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OPTIMIZATION TECHNIQUES IN ECOSYSTEM AND LAND USE PLANNING

Edited and Coordinated

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ABSTRACT

This paper reviews a series of techniques which have been used by the systems planner to aid him in best achieving his objectives in light of the physical limitations imposed upon him by the system. These techniques have been collectively called optimization techniques. The report briefly reviews these techniques from a mathematical viewpoint and then proceeds to summarize the applications of these techniques to ecosystem problems. Another section speculates on other possible applications of optimization techniques in ecosystem problems.

Finally, the paper reviews the general application of optimization techniques to land use planning and tries to develop a general procedure for maximizing the "equality of life" in a community by the proper allocation of land in the community between the urban, agricultural, and range sections. This problem represents an interdisciplinary approach to the solution of land resource planning problems and combines the inputs of the various disciplines into an optimization framework.

1. INTRODUCTION

Until very recently, ecosystem planning has been done in a very ad hoc fashion. The planner, by finding out a great deal about the system and by trying various management approaches, was eventually able to come out with a decision-making framework which often proved adequate to his needs. However, the planner recently has been called upon to make decisions about systems which are too large and complex for him to understand and which are changing too rapidly to allow his ad hoc procedure for decision making to adequately keep up with the system changes.

At about the same time as this problem began confronting ecosystem planners, a group of techniques was being developed to help handle similar problems in industrial systems. These techniques go under the common name of *operations research*. They are basically techniques which allow the planner to insert his objectives into a mathematical framework which results in a decision based both on the planner's objectives and the physical constraints imposed by the system. Recently these techniques have also come to be applied in ecosystem management situations. While they often do not give any advantage in situations where a person has been making decisions about a system for a long time, they help supply direction in newly instituted systems. Of course, they are no panacea and the decisions resulting from operations research application must be reevaluated and often altered.

A technical report by Swartzman (1970) entitled "Some Concepts of Modelling" addresses problems in the areas of simulation and outlines the approach that we in the U.S. IBP Grassland Biome are using in our ecosystem simulation modelling. It is our hope to produce a report concerning optimization techniques similar to the modelling paper. We also will attempt to "exercise" these techniques on an example problem--specifically one dealing with land use planning.

It is recognized that a great deal of the complexity facing the ecosystem planner does not arise solely from the natural ecosystem itself but from human pressures on the ecosystem. For example, the game hunter regulating agency must consider not only the big game population dynamics but also the effects of hunting pressure, of the types of hunters, of road building, etc. Human systems have been studied by sociologists, political scientists, economists, psychologists, and human ecologists, and it is the input of these individuals to the development of a complete understanding of the system that we feel is presently needed.[1/]

In order to be able to get the inputs of sociologists, political scientists, economists, and environmental lawyers to problems of ecological and land use planning, we organized a series of day-long seminars which were held at monthly intervals. In all there were four workshops. We

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[1/] It has been pointed out to the author by one of the reviewers that the terms systems planner or manager are not clearly defined. In general, when one thinks of management, one thinks of the class of authorities or managers who make decisions in their own interests. This has frequently been termed Freudian revisionism in the sense that frequently these "managers" make decisions without trying to ascertain the public interest with respect to these decisions and by trying to take advantage of the insecurity of the individual in a mass society utilizing his fears and feelings of inadequacies (which he has learned from Freud) to manipulate the public. On the other end is the conscientious authority or planner who tries to make decisions with the best interests of the public at heart. We wish to remove all lack of clarity from our preceding statements by putting ourselves on the side of the public interests. We hope that in the future more and more decisions will be made with a real interest in the needs of the public. Specifically, we hope for greater public participation in choosing between alternative futures.

understood that initial communication difficulty would result. Not only were the social scientists unfamiliar with the optimization techniques which we hoped to utilize, but frequently they were unfamiliar with the work of social scientists in related areas and were certainly unaware of the work done by ecologists in this area. To develop a sounder communications base concerning the techniques we were considering, we devoted the first two workshops to a review of optimization techniques and some of their applications in resource management. The review of optimization techniques appears in Section 3 of this report while the applications are reviewed in Section 4.

Plenty of time was allowed for small group discussions to try to generate new applications for the techniques in the areas of expertise of the participants during the first two sessions.

To deal with the problem that the social and biological scientists were relatively unfamiliar with the work done by other participants in different areas of social and biological sciences, we spent the last two weeks trying to relate optimization techniques to a specific problem which would involve the inputs from each of the "experts" who had been involved in the workshop. This problem was a general land allocation problem in a western country. It is introduced as a potential application of operations research to a large-scale planning problem and the work carried out in trying to define and formulate this problem is described in Section 6.

2. OPERATIONS RESEARCH USE IN ECOLOGICAL SYSTEMS

The question arises at this point "Why might operations research techniques prove useful in ecosystem and land use planning?" It was recognized in the introduction that operations research techniques have been successfully applied in industrial systems where a great deal of data could be collected and the results of the system very easily monitored. However, what application will these have in ecological systems where data is frequently difficult to collect and in land use planning where interaction might be necessary between experts in such diverse fields as sociology, ecology, economics, law, and political science?

Part of the answer to this question might come from an examination of how models are built. The first step in the building of a model is frequently to assemble the ideas about the system and the past data that have been collected concerning the system and the hypotheses formulated about the system into a "word" model of the system. This word model is initially largely quantitative in nature--frequently containing many abstract ideas which, although intuitively peering, are not easily quantifiable. This word model might not be overly difficult to formulate for an industrial system such as an assembly line process, but when dealing with complex ecological and human social systems, the word model might be difficult indeed to formulate. In fact, it may happen that not one word model but several word models exist which describe segments of the system without any single word model adequately describing the whole system. This then is very frequently the difficulty confronting an ecological or land use planner when he comes to make decisions about complex systems. He cannot get the information he needs from any one expert because the fields of expertise have not, up until recently, dealt with complex whole systems.

He may find, in fact, that he does not have the adequate framework to combine the word models that he has received by talking with experts in the various fields dealing with the system he must make decisions about.

We offer the mathematical quantification techniques of operations research as a possible framework for combining the many word models formulated by experts in various subsystems within a complex system. The advantages of such techniques are that they focus specifically on the planner's problem of making a decision about a system. Therefore, when the planner tries to combine the word models of the various experts that he has consulted, he can, hopefully, have them focus on the factors which are most important to him in his decision making. Also, the general jist of the optimization (operations research) techniques are fairly simple to understand and can provide a common "language" for communication between the experts and the previously separate fields of expertise. It is postulated in this paper and an attempt is made to show (see Section 6) that optimization techniques can provide a viable framework for decision making in large-scale, complex ecosystems. There are, understandably, many difficulties with the use of optimization techniques in such large-scale planning problems. When moving from a highly articulate and information-filled word model or a group of word models to a general quantitative model as is done in optimization modelling, a great deal of information must be sacrificed. This may be due both to the fact that many of the variables in the word model are not easily collectable and also that many of the variables are not considered important enough to be introduced into the quantified model although they do have a definite effect when added altogether on the system. There are also several other detrimental aspects to the use of a highly precise mathematically quantified technique as research techniques for large-scale systems planning. It is frequently difficult to separate the objectives of a systems planner from the system itself. However, there is no provision in the optimization techniques, per se, for separating a system from the objectives of the system.

Another important difficulty is that making a decision which will effect the system frequently changes the system. With this in mind, the new system resulting from a combination both of the original system and decisions made with respect to it must be reevaluated after the decisions resulting from the optimization techniques are instituted.

So we see that these techniques when applied to ecosystems are still a long way from the continuous monitoring and instantaneous decision making that is presently going on in our space programs. The main benefit of the optimization techniques is that a great deal can be learned from the structuring of the problem and that the solution may be seen as a comparison point against techniques that are presently used to handle the real life problem. If we know the strengths and weaknesses of existing solution techniques, we can move on to more sophisticated and more appropriate techniques in the realm of ecosystem decision theory and practice.

3. REVIEW OF OPTIMIZATION TECHNIQUES

In the following section we review in a fairly general fashion the operations research techniques which we deemed potentially applicable to ecosystem problems. We apologize initially for the discontinuous writing style, but this section represents the efforts of the various individuals in the workshop group who spoke on these various techniques.

Table 3.1 below gives a brief summary of the various techniques to be discussed in this section. It is for purposes of comparison and gives in brief form the objectives of the various optimization techniques.

Table 3.1 also tells whether the technique is linear or nonlinear, static or dynamic, and deterministic or stochastic.[2/] This can be used as a gauge of difficulty of solution. Difficulty of solution generally increases continuously in moving from linear, static, deterministic models to nonlinear, dynamic, stochastic models. Notable exceptions are queueing theory problems which, while nonlinear, dynamic, and stochastic are sometimes easier to solve than some linear programming problems.

In Table 3.1 the *objective function* refers to the objectives set by the planners. In optimization techniques we look for those values of the

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2/ By linear it is meant that all the functions which represent the system or the objective function must be straight lines, while nonlinear formulations are not limited to straight lines alone. By static it is meant that the formulation does not include time as a specific manipulable variable, whereas dynamic optimization techniques have the ability to recognize explicitly that the objectives and/or the system change over time. A deterministic formulation does not allow for probabilistic elements in any of the variables in the system, whereas a stochastic technique allows for this. Because of the somewhat mathematical nature of the descriptions of the techniques in this section, the reader who is unfamiliar with mathematical notation may want to focus his or her attention on Table 3.1 and read only the introduction to each of the following sections in Section 3.

Table 3.1. Relationship between optimization techniques.

Type of Technique	Objective	Linear or Nonlinear	Static or Dynamic	Deterministic or Stochastic
1. Linear programming	To maximize or minimize objective function subject to constraints.	Linear	Static	Deterministic
2. Nonlinear programming	To maximize or minimize objective function subject to constraints.	Nonlinear	Static	Deterministic
3. Stochastic programming	To maximize or minimize objective function subject to constraints.	Either	Static	Stochastic
4. Dynamic and recursive programming	To maximize or minimize objective function over time subject to constraints and recursion relations which relate to states earlier in time.	Either	Dynamic	Either
5. Simulation optimization	To maximize or minimize objective functions at discrete stages over time with constraints supplied by a simulation.	Either	Static optimization with dynamic simulation	Either
6. Operational (simulation) gaming	To test various management procedures using a simulation of a system to find the one that gives the highest value of an objective function over time.	Either	Dynamic	Deterministic
7. Experimental optimization	To determine the optimal value of an objective function by performing a series of experiments on the pertinent variables.	Either	Static	Either
8. Game theory	To find the best strategies of each of the players in a "game."	Either	Either	Either
9. Calculus of variations	To find the optimal control which maximizes the objective function when the variables are given by differential equations.	Either	Dynamic	Deterministic
10. Statistical optimization	To find the "best" estimators to represent sampled populations.	Either	Static	Stochastic
11. PERT and CPM	To plan a complex job for completion in the minimum time possible.	Linear	Dynamic	CPM-deterministic PERT-stochastic
12. Queueing and inventory theory	To plan a service facility which services random demands for service in the best fashion with respect to selected management criteria.	Nonlinear	Dynamic	Stochastic
13. Quasilinearization	To linearize a nonlinear problem to a series of linear problems. It is often used in conjunction with dynamic programming or calculus of variations to linearize nonlinear functions.	Converts nonlinear to linear	Dynamic	Deterministic
14. Benefit-cost analysis	To compare various selected management procedures with respect to their benefit-cost ratios.	Nonlinear	Static	Deterministic

controllable variables which give the maximal value of the objective function. The maximal value of the objective function is obtained by manipulating the variables upon which the objective function depends. This manipulation, generally called an algorithm, to get the best combination of controllable variables with regard to selected objectives (quantified in the objective function) is the essence of optimization techniques.

3.1 Linear Programming

3.1.1 *History.* Linear programming was the first of the "programming" optimization techniques to be developed and the first one with a computer algorithm worked out. It was developed in 1947 by Dantzig, who also worked out the first solution algorithm, which is still the most commonly used algorithm, called the Simplex method. Some references on the techniques are Spivey (1963), Hadley (1962), and Dantzig (1963). The main use of linear programming has come since its implementation on the digital computer, which has reduced solution times from the order of 120 man-days to one or two seconds for the same problem.

3.1.2 *The linear programming formulation.* Basically, the technique may be looked at as giving the maximum or minimum value of some *objective function* which is a linear function of a set of variables which may be varied (controlled) by implementer of the linear programming technique. There are also a series of *constraints* to the problem which limit the size that the variables may take. In linear programming the constraints are also linear functions of the controllable variables. There are also, in many physical applications of linear programming, the so-called non-negativity constraints which restrain the variables from taking negative values. The linear programming format may be stated in matrix notation as in equation (3.1.1) below or in subscript notation as in (3.1.2).

$$\max\ (\min)\quad \underset{\sim}{C}^T \underset{\sim}{x}$$

$$\underset{\sim\sim}{Ax} \begin{Bmatrix} \leq \\ \\ \geq \end{Bmatrix} \underset{\sim}{b} \tag{3.1.1}$$

$$\underset{\sim}{x} \geq \underset{\sim}{0}$$

$$\text{maximize (min)} \quad \sum_{i=1}^{n} c_i\, x_i$$

where

$$\sum_{i=1}^{n} a_{j,i}\, x_i \leq b_j \tag{3.1.2}$$

and $x_i \geq 1$ for all i.

The controllable variables are represented by the vector $\underset{\sim}{x}$. The coefficients in the linear objective function of the variables in $\underset{\sim}{x}$ are noted by $\underset{\sim}{C}^T$, where the $\underset{\sim}{C}^T$ is the transpose of the row vector $\underset{\sim}{C}$ and is therefore a column vector of constant coefficients. The matrix $\underset{\sim}{A}$ is the constraint matrix which, when multiplied by the variable vector $\underset{\sim}{x}$, gives a linear combination of the variables which must be greater than or less than certain constants which are represented in vector form by $\underset{\sim}{b}$. The non-negativity constraints are represented by the vector $\underset{\sim}{x}$ being greater than or equal to the vector $\underset{\sim}{0}$ which implies that each of the variables in the $\underset{\sim}{x}$ vector is greater than zero.

Solution to linear programming problems may be obtained using a graphical solution technique when there are only two or three variables. Consider the following objective function which is to be maximized:

$$5x_1 + 3x_2$$

We want to select values of x_1 and x_2 which will give us the maximum value for f(x). However, there are also some constraints on the selection of values of x_1 and x_2. These constraints are:

$$3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

These two inequalities indicate that the values of x_1 and x_2 to be chosen to maximize the objective function above must simultaneously satisfy the inequalities that are listed in the two equations above.

We may represent the two inequality constraints in a graph of x_1 against x_2. The graph showing these constraints appears in Fig. 3.1.1 below:

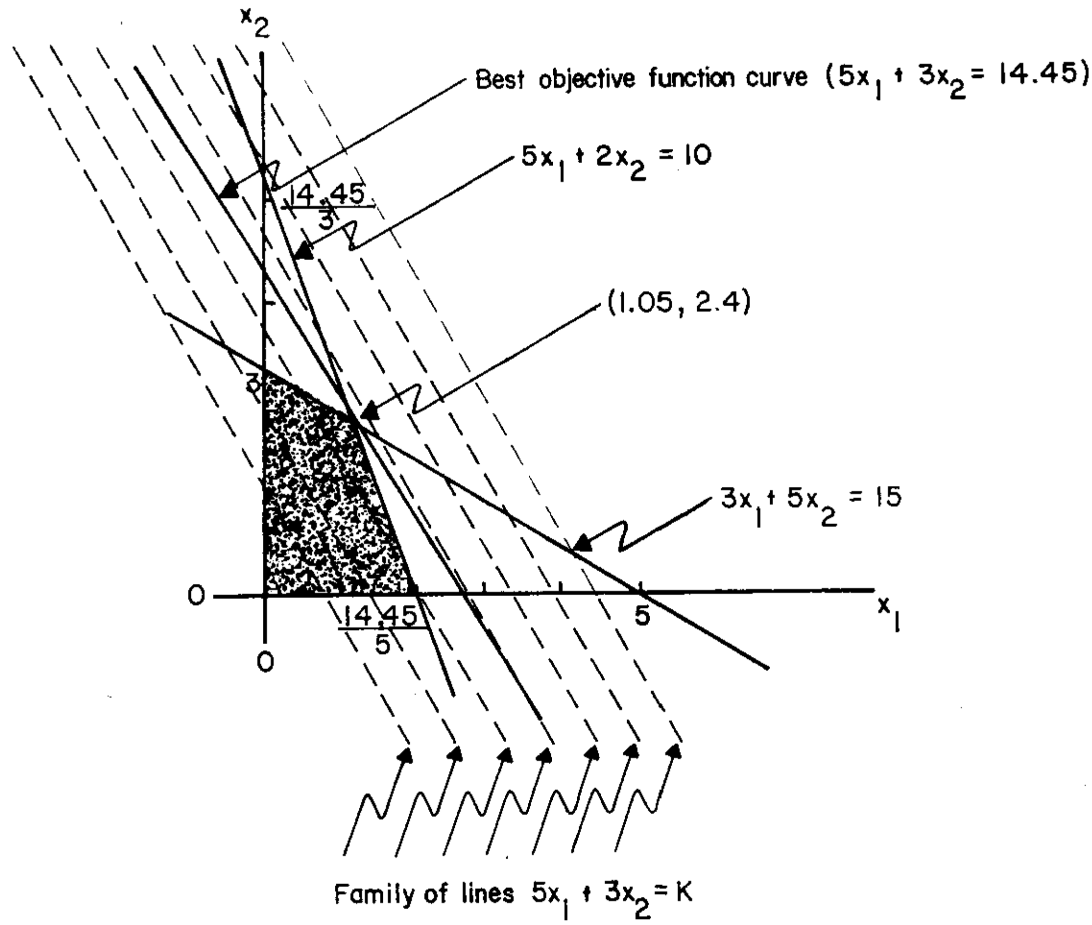


Fig. 3.1.1. Graphical solution to example linear programming problem.

The objective function, which is also linear, may be graphed on the same set of axes. It is a family of straight lines. The particular line which gives the maximum value of the objective function (that is the maximum value of $5x_1 + 3x_2$) and also satisfies the constraints, turns out to be the point within the shaded area, represented by the combination of the constraint equations, that gives the maximum value for the objective function. In linear programming this point is always one of the vertex points of the *convex set*[3/] which is bounded by the constraint equations. You may see from the graph above that some of the objective function lines lie above this

[3/] A convex set is a surface such that a line connecting any two points contained in the surface lies entirely within the surface.

convex set and some intersect it. The line that gives the maximum value is the one that intersects the convex set furthest to the right. In this problem it turns out that the best line for the objective function, that is, the line with the highest value of the objective function which still intersects the convex set determined by the constraints is the line $5x_1 + 3x_2 = 14.45$ which intersects the convex set at only one point--the point $x_1 = 1.05$, and $x_2 = 2.4$. Notice that this point is one of the vertices formed by the boundary of the convex set. In this particular case it represents the intersection point of the two lines $5x_1 + 2x_2 = 10$ and $3x_1 + 5x_2 = 15$.

The Simplex solution algorithm for linear programming problems uses the fact that the best solution for the objective function in a linear programming problem *always* occurs at one of the vertices of the boundary determined by the constraint equations. Thus, it consists of moving from vertex to vertex in such a way as to always increase the value of the objective function until the vertex which gives the maximum value of the objective is found. Although in this problem the solution was quite trivial, in many problems there are many more than two constraints and two variables. This results in problems which cannot be graphed in two or even three dimensions and which have a great many vertices. This is why the computer is of great value in the solution of such kinds of problems. Several good computer programs for solving linear programming problems are available in most computer center library files. In the U.S. IBP Grassland Biome Program we have found two specific programs to be quite useful. These are LINPROG and a simpler program entitled OPTIMIZ.

Linear programming problems have the specific simplicity that the objective function and constraints are always linear functions of the variables. This means that the coefficients are not direct functions of the variables and that the variables appear in the equations in first order (i.e., there are no square terms or square roots, etc.). Also, the equations are assumed to be deterministic, and the problems are static in that there are no variables which can change over time. If these conditions are not satisfied, then the problem is no longer linear.

3.1.3 *Duality and sensitivity analysis.* Two other concepts in linear programming which are of some importance are *duality* and *sensitivity analysis*. The concept of duality says that for every maximization problem there is a corresponding minimization problem which has exactly the same solution value as the maximization problem, and vice versa. Furthermore, the coefficients in the dual problem bear an interesting relationship to the coefficients in the original (primal) problem. Equation (3.1.3) below shows, in vector notation, the relationship between the coefficients in the primal and dual problem. The variables $\underset{\sim}{u}$ in the dual problem usually bear no physical relationship to the variables in the primal problem. Furthermore, it is only the *value* of the objective function that is the same in the solution to the primal and the dual linear programming problems, not the values of the variables. Another interesting relationship is that the number of variables in the primal problem is equal to the number of constraint equations in the dual problem, and vice versa. Also, any less-than-or-equal-to constraint ($\leq$) in the primal problem gets changed to a greater-than-or-equal-to constraint ($\geq$) in the dual problem, and vice versa.

Primal	Dual
$\max \quad \underset{\sim}{c}^T \underset{\sim}{x}$	$\min \quad \underset{\sim}{b}^T \underset{\sim}{u}$
$\underset{\sim}{A}\underset{\sim}{x} \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} \underset{\sim}{b}$	$\underset{\sim}{A}^T \underset{\sim}{u} \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} \underset{\sim}{c}$
$\underset{\sim}{x} \geq \underset{\sim}{0}$	$\underset{\sim}{u} \geq \underset{\sim}{0}$

(3.1.3)

Duality usually plays an important role in sensitivity analysis. Sensitivity analysis is performed to find out how much the objective function can be changed without changing the point at which the optimal solution to the problem occurs. It is sometimes also used to see how much the $\underset{\sim}{b}$ vector in the constraints can be changed without changing the value of the objective function. Since the $\underset{\sim}{b}$ vector in the primal problem is in the objective function in the dual problem, changing the coefficients in the objective functions in the dual problem is equivalent to changing the $\underset{\sim}{b}$ vector in the constraints of the primal problem. This tells us how sensitive the solution is to changes in the $\underset{\sim}{b}$ vector.

More realism may be often included in a problem formulation by making some of the constraints or objective function nonlinear, by adding random elements into the problem, or by including time as an explicit variable in the problem. This generally increases by manyfold the complexity of the solution technique. Thus, in choosing a programming formulation there is frequently the decision to be made between a simple easily solved problem or a more realistic problem which would be more difficult to solve. A discussion of some of the difficulties in solving more complex types of programming problems will be included in a discussion of some of the other programming techniques.

3.2 Nonlinear Programming

Nonlinear programming problems are similar to linear programming problems except that the objective function, the constraint equations, or both may be nonlinear. That is, they may be made of curvilinear functions rather than solely linear functions. The addition of nonlinear terms to the objective function or constraint adds a great deal of complexity to the mathematical techniques for solution of the problem. In fact, in some nonlinear programming problems there is no technique which can assure a single best solution. In some nonlinear programming problems no such best solution exists. References on mathematical solution techniques for nonlinear programming problems are Hadley (1964) and Kunzi, Tzschach, and Zehnder (1968).

The simplest extention to a nonlinear from a linear programming problem is the quadratic programming problem. In this problem the objective function has quadratic (second-order) terms as well as linear (first-order) terms. The constraints, however, are all linear. As an example of a quadratic programming problem which will introduce some of the difficulties of solution, let us consider the following two-dimensional problem which we will solve graphically in Fig. 3.2.1.

$$\text{minimize} \quad x_1^2 + x_2^2$$

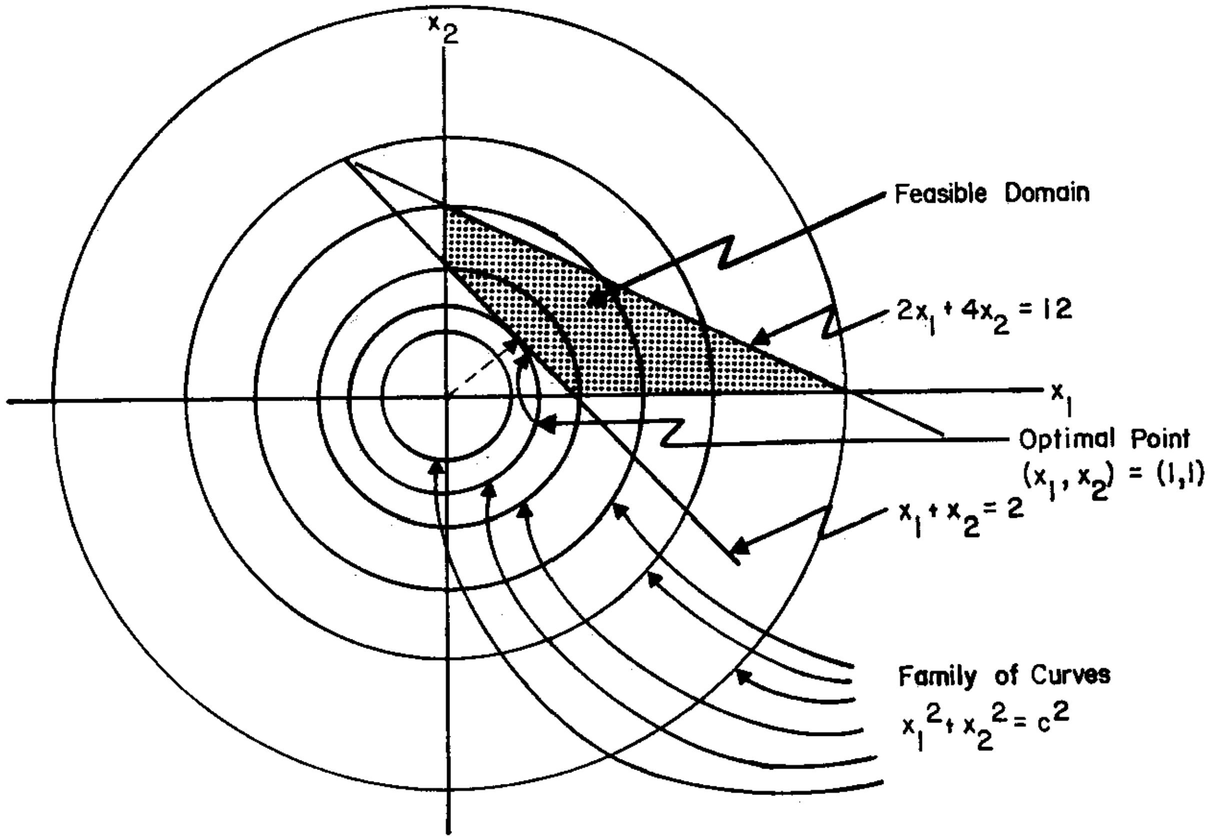


Fig. 3.2.1. Sample quadratic programming problem with solution.

where

$$\begin{aligned} x_1 + x_2 &\geq 2 \\ 2x_1 + 4x_2 &\leq 12 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned} \qquad (3.2.1)$$

The objective function, $x_1^{\ 2} + x_2^{\ 2}$, is a circle and the minimum value of it is the point $x_1 = 1$ and $x_2 = 1$, giving a minimum value of the objective function of 2. The solution in this case was done graphically with the radius of the optimal circle and the optimal point determined geometrically. It would also be possible to approximate the optimal value graphically by measuring the radius of the optimal circle. The optimal solution does not occur at a vertex of the feasible domain. This makes it extremely difficult to develop a simple algorithm to solve these problems since the solution is equally likely to be anywhere along the boundary of the feasible domain. There are several algorithms that have been worked out to directly solve quadratic programming problems.

Considering the simplest imaginable nonlinear programming problem given above it is clear that finding a solution to a nonlinear programming problem is often an extremely difficult task.

A great number of algorithms have been developed to solve nonlinear programming problems. The main difficulty in these algorithms is finding an initial solution inside the feasible domain. This may be extremely difficult to do when the constraints are not linear. One algorithm which has been implemented on a digital computer is the technique of Fiacco and McCormick (1964). This technique attempts to solve a nonlinear programming problem with constraints by converting it into an unconstrained maximization problem which has the constraints in them and some parameters which are then allowed to go to zero in the technique to eventually give the solution to the original problem. This technique has all the disadvantages of most of the other algorithms and does not assure a solution to many nonlinear programming problems. It will, however, assure a solution within a finite number of steps if certain general conditions about the shape of the constraint and objective function curves are obeyed. Heuristically speaking, these conditions are that the objective function must be convex (that is, it must be as smoothly shaped as a bowl and must be curved so that if it were a bowl it would hold water). Also, the constraints must all be concave functions (they must be smoothly curved and must be like an upside down bowl, heuristically speaking), and they must have first and second derivatives that exist at all points (i.e., they must be continuous functions). These are somewhat constrictive functions and remove all winding and jagged curves from consideration when a solution to the problem is desired.

3.3 Stochastic Programming

The general statement of a stochastic programming problem is very similar to a linear or nonlinear programming problem. The difference is that some of the coefficients either in the A matrix, the b constraint vector, or the C objective function vector may be random in nature. In general, the assumption is made that the random elements have a known mean and a known variable, with a normal distribution.

3.3.1 Finite random variables. A solution algorithm to a stochastic programming problem exists when each coefficient on the objective function can take on only one of a finite number of known values. It is possible to solve these problems because there are only a finite number of possible combinations of the variable values. This procedure is covered in Hillier and Liebermann (1967). This is essentially a combination of expected value problems with linear programming so that the objective function is to minimize

$$f_{min} = \sum_{k=1}^{l} p_k \underset{\sim}{C}_k \underset{\sim}{x} \qquad (3.3.1)$$

subject to $\underset{\sim}{A}_k \underset{\sim}{x} \leq \underset{\sim}{b}_k$, where p_k is the probability that the vector $\underset{\sim}{C}_k$ takes on a given set of values.

This is an intuitively simple approach, but it may be computationally difficult if l is large. It may also result in values of $\underset{\sim}{x}$ which may be the best "average" values but which may not meet the requirements of any particular set of parameter values.

3.3.2 Stochastic elements in the constraint $\underset{\sim}{b}$ vector. An algorithm called the chance-constrained procedure deals with problems where there is variation only in the right-hand side of the constraints (the $\underset{\sim}{b}$ vector). The procedure essentially is to replace the original random problem by another deterministic problem having an additional term added to the $\underset{\sim}{b}$ vector.

$$f_{min} = \sum_{i=1}^{n} c_i x_i$$

subject to

$$\sum_{i=1}^{n} a_{ij}\, x_i \leq \bar{b}_j + K_{\alpha_j}\, \sigma_{b_j} \quad , \quad \text{for } j = 1, 2, \ldots, m, \tag{3.3.2}$$

and

$$x_i \geq 0 \quad , \quad \text{for } i = 1, 2, \ldots, n$$

where a_{ij} are the coefficients of the constraint equations, the x_i are the controllable variables, $\bar{b}_j$ is the mean value of the right-hand side for constraint i, and c_i are the objective function coefficients. The usual values b_j are replaced by the mean values of the variable population $\bar{b}_j$. The K_{α_j} are the factors added on to $\bar{b}_j$ to assure satisfying constraint j with probability 1-α. The σ_{b_j} is the standard deviation of the random variable b_j. After the addition of K_{α_j} to b_j the solution algorithm is the same as a deterministic problem.

3.3.3 Probabilistic coefficients in the objective function. This procedure is outlined by Sinha (1963). If the only probabilistic coefficients on the problem are in the objective function, coefficients $\underset{\sim}{C}$, the problem can be reformulated as

$$\text{minimize} \quad f_{min} = \underset{\sim}{C}^T \underset{\sim}{x} - (\underset{\sim}{x}^T \underset{\sim}{v}\underset{\sim}{x})^{\frac{1}{2}} \tag{3.3.3}$$

subject to

$$\underset{\sim}{A}\underset{\sim}{x} \leq \underset{\sim}{b}$$

and

$$\underset{\sim}{x} \geq 0$$

Here $\underset{\sim}{A}\underset{\sim}{x}$, $\underset{\sim}{b}$, and $\underset{\sim}{C}$ are as usually defined, and $\underset{\sim}{v}$ is the covariance matrix of the random variables $\underset{\sim}{C}$. This is a quadratic programming problem and can be solved by the usual quadratic programming solution techniques if the original problem was a linear stochastic programming problem.

3.3.4 Probabilistic coefficients in the $\underset{\sim}{A}$ matrix. If a stochastic linear programming problem has random elements only in the $\underset{\sim}{A}$ matrix (the constraint coefficients), then it can be reformulated as a nonlinear programming problem by a method of Charnes and Cooper (1961). The original problem is reformulated to

$$\text{minimize} \quad \underset{\sim}{C}^T \underset{\sim}{x}$$

subject to

$$\underset{\sim}{A}\underset{\sim}{x} + \underset{\sim}{k}(\underset{\sim}{x}^T \underset{\sim}{v}\underset{\sim}{x})^{\frac{1}{2}} \geq \underset{\sim}{b} \qquad (3.3.4)$$

and

$$\underset{\sim}{x} \geq 0$$

where the elements in $\underset{\sim}{A}$ are now the mean values of the matrix coefficients, $\underset{\sim}{v}$ is their covariance matrix, $\underset{\sim}{b}$ is the right-hand side of the constraints, and $\underset{\sim}{k}$ is related to the probability that the constraints are satisfied, assuming a normal distribution of the a_{ij}'s of the $\underset{\sim}{A}$ matrix.

3.3.5 Monte Carlo simulations. If the problem is more complex than any of those stated above, it can still be approached by Monte Carlo simulation. This consists of a series of runs of the problem as a deterministic problem, with the coefficients for each run of the problem generated using a random number generator. The problem is rerun many times to get an "average" solution.

3.4 Dynamic Programming

3.4.1 History and basics. Dynamic programming was developed by Richard Bellman during the postwar period and the first solution algorithms were worked out in 1955. The techniques, while frequently clumsy and inapplicable to very large problems, have remained in fairly common use due largely to the ingenuity of Bellman and his cohorts in structuring medium-sized problems into a simpler form to allow for a dynamic programming solution. It might be initially mentioned that dynamic programming techniques, unlike other programming techniques, do not relate to specific solution algorithms, but rather might be more adequately described as relating to a philosophy of solution techniques. Several good accounts of some of the techniques appear in Bellman and Dreyfus (1962), Bellman (1957), White (1969), and Howard (1960).

Basically, the techniques may be looked at as optimizing (maximizing or minimizing) "the total cost" over a *path* or *policy* which moves between *states* subject to *decisions* at each stage of the process. The optimal policy is that set of decisions which result in a minimal (or maximal) cost over the time period of the problem. The decision stages are generally at discrete time intervals. Thus, dynamic programming problems are dynamic in that they offer solutions to objective functions over a period of time.

The major principle which guides solution techniques in dynamic programming is called the "principle of optimality." Basically, this principle is equivalent to the *Markovian property* which says that whatever state you go to depends only on what state you are in and not on the previous history of the problem. The principle of optimality says that an optimal policy has the property that, whatever the initial state and decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. Two commonly used dynamic programming algorithms are the *method of successive approximations* and the *method of approximation in policy space*.

Heuristically, the first technique consists of finding the set of decisions that gives you the minimum transfers from the beginning state to the end state in 1, 2, 3, 4, 5, ..., n steps or decisions and then picking the minimum of these. If there are n possible states, completion of the algorithm is guaranteed in n-1 steps or decision stages. Of course, if n is large, this could be a very large number of steps. Also, the computation of the minimal transfer for n steps is a very tedious task if n is large. In the method of approximation in policy space you pick any policy that gets from the beginning state to the end state, and you keep refining it by a mechanical technique (an algorithm). This technique also assures conversion of the algorithm in n-1 steps.

The following review of dynamic programming is slightly more mathematical. It goes through several of the basic considerations in dynamic programming and solves an example problem using both the method of successful approximations and method of approximation in policy space. It may be skipped by the person who does not have a high degree of mathematical expertise without losing a great deal of understanding in the discussion to follow.

