Technical Report No. 202
SURFACE ENERGY AND WATER BUDGET
OVER THE GRASSLAND, 1970

Guillermo F. Almeyda and James L. Rasmussen
Department of Atmospheric Science
Colorado State University
Fort Collins, Colorado

GRASSLAND BIOME
U.S. International Biological Program

December 1972
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title Page</td>
<td>1</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>ii</td>
</tr>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>1.0 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>2.0 Formulation of the Energy and Water Budgets</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Energy Budget</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Water Budget</td>
<td>5</td>
</tr>
<tr>
<td>3.0 Data</td>
<td>7</td>
</tr>
<tr>
<td>3.1 Source of Data</td>
<td>7</td>
</tr>
<tr>
<td>4.0 Analysis of the Components of the Energy Budget</td>
<td>10</td>
</tr>
<tr>
<td>4.1 Radiation</td>
<td>10</td>
</tr>
<tr>
<td>4.1.1 Solar Radiation (Q + q)</td>
<td>10</td>
</tr>
<tr>
<td>4.1.2 Albedo</td>
<td>11</td>
</tr>
<tr>
<td>4.1.3 Longwave Radiation</td>
<td>14</td>
</tr>
<tr>
<td>4.1.3.1 Upward Directed Longwave Radiation from the Surface (I↓)</td>
<td>14</td>
</tr>
<tr>
<td>4.1.3.2 Downward Directed Longwave Radiation from the Atmosphere (I↑)</td>
<td>15</td>
</tr>
<tr>
<td>4.1.3.3 Net Longwave Radiation I = I↓ – I↑</td>
<td>17</td>
</tr>
<tr>
<td>4.1.3.4 Net Radiation – Computed vs. Observed</td>
<td>18</td>
</tr>
<tr>
<td>4.2 Transfer of Sensible Heat</td>
<td>24</td>
</tr>
<tr>
<td>4.3 Change in Heat Storage in the Soil</td>
<td>30</td>
</tr>
<tr>
<td>4.4 Evaluation of the Latent Heat Exchange as a Residual and Summary of</td>
<td></td>
</tr>
<tr>
<td>the Terms in the Energy Equation</td>
<td>32</td>
</tr>
<tr>
<td>4.5 The Ratio of Sensible to Latent Heat Transfer</td>
<td>33</td>
</tr>
<tr>
<td>5.0 Water Budget Evaluation of Evaporation</td>
<td>35</td>
</tr>
<tr>
<td>6.0 Summary and Conclusions</td>
<td>40</td>
</tr>
<tr>
<td>Literature Cited</td>
<td>42</td>
</tr>
<tr>
<td>Appendix I. List of Symbols</td>
<td>44</td>
</tr>
</tbody>
</table>
ABSTRACT

The components of the energy budget at the surface were studied for grassland terrain. This study was performed for the sites of Dickinson (North Dakota) and Cottonwood (South Dakota), located in the U.S. IBP Grassland Biome, for the growing season of 1970 (May to October). In addition to the energy budget, the water budget and the Thornthwaite-Mather water balance were computed from data from the same sites for the purpose of comparison. Radiosonde data from stations close to the two sites were used to obtain the daily magnitudes of the downward infrared radiation from the atmosphere as well as the sensible heat transfer from the surface.

The latent heat transfer calculated as a residual from the energy budget was tested against the evapotranspiration obtained from the water budget and compared to the evapotranspiration calculated from the Thornthwaite-Mather water balance. Good quantitative agreement was attained. This indicates that the easily computed Thornthwaite-Mather water balance can be used as a first approximation of the water balance in a region where no soil water measurements exist.

The study of the radiation budget shows that the net radiation at the surface is largely determined by the incoming solar radiation. In late spring and early summer the net radiation goes equally into sensible and latent heat while it is unequally distributed at other times. The net heat flux into the ground is small.
1.0 INTRODUCTION

The understanding of grassland climatology is important to man and his environment. The harmony that exists in the ecology is governed to a certain extent by abiotic phenomena such as solar radiation, precipitation, wind, and temperature. Their role is to act as driving forces primarily through energy exchanges.

The energy budget of the grassland surface is composed of basic components which interact to determine the environment in which the biome exists. The components include solar and terrestrial radiation, sensible and latent heat, and the storage of heat in the earth's surface layer. It is the objective of this work to study these energy budget components over grassland terrain.

This work is designed to be incorporated into the larger context of the U.S. International Biological Program (IBP) Grassland Biome study. The IBP has a global objective "to examine the basis of productivity in human welfare." The U.S. IBP Grassland Biome study is to develop and test ecological theory; a second goal of the U.S. IBP Grassland Biome study is to develop models useful for resource management decisions.

This paper summarizes a research program aimed at determining the energy and water budgets of grassland locations using existing meteorological data. The components of these budgets are summarized such that they could be used to define the driving forces of the ecosystem in ecosystem models.

1.1 Background

The study of the energy and water budgets of the earth's surface have been one of the central themes of micrometeorology. Geiger's
(1959) text is the classic summary of the problem. The lengthy review article by Miller (1965) puts the relative importance of the components of the energy budget and water budget in perspective and quantifies them in an "order of magnitude" fashion of the grassland setting. Sellers (1965) treats the problem on a large scale and from the point of view of the physical climatologist. Invariably, the point is made regarding the difficulty of measuring the various components of the budgets, particularly in terms of the energy budget. Lettau and Davidson (1957) have reported the results of a sophisticated measurement program carried out over grassland terrain at O'Neill, Nebraska in 1953. The results of these studies provide a background on which this study is based. It is the intent of the program reported herein to use the standard meteorological observations coupled with some special observations taken at the IBP Grassland Biome locations.

The standard observations do not allow us to pursue the problem from the viewpoint of classical turbulence. Direct calculation of the interaction between the lowest atmosphere and the underlying surface result in fluxes of heat, momentum, and moisture which require observations not available from the standard network and also not available from the IBP network. Priestly (1959) gives a thorough description of the physical process and examples of the required data.

