Fault Tolerant Properties of Kinematically Redundant Manipulators

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Abstract

The dexterity of kinematically redundant manipulators has been quantified and utilized for satisfying a variety of performance criteria. In this work, the degradation of dexterity due to a joint failure is analyzed. A definition of fault tolerance based on a worst-case measure of the remaining dexterity is presented and the properties of optimal fault tolerant configurations are discussed.

I. Introduction

Kinematically redundant manipulators are inherently more dexterous than conventional non-redundant manipulators due to the increased number of degrees of freedom. Researchers have analyzed this dexterity and have defined a variety of measures to attempt to quantify this somewhat qualitative property [7,13,14]. These measures have been applied as criteria in the design process and manipulator configurations which are optimal in terms of manipulability [13], isotropy [8], and singularity properties [5] have been determined. A large body of work has further illustrated how to control redundant manipulators that are constrained to follow a specified end effector trajectory while simultaneously optimizing various secondary criteria including joint range availability [9], singularity avoidance [13], joint torque [6], and obstacle avoidance [1,10,12], as well as the various measures of dexterity [2,7,13,14].

One aspect of redundant manipulators that has not been analyzed, however, is the consequences of being unable to control one or more of the degrees of freedom. This loss of a degree of freedom may be a result of a mechanical failure of the actuator, damage to the control electronics, or corruption in the sensing circuitry (experience has shown that the loss of joint position information due to contamination of the optical encoders is not an uncommon occurrence in dirty environments [4]). It is obvious that if the number of failed joints is less than or equal to the degree of redundancy then the end effector can still be arbitrarily positioned and oriented, however, the degree of dexterity may be severely compromised. For example, consider a seven degree-of-freedom manipulator which has been designed to have a three degree-of-freedom positioning component and a four degree-of-freedom wrist for specifying orientation. Loss of one of the positioning joints will render the manipulator virtually useless in the desired workspace even though it is still theoretically equivalent to a conventional non-redundant manipulator.

In this work, a measure of fault tolerance for redundant manipulators is derived based on the remaining dexterity following the loss of a degree of freedom. Using this measure as a criterion, a technique for calculating optimal fault tolerant configurations for redundant manipulators will be presented. Finally, the properties of these configurations will be analyzed in order to assist designers in determining the number of degrees of freedom required to maintain a minimum level of dexterity under a worst-case scenario.

II. A Definition of Fault Tolerance

The dexterity of manipulators is frequently quantified in terms of the properties of the Jacobian matrix which relates end effector velocities to joint angle velocities. The Jacobian will be denoted by the m by n matrix J where m is the dimension of the end effector space and n is the number of degrees of freedom of the manipulator. For redundant manipulators $n > m$ and the quantity $n - m$ is the degree of redundancy. The Jacobian can be written as a collection of columns

$$J_{m\times n} = [\, j_1 \quad j_2 \quad \cdots \quad j_n \,] \tag{1}$$

where j_i represents the end effector velocity due to a rotation at joint i. For an arbitrary single joint failure at joint f, assuming that the failed joint can be locked, the resulting m by $n-1$ Jacobian will be missing the fth column, where f can range from 1 to n. This Jacobian will be denoted by a preceding superscript so that in general

$${}^{f}J_{m\times n-1} = [\, j_1 \quad j_2 \quad \cdots \quad j_{f-1} \quad j_{f+1} \cdots \quad j_n \,]. \tag{2}$$

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The properties of the Jacobian are typically illustrated through the use of the singular value decomposition (SVD) which can be defined as

$$J = UDV^T \tag{3}$$

where U is an m by m orthogonal matrix of the output singular vectors, V is an n by n orthogonal matrix of the input singular vectors, and D is a diagonal matrix of the form

$$D_{m\times n} = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \ddots & \vdots & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 & \vdots & & \vdots \\ 0 & \cdots & 0 & \sigma_m & 0 & \cdots & 0 \end{bmatrix} \tag{4}$$

where the σ_i are the singular values which are typically ordered from largest to smallest. Most local dexterity measures can be defined in terms of simple combinations of these singular values, for example their product (determinant), sum (trace), or ratio (condition number). The most significant of the singular values is σ_m, the minimum singular value, since it is by definition the measure of proximity to a singularity and tends to dominate the behavior of both the manipulability (determinant) and the condition number. The minimum singular value is also a measure of the worst-case dexterity over all possible end effector motions.

