Anderson (1967). He concluded that meander length is related to the Froude number and that no unique relationship exist between meander length and discharge.

Adachi (1967) and Hayashi (1970) used small amplitude oscillation techniques to explain the origin of meandering. Engelund and Skovgaard (1973) developed a three-dimensional model to analyze the hydrodynamic stability of a straight alluvial channel. Parker (1976) used a perturbation technique involving the ratio of sediment transport to water transport in a straight reach. He concluded that existence of sediment transport and friction are necessary for occurrence of instability. In the cases where the channel width is known, he obtained a relationship for differentiating meandering and braided regimes. He observed meandering in ice (Parker, 1975) and suggested that in absence of sediment load the origin of sinuosity is purely hydrodynamic. Other evidences of meandering in ice, in bedrock, density currents and flow of the Gulf Stream were reported by several researchers: Leopold and Wolman (1960), Leopold et al. (1964), Dury (1965), Gorycki (1973), Parker (1975), Zeller (1967). New theories include Parker et al. (1982). Though several theories were proposed, they are not always supported by experimental data, Chang et al. (1971).

III. VARIABLES AND EQUATIONS

The detailed analysis of alluvial channels is complex, and one major difficulty in research is the definition of variables. Discharge varies with time while most theories are limited to steady-flow conditions. The motion of dominant discharge, for example, is still subject to interpretation. Also, the representative size fraction to define the roughness of a stream varies among researchers. Common reference is made to d_{50} and d_{65} but under certain conditions, some authors suggest d_{84} or d_{90}. Furthermore, the presence or absence of bed forms in alluvial streams are extremely important regarding the total resistance to flow. Gregory and Madew (1982) made a step forward in the rationalization of the variables, and they summarized the significance of flows for various recurrence intervals. However, more work has to be done to define the representative bed material size and water discharge of an alluvial streams. For this reason, throughout this paper these two variables are considered without any specific reference to a particular definition (such as mean annual discharge, dominant discharge or d_{65} for example).

Hey (1978, 1982a) presented an analysis of variables, degrees of freedom and governing equations for gravel rivers. He considers that the sediment discharge, water discharge, and sediment size, are independent variables, while velocity, hydraulic radius, slope, wetted perimeter, maximum flow depth, sinuosity and meander arc length are dependent variables.

Hey (1982a) states that the governing equations for gravel rivers are: 1) continuity, 2) flow resistance, 3) sediment transport, 4) bank erosion, 5) bar deposition, 6) sinuosity and 7) riffle spacing. Unfortunately, many of these equations are not adequately defined, therefore restricting the utility of this approach.

He also points out that further research to develop general theoretically based process equations remain a priority. A step forward had been done by Kellerhals (1967) by combining an empirical Lacey type equation with a threshold type equation and a power form of resistance equation. The equation derived seems to be dependent on the data on which it was derived. Smith (1974) used conservation principles and a sediment trasnport law to define the hydraulic geometry of steady-state channels. His relationships are similar to those found by Leopold and Maddock (1953), though his assumptions are restrictive. Li, Simons and Stevens (1976) derived hydraulic geometry relations for both at-a-station and downstream cases. Their results theoretically support those suggested by Leopold and Maddock (1953). An analysis of steady flow conditions in alluvial channels is found in Holtorff (1982a), however, no alluvial geometry relationships were obtained. Bray (1982b) proposed other methods for gravel-bed rivers among which his so-called threshold method which is based on Lacey equation, Manning-Strickler resistance relationship and Neill's threshold equation. The results obtained with the derived equations for width, depth, velocity and slope compare fairly well with observed data though they cannot be regarded as theoretically based relationships.

From the literature review meandering has been observed on ice, bedrock and in the Gulf Stream and previous analysis suggest that secondary flow in bends plays a leading role in meandering.

The major question of interest in this paper is to define the hydraulic geometry of alluvial streams (top width w, average depth h, average velocity $\bar{U}$ and slope S) for a given discharge Q over sediments of a given size d_s. Therefore, three types of conditions are suggested to describe alluvial streams:

a) continuity and flow resistance,

b) threshold condition,

c) flow in bends.

The first two conditions are often referred to in the literature, while the last condition for flow in bends is a new element in this type of analysis.

3.1 Continuity

The continuity equation for steady channel flow is:

$$Q = w\, h\, \bar{U} . \tag{1}$$

in this equation, w is the channel top-width, h is the mean flow depth and $\bar{U}$ is the average velocity across the section.

3.2 Flow Resistance

A resistance to flow relationship for alluvial streams is very complex. The Keulegan equation (1938) is a theoretically sound relationship to represent resistance in uniform rough channels. When the mean flow depth is nearly equal to the hydraulic radius, one can write:

$$C = \sqrt{\frac{8g}{f}} = 32.6 \log \left(\frac{12.2\ h}{d_s}\right) \tag{2}$$

Unfortunately, flow resistance is not so simple due to bed forms, non-uniformity of cross sections and of sediment gradation, (Simons et al., 1977, 1979; Gladki, 1979). Modifications of the original equation were proposed by Burkham and Dawdy (1976), Hey (1979), Bathurst (1978, 1982), and Bray (1979, 1982a). Also, some authors have shown departures from the original log-law and power laws that were proposed by Leopold and Wolman (1957), Kellerhals (1967), Church (1972), and Day (1977). The Darcy-Weisbach friction factor f is given by:

$$\frac{1}{\sqrt{f}} = b\ \left(\frac{h}{d_s}\right)^a \tag{3}$$

Kellerhals suggested $a = 0.25$, while for $0.7 < (h/d_s) < 10$, Leopold and Wolman found $a = 0.5$ and further analysis by Church showed that $0.43 < a < 3.35$. Though most of these studies were carried on gravel bed rivers, it must be remembered that for the well-known Manning-Strickler relationship, $a = 1/6 \cong 0.167$.

