AN INTEGRATED ECONOMIC HYDROLOGIC MODEL
FOR GROUNDWATER BASIN MANAGEMENT

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May 1994

Colorado
State
University

Colorado Water Resources
Research Institute

Completion Report No. 186

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ABSTRACT

AN INTEGRATED ECONOMIC-HYDROLOGIC MODEL
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Most previous works on optimal long run ground water basin management have tried to address the issue in part-focusing either on the economic or the hydrologic aspect of the problem within an optimization framework. Very few attempts have been made to incorporate serious economic considerations and complex aquifer hydraulics within an integrated optimal decision model. So far such attempts have enjoyed only limited success due to the mathematical complexity of the optimization problem and the requirement for sophisticated computing facilities. Therefore, the need for a scheme for integrated economic-hydrologic groundwater management scheme still exists.

This study presents a simple and computationally efficient integrated groundwater management scheme which combines long run optimal resource allocation rules with realistic aquifer response through the use of discrete kernels. A conjugate gradient based nonlinear programming algorithm is used to solve the model. The algorithm uses an augmented Lagrangian based penalty function technique to automatically update penalties and multipliers. The unique combination of the response matrix and the conjugate gradient method allows the integrated model to be defined and solved in an economic and efficient manner (in terms of memory requirement and computational time). This approach also allows explicit identification of direct, spatial and temporal costs of pumping groundwater from a confined aquifer. When drawdown is not a significant part of the saturated thickness, the technique can also be applied for optimal management of unconfined aquifers.

This method has been applied to a realistic groundwater basin designed after the Arapahoe aquifer of the Denver basin system. Three case studies and additional discussions on operational management are presented to demonstrate that a diverse group of problems could be investigated using this decision making tool.

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CHAPTER 1

INTRODUCTION

Groundwater reserves are being increasingly exploited all over the world for meeting irrigation, industrial and municipal demands. This demand for groundwater is likely to continue to grow quite rapidly in future due to a number of reasons. First, in places where the reserve is not already overexploited, groundwater serves as a source of dependable low-cost good quality water. Second, population growth increases the overall water demand for both consumptive and non consumptive uses, and groundwater being the single largest source of fresh water (except for the glaciers in the polar areas; Heath, 1984) must contribute its due share. Third, historically surface water sources have been developed first. This was mainly due to lack of understanding of the complex groundwater hydraulics and lack of adequate data to evaluate physical and economic feasibility of groundwater based projects. Now that most of the potential surface water sources have been developed and new developments are likely to face stiff resistance from different environmental protection groups, attention has shifted to developing groundwater. It is therefore very important that different groundwater development scenarios should be examined carefully and

guidelines should be prepared so as to ensure long run optimal development of this valuable resource.

Groundwater management could be characterized as either a renewable (when the aquifer system gets annual recharge from precipitation and seepage) or a nonrenewable resource management problem. In any case, the problem has a number of attributes which make the task difficult. First, groundwater flow through different geologic formations is a complicated process to model. This is the physical aspect of the problem. Second, in many places groundwater is being extracted as a "common pool" resource. Because of the absence of adequate property rights, users are likely to draw as much water as necessary without taking into consideration the spatial and inter-temporal effects of such withdrawal. Under a common pool scenario, external costs are imposed on all the related parties (both users and non users) through draw-down induced increased pumping cost, water quality deterioration, and land subsidence. Since externalities do not enter into private cost benefit calculation, they lead to nonoptimal resource extraction pattern. The market mechanism is incapable of correcting these undesirable outcomes and so separate institutional measures become necessary.

Thus managing groundwater has two major components. As mentioned by Young (1992), these are: "managing the water" and "coordinating the people." Any comprehensive and dynamic management model must address both the issues simultaneously. The first component involves modeling the groundwater flow

through porous media along with its linkages with other hydrologic systems. Two most widely used numerical methods for groundwater flow modeling are based on finite difference and finite element techniques which are well documented in the literature (Huyakorn and Pinder, 1983). These methods can model the aquifer responses due to external excitations with a reasonable degree of accuracy. The second management component involves understanding and modeling the fundamental human motives which prompt him to employ groundwater solely or in conjuncture with the surface water for consumptive and non-consumptive uses. This part confronts the modeler with a significant challenge because the system of "users", unlike the physical system, can make its own decisions which are considerably more difficult to predict.

