THESIS

A SPATIAL STOCHASTIC PROGRAMMING MODEL FOR TIMBER AND CORE AREA MANAGEMENT UNDER RISK OF STAND-REPLACING FIRE

Submitted by

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ABSTRACT

A SPATIAL STOCHASTIC PROGRAMMING MODEL

FOR TIMBER AND HARBITAT CORE AREA MANAGEMENT

UNDER RISK OF STAND-REPLACING FIRE

Forest harvest scheduling has been modeled using deterministic and stochastic programming models. Past models seldom address explicit spatial forest management concerns under the influence of natural disturbances. In this research study, we employ multistage full recourse stochastic programming models to explore the challenges and advantages of building spatial optimization models that account for the influences of random stand-replacing fires. Our exploratory test models simultaneously consider timber harvest and mature forest core area objectives. Each model run reports first-period harvesting decisions for each stand based on a sample set of random fire. We integrate multiple model runs to evaluate the persistence of period-one solutions under the influence of stochastic fires. Follow-up simulations were used to support multiple comparisons of different candidate forest management alternatives for the first time period. Test case results indicate that integrating the occurrence of stand-replacing fire into forest harvest scheduling models could improve the quality of long-term spatially explicit forest plans.

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1 Introduction

Natural disturbances such as fire, wind, insects and diseases interact with forest management activities across and beyond forest planning horizons. Disturbances can influence timber flow, economic return, forest age-class distribution, and forest spatial structure. A number of research studies have highlighted the importance of accounting for the influence of such stochastic disturbances in forest planning models.

In many forests, wildland fire is one of the major disturbance factors influencing long-term timber supplies. Interest in developing forest-wide harvest schedules that account for the risk of wildland fire blossomed in the 1980s beginning with a study by Reed and Errico (1986) and has continued into the present. Reed and Errico formulated a deterministic non-spatial harvest scheduling model with flow constraints to maximize the return from harvests under the influence of mean value fire disturbance rates. They later enhanced this model (Reed and Errico 1989) by accommodating the possibility of partial salvage, multiple timber types, accessibility constraints and variable recovery costs. Pasalodos-Tato et al. (2010) developed a deterministic non-spatial model to evaluate the interaction between fire and timber management in stands of Maritime Pine (*Pinus pinaster*).

Gassmann (1989) developed a non-spatial stochastic programming model with seven recourse stages to evaluate optimal forest-wide harvesting in the presence of fire. Because solving a stochastic problem with many recourse stages is computationally challenging, the last three stages in this model were deterministic with only one realization of fire occurrence. Gassmann found that by modeling four stages of recourse, the period one solution tended to stabilize. He also suggested using penalty terms in the objective function instead of enforcing timber flow through constraints. Similarly, Boychuck and Martell (1996) developed a non-spatial multistage stochastic programming model with ten discrete planning periods, the first four modeled with recourse. They examined the impacts of timber flow limitations and fire severity.

Armstrong (2004) developed a non-spatial stochastic simulation model to test deterministic annual allowable cut (AAC) solutions in Alberta, Canada. After determining an AAC using a linear programming model, Armstrong evaluated the results using Monte Carlo simulations by assuming the proportion of area burned in each period is random. The study compared the effects of fire under different harvest schedules in different types of forest. Konoshima *et al* (2008) tested a spatially explicit model in a regularly shaped landscape with seven hexagonal stands assuming that harvesting a stand produces higher fire spread rates across the stand.

Spring and Kennedy (2005) used a stochastic dynamic programming (SDP) model to study the trade-offs between producing timber and protecting endangered species under the threat of random fires. Ferreira *et al.* (2011) developed another SDP model which assumed harvesting happens before fire in each stand. Markov chain models describing stand transitions of mixed loblolly pine-hardwood forests under the influence of natural disturbances have been used to study the trade-offs between landscape diversity and timber objectives (Zhou and Buongiorno 2006). This type of model also has been used to search for the optimal harvest schedule for a forest subject to random wildfires (Campbell and Dewhurst 2007)**.**

As Boychuck and Martell (1996) summarized, the effects of fire disturbance on timber supplies over time appear to vary considerably depending on a number of factors. In multi-stand or multi-strata harvest scheduling models with forest-level constraints such as non-declining flow, a frequent outcome is that "attempting implementation of mean value problem solutions in a stochastic system leads to infeasibility with high probability" (e.g., see Pickens and Dress 1988 and Hof et al. 1988). Boychuck and Martell observed, however, that mean value problem solutions generally provided fair approximations to stochastic programming problem first-period solutions. In systems where periodic re-planning occurs, they suggest that mean value solutions may even be good approximations where allowances are made for fire risk by harvesting less than the solution indicates (i.e., by retaining a timber supply buffer stock). Boychuck and Martell indicate that more complex stochastic programming methods "would be justified in areas with a tight timber supply, lacking sufficient overmature areas, having high and highly variable fire losses, and where harvest quantity declines are particularly unwanted."

Many forest planning problems, however, include objectives besides allowable cut or financial returns from timber harvests. These non-timber objectives are often spatially explicit and many require spatial optimization methods to account for landscape patterns and arrangement effects (e.g., see Hof and Bevers 1998, 2002; Murray 2007). Forest planning problems with harvest area ("adjacency") constraints (e.g. Goycoolea et al. 2005) or habitat requirements for species that dwell in the interior ("core area") of mature forests (e.g., Öhman and Eriksson 1998) are common examples. We note that, so far, few studies on the subject of forest planning under fire risk have been conducted with spatially explicit models. We also note that mean value approaches to modeling fire disturbance may be untenable in the stand-based or cut block approaches to forest-wide harvest scheduling typical in mathematical programming formulations of these problems.

In this paper, we explore the use of a spatial multi-stage stochastic programming model to select optimal harvest schedules for timber and core area joint production under the influence of wildfires. This model is similar in many respects to the model III harvest scheduling formulation (Gunn 1991), which models forest growth and harvesting by balancing the area of forest entering and leaving each state (age class) in each stage (period) (Boychuck and Martell 1996). Our formulation is revised to maintain stand boundaries and to produce mature forest core area. We tested this model through an artificial forest to examine model performance under different disturbance assumptions. We also studied the resulting trade-offs between timber and core area production.

2 Methods

We first introduce a deterministic even-aged harvest scheduling model. This model depicts the development and management of stands in a simulated forest without the appearance of any random disturbance. Management decisions are simplified to just harvest each stand entirely or do nothing in each scheduling period. We then incorporate random samples of fire disturbance into the model and reformulate it using sample average approximation (SAA) method (Kleywegt et al 2001; Bevers 2007). Management actions and fires in this revised model are assumed to have a fixed sequence of occurrence. An enhanced formulation is then presented by modeling that sequence as a random event. In the next step, we introduce the concept of influence zone (Hoganson et al 2005) as well as describe the construction of mature forest core area (Wei and Hoganson 2007). We then incorporate core area into forest planning, forming a multistage full recourse stochastic model, to address both timber harvest and mature forest core area conservation in the presence of random wildfire.

2.1 Formulating a deterministic timber harvesting model

We consider the stand as the smallest management unit. We assume a stand could be in age class 1, 2 or 3 at the start of each period before the implementation of any management activity. Harvest and “do nothing” are the only two available management options for each stand. Without being harvested, a stand will stay in the same age class within the same period and will grow into an older age class at the beginning of the next planning period; the age of harvested stands will be reset to zero in the same period and will be advanced to age class one when entering the next planning period. Stands older than the defined maximum age are assumed to stay within the maximum age class without harvesting (See figure 1 below).