The increase of "a" as the ratio h/d_s decreases can be predicted from the logarithmic law. Evidence can be given whether from plotting both functions on a log paper, or mathematically in the following way. If we assume the Keulegan equation for turbulent rough flow to be valid, the parameters a and b of a power relationship can be derived analytically when both functions and their slopes are equal such that:

$$b\left(\frac{h}{d_s}\right)^a = 4.68 \ln \left(12.2 \frac{h}{d_s}\right) \tag{4}$$

and, the first derivative is

$$a\, b\left(\frac{h}{d_s}\right)^{a-1} = 4.68 \frac{d_s}{h} \tag{5}$$

Combining these two equations gives

$$a = \frac{1}{\ln\left(\frac{12.2\ h}{d_s}\right)} \tag{6}$$

$$b = \frac{4.68}{a} \left(\frac{d_s}{h}\right)^a \tag{7}$$

Equation 6 has been plotted in Figure 1, and compared with Chézy and Manning-Strickler equations. It must also be noticed that when h/d_s goes to infinity, the exponent a tends to zero, which corresponds to the Chézy equation. One further observes that for a wide range of flow conditions, the exponent value differs only slightly from the Manning-Strickler equation and therefore support its wide use in common practice. For ratios of h/d_s varying from 1 to 10, however, the exponent a of the power relationship varies respectively from 0.40 to 0.20. Thus, when the relative roughness is very large, such as in gravel beds, the commonly used Manning-Strickler equation $a = 0.17$ should not represent adequately the flow conditions. Henceforth, Manning equation must be used with great care when dealing with flows having large roughness elements compared to flow depth. Therefore, the following power-equation with a variable exponent has been selected for this study.

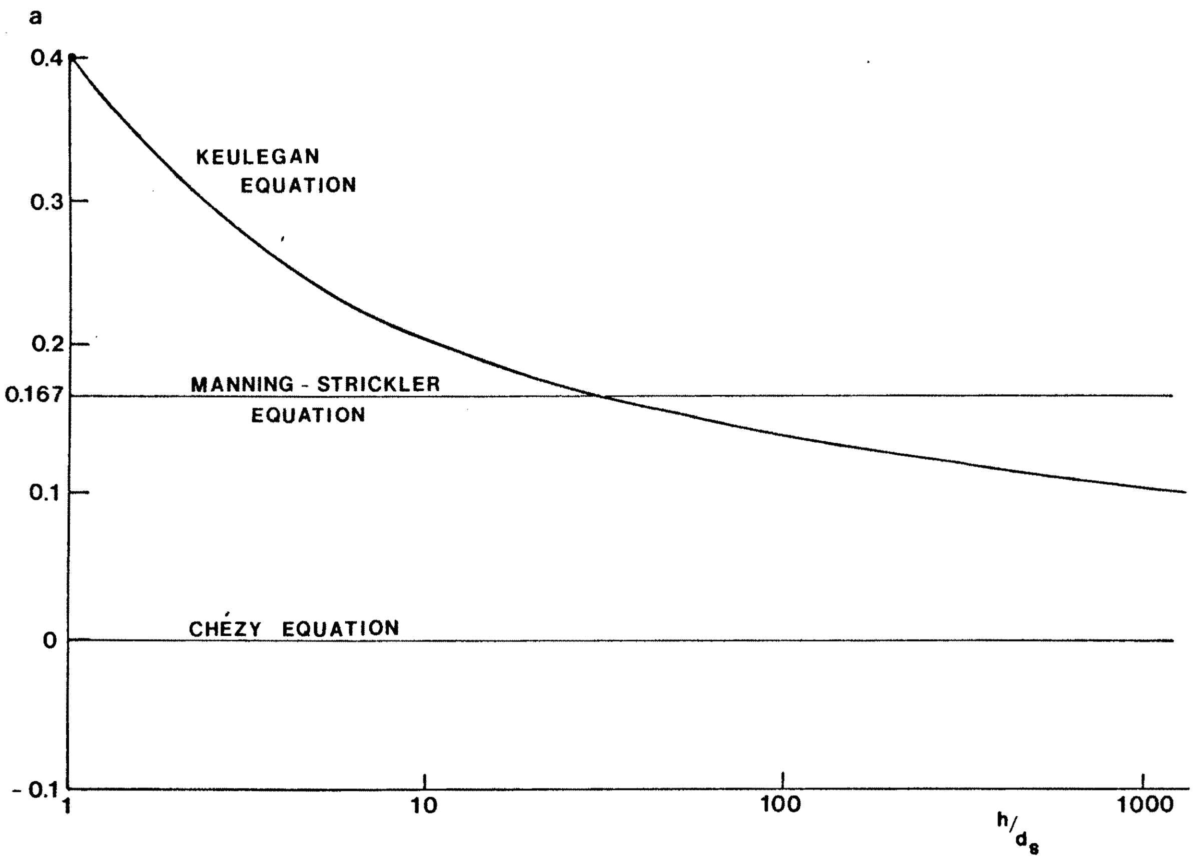

Fig. 1. Exponent a vs. log h/d_s.

$$\bar{U} \;\alpha\; \left(\frac{h}{d_s}\right)^a h^{1/2} S^{1/2} \tag{8}$$

In Eq. 8, only the functional relationship is considered and the equality sign has been replaced by the proportionality sign.