If one assumes that the manipulator is initially in an isotropic configuration, then all of the σ_i will be equal and without a loss of generality can be considered to be equal to 1. In this case, if ${}^f\sigma_m$ denotes the minimum singular value of ${}^fJ$ then ${}^f\sigma_m$ is a measure of the worst-case dexterity if joint f fails. If all joints are equally likely to fail, then a measure of the worst-case fault tolerance of the current manipulator configuration is given by the minimum ${}^f\sigma_m$ over all f. In order to guarantee that the minimum ${}^f\sigma_m$ is as large as possible, they should all be equal. Thus, in this work, the definition of an optimal fault tolerant configuration is one in which all of the ${}^f\sigma_m$ are equal for $1 < f < n$. Physically, this can be interpreted as attempting to balance the use of all joints so that they contribute equally to the motion of the end effector.

Using the above definition of an optimal fault tolerant configuration, one can identify the structure of the Jacobian required to obtain this property. In terms of the SVD of J, the matrix of output singular vectors, U, simply represents a rotation of the end effector coordinate frame so that it does not affect the configuration of the manipulator and can be arbitrary set to identity without a loss of generality. In addition, since the manipulator is initially in an isotropic configuration with all $\sigma_i = 1$ only the matrix V^T needs to be considered. If V^T is partitioned between the m and $m+1$ rows in the following manner

$$V^T = \begin{bmatrix} v_{1,1} & v_{2,1} & \cdots & v_{n,1} \\ v_{1,2} & v_{2,2} & \cdots & v_{n,2} \\ \vdots & \vdots & \cdots & \vdots \\ v_{1,m} & v_{2,m} & \cdots & v_{n,m} \\ \hline v_{1,m+1} & v_{2,m+1} & \cdots & v_{n,m+1} \\ \vdots & \vdots & \cdots & \vdots \\ v_{1,n} & v_{2,n} & \cdots & v_{n,n} \end{bmatrix} \tag{5}$$

then the $n-m$ rows v_{m+1}^T to v_n^T will span the null space and the upper m rows v_1^T to v_m^T will be equivalent to J so that

$$j_i = \begin{bmatrix} v_{i,1} \\ v_{i,2} \\ \vdots \\ v_{i,m} \end{bmatrix}. \tag{6}$$

The magnitude of the contribution of an individual joint i to the motion at the end effector is given by the norm of j_i. Since the definition of an optimal fault tolerant configuration requires that each joint contribute equally to the motion at the end effector, this translates into a constraint that all of the norms of the columns of the Jacobian be equal. In the following discussion, the scalar R_i will be used to denote the portion of joint i's motion that is transformed into the range space of J, so that

$$R_i = \|j_i\|^2 = \sum_{k=1}^{m} v_{i,k}^2. \tag{7}$$

The optimal fault tolerant criteria can be alternatively described as requiring each joint to contribute equally to the null space of the Jacobian transformation. Physically, this means that the redundancy of the robot is uniformly distributed among all the joints so that a failure at any joint can be compensated for by the remaining joints. An individual joint's contribution to the null space, denoted by N_i, is given by

$$N_i = \sum_{k=m+1}^{n} v_{i,k}^2. \tag{8}$$

It is easy to see that the condition that all of the R_i be equal is equivalent to all of the N_i being equal since

$$R_i + N_i = 1 \tag{9}$$

due to the fact that V is an orthogonal matrix. While these conditions are mathematically equivalent, one or the other may be computationally preferable depending on the degree of redundancy relative to the dimension of the end effector space.

III. Calculation of Optimal Fault Tolerant Configurations

The preceding section has shown that a necessary and sufficient condition for identifying an optimal fault tolerant configuration is that the Jacobian have columns of equal norm as well as orthogonal rows. In order to generate a Jacobian that has these properties, one can use a procedure that is similar to a method which is used to calculate the SVD [11]. The algorithm discussed here starts with an arbitrary orthogonal matrix (which can be considered V^T in (5)) and applies transformations until all of the N_i (or R_i) are equal. In order to not destroy the orthogonality of the original matrix while making all of the N_i equal, the transformations that are applied are a combination of Givens rotations. Givens rotations are orthogonal transformations of the form