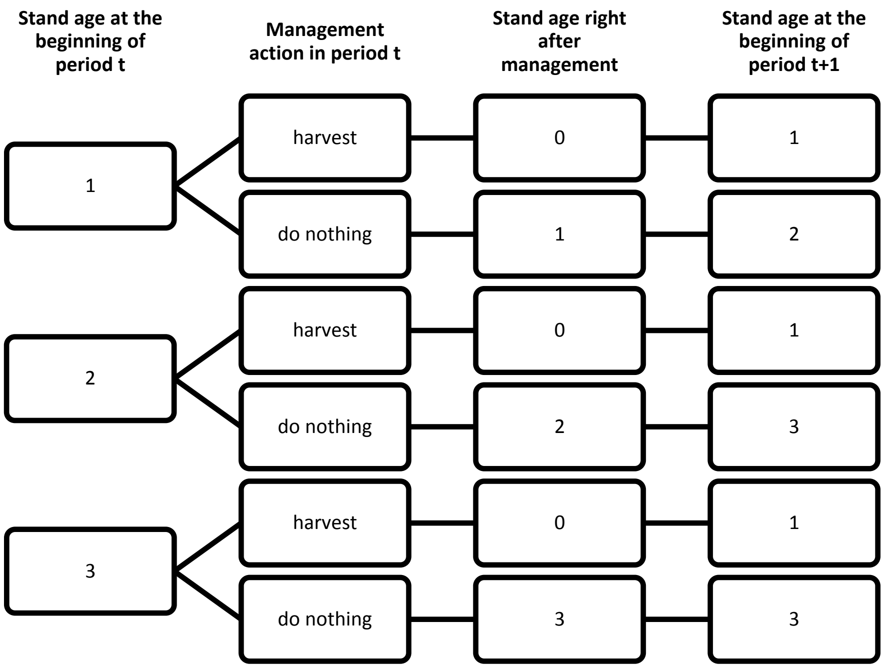


Fig 1: Illustration of a network representing the transition of stand age between two consecutive periods (deterministic model)

This model coordinates stand level decisions across time and space to maximize forest level timber-based economic returns. The formulation, as presented below, is constructed to maintain the integrity of stand boundaries by not merging stands with identical age classes and stand types as model III formulations do. Area balance constraints are used to track how timber harvests and fire in each stand impact stand age class over time.

Maximize:

$$\sum_i \sum_j \sum_k \sum_t v_{ijkt} X_{ijkt} - q \sum_{t \in \{2\ to\ T\}} Q_t \qquad (1.1)$$

Subject to:

$$\sum_k X_{ij_1k1} = 1 \qquad \forall i \qquad (1.2)$$

$$\sum_{j'} \sum_{k \in K_{ij'j}} X_{ij'k(t-1)} - \sum_k X_{ijkt} = 0 \qquad \forall i, j \in J_{it}, t \in \{2\ to\ T\} \qquad (1.3)$$

$$Q_t \geq \sum_i \sum_j \sum_k c_{ijk} X_{ijk(t-1)} - \sum_i \sum_j \sum_k c_{ijk} X_{ijkt} \quad \forall\, t \in \{2\ to\ T\} \qquad (1.4)$$

Where:

i indexes forest stands.

j, j' index stand ages. In equation *(1.2)*, j_1 denotes the current age class of stand *i* (age class at the start of period 1)

k indexes management actions: either harvest or do nothing.

t indexes time period.

X_{ijkt} is a binary decision variable indicating, when set to 1, the selection of management option *k* for stand *i* at age class *j* during period *t*.

Q_t is a set of bookkeeping variables to track declines in timber production between two consecutive periods.

T denotes the total number of planning periods across the entire planning horizon.

J_{it} denotes the set of possible age classes for stand *i* at period *t*.

$K_{ij'j}$ denotes the set of management options that can advance stand *i* from state j' in period t-1 to state *j* in period t.

a_i is area (Acres) of stand i

c_{ijk} denotes timber yield from managing stand *i* following prescription *k* when this stand is at age class *j*. c_{ijk} is zero if the "do nothing" prescription is assigned to stand *i*.

p_i is the predefined price per cord of timber for stand i which varies by stand-type and stand age class as in table 1 below

Table 1: Prices of timber that vary by stand type and stand age class

	Price per cord of timber ($/cord)		
	Stand type 1	Stand type 2	Stand type 3
Age class 1	*0*	*0*	*0*
Age class 2	*27*	*28*	*25*
Age class 3	*27*	*28*	*25*

q is a positive coefficient which is used as the penalty for each unit of timber decline between each pair of consecutive periods.

r is the predefined discount rate for timber value (r=0.04)

v_{ijkt} denotes the discounted net revenue of managing stand *i* at age class *j* following prescription *k* in period *t*. v_{ijkt} is zero if the "do nothing" prescription is assigned to stand *i*.

$$v_{ijkt} = \frac{1}{(1+r)^{40t-20}} a_i c_{ijk} p_i$$

The objective function *(1.1)* maximizes the total return of managing a forest for *T* periods. It includes two components: the discounted profits from harvesting timber, and a penalty on any decline in timber production between the *(T-1)* pairs of consecutive periods. Constraint *(1.2)* requires that exactly one stand management activity, including "do nothing", is selected for each stand at period one. Constraint *(1.3)* links the harvesting options between two consecutive periods for each stand. This is similar to the area balance constraint used in model III formulations. However, because this constraint is built for each stand, it maintains the stand boundaries when tracking the age of a stand. Constraint *(1.4)* requires that there is no decline in timber production from between two consecutive periods. This formulation is used as a base for further enhancement. In the next step, we add new decision variables and constraints to model the influence of stand-replacing fires.

2.2 Adding random samples of fire disturbance

We incorporate random stand-replacing fire events into our harvest scheduling model with the assumption that *fires always occur after management activities within each planning period*. Simulated fires destroy randomly selected stands based on fire probabilities that vary by age classes and reset stand age to zero within the period of occurrence. The simulation of fire disturbance is illustrated with an example as in figure 2 below

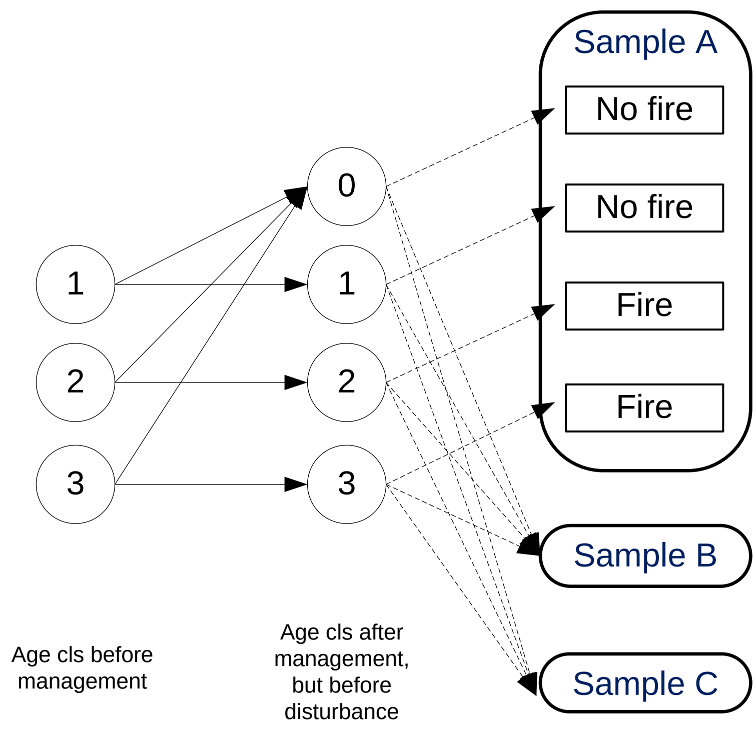


Fig 2: Illustration of how to simulate fire disturbance in a stand in each planning period

In this example, we draw three random samples *A*, *B* and *C* to reflect the fire occurrences in a stand depending on the age class after management. The number in each circle represents stand age class; arrows represent the possible stand age class changes in the same planning period. “Harvest” resets stand age to zero; “do nothing” does not change stand age class in the same period. We assume the probability of fire in each stand depends on its age class. Two samples *A* and *B* were built based on random draws to reflect the fire occurrences in this stand depending on its possible age classes. We assume sample *A* represents a case based on random draws that 1) there is no fire if this stand is in age class zero or one; 2) fire happens if this stand is in age class two or three. Different samples *B* and *C* could reflect different fire occurrences depending on the age class of a stand after management. The transition of stand age class with random fire occurrence is illustrated in figure 3.

