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A STUDY OF THE DRY WEIGHT RANK METHOD OF BOTANICAL ANALYSIS

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ABSTRACT

The dry weight rank method was applied to data from eight grassland and shrub communities. These varied from the complex, rapidly changing early stages of old field succession in Tennessee to the simple, climax vegetation of the salt desert shrub community of western Colorado. Through computer analysis of these data sets, the variation in the results due to plot size, observer variability, and analysis options was determined. It was found that there was no statistically significant difference in predictions due to plot size or observer variability. Effect of using each of the analysis options is presented. The field and analytical study results indicate that dry weight rank is a suitable method of determining the botanical composition of the variety of communities to which it was applied. Suggestions are given for field use of the method. Subsequent studies were made with the ranking of species in sets of simulated plot data with known distributional characteristics. These studies suggested it was questionable if dry weight rank estimates converged and if the rise of the technique actually reduced variance estimates as compared to clipping alone. Our overall conclusion after widespread field usage, data analysis, and simulation studies was that the dry weight rank method, as we applied it, was inadvisable for further widespread use in our studies.

1.0 INTRODUCTION

Native plant species seldom grow in pure stands over appreciable land areas and these species vary with regard to their palatability, digestibility, toxicity, and nutrient value to grazing animals. Determination of plant composition and yield is not only justifiable, but also economically imperative in ecological and agricultural researches. A knowledge of plant composition, yield, and how these may be maintained or altered, is necessary for proper grassland management.

Early in the US/IBP Grassland Biome study numerous methodological investigations were undertaken on techniques to be utilized in the field, the laboratory, or in both types of studies in the program. These studies were reported to scientists within the program, and sometimes the methods were adapted for utilization throughout the Biome. In some instances, particularly with the present case, although the method was adapted it was later replaced by other techniques.

1.1 Available Botanical Analysis Methods

Some of the more common methods of determining botanical composition and yield are weight estimate, hand separation, constituent differential, and dry weight rank. These vary in accuracy, precision, and time requirements.

1.1.1 Hand separation

Hand separation is the most accurate, and for this reason it is used as the "standard" to which the other methods are compared. The method involves clipping all plots by plant species, oven-drying the samples, and determining from the dry weight the part of the total yield which is contributed by each species. Hand separation is accurate for determining the plant composition of each plot that is sampled, but many plots are

required to obtain a reasonably good prediction of the species proportions of a large area. In most plant communities, the number of plots required is large with the result that the hand separation method is monetarily costly and time consuming.

1.1.2 Procedures involving double sampling

The three remaining methods utilize a double-sampling procedure in which a fast but less accurate method is combined with a slower but more accurate procedure (National Academy of Sciences--National Research Council 1962).

The constituent differential method of estimating species composition consists of measuring the constituent concentration of a large sample from each plot and the species components from small samples taken at random from plots treated alike (Cooper et al. 1957). Species composition of the large sample is then computed using appropriate formulae. Oven-dry matter, calcium, or crude protein can be used as the constituents. When dry matter is used, not only must the samples be handled rapidly to avoid moisture loss, but also time of day, precipitation, and humidity may introduce error.

In the weight estimate method, observers undergo intensive training before sampling an area and weight of each plant species to the nearest 10 g is estimated on a plot basis (Pechanec and Pickford 1937). A variation of this method requires the assigning of percentages of the total weight to each species. Between 10% and 20% of all plots are both weight-estimated (grams or percentage) and hand-separated. A regression is run to derive an equation that will adjust for any error that might exist. This method requires much initial training and also continued training to maintain a reasonable degree of accuracy.

The dry weight rank method should not be confused with ranked-set sampling (McIntyre 1952). Although similarly named, the two do not determine the same values. Ranked-set sampling is a method of stratification of a population into subpopulations for increased sampling efficiency. Dry weight rank was developed and subsequently improved upon by Mannetje and Haydock (1963).

Field use of the dry weight rank method is simple. Observers need only to distinguish relative, not absolute, amounts of vegetation by species. A rank of 1 is assigned to the species which in the observer's opinion contributes the most to the total dry weight of the plot. The species which contributes the next highest amount to the total plot weight receives a rank of 2. This pattern of ranking continues until the plant species which contributes the least amount to the total weight receives a rank which is equal in magnitude to the total number of species on the plot. Some plots are both ranked and hand-separated in order to determine the relationship between ranking and actual weight proportions. The field and lab procedures and mathematical foundations are explained more fully later.

Dry weight rank can only determine the percent or proportion of the total vegetation dry weight that is contributed by each species. Some independent determination of the total herbage must be used with dry weight rank in order to get the weight yield by plant species.

Ranking, if it gives comparable results, is more advantageous to use than (i) hand separation because it is faster, (ii) constituent differential because it requires less laboratory work, or (iii) weight estimate because it requires less training and subjectivity. Once adequate multipliers have been derived, dry weight rank is nondestructive and with certain types of studies this is desirable.

1.1.3 Previous use of the dry weight rank method

Mannetje and Haydock (1963) found significant differences between exact-rank dry weight rank and hand separation for 3 of the 12 botanical components. The most serious of these was the underestimation by 4% of a component which comprised 69% of the total dry weight by hand separation and 65% by dry weight rank. No differences between mean values for four quadrat sizes from 1 up to 25 dm^2 were detected for any pasture component. From their data, the minimum number of quadrats required could not be determined, but Mannetje and Haydock felt that good results could be obtained by taking 50 to 100 quadrat estimates per acre. They were able to find affirmative answers to their questions about dry weight with the result that they were optimistic about future use of this method. It is now used widely in Australia (L. T. Mannetje, personal communication with G. M. Van Dyne).

Zorich (1966) evaluated the previously mentioned four methods of determining species composition on two perennial grass pastures in Oregon. On fescue pastures each of the methods proved to be equally good at estimating the botanical composition. Each of the four methods did not prove to be equally good when tested on the ryegrass pastures. Dry weight rank was especially poor in that it generally underestimated the ryegrass percentages. But Zorich used rank multipliers derived and reported by Mannetje and Haydock (1963). Zorich found the constituent differential method to be the most promising.

Opstrup (1968) made use of dry weight rank in determining the species composition of *Festuca* and *Andropogon* communities in Tennessee. The estimated values for each replicate were based on a combination of 20

clipped plots and 50 "ranked only" plots. The actual values were based on the 20 clipped plots. Opstrup concluded that estimates of species composition can be improved appreciably by using the dry weight rank method as an adjunct to clipping plots.

These studies suggest considereable gain in efficiency can be made using the dry weight rank method, but leave many questions unanswered regarding the adaptability, reliability, and precision of the method.

1.2 Rationale for Evaluation of a Statistical Estimator

Determination of fractional vegetation biomass in different plant species occurring on a geographical area can be regarded as a problem in determination of the best "statistical estimator" for a set of variables which are in general unobservable. The unobservable variables represent the means of the populations of vegetation biomass densities for plant species occurring on the area. Various observations can be made on the area and the observations can be interpreted as estimates of the unobservable variables.

We will regard one such observation (made by clipping the vegetation on a unit area and separating the material into species categories and weighing each category) as a "standard" method based upon sampling of a population which is for practical purposes infinite in extent. There will be variations in this estimator because of inherent variation of the population in space, and also components of variation because of the physical procedures involved in determining the weights. All of these sources will sum up to a certain total variability in the procedure. The statistical characteristics of the resulting species' weight estimates can be calculated

by taking several samples and deriving a mean vector and a covariance matrix for the estimates according to the standard statistical formulae.

Another method of estimating this vector of biomass fractions is the dry weight rank technique described in section 1.1.3. This technique has the disadvantage that the intitial numerical values obtained from each unit area or plot observed (ranks of each species) cannot be regarded as direct estimates of the biomass composition. Instead it is necessary to observe a number of plots and then go through a numerical technique involving the clipping of some of the plots to derive a vector which can be regarded as an estimate of the true population values. Thus it is not possible (except by subsampling) to utilize the dry weight rank technique to get an estimate of the covariance matrix; only an estimate of the mean vector can be determined. In dry weight rank technique the cost or personnel time requirement for sampling a plot is a small fraction of that for the clipping estimate; dry weight rank has an economic advantage over the clipping method if the characteristics of the estimate produced are acceptable.

1.2.1 The problem of bias

In the statistical sense an estimator for the characteristics of a population having some known or unknown statistical distribution function should have certain characteristics in order for it to be useful. One of these characteristics is unbiasedness. Intuitively unbiasedness means that if the estimator is repeated many times in statistically independent trials, the mean of the several estimators should approach the true population parameter as the number of trials increases. Mathematically, the expected value of an estimator should be equal to the population parameters being estimated. Another characteristic which a "good"

estimator should have is that of minimum variance over a certain class of possible estimators. Generally the minimization is performed with respect to various mathematical methods of making an estimation; however, here we are concerned with the variance also caused by the physical procedures involved in making the estimate. In particular, we are concerned with minimizing variance over only two possible estimation procedures.

1.2.2 Simulated plot data

The difficulty in evaluating unbiasedness or the relative variance of an estimation procedure is that the characteristics of the true population being sampled are rarely if ever known. This is certainly true in the case of estimating biomass fractions. In addition, there is a difficulty involved in the cost of producing estimates. It is an expensive procedure to go into the field and clip vegetation, and it is also expensive, though less so, to rank vegetation in the field. For these reasons we have decided to use a computational technique to produce possible estimates of biomass fractions based upon the two methods described above and based upon some assumptions about the statistical properties of the field samples. We shall assume that the amounts of biomass in the various species' categories, as measured by the clipping technique, are distributed according to a multivariate normal distribution function. Making this assumption, we can use a numerical procedure to generate data which are statistically indistinguishable from field data if the assumption is valid. Thus we have a relatively inexpensive method of producing great numbers of estimates based upon the clipping technique. By applying the dry weight rank procedure to groups of these artificial data, we also have a great number of corresponding estimates based on

this alternate technique. By repeatedly evaluating biomass fraction estimates based on the two techniques, we can examine the statistical properties of both estimators. The estimate based on clipping is a simple average of the normally distributed numbers produced, and its statistical properties are well known. Thus we use it as a basis with which to compare the hypothetical dry weight rank technique. Certain errors involved in the assumption of normal distribution for biomass estimates are discussed later in this paper.

2.0 OBJECTIVES

There are three primary objectives of the present research of field studies of dry weight rank method: (i) How well does the method predict percentage composition by species for an array of grassland communities? (ii) what observer variability is there? (iii) how are predictions affected by the various analysis options? Incidental to the primary objectives some of the data were used to determine: (iv) the effect of plot size on percentage composition prediction and (v) the prediction difference between field and laboratory ranking. This paper also reports on subsequent investigation into the efficacy of the dry weight rank technique by utilizing computer simulation techniques based on traditional statistical concepts of the "goodness" of a statistical estimator. Specific objectives were (i) to make a decision on the use of dry weight rank method as a biomass estimator technique in the Grassland Biome program of the International Biological Program and (ii) to investigate in general the properties of the dry weight rank technique.

3.0 METHODS

3.1 Field Sites and Methods

We applied the dry weight rank method to several pasture and range communities that varied from the early stages of old field succession in eastern Tennessee to the climax vegetation of the salt desert shrub community of western Colorado. Pertinent information about all sampled areas is given in Table 1. Species lists, including standing crop in pounds per acre for each species, for communities are given in Appendix A.

The range of tests of dry weight rank to which any one data set could be, and was subjected to, varied. Sets from Montana and South Dakota were collected for purposes other than dry weight rank. These two sets could be used only to test the prediction ability of perfect, known error, and randomly assigned ranks by assigning these ranks to the existing weight data. Tennessee data have been used to test the difference between using nonlinear programming and Lagrangian multiplier, least squares fitting of rank multipliers and the effect of lab vs. field ranking. Sandhills grass community data from Akron, Colorado, were used for only a few analyses because of the small numbers of ranks and species which precluded this data from being considered as representative. The remaining four Colorado data sets were utilized in most of the experimentation of analysis because they had many species and ranks, and data were replicated for one community.