Thornthwaite and Mather (1957) attempted to use only the parameters of air temperature and precipitation to describe the energy water balances of the earth's surface. One cannot, however, deduce the rate and interaction of the various components of the balance through the use of such simplified models. Numerous extensions of Thornthwaite's work may be found in the literature (Sellers, 1965; Penman, 1948).
Thornthwaite-Mather model is convenient as a comparison because it has been evaluated for practically all climate types. Rasmussen (1971) has evaluated the Thornthwaite-Mather water budget model for each of the Grassland Biome sites.

The requirement for abiotic data in the grassland ecosystem study is summarized by Lewis (1971). The requirement for a complete energy budget is twofold. First, it allows us to seek out those links in the energy budget which most influence the ecosystem or differentiate between ecosystems. Second, the complete budget offers a check on the measurement of the various components.

There is some evidence of current climatic fluctuations in certain components of the energy budget. Over the grassland, for example, solar radiation has been observed to be quite variable and has a decreasing secular trend (Rasmussen, Bertolin and Almeyda, 1971). The question is open as to whether these rather small but systematic variations initiate response at the earth's surface. This type of question is best addressed from the point of view of the complete energy budget where the variations can be assessed with regard to relative magnitudes of various components.
2.0 FORMULATION OF THE ENERGY AND WATER BUDGETS

The interaction between the ecosystem and the atmosphere is centered in the exchanges of energy and water across the earth-atmosphere interface. The energy and water budgets of the earth-atmosphere interface are in turn connected through the process of evapotranspiration. We wish to evaluate the balance of energy and water for the grassland and study the exchanges using data collected in the IBP program. Appendix I contains a complete list of symbols.

2.1 Energy Budget

The energy budget for the earth's surface may be written

\[ R_n = G + H + LE \]  

(1)

where

- \( G \) = rate of change of heat stored in the ground
- \( H \) = rate of sensible heat transferred to the atmosphere
- \( LE \) = heat used in the evapotranspiration process
- \( R_n \) = net radiation

The net radiation is composed of the components

- \( Q \) = direct solar radiation
- \( q \) = diffuse solar radiation
- \( I \) = net infrared radiation = \( I^+ - I^- \), the upward and downward directed components, respectively

\( R_n \) may be expressed

\[ R_n = (Q + q) (1 - \alpha) - I \]  

(2)

where \( \alpha \) is the albedo of the surface.

The overall goal of this study is to evaluate the evapotranspiration (LE) for the Grassland Biome and to investigate the role played
by various components of the budgets in the evapotranspiration regime. Rewriting equation (1) and incorporating (2) we obtain

\[ LE = (Q + q) (1 - \alpha) - I - G - H \]  \hspace{1cm} (3)

The method used in this study was to evaluate the terms on the right in equation (3) and compute LE as a residual. Independent estimates of LE were obtained from the terrestrial water budget analysis.

2.2 Water Budget

The water of the surface and soil may be partitioned as follows:

\[ P - R - E - \Delta W = 0 \]  \hspace{1cm} (4)

where

- \( P \) = precipitation rate
- \( R \) = runoff
- \( E \) = evapotranspiration
- \( \Delta W \) = rate of change of soil water storage \((W_2 - W_1)\)

Subscripts 1 and 2 refer to the end and beginning of the time period and study. The depth is assumed great enough so that the vertical transports of water in the soil are small. Equation (4) may be evaluated for evapotranspiration:

\[ E = P - R - \Delta W \]  \hspace{1cm} (5)

Equation (5), then, would afford a check on the results of equation (3) providing the terms on the right-hand side are evaluated with enough accuracy.

This paper will follow the line of investigation outlined above, that is, to calculate the evaporation as a residual from the energy budget at the earth's surface and compare the calculated evaporation
to an independently estimated value obtained from the water budget. The magnitudes and roles played by the various terms in the equations in the secular change in evapotranspiration over the grassland will be evaluated using the data set described in the next section.
3.0 DATA

This study is centered on the analysis of data collected in the abiotic portion of the U.S. IBP Grassland Biome program. Certain limitations to the data set will become evident. There has been an attempt to utilize in this study other sources of data, particularly meteorological data, so that a reasonable evaluation of all the terms on the right-hand side of equations (3) and (5) are possible.

3.1 Source of Data

Extensive measurements of meteorological and soil parameters were planned to be taken at various sites across the grassland region of the U.S. during the late spring and summer of 1970. These data were to be used in the development of the abiotic driving forces in the numerical modelling of the grassland ecosystem (Lewis, 1971). The data that were actually realized from this observational scheme were sometimes of a questionable quality and breaks in the record were common. There is evidence that the truly important abiotic inputs into the ecosystem model are the rather long period variations (day to weeks), and the more standard meteorological observational material may be used to define these inputs (Rasmussen, 1971; Lewis, 1971). The stations of Cottonwood, South Dakota (43°58'N. Lat, 101°52'W. Long), and Dickinson, North Dakota (46°53'N. Lat, 102°49'W. Long), have particularly good records and will be used in the following analysis (Lewis et al., 1971; Whitman, 1971). Future work will be to study all available data and formulate a comprehensive study of the whole grassland region. Data archived from later years (1971-1972) will provide such opportunity.
Table I shows the parameters measured at the two sites and at nearby standard meteorological stations (Figure 1). As will be evident later, the temperature and humidity measurements in the free atmosphere were required to calculate the energy budget given the incomplete surface data at Dickinson and Cottonwood. The radiosonde data from Bismarck and Rapid City (Figure 1) were assumed to be representative of the state of the atmosphere over the surface observation sites. In the analysis summation of each term in the energy budget over time scales of a week or so will be made. The spatial separation of the station pairs, therefore, is minimized.

It is apparent from Table I that different parameters are measured at the two sites, and the equivalent parameter is often measured differently. For example, the air temperature is measured at different heights, and the heat flux into the soil must be calculated from the soil temperature profile at Cottonwood while it was measured directly at Dickinson. Because of the nonstandard nature of the data sources, it will be necessary to compare the results of components of the energy budget calculated in different ways.