The actions of groundwater users observed in reality are outcome of complex interaction among different economic, social, and legal factors. However, mathematical representation of the human dynamics is performed by simplification and idealization of the actual system. Thus concepts such as "consumer surplus" and "producer surplus" from applied welfare economics (Just et al., 1982) are used to define the objective function for the optimal management model. The general idea is to formulate the objective function from the society's point of view (or from appropriate agent's point of view) and then, to generate a policy that will maximize the value of the objective function. Groundwater users are described in the model as economic agents who act

according to their perceived self interest to maximize their utility or profit as the case may be. Other social and legal considerations can be included through a set of model constraints.

Hence the combined groundwater management model (with hydrologic and economic subsystems) essentially becomes an optimal control model where the sum of discounted net benefits is maximized over the planning horizon subject to all the physical and institutional constraints. Although the conceptual formulation of the optimal groundwater management model seems simple enough, its actual implementation encounters a number of theoretical and practical difficulties. This is why incorporation of physical and institutional considerations into a single optimization model is still an active research area and there is considerable room for improvement. As will be discussed in the next chapter, very few studies have been done so far which attempt to integrate a realistic groundwater simulation model with socio-economic management objectives. This study will attempt to contribute in this area by proposing and examining a specific "integrated groundwater management" methodology which can potentially become a powerful decision making tool for a certain class of groundwater management problems.

CHAPTER 2

LITERATURE REVIEW AND STUDY OBJECTIVES

The literature related to optimal groundwater management could be divided into three major groups:

1] Research carried out mostly by economists.

2] Research carried out mostly by engineers.

3] Research carried out by interdisciplinary teams.

2.1 Group one: economic approaches

The primary preoccupation of the first group has been to develop a set of decision rules for managing groundwater based on static or dynamic economic optimality concepts. Among earlier researchers, Renshaw (1963) examines groundwater as common pool resource. He outlines the two major concerns of a common pool situation as overextraction and external costs imposed on all pumpers due to the same. Using some simple cost-benefit calculations, he concludes that optimal mining (where recharge is negligible) and optimal extraction (where recharge is significant) would generate a substantial increase in economic return over the common property regime. He also argues that a pricing mechanism should be installed for rationing the overdraft.

Burt (1964, 1966, 1967) in a series of papers lays down the foundation of optimal inter-temporal groundwater management using the concept of dynamic programming in discrete time (DP). In the first two papers he elaborates on DP formulation because "the methodology has the virtue of generality and completeness for empirical estimation of optimal groundwater policies." However, in his 1967 paper Burt presents an "approximately optimal decision rule" and backs away from DP saying that "it (DP) is fairly demanding in the amount of resources required to obtain the estimated policies." Burt also introduces the concept of a conditional decision rule when groundwater recharge is treated as random variable instead of deterministic one. Although very strong in economic contents, the major limitation of these papers is that the groundwater aquifer was unrealistically treated as a single homogeneous and isotropic cell.

Domenico et al. (1968) uses a continuous time analytical model to develop the decision rules for long run groundwater management. The objective was to maximize the discounted present value of net benefit. They do not use the concept of optimal control but rather try to combine some economic intuition and marginal analysis (as always done in static optimization problems). As usual the hydraulic response of the aquifer has been dealt with only superficially.

Burt (1970) continues on the issue of optimal allocation of groundwater over time, this time introducing the issue of institutional constraints which "prevent imposition of a

criterion based strictly on economic efficiency." First part of the paper discusses how such considerations could be accommodated in a dynamic programming setting. He recognizes the very important point missed by many earlier and later researchers that under a set of realistic institutional constraints, "There is no reason to expect $G(x,s)$ (the return function) to be a nicely behaved function for the purpose of optimization, i.e., concave with first partial derivatives." He then concludes that "the only feasible means of deriving an estimated optimal policy (optimal subject to the definition of $G(x,s)$) is to use discrete variable dynamic programming."

Burt also presents an interesting discussion on relative merits of centrally administered water pricing and negotiable water rights - two major institutional policies usually recommended by economists as remedy to common pool externality. In the rest of the paper Burt extends the work by Domenico et al. (as discussed above) by examining the impact of variable marginal productivity of water on equilibrium storage when the length of planning period is itself a decision variable.

