DISSERTATION

DEVELOP A MULTISTAGE STOCHASTIC PROGRAM WITH RECOURSE FOR SCHEDULING PRESCRIBED BURNING BASED FUEL TREATMENTS WITH CONSIDERATION OF FUTURE WILDLAND FIRES AND FIRE SUPPRESSIONS

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ABSTRACT

DEVELOP A MULTISTAGE STOCHASTIC PROGRAM WITH RECOURSE

FOR SCHEDULING PRESCRIBED BURNING BASED FUEL TREATMENTS

WITH CONSIDERATION OF FUTURE WILDLAND FIRES AND FIRE SUPPRESSIONS

In this study, I present a multistage stochastic linear program with recourse for scheduling prescribed burning based fuel treatments under the influences of random future windland fires and fire suppressions across multiple planning periods. Prescribed burning decreases future wildfire's spread rate and intensity. Future wildfire uncertainties are characterized by sequences of independent and identical (i.i.d.) fire samples across the entire planning horizon. Each simulated sample fire ignites at a random location and spreads for a random duration under the influence of a randomly selected wind direction and speed. This stochastic program explicitly addresses the spatial and temporal relationships between fire behavior, prescribed burning, and suppression in multiple fire-planning periods. It uses sample average approximation and minimizes the sum of average discounted management cost plus average discounted fire loss across a planning horizon. Test cases are designed to examine fire-and-management situations on an artificial forested landscape, and are focused on selecting good quality first period prescribed burning locations. Results provide a wide range of optimal solutions for allocating the first period prescribed burning to handle risks from future wildfires.

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1 Introduction

Wildfire is a natural component of many terrestrial ecosystems. It has beneficial effects on many ecosystem processes and also posts threat to human life, property and natural resources (King et al. 2008). During the past two decades, there has been escalation of extreme wildfire behaviors and associated fire management costs. For example, the annual wildfire program spending for USDA Forest Service (USFS) and the Department of the Interior (DOI) increased from $2.3 billion in 2001 to $2.5 billion in 2005 (Alkire 2004). Mitigating the impact of large detrimental fires efficiently is an important component of wildland fire management program.

The National Cohesive Wildland Fire Management Strategy (NCWFM) developed in 2009 is an example fire management program that comprehensively addresses wildland fire management issues across the USA. In the NCWFM 2014 report (http://www.forests-andrangelands.gov/strategy), four imminent challenges are identified: managing vegetation and fuels; protecting homes, communities, and other values at risk; managing human-caused ignitions; and effectively and efficiently responding to wildfire. It suggests various management actions being employed and leveraged to address these challenges to improve the effectiveness and efficiency in managing wildland fire.

An unintended consequence of aggressive fire suppression since the 20th century is the accumulation of forest-fuels that increases wildfire risk in both extent and intensity (Conard et al. 2001, Agee and Skinner 2005, Cohen 2010). Fuel treatment represents a process of altering the quantity and structure of fuels to reduce wildfire risk (Pyne et al. 1996, Finney 2001). Fuel treatment becomes increasingly important in wildfire management across many forested landscapes (Collins et al. 2010).

Fuel treatment in forest stands can change fire behaviors and reduce negative fire impacts (Fulé et al. 2001, Martinson et al. 2002, Fiedler et al. 2004, Skinner 2005, Ritchie et al. 2007, Strom and Fulé 2007, Schmidt et al. 2008, Stephens et al. 2009). It alters fuel structures and reduces fire potential (Laverty and Williams 2000, Radeloff et al. 2005, Ager et al. 2007b, Contreras et al. 2012), slows fire spread rate in some cases (Gonzalez et al. 2008), reduces fire intensity and severity (Reinhardt et al. 2008, Mell et al. 2010), and potentially reduces fire sizes (Bevers et al. 2004, Hirsch et al. 2004, Loehle 2004). Commonly used fuel treatment methods include prescribed burning, mechanical thinning, and harvesting (Loehle 2004). Fuel breaks created by treatments can facilitate the establishment of fire control lines (Agee et al. 2000, Finney 2001, Finney and Cohen 2003) and also improve safety for firefighters (Moghaddas and Craggs 2008). The effect of fuel treatment however, is transient instead of permanent. Therefore, it is important to coordinate treatments in a landscape with respect to their size, location, and timing (Collins et al. 2010). Without spatial coordination of treatment units, large fires can more easily circumvent treated areas and travel through a forest (Salazar and González-Cabán 1987, Dunn 1989, Finney et al. 2005).

Total area treated, or the percentage of area treated on a landscape is important in altering wildfire behaviors. Treatment effects may not be significant if the area treated is too small because the chance a future fire spreading into any treated area may be low. Some studies suggest treating 20% of the total landscape areas to have a more consistent effect in reducing fire size and behavior (Ager et al. 2007a, Finney et al. 2008, Schmidt et al. 2008). If more areas in a landscape are treated, fire size and behavior can be further decreased (González et al. 2005, Parisien et al. 2007, Kim and Bettinger 2008, Schmidt et al. 2008). However, the marginal rate of reduction may diminish when the proportions of the landscape treated are beyond a threshold (Ager et al. 2007a, Schmidt et al. 2008). Treating an entire forest is often impractical (Lynch et al. 2002, Finney and Cohen 2003) due to funding limitation and potential conflicts with other management objectives such as habitat protection or aesthetic concerns.

Locating fuel treatments in a landscape is also important because it can change the spatial arrangement of landscape fuels and consequently influences patterns of fire spread (Green 1983, Davis and Burrows 1994, Turner and Romme 1994). Research shows even randomly located treatments can reduce fire spread rate given that a reasonable proportion of a landscape is treated (Finney 2003). However, regular treatment patterns often outperform random patterns in reducing fire spread and area burned (Schmidt et al. 2008), especially if treatments can only be scheduled in a small fraction of a landscape (Finney 2003, Loehle 2004), or if fire intensity is high (Kim et al. 2009). Finney (2001) suggests implementing treatments that overlap in the heading fire spread direction to reduce fire spread rate. Loehle (2004) suggests fragmenting fuel complex by allocating treatments analogous to ship bulkhead. Palma et al. (2007) laterly suggest allocating treatments to disrupt critical fire spread paths. Other studies indicate that forming treatments as linear barriers (Price 2012) or parallel strips perpendicular to major fire spread directions (Fujioka 1985, Finney 2007) can effectively retard fire growth.

Fuel treatments also need to be temporally coordinated. The effectiveness of fuel treatments deployed on a landscape would reduce over time because fuel load increases as tree grows (Agee and Skinner 2005, Collins et al. 2009). Therefore, periodically rescheduling fuel treatments on a landscape is needed to maintain their effectiveness.

Fuel treatment planning represents a pressing need for many land management agencies to improve their fuel treatment program efficiencies (Black 2004, Collins et al. 2010). Scheduling fuel treatments efficiently and effectively in a landscape represents a type of challenging forest management decision that requires careful consideration of many influencing factors, and also requires empirical knowledge and site specific evidences as suggested by researchers (Carey and Schumann 2003, Fernandes and Botelho 2003, Graham et al. 2004). Fuel treatment strategy can vary depending on management goals (Weatherspoon and Skinner 1996).

The strategic placement of fuel treatments across landscapes can be supported by using decision tools such as optimization models. Some optimization models have been developed to configure spatial treatment layouts for one or many fire events. Hof et al. (2000), and latterly Hof and Omi (2003) developed mixed integer programming (MIP) models for scheduling treatments to delay the spread of a targeted fire from its ignition location to one or more preselected protecting locations. Konoshima et al. (2010) developed a dynamic programming model that can recognize numerous spread patterns and associated probabilities of a single fire, and based on these spread patterns and probabilities to optimize fuel treatment and harvest across a hypothetical landscape. Wei et al. (2008) developed a MIP model that uses a fire probability distribution map pre-calculated through simulating a large number of random fires to optimize fuel treatment allocation to break fire probability accumulation pathways. This method uses linear approximation to track the accumulation of fire probabilities across a landscape. Wei (2012) built another MIP model to schedule fuel treatment to provide control opportunities for a set of systematically selected future fires. Fire ignitions are modeled simultaneously from many possible locations of a landscape. This model schedules treatments in one planning period and assumes no interactions between multiple fires.

Other optimization models locate fuel treatment based on modification of landscape fuel connectivity. Percolation theory (Stauffer and Aharony 1991, With 2002) indicates that randomly treating a fraction of the landscape up to a "percolation threshold" could form connected fuel breaks to obstruct the spread of fires. Bevers et al. (2004) designed a shortest path network optimization model to measure the continuity of fuel breaks. They discovered if treatments were randomly allocated, more than half of a forest would need to be treated to form continuous fuel breaks on most tested landscapes. Instead of randomly allocating treatments in a landscape, Minas et al. (2014) developed a MIP model to generate spatial fuel patterns so as to reduce the connectivity of "old fuel cells" in a landscape. The total number of connected pairs of "old fuel cells" is minimized across all time periods to inhibit fire spread. Wei and Long (2014) developed a spatial optimization model to fragment high fire hazard fuel patches to minimize the expected future fire losses weighted by the ignition probability of each fire. Post-optimization simulations (Wei and Long 2014) suggest that scheduling fuel treatments to fragment fuel patches have similar effect as scheduling fuel treatments to slow the spread of a large number of long duration sample fires.

Optimization models have been used to search through a large set of treatment alternatives and support many tradeoff analyses. Selecting good fuel-treatment mosaics through these models however, still remains challenging (Martell 2007) because comparing a large number of candidate treatment plans requires a lot of computing power, especially when multiple objectives and constraints are added into the models. For this reason, heuristics are often combined with optimization model to find near-optimal solutions (Borges et al. 2002), and they become more popular in wildfire management in forested landscape (Thompson et al. 2000, Calkin et al. 2005, González et al. 2005)

Optimization-via-simulation is a type of models using heuristics to optimize fuel treatment scheduling. This type of models searches for good solutions of a given system iteratively (Gosavi 2003). For example, Finney et al. (2008) integrated three models into a simulation-optimization system: a forest and fuel dynamics model (Crookston and Stage 1991, Reinhardt and Crookston 2003) for simulating forest vegetation changes over time and comparing different treatment strategies; a spatial model (Finney 2002, 2004, Finney 2007) for choosing the location of treatment units using topologically optimal or random selection logic; and a fire growth simulation model (Finney 2002) for evaluating how treatments would modify fire growth rate, fire sizes, and conditional burn probability. This system runs iteratively to identify intersections between the fire spread paths and the stands where treatments would slow fire spread the most, and accordingly suggests treatments on those stands. In another research, Rytwinski and Crowe (2010) ran a stochastic fire simulation model repeatedly to compare fire risks of different fuel-break solutions identified from a meta-heuristic search algorithm. This algorithm starts from a randomly selected or a user-defined solution; iteratively creates new solutions through weighted linear combinations of previous solutions found based on scatter search (Glover 1998), and stops when a pre-determined number of optimization iterations have been performed. González-Olabarria and Pukkala (2011) used simulated annealing to iteratively search for better forest management schedules to maximize timber incomes and improve landscape fire resistance. In each iteration, a fire spread model is used to calculate the probability of fire occurring in each management-stand in a forest following a selected harvest schedule. The schedule is then revised based on the updated fire probability map in the following iteration. This process is repeated until the fire probability distribution in a forest stop to change significantly. Optimization-via-simulation can be used to effectively handle complex problem by breaking it into smaller and solvable components. However, this approach may stop at a sub-optimal solution and it can be difficult to quantify the quality of a discovered solution.

Fire suppression and fuel treatment are often related (Martell 2007). Although fuel treatment alone may not be able to stop fires from burning or spreading (Finney 2003), it can improve the effectiveness of suppression effort (Minas et al. 2013). Schaaf et al. (2004) evaluated five combinations of fire suppression and fuel treatment programs on the Angeles National Forest in western US, and suggested that using a low intensity fire suppression program together with a moderate intensity fuel treatment program would provide the most cost-beneficial fire protection strategy for their study area. Some decision models were also built to address the complementary effects between fuel treatment and suppression. For example, Mercer et al. (2008) developed an integer programming model to evaluate tradeoffs between expenditures for fuels management and suppression resources on representative fires. The effect of fuel treatment is incorporated into a suppression dispatch model to minimize the expected cost of fire escapes.

The probability of fire escape is predicted as a function of fuel treatment amount and the number of initial attack resources dispatched to the fire. Minas et al. (2013) incorporated fuel treatment and suppression decisions into a single MIP model to maximize their joint effects on wildfire control. Their model does not directly model fire spread. Instead, it uses a pre-calculated "location-specific fire escape time" as the time taken for a fire to reach a pre-defined threshold size (e.g. five hectares) and deemed as escaped.

It is challenging to study fuel treatment impact on fire suppression, especially when both of those management actions are simultaneously considered along with wildfires. In an overview of methods for incorporating wildfires into forest planning models, Bettinger (2010) pointed out this challenge, as many studies only incorporated wildfires into a planning process either before or after the schedule of management activities. Modelling wildfires in a spatially explicit way is also a challenging task. Studies that explicitly incorporate fire behaviors into the selection of optimal plans are rare, and they can only deal with small landscapes (Konoshima et al. 2008) or a limited number of fire samples (Kim et al. 2009).

In this dissertation, I introduce a multistage stochastic linear program with recourse for planning fuel treatments to mitigate the risk from wildland fires in a multiple planning period horizon. This program focuses on the use of only prescribe burning, with consideration of random future wildfire and also simplified fire suppression. Fuel treatments can also be implemented through mechanical methods. However, prescribed burning and wildland fire use (http://www.fs.fed.us/fire/fireuse/index.html) are suggested as the primary fuel treatment methods in the wildland; while mechanical fuel reduction treatments are more appropriate in

WUI areas (Reinhardt et al. 2008). The stochastic program presented here explicitly captures the spatial and temporal interactions between fire behavior, prescribed burning, and suppression. Random sample fires are employed in this program using a sample average approximation formulation (Kleywegt et al. 2002) to minimize the sum of average discounted management cost plus average discounted fire loss for all planning periods. A set of hypothetical testing problems is designed to examine fire-and-management situations in an artificial forested landscape across three fire-planning periods, and is focused on selecting good quality first period prescribed burning layouts. Test cases are solved using IBM's ILOG-CPLEX v.12.6 on a 64-bit workstation equipped with a quad-core 2.53GHZ processor and 8GB of memory, with optimality gap set to 1%.

2 Methods

2.1 Model structure

This stochastic program follows the general structure of multistage stochastic linear program with recourse proposed by Birge and Louveaux (2011). It models prescribed burning based fuel treatment and suppression decisions in multiple planning periods (or multiple stages) to mitigate risks from wildfires. Wildfires are modeled as uncertain events represented by random sample fires across the entire planning horizon. The design of this program is illustrated by the branching tree in Figure 1. This design ensures that prescribed burning decisions made in the first period (or stage) would be identical for all *DFS* samples, where each sample is represented by a sequence of prescribed burning decisions, random fire events, and fire suppression decisions across all planning periods. Decisions after the first stage are recourse decisions. My interest lies in the quality of the first period prescribed burning decisions, which have to be made before future fire uncertainties can be revealed. I do not increase the number of *DFS* samples after the first period to limit the model size, which also helps reduce computing difficulty when solving this stochastic program. It will be an interesting future study to explore how adding more *DFS* samples in the later planning periods may help better represent the stochastic fire situation after period two and improve the quality of the first period prescribed burning decision. *DFS* samples are incorporated into an sample average approximation formulation (Kleywegt et al. 2002) with the objective to minimize the sum of average discounted management cost plus average discounted fire loss across all planning periods.

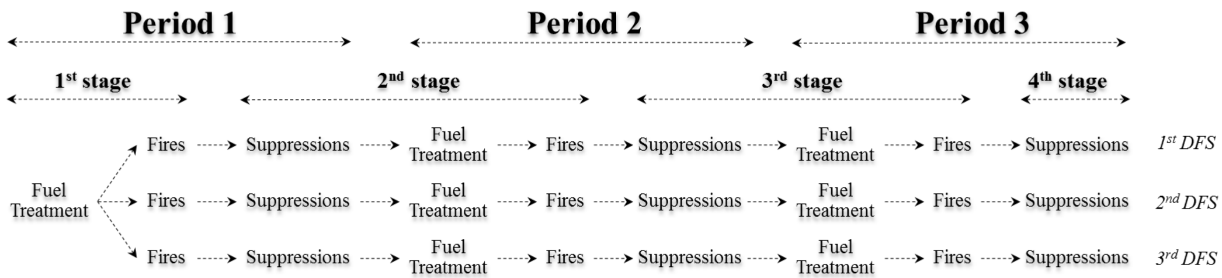


Figure 1: Illustration of a multistage stochastic program with three planning periods and three *DFSs* (e. g. $N = 3$). Each *DFS* represents a sequence of fire management decisions (prescribe burning and suppression) and fire events across three planning periods. The term "Fires" used in this figure may include zero or multiple fire occurrences within a planning period, which are exogenously determined by random draws. Prescribed burning is scheduled at the beginning of each planning period before any random sample wildfire in that same period is realized. Suppression can also be implemented to each fire as recourse action.

2.2 Model assumptions

In this model, raster "cell" is the smallest modeling unit for fire suppression, forest age-class transition, and fire spread. Forest "stand" includes one or multiple cells covering a forested area with homogeneous vegetation characteristics. Stand is the smallest modeling unit to schedule prescribed burning. Prescribed burning decision is made for an entire stand by treating all cells in the stand at the beginning of a planning period. Treated areas have beneficial effects of reducing future fire spread rate and intensity that last for certain period of time (Figure 4). Fire suppression is simplified as building fire control lines in cells where crown fire could not occur, and is assumed to be able to stop fire spread in cells where fire control lines are constructed. This model captures the possible impact of wildfire and prescribed burning to create suppression opportunities and how suppression may take places to stop the spread of surface fire in recently burned or treated areas. However, fire suppression scheduling itself is not the focus of this model.

Fire uncertainties are modeled by using sample fires in each *DFS*. Random draws are used to determine the ignition location (i.e. in a cell), the active fire spread duration, and the combination of wind direction and speed during the duration of a fire. Similar to prescribed fire, wildfire also consumes fuels (Figure 2) that can help reduce future fire spread rate and intensity. The beneficial effects from wildfire, however, may last for a different period of time. A sample fire is allowed to spread between cells (Figure 3) within its maximum spread range (*MSR*) pre-calculated by the processing algorithm (Figure 4). When spreading under certain wind condition, it may become crown fire or stay as surface fire in different cells (Figure 5). Fire spread rate and fire line intensity modeled in this stochastic program would be based on surface fire behaviors. In cells where surface fires spread into crown, I assume the forest will be destroyed.

Areas treated by prescribed fire or burned by wildire

→ Future fires in these areas would have decreased rates of spread within certain periods.

→ Future fires in these areas would have decreased fire line intensities within certain periods.

Figure 2: Prescribed fire and wildfire both have beneficial effects of reducing fuel loads, but their effects may last for different periods of time.