Field procedures included randomly locating plots in experimental areas, ranking species by one or more observers, and clipping current annual growth. Details are reported by Hughes (1969).

Table 1. Description of plant communities with geographic and precipitation data for each.

Plant community	Location and/or nearby town	Elevation (feet above sea level	Annual precipitation (inches)	Number and size of plots	Number of species (all plots)	Average number and range of ranks per plot	Standing crop (mean and standard deviation in lb/acre)
Shortgrass mixed prairie	Cottonwood, S. D.	2400	15	168 - 6 ft^2	21	6.1 3 - 11	1795 ± 779
Shortgrass mixed prairie	Red Bluff Ranch, Norris, Mont.	5000	18	108 - 2 ft^2	31	9.5 6 - 15	1064 ± 606
Old field secondary succession	Oak Ridge, Tenn.	900	56	238 - 0.5 m^2	83	3.4 1 - 6	
Shortgrass sandhills	Eastern Colorado Range Exp. Sta., Akron, Colo.	4275	15	36 - 4.8 ft^2	5	4.1 2 - 5	856 ± 307
Shortgrass mixed prairie	Pawnee National Grassland, Nunn, Colo.	5100	12	30 - 50 × 50 cm 30 - 1 × 2 ft 30 - 25 × 25 cm 30 - 15 × 30 cm	30	4.7 1 - 7	1358 ± 997 1052 ± 674 2974 ± 2527 3452 ± 2779
Brush openings in pinyon-juniper	Little Hills, Meeker, Colo.	6100	14	40 - 1 m^2	32	7.2 3 - 12	883 ± 332
Salt desert	Badger Wash, Loma, Colo.	4500	8	40 - 1 m^2 40 - 1 m^2	15 15	3.8 1 - 7	592 ± 506 427 ± 429
Oak brush	Hesperus, Colo.	7600	19	40 - 1 m^2	29	7.3 3 - 14	1503 ± 1043

3.2 Least Squares, Lagrangian Multiplier Method

Dry weight rank has two sets of *knowns*: (i) the observed dry weight proportion of each plant species and (ii) the proportion of times that each species was assigned each rank. The observed dry weight proportion of each species is that fraction of the total dry weight of clipped vegetation of a set of plots which is attributable to each species. The weight fractions for all species will sum to 1.

For greater clarity, the proportion of times that a species receives a particular rank will be explained by an example. Let the species be blue grama and the rank be 1. Simply divide the number of plots on which blue grama receives a rank of 1 by the total number of plots taken. If blue grama receives rank 1 on 25 of 100 plots, then the proportion of times that blue grama receives first rank is 0.25. For a rank that occurs on all plots, the sum of the proportions for that rank will be 1. For example, rank 1 will occur on all plots because all the plots have at least some vegetation, if only a small quantity of one species. However, the sum of the rank proportions will probably not be 1 for some of the higher ranks. Possibly only one plot in a set of 100 would have as many as 10 ranks. The species receiving the rank of 10 would have a rank proportion for rank 10 of 0.01. This would also be the sum of the rank proportions for rank 10 because this value for each of all the other species is 0. Note that individual plot identity is lost and replaced by plot set identity with reduction of ranks and weights to rank and weight proportions.

This method has one set of *unknowns*--the multipliers by which the rank proportions for a species can be multiplied to obtain an accurate prediction of dry weight proportion for that species. Mannetje and

Haydock (1963) give the following mathematical equation showing the relationship between the knowns and the unknowns:

$$k_1x_{1ij} + k_2x_{2ij} + k_3x_{3ij} = p_{ij}$$

Only ranks 1, 2, and 3 are considered in their equation--more ranks are possible--the only requirement is that there be as many or more species than ranks ($i \geq j$).

Rank multipliers, the unknowns, are the k's. The proportion of times that i received a rank of 1 in the j^{th} set of plots is denoted by x_{1ij}. The rank proportions for this species for ranks 2 and 3 are, respectively, x_{2ij} and x_{3ij}. The proportion of the total dry weight of plot set j contributed by species i is given as p_{ij}. In solving for the values of $k_1, k_2, \ldots, k_n$, the constraint is made that these multipliers must sum to 1 or 100 (depending upon whether the rank proportions are given in terms of decimal proportions or percentage proportions).

With a least squares fit, the multipliers are obtained so as to minimize the weight proportions sum of squares deviation,

Σ(Observed weight proportions - Predicted weight proportions)^2.

The field data are converted into a matrix $\bar{R}$ and a vector $\bar{W}$. The $\bar{R}$ matrix contains the proportions of times that the i^{th} species receives the j^{th} rank, and the $\bar{W}$ vector is the proportion of the total dry weight that is attributed to the i^{th} species. The subscript i varies from i = 1, 2, ..., n where n is the total number of species that occurs on a set of plots and j takes on values between j = 1, 2, ..., m where m is the maximum number of ranks (species or species groups) that occurs on any one plot in a particular set of plots. The $\bar{R}$ matrix and the $\bar{W}$ vector are the numerical representations of the two known sets of values for the dry weight rank method. Using matrix algebra techniques, this matrix and vector will be manipulated to obtain a least squares estimate

of the rank multipliers. The transpose of $\bar{R}$ is obtained and will be symbolized by $\overline{RT}$. The dimensions of $\overline{RT}$ are m by n. A new matrix and vector will be formed: (i) $\bar{C}$ is an m by n matrix, which is the product of the premultiplication of $\bar{R}$ by $\overline{RT}$, and (ii) $\bar{D}$ is an m × 1 vector. the product of $\overline{RT}$ and $\bar{W}$. The constraint that the set of multipliers must sum to 1 is now applied.

This is accomplished by adding an equation of constraint which, in this case, is a row of 1's adjoined at the bottom of $\bar{C}$. However, this additional row of values for $\bar{C}$ has made the system of equations unsolvable. The system is made solvable by use of Lagrange's method (Taylor 1955) which requires the adjoining of a column of 1's to the right side of the original matrix. The coefficient, in the final solution, for this additional column of 1's is known as the Lagrange Multiplier. Other than to make the set of constrained equations solvable, its exact meaning or usefulness in this particular problem is unknown. The new element created by adjoining the row and column of 1's, c_{m+1m+1}, is given a value of 0. The constraint is also applied to $\bar{D}$, the weight vector, by creating a new element d_{m+11} whose value is 1. The vector of multipliers, $\bar{E}$, is obtained by pre-multiplying $\bar{D}$ constrained by the inverse of $\bar{C}$ constrained. In equation form, the solution is described:

$$\bar{E} = (\overline{RT}\cdot\bar{R} \text{ constrained})^{-1} \cdot (\overline{RT}\cdot\bar{W} \text{ constrained})$$

$$= \bar{C}^{-1} \text{ constrained} \cdot \bar{D} \text{ constrained}$$

Appendix B gives a sequential listing of the matrices obtained in the solution of rank multipliers for the simple five-species, five-rank eastern Colorado sandhills grass community.

Multipliers are first used to predict weight proportions for the data set from which they were derived. These predicted weight proportions by species are compared to the actual weight proportions by use of a

statistic which would be the square of the correlation coefficient (R) if there were no constraints. In the case at hand, R^2 (coefficient of determination) is a biased estimate of the variance of the predicted weight proportion which can be attributed to its linear regression on the actual weight proportions:

$$R^2 = \frac{\left(\Sigma(\hat{W}_i \cdot W_i) - \frac{\Sigma\hat{W}_i \cdot \Sigma W_i}{N}\right)^2}{\left(\Sigma(\hat{W}_i)^2 - \frac{(\Sigma\hat{W}_i)^2}{N}\right) \cdot \left(\Sigma(W_i)^2 - \frac{(\Sigma W_i)^2}{N}\right)}$$

where $\hat{W}_i$ = predicted weight proportion of the i^{th} species, W_i = actual weight proportion of the i^{th} species, and N = the number of species.

A more valid test of the multipliers was made when they were used to predict weight proportions of an independent set of data. Independent data sets were collected from the same area and vegetation type from which the multipliers were derived. A coefficient of determination (R^2) was calculated by the previously mentioned equation.

The "goodness" of the R^2 must be tested in a manner different from the common practice because of a small number of degrees of freedom. Through analysis of several data sets, a trend has become noticeable. If an R^2 of 0.95 or greater is obtained from the dependent data set, an R^2 of 0.85 or greater can be expected when the same multipliers are used on an independent data set. Such an R^2 calculated for the independent test is "statistically good." For most of our data sets considered, an R^2 in the range of 0.4 to 0.6 is significant at the 1% level.

3.3 Double Sampling and the Optimum Ratio

Dry weight rank utilizes a double sampling procedure. Specifically, ranking is very fast on a plot basis and clipping is relatively slow, but

also very accurate, in predicting botanical composition. Unlike double-sampling for weight estimation, the two methods incorporated in dry weight rank do not independently estimate the dry weight proportions. Ranking is dependent upon the multipliers that are obtained from the plots that are both ranked and clipped. The fast method is meaningless without the slow method.

In using a double sampling procedure, time cost should be minimized, but a reasonable level of accuracy must also be maintained. Calculation of the optimum ratio followed from Wilm, Costello, and Klipple (1944) as modified by Van Dyne, Glass, and Opstrup (1968). The optimum ratio incorporates "accuracy" in the form of the R^2 statistic and also time costs by considering the time required to perform the fast, fixed, and slow processes. The fast process involves ranking only, the fixed process is plot location, and the slow process is plot clipping and sample processing. Table 2 gives process time costs for the seven of the data sets included in this study. The equation for optimum ratio (OR) is:

$$OR = \sqrt{\frac{R^2}{1 - R^2} \cdot \frac{CS}{CT + CF}}$$

where OR = number of plots to be ranked only for each plot that is both ranked and clipped, CS = the total cost of ranking and clipping a plot and sample processing, CT = time cost of locating a plot, CF = time cost of only ranking a plot, and R^2 = the coefficient of determination between observed and predicted species dry weight proportions.

3.4 Nonlinear Programming Method

Nonlinear programming was employed to solve for the rank multipliers using as the objective function:

Table 2. Cost factors given in man-minutes for the slow, fixed, and fast processes of the dry weight rank method for seven shrub and grassland communities.

Community	Time cost		
	Slow	Fixed	Fast
Shortgrass mixed prairie, S. D.	22	1	2
Shortgrass mixed prairie, Mont.	45	1	3
Shortgrass sandhills, Colo.	45	1	3
Shortgrass mixed prairie, Colo.	35	1	3
	28	1	2
	23	1	2
	22	1	2
Pinyon-juniper	45	2	3
Salt desert	19	1	1
	15	1	1
Oak brush	75	1	2

$$Q_{min} = \sum_{i=1}^{n} (w_i - \sum_{j=1}^{m} k_j r_{ij})^2$$

with the constraints that,

$$\sum_{j=1}^{m} k_j = 1$$

$$k_j \geq 0$$

where Q_{min} = minimum sum of squares of deviations between observed and predicted species weight proportions, w_i = observed weight proportion of the i^{th} species, k_j = rank multiplier for the j^{th} rank, r_{ij} = proportion of times that the i^{th} species received the j^{th} rank, n = number of plant species, and m = number of ranks.

The objective function is formed such that a set of rank multipliers K, is determined while at the same time the sum of squares of deviations between observed and predicted dry weight proportions is held at a minimum, Q_{min}. These multipliers are subjected to the constraints that each multiplier is equal to or greater than 0 and that, when added, the multipliers sum to 1.