Gisser and Mercado (1972) use a two-cell aquifer model and a linear-parametric economic optimization model to estimate seasonal groundwater use patterns. Then using simple yearly water budgeting, they project the results for a number of years in the future. Despite the authors's claim that "we provide a complete integration of the agricultural demand function with the hydrologic model in the Pecos River basin,"

the two cell aquifer model is only slightly improved version of the single cell models used by previous researchers. The economic model is also not a dynamic one (even if linearity assumptions were valid), and so, their claim of "complete integration" could not be taken seriously.

Gisser and Sanchez (1980) presents one of the first groundwater management models rigorously based on formal optimal control methodology. They however used a continuous time version to simplify the analysis and used a single cell aquifer model as has been the tradition with economists. They simulate two extreme groundwater pumping scenarios - the pure competitive extraction and the socially optimal extraction. The competitive scenario has been defined as the situation where "instead of maximizing present value, farmers simply pump water each year, satisfying the condition that the marginal cost of pumping equals the value of the marginal physical product (VMP) of water." Using steady state analysis the authors conclude that if the aquifer has a relatively large storage capacity (in fact their hypothetical aquifer is so large that the aquifer does not have a bottom and natural recharge is small compared to the storage capacity of the aquifer), then the difference between the competitive and socially optimal strategies become negligible. The conclusion is contrary to the commonly held notion about merit of optimal control and it would be interesting to determine if such claim remains valid when a more realistic aquifer simulation is incorporated.

Gisser (1983) continues to elaborate the point made in Gisser and Sanchez (1980) that for a large aquifer in a semiarid area, optimal control will be superfluous. Making a general allegation that "Water economists have generally neglected to examine the real life aquifers," he draws a few specific conclusions. One, the externalities due to common property extraction is negligible. Two, by giving property rights to groundwater users and allowing potential new comers to bargain with incumbents for exchange of rights would lead to a Pareto optimal outcome. Three, for a Pareto optimal allocation, social surplus should be estimated from the aggregate demand curve for water by all potential users, not by only the current owners of water rights (this is in accord with the idea of negotiable water rights plus new entry).

On a separate issue of stream-aquifer interaction, Gisser supports the New Mexico Underground Water Law which takes the position that "Groundwater appropriation will be permitted, provided that the immediate and potential effects on the flow of the Rio Grande are offset by the retirement of usage under existing surface rights." However he also comments that when pumped groundwater is returned to the stream (the non-consumptive part of acceptable quality) by some institution, they should be allowed to sell the augmentation which is not allowed under water law in New Mexico.

Since mid eighties, economists have changed their focus from generating optimal extraction path to studying tradeoffs among different management policies, occasionally within a

dynamic game theoretic framework. Eswaran and Lewis (1984, 1985) present the basic idea that optimal extraction patterns resulting from open loop and feedback Nash equilibria in a oligopolistic resource market are likely to differ. The idea has been further explored by Negri (1989) where he isolates two sources of dynamic inefficiency in common pool aquifer, namely the pumping cost externality and strategic externality. The latter results from failure to revise the optimal policy in the light of current value of the state variable(s). When such feedback is incorporated, the resulting policy is called "subgame perfect" in the game theory literature.

Dixon (1988) works on the same theme as above and presents some empirical findings using data from Kern County, California. He asserts that "farmers (could) do better in the collusive solution than when they compete with each other over groundwater extractions." However they still do not cooperate simply because of the absence of property rights.

Dixon also reports that "The difference between the social optimum (which is also the collusive outcome in his case) and the myopic solution is not large over a substantial portion of the parameter space tested." This interestingly concurs with the previous assertion made by Gisser (1983) that for an aquifer with large storage, optimal policies may not significantly differ from an uncontrolled withdrawal pattern.

The final work to be discussed in this group is by Eheart and Barcay (1990). Their main concern is different groundwater permitting schemes such as non-negotiable permit, trading of

long-term permit and trading of both long and short term permits. The conclusion is that "considerable increase in economic efficiency may be realized from long run permit trading and improving the accuracy of weather and crop yield forecast."

In summary, the papers discussed above present a number of analytical frameworks for optimal groundwater management, mainly from economic point of view. Most of the discussion is in continuous time format (which simplifies the analysis) with restrictive assumptions on marginal benefit, marginal cost, and other system components. These works also present a variety of interesting policy tools that could be used to correct the common pool externalities. Unfortunately, the hydraulic response of the aquifer to be managed has been either overlooked or dealt with only superficially by this group which casts doubt on many of the conclusions.