3.4.2 Optimal path problem. Given a set of points $\{p_i\}$ $i = 1, \ldots, n$ with an associated cost matrix $[c_{ij}]$, where c_{ij} is the cost of moving directly from p_i to p_j in one move $(0 \leq c_{ij} < \infty)$. We wish to know the minimal cost of moving from any point p_i to point p_n (the end) using as many moves as required and the optimal sequence of moves to obtain this cost.

Alternatively, the problem may be looked at as an n-stage decision process in which you begin in one state, x_i, and end in another, x_n. Each decision results in a transfer between states which has a cost associated with it. The objective is to minimize the total cost of the process, this being some function (not necessarily the sum) of the costs c_{ij} associated with each of the decisions.

Usually the assumption is made that the stages are related to each other in *Markovian* fashion, i.e., the state you are in at stage n depends only on where you were at stage n-1 *and* your decision there--not on previous stages (not directly).

3.4.3 Principle of optimality in sequentially controlled systems.

Optimal policy. Whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the stage resulting from the first decision (Bellman 1957).

The functional equation. Denote the x_i of above by i. The decision d(i) moves us from state i to state j with cost c_{ij}. Let the cost associated with the *optimal policy* be denoted by f(i). Invoking the *principle of optimality*, the cost of going from i to j and then optimally from j (i.e., to give us minimum total cost) to r (where r is the final state) is c_{ij} + f(j). Then choose the j which minimizes this cost of going from i to r.

$$f(i) = \min_j \; [c_{ij} + f(j)] \tag{3.4.1}$$

This is called the *forward functional equation.*

Similarly, the optimal policy to get from point r to point i through some j (one step before i) has a cost f(i) given by:

$$f(i) = \min_j \; [f(j) + c_{ji}] \tag{3.4.2}$$

This is termed the *backward functional equation.*

Combining (3.4.1) and (3.4.2) to get between any two points i and j in an unspecified number of steps, the optimal (minimum) cost would be:

$$f(i,j) = \min_s \; [f(i,s) + f(s,j)] \tag{3.4.3}$$

3.4.4 Computational algorithms (solution techniques). For demonstration let us develop the method of successive approximations mathematically.

Method of successive approximations. Define $f_1(i) = c_{in}$ (cost to go from i to n in one step). Also define

$$f_n(i) = \min_j \; [c_{ij} + f_{n-1}(i)] \qquad \text{(minimum cost to go from i to n in n steps)} \tag{3.4.4}$$

i.e.,

$$f_2(i) = \min_j \; [c_{ij} + f_1(j)]$$

At each stage determine the optimal policy $S_n(i)$ which is the decision network (s) which gives minimum cost.

The sequence $\{f_n(i)\}$ converges to a unique solution of equation (3.4.1) in at most n-1 steps if there are n states.

For example, consider the following "cost" matrix.

c_{ij}

i \ j	1	2	3	4	5
1	0	20	5	∞	∞
2	20	0	10	8	20
3	5	10	0	∞	40
4	∞	8	∞	0	6
5	∞	20	40	6	0

Using method A we have the following:

	i = 1	2	3	4	5
$f_1(i)$	∞	20	40	6	0
$s_1(i)$	5	5	5	5	5
$f_2(i)$	40	14	30	6	0
$s_2(i)$	2	4	2	$\left.\begin{matrix}4\\5\end{matrix}\right\}$	5
$f_3(i)$	34	14	24	6	0
$s_3(i)$	2	$\left.\begin{matrix}2\\4\end{matrix}\right\}$	2	$\left.\begin{matrix}4\\5\end{matrix}\right\}$	5
$f_4(i)$	29	14	24	6	0
$s_4(i)$	3	$\left.\begin{matrix}2\\4\end{matrix}\right\}$	$\left.\begin{matrix}2\\3\end{matrix}\right\}$	$\left.\begin{matrix}4\\5\end{matrix}\right\}$	5

Optimal solution from $1 \to 5$ is 1-3-2-4-5 and the cost is 29.

How do we get the above table?

$$f_1(i) = c_{in}$$

$$f_2(i) = \min_j \; [f_1(j) + c_{ij}]$$

Form $f_1(j) + c_{ij}$ and take the minimum overall j for each i.

$i \backslash j$	1	2	3	4	5
1	∞	40	45	∞	∞
2	∞	20	50	14	20
3	∞	30	40	∞	40
4	∞	28	∞	6	6
5	∞	40	80	12	0

Similarly $f_3(i) = \min_j \; [f_2(j) + c_{ij}]$.

Form $f_2(j) + c_{ij}$ and take the minimum for each i.

$i \backslash j$	1	2	3	4	5
1	40	34	35	∞	∞
2	60	14	40	14	20
3	45	24	30	∞	40
4	∞	22	∞	6	6
5	∞	34	70	12	0

Similarly the $f_3(j) + c_{ij}$ matrix is:

$i \backslash j$	1	2	3	4	5
1	34	34	29	∞	∞
2	49	14	34	14	20
3	29	24	24	∞	40
4	∞	22	∞	6	6
5	∞	34	64	12	0

Notice that in this simple problem the solution technique is still somewhat cumbersome. Problems with more variables take orders of magnitude more time. Also, it becomes clear that if the number of states is large the computations can become extremely tedious.

3.5 Simulation-Optimization

The simulation-optimization (SIM-OPT) approach is an ad hoc approach to dynamic systems optimization which takes a simulation of a system and then interrupts it at equal intervals by optimization decision programs (these may be linear, nonlinear, or stochastic programming) which make optimization decisions on how to add to, subtract from, or modify the variables in the simulation while optimizing the objective function at each decision stage. The solution of the optimization stage then determines the initial conditions for the variables in the simulation during the next time period. Analogously, values of the variables at the end of a simulation period determine some of the constraints (particularly in the $\underset{\sim}{b}$ vector) in the optimization problem for that decision stage. The simulation-optimization combination can be represented as in equation (3.5.1) below:

$$\begin{array}{lll} \underline{\text{SIM}} & & \underline{\text{OPT}} \\ & \text{maximize} & \underset{\sim}{c}^T \underset{\sim}{x} \\ \dot{\underset{\sim}{x}} = \underset{\sim}{D}\underset{\sim}{x} + \underset{\sim}{E}\underset{\sim}{z} & & \underset{\sim}{A}\underset{\sim}{x} \begin{Bmatrix} \le \\ \ge \end{Bmatrix} \underset{\sim}{b} \\ & & \underset{\sim}{x} \ge \underset{\sim}{0} \end{array} \qquad (3.5.1)$$

The variables are here represented by the vector $\underset{\sim}{x}$. The vector $\underset{\sim}{z}$ is a vector of the driving forces for the simulation model. The optimization problem is written in the standard programming format. In initial applications we are using the optimization problem as a linear programming problem, but this can be altered by not requiring the $\underset{\sim}{C}$ vector or the $\underset{\sim}{A}$ matrix to be constant coefficients, but by allowing them to become functions of the variables $\underset{\sim}{x}$ or $\underset{\sim}{z}$.

The advantage of this technique is that it makes provision for time variation by allowing the simulation to control the dynamics of the system. The main disadvantage is that it does not optimize over the whole time period of the problem, but only at specific stages. Thus, a problem which requires long-term considerations to be optimized often does not get optimized by this technique, and the solution that comes out of the SIM-OPT technique is sometimes very different from the solution that would come out of a dynamic optimization procedure where the long-term objectives were included as part of the total objective function. At present a great deal of thought is going into how long-term dynamic considerations can be included in the static objective functions presently being used in the SIM-OPT procedure.

3.6 Operational Gaming

Basically, operational gaming consists of playing with a simulation model which has already been developed. This is accomplished by allowing certain manipulation knobs to be inserted into the model which allow decisions made by the game players which are reflected in changes in the dynamic behavior or output of the model. These have been used in management in five general areas: (i) to help develop a model by seeing how it responds to various decisions and manipulations, (ii) to help find an actual decision solution or optimal decision to be made with an existing model or about an existing model, (iii) to help evaluate proposed solutions to problems that deal with the situation modelled by the game, (iv) to try to convince managers that certain decision policies are desirable by allowing them to play the game, and (v) to test the decision-making abilities of certain individuals by evaluating their game performance.

Simulation gaming may also be put into a game theory context (in principle the two are not the same thing; see Section 3.8) by having an objective function which is then evaluated under various decisions by the participants. Thus, the payoff would consist of the value of the objective function which is incurred by the combined decisions of the various participants in the game. The game would be looked at as an n-person game having a payoff depending both on the simulation and the decisions of the participants. If the simulation were deterministic, the payoff would be deterministic; whereas if the simulation had random elements, the game would be a stochastic game.

Although operational gaming has frequently proven useful in educational situations, its worth in solving real-life problems has not been proven. It offers, however, a good forum for the interaction between people from various disciplines who have not had the experience in evaluating the impact of decisions that they have made in their area of expertise on some of the other areas.

3.7 Experimental Optimization

In some situations in which the relationship between certain independent and dependent variables is not known and an optimal solution to the performance of the system is desired, the optimal solution may be found by the use of *experimental optimization*. This technique consists of conducting experiments on the system under study in such a way as to locate an optimal solution to the problem at hand. There are basically four approaches that have been adopted to experimental optimization. These are the random approach, the factorial approach, the single-step approach, and the steepest-descent approach.

To illustrate the differences between these approaches in a heuristic fashion, let us consider an experiment to find the combination of two independent variables, x_1 and x_2, which give the optimal output of the system. In the random design approach, the ranges on the x_1 and x_2 scales are selected within which the optimal values of x_1 and x_2 are believed to lie. Values of x_1 and x_2 are then selected at random, and observations on the system are made at each pair of values selected. The pair which yields the best performance is selected as the optimal solution, or else another set of experiments in more detail is conducted within the area of the optimal solution of the first experiment.

In the factorial design, the scales on x_1 and x_2 are chosen. Then, x_1 and x_2 are each divided into subintervals, each subinterval assumed to be a unit over which the output of the system does not change a great deal. Observations are made of each combination of subintervals on the x_1 and the x_2 scale. This would be equivalent to looking at observations on the x_1 scale at finite intervals and observations along the x_2 scale at finite intervals to produce a grid.

The above approaches are known as simultaneous approaches since the system is monitored at several combinations of variables before any optimization decisions are made. In the next two techniques, the single-step and steepest-descent techniques, the information about previous observations goes into choosing where the next observation should be taken. As a result, these techniques are known as sequential techniques.

In the single-step method it is first decided which of the independent variables would be expected to have the greater effect on system performance. An estimate of the optimal solution is first made, and the first observation is made at this point. One then moves out from this point along the scale of the variable that is assumed more important for the output, holding the other variable constant. The interval of the move is the smallest one within which significant change in performance is believed possible. If the observation shows improved performance, movement continues in the same direction along that scale; otherwise, movement is started in the opposite direction. When a reduction on the objective function is obtained, one goes back to the point giving the value of the objective best and then begins to move along the other variable direction. Then when no further improvement is gained along this axis, movement again returns to the axis of the original variable. It is quite apparent that with only two variables this technique is quite feasible, while with many variables it becomes cumbersome and rather complicated.

In the steepest-descent procedure, a similar approach is applied, but instead of a single observation being made, five observations in the form of a rectangle with the solution deemed optimal being in the center are made. A least-squares technique is then used to try to get the direction which gives the greatest improvement in the value of the objective function. Then, as in the single-step method, movement continues along this direction until no further improvement is obtained.

Of the four procedures, the steepest-descent procedure is considered the most efficient, with the single-step, factorial, and random designs following. However, it is important to realize that the single-step and steepest-descent techniques are less likely to arrive at the optimal solution if a local optimum exists as well as a global optimum. This insecurity as far as obtaining the best possible solution is balanced by the increased speed in getting to some solution. Thus, when a good idea about the optimal solution is available, the steepest-descent and single-step techniques are preferable, whereas when the optimum might range over quite a broad range of the variables, the random and factorial designs are superior.

These techniques are discussed in greater detail in Ackoff and Sasieni (1967).

3.8 Theory of Games

3.8.1 Basic ideas. The mathematical techniques which are utilized in the theory of games were developed by John von Neumann in the 1920's and were first expanded to include possible applications in von Neumann and Morgenstern (1944).

The theory of games deals with games in which any number of players have some choice about the strategies which they may play, either in a single move or in a series of moves, with the combined result of the strategies of all the players resulting in a *payoff* for each of the players. Games can be subcategorized in various ways. One categorization is the number of players in the game. A *two-person game* has two players and an *n-person game* n players. Another categorization relates to the number of strategies available to each of the players. A game is called a finite game if there are a finite number of strategies for each player, and it is an infinite game if any one of the players has an infinite number of strategies. Games may further be described as games of chance or non-chance. A game is called a zero-sum game if the sum of the payoffs for all of the players is equal to zero and a nonzero-sum game if the sum of the payoffs is not equal to zero. A game is called *antagonistic* if the players are "out to get each other" and *cooperative* if they cooperate with each other. In a *deterministic* game the payoff resulting from any combination of moves by the players is deterministic. Alternatively, if the payoff is unknown or has some random elements about it, it is called a *stochastic* game. A game in which there are no simultaneous moves and in which all previous moves have been known to all of the players is termed a game of *perfect information*. A game in which one of these two qualifications is not satisfied is called a game with *imperfect information*.

Examples of games with no chance moves and perfect information are chess and tic-tac-toe. A game with no chance moves but imperfect information is paper-stone-scissors because the moves of all players are simultaneous. A game involving chance moves and perfect information is backgammon, and a game involving chance moves and imperfect information is poker.

It turns out that any *finite, zero-sum game* with *complete information* can be *normalized* into a single payoff matrix involving many strategies, but only one move. This is equivalent to taking a series of moves by each of the players and combining them into one single strategy for the game. A solution to a game of this sort is assured using either a linear programming algorithm or an alternative game algorithm if the payoff matrix is not too large to handle.

As an introduction to game theory and solution techniques, let us consider the following simple problem.

Consider a two-person game:

Player A chooses variable s from -1 0 1

Player B chooses variable t from -1 0 1

B pays A $s(t-s) + t(t+s)$

What strategies should be played by both players? The payoff matrix for this game is shown below.

		B		
		-1	0	1
	-1	2	-1	-2
A	0	1	0	1
	1	-2	-1	2

(3.8.1)

The game is zero sum since A's winnings equal B's losings and vice versa. Call A the *maximizing* player. He chooses s = 0 because he cannot lose with this choice. His minimum gain is 0. Similarly B, the *minimizing* player, chooses t = 0 because he prevents A from winning and he minimizes the payoff to A. The game outcome, called the saddle point, is A = 0, B = 0, payoff = 0. Both are happy with these moves.

min-max A's moves = max-min B's moves

The solution in this case is called the *saddle point.*

Not all games have saddle points. However, all *finite* games do have *optimal strategies* for both players if mixed strategies are admitted.

Consider the following payoff matrix (where B pays A):

		B		
		1	2	3
	1	1	-1	3
A	2	3	5	-3
	3	6	2	-2

(3.8.2)

Suppose A chooses 1, 2, and 3 with probabilities 2/3, 1/3, and 0; and B chooses 1, 2, and 3 with probabilities 0, 1/2, 1/2.

The expected payoff is then:

$$(2/3 \cdot 0 \cdot 1) + (2/3 \cdot 1/2 \cdot -1) + (2/3 \cdot 1/2 \cdot 3)$$

$$+ (1/3 \cdot 0 \cdot 3) + (1/3 \cdot 1/2 \cdot 5) + (1/3 \cdot 1/2 \cdot -3)$$

$$+ (0 \cdot 0 \cdot 6) + (0 \cdot 1/2 \cdot 2) + (0 \cdot 1/2 \cdot -2) = 2$$

Any other strategy for A (given B plays his optimal strategy) will be less advantageous for him (for example, if A plays strategy 3, then B can make an expected payoff of 0 if he sticks to his 2-3 strategy). The same is true for B.

3.8.2 Linear programming solution. The payoff matrix $\{a_{ij}\}$ is given and is an $N \cdot M$ matrix. If A plays a mixed strategy $\{x_1, x_2, \ldots, x_N\}$ (x_i is the probability of playing strategy i), he receives at least the minimum (over all of B's strategies) of $\sum_i a_{ij} x_i$. We will let this minimum be equal to ν. Therefore,

$$\sum_{i=1}^{N} a_{ij} x_i \geq \nu \qquad \text{for all } j \tag{3.8.3}$$

and

$$\sum_{i=1}^{N} x_i = 1 \qquad x_i \geq 0 \quad \text{for all } i$$

since the x_i's are probabilities. We wish to maximize ν. If ν is positive, divide by ν (let $x_i' = \frac{x_i}{\nu}$) and our problem becomes:

$$\text{minimize} \quad \sum_{i=1}^{N} x_i'$$

subject to

$$a_{1j} x_1' + \ldots + a_{Nj} x_N' \geq 1 \qquad \text{for all } j \tag{3.8.4}$$

$$\sum_{i=1}^{N} x_i' = \frac{1}{\nu}$$

Then maximizing ν is equivalent to minimizing $\sum_{i=1}^{N} x_i'$.

Similarly for Player B, we wish to maximize $\sum_{j=1}^{M} Y_j$ subject to $a_{i1} Y_1 + a_{i2} Y_2 + \ldots + a_{iM} Y_M \leq 1$ for all i. Here Y_j is the probability of B playing strategy j.

The problems are duals of each other and have the same solutions. The x_i's $= \frac{x_i}{\nu}$ are A's optimal probabilities, and Y_j''s $= \frac{Y_j}{\nu}$ are B's optimal probabilities.

3.9 Calculus of Variations

Problems of the calculus of variations are concerned with the determination of maximum or minimum values of objective functions which are expressed as an integral of the objective function over time by seeking the control (combination of variables) for which the objective function assumes the extremum (maximum or minimum). The classical formulation of this problem is

$$I(y) = \int_0^L F(x, y, \dot{y})dx \tag{3.9.1}$$

where one wishes to minimize or maximize I by varying (controlling) the variable y of the time period L.

Examples of problems which can be posed in this framework include that of finding the shortest distance between two points and that of determining the grassland ecosystem management policy which maximizes the net return from a given resource.

One of the most important distinctions of the calculus of variations approach from the others that are discussed is that the objective function I represents the present worth net gain over the entire time interval from now until L years into the future. The other techniques are almost all oriented towards finding the management policy which provides the maximum return at one time in the future. Excellent references on the calculus of variations include Bliss (1946) and Courant and Hilbert (1953).

3.10 Statistical Optimization

By *statistical optimization* we mean the variety of techniques used in sampling and estimation to give descriptive parameters of the population samples which are optimal representatives of the population with respect to criteria chosen by statisticians. Some of these techniques commonly used are least-squares estimators, maximum-likelihood estimators, Bayesian estimators, stratified sampling, etc.

3.11 Critical Path Analysis--CPM and PERT

Critical Path Analysis is a methodology used for planning, scheduling, and controlling a project. The best known and most often used of these techniques are *CPM* (Critical Path Method) and *PERT* (Program Review and Evaluation Technique). Originally, CPM and PERT were developed separately and for slightly different uses; today their differences are seen to be rather nominal.

3.11.1 Underlying ideas. The basic idea of critical path analysis is to regard the relations of all the jobs that must be accomplished in a project as a network. This network is made up of arrows that represent individual jobs or activities. The arrows are arranged in the logical sequence in which these jobs must be carried out.

When the logical network planning is complete, the next step is to assign the time required to complete each individual activity. Critical Path Analysis of this simple network then provides:

i. the longest sequence of activities from the beginning to the end of the project (this is the critical path), and

ii. the amount of idle time associated with noncritical activities.

In addition to the above items, PERT analysis provides probabilities of completing either particular activities or completing the project.

Since the critical path is defined as that path through the network which requires the longest duration of time to complete, the particular sequence of jobs along this path control the duration of the entire project. Therefore, if there is a delay in any one of the jobs along this path, there is a corresponding delay in completing the project. Likewise, if the project is to be completed in any less time, time can only be gained by shortening the duration of any of the activities on the critical path.

The fundamental basis of CPM and PERT is the network diagram. This is a graphical model of the entire project and shows each step (job, activity, or operation) and the relation between steps. The network is made up of arrows and is called an arrow diagram.

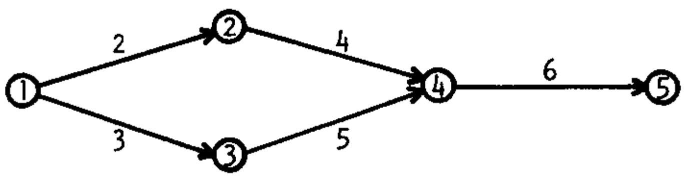
The arrow diagram to which circles have been added has further implications. In this diagram the arrows represent the activities, operations, or jobs which require *time* for completion. Action is occurring along an arrow. Also the circles or nodes have a physical significance. They represent *events* or instants in time that mark the completion of a certain activity and simultaneously the initiation of another activity. The events are markers and consume no time.

Attention to activities or events is simply a point of view. While a foreman may be more interested in the activities, an administrator may prefer to follow the progress of a project by when certain events or milestones are reached. Traditionally, CPM is considered to be activity-oriented, while PERT is regarded as being event-oriented.

Once the network is complete, the next step is assigning the durations of the various activities.

The critical path is that sequence of activities with the longest total duration. Its chain of activities determine the overall project time. Activities on the critical path are critical, not by virtue of their difficulty but simply because any delay in them will cause the same delay in completing the project.

Isolating the critical path is accomplished by a forward/backward pass solution. Below is an example of a CPM network diagram showing stages of activity and activity completion times.



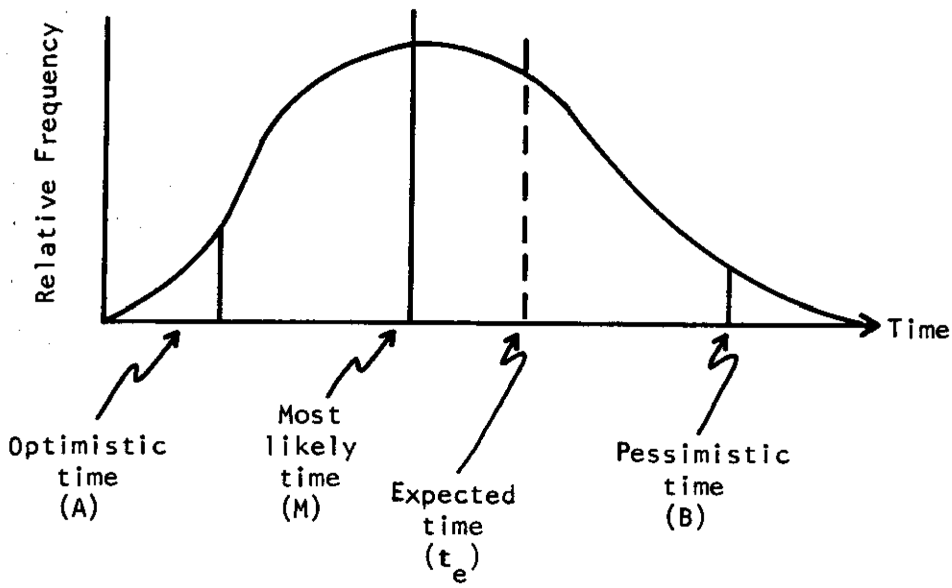
Activities are identified by the pair of numbers (ij), designating the tail and head of the arrow representing the particular activity. The assigned durations are marked on each arrow.

The method for performing the calculations directly on the network diagram for solving CPM problems is discussed in Horowitz (1967).

3.11.2 PERT. PERT is a variant of critical path methodology that is useful for planning, scheduling, and analyzing projects that have never been undertaken before and necessarily involve considerable uncertainty in assigning the duration of activities. Recall that CPM involved assigning durations for which the time required was already known.

Three estimates of activity duration are required for PERT. These are the optimistic, the most likely, and the pessimistic times.

To calculate the *expected* activity duration, the three time estimates are assumed to be drawn from a beta distribution. This distribution was used purely for its convenient properties. The appropriateness of the beta distribution has been examined and is generally accepted. The beta distribution is usually skewed and appears as below:



Once the expected time has been calculated, isolation of the critical path is arrived at just as it is in CPM, with the forward/backward pass solution. In addition to the critical path and the float, probabilities for completing an *individual* activity, according to a scheduled time and probabilities of reaching a point in time (the *event* at the end of the particular activity), can be calculated.

The probability of reaching a particular point in time (an event or milestone) is arrived at by summing up all the expected times for the activities terminating at the particular point and their standard deviations. The expected time for the *event* is taken as the sum of the expected times for the preceding *activities*, and the standard deviation for the expected time for the *event* is calculated by squaring and summing all the standard deviations of the preceding activities and taking the square root of the sum.

3.11.3 CPM and linear programming. Critical Path Analysis is based on network flow theory which is a specialized application of graph theory. A *graph* is a set of junction points called nodes with certain pairs of nodes connected by lines called branches.

A project can be represented by a time-oriented graph, letting the nodes represent *events* in time and the branches represent *activities* requiring time to complete. The interrelationship among events and activities comprise the network in which time "flows."

Knowing the time required to complete each activity, the network can be drawn showing the events (nodes), activities (arrows), and their durations (number on arrows). Nodes are also numbered.

A linear programming solution of this network will provide the longest sequence of events in the project (the critical path). Thus, linear programming can be used to solve CPM and more generally network flow problems.

$$\text{maximize} \quad f(y) = 16x_{12} + 20x_{13} + 30x_{16} + 15x_{25} + 10x_{35}$$

$$+ 3x_{45} + 16x_{46} + 12x_{56}$$

The linear programming solution is not as satisfactory as the forward/backward pass solution because it does not provide information on the free time associated with the noncritical activities. Hadley (1962) and Moder and Phillips (1970) provide further reading on network flow and linear programming solutions.

3.12 Queueing and Inventory Theory

Queueing and inventory theory are usually applied to problems of how to optimally plan a facility to handle demands for a service which can be recognized as occurring at discrete time intervals, but which does not occur at deterministic times. In other words, the demands occur with a certain frequency, but do not occur at known time intervals. Such demands are termed *stochastic* or random. It is desired, then, to know how the service facilities should be planned in order to handle the demands as expediently as possible. Examples of systems which involve queues are check-out counters in a supermarket, runways in an airport, parking spaces in a parking lot, cashiers in a department store, maintenance crews in a factory, doctors in a clinic, beds in a hospital, arrivals of a train, etc.

The general inventory problem is to decide how much inventory should be kept on stock and how often new inventory should be ordered to handle a stochastic demand. The general consideration in both kinds of problems is the balancing of the cost of having a better facility, that is, having more servers, or having a larger inventory on stock against the cost of customer inconvenience, that is, customers waiting a long time, customers being turned away because the facilities are all full, customers coming into a store and finding they cannot get what they want, etc.

Queueing theory concerns itself with the numbers of servers to be put on the service and the way in which the arrivals are to be serviced (that is, should they be served as "first come, first served"; should some priority be put on when they should be served; should the arrivals all wait in the same line and then the first one goes to any one of the servers that is available; or should they each wait in separate lines, each line going to one server, etc.). It turns out that in many situations the arrivals may be looked at as coming at random; that is, one can assume it is equally likely for the arrivals to occur at any point in time. Under this assumption, which has been tested and found acceptable in many real life systems, the arrival process may be statistically represented by a *Poisson process*. For the Poisson arrival process, the probability of having n arrivals in any finite time interval of length t is given by

$$\frac{e^{-\lambda t}\,(\lambda t)^n}{n!} \tag{3.12.1}$$

where λ is the arrival rate (arrivals/unit time).

Similarly, the assumption is frequently made about the servers that the probability of a service being completed at any time interval is independent of how long the service has been going on. This assumption is known as the *negative exponential* assumption. Under the negative exponential assumption, the probability that a service is completed in less than some time period t is equal to $1 - e^{-\mu t}$. Here, μ is the service rate of the server (number of services completed/unit time).

If there is only one server and a single stream of arrivals and if the service rate is faster than the arrival rate, the line length can theoretically be kept finite; whereas if the arrival rate is faster than the service rate, then theoretically the line grows to infinite size. For the case where the arrival process is assumed to be Poisson and the service process is assumed to be a negative exponential, the expected waiting time for each arrival, the average length of the line, and the percentage of time that the server will be idle have been theoretically worked out.

In the case where either one of these assumptions does not hold, as for example if the arrivals were scheduled in some fashion (which would eliminate the randomness of the arrival stream), information about waiting times, average length of line, and idle periods is more difficult to derive theoretically. However, in such cases, it is possible to simulate (Monte Carlo simulation) the arrivals and service under certain assumptions about the probability distribution of arrivals and servers. Several runs of the simulation will then give estimates for the average waiting times, as well as the percentage of the time the server is idle, etc. However, simulation has been overused in many cases where theoretical solutions could have been obtained with less difficulty.

Since most inventory problems involve the arrivals of demands to a fixed stock, the inventory problem may be looked at as a special case of the queueing problem in which the service time is either zero or infinity. It would be zero if the stock were available and infinity if the stock were not available. Then there is interest in the distribution of the probability that the stock will not be available.

Queueing theory and many of its applications are discussed in Haight (1967), Cox and Smith (1961), and Ackoff and Sasieni (1967).

3.13 Quasilinearization

Quasilinearization is a technique of numerical analysis for converting the solution of a nonlinear problem into the limit of a sequence of solutions of a sequence of linear problems. It can be viewed as an extension of the Newton-Raphson technique for finding roots of equations to function spaces; or alternatively, it can be derived from a maximum (minimum) principle for convex (concave) functions.

References of interest are Bellman and Kalaba (1965) and Lee (1968).

3.13.1 Newton-Raphson technique. Find r such that $f(r) = 0$, given $f(x)$. To solve this we replace $f(x)$ by $f(x_0) + f'(x_0)(x - x_0)$ which is linear (where $f'(x_0)$ is the slope of $f(x)$ at $x = x_0$). This is the two term Taylor series approximation to f. Choose x_0 and find x such that

$$f(x_0) + f'(x_0)(x - x_0) = 0 \tag{3.13.1}$$

Call this $x = x_1$ and then solve

$$f(x_1) + f'(x_1)(x - x_1) = 0 \tag{3.13.2}$$

for $x = x_2$ and keep this up eventually giving,

$$f(x_n) + f'(x_n)(x_{n+1} - x_n) = 0 \tag{3.13.3}$$

or

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Under appropriate restrictions on f this technique converges to r and does so quadratically.

Pictorially

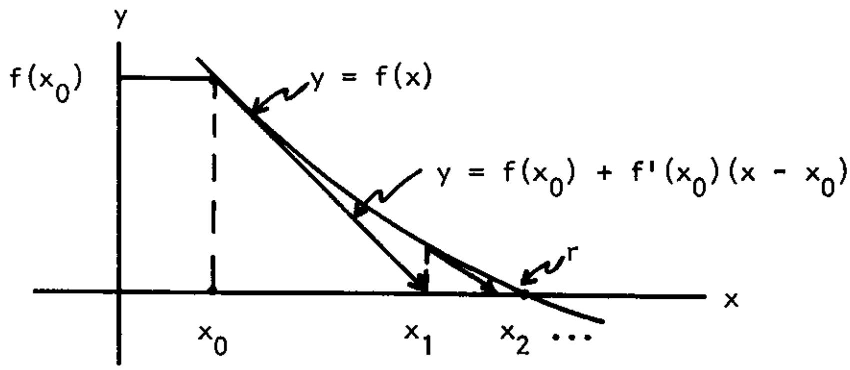


Fig. 3.13.1. Linearization by Newton-Raphson technique.

The key is to replace f by a linear approximation. The technique can be extended to a system of n first-order differential equations.

Quasilinearization is useful because problems of parameter estimation and certain classes of optimization problems can be viewed in the context of multipoint boundary value problems.

3.13.2 Example. As an illustration consider the following parameter estimation problem (scalar problem):

Example: $y(x_0) = m_0$ (Initial conditions)

$$\left.\begin{aligned} y(x_1) &= m_1 \\ y(x_2) &= m_2 \end{aligned}\right\} \text{ Measured conditions}$$

and

$$y' = aye^{by}$$

What estimated values of a and b should we pick?

We consider the problem

$$y' = aye^{by}$$

$$a' = 0$$

$$b' = 0$$

subject to

$$y(x_0) = m_0$$

$$y(x_1) = m_1$$

$$y(x_2) = m_2$$

The first differential equation is nonlinear, and we will replace it by the quasilinearization formula

$$y_1 = y \qquad f_1 = aye^{by}$$

$$y_2 = a \qquad f_2 = 0$$

$$y_3 = b \qquad f_3 = 0$$

$$J = \begin{pmatrix} ae^{by} + abye^{by} & ye^{by} & ay^2e^{by} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where J is the Jacobian or matrix of partial derivatives so that

$$y^{n+1'} = a^n y^n e^{b^n y^n} + (a^n e^{b^n y^n} + a^n b^n y^n e^{b^n y^n})(y^{n+1} - y^n)$$

$$+ \; y^n e^{b^n y^n} (a^{n+1} - a^n) + a^n (y^n)^2 e^{b^n y^n} (b^{n+1} - b^n)$$

$$a^{n+1'} = 0$$

$$b^{n+1'} = 0,$$

which can be solved. If more intermediate measures are available, $y(x_i) = c_i$, and $i = 1, \ldots, k$. Then least squares could be employed or some other suitable measure of fit. The above example is discussed by Leary (1970). Problems of convergence are rather difficult because the operators are quite complicated. As Lee (1968) points out, in actual practical problems the computations are simply carried out and convergence is based on the numerical behavior.

Problems of storage in large systems can apparently be avoided to some degree and this is discussed by Bellman and Kalaba (1965). This point, however, has been contested.

In certain cases, quasilinearization yields a convergent sequence provided the interval in question is sufficiently short. Quasilinearization may be combined with dynamic programming or calculus of variations with quasilinearization giving the optimal path for a subinterval and dynamic programming or calculus of variation yielding the overall optimal path.

3.14 Benefit-Cost Analysis

The classic definition of economics is "the problem of allocating scarce resources among competing ends." For analyzing the optimal allocation of resources this rather simplified definition of economics has special meaning. The focus of this review of benefit-cost analysis will be to show how this technique can be applied to determine an optimal utilization of resources.

3.14.1 Efficient resource allocation. Economists contend that an "optimal" resource allocation exists when a resource is allocated so that the value of its marginal product (contribution) in one use is equal to the value of its marginal product in any other use. Value of marginal product (VMP) can be defined as the marginal product of a resource (its contribution to output) times the price of the produce being produced or:

$$VMPa = MPa \cdot PX \tag{3.14.1}$$

where VMPa = value of marginal product of factor a in the production of product X.

MPa = marginal product of a.

PX = price of product that factor a (resource a) is being utilized to produce.

It follows that if the value of the output resulting from the last unit of a resource applied to use "X" is greater than the value of the output resulting from the last unit of that resource applied to use "Y", economic efficiency of resource allocation would be improved upon by switching some of the resource from use "Y" to use "X." In terms of a grassland area use "Y" might be agricultural use and use "X" might be urban use. To optimize resource allocation, water should be allocated so that value of the marginal product of water in agricultural use is equal to the value of water's marginal product in urban use.

Benefit-cost analysis is a method for evaluating resource allocation changes in terms of economic efficiency. Conceptually, benefit-cost analysis requires data similar to VMP's to evaluate resource reallocation policies in terms of economic efficiency. Operationally, benefit-cost analysis is much more general than comparisons of VMP's, but is very similar theoretically.

Various optimizing and impact analysis techniques such as linear programming, simulating studies, and input-output models can provide data to utilize in the benefit-cost analytical framework. In the description which follows, emphasis will be on describing the framework rather than specifically showing how other optimizing techniques may be applied in conjunction with benefit-cost analysis (Dobbs, Paananen, and Rechard 1971).