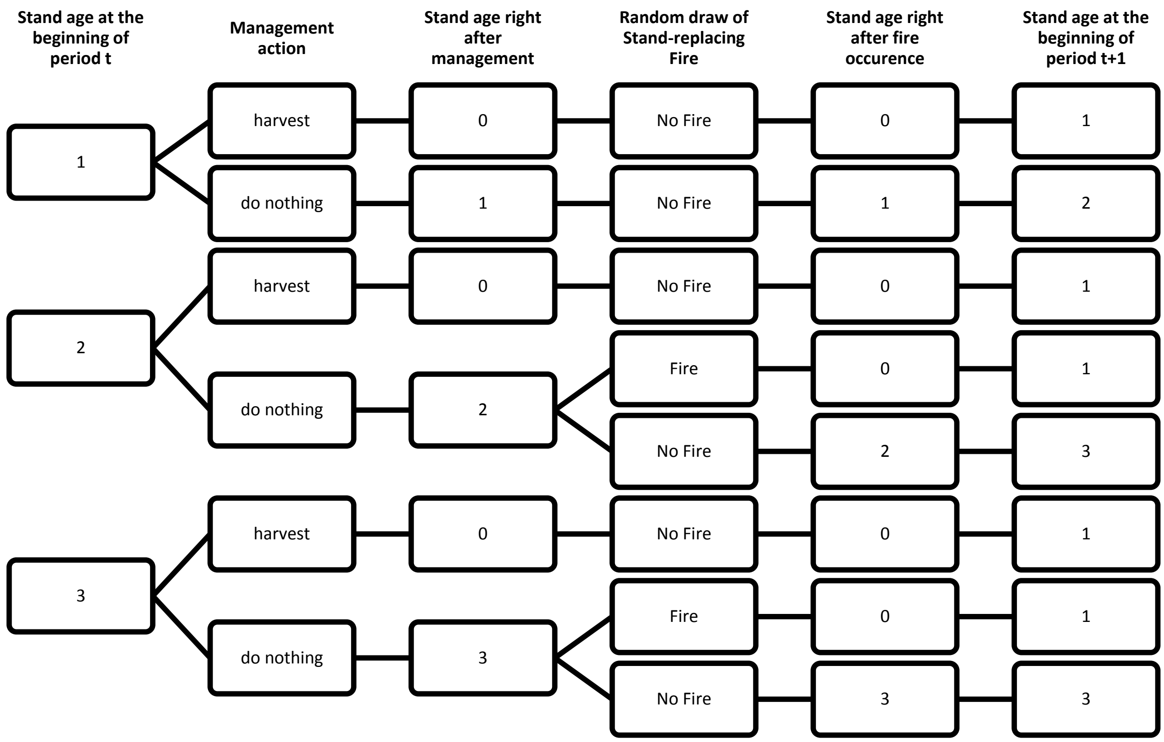


Fig 3: Illustration of a network representing the transition of stand age between two consecutive periods (when adding random samples of fire disturbance with fixed sequence of occurrence between management and fire). In each period, the occurrence of either harvesting or stand-replacing fire will reset stand age class to zero within that period. Stand will then has age class one at the beginning of the next period. Within each period, there are no changes on stand age class if both "do nothing" and "no fire" happen.

With the above assumption, we built an enhanced formulation using SAA method as presented below

$$Max \quad \sum_{t\in\{1,T\}} \sum_{(n)_{t-1}} \frac{1}{N^{t-1}} \sum_i \sum_j \sum_k v_{ijkt} X_{ijk(n)_t} - q \sum_{t\in\{2\ to\ T\}} \frac{1}{N^t} Q_t \qquad (2.1)$$

St.

$$\sum_k X_{ij_1k(n)_0} = 1 \qquad \forall i \qquad (2.2)$$

$$D_{ij'h(n)_t} = 0 \qquad \forall i,\ j',h,\ (n)_t \in \widetilde{N}^{\circ\circ}_{ij'ht}, t \in \{1,T\} \qquad (2.3)$$

$$\sum_j \sum_{k\in K'_{ijj'}} X_{ijk(n)_{t-1}} - \sum_h D_{ij'h(n)_t} = 0 \quad \forall i,\ j',(n)_t \notin \widetilde{N}^{\circ\circ}_{ij'ht}, t \in \{1,T\} \qquad (2.4)$$

$$\sum_{j'} \sum_{h\in H'_{ij'j}} D_{ij'h(n)_t} - \sum_k X_{ijk(n)_t} = 0 \qquad \forall i,j,(n)_t \in \widetilde{N}_t,\ t \in \{1,T-1\} \qquad (2.5)$$

$$Q_t \geq \sum_i \sum_j \sum_k c_{ijk}X_{ijk(n)_t} - \sum_i \sum_j \sum_k c_{ijk}X_{ijk(n)_{t-1}}$$

$$\forall\ (n)_t \in \widetilde{N}_t, t \in \{1,T-1\} \qquad (2.6)$$

Where:

h indexes the type of fire disturbance which is either "no fire" (h=0) or "stand-replacing fire" (h=1)

$(n)_t$ indexes each random instance in the set $\widetilde{N}_t$ in period *t*. $(n)_0$ represents the current forest state (at the very beginning of the first period)

$X_{ijk(n)_t}$ is a binary decision variable indicating, when set to 1, the selection of management option *k* for the random instance $(n)_t$ of stand *i* at age class *j* at the start of period *t+1*. Management decisions are made before knowing the occurrence of any fire disturbance within the same period. Period one decisions are denoted by $X_{ij_1k(n)_0}$

$D_{ij'h(n)_t}$ is a binary variable indicating, when set to 1, the fire disturbance of type h if stand *i* is at age class *j* in random instance $(n)_t$ during period *t* after implementation of the management activity selected for that time period.

$H'_{ij'j}$ is the set of random fire disturbances that cause a transition in stand *i* from state j' in a given time period to state *j* at the start of the following time period.

$K'_{ijj'}$ is the set of management activities for stand *i* that cause a transition from state *j* to state j' upon implementation in a given time period.

N is the number of new samples generated for each existing stand sample state in each time period. We make a three period example to illustrate the sampling notation used in our sample-based stochastic programming model as describe in table 2 below.

Table 2: Illustration of sampling notation

Samples set, index of each sample and the simplified denotations		
Period 1 sample set $\tilde{N}_1$	**Period 2 sample set** $\tilde{N}_2$	**Period 3 sample set** $\tilde{N}_3$
$(1)_1$	$(1,1)_2$ denoted as $(1)_2$	$(1,1,1)_{3,}$ denoted as $(1)_3$
		$(1,1,2)_{3,}$ denoted as $(2)_3$
	$(1,2)_2$ denoted as $(2)_2$	$(1,2,3)_{3,}$ denoted as $(3)_3$
		$(1,2,4)_{3,}$ denoted as $(4)_3$
$(2)_1$	$(2,3)_2$ denoted as $(3)_2$	$(2,3,5)_{3,}$ denoted as $(5)_3$
		$(2,3,6)_{3,}$ denoted as $(6)_3$
	$(2,4)_2$ denoted as $(4)_2$	$(2,4,7)_{3,}$ denoted as $(7)_3$
		$(2,4,8)_{3,}$ denoted as $(8)_3$

We assume random fires can occur in each of the three periods. Suppose two random sample states are drawn for a stand in period 1 and two more samples are drawn for each resulting state in subsequent periods. In this example, we would have two sample forest states at the end of period one, four at the end of period two and eight at the end of period three for this one forest stand. We index sample states from *1* to *n* at the end of period *t* using the abbreviation $(1)_t$, $(2)_t, \ldots \ldots (n)_t$ for simplicity. Each stand state at the end of the three-period planning horizon reflects a sampled succession pathway up to the end of period three. For example, the state indexed by $(5)_3$ represents a sampled potential forest succession pathway of $(2)_1 \rightarrow (3)_2 \rightarrow (5)_3$ for the stand.