2.2 Group two: engineering and hydrological approaches

Members of the second group are mainly engineers and geo-hydrologists who focus on the actual response of the system rather than the economic issues. Models in this group usually deal with rather simplistic objective functions such as minimizing the pumping cost, maximizing the average potential head across the basin, or minimizing the deviation of potential heads from a set of target values. All such objectives lack proper economic justification and merely

serves as the performance criteria needed in the optimization process. However, such objectives could be readily modified to reflect more sophisticated economic objectives.

This group, however, is very strong in the computational side. Most of the works include a detailed groundwater simulation model along with a thorough description of a specific numerical solution algorithm for the optimal control problem. Empirical findings about relative merits of the simulated management policies as well as the solution algorithm used are often reported. Solution algorithms used by this group encompass the entire spectrum of numerical analysis and mathematical programming. So for detailed description of these techniques, references provided by the respective papers should be consulted.

One preliminary point warrants mention. Two different approaches have been used in the literature that allow incorporation of a general (multilayer, multicell, heterogeneous and anisotropic) groundwater flow model within an optimization framework. The first approach is called "embedding" where the governing flow equations and boundary conditions are directly included in the optimization model. These equations then become equations of motion and system constraints. The second approach is called "response matrix" method where a simulation model is repeatedly used to generate discrete kernels (also called influence or transfer coefficients) which are then used in the optimization model.

Models for optimal groundwater management abound in the engineering and geo-physical literature. Major works before 1983 have been summarized by Gorelick (1983). So only the most recent works will be reviewed.

Willis and Finney (1985) presents a quasilinearization based optimization method which could be used to solve optimal unconfined aquifer management problems with nonlinear hydraulics. They use the embedding approach to directly incorporate the governing flow equations into the optimization model. But to reduce the size of the problem, they use the Taylor series expansion of the nonlinear equations, and by dropping second and higher order terms, achieve quasilinearization.

This method performed equally well when compared to MINOS (a nonlinear optimization program; Murtagh and Sanders, 1980) in terms of CPU time used, but did better in terms of memory requirement due to smaller size of the program description. However, the authors really aim to develop only a seasonal optimal pumping schedule rather than a long run optimal management plan of the basin.

Wanakule et al. (1986) try a different approach to improve the computational efficiency by a combined simulation-optimization approach. Here the original embedding method is separated into its two basic components: an ADI (alternating direction implicit) based finite difference flow simulation model and a generalized reduced gradient (MINOS uses similar approach) based optimization model. So solving the discrete

time optimal control problem now becomes a two step process. First, simulation is used to express the state variable (head) as an implicit function of the control variable (groundwater withdrawal). This reduces the number of model constraints significantly. Second, a nonlinear optimization algorithm is used to solve the reduced problem.

The technique has been used to develop pumping policy for a five year period for the Edwards basin underlying San Antonio, Texas. The objective was to "maximize the sum of heads at pumping nodes subject to flow bounds, head bounds, and demand constraints." It took five hours of CPU time on a Cyber 170/750 which is not quite satisfactory if the method were to be implemented on a personal computer. The authors report that about 80% of the time was spent in the simulation model and a response matrix approach (for the hydraulic part) could have saved a lot of computations.

Yazicigil and Rasheeduddin (1987) present an implicit finite difference based embedding scheme for seasonal management of a multi-layer aquifer system. Linear programming (LP) is used for optimization where the objective is to maximize the sum of hydraulic heads which the authors describes as "linear surrogate for minimizing pumping costs." The authors also perform weighing and epsilon constraint based multiobjective analysis to develop trade-off curves for different water withdrawal policies. However due to the use of LP based optimization, only a subclass of general optimal management problem could be addressed without undue simplification.

Jones et al. (1987) introduce a new technique in the field of optimal groundwater management - the method of differential dynamic programming (DDP). The method has been used successfully in other areas of water resources management, such as for optimal reservoir operation (Murray and Yakowitz, 1979). This is a clever innovation derived from a number of previously used methods - dynamic programming (DP), quadratic programming (QP) and quasi- linearization. The procedure is mathematically involved and could not be described briefly. The algorithm allows stagewise decomposition of the problem and thereby significantly reduces the dimensionality problem. The authors solve two hypothetical problems, the second one being an unconfined basin divided into 108 cells with eight pumping nodes. With an objective of minimizing the cumulative pumping cost subject to a set of pumping constraints, this problem was solved for 12 stages (3 years) on an IBM 3090 which took less than five minutes. This is a clear indication of the computational superiority of the method.