![Map of IBP Grassland Biome sites and meteorological stations.](image)

Figure 1. Location of the IBP Grassland Biome sites and meteorological stations nearby considered in the study.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Location</th>
<th>Observation Time</th>
<th>Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Temperature</td>
<td>Cottonwood</td>
<td>Hourly</td>
<td>Hygrothermograph: Standard U.S. Weather Bureau Metalation Shelter at 1.0 m</td>
</tr>
<tr>
<td>Soil Temperature</td>
<td>Cottonwood</td>
<td>Daily at 8</td>
<td>Thermocouples at 10, 20, 50, and 150-cm depth</td>
</tr>
<tr>
<td>Incoming Solar Radiation</td>
<td>Cottonwood</td>
<td>Daily</td>
<td>Epply Pyrhelio graph</td>
</tr>
<tr>
<td>Atmospheric Sounding</td>
<td>Cottonwood</td>
<td>Twice Daily</td>
<td>Radiosonde</td>
</tr>
<tr>
<td>Air Temperature</td>
<td>Dickinson</td>
<td>Hourly</td>
<td>Shielded thermocouples at 2.5 cm</td>
</tr>
<tr>
<td>Net Soil heat flux</td>
<td>Dickinson</td>
<td>Hourly</td>
<td>Thornthwaite soil heat flux at 2.5 cm depth</td>
</tr>
<tr>
<td>Net Radiation</td>
<td>Dickinson</td>
<td>Hourly</td>
<td>Beckman-Whitley net radiometer at 130-cm height above grazed terrain</td>
</tr>
<tr>
<td>Incoming Solar Radiation</td>
<td>Bismarck</td>
<td>Daily</td>
<td>Epply pyranometer</td>
</tr>
<tr>
<td>Atmospheric Sounding</td>
<td>Bismarck</td>
<td>Twice Daily</td>
<td>Radiosonde</td>
</tr>
</tbody>
</table>
4.0 ANALYSIS OF THE COMPONENTS OF THE ENERGY BUDGET

4.1 Radiation

The net radiation at the earth's surface is, as described in Section 2.1.

\[ R_n = (Q + q)(1 + \alpha) - I^+ + I^+ \]  

(6)

\( R_n \) is measured at Dickinson but not at Cottonwood (Table I). The incoming solar radiation \((Q + q)\) is measured at Cottonwood and also at Bismarck, a station close to Dickinson. The analysis will be designed to estimate \( R_n \) for Cottonwood by computing the longwave radiation terms. A convenient check on the computational method and results will be to evaluate \( R_n \) using the Bismarck value of \( Q + q \), computing \( R_n \) for Dickinson, and comparing this computed value to the observed value.

4.1.1 Solar Radiation \((Q + q)\)

The daily course of incoming solar radiation over the period of study for Bismarck (Dickinson) is shown in Figure 2. A slightly variable trace depicting the alternating fair sky, i.e., cloudy sky characteristics of the weather events superimposed upon the seasonal variation, is obvious. Figure 3a and 3b show the frequency distribution of daily \( Q + q \) for each month for Dickinson and Cottonwood, respectively.

The average daily solar radiation received at Bismarck was 582 langleyes/day with standard deviation of 179 langleyes/day. The general descriptive picture of the solar radiation climate, therefore, is one with a rather intense solar radiation and high variability. The month of May at Bismarck deviates from this picture showing a rather uniform distribution.
Figure 2. The daily course of incoming solar radiation \((Q + q)\) of Bismarck (Dickinson) May to October, 1970.

4.1.2 Albedo

The surface albedo obtained from satellite in the year 1969 (Raschke et al., 1971) was used in the study of the energy balance at Dickinson and Cottonwood. The measurements were obtained for the minimum albedo, or clear sky, at noon (local time) for a period of time for each summer month. The values used here were taken from analyzed charts drawn for North America. Table II shows the monthly surface albedo as estimated from satellite data. These values agree closely with other measurements (e.g., Miller, 1965).
Figure 3a. The frequency distribution of incoming solar radiation at Dickinson for the growing season 1970.
Figure 3b. The frequency distribution of incoming solar radiation at Cottonwood for the growing season 1970.

TABLE II

Integral Albedo for Cottonwood and Dickinson Sites, 1969.

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Cottonwood</th>
<th>Dickinson</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 16 - 31</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>June 1 - 15</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>July 1 - 15</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>August 1 - 15</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>Average</td>
<td>17</td>
<td>17</td>
</tr>
</tbody>
</table>
The albedo is greater in the early season during the growth period and decreases after June as the grass dries. The average albedo for both sites was 17 percent and little variation around the average is noted. The albedo was carried as a constant in the analysis. Some study of the effect of systematic variation of albedo will be included in the analysis of the computed vs. observed net radiation.

4.1.3 Longwave Radiation

4.1.3.1 Upward Directed Longwave Radiation from the Surface (I+)

In order to evaluate this term of equation (6), one must know the radiative temperature of the surface. The active radiating surface of the grassland is a complex system as one views the problem as an areally integrated value. For purposes of this paper, we wish to evaluate the outward flux of infrared radiation from the surface on a daily basis and in turn we will sum these daily values over a week or so. The air temperature at 2.5 cm is assumed to be representative of the surface condition in the grassland setting. The fourth power of the average daily temperature (average of 24 hourly values) is assumed to be proportional to the outward flux of energy. The Stefan-Boltzman relationship was evaluated to obtain I+.

\[ I^+ = \varepsilon \sigma T^4_{2.5 \text{ cm}} \]

\( T_{2.5 \text{ cm}} \) denotes the mean daily temperature at 2.5 cm, \( \varepsilon \) is the emissivity (chosen for this paper to be 1.0) and \( \sigma \) is Stefan-Boltzman constant. One may assume the radiative flux to be represented by the fourth power of the mean temperature since the relationship between I+
and $T$ is relatively linear over the temperature range which concerns us.

![Graph](image)

Figure 4. Daily course of upward infrared radiation from the surface at Dickinson for the 1970 growing season.

Figure 4 shows the time series of the total flux from the surface. The daily values vary from day to day, increasing in magnitude early in the season and reaching a minimum of 985 langleys on 8 July 1970 and then decreasing thereafter. The mean for the season was 862 langleys/day and the standard deviation was found to be $\pm 67$ langleys/day.

