Raindrop Axis Ratios and Size Distributions in Florida Rainshafts: An Assessment of Multiparameter Radar Algorithms

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Abstract—Eleven penetrations of rainshafts by the University of Wyoming King Air (WKA) aircraft equipped with a two-dimensional (2-D) optical array probe are studied in coordination with multiparameter radar measurements from the National Center for Atmospheric Research (NCAR) CP-2 radar collected in a multicellular storm that occurred on August 8, 1991, of the Convective and Precipitation/Electrification (CaPE) experiment. A comparison is made between the mass-weighted mean diameter ($D_m$) and rainrate ($R$) computed from the nine-size spectra and their estimates from multiparameter radar algorithms based on $Z_D$ and $Z_R$. It was found that $D_m$ could be estimated with a mean bias of 0.07 mm and a standard deviation of 0.35 mm. Rainrates (in the range of 10–60 mm h$^{-1}$) could be estimated from $Z_D$ and $Z_R$ with a mean bias of 1–4% and fractional standard error (FSE) of 30–40% depending on the estimator used. Raindrop axis ratios are analyzed as a function of volume equivalent spherical diameter ($D_{eq}$) in the range 2–6 mm. The mean axis ratio versus the $D_{eq}$ relationship was found to be consistent with previous data from the High Plains (from Colorado and Montana). A study of fluctuations of axis ratio (about their mean value) showed that most drops have axis ratios close to their mean values with oscillation amplitudes to be typically ±10% in axis ratio, again consistent with the earlier High Plains results.

Index Terms—Multiparameter, radar, raindrop, size distributions.

I. INTRODUCTION

POLARIMETRIC radar rainrate algorithms based on reflectivity ($Z_h$), differential reflectivity ($Z_{dr}$), and specific differential phase ($K_{dp}$) offer physically based approaches to the measurement of rainfall. These algorithms are generally derived based on 1) equilibrium raindrop shapes via an axis ratio ($a/b$) versus the $D_{eq}$ (volume-equivalent spherical diameter) relation [1], [2] and 2) either exponential [3] or gamma drop-size distribution (dsd) [4] or experimentally measured distributions. Quantitative assessments of the improvements of polarimetric algorithms over $Z-R$ relations using simulations of dsd (fluctuations and radar measurement errors) have been performed by several investigators [5], [6] based on 1) and 2) above. The mean axis ratio ($a/b$) versus the $D_{eq}$ relation is critical for deriving algorithms based on $Z_{dr}$ or $K_{dp}$.

There has been much debate on the extent that the mean axis ratio versus the $D_{eq}$ relation can be biased by raindrop oscillations (e.g., [7], [8]), and quantitative data are lacking for large drops under conditions of moderate-to-high rainrates. Chandrasekar et al. [7] found that most of the drops had axis ratios close to the mean, with small oscillation amplitudes, typically 10%, in their aircraft-based study from the High Plains (from Colorado and Montana) (rainrates 1–15 mm h$^{-1}$) using two-dimensional (2-D)-Particle Measuring Systems, Inc. (PMS) probe images. In the High Plains data, ice cores in partially melting drops may have suppressed oscillations for drops with $D_{eq} > 4$ mm. In the Florida data set reported here, the 0 °C level is much higher [near 4.8-km altitude mean sea level (msl)] and the penetration altitude is at a much warmer temperature (~15 °C). Thus, the possibility of partially melted drops at the penetration altitude should be extremely small.

One goal of this paper is to extend the Chandrasekar et al. [7] drop axis ratio results to the much higher rainrate conditions (20–60 mm h$^{-1}$) in the Florida environment. A second goal is to quantitatively assess the ability of radar-measured $Z_{dr}$ to predict the mass-weighted mean diameter ($D_m$) of the dsd (assuming equilibrium shapes) by direct comparisons with $D_m$ from dsd’s measured by the 2-D-PMS probe. Effects of large amplitude drop oscillations due to collisional forcing, as suggested by Beard et al. [8], at high rainrates should negatively impact such a comparison. In fact, such comparisons between radar-measured $Z_{dr}$ and $Z_{dr}$ computed from dsd data from a distrometer in light rainrate conditions forced Goddard and Cherry [9] to empirically adjust the mean ($a/b$) versus the $D_{eq}$ relation (for $D_{eq} < 3$ mm) to remove a 0.1-dB bias in the $Z_{dr}$ comparisons. Later work revealed that their adjustment was in the right direction and that the cause of the upward shift toward sphericity in mean axis ratio was due to raindrop oscillations [7], [10]. A third related goal is the quantitative assessment of several rainrate algorithms based on $Z_h$ alone and on $Z_{dr}$ and $Z_{dr}$. These assessments will also be placed in context with theoretical assessments based on simulations of dsd and radar measurement errors. There is a need for such assessment, especially under the higher rainrate conditions reported here. Previous quantitative assessments using ground-based distrometers were generally made under light rainfall conditions [9], [11]. The one aircraft-based experiment using a 2-D-PMS probe and the Chilbolton radar [12] also involved light
rainrate (<10 mm h\(^{-1}\)) conditions. More recently, quantitative assessments of polarimetric rainrate algorithms have been conducted in heavier convective rainfall using raingages [6], [13]–[15].