DDP is a major improvement of the original embedding approach. However, it still requires convexity of the objective function for guaranteed convergence. It may also require second order Taylor series approximation for a problem which could not be accommodated within a linear quadratic control model (LQCM). This may simply render the method as infeasible for a more general class of problems.

Makinde-Odusola and Marino (1989) employ a hybrid groundwater simulation model and dynamic programming to solve for the optimal pumping pattern for a confined aquifer. Due to the dynamic programming approach, the method also generates feedback policies using "feedback rule coefficients." The approach is similar to the response matrix method in spirit in the sense that once all the feedback coefficients are estimated, the feed back policies can be generated without using the simulation model (provided parameters in the objective function remain the same). The main limitation of the proposed method is that the objective function has to be quadratic. The objective function for this study was to minimize the sum of squared deviations of the heads from a set of "target" levels. The authors present a lengthy discussion on merits of such an objective, but remain silent as to how such target levels could be obtained.

Dougherty and Marryott (1991) introduce another new technique - the method of simulated annealing. This is a heuristic (meaning that the reasoning is based on intuition and experience rather that on rigorous mathematical analysis), probabilistic optimization method for large-scale systems.

Conceptually, this method is not "greedy" (not likely to get trapped in a local minimum) and should eventually settle at the global minimum. It also allows the objective function to be discontinuous and nonconvex which is a big plus over other gradient based methods. Due to the practical limitation of CPU time allowed for a problem, the method will usually

terminate at a near optimal point. However, this is not a major drawback given the fact that other gradient based methods are not even applicable in the most general formulation of the discrete time control problem. Since no direct comparison of performance with other optimization techniques is available, the method requires further evaluation. The authors do mention that when "practical algorithmic guidance that leads to enormous computational savings" is provided, it can "sometimes make simulated annealing competitive with gradient-type optimization methods."

The final work to be discussed in this group has been reported by Culver and Shoemaker (1992). They use the previously mentioned DDP algorithm and a finite element flow and transport model to develop optimal groundwater remediation policies. Their main contribution is incorporation of the water quality issue and some analysis on the computational efficiency of the algorithm. As mentioned before, DDP is not a general method applicable for all problems. In this instance, first and second order Taylor series expansions for the objective function and the equation of motion were necessary. So, the method is not applicable when a nonconvex and discontinuous functions or equations are encountered.

To summarize, engineers and geo-hydrologists have approached the optimal groundwater management problem from hydraulic point of view. Their main preoccupation has been to develop a computationally efficient solution algorithm for the

discrete time optimal control problem. Since the mid eighties, a number of innovative approaches have been proposed and used to solve a variety of management problems. Among these, the DDP seems to have the superiority of computational efficiency. However, this is an embedding based approach, and therefore, require considerable effort during problem preparation (particularly when second order Taylor's series approximations are needed). The other promising method is simulated annealing which could be used to solve a problem with nonconvexity and discontinuity. But simulated annealing is really not a main stream optimization method due to its heuristic, probabilistic nature. Moreover it seems to be computationally inefficient. So, there is still room for introducing new methods in this field which would be computationally efficient, theoretically well founded, and yet relatively easy to implement.

2.3 Group three: team approaches

This group of studies are done by interdisciplinary teams, usually comprised of hydrologists and economists. Therefore the dichotomous management model with separate economic and hydrologic analysis is integrated into a single control problem through incorporation of economically meaningful objective functions and constraints, and true aquifer simulations. Both the embedding and response matrix approach have been used for the latter part.

Bredehoeft and Young (1970) and Young and Bredehoeft (1972) use finite difference based aquifer simulation linked with a linear programming based optimal management model. The first paper investigates the temporal allocation of groundwater for a hypothetical basin. It also studies the effects of two policy tools - use taxes and quotas. The second work develops a seasonal two-step planning and operational model for the South Platte basin in Colorado. The study also examines the effect of stream-aquifer interaction under different pumping capacity and location assumptions. Due to the simulation approach, optimality of the results is not guaranteed. The main conclusion from the study is that "centralized control of pumping by some institution would probably produce a higher value of production than unregulated development would." This study has been later extended by Bredehoeft and Young (1983) by introducing risk aversion into the decision making process. The latter study shows that the actual installed pumping capacity could be more accurately explained if it were assumed that farmers not only maximize their expected net income but also tries to keep the variability of the expected income to a minimum.