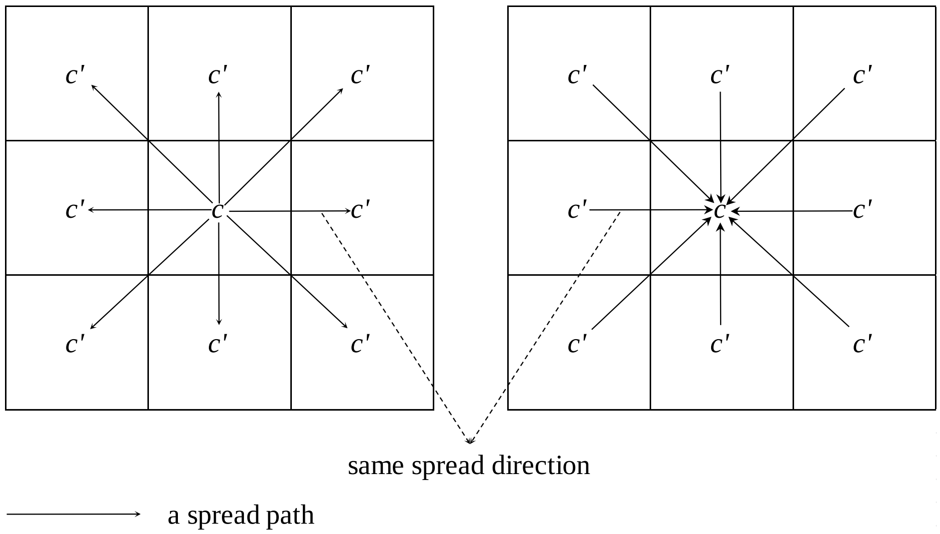


Figure 3: Illustration of the 16 possible spread-paths (in eight possible spread directions) from or toward a cell (c). For each spread direction, fire would spread in a cell (c) with a specific spread rate. Details of the spread-rate calculations will be described later in sections **2.4** and **3.1.**

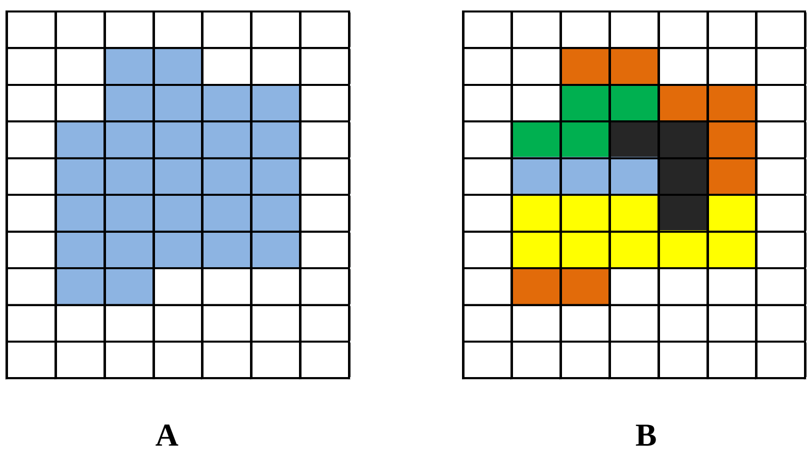


Figure 4: An example to illustrate the spread pattern of a fire in a rasterized landscape under different assumptions: **A**: The fire's *MSR* (blue cells) pre-calculated by the processing algorithm with assumptions that fire spreads freely without influences from previous fires, prescribed burnings, and suppressions (For more details of this algorithm, see **Appendix**). In the stochastic program, the spread and suppression of a fire will be modeled inside its *MSR*. **B**: I assume fire spreads more slowly in areas recently burned by wildfire (e.g. yellow cells) or treated by prescribed fire (e.g. green cells), and can be stopped by cells with fire control lines constructed (e.g. black cells). Therefore, a fire may not be able to burn the orange cells within its *MSR* during the same duration under the influence from previous fires, fuel treatments, and suppressions.

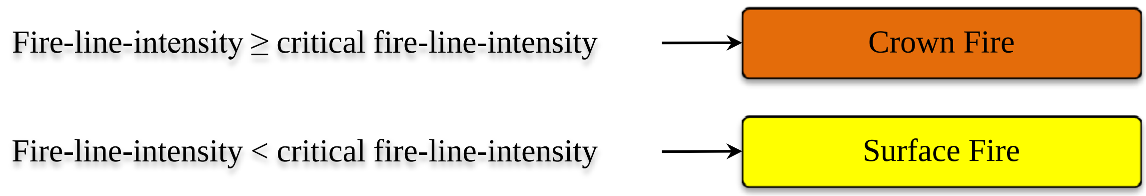


Figure 5: Fire line intensity is used as a criterion to predict crown fire activities. I model fire line intensity as a function of fire spread rate along each spread direction. Crown fire is assumed to occur when fire line intensity is beyond a critical threshold. Areas recently treated by prescribed fire or burned by wildfire would have decreased fire line intensity. Consequently, the likelihood of crown fire in these areas will be reduced. More details on the calculations of fire line intensity and the critical threshold of fire line intensity will be described later in sections **2.4** and **3.1.**

In this model, the value of forest in a cell to be protected from wildfire (referred to as “cell value”) is assumed to be related with forest age-classes (referred to as “cell age-classes”). Fire loss is measured each time a cell is burned by wildfire with the amount of loss depending on both the fire line intensity in that cell and the cell age-class. Cell age-class transition is tracked during the planning horizon and is only influenced by crown fires (Figure 6). A crown fire would destroy the value of forest in a cell and also reset age-class of the burned cell to zero. A surface fire may cause partial loss of cell value but not change the age-class of the burned cell. Upon entering the next period, cell age-class will increase by one. Within a planning horizon, a cell may be burned by multiple fires with various losses.

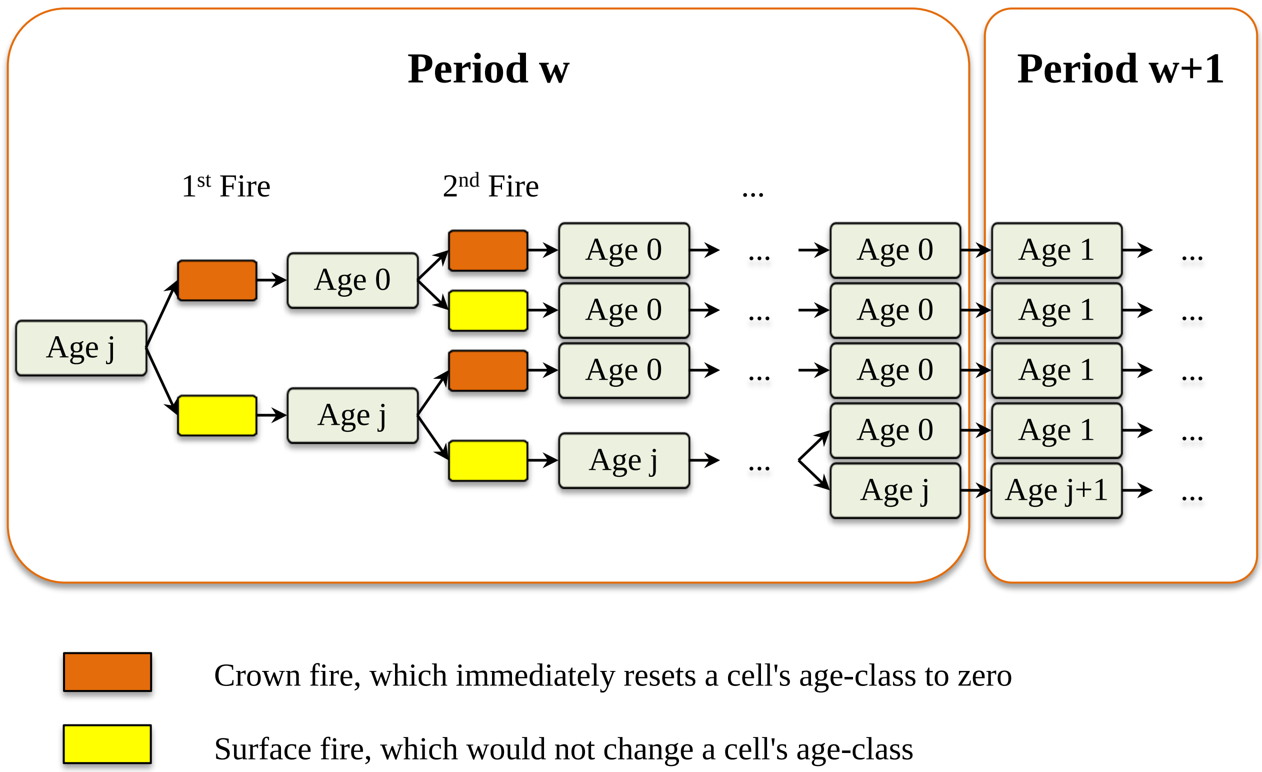


Figure 6: An example of the age-class transition of a cell in two planning periods if it gets burned by multiple fires.

2.3 Notations

I use capital letters to denote most of the parameters and sets. Some parameters are denoted by Greek letters. Lower-case letters are used to represent indices or decision variables. Abbreviations are defined to standardize some of the descriptions in the method, and are represented by both capital and lower-case letters. Notations are presented in alphabetical order.

Abbreviations

BEs Denotes the beneficial effects from prescribed burning based fuel treatment or from wildfire. For example, areas recently treated by prescribe fire or burned by wildfire can decrease future fire line intensity and fire spread rate. I assume the beneficial effects from prescribed burning will last for $\ddot{W}$ planning periods including the period when it is implemented (i.e. areas treated by prescribed fire in period w will have *BEs* lasted in period w and period $w+1$ if $\ddot{W}$ is set to two). I assume the beneficial effects from wildfire will last for $\dddot{W}$ planning periods also including the period when fire occurs.

DFS Denotes a sequence of management decisions and fire events across all planning periods (as described in section **2.1**). In this study, management decisions include fuel treatment modeled as prescribed burning, and fire suppression modeled as fire-control-line construction. Wildfires are uncertain events represented by random sample fires.

FT1 Denotes the first period fuel treatment solution that includes a set of stands selected for prescribed burning at the beginning of the first period. Although this multi-stage stochastic program models prescribed burning decisions in multiple planning periods, the focus is to improve quality of the first period decision because this is the immediate decision a manager has to make without waiting for the reveal of any future fire situation.

MFAT Denotes "minimum fire arrival time" to each location (i.e. a cell) of a landscape. I use a set of equations to calculate the minimum travel time when each sample fire spreads. Details will be presented in section **2.4.**

MSR Denotes "maximum spread range" of a fire pre-calculated by the preprocessing algorithm. Details on this algorithm are presented in the **Appendix.**

TFS Denotes a sequence of testing fires across all planning periods. In this study, a set of 300 i.i.d. *TFSs* are randomly simulated based on historical data of fire ignition and wind (described later in section **3.2**). This set would then be fixed to test the performance of different *FT1s*. *TFS* and *DFS* are different. Sample fires in each *DFS* are not fixed. Each time running the stochastic program, sample fires in each *DFS* will be randomly redrawn; therefore, each stochastic run may result in a different optimal *FT1*.

Indices:

a Index of a stand.

a_c Index of the stand that contains a raster cell c.

c, c' Indices of raster cells. Cell's index starts from the top-left to the bottom-right of a rasterized testing landscape.

i, i', i'' The occurrence order of sample fires in a planning period. In a specific planning period, a fire indexed by $i\ (or\ i', i'') = 1$ occurs immediately before the fire indexed by $i\ (or\ i', i'') = 2$.

j Index of age-class of the forest in a raster cell; age class is used to estimate the forest value to be protected and also the critical threshold of fire line intensity in each cell.

n, n' Indices of *DFS* samples.

w Index of a planning period.

(w, i, n) An ordered set denotes the three attributes of a sample fire: w is the planning period when this fire occurs; i is the occurrence order of this fire in period w; and n is the *DFS* in which this fire belongs.

Parameters:

$\beta_{c',c}$ Half of the distance for a fire to spread from the center of cell c' to the center of its adjacent cell c.

$\beta_{c',c} = \frac{cell\_size}{2}$ if cell c and cell c' share an edge

	c'	
c'	c	c'
	c'	

$\beta_{c',c} = \frac{\sqrt{2} \times cell\_size}{2}$ if cell c and cell c' share a vertex

c'		c'
	c	
c'		c'

where *cell-size* is the size length of a rater cell.

φ A small positive number which is arbitrarily set. When a fire control line is built in a cell, the *MFAT* of this cell is calculated by the sum of φ and the fire's active spread duration indicating that this fire would not burn the corresponding cell.

$\dddot{C}_a$ The total number of cells within stand a.

$\ddddot{C}_c$ The total number of adjacent cells of the cell c.

$E_{critical_{(w,i,n),c,j}}$ The pre-calculated critical threshold of fire line intensity in cell c when this cell is in age-class j at occurrence time of fire (w, i, n). I assume if fire (w, i, n) burns cell c with the estimated fire line intensity meeting or exceeding this threshold (e.g. $e_{(w,i,n),c} \geq E_{critical_{(w,i,n),c,j}}$), it would become crown fire in cell c.

$E_{(w,i,n),c}$ The pre-calculated fire line intensity of fire (w,i,n) when it first ignites and spreads in cell c and this cell has not been treated by prescribed fire within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods. This parameter is set to zero if cell c is not the ignition cell of fire (w,i,n).

$E'_{(w,i,n),c}$ The pre-calculated fire line intensity of fire (w,i,n) when it first ignites and spreads in cell c and this cell has been treated by prescribed fire within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods. This parameter is set to zero if cell c is not the ignition cell of fire (w,i,n).

$E_{(w,i,n),c\leftarrow c'}$ and $E'_{(w,i,n),c\leftarrow c'}$

$E_{(w,i,n),c\leftarrow c'}$ is the pre-calculated fire line intensity in cell c if fire (w,i,n) spreads from c' into c at spread rate $ROS_{(w,i,n),c\leftarrow c'}$; while $E'_{(w,i,n),c\leftarrow c'}$ is the pre-calculated fire line intensity in cell c if fire (w,i,n) spreads from c' into c at spread rate $ROS'_{(w,i,n),c\leftarrow c'}$ ($E'_{(w,i,n),c\leftarrow c'} < E_{(w,i,n),c\leftarrow c'}$).

$G_{(w,i,n),c}$ A binary parameter, which is set to one if fire (w,i,n) ignites in cell c. This parameter will be set to zero if c is not the ignition cell of fire (w,i,n).

$H_{(w,i,n)}$ The active spread duration of fire (w,i,n) determined exogenously through random draw.

J_{c_1} Age-class of the forest in cell c at the beginning of the first period.

$L_{(w,i,n)}$ The time of occurrence (i.e. year) of sample fire (w,i,n).

L'_w The time (i.e. year) at the beginning of the planning period w (e.g. in case using 10-year planning period: $L'_{w=1} = 0$, $L'_{w=2} = 10$, and $L'_{w=3} = 20$). This parameter helps calculate the discounted cost of fuel treatment which is assumed to be scheduled at the beginning of each planning period.

M A large positive number (Big M).

N The total number of *DFS* samples. Throughout this dissertation, I use the term "sample size" to represent N.

P_{FT} A predefined per-cell based treatment cost if that cell has not been treated by prescribed fire within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods.

P'_{FT} A predefined per-cell based treatment cost if that cell has been treated by prescribed fire within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods. I assume $P'_{FT} < P_{FT}$.

P_{SUP_c} A predefined cost for building fire control line in cell c during suppression of a fire.

R An adopted annual discount rate.

$ROS_{(w,i,n),c\leftarrow c'}$ and $ROS'_{(w,i,n),c\leftarrow c'}$

$ROS_{(w,i,n),c\leftarrow c'}$ is the estimated spread rate of fire (w, i, n) in cell c when this fire spreads into c from its adjacent cell c' and when c has not been treated by prescribed fire within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods. Eight values of $ROS_{(w,i,n),c\leftarrow c'}$ are pre-calculated to account for the eight possible spread paths into cell c. If cell c has been treated by prescribed fire within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods, the spread rate in this cell is assumed to be reduced to $ROS'_{(w,i,n),c \leftarrow c'}$ with $ROS'_{(w,i,n),c \leftarrow c'} < ROS_{(w,i,n),c \leftarrow c'}$.

$ROS_{(w,i,n),c' \rightarrow c}$ and $ROS'_{(w,i,n),c' \rightarrow c}$

$ROS_{(w,i,n),c' \rightarrow c}$ is the estimated spread rate of fire (w,i,n) in cell c' when this fire spreads from c' to its adjacent cell c and when c' has not been treated by prescribed fire within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods. Eight values of $ROS_{(w,i,n),c' \rightarrow c}$ are pre-calculated to account for the eight possible spread paths from cell c'. If cell c' has been treated by prescribed fire within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods, the spread rate in this cell is assumed to be reduced to $ROS'_{(w,i,n),c' \rightarrow c}$ with $ROS'_{(w,i,n),c' \rightarrow c} < ROS_{(w,i,n),c' \rightarrow c}$.

$V_{c,j}$ A pre-calculated value to be protected in cell c when the forest in this cell is in age-class j. The value to be protected in a cell is assumed to be lost if it is burned by a crown fire.

$V_{(w,i,n),c,j}$ Fire loss in cell c if the forest in this cell is in age-class j at occurrence time of fire (w,i,n) and this fire burns as surface fire in c; I assume $V_{(w,i,n),c,j} < V_{c,j}$. $V_{(w,i,n),c,j}$ could also be set to zero to indicate fire would be not harmful and would cause zero loss.

W The total number of planning periods in the entire planning horizon.

$\ddot{W}$ The number of continuous planning periods in which the *BEs* from fuel treatment would last.

$\dddot{W}$ The number of continuous planning periods in which the *BEs* from wildfire would last.

Sets:

$\hat{A}$ The set of all stands in a landscape.

$\hat{C}$ The set of all cells in a landscape.

$\hat{C}_{Active_{(w,i,n)}}$ The set of flammable cells inside the *MSR* of fire (w, i, n). The *MSR* of each sample fire is pre-calculated by the preprocessing algorithm.

$\hat{C}_a$ The set of all cells in stand a.

$\hat{C}_c$ The set of adjacent cells to cell c (sharing an edge or a vertex with c). This set does not include non-flammable cells.

$\hat{C}_{Ignition_{(w,i,n)}}$ The ignition cell of fire (w, i, n) exogenously selected by random draws based on the historical ignition frequency in each flammable cell in the entire landscape.

$\hat{C}_{InActive_{(w,i,n)}}$ The set of cells that are either non-flammable or outside the *MSR* of fire (w, i, n); $\hat{C}_{InActive_{(w,i,n)}} = \hat{C} - \hat{C}_{Active_{(w,i,n)}}$.