3.5 Procedure for Generating Simulated Data

In order to study the variance of the biomass percentage estimates made by the dry weight rank technique, in comparison with those made on a straightforward clipping procedure, the following algorithm was designed:

(i) Generate a set *M* of m vectors v_i (h × 1), i = 1, ..., m using a multivariate normal vector generator as described by Naylor et al. (1966). Each vector is drawn from a multivariate normal population having a mean vector μ (n × 1) and covariance matrix C (h × h). If

a component of ν_i is generated as less than zero, then substitute zero for that component. Thus we are using a truncated multivariate normal distribution. Each component of the vectors ν_i is interpreted as the biomass measured on a clipped plot in an area of homogeneous vegetation statistically described by mean μ and covariance matrix C (multivariate normal, truncated). Normalize the vectors ν_i by dividing each vector by the sum of its components. Thus each component of ν_i is a number between 0.0 and 1.0 and the components sum to 1. Now ν_i can be interpreted as a vector giving the fractional amount of biomass in each of h species. *M* represents all sampled plots whether ranked only or clipped and ranked.

(ii) For a subset $C \subset M$ consisting of n of the m vectors in *M*, generate vectors γ_i, i = 1, ..., n in the following manner. Let γ_{ij} and ν_{ij} be the j^{th} component of the vectors γ_i and ν_i respectively. Let $\gamma_{ij} = 1$ if ν_{ij} is the largest component in ν_i, let $\gamma_{ij} = 2$ if ν_{ij} is the second largest component of ν_i, etc. Thus γ_i is a vector of the *ranks* of the relative species biomasses in ν_i with the largest biomass being given rank 1. The vector γ_i will be called a *rank vector*. *C* represents those plots which are both clipped and ranked. *M - C* represents plots which are ranked only.

(iii) Form a matrix R_1 (h × h) whose components are r_{ij}. Let r_{ij} be $\frac{1}{n}$ times the number of times the i^{th} component of the vectors in *C* received the j^{th} rank. R_1 is called a *rank matrix*.

(iv) Calculate the *average weight vector based on clipping* ω_1 (k × 1): $\omega_1 = \frac{1}{n} \sum_{i=1}^{n} \nu_i$.

(v) Form the *augmented rank matrix* R_1^* (k+1 × k+1) by adding a row and a column of ones, with a zero in the last diagonal position, to R_1:

$$R_1^* = \begin{bmatrix} R_1 & \delta \\ \delta' & 0 \end{bmatrix} \qquad \text{where} \quad \underset{(h\times 1)}{\delta} = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$

Reasons for this augmentation will be discussed below.

(vi) Find a *vector of multipliers* ε^* $(k+1 \times 1)$ such that $R_1^* \varepsilon^* = \omega_1^*$ where ω_1^* $(k+1 \times 1)$ is formed from ω_1 by adding a 1 as the $k+1^{st}$ component

$$\omega_1^* = \begin{bmatrix} \omega_1 \\ 1 \end{bmatrix}.$$

If R_1^* is non-singular, then $\varepsilon^* = R_1^{*-1} \omega_1^*$. There is no guarantee that R_1^* will be non-singular; in fact a certain proportion of the possible R_1^* matrices are certain to be singular. This point will be discussed later.

(vii) Form a matrix R_2 $(h \times h)$ in a manner analogous to the formation of R_1 in (iii); however, R_2 is to be based on the vectors in $M - C$ rather than C. Thus R_2 is a summary of the ranking information on the ranked only plots. Also form R_2^* analogously to the formation of R_1^* in (v).

(viii) Calculate $\omega_2^* = R_2^* \varepsilon^*$. The ω_1 and ω_2 (formed from ω_1^* and ω_1^* by deleting the last component) are two simulated estimates of the fractional biomass in an area of homogeneous vegetation. The ω_1 is based on clipping n plots; ω_2 is based on clipping and ranking the same n plots, then ranking m - n more plots and applying the dry weight rank technique.

(ix) Repeat (i) through (vii) for k times, each time using the same mean vector μ, and covariance matrix C, but generating independent sets M. Each time (i) through (vii) are done, a *simulation* has been performed for a total of k simulations. After each simulation, calculate the mean and variance of each component of ω_1 and ω_2 based on all the simulations done so far.

The algorithm described above was programmed in FORTRAN for the CDC 6400 computer and run for various combinations of μ, C, m, n, and k.

4.0 RESULTS

The subjection of every data set to all possible options and facets of analysis was considered neither practical nor necessary. Only a few example results are presented herein from the many analyses that were made.

4.1 Dependent testing

A gross answer to the question "How well does dry weight rank work on the vegetation types considered?" is given in Tables 3, 4, 5, and 6. The tables contain results of previously explained dependent testing. In each table the column of most validity and usefulness here is the one presenting the coefficient of determination (R^2) obtained when the data were used in simple form (unweighted) with the rank matrix columns not forced to sum to 1. The remaining columns in these tables will be explained later.

Analysis of variance was computed from R^2's for the data sets given in Tables 3, 4, 5, and 6 and showed no significant difference at the 5% level between results obtained from the three rankers or between the four shrub communities of western Colorado. Similarly, there was no statistically significant difference which can be attributed to the differences between the two rankers or the four plot sizes used on the shortgrass prairie of the Pawnee National Grassland in Colorado. There is no significant difference between field ranking versus laboratory ranking or between the three rankers for the Tennessee data. There is a highly significant difference between the 14 field locations in Tennessee. This is because the field locations varied greatly with respect to plant complexity. In field locations with a minimum of one or two

Table 3. Western Colorado shrub. Comparison of three rankers, three shrub communities, and four analysis options giving the coefficient of determination (R^2) and optimum ratio (OR).

Shrub community and ranker	Rank matrix columns may not sum to 1				Rank matrix columns sum to 1			
	Simple data		Weighted data		Simple data		Weighted data	
	R^2	OR	R^2	OR	R^2	OR	R^2	OR
Pinyon-juniper								
A	0.950	13	1.0	847	0.948	13	1.0	847
B	0.963	15	1.0	715	0.967	16	1.0	707
C	0.983	23	1.0	1163	0.983	23	1.0	1163
Salt desert 2								
A	0.975	19	1.0	1273	0.979	21	1.0	1296
B	0.945	13	0.988	28	0.945	13	0.989	29
C	0.916	10	1.0	1410	0.926	11	1.0	1393
Salt desert 3								
A	0.866	7	1.0	480	0.861	7	1.0	481
B	0.946	11	1.0	1135	0.974	17	1.0	479
C	0.930	10	1.0	1274	0.933	10	1.0	1253
Oak brush								
A	0.962	25	1.0	5030	0.962	25	1.0	5022
B	0.913	16	1.0	2304	0.913	16	1.0	2304
C	0.926	18	1.0	5415	0.923	18	1.0	5426

Table 4. Pawnee National Grassland. Comparison of two rankers, four plots sizes, and four analysis options giving the coefficient of determination (R^2) and optimum ratio (OR).

Ranker and plot size	Rank matrix columns may not sum to 1				Rank matrix columns sum to 1			
	Simple data		Weighted data		Simple data		Weighted data	
	R^2	OR	R^2	OR	R^2	OR	R^2	OR
Ranker A								
50 × 50 cm	0.996	46	1.0	9694	0.996	47	1.0	9695
1 × 2 ft	0.992	35	1.0	103781	0.991	33	1.0	104539
25 × 25 cm	0.951	12	1.0	977	0.946	12	1.0	856
15 × 30 cm	0.973	17	1.0	2965	0.945	11	1.0	2965
Ranker B								
50 × 50 cm	0.982	22	1.0	9446	0.985	24	1.0	9434
1 × 2 ft	0.984	24	1.0	58209	0.984	24	1.0	58297
25 × 25 cm	0.988	25	1.0	19083	0.988	25	1.0	19177
15 × 30 cm	0.967	15	1.0	4023	0.966	14	1.0	4023

Table 5. Eastern Colorado sandhills. Comparison of three rankers and true ranks giving four analysis options.

Ranker and ranks	Rank matrix columns may not sum to 1				Rank matrix columns sum to 1			
	Simple data		Weighted data		Simple data		Weighted data	
	R^2	OR	R^2	OR	R^2	OR	R^2	OR
Ranker								
A	0.983	25	1.0	905	1.0	2732	1.0	2372
B	0.958	16	0.999	188	1.0	2732	1.0	2372
C	0.914	11	0.999	87	1.0	2732	1.0	19,895,331
True	0.994	43	1.0	747	1.0	2732	1.0	2372

Table 6. Tennessee: Comparison of three rankers, laboratory versus field ranking and 14 field locations giving the coefficient of determination (R^2) between predicted and observed dry weight proportions.

Field location	No. of plots	No. of species	No. of ranks	Field			Lab		
				Ranker			Ranker		
				A	B	C	A	B	C
A	10	22	5	0.903	0.938	0.950	0.947	0.878	0.867
B	10	22	6	0.866	0.963	0.700	0.979	0.965	0.964
C	11	22	4	0.780	0.808	0.747	0.784	0.766	0.749
D	12	25	4	0.964	0.968	0.938	0.974	0.968	0.940
E	12	26	4	0.879	0.884	0.893	0.918	0.905	0.863
F	16	24	4	0.925	0.930	0.926	0.916	0.911	0.881
G	18	23	3	0.978	0.994	0.859	0.978	0.968	0.962
H	21	21	3	0.952	0.972	0.942	0.965	0.821	0.985
I	18	17	3	0.951	0.888	0.927	0.933	0.770	0.932
J	24	23	3	0.980	0.971	0.986	0.986	0.976	0.988
K	23	24	3	0.860	0.941	0.826	0.835	0.948	0.901
L	13	14	3	0.994	1.000	0.999	1.000	1.000	0.999
M	25	17	2	0.979	0.974	0.989	0.995	0.973	0.987
N	25	7	1	0.990	0.940	0.967	0.976	0.935	0.974

species per plot, the R^2's ranged in the 0.90's whereas locations with a minimum of four to six species per plot gave R^2's in the 0.70 to 0.90 range for the dependent test. Plot sets with a minimum of one or two species are unrealistic and probably should not be considered in the analysis. If these were not considered, there would probably be no difference due to field locations.

These results suggest that dry weight rank is relatively insensitive to reasonable amounts of ranker and community variability. Plot size within the range of sizes used has no adverse effect on accuracy. Also, plots can be clipped and brought to the laboratory for ranking without loss of prediction accuracy.

4.2 Forcing the Rank Matrix Columns to Sum to 1

Complexity of communities in the present studies ranged from the salt desert shrub community with 16 plant species to the pinyon-juniper with 32 plant species (Table 1). Individual plots within a community were quite variable with regard to the number of plants to be ranked. Maximum variability occurred in the oak brush community with plots having between 3 and 15 rankable plant species. For this reason, all ranks could not be filled on each plot. Mannetje and Haydock's (1963) recommendation (but without justification) with regard to the simple community with which they worked was that "when the three ranks are not all filled in one or more quadrats, the calculations must be carried out on number instead of proportion of quadrats." Their recommendation was not followed, but two alternative ways of dealing with unfilled ranks were tested.

In a properly filled rank proportion matrix, the columns sum to 1 because every rank occurs on every plot. This matrix, for each of the

communities now being discussed, never contained a complete set of columns that summed to 1. In fact, columns beyond rank 3 seldom sum to 1. Rank matrix columns were forced to sum to 1 by basing calculations for a particular rank upon the number of plots that received that rank instead of the total number of plots in a data set. To determine the effect that this has on prediction, R^2 from data that were forced to have all columns sum to 1 is compared in Tables 3, 4, 5, and 6 to the same data that were not constrained in this manner.

There is no consistent significant difference between the constrained and non-constrained data with regard to prediction ability.