3.3 Longitudinal Threshold

Stability of alluvial channels can be described by the relative magnitude of shear forces exerted on the bed and the resistive forces to motion of individual grains. For noncohesive sediments, the ratio of these two forces is a characteristic of an alluvial channel and similar ratios can be expected for similar channels. This ratio is defined by the Shields number and, for turbulent rough flows:

$$\frac{\tau_o}{\gamma(G-1)\ d_s} = k_\ell \tag{9}$$

in which τ_o: longitudinal bed shear stress

k_ℓ: longitudinal Shields number

γ : specific weight of water

G : density of grain.

The coefficient k_ℓ is the Shields number. When this number reaches a certain critical value, it represents the incipient motion of the bed material. As the Shields number increases (above the critical value) we should expect an increase in the rate of sediment transport. Therefore, the Shields number k_ℓ is also an indicator of the rate of sediment transport, and is proportional to the sediment load Q_s.

From the equilibrium condition of a steady uniform flow, the bed shear stress is:

$$\tau_o \;\alpha\; \gamma h S \tag{10}$$

In natural rivers the density of grains remain fairly constant such that the equation for longitudinal threshold is obtained from Eqs. 9 and 10:

$$hS \propto d_s \, k_\ell \tag{11}$$

This equation is a descriptive equation for longitudinal stability of alluvial channels under turbulent rough flow conditions. It may be noted that similar results are obtained from the ratio of fall velocity to shear velocity.

3.4 Transversal Threshold

As stated previously, several authors concluded that a meandering river is more stable than a straight one. Thus, consideration must be made to the very complex problem of flow in bends.

Analytical treatment of flow in bends is generally based on the Navier-Stokes equations modified by Reynolds for turbulent flows. Secondary flow involve centrifugal force, pressure, shear stress and inertia. For a complete treatment, none of these can be neglected but these equations cannot be solved analytically. Odgaard (1981) studied the transverse slope in a bend and the following first order approximation has been proposed by Kondrat'ev (1933), Rozovskii (1957), and Yen (1972):

$$\frac{\bar{u}^2}{r} = g\, S_t - \frac{1}{\rho}\, \partial\, \frac{\tau_t}{\partial z} \tag{12}$$

in which $\bar{u}$: local longitudinal velocity

r : radius of curvature

S_t: transverse water surface slope

g : gravitational acceleration

ρ : mass density of fluid

τ_t: transverse bed shear stress

z : vertical coordinate.

Equation 12 neglects spatial derivatives in a steady turbulent flow. It expresses the equilibrium condition between centrifugal acceleration, radial pressure gradient and vertical shear stress gradient. After integration of Eq. 12 over the depth h, simplified force equilibrium conditions are shown in Figure 2. In a broad sense, the pressure force F_p balance the sum of centrifugal force F_c and shear force F_s. Also, moment equilibrium around the point A gives:

$$\frac{F_c}{F_s} \propto \rho \frac{h\bar{U}^2}{r\tau_t} \propto \frac{d}{c} \tag{13}$$

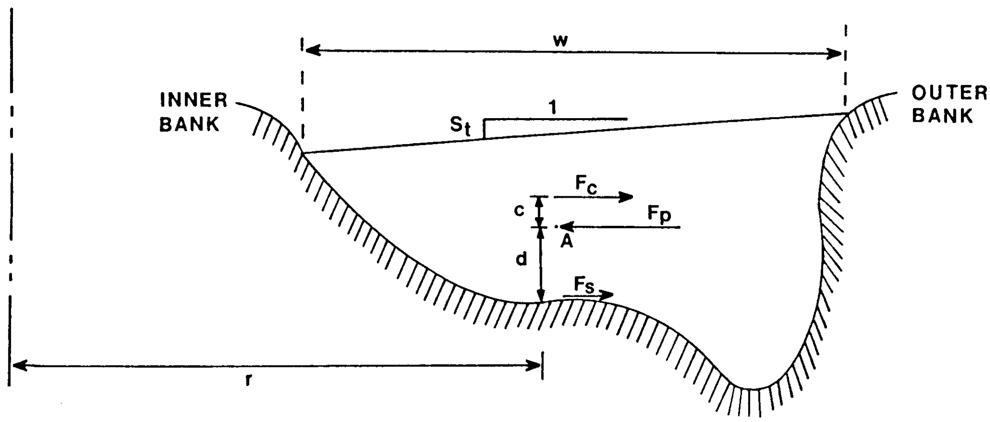

Figure 2. Simplified force equilibrium in a bend.

This simplified relationship just tells that the centrifugal force generating motion, is proportional to the shear force abating the motion and dissipating energy. For similar channels, one must expect that the force ratio should be constant and equal to the ratio d/c. The transverse stability of a stream can be analyzed. An equilibrium criterion for lateral stability can be defined from the ratio of transverse shear forces to resistive forces of individual gains. The resulting criterion has the same form as the Shields number (Eq. 9), except that the longitudinal shear stress τ_o has been replaced by the transversal shear stress τ_t.