$\hat{J}_{(w,i,n),c}$ The set of age-classes which: forest in cell c can only be in one of these age-classes at occurrence time of fire (w, i, n). For example, $\hat{J}_{(1,i,n),c} = \{0, J_{c_1}\}$; $\hat{J}_{(2,i,n),c} = \{0,1, J_{c_1} + 1\}$; $\hat{J}_{(3,i,n),c} = \{0,1,2, J_{c_1} + 2\}$ (illustrative example will be given in section **2.4**).

$\widehat{W}_{j,w}$ A set of planning periods which includes the total of j number of periods counting back from period w. For example, $\widehat{W}_{1,3}$would include only period 3, while $\widehat{W}_{2,3}$would include both period 2 and period 3.

$\widehat{W}_w$ A set of planning periods that, if fuel treatment is implemented in these periods then its *BEs* will last into period w. This set includes periods from $(w - \ddot{W} + 1)\ to\ (w - 1)$.

$\widehat{W}'_w$ A set of planning periods that, if a fire occurs in these periods then its *BEs* will last into period w. This set includes periods from $(w - \dddot{W} + 1)\ to\ (w - 1)$.

Variables:

In this study, “a spread path” is defined as the path connecting the center of a cell to the center of an adjacent cell. “A spread route to a destined cell” may include multiple connected spread paths for a fire to spread from the ignition cell to that destined cell. A fire can reach the center of a cell by following different spread routes as illustrated in Figure 7. This model tracks all the possible fire spread routes to a cell, and finds the fastest route indicated by the *MFAT* of that cell. If the *MFAT* of a cell is less than the active fire spread duration then that cell is defined as “burned”. Throughout this dissertation, the term “burned” is only referred to wildfire to avoid the confusion when prescribed fire is used.

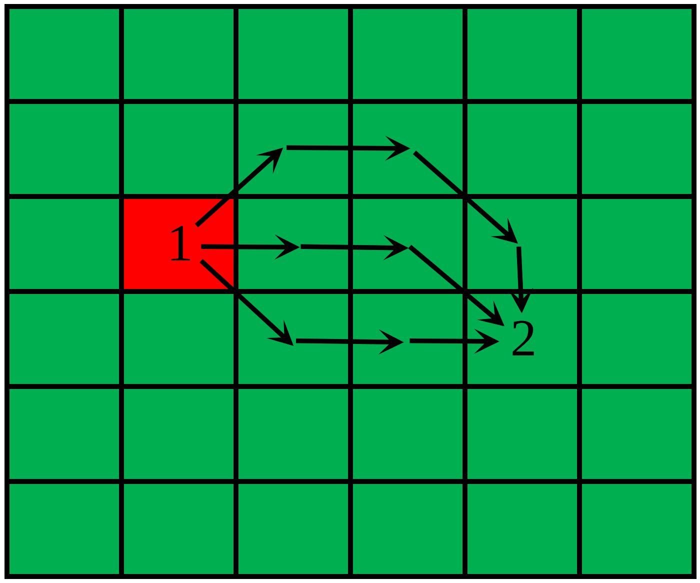


Figure 7: An illustrative example of fire spreading in a rasterized landscape. The fire is assumed to ignite in cell number 1. It can spread to cell number 2 by following different spread routes. In this example, I only draw three of the many possible routes for the fire to spread from the ignition cell 1 to cell 2.

$b_{(w,i,n),c,c'}$ A binary variable receiving a value of one if fire (w,i,n) successfully spreads from cell c into its adjacent cell c', and the spread path from c to c' must belong to the fastest spread route of this fire to c'; otherwise, $b_{(w,i,n),c,c'} = 0$.

$d_{(w,i,n),c}$ A binary variable receiving a value of one if fire (w,i,n) burns cell c; otherwise, $d_{(w,i,n),c} = 0$.

$e_{(w,i,n),c}$ A continuous variable to calculate the fire line intensity in cell c if it is burned by fire (w,i,n) when this fire spreads following its fastest spread route into cell c. If fire (w,i,n) does not burn cell c then $e_{(w,i,n),c} = 0$.

f_{FL_n} A continuous variable to calculate the total discounted fire loss for the n^{th} *DFS.*

f_{FT_n}	A continuous variable to calculate the total discounted cost from prescribed burning for the n^{th} *DFS*.
$f_{FT_{w,n}}$	A continuous variable to calculate the total discounted cost from prescribed burning scheduled in period w for the n^{th} *DFS*.
f_{SUP_n}	A continuous variable to calculate the total discounted cost from fire-control-line construction for the n^{th} *DFS*.
$k_{(w,i,n),c,w' \leq w}$	When fire (w, i, n) occurs we need to track the age-class of cell c to identify the fire loss. The age class of c is determined by crown fire occurrences in this cell before fire (w, i, n) starts. This binary variable tracks if any crown fire has occurred in cell c in period w' before occurrence time of fire (w, i, n). It would be set to one if at least one crown fire has burned cell c.
$o_{(w,i,n),c}$	A binary variable receiving a value of one if either fire (w, i, n) does not burn cell c or it burns as surface fire in cell c. If fire (w, i, n) burns as crown fire in cell c then $o_{(w,i,n),c} = 0$.
$p_{(w,i,n),c}$	A binary variable receiving a value of one if at occurrence time of fire (w, i, n), cell c has been treated by prescribed fire within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods; otherwise, $p_{(w,i,n),c} = 0$.
$q_{(w,i,n),c,j}$	A binary variable receiving a value of one if the forest in cell c is in age-class j at occurrence time of fire (w, i, n); otherwise, $q_{(w,i,n),c,j} = 0$.

$r_{(w,i,n),c}$ A binary variable receiving a value of one if fire control line is built in cell c to protect that cell from being burned by fire (w,i,n); otherwise, $r_{(w,i,n),c} = 0$.

$s_{w,a,n}$ For the n^{th} *DFS*, this integer variable calculates the total number of cells in stand a in period w that have not been treated by prescribed fire within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods.

$t_{(w,i,n),c}$ A continuous variable to track the *MFAT* of cell c, which is calculated based on the fastest route for fire (w,i,n) to spread into the center of c.

$u_{(w,i,n),c,j}$ A binary variable receiving a value of one if cell c in age-class j is burned by fire (w,i,n). If either cell c is not in age-class j or fire (w,i,n) does not burn this cell then $u_{(w,i,n),c,j} = 0$.

${v_{Crown}}_{(w,i,n),c,j}$

A binary variable receiving a value of one if fire (w,i,n) burns as crown fire in cell c and this cell is in age-class j at occurrence time of fire (w,i,n); otherwise, ${v_{Crown}}_{(w,i,n),c,j} = 0$.

${v_{Surface}}_{(w,i,n),c,j}$

A binary variable receiving a value of one if fire (w,i,n) burns as surface fire in cell c and this cell is in age-class j at occurrence time of fire (w,i,n); otherwise, ${v_{Surface}}_{(w,i,n),c,j} = 0$.

$x_{w,a,n}$ A binary variable receiving a value of one if prescribed burning is implemented at the beginning of period w in stand a in the n^{th} *DFS*; otherwise, $x_{w,a,n} = 0$.

$y_{(w,i,n),c}$ A binary variable receiving a value of one if either fire control line has been built in cell c or the *MFAT* for fire (w,i,n) arriving the center of cell c is greater than that fire's active spread-duration; otherwise, $y_{(w,i,n),c} = 0$.

$z_{w,c,n}$ A binary variable receiving a value of one if at the beginning of period w in the n^{th} *DFS*, cell c is identified as not being treated by prescribed fire within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods; otherwise, $z_{w,c,n} = 0$.

2.4 Model formulation

Minimize:

$$\frac{1}{N}\sum_n \left(f_{FT_n} + f_{SUP_n} + f_{FL_n}\right) \qquad (1)$$

Subject to:

$$x_{w=1,a,n} = x_{w=1,a,n'} \qquad \forall n, n' \qquad (2)$$

$$r_{(w,i,n),c} \le o_{(w,i,n),c} \qquad \forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (3)$$

$$d_{(w,i,n),c} + r_{(w,i,n),c} \le 1 \qquad \forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (4)$$

$$t_{(w,i,n),c} \ge \left(H_{(w,i,n)} + \varphi\right) \times r_{(w,i,n),c}$$
$$\forall\, c \in \left(\hat{C}_{Active_{(w,i,n)}} \backslash \hat{C}_{Ignition_{(w,i,n)}}\right), i, n, w \qquad (5)$$

$$t_{(w,i,n),c} = 0 \qquad \forall\, c = \hat{C}_{Ignition_{(w,i,n)}}, i, n, w \qquad (6)$$

$$d_{(w,i,n),c} = 0 \qquad \forall c \in \hat{C}_{InActive_{(w,i,n)}}, i, n, w \qquad (7)$$

$$d_{(w,i,n),c} \ge \frac{H_{(w,i,n)} - t_{(w,i,n),c}}{M} \qquad \forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (8)$$

$$y_{(w,i,n),c} \geq \frac{t_{(w,i,n),c} - H_{(w,i,n)}}{M} \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (9)$$

$$d_{(w,i,n),c} + y_{(w,i,n),c} \leq 1 \qquad \forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (10)$$

$$d_{(w,i,n),c} + r_{(w,i,n),c} + y_{(w,i,n),c} \geq d_{(w,i,n),c'}$$

$$\forall c \in \hat{C}_{Active_{(w,i,n)}}, c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (11)$$

$$b_{(w,i,n),c,c'} \leq d_{(w,i,n),c}$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (12)$$

$$b_{(w,i,n),c,c'} + b_{(w,i,n),c',c} \leq 1$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (13)$$

$$\sum_{c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}} b_{(w,i,,n),c',c} = d_{(w,i,n),c} - G_{(w,i,n),c}$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (14)$$

$$p_{(w,i,n),c} \leq x_{w,a_c,n} + \sum_{i'<i} d_{(w,i',n),c} + \sum_{w' \in \widehat{W}_w} x_{w',a_c,n} + \sum_{w'' \in \widehat{W}'_w} \sum_{i''} d_{(w'',i'',n),c}$$

$$\forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (15)$$

$$p_{(w,i,n),c} \geq \frac{x_{w,a_c,n} + \sum_{i'<i} d_{(w,i',n),c} + \sum_{w' \in \widehat{W}_w} x_{w',a_c,n} + \sum_{w'' \in \widehat{W}'_w} \sum_{i''} d_{(w'',i'',n),c}}{M}$$

$$\forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (16)$$

$$\begin{aligned} t_{(w,i,n),c} \leq t_{(w,i,n),c'} \quad & + \frac{\beta_{c',c}}{ROS_{(w,i,n),c \leftarrow c'}} + p_{(w,i,n),c} \times \left(\frac{\beta_{c',c}}{ROS'_{(w,i,n),c \leftarrow c'}} - \frac{\beta_{c',c}}{ROS_{(w,i,n),c \leftarrow c'}} \right) \\ & + \frac{\beta_{c',c}}{ROS_{(w,i,n),c' \to c}} + p_{(w,i,n),c'} \times \left(\frac{\beta_{c',c}}{ROS'_{(w,i,n),c' \to c}} - \frac{\beta_{c',c}}{ROS_{(w,i,n),c' \to c}} \right) \\ & + M \times \left(1 - d_{(w,i,n),c'} + r_{(w,i,n),c} \right) \end{aligned}$$

$$\forall\, c \in \left(\hat{C}_{Active_{(w,i,n)}} \backslash \hat{C}_{Ignition_{(w,i,n)}} \right), c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (17)$$

$$t_{(w,i,n),c} \geq t_{(w,i,n),c'} \quad + \frac{\beta_{c',c}}{ROS_{(w,i,n),c \leftarrow c'}} + p_{(w,i,n),c} \times \left(\frac{\beta_{c',c}}{ROS'_{(w,i,n),c \leftarrow c'}} - \frac{\beta_{c',c}}{ROS_{(w,i,n),c \leftarrow c'}} \right)$$

$$+\frac{\beta_{c',c}}{ROS_{(w,i,n),c'\to c}} + p_{(w,i,n),c'} \times \left(\frac{\beta_{c',c}}{ROS'_{(w,i,n),c'\to c}} - \frac{\beta_{c',c}}{ROS_{(w,i,n),c'\to c}}\right)$$

$$-M \times \left(1 - b_{(w,i,,n),c',c}\right)$$

$$\forall\, c \in \left(\hat{C}_{Active_{(w,i,n)}} \backslash \hat{C}_{Ignition_{(w,i,n)}}\right), c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \quad (18)$$

$$e_{(w,i,n),c} \geq \sum_{c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}} b_{(w,i,,n),c',c} \times E'_{(w,i,n),c\leftarrow c'} + E'_{(w,i,n),c,j}$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \quad (19)$$

$$e_{(w,i,n),c} \leq \sum_{c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}} b_{(w,i,,n),c',c} \times E_{(w,i,n),c\leftarrow c'} + E_{(w,i,n),c}$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \quad (20)$$

$$e_{(w,i,n),c} \geq \sum_{c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}} b_{(w,i,,n),c',c} \times E_{(w,i,n),c\leftarrow c'} + E_{(w,i,n),c}$$

$$-M \times p_{(w,i,n),c}$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \quad (21)$$

$$e_{(w,i,n),c} \leq \sum_{c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}} b_{(w,i,,n),c',c} \times E'_{(w,i,n),c\leftarrow c'} + E'_{(w,i,n),c}$$

$$+M \times \left(1 - p_{(w,i,n),c}\right)$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, c' \in \hat{C}_c \cap \hat{C}_{Active_{(w,i,n)}}, i, n, w \quad (22)$$

$$o_{(w,i,n),c} \geq \frac{\sum_j E_{critical_{(w,i,n),c,j}} \times q_{(w,i,n),c,j} - e_{(w,i,n),c}}{M} \qquad \forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \quad (23)$$

$$1 - o_{(w,i,n),c} \geq \frac{e_{(w,i,n),c} - \sum_j E_{critical_{(w,i,n),c,j}} \times q_{(w,i,n),c,j}}{M} \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \quad (24)$$

$$z_{w,c,n} \geq x_{w',a_c,n} \qquad \forall c \in \hat{C}, n, w, w' \in \widehat{W}_w \quad (25)$$

$$z_{w,c,n} \geq d_{(w'',i'',n),c} \qquad \forall c \in \hat{C}, i'', n, w, w'' \in \widehat{W}'_w \quad (26)$$

$$z_{w,c,n} \leq M \times \left(x_{w',a_c,n} + \sum_{w'' \in \widehat{W}_w} \sum_{i''} d_{(w'',i'',n),c}\right)$$

$$\forall c \in \hat{C}, n, w, w' \in \widehat{W}_w \quad (27)$$

$$s_{w,a,n} \leq \sum_{c \in \hat{C}_a} z_{w,c,n} \qquad \forall a \in \hat{A}, n, w \qquad (28)$$

$$s_{w,a,n} \leq M \times x_{w,a,n} \qquad \forall a \in \hat{A}, n, w \qquad (29)$$

$$\sum_{a \in \hat{A}} \frac{1}{(1+R)^{L'_w}} \times \left(P_{FT} \times \ddot{C}_a \times x_{w,a,n} - (P_{FT} - P'_{FT}) \times s_{w,a,n}\right) = f_{FT_{w,n}} \qquad \forall n \qquad (30)$$

$$f_{FT_{w,n}} - f_{FT_{w-1,n}} \leq 0 \qquad \forall n, w \geq 2 \qquad (31)$$

$$\sum_w f_{FT_{w,n}} = f_{FT_n} \qquad \forall n \qquad (32)$$

$$\sum_w \sum_i \sum_{c \in \hat{C}_{Active_{(w,i,n)}}} \frac{1}{(1+R)^{L_{(w,i,n)}}} \times P_{SUP_c} \times r_{(w,i,n),c} = f_{SUP_n} \qquad \forall n \qquad (33)$$

$$k_{(w,i,n),c,w'} \geq 1 - o_{(w',i',n),c}$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, i' < i \; if \; w' = w, n, w, w' \leq w \qquad (34)$$

$$k_{(w,i,n),c,w'} \leq \sum_{i':\, i'<i \; if \; w'=w} \left(1 - o_{(w',i',n),c}\right)$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w, w' \leq w \qquad (35)$$

$$q_{(w,i,n),c,j} = 0 \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, j \notin \hat{J}_{(w,i,n),c}, n, w \qquad (36)$$

$$q_{(w,i,n),c,j=0} = k_{(w,i,n),c,w'=w} \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (37)$$

$$q_{(w,i,n),c,j} \geq k_{(w,i,n),c,w'=w-j} - \sum_{w'' \in \widehat{W}_{j,w}} k_{(w,i,n),c,w''}$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, j \in \hat{J}_{(w,i,n),c} \backslash \left\{0, J_{c_1} + w - 1\right\}, n, w \qquad (38)$$

$$\sum_{j \in \hat{J}_{(w,i,n),c}} q_{(w,i,n),c,j} = 1 \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (39)$$

$$u_{(w,i,n),c,j} \geq d_{(w,i,n),c} + q_{(w,i,n),c,j} - 1 \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, j, n, w \qquad (40)$$

$$u_{(w,i,n),c,j} \leq \frac{q_{(w,i,n),c,j} + d_{(w,i,n),c}}{2} \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, j, n, w \qquad (41)$$

$$v_{Surface_{(w,i,n),c,j}} + v_{Crown_{(w,i,n),c,j}} = u_{(w,i,n),c,j} \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, j, n, w \qquad (42)$$

$$v_{Crown_{(w,i,n),c,j}} \geq q_{(w,i,n),c,j} - o_{(w,i,n),c} \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, j, n, w \qquad (43)$$

$$v_{Crown_{(w,i,n),c,j}} \leq \frac{1+q_{(w,i,n),c,j}-o_{(w,i,n),c}}{2} \qquad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, j, n, w \qquad (44)$$

$$\sum_w \sum_i \sum_{c\in\hat{C}_{Active_{(w,i,n)}}} \sum_j \frac{1}{(1+R)^{L_{(w,i,n)}}} \times$$
$$\left(V_{(w,i,n),c,j} \times v_{Surface_{(w,i,n),c,j}} + V_{c,j} \times v_{Crown_{(w,i,n),c,j}}\right) = f_{FL_n} \qquad \forall n \qquad (45)$$

Objective function

Equation 1 minimizes the sum of average discounted prescribed burning cost, average discounted fire suppression cost, and average discounted fire loss across all modeled *DFS* samples.

Model fire management decisions

Equation 2 guarantees the same first stage prescribed burning decision *(FT1)* to be applied for all *DFS* samples. It reflects the "non-anticipativity" property of this stochastic program, which requires a consistent *FT1* to be made before realizing the random outcome from future fire-and-management situations. Fire control lines can only be built in cells where fire line intensities are low that a fire could not transit into a crown fire under the influence of wind speed and direction associated with that fire (Equation 3). I assume fire control line in a cell will always hold and save that cell from being burned under the modeled fire line intensity (Equation 4). The *MFAT* of a cell will be set to an arbitrarily selected value greater than the predefined fire spread duration to indicate a successful establishment and holding of fire control line in that cell (Equation 5).