4.3 Weighting the Data

The effect of many ranks and species, and the resulting unfilled ranks, on the accuracy of prediction was also tested by another means. This was accomplished by using the weight proportion of each species as a weighting factor. The rank proportions and actual weight percentage for a species were multiplied by the weighting factor. In effect, the data were weighted in favor of the heavier species. Hopefully, the accuracy of prediction would be improved by making the important species numerically more important and the insignificant species only traces. Comparison of weighted versus nonweighted (simple) data is given in Tables 3, 4, 5, and 6.

The tables indicate that weighting the data significantly increases the value of R^2. A value of 1 is common for this statistic. It should be noted that the R^2 is calculated from the weighted data, before it is reconverted into usable dry weight percentages. Converted percentages, actual percentages, and unweighted data percentages along with R^2 relating these are given in Table 7.

Table 7. Comparison of actual dry weight percentages (Column 1) to predicted percentages using simple data (Column 2) and weighted data (Column 3).[a/] Predictions from weighted data have been unweighted to put them into form comparable to Columns 1 and 2. Values are from a dependent test of dry weight rank from Badger Wash, Ranker A.

Species	Actual (%)	Predicted (%)	
		Simple	Weighted
Atriplex confertifolia	46.9	38.3	46.9
Chrysothamnus viscidiflorus	22.5	26.3	22.4
Gutierrezia sarothrae	9.93	6.90	9.47
Hilaria jamesii	5.16	12.0	13.4
Atriplex nuttallii	4.88	2.34	2.26
Eriogonum sp.	4.01	3.54	4.85
Oryzopsis hymenoides	1.76	6.13	10.1
Salsola kali	1.76	-3.38	-6.27
Unidentified forb	1.49	3.50	2.88

[a/] R^2 between actual and simple = 0.866

R^2 between actual and weighted = 0.828

R^2 between actual and weighted (before unweighting) = 1.00

It seems that weighting increases the accuracy of prediction for the three or four dominant species at the expense of the lighter, rarer ones. This loss of accuracy on the lighter species probably is not important for two reasons. At best, prediction of these species may be far from accurate with any method. If the amount of herbage is the objective of determination, these light species are unimportant.

4.4 Subset Matrix Alterations

Subset rank matrices were constructed to be the same size as the one containing all the data from a set. If for a given subset a particular rank or species never occurred, the column corresponding to the rank or the row corresponding to the species was made up completely of zeros. Some subsets contained many such columns and rows. It was decided to test effect on R^2 of removal of these columns and rows. An example of the effect of collapsing the rank matrix on the R^2 value obtained from subsets is given in Table 8.

Some of the smaller but unaltered subsets gave extremely high R^2 values. This is because species not occurring in the subset had zero weight and this could be predicted with 100% accuracy. Collapsing the matrix as shown by the lower R^2's in the right-hand column of Table 8 eliminates this problem.

4.5 Imposition of Known Error and Random Ranking

Two data sets were altered so that known error could be injected into a correctly ranked data set. Then this newly created, known error data were analyzed. The correct data are known as 0% ranks and the error sets are 5%, 10%, and 15% ranks. The 5% ranks will be used as an example to explain the method of creating known error.

Table 8. Comparison of results given in terms of coefficient of determination (R^2) and optimum ratio (OR) obtained when subset rank matrices were collapsed opposed to subset rank matrices that were not collapsed. Data from Badger Wash, Ranker C.

Collapsed matrix			Non-collapsed matrix		
No. of plots	R^2	OR	No. of plots	R^2	OR
17	0.661	5	15	0.970	15
18	0.604	4	17	0.946	11
24	0.850	8	18	0.880	7
25	0.939	13	18	0.940	11
27	0.485	3	18	0.936	10
31	0.950	15	20	0.923	10
33	0.947	14	27	0.888	8
35	0.899	10	28	0.892	8
37	0.951	15	33	0.839	6
40	0.930	12	40	0.930	10

Weights of all species on each plot of the 0% ranks data were compared to each other. If two weights were within 5% of each other in magnitude, the ranks corresponding to these weights were switched one-half of the time. Theoretically, this simulates the ability of a ranker to correctly rank 50% of the time when a $\leq$5% difference occurs in the weights of two plant species. The 10% and 15% ranks similarly simulate the ability of a ranker to correctly rank one-half the time plants which differ in weight by 10% and 15%.

The R^2 values increase with an increase in ranking error (Table 9). This increase is illogical even though it is not statistically significant. A check of the data sets indicates that ranking has actually been changed little at the extreme 15% error level. Even so, logically the trend of values should be reversed.

The known error results led to further analysis of a similar type. Ranks were randomly assigned to species for a set of data. The results of this random ranking and an independent test of the "random" multipliers show a reasonably good least-squares fit of the multipliers which can be obtained through the random ranking, but the multipliers thus obtained are inadequate in the independent test (Table 10). The independent test and examination of predicted and observed weight proportions does indicate, as expected, that the frequency of occurrence of a species, i.e., the number of times that it receives all ranks in a data set, is correlated with its dry weight proportion for that data set. The R^2 obtained from the independent test of the random multipliers for the two data sets is 0.47 in both cases which is significant at the 1% level for the shortgrass community.

These results suggest that if the rank matrix were properly constructed, percentage composition could be predicted if frequency data alone were used.

Table 9. The effect of known error on ability to predict weight proportions as measured by coefficient of determination (R^2) and optimum ratio (OR).

Percentage error	Montana (108 plots)		South Dakota (58 plots)	
	R^2	OR	R^2	OR
0	0.995	48	0.997	45
5	0.996	53	0.998	60
10	0.996	53	0.998	57
15	0.997	62	0.999	79

Table 10. Comparison of random ranking to some standard of ranking for the data set using the coefficient of determination (R^2) and the optimum ratio (OR) as comparison statistics.

Community	Ranker	Deriving data			Independent data		
		No. of plots	R^2	OR	No. of plots	R^2	OR
Salt desert	True	40	0.933	10	40	0.747	5
	Random	40	0.730	6	40	0.477	3
Montana shortgrass	True	40	0.984	26	68	0.802	6
	Random	40	0.912	11	68	0.474	3

used. Similarly, dry weight rank deletes correlation between the ranks assigned and the weights that each represents and replaces it with a correlation between rank proportions and weight proportions for each species. By doing this, the method is insensitive to a few errors on the part of the ranker. In fact, it has also been observed that the ranker can be consistently biased and yet the method continues to give good predictions. For example, plant litter was included as a species on the plots taken on the shortgrass prairie near Nunn, Colorado. It accounted for 38% of the total weight for the 50 × 50 cm set of plots while blue grama amounted to 31%. The rankers consistently but wrongly ranked blue grama first on 27 of the 30 plots while they similarly ranked litter second on 23 plots. The ranks should have been reversed on almost every plot, and yet dry weight rank was able to fit multipliers to the data so that litter was predicted to be 38% of the total weight. Granted, this incorrect ranking might not work as well in a community dominated by more than two or three plant species. This does indicate, however, that incorrect ranking is not always penalized and that when penalized, it is in varying degrees.

4.6 Solution of Negative Predictions and Predictions That Do Not Sum to 1

In every data set analyzed, a few negative weight proportions have been predicted, but intuitively and biologically they are impossible. These negative values are an artifact of the mathematics involved in the solution for the rank multipliers. The constraint that the multipliers sum to 1 has forced some of the multipliers to be negative to off-set the positive multipliers that sum to greater than 1. These negative multipliers are generally for the higher ranks (usually rank 4 or larger).

Forcing the multipliers to sum to 1 has not in like fashion forced the predicted dry weight proportions to sum to 1. This also is disturbing even though usually only species of minor importance are thus affected.

The negative predicted weights cannot be allowed to remain negative. Column 1 of Table 11 gives the observed weight proportions by species; column 2 is the predicted proportions, including some negative predictions; column 3 indicates the predicted proportions that were obtained using the first technique for eliminating negative proportions. The correction technique is as follows: (i) Column 2 was searched for the most negative value that it contains which is in this example -0.1581, (ii) this amount was added to each entry in that column, and (iii) the column thus created was scaled so that it summed to 1. Column 3 is merely a scaling of column 2 and thus its R^2 value of 0.801 with the observed values of column 1 is the same as that between column 1 and column 2. Column 3 shows that accuracy of predictions for important species has been reduced even though the R^2 has been maintained. Negative predictions must be eliminated in some other way.

Fortunately, negative predictions are usually made for species that individually make up less than 1% of the total weight. This percentage is smaller than the resolution of the dry weight rank method. For this reason, all negative predictions could be increased to zero or an arbitrarily small positive value. The results of this alteration are given in column 4 of Table 11. The R^2 of 0.849 between column 1 and column 4 is a slight improvement over the R^2 of 0.801 between column 2 and column 3. The predicted weight proportions are acceptable. All species below the resolution of the method (perhaps 5% of the total weight) could be lumped into a miscellaneous category. Acceptable weight percentages could be

Table 11. Comparison of observed species weight percentages (Column 1) and three sets of predicted percentages (Columns 2, 3, and 4).[a/] Column 2 is the set of predictions obtained with simple data and rank matrix not forced to sum to 1. Column 3 is a scaling of Column 2 such that all values are equal to or greater than 0 and they sum to 1. Column 4 is identical to Column 2 except that all negative values have been replaced by 0.00. Values are from an independent test of 50 × 50 cm Pawnee National Grassland plots using multipliers from 25 × 25 cm plots. The various predictions are compared to the observed percentages using the R^2 statistic.

Species	Column			
	1	2	3	4
1	38.9	59.0	16.7	59.0
2	31.2	18.9	7.74	18.9
3	21.0	19.9	7.98	19.9
4	3.11	2.76	4.15	2.76
5	2.29	0.494	3.64	0.494
6	1.21	2.77	4.15	2.77
7	0.669	-4.16	2.60	0.00
8	0.476	0.580	3.66	0.580
9	0.286	1.78	3.93	1.78
10	0.216	-0.377	3.44	0.00
11	0.119	-0.562	3.41	0.00
12	0.108	3.96	4.41	3.96
13	0.097	1.69	3.91	1.69
14	0.092	0.958	3.74	0.958
15	0.059	0.580	3.66	0.580
16	0.043	1.40	3.84	1.40
17	0.038	0.958	3.74	0.958
18	0.027	2.44	4.07	2.44
19	0.022	-15.8	0.000	0.00
20	0.022	1.69	3.91	1.69
21	0.005	1.69	3.91	1.69
22	0.005	-0.562	3.91	0.562

[a/] R^2 between Column 1 and 2 = 0.801

R^2 between Column 1 and 3 = 0.801

R^2 between Column 1 and 4 = 0.849

summed and subtracted from 100%. The percentage resulting from this subtraction would be given to the miscellaneous group. For the example given here, the latter method is preferable for handling negative predicted dry weight percentages and predictions that do not sum to 1.

4.7 Nonlinear Programming vs. Least Squares, Lagrange Multiplier Method

A data set was subjected to analysis by both the method of nonlinear programming and the method of least squares, Lagrangian multipliers. Rank multipliers derived were almost exactly the same and the two R^2's obtained were 0.866 and 0.867. These results indicate that the two analyses give results that are not significantly different and thus either method would work equally well.

4.8 Reducing the Number of Rank Levels

The three western Colorado shrub community data sets for Ranker C were subjected to analysis to determine the results if only the important plant species were ranked. Systematically, the number of ranks was reduced by one rank at a time until only four ranks were left. Two separate simulations were run. In the first, weights for the omitted ranks were allowed to remain in the weight vector. This would indicate that all vegetation on a plot was clipped, but that only important species were ranked. In the second simulation, weights were omitted at the same time that corresponding ranks were omitted, thus simulating the case where only important species were ranked and clipped.