$\widetilde{\mathrm{N}}_{\mathrm{t}}$ denotes the set of randomly generated forest stand sample states at the end of period *t*. Using the example in table 2, if *N* is set to two, $\widetilde{\mathrm{N}}_1$ would include two sampled states in period one for each stand. Branching from each period-one state creates two new states for each stand. Therefore, $\widetilde{\mathrm{N}}_2$ will include four sample states. $\widetilde{\mathrm{N}}_3$ includes eight sampled ending states and each of them indicates a sampled forest stand succession pathway across three periods.

$\widetilde{\mathrm{N}}^{\circ\circ}_{\mathrm{ij'ht}}$ denotes the period *t* sample set in which fire type *h* does not occur in stand *i* when the stand is in age class j'. For each sample $(n)_t$ in set $\widetilde{\mathrm{N}}^{\circ\circ}_{\mathrm{ij'ht}}$, the value of $D_{ij'h(n)_t}$ is set to zero exogenously; the values of other $D_{ij'h(n)_t}$ variables are determined by management activities selected by the model.

Objective function *(2.1)* is the revision of *(1.1)* using sample average approximation method. Constraint *(2.2)* requires one and only one stand management activity, of either "harvest" or "do nothing" option, can be selected for each stand at period one. The age class balance constraints *(1.3)* for each stand are split into two types of constraints *(2.3)* and *(2.4)*.

Example of how those constraints are designed to reflect the fire occurrences in sample *A* (in figure 2) is presented as below (for simplicity we omit the subscripts of the stand and the sample index from each decision variable).

$$D_{age\_0,\, fire} = 0$$

$$D_{age\_1,\, fire} = 0$$

$$D_{age\_2,\, no\_fire} = 0$$

$$D_{age\_3,\, no\_fire} = 0$$

$$X_{age\_1,\, cut} + X_{age\_2,\, cut} + X_{age\_3,\, cut} = D_{age\_0,\, no\_fire,} + D_{age\_0,\, fire}$$

$$X_{age\_1,\, no\_cut} = D_{age\_1,\, no\_fire} + D_{age\_1,\, fire}$$

$$X_{age\_2,\, no\_cut} = D_{age\_2,\, no\_fire,} + D_{age\_2,\, fire}$$

$$X_{age\_3,\, no\_cut} = D_{age\_3,\, no\_fire} + D_{age\_3,\, fire}$$

Constraint (2.5) advances each stand into an older age class as it moves into the next planning period. Constraint (2.6) tracks the average declining timber flow across all samples between every pair of consecutive periods.

2.3 Modeling random sequences between management and fire

It is hard to guarantee that we can always make scheduling decisions ahead of time before the occurrence of any disturbance event at the beginning of each period like, for example, harvesting to liquidate the timber value before it can be destroyed by a stand-replacing fire. Wildfires in reality may occur before any implementation of management activities which will make the planned harvesting become impractical. Thus, the assumption that harvesting always precludes fire may overestimate the allowable cut.

In this step, we develop a more general formulation which assumes the sequence between harvesting and fire within a time period is also random. We use two random draws to create samples of fire occurrences for each stand at each existing forest state. The first draw indicates the occurrence of fire. If there were a fire, the second random draw determines *the sequence between this fire and any planned harvesting*. The key thing of this enhanced formulation is the use of the two sets of variables indicating the planned (or scheduled) management activities (denoted by X) and the implemented management activities (denoted by W). Management decisions and fire occurrences can interact within a stand during each planning period. The idea here is that scheduled stand management activities are selected at the start of an initial planning period, prior to observing subsequent random stand-replacing fires. As a result of random wildfires, some of the stands scheduled for treatment in each time period are burned first and cannot be treated as planned in those sample paths. In other cases, treatments are accomplished prior to fire disturbance. And in other cases, no disturbance occurs regardless of treatment or non-treatment. Each sample path leads to an opportunity for the selection of recourse management schedules in the following planning period that potentially could compensate for losses incurred from the preceding sequence of fire disturbances.

In each sample, implemented management variables (W) are used to track whether "harvest" could be implemented as a consequence of the within-period timing of fire occurrence. Table 3 lists six possible scenarios of fire occurrence and forest stand management (mgmt) in a given time period. The resulting fire occurrences and implemented management activities are shown for each scenario. Even there is a fire as the result of the first random draw, it will actually not happen if being precluded by an implemented harvesting.

Table 3: Possible interactions between the planned management decisions and the fire occurrence in each stand

Scenario No	**Sampled fire occurrence**	**Planned mgmt $X_{ijk(n)_{t-1}}$**	**Sequence between mgmt and fire**	**Resulting mgmt $W_{ijk(n)_t}$**	**Resulting fire occurrence**
1	*No*	*do nothing*	*N/A*	*do nothing*	*No*
2	No	harvest	N/A	harvest	No
3	Yes	do nothing	mgmt, fire	do nothing	Yes
4	Yes	harvest	mgmt, fire	harvest	No
5	Yes	do nothing	fire, mgmt	do nothing	Yes
6	Yes	harvest	fire, mgmt	do nothing	Yes

There's only one scenario (No 1) where stand age class will be preserved within a time period because of "do nothing" activity and "no fire" occurrence. Every other scenario will lead to the same transition of stand age class to the value zero right after the occurrence of management activity and fire no matter what happens first. That zero stand age class will then become one when entering the next time period (see figure 4)

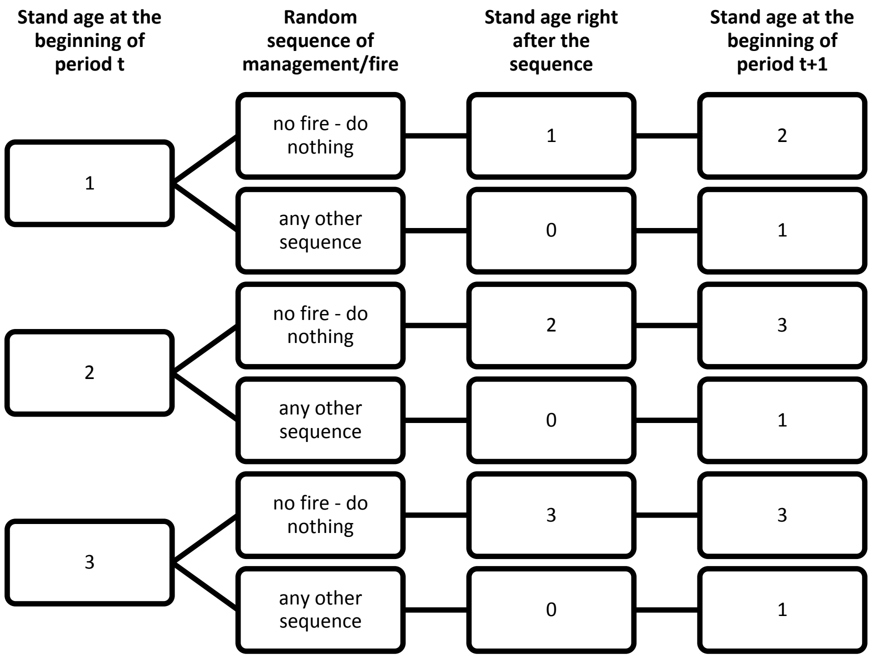


Figure 4: Illustration of a network representing the transition of stand age between two consecutive periods (with random sequences between management and random stand-replacing fire)

Following those assumptions above, we built an enhanced SAA formulation as follow

$$Max \quad \sum_{t\in\{1,T\}} \sum_{(n)_t} \frac{1}{N^t} \sum_i \sum_j \sum_k v_{ijkt} W_{ijk(n)_t} - q \sum_{t\in\{2\ to\ T\}} \frac{1}{N^t} Q_t \qquad (3.1)$$