A secondary goal is an assessment of a new algorithm to compute X-band-specific attenuation for dual-wavelength reflectivity ratio [16] by comparing radar estimates with specific attenuation computed directly from the measured dsd.

The data reported herein were obtained on August 8, 1991, in east-central Florida during the Convective and Precipitation/Electrification (CaPE) experiment. A multicellular storm was the target of intensive observations by the National Center for Atmospheric Research (NCAR) CP-2 radar and the University of Wyoming King Air aircraft (WKA). The WKA equipped with a 2-D-PMS probe made a series of 11 rainshaft penetrations over a period of 30 min at altitudes between 250 and 550 m. The storm, located at a range of 60 km from the CP-2 radar, was scanned with good time resolution [Range Height Indicator (RHI) or Plan Position Indicator (PPI) scan covering the storm every 2 min]. We focus on radar measurements of \(Z_{hi}\) and \(Z_{ek}\) and the dual-wavelength radar estimate of specific attenuation \(\Delta n_e\) at the 10-GHz frequency. The excellent navigation, typically within 100 m, made possible by the Global Positioning System (GPS) receivers on the WKA, ensured that aircraft and radar data could be accurately aligned in space. This requirement is critical for the quantitative assessments reported here in typical Florida cells with diameters \(\sim 5\) km.

This paper is organized as follows. Section II gives a radar overview of the storm over a period of about 30 min via constant altitude PPI sections. Section III describes the results of 2-D-PMS probe image analysis as related to raindrop axis ratios and inferred oscillation amplitudes. Section IV discusses details of four penetrations by comparing \(Z_{hi}\) and \(Z_{ek}\) from radar and computed from the probe data as a function of time (or, equivalently, position) along the track. Also given are up/down draft speeds and the penetration-averaged dsd. Section V discusses the quantitative assessments of radar-based inferences against probe data. Section VI concludes the paper with a summary of results. The Appendix details the radar data analysis procedures.

II. RADAR DATA

The storm of interest developed around 1730 UTC along a sea-breeze cloud line (visible as towering cumulus clouds from 1430 UTC) near the east-central Florida coast. The Appendix describes details of the multiparameter radar analysis methods used in this work. All radar measurements were obtained with the NCAR CP-2 radar, whose location is the grid origin in all subsequent figures. The environmental sounding taken at around 1700 UTC near the location of the storm showed the 0 °C level at 4.8 km and cloud base at 1.2 km. Henceforth, all times will be UTC and all altitudes will be above msl unless otherwise indicated.

Fig. 1 shows a time sequence of constant altitude PPI sections at 4-km (4 °C) altitude from 1734 to 1811, spaced roughly 7 min apart. Data from this altitude level enable a clearer depiction of individual cells within the complex in terms of positive \(Z_{hi}\) columns, i.e., areas of \(Z_{hi} \geq 1\) dB near the 0° level. Panels (a)–(f) show contours of \(Z_{hi}\) with differential reflectivity \(Z_{ek}\) as grayscale filled contours; only \(Z_{ek} \geq 1.0\) dB at this level is shown. These panels also show superimposed the WKA tracks at altitudes of 250–550 m as straight-line segments. These segments (identified by time) correspond to the spatial distances over which the dsd’s have been averaged for later analysis [described in Section IV(E)].

Prior to 1730 UTC, the storm could be associated with a single cell. The storm was isolated and appeared to have been initiated along a sea-breeze cloud line. After 1730, the storm displayed a multicellular structure, as seen in Fig. 1; separate cells are identified as the NW, S, and SW cells in panels (a), (d), (e), and (f). The signature of positive \(Z_{ek}\) (\(\geq 2.0\) dB) in the CAPPI sections at altitudes near 4 km is indicative of updrafts at these levels, as confirmed by aircraft and multiple-Doppler studies in similar Florida cells [17]–[20]. For example, positive \(Z_{ek}\) columns are noted in Fig. 1(a), (c), (d), and (e); this signature is typically due to a low concentration of large raindrops either ascending, descending, or being suspended in the updraft, depending on the terminal fallspeed [21], [22].

Fig. 1(a)–(f) show that the WKA made 11 penetrations of rainshafts between 1745 and 1813. The penetrations at 1745, 1756, 1804, and 1813 are detailed in Section IV-A and B. The 1745 penetration [see Fig. 1(b)] was made during the mature phase of the NW-cell marked in Fig. 1(a). The 1756 penetration [Fig. 1(d)] was also made during the mature phase around 6 min after the collapse of the positive \(Z_{hi}\) column in Fig. 1(c). The penetrations at 1804 and 1806 were made near the SW-cell [see Fig. 1(e)] during its vigorous growth phase. The last two penetrations at 1811 and 1813 were made during the mature phase of the SW-cell after the positive \(Z_{hi}\) column had weakened [see Fig. 1(f)].