Reducing ranks also reduced the R^2 (Table 12) probably because of at least two factors. If the full weight vector is maintained, some species which never receive a rank as high as the cut-off rank level have a predicted

Table 12. Changes in coefficient of determination (R^2) as a result of decreasing the number of rank levels used on three shrub communities. Column 1 in each case is the R^2 obtained when ranks were dropped but the weight vector was maintained. Column 2 is the R^2 obtained by dropping ranks and weights corresponding to these ranks.

Number of ranks	Pinyon-juniper		Desert 2		Desert 3		Oak brush	
	1	2	1	2	1	2	1	2
14							0.926	0.926
13							0.924	0.925
12	0.983	0.983					0.923	0.923
11	0.983	0.983					0.917	0.917
10	0.983	0.983					0.917	0.917
9	0.983	0.983					0.916	0.917
8	0.968	0.968					0.916	0.916
7	0.965	0.966	0.916	0.916			0.914	0.915
6	0.929	0.956	0.869	0.867	0.930	0.930	0.908	0.909
5	0.927	0.952	0.864	0.864	0.926	0.926	0.902	0.901
4	0.915	0.945	0.860	0.858	0.927	0.927	0.897	0.894

percentage of zero, and yet the observed weight vector has a positive value for these species. Thus, the variance between the observed and predicted percentages increases and the R^2 is lowered. A second factor, which was mentioned by Mannetje and Haydock (1963), is also causing this reduction in accuracy of prediction. They noted that as the number of ranks is increased, the sensitivity of the method increases. A small number of ranks will not do an adequate job of predicting the percentage composition, especially in plant communities with many species.

4.9 Laboratory vs. Field Ranking

There are several advantages of ranking species after plots have been clipped and samples brought into the laboratory if accuracy is comparable. Field time often is more costly than laboratory time if one considers adverse weather and travel expenses. Field work should be limited to those activities that can be accomplished only, or less expensively, in the field. If plots contain tall, thick vegetation, laboratory ranking is also more likely to reveal minor species that would go undetected in the field. In addition, more representative subsamples of each plot can be taken in the laboratory for species separations with a corresponding reduction in time requirements. The laboratory ranking in this test utilized a 10% aliquot of each field plot taken. There is a 0.99 correlation between predictions for each of the two ways of ranking and the observed dry weight proportions (Fig. 1). The indication is that laboratory and field ranking are equally good and thus plots should be ranked where it is the cheapest and most convenient.

4.10 Effects of Dominant and Troublesome Species

Plant litter was considered as a category to be ranked in the shortgrass prairie on the Pawnee National Grassland, and because it was

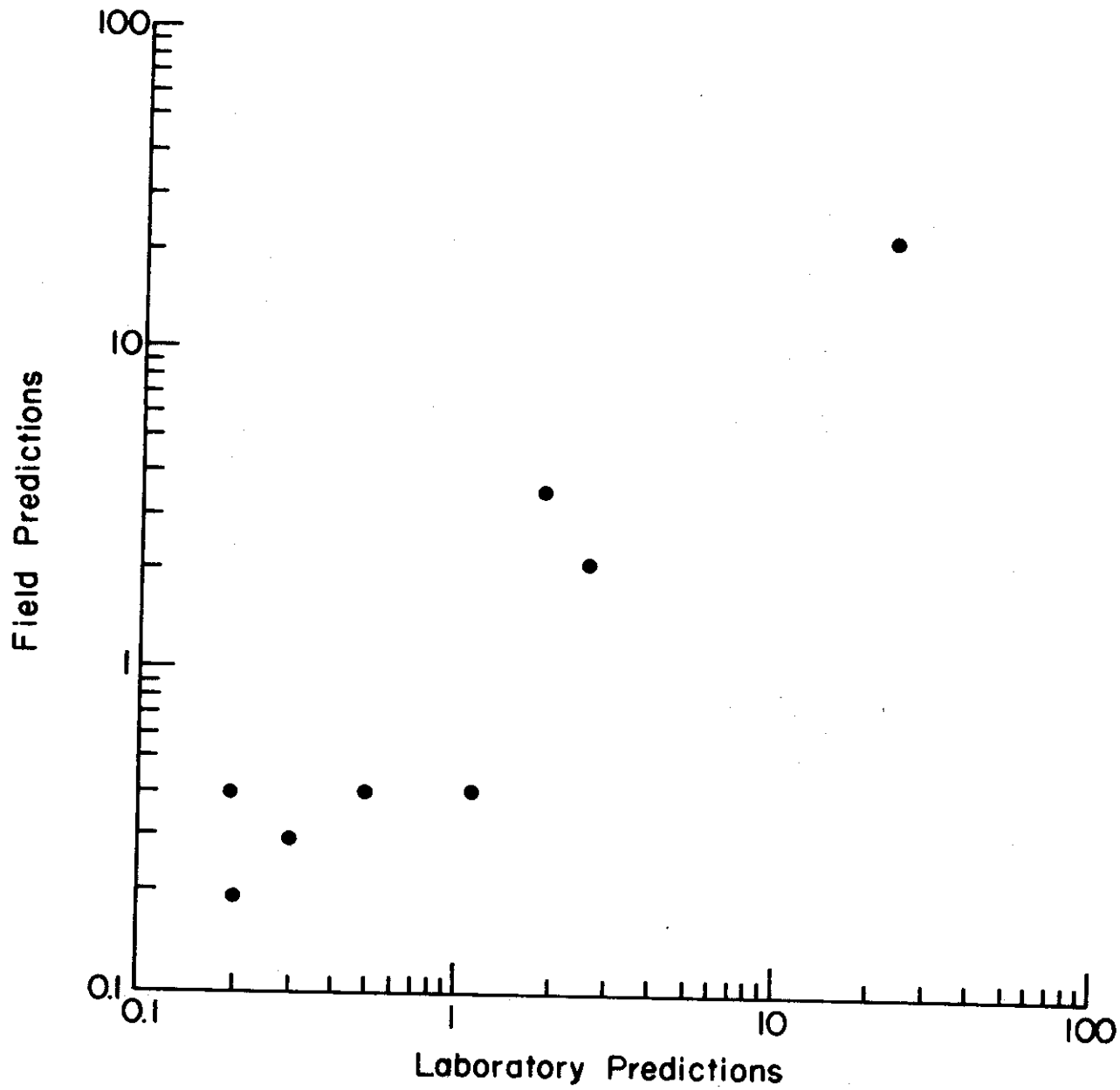

Figure 1. Predicted dry weight percentages by species for laboratory ranking of 1964 Tennessee data is plotted on the x-axis with predictions from field ranking of the same plots given on the y-axis. Predictions too small to round to 0.1% have been omitted.

always the most important component, it could affect results obtained using the dry weight rank method. The R^2 were calculated between observed and predicted dry weight proportions for plot sets with litter and for these same data sets with the litter category eliminated. In every comparison, the plot set with litter included has an R^2 higher than the same plot set with litter excluded (Table 13).

Prediction of litter was highly accurate with the result that inclusion of litter increased the sums of squares due to regression and consequently the R^2. Litter serves as the example for any plant species that is clearly dominant and maintains its rank position. Its percentage of the total composition can be predicted with good accuracy and thus such species contribute significantly to the regression sum of squares and to the R^2.

4.11 Independent Testing

The usefulness of the dry weight rank method lies in the ability of a derived set of multipliers to be used on subsequently collected rank data and accurately predict dry weight proportions. Several data sets were subjected to a test of the previously derived multipliers. To give equal basis for comparison, multipliers were fitted to a subset of unweighted data in which the columns of the rank matrix were not forced to sum to 1. These multipliers were then used to obtain predicted weights for the independent data set, i.e., the plots remaining after the original subset was taken.

The vegetation types tested in this manner give reasonably good results with R^2's for the independent data set in the range of 0.768 to 0.982 (Table 14). These R^2's are sound and are based upon one dependent

Table 13. Comparison of results with plant litter included and with it excluded on the same plots for the Pawnee National Grassland data using coefficient of determination (R^2) and optimum ration (OR) as comparison statistics.

Plot size	With litter		Without litter	
	R^2	OR	R^2	OR
50 × 50 cm	0.996	46	0.993	34
1 × 2 ft	0.992	35	0.952	14
25 × 25 cm	0.951	12	0.889	8
15 × 30 cm	0.971	17	0.924	9

Table 14. Test of multipliers from deriving data on independent data using coefficient of determination (R^2) and optimum ratio (OR) as testing statistics.

Community and location	Deriving data			Independent data		
	No. of plots	R^2	OR	No. of plots	R^2	OR
Oak brush	15	0.873	13	25	0.870	13
Salt desert	40	0.870	8	40	0.768	6
Pinyon-juniper	23	0.948	13	17	0.862	8
Montana (0%)	40	0.984	26	68	0.802	6
Montana (15%)	27	0.995	48	27	0.942	14
Montana (15%)	27	0.995	48	27	0.968	18
Montana (15%)	27	0.995	48	27	0.943	14
South Dakota	58	0.996	45	51	0.982	20
Pawnee National Grassland	30	0.951	12	30	0.801	6
Eastern Colorado	18	0.966	18	18	0.920	12

and one independent variable. Dry weight rank may need more testing on independent data sets. The independent data sets used here were small in size, from the same general plot locations, and collected at the same time as the data from which the multipliers were derived. Further testing should involve independent data collected throughout the year, collected in other areas having the same vegetation type, and collected over several years. This, however, is a general problem in applying prediction to to independent data sets.

4.12 Tests for Bias in Simulation Studies

In any computer simulation procedure it is essential to determine that the computer program (and the algorithm itself) are behaving correctly according to the desired characteristics of the simulation. This is especially important in this case because of the truncated normal distribution function utilized in the simulation.

Though the assumption of normal distribution in field data of the type being simulated is frequently made, this distribution can only approximate the actual distribution of data of this type. This is because any variable distributed normally will have a finite probability of having a negative value which is impossible for biomass estimates. The large variances usually experienced in field situations make this a practical as well as theoretical problem. Thus it was necessary to truncate the distribution function by assuming that the biomass value generated was zero whenever the normal distribution number generator gave a value less than zero. This assumption is not as far from reality as it might seem for there are frequent cases in field data where one or more of a set of species will not be found in each of the plots measured. A more logical procedure would have been to statistically evaluate the precise

distribution function of a measured set of data and determine the approximate percentage of zero biomasses for each of the given groups of species and generate the simulation data accordingly. However this would have involved more time and money than was available at the outset of this study.

The unbiasedness of the dry weight rank and clipping estimates has been evaluated by generating data sets according to a specific set of distribution parameters. The approach of the estimates to the true population parameters (the set) was tested by developing a graph of parameter values as a function of number of simulations. When the estimates were examined on a fine scale there was some question as to whether or not the dry weight rank estimate was converging exactly. However, any bias in the estimate is small compared to the errors usually inherent in plant biomass estimations. We have assumed that the dry weight rank procedure is unbiased for practical purposes. We assumed a covariance matrix with zero non-diagonal entries because of computational difficulties associated with the determination of parameters for the truncated distribution. For a data set generated with non-zero covariances, the parameter determination would have involved further investment in computer time.

4.13 Reduction in Variance with the Dry Weight Rank Method

For subsequent analysis a mean vector and covariance matrix calculated from actual field data was utilized for input to the simulation program. This is given in Table 15 and is taken from data reported by Hughes (1969). This data involves six species of plants. The variances for some of the species were quite high so there was considerable truncation involved in the simulation. The cost of a simulation run for a reasonable number of simulations turned out to be quite high as a result of the fact that

Table 15. Statistical parameters used in generating truncated multivariate normal random vectors. Taken from Hughes (1969). Biomass is in mg/ha.