St.

$$\sum_k X_{ij_1k(n)_0} = 1 \qquad \forall i \qquad (3.2)$$

$$W_{ijk(n)_t} = 0 \qquad \forall i, j, k = 1, (n)_t \in \tilde{N}^{\circ}_{ijkt}, t \in \{1, T\} \qquad (3.3)$$

$$D_{ij'h(n)_t} = 0 \qquad \forall i, j', h, (n)_t \in \tilde{N}^{\circ\circ}_{ij'ht}, t \in \{1, T\} \qquad (3.4)$$

$$X_{ijk(n)_{t-1}} - W_{ijk(n)_t} = 0 \qquad \forall i,j,k,(n)_t \notin \widetilde{N}^{\circ}_{ijkt}, t \in \{1,T\} \qquad (3.5)$$

$$\sum_j \sum_{k \in K'_{ijj'}} W_{ijk(n)_t} - \sum_{h \in H_{ij'}} D_{ij'h(n)_t} = 0$$

$$\forall i,j',(n)_t \notin \widetilde{N}^{\circ\circ}_{ij'ht}, t \in \{1,T\} \qquad (3.6)$$

$$\sum_{j'} \sum_{h \in H'_{ij'j}} D_{ij'h(n)_t} - \sum_k X_{ijk(n)_t} = 0 \qquad \forall i,j,(n)_t \in \widetilde{N}_t, t \in \{0,T-1\} \qquad (3.7)$$

$$Q_t \geq \sum_i \sum_j \sum_k c_{ijk} W_{ijk(n)_t} - \sum_i \sum_j \sum_k c_{ijk} W_{ijk(n)_{t-1}}$$

$$\forall \ (n)_t \in \widetilde{N}_t, t \in \{2 \ to \ T\} \qquad (3.8)$$

Where:

$X_{ijk(n)_{t-1}}$ is a binary variable indicating, when set to 1, the planned harvesting for stand *i* at age class *j* at the start of period *t*. This decision applies for all subsequent branches of the sample path.

$W_{ijk(n)_t}$ is a binary variable indicating implementation, when set to 1, of a scheduled harvest *(k=1)* for stand *i* in period *t* in sample $(n)_t$.

$H_{ij'}$ is the set of random fire disturbances that can occur in stand *i*, state j', including "no fire" and "stand-replacing fire"

$\widetilde{N}^{\circ}_{ijkt}$ denotes the subset of samples for which harvesting *(k =1)* is precluded by a fire in stand *i* at age class *j* during period *t*.

Objective function *(3.1)* summarizes and maximizes the total discounted revenue only from the implemented harvesting decisions. Equation *(3.2)* works as described in the previous formulation. Equation *(3.3)* exogenously forces the implemented management action

$W_{ij(k=1)(n)_t}$ to take the value of zero in cases if harvesting is precluded by fire; otherwise, the value of $W_{ijk(n)'_t}$ is determined endogenously by the model-selected management actions $X_{ijk(n)_{t-1}}$ through equation *(3.5)*. Equation *(3.4)* is used to eliminate disturbances that will not happen according to the random drawn in the sample. Equation *(3.6)* maintains the age balances for each stand within a planning period. It transfers stands within each time period and sample path from management implementation to disturbance variables. Equation *(3.7)* transfer stands from disturbance variables in one time period and sample path to planned management variables at the start of the next time period. When moving into the next planning period, equation *(3.7)* increases the age class of every stand by one.

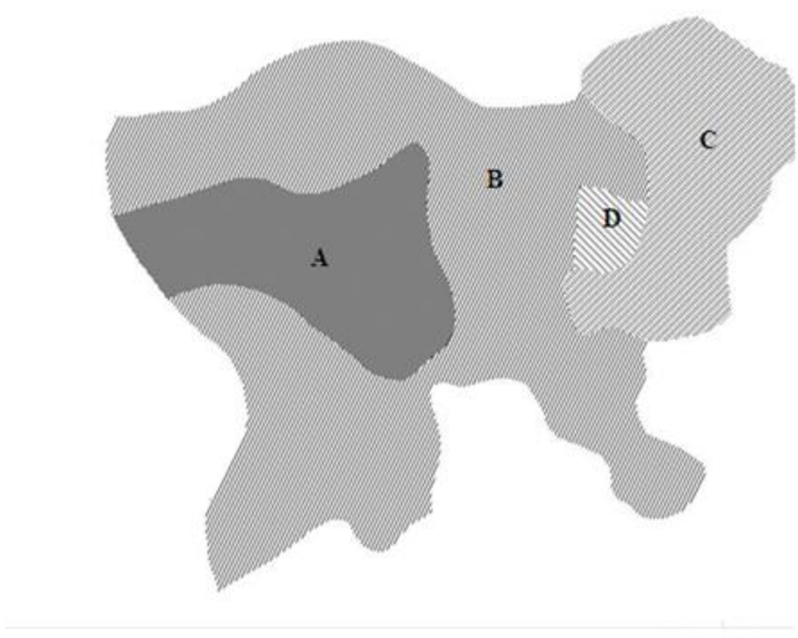
2.4 Modeling forest core area

Core area, the area of forest that is free of edge effects from surrounding habitats, (Zipperer 1993; Baskent and Jordan 1995) is an important spatial measure describing forest ecological conditions. Preserving mature forest core area has been increasingly concerned in landscape forest planning (Zipperer 1993; Baskent and Jordan 1995; Fischer and Church 2003) because of its important role to protect forest interior habitats. Core area was integrated into forest planning through dynamic programming (Hoganson *et al*. 2005), mixed integer programming (Wei and Hoganson 2007), and heuristic models (Ohman and Eriksson 1998, 2002; Ohman 2000). Core area can be modeled by tracking the states of many pre-defined influence zones across time (Hoganson *et al*. 2005). While core area has been modeled in this fashion before, it has not been modeled in the presence of wildfire.

2.4.1 Identifying Influence zone

We first introduce the concept of influence zones which are the areas capable of producing core area. The concept of influence zones can be used to account for how forest management could preserve mature forest core area across a planning horizon. Influence zones are delineated through a separate GIS process (Hoganson et al. 2005). Each influence zone can be considered as a portion of the forest where a unique set of stands influence whether the area in the zone will produce core area of mature forest in a given time period. An example forest of four stands (A, B, C and D) (Fig 5a) (Wei and Hoganson 2006) illustrates this concept and the associated modeling method.

5a



5b

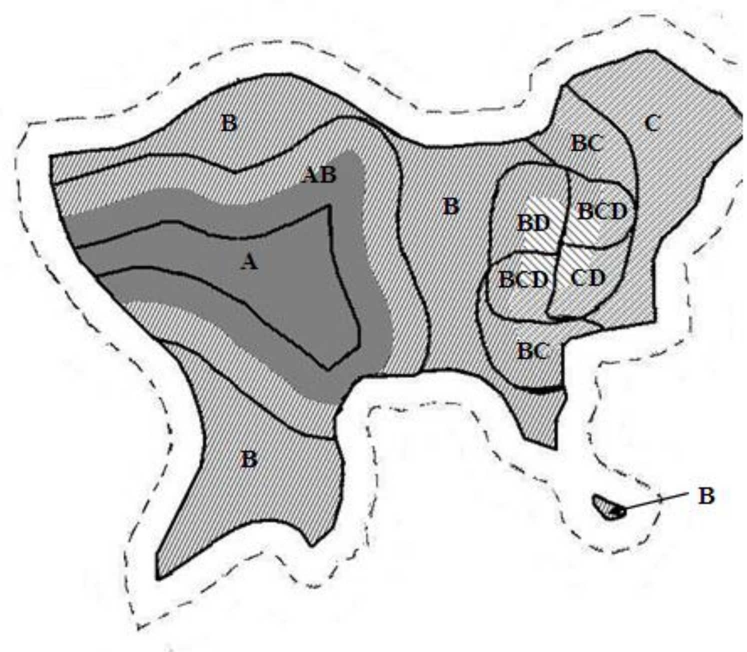


Fig 5: A forest is composed of four stands A, B, C and D. Buffering the boundary of each stand creates eight influence zones. Whether each influence zone will become core area at the end of each planning period depends on the age class of every stand in the zone.