The stability and scour of the outer bank in alluvial bends is linked to secondary flows. Since bank material might differ from bed material, the transversal threshold condition should preferably be function of the bank material sediment size d_b, and the transversal shear stress τ_t. The transverse Shields number k_t is then defined:

$$\frac{\tau_t}{\gamma(G-1)\ d_b} = k_t \tag{14}$$

By introducing the ratio of bank to bed material $R_d = d_b/d_s$, the transverse threshold condition is obtained from the integrated form of Eq. 12, and from Eq. 14:

$$\frac{h\ \bar{U}^2}{r} \alpha \frac{\tau_t}{\rho} \alpha\ g\ d_b\ k_t = g\ d_s\ R_d\ k_\ell \tag{15}$$

This simple relationship describes bank stability in bends.

Like for the parameter k_ℓ defined previously, the parameter k_t represents the transversal Shields number. A critical value represents the incipient motion and increasing values of k_t (above the critical value) indicate an increasing rate of sediment transport in the transverse direction. Equation 15 introduces a new variable which was not considered previously: the radius of curvature r. Therefore, an additional equation must be provided to solve the set of equations.

3.5 Similitude in Bends

As mentioned by Quick (1974), some writers remarked that it is difficult to tell the size of a river from aerial photographs of their meanders. This simply means that there exist similitude between various plan views of meanders. The plan view of meanders is described by two variables: the river width and the radius of curvature. Similar meandering channels have the same ratio of width to radius of curvature. This is:

$$r \propto w \tag{16}$$

This equation finds theoretical support from the variation of centrifugal force along a cross section. The magnitude of this force being inversely proportional to the radius of curvature, it varies from the right bank to the left bank. Therefore, similar r/w ratios correspond to similar centrifugal force distributions over the section. Bagnold (1960) points out that minimum resistance occurs when the radius of curvature bears a certain critical ratio to the channel width. Leopold and Wolman (1960) and Hickin (1974) show considerable evidence that when a stream develop meander patterns, the ratio r/w tend to a common value between 2 and 3.

IV HYDRAULIC GEOMETRY RELATIONSHIPS

Five equations can be used to determine the hydraulic geometry of alluvial channels: 1) continuity (Eq. 1), 2) flow resistance (Eq. 8), 3) longitudinal threshold (Eq. 11), 4) transverse threshold flow (Eq. 15), and 5) bend geometry (Eq. 16). In these equations, the rate of sediment transport is indicated by the factors k_ℓ (longitudinal direction) and k_t (transversal direction), and the sizes of bed material and bank material are treated separately.

For a given condition of discharge Q and sediment size d_s, these five equations were combined to obtain the following hydraulic geometry relationships, (see detailed derivations in Appendix A for flow depth, channel width or radius of curvature, velocity and slope).

$$h \alpha Q^{\frac{1}{2+3a}} d_s^{\frac{6a-1}{4+6a}} (R_d k_t)^{\frac{1}{2+3a}} k_\ell^{\frac{-3}{4+6a}} \tag{17}$$

$$w \alpha r \alpha Q^{\frac{1+2a}{2+3a}} d_s^{-\frac{1+4a}{4+6a}} (R_d k_t)^{-\frac{1+a}{2+3a}} k_\ell^{\frac{1}{4+6a}} \tag{18}$$

$$\bar{U} \alpha Q^{\frac{a}{2+3a}} d_s^{\frac{1-a}{2+3a}} (R_d k_t)^{\frac{a}{2+3a}} k_\ell^{\frac{1}{2+3a}} \tag{19}$$

$$S \alpha Q^{-\frac{1}{2+3a}} d_s^{\frac{5}{4+6a}} (R_d k_t)^{\frac{-1}{2+3a}} k_\ell^{\frac{7+6a}{4+6a}} \tag{20}$$

These relationships depend upon the value of the parameter a which may vary from 0 to roughly 0.4. The exponents of each equation are computed for three cases. The Chézy equation correspond to the case when a = 0, the Manning equation correspond to a = 1/6, and for very high relative roughness (a = 1/3).

In the following, all the variables (Q, d_s, R_d, k_t and k_ℓ) are analyzed. Also, for stable alluvial channels, one may consider the cases in which the ratio of bank to bed material sizes is the same and that incipient motion for turbulent rough conditions is given by constant values of k_ℓ and k_t. Therefore, for most channels, the hydraulic geometry relationships can be described only as a function of two variables, namely Q and d_s.

4.1 Flow Depth Relationships

In Table I, the flow depth relationships given by Eq. 17 show a slight decrease in the exponent of water discharge with increasing relative roughness (coefficient a) and is independent of sediment size when Manning relationship applies. The exponents of R_d and k_t are similar to the exponent of water discharge. This indicates that for increasing bank roughness increases the flow depth. On the other hand, the negative values of the exponent of k_ℓ show that for an increase in k_ℓ, corresponding to an increase of sediment load, the flow depth decreases. This is in agreement with qualitative principles in fluvial geomorphology (Schumm, 1977). Several authors defined the flow depth uniquely as a function of discharge and the exponent varies from 0.30 to 0.50. When parameters Q and d_s are considered, both values of exponents are in agreement with those derived theoretically. The most interesting results are those equations for shallow and deep gravel-bed channels (Charlton, 1982). Both equations are in perfect agreement with Eq. 17.

4.2 Channel Width Relationships

Channel width relationships in Table II show a slight increase of the exponent of discharge with increasing relative roughness. On the other hand, Eq. 18 gives negative exponents for sediment size. The same trend was obtained by the few researchers who included sediment size in their analysis but the exponent for d_s is generally smaller than those given by Eq. 18. The exponents of discharge compares fairly well with those of Eq. 18 though the variation of "e" with relative roughness could not be verified by Lacey type of equations.