Mean Vector					
339.69846	27.75077	23.34308	10.63231	6.40154	11.32615
Variance-covariance matrix					
9243.966	-878.342	-2622.284	74.051	-37.863	333.115
-878.342	1892.016	-354.069	-27.072	139.458	-84.300
-2622.284	-354.069	2235.391	-128.037	-43.181	-102.324
74.051	-27.072	-128.037	381.849	-27.078	267.836
-37.863	139.458	-43.181	-27.078	35.274	-24.362
333.115	-84.300	-102.324	267.836	-24.362	739.632
Correlation matrix					
1.00000	-0.21003	-0.57686	0.03941	-0.06631	0.12740
-0.21003	1.00000	-0.17217	-0.03185	0.53983	-0.07126
-0.57686	-0.17217	1.00000	-0.13858	-0.15378	-0.07958
0.03941	-0.03185	-0.13858	1.00000	-0.23331	0.50398
-0.06631	0.53983	-0.15378	-0.23331	1.00000	-0.15083
0.12740	-0.07126	-0.07958	0.50398	-0.15083	1.00000

a 6 × 6 matrix had to be inverted for each simulation. It was also for this reason that data with more than six species were not chosen for this simulation. Three hundred simulations (k = 300) were chosen as a compromise between too many simulations with resulting high computer costs and too few simulations with inadequate degrees of freedom for statistical testing of resulting variances.

Table 16 shows results of simulation runs presented as ratios of the variances (variance of dry weight rank estimate/variance of clipping estimate) of estimates resulting from 300 simulations. The table shows the results for two of the six species; results for the other four species were not qualitatively different. However, we will discuss only the first, which is the most abundant, since the other tables depict essentially the same results. As described earlier the objective of the dry weight rank technique is to achieve an estimation of biomass fractions having a lower variance than the estimates based on clipping techniques for equivalent amounts of work put into the two techniques. For purposes of the data presented in Table 16 we have assumed that there is a ratio of 10:1 in the cost of ranking a field plot over that of clipping it. This ratio may be in error; however, we will show later that the cost ratio does not affect our conclusions.

A ratio of variances calculated in the manner described can be tested by comparison with percentile points of Fisher's F distribution with 300 df for each parameter. Consider a null hypothesis that the variances due to the two techniques for equal amounts of work done are equal. Consider two alternate hypotheses, one that there is a significant reduction in the variance because of the dry weight rank technique, and the second that the clipping-only technique gives a significantly lower variance

Table 16. Variance ratios of dry weight rank and clipping estimates predicted for two of the six species with various combinations of clipping and ranking. Variances were computed from 300 independent trials and are based on equal costs expended in both ranking and clipping with a 10:1 cost ratio. Costs are shown in inset in terms of equivalent number of plots clipped.

Species	No. of clipped and ranked	No. of ranked only			
		50	100	150	250
Sp. 1	5	2.11 [10]	2.40 [15]	4.34 [20]	5.51 [30]
Sp. 3		5.41	3.41	7.21	0.922
Sp. 1	10	1.17 [15]	1.97 [20]	2.62 [25]	--
Sp. 3		4.65	7.40	3.78	--
Sp. 1	15	1.59 [20]	1.90 [25]	1.73 [30]	--
Sp. 3		11.20	5.03	17.09	--

of the estimates. The 90th percentile point of the F distribution with 120 df for both parameters is about 1.18. On comparison with the data of Table 16 we must reject the null hypothesis that the two methods give equal variance, reject the first alternate hypothesis that dry weight rank is an improved technique, and accept the second alternate hypothesis that clipping will give an improved estimate, over dry weight rank, of the biomass fractions for a given amount of time in sampling.

4.14 Cost Considerations

This conclusion is based upon the assumption of a 10:1 ratio in the cost of ranking over the cost of clipping. Variance ratios for the other five species support the conclusion more strongly than the first species. The indication is sufficiently strong that there can be little doubt that a correct rejection of the null hypothesis has been made.

If the cost ratio is different from 10:1 for the two techniques, the previous conclusion may not be valid. Suppose the cost of ranking a plot is less than one-tenth that of clipping the plot. If the ratio is sufficiently large that it may be considered infinite, then the cost of getting a dry weight rank estimate is the same as the cost of clipping the plots on which the dry weight rank multipliers are based, i.e., ranking requires no additional expenditure. The question is: "Does ranking plots give any improvement in the biomass estimates over the estimate based on the clippped plots alone?" Table 17 is structured similarly to Table 16 in that it shows ratios of the variances of the estimates by the two techniques. However, in Table 17 the variances for the clipping procedure are calculated from estimates based on the clipped plots required to do the ranking. Basically a similar result can be seen as in Table 16: Though it costs no more to do the ranking, the biomass estimators are

Table 17. Variance ratios of dry weight rank and clipping estimates predicted for two of the six species with various combinations of clipping and ranking. Variances were computed from 300 independent trials and are based on infinite cost ratios.

Species	No. of clipped and ranked	No. of ranked only			
		50	100	150	250
Sp. 1	5	1.16	1.10	0.949	1.03
Sp. 3		2.78	1.33	1.85	0.992
Sp. 1	10	1.03	0.941	1.10	--
Sp. 3		4.65	7.40	3.78	--
Sp. 1	15	1.10	1.05	1.02	--
Sp. 3		9.15	3.02	9.11	--

not improved as a result. The variance ratio is not always significant in this table at the 90% level. In one case (5:150 clip: rank ratio) a variance ratio is less than one. Notice that the ratio is not so low as to be significant in the opposite direction ($F = 1.0/1.18 = 0.85$) and supports the first alternate hypothesis (dry weight rank better than clipping). On the basis of the other variance ratios, this can be regarded as a chance occurrence caused by random fluctuations in the test statistic.

The data of Table 17 suggest the hypothesis that the variance of the dry weight rank estimates approaches the clipping estimate asymptotically from above as the number of ranked plots increases. Fig. 2 gives a plot of the standard deviations of biomass estimates as a function of the number of clipped plots and shows this trend more clearly. Thus dry weight rank, if this hypothesis is correct, only adds noise to the clipping estimates. Since the clipping estimate is based on a sufficient statistic for the population mean, the implication is that ranking of plots carries no new information about the mean (see Mood and Graybill (1963) for a definition of sufficient statistic). This hypothesis might be difficult to test by the simulation techniques used herein; it would certainly be expensive in terms of computer time. Analytical studies might be more appropriate.

5.0 CONCLUSIONS

Dry weight rank is reportedly a fast, accurate, nondestructive, botanical mensuration technique. Several questions must be considered in its use, however.

5.1 Field Application

As the Pawnee results indicate, plot size is not critical, but plots should be large enough to insure inclusion of several plant species.

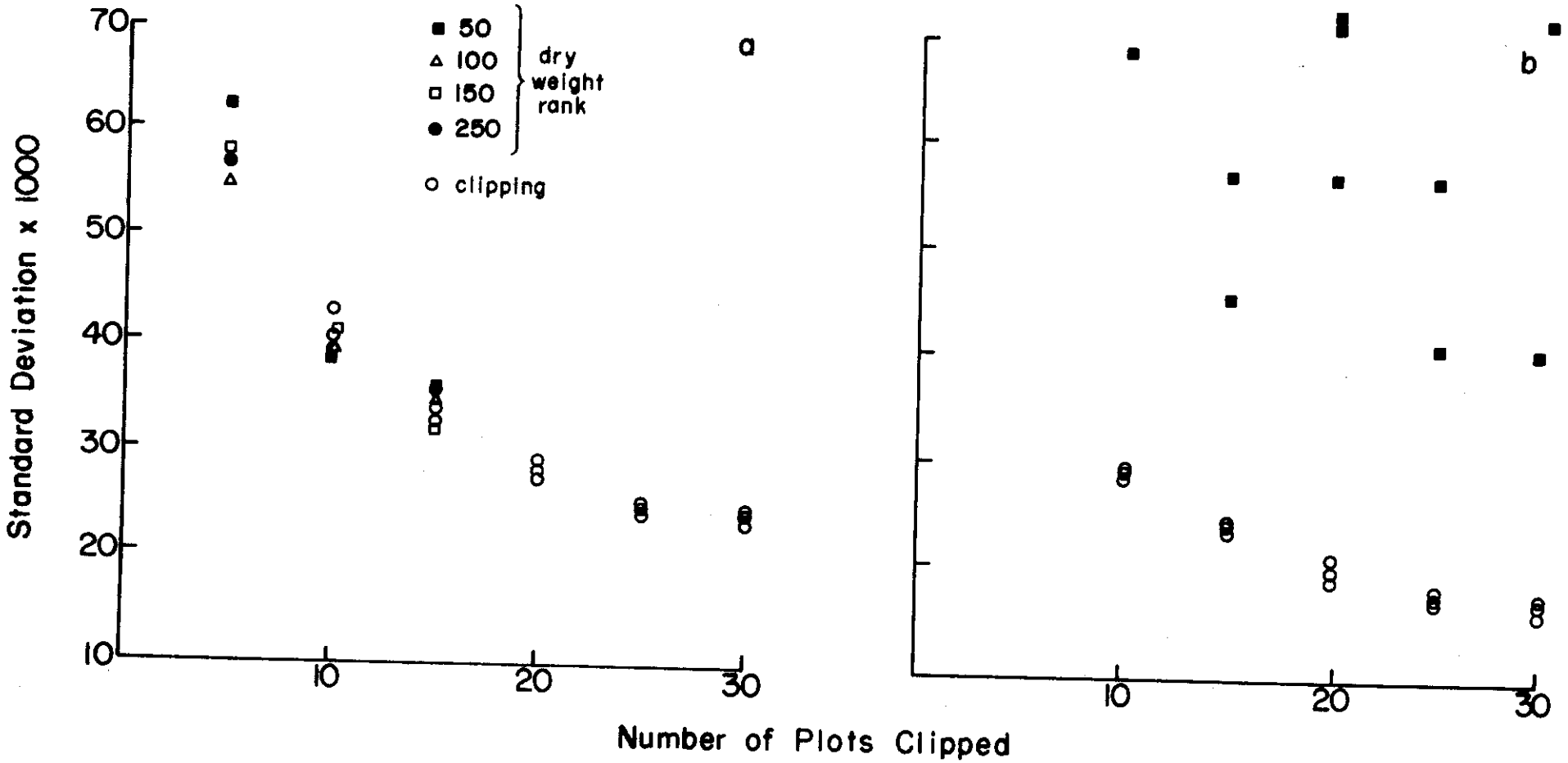

Figure 2. Standard deviations of biomass fraction estimates based on dry weight rank and clipping only for equal costs based on 10:1 cost ratio: a. sp. 1; b. sp. 3.

All plots will be ranked and clipped in a preliminary study because several parameters must be accurately determined. Between 50 and 100 plots will be required, depending upon the complexity of the community. The previously explained time data must be recorded.

One value of dry weight rank method is that the samples may be clipped in the field to be sorted later in the lab when time permits. This increases the speed of field work if the trained personnel are utilized in the field for ranking plots only and untrained personnel for clipping plots which have previously been ranked. Thus, one man ranking might be able to rank 20 to 50 times as many plots as one man clipping. He would need only to rank unclipped plots, for these studies have shown no significant difference in ranking in the field and in the lab.

To determine if dry weight rank gives the desired results for the allowable expenditure of time or money one could use one-half of the data for an analysis to determine multipliers.

5.2 Analysis Options

It seems that the best combination of analysis options will vary with the objective one has in mind and with the data set in question. The analysis options and expected consequences of using each are given in Table 18.

The R^2 value obtained using one-half of the data from a preliminary sample gives an indication of how well the multipliers fit the data from which they were derived. This R^2 should be above 0.85. The multipliers must now be tested on the independent data, i.e., the remaining one-half of the initial data set. According to Kozak (1966), the minimum conceivable

Table 18. Summary of the options of the dry weight rank method that were investigated in this study.