By buffering outward from the boundaries of each stand for a predefined distance, a set of influence zones {A, B, C, AB, BC, BD, CD, and BCD} can be identified (Fig 5b). Using influence zone AB as an example, the ages of both stand A and B must satisfy the age requirement of mature forest for this influence zone to be classified as mature forest core area.

2.4.2 Identifying core area in the deterministic model

The influence zone concept has been modeled in a deterministic context, in which harvesting decisions determine whether each influence zone would become core area at the end of each period. We build one constraint for every stand in each influence zone I_z during each planning period *t*. In this model, core area is tallied at the end of each planning period *t*.

$$Y_{zt} - \sum_{j \in J''} \sum_{k \in P'} X_{ijkt} \quad \leq 0 \qquad \forall\, z, i \in I_z, t \qquad (4.1)$$

Where:

Y_{zt} is a binary variable indicating, when set to 1, influence zone *z* becomes core area at the end of period *t*.

z indexes influence zones within a forest.

I_z is the set of stands that create influence zone z.

J'' denotes the set of stand age classes that satisfy the requirement of core area.

P' denotes the set of management activity options that are satisfy the requirement for core area production.

Equation (4) is used to track whether influence zone *AB* will be core area at the end of period *t*. Two conditions are required for any influence zone *z* to be core area at the end of each period: 1) all stands *i* associated with the influence zone *z* need to satisfy the age class requirement for mature forest core area; 2) management option $k \in P'$ maintain the ages of all stands within influence zone z. When both conditions are satisfied, Y_{zt} will be set to one to indicate that influence zone *z* contributes to forest core area at the end of period *t*. The total core area maintained at the end of each period could be tracked in the objective function or through bookkeeping constraints.

To illustrate, we use influence zone *AB* in figure 5b above for example with the assumption that only stands in age class two or three can contribute to mature forest core area; and both stands *A* and *B* can be harvested (k=1) or not (k=0) during period *t*.

Equation (4.1) can then be presented as

$$Y_{AB,t} - X_{A,2,0,t} - X_{A,3,0,t} \leq 0$$

$$Y_{AB,t} - X_{B,2,0,t} - X_{B,3,0,t} \leq 0$$

According to these two constraints, influence zone *AB* will become mature forest core area only if one of the variables $X_{A,2,0,t}$ and $X_{A,3,0,t}$, and one of the variables $X_{B,2,0,t}$ and $X_{B,3,0,t}$ are set to one (no harvesting for both stands).

2.4.3 Identifying core area in the stochastic model

Random fire disturbances could also influence core area production. For example, a stand-replacing fire might destroy the overstory trees within a stand and reset the stand age to zero. This would destroy core area within this stand and also stands for which this stand provides buffer. To account for these effects, we model the impact of fire on core area for each influence zone *z* at the end of each period *t*. Management decisions interact with fires to determine stand age at the end of each planning period. We substitute a new set of constraints (4.2) for (4.1).

$$Y_{z(n)_t} - \sum_{j \in J''} \sum_{h \in H''} D_{ijh(n)_t} \leq 0 \quad \forall z, i \in I_z, (n)_t \in \tilde{N}_t \qquad (4.2)$$

Variable $Y_{z(n)_t}$ tracks whether influence zone *z* at the end of period *t* would contribute core area according to the sample indexed by $(n)_t$. H''denotes the subset of disturbances in stand *i* that could maintain the stand age and satisfy core area requirements. For example, when there are only two types of disturbances, "stand-replacing fire" or "no fire", H'' would only include the "no fire" option. Because management $X_{ijk(n)_t}$ will influence $D_{ijh(n)_t}$ through equations *(3.5)* and *(3.6)*, equation (4.2) can be used to replace equation (4.1) in the stochastic model. For example if $D_{age=2,no\_fire} = 1$ then equation (3.6) will force $W_{age=2,do\_nothing} = 1$. It means that both equation (4.1) and equation (4.2) can be simplified by $Y_{z(n)_t} - 1 \leq 0$ and also means that influence zone z will become core area because the objective function (will be presented later in equation (5.1) is designed to maximize the total production of core area for more benefit.

2.4.4 Integrating core area into the stochastic model

In this study, SAA formulation is used to track the amount of core area produced in each sample at the end of each planning period. Average minimum core area is our performance measure. To address this, a *Max(Min())* approach (e.g., Bevers 2007) is used. This is accomplished as follows:

1) For each sampled forest succession pathway, calculate the minimum core area produced from period one to T;
2) Build a bookkeeping constraint to average the core area minima calculated in step (1) across all sampled succession pathways;
3) Value the average minimum core area calculated from step (2) in the objective function.

Based on the above discussion, we built an integrated model to incorporate the impact of fire into a spatially explicit harvest-scheduling model with core area concerns. This new model combines constraints (3.2) to (3.7) to model fire occurrence and stand age class succession. It uses constraint (4.2) to track whether each influence zone produces forest core area at the end of each period given fire disturbances and management actions. It uses objective function (5.1) below to maximize the total weighted return from timber, the average minimum core area along all sampled forest succession pathways, and a penalty for average timber production declines between any two consecutive periods across all samples. It also uses bookkeeping constraints (5.9), (5.10) and (5.11) below to support the *Max(Min)* model format. By using spatially explicit constraints to track core area preservation, this model maximizes the total benefits from timber harvesting and core area conservation in a forest over a given planning horizon.

$$Max \quad \sum_{t\in\{1,T\}} \sum_{(n)_t} \frac{1}{N^t} \sum_i \sum_j \sum_k v_{ijkt} W_{ijk(n)_t} - q \sum_{t\in\{2\ to\ T\}} \frac{1}{N^t} Q_t + q' V_a \qquad (5.1)$$

$$V_{(n)_t} = \sum_{z\in Z} o_z Y_{z(n)_t} \qquad \forall\ (n)_t \in \widetilde{N}_t, t \in \{1, T\} \qquad (5.9)$$

$$V^{\circ}{}_{(n)_T} \leq V_{(n)_t} \qquad \forall\ (n)_T, (n)_t \in P_{(n)_T}, t \in \{1, T\} \qquad (5.10)$$

$$V_a = \frac{1}{(N)^T} \sum_{(n)_T} V^{\circ}{}_{(n)_T} \qquad (5.11)$$

The area of influence zone *z* is denoted as o_z. Constraint (5.9) calculates $V_{(n)_t}$which is the total core area produced in each randomly drawn sample at the end of planning period *t*. Equation (5.10) identifies $V^{\circ}{}_{(n)_T}$ which is the minimum total core area preserved in the forest along each sampled forest succession pathway. The term $P_{(n)_T}$ denotes the set of samples along the succession pathway ending with $(n)_T$. Using the example in table 2, $P_{(5)_3}$ represent a sample set $\{(2)_1, (3)_2, (5)_3\}$. Equation (5.11) calculates V_a that denotes the average $V^{\circ}{}_{(n)_T}$ across all sampled pathways.

2.5 Selecting the first period solution

A primary purpose for using a multistage stochastic harvest scheduling model is to identify first-period harvests that perform well over a range of plausible future conditions (Hoganson and Rose 1987). Harvesting decisions for the first period need to be implemented immediately and period-one decisions may also have longer-term impacts on future timber production and spatial forest structure. The quality of the first-period decisions can be used to evaluate overall model performance.