3.14.2 Benefit-cost analyses and efficiency criterion. Benefit-cost formulations vary for measuring efficiency of resource allocation according to the purpose of the study. The two most common formulations are (i) discounted present value and (ii) benefit-cost ratio. Several economists hold that the discounted present value is the criterion of economic efficiency (Dobbs et al. 1971). Baumol (1965) contends that projects having the largest discounted present value make the largest contribution to income. He argues that, setting aside considerations of risk and uncertainty, an investor would choose the project which produced the greatest net benefits. For grassland area planning the planning authority also would maximize income of the area by implementing the project with largest net benefits.

3.14.3 The formulations. The two approaches to benefit-cost analysis can be represented symbolically as follows:

$$\frac{B}{C}\text{ ratio} = \frac{\sum_{t=0}^{n} \frac{Bt}{(1+R)^{t}}}{\sum_{t=0}^{n} \frac{Ct}{(1+R)^{t}}} \qquad (3.14.2)$$

Discounted present value:

$$PV = \sum_{t=0}^{n} \frac{Bt - Ct}{(1+R)^t} \qquad (3.14.3)$$

where Bt = benefit in time period t.

Ct = cost at time period t. (Note: Co is the initial cost outlay of costs at start of project.)

t = number of years after initial cost or benefit.

R = discount rate.

Utilizing the discounted present value criterion, if the PV of the project has a positive value, the project would be considered a "good" project. In choosing between mutually exclusive projects, the project with the higher PV should be chosen.

With the application of the benefit-cost ratio criterion, a ratio with a value greater than one would be considered a "good" project. A choice between mutually exclusive projects would require an evaluation of relative B/C ratios, assuming both ratio values are in excess of a value of one.

3.14.4 Benefit-cost analyses efficiency criterion. Benefits and costs of resource projects and policies occur over time and consequently must be evaluated on a comparable time basis (see Dobbs et al. 1971) for further discussion regarding benefit-cost efficiency criterion. This section draws on their review of benefit-cost analysis. As a consequence, costs and benefits are discounted to their present value in any formulation of benefit-cost analysis. Considerable controversy exists regarding which interest rate (discount rate) should be utilized in the analysis of alternative policies.

The discount rate controversy revolves around a question of economic efficiency. Present federal policy requires that a discount rate based upon long-term borrowing rates be utilized in the evaluation of federal projects (specifically, the rate is defined as the average yield during the preceding fiscal year on interest bearing marketable securities with 15 years to maturity). This rate has been criticized because discount rates are normally lower than private sector borrowing rates. Federal borrowing rates are lower because federal securities involve less risk and uncertainty. Critics argue that the use of lower government bond rates can lead to an inefficient allocation of resources. The use of subsidized capital leads to a misallocation of resources from higher value private use to lower value public use due to the lower interest on government bonds. Critics argue that the "appropriate" interest rate should reflect the opportunity cost of capital (potential higher returns in the private sector).

Advocates of the lower government bond rate build their case on "social time preference" which means that society is willing to forego present consumption in favor of future consumption to a greater degree than is indicated by the market rates of interest.

The appropriate rate of interest is an unsettled question. Its relevance is that justification of public projects is sensitive to the interest rate chosen. However, if two projects are being compared utilizing the same discount rate and if it has previously been decided that a certain number of dollars are going to be spent in the project area (suboptimization), the appropriate discount rate has less significance.

4. APPLICATION OF OPTIMIZATION TECHNIQUES TO ECOSYSTEM MANAGEMENT

4.1 Introduction

The optimization and operations research techniques described in the previous sections have been, as mentioned earlier, applied to some natural resource management situations. They also have potential applications in the planning of a large-scale program of the magnitude of the U.S. IBP Grassland Biome Program.

In resource management, various optimization or operations research techniques have been applied to the areas of forest management, big game management, regional produce distribution, agricultural planning, ecosystem function, rangeland sites, animal nutrition, pest control, reservoir system planning, stratified sampling, government planning, and multiple use of natural resources.

4.2 Resource Applications

Table 4.2.1 shows which types of programming techniques, to the knowledge of the authors, have been applied in which resource management areas. A check in a box denotes that there have been applications of the programming techniques in this specific area while a blank denotes that there are none known to the authors. An R in the box denotes that by and large the applications in the literature in that area are real--that is, the models are based on real data and have been or expect to be applied to particular resource problems. An H in the box denotes that the applications are by and large hypothetical and present a framework for an applied solution to problems while not yet grappling with the actual applications to specific systems.

Table 4.2.1. General application of programming techniques to resource management.

	Linear Program	Quadratic Program	Stochastic Program	Nonlinear Program	Dynamic Program
Forest management	√R,H				√H
Farm planning	√R	√R			
Game & fish management	√R			√H	√H
Pest control					√H
Animal nutrition	√R	√R	√R	√H	
Ecology (classical)	√H				
Social system planning	√H				

R = Real applications.
H = Hypothetical applications.
√ = Applications of programming techniques.

It can be seen from Table 4.2.1 that the most widely applied technique is linear programming. It has been extensively applied in each of the resource management areas considered except for pest control, where consideration of pest population dynamics over time is essential to pest management. Furthermore, in four of the six areas having linear programming applications, the applications are to real-life problems.

The application area utilizing the widest variety of programming techniques was in animal nutrition, a category which concerned various feeder lot and laboratory animal studies. A wide variety of techniques has also been applied in game and fish management, although many of these have been hypothetical in nature. This is probably due to the difficulty of obtaining data on game and fish populations dynamics, which is essential to the use of quantitative techniques in management of these game populations. All the applications in farm planning have been real and in fact have been applied to specific farm situations.

It is not possible from this table to ascertain which areas have the most important applications and the broadest number of applications of programming techniques. The largest number of application papers in the literature found by the authors was in the area of forest management--most of these were linear programming applications, including several that were general computer programs which could be applied to a large number of forest management situations. Only a small number of the papers in farm planning were reviewed by the authors, and it is expected that a large number of applications, mostly in the areas of linear programming and quadratic programming, have also been implemented in farm situations. In the other areas the number of applications was very scanty and except for perhaps the area of animal nutrition the problems are so complex and the attempts at application so new that very few applied problems have been tackled.

Table 4.2.2 reviews in brief some of the applications of optimization techniques to resource management situations. The applications are arranged according to the same categories as in Table 4.2.1. The table gives in brief the objective function as well as the number of variables involved in the problem to give some indication of the size of the problem. The comments indicate general information about the application such as whether real data is used to set up the problem or the example, whether the problem was applied, or other information to give a better understanding of the particular application.

As mentioned earlier, the largest number of applications reported in this review were in the area of forest management. The sizes of the problems were quite variable, ranging from four variable problems in the ecologically-based Wilson and Barea papers to the 28,000 or so variable problems of Liittchwager and Tcheng. Most of the direct resource applications dealt with the maximization of profit or the minimization of cost in development of and/or allocation of the resource. The emphasis of profit and cost in the objective function has been the traditional pattern in operations research techniques. However, it is interesting to note that there are a large number of applications which do not deal with money specifically. This is evidenced in forest management, in the papers of Loucks and Liittchwager and Tcheng, in animal nutrition in the paper of Bracken, in all of the papers in game and fish management except the Rothschild and Balsiger fish management paper which does have monetary

Table 4.2.2. Some applications of optimization techniques to resource management situations.

Areas	Author(s)	Type of Program	Objective	Number of Variables	Comments
Forest management	Hool 1966	Dynamic	Maximize profit or forest yield in the face of risk in future growth	288	A general framework (examples given)
	Liittschwager and Tcheng 1967	Linear (decomposition)	Maximize yield over 24 years by yearly cuttings in over 1000 wood lots	~28,000	The problem was real, but the solution was not applied to the problem
	Loucks 1964	Linear	Minimize cut area while assuring a guaranteed yield	43	A hypothetical example
	Navon 1971	Linear	Optimize expected future yield in converting from irregular to regular aged stand	--	Provides a program called Timber RAM for use in forestry management problems
	Nautiyal and Pearse 1967	Linear	Maximize present worth of a forest while converting from an irregular to regular aged stand in a given number of years	--	Provides a general framework (no example provided)
	Schreuder 1968	Dynamic	Maximize discounted net revenue in developing a large virgin stand over 75 years (includes road building, tree preparation, cutting cost, etc.)	75	A hypothetical example using real data for constraints
	Wardle 1965	Linear	Maximize net discounted revenue by optimizing felling and planting program in area having both hardwoods and conifers	12	A real applied example, implemented, no validation or follow-up in this paper
Farm management	Barker 1964	Linear	Maximize farm profit by proper allocation and expansion of resources	7	Applied to two farm situations
	Heady and Whittlesley 1965	Linear	Allocate crops to regions in the U.S. to minimize production and transportation costs	2682	The problem was formulated with real data, its large size precluded application
	Haryama and Fuller 1965	Quadratic	Decide on where to produce milk and how to transport it between competing markets to maximize net revenue	28	Real data used (not applied)
	Rickards and McCarthy 1966	Linear	Maximize revenue by allocating cattle, sheep, and crops among selected soil types on large farms	45	Program was implemented
Animal nutrition	Bracken 1963	Quadratic	Minimize radionuclide intake of cattle while satisfying their diet needs	9	A real problem implemented in the laboratory
	van de Panne and Popp 1963	Stochastic	Minimize cost of feed while satisfying animal nutrient requirements	4	A real problem implemented in a feeder lot
Game and fish management	Davis 1967	Dynamic	Maximize the "value" of killed deer, surplus deer, and game clearings over a 20-year period	10	Assumes complete control over deer kill (a hypothetical example)
	Mann 1968, 1971	Dynamic	Minimize expected discounted population loss over a given time period by proper harvesting of males and females	20 for 10 decision stages	Assumes complete control over game populations (hypothetical)
	Rothchild and Balsiger 1971	Linear	Allocate salmon catch among days of run to maximize value of landings while allowing adequate escapement	18	Real data (not implemented)
	Swartzman 1970, 1972	Nonlinear	Maximize harvest over a state without decimating herds	~20,000	Does not assume control of herd populations, but gives them as a function of agency controls (not applied, data not real)
Estuary productivity	Patten and Van Dyne 1968	Nonlinear	Find combination of source sample and incubation conditions to optimally estimate plankton productivity	49	Real data (not applied)
Social insects	Wilson 1968	Linear	Show that allocation of individuals among castes optimizes future survival of queens	4	Real data
Ecology	Barea 1963	Linear	Show that potential energy is maximized in climax situation	4	Hypothetical example
Pest control	Mann 1968, 1971	Dynamic	Minimize loss of cropland to pests over given time period by eliminating pests to a certain level at each decision stage	10 for a 10 stage period	Hypothetical example, assume complete control over pest elimination
Social planning	Charnes Kirby, and Walters 1970	Linear	Maximize government revenue minus costs while maintaining education and health facilities	517	Hypothetical example given, general simplified framework for social planning
Dam planning	Avi-Itzhak and Ben-Tuvia 1962	Queueing theory	Determine optimal size of reservoir and optimal pumping capacity to minimize pumping cost per cubic meter of water pumped	--	Real example, applied and implemented

considerations in the objective function, in all the ecological applications including Patten and Van Dyne, Wilson, and Barea, and in the pest control paper by Mann. The large-scale social planning paper by Charnes, Kirby, and Walters will be discussed in greater detail shortly. It is interesting to note initially, while we are on the topic of considerations in the objective function, that this paper, while having the maximization of government revenue minus costs as its objective function also has the value of education and health facilities included in the objective function.

It is interesting to notice from Table 4.2.2 that most of the applications, while depending in some way on real data, have not been applied to specific situations.

Assuming that the applications reviewed in Table 4.2.2 offer a representative sample of the use of programming techniques in resource management, it can be noted that by-and-large the large number of applications occur in the linear programming area. The second most utilized technique is dynamic programming with nonlinear programming (including quadratic programming) a close third. The other techniques, stochastic programming and queueing theory, show only one application in this review. This tends to indicate that there has been very little application in resource management of techniques outside of linear, nonlinear, and dynamic programming. In the next section (Section 5), we will discuss some of the areas where we feel the other techniques can be applied to resource management problems.

4.3 Large-Scale System Planning

Charnes, Kirby, and Walters (1970) deal with a large-scale horizon plan for social development as a linear programming model. They look for the solution that will optimize the "living conditions" in a country or a large regional area with minimal costs. The problem deals with three ethnic groups (a native group, a mixed group, and a foreign group) living in three major areas (urban, agricultural, and undeveloped). The model tries to select how much of various resources, including electric power, educational facilities, construction facilities, capital investment, and health and sanitation facilities, should be utilized and which industries of 21 industries ranging from tourism to education should be encouraged in order to optimize living standards.

Some of the goals are introduced as part of the objective function (e.g., the total net economic value, which is revenue costs). Others are introduced in the form of constraints (for example, the percent of unemployment in the population must remain below a specified level; the percent of the population attaining the various education levels must be at least at a specified level; the health and sanitation facilities available in the country must increase at least at a specified rate and at the end of the planning horizon the sanitation facilities must be at least of a certain quality, etc.).

The problem is applied to a simple situation in a hypothetical country named Zandel with some interesting implications arising in the final solution. For example, one of the conclusions was that political and sociological constraints must be introduced into the model. If the goals are solely economic, the solutions that result would be totally unacceptable on the basis of social or political realities. The Charnes problem, while not a specific example of the application of linear programming to ecosystem management, gives an idea of the kinds of considerations that have to be made when sociological, political, and economic factors have to be combined with ecological factors in large-scale planning types of problems.

An application of the simulation-optimization approach which is currently being developed by the modelling group at the U.S. IBP Grassland Biome is to an Australian land use planning problem. The simulation is controlled by precipitation and temperature and gives the dynamics of herbs and shrubs and the energy dynamics of the herbivores dependent upon this vegetation. At six-month intervals an optimization procedure determines how many cattle, sheep, and kangaroos should be harvested from each of the private ranch holdings and how many should be transferred to a common land area which is controlled by the government. The optimization procedure also tries to plan for the pessimistic case of drought situations by limiting the number of animals to be held on any specific site. The major problem with this approach is how to get the incentive for keeping animals in an area when there is no direct financial benefit at any one specific time period for keeping the cattle in terms of financial gain. The trick is that cattle that are kept have future worth both in terms of selling price and in terms of reproductive value. The concept that is presently being used to take this information into consideration is analogous to the concept of the present worth or discounted worth of money which would not be invested at the present.

5. FURTHER POSSIBLE APPLICATIONS OF OPTIMIZATION TECHNIQUES

As is apparent from the previous section, the majority of applications of optimization techniques in resource management have used linear, nonlinear, and dynamic programming. There is a sparsity of applications of the other techniques to resource problems. It seems from the previous discussion that the applications of the programming techniques to more complex problems will follow basically along the same lines that the applications to date have. As the availability of information about large-scale systems and our ability to simulate these systems increases, these techniques should become more and more helpful in making management decisions about the system.

With this in light, this discussion will focus on some of the possible applications of techniques other than programming techniques. We will talk about the possible applications of queueing theory, the simulation-optimization approach, game theory, and experimental optimization.

The simulation-optimization approach, as mentioned earlier, is being experimented with by the U.S. IBP Grassland Biome modelling group in an Australian grasslands problem. Assuming this technique will be shown to be effective in relating to that problem, the technique might well be applied to many other management problems--in specific, to larger-scale regional management problems. The approach might be combined with a grid display device which would be able to graphically display the output of a simulation-optimization run in such a way that the managers would be able to visualize the effects of the decisions on the simulation. Such an interactive display and simulation-optimization technique might be applied to such a model as a U.S. Grassland Biome model in which the grasslands are divided up into a grid network, each area within the grid having similar characteristics with respect to the abiotic and biotic components of the environment. Thus, a particular grid area might be defined by isopleths of precipitation, isotherms of temperature, and perhaps the dominant plant species in the region. The simulation would simulate population changes of major species within each of the grid subregions as well as transfer of animals, plants, goods, and services between the regions. The optimization would then allocate and manage resources in the region at given intervals. It should be emphasized that an entire grasslands region model could not be solely an ecological model, but would have to have human ecological considerations. Thus, not only migrating animals would cross between the grids, but the human goods and services alluded to earlier would also.

We see queueing theory as having many possible applications in the areas of resource management. It could be especially applied in the area of fish management, forest management, and agricultural situations. In fish and fisheries management the fish could be looked at as arriving at random at various points along the river system. We could then look at the service processes of fish harvest (fishing) or the safe passage of fish over large falls areas (fish sluicing). In fishing the problem could be looked at as a series of servers at various points along the river, all of whom take the fish out of circulation. In essence, we want to know what is the optimal set of servers, such that the fishing catch is maximized while assuming that the fish supply in the coming years is replenished and the cannery limits are not exceeded.

In the fish sluicing problem we could look at the fish as forming a line (*a queue*) at any of the dams or falls where they cannot get up and have to use the sluice. The waiting period is then the amount of time that it takes the fish to find out about the sluice. Obviously, if there is only one small sluice, the waiting period is going to be very long and the chance of losing the fish by having them kill themselves on the falls is very large. This problem presents an interesting case to queueing theory because it is quite apparently not a "first-come, first-serve" queueing system.

In forest management we could look at fire fighting as being a queueing system. The fires could be looked at as random arrivals for servicing, the servicing being the putting out of the fires. The queueing problem is then how many forest fire fighters (servers) should be kept on hand to potentially fight the fires in order to have all the fires potentially put out or all the fires of a certain size potentially put out, while not having people sitting around doing nothing for long periods of time.

In agricultural crop planting planning, you could look at the growth of plants as being serviced by thunderstorms in the Great Plains region. Since the thunderstorms do not arrive at regular intervals, there is good reason to believe that they could probably be represented by a random process with a mean arrival rate changing from month to month through the summer period. The question then becomes: Knowing the growth response of the agricultural crops to various amounts of watering, what kinds of crops should be planted in various regions that have different rainfall patterns, and at what times do we plant them so as to get the maximum expected amount of yield from this arrival process of rainfalls? Some noteworthy events that might occur in this process, which are analogous to events which occur in a queueing system, are that a thunderstorm may arrive without a crop in the ground to "service," or that the crop may be planted (the server may be put on the job) but no thunderstorm may arrive for a long time to be able to be serviced (converted into more of the crop) by the crop.

Game theory might be applied in many situations which can be looked at as man against nature. For example, the installation of dams may be looked at as a game against nature. The manager has the alternative of putting various size dams or various numbers of dams, perhaps, on a river system. Nature has the strategies of producing various amounts of snow melt and rainfall during the winter and wetter months of the spring to produce various amounts of potential flooding. The payoffs then refer to the combinations of the strategies of the manager and nature (even though nature's strategy is a random type of strategy).

In many resource situations the key conflicts are becoming recreation vs. reclamation, development vs. wilderness, and Sierra Club vs. industry. These kinds of conflicts can be looked at as games in which both of the antagonists, for example the developer and the conservationists, have a number of strategies. The developer and the conservationists have different strategies which can then be related to each other in terms of a payoff or a probability distribution of various payoffs, dependent upon the strategies of the antagonistic groups.

As an application of stochastic programming we might look at an irrigation scheme where the question is whether or not to irrigate a particular land plot. The effects of the irrigation are subject to the basic uncertainty of whether or not it would have rained anyway on that plot. There is a cost associated with the irrigation which must then be balanced in the stochastic optimization against the uncertain cost, or the uncertain payoff, resulting from the irrigation and/or rainfall that might occur on that particular day. This problem could be extended over a long time period to a stochastic dynamic programming problem.

Another possible application of stochastic programming would be to determine the proper stocking rate for rangelands with probabilistic components. This problem has been approached intuitively for a long time, but has only been solved by general rules of thumb. Basically, the problem is what is the optimal stocking level giving probabilistic future forage production? Overestimating the carrying capacity of the range will result in penalties due to such things as the necessity of buying supplemental feed or selling livestock at a loss. Underestimating the future forage production will result in penalties due to unutilized resources that could have been profitably employed. Stochastic programming gives us a good way of evaluating this problem as well as a more general view of evaluating what discount rates should be applied with different degrees of uncertainty in such an optimization system. If a direct analytical solution to the technique to the problem would not be found, an experimental technique such as the Monte Carlo approach (see page 22) would offer another possible way of solving the problem, which although a brute-force technique would give some insight into solving the problem.

As an example of an experimental optimization approach, we might perform experiments to try to find the optimal combination of grazing treatments (light, moderate, or heavy grazing), nutrient treatments, and irrigation treatments on land plots to maximize beef yields. The statistical setup for an operation like this would try to standardize the site factors (elevation, slope, soil type, etc.) in order to be able to separate them from the effects of the treatments that we are interested in--that is, grazing, nutrient, and irrigation treatments. An approach of this sort might well be applied on the U.S. IBP Grassland Research Site at Nunn, Colorado (commonly called the Pawnee Site).

A possible use of calculus of variations is in a grassland management problem. Let us consider the objective function Q where

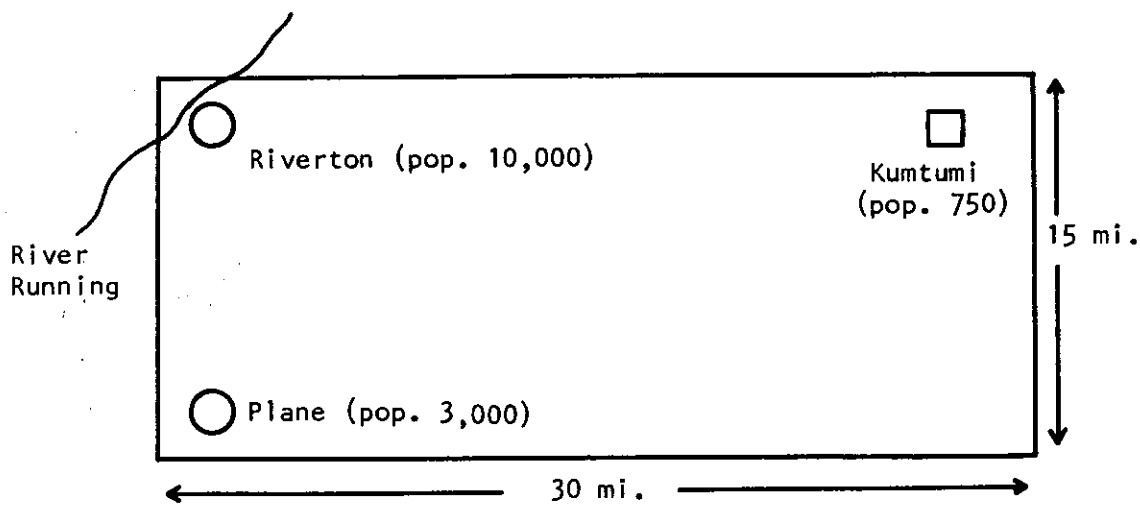
$$Q = \int_0^L \frac{\sum_{i=1}^{n} Y_i H_i - \sum_{i=1}^{m} C_i U_i}{(1+R)^t} dt \tag{5.1}$$

in which the Y_i's and C_i's are yields associated with harvests and costs associated with management policies, respectively. The H_i's are harvests, the U_i's are management units, and R is a discount rate. L is the period of time over which the management effort is to provide maximum return. Q then represents the net return (stated as the present value of future assets) as a result of the management program identified by the various variables in the expression. The range manager has at his disposal the option of varying the harvests and the management units in an attempt to increase Q to its maximum value. The Q may be interpreted as the net cash yield of the harvesting policy over L years. As stated, therefore, this problem is precisely one of calculus of variations. It is also essentially impossible to solve the problem in this general form. One can, however, gain very useful insight into the directions for management to improve, i.e., increase Q by making a number of assumptions regarding the H's and U's and continuing the analysis.

One nice assumption (mathematically) regarding these variables H and U which might simplify the solution is that they are piece-wise linear, that is, that one can change harvest rates and management units only at specific periods of time and that from one period of time to another these are either constant or they change linearly. In this formulation the calculus of variations problems smacks of the quasilinearization study discussed in Section 3.14. One is tempted to apply some dynamic programming technique to this problem in an attempt to solve it when it is posed in this piecewise linear form.

6. A SPECIFIC PROBLEM DEVELOPED USING OPERATIONS RESEARCH TECHNIQUES

As an example of a general problem which utilizes both an operations research approach for solution and an interdisciplinary approach for the structuring of the problem, let us consider the following hypothetical situation:



The *River Running* flows through the corner of a region in Upper-Agraria County (the real names have been changed to protect the area). Three towns are in the region--Riverton is a major distribution center and lies in the rich agricultural belt irrigated from River Running. Average annual rainfall is 12 to 14 inches, and when not irrigated it supports shortgrass prairie and has been traditionally a ranching area (largely heavily grazed). Riverton's business leaders have been promoting industry in the region and have held out promises of development of a subterranean aquifer to supply water needs for industry and irrigation. They have met with resistance from local ranchers who, although they are making a marginal living, swear by their life style. It is expected that if the new irrigation project is initiated the tillable land range could be extended as far as Plane. Friction between business leaders, farmers and ranchers, as well as a concern over how growth in the area should be planned has resulted in the formation of a regional planning commission composed largely of business leaders, a few ranchers, and several professors from Riverton Community College.

As an initial structuring of the above problem, the commission desires a decision-making process which will tell how to allocate the land available in this area to combinations of various land use categories. For purposes of simplification, the following three land use categories were chosen: (i) urban, (ii) irrigated forage and food crop, and (iii) range.

Of course, there are technological limitations on how much of the total land area may be put into any one of these categories. For example, the problem statement specifies that the irrigated area could extend only as far as Plane, which implies that approximately 40% of the area at the maximum could be put into irrigation.

We might look at the planning commission as trying to give that land allocation decision which would best improve the *quality of life* of the individuals living in this area. With this in mind, the allocation system would best try to satisfy the community needs of as many groups living in this area as possible and to balance off conflicting interests between the ranchers, farmers, and businessmen in the region. To decide what the quality of life means to the community, one approach is to try to get variables or indicators which relate to the quality of life. These might be conceptualized as ecological variables, economic variables, political variables, and socio-cultural variables. The optimization strategy would require the effects on the variables in each of these sectors of putting various amounts of acreage into the various land uses. For example, the effect of increasing urban acreage on such indicators of quality of life as population density or pollution must be estimated. The land allocation system would have to balance off such effects as the increase in population density resulting from increase in urban acreage (which might be seen as desirable up to a point for quality of life), against the increasing pollution resulting from the same urban acreage increase. Furthermore, acreage increases in one land use must be reflected by acreage reduction in other land uses with their concomitant effects on quality of life indicators.

Such a decision fits in fairly naturally to optimization (operations research) techniques. As stated earlier, these are quantitative techniques for getting the best for the least, subject to systems constraints. The "best" could be looked at as the maximum quality of life for the community over *all* the land use areas. The systems constraints would be constraints of the type that the population density has to be less than or equal to a certain maximum allowable population density or that the per capita income has to be greater than or equal to a certain acceptable minimum. The quality of life indicators then have to be translated into an objective function. In this case it is to maximize the total *value* of the quality of life indicators to the community.

It may not be clear at this point just how these objectives can be mathematically stated, or these values put into a standard form (i.e., the values of different types of variables standardized to the same scale). The next section will discuss some of these techniques and some of the applications of these techniques which should give a better feeling for just how the objectives of the community and the physical constraints may be put into the various operations research technique frameworks. The Riverton problem will also be developed as an optimization problem in greater detail in a later section.

6.1 Possible Optimization Techniques in a Land-Allocation Problem

We may utilize operations research techniques to offer optimization decisions to various problems arising with respect to the Riverton problem.

The programming techniques (*linear*, *nonlinear*, and *stochastic*) might be utilized to give a decision on the land-use allocation to be practiced in order to maximize the quality of life to the community over some time period of interest subject to the physical constraints presented in the problem statement. In the nonlinear programming approach the variables which are used as quality of life indicators are expressed as a function (generally nonlinear) of the acreage put to each land use. This then expresses the *impact* of land-use allocation on our objective function which is to maximize the *quality of life* over the time period of interest.

A linear programming approach would be a simplification of the nonlinear programming problem. This would result in a loss of realism, but would allow a much simpler solution to give an initial "ball-park" estimate of the type of allocation desired. A *dynamic* programming problem involving all the intricacy of the Riverton problem would likely be unsolvable. However, it would have the advantage of including time as an explicit variable. This might be important since the static programming techniques (linear, nonlinear, and stochastic programming) would allow all the land-allocation decisions to be made at only one time, whereas the dynamic programming technique would allow for land-allocation decisions to be made at distinct stages throughout time.

As a compromise to these two techniques, either the *simulation-optimization* approach or a *simulation gaming* approach might be used to improve on the decision process in the static programming approaches without involving the solution complexity of the dynamic programming approach. In the SIM-OPT approach care would have to be exercised to make sure that the optimization objective function had some look-ahead elements so that all the resources are not utilized in one fell swoop to satisfy a short-sighted objective function. The simulation gaming approach, while not an optimization approach per se, would allow the decision maker to start with the best decision from the static optimization problem and then try to improve on it by trying variations on the theme, mostly by approaches which change land allocation over time rather than all at one time.

Stochastic programming, while it incorporates the fact that the land-allocation decisions are made under uncertainty about future developments in the area, adds an order of magnitude to the solution complexity. It might replace the *linear programming* problem to give an idea of the sensitivity of the decisions to uncertainty about the future. In order to linearize some of the complex functions in the objective function, *quasilinearization* techniques might be utilized.

Calculus of variations is another optimization technique which allows for the possibility of including time as an explicit variable. However, it shares with dynamic programming the problem of solution complexity in the case of a large size problem.

PERT, CPM, and queueing theory, while not aiding in land-allocation decisions, can be utilized in implementing the decisions. PERT and CPM might be used, for example, in how best to undertake a large-scale irrigation project if this were decided upon. Queueing theory would be used to provide adequate service facilities, especially in the cities. In line with optimizing quality of life, queueing theory might be utilized to provide convenience of service for as many people as possible. This would be involved in transportation, hospitals, communications, etc.

Game theory might be used to resolve some of the political unrest in the area. By expressing the various political strategies of the "antagonists" in the Riverton area, the "political payoff" might be evaluated, and various land allocation strategies could be compared from their standpoint of political reality. Game theory might provide the avenue for evaluating the political expediency of land-use systems chosen to optimize quality of life or some such esoteric measure. A neglect of the game theory aspect of the political arena would make a land-use allocation system idealistic at best. *Game theory* might be combined with a *simulation game* where the payoff of a combination of strategies is the value of the objective function resulting from a run of the simulation with the combination of game strategies introduced as management 'knobs' into the simulation.

6.2 A Formulation Using Nonlinear Programming

As an example of the types of considerations necessary in formulating a planning problem in an operations-research framework, let us formulate the Riverton land-allocation problem as a nonlinear programming problem. First, let us give some notation.

Let A_i = the number of acres put to land-use i, where i = 1 for urban use; i = 2 for irrigated forage or farm crop use; and i = 3 for rangeland use.

Let D_i = the dollar investment per acre by the *public* sector in land-use i. This is *not* a variable, but is considered a constant for purposes of problem simplification.

V_{ik} = the *value* per acre of quality of life indicator k in land-use type i over m years after land "reapportionment."

These are the economic, political, ecological, and sociocultural measures of quality of life that have been standardized to the same "value units." They will be a function of the number of acres put to land-use type i (A_i). This function represents the impact of land allocation on the variables which in turn are related in the objective function to the value of the quality of life.

Let the total acreage available be A acres.

Let C_i = the value in value units of having a dollar investment of D_i in land-use type i.

Let R = discount rate on dollar investment D_i.

In our nonlinear programming problem our objective function is to maximize *quality of life* over the m year time period after the land-allocation decision is made. This may be expressed mathematically:

$$\text{maximize} \sum_{k=1}^{19} \sum_{i=1}^{3} A_i V_{ik} - \sum_{i=1}^{3} A_i C_i D_i \left(\frac{1}{1+R}\right)^m \qquad (6.2.1)$$

The 19 values considered are the following:

Economic values.

i. income per capita = $V_{i,1}$

ii. employment stability = $V_{i,2}$

iii. net regional product change = $V_{i,3}$

iv. income distribution = $V_{i,4}$

Ecological variables.

i. ecological degradation (impact of man on nature) = $V_{i,5}$

ii. environmental quality index = $V_{i,6}$

iii. percentage use of renewable resources = $V_{i,7}$

iv. percentage annual use of nonrenewable resources = $V_{i,8}$

v. man-initiated energy consumption (system stability index) = $V_{i,9}$

Sociocultural variables.

i. population size = $V_{i,10}$

ii. social differentiation = $V_{i,11}$

iii. cultural heterogeneity = $V_{i,12}$

iv. social psychological (social solidarity) = $V_{i,13}$

v. information gap = $V_{i,14}$

Political variables.

i. scope of government services = $V_{i,15}$

ii. uses of government services = $V_{i,16}$

iii. political participation = $V_{i,17}$

iv. property tax base = $V_{i,18}$

v. political power advantage = $V_{i,19}$

These variables will be defined and described in a later section of this report. The dollar investment per acre in the public sector, D_i while considered a constant in this model, could be included as an economic variable and would then become another V_{ik}.

Of course, more variables could be considered to make the problem more realistic, but let us initially work with the variables chosen. They represent the educated guesses of individuals involved in the optimization workshops and (as will be seen) should not be taken as definitive.

We now come to the constraints posed on the optimization problem by the physical realities of the problem as well as some of the planning necessities.

$\sum_{i=1}^{3} A_i \leq A$ (Assures that no more than the total acreage is utilized in some land use) (6.2.2)

$A_1 + A_2 \leq .4A$ (Notes that no more than 40% of the acreage can be put to intensive water use, i.e., industry, urban, and agriculture; this is due to the irrigation not extending beyond Plane in the problem statement) (6.2.3)

All the values were constrained to be in "value-units" between +10 and -10. Sometimes the curves of some variables dipped below -10 (because having the variables reach that quantity was considered "unacceptable for planning." This is expressed as the constraint:

$V_{i,k} \geq -10$ for each i and k (6.2.4)

This assures that none of the variables ever reach "unacceptable" quantities.

This represents the general mathematical structuring of the Riverton problem as a nonlinear programming problem. We must now concern ourselves with the following problems:

i. How large is D_i in each land-use area and how will this be allocated (on the average)?

ii. How do we standardize the variables to the same units (value units) to put them into the objective function of the nonlinear programming (NLP) problem?

iii. What is the impact of land allocation on each of the variables? The variables must be expressed as a function of A_i (e.g., how is the population density in the city, $V_{1,10}$, affected by acreage of urban development A_1), and a set of curves of this sort must be developed for each variable.

iv. What is the political milieu of Riverton?

With the dependent variables ($V_{i,k}$'s) related to the independent variables (A_i's) and standardized to value units, a nonlinear programming algorithm may be applied to try and find the "optimal" land-allocation system.

Remember, however, a solution is not assured in all nonlinear programming problems. Significant simplification and alteration of the functions might be necessary before a solution is obtained. It may be necessary to either linearize the problem or simplify the number of variables to make the problem tractable.

With this in mind, let us discuss each of the other problems (*i* through *iv*) discussed above.

6.3 Dollar Investment D_i

In talking about dollar investment we are referring to dollar investment by the public sector. Expenditures in almost all areas are made mainly on education, police protection, highways, sanitation facilities, and fire protection. Also, payment is included on loans outstanding. Since these expenditures are paid for by taxes (local, state, and federal) and since the ratios of expenditures to taxes in the western U.S. are more or less the same from region to region (not changing significantly in a given direction from range to agriculture to urban areas in the West), it seems meaningless to include the effect of tax expenditure as significantly different from one land-use type to another.

D_i might more meaningfully be considered the cost of the loans taken out by the area to finance:

i. the irrigation projects in the agricultural area, and

ii. sanitation, power, and water development for industry in the new urban area.

These are discounted over the m years of the problem at interest rate R as seen in equation (6.2.1). This is what was done with dollar expenditure D_i in equation (6.2.1).

The following assumptions were made about these D_i's:

i. To have agricultural land we must have irrigation. The dollar cost D_2 per acre of irrigation is a function of the number of acres put to irrigation A_2.

ii. To have more urban land we must have power, sanitation, and water development. The dollar cost per acre of this development D_1 is a constant. Increased urban areas will be assumed to attract industry and increase *job stability*.

iii. Range expenditures D_3 will be assumed negligible.