4.1.3.2 Downward Directed Longwave Radiation from the Atmosphere ($I^+$)

The evaluation of this term demands that one knows the magnitudes (mixing ratio) of the variable atmospheric constituents present at any time. For this purpose the main atmospheric constituents to be considered are the water vapor, carbon dioxide, and ozone.

$$I^+ = I^+ (H_2O) + I^+ (CO_2) + I^+ (O_3)$$  \hspace{1cm} (7)
Of these three, the water vapor varies the most from day to day, while the others remain quite constant for a longer period of time.

Staley and Jurica (1972) have formulated the radiative transfer equations to solve (7) for $I$, given the humidity data from atmospheric soundings and the profile of $CO_2$ and $O_3$ from climatological sources. A computer program was made available to the author by Dr. S. Cox of Colorado State University, which had been coded to perform the computation of the infrared radiation emission of the atmosphere using the Staley formulation.

The total downward longwave radiation at Dickinson and Cottonwood were approximated by using the 0000Z radiosonde data of Bismarck and Rapid City, respectively, and climatological ozone data obtained at Bradford, Massachusetts were used. The carbon dioxide mixing ratio was considered constant throughout the entire season (Johnson, 1954).

Figure 5 shows the daily time series of the incoming infrared radiation from the atmosphere, assuming clear skies. Daily variations are slightly more pronounced than the variations of infrared flux from the surface; the seasonal course is similar to the upward flux with a maximum of 820 langley/day reached on 27 July 1970. One should note the two secondary maxima with one in the first week of June and the other in the first week of September. The mean for the season is 680 langley/day with a standard deviation of ±72 langley/day.
Figure 5. Daily course of downward longwave radiation received at the surface at Dickinson.

### 4.1.3.3 Net Longwave Radiation $I = I_+ - I_-$

Figure 6 shows the net longwave or "effective" terrestrial radiation. The result shows a rather uniform distribution across the season with a mean of 181 langleys/day directed outward with a standard deviation of 31 langleys/day. A positive correlation between the daily upward and downward fluxes must exist in order to explain the rather uniform distribution of $I$. The correlation coefficient was found to be 0.90. Based on this rather high correlation it is of interest to point out that the infrared radiation budget is rather well defined by only one of the two components.

Figure 7 shows the relationship between the daily net longwave radiation and the daily net total radiation. There is no correlation between the parameters. The results suggest that for a first approximation over grassland, the daily net radiation follows the daily incoming shortwave radiation adjusted with a relatively constant value for net outgoing longwave radiation. This simple approach becomes more valid when time periods longer than one day are considered.
Figure 6. Net longwave radiation (Dickinson).

Figure 7. Observed net radiation vs. the net effective outgoing radiation (I).

4.1.3.4 Net Radiation - Computed vs. Observed

Figures 8 and 9 show the daily sequence of net radiation obtained from the computations and observations, respectively. The general seasonal trend is dramatically influenced by the daily sky character. A few values of negative net radiation were obtained. As
Figure 8. Computed daily net radiation, Dickinson, 1970.

Figure 9. Observed daily net radiation at Dickinson (dots). Daytime and nighttime components are shown by solid lines, 1970.
usual, these negative events are related to the passage of synoptic
scale weather systems with cloudy day (light hours) followed by clear-
ing skys at night.

For example, Figure 10 shows the cloud cover observed from satel-
lite for the daytime hours of 23 July 1970, a day of negative net
radiation. The sequence of weather with the cloud cover during the
day giving way to clear skies at night produce the observed negative
radiation value.

Figure 11 summarizes the calculated vs. observed daily net radia-
tion for Dickinson. The correlation coefficient of .83 is realized
from this analysis. Choosing different values of albedo, one obtains
the regression lines shown in Figure 12. Table III shows the change in
correlation for the varying choice of albedo. In general, the correla-
tion increases slightly as the albedo decreases. Problems with regard
to the station separation between Bismark and Dickinson contribute to
the analysis errors on a daily basis. Summing over periods of a week
or so, the scatter diminishes as shown in Figure 13. It may be con-
cluded that one may use the solar radiation computed with the above
technique to estimate $R_n$ for the purposes of climatological work for
grassland locations. More precise estimates of albedo and systematic
on-site measurements of all quantities are necessary to accurately
evaluate the result.

At Cottonwood the computation of $R_n$ cannot be verified by direct
observation; the analysis followed the same procedure as for Dickinson
and will be incorporated into the energy budget in the later sections.
Figure 10. Satellite picture from TIROS-1 orbit 2269, Long 95 W, Lat 55N, July 23, 1970, 1400 LST. Location of Dickinson is noted by circle.
Figure 11. Calculated net radiation ($R_n$) vs. observed net radiation ($R_{no}$) at Dickinson, summer, 1970. Assumption of 15% albedo.

Figure 12. Effect of albedo variations on the calculated ($R_{nc}$) vs. observed ($R_{no}$) relationship at Dickinson, growing season, 1970.
Cloud layers have a dual effect upon the radiation fluxes. During the day the cloud layers reflect a large part of the incoming solar radiation, thus reducing the increase of temperature of the ground during the daytime. On the other hand, when the sky is overcast, the clouds act in reducing the cooling of the air below the cloud base by trapping the outgoing longwave radiation. Because of these two opposite effects, the net effect of clouds on the average temperature at the surface and below the cloud base over a 24-hour period may be small. When clouds (see Figures 3a and 3b for overcast days) were introduced in the program to compute the downward infrared radiation, the result on the calculated net radiation at the surface was to decrease its correlation with the observed net radiation. A correlation coefficient of 0.68 was found. This correlation did improve (to 0.76) when a weighing factor that considers the amount of cloud cover was introduced. However, when clear skies were assumed the correlation coefficient increased to 0.83.
There are limitations to the calculated downward longwave radiation when the effects of clouds are considered. This includes the data measured for cloudiness. It is done only once a day at 0500 LST. Bias could be introduced by considering this value as a representative mean for the 24-hour period. It should be pointed out that over the area of study, extreme change of convective action is observed during the day.