In our SAA model, randomly generated fires are used to estimate the expected consequences of stand-replacing fires and recourse management actions. Models built on different independent random fire samples with identical probability distributions could suggest different period-one harvest schedules. Larger sample sizes can be used to better inform the model about future fires up to a point. More samples also increase model complexity, however, and make the model more difficult to solve. An alternative approach is to build multiple SAA models using different sets of independent, identically distributed fire samples. Solutions from the R different models can be used to calculate the "persistence" (Bertsimas et al. 2007) of first-period harvest decisions. For example, if more than half of the R models choose to harvest stand α in the first period, we might select this stand for harvesting. Note this persistence calculation reflects a heuristic design and may lead to suboptimal solutions (Bevers 2007).

To better understand the quality of the first-period solutions, we created and ran multiple models with each model built on a different independent and identically distributed set of randomly drawn fires, as described above. We then evaluated the quality of all resulting period-one solutions by simulating the solutions with new samples and comparing results using Tukey's multiple comparison method (see Goldsman and Nelson 1998). We used the results of these statistical tests to select first-period harvests.

3 Test Cases and Results

We built a computer-simulated forest with eleven stands as the test site (fig 6).

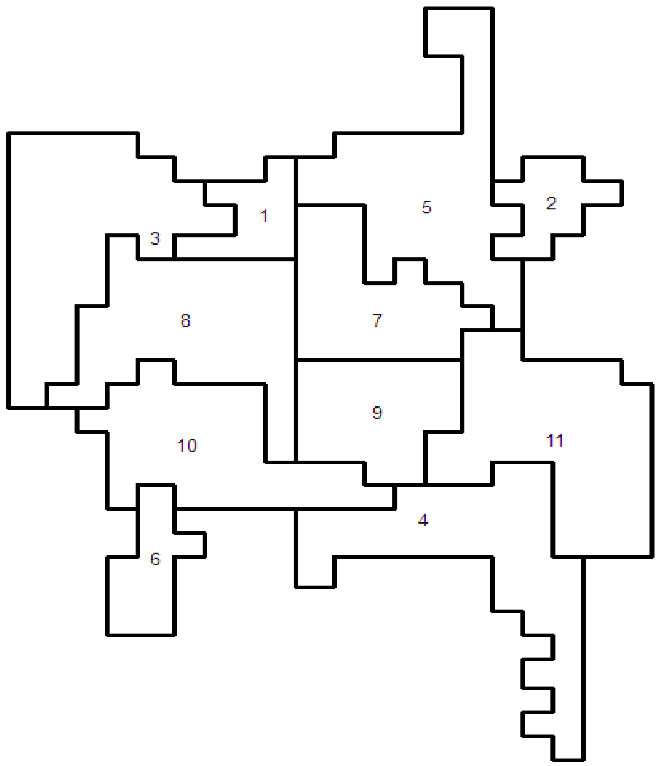


Fig 6. A forest composed of eleven stands used as the test site for our SAA model.

We divided a 120-year planning horizon into three 40-year planning periods. At the beginning of each period, a stand is in either age-class one (1 to 40 years), two (41 to 80 years), or three (81+). At the beginning of the planning horizon, stands 1, 4, 7 and 10 are assumed in age class one; stands 2, 5, 8 and 11 are in age class two; and stands 3, 6, and 9 are in age class three. Stands older than 40 years (in either age class two or three) are considered mature forest for core area calculations. Areas of mature forest 50m away from the boundary of a young forest (≤ 40 years old) or the boundary of the forest are counted as interior forest habitat, or core area.

Two management alternatives are assumed available for each stand in each planning period: "do nothing" or clearcut. Clearcuts reset the stand age back to zero (age class zero) in the period of harvest. We assume clearcutting a stand in age class one generates no financial returns. Stand-replacing fire is assumed to occur randomly in each stand in each period following known probability distributions (defined below). Stand-replacing fire sets the stand age back to zero in the period the fire occurs. The sequence between fire and clearcut is also random.

Two levels of hypothetical fire probability are modeled. Under the low fire probability assumption, the chance of fire is 0.1 for age class one stands, 0.2 for age class two stands and 0.3 for age class three stands in each 40-year planning period. Under the high fire probability assumption, fire probabilities for stands in age class one, two and three are 0.2, 0.4 and 0.6, respectively, in each 40-year planning period. The chances of fire occurring before or after harvesting are assumed to be 0.5 each. We assume stand-replacing fire and clearcutting are mutually exclusive in each stand in the same period. Random fire occurrences are reflected in the SAA model by setting the value of disturbance variables *(D)* and decision variables *(W)* as described in equations *(3.3)* and *(3.4)*.

3.1 Solution persistence under high fire risk assumptions

Tests under the assumption of high fire probability show that persistence of first-period harvests improves with increasing sample size *N*. Under an assumed core area price of $500/acre, increasing sample size *N* from one to 15 caused decisions for stands 8 and 9 to switch from "do nothing" to "clearcut" (table 4). When *N* is set to 15, all SAA model runs consistently support a period-one decision for every stand in the forest: harvesting stands 2, 3, 5, 6, 8, 9 and 11, and doing nothing to the other stands 1, 4, 7, and 10.

Table 4: The persistence of clearcut decisions for each stand in period one by using sample size *N* from 1 to 15. This test case assumes high fire risk for each stand of the forest. No penalty is imposed for declines in timber production. Shaded cells represent the decision to clearcut the corresponding stand based on solution persistence.

StandID	1	2	3	4	5	6	7	8	9	10	11
Core area price = $500/acre											
N=15	0%	100%	100%	0%	100%	100%	0%	100%	100%	0%	100%
N=10	0%	100%	100%	0%	97%	100%	0%	100%	100%	0%	100%
N=6	0%	100%	100%	0%	93%	100%	0%	77%	100%	0%	100%
N=5	0%	93%	97%	3%	87%	100%	0%	67%	97%	0%	100%
N=4	0%	97%	97%	10%	63%	100%	3%	63%	100%	0%	100%
N=3	0%	83%	83%	10%	77%	93%	0%	53%	90%	0%	90%
N=2	3%	80%	80%	20%	43%	93%	7%	63%	73%	20%	77%
N=1	23%	73%	57%	37%	60%	77%	13%	43%	47%	13%	63%
Core area price = $1000/acre											
N=15*	0%	43%	63%	0%	20%	100%	0%	0%	73%	0%	87%
N=10	3%	83%	63%	0%	47%	100%	0%	3%	43%	0%	63%
N=6	0%	60%	73%	0%	30%	100%	0%	0%	73%	0%	77%

*The relative gap between the best solution found and the best possible solution is set to 5% to prevent the model from running out of memory. All other runs used a relative gap of 1%.

Under the high fire probability assumption, the trade-offs between producing timber and maintaining forest core area also become clearer as sample size *N* increases (See table 5).

Table 5: A statistical summary of 30 runs based on different random draws. Each run is based on either a core area price of $500/acre or $1000/acre. No penalty is imposed for declines in timber production. High fire risk is assumed for each stand in the forest.

	Average value across samples					STD across samples				
	Average	Timber yield				Average	Timber yield			
Sample	min core	P1	P2	P3	Obj.	min core	P1	P2	P3	Obj.
size	(acres)	(cords)			($)	(acres)	(cords)			($)
Core area price = $500/acre										
N=15	0	4621	1846	3886	63758	0	165	97	135	2131
N=10	0	4628	1807	3889	63992	1	288	149	199	3216
N=6	2	4449	1657	3622	64705	4	491	326	537	3104
N=5	6	4328	1497	3398	67189	6	631	411	654	6008
N=4	7	3992	1556	3305	65623	6	638	465	680	5524
N=3	11	3849	1234	3251	68006	8	921	523	953	8194
N=2	14	3419	1218	3100	67059	8	928	607	871	8860
N=1	17	3690	1590	3028	75732	14	1415	810	1678	####
Core area price = $1000/acre										
N=15*	15	2428	517	1822	78224	2	463	187	337	4071
N=10	16	2309	423	1745	79578	3	504	169	388	5868
N=6	17	2474	519	1862	82680	3	593	257	484	5901

*The relative gap between the best solution found and the best possible solution is set to 5% to prevent the model from running out of memory. All other runs used a relative gap of 1%.