Model wildfires

Each fire starts at time zero from its ignition cell (Equation 6) and will not burn non-flammable cells or cells lying outside its *MSR* (Equation 7). A cell is considered as burned by a fire ($d_{(w,i,n),c} = 1$) if that fire arrives the center of the cell within its predefined active fire spread duration (Equation 8); otherwise that cell is considered unburned ($y_{(w,i,n),c} = 1\ and\ d_{(w,i,n),c} = 0$) (Equations 9 and 10). A cell must be burned ($d_{(w,i,n),c} = 1$) if any of its adjacent cells lying inside the fire's *MSR* was burned ($d_{(w,i,n),c'} = 1$), unless fire control line is built in it ($r_{(w,i,n),c} = 1$), or fire cannot spread into it within the predefined active fire spread duration (Equation 11). After a cell is burned ($d_{(w,i,\mathrm{n}),\mathrm{c}} = 1$), fire can spread from it into any of its adjacent cells ($b_{(w,i,n),c,c'}$ variable is free to be 0 or 1) (Equation 12). The potential of a fire burning back after spreading from one cell to another cell is not modeled (Equation 13). Equation 14 ensures that a non-ignition cell can only be burned ($d_{(w,i,n),c} = 1$) by the fire spreading from exactly one of its adjacent cells ($\sum_{c' \in C_{c_{w,a,n}}} b_{(w,i,,n),c',c} = 1$). Equation 14 also assumes that a fire will not spread back to its ignition cell.

In this model, only the fastest route for a fire to spread from its ignition cell to a flammable cell lying inside its *MSR* is recorded by tracking the *MFAT* of that cell. Equations 15 and 16 work together to track whether a cell has been "treated by prescribed fire within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods" at the time a sample fire (w, i, n) starts. The *MFAT* of each cell (c) is calculated in Equations 17 and 18 by tracking all the possible spread paths from its adjacent cells toward it. In the spread path from cell c' to c, fire would spread with spread rate $ROS_{(w,i,n),c \leftarrow c'}$ in cell c and spread rate $ROS_{(w,i,n),c' \rightarrow c}$ in its adjacent cell

c' under the assumption that both cells c and c' have not been treated within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods. If a cell has been treated within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods, spread rate in that cell would decrease (to be $ROS'_{(w,i,n),c \leftarrow c'}$ and $ROS'_{(w,i,n),c' \rightarrow c}$). The two Equations 17 and 18 work together as follows:

- Equation 17 identifies the "upper bound" for the *MFAT* of cell c. Fire cannot arrive the center of cell c later than the *MFAT* of any of its adjacent cells (c') plus the spread time from the center of c' to the center of c. If the fire does not burn cell c' ($d_{(w,i,n),c'} = 0$) or if fire control line is constructed in cell c ($r_{(w,i,n),c} = 1$), the "Big M" will guarantee that the "upper bound" will not be set.
- Equation 18 identifies the "lower bound" for the *MFAT* of cell c. Fire cannot arrive the center of cell c earlier than *the MFAT* of any of its adjacent cells (c') plus the spread time from the center of c' to the center of c. If the fire cannot spread from c' to c ($b_{w,a,c',c,n} = 0$), the "Big M" will guarantee that the "lower bound" will not be set.
- The exact *MFAT* of cell c can be identified when the "upper bound" and the "lower bound" are set and converged (equal values). Otherwise, the *MFAT* of cell c will be assigned an arbitrary value greater than the sample active fire spread duration to indicate fire would not burn that cell.

Only one of the eight possible spread paths from adjacent cells (c') to c is part of the fastest fire spread route to cell c. The fire line intensity in cell c would be calculated based on the spread path that belongs to the fastest fire spread route (Equations 19, 20, 21, and 22). If cell c has not been treated within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning period

($p_{(w,i,n),c} = 0$), the fire line intensity in cell c would be $E_{(w,i,n),c \leftarrow c'}$. If that cell has been treated within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods ($p_{(w,i,n),c} = 1$), the fire line intensity would be decreased to $E'_{(w,i,n),c \leftarrow c'}$. The fire line intensity in cell c is then compared to the critical threshold of fire line intensity in that same cell at its current age-class (age-class of cell c at occurrence time of fire (w, i, n)) to decide whether a fire (w, i, n) would burn cell c as crown fire ($o_{(w,i,n),c} = 0$ and $e_{(w,i,n),c} \geq \sum_j E_{critical_{(w,i,n),c,j}} \times q_{(w,i,n),c,j}$), or as surface fire ($o_{(w,i,n),c} = 1$ and $0 < e_{(w,i,n),c} < \sum_j E_{critical_{(w,i,n),c,j}} \times q_{(w,i,n),c,j}$), or it would not burn cell c ($o_{(w,i,n),c} = 1$ and $e_{(w,i,n),c} = 0$) (Equations 23 and 24).

Estimate fire damages and consequences of prescribed burning and fire suppression

In this model, the smallest treatment unit for prescribed burning based fuel treatment is a forest stand and the cost of treating a stand is calculated by the total costs of treating all cells within it, assuming cells "treated within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods" would have lower treatment cost ($P'_{FT} < P_{FT}$). At the beginning of each planning period, cells that have been "treated within $\ddot{W}$ planning periods or burned within $\dddot{W}$ periods" are tracked by Equations 25, 26, and 27, and the total number of such cells ($s_{w,a,n} > 0$) is identified for each stand (Equation 28) only when prescribed burning is implemented in the stand ($x_{w,a,n} = 1$) (Equation 29); otherwise, $s_{w,a,n}$ is set to zero. For each *DFS*, the total discounted cost of prescribed burning in each planning period is calculated (Equation 30), and is constrained to be non-increasing while moving from one period to the next (Equation 31). This management rule helps more evenly distribute treatment workload across time. The total discounted cost of prescribed burning for each *DFS* is calculated in Equation 32 by summing up prescribed burning costs in all planning periods. The total discounted cost for building fire control lines is calculated for each *DFS* as in Equation 33.

In this model, forest age-class of a cell is identified at the time immediately before the occurrence of each fire (by $q_{(w,i,n),c,j}$ variable). Each time when a fire (w, i, n) occurs, past fire situations in each cell will be tracked (Equations 34 and 35) and used to identify the forest age class of that cell at the occurrence time of fire (w, i, n) (Equations 36, 37, 38, and 39). The logic of the four Equations 36, 37, 38, and 39 can be described by the following example. In this example, I assume the age-class of forest in the cell c is J_{c_1} at the beginning of the first period. I use an example set of three fires in three continuous planning periods denoted by fire $(1, i, n)$, fire $(2, i, n)$, and fire $(3, i, n)$; this model identifies the age-class of cell c at the time immediately before the occurrence of each fire. Each of those three fires can occur before or after the occurrences of the other fires (see also Figure 6 for the illustration of cell age-class transition under the influences of fires). The set of equations used to identify the age class of cell c at the occurrence time of each of those three fires are listed below (see also Figure 8 for illustration of possible age-classes of a cell at different times during a planning horizon).

For fire $(1, i, n)$ in the 1st period: Age-class of cell c at the time immediately before the occurrence of fire $(1, i, n)$ can be either 0 or J_{c_1} ($\hat{J}_{(1,i,n),c} = \{0, J_{c_1}\}$)

$$q_{(1,i,n),c,j} = 0 \qquad \forall j \notin \hat{J}_{(1,i,n),c}$$

$$q_{(1,i,n),c,j=0} = k_{(1,i,n),c,1}$$

$$q_{(1,i,n),c,j=0} + q_{(1,i,n),c,j=J_{c_1}} = 1$$

For fire $(2,i,n)$ in the 2[st] period: Age-class of cell c at the time immediately before the occurrence of fire $(2,i,n)$ can only be 0, 1, or $J_{c_1}+1$ ($\hat{J}_{(2,i,n),c}=\{0,1,J_{c_1}+1\}$)

$$q_{(2,i,n),c,j}=0 \qquad \forall j \notin \hat{J}_{(2,i,n),c}$$

$$q_{(2,i,n),c,j=0}=k_{(2,i,n),c,2}$$

$$q_{(2,i,n),c,j=1}\geq k_{(2,i,n),c,1}-k_{(2,i,n),c,2}$$

$$q_{(2,i,n),c,j=0}+q_{(2,i,n),c,j=1}+q_{(2,i,n),c,j=J_{c_1}+1}=1$$

For fire $(3,i,n)$ in the 3[rd] period: Age-class of cell c at the time immediately before the occurrence of fire $(3,i,n)$ can only be 0, 1, 2, or $J_{c_1}+2$ ($\hat{J}_{(3,i,n),c}=\{0,1,2,J_{c_1}+2\}$ (see also illustration in Figure 8)

$$q_{(3,i,n),c,j}=0 \qquad \forall j \notin \hat{J}_{(3,i,n),c}$$

$$q_{(3,i,n),c,j=0}=k_{(3,i,n),c,3}$$

$$q_{(2,i,n),c,j=1}\geq k_{(3,i,n),c,2}-k_{(3,i,n),c,3}$$

$$q_{(3,i,n),c,j=2}\geq k_{(3,i,n),c,1}-k_{(3,i,n),c,2}-k_{(3,i,n),c,3}$$

$$q_{(3,i,n),c,j=0}+q_{(3,i,n),c,j=1}+q_{(3,i,n),c,j=2}+q_{(3,i,n),c,j=J_{c_1}+2}=1$$

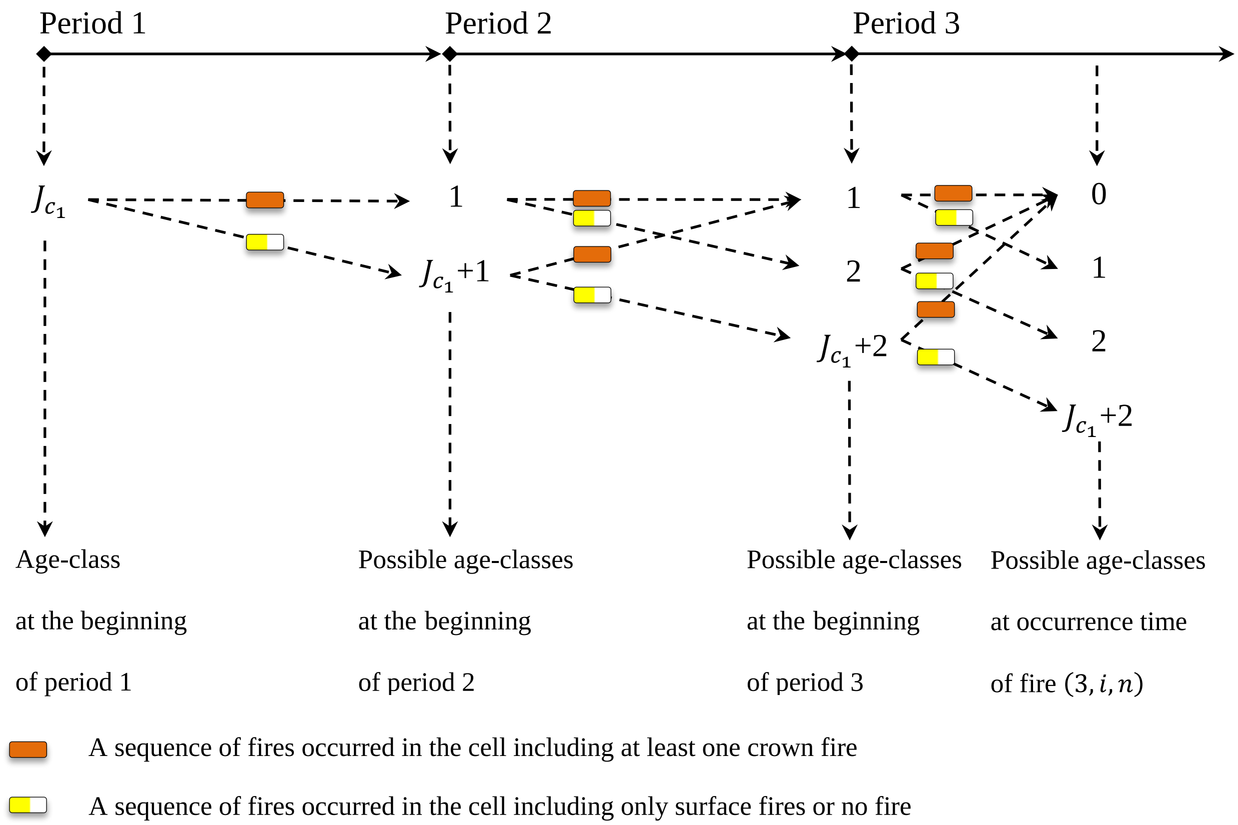


Figure 8: Possible age-classes of forest in a cell at different times during a three-period planning horizon, assuming the age-class of forest in this cell is J_{c_1} at the beginning of the first period.

Equations 40 and 41 works together to guarantee that if a fire burns a cell (c) then it only burns that cell at its exact (or current) age-class (the age-class immediately before the occurrence of this fire). A fire occurrence can lead to only one of the three situations of a cell (c) at its current age-class: not being burned by the fire ($d_{(w,i,n),c} = 0$); being burned by the fire as surface fire in this cell ($v_{Surface_{(w,i,n),c,j}} = 1$); or being burned by the fire as crown fire in this cell ($v_{Crown_{(w,i,n),c,j}} = 1$) (Equation 42). Equations 43 and 44 are used to track one of those situations, when a fire burns as crown fire in a cell (c) at its current age-class (j) ($v_{Crown_{(w,i,n),c,j}}$ receives the value of one only for the case when $o_{(w,i,n),c} = 0$ and $q_{(w,i,n),c,j} = 1$). Those two equations 43 and 44 work together with equation 42 to identify the exact fire situation in each cell at its current age-class when a fire occurs. This can help calculate the exact fire loss for each fire. For each *DFS*, the total discounted fire loss is calculated by Equation 45.

3 Test cases

3.1 Test-case assumptions

An artificial landscape is designed for the purpose of testing this stochastic program (Figure 9). The landscape includes 64 raster cells with side length of 150m. It is delineated into 12 stands, with each stand covering a forested area of homogeneous vegetation characteristics at the start of the planning periods. The landscape also includes non-flammable areas (i.e. Open water with “Stand-ID” = 0).

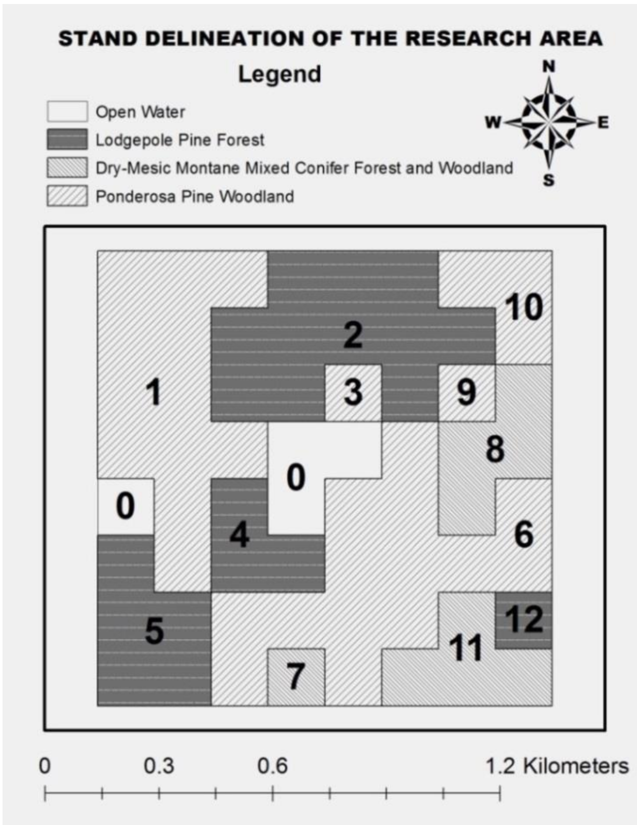


Figure 9: An artificial landscape used to build test cases, which is delineated into 12 stands. The number in each stand represents its “Stand-ID”.

Sample fires and management decisions are modeled for three planning periods ($W = 3$); each period lasts for 10 years. All flammable cells in the landscape are assumed to be at age-class three at the beginning of the first period ($J_{c_1} = 3$). The *BEs* from prescribed fire or wildfire would be assumed to both last for two continuous periods, in which following wildfires would have spread rate and intensity reduced by 50%. The annual discount rate is set to 4% ($R = 0.04$).

Each planning period may have zero to multiple sample fires from random draws. Random numbers are also drawn to determine ignition locations, fires' occurrence order, active fire-spread duration of each fire, and a combination of wind direction and speed influencing the spread of each fire. The random-draw process includes following steps:

- Step 1: Ignition in the artificial landscape is assumed to follow the average ignition frequencies of Larimer County based on historical fire data from Short (2014), which is calculated to be 0.0078125 for each flammable raster cell (150m side length) during each 10-year-planning period. To decide whether a sample fire would ignite in a flammable cell in each planning period, a random number from 1 to 10,000 will be drawn. A number ≤ 78 indicates an ignition in the cell; a number > 78 indicates "no ignition".
- Step 2: After identified the ignition locations of sample fires in a planning period (in step 1), the occurrence orders of all fires in this period will be randomly decided and evenly distributed across time in that same period. For example, if there are three fires in a planning period, each fire will be randomly assigned an order of one, two, or three occurred in year 2.5, 5 or 7.5.
- Step 3: For each sample fire, a random number in the range from 360 to 1440 minutes (6 to 24 hours) is drawn and used as the fire's active spread duration.

- Step 4: For each sample fire, a random number between 1 and 1000 is drawn to assign a combination of wind direction and speed during that fire (Table 1). Wind direction and speed are assumed to follow the historical pattern of a 10-year RAWS data (collected during April-October of 2003-2013 from Red Feather Lake station in Colorado - Western US).

Table 1: Random numbers are used to decide wind direction and speed influencing the sample fires. Wind direction and speed in the testing landscape are assumed to follow the historical pattern of a 10-year RAWS data collected from Red Feature Lake station.

Random Number	**Wind Direction**	**Wind Speed (mph)**	**Azimuth (degree)**	**Cumulative Percentage (%)**
1-20	N	4.5	0	2.0
21-36	NNE	4	22.5	3.6
37-67	NNW	5	337.5	6.7
68-114	NE	4.9	45	11.4
115-147	ENE	4.6	67.5	14.7
148-194	E	5	90	19.4
195-270	ESE	5.3	112.5	27.0
271-320	SE	5.2	135	32.0
321-341	SSE	4.4	157.5	34.1
342-358	S	4.7	180	35.8
359-390	SSW	5.5	202.5	39.0
391-522	SW	5.2	225	52.2
523-653	WSW	6.6	247.5	65.3
654-808	W	7.8	270	80.8
809-934	WNW	8	292.5	93.4
935-1000	NW	6.1	315	100.0

For each sample fire, the fire spread-rate and associated fire line intensity in each cell are pre-estimated:

- Step 1: FLAMMAP (v1.5) calculates the spread rate and associated fire line intensity in each cell for the max spread direction of the fire in that cell (the spread direction in which fire would travel the fastest). It also reports the dimension of the assumed elliptical shape of fire-spread in each cell.
- Step 2: Base on the elliptical dimension of fire spreading, the spread rate and associated fire line intensity in each cell for the max spread direction (in step 1), the spread rate and associated fire line intensity in each cell for the direction of interest can be calculated (describe later).