In view of the limitation and realizing that most of the days were clear (see Figures 3a, b), the effects of cloudiness were not considered in this study.

![Diagram](image)

**Figure 13.** Calculated net radiation vs. observed net radiation at Dickinson summed over periods of time for which the water balance is sampled. Periods average approximately one week in length.

### 4.2 Transfer of Sensible Heat

The problem of estimating the exchange of sensible heat between the earth's surface and the atmosphere is exceedingly difficult to resolve. The instrumentation necessary to evaluate the exchange directly was not available. Indeed, even data necessary to apply micrometeorological models, e.g., classical aerodynamic methods, were unavailable.
The vertical transfer of sensible heat from the surface to the boundary layer of the atmosphere during the day may be written

\[
\int \text{Hdt} = C_p \int \int \frac{dT}{dt} \frac{\delta p}{g} - \int \int \frac{\partial}{\partial p} (1 - \alpha).
\]

Mor. \quad Mor. \quad p_o \quad Mor. \quad p_o

\[
(Q + q) \frac{\delta p}{g} + \int \int \frac{3}{\partial p} (I) \frac{\delta p}{g}
\]

Mor. \quad p_o \quad (8)

where \( t \) is time, \( T \) is temperature, \( C_p \) is the specific heat at constant pressure, \( p \) is pressure, \( p_o \) is surface pressure, and \( g \) is the acceleration of gravity.

The second and third terms on the right side of (8) are the convergence of short and longwave radiation, respectively; where \( \alpha \) is albedo, \((Q + q)\) is the direct and diffuse solar radiation, and \( I \) is the net infrared radiation at any layer.

Let us assume following Yamamoto (1962) that heating of the atmosphere boundary layer due to solar radiation is negligible. It follows then that (8) may be written

\[
\int \text{Hdt} = C_p \int \int \frac{\partial T}{\partial t} \frac{\delta p}{g} + C_p \int V_H \cdot V_H T \frac{\delta p}{g}
\]

Mor. \quad Mor. \quad p_o \quad p_o

\[- C_p \int \omega \frac{\partial T}{\partial p} \frac{\delta p}{g}\]

P \quad p_o \quad (9)
where the first term in the right hand side of (8) has been expanded into local derivatives. Here \( V_H \) is horizontal wind vector, \( \nabla \times \) is the horizontal gradient operator, and \( \omega \) is \( \frac{\partial p}{\partial t} \).

The terms of equation (9) were evaluated using the atmospheric sounding data from Bismarck (Dickinson) and Rapid City (Cottonwood) and then analyzed from charts originated by the U.S. Weather Bureau.

Soundings taken at Bismarck and Rapid City at 0500 and 1700 LST were used in the analysis. The integrals of equation (9) were evaluated by analysis techniques as described in Saucier (1955) or Pettersen (1956).

Figures 14a and 14b illustrate the magnitudes of the contributions to the net result by the three terms. Figure 14a shows an example of a day with extremely strong horizontal advection of temperature. The hatched area is the warming allotted to vertical sensible heat flux after the warming due to advection (dotted area) has been subtracted. Figure 14b shows an example of the contribution due to heating by subsidence (dotted area). This is a typical temperature structure of anticyclonic and subsiding air. The choice of the limits of the dotted area are subjectively chosen since no vertical motion measurements or computations are available. The limit, however, fits the sounding, and the classical subsidence inversion maintains itself against the sensible heat flux from below.

The results of the computation of sensible heat transport into the atmosphere, summed over approximately weekly intervals, are shown in Figures 15a and 15b for Bismarck (Dickinson) and Rapid City (Cottonwood), respectively, for the summer of 1970. The tabulated results are given in Table IV.
Figure 14a. Warming of the lower atmospheric layer due to sensible heat flux from the surface (hatched area) and horizontal advection (dotted area). May 12, 1970 at Rapid City, South Dakota.

Figure 14b. Warming of the lower atmospheric layer due to sensible heat from the ground (hatched area) and to subsidence aloft (dotted area), June 3, 1970 at Bismarck, North Dakota.
Figure 15a. The components of the energy budget for weekly period of time at Dickinson for the growing season of 1970.

Figure 15b. The components of the energy budget for week or longer period of time at Cottonwood for the growing season of 1970.
<table>
<thead>
<tr>
<th>Date</th>
<th>R (_{\text{no}}) (10(^3)ly)</th>
<th>LE (10(^3)ly)</th>
<th>H (10(^3)ly)</th>
<th>G (10(^3)y)</th>
<th>(\beta = H/LE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/27-6/14</td>
<td>6.57</td>
<td>2.73</td>
<td>3.19</td>
<td>0.65</td>
<td>1.17</td>
</tr>
<tr>
<td>6/15-6/21</td>
<td>3.00</td>
<td>1.60</td>
<td>1.28</td>
<td>0.16</td>
<td>0.80</td>
</tr>
<tr>
<td>6/22-6/28</td>
<td>3.34</td>
<td>1.74</td>
<td>1.44</td>
<td>0.16</td>
<td>1.13</td>
</tr>
<tr>
<td>6/29-7/5</td>
<td>3.04</td>
<td>1.36</td>
<td>1.54</td>
<td>0.14</td>
<td>1.13</td>
</tr>
<tr>
<td>7/6-7/12</td>
<td>2.93</td>
<td>1.36</td>
<td>1.40</td>
<td>0.17</td>
<td>1.03</td>
</tr>
<tr>
<td>7/13-7/19</td>
<td>2.74</td>
<td>1.30</td>
<td>1.43</td>
<td>0.01</td>
<td>1.10</td>
</tr>
<tr>
<td>7/20-7/27</td>
<td>3.00</td>
<td>1.26</td>
<td>1.65</td>
<td>0.11</td>
<td>1.32</td>
</tr>
<tr>
<td>7/28-8/3</td>
<td>2.32</td>
<td>1.60</td>
<td>0.76</td>
<td>-0.01</td>
<td>0.47</td>
</tr>
<tr>
<td>8/4-8/10</td>
<td>2.95</td>
<td>1.33</td>
<td>1.53</td>
<td>0.09</td>
<td>1.15</td>
</tr>
<tr>
<td>8/11-8/17</td>
<td>2.43</td>
<td>0.87</td>
<td>1.48</td>
<td>0.08</td>
<td>1.70</td>
</tr>
<tr>
<td>8/18-8/23</td>
<td>1.98</td>
<td>0.63</td>
<td>1.35</td>
<td>0.00</td>
<td>2.14</td>
</tr>
<tr>
<td>8/24-8/30</td>
<td>1.88</td>
<td>0.66</td>
<td>1.18</td>
<td>0.04</td>
<td>1.80</td>
</tr>
<tr>
<td>8/31-9/7</td>
<td>1.61</td>
<td>0.24</td>
<td>1.32</td>
<td>0.05</td>
<td>5.60</td>
</tr>
<tr>
<td>9/8-9/13</td>
<td>0.95</td>
<td>0.54</td>
<td>0.59</td>
<td>-0.19</td>
<td>1.10</td>
</tr>
<tr>
<td>9/14-9/20</td>
<td>0.91</td>
<td>0.12</td>
<td>0.84</td>
<td>-0.06</td>
<td>6.75</td>
</tr>
<tr>
<td>9/21-9/27</td>
<td>0.83</td>
<td>0.23</td>
<td>0.69</td>
<td>-0.09</td>
<td>3.10</td>
</tr>
<tr>
<td>9/28-10/2</td>
<td>0.77</td>
<td>0.26</td>
<td>0.57</td>
<td>-0.06</td>
<td>2.22</td>
</tr>
<tr>
<td>Total 5/27-10/2</td>
<td>41.26</td>
<td>17.84</td>
<td>22.2</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>Percentage</td>
<td>100</td>
<td>43.1</td>
<td>53.8</td>
<td>3.1</td>
<td></td>
</tr>
</tbody>
</table>
4.3 Change in Heat Storage in the Soil