Table I. Flow Depth Relationships

$$h \propto Q^e d_s^{\,i} (R_d k_t)^j k_\ell^m \qquad \text{(Eq. 17)}$$

	h/d_s	a	e	i	j	m
Chézy Type	∞	0	0.50	-0.25	0.50	-0.75
Manning Type	30	1/6	0.40	0	0.40	-0.60
Very Rough	2	1/3	0.33	0.17	0.33	-0.50

Observed	e	i	Remarks
Bray (1982b)	0.428	-0.285	gravel beds (semi-empirical)
Bray (1982b)	0.397	0.008	gravel beds (regression)
Bray (1982b)	0.331	-0.025	gravel beds (regression)
*Hey (1982b)	0.38	-0.16	gravel-bed rivers
**Charlton (1982)	0.42	-0.14	deep gravel-bed channels $3 < h/d_s < 80$
**Charlton (1982)	0.25	0.33	shallow gravel-bed channels $h/d_s < 3$
Hey (1978)	0.46	-0.15	Fixed bed, coarse material
Engelund and Hansen (1967)	0.317	0.21	With sediment transport
Kellerhals (1967)	0.400	-0.120	gravel beds
†Lacey (1929)	0.33	-0.167	regime equation
Bray (1982b)	0.333	--	gravel-bed rivers
Parker (1982)	0.33-0.50	--	gravel-bed rivers
Ackers and Charlton (1970)	0.44	--	separation straight-meandered
Blench (1969)	0.33	--	regime equation
Henderson (1966)	0.36	--	
Leopold and Miller (1956)	0.30	--	ephemeral streams
Leopold and Maddock (1953)	0.40	--	downstream
Langbein (1964)	0.37	--	theoretical

†Hydraulic radius instead of mean flow depth.
*Maximum flow depth instead of mean flow depth.
**Charlton used two sediment sizes (d_{65} and d_{90}).

Table II. Channel Width and Radius of Curvature Relationships

$$r \propto w \propto Q^e \, d_s^{\,i} (R_d \, k_t)^j \, k_\ell^{\,m} \qquad \text{(Eq. 18)}$$

	h/d_s	a	e	i	j	m
Chézy Type	∞	0	0.50	-0.25	-0.50	0.25
Manning Type	30	1/6	0.53	-0.33	-0.47	-0.20
Very Rough	2	1/3	0.56	-0.39	-0.44	0.17

Observed	e	i	Remarks
*Hey (1982b)	0.41	-0.15	gravel-bed rivers
Bray (1982b)	0.496	-0.241	gravel beds (regression)
Bray (1982b)	0.528	-0.070	gravel beds (regression)
Hey (1978)	0.46	-0.15	Fixed bed, coarse material
Engelund and Hansen (1967)	0.525	-0.316	With sediment transport
Henderson (1963)	0.50	-0.15	
Blench	0.50	-0.25	cited in Engelund (1967)
Bray (1982b)	0.527	--	gravel-bed rivers
Parker (1982)	0.38-0.45	--	gravel-bed rivers
Charlton (1982)	0.45	--	gravel-bed rivers
Ackers and Charlton (1970)	0.42	--	separation straight-meandered
Blench (1969)	0.50	--	regime equation
Kellerhals (1967)	0.50	--	gravel beds
Carlston (1965)	0.47	--	field data
Langbein (1964)	0.53	--	theoretical
Leopold and Maddock (1953)	0.50	--	downstream geometry
*Lacey (1929)	0.50	--	regime equation

*Wetted perimeter instead of channel width.

The exponent of R_d and k_t is negative. This indicates that the bank material has a significant influence on the channel width. It is widely agreed that rough banks will reduce the channel width, and this is well predicted by Eq. 18. On the other hand, the sediment load appears to have only a slight influence on the channel width. Indeed, the exponents of k_ℓ are shown to be relatively small. Equation 18 tells that an increase in sediment load should give a small increase in channel width. This supports qualitative concepts in channel adjustments (Schumm, 1977).

4.3 Velocity Relationships

In Table III, the velocity relationships given by Eq. 19 show a large decrease in "i", and a slight increase in "e" with increasing relative roughness. Exponents of discharge are in the same range as those obtained from field investigation, while Eq. 19 seems to slightly overpredict the exponent of the sediment size. This analysis clearly indicates that the channel width and the velocity are not only function of discharge. The sediment size appears to be an important factor in such relationships for alluvial channels, and this is well supported by experimental data.

The exponent of R_d and k_t is shown to be unsensitive to the relative roughness of the channel. Increased bank roughness correspond to slightly higher water velocities. Similarly, the rate of sediment transport is proportional to the velocity, as one might naturally expect.