The following inputs are used for running FLAMMAP:

- 1st input: a landscape file (LCP file) is created to represent the topography and fuel condition of the testing landscape (details described in Table 2). This LCP file includes five GIS raster themes (Elevation, Slope, Aspect, Fuel Model, and Canopy Cover). Elevation, Slope, Aspect, and Fuel Model are created by using LANDFIRE data (http://www.landfire.gov) of a real landscape located in Larimer County – Colorado with the coordinate extents from upper left: 40.869494, -105.5856 to lower right: 40.859013, -105.5719. Canopy Cover is artificially created for each raster cell with a random range between 80% and 100% to mimic a potential forest condition with substantial risk of detrimental crown fires.

Table 2: An LCP file is used to represent topography and fuel condition of the testing landscape.

Raster Themes	Cell Value
Elevation	2,455–2,587m
Slope	5-90%
Aspect	0, 45, 90, 135, 180, 225, 270, 315, 360
Fuel Model	98 (Open Water) 122 (Mixed Conifer Forest and Woodland) 165 (Ponderosa Pine Woodland) 183 (Lodge pole Pine Forest)
Canopy Cover	80-100%

- 2nd input: Foliar moisture content (*FMC*) is set to 100% as default. Although *FMC* can vary with tree-species and time of year, the range of *FMC* for most species straddles 100% (Agee et al., 2002). Scott and Reinhardt (2001) show relative insensitivity of crown fire initiation to this parameter.
- 3rd input: Wind direction and speed during the active spread-duration of the sample fire are decided by the random-draw process described earlier.

FLAMMAP can identify the max spread direction of a fire spreading in each cell. It models fire spread in each cell by an elliptical shape with the ellipse's major axis following the max spread direction. The following outputs from FLAMMAP are exported:

$\lambda_{(w,i,n),c}$ $\psi_{(w,i,n),c}$ Parameters describing the elliptical shape of fire (w,i,n) spreading in cell c; where $\lambda_{(w,i,n),c}$ denotes half the distance between two foci, and $\psi_{(w,i,n),c}$ denotes half the length of the ellipse's major axis.

$E_{(w,i,n),c}$ — Fire line intensity in cell c when fire (w, i, n) spreads following the max spread direction in that same cell.

$ROS_{(w,i,n),c}$ — Spread rate in cell c when fire (w, i, n) spreads following the max spread direction in that same cell.

To calculate the spread rate and associated fire line intensity in each cell for the direction of interest, the following parameters are also needed (see also illustration in Figure 10):

$\theta_{(w,i,n),c}$ — The angle between the max spread direction in cell c and the spread direction when fire (w, i, n) spreads from the center of c' to the center of c.

$\theta_{(w,i,n),c'}$ — The angle between the max spread direction in cell c' and the spread direction when fire (w, i, n) spreads from the center of c' to the center of c.

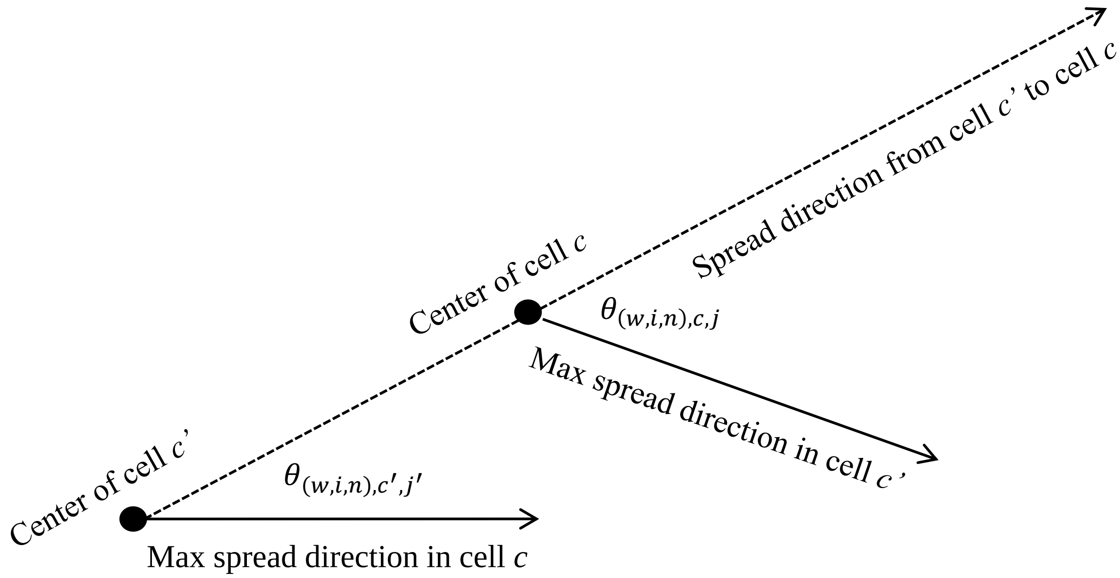


Figure 10: Illustration of the angle between the max spread direction and the spread direction of interest from c' to c (See also Figure 5 for illustration of all eight possible spread directions from or toward a cell).

Fire spread rate and intensity for the direction of interest would then be calculated based on Wei et al. (2011) as follows:

Spread rate in cell c when fire (w, i, n) spreads from c' to c is calculated by:

$$ROS_{(w,i,n),c\leftarrow c'} = \frac{(\psi_{(w,i,n),c})^2 - (\lambda_{(w,i,n),c})^2}{\psi_{(w,i,n),c} - \lambda_{(w,i,n),c} \times cos(\theta_{(w,i,n),c})}$$

$$if\ 0 \leq \theta_{(w,i,n),c} \leq \pi/2 \qquad (46)$$

$$ROS_{(w,i,n),c\leftarrow c'} = \frac{(\psi_{(w,i,n),c})^2 - (\lambda_{(w,i,n),c})^2}{\psi_{(w,i,n),c} - \lambda_{(w,i,n),c} \times cos(\pi - \theta_{(w,i,n),c})}$$

$$if\ \pi/2 \leq \theta_{(w,i,n),c} \leq \pi \qquad (47)$$

Spread rate in cell c' when fire (w, i, n) spreads from c' to c is calculated by:

$$ROS_{(w,i,n),c'\to c} = \frac{(\psi_{(w,i,n),c'})^2 - (\lambda_{(w,i,n),c'})^2}{\psi_{(w,i,n),c'} - \lambda_{(w,i,n),c'} \times cos(\theta_{(w,i,n),c'})}$$

$$if\ 0 \leq \theta_{(w,i,n),c'} \leq \pi/2 \qquad (48)$$

$$ROS_{(w,i,n),c'\to c} = \frac{(\psi_{(w,i,n),c'})^2 - (\lambda_{(w,i,n),c'})^2}{\psi_{(w,i,n),c'} - \lambda_{(w,i,n),c'} \times cos(\pi - \theta_{(w,i,n),c'})}$$

$$if\ \pi/2 \leq \theta_{(w,i,n),c'} \leq \pi \qquad (49)$$

Base on the spread rate and associated fire line intensity for the max spread direction derived from FLAMMAP, I calculate fire line intensity in each cell for the spread direction of interest as follows:

Fire line intensity in cell c when fire (w, i, n) spreads from c' to c is calculated by:

$$E_{(w,i,n),c\leftarrow c'} = E_{(w,i,n),c} \times \frac{ROS_{(w,i,n),c\leftarrow c'}}{ROS_{(w,i,n),c}} \qquad (50)$$

Fire line intensity in cell c' when fire (w, i, n) spreads from c' to c is calculated by:

$$E_{(w,i,n),c'\to c} = E_{(w,i,n),c'} \times \frac{ROS_{(w,i,n),c'\to c}}{ROS_{(w,i,n),c'}} \qquad (51)$$

Equations from 46 to 51 are used to estimate the spread rate and associated fire line intensity from or toward each cell when it has not been treated by prescribed fire within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods. If that cell has been treated by prescribed fire within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods, the spread rate and associated intensity in that cell are assumed to be decreased by 50% in the test cases.

$$ROS'_{(w,i,n),c\leftarrow c'} = \frac{1}{2} ROS_{(w,i,n),c\leftarrow c'} \tag{52}$$

$$ROS'_{(w,i,n),c'\rightarrow c} = \frac{1}{2} ROS_{(w,i,n),c'\rightarrow c} \tag{53}$$

$$E'_{(w,i,n),c\leftarrow c'} = \frac{1}{2} E_{(w,i,n),c\leftarrow c'} \tag{54}$$

$$E'_{(w,i,n),c'\rightarrow c} = \frac{1}{2} E_{(w,i,n),c'\rightarrow c} \tag{55}$$

For each sample fire, the critical threshold of fire line intensity for transition from surface fire to crown fire in each cell is calculated based on Van Wagner (1977) as a function of canopy base height (*CBH*) and foliar moisture content (*FMC*).

$$E_{critical_{(w,i,n),c,j}} = \left(0.01 \times CBH_{(w,i,n),c,j} \times \left(460 + 25.9 \times FMC_{(w,i,n),c}\right)\right)^{1.5} \quad \forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \tag{56}$$

where:

$CBH_{(w,i,n),c,j}$ Denotes the "canopy base high" of forest in cell c at age-class j at the occurrence time of fire (w,i,n).

$FMC_{(w,i,n),c}$ Denotes the "foliar moisture content" of cell c at the time fire (w,i,n) starts.

As Scott (2012) suggested, *CBH* is among "the least reliable fire modeling inputs to estimate, so adjustment of this parameter may be necessary to obtain reasonable fire modeling results”. Estimating *CBH* is more than measuring the lowest crown base height or the average crown base height in a stand (Scott and Reinhardt 2001). Rather, the vertical distribution of fuel load needs to be considered. Various definitions of *CBH* have been introduced and listed in Table 3; many of them follow Van Wagner’s suggestion (1993) that *CBH* is the lowest height above the ground at which there is sufficient canopy fuel to propagate crown fire. Measured *CBH* can vary by different measurement methods, and can greatly impact fire modeling results. For the purpose of testing this stochastic program, I assigned *CBH* randomly in a cell according to forest age classes in the cell:

- Age-class $j = 1$ (1-10 years): $CBH_{(w,i,n),c,1} = 1 - 2m$.
- Age-class $j = 2$ (11-20 years): $CBH_{(w,i,n),c,2} = 2 - 3m$.
- Age-class $j \geq 3$ (≥ 21 years): $CBH_{(w,i,n),c,3} = 3 - 4m$.

Table 3: Definitions of *CBH* from different sources.

Source	Definition
	The average crown base height in the stand
	The lowest crown base height in the stand
Fulé et al. (2002), Hoffman et al. (2007)	The lowest 20th percentile of all crown base heights in the stand
Sando and Wick (1972)	The height at which a minimum bulk density of fine fuel (100 lb/acre/ft, 0.037 kg/m^3) is found
Beukema et al. (1997)	The height at which a minimum bulk density of fine fuel (30 lb/acre/ft, 0.011 kg/m^3) is found
Cruz et al. (2003)	*CBH* is calculated by an allometric equation as a linear function of stand height and basal area

In the test cases, I only model the effect of prescribed fire on modifying surface fuels, and assume that surface fire and prescribed fire would not change canopy characteristic. Within a 10-year planning period, after a cell is burned by a crown fire, forest age-class will be reset to zero. I assume any following fire in that same period would be surface fires by assigning a very large positive value (Big M) to the *CBH* of forest at age class zero ($CBH_{(w,i,n),c,0} = M$).

Various sources of information are also collected to estimate relative prescribed burning cost, suppression cost, and value to be protected from fire in each cell in the testing landscape (Table 4). Prescribed burning cost is set to be one ($P_{FT} = 1$) for every cell that has not been treated within $\ddot{W}$ planning periods and burned within $\dddot{W}$ planning periods. Treatment cost in a cell would be assumed to be reduced by 50% if that cell has been treated within $\ddot{W}$ planning periods or burned within $\dddot{W}$ planning periods ($P'_{FT} = 0.5$). Suppression cost (the cost for building fire control line in each cell) is set to two ($P_{SUP_c} = 2$), and is assumed to be the same for every cell. In the test cases, I assume surface fires would cause zero fire loss (all $V_{(w,i,n),c,j}$ parameters are set to zero). For crown fires, two assumed per cell value losses are used:

- **Low**: $V_{c,j\geq 3} = 4$, and $V_{c,j<3} = 0$.
- **High**: $V_{c,j\geq 3} = 8$, and $V_{c,j<3} = 0$.

Table 4: Fuel treatment cost, suppression cost, and potential forest value to be protected have been estimated in different ways by various sources.

	Cost or Value	Source
Fuel treatment in general forest	\$130-\$1,100/acre	Buckley and Podolak (2014)
Prescribed fire treatment Mechanical treatment	\$125-\$490/acre \$700-\$2,084/acre	Hartsough et al. (2008)
Slash reduction burning Prescribed natural fire Management ignited fire	\$167/acre \$104/acre \$78/acre	(Cleaves et al. 1999)
Suppression for large fires	\$101-\$781/acre-burned	Buckley and Podolak (2014), Dale (2009)
Suppression for similar-sized fires and conditions in untreated areas Suppression for similar-sized fires and conditions in treated areas	\$706-\$825/acre-burned \$287-\$327/acre-burned	Fitch (2013)
Suppression for large fires	\$370-\$826/chain (*)	Smith (1987)
Forest timber value	\$3,700-\$4,300/acre	Calculated based on the estimated net volume of saw timber (Smith et al. 2009), and saw-timber price (RISI, 2014 - http://www.risiinfo.com)
Forest ecosystem value	\$392/acre	Costanza et al. (1998), Krieger (2001)
Wilderness preservation value	\$1,246/acre	Loomis et al. (1996)

(*): $\text{length in chains} = 16.82 \times (\text{area in acres burned})^{0.5}$

3.2 Test-case designs

Fuel treatment decisions at period one (*FT1s)* can be generated by the stochastic program using different sample sizes and assumptions. Two test cases are designed to answer the following questions:

- 1) Test case one focuses on answering the question: "Does changing sample size have significant impact on the overall quality of the *FT1s* suggested by the stochastic program?"
- 2) Test case two focuses on answering the question: "Among the *FT1s* suggested by the stochastic program, are some of the solutions significantly more efficient than the others?"

For both test cases, I use a set of 300 i.i.d. testing fire sequence samples (*TFSs)* to measure and compare the performance of different period one fuel treatment schedules. These *TFSs* are generated by repeatedly and randomly drawing fires based on historical data of wind and fire (as described in section **3.1**). They represent a set of many possible fire situations across a three-period planning horizon in the testing landscape.

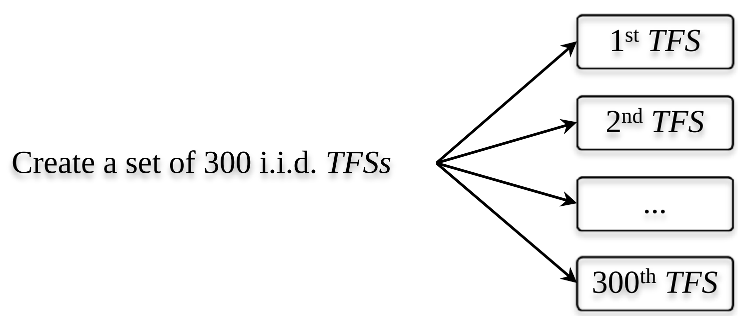


Figure 11: A set of 300 i.i.d. *TFSs* is generated to represent a set of possible fire situations across a three-period planning horizon in the testing landscape. These samples are used to measure and compare the quality of *FT1s* in both test cases one and two.

Test case one: evaluate the impact of changing sample size on the performance of FT1s from the stochastic program

I first use the stochastic program to find many *FT1s* based on random sets of fire sequence samples of fixed size *N*. The performance of these *FT1s* is evaluated by testing against 300 i.i.d. *TFSs* through multiple model runs as illustrated in Figure 12. In each run, a known *FT1* is used to hardcode the first stage fuel treatment solution, and the stochastic model is allowed to make recourse decisions in all later stages to adapt to a *TFS*. The optimal objective function value is reported for each run. The mean of the optimal objective values from the 300 runs are calculated. Different sample size *N* may create solutions with various mean of optimal objective function value when testing against the 300 i.i.d *TFSs*. Paired-t-tests are used to compare these means at the 0.05 level of significance.

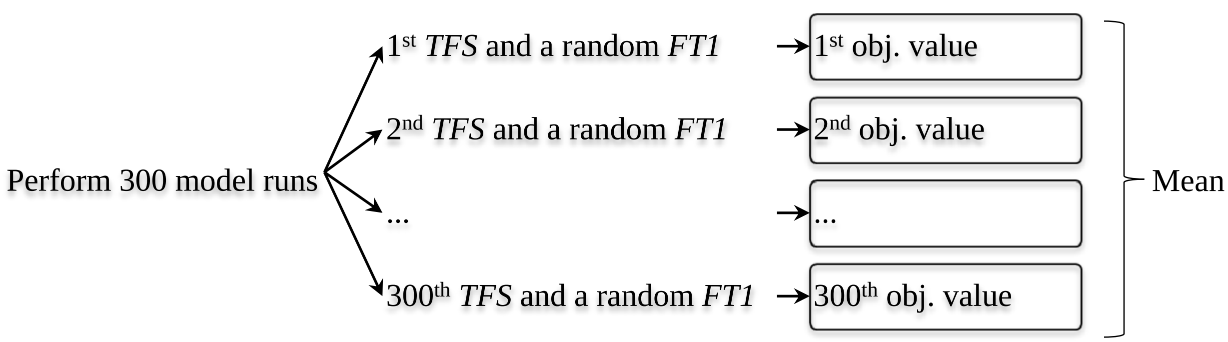


Figure 12: The process to calculate the mean that represents the overall quality of *FT1s* generated by the stochastic program using a specific sample size *N*. Here, each run uses an i.i.d. *TFS* belonging to the fixed set of 300 i.i.d. *TFSs*, and an i.i.d. *FT1* randomly generated by the stochastic program using sample size *N*.

Test case two: Identify the best FT1 and the alternative FT1s generated by the stochastic program using a specific sample size

For this test case, I select a fixed sample size *N* to populate all stochastic program runs. Three hundred runs are conducted with the selected sample size to find 300 *FT1s*. Duplicated *FT1s* may come out from these runs, so the number (denoted by *U*) of unique set of stands selected for fuel treatment in the first period is less than 300. The performance of each of the *U* unique *FT1* is then evaluated according to the process illustrated in Figure 13, where that *FT1* is hardcoded and tested against all 300 i.i.d. *TFS* samples. I use the mean of objective function values to represent the quality of each *U* unique *FT1*. Paired-t-tests are used to compare the means between the *U* unique *FT1s* at the 0.05 level of significance. I consider:

- The unique *FT1* that results in the lowest mean is “the best *FT1*”.
- The unique *FT1*s that result in the means not significantly different with the lowest mean at 95% confidence level and the difference is less than 5% are “the alternative *FT1s*”. For comparison among alternatives, a lower mean represents a better quality *FT1*.
- The best and the alternative *FT1s* are considered as high quality solutions. All the other unique *FT1s* are considered as low quality.