6.4 The Political Milieu in Riverton

Riverton has a council-manager form of government as contrasted to the mayor-council or commission form of government. In the council-manager form, the city council hires a professionally trained city manager who is charged with running the daily activities of the city government. The manager is usually a very important political actor as well as administrator. Most often, due to the manager's training and experience, he is able to propose policies for adoption (or rejection) by the council, and in effect, decides for the council and the community the direction to be taken. While the council has the final say-so in city decisions, the manager probably controls the agenda and thus, the range of alternatives.

Especially with regard to the issue of whether or not to irrigate, the manager would probably play a critical role. Due to his expertise and the expertise of either the city planning office or a private consulting firm, the manager would know more about the costs, effects, and so on of the irrigation plans than the city council members or leading citizens. In all likelihood the business leaders who were promoting the irrigation plans would be working with the manager, in some capacity, to advance the irrigation of the Riverton area.

Opposition to the irrigation of the area, of course, is likely to come from the ranchers, as well as a generally conservative populace. Indeed, even the promoters of irrigation are likely to search long and hard for a plan that will not go "too far" in changing Riverton. In communities between 10,000 and 50,000 population the populace is generally less tolerant of new-comers and "different political ideas" and less likely to want the growth of governmental services than citizens in larger communities (Key 1961). The white middle-class conservative views dominate in Riverton to the extent that even blue-collar workers in Riverton vote and hold opinions more closely to that of their white-collar friends than their blue-collar counterparts in the metropolitan areas.

While conservative in many ways, the populace of Riverton probably has a strong sense of political community and need to fulfill their civic obligations. Voter turnout is usually fairly high *vis-á-vis* other similar sized cities and is usually around 40%.

Other types of political participation, such as campaigning, and especially talking to political representatives and others, are also likely to be fairly high. Assuming Riverton has an old population, that is fairly well educated, and a reasonable percentage of professional and skilled laborers, then political participation will be quite high.

Riverton governmental services probably reflect this mixture of conservatism and high participation by demanding "efficiently" operated "good" government. The services provided are likely to be the common ones, e.g., police, fire, sanitation, streets and roads, water and sewerage, and maybe some public health activities. In addition, Riverton citizens would probably provide for and heavily use library facilities and parks and other recreational and cultural programs. Indeed the latter, in the form of well-kept city facilities, may be a source of community pride. The welfare and health needs of the poor Riverton citizens are probably handled by private institutions, like the church and civic organizations and/or the national and state governments. In sum, the allocation of governmental services is likely to be in maintaining the physical parts of the city and in providing cultural and recreational programs but not in supporting the public welfare and health areas.

The educational system of Riverton is probably independent of the city government. That is, the school officials are elected to their positions separately from the city officials, and the school officials are independent of the city government in deciding on educational policies and setting tax levies for the school system. Educational systems are also a source of pride in most small towns to the extent that changes, especially consolidation of school systems, are strongly opposed.

An increasingly popular way to summarize the governmental functions of local areas is the following scheme given in Table 6.4.1.

Given the expected conservative nature of Riverton and its small size, one would not expect all of these programs to be performed in the area; and indeed, even if all were in operation, one would expect the nongovernmental groups in the area to perform many of these programs. For example, the docks for water recreation on River Running would probably be owned and operated by private firms. However, given the kinds of changes that could be expected from irrigation of the area, one could see how the government would be expected to expand the number of services and increase the intensity of existing programs to meet the demand.

6.5 Variable Standardization in the Riverton Problem

In the following section the variables ($V_{i,1}$ to $V_{i,19}$) are discussed from the standpoint of how they are standardized as quality of life indicators. The units chosen were "value units" ranging on a scale from +10 to -10. There are several reasons why value units have been chosen instead of dollars--the most commonly used unit in mathematical programming techniques.

Table 6.4.1. Governmental services provided in Riverton area (from George Washington Univ. 1967, PPB Note 3).

Governmental Function	Programs
Personal Safety	Police, fire, jails, traffic control, civil defense, court system
Health	Boards of health, inspection services, ambulance services, hospital, custodial or nursing homes
Education	Primary and secondary educ., vocational educ., special educ., evening and part-time educ., post-high school educ.
Satisfactory home and community environment	Sewers, utilities, waste collection and disposal, pollution control, planning and zoning
Economic satisfaction and satisfactory work opportunities	Welfare, land improvement, urban renewal, public housing, civil rights, industrial development
Leisure-time opportunities	
Physical recreation	Swimming pools, parks, playgrounds, golf courses, water areas
Cultural recreation	Libraries, orchestras and bands, art galleries, cultural and scientific facilities, zoo
Transportation	Roads, bridges, parking, airports, river transportation facilities

These are listed below:

i. Dollars are not linear. With regard to quality of life \$100K may be worth more than twice \$50K if what you need costs \$75K.

ii. Dollars circumvert the value problem illegally. We all can convert our values into dollars, but then we really have to standardize our dollars to each other which gets back to the original value problem. Thus, although we think we are standardized when we talk about dollars, we really are not.

iii. Many variables do not translate into dollar units, e.g., social solidarity. We can give value units to this variable, but dollar units are meaningless and offensive.

Although the value standardizations we have chosen are arbitrary and would likely have to be reevaluated and modified for application to a specific problem, they do represent a step toward trying to standardize ecological, political, economic, and sociological variables to "quality of life," and these variables should be considered in planning decisions.

6.5.1 Ecological variables. There are five ecological-related variables which are considered in relation to the Riverton problem. These are:

i. ecological degradation (impact of man on nature),

ii. environmental quality index,

iii. percentage use of renewable resources,

iv. percentage utilization of nonrenewable resources, and

v. man-initiated energy consumption (a measure of system stability).

Each of these variables is represented by an indicator which is seen as a measure of the variable. This is done since a true measure of the variables above would be extremely complex and would likely be impossible to obtain.

Ecological degradation-$V_{i,5}$. This is a measure of man's impact on the natural environment. It is indexed by the pollution, erosion (or range quality on the range), and biotic diversity.

Mathematically we might say that the degradation

$= a_1$ (pollution level) + a_2 (erosion level) + a_3 (biotic diversity).

Since three separate indicators are used, we divided each indicator into three levels, 1, 2, and 3:

For pollution	1 = low	2 = moderate	3 = heavy
For erosion	1 = light	2 = moderate	3 = severe
For diversity	1 = high	2 = medium	3 = low

We can substitute range condition for erosion on the range area since they are usually inversely proportional.

Since not all indicators apply in all land-use regions, in the urban area let $a_2 = a_3 = 0$ and in the range area let $a_1 = 0$, i.e., pollution is relatively unimportant on the range. Erosion and biotic diversity are unimportant in the city. Urban pollution refers to air, water, and noise. Irrigated land pollution is mainly pesticides and superfluous water nutrients (eutrophic water systems).

For each of these indicators the levels (i.e., 1, 2, or 3) will have to be defined.

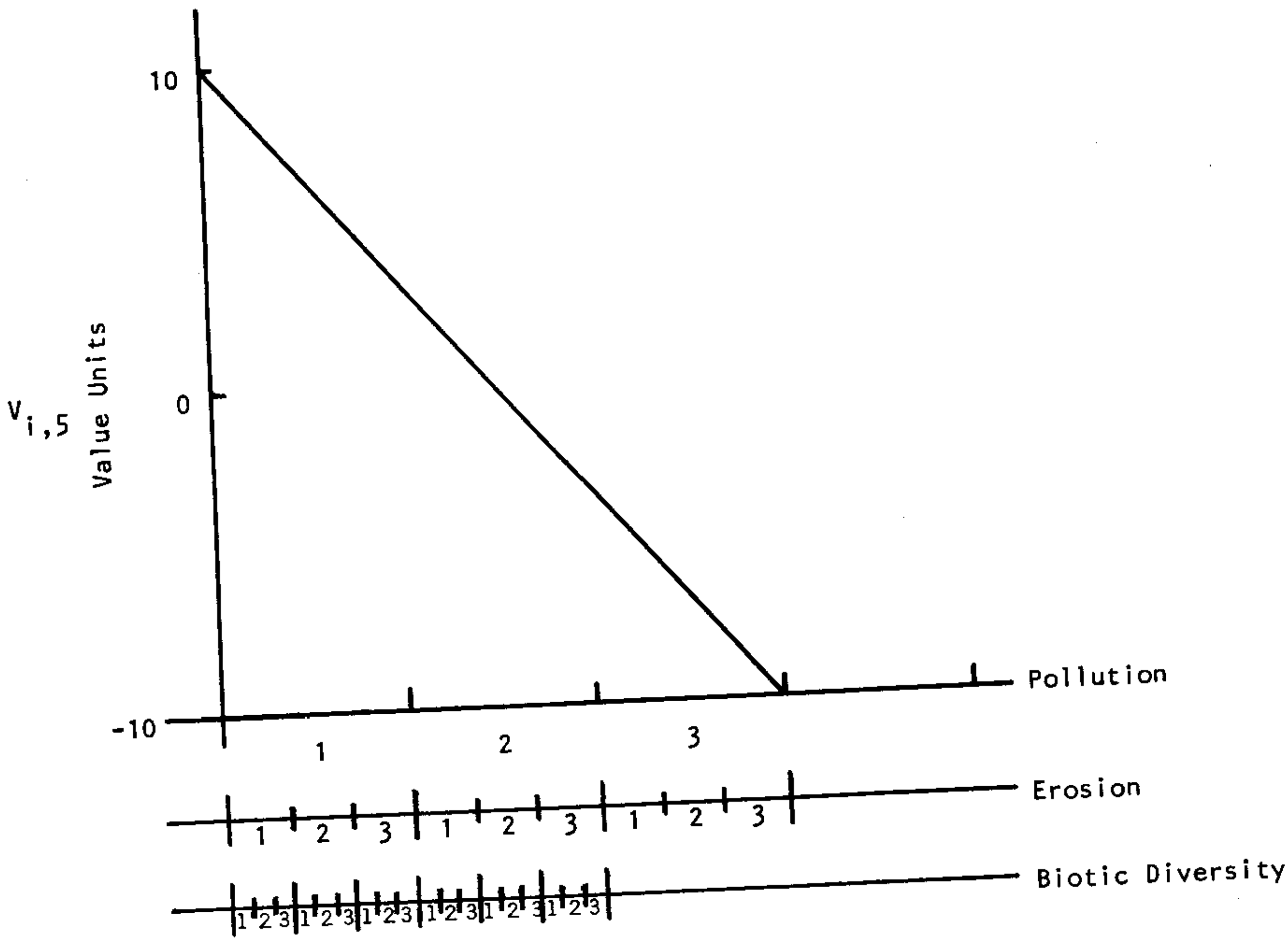


Fig. 6.5.1. Standardization of ecological degradation - $V_{i,5}$.

Now let us interpret this curve. Consider first the urban area. Since erosion and biotic diversity are not important we have only three levels. We take the average reading for each level (e.g., high pollution = -6.7 value units, low pollution = +6.7 value units). In the range area (a_1 = 0) we use the *average* (2) pollution value. Thus, our range is cut from +3.5 to -3.5. For the irrigated area the value of any combination of pollution level, erosion level, and biotic diversity level may be read from the figure. Here, lack of degradation is seen as very desirable, while high degradation is seen as extremely undesirable (-10 in the extreme, i.e., on irrigated land).

Environmental quality. The indicator used to define this is the macro-diversity of the environment. In the urban area this is the differences of housing styles, public buildings, parks, recreation areas, etc. In irrigated land it is the differences in farm sizes, crop diversity, existences of fence rows, streams, woodlots, etc. On the range it is the differences in terrain, developed ponds, etc.

In each area we use the information statistic to give the macro-diversity of the environment. This is $-\Sigma p_i \log_n p_i$ where the p_i's are the proportion of the total land-use area put into the various subcategories mentioned above.

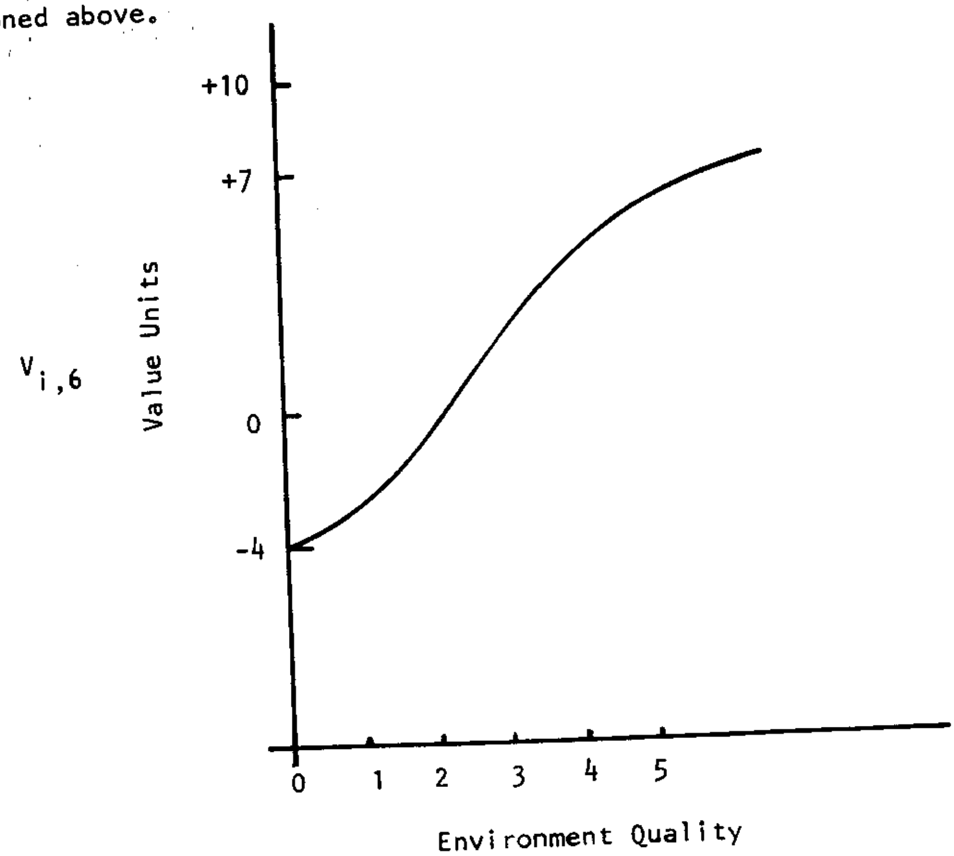


Fig. 6.5.2. Standardization of environmental quality - $V_{i,6}$.

High diversity is quite desirable (+7). Low diversity, while undesirable, is not *very* undesirable (-4 at the minimum). It is postulated that a high diversity is esthetically desirable and therefore raises quality of life.

The choice of which subcategories are to be included as indicators of the diversity of the environment affects the value of the diversity index resulting from the computation of the environmental macrodiversity. For example, the diversity in an industrial city with many small industries all employing workers in the same cultural and income categories would be larger than a very similar city with only one factory in it, if in the choice of subcategories for environmental diversity the distinction is made between what is produced in the various factories. For this reason, great care must be taken in choosing indicators of system diversity which are concomitant with the hypothesis that high diversity is esthetically desirable.

Percentage use of renewable resources. This is indicated by the percentage utilization relative to the maximum feasible use to maintain the resource as renewable. Thus, in forage we want to compare the amount being utilized relative to the potential utilization at "maximum sustained yield." Since there may be more than one renewable resource policy, the average percent utilization is used here.

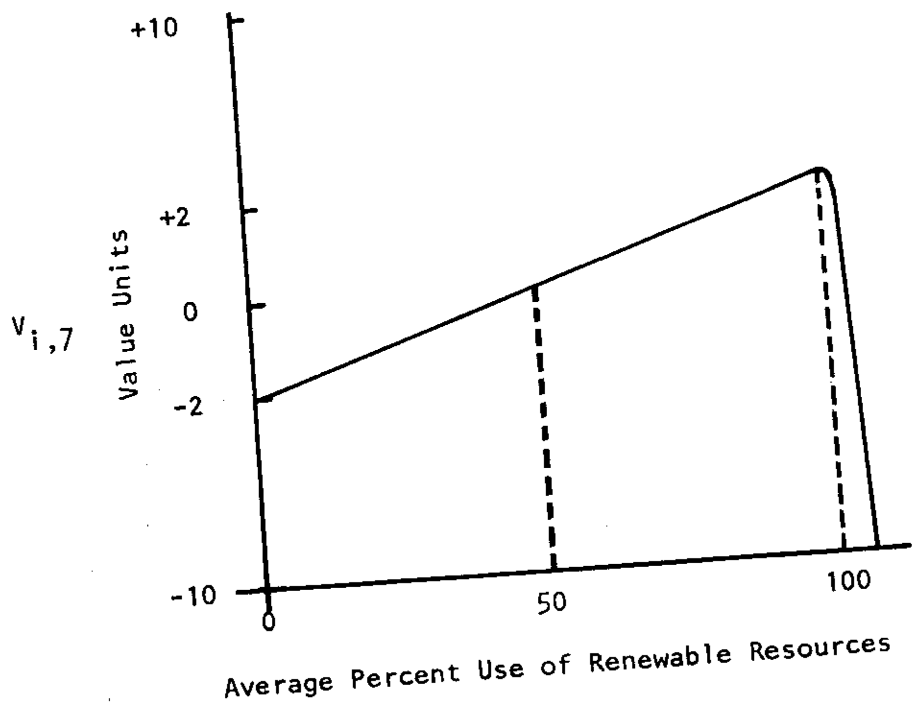


Fig. 6.5.3. Standardization of renewable resources use - $V_{i,7}$.

Within the range 0 to 100% utilization, increasing utilization is seen as having value. A 50% utilization is the "breakeven" point. Above this 50% utilization value is positive, and below it is negative. Above 100% utilization the resource is no longer renewable, which is seen as extremely undesirable.

Percentage use of nonrenewable resources-$V_{i,8}$. Oil, gas, nonrenewable aquifer, minerals, etc., will be defined in terms of the average percent annual use of the remaining nonrenewable resources. Beyond .5% annual use value drops; it is positive below about 1.5%. Utilization above 5% will not be tolerated. Below .5% value also drops, since it is assumed that man has become in some sense dependent upon his nonrenewable resources and must continue some harvesting to maintain his system from breakeven (see energy considerations variable). A high percent utilization is seen as extremely negative in value.

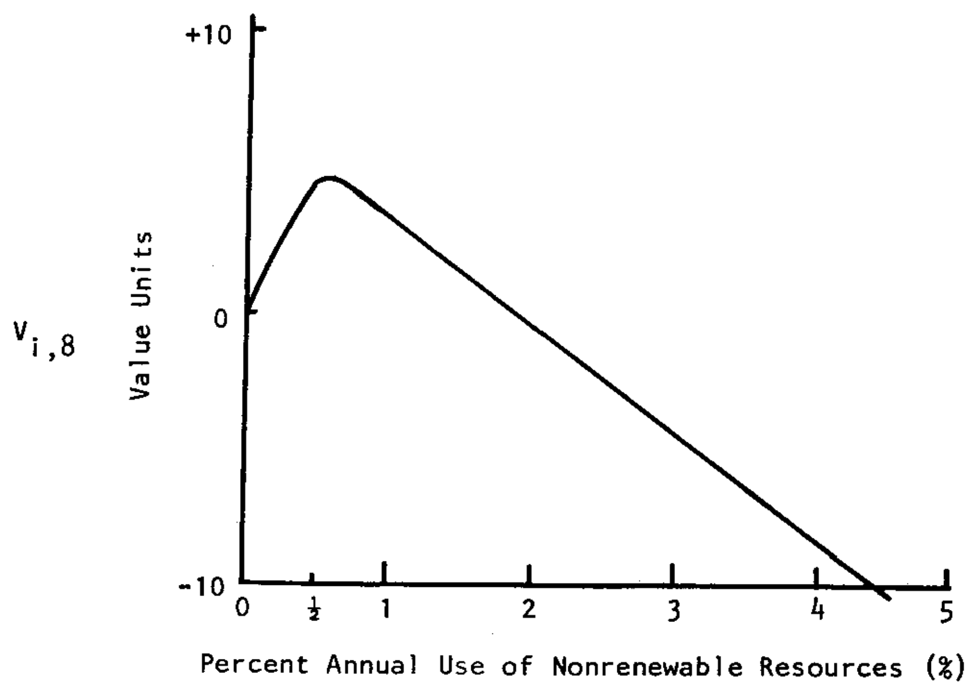


Fig. 6.5.4. Standardization of nonrenewable resource use - $V_{i,8}$.

System stability potential--man-controlled energy consumption-$V_{i,9}$. We consider the ratio of man-controlled energy inputs to solar input utilization an important ecological indicator (Odum 1971). The larger the ratio, the more man must *depend* on his energy input to maintain stability (fossil fuels are his major source of exogenous energy input). For example, in a city stability is extremely dependent upon machines which run on fossil fuels. Loss of these fuels is a source of potential system instability.

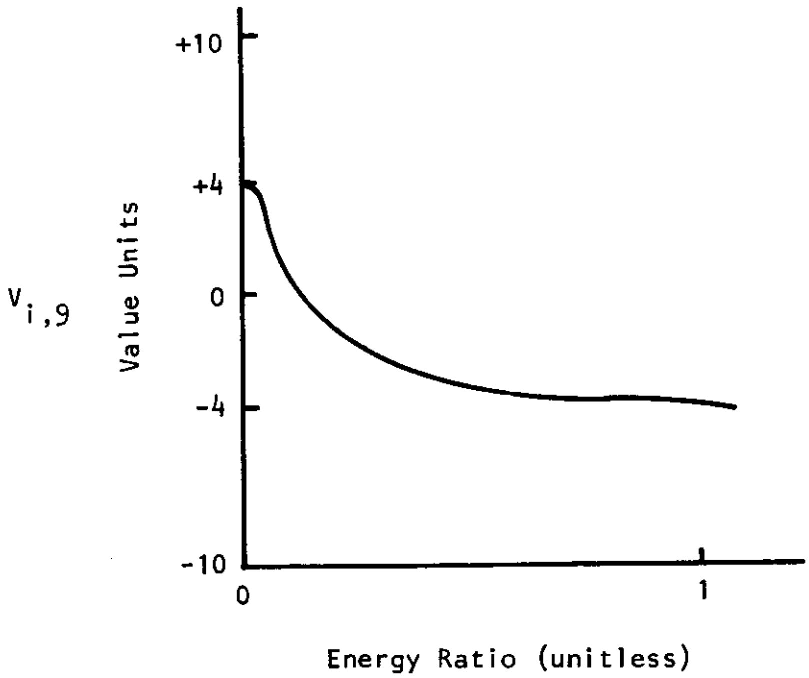


Fig. 6.5.5. Standardization of man-controlled energy consumption variable - $V_{i,9}$.

The range is +4 to -4. For very small ratios the value is positive; for larger ratios the value is negative. The value drops off rapidly (exponentially) with increase in energy ratio, but the rate of value drop-off rapidly diminishes and stabilizes at -4. We might say that above a certain ratio man is "hooked," to his own detriment, on fossil fuels.

6.5.2 Sociocultural value standardization curves. The five values that were chosen from the sociocultural area are:

i. population size,

ii. social differentiation,

iii. cultural heterogeneity,

iv. social psychological, and

v. information gap.

Also considered was *social stratification*, which will be equated with the economic variable *income distribution* and will be treated in the economic variables section.

For each of the value standardization curves the units for the factors on the x axis (abcissa) are defined. The y axis (ordinate) is always standardized to value units (v.u.) with a total maximum range of +10 to -10. Each curve also includes an explanation of the reasoning behind it.

Population size-$V_{i,10}$. The units equal population per square mile.

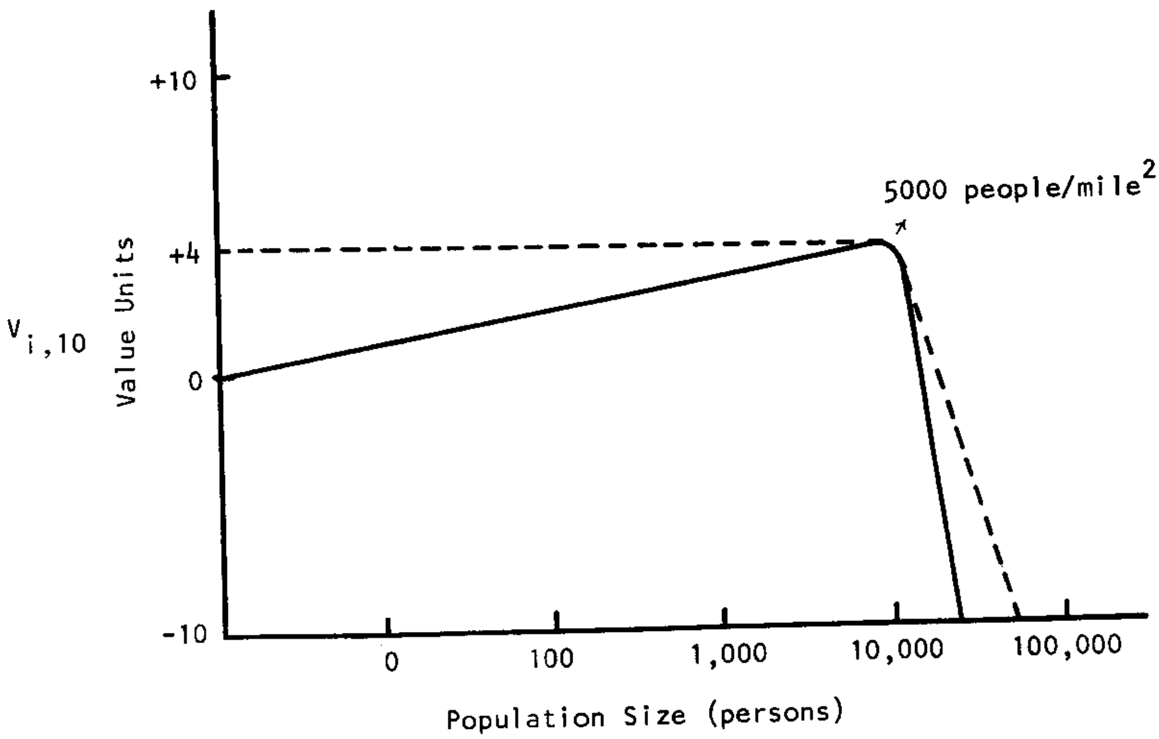


Fig. 6.5.6. Standardization of regional population size variable - $V_{i,10}$.

For low population densities, value is low since resources cannot be properly utilized. At very high densities conflict and aggression develop. This is seen as of extreme negative value (-10 at about 20,000 people/mile^2). This curve is for urban populations. For rural populations the optimal density is much lower--about 60 to 100/mile^2. Also, instituting irrigated agriculture into range area will affect urban population densities which increase to "support" the agricultural sector.

Social differentiation-$V_{i,11}$. The degree of social differentiation is operationally defined as the number of separate job categories listed for the area. This ranges to the "jack-of-all-trades" tribal society with around 10 to 20 separable jobs to the "mass" industrial society with approximately 40,000+ jobs.

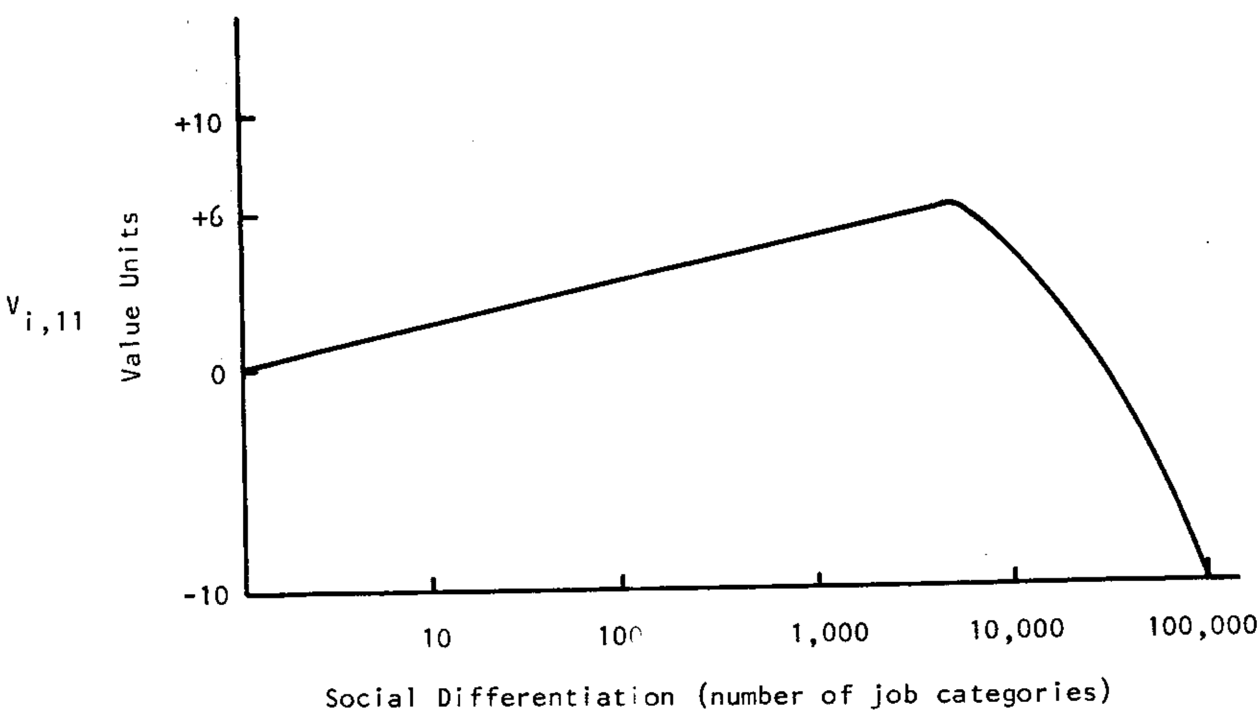


Fig. 6.5.7. Standardization of social differentiation variable - $V_{i,11}$.

With a small social differentiation there are not enough "information resources" to logistically handle social systems of the sort in the U.S. Increase in social differentiation (specialization) is seen as desirable. With too much specialization, however, value drops; people become irreplaceable with no job interchange possible. *Trust* in the interchange of job services *breaks down* as one loses sight of just what the other guy is doing. This leads to unconstrained conflict (seen as negative in value). We must emphasize that conflict per se is an important asset to a social system since it promotes the search for "variety." What is bad is a system without conflict resolution capacities by which "variety" or alternates are selected and introduced. The elimination of conflict or the presence of unconstrained conflict is a survival problem for a variety seeking system in a changing environment.

Cultural heterogeneity-$V_{i,12}$. Cultural heterogeneity, operationally defined as differences in life styles including language, ideology, techniques (child rearing, food preparation, etc.), was initially defined as the number of significantly different life styles in a community.

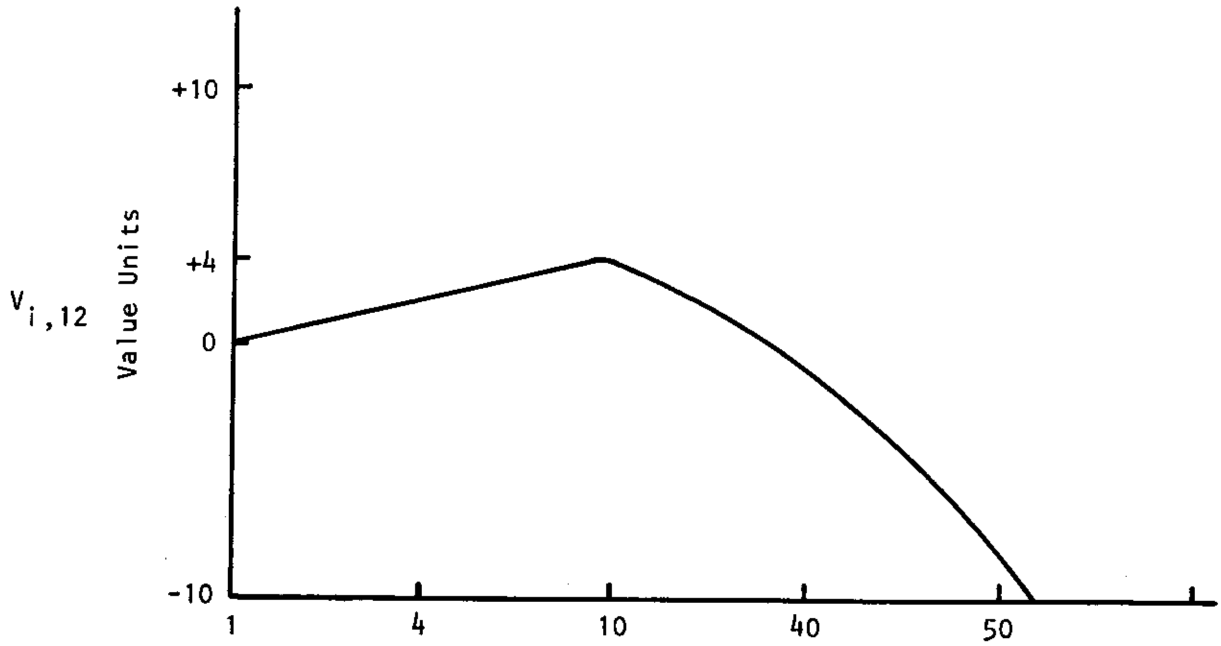


Fig. 6.5.8. Standardization of cultural heterogeneity variable - $V_{i,12}$.

A uniform life style is seen here as having no value (i.e., in a planning situation we would plan neither for nor against it). There is some value in having a good mix of 3 to 10 life styles or so (+4 = maximum value). However, with 40 or more languages, major techniques, idealogies, etc., *conflict* frequently develops. In some systems (e.g., some cities which form ethnic neighborhoods) conflict may not directly develop. Thus, this value curve is dependent on the land use system within which it exists.

Cultural heterogeneity might be alternatively defined as "variety" in the information theory sense of the word. In modern systems theoretical language, only variety can destroy variety. For our purposes here, this translates into the statement that for every change in the environment upon which a given social system depends, there must be an option in the system by which to match that change or the social system ceases to be an irreversible thermodynamic entity and tends to entropy. This is often stated as Ashby's law (principle) of requisite variety. A pluralistic society satisfies Ashby's principle better than an homogeneous culture. There are more options stored in the system.

Sociopsychological variable-$V_{i,13}$. The sociopsychological variable is defined as how we relate to other individuals in our environment. The relation ranges from alienation (a sense of apartness) to solidarity (an ability to work, relate, and recreate with the individuals around us).

We did not get around to putting units on this. It may not be easily quantifiable so let us suggest five categories.

i. high alienation *ii.* low alienation *iii.* wavering *iv.* low solidarity *v.* high solidarity	Measured by suicide rates, crime rates, unemployment rates, and dropout rates; also feelings of powerlessness, estrangement, apathy to the fate of others.

The value scale will be for the *mass society* (range of +3 to -3).

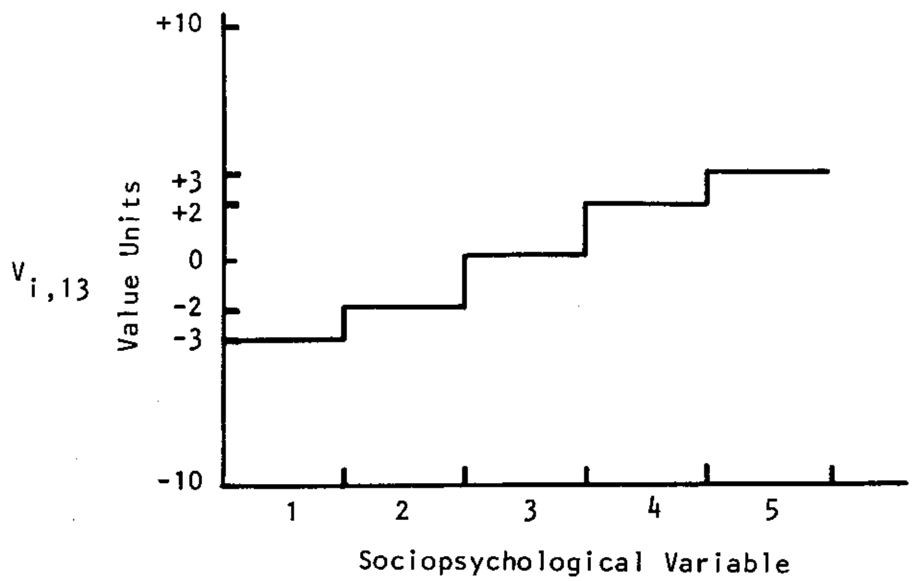


Fig. 6.5.9. Standardization of sociopsychological variable - $V_{i,13}$.