III. AIRCRAFT DATA

The principal data source from the WKA was from the 2-D-PMS precipitation probe, which was mounted with horizontal optical axis so that the elliptical cross section of the drops could be imaged. The analysis method used here follows Chandrasekar et al. [7]. The 2-D-PMS precipitation probe has a resolution of 0.2 mm. For each entire image of the drop within the scan area (the minimum \(D_{eq}\) is chosen as 0.75 mm), the major (2b) and the minor (2a) axes of the principal elliptical cross section are determined via 1) a direct method for \(0.75 \leq D_{eq} \leq 2\) mm and 2) a 2-D filtering method for \(D_{eq} \geq 2\) mm. The missing scans at the leading edge of the image are compensated for using the method described in Xiao et al. [23]. The horizontal resolution was recalculated from the measured airspeed and the recorded clock rate, and the adjusted horizontal resolution was used to calculate the horizontal size of the images. The volume equivalent spherical diameter \(D_{eq}\) was estimated from \(D_{eq} = 2(ab)^{3/2}\). In the size distribution plots, all drops with \(D_{eq} \geq 0.75\) mm are shown.

Fig. 2 shows the average distribution from all 11 penetrations with a total sample volume of 62 140 l. The dashed straight line, which is a fit to the data, has a slope of 15.05 cm\(^{-1}\) and an intercept of 2155 m\(^{-3}\)mm\(^{-1}\) (which is around a factor of four less than the Marshall–Palmer [24]
Fig. 1. (a) Constant altitude PPI section at 4-km height showing contours of $Z_h$ (starting at 10 dBZ with increments of 10 dB) with grayscale overlay of $Z_{dr}$ (only values exceeding 1 dB are shown with darker shades representing more positive values). The CP-2 radar location is at the grid origin. (b)-(f) As in (a), except for times as indicated from 1742–1811 UTC. Straight line segments represent the WKA penetrations at the indicated times.
value of 8000). Later, we will compare this averaged slope of 15.05 cm\(^{-1}\) with the slopes from individual penetrations at 1757 and 1813 to illustrate the occurrence of parallel exponential slopes at different rainrates.

A. Raindrop Axis Ratios

The relation between axis ratio (ratio of minor to major axes \(a/b\) and \(D_{eq}\) for raindrops is fundamental to the interpretation of \(Z_{bh}\) and to the rainrate algorithm based on \(Z_{bh}\) and \(Z_{atk}\). We note here, in passing, that it is also fundamental to specific differential phase (\(K_{dp}\)) and on rainrate algorithms using \(K_{dp}\) or \(K_{hp}\) combined with \(Z_{bh}\). As shown by Chandrasekar et al. [7], it is possible to accurately estimate (within a few percent for \(D_{eq} \geq 2\) mm) axis ratios of drops from 2-D-PMS probe images. The first results from Chandrasekar et al. [7] were from rainshafts in the High Plains with rainrate ranging 1–15 mm h\(^{-1}\). The Florida results presented here in rainshafts with significantly higher rainrates (10–60 mm h\(^{-1}\)) and in warm-based clouds greatly expand the database on raindrop axis ratios.

Fig. 3(a) shows the mean axis ratio \((a/b)\) versus \(D_{eq}\) for all drops (numbering 3523) with \(D_{eq} \geq 2\) mm from all the penetrations between 1745 and 1815. The vertical bars represent the 95% confidence interval for \((a/b)\). Also shown are the mean \((a/b)\) from Chandrasekar et al. [7] as well as the model fit for equilibrium-shaped drops from Beard and Chuang [2] and the straight-line fit to the wind-tunnel results of Pruppacher and Beard [1]; i.e., \(a/b = 1.03 - 0.062D_{eq}\), where \(D_{eq}\) is in mm. Between 2 and 2.7 mm, the Florida results show a larger \((a/b)\) compared to the Beard and Chuang model results as well as to the earlier Chandrasekar et al. [7] results. Because the 2-D-PMS probe has a resolution of 0.2 mm, the axis ratio estimates for the smaller drops (\(D_{eq} \sim 2\) mm) will be less precise than for the larger ones (\(D_{eq} \geq 3\) mm). Thus, we are not confident about the magnitude of the upward shift in axis ratio at \(D_{eq} = 2.2\) mm. However, this trend toward higher \((a/b)\) for \(2 \leq D_{eq} \leq 2.7\) mm is qualitatively consistent with the empirical adjustment made by Goddard and Cherry [9] and with the laboratory results of Kubesh and Beard [25].

For \(3 \leq D_{eq} \leq 4.5\) mm, Fig. 3(a) shows that the mean axis ratios are in very good agreement with Beard and Chuang’s [2] model and fall within their upper and lower bound results [only their mean model curve is shown in Fig. 3(a)]. For \(4.5 < D_{eq} \leq 5.7\) mm, the Florida results are very close to the lower bound result of Beard and Chuang [2] and to the empirical fit recommended by Clift et al. [26].