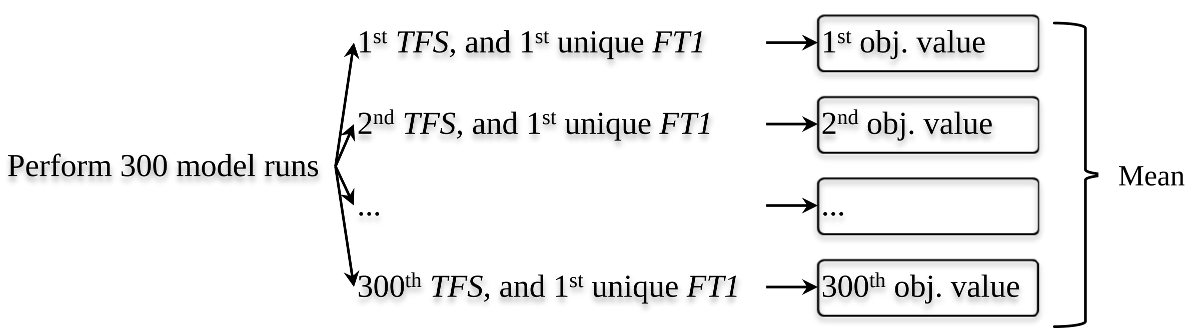


Figure 13: Illustration of the process to calculate the mean that represents the quality of a unique *FT1* (i.e. the 1^{st} unique *FT1* in this Figure).

4 Results

Testing results are reported for the two test cases based on two sets of hypothetical ratios between prescribed burning cost, suppression cost, and forest value to be protected (*FVP*). The detail of these ratios was described in section **3.1**. Per cell based *FVP* in the high *FVP* value assumption is assumed to be twice of the *FVP* in the low *FVP* value assumption.

4.1 No fuel treatment and suppression

I used an average historical ignition frequency of 0.0078125 per flammable cell on the testing landscape. One hundred and one out of the 300 randomly drawn *TFSs* have no fire; the other 199 *TFSs* include between one and five fires across all three modeled planning periods. Under the influence of the randomly drawn wind conditions, free-burning fires have an average size of 10 cells with random active spread durations from 6 to 24 hours without the impact from fuel treatments, suppressions, and previous fires.

I estimated the fire losses in the set of 300 i.i.d. *TFSs* when all fires are allowed to burn without interference from both fuel treatment and suppression. The mean and standard deviation of discounted fire losses are estimated based on the objective function values obtained from the 300 model runs. I use the term "*NoFS*" to represent these estimations. Under the low *FVP* assumption, the mean discounted fire loss for the 300 *TFSs* is 42.8 with standard deviation of 39.8. The mean and standard deviation under the high *FVP* assumption are about doubled at 85.6 and 79.7 respectively. Standard deviations of the objective function values are large under both assumptions of *FPV*s.

4.2 The impact of sample size on the quality of the first period prescribed burning solutions and on model complexity

Results from test case one are presented in Figure 14 for both assumptions of low and high *FPVs* including the means (15A), the standard deviations (15B), and the 95% confidence interval (15C) of each mean calculated from the 300 objective function values as described in section **3.2**. I use “mean” to compare the quality of the *FT1s* generated by the stochastic program using a specific sample size *N*. A sample size that leads to a lower objective function mean is considered as producing better quality and more robust *FT1s* to dealing with various fire situations represented by the 300 i.i.d. *TFSs*, as described by Ben-Tal and Nemirovski (1999).

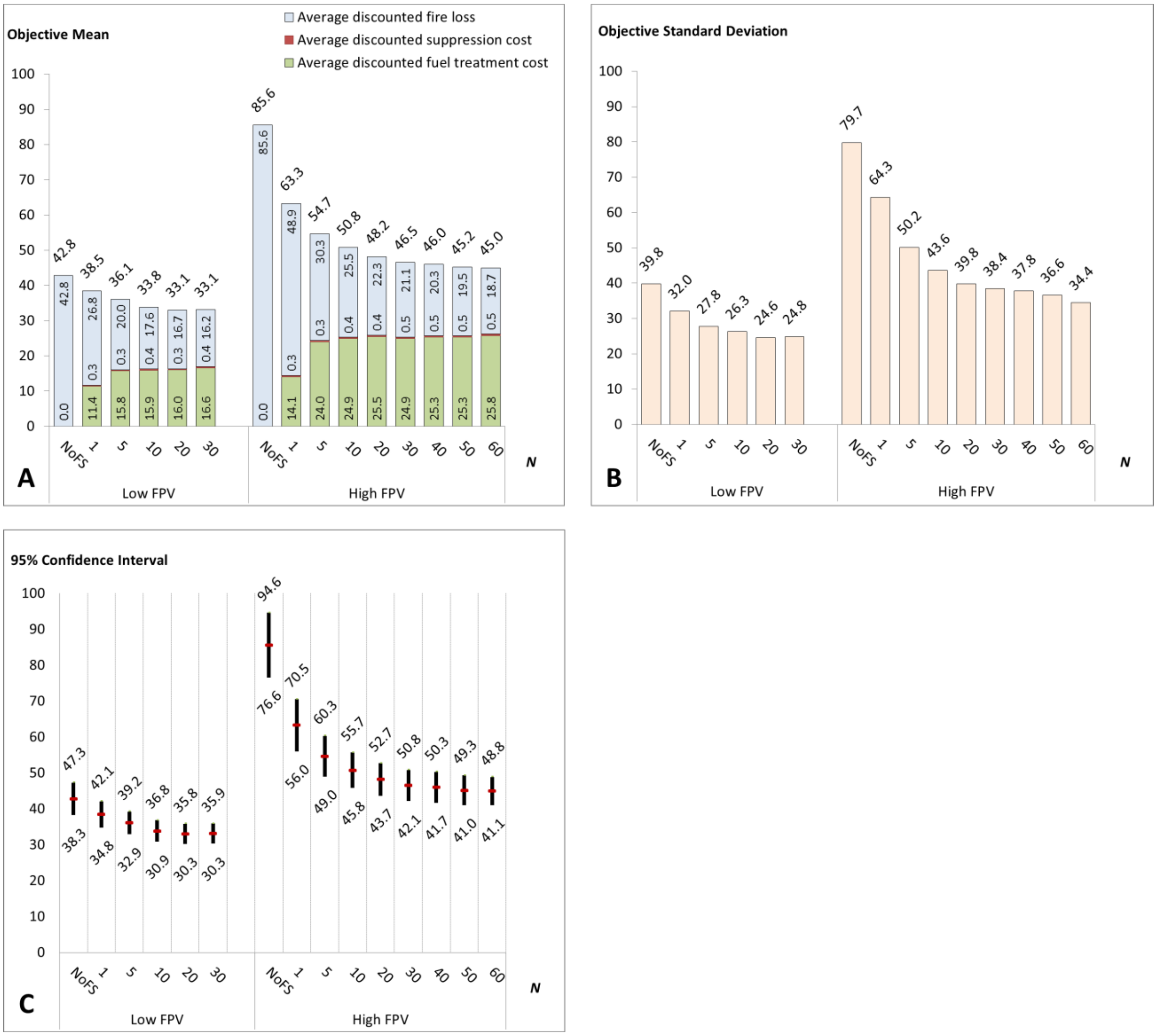


Figure 14: Results comparing the overall performance of *FT1s* generated by the stochastic program using different sample sizes *N*. (A): Objective function value means, (B): Objective function standard deviations, and (C): 95% confidence intervals of the objective function means.

Results indicate that when sample size increases, the overall quality and robustness of the *FT1s* generated by the stochastic program improve. In most of the cases, using a larger sample size leads to a lower objective function mean (15A), a lower standard deviation (15B), and also a narrower 95% confidence interval (15C). Under the assumption of low *FVP*, although models built on sample size one to five lead to lower means and standard deviations comparing to *NoFS*, statistical tests indicate that the differences among those means are not significant; only models with sample size ≥10 can lead to significantly lower objective function means in comparison with *NoFS*. Under the high *FVP* assumption, the improvement of solution quality due to increasing sample size is more obvious. All the models with sample size ≥1 lead to significantly lower objective function means compared to *NoFS*; reductions in the mean are also significant when increasing sample size from one to more than 10, and from five to 60.

Although increasing sample size can improve the overall quality and robustness of the *FT1s* generated by the stochastic program, this effect diminishes when sample size grows as indicated by less differences between the means, less differences between the standard deviations, and more overlaps between the 95% confidence intervals. Under the assumption of low *FPV*, I saw 12.0% reductions of the objective function mean, and 18.1% reduction of standard deviation when sample size increases from 1 to 10; the corresponding reductions under high *FPV* assumption are 19.8% and 32.2% respectively. Quality of the *FT1s* across many stochastic program runs based on larger sample sizes is more consistent as indicated by the smaller difference of the objective function means. For example, the maximum difference of the means is less than 2.5% when comparing between models using sample size ≥10 under low *FPV* assumption, and comparing between models using sample size ≥40 under high *FPV* assumption. Results indicate little improvement of solution quality when sample size increases from 20 to 30 or from 50 to 60 respectively for the two assumptions of low or high *FPV*.

Increasing sample size would also increase model complexity, reflected partially by longer computing time required to solve the stochastic program (Table 5). Solution quality however, shows little difference between large sample size model runs. For example, under the low *FPV* assumption, increasing sample size from 20 to 30 would increase solution time 3.5 times (423 to 1491 minutes), but solutions have about the same objective function means (33.1) and little difference in objective function standard deviations (24.6 and 24.8).

Table 5: Total solving time estimated for 300 runs of the stochastic program using each different sample size.

Sample size	1	5	10	20	30	40	50
Solving time (minutes) under low ***FPV*** assumption	5	38	158	423	1491	N/A	N/A
Solving time (minutes) under high ***FPV*** assumption	5	17	46	179	413	827	1344

4.3 Comparison of the first period prescribed burning solutions

Fuel treatment decisions need to be made across space and time. Treatment decisions at the first stage (or first period) needs to be carried out immediately before the reveal of future fire conditions, and may have impact on future fire behaviors and management recourse decisions at later stages. A good *FT1* should consider the future fire situations and support future management activities. Different sets of random sample fires used by the stochastic program may suggest different *FT1s*; changing sample size *N* can also suggest different *FT1s*. Selecting a good *FT1* is often challenging.

Repetitively running the stochastic program using larger sample sizes can increase the chance of finding a good quality *FT1*. However, increasing sample size also makes the stochastic program more complex and consequently more difficult to solve. As indicated by results from test case one, solution quality will increase in a diminishing manor when sample size grows. This makes it possible to obtain a good set of *FT1s* with a moderate sample size. The *FT1s* selected by using these sample sets may still have reasonably good quality.

Under the assumption of low *FPV*, I selected a sample size of 30 *DFSs* to run the stochastic program 300 times and found 90 unique *FT1s*; each of them represent a unique set of stands selected for first period prescribed burning. Under the high *FPV* assumption, the sample size of 60 *DFSs* was selected to run the stochastic program 300 times which also identified 90 unique *FT1s*. The performance of each unique *FT1* was then evaluated through paired-t-tests with 95% confidence to remove all low quality solutions. I only focus on studying the remaining high quality *FT1s* that include the best solution found and the alternative solutions that have less than 5% difference compared with the discovered lowest objective function mean from the best solution. These high quality *FT1s* are listed in the Table 6 and also illustrated in Figure 15 and Figure 16.

Table 6: The best *FT1* and the alternative *FT1s* under different *FPV* assumptions.

	No	Treated Stands	Chance (%)	Treatment Amount (%)	95% Confidence Interval Lower bound	Mean	Upper bound
Low *FPV*	1	3, 4, 8, 9	14.0	15.0	30.2	33.0	35.9
	2	3, 4, 7, 8	0.7	15.0	30.5	33.3	36.2
	3	4, 7, 8, 9	1.7	15.0	30.3	33.2	36.1
	4	3, 4, 8, 12	1.0	15.0	31.0	33.9	36.9
	5	3, 4, 7, 8, 9	0.3	16.7	30.5	33.3	36.0
	6	3, 4, 8, 10	0.7	18.3	30.7	33.5	36.4
	7	3, 4, 7, 8, 9, 12	3.3	18.3	30.3	33.0	35.7
	8	4, 8, 9, 10	2.0	18.3	30.7	33.7	36.6
	9	3, 4, 8, 9, 10	1.0	20.0	30.9	33.7	36.4
	10	4, 8, 9, 11	0.3	20.0	31.3	34.1	36.9
	11	3, 4, 8, 9, 11	0.7	21.7	31.5	34.1	36.8
	12	1, 3	3.0	21.7	31.0	34.2	37.3
	13	6 (**best *FT1***)	16.7	23.3	30.1	32.6	35.1
	14	1, 3, 9	3.7	23.3	30.9	33.9	36.9
	15	1, 3, 7	1.0	23.3	31.1	34.2	37.2
	16	6, 9	2.7	25.0	30.6	33.0	35.4
	17	3, 6	3.3	25.0	30.7	33.1	35.5
	18	3, 6, 9	1.0	26.7	31.2	33.5	35.9
High *FPV*	1	6	3.0	23.3	41.3	46.3	51.4
	2	6, 9	3.3	25.0	41.3	46.1	51.0
	3	3, 6	3.0	25.0	41.4	46.2	51.0
	4	3, 6, 9	4.0	26.7	41.5	46.2	50.8
	5	6, 8	1.0	30.0	41.7	46.2	50.7
	6	6, 8, 9	3.0	31.7	41.8	46.2	50.6
	7	1, 3, 4, 8, 9	3.7	35.0	42.1	46.3	50.5
	8	1, 3, 4, 7, 8, 9	0.3	36.7	42.1	46.0	49.9
	9	1, 6	7.7	43.3	41.2	44.3	47.4
	10	1, 6, 9 (**best *FT1***)	4.7	45.0	41.4	44.2	47.0
	11	1, 3, 6	5.3	45.0	41.5	44.3	47.0
	12	1, 3, 6, 9	3.3	46.7	41.7	44.2	46.7
	13	1, 6, 10	0.3	48.3	42.4	45.1	47.7
	14	1, 4, 6	0.7	48.3	43.0	46.0	49.0
	15	1, 6, 8	1.7	50.0	42.0	44.3	46.6
	16	1, 3, 6, 10	0.3	50.0	42.7	45.0	47.3
	17	1, 6, 9, 10	0.3	50.0	42.7	45.1	47.5
	18	1, 6, 8, 9	4.3	51.7	42.2	44.3	46.3
	19	1, 3, 6, 8	1.7	51.7	42.5	44.5	46.5
	20	1, 3, 6, 8, 9	2.0	53.3	42.8	44.6	46.4
	21	1, 6, 8, 9, 10	0.3	56.7	44.0	45.8	47.6
	22	1, 3, 4, 6, 8, 9	0.3	58.3	44.6	46.3	47.9

Chance = (total number of duplication of a unique FT1)/(total number of runs)×100

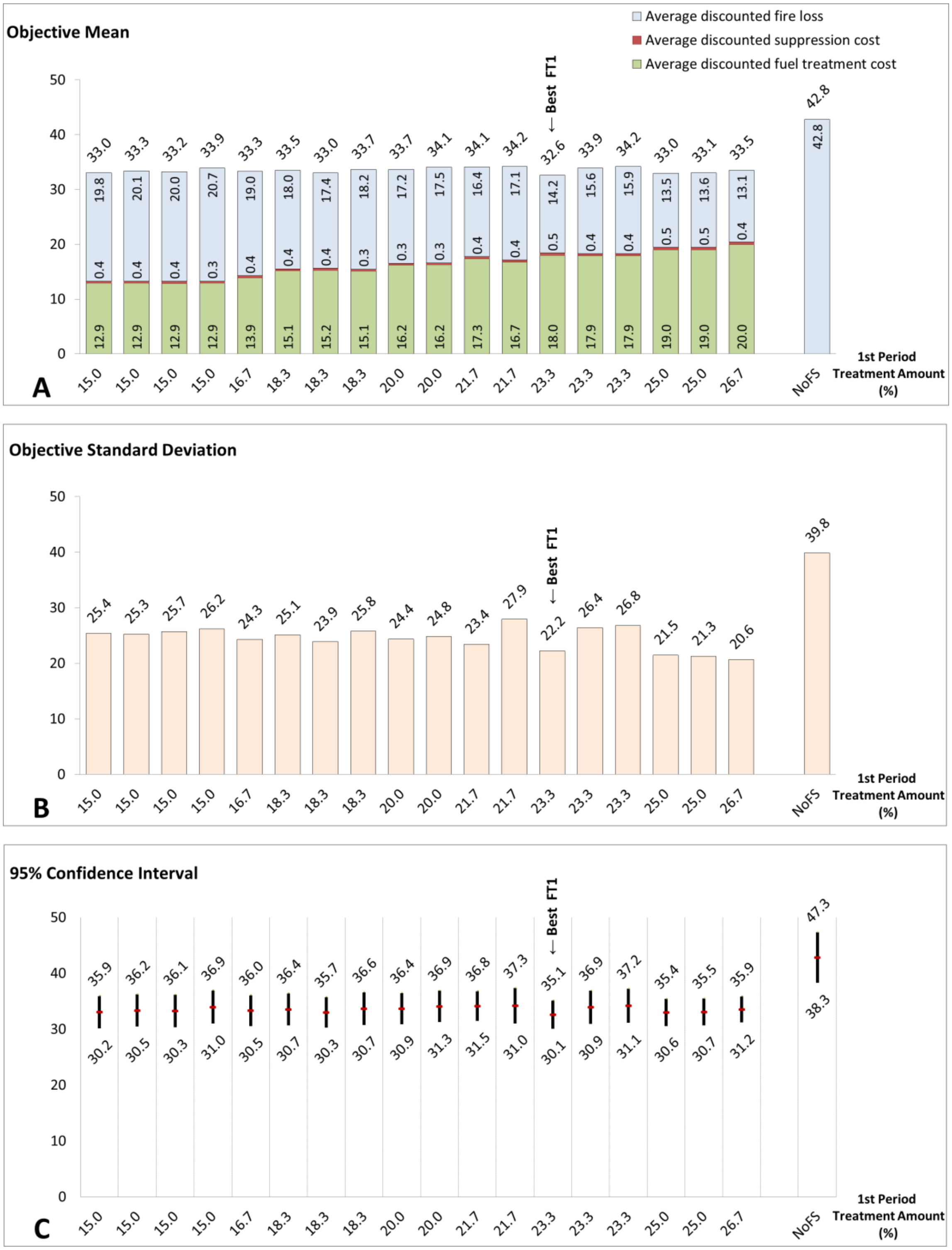


Figure 15: The best and the alternative *FT1s* under the assumption of **Low *FPV***. Results include: (A): Objective means, (B): Objective standard deviations, and (C): 95% confidence intervals of the means.