We first tested the assumption of $500/acre core area price with no penalty for timber production declines. When N is set to six, 30 model runs suggest maintaining an average of two acres of core area across the planning horizon (table 5). By increasing N to ten, most model runs found no benefit in maintaining any core area (table 5) and suggest all stands at age class two and three should be harvested during the first period to maximize expected returns (table 4). The benefits of using larger sample sizes are also reflected by the decreased standard deviations of the average minimum core area, the timber productivity of each period, and the objective function value across the 30 tested SAA model runs (table 5).

We assumed stands in age classes two or three are mature forest and can contribute to forest core area when they occur away from edges. However, older forest could be more susceptible to fires. Under this assumption, higher fire probability increases the cost of maintaining core area. Test results show, when core area price is set at $500/acre under the high fire probability assumption, no core area should be maintained across the planning horizon (table 5). With the core area price doubled to $1000/acre, SAA model runs suggest maintaining about 15 acres of core area (table 5) by delaying harvests of stands above age class two (table 4) during the first 40-year planning period.

3.2 Comparing the quality of first period solutions

Forest management occurs across space and time. Management decisions for period one need to be carried out without knowing future fire conditions with certainty. Decisions for later periods, on the other hand, can be adjusted based on the actual management activities and fire occurrences in earlier periods. A good period one solution should help facilitate adjustment of future forest management activities. Different sample sets used by the SAA model may suggest different period-one solutions. Changing sample size *N* also causes the model to select different period-one solutions as described in table 4. While we expect better solutions to be derived from models built on larger sample sizes, the problem of selecting a single period-one solution from a set of differing solutions remains.

We first revised the SAA model to find the optimal period-one solution under the assumption of no fire risk, $500/acre core area price, and no penalty for timber production declines. This leads to a deterministic period-one decision. Results indicated that only stand 6 should be harvested in period one to maximize the total return from timber and core area. Delaying harvests is preferred in this case without the risk of fires. We compared this solution with the three other period-one solutions listed in table 4 at the same core area price with sample size N=1, 2 or 3.

For this comparison, we reran the SAA model 300 times with independent, identically distributed fire occurrence samples. In each of the 300 runs, we hardcoded the selected period-one solution, simulated one random forest succession pathway across the three 40-year planning periods, and allowed the model to make recourse decisions for periods two and three to adapt to the simulated fires. The objective function value, timber production and amount of core area were reported and saved for each sample.

We used Tukey's multiple comparison method to compare performance of the four solutions. Results in table 6 show all period-one solutions selected by our SAA models are significantly better than the deterministic solution, producing higher average objective function values. Increasing sample size *N* in the SAA model also led to higher average objective function values across our 300 independent samples. However, the differences are not statistically significant at the 95% confident level.

Table 6: Multiple comparison of the performance of different period one solutions from the deterministic approach and from stochastic programming with sample sizes N=1, 2 or 3. In the stochastic model, we assume $500/acre core area price, no penalty for declining timber production, and high fire probability. The confidence interval around the average difference in objective function value is calculated based on 95% confidence.

Comparison	Differences of Objective function values ($)		
	Average	Lower bound	Higher bound
N1 To Deterministic	23803	20526	27080
N2 to Deterministic	24995	21718	28272
N3 to Deterministic	25845	22568	29122
N2 to N1	1192	-2085	4469
N3 to N1	2042	-1235	5319
N3 to N2	850	-2427	4127

3.3 Influence of the declining timber flow penalty and fire probability

We tested two variations of forest management assumptions regarding the penalty on timber declines and core area prices: 1) $500/acre core area price with a penalty of $90/cord for timber production declines between two consecutive periods; 2) $500/acre core area price, and no penalty for timber declines. In these tests, we also assumed the chance of stand-replacing fire is low (as previously defined). We used two sample sizes to build the SAA model: $N=6$ and $N=10$. Increasing N from six to ten created a much larger MIP model. To prevent our computer from running out of memory while using the CPLEX solver, we set the maximum computing time for solving each SAA model to one hour and we allowed the solver to stop when a feasible integer solution within five percent of the best possible solution was found.

Results show that a $90/cord penalty effectively prevents timber production from declining between two consecutive planning periods in this test case (table 7). Applying this penalty lowers the overall harvest level substantially across the planning horizon, especially during the earlier periods. Lower harvesting levels also allow the model to increase the average minimum core area by more than 20 percent across the 120 years.

Table 7: Model performance with and without a penalty for declines in timber production assuming a core area price of $500/acre. This test case also assumes low fire risk in each stand of the forest.

	Average across samples					STD across samples				
Sample size	Average min core (acres)	Timber yield (cords) P1	P2	P3	Obj. ($)	Average min core (acres)	Timber yield (cords) P1	P2	P3	Obj. ($)
$90/cord penalty for timber decline										
*N10	57	20	224	1002	85473	5	75	145	231	6958
N6	55	90	318	1115	85215	7	186	316	345	10886
No penalty for timber decline										
N10	45	1750	648	1413	91627	9	617	281	527	7754
N6	46	1672	675	1544	91400	10	720	347	602	7643

*Computing time is set to one hour to prevent "out of memory" error. The average relative gap is 4.12% between the best solution found and the best possible solution for all 30 runs.

Fire probability played an important role in timber and core area production (compare results in table 5 and table 7). Under the assumption of $90/acre core area price without the declining timber penalty, doubling the expected fire probability in each age class causes the model to increase timber harvest levels by 2.6 to 3.9 times during each of the three planning periods. Higher fire probability also makes the conservation of forest core area too risky to be justified and decreases the overall objective function value by more than 30 percent in this test case.

4 Conclusion

Stochastic disturbances such as fire, insect, disease or wind can have a significant impact on long-term forest management. Ignoring the effects of these disturbances in forest plans could lead to decisions and conclusions that are distanced from reality. Spatial concerns such as core area, edge effects, adjacency and patch connectivity pose additional challenges to building disturbance impacts in forest management models. Without spatial considerations, stochastic disturbances might be modeled adequately using mean probabilities of different scenarios, or essentially as average fractions of land being disturbed. Under a spatial context, using average disturbance rates seems less likely to be adequate for approximating system behavior and optimal management. Even in stochastic programming models it can be difficult to select adequate representative scenarios from the enormous number of possible spatial disturbances patterns and forest spatial structures; sample-based methods might be required, like those used in this study. Forest spatial models often use binary variables to track and form desired spatial structures such as core area, or to prevent undesired spatial conditions such as violations of adjacency constraints or harvest block size restrictions. Both scenario- and sample-based stochastic programming approaches can support these binary model formulations.

Harvest decisions for later time periods can often be adjusted through recourse actions, whereas implementation of many first-period decisions needs to begin immediately. A primary purpose in multistage stochastic programming is to improve the quality of the first-stage, or first-period, decisions while taking recourse opportunities into account. This research demonstrates that integrating fire risk explicitly in a harvest scheduling model can lead to better first period decisions compared with using a deterministic model.

We examined the benefit of using larger sample sizes with a number of test cases. In our models, larger sample size improves the persistence of the period one solution. It also reduces the variations between solutions, as indicated by lowering the standard deviation of objective function values across many SAA model runs. However, increasing the sample size also increased model complexity and model size. With a relatively small 11-stand and three-period harvest scheduling problem, we experienced the “out of memory” error occasionally when solving the model on a workstation with 6GB of memory. Installing more memory in the computer could be a simple but expensive fix when solving larger problems. During our tests, we found that using computers with more memory, i.e. 32GB, allowed us to solve slightly larger problems, but computing time sometimes became too long (a week) to be suitable for testing purposes. More efficient modeling or solution approaches might also be formulated. For example, fires occurring during later periods might have less impact on the quality of first-period decisions than fires in earlier periods. In this case, we could allocate more samples to earlier periods and fewer samples to later periods. Decomposition methods could also help with the solution time.

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