Fig. 3(b) shows a histogram of \(a/b - \langle a/b \rangle\) for all drops with \(D_{eq} \geq 1.5\) mm. The measured axis ratio for each drop was divided by the mean axis ratio in the corresponding size \(D_{eq}\) interval to obtain the values that make up the histogram. The histogram shape in Fig. 3(b) is very similar to the results of Chandrasekar et al. [7], with the mode of the distribution being close to unity. One straightforward explanation is that most of the drops have axis ratios close to the mean [shown in Fig. 3(a)] with small oscillation amplitudes, typically, \(\pm 10\%\) in axis ratio. Thus, while raindrops do oscillate, the mean axis ratio of drops with \(D_{eq}\) in the range 3–5.7 mm is not altered significantly from the equilibrium shape results of Beard and Chuang [2]. This result contradicts the hypothesis of Beard et
al. [8] regarding collisional forcing of large amplitude drop oscillations and the resulting upward shift in mean axis ratio at higher rainrates. Recently, Tokay and Beard [27] argued that collisions cannot be a major source of drop oscillations since they observed significant numbers of oscillating drops, even though their estimated influence of collisions was clearly negligible.

IV. RADAR/AIRCRAFT INTERCOMPARISONS

Because of the high degree of navigational accuracy provided by GPS receivers on the WKA, it was possible to compare radar measured $Z_h$ and $Z_{kr}$ along the aircraft track with corresponding calculations from 2-D-PMS probe data. We illustrate such detailed comparisons for four penetrations at 1745, 1756, 1804, and 1813 [see Fig. 1(b), (d), (e), and (f)].

A. 1745 and 1756 Penetrations

The NW-cell marked in Fig. 1(a) was penetrated on its north side by the WKA at 1745. Fig. 4 summarizes the comparison between radar and processed 2-D-PMS probe data. Fig. 4(a) is generated as follows. Since the radar $Z_h$ is interpolated to a Cartesian grid with grid spacing of 0.25 km, values of $Z_h$ closest to the WKA track [shown in Fig. 1(b)] are extracted for each second of flight time, beginning at the start of the penetration noted in Fig. 4(a). Since the WKA penetration time (1745) fell in between two radar volumes, $Z_h$ data from two volumes (centered at 1744 and 1747) were averaged (in linear sense, i.e., units of mm$^3$m$^{-3}$) and plotted in Fig. 4(a) versus aircraft penetration time (the WKA airspeed was approximately 80 ms$^{-1}$). Similarly, $Z_{kr}$ from radar is plotted in Fig. 4(b). Also plotted in Fig. 4(a) and (b) are $Z_h$ and $Z_{kr}$, respectively, computed from the 2-D-PMS probe measurements as follows. Starting from 1745:30, bins of 500 raindrops are accumulated sequentially in time and a size spectrum is obtained for each bin from which $Z_h$ and $Z_{kr}$ are computed and plotted versus the center time of the bin. The $Z_h$ is approximately computed as $\langle D_{eq}^2 N(D_{eq}) \rangle$ over the bin of 500 drops where the drops are assumed to be spherical (later we use rigorous scattering methods for oblate shapes to calculate $Z_h$). The $Z_{kr}$ is also approximately computed by first calculating the reflectivity-weighted mean axis ratio for the bin of 500 drops and then, using a simple relation from Jameson [28] to calculate $Z_{kr}$, $10^{-\alpha_D(Z_{kr})} = \left(\frac{a}{b}\right)^{7/3}$. The straight-line fit to Pruppacher and Beard [1], i.e., $a/b = 1.03 - 0.0062 D_{eq}$ ($D_{eq}$ in mm), is used for these figures [see Fig. 3(a)]. These approximations are sufficient to show that good correlation can be achieved between the radar and 2-D-PMS measurements, in spite of large sample volume differences. Also, the axis ratios from the consecutive bins of 500 drops are not used to calculate $Z_{kr}$ since such a small sample cannot accurately estimate the mean axis ratio, especially for the large drops (>3 mm). Over the 3.2-km penetration segment, the $Z_h$ gradients from radar are less than $\sim$8 dBkm$^{-1}$. There is a general tendency for the radar measurements to “smooth” the 2-D-PMS-based data. This spatial smoothing effect is caused by the antenna beam pattern (at a range of typically 60 km to the cell, the 3-dB beamwidth of around 1° produces a cross-beam width of 1 km). Fig. 4(c) shows the up/downdraft speed from aircraft data with downdrafts up to $-5$ ms$^{-1}$ during this penetration. Fig. 4(d) shows the averaged size distribution for $D_{eq} \geq 0.75$ mm. The dashed line is a straight-line fit to the exponential distribution $N(D_{eq}) = N_0 \exp(-\Delta D_{eq})$. The rainrate $R$, the mass-weighted mean diameter ($D_m$) defined as $\langle D_{eq}^2 N(D_{eq}) \rangle / \langle D_{eq}^2 N(D_{eq}) \rangle$, and $\Delta$ are, respectively, 32.6 mm$^{-1}$, 2.6 mm, and 14.7 cm$^{-1}$.