4.4 Slope Relationships

The slope relationships (in Table IV) also seem to depend on several parameters. As computed from Eq. 20, the discharge exponent increases gradually (while the sediment exponent decreases) with

Table III. Velocity Relationships

	$U \alpha Q^e d_s^{\,i} (R_d k_t)^j k_\ell^m$				(Eq. 19)
	h/d_s	a	e	i	j	m
Chézy Type	∞	0	0	0.50	0.00	0.50
Manning Type	30	1/6	0.07	0.33	0.07	0.40
Very Rough	2	1/3	0.11	0.22	0.11	0.33

Observed	e	i	Remarks
Bray (1982b)	0.071	0.285	gravel beds semi-empirical
Bray (1982b)	0.107	0.233	gravel beds (regression)
Bray (1982b)	0.140	0.095	gravel beds (regression)
Hey (1978)	0.08	0.30	Fixed bed, coarse material
Kellerhals (1967)	0.100	0.120	gravel beds
Lacey (1929)	0.167	0.167	regime equation
Bray (1982b)	0.140	--	gravel beds
Blench (1969)	0.17	--	regime equation
Langbein (1964)	0.10	--	theoretical
Leopold and Miller (1956)	0.20	--	ephemeral streams
Leopold and Maddock (1953)	0.10	--	downstream

Table IV. Slope Relationships

$$S \propto Q^{e} \, d_s^{\,i} (R_d \, k_t)^{j} \, k_{\ell}^{\,m} \qquad \text{(Eq. 20)}$$

	h/d_s	a	e	i	j	m
Chézy Type	∞	0	-0.50	1.25	-0.50	1.75
Manning Type	30	1/6	-0.40	1.00	-0.40	1.60
Very Rough	2	1/3	-0.33	0.83	-0.33	1.50

Observed	e	i	Remarks
Bray (1982)	-0.428	1.285	gravel beds (semi-empirical)
Bray (1982)	-0.375	0.937	gravel beds (regression)
Bray (1982)	-0.334	0.586	gravel beds (regression
*Charlton (1982)	-0.42	1.14	deep gravel-bed channels $3 < h/d_s < 80$
*Charlton (1982)	-0.25	0.67	shallow gravel-bed channels $h/d_s < 3$
Hey (1978)	-0.46	1.15	Fixed bed, coarse material
Kellerhals (1967)	-0.400	0.920	gravel beds
Henderson (1966)	-0.44	1.14	separation single-thread to braided
Engelund and Hansen (1967)	-0.212	0.527	With sediment transport
Henderson (1961)	-0.46	1.15	
Lacey (1929)	-0.167	0.83	regime theory
Hey (1982a)	-0.68	--	stable ripple sites
Ackers (1982)	-0.21	--	separation straight to meandered
Bray (1982b) (-0.19→-0.68)	-0.342	--	gravel-bed rivers
Parker (1982)	-0.02→-0.46	--	gravel-bed streams
Ackers and Charlton (1970)	-0.12	--	separation straight to meandered to braided
Leopold and Wolman (1957)	-0.44	--	separation meandering to braided
Lane (1937)	-0.25	--	separation meandering to braided

*Charlton used two sediment sizes (d_{65} and d_{90}).

increasing relative roughness. When compared to field analysis with two parameters, there exist an excellent agreement between observed relationships and Eq. 20. The most striking example is given by Charlton (1982). Indeed, after classification between shallow and deep gravel-bed channels, the regression equations obtained from experimental data correspond precisely to the exponents given by Eq. 20. Furthermore, when sediment size is not included in the analysis of field data, the exponent of discharge is shown to vary largely ($-0.68 < e < -0.02$).

The slope is inversely proportional to R_d and k_t. On the other hand, it is highly dependent on the rate of sediment transport. Equation 20 shows that the slope increases with increasing sediment load. This supports qualitative geomorphologic principles reported by Schumm (1976).

It is concluded from this analysis that hydraulic geometry relationships are a complex function of several variables including discharge, bed and bank material sizes, and rates of sediment transport. The large scatter observed in hydraulic characteristics (Park, 1977) can be explained by the fact that geometry relationships are not uniquely depending on the discharge. In this study, bank and bed materials are treated separately and the rate of sediment transport is related to two Shields numbers for both longitudinal and transversal components. The results of this analysis are in agreement with previous qualitative studies in alluvial rivers (Simons et al., 1972; Schumm, 1977 and 1982). Other studies including sediment transport (Inglis, 1949; Shahjahan, 1970; and Parker, 1976) have been considered. The formation of meanders and the corresponding hydraulic geometry relationships appear to be fundamentally an hydrodynamic problem. In this view, the sediment transport capacity is linked to the resulting hydraulic conditions and determines the equilibrium condition for sediment transport. It is also recognized that when the sediment input in an alluvial reach is different than the equilibrium sediment transport capacity, transient conditions will be imposed to the system until a new equilibrium is reached. The proposed set of equations (Eq. 17 to 20) derived from basic principles globally describes the hydraulic geometry very well and could be used to support existing empirical relationships as well as to guide further investigations.

V. SMOOTH CHANNELS

In the case of smooth channels, the resistance to flow relationship is not dependent upon the sediment size. Therefore, the threshold condition for incipient motion of sediments is not required. The friction term of flow in bends is much smaller than for turbulent rough flows, the velocity should increase, and the pressure gradient across the transverse direction should be predominant, showing significant superelevation in the outer bend.