Alienation is seen as bad, solidarity as good. It is not too heavily weighted because mass society does not stress solidarity too much and places emphasis on technological alternatives to personal communication (as, e.g., TV, stereo, etc.).

Information gap-$V_{i,14}$. Information gap is defined as how far behind (or ahead) the *information resource* ability of a society (the ability to report, make decisions, implement, and evaluate them) is compared with its *information needs*. Both information needs and resources are defined in terms of *information half-life*. This is considered the amount of time for half the information to become obsolete. We define the *information gap* as the

$$\frac{\text{information needs half-life} - \text{information resources half-life}}{\text{information needs half-life}}$$

This is positive if the half-life of needs exceed the half-life of resources and negative if the reverse is true. (Note: A short half-life for information needs implies faster circulation of information and a need for a shorter half-life of information resources). Since *information gap* is normalized by information needs, its maximum is +1. We limit it in practical situations to fall between +1 and -5. Thus, information gap is not allowed to fall below -5 (i.e., information resource half-life cannot be more than six times the information need half-life).[4/]

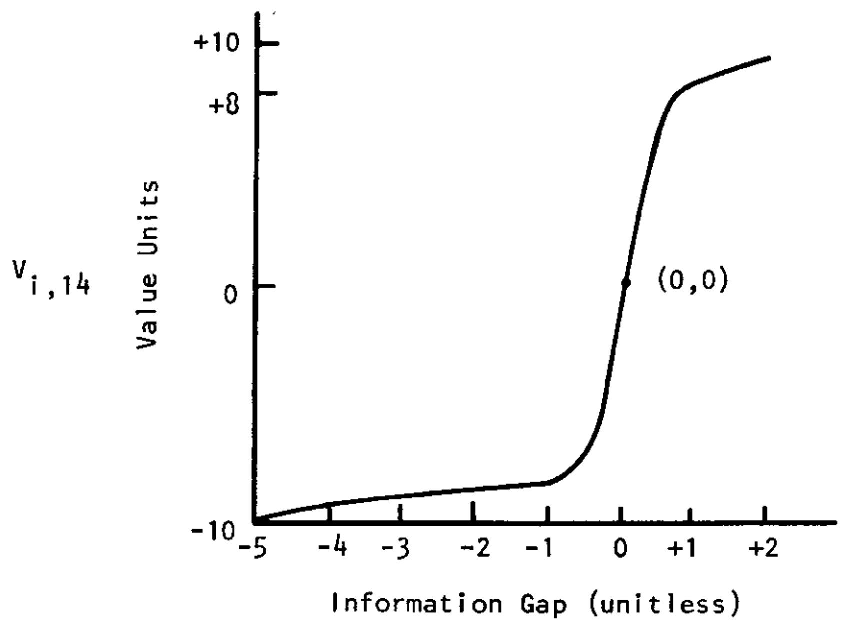


Fig. 6.5.10. Standardization of information gap variable - $V_{i,14}$.

[4/] This discussion of informational half-life and the concept of information gap is entirely original in this report and is the joint product of T. R. Young and Gordon Swartzman.

A premium is put on good information resources relative to information needs (information gap = +1 = value = +10). At 0 information gap the resources are keeping up with the needs, and this is seen as quite desirable (value = +8). Below this the information gap is negative (needs exceed resources); and the system, if continued this way for a long while (relative to information needs), will often self-destruct.

6.5.3 Economic variable standardization curves. There are four variables which have been chosen for consideration from the economic sector.

i. income per capita,

ii. employment stability,

iii. net regional product change, and

iv. income distribution.

Another variable, *income stability*, was omitted because it purportedly can be inferred from *per capita income* and *income distribution.*

Income per capita-$V_{i,1}$. Income per capita is defined as the dollar income per individual.

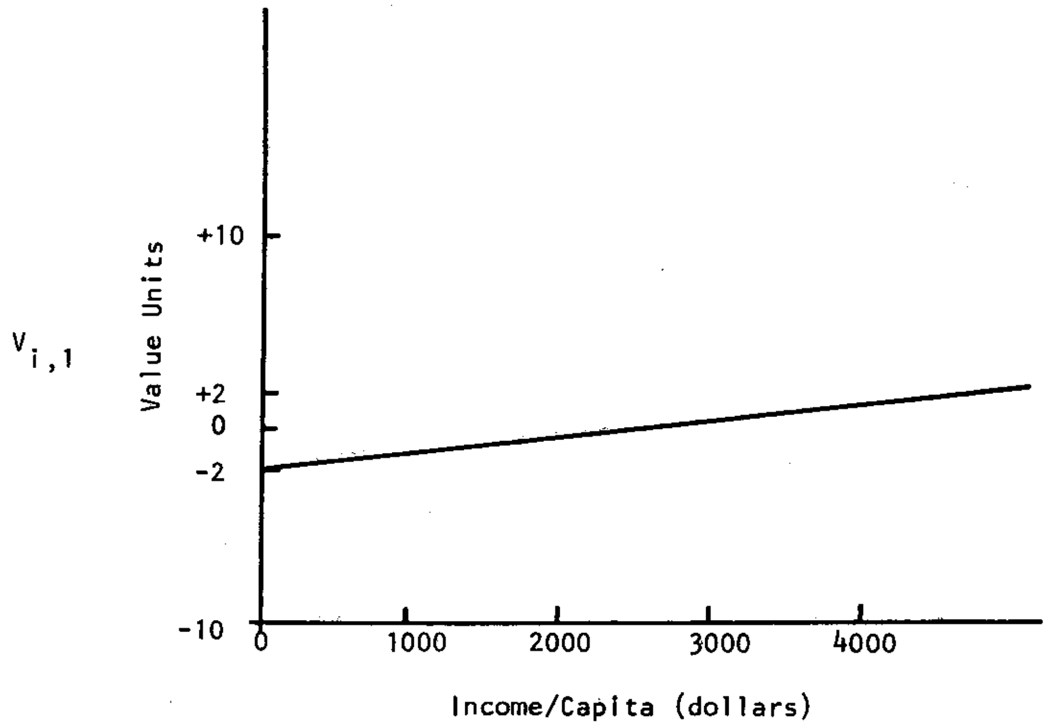


Fig. 6.5.11. Standardization of income per capita variable - $V_{i,1}$.

Here the income per capita value increases linearly with increased income. The 0 value point is the mean per capita income (about $2500) in the U.S. today. Income per capita is seen as relatively unimportant since it may often be misleading as an indicator of life quality.

Employment stability-$V_{i,2}$. This is defined as the percentage of the work force employed.

Fig. 6.5.12. Standardization of employment stability variable - $V_{i,2}$.

The 0 value is 96% employment--the national average. A 98% employment is seen as optimal having a value of +6 value units. Below a 96% employment, the value is negative with no less than 85% employment allowed (i.e., employment stability $\geq$ 85%). Above 98% employment the value drops rapidly since there is no surplus labor force to meet fluctuations in demand.

Net regional product change after m years-$V_{i,3}$. This is defined as the *percentage* change in *net regional product* m years after land uses are allocated.

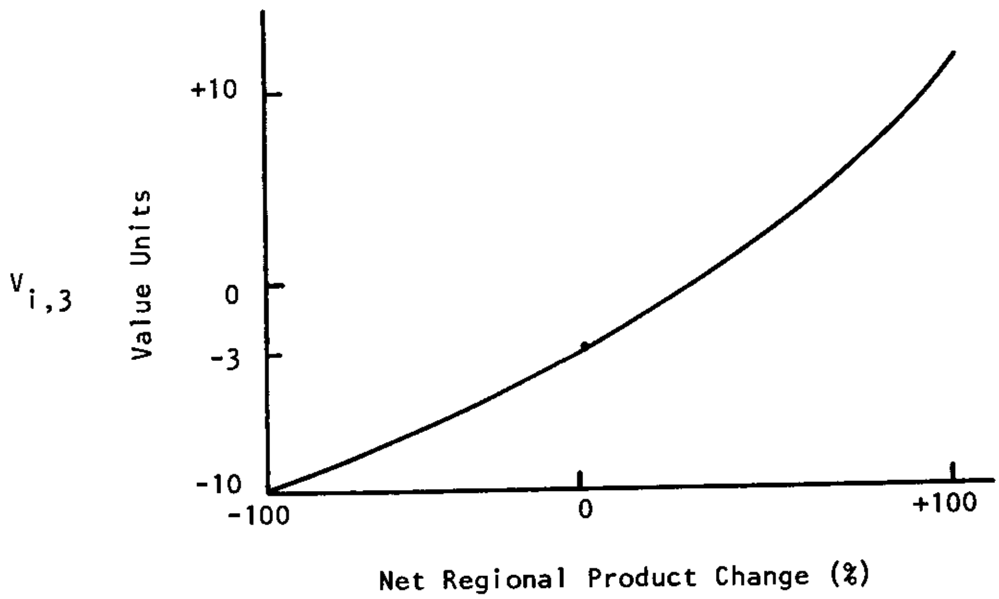


Fig. 6.5.13. Standardization of net regional product change variable - $V_{i,3}$.

Net regional product change is seen as very important as evidenced by the full use of the +10 to -10 scale. A 0% change in net regional product over m years is seen as a negative value (-3). The range +100 to -100% change has been considered because it is a believable maximum range for regional product change over m years (m is not greater than 10 years).

Income distribution-$V_{i,4}$. This is defined as the coefficient of variation of incomes $\left(\frac{\hat{\sigma}}{\hat{\mu}}\right)$.

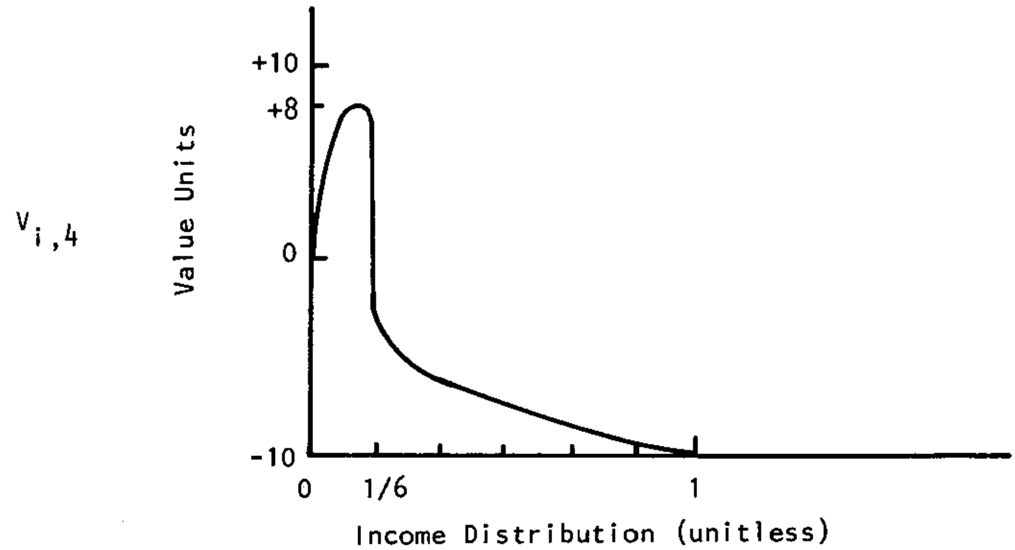


Fig. 6.5.14. Standardization of income distribution variable - $V_{i,4}$.

The larger the coefficient of variation, the more dispersed are the incomes. For example, under an assumption of normal income distribution, a coefficient of variation of 1/4 will result in 95% of the incomes lying between 1/2 and 1 1/2 times the mean income, while a coefficient of variation of 1/6 will result in 95% of the incomes between 2/3 and 4/3 the mean income. The 0 value point is taken as income distribution (coefficient of variation) equal to 1/6. Higher coefficients of variation (more dispersal of income) are worth negative values. Value increases rapidly for less income dispersal than 1/6 and reaches a peak of +8 at coefficient of variable equal to 1/10. For lower coefficients of variation, value drops severely; this would imply too uniform a society.

6.5.4 Standardization of political variables. The political variables in the Riverton problem have been chosen to reflect the political considerations to be made in the optimization framework. These are:

i. scope of government services,

ii. uses of government services,

iii. political participation (a measure of how open and accessible the political system is to individuals and groups),

iv. property tax base, and

v. political power advantage (a measure of the ruling structure).

Scope of government services-$V_{i,15}$. The scope of government services in an area, then, will be measured by the number of services provided to the community by the regional government.

In looking at how the number of services or the scope of government services are related to the other variables, we have standardized them to the same value units to which the other variables have been standardized.

Obviously, the minimal services are going to be performed by any local government. Oftentimes, these few services are all the people want, especially if they are inexpensive (low tax rate) and of fairly high quality, say due to the luck of attracting an ambitious young city manager who wants to do well to be able to progress professionally. Again these attributes of a city often make it very attractive to people.

However, an increase in the number of governmental programs also increases the attractiveness of the area as a place to live. The value of having a wide variety of public services open to all people is often preferable to the private performance of the services. Especially if or when a city is able to provide facilities in the leisure-time category, the attractiveness of the city increases.

Over time the number of services is likely to increase. In large measure this is simply due to more people in an area with a greater variety of needs. Also, one may discern a general trend to make more demands on the public sector even if population size and characteristics change very gradually. Thus, more services would be expected given no significant change, like irrigation; but the impact of irrigation would increase the rate of increased governmental programs.

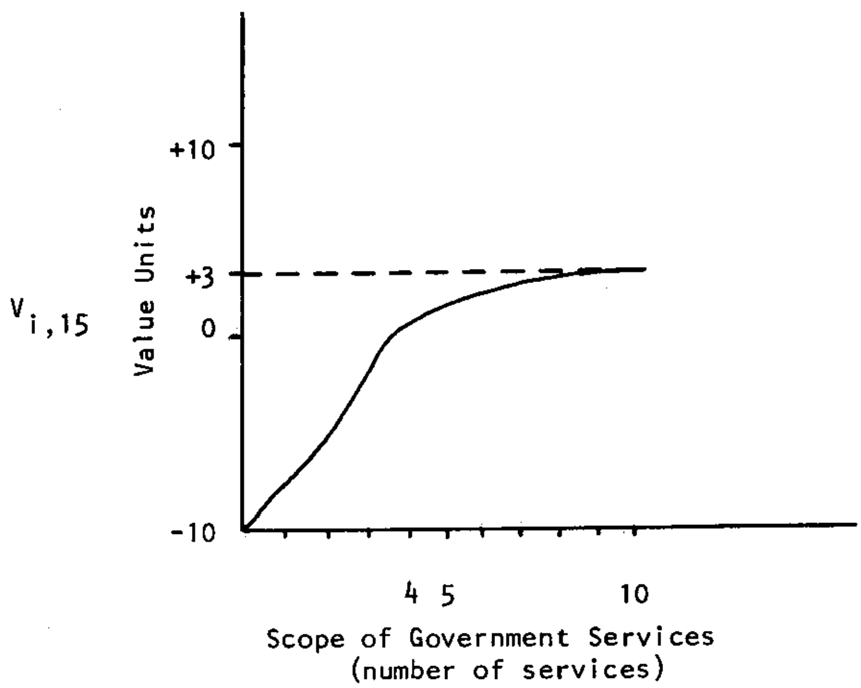


Fig. 6.5.15. Standardization of scope of government services variable - $V_{i,15}$.

The 0 value point is the point at which four services are provided. These are assumed to be education, police protection, sewage and street building, and repair. As the number of government services increases beyond this point, value increases to a maximum of +3. However, if the number of services should drop below this point (i.e., if some of these "essential" services are dropped), then the value drops sharply to a minimum of -10 at zero services provided.

Uses of government services-$V_{i,16}$. By uses of government services we mean how heavily the services provided are utilized. We measure this by the percentage of the population utilizing the services that are provided.

Uses of governmental services are likely to be fairly high in Riverton. One reason is that there are probably not many alternative things to do in a town that small. Also, of course, the public facilities serve as places for social gatherings.

The personal safety and health services, especially inspection of homes for building code violations, are done in a very informal and personal manner. For example, in New York City a disorderly drunk is likely to be poorly treated whereas in Riverton he may be given a ride home by the local police. In meeting home safety requirements inspectors in Riverton seldom hassle Mrs. Murphy even if her boarding house does not meet all of the health standards.

General usage of governmental services then includes both calling for help from, say, the police, as well as having a picnic at the local park. Probably 20 to 30% of the populace has made use of the government for problem solving, e.g., calling the police and using the parks; and, maybe another 30 or 40% have only used the park facilities. The combined usage may be from 50 to 70%. The value curve for this variable is given below.

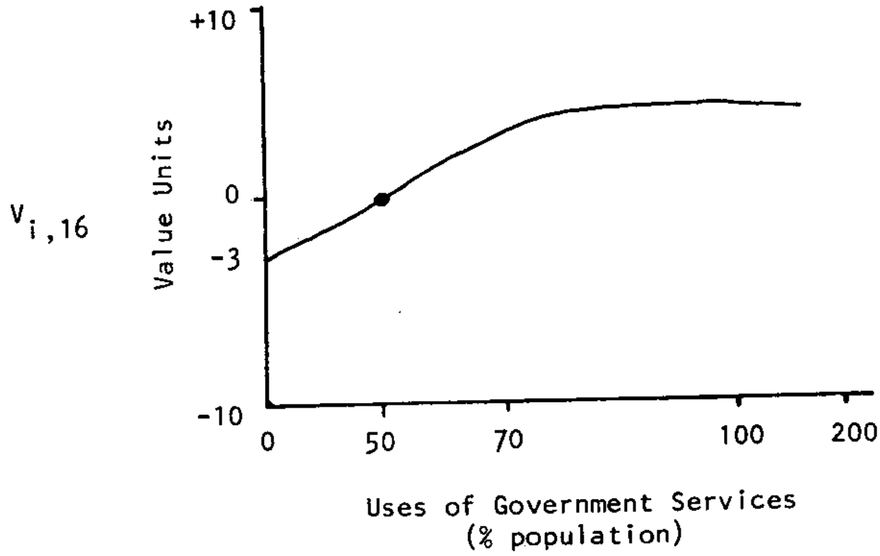


Fig. 6.5.16. Standardization of uses of government services variable - $V_{i,16}$.

An examination of the above figure shows that increased use of government services is of increasing value. The breakeven point, the point at which the use of government services is considered of either positive or negative value, is 50% usage. Below this usage the value is negative with a minimum of -3 value units. Above this usage the value is positive reaching a maximum of +3 at about 70% usage, and then further increase in usage does not increase the value. Apparently, it is extremely rare to find areas where usage of government services extends beyond 70% of the population.

Political participation-$V_{i,17}$. Political participation is measured by the percentage of the voting age population which votes in local elections. A very low voting percentage is seen as a measure of a closed system, where no real political choice is available to the voters, while a high degree of participation is probably indicative of an open political system. While normally high for a town the size of Riverton, due to the manager form of government, the closed elite system and the generally non-tolerant nature of the populace makes it not so here. Poor and uneducated people normally do not like the manager form of government and hence do not participate as much as in other forms for a couple of reasons. First, government is in the hands of an expert, the city manager, and is not run by people like one's neighbor. Secondly, councilmen are elected in at-large elections and may not live in the poorer sections in town. Together there seems to be powerful detriments to poor peoples' participation in politics in small-city manager cities. The impact of the closed elite system and non-tolerant attitudes is fairly obvious: it does not promote much hope in getting one's views heard or respected; even one's vote may be considered meaningless.

The goal of widespread political participation is that of self-realization of one's own goals in society. In the process of political participation one would hope for continued personal self-development as well. Then as one's political efficacy increases, his satisfaction with participation and its results will increase. In actuality a voting rate of 30 to 40% of the adults of voting age in local elections is about par. The figure below shows political participation standardized to value units.

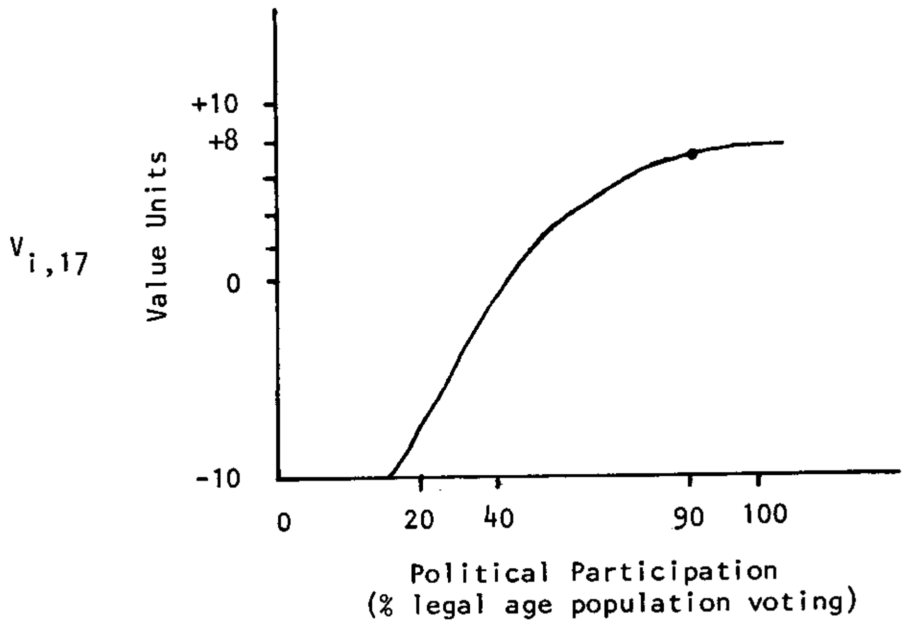


Fig. 6.5.17. Standardization of political participation variable - $V_{i,17}$.

Political participtation is seen as extremely important to the decision maker. The average political participation for a local or regional election is about 40%, and this is the 0 value point. Above this point the value of participation increases quite rapidly until a +8 value is reached at 90% participation after which there is no further increase in value. Below 40% participation the value goes negative and drops extremely rapidly. A -10 value is reached at 20% participation below which an extremely closed system is assumed to exist and such a situation will not be tolerated by the planners.

Property tax base-$V_{i,18}$. The property tax base is defined as the dollar per capita property value in the region. It is a direct measure of the size and type of buildings that exist in the area. In a residential (solely residential) area the property tax base is generally low. It increases as commercial areas are built and is generally highest in the areas which have industrial development. The expected range of property taxes is from $25 to $75 per capita for a city the size of Riverton.

Roughly one would expect the largest tax base from an industrial town, next largest from a commercial trade center town, and the smallest tax base from a largely residential community, like a "bedroom suburb." This is an interesting index due to its consequences on a number of things.

Since property taxes (rather than user charges, and state, sales, and income taxes) still account for the bulk of local revenues, a city and school system needs the tax base to raise money to spend on the increasing demands for services. Oftentimes, the tax base may not change, but the tax rate does increase. To avoid this problem the traditional thing to do was to seek new industry for the town. Unfortunately, few city officials were able to see the consequences. If at first they solved their problem of a declining tax base, they were unable to meet the new needs for local services such as police and fire protection for the industry and its employees. School officials had more kids per classroom with which to deal. Public utilities had new demands for water and sewerage, and so on until the costs of public services again outdistanced the means of raising revenues.

To some extent reliance on property taxes is decreasing while the use of sales taxes and user charges are increasingly popular ways of raising local revenues. In addition, state aid to localities is a larger part of total revenues for localities than in the recent past. Even with the alternative means of raising revenue the cities have had to seek both federal aid as well as to increase their own indebtedness to meet new demands.

The property tax base not only has a limiting impact on what the government may provide, but also it obviously has a large impact on the environment and life styles of Riverton citizens. A heavily industrialized town will have more pollution problems in need of a solution than a commercial or residential town. Thus, there seems to be an inverse relationship between revenue potential and pollution potential.

Life styles can also be affected by the tax base. Assuming one is in a largely residential community that has demands for high quality educational systems, libraries, parks, etc., then the tax rate will be very high. As one has to pay more in taxes, the economic ability to do other things declines. Changing the tax base from a residential to a commercial or industrial mix may reduce property taxes, initially, yet alter the types of people one may find as a neighbor.

The figure below shows the value standardization curve for this variable.

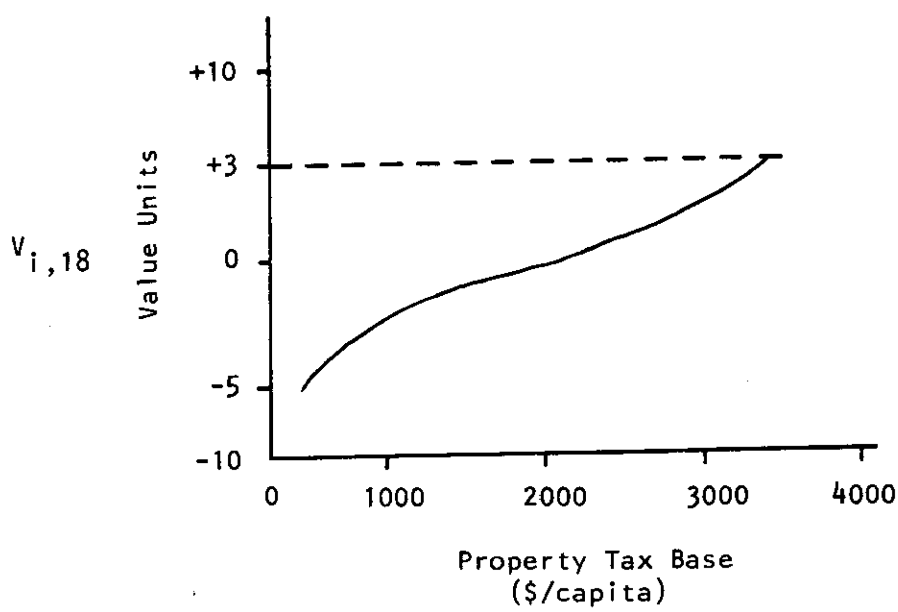


Fig. 6.5.18. Standardization of property tax base variable - $V_{i,18}$.

The average property tax base per capita is \$50, and this is the 0 value point. Both above and below this, between \$30 and \$60, value does not change a great deal. However, above \$60 value rises a bit more sharply to a maximum of +3 value units, and below this range value drops off quite sharply to a minimum of -5 at about \$10 per capita. The curve does not go below this point because per capita property tax base below \$10 is not expected.

Political power advantage-$V_{i,19}$. When we refer to political power advantage we are trying to infer the structure of the ruling body in the community. We can look at the ruling body of various communities as ranging along a continuum between rule by a traditional elite (as for example a king and aristocracy or a political machine) to rule by a competitive elite who are more reflective of a highly participatory system of democracy. Thus, we define political advantage as the percentage of the ruling organization composed of individuals who are the competitive elite category.

It is likely that the people occupying the roles of authorities in the city government, or the school board, the utilities commission, and other public bodies are drawn from the traditionally important families in Riverton. This elite group has run the public institutions for many years and will continue to do so for some time to come. Indeed the bankers, owner of the largest feed store, large farm owners who live in town, and lawyers will occupy most, if not all of these positions. Opposition to these traditional elites at election time is weak or nonexistent and obviously with very little hope of ever winning. Thus, the small store owner, small farmers and ranchers, store clerks, farm and ranch laborers, and others tend to have few, if any, political authorities recruited from their ranks.

The upshot of this is likely to be a nearly self-perpetuating elite that "knows what is best for Riverton" even without consulting with people outside of their own elite group. Of course, as suggested above, this is probably the accepted way of doing things and is unlikely to be strongly opposed unless a change occurs, like irrigation. In one sense of political representation, i.e., an authority standing for the wishes of his constituents, the system tends to be closed. Also, the recruitment to positions of authority is likely to be closed.

On the other end of the spectrum is the open system where a large proportion of the people have a realistic chance of occupying authority roles and standing for the wishes of a large proportion of the populace. Access to the authorities to express demands is greater in the open than in the closed system as is the multiplicity and range of demands. While the closed system is usually not physically repressive of minorities, the South excluded, the open system being more tolerant of a wider variety of people is not likely to be as repressive as is the closed system.

An open political system is preferable to a closed system on a number of counts, chief among which is the probabilities of which variety can be acquired and introduced into the system under changed environmental conditions. A closed political system will not adopt requisite changes inimical to narrow establishment interests of the power elite.

The figure below shows the value standardization curve for political power advantage.

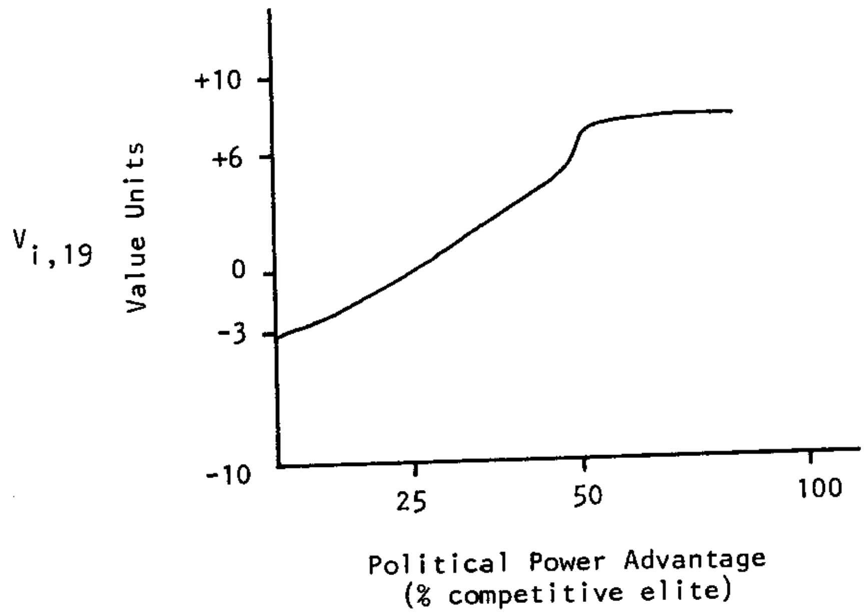


Fig. 6.5.19. Standardization of political power advantage variable - $V_{i,19}$.

It will be noticed that a competitive power situation is considered more desirable than a traditional one. As more than 50% of the ruling body is composed of competitive elite, they can control the ruling situation. Therefore, around 50% competitive elite the value rises quite steeply to a maximum of +6 value units. Since the competitive elite has control of the situation, little further increase in value is evidenced above 50%. The breakeven point in value is 25% competitive elite. This is the point at which the competitive elite can begin to assert themselves in the ruling situation while the traditional elite still maintains control. Below 25% competitive elite value is negative going to a minimum of -3 value units.

6.6 Impact of Decisions Upon Variables

Each of the variables described in the previous section can be related back to the independent variables (the A_i's). In this section we will show impact of the land allocation decision upon the variables which appear as values to the planner in the objective function of the nonlinear programming problem. In some cases the variables are interrelated. That is, some of the variables may depend on other variables which in turn may be related to land allocation. Thus, each of the variables are at least indirectly related to the land-allocation decision resulting from the objective function. Stated in another way, each of the variables are a function of the amounts of acreage put to the various land uses.

In this section we will try to develop, in a rough fashion, the relationship between each of the 19 variables outlined in the previous sections and the land acreage in urban land, irrigated agricultural land, and rangeland, respectively. Table 6.6.1 shows the general effect of land allocation on each of these 19 variables.

6.6.1 Economic variables. Let us examine the effect of the acreage allocation between the three land-use types on the economic variables--per capita income, income distribution, job stability, and net regional product change.

Per capita income. The following three figures show the changes in per capita income with the change in the percentage of the total acreage put into urban, rural, rural irrigated, and range areas, respectively.

Table 6.1. Table showing general effect of land allocation upon value to the community via "quality of life."

Variable	Relative Importance (value range in value units)	General Influence of Increase in Variable On Value	Effect of Increasing Land Usage In: Urban	Agriculture	Range
		Economic			
1. Income per capita - $V_{i,1}$	4	+	Increasing	Increasing	Increasing
2. Employment stability - $V_{i,2}$	16	+	Increasing	Increasing and then decreasing	Increasing and then decreasing
3. Net regional product change practice - $V_{i,3}$	20	+	Increasing and then decreasing	Increasing	Increasing
4. Income distribution - $V_{i,4}$	18	+ for low / - for high	Increasing and then decreasing	Constant	Constant
		Ecological			
5. Ecological - $V_{i,5}$	20	-	Increasing	Increasing	Increasing and then decreasing
6. Environmental quality index - $V_{i,6}$	11	+	Increasing	Increasing and then decreasing	Decreasing and then increasing
7. Percentage use of renewable resources - $V_{i,7}$	4	+ under 100 / - over 100	Increasing	Increasing over 100	Increasing over 100 and then decreasing under 100
8. Annual % usage of nonrenewable resources - $V_{i,8}$	14	-	Increasing	Not applicable	Not applicable
9. Man-initiated energy consumption - $V_{i,9}$	8	-	High constant	Increasing and then decreasing	Increasing
		Socioculture			
10. Population size - $V_{i,10}$	14	+ then -	Increasing	Increasing and then decreasing	Decreasing
11. Social differentiation - $V_{i,11}$	16	+ then -	Increasing	Increasing and then decreasing	Increasing
12. Cultural heterogeneity - $V_{i,12}$	14	± then -	Increasing, decreasing, and then increasing	Decreasing and then increasing	Increasing and then decreasing
13. Sociopsychological (solidarity) - $V_{i,13}$	6	+	Decreasing	Decreasing and then increasing	Decreasing
14. Information advantage - $V_{i,14}$	20	+	Decreasing	Decreasing and then increasing	Decreasing and then increasing
		Political			
15. Scope of government services - $V_{i,15}$	13	+	Increasing*	*	*
16. Uses of government services - $V_{i,16}$	6	+	Increasing*	*	*
17. Political participation - $V_{i,17}$	18	+	Increasing	Increasing	Increasing
18. Property tax base - $V_{i,18}$	8	+	Increasing*	*	*
19. Political power advantage - $V_{i,19}$	9	+	Increasing*	*	*

* Applies to whole Riverton area.

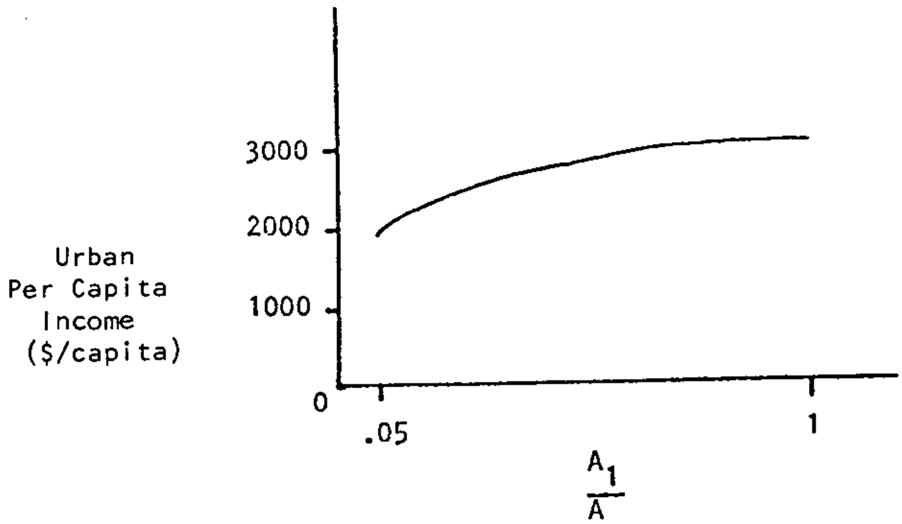


Fig. 6.6.1. Urban per capita income vs. urban area.