The WKA penetration in the region of most uniform reflectivity of all 11 penetrations occurred between 1756:00 and 1756:54 [see Fig. 1(d)]. Fig. 5 shows the results of the radar/aircraft comparison, similar to Fig. 4. The $Z_h$ comparison is excellent, while $Z_{kr}$ from the 2-D-PMS probe data slightly underestimates the radar $Z_{kr}$. Again, weak downdrafts prevail. The rainrate calculated from Fig. 5(d) is 53.4 mm$h^{-1}$ [only the 1751 penetration, see Fig. 1(c), had a higher value of 57.8 mm$h^{-1}$, which is the largest rate of all the penetrations]. In fact, both the 1751 and 1756 penetrations occurred within the same general area of the cell. The parameter $D_m = 2.31$ mm, while $\Delta = 15.9$ cm$^{-1}$.

B. 1804 and 1813 Penetrations

Two WKA penetrations at 1804 and 1813 are next discussed. At 1804, the WKA penetrated a high $Z_{kr}$ region ($Z_{kr} > 3$ dB) with high $Z_h$ ($\sim$50 dBZ). The timing of this penetration was during the growth phase of the SW-cell.

The radar/aircraft comparisons are shown in Fig. 6. In Fig. 6(a), note the $Z_h$-gradient from radar around 15 dBkm$^{-1}$ nearly coincident with the $Z_{kr}$-gradient from the 2-D-PMS data. The same holds true for the $Z_{kr}$-gradient in Fig. 6(b). The up/downdraft speed is shown in Fig. 6(c), again with weak downdrafts around 0 to $-2$ ms$^{-1}$. The size distribution in Fig. 6(d) shows a “flat” tail for $D_{eq} > 3.5$ mm. The rainrate is 16.8 mm$h^{-1}$ and $D_m = 3.24$ mm. Personnel aboard the WKA reported strong evidence of drop sorting with big raindrops on the southwest side of the storm complex [see Fig. 1(e)].

Because of an excess of big drops in the distribution, the $Z_h$ is large but $R$ is much smaller than expected from the usual $Z_h$-$R$ relations. It is precisely under such circumstances that knowledge of $Z_{kr}$ is important.

The WKA made a repeat penetration of the high $Z_{kr}$ region at 1806 [see Fig. 1(e)]. The shape of the distribution (not shown here) was nearly identical to Fig. 6(d). The rainrate was 8 mm$h^{-1}$, the lowest value of all the penetration segments, while $D_m = 3.26$ mm. Again, an excess of big drops gives a high $Z_{kr}$ ($)\sim$45 dBZ) and lower $R$ than expected from $Z_h$-$R$ relations.

The WKA made its last two penetrations at 1811 and 1813, closer to the cell’s center and during its mature phase [see Fig. 1(f)]. Fig. 7 shows the radar/aircraft comparison for the 1813 penetration. The correlation between radar- and aircraft-derived $Z_h$ and $Z_{kr}$ is comparable to the earlier penetrations. From the size spectrum in Fig. 7(d), the rainrate is 27.8 mm$h^{-1}$, $D_m = 2.5$ mm, and $\Delta = 15.8$ cm$^{-1}$.

This slope is nearly identical to the 1756 penetration (15.9 cm$^{-1}$), whereas the rainrate at 1756 (53 mm$h^{-1}$) is nearly twice the value at 1813 (27.8 mm$h^{-1}$). This result contradicts the Marshall–Palmer [24] relation between $\Delta$ and $R$ given
as $\Lambda = 41R^{-0.21}$, which yields $\Lambda$ of 17.8 (20.4 cm$^{-1}$) for $R = 53$ (28 mmh$^{-1}$). A large number of dsd’s summarized by Hu and Srivastava [29] show this tendency for parallel exponential slopes (at large $D_{eq} \geq 2$ mm), with an average slope around 20 cm$^{-1}$ for $R$ ranging from 20 to 200 mmh$^{-1}$. Such “equilibrium” distributions are thought to occur when the binary processes of drop breakup and coalescence are in balance [29]. Even when all 11 penetrations are averaged (Fig. 2), the slope $\Lambda = 15.05$ cm$^{-1}$ is only slightly less than the slope for the individual 1756 and 1813 penetrations. This slight decrease is caused by the inclusion of “flat” tail spectra at 1804 [see Fig. 6(d)] and 1806.
Fig. 5. As in Fig. 4, except comparison is for the 1757 penetration [see Fig. 1(d)]. (d) The dashed line corresponds to $N(D_{eq}) = 5.290 e^{(-1.59D_{eq})}$. The still-air rainfall rate is 53.4 mm h$^{-1}$, $D_{eq} = 2.31$ mm and sample volume is 7014 l.