Blasius power law can be used to describe turbulent smooth flows. A condition for transverse degradation (or stability) of bank material (ice, bedrock or others) is given by constant shear strength. Including continuity equation and the geometrical similarity of bends (Eq. 16), the governing equations for smooth flows are:

$$Q = w\ h\ \bar{U} \tag{1}$$

$$S\ \alpha\ \left(\frac{1}{\bar{U}h}\right)^{0.25}\ \frac{\bar{U}^2}{gh} \tag{21}$$

$$\frac{h\ \bar{U}^2}{r}\ \alpha\ g\ h\ S_t = \text{constant} \tag{22}$$

$$\gamma \; h \; S \quad \text{is constant} \tag{23}$$

$$w \; \alpha \; r \tag{16}$$

These equations can be combined (see Appendix B for derivations) to give the following hydraulic geometry relationships

$$h \; \alpha \; Q^{7/17} \quad (\text{or } Q^{0.41}) \tag{24}$$

$$w \; \alpha \; Q^{9/17} \quad (\text{or } Q^{0.53}) \tag{25}$$

$$\bar{U} \; \alpha \; Q^{1/17} \quad (\text{or } Q^{0.06}) \tag{26}$$

$$S \; \alpha \; Q^{-7/17} \quad (\text{or } Q^{-0.41}) \tag{27}$$

When compared with exponents given in Tables I, II, III and IV (except for the influence of sediment size), the values derived theoretically compares fairly well with the relationships for rough channels using Manning Equation. Henceforth, one may understand why meandering on smooth surfaces, such as ice, looks similar to meandering in sediment channels.

VI. NOTES ON CHANNEL ADJUSTMENT

Channel adjustment from nonequilibrium conditions has been described by Schumm (1972, 1977, 1982) and Simons et al. (1977). The authors wish to point out just a few results from the downstream geometry relationships (Eqs. 17 to 20).

6.1 Sediment Load

The alluvial reach is in equilibrium if the upstream sediment load is equal to the sediment transport capacity. If in exceedance, part of the sediment load will deposit in the upstream reach, thus decreasing flow depth and increasing slope. From Eq. 20, the reach can stabilize itself with an increase in bed material size (if this material is available) otherwise, the river might also reduce its water discharge per channel by braiding. Then the total water discharge might flow in several channels and provide new equilibrium to the reach.

If the sediment transport capacity exceeds the available load, erosion might occur in the upstream reach thus reducing slope. Equation 20 states that new equilibrium could be reached with smaller bed material size or by meandering.

6.2 Low Flows

The at-a-station relationship for channel width has usually a smaller exponent (around 0.1) than in the case of the downstream equation. Therefore, at low flows the channel width remains fairly the same, while the downstream relationship (Eq. 18) indicates that for low discharge, the radius of curvature decreases significantly. Thus, this indicates that in some cases, streams could show meandering thalweg within the stream width. This could support Karcz (1971) analysis.

6.3 Bed Versus Bank Stability

The longitudinal stability of an alluvial channel was previously described by Eq. 11. Similarly, transversal equilibrium is defined by Eq. 15. In these two equations, two Shields numbers were defined for longitudinal and transversal conditions the ratio R of these Shields numbers is:

$$R = \frac{k_\ell}{k_t} \alpha \frac{hS}{d_s} \frac{r\,g\,d_s}{h\bar{U}^2} R_d = \frac{g\,r\,S}{\bar{U}^2} R_d \tag{28}$$

From Eq. 8, the velocity can be written in terms of the other variables:

$$R \; \alpha \; \frac{g \; r \; S \; R_d}{h \; S} \left(\frac{d_s}{h}\right)^{2a} = R_d \frac{r}{h} \left(\frac{d_s}{h}\right)^{2a} \qquad (29)$$

For a given longitudinal Shields number k_ℓ, the bank stability decreases when the transversal sediment transport rate (proportional to k_t) increases. In other words, the stability of banks is proportional to R. It is shown from Eq. 29 that bank stability increases with increasing bank material sizes, radius of curvature and bed sediment size. Bank stability is very sensitive to flow depth and decreases at high stages.

VII. SUMMARY AND CONCLUSION

In this report five basic equations are used to obtain the hydraulic geometry relationships. These are: 1) continuity, 2) flow resistance, 3) longitudinal threshold, 4) transverse threshold, and 5) similitude in bends. The threshold conditions are those written in terms of Shields numbers.

The hydraulic geometry relationships for turbulent flow were theoretically derived both for smooth and rough conditions. For smooth flows, the hydraulic geometry is only function of water discharge while for rough flows, the sediment size plays an important role. These theoretical relationships compare very well (particularly for gravel-bed streams) with many empirical relationships suggested by various investigators (Tables I, II, III and IV).

Some channel adjustment conditions are discussed for the cases where the upstream sediment input is different than the transporting capacity. Also, a criterion to describe relative stability of banks and bed is defined.

In conclusions, this research lead to the derivation of hydraulic geometry relationships from five fundamental principles. The derived morphologic relationships account for the variation of bed and bank materials. Also, a parameter describing the rate of sediment transport is included in the analysis. The results obtained support qualitative morphologic analyses reported by Schumm and Simons. Under some conditions in alluvial streams, the number of variables can be reduced to two: water discharge and bed material size. The theoretically derived relationships compare very well with empirical equations reported in the literature.