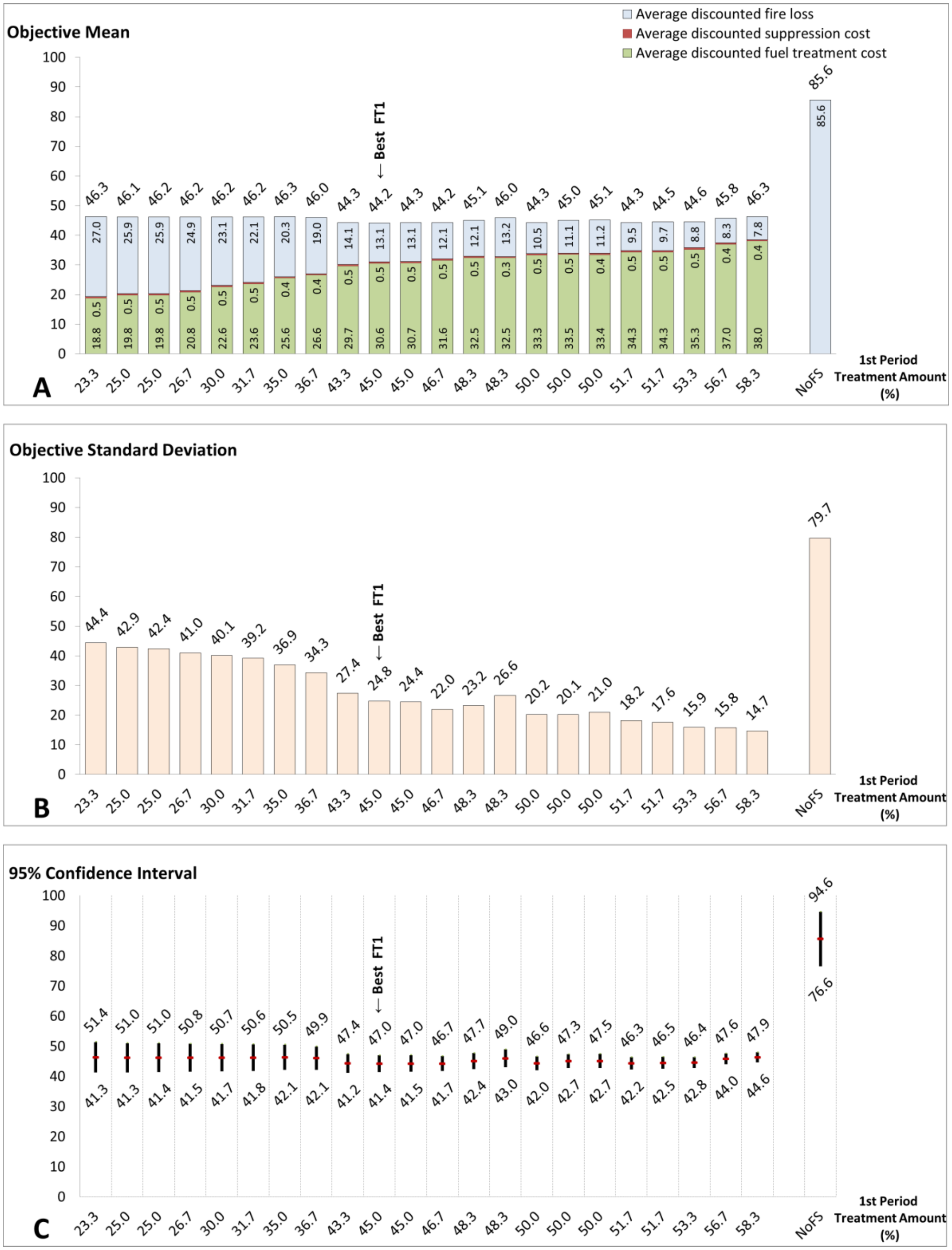
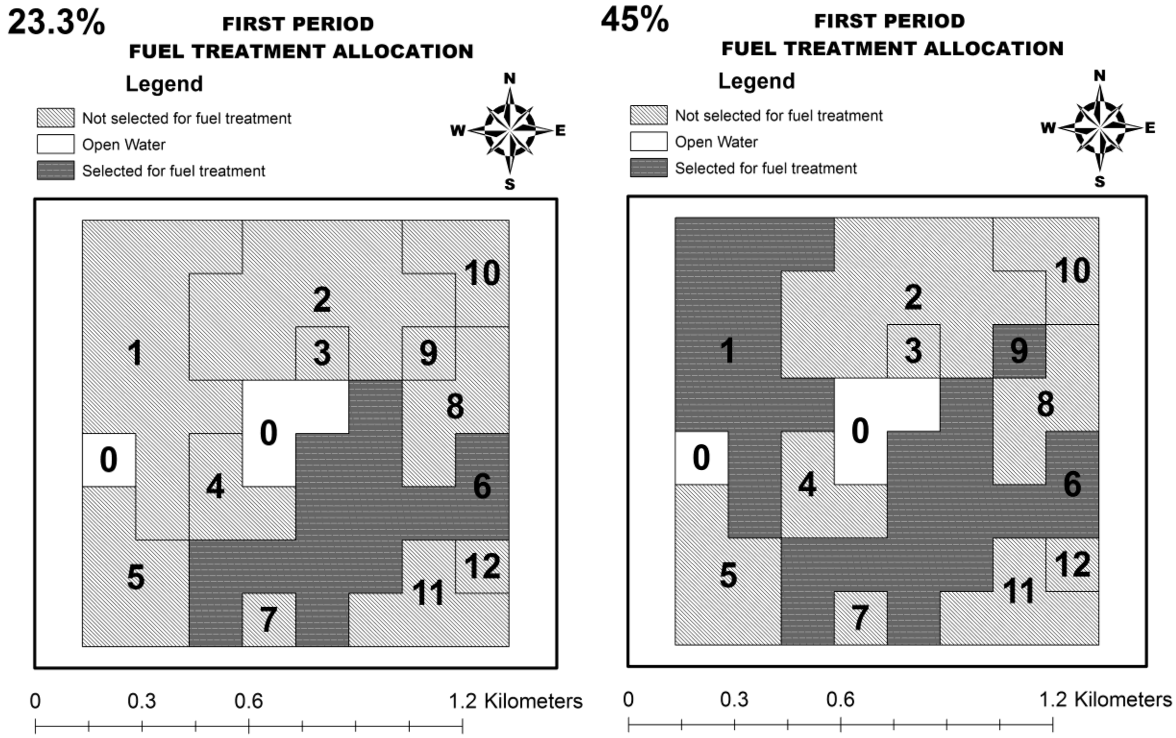


Figure 16: The best and the alternative *FT1s* under the assumption of **High *FPV***. Results include: (A): Objective means, (B): Objective standard deviations, and (C): 95% confidence intervals of the means.

<u>Study the best FT1</u>

I simply choose the *FT1* that leads to the lowest objective function mean to be the best solution. Under the assumption of low *FPV*, the best *FT1* decreases the objective function mean from 42.8 (*NoFS*) to 32.6, an approximately 23.8% reduction. In this *FT1*, prescribed burning is only scheduled in stand six covering 23.3% of the tested landscape. This stand is adjacent to the open water at the center of the landscape. Scheduling prescribed fire in it separates the landscape into two disconnected patches (Figure 17A). Under the assumption of high *FPV*, implementing the best *FT1* also decreases the objective function mean significantly in comparison with *NoFS*. Objective function mean decreases from 85.6 to 44.2, an approximately 48.4% reduction rate. The total area of the first period prescribed burning as suggested by this solution covers 45% of the total landscape area, which is approximately doubled comparing with the low *FPV* test case. Treated area includes stand one, six and nine. These treated stands also connect with the open water and separate the whole landscape into four patches (Figure 17B). This treatment arrangement is more effective in reducing landscape fire loss.



A: **Low *FPV*** **B**: **High *FPV***

Figure 17: Stands selected for the first period fuel treatments as suggested by the best *FT1* under low and high assumed forest protected value.

The chance of identifying the best *FT1* from just one stochastic program run is 16.7% and 4.7% respectively (Table 6) associated with the two assumptions of low and high *FPVs*. This is partially due to the low historical fire frequency used for the tests. As mentioned earlier, the random draws resulted in 101 over the total of 300 i.i.d. *TFSs* not having any fire. It indicates that no treatment would be identified as the optimal solution in at least 30% of the stochastic model runs.

Alternative FT1s

Alternative *FT1s* are identified for both assumptions of low and high *FPVs* as presented in Table 6 and also illustrated in Figure 15 and Figure 16. The Pair-t-test selection method guarantees that the mean from any alternative *FT1* and the lowest mean from the best *FT1* are not significant different at the 95% confidence level, and the difference must be less than 5%. This is illustrated by the overlaps of all the 95% confidence intervals (16C and 17C). Under the assumptions of low and high *FPVs*, our selection method respectively identifies 17 and 21 alternative *FT1s*. These high quality solutions with various treatment patterns and amounts can give fire managers more flexibility in choosing what would fit their plan the best. Figure 18 and 19 below illustrate some of the high quality treatment plans for the first period. The complete set of alternative *FT1s* for each *FPV* assumption was listed earlier in Table 6.

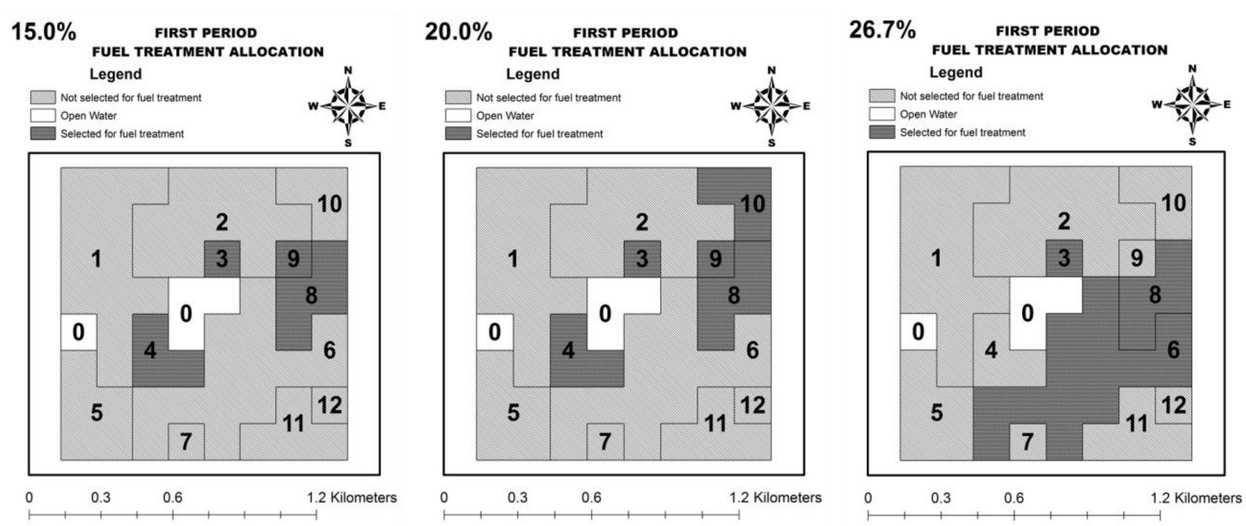


Figure 18: Some alternatives *FT1s* under **Low *FPV*** assumption

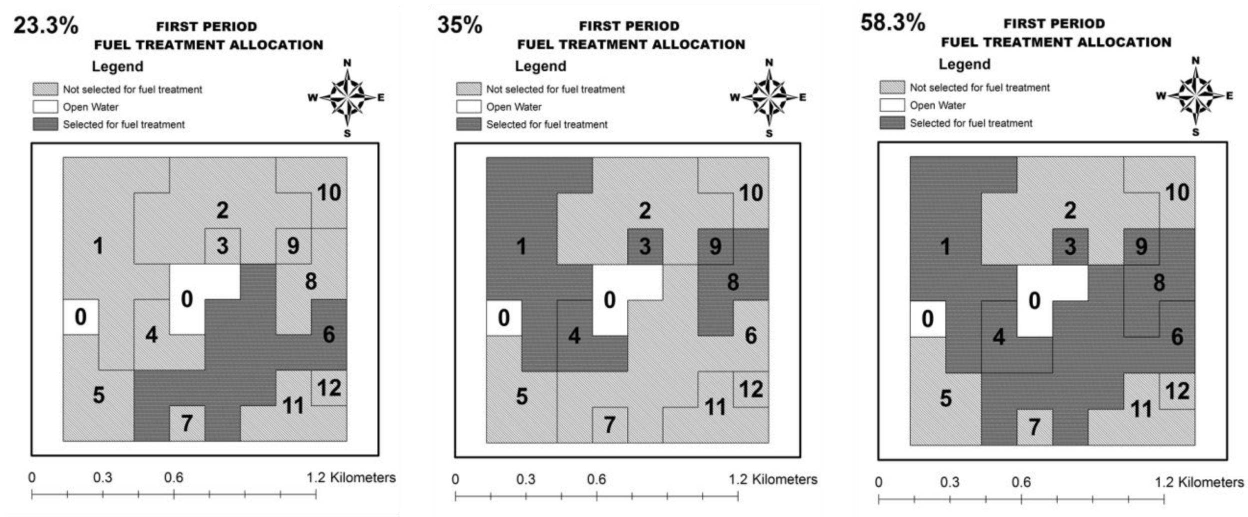


Figure 19: Some alternatives *FT1s* under **High *FPV*** assumption

Results indicate that a good prescribed burning plan should treat between 15.0% and 26.7% of the total landscape areas during the first period when the *FPV* is low. When the *FPV* is doubled (the high *FPV* case), treated areas in the first period should cover 23.3% to 58.3% of the total landscape. As illustrated in Figure 15A and 17A, while the objective function means remains at less than 5% difference, in most of the cases increasing the total area of the first period prescribed burning would decrease the average discounted loss from future fires. Larger area of prescribed burning applied in the first period leads to solutions with objective function means having lower standard deviations and narrower confidence intervals. This is more obvious when testing under the assumption of high *FPV* (17B and 17C).

Suppression is allowed in all planning periods by building fire-control-lines. Within the alternative *FT1s*, increasing the total area of prescribed burning does not necessarily reduce the total cost of suppression across all periods (16A and 17A). Modeling results under different assumptions suggest that some fire suppression activities should be applied in the tested landscape, with the average discounted cost of building fire control lines accounting for a small proportion of the total objective function mean: from 0.48% to 1.05% under the low *FPV* assumption (16A), and from 0.72% to 1.14% under the high *FPV* assumption (17A). There are several reasons that may contribute to the low suppression cost. Firstly, our test cases assume the per-cell based cost for control line construction is twice of the per-cell based cost for prescribed burning, which makes suppression a more expensive and less appealing decision in comparison with prescribed treatment. Secondly, suppressions are only allowed in cells where fires cannot reach forest canopy (surface fires), and these fires in the test cases are assumed to cause no *FPV* loss; therefore, the effect of suppression is only to prevent fire from spreading into other cells. With an average *MSR* of 10 cells for each sample fire, the number of cell can be saved by suppression is small. Furthermore, due to the assumed beneficial effects of prescribed burning and previous fires (decrease fire spread rate and intensity by 50%) the spread range of each sample fire would be even smaller and dampen the need for suppression to be aggressively implemented after encountering areas previously treated or burned. A study from Schaaf et al. (2004) in Angeles National Forest in southern California also suggests that a small and less costly fire suppression program, matched with a moderate intensity prescribed burning program would provide the most cost-beneficial fire management strategy for their study area.

5 Conclusion and Discussion

Wildfire, along with the uncertainties associated with it, creates significant challenges in long-term wildland fire management. Decision models have been used to support wildfire managers on estimating fire impact and suggesting fire planning strategies. Fire-risks have been integrated into planning processes in either deterministic or stochastic formulations. Deterministic models often lack the ability to adequately capture the stochastic nature of wildfires and may lead to biased management decisions (Hof and Omi 2003). Modelling wildfire decisions under a stochastic framework often has the advantage of providing more robust decisions to account for the uncertainties in wildfire management. In building stochastic programming models to support wildfire decisions, we can either draw random fire samples or construct representative fire scenarios to reflect the important future stochastic pattern of wildfire occurrence in a landscape. The number of possible scenarios can be enormous when modeling the spatial and temporal interactions between wildfire and wildfire decisions, which makes it difficult to construct and select adequate representative scenarios. In this study, I use random fire samples to build a stochastic program and several test cases in supporting long term prescribed burning based fuel treatment decisions.

In this study, I integrate wildfire occurrence and behavior, prescribed burning, and fire suppression into a prototype multistage stochastic program that can explicitly model the spatial and temporal interactions between these components. Future forest fuel load can be decreased by wildfire or prescribed burning. Within certain periods after these events, there may be less devastating fire behavior and potentially safer and more effective fire suppression activity in the previously burned or treated areas. The expected outcome of fuel and fire management across a multi-period planning horizon is minimized in a sample average approximation based formulation, where each sample represents a sequence of management decisions and randomly simulated sample fires across all periods. The spatial locations of prescribed burning areas in the first period is the primary interest of implementing this model because the first period (first stage) prescribed burning decision needs to be implemented immediately, whereas recourse decisions can be adjusted accordingly during the later stages of the decision process. The design of this multistage stochastic program would improve the robustness of the first period prescribed burning decision by accounting for uncertainties of future prescribed burning, fire behavior, and simplified suppression activities.

Replicating the stochastic model runs built on different i.i.d. *DFSs* may suggest different layouts for prescribed burning in the first period. I use Monte Carlo simulations and paired-t-tests to compare quality of the first period prescribed burning solutions at 95% confidence. Tests show that a high quality solution (as presented in Table 6) can often be discovered by comparing and evaluating the pool of many candidate solutions acquired from multiple stochastic program runs.

Test cases in this study are built on a 12-stand forested artificial landscape with sample fires simulated randomly. I use a three-period fire planning horizon for the tests. Prescribed burning is modeled as the only method used for fuel treatment. Fire management activity is simplified as the construction of fire-control-line in a cell with a predefined cost. I solved a series of test cases under two different hypothetical assumptions of ratios between prescribed burning cost, suppression cost, and *FPV*. Results provide various optimal layouts for allocating prescribed fires in the first planning period. Results suggest that a good plan for the first period prescribed burning should cover between 15.0% and 26.7%, or between 23.3% and 58.3% of the tested landscape respectively depending on whether the assumed *FPV* is low or high. Some of the discovered first period prescribed-burning solutions are significantly better than the others when tested against a fixed set of 300 i.i.d *TFSs*. These indicate that a single stochastic run may not always find a high quality first period prescribed burning solution (Table 6). It is interesting to see whether the chance of encountering a good quality first period solution might increase when testing in a landscape with a higher fire frequency. I leave it to future work because of our current computer limits.