The daily flux of heat into the soil was observed directly at Dickinson while it had to be calculated at Cottonwood. The calculation required differencing of temperature profiles from the surface to 150 cm over weekly periods and multiplying by the specific heat of soil.

The time series of the measured heat flux into the ground for the summer of 1970 at Dickinson is shown in Figure 16. It is observed to be a regime of relatively large variability. The seasonal trend shows mostly positive daily values from early in the season until the end of the first week of August, thereafter becoming negative. The mean of the net heat flux from 19 June until October 15 was 2.1 langleys with a standard deviation of ±18.8 langleys. The weekly sequence is plotted in Figure 15a and tabulated in Table IV.

The heat storage in the ground at Cottonwood was calculated for longer periods of time (approximately weeks) that corresponded to the soil water measurements.

The analysis technique involved least-squares fitting of fourth-order curves to the daily temperature series at each of the 10-, 20-, 50-, and 150-cm depths. The fitted curves were then interpolated at the dates desired for the corresponding water balance analysis.

The seasonal curve is shown on Figure 15b and tabulated in Table V. Positive values are observed early in the season until the end of the first week of August, changing thereafter to negative. As is the case for Dickinson, the ground at Cottonwood may be characterized as retaining energy as net heat flux early in the summer season and giving it off to the atmosphere later in the summer. The magnitudes are small compared to the other terms of the energy budget.
Figure 16. Sequence of daily net heat flux at Dickinson for the growing season, 1970.

<table>
<thead>
<tr>
<th>Date</th>
<th>R_{nc}(10^3 J/yr)</th>
<th>LE(10^3 J/yr)</th>
<th>H(10^3 J/yr)</th>
<th>G(10^3 J/yr)</th>
<th>( \beta = H/LE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 5/8-5/22</td>
<td>3.20</td>
<td>1.07</td>
<td>1.94</td>
<td>0.19</td>
<td>1.80</td>
</tr>
<tr>
<td>2 5/23-6/8</td>
<td>6.15</td>
<td>2.44</td>
<td>3.41</td>
<td>0.30</td>
<td>1.40</td>
</tr>
<tr>
<td>3 6/9-6/22</td>
<td>4.94</td>
<td>2.41</td>
<td>2.36</td>
<td>0.17</td>
<td>0.98</td>
</tr>
<tr>
<td>4 6/23-7/7</td>
<td>5.77</td>
<td>2.61</td>
<td>3.01</td>
<td>0.15</td>
<td>1.15</td>
</tr>
<tr>
<td>5 7/8-7/10</td>
<td>1.22</td>
<td>0.65</td>
<td>0.55</td>
<td>0.02</td>
<td>0.83</td>
</tr>
<tr>
<td>6 7/11-7/21</td>
<td>3.74</td>
<td>1.23</td>
<td>2.44</td>
<td>0.07</td>
<td>1.98</td>
</tr>
<tr>
<td>7 7/22-8/4</td>
<td>4.29</td>
<td>1.40</td>
<td>2.87</td>
<td>0.02</td>
<td>2.04</td>
</tr>
<tr>
<td>8 8/5-8/20</td>
<td>4.78</td>
<td>1.37</td>
<td>3.48</td>
<td>-0.08</td>
<td>2.54</td>
</tr>
<tr>
<td>9 8/21-9/4</td>
<td>3.51</td>
<td>0.77</td>
<td>2.89</td>
<td>-0.15</td>
<td>3.76</td>
</tr>
<tr>
<td>10 9/4-10/2</td>
<td>5.44</td>
<td>1.95</td>
<td>3.93</td>
<td>-0.44</td>
<td>2.01</td>
</tr>
<tr>
<td>Total 9/8-10/2</td>
<td>43.06</td>
<td>15.93</td>
<td>26.9</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Percentage</td>
<td>100.0</td>
<td>37.0</td>
<td>62.5</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>
4.4 Evaluation of the Latent Heat Exchange as a Residual and Summary of the Terms in the Energy Equation

The latent heat exchange (LE) was determined from the summation of the terms of the energy budget described previously. The daily values of each term were accumulated over periods of time corresponding to the measurements made of the soil water. These time increments average out to be about 8 days in length. Summation over this length of time help to smooth out errors, a factor realized previously in the net radiation computation (Section 4.1.3.4, Figure 13). This summation is particularly of benefit in the sensible heat transfer term since the computation is difficult to perform and contains large fluctuations.