APPENDIX I. BIBLIOGRAPHY

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APPENDIX A

Derivation of Hydraulic Geometry Relationships for Rough Channels

The five basic equations are:

$$Q = w\ h\ \bar{U} \tag{1}$$

$$\bar{U}\ \alpha\ \left(\frac{h}{d_s}\right)^a h^{1/2}\ S^{1/2} \tag{8}$$

$$hS\ \alpha\ d_s\ k_\ell \tag{11}$$

$$\frac{h\bar{U}^2}{r}\ \alpha\ d_s\ R_d\ k_t \tag{15}$$

$$r\ \alpha\ w \tag{16}$$

<u>Flow Depth Relationship</u>

From Eqs. 1 and 16,

$$h = \frac{Q}{\bar{U}w}\ \alpha\ \frac{Q}{\bar{U}r}$$

From Eq. 15

$$h^2\ \alpha\ \frac{Q\ d_s\ R_d\ k_t}{\bar{U}^3}$$

From Eq. 8

$$h\ \alpha\ \frac{Q\ d_s\ R_d\ k_t}{h\left(\frac{h}{d_s}\right)^{3a} h^{3/2}\ S^{3/2}}$$

From Eq. 11

$$\boxed{h\ \alpha\ Q^{\frac{1}{2+3a}}\ d_s^{\frac{6a-1}{4+6a}}\ (R_d\ k_t)^{\frac{1}{2+3a}}\ k_\ell^{\frac{-3}{4+6a}} \qquad (17)}$$

✓

<u>Channel Width Relationship</u>

From Eqs. 15 and 16

$$w\ \alpha\ r\ \alpha\ \frac{h\ \bar{U}^2}{d_s\ R_d\ k_t}$$

From Eq. 1

$$w \; \alpha \; \frac{h \; Q^2}{d_s \; w^2 h^2 \; R_d \; k_t}$$

$$w^3 \; \alpha \; \frac{Q^2 \qquad k_\ell^{\frac{3}{4+6a}}}{d_s \; R_d \; k_t \; Q^{\frac{1}{2+3a}} \; d_s^{\frac{3a-0.5}{3a+2}} \; (R_d k_t)^{\frac{1}{2+3a}}}$$

$$\boxed{w \; \alpha \; \frac{Q^{\frac{1+2a}{2+3a}} \; k_\ell^{\frac{1}{4+6a}}}{(R_d \; k_t)^{\frac{1+a}{2+3a}} \; d_s^{\frac{1+4a}{4+6a}}} \qquad (18)}$$

Velocity Relationship

$$\bar{U} = \frac{Q}{hw} \qquad (1)$$

From Eqs. 17 and 18,

$$\bar{U} \; \alpha \; \frac{Q \; d_s^{\frac{1+4a}{4+6a}} \; (R_d \; k_t)^{\frac{1+a}{2+3a}} k_\ell^{\frac{3}{4+6a}}}{Q^{\frac{1}{2+3a}} \; d_s^{\frac{3a-0.5}{3a+2}} \; (R_d \; k_t)^{\frac{1}{2+3a}} \; Q^{\frac{1+2a}{2+3a}} \; k_\ell^{\frac{1}{4+6a}}}$$

$$\boxed{\bar{U} \; \alpha \; Q^{\frac{a}{2+3a}} \; d_s^{\frac{1-a}{2+3a}} \; (R_d \; k_t)^{\frac{a}{2+3a}} \; k_\ell^{\frac{1}{2+3a}} \qquad (19)}$$

Slope Relationship

$$S \; \alpha \; \frac{d_s \; k_\ell}{h} \qquad (11)$$

From Eq. 17

$$S \;\alpha\; \frac{d_s \; k_\ell \; k_\ell^{\frac{3}{4+6a}}}{Q^{\frac{1}{2+3a}} \; d_s^{\frac{3a-0.5}{2+3a}} \; (R_d \; k_t)^{\frac{1}{2+3a}}}$$

$$\boxed{S \;\alpha\; Q^{\frac{-1}{2+3a}} \; d_s^{\frac{2.5}{2+3a}} \; k_\ell^{\frac{7+6a}{4+6a}} \; (R_d \; k_t)^{\frac{-1}{2+3a}} \qquad (20)}$$

APPENDIX B

Derivation of Hydraulic Geometry Relationships for Smooth Channels

The five basic equations are:

$$Q = w\ h\ \bar{U} \tag{1}$$

$$S \ \alpha \left(\frac{1}{\bar{U}h}\right)^{1/4} \frac{\bar{U}^2}{gh} \tag{21}$$

$$\frac{h\bar{U}^2}{r} \ \alpha \ g\ h\ S_t = \text{constant} \tag{22}$$

$$\gamma\ h\ S \ \text{ is constant} \tag{23}$$

$$w \ \alpha \ r \tag{16}$$

Flow Depth Relationship

$$w \ \alpha \ r \ \alpha \ h\ \bar{U}^2 \qquad \text{(From Eqs. 16 and 22)}$$

$$Q \ \alpha \ h^2\bar{U}^3 \qquad \text{(From Eq. 1)}$$

$$\frac{1}{h} \ \alpha \left(\frac{1}{\bar{U}h}\right)^{1/4} \frac{\bar{U}^2}{h} \qquad \text{(From Eqs. 21 and 23)}$$

$$\bar{U}^{7/4} \ \alpha \ h^{1/4}$$

$$Q \ \alpha \ h^2\ h^{3/7}$$

or

$$\boxed{h \ \alpha \ Q^{7/17} \qquad (24)}$$

Velocity Relationship

$$\boxed{\bar{U} \ \alpha \ h^{1/7} \ \alpha \ Q^{1/17} \qquad (26)}$$

Channel Width Relationship

$$w \ \alpha \ h\bar{U}^2 \ \alpha \ Q^{7/17}\ Q^{2/17}$$

$$\boxed{w \; \alpha \; Q^{9/17} \qquad (25)}$$

Slope Relationship

$$S \; \alpha \; \frac{1}{h} \; \alpha \; \frac{1}{Q^{7/17}}$$

$$\boxed{S \; \alpha \; Q^{-7/17} \qquad (27)}$$