$$Q = \begin{array}{c} \left[\begin{array}{ccccc} I & & & & \\ & \cos(\theta) & & -\sin(\theta) & \\ & & I & & \\ & \sin(\theta) & & \cos(\theta) & \\ & & & & I \end{array}\right] \begin{array}{c} \\ i \\ \\ k \\ \\ \end{array} \\ \begin{array}{ccccc} \quad & i & \quad & k & \quad \end{array} \end{array} \tag{10}$$

where all other elements not shown are zero. This transformation can be geometrically interpreted as a plane rotation of θ in the $i-k$ plane. Each of the individual Givens rotations operates on two columns, i and k, with the rotation angle, θ chosen to make $N_i = N_k$. After applying a Givens rotation the new columns of the Jacobian, j_i' and j_k' are given by

$$j_i' = j_i \cos(\theta) + j_k \sin(\theta) \tag{11}$$

and

$$j_k' = j_k \cos(\theta) - j_i \sin(\theta). \tag{12}$$

One can show that the constraint on the resulting column norms being equal can be satisfied by choosing a θ which solves the following quadratic equation

$$\tan^2\theta + \left[\frac{\sum_{l=1}^{m} 4 j_{i,l} j_{k,l}}{\sum_{l=1}^{m} j_{i,l}^2 - j_{k,l}^2}\right] \tan\theta - 1 = 0. \tag{13}$$

The above discussion shows how to determine a single Givens rotation which will make two column of J have equal norms. It still needs to be shown how these elementary transformations can be combined to make all of the column of J have equal norms. If the Givens rotation that makes columns i and k equal is denoted Q_{ik}, then the product of a set of $n(n-1)/2$ rotations denoted by

$$Q_l = \prod_{i=1}^{n-1} \left(\prod_{k=i+1}^{n} Q_{ij} \right) \tag{14}$$

will be referred to as a sweep [3]. Unfortunately, a single sweep will not, in general, make all of the columns of J equal since subsequent rotations can destroy the equality produced by previous ones. However, it is clear that the procedure must converge so that

$$V^T = \prod_{l=1}^{s} Q_l \tag{15}$$

where s is the number of sweeps. Convergence of the algorithm is based on completing an entire sweep with all of the columns of J being equal within some desired tolerance. If any two columns are equal to within this tolerance during a sweep then that rotation need not be performed.

IV. Examples of Optimal Fault Tolerant Configurations

The simplest example of an optimal fault tolerant configuration is given by the following Jacobian

$$J = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \end{bmatrix} \tag{16}$$

which represents the configuration of a planar three degree-of-freedom manipulator. The null space for this manipulator is given by

$$v_3 = \begin{bmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \end{bmatrix} \tag{17}$$

which illustrates that each joint contributes equally to the null space motion thus distributing the redundancy proportionally to all degrees of freedom. Geometrically, it is easy to see that the three vectors j_1, j_2, and j_3 are all 120 degrees apart which results in a balanced coverage of the planar workspace. If the three possible joint failures are considered, one can show that

$$^f\sigma_2 = \sqrt{\frac{1}{3}} \tag{18}$$

for $f = 1$ to 3, which satisfies the optimal fault tolerant criterion. The end effector motion which suffers most from a failure at one of the joints is, intuitively, in the direction of the column of the Jacobian which is associated with that joint, so that

$$^f u_2 = \frac{j_f}{\|j_f\|}. \tag{19}$$

It is instructive to consider the effects of adding another degree of freedom to this planar manipulator and recalculating an optimal fault tolerant configuration. For a planar four degree-of-freedom manipulator, one can see by inspection that the following orthogonal matrix

$$V^T = \left[\begin{array}{cccc} \sqrt{\frac{1}{2}} & 0 & \sqrt{\frac{1}{2}} & 0 \\ 0 & \sqrt{\frac{1}{2}} & 0 & \sqrt{\frac{1}{2}} \\ \hline \sqrt{\frac{1}{2}} & 0 & -\sqrt{\frac{1}{2}} & 0 \\ 0 & \sqrt{\frac{1}{2}} & 0 & -\sqrt{\frac{1}{2}} \end{array}\right] \tag{20}$$

satisfies all of the conditions of an optimal fault tolerant configuration. Physically one can interpret this configuration as simply having joints three and four duplicate the responsibilities of joints one and two, in effect adding another actuator to each of the actuators of the original non-redundant manipulator. The worst-case dexterity for an arbitrary joint failure in this case is given by

$$^{f}\sigma_2 = \sqrt{\frac{1}{2}} \tag{21}$$

for $f = 1$ to 4, which should be obvious from a physical point of view since the joints work in pairs to provide the two end effector degrees of freedom.