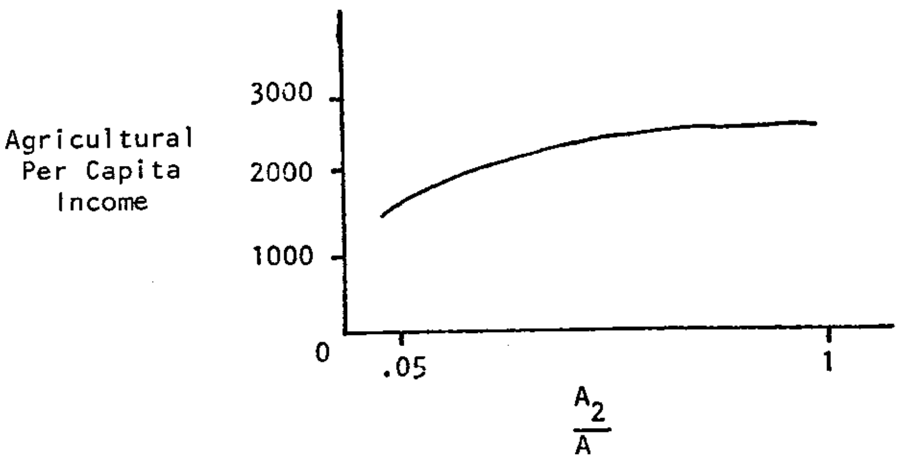


Fig. 6.6.2. Agricultural per capita income vs. irrigated area.

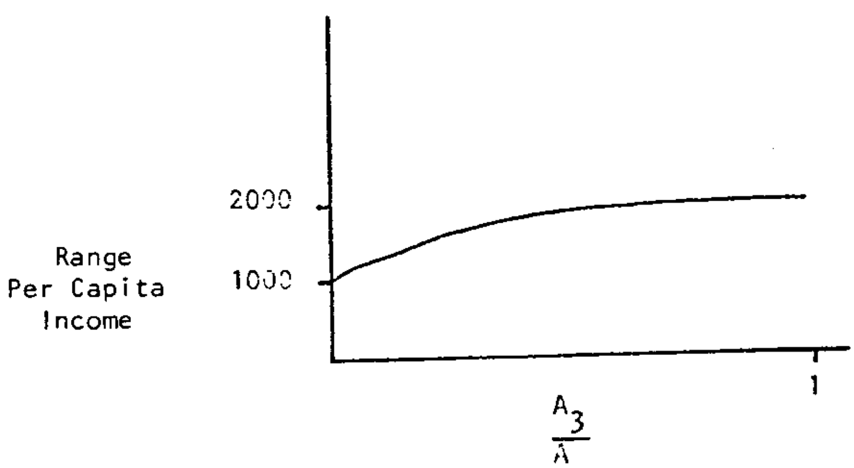


Fig. 6.6.3. Range per capita income vs. range area.

It is postulated in each of these figures that increasing the land use in any one area increases the per capita income. The per capita income in the urban area is the highest overall. Its range is \$2000 to \$3000 per capita. The hypothesis here is as the size of the urban area is increased, the number of professionals and managerial types that move into the area increase, jobs are more available, and a general booming economy results in higher per capita income.

In the agricultural areas incomes are lower than in the urban areas. Increasing the acreage in agriculture implies an increase in the size of the holdings, which means larger per capita income for the individuals involved, although population densities probably drop. Population density will be discussed with the sociological variables.

In the range area increased amounts of acreage in range also tends to increase the size of the land holdings which increases the per capita income of the people involved.

Income distribution. The impact of land allocation upon income distribution is given in the following three figures for each of the land-use areas.

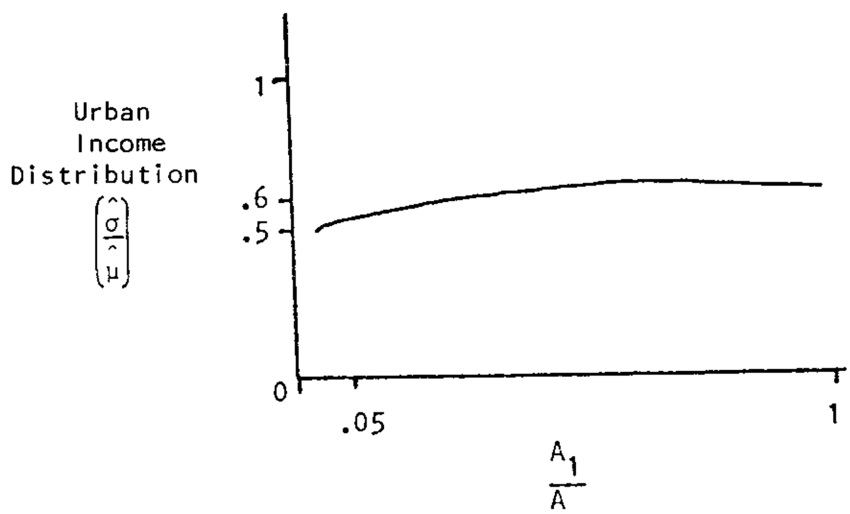


Fig. 6.6.4. Urban income distribution vs. urban area.

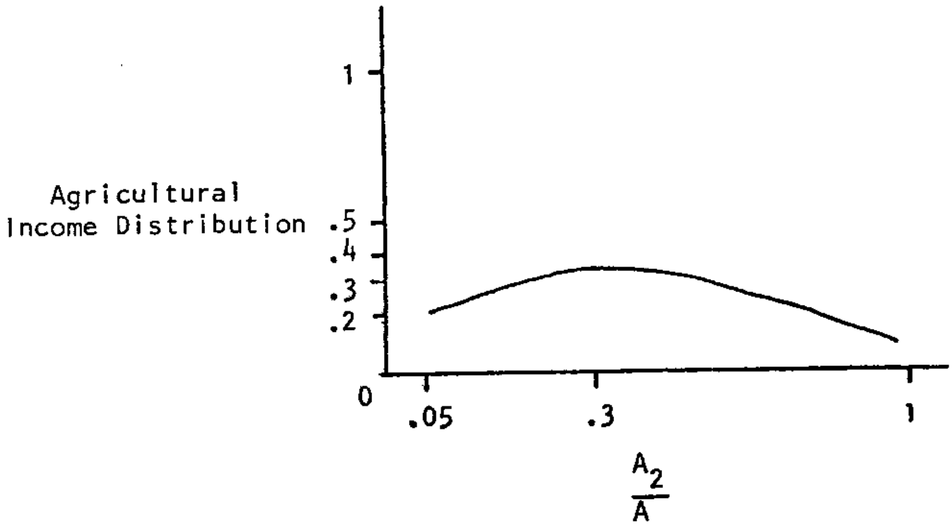


Fig. 6.6.5. Agricultural income distribution vs. irrigated area.

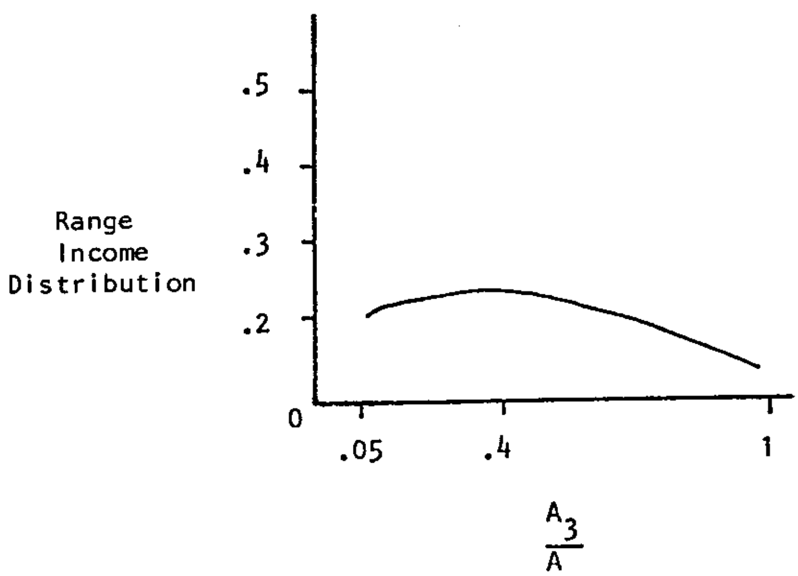


Fig. 6.6.6. Range income distribution vs. range area.

In the urban areas income distribution is the highest of the three areas. However, it is not greatly affected by the change in the size of the city. As the city size grows, there is a slight increase in income distribution as professionals and laborers come in to work in the industrial areas; however, income distribution is fairly high to start with, and does not increase greatly.

In the agricultural area the income distribution increases with initial distribution of agriculture as laborers and farm specialists enter the area. However, with increasing agricultural use, the land holdings are increased with a tendency for a land-holding class and a laboring class, without a great deal between, reducing the distribution of incomes. A similar pattern occurs on the range where increasing the size of the range area initially increases the distribution as specialists come in, but further increase results in a laboring class and an owner class with very little income distribution between them.

Job stability (percentage of labor force working). The figure below shows the change in job stability as a result of land allocation change.

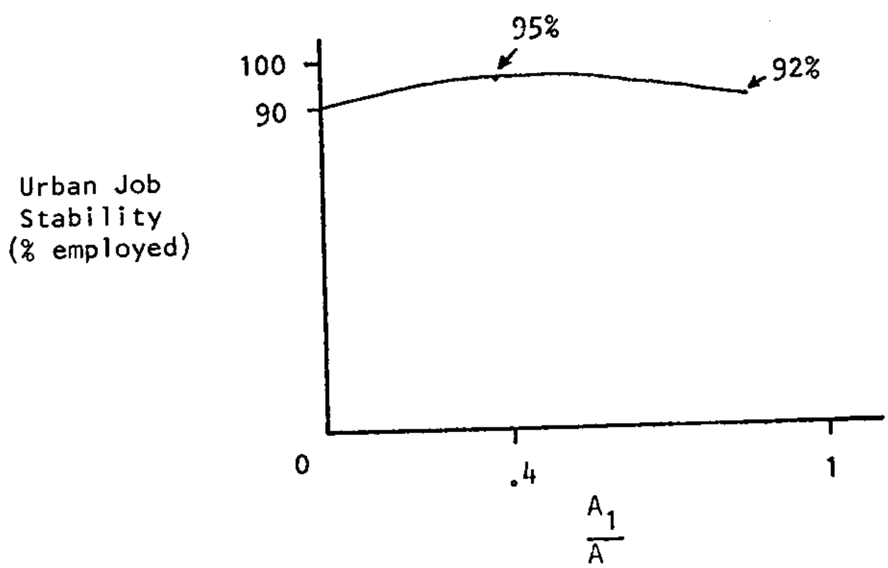


Fig. 6.6.7. Urban job stability vs. urban area.

In the urban areas it is noticed that job stability increases originally as urban size increases because of the input of factories which increase the number of jobs available, but then as immigration increases, a larger percentage of the population is unemployed. Employment overall is quite high in the rural and range areas, and it is not clear that there is any effect upon job stability by changing the amount of acreage in the various land uses. In the agricultural area employment is assumed constant at 92%, whereas in the range area employment is assumed constant at 90%. We are open to any contrary ideas about changes in job stability with increasing acreages in agriculture or range as these are quite tentative hypotheses.

Net regional product change. The change in the net regional product is seen as an extremely important economic variable and is reflected as a function of land allocation in the three curves below.

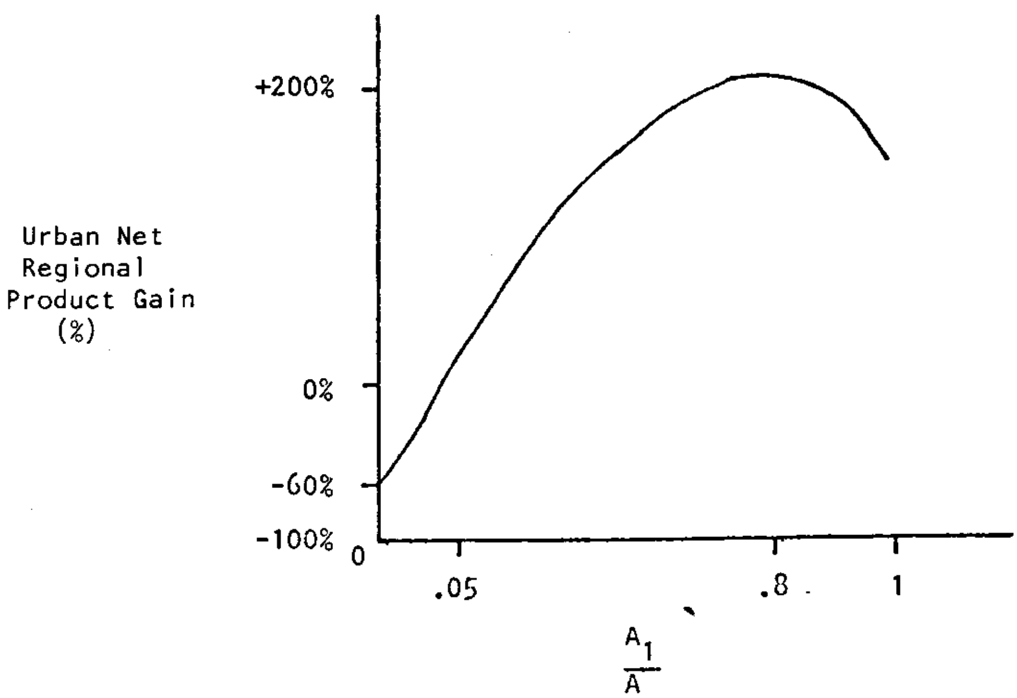


Fig. 6.6.8. Urban net regional product change vs. urban area.

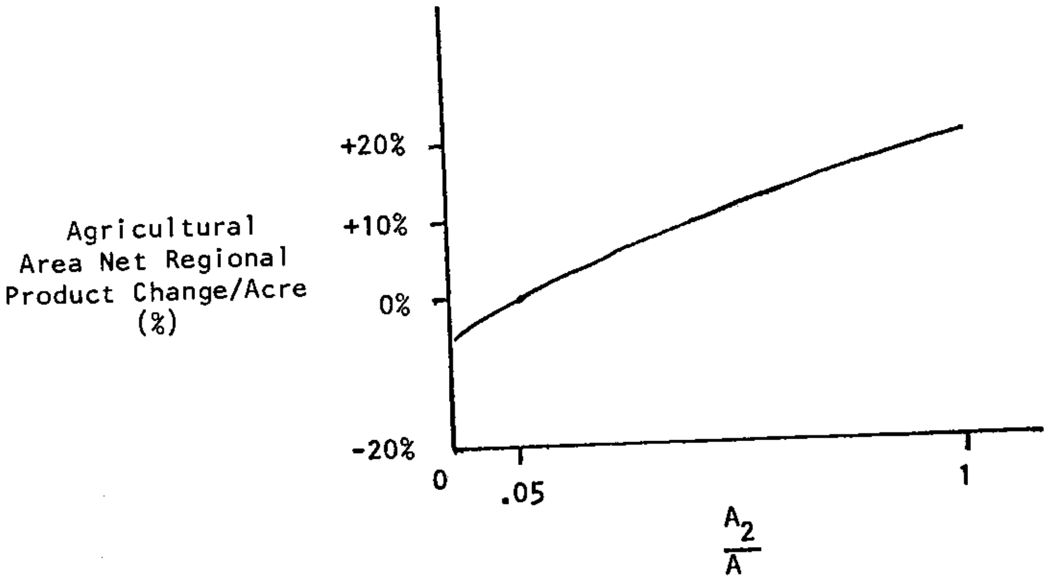


Fig. 6.6.9. Agricultural net regional product change vs. irrigated area.

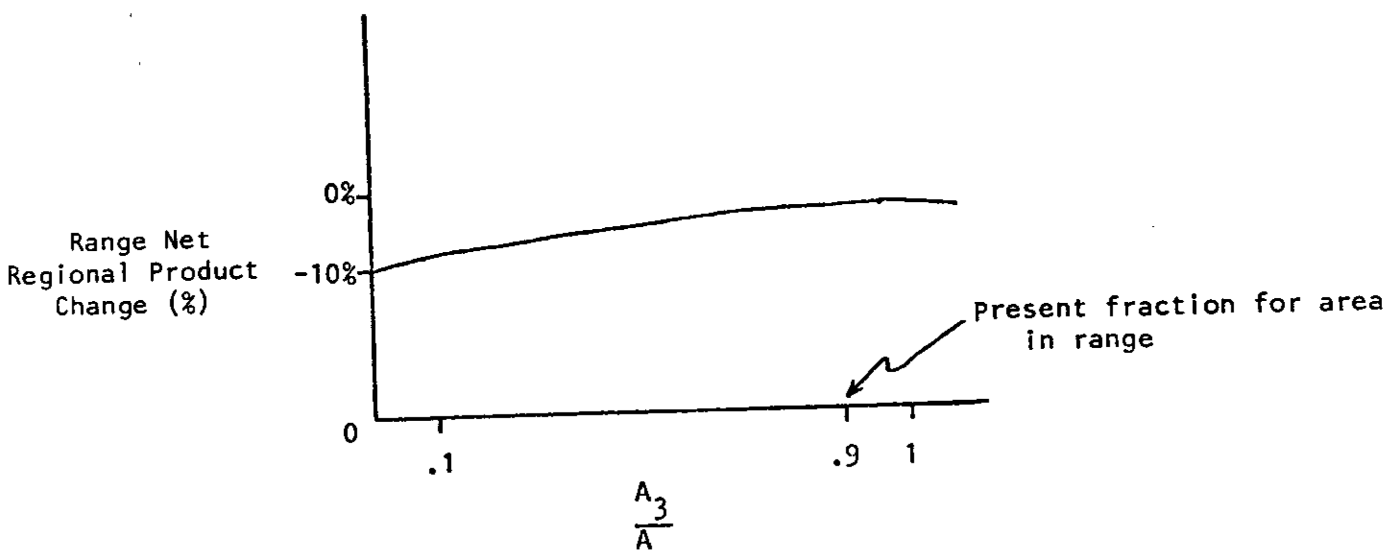


Fig. 6.6.10. Range net regional product change vs. range area.

In the urban areas increasing population initially has a very large effect on the net regional product change per acre. In fact, at approximately

40% of the total urban area, there is a 100% increase in net regional product over 5% urban area. This reflects a population change from about 10,000 to approximately 100,000. The major change is in the industrialization of the area which greatly increases the net regional product. In fact, a 100% change may be an underestimate.

Reducing the urban area below 5% means reducing the net regional product as the remnants of urbanization leave the area. However, it is not likely that the city will drop in population because the major thrust of the planning concerns whether or not to expand the urban areas presently existing.

In the agricultural area where approximately 5% of the acreage in the Riverton area is now put to agriculture, increasing the acreage of agriculture will increase the net regional product per acre by about 20% when the agricultural acreage is increased to 40% of the total acreage, which is the maximum it is allowed to increase under the optimization programming framework. Here again, no consideration is made of the curve below the 5% level because reducing the amount of acreage in agriculture is not a realistic planning alternative.

With the range area where presently 90% of the acreage is in range, reducing the acreage in range will reduce the size of the land holdings and will reduce the income or the net regional product per acre by about 10% if range area is reduced to about 10% of the area. Further increasing the range beyond this 90% level seemed to have no major effect on the net regional product per acre.

6.6.2 Ecological variables. Let us now consider the effects of the land allocation and other variables upon the ecological variables.

Ecological quality index. The three figures below show the effects of land allocation upon the environmental quality index. It is noticed that in the urban area, the population is considered a more important direct influence upon environmental quality than the amount of acreage in the city. Therefore, this is used as the independent variable in this figure.

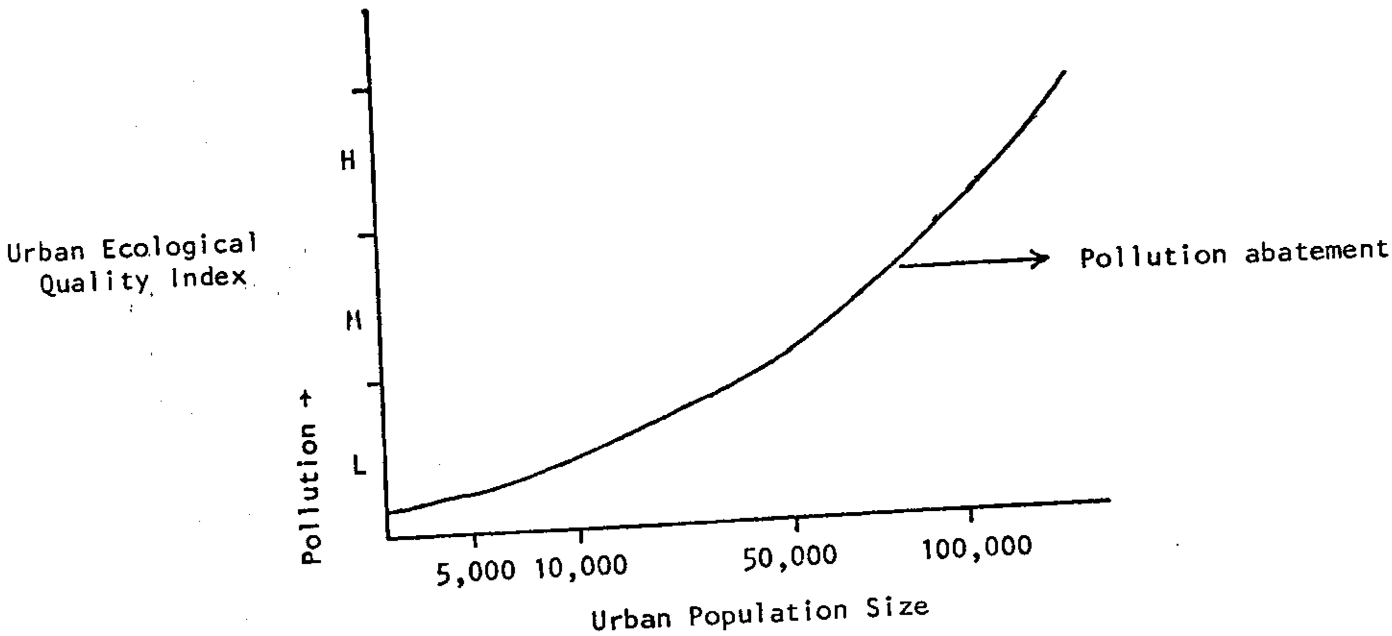


Fig. 6.6.11. Urban ecological quality index vs. urban population.

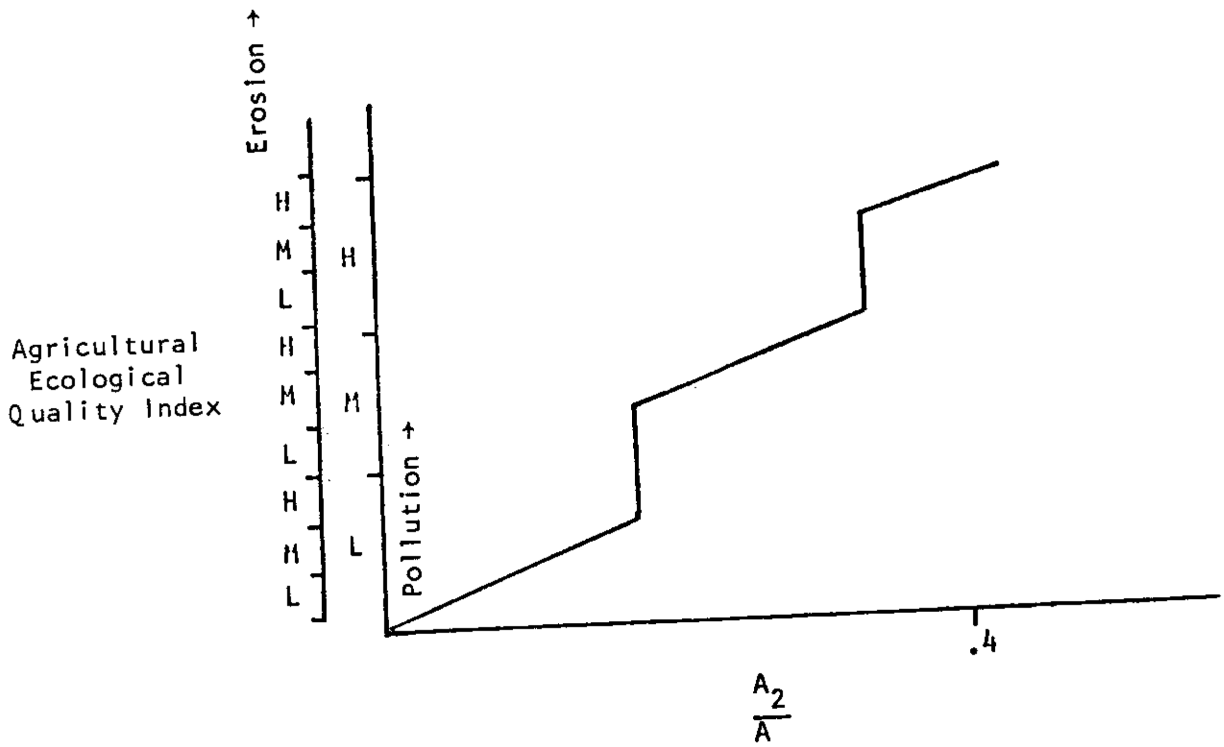


Fig. 6.6.12. Agricultural ecological quality index vs. irrigated area.

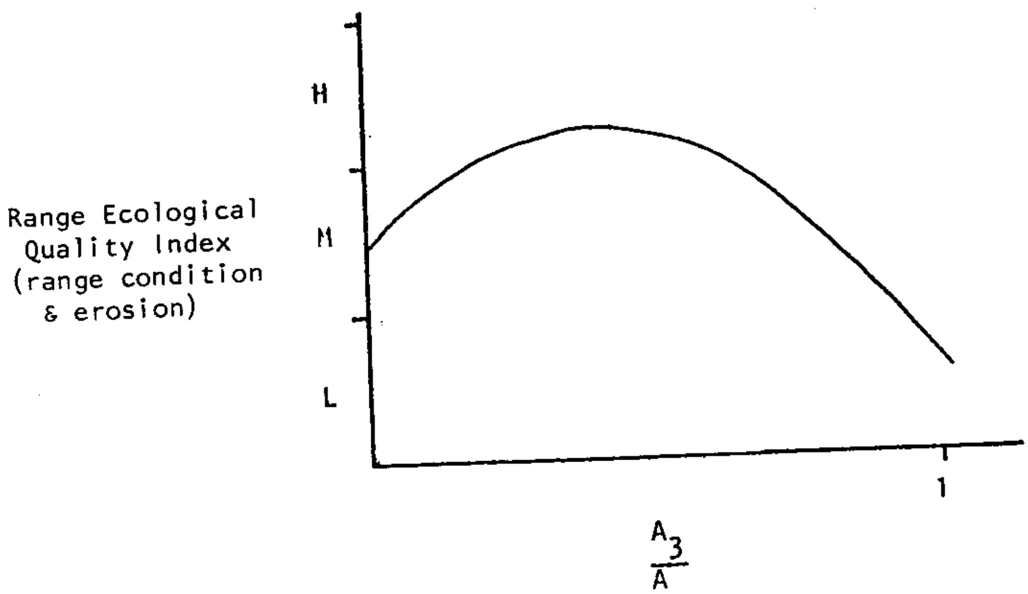


Fig. 6.6.13. Range ecological quality index vs. range area.

In the urban area as population density increases, the pollution, which is the major factor determining environmental quality in the city, increases exponentially. We are not considering the case of cities beyond 100,000 where some pollution abatement activity might be necessary. Actually, considering pollution abatement, it would be a factor pushing the entire curve to the right, rather than eliminating pollution in large cities. The only real solution to pollution that is apparent from this curve is a low population density.

In the agricultural areas the major pollution source to be considered is water pollution; air pollution is assumed negligible. Also, erosion resulting from over-intensive use of the land is a factor to be considered. The curve is piece-wise discontinuous because it is assumed that you cannot go from low pollution and moderate erosion to low pollution and high erosion, but rather from low pollution and moderate erosion to moderate pollution and moderate erosion. The reasoning with this curve is that as the acreage in agriculture increases, the land passes from small land-holders to larger land-holders with larger land units. The large landowners are assumed to use less intensive agricultural procedures in getting their crop. This, as mentioned earlier, results in less yield per acre and a lower gross regional product per acre. However, it does, in general, result in more erosion and more pollution due to the increased use of fertilizers, machinery, and more complete usage of the land. For example, wood lots are cleared, and gullies are generally filled in large farms. This results in a potential erosion situation much more serious than where these practices are not done. Here again, it is to be noticed that the curve is not considered beyond 40% of the acreage put to agriculture, because this is a limitation set upon the planning procedure as a constraint by the nonlinear programming framework.

In the range area the two major factors are range condition and erosion. However, since range condition and erosion are very highly correlated (that is, good range condition generally results in very little erosion, whereas poor range condition generally results in high erosion), the two will be considered together. The H category (standing for high) refers to high erosion with poor range condition, whereas the L category refers to low erosion and good range condition. The explanation for the curve is as follows: with small amounts of area in range, landholdings are small. There is little economic pressure and also little done in the way of land conservation practices. Therefore, grazing is assumed to be moderate, range condition moderate, and erosion moderate. With increased acreage in range the economic pressure and competitiveness is increased. However, since the landholdings are still small, this pressure is generally evidenced not in land conservation practices but in more intensive grazing, which reduces the range condition and generally increases the erosion. With increased acreage in range the operations tend to become larger, with operations frequently being bought up as tax losses. This results in institution of conservation practices, less intensive grazing with a result in increase in range condition, and a sharp reduction in the amount of erosion.

Environmental degradation. The environmental degradation, or landscape degradation, is a measure of the diversity in the area. In the urban area we are referring to diversity of types of structures in which people live, work, play, or buy; whereas in the range and agricultural areas we are referring to the biotic habitat diversity. These are all measured by the information statistic for diversity as outlined in the value standardization section presented earlier. It should be initially mentioned that these curves represent only the general response of diversity to changes in the amount of acreage of the various land uses. We are using the information that diversity indices found in the field lie mostly within the range of 1.5 to 4. We have actually done no studies to show that diversities of the types of structures in a city would also fall in this range. Thus, our curves represent in some ways, mere inference, or hypothesis.

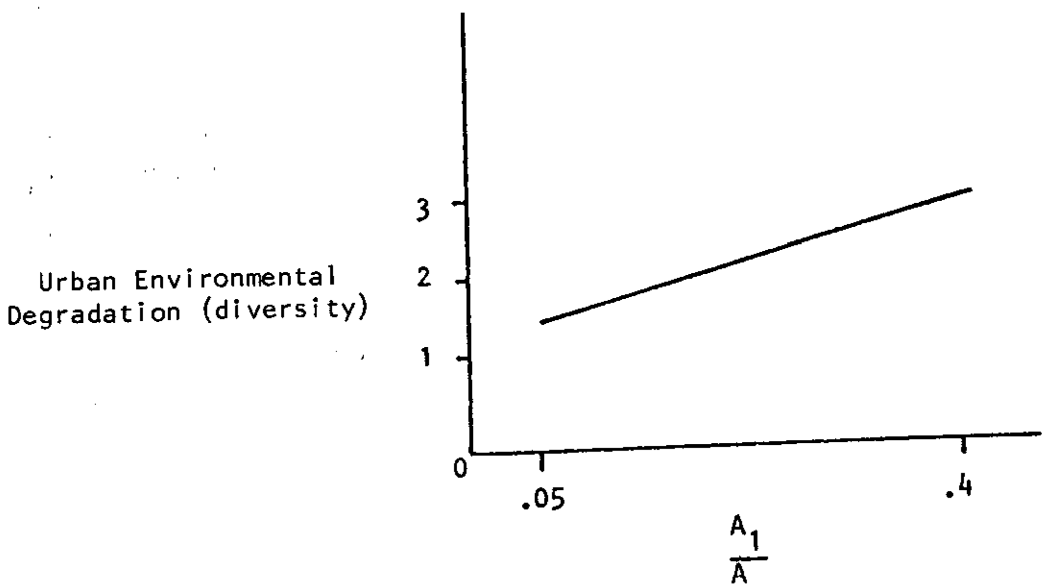


Fig. 6.6.14. Urban environmental degradation vs. urban area.

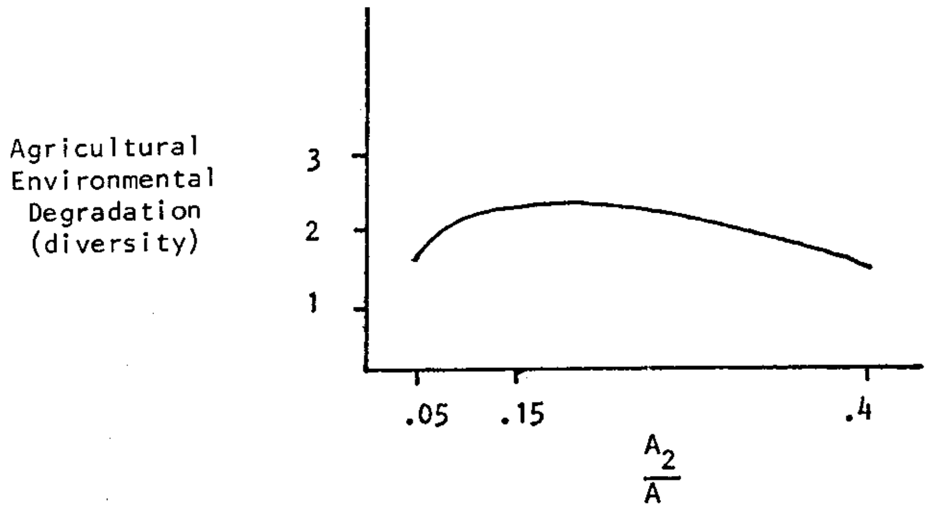


Fig. 6.6.15. Agricultural area environmental degradation vs. irrigated acreage.

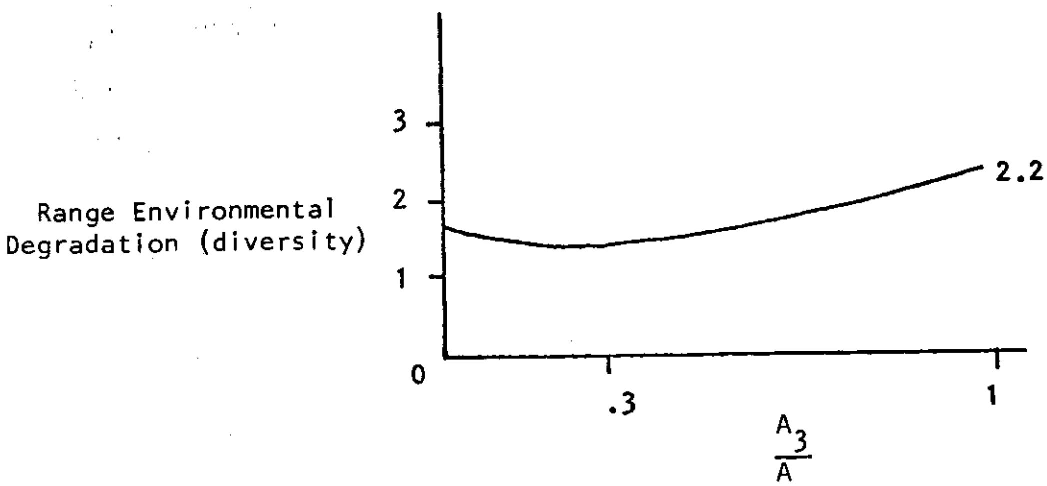


Fig. 6.6.16. Range area environmental degradation vs. range area.