V. Pénétration-Averaged Comparisons

For all 11 penetration segments from 1745–1816, plots similar to those shown in Figs. 4–7 were examined to select those segments where the radar $Z_h$ gradients were less than 10 dB km$^{-1}$ to avoid biases due to antenna beam smoothing. Except for the penetration segments at 1804 and 1806, the remaining nine segments typically had $Z_h$ gradients $\leq 5$ dB km$^{-1}$. Henceforth, our analysis will exclude the 1804 and 1806 segments. From the size spectrum obtained over the duration of each of the nine penetration segments [for example, such as those shown in Figs. 4(d) through 7(d)], we computed using the $T$-matrix method [30] parameters, such as reflectivity at horizontal polarization ($Z_h$) and specific attenuation at X-band (10 GHz) at horizontal polarization.
The raindrops are modeled as oblate spheroids with the symmetry axis of the spheroids oriented vertically, and the dielectric constant of water at a temperature of 25 °C is obtained at 3 and 10 GHz \cite{31}. From each of the nine size spectra, we also computed rainrate $R$ (in still air) and mass-weighted mean diameter ($D_m$). The penetration-averaged radar parameters were computed as follows from radar data shown, for example, in Figs. 4(a) and (b) through 7(a) and (b). For each segment, the radar $Z_h$ along the track is averaged (in linear sense, i.e., converted to mm$^6$m$^{-3}$) to give $\langle Z_h \rangle$. The average $Z_{kr}$ is computed as $\langle Z_h \rangle - \langle Z_a \rangle$, where $Z_h$ and $Z_a$ are separately averaged in linear sense. The specific attenuation $A_k$ is averaged in units of dBkm$^{-1}$. Several algorithms are used to compute penetration-averaged rainrate $\langle R \rangle$ based on $Z_h$ alone and based on $Z_h$ and $Z_{kr}$. For example, $\langle R \rangle$ from algorithms based on $Z_h$, such as $R = aZ_h^b$, is obtained as $\langle aZ_h^b \rangle$ rather than $a\langle (Z_h) \rangle^b$ and similarly for $\langle R \rangle$ based on $Z_h$ and $Z_{kr}$.
Fig. 7. As in Fig. 4, except comparison is for the 1813 penetration [see Fig. 1(f)]. (d) The dashed line corresponds to \(N(D_{sv}) = 2436 \exp(-1.58D_{sv})\). The still-air rainfall rate is 27.8 mm h\(^{-1}\), \(D_{sv} = 2.5\) mm and sample volume is 8312 l.

Fig. 8 shows the scatter plot of penetration-averaged \(Z_{th}\) (or \(\langle Z_{th}\rangle\)\(\text{radar}\)) and computed from the size distribution \(\langle Z_{th}\rangle_{\text{dsd}}\). The bias \((B)\) is defined as

\[
B = \frac{1}{N} \left\{ \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} x_i \right\}
\]

and the fractional standard error (FSE) as

\[
\text{FSE} = \left[ \frac{1}{N} \sum_{i=1}^{N} (y_i - x_i)^2 \right]^{1/2} \div \left[ \frac{1}{N} \sum_{i=1}^{N} x_i \right]
\]

where \(N\) is the number of penetration segments \((N = 9\) for this and all subsequent scatter plots), \(y_i\) is penetration-averaged ordinate values based on radar measurements, and \(x_i\) is abscissa values based on penetration-averaged size distribution calculations. In Fig. 8, the bias is only \(-0.41\) dB. As discussed in the Appendix, the CP-2’s radar constant was adjusted using the technique described in Chandrasekar et al. [5] by using time series data from August 24, 1991. The excellent bias result in Fig. 8 is due to this adjustment scheme, which compares \(R(Z_{th}, Z_{kb})\) with \(R(K_{th})\) in rainfall. Note that the CP-2
Fig. 8. Scatter plot of (penetration-averaged) radar measured $Z_h$ versus $Z_h$ computed from (penetration-averaged) dsd. The August 24th data were obtained in a special time series mode, where the complex video returns were stored for each transmitted pulse and for each resolution volume. Note that the radar’s signal processor could not compute $K_d$ in real-time during CaPE. The August 24th data were obtained in a special time series mode, where the complex video returns were stored for each transmitted pulse and for each resolution volume.

The specific attenuation (at 10 GHz) scatter plot is shown in Fig. 9. The $B = -0.17$ dB/km$^{-1}$ and FSE = 40% (corresponds to rms error of 0.49 dB/km$^{-1}$). As described in the Appendix, a new range-filtering algorithm was used to estimate $A_{\text{specific}}$ [16]. Liu et al. [32] estimated the accuracy of specific attenuation estimates using this algorithm to be around 0.17 dB/km by using vertical pointing data from August 24, 1991, in homogeneous light rain conditions (where the mean $A_{\text{specific}}$ should tend to zero, thus, revealing errors due to algorithm bias and statistical fluctuations). Note that radar constant biases will not contribute to the error. In the narrow rainshafts (typically 2–4 km) of this present case, the cumulative attenuation is not large (typically 10 dB), which makes it difficult to estimate $A_{\text{specific}}$ accurately. Aydin et al. [14] estimated the accuracy of the algorithm to be around 0.25 dB/km$^{-1}$ (based on data from Colorado using the CP-2 radar), which is consistent with the results in Fig. 9. We believe these are the first results comparing radar estimates directly with aircraft dsd measurements. These results give confidence in the new algorithm of Hubbert and Bringi [16], which is an iterative range-filtering technique using finite impulse response filters.