Table IV and Figure 15a give the smoothed time sequence for each of the terms of the energy budget at Dickinson. Table V and Figure 15b give the corresponding analysis for Cottonwood. The latent heat transfer is the residual of the other terms of the budget.

Considering the Dickinson data (Figure 15a and Table IV), the latent heat transfer is of the same magnitude as the sensible heat flux until 10 August when it discontinuously falls to about half the magnitude of the sensible heat flux. Up until 10 August the latent heat transfer accounted for about 48 percent of the net radiation. After 10 August the latent heat transfer explains 31 percent of the net radiation. The energy budget is summarized for the whole season in the last row of Table IV.

A similar sequence is apparent for the Cottonwood analysis (Figure 15b, Table V) except that the separation between the latent and sensible heat transfers occurs a month earlier. The seasonal summary
for Cottonwood shows a greater dominance of sensible heat transfer
over the latent heat transfer than was realized at Dickinson.

For both stations the heat flux into the soil was small compared
to the other terms in the weekly as well as seasonal time frames.

4.5 The Ratio of Sensible to Latent Heat Transfer

The ratio of sensible to latent heat transfer, the Bowen ratio, is
a parameter often used to describe climate characteristics (Waggoner,
1965; Sellers, 1965) and is written

\[ \beta = \frac{H}{LE} \]

Figures 17a and 17b show the plots of H against LE as tabulated in
Tables IV and V for Dickinson and Cottonwood, respectively. The ratio
for Dickinson roughly fits the slope \( \beta = 1.0 \) over most of the range.
As the evaporation decreases in the latter part of the summer, the ratio
increases dramatically.

Cottonwood has a much more scattered relationship between H and LE
showing no systematic variation as the season progresses. The Bowen
ratio varies widely around the average value, lying between \( \beta = 1.0 \)
and \( \beta = 2.0 \). Dickinson's precipitation regime was one of a relatively
wet period in July preceded and followed by dry weather (Table VI). The
relatively wet soil continuously drying out with time provides the
seasonal variation in \( \beta \). Cottonwood had periodic precipitation periods
(Table VII) throughout the summer. The \( \beta \) response, therefore, is not
systematic, but follows each wetting of the soil.
Figures 17a, b. The sensible heat transfer plotted against transfer for Dickinson and Cottonwood, respectively, for the growing season, 1970.
5.0 WATER BUDGET EVALUATION OF EVAPORATION

The water budget may be expressed

\[ E = P - R + \Delta W \]

as developed in Section 2.2 previously. The components of the budget were measured at Dickinson and Cottonwood and compiled in the reports by Whitman (1971) and Lewis et al. (1971). The observation techniques are described in these reports. Observations of soil water content were normally taken on a weekly basis, but sometimes, particularly at Cottonwood, the period covers two weeks.

Figure 18 shows the soil water content measured at Dickinson over the summer season, 1970. The curve shows the response in soil water to large precipitation events of July shown in Figure 19. Assuming no surface or subsurface runoff, the water balance is tabulated in Table VI for Dickinson along with the estimate of evaporation from the energy budget. Figure 20 summarizes the two estimates of evaporative water loss as well as the corresponding evaluation of the Thornthwaite-Mather water balance model (Rasmussen, 1971).

![Graph of soil water content](image)

**Figure 18.** Soil water content for the first 50-cm depth at Dickinson for the growing season of 1970.
Figure 19. Daily course of precipitation recorded at Dickinson during the growing season of 1970.

**TABLE VI**
The Observed and Computed Values of the Components of the Water Budget for Dickinson, 1970. The values are rounded off.

<table>
<thead>
<tr>
<th>Date</th>
<th>ΔW(cm)</th>
<th>P(cm)</th>
<th>E(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/27-6/14</td>
<td>0.13</td>
<td>6.959</td>
<td>7.086</td>
</tr>
<tr>
<td>6/15-6/21</td>
<td>2.67</td>
<td>0.786</td>
<td>3.450</td>
</tr>
<tr>
<td>6/22-6/28</td>
<td>1.03</td>
<td>0.254</td>
<td>1.280</td>
</tr>
<tr>
<td>6/29-7/5</td>
<td>1.65</td>
<td>0.000</td>
<td>1.650</td>
</tr>
<tr>
<td>7/6-7/12</td>
<td>0.79</td>
<td>0.305</td>
<td>1.095</td>
</tr>
<tr>
<td>7/13-7/19</td>
<td>0.12</td>
<td>2.439</td>
<td>2.559</td>
</tr>
<tr>
<td>7/20-7/27</td>
<td>-2.07</td>
<td>3.556</td>
<td>1.486</td>
</tr>
<tr>
<td>7/28-8/3</td>
<td>-1.83</td>
<td>5.054</td>
<td>3.220</td>
</tr>
<tr>
<td>8/4-8/10</td>
<td>1.81</td>
<td>0.584</td>
<td>2.394</td>
</tr>
<tr>
<td>8/11-8/17</td>
<td>1.50</td>
<td>0.000</td>
<td>1.500</td>
</tr>
<tr>
<td>8/18-8/23</td>
<td>0.43</td>
<td>0.381</td>
<td>0.811</td>
</tr>
<tr>
<td>8/24-8/30</td>
<td>0.74</td>
<td>0.000</td>
<td>0.740</td>
</tr>
<tr>
<td>8/31-9/7</td>
<td>-0.47</td>
<td>0.152</td>
<td>-0.318</td>
</tr>
<tr>
<td>9/8-9/13</td>
<td>-0.31</td>
<td>1.473</td>
<td>1.16</td>
</tr>
<tr>
<td>9/14-9/20</td>
<td>-0.32</td>
<td>0.635</td>
<td>0.315</td>
</tr>
<tr>
<td>9/21-9/27</td>
<td>-1.19</td>
<td>1.803</td>
<td>0.613</td>
</tr>
<tr>
<td>9/28-10/2</td>
<td>1.14</td>
<td>0.000</td>
<td>1.14</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>6.10</td>
<td>24.381</td>
<td>30.499</td>
</tr>
</tbody>
</table>
Figure 20. The evaporation calculated as a residual from the energy and water budget, respectively. In addition, the potential evapotranspiration obtained from Thornthwaite-Mather method is included for Dickinson, 1970.