The configuration represented by (20), however, is only one of an entire family of configurations which can be represented by the matrix

$$V^T = \begin{bmatrix} \sqrt{\frac{1}{2}} & 0 & \sqrt{\frac{1}{2}}c\alpha & -s\alpha \\ 0 & \sqrt{\frac{1}{2}} & s\alpha & \sqrt{\frac{1}{2}}c\alpha \\ \hline \sqrt{\frac{1}{2}} & 0 & -\sqrt{\frac{1}{2}}c\alpha & s\alpha \\ 0 & \sqrt{\frac{1}{2}} & -s\alpha & -\sqrt{\frac{1}{2}}c\alpha \end{bmatrix} \tag{22}$$

where $s\alpha$ and $c\alpha$ are the sine and cosine of an arbitrary parameter describing this family. This equation points out an important property regarding the duplication of all actuators in a redundant robotic system. From a fault tolerance point of view, the same result can be achieved by combining two non-redundant manipulators of arbitrary design, as long as they are both at an isotropic configuration. The important point to be noted is that it is not necessary to exactly duplicate the effects of each actuator but to duplicate the coverage of the end effector motion space. One of the implications of this observation is that manipulators can be modularly designed using isotropic components, any number of which can be combined in order to achieve an arbitrary degree of fault tolerance.

For a manipulator that must position it's end effector in three-dimensional space, the simplest redundant manipulator would possess four joints. An optimal fault tolerant configuration for such a manipulator is given by

$$V^T = \begin{bmatrix} -\sqrt{\frac{3}{4}} & \sqrt{\frac{1}{12}} & \sqrt{\frac{1}{12}} & \sqrt{\frac{1}{12}} \\ 0 & -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ 0 & 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \hline \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}. \tag{23}$$

The worst-case dexterity for an arbitrary joint failure is given by

$$^{f}\sigma_3 = \frac{1}{2} \tag{24}$$

for $f = 1$ to 4. Thus the addition of a fourth joint to a three degree-of-freedom non-redundant positioning manipulator can guarantee that half of the dexterity can be maintained after an arbitrary single joint failure if the manipulator operates at the optimal configuration. One can show that in general, the worst-case dexterity of a redundant manipulator that experiences a single joint failure is governed by the inequality

$$^{f}\sigma_m \leq \sqrt{\frac{n-m}{m}} \tag{25}$$

where the best case of equality occurs if the manipulator is in an optimal fault tolerant configuration. The above inequality makes sense from a physical point of view since it represents the ratio of the degree of redundancy to the dimension of the task space that it must cover. This inequality can be used to determine the degree of redundancy required to maintain a minimum amount of dexterity in the event of a single joint failure.

V. Disscusion and Conclusions

This paper has quantified the effect of a joint failure on the remaining dexterity of a kinematically redundant manipulator. The basic assumption has been that once a failure is identified (with the faulty joint being locked) one would still like to operate the manipulator in a somewhat limited capacity, i.e. graceful degradation. Since one must assume that all failures are equally likely (otherwise more effort would be put into the design of the component most likely to fail) the task of maximizing the remaining dexterity requires that the motion responsibilities of all joints be balanced. This has implications both on manipulator design and on the nominal operating point within the workspace. It has been shown that the remaining dexterity following a joint failure is governed by (25) which implies that a single joint failure in a poorly designed manipulator can render the machine useless, regardless of the degree of redundancy. On the positive side, however, the three-dimensional example in section IV shows that with the addition of one fourth of the total number of actuators, one can guarantee half of the original dexterity following an arbitrary joint failure.

One final point concerning a limitation on the algorithm presented in section III should be emphasized. While the algorithm is fully general and can calculate an optimal fault tolerant configuration for an arbitrary dimension of the task space (m) and an arbitrary number of degree-of-freedom manipulator (n), it makes no assumption about the type of actuators that are available. Thus, for a task space that includes both positioning and orientation, the algorithm may provide a design that requires a fully general screw motion for the actuator rather than a simple rotational or prismatic joint. Modifications to the algorithm which limit the desired manipulator actuators to either rotary or prismatic joints are currently being implemented.

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