Jameson [33] has derived a third-order polynomial fit relating $D_m$ to $Z_{\text{eq}}$ (in linear scale) based on gamma dsd’s and the $a/b = 1.03 - 0.002D_{\text{eq}}$ relation for axis ratio. The solid line in Fig. 10 shows this polynomial relation. This figure also shows a scatter plot of $D_m$ dsd (computed for each of the nine WKA size spectra) versus $Z_{\text{eq}}$ radar (averaged $Z_{\text{eq}}$ along the track from radar measurements) in linear scale. Note that the experimental mean axis ratio versus $D_{\text{eq}}$ curve in Fig. 3(a) is not used in constructing Fig. 10. The experimental data are in good agreement with Jameson’s [4(b)], which gives the average fractional error of the estimate of $D_m$ inclusive of gamma distribution fluctuations and $Z_{\text{eq}}$ measurement errors ($\Delta Z_{\text{eq}}$). Using an average value of $D_m$ dsd = 2.55 mm from all nine penetrations in Jameson’s Fig. 4 gives $\Delta D_m/D_m = 10.1\%$ (for $\Delta Z_{\text{eq}} = 0.1$ dB) and 17.9% (for $\Delta Z_{\text{eq}} = 0.2$ dB). Note that the experimentally-derived FSE of 13.7% falls within the two simulated values. These results are comparable to the only two known earlier ground-based results of Aydin et al. [11] and Goddard and Cherry [9] in light rainfall, who obtained FSE’s in the range 10–15% for the prediction of median volume diameter ($D_3$) from radar $Z_{\text{eq}}$ measurements.

Several rainrate algorithms were tested next. These include the following:

1) $Z_h-R$ relation from Sekhon and Srivastava [34] used by Jameson [33] in his simulations. $R = 0.0146(Z_h)^{0.4741}$; 
2) sixth order polynomial fit of $Z_h/R$ versus $Z_{\text{eq}}$ by Jameson [33]; and 
3) relation proposed by Gorgucci et al. [15] that fits the parameters of a relation $R = C_2Z_h^{1.07}Z_{\text{eq}}$ based on a cumulative distribution function (cdf) matching criteria for rainfall by using data from radar and raingages for a 1-h convective event that occurred on July 26, 1991, during CaPE.
Their fitted equation is \( R(Z_{th},Z_{bf}) = 0.01(Z_{th})^{0.014} 10^{-0.37r_{zd}} \). Note that in the above equation \( Z_{th} \) is in \( \text{mm}^2\text{m}^{-3} \), while \( Z_{bf} \) is in decibels. The above three rainfall estimates are termed \( R(Z_{th})_{SS} \) (for Sekhon–Srivastava [34]), \( R(Z_{th},Z_{bf})_{JAM} \) (for Jameson [33]), and \( R(Z_{th},Z_{bf})_{cdf} \) (from Gorgucci et al. [15]).

Fig. 11(a)–(c) show the scatter plot of radar-based rain-rates versus \( (R)_{dsd} \) using the three estimators. In Fig. 11(a), the \( B = 20.6 \text{ mmh}^{-1} \) and FSE = 70.4%. From the error simulations of Jameson [33] for the average fractional error \( (\Delta R/R) \) versus \( D_m \), the \( R(Z_{th})_{SS} \) estimator gives a FSE = 75% at \( D_m = 2.55 \) mm in good agreement with the measured FSE of 70.4%. The \( Z_{th} \) measurement error in the simulations were assumed to be \( \pm 1.0 \) dB. In Fig. 11(b), the \( B = 0.33 \text{ mmh}^{-1} \), a drastic reduction from the value of \( 20.6 \text{ mmh}^{-1} \) in Fig. 11(a), which shows the added information contributed by \( Z_{bf} \). The FSE is also reduced to 37.1% in good agreement with simulated FSE of 42% in Jameson [33] at \( D_m = 2.55 \) mm. Finally, in Fig. 11(c), we show the results for \( (R(Z_{th},Z_{bf}))_{cdf} \). The \( B = 1.4 \text{ mmh}^{-1} \) and FSE = 31.8%; this FSE being the smallest among the three estimators considered, perhaps, because the Gorgucci et al. [15] estimator is based on cdf matching for a rainfall event from the same CaPE regime, as in the present study (convective rainfall events from July 26 and August 8, respectively). The results obtained here for the FSE using \( R(Z_{th},Z_{bf}) \) are consistent with the raingage comparison reported by Aydin et al. [13], who obtained an FSE of 38% in a rainfall event lasting 30 min, with rainrates in the range 20–40 mmh\(^{-1}\).

Two recent studies involving \( K_{dp} \) rainrate estimators were compared against raingage accumulations over periods of 1–2 h in convective events known to contain small hail. In the Aydin et al. [14] study, one event with one weighing bucket raingage was analyzed for rainfall accumulation over an 80-min period using \( R(K_{dp}) \) from the Colorado State University (CSU)-CHILL radar and \( R(A_x) \) from the CP-2 radar. Both algorithms predicted the total gage accumulation of 73 mm to within 10%. The Ryzhkov and Zrnic [6] study involved 2-h rainfall accumulations over 42 gages in a squall-line event with small hail. They report an FSE of around 20% for \( R(K_{dp},Z_{bf}) \) estimators when comparing 2-h rainfall accumulations. Because \( K_{dp} \) is independent of the radar constant and \( Z_{bf} \) can be accurately calibrated to within 0.1 dB (because it is a differential power measurement), estimators using both \( K_{dp} \) and \( Z_{bf} \) should outperform estimators that use \( Z_{th} \) and \( Z_{bf} \). This was pointed out nearly two decades ago by Seliga and Bringi [35].