Option	Consequence of use
1. Weighted data	High accuracy of prediction of heavy species, low accuracy of light ones.
1. Simple data	Moderate accuracy of prediction of all species
2. Rank matrix columns sum to 1	No consistent statistically significant difference between sums to 1 and not sums to 1
2. Rank matrix columns do not sum to 1	
3. Rank in field	No statistically significant difference between ranking locations
3. Rank in lab	
4. Rank important species	R^2 declines as the number of species ranked is reduced
4. Rank all species	
5. One ranker	No statistically significant difference between rankers
5. Many rankers	
6. Lagrangian multiplican, least squares method	Exactly the same results for the one data set tested using both methods
6. Nonlinear programming	
7. Rows and columns of rank matrix with all zero elements should be removed.	
8. Negative predictions should be rounded to zero and minor species (each <5% of total weight) should be lumped into a miscellaneous category.	

R^2 for any data set at this second stage of testing is 0.4. One other important parameter, the optimum ratio, should be calculated to give an estimate of the number of plots which can be ranked for each plot that is both ranked and clipped.

The total number of plots that must be taken to give an acceptable accuracy must also be determined according to usual procedures.

In using the dry weight rank method, in order to obtain estimates of variances of percentage composition of individual species, it is necessary to locate plots in replicates. Thus, independent estimates of species composition can be made for each replicate.

Some decision must be made concerning exactly how the plot set from which the multipliers were derived will be merged with the ranked-plot plots. The answer to this question remains unanswered.

As the number of ranks increases and the vegetation type becomes more complex, the accuracy of the method declines for a given sample size. This is to be expected even with hand separation. Dry weight rank should be abandoned if the expenditure exceeds the value of the information gained.

5.3 Limitations of the Simulation Technique

In attempting to derive a general scientific result by inductive rather than deductive means, there is always the danger that a special situation has been utilized to reach the conclusions. Thus there is a possibility that the validity of the assumptions concerning the multivariate normal distribution or the errors introduced by the use of its truncated form might alter the conclusions. We feel that the specific case simulated and the assumptions made are sufficiently close to the real field situation that the probability of an error due to these causes is minimal.

One problem of the form of the dry weight rank technique used in this analysis should be discussed. This is that the matrix of rank proportions (R_1) is sometimes singular. The reader can experiment with some possible R_1 matrices for small data sets and verify this fact. The simulation procedure handled this problem by simply discarding such singular matrices when they were generated and regenerating the data set. The nonlinear programming method of deriving the multipliers does not suffer from this disadvantage and there are indeed methods of manipulating the R_1 matrix to avoid the problem. The methods involve use of a "generalized" or "least squares" inverse (Graybill 1969) instead of the traditional inverse of R_1. The problem does not frequently occur in a field application of the dry weight rank technique because of various conventions used to normalize R_1. The exact proportion of the possible R_1 matrices which are singular is a very knotty problem in matrix algebra. We are assuming in our conclusions that the method used to resolve the problem of singular R_1 matrices is not significantly affecting the result.

A final comment concerning the cost of this study is in order. The computer program required large amounts of time for execution in each one of 300 simulations. The variance ratio in the 50:150 column of Table 16 cost about $40 to produce. Total computational costs for the project were about $600.

5.4 Programmatic Application and Rejection

The purpose of this report is to document the background planning, analysis, and preliminary study activities that went into the examination of the dry weight rank method of botanical analysis which was utilized widely throughout the Grassland Biome study during one phase of work.

Much of the information in this report was presented to the participants at the 12-14 November 1970 annual review meeting of the US/IBP Grassland Biome study. Botanical investigators adapted the methodology for field studies in 1971 for further tests. The extreme difficulty of working with estimates of plant biomass densities by species via the clipping technique makes the dry weight rank method an extremely attractive proposition. While laboratory and field applications gave promising results, the simulation studies suggest variance estimates are not reduced below those by clipping. We think that the dry weight rank method is a false hope and other means must be sought to alleviate experimental difficulties in this area. Difficulties were encountered in processing and analyzing data and interpreting results. Subsequently the weight-estimation sampling procedure was utilized as a botanical analysis method in Grassland Biome investigations.

6.0 ACKNOWLEDGMENTS

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8.0 APPENDICES

APPENDIX

SPECIES LISTS AND STANDING CROP BY SPECIES FOR THE PLANT COMMUNITIES INVESTIGATED. DATA ARE MEANS AND STANDARD DEVIATIONS

South Dakota mixed prairie species	lb/acre ± SD	Montana shortgrass species	lb/acre ± SD	Eastern Colorado sandhills species	lb/acre ± SD	Pawnee National Grassland species	lb/acre ± SD
Buchloe dactyloides	630 ± 523	*Bouteloua gracilis*	157 ± 217	*Bouteloua gracilis*	356 ± 178	Litter	866 ± 608
Agropyron smithii	331 ± 269	*Stipa comata*	154 ± 194	*Stipa comata*	233 ± 206	*Bouteloua gracilis*	694 ± 329
Bouteloua gracilis	213 ± 165	*Oxytropis sericea*	109 ± 219	Other grasses	125 ± 208	*Opuntia polyacantha*	467 ± 737
Invaders	134 ± 152	*Koeleria cristata*	99 ± 94	*Calamovilfa longifolia*	116 ± 135	*Aristida longiseta*	69 ± 347
Forb decreasers	71 ± 112	*Agropyron spicatum*	61 ± 61	Forbs	26 ± 38	*Artemisia frigida*	51 ± 279
Carex stenophylla	66 ± 75	*Antennaria rosea*	49 ± 88			*Thelesperma trifidum*	29 ± 159
Woody increasers	40 ± 194	*Artemisia frigida*	37 ± 84			*Carex heliophila*	27 ± 42
Stipa viridula	31 ± 133	*Aristida longiseta*	33 ± 99			*Schedonnardus paniculatus*	16 ± 76
Sphaeralcea coccinea	26 ± 186	*Phlox hoodii*	30 ± 54			*Sphaeralcea coccinea*	15 ± 16
Forb increasers	14 ± 44	*Calamagrostis montanensis*	30 ± 39			*Psoralea tenuiflora*	12 ± 49
Symphoricarpos occidentalis	5 ± 31	*Liatris punctata*	20 ± 43			*Gutierrezia sarothrae*	10 ± 43
Aristida longiseta	4 ± 26	*Phlox albomarginata*	19 ± 100			*Buchloe dactyloides*	7 ± 40
Artemisia ludoviciana	4 ± 17	*Erigeron compositus*	16 ± 51			*Verbena bracteata*	6 ± 32
Opuntia sp.	4 ± 13	*Carex eleocharis*	14 ± 14			*Festuca octoflora*	6 ± 22
Bouteloua curtipendula	3 ± 16	*Aster commutatus*	13 ± 45			*Eriogonum effusium*	5 ± 26
Sporobolus coccinea	2 ± 7	*Erigeron caespitosus*	13 ± 29			*Oenothera coronopifolia*	3 ± 18
Carex filifolia	<1 ± 3	*Chrysopsis villosa*	10 ± 39			*Muhlenbergia torreyi*	2 ± 12
Poa secunda	<1 ± 2	*Carex filifolia*	10 ± 17			*Erigeron canadensis*	2 ± 11
Artemisia frigida	<1 ± 1	*Gutierrezia sarothrae*	9 ± 43			*Lepidium densiflorum*	2 ± 3
Other increasers	<1 ± 1	*Artemisia ludoviciana*	8 ± 45			*Gaura coccinea*	1 ± 7
Carex spp.	<1 ± <1	*Astragalus striatus*	7 ± 45			*Plantago purshii*	2 ± 8
		Plantago purshii	4 ± 4			*Euphorbia glytotsperma*	1 ± 2
		Potentilla hippiana	3 ± 28			*Cirsium undulatum*	<1 ± 3
		Senecio canus	3 ± 17			*Solanum triflorum*	<1 ± 3
		Oxytropis lambertii	2 ± 15			*Cryptantha minima*	<1 ± 3
		Poa secunda	2 ± 6			*Mirabilis linearis*	<1 ± 1
		Solidago missouriensis	1 ± 10			*Chenopodium leptophyllum*	<1 ± 1
		Arenaria congesta	1 ± 5			*Salsola kali*	<1 ± 1
		Cryptantha bradburiana	1 ± 3			*Carex filifolia*	<1 ± 1
		Festuca idahoensis	<1 ± 3				
		Sphaeralcea coccinea	<1 ± 1				

Appendix I. nt.

Little Hills species	lb/acre ± SD	Desert 2 species	lb/acre ± SD	Desert 3 species	lb/acre ± SD	Oak brush species	lb/acre ± SD
Artemisia tridentata	175 ± 279	*Atriplex confertifolia*	320 ± 479	*Atriplex confertifolia*	200 ± 452	*Quercus gambellii*	995 ± 1176
Symphoricarpos sp.	163 ± 203	*Gutierrezia sarothrae*	87 ± 171	*Chrysothamnus viscidiflorus*	96 ± 96	*Artemisia dracunculus*	105 ± 149
Poa fendleriana	125 ± 160	*Hilaria jamesii*	49 ± 84	*Gutierrezia sarothrae*	42 ± 83	*Poa pratensis*	74 ± 92
Chrysothamnus viscidiflorus	97 ± 105	*Oryzopsis hymenoides*	41 ± 108	*Hilaria jamesii*	22 ± 54	*Stipa comata*	56 ± 103
Stipa comata	78 ± 125	*Aster arenosus*	27 ± 50	*Atriplex nuttallii*	21 ± 132	*Chrysothamnus depressus*	52 ± 141
Koeleria cristata	52 ± 68	*Chrysothamnus viscidiflorus*	21 ± 50	*Eriogonum* sp.	17 ± 29	*Eriogonum racemosum*	44 ± 57
Oryzopsis hymenoides	46 ± 83	*Sphaeralcea coccinea*	19 ± 34	*Salsola kali*	7 ± 47	*Bouteloua gracilis*	31 ± 50
Agropyron smithii	39 ± 78	*Sitanion hystrix*	9 ± 23	*Oryzopsis hymenoides*	7 ± 20	*Gutierrezia sarothrae*	28 ± 85
Amelanchier sp.	25 ± 149	Unidentified forb	4 ± 22	Unidentified forb	6 ± 40	*Cryptantha virgata*	17 ± 39
Artemisia ludoviciana	22 ± 77	*Elymus salinus*	3 ± 14	*Asclepias* sp.	3 ± 16	*Agropyron smithii*	15 ± 43
Gutierrezia sarothrae	14 ± 36	*Lesquerella* sp.	3 ± 12	*Elymus salinus*	2 ± 9	Unidentified forb	13 ± 17
Bromus sp.	7 ± 38	*Opuntia polyacantha*	2 ± 15	*Astragalus* sp.	1 ± 5	*Lithospermum ruderale*	11 ± 55
Agropyron sp.	7 ± 24	*Bromus tectorum*	2 ± 7	*Lesquerella* sp.	1 ± 3	*Poa compressa*	11 ± 25
Sphaeralcea coccinea	7 ± 11	*Erigeron* sp.	1 ± 6	*Sphaeralcea coccinea*	1 ± 2	*Cirsium* sp.	8 ± 31
Festuca idahoensis	5 ± 29	Unidentified forb	<1 ± 1	*Sitanion hystrix*	<1 ± <1	*Vicia americana*	8 ± 16
Poa compressa	5 ± 21					*Sitanion hystrix*	7 ± 16
Erigeron sp.	3 ± 13					*Carex* sp.	7 ± 15
Sitanion hystrix	3 ± 9					*Orthocarpus purpureo-albus*	4 ± 13
Tetrademia canescens	2 ± 12					*Achillea lanulosa*	4 ± 11
Unidentified forb	2 ± 8					*Lupinus aduncus*	4 ± 11
Achillea lanulosa	1 ± 7					*Sphaeralcea coccinea*	2 ± 11
Phlox hoodii	1 ± 5					*Festuca arizonica*	1 ± 7
Unidentified forb	1 ± 55					*Cilia congesta*	<1 ± 3
Unidentified forb	1 ± 4					*Calochortus* sp.	<1 ± 2
Lupinus sp.	1 ± 4					*Poa fendleriana*	<1 ± 2
Chrysothamnus nauseosus	<1 ± 3					*Tragopogon* sp.	<1 ± 1
Senecio sp.	<1 ± 2					Unidentified forb	<1 ± 1
Unidentified forb	<1 ± 2					*Koeleria cristata*	<1 ± <1
Tragopogon sp.	<1 ± 2						
Aster sp.	<1 ± 2						
Agropyron spicatum	<1 ± <1						
Bromus tectorum	<1 ± <1						