Alternative first-period fuel treatment selections will give fire managers more flexibility in choosing their fire management plans. Scheduling prescribed fire in less areas will lower fuel treatment cost, but may increase the expected fire loss. Due to these tradeoffs, there may be less significant change in the expected objective function value. Scheduling fuel treatment in a large portion of a landscape may decrease fire loss significantly. However, intensively scheduling fuel treatment across a landscape is costly and may not be practical (Laverty and Williams 2000, GAO 2003). In this case, effectively selecting a smaller portion of the landscape for treatment can be a better option.

In this study, the stochastic program is tested by varying the number of i.i.d. *DFSs*. As expected, results show the benefit of using larger sample sizes to improve solution robustness and efficiency, as indicated by smaller values of objective function means and standard deviations across many replicated model runs. Results indicate that sample size of 10 or 30 could be used to find good quality first period prescribed burning solutions under the two assumptions of low or high *FPV*. The benefit of increasing sample size tends to diminish as sample size increases, which allows us to use a moderate sample size to obtain a reasonably good set of solutions. Using larger sample sizes would significantly increase model complexity and solution time.

I acknowledge that running this stochastic programming model on a larger landscape with more planning periods may require us to use large sample sizes to obtain solutions with good quality. Models built on larger landscapes are also difficult to solve. For example, I tested a 20-by-20 cell landscape including 70 stands; using the same assumptions as the presented test cases, our computer can only solve a problem with sample size up to 15 *DFSs*. Solving larger problems with more complicated forest fuel structures and fire conditions might be possible by using computer with more memory, developing more efficient modeling formulations, designing proper heuristics, or employing other solution approaches such as decomposition methods. These are potential future research.

Another limitation from this stochastic program is the simplification of fire behavior. Fire spread rate and fire line intensity are modeled based on surface fires. I assumed crown fire occurred only if surface fire could spread into crown. A detailed fire modelling approach may model the spread of crown fire across forest canopy directly, and also the effect of spotting fire (Rothermel 1991, Scott and Reinhardt 2001). Active crown fires can spread faster than surface fires (Scott and Reinhardt 2001). Therefore, only modelling fire spread on surface may underestimate the actual fire spread rate, especially when a severe fire spreads under extreme weather conditions.

The stochastic program in this study only addresses prescribed-fire based fuel treatment. This program however, could be readily adapted without significantly altering its structure to consider other fuel treatment methods. Treatment methods which have different effects on fire spread rate and intensity that last for different duration can be modeled by simply modifying the parameters ($ROS'_{(w,i,n),c'\rightarrow c}$, $ROS'_{(w,i,n),c\leftarrow c'}$, $E'_{(w,i,n),c'\rightarrow c}$, $E'_{(w,i,n),c\leftarrow c'}$, $\ddot{W}$). Other treatment methods can increase *CBH* in a forest, such as mechanical treatments. Base on the correlation between *CBH* and the critical threshold of fire line intensity (as in Equation 56), the effect of fuel treatment on increasing *CBH* can be modeled by increasing the critical threshold of fire line intensity. For example, we can add a variable to track fuel treatment activity (i.e. $p'_{(w,i,n),c}$ in Equations 57 and 58), use that variable to calculate the increase of the critical threshold of fire line intensity after fuel treatment, and consider that increase when comparing between fire line intensity and the critical intensity threshold (Equations 59 and 60 are used to replaced Equations 23 and 24). Incorporating various fuel treatment methods into a single model is a challenging and interesting task in future research.

$$p'_{(w,i,n),c} \leq x_{w,a_c,n} + \sum_{w' \in \widehat{W}_w} x_{w',a_c,n} \qquad \forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (57)$$

$$p'_{(w,i,n),c} \geq \frac{x_{w,a_c,n} + \sum_{w' \in \widehat{W}_w} x_{w',a_c,n}}{M} \qquad \forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (58)$$

$$o_{(w,i,n),c} \geq \frac{\sum_j E_{critical_{(w,i,n),c,j}} \times q_{(w,i,n),c,j} + \Delta \times p'_{(w,i,n),c} - e_{(w,i,n),c}}{M}$$

$$\forall c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (59)$$

$$1 - o_{(w,i,n),c} \geq \frac{e_{(w,i,n),c} - \sum_j E_{critical_{(w,i,n),c,j}} \times q_{(w,i,n),c,j} - \Delta \times p'_{(w,i,n),c}}{M}$$

$$\forall\, c \in \hat{C}_{Active_{(w,i,n)}}, i, n, w \qquad (60)$$

where:

Δ is a parameter denoting certain increase in the critical threshold of fire line intensity when fuel treatment effect lasts.

$p'_{(w,i,n),c}$ is a binary variable receiving a value of 1 if at the time immediately before the occurrence of fire (w, i, n) cell c has been treated within $\ddot{W}$ planning periods; otherwise, $p'_{(w,i,n),c} = 0$.

In this study, fire suppression is not modeled with details. A stochastic model from Belval et al. (2015) includes many more realistic aspects of fire suppression activities such as crew movement, crew safety, line production, and line quality. That model also captures the interaction between fire behavior and suppression in a two-stage problem. However, including that level of details in suppression of even a single fire would demand long computing time and large capacity of computer memory. The preprocessing algorithm used in my study greatly helps improve the computation efficiency. However, modelling fire behavior, fuel treatment, and suppression in details may significantly reduce the sample size that can be dealt by this stochastic program.

As the final remark, the purpose of this study is to demonstrate that stochastic program could be used to provide insight about the magnitude and spatial allocation for the first period prescribed burning. The flexibility of this stochastic program would allow fire managers to study different management assumptions and to meet different management interests. Results demonstrate that scheduling prescribed fire at the beginning of a planning horizon can effectively lower the total fire loss and management cost across all planning periods. However, many insights from this study are based on testing a prototype model on an artificial landscape. The main purpose of the test cases is to validate the model logic and performance. Depending on specific future fuel treatment decisions, more detailed modeling and modifications may be required to apply this prototype model to a real world landscape. To improve the accuracy and reliability of the presented program, it is also necessary to have more supports such as site-specific analysis, or experimental evidences.

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Appendix

A preprocessing algorithm to calculate the maximum spread range (*MSR*) of each sample fire

This algorithm is used to calculate the *MSR* for each sample fire within an assumed duration, under a modeled wind condition, and without interference from fuel treatments, suppressions, and previous fires. A fire cannot spread beyond its *MSR* in future modeling when the effects of the previous fires, fuel treatments, or suppressions are to reduce fire spread rate and contain fires. Therefore in the stochastic program, suppression and fire spread will not be modeled outside of the *MSRs*. This can reduce computer memory requirement and also speed up solution time of the stochastic program.

Beside notations described in section **2.3**, this preprocessing algorithm also uses the following notations:

$iter$	Index of a simulation iteration.
$\hat{C}_{iter}$	The set of "candidate cells" that can be burned in the iteration denoted as "$iter$". This set includes all flammable cells that are adjacent to the "already burned cells" identified from the previous iteration denoted as "$iter - 1$".
$\hat{C}'_{iter}$	The set of "burned cells" identified at the end of iteration "$iter$".
$\hat{C}^{*}_{iter}$	The cell which the fire will spread to the first (the earliest) at the end of iteration "$iter$".

$ITER_{max}$ The maximum number of possible iterations required to determine the *MSR*, which is equal to the total number of flammable cells in the landscape.

T_c The *MFAT* of cell c.

$T^*{}_{iter}$ The minimum value among all possible fire arrival times for cells in set $\hat{C}_{iter}$.

This algorithm uses an iterative process to model fire spread. Fire can spread between adjacent cells sharing an edge or a vertex. At the end of each iteration, only one cell with the earliest fire arrival time would be identified as the cell that the fire will spread to in this iteration. The *MSR* of the simulated sample fire includes all the burned cells identified in the last iteration. The algorithm is described below (see also a flowchart of this algorithm in Figure 20, and an illustrative example in Figure 21).

<u>Select a sample fire</u> (1 step)

- Step 1: Select a randomly drawn sample fire (w, i, n) with its pre-determined active spread duration ($H_{(w,i,n)}$), and its pre-estimated spread rates ($ROS_{(w,i,n),c\leftarrow c'}$, and $ROS_{(w,i,n),c'\rightarrow c}$); then proceed to step 2.

<u>Initiate the Mode</u> (5 steps)

- Step 2: Set the current iteration to one: $iter = 1$
- Step 3: The ignition cell is identified as the only cell to be burned at the end of the first iteration: $\hat{C}^*{}_{iter=1} = \hat{C}_{Ignition_{(w,i,n)}}$.

- Step 4: Set *MFAT* of the ignition cell to zero: $T_{c=\hat{C}_{Ignition_{(w,i,n)}}} = 0$.
- Step 5: Set "burned cells" at the end of iteration one to include only the ignition cell: $\hat{C}'_{iter} = \left\{\hat{C}_{Ignition_{(w,i,n)}}\right\}$
- Step 6: Set "candidate cells to be burned" in the next iteration (iteration two) to include all flammable cells that are adjacent to the ignition cell: $\hat{C}_{iter+1} = \hat{C}_{c=\hat{C}_{Ignition_{(w,i,n)}}}$. Then proceed to step 7.

<u>*Check algorithm stopping conditions*</u> *(1 step)*

- Step 7: Stop the algorithm if either: 1) *MFAT* of the cell identified as burned earliest at the end of the current iteration is greater than the fire duration ($T_{c=\hat{C}^*_{iter}} > H_{(w,i,n)}$). In this case, the sample fire only burns part of the landscape at the end of its duration; or 2) the total number of iterations reaches the maximum ($iter = ITER_{max}$). In this case, all flammable cells in the landscape are burned by the fire before the end of its duration. The *MSR* is identified when the algorithm stops, which includes all the burned cells in the last iteration ($\hat{C}_{Active_{(w,i,n)}} = \hat{C}'_{iter}$). If stopping criteria are not met yet, proceed to step 8.

<u>*Iterative Process*</u> *(6 steps)*

- Step 8: Increase the current iteration by one: $iter = iter + 1$.
- Step 9: Identify all possible fire spread paths that connect a "burned cell" at the end of the previous iteration ($c' \in \hat{C}'_{iter-1}$) to a "candidate cell to be burned" in the current iteration ($c \in \hat{C}_{iter}$) when these two cells are adjacent (sharing an edge or a vertex); Calculate all

possible fire arrival times of the candidate cells, each corresponding to a unique fire spread route to a candidate cell; and compare to find the minimum value:

$$min\left\{T_{c'} + \frac{\beta_{c',c}}{ROS_{(w,i,n),c\leftarrow c'}} + \frac{\beta_{c',c}}{ROS_{(w,i,n),c'\rightarrow c}}: \forall c \in \hat{C}_{iter}, \qquad c' \in \hat{C}'_{iter-1}\right\}$$

- Step 10: Select the candidate cell with the minimum fire arrival time as the cell that will be burned the earliest at the end of the current iteration:

$$\hat{C}^*_{iter} = c \ if \ T^*_{iter} = T_{c'} + \frac{\beta_{c',c}}{ROS_{(w,i,n),c\leftarrow c'}} + \frac{\beta_{c',c}}{ROS_{(w,i,n),c'\rightarrow c}}: \forall c \in \hat{C}_{iter}, \qquad c' \in \hat{C}'_{iter-1}$$

- Step 11: Update *MFAT* of the cell identified as being burned the earliest:

$$T_{c=\hat{C}^*_{iter}} = T^*_{iter}$$

- Step 12: Update "burned cells" at the end of the current iteration to also include the cell identified as burned the earliest: $\hat{C}'_{iter} = \hat{C}'_{iter-1} \cup \hat{C}^*_{iter}$.
- Step 13: Update "candidate cells to be burned" in the next iteration to include all flammable cells that are adjacent to the "burned cells" (identified in step 12):

$$\hat{C}_{iter+1} = \left(\hat{C}_{iter} \backslash \hat{C}^*_{iter}\right) \cup \left(\hat{C}_{c=\hat{C}^*_{iter}} \backslash \left(\hat{C}_{c=\hat{C}^*_{iter}} \cap \hat{C}'_{iter}\right)\right)$$

Then go back to step 7.

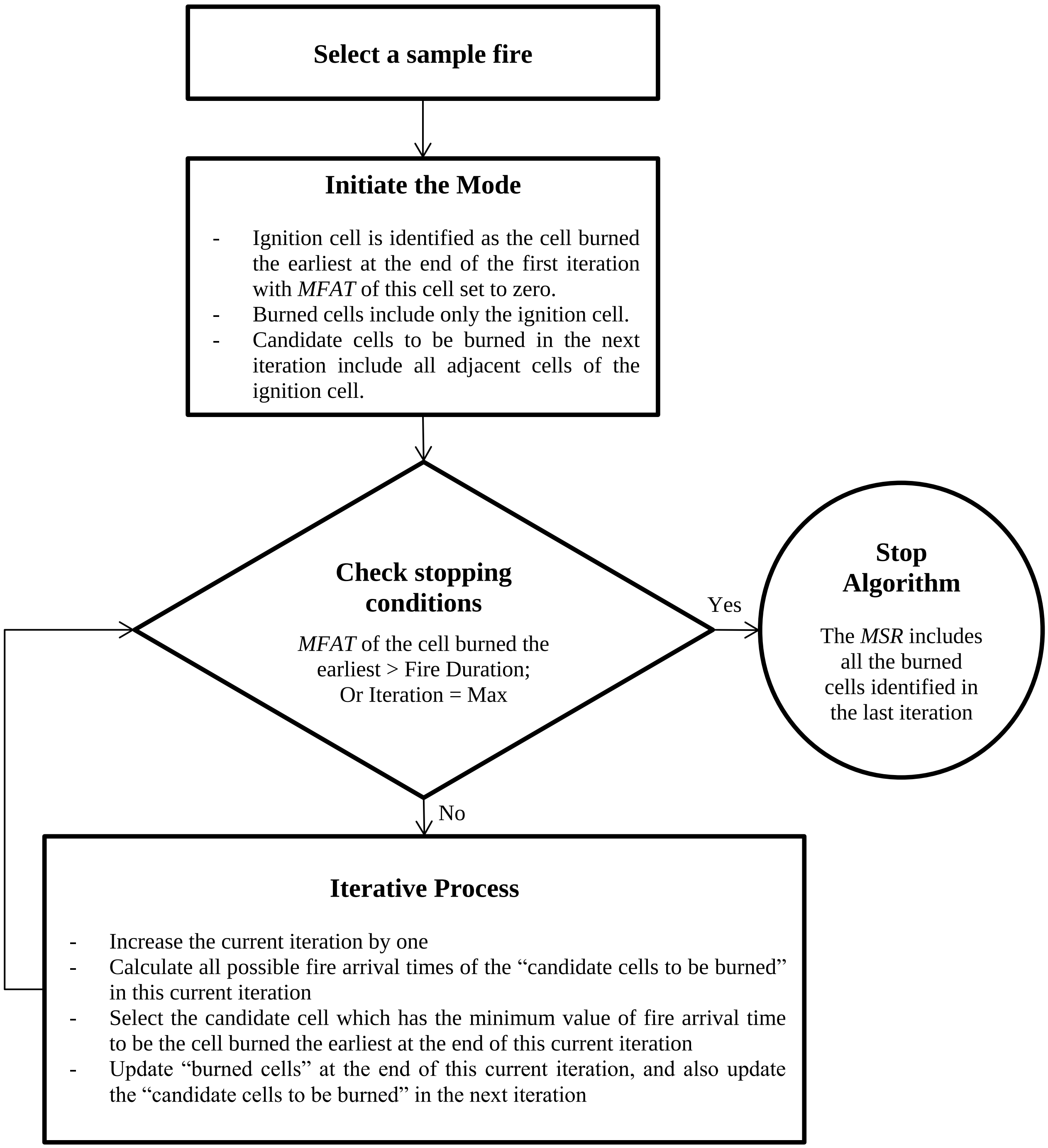

Figure 20: A flowchart of the preprocessing algorithm used to calculate the *MSR* of a selected sample fire.

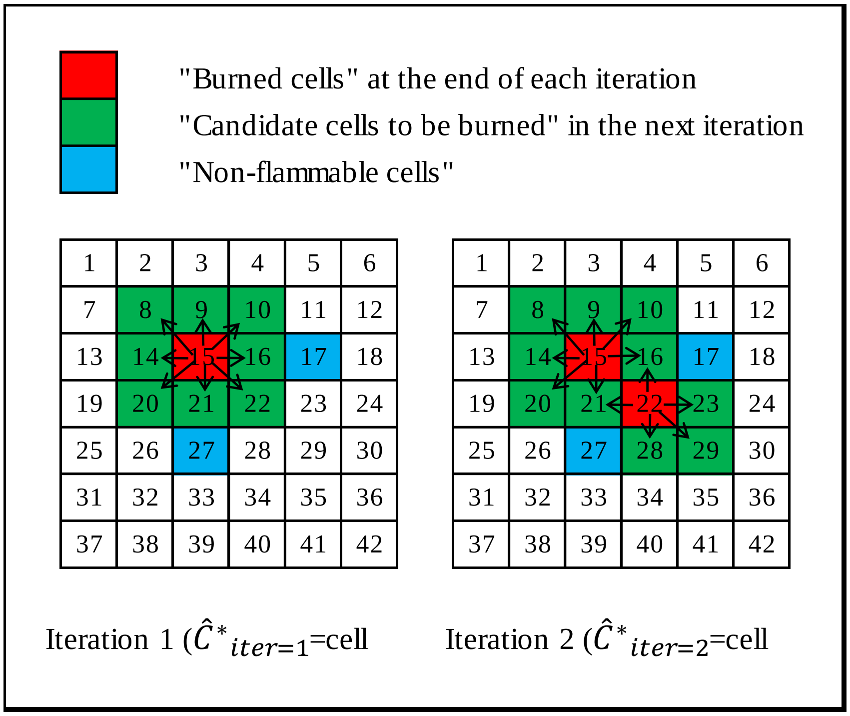

Figure 21: An example to illustrate the preprocessing algorithm. In this example, a rectangular-landscape including 42 cells is used to examine fire-spread-pattern for the first two iterations, assuming stopping criteria are not met yet. This landscape includes two non-flammable cells (i.e. open water represented by the two blue cells 17 and 27) and 40 flammable cells (i.e. forest), with the number in each cell representing the cell's ID. The arrows represent all possible fire spread paths to the "candidate cells to be burned" (green) in the next iteration. In this example, cell 15 is assumed to be the fire's ignition cell, which is realized as the cell burned the earliest at the end of iteration one. At iteration two, fire can spread from cell 15 to its eight "candidate cells to be burned" (8, 9, 10, 14, 16, 20, 21, and 22). I assume this fire would spread from cell 15 to cell 22 faster than to the other seven cells. Therefore, cell 22 would be identified as the cell burned the earliest at the end of the second iteration.