In small towns the diversity of structures is assumed to be rather low with the types of commercial, industrial, and residential establishments being fairly uniform and similar. As the size of the urban area grows, the diversity increases with different kinds of commercial industries coming in, ghettos being formed, neighborhoods being formed, architectural influences from areas playing a role, and different and varied kinds of commercial establishments being instituted. Notice that no more than 40% of the area is considered to be urban, which puts a maximum limit on the population density of about 100,000 (as will be seen from the population density curves in the sociological variable section). With further increase in urban density the factors that were outlined above would increase even more rapidly, and very large diversities of urban structure might be expected. By urban structure we mean not only buildings, but also parks, recreation areas, etc.

In the agricultural area instituting more agricultural acreage through irrigation will initially increase the habitat diversity. This means that the degradation of the landscape will be decreased initially. However, with increasing acreage in agriculture, the diversity will drop because the previous areas will be replaced by an area with large stands of uniform crops which support basically the same kinds of wildlife. The smaller plots with the larger diversity would have a tendency to have larger diversity of types of agriculture which also would increase the habitat diversity.

In the range area the initial increase in the size of the range would result in more intensive grazing and poorer management practices which would tend to decrease the diversity from open range situation. However, as conservation practices, such as pond formation and fencing increase with the increase in acreage and larger landholdings, the diversity would increase.

Utilization of renewable resources. By renewable resources we mean scenery, cultural milieu (especially in the city), sunlight, water (where it is renewable), etc. The following three figures show the effect of the percentage of the renewable resource which is utilized with respect to the total amount of renewable resource which could be utilized without destroying the renewability of the resource, as a function of the amount of acreage put to the various land uses.

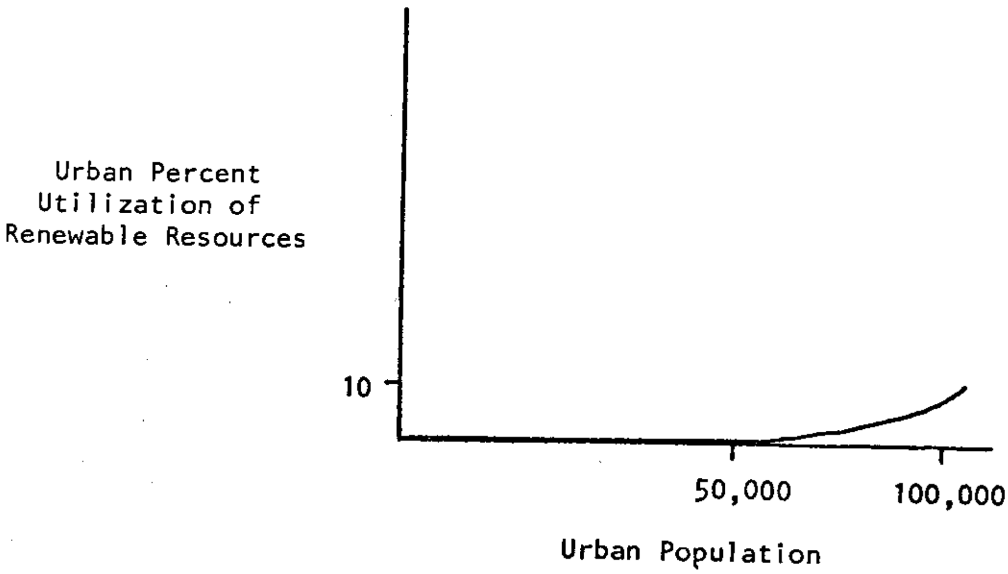


Fig. 6.6.17. Percent utilization of renewable resources in urban area vs. urban population.

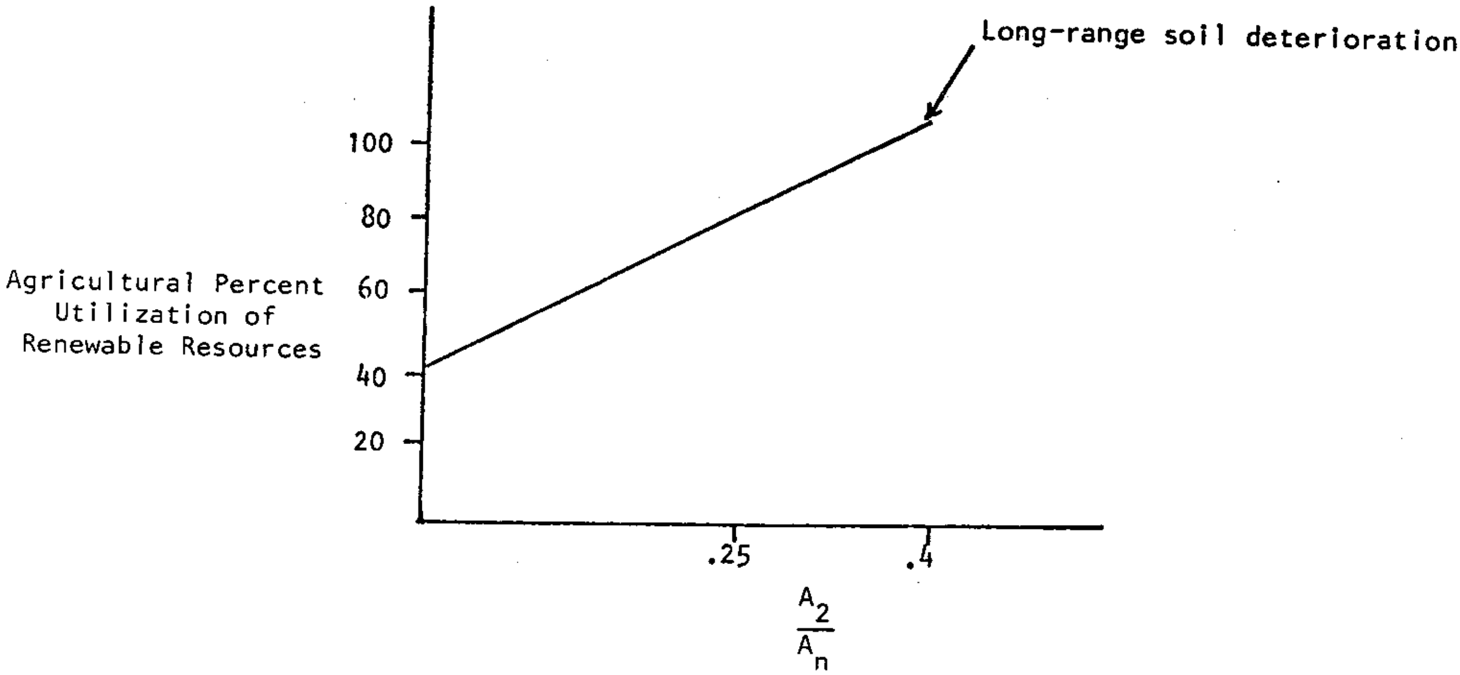


Fig. 6.6.18. Percent utilization of renewable agricultural resources vs. agricultural area.

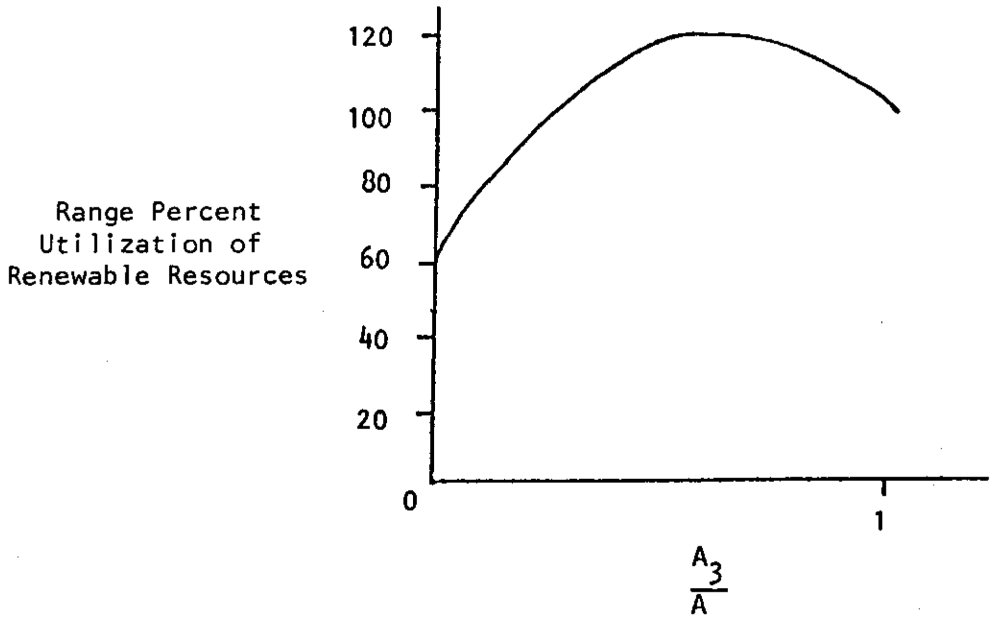


Fig. 6.6.19. Percent utilization of range renewable resources vs. range area.

In the city the major renewable resources are the cultural facilities. A city of under 50,000 is assumed to have little in the way of cultural facilities; and therefore, the percentage use of utilizable resources is simply zero. In cities in the 50,000 to 100,000 population range, the percentage of renewable resources utilized goes up to about 10%. That is, we feel that cultural facilities such as museums which are in these areas are not heavily used.

In the agricultural areas the utilization per acre of sunlight, which is the major renewable resource, and of water, which is also considered a partially renewable resource, generally increase with increased amounts of acreage in agriculture. The reason for this is that fertilizer treatments increase with larger landholdings (which are assumed to occur with increased agricultural use), and fertilizer treatments result in larger crop yields and more efficient utilization of sun and water energy. With very large land-holdings, because of the intensity of management, there is a tendency for long-range soil deterioration which results in a percentage use of the renewable resources greater than 100%.

In the range area the major renewable resources are the forage crops which grow generally unaided by fertilizer or extra water treatment. There is an initial rise in usage above 100% with increased acreage because the extreme intensity of the management and the large number of cattle that are generally grazing on the plots result in over-utilization of the resource. This implies that the resource now is not as renewable as it was before (see the value standardization curve). With increased acreage and increased size of landholdings, conservation practices and less intense grazing result in a very high utilization of the resource compared with what is feasible. Since the range is so large, it can be managed experimentally until a management combination is reached which results in extremely efficient utilization of the renewable resources. We thus hypothesize that as the entire acreage goes to range, the percent utilization of the renewable resource goes to 100%.

Percentage use of nonrenewable resources. By percentage of non-renewable resources we refer to the resources such as oil, gas, and water which are not renewable. We look at the percentage of the remaining resources used per year on the average over the time period at which we are looking. As an example of what utilization of nonrenewable resources means, a 3% annual utilization of resources will result in 50% depletion of the resource stock in less than 22 years.

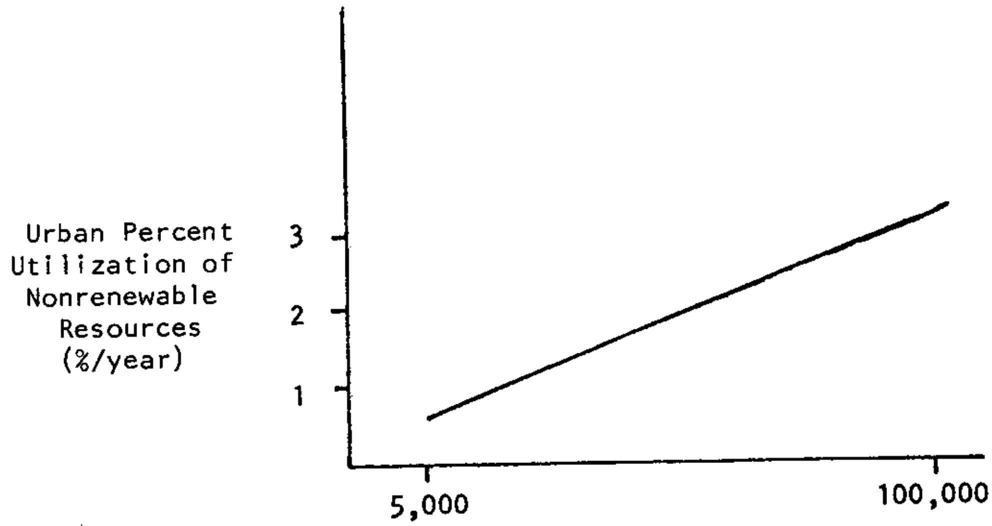


Fig. 6.6.20. Percent annual utilization of nonrenewable resources vs. urban population.

Since most of the nonrenewable resources are not directly connected with a particular land use in that they may occur in any one of the land-use regions and since the majority of the nonrenewable resources are concerned with the large population density areas, we are looking at the use of nonrenewable resources for the entire region and are considering the population density in the urban area as a measure of the nonrenewable resource demand. We see that as population density grows from a population of about 5,000 to a population of 100,000 the annual consumption of nonrenewable resources increases from about .5% to 3%. Of course, this trend cannot continue because in the long run the nonrenewable resource would soon be depleted in the large urban demand areas. Also, this curve is somewhat unrealistic because the nonrenewable resources may well not come from the area in which the urban density that we are talking about exists, but would come from other areas and be paid for with money, which is also potentially a nonrenewable resource.

Man-initiated energy consumption. We defined man-initiated energy consumption as the ratio of the energy utilized by man through fossil fuels to the energy utilized from the sun.

The curves below show our hypothesis on how this ratio would change with land-use allocation.

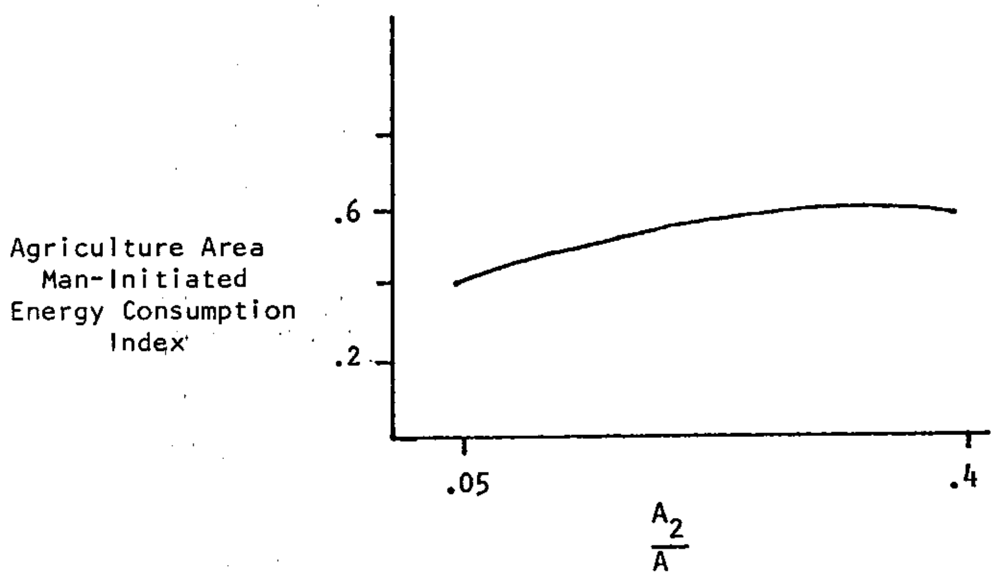


Fig. 6.6.21. Agriculture area man-initiated energy index vs. irrigated area.

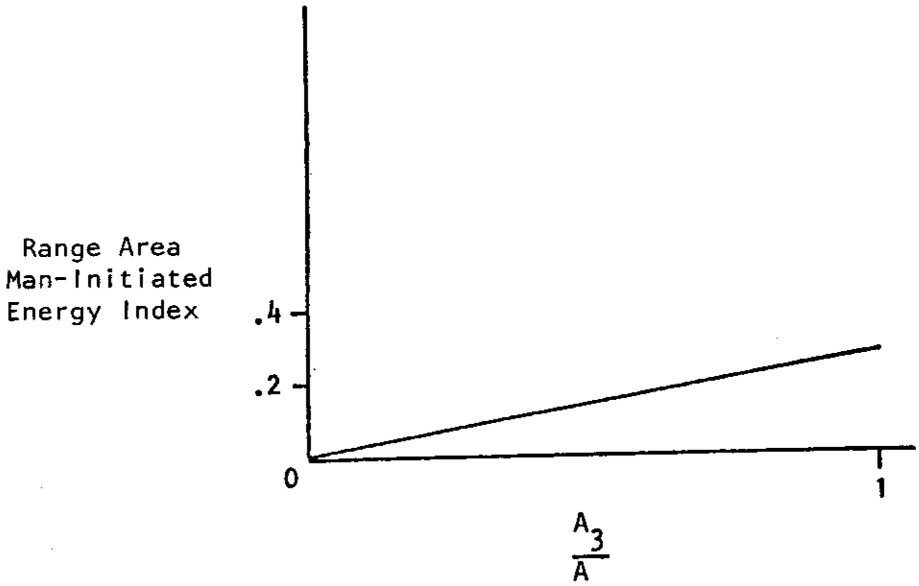


Fig. 6.6.22. Range area man-initiated energy index vs. range area.

In the urban areas where almost the entire energy consumption depends on fossil fuels for machinery, heat, electricity, etc., the ratio is assumed to be arbitrarily 1000. The only sun utilization that we can imagine in the urban area is for grass growing and solar energy for heating.

In the rural agricultural areas the fossil fuel inputs are mainly through fertilizers and machinery. The use of fertilizer greatly increases the efficiency of the utilization of the sun's energy. However, a great deal of energy is also spent using fossil fuels for machinery to keep pests away and to mechanize the planting and harvesting processes. Therefore, with increased agricultural intensity resulting from increased area in agriculture, the ratio of man-initiated energy to sun-utilized energy increases due to the increased use of mechanical devices over human labor. Notice that the ratio is still less than 1, which indeed shows us that our agricultural areas are giving us the ability to support the large imbalance of fossil fuel to sunlight ratios in the city. Also, this curve points out the fallacy in thinking that we actually get something for nothing when we mechanize because the actual input in terms of fossil fuels that is put into the increased crop yield is actually greater than the increase in crop yield that we receive from an energy standpoint (the ratio goes from .4 to .6 with increased mechanization which implies less energy efficiency).

In the range area where very little is done in the way of fertilizer or water treatments to the grasses, the energy consumption from fossil fuels is very low. However, we hypothesize that with increased size of the range, the more intensive conservation practices are done at the expense of larger fossil fuel input from human elements (for example, jeeps, fence post holes, seeding for hay, etc.), raising the ratio of human to sunlight energy input utilized.

6.6.3 Political variables. In considering the effects of land-allocation decisions upon the five political variables, we have decided to treat the region as a whole with the major political decisions and political results emanating from the county seat (Riverton). The feeling is that the implications for political variables in the urban area will affect the political situation throughout the whole region.

Also, it was felt that the land allocation itself would not be a good measure of political variables. Therefore, a more direct measure was sought. Social differentiation, defined as the number of different job categories available, was chosen as a measure of the political variables. Notice that this variable itself is a function of the land-allocation system; and therefore, indirectly the political variables are all influenced by land allocation.

Scope of government services.

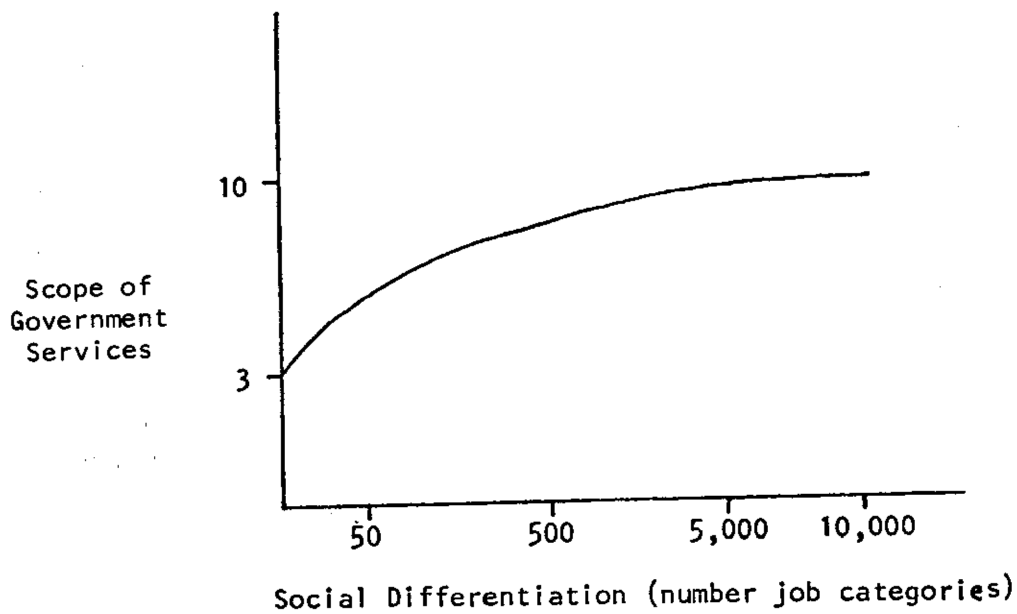


Fig. 6.6.23. Scope of regional government services vs. social differentiation.

As social differentiation in the urban area increases, the scope of government services is assumed to increase from 3 to 10. This means a move from the basic facilities, of police, education, and highways to the initial addition of sewage, fire protection, and parks and recreation facilities, and finally to the fringe benefits of welfare, hospitals, and library systems. In our problem social differentiation is never assumed to increase greatly enough to add such services as museums operated on a public basis.

Uses of government services.

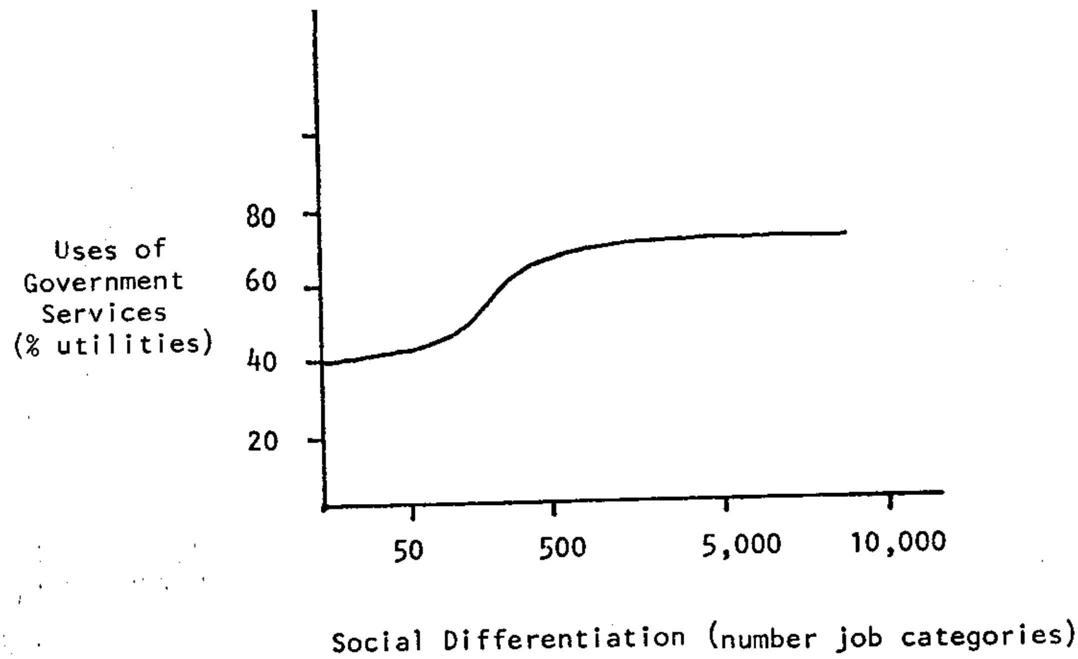


Fig. 6.6.24. Regional uses of government services vs. social differentiation.

As the social differentiation increases, the percent of the population using the government services is assumed to increase from about 40 to 70%. The reasoning here is that increased social differentiation generally results in a larger scope of government services along with more information about the government services available. This results in a larger use of the government services, both by the professionals that make the social differentiation larger and by the original local people who benefit from the better educational system resulting from the higher social differentiation.

Political participation.

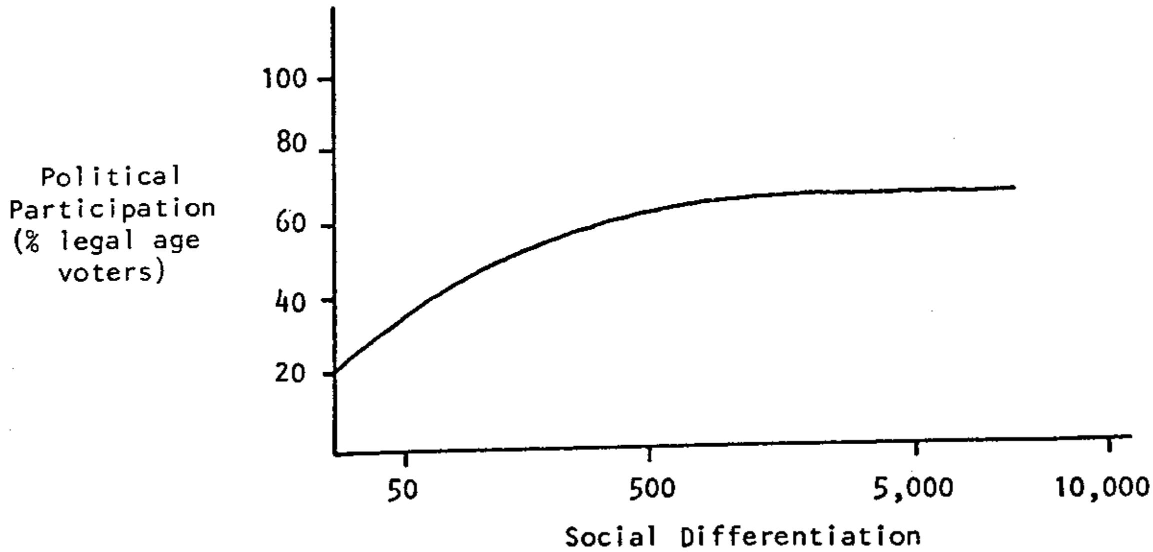


Fig. 6.6.25. Regional political participation vs. social differentiation.

Political participation in areas of small social differentiation generally is low. We have assumed that it is in the range of 30 to 40%. As the social differentiation increases, the political participation in terms of the percentage of the population voting starts to increase quite rapidly, until it levels off at about 60% participation, which is about the highest participation to be expected in any regional election.

Property tax base.

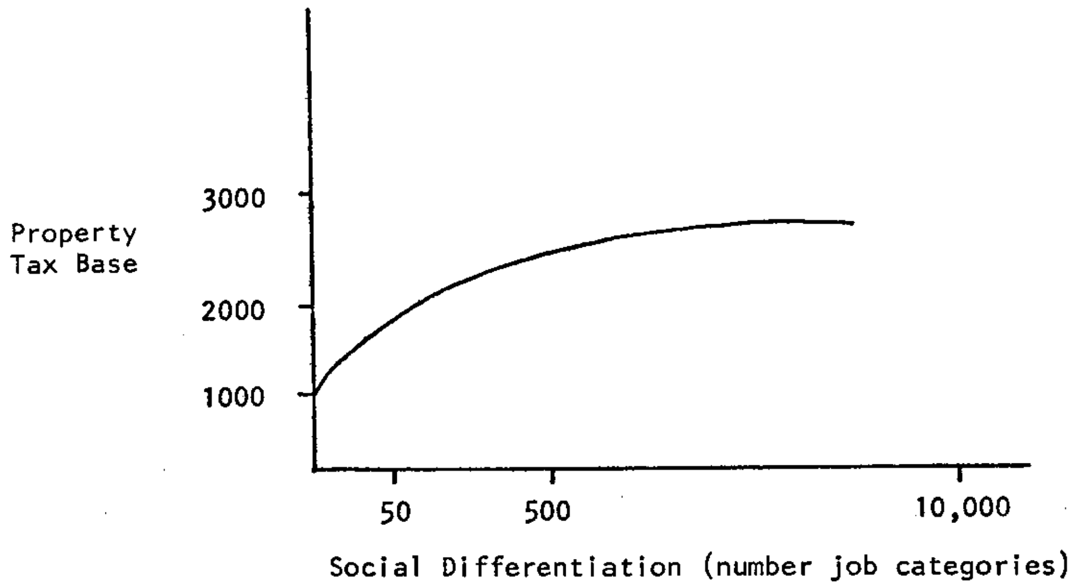


Fig. 6.6.26. Property tax base vs. social differentiation.

As social differentiation increases, there is a rapid increase in property tax rate from an initial value of $1000 per capita in low social differentiation areas, eventually leveling off at around $3000 in areas of high social differentiation. The reasoning here is that the social differentiation, a mix of the types of occupation, is correlated with the ages of the people and their professions. Higher social differentiation gives a greater likelihood of professionals and skilled workers who are more likely to build more expensive homes than unskilled workers. Also, there is an attraction of industry to the area which further increases the property tax base.

Notice that the curves for uses of government services, scope of government services, and property tax base are quite similar in form. This is because the reasoning that we have used to show how social differentiation affects each of these factors is quite similar and the increases in each of these factors is quite similar as well. However, they are quite separate variables and can be easily distinguished by the value standardization curves which have been drawn in Section 6.3.

Political power advantage.

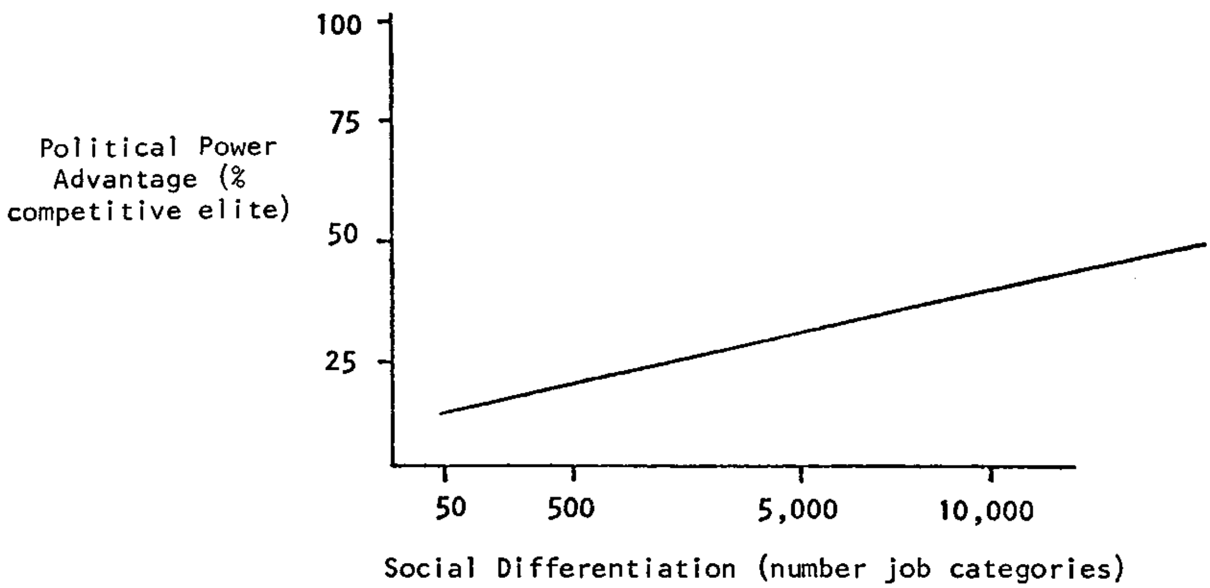


Fig. 6.6.27. Political power advantage vs. social differentiation.

The political power advantage increases with increased social differentiation. As the number of jobs in the area increase, the number of types of people increases and the traditional elite have a tendency to be replaced by the competitive elite. However, these changes are very slow as pointed out by the very small slope of the curve. It takes extremely high social differentiation (likely much higher than we will experience for the Riverton area) to change the traditional power elite to a majority of competitive elite in the 20-year period or so to which we are referring in our planning problem.

6.6.4 Sociocultural variables. In this section we will consider the effects of land allocation upon the sociological variables. Due to the unique nature of some of these variables the curves are of a tentative and hypothetical nature. Criticism and resultant improvement of these curves is desired.

Population density. The following three curves show our hypothesis on projected population change with land allocation in urban, agricultural, and rangeland uses.

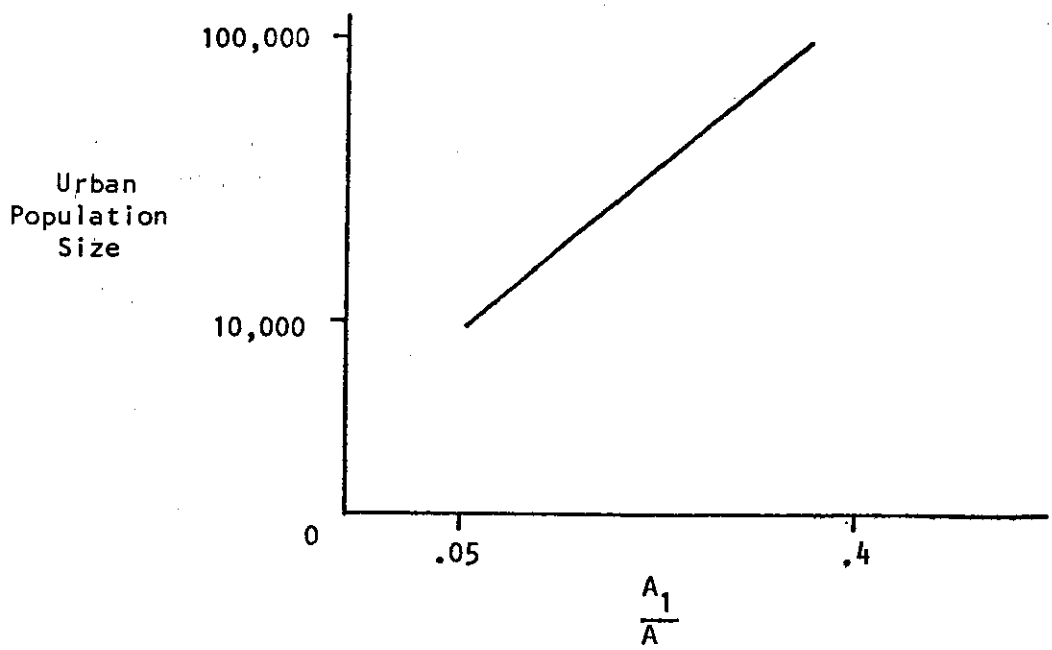


Fig. 6.6.28. Urban population size vs. urban area.

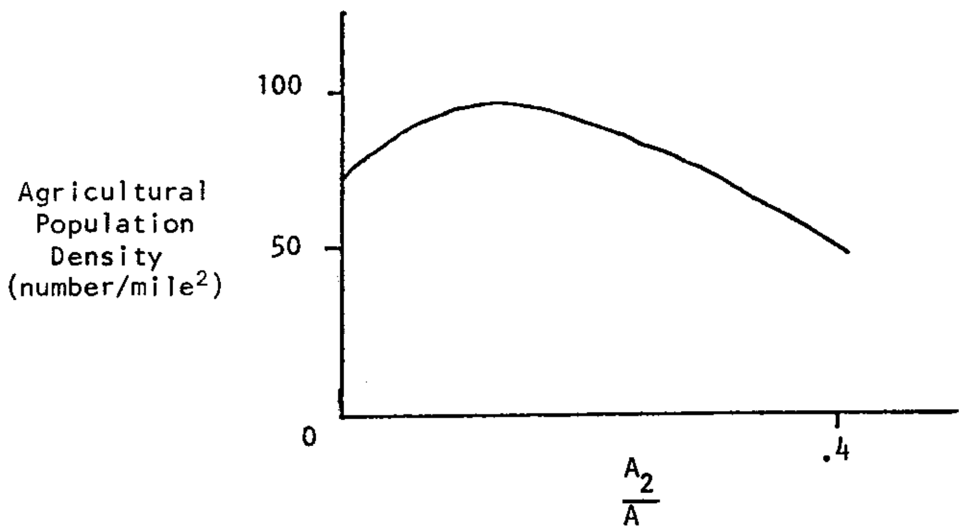


Fig. 6.6.29. Agricultural area population density vs. irrigated acreage.