VI. CONCLUSION

Coordinated analysis of aircraft 2-D-PMS probe observations of raindrops with multiparameter radar measurements in a multicellular storm have yielded valuable results concerning the accuracy of multiparameter radar estimates of \( D_m \) and rainrate and the accuracy of estimating \( A_x \) from dual-frequency reflectivity data. By analyzing nine aircraft penetrations in rainshafts (with rates in the range 10–60 mmh\(^{-1}\)), it was determined that, using \( Z_{bf} \), the mass-weighted mean diameter of the size distribution (\( D_m \)) could be estimated with a mean bias of 0.07 mm and a standard deviation of 0.35 mm. These results are in excellent agreement with Jameson’s [33] simulations for the average fractional error in estimating \( D_m \) (using \( Z_{bf} \)), considering gamma size distribution fluctuations and measurement errors. Several rainrate algorithms were considered, such as \( R(Z_{th}) \) and \( R(Z_{th},Z_{bf}) \). As expected, the \( R(Z_{th}) \) estimator gave FSE = 70% and normalized bias = 55%. The \( R(Z_{th},Z_{bf}) \) estimator from Gorgucci et al. [15] gave FSE = 32% and normalized bias = 4%. The results using the \( R(Z_{th},Z_{bf}) \) estimator from Jameson [33] gave FSE = 37% and normalized bias = 1%, which are in excellent agreement with his simulations. The dual-frequency radar estimates of \( A_x \) using a new, adaptive
filtering algorithm described by Hubbert and Bringi [16] were directly compared with $A_x$, computed from size distributions yielding FSE = 40% and normalized bias = 14%. The rms error corresponds to 0.5 dBkm$^{-1}$, which is quite good considering the small cumulative attenuation ($\sim 10$ dB) in rainshafts considered here.

Raindrop axis ratios of nearly 3500 drops (with $D_{eq} > 1.5$ mm) were estimated from the 2-D-PMS probe images and compared with the earlier results of Chandrasekar et al. [7] from the High Plains. Good consistency was obtained for the mean axis ratio versus $D_{eq}$ relation between the High Plains (rainrate 1–15 mmh$^{-1}$) and Florida (rainrate 10–60 mmh$^{-1}$) data sets and with the model results of Beard and Chuang [2] for equilibrium-shaped drops. Drop oscillations were studied via a histogram of measured $(a/b)/[a/b]$ for all drops with $D_{eq} > 1.5$ mm. These showed excellent consistency between the High Plains and Florida data sets. Most of the drops were observed to have axis ratios very close to their mean values (the mode of the histogram was close to unity), with oscillation amplitudes to be typically ±10% in axis ratio for rainrates in the range 10–60 mmh$^{-1}$.

**APPENDIX**

The NCAR CP-2 radar used in this study is described, for example, in Bringi and Hendry [36]. The raw data from the radar analyzed here consists of range profiles of horizontally polarized reflectivity ($Z_h$), the differential reflectivity ($Z_{dh}$), and the dual-frequency reflectivity ratio (DFR) at the two frequencies (10 and 3 GHz). The accuracy in estimates of $Z_h$ (±1 dB) and $Z_{dh}$ (±0.25 dB) are standard (see, for example, Doviak and Zrnić [37]). From the range profiles of DFR, the specific attenuation at X-band is calculated using the ice particles and/or because of their lower density. This systematic overprediction or underprediction of $R(Z_h, Z_{dh})$ from $R(K_{dp})$ away from the 1:1 line can be attributed to the radar constant. Thus, by fine-tuning the radar constant, it is possible to correct for any systematic overprediction or underprediction of $R(Z_h, Z_{dh})$ relative to $R(K_{dp})$. Time series data were collected in a special data acquisition mode on August 24, 1991, in a squall-line-type rainfall event from which $K_{ph}$ could be derived. This procedure resulted in an adjustment to the nominal radar constant of 3.73 dB, i.e., from 73.3 to 69.57 dB. Wilson et al. [39], using a scheme of intercomparing reflectivities from the three NCAR radars (CP-2, 3, and 4) in carefully selected precipitation echoes, found that the CP-2 radar constant should be adjusted downward by 1.5 to 3.0 dB, which is consistent with the direction and magnitude of our adjustment of 3.73 dB.

The multiparameter fields $Z_h$, $Z_{dh}$, and $A_x$ are interpolated onto a Cartesian grid by using the NCAR REORDER software package. A relatively fine spacing was used (250 m in the three directions). A Cressman weighting scheme was used with a 250-m horizontal radius and a 500-m vertical radius for data acquired in PPI mode; for data in RHI mode, the horizontal radius was set to 500 m and the vertical radius to 250 m. The NCAR CEDRICK software was used to further analyze and display the fields.

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