The corresponding analysis for the Cottonwood data is presented in Table VII and in Figure 21.

The general agreement between all three curves is quite remarkable. It appears that the evapotranspiration calculated from the energy budget is a little more sensitive to short-period fluctuations observed in the water balance. For Cottonwood the energy budget gives an underestimate for most periods, particularly late in the season. The result could be error due to the choice of two of a value for albedo; other factors could be misrepresentation of Rapid City due to elevation difference or some topography effect such as slope variations.

The results for Dickinson are presented in the scatter diagram format shown in Figure 22. It is apparent that a high correlation exists between the estimates, and this provides confidence in the result.
TABLE VII

The Observed and Computed Values of the Components of the Water Budget for Cottonwood, 1970.

<table>
<thead>
<tr>
<th>Date</th>
<th>ΔW(cm)</th>
<th>P(cm)</th>
<th>LE(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/8-5/22</td>
<td>0.78</td>
<td>1.650</td>
<td>2.43</td>
</tr>
<tr>
<td>5/23-6/8</td>
<td>3.05</td>
<td>0.889</td>
<td>3.94</td>
</tr>
<tr>
<td>6/9-6/22</td>
<td>2.85</td>
<td>2.184</td>
<td>5.03</td>
</tr>
<tr>
<td>6/23-7/7</td>
<td>3.00</td>
<td>0.610</td>
<td>3.61</td>
</tr>
<tr>
<td>7/8-7/10</td>
<td>-3.49</td>
<td>4.851</td>
<td>1.36</td>
</tr>
<tr>
<td>7/11-7/21</td>
<td>1.41</td>
<td>1.626</td>
<td>3.04</td>
</tr>
<tr>
<td>7/22-8/4</td>
<td>2.09</td>
<td>1.473</td>
<td>3.56</td>
</tr>
<tr>
<td>8/5-8/20</td>
<td>-0.063</td>
<td>3.734</td>
<td>3.10</td>
</tr>
<tr>
<td>8/21-9/4</td>
<td>0.95</td>
<td>0.305</td>
<td>1.25</td>
</tr>
<tr>
<td>9/5-10/2</td>
<td>0.40</td>
<td>4.115</td>
<td>4.51</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10.41</strong></td>
<td><strong>21.437</strong></td>
<td><strong>31.83</strong></td>
</tr>
</tbody>
</table>

Figure 21. The evaporation calculated as a residual from the energy and water budgets, respectively. In addition, the Thornthwaite-Mather potential evapotranspiration is included for Cottonwood, 1970.
Figure 22. Plot of the evaporation obtained from the energy budget against the evaporation obtained from the water budget. Dashed line is the perfect correlation for Dickinson, 1970 data.
6.0 SUMMARY AND CONCLUSIONS

Standard meteorological data sources have been used to estimate the components of the energy and water budget of the earth's surface. Good quantitative agreement between evaporation calculated on a weekly basis from the energy budget and from the water budget was attained.

The study of the radiation budget showed that the net radiation is largely determined by the incoming solar radiation adjusted for constant net infrared radiation flux.

The study showed that the net radiation was balanced by a relatively even distribution between sensible and latent heat transfers at Dickinson, North Dakota with a systematic variation in the ratio of the two fluxes (Bowen ratio) over the season. At Cottonwood, South Dakota the net radiation was balanced by latent and sensible heat transfers amounting to 37 and 62 percent, respectively. In both cases the net heat flux into the ground was small on both weekly and seasonal time scales.

The 1970 summer season for these sites was characterized by a summer season soil-water depletion. Good agreement was attained in the results of the energy budget, water budget, and Thornthwaite-Mather model describing the rate of depletion as a function of time.

The analysis suggests that the easily obtained Thornthwaite-Mather water balance would be a useful first approximation for the water balance of a region where no soil water measurement exists.

Energy and water budget computations such as reported in this paper should be expanded to cover all the IBP sites so that intercomparisons
can be made to develop site characteristics. In addition, the parameters should be included in the development of the ecological models. These evaluations of the physical processes at the earth's surface have the advantage of fitting into the balanced systems, and they have been checked using an independent technique.
LITERATURE CITED


APPENDIX I

LIST OF SYMBOLS

\(E_n\) —— net radiation at the surface
\(G\) —— rate of change of heat stored in the ground
\(H\) —— rate of sensible heat transferred to the atmosphere
\(LE\) —— heat used in the evapotranspiration process
\(Q\) —— direct solar radiation
\(q\) —— diffuse solar radiation
\(I\) —— net infrared radiation
\(I^\uparrow\) —— upward longwave radiation from the earth’s surface
\(I^\downarrow\) —— downward longwave radiation from the atmosphere
\(I^\downarrow(H_2O)\) —— downward longwave radiation due to water vapor
\(I^\downarrow(CO_2)\) —— downward longwave radiation due to carbon dioxide
\(I^\downarrow(O_3)\) —— downward longwave radiation due to ozone
\(a\) —— surface albedo
\(P\) —— precipitation
\(R\) —— runoff
\(E\) —— evapotranspiration
\(\Delta W\) —— rate of change of soil water storage
\(t\) —— time
\(T\) —— temperature
\(P\) —— pressure
\(P_0\) —— surface pressure
\(Mor.\) —— morning sounding time
\(Aft.\) —— afternoon sounding time
\(g\) —— acceleration of gravity
\(V_H\) —— horizontal wind vector
LIST OF SYMBOLS (Continued)

\( \omega \)  
change of total pressure with time

\( C_p \)  
specific heat at constant pressure

\( \nabla_H \)  
horizontal gradient operator

\( T_{2.5\text{ cm}} \)  
temperature at 2.5 cm height

\( \sigma \)  
Stephan–Boltzmann constant

\( \beta \)  
Bowen ratio