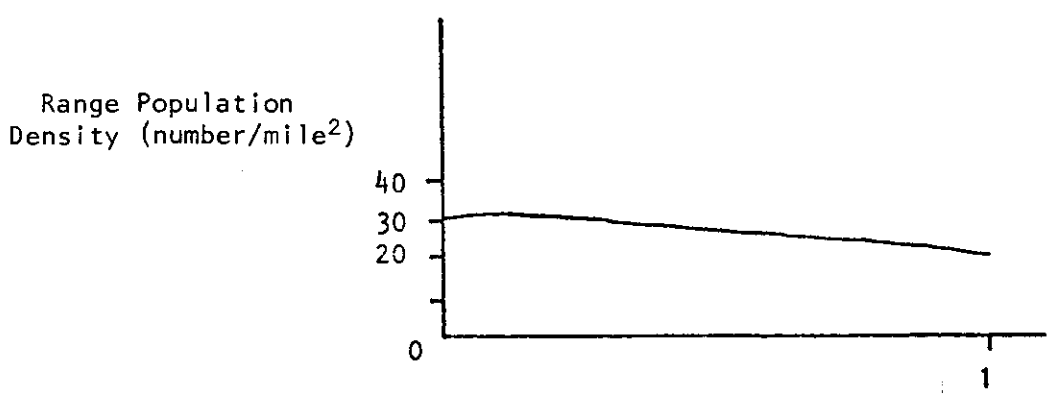


Fig. 6.6.30. Range area population density vs. range area.

On the urban areas increasing urban land use will greatly increase the population from 10,000 at .05 of the total area, A, in urban land to 100,000 with .4 of area A in urban land.

In the agricultural sector densities are low. Initially irrigation will increase densities, but with increasing acreage densities will drop as the land goes to larger farms.

The range area is similar to the agricultural area, except the curve is flatter and the populations more sparse.

Social differentiation.

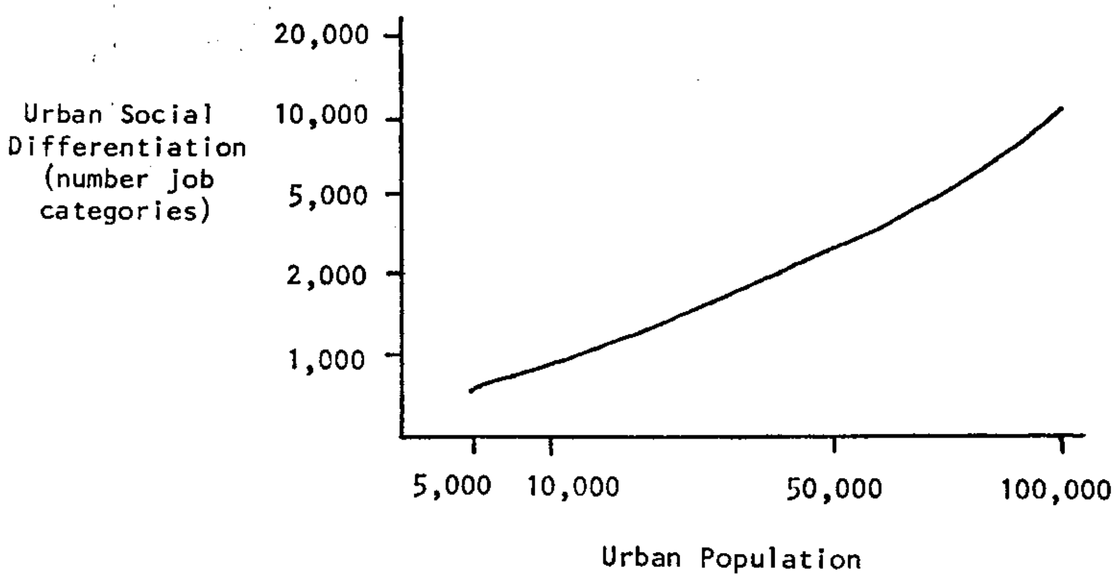


Fig. 6.6.31. Urban social differentiation vs. urban population.

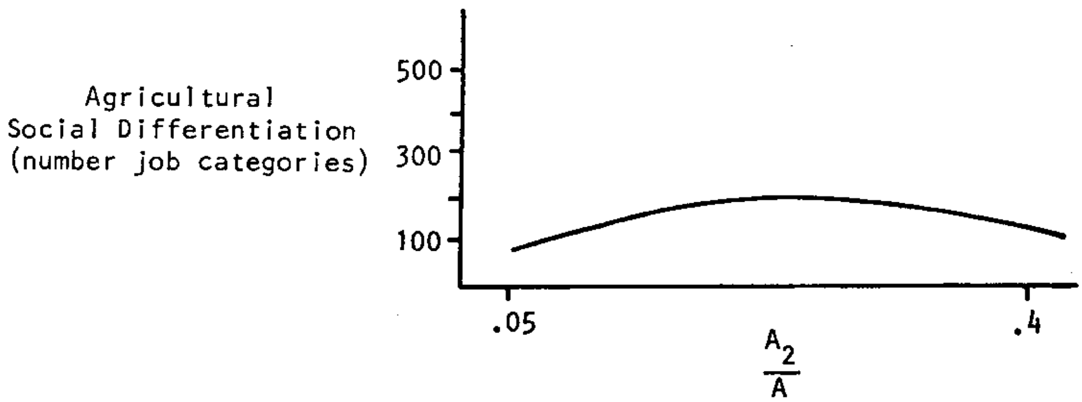


Fig. 6.6.32. Agricultural social differentiation vs. irrigated area.

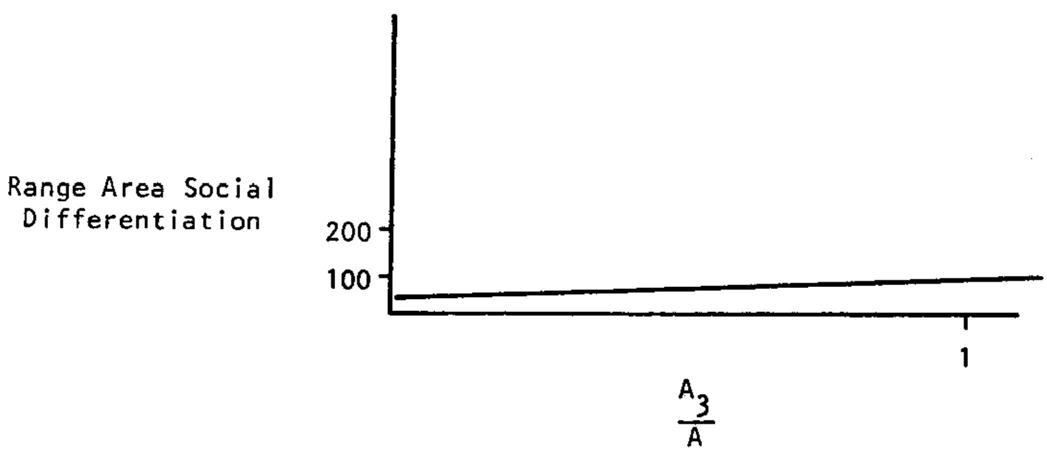


Fig. 6.6.33. Range area social differentiation vs. range area.

The three curves above suggest how social differentiation is affected by land allocation. Social differentiation refers to how many different types of job possibilities exist in the area as a measure of the potential job-interchangeability of the area and the production diversity of the area.

In the urban area population was considered a better indicator of social differentiation than area in urban land. As population rises, social differentiation increases with larger percentage increases occurring with higher populations.

In the agriculture and range areas increased area initially increases social differentiation as specialists come into the area to handle expanding land-use problems. The consolidation of farms results in fewer jobs in the agricultural area and a stabilization of job numbers in the range area.

Cultural heterogeneity.

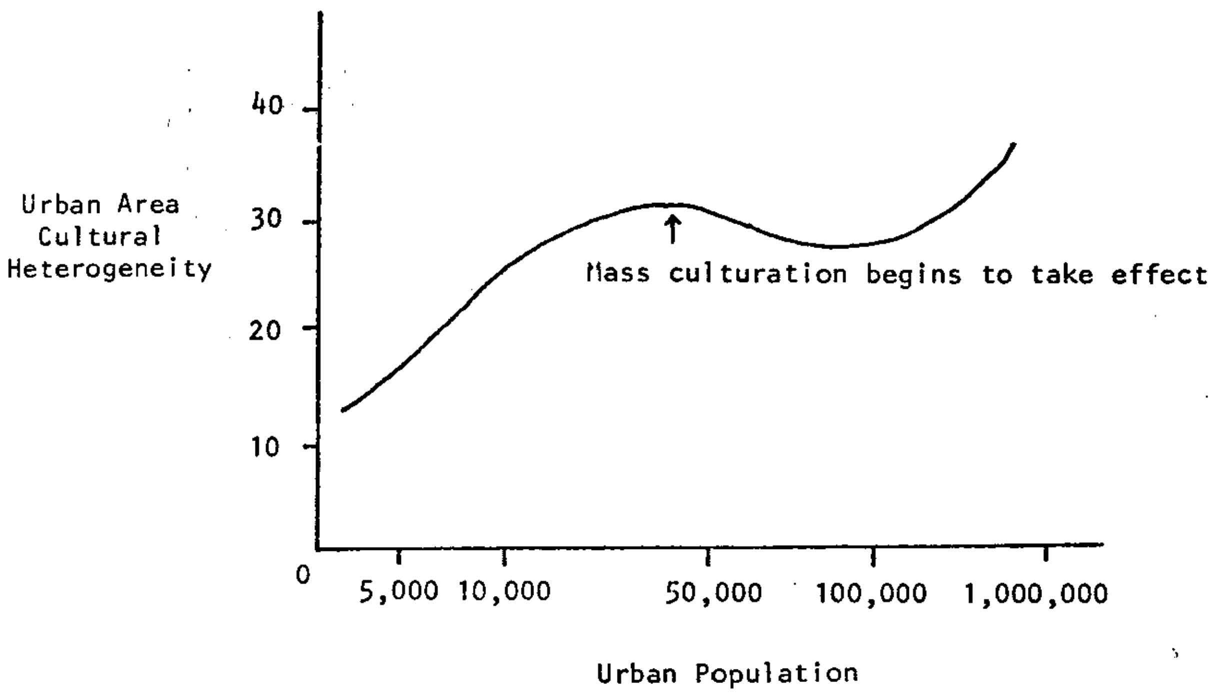


Fig. 6.6.34. Urban area cultural heterogeneity vs. urban population.

As the city grows, persons with different ways of life come in and the urban area becomes pluralistic. The interaction tends to produce homogeneity (similar tastes, shared views, mutual outlooks). As mass media develops this process continues (McDonald 1964). However, in the post-industrial world, neo-Gemeinschaft occurs: a myriad of microcommunities each with sufficient resources to support widely varying life styles. Neo-Gemeinschaft is a term referring to the "retribalization" of society into small primary groups each with a unique ethos, but one incorporating that advanced technology which permits human dimensions of experience. Neo-Gemeinschaft stands in contrast to "Gesellschaft," wherein human capacities are organized by mere technical considerations (the plastic world of middle-class America), as well as in contrast to the anti-technology stance of old Gemeinschaft (from Young 1971).

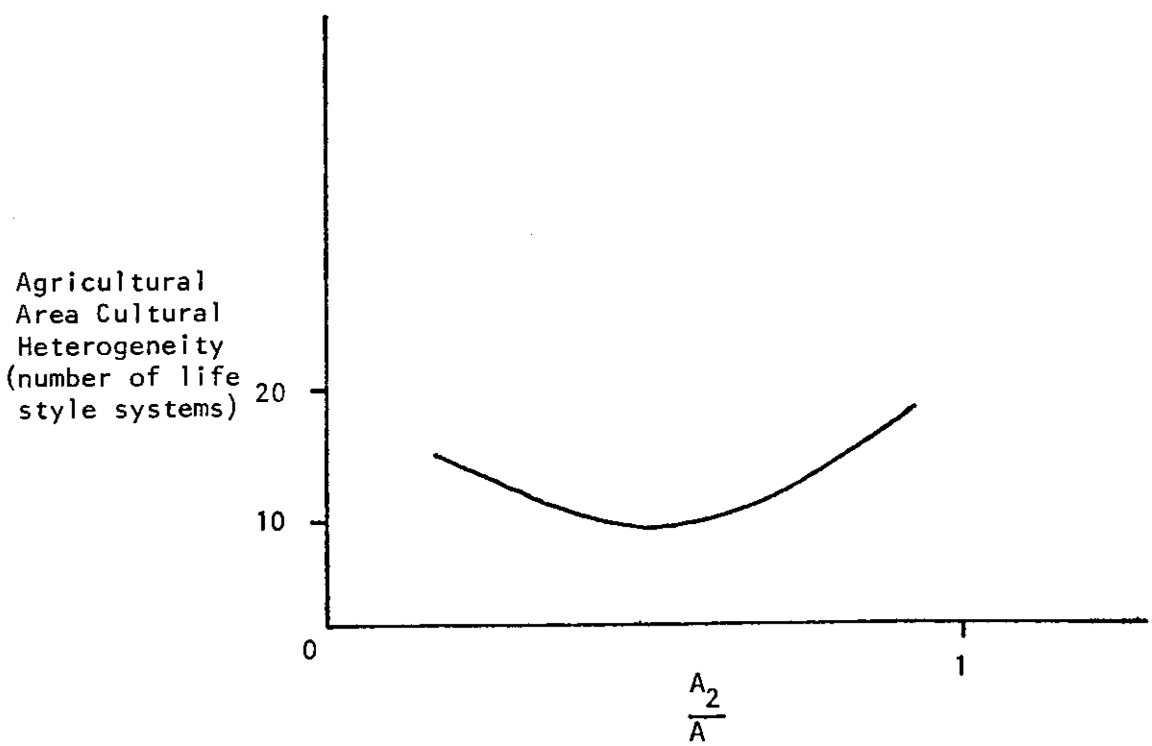


Fig. 6.6.35. Agricultural area cultural heterogeneity vs. irrigated area.

In the agricultural area there are a great number of life styles based upon previous ethnic, class, age, and occupational differentiations.

Interaction wipes out many of these (language, world view, skill, income), but technology coming with larger farms permits the trend to be reversed to greater heterogeneity.

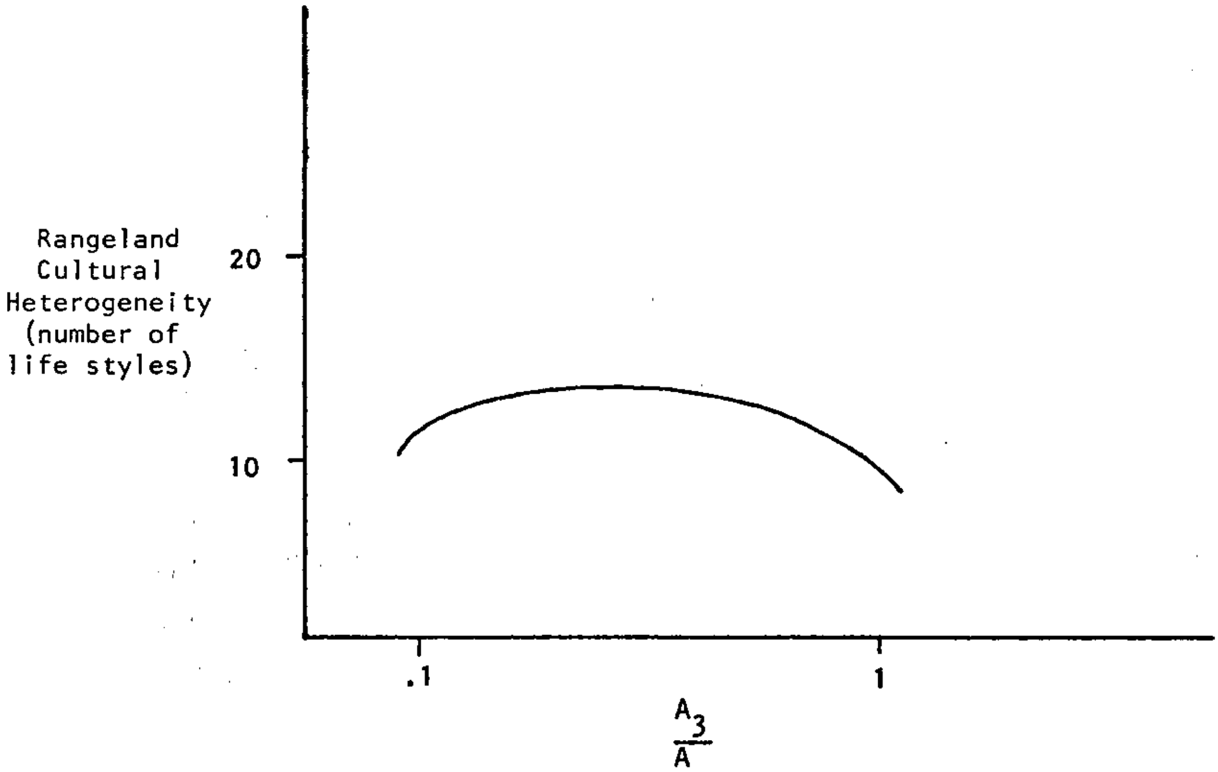


Fig. 6.6.36. Rangeland cultural heterogeneity vs. range area.

As rangeland increases initially, the smaller holdings which depend largely on manual labor are replaced by larger holdings which are more dependent upon machines. This results in the influx of a group of personnel who know how to both fix and operate machines, thereby, initially increasing the cultural heterogeneity in the range area.

As rangeland increase continues, living units consolidate, power accumulates to large ranches, and persons with different life styles are defined as anathema. There is limited dependency upon outsiders, and life style becomes homogeneous.

Social psychological-solidarity.

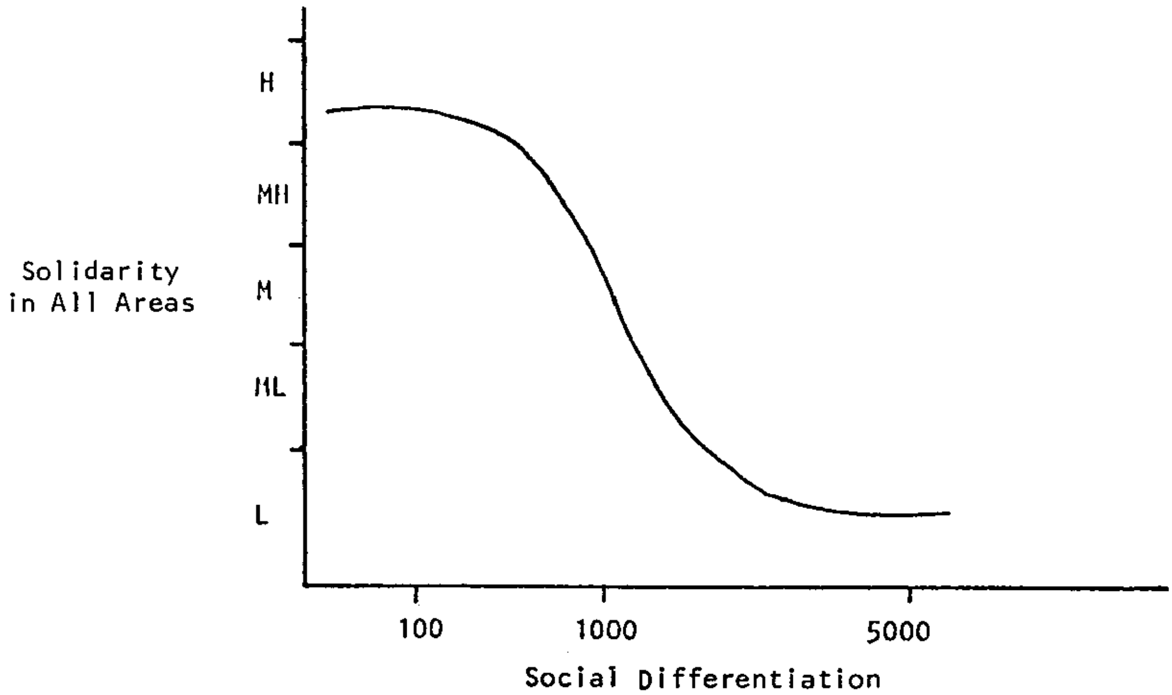


Fig. 6.6.37. Solidarity vs. social differentiation.

As social differentiation increases, solidarity decreases. As social differentiation increases, fragmentation of community occurs, human intimacy declines, and contact is brief and faceless. Corporations gain power--the individual is an impotent atom confronted by schools, businesses, churches, and military bureaucracies having a virtual monopoly on mass media and the arts of management, persuasion, and illusion. The citizen lives comfortably but with little real choice or challenge. This scenario is not necessary, but typical given existing political and economic parameters.

Information advantage.

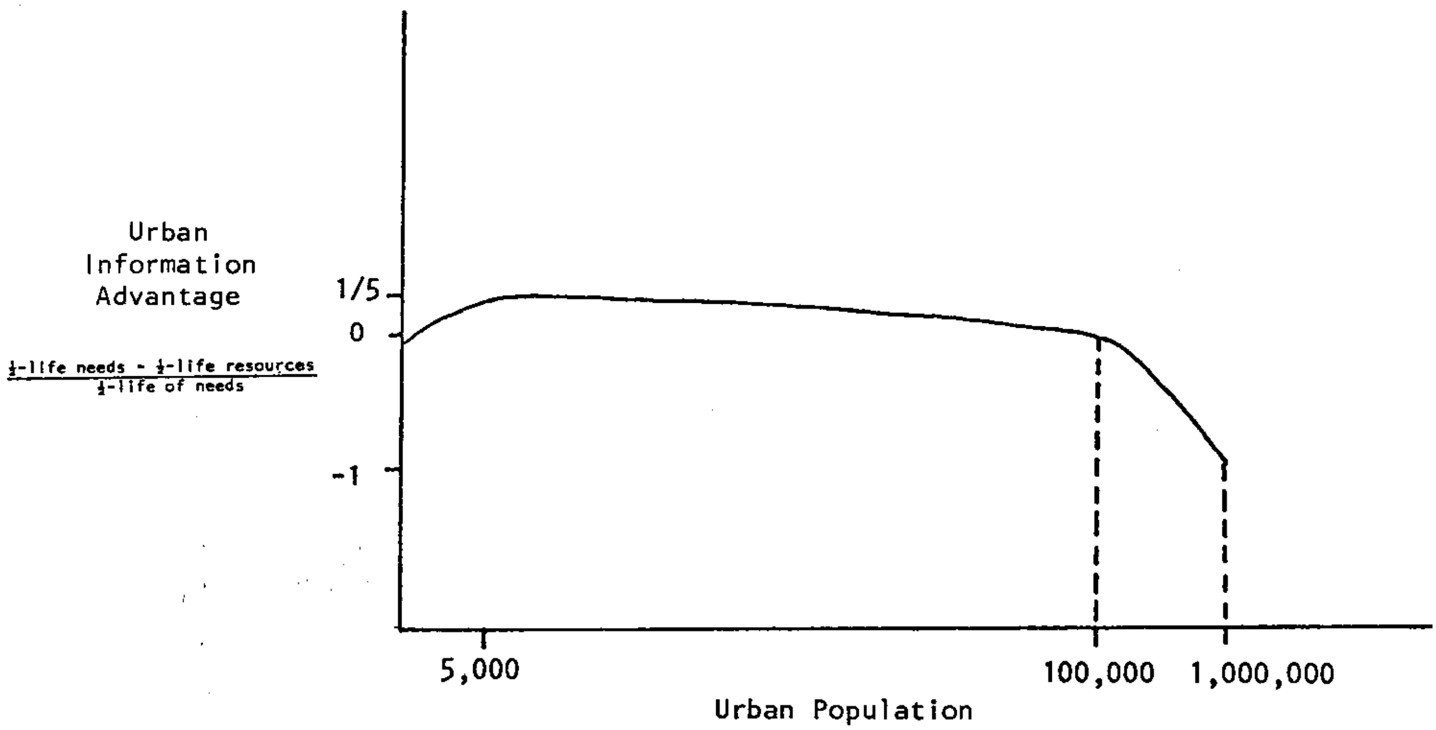


Fig. 6.6.38. Urban information advantage vs. urban population with increased population.

Here the information needs increase in an exponential fashion. Since such use overtaxes existing information systems vital to the system, new sources and forms of information flow are required. For example, extensive agriculture has different information needs from intensive horticulture. Urbanization, if it approaches very large urban populations, would require the conversion from extensive to intensive horticulture if not to food synthesis. Information half-life needs drop to hours at times. Media frequently grow to meet the expanding needs except in very large conglomerates where overcomplexity breeds information overload.[5/]

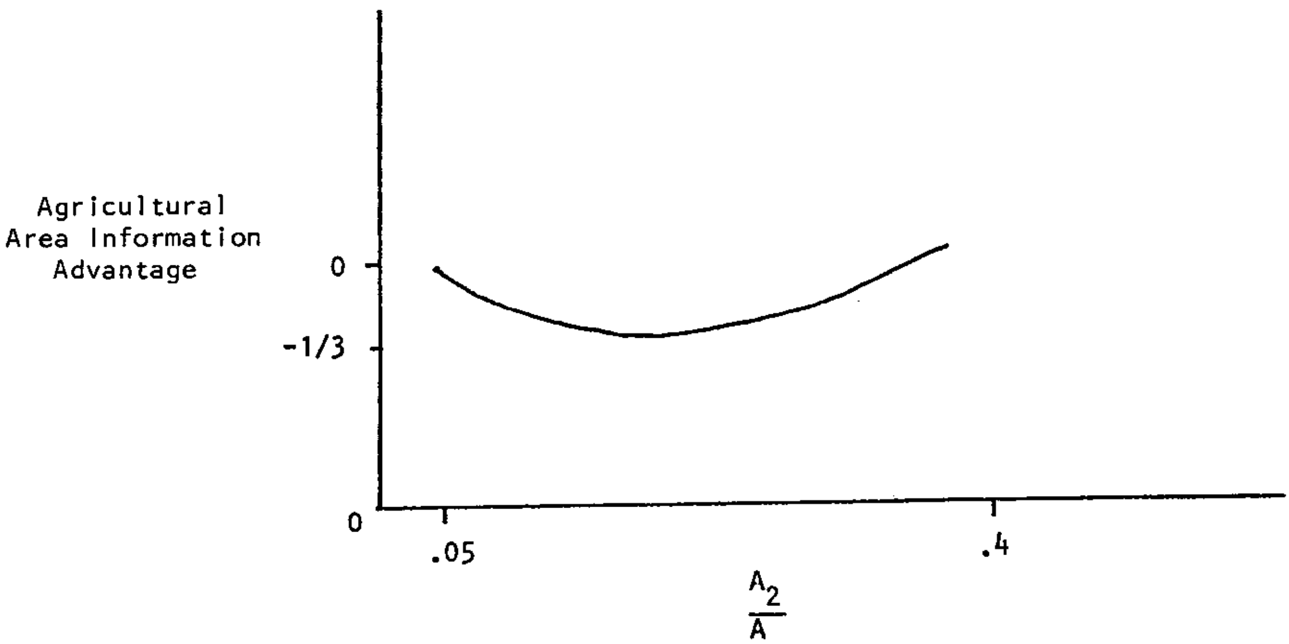


Fig. 6.6.39. Agricultural area information advantage vs. irrigated area.

As land in agriculture grows, farms get larger and newer management procedures are needed to compete. The larger farmers are better able to keep up with information needs than smaller farmers. Information needs shorten half-life to about one month (during summer months), and only larger farms can really be informed enough to keep up. With small farms and little agriculture,

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[5/] This discussion of informational half-life and the concept of information gap is entirely original in this report and is the joint product of T. R. Young and Gordon Swartzman.

information needs are much lower and the gap is negligible. Thus, the curve shows a dip as increased agricultural land results in a mix of small and large farms.

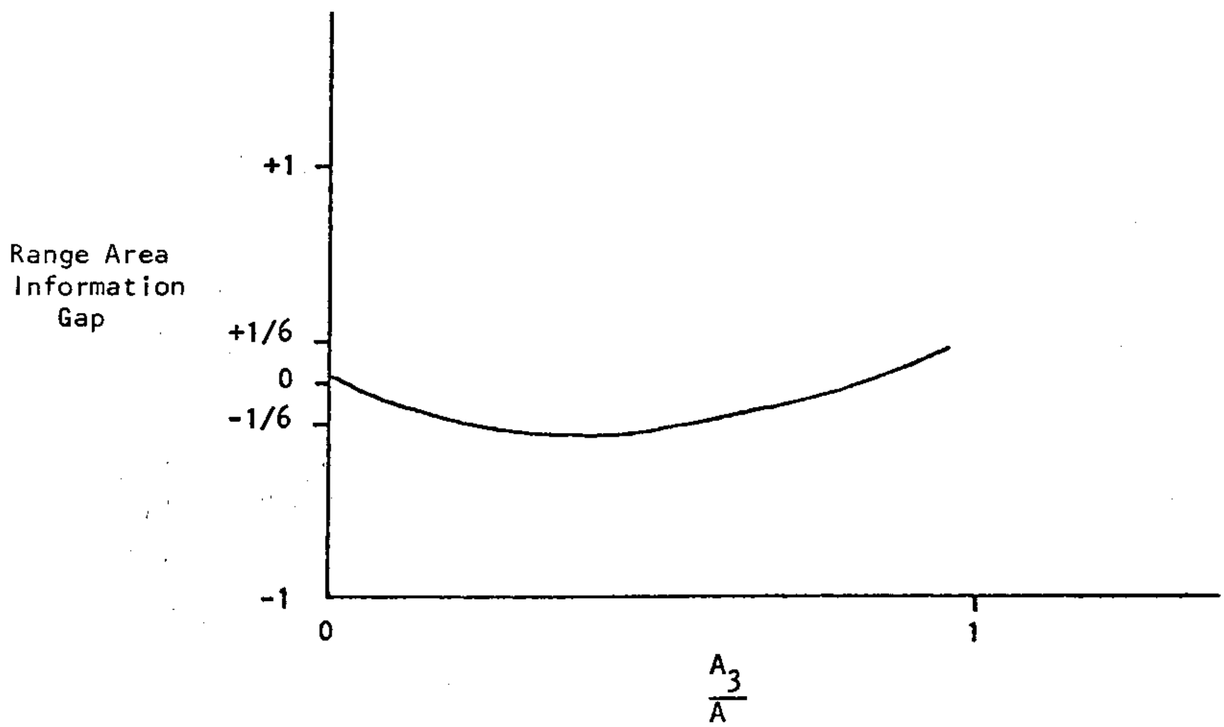

Fig. 6.6.40. Range area information gap vs. range area.

As the amount of land used increases there is need for more and faster information about quality and quantity of herd and range conditions in order to best match each set of conditions for competition with other ranchers. Half-life needs go from six months to two years. Most ranchers can keep up with this, but some cannot. Therefore, the information gap increases with increased acreage (information advantage is reduced). Further increase in rangeland results in consolidation of ranches and a new information steady state since only the better informed ranchers can grow.

7. CONCLUSIONS

From this report it seems apparent that optimization techniques have wide application in natural resource management and that they have already been applied to many problems in this area. Furthermore, it will be necessary to use an interdisciplinary approach in applying these techniques in larger-scale problems as coordinated decisions become necessary in the complex and little understood areas of regional and governmental planning and regulation. These call for decisions combining the data and thinking of ecologists, economists, political scientists, sociologists, etc. We will have to reckon with the necessity of intermeshing interaction of these areas and with the lack of "practice" in having people from these areas working together. Furthermore, optimization techniques, by providing a framework which is not too closely bound to any of the disciplines when social scientists combine with biological scientists in a interdisciplinary approach, could provide a common language of communication for people of varying disciplines. This would be especially efficacious if combined with a diagram or flow chart of the system which could be used as an intermediary to go from the word models of the experts in each of the disciplines to an overall quantified systems model which could then be utilized in decision making using optimization techniques.

The potential application of these techniques to large-scale planning decisions will expand as computer sizes grow and as funding becomes available for interdisciplinary "core" groups to come together in handling such problems. There will also be a great need for the planner and the analyst to work hand-in-hand so that both the planner's objectives and his understanding of the system can be implemented in the optimization model.

From the "Riverton" problem we have learned that a lot of input is still needed from the value standardization area before meaningful decisions can be made using operations research techniques. This again will have to bring the large bank of data and thinking about value scale systems together which have heretofore been scattered amongst separate disciplines.

The Riverton problem represents an initial attempt at utilizing optimization techniques to the planning of land allocation systems which involves interaction between social scientists and biological scientists as well as other experts such as environmental lawyers, etc. Although it does represent the combined effort of a group of experts in their related fields, it is still missing a great deal in getting at the total picture in land allocation planning decisions. For example, one indicator of quality of life which is definitely missing in the problem, as it is presented in this paper, is an index of the health of the community. This would be influenced by such variables as population density and pollution index and might be measured by the incidence of infectious diseases or the average life span of a community member. The term *quality of life* has been the measure in the objective function of that problem. An index of quality of life has been discussed a great deal in recent months and a discussion of this will be part of the AAAS National meeting in Philadelphia in 1971. Some of the ideas in this paper for quality of life indicators are unique and perhaps merit some consideration. It is the hope of the author and other contributors to this report that the part of the report dealing with quality of life indicators gets an interaction and criticism from the scientific community so that our ideas on the subject can be updated and our measures improved.

We can discuss two distinguishable major categories for classifying optimization techniques. They are the decision-making techniques, like nonlinear programming and game theory, and the implementing techniques, such as PERT, queueing theory, and gaming. The first type aid us in making decisions about how to operate and change systems subject to physical realities and desires of the public. The others are "efficiency" techniques. They enable us to accomplish a task in "optimal" fashion once the needs of the task have been specified and it has been decided to do the task. There are places where this distinction breaks down on a particular technique; e.g., dynamic programming can be an implementing technique, and queueing theory can be a decision-making technique.

Operational and simulation gaming will also play a major role both in the training of public servants to handle systems with which they have not previously studied or had contact and in comparing new management alternatives with existing decision-making techniques. In this fashion games act as a kind of sensitivity analysis in trying new management procedures, that is, the sensitivity of the system to new management procedures.

It is important not to overrate the power of operations research techniques in management decision making. Although these techniques offer an extremely precise solution algorithm, the precision of the technique frequently greatly outstrips the accuracy both in formulation of the goals of the manager in the objective function, especially in the area of the values of ordinarily non-quantifiable quantities (as ecological quality) and in the parameters of the system. It is also to be mentioned that as realism is increased (e.g., as the optimization generally changes from a linear programming to a nonlinear programming framework), the feasibility of finding a solution using a mathematical algorithm is reduced. In fact, in many nonlinear programming problems no solution may even exist.

In conclusion, it might be said that the use of operations research techniques in resource management and decision making will be more or less effective in helping the public make decisions about their future systems depending upon the following criteria:

i. the ability to collect information and integrate this information about the system into an optimization model,

ii. the ingenuity of the analyst in interpreting the results that come from the solution to the optimization problem and in conducting a sensitivity analysis on this solution,

iii. how frequently and how well the results are reevaluated after the decisions have been made to see how the decisions have changed the model of the system,

iv. whether there exist similar problems in which decisions have been made so that comparison of the optimal decision with previous decisions (that have been made on similar systems) is available, and

v. how much interaction can be obtained from the scientific community in the form of constructive criticism so that the implementation of the techniques in large-scale "interdisciplinary" planning problems can be made more relevant to the problems at hand.

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