APPENDIX II

THE VARIOUS MATRICES AND VECTORS OBTAINED IN THE STEPS LEADING TO A LEAST SQUARES, LAGRANGIAN MULTIPLIER FIT OF THE RANK MULTIPLIERS ARE LISTED. THE COMPLETE SET OF EASTERN COLORADO SANDHILLS DATA WHICH APPEARS IN APPENDIX III HAS BEEN USED. FIVE SPECIES AND FIVE RANKS ARE INCLUDED.

$$W = \begin{bmatrix} 0.41598 & 0.13535 & 0.27206 & 0.14638 & 0.03024 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.44444 & 0.30556 & 0.25000 & 0.22220 & 0.00000 \\ 0.19444 & 0.11111 & 0.19444 & 0.16667 & 0.00000 \\ 0.27778 & 0.41667 & 0.11111 & 0.05556 & 0.00000 \\ 0.08333 & 0.16667 & 0.16667 & 0.19444 & 0.08333 \\ 0.00000 & 0.00000 & 0.25000 & 0.33333 & 0.30556 \end{bmatrix}$$

$$C = R^T R = \begin{bmatrix} 0.31944 & 0.28704 & 0.19367 & 0.06404 & 0.00694 \\ 0.28704 & 0.30710 & 0.17207 & 0.07407 & 0.11389 \\ 0.19367 & 0.17207 & 0.20293 & .15432 & 0.09028 \\ 0.06404 & 0.07407 & 0.15432 & 0.17978 & 0.11806 \\ 0.00694 & 0.01389 & 0.09028 & 0.11806 & 0.10031 \end{bmatrix}$$

C adjoined =

$$\begin{bmatrix} 0.31944 & 0.28704 & 0.19367 & 0.06404 & 0.00694 & 1.00000 \\ 0.28704 & 0.30710 & 0.17207 & 0.07407 & 0.01389 & 1.00000 \\ 0.19367 & 0.17207 & 0.20293 & 0.15432 & 0.09028 & 1.00000 \\ 0.06404 & 0.07407 & 0.15432 & 0.17978 & 0.11806 & 1.00000 \\ 0.00694 & 0.01389 & 0.09028 & 0.11806 & 0.10031 & 1.00000 \\ 1.00000 & 1.00000 & 1.00000 & 1.00000 & 1.00000 & 0.00000 \end{bmatrix}$$

D adjoined =

$$\begin{bmatrix} 0.29897 & 0.27990 & .19250 & 0.07621 & 0.02144 & 1.00000 \end{bmatrix}$$

$$D = R^T W = \begin{bmatrix} 0.29897 & 0.27990 & 0.19250 & 0.07621 & 0.02144 \end{bmatrix}$$

$C = R^T R$ adjoined inverse =

$$\begin{bmatrix} 119.97342 & -62.61267 & -147.40834 & 108.23777 & -18.19018 & 2.39073 \\ -62.61267 & 40.23486 & 64.30478 & -50.49049 & 8.56352 & -0.82761 \\ -147.40834 & 64.30478 & 215.73079 & -157.24184 & 24.61461 & -3.25093 \\ 108.23777 & -50.49049 & -157.24184 & 140.12914 & -40.63458 & 1.67803 \\ -18.19018 & 8.56352 & 24.61461 & -40.63458 & 25.64663 & 1.00979 \\ 2.39073 & -0.82761 & -3.25093 & 1.67803 & 1.00979 & -0.01101 \end{bmatrix}$$

$E = (R^T R)^{-1} RT = CD =$

$$\begin{bmatrix} 0.2173691 & 0.4288609 & 0.7486583 & -0.5544167 & 0.1595285 & -0.0041661 \end{bmatrix}$$

APPENDIX III

EASTERN COLORADO SANDHILLS, JUNE 1967,
BASED ON RANKER A.

THIS IS THE ORIGINAL DATA FROM WHICH THE FOLLOWING
10 RUN(S) WILL BE MADE

DRY WEIGHT IN LB/ACRE

Plot	BOGR	CALO	STCO	ZZ grass	Forbs	Total
1	374.00	0.	0.	300.00	12.00	686.00
2	264.00	328.00	160.00	92.00	4.00	848.00
3	392.00	339.00	0.	12.00	0.	742.00
4	346.00	114.00	284.00	234.00	0.	978.00
5	340.00	224.00	628.00	342.00	0.	1534.00
6	140.00	48.00	126.00	156.00	88.00	558.00
7	140.00	294.00	346.00	0.	76.00	856.00
8	518.00	240.00	132.00	46.00	32.00	968.00
9	324.00	6.00	582.00	20.00	4.00	936.00
10	216.00	214.00	100.00	158.00	0.	688.00
11	266.00	146.00	206.00	128.00	0.	746.00
12	784.00	116.00	268.00	0.	20.00	1188.00
13	310.00	0.	144.00	732.00	80.00	1266.00
14	160.00	42.00	710.00	146.00	16.00	1074.00
15	536.00	64.00	208.00	0.	26.00	834.00
16	312.00	0.	104.00	76.00	0.	492.00
17	378.00	482.00	222.00	0.	26.00	1108.00
18	536.00	8.00	466.00	26.00	16.00	1052.00
19	338.00	0.	0.	88.00	0.	426.00
20	426.00	0.	0.	52.00	0.	478.00
21	616.00	0.	490.00	0.	8.00	1114.00
22	304.00	0.	70.00	0.	0.	374.00
23	436.00	16.00	622.00	78.00	0.	1152.00
24	136.00	164.00	182.00	0.	20.00	502.00
25	606.00	0.	58.00	22.00	0.	686.00
26	148.00	6.0	592.00	218.00	0.	964.00
27	200.00	0.	208.00	852.00	4.00	1264.00
28	386.00	112.00	266.00	0.	10.00	774.00
29	260.00	0.	90.00	0.	180.00	530.00
30	198.00	432.00	8.00	0.	66.00	704.00
31	366.00	98.00	64.00	2.00	30.00	560.00
32	892.00	108.00	0.	614.00	60.00	1674.00
33	136.00	314.00	246.00	52.00	14.00	762.00
34	274.00	120.00	108.00	0.	10.00	512.00
35	204.00	50.00	434.00	0.	90.00	859.00
36	480.00	89.00	262.00	66.00	40.00	936.00
Ave. wt.	536.17	115.89	232.94	125.33	25.89	856.22
SD	177.94	134.97	205.83	207.79	38.21	306.99

COSTS OF THE SLOW, FIXED, AND FAST PROCESSES

45.0000 1.0000 3.0000

APPENDIX III (cont.)

RANKS ASSIGNED

Plot	BOGR	CALO	STCO	ZZ Grass	Forbs
1	2	-0	-0	1	3
2	3	1	2	4	5
3	2	1	-0	3	4
4	3	1	2	4	5
5	2	3	1	4	5
6	2	3	1	5	4
7	3	2	1	5	4
8	1	3	2	4	5
9	2	-0	1	-0	3
10	1	2	3	4	5
11	1	4	2	3	5
12	1	2	3	4	5
13	2	-0	4	1	3
14	3	4	1	2	5
15	1	3	2	5	4
16	1	-0	2	3	-0
17	3	1	2	-0	4
18	1	4	2	3	5
19	1	-0	-0	2	-0
20	1	-0	-0	2	3
21	1	-0	2	-0	3
22	1	-0	2	-0	3
23	2	4	1	3	-0
24	3	2	1	-0	4
25	1	-0	3	2	-0
26	3	-0	1	2	4
27	3	-0	2	1	4
28	2	3	1	-0	4
29	1	-0	2	-0	3
30	2	1	4	-0	3
31	1	3	2	-0	4
32	2	1	-0	3	4
33	3	1	2	4	5
34	1	3	2	-0	4
35	2	4	1	-0	3
36	1	4	3	2	5

APPENDIX III (cont.)

THE PLOTS USED

No.	Plots																																						
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	4	7	8	11	12	13	14	17	19	20	22	23	28	30	31	32			
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	5	7	8	9	10	12	14	16	17	25	29	30	31	32	34	36			
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	4	5	8	9	11	14	15	16	17	18	19	21	24	25	27	28	29	30	31	33	36			
24	0	0	0	0	0	0	0	0	0	0	0	0	2	3	5	8	10	11	12	13	14	16	18	19	20	21	22	23	24	25	27	28	30	32	33	34			
25	0	0	0	0	0	0	0	0	0	0	0	1	4	5	6	7	8	12	14	15	16	18	19	21	24	25	26	27	28	29	30	31	32	33	35	36			
27	0	0	0	0	0	0	0	0	0	2	5	6	8	10	11	12	13	14	15	16	19	20	21	23	24	25	26	27	28	29	30	31	32	34	35	36			
29	0	0	0	0	0	0	0	1	2	4	5	6	7	8	9	10	12	14	15	16	17	18	19	20	21	23	25	27	28	29	30	31	32	34	35	36			
31	0	0	0	0	0	1	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18	21	23	24	25	26	27	28	29	30	31	32	33	34	35	36			
33	0	0	0	1	2	3	4	6	7	8	9	10	11	12	13	14	15	16	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36			
36	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36			

APPENDIX III (cont.)

THIS WAS AN UNWEIGHTED REGRESSION

No. of plots	Simple R^2	Simple optimum ratio	Rank multipliers					Lagrangian multiplier
17	.9937	42	.17570	.50293	.72995	-.38046	-.02812	-.00732
18	.9657	18	.77206	.15998	-.07593	.13678	.00711	.00608
22	.9869	29	.40019	.31265	.45483	-.17589	.00821	-.00394
24	.9874	30	.40811	.31253	.65582	-.38445	.00798	-.00989
25	.9628	17	-.22531	.67678	1.04952	-.97389	.47289	.01343
27	.9750	21	.91265	.14037	-.53762	.73227	-.24766	-.00305
29	.9932	26	.90246	.14359	-.32690	.30231	-.02145	.01052
31	.9953	49	.11785	.36484	.88416	-.62999	.26315	.00605
33	.9741	21	.29105	.40027	.68124	-.59520	.22265	-.00070
36	.9830	25	.21737	.42886	.74866	-.55442	.15953	-.00417

PREDICTED AND OBSERVED WEIGHTS FOR LAST RUN

.41481	.41597
.14309	.13535
.29146	.27206
.11986	.14638
.05110	.03024

R MATRIX COLUMNS MAY NOT SUM TO 1.0

APPENDIX III (cont.)

MULTIPLIERS FROM 18 PLOTS TESTED ON THE OTHER 18 PLOTS

No. of plots	Simple R^2	Simple optimum ratio	Rank multipliers				
18	.9199	11	.77206	.15998	-.07593	.13678	.00711

PREDICTED AND OBSERVED WEIGHTS FOR THIS RUN

.36648	.42468
.14771	.09487
.28894	.27681
.12467	.17888
.03368	.02476

UNWEIGHTED R MATRIX COLUMNS MAY NOT SUM TO 1.0

PREDICTED DRY WEIGHT PROPORTIONS WERE CALCULATED WITH AN UNWEIGHTED RANK MATRIX