Young et al. (1986) conduct an interesting study where different institutional alternatives for groundwater management have been evaluated (using the same LP based simulation discussed above). They specifically examined the merit of an "augmentation plan" which require that "before each growing season, groundwater users must acquire or develop

augmentation surface water." This augmented water could be claimed by users with senior surface water rights in case of a shortage. The study concluded that this quasi-market mechanism is likely to perform better than zero pumping or unrestricted pumping scenario. Another new addition to this study is the use of response matrix instead of a full blown finite difference flow simulation model inside the LP based seasonal optimization.

Noel et al. (1980) and Noel and Howitt (1982) present probably the first true intertemporal conjunctive surface water and groundwater management model. They use LQCM (linear quadratic control model) formulation and examine relative performances of social optimal policy, pro-rata quota system, pumping tax policy, and laissez-faire policy. They also study the effect of increases in energy cost on groundwater extraction. For the hydraulic response part, a finite element simulation model has been used to create a database from which equation of motion for the basin is evaluated during the optimization process. Noel et al., based on the case study of Yolo County district in California, conclude that either quota (here total allowable extraction was restricted to long run average recharge) or time variant pumping tax would significantly increase the net social gain compared to the free extraction scenario. Note that this contradicts conclusions stated by Gisser (1983) and Dixon (1988) that for aquifers with large storage capacities, socially optimal and competitive extraction schemes are likely to perform comparably.

Finally, Casola et al. (1986) also preset a LQCM similar to Noel et al. Their model differs from Noel's in two minor ways. First they use a finite element based flow simulation model which is embedded in the control model. Second, they incorporate the stock effect of pumping by using a cost function which depends on both the pumping head and current stock size (Noel uses a separate term for stock effect in the cost function). Like Noel et al., Casola et al. also use MINOS to solve the control problem. Based on the case study of the Beryl-Enterprise area in Utah, the authors conclude that "a common property situation probably exists with the result that a greater than optimal amount of water is being extracted in the basin at the present time." They also recommend that transferable water permits should be introduced to correct the externality, but do not provide any empirical analysis to support the recommendation.

To sum up, it could be said that works that integrate true groundwater flow simulation and meaningful economic objectives within a dynamic optimization scheme are still very few in numbers (only Noel et al. and Casola et al. fall into this category). And when such attempts were made, LQCM was chosen to simplify the analysis which precludes the possibility of addressing more general class of problems.

2.4 Objectives of the current study

It is clear from the review above that although the theory of optimal renewable and nonrenewable resource extraction is well established, there is a dearth of empirical investigation of the same in relation to long run groundwater extraction. Most studies done by economists are based on simple analytical formulations and little attention is given to the time and location dependent aquifer response. On the other hand, studies conducted by engineers and mathematical modelers explore the numerical solution algorithms to a great extent but make no serious attempt to incorporate an economically meaningful objective function into the optimization problem. And so far attempts to integrate these two aspects of the problem have had only limited success.

Therefore, this study will attempt to extend the current body of work on optimal groundwater management by proposing a different integrated modelling strategy which promises to overcome most of the difficulties mentioned above. Specific objectives of this study are outlined below.

1. This study will develop an integrated groundwater management model capable of incorporating both nonlinear objective function and constraints, focusing on agricultural water use. So the model will allow more realistic nonlinear production functions for the major crops as opposed to linear ones used in all previous integrated model studies. Real agronomic and economic data from northern Colorado and the

Denver groundwater basin area will be used to build the economic submodel for this study.

2. This study will incorporate a discrete kernel based linear response matrix of the groundwater aquifer within the nonlinear optimization model to investigate a number of long run extraction policies. This will eliminate the need for embedding a groundwater model within the optimization model. Use of the response matrix will considerably increase the computational effciency and lower runtime memory requirement of the optimal control/nonlinear programming problem. For confined aquifers, discrete kernels are accurate representation of the aquifer response. However, the method could also be used to study problems related to unconfined aquifers when the resulting drawdown does not become a significant part of the original saturated thichness of the aquifer.

For this study, a hypothetical groundwater basin will be used to generate the discrete kernels starting from the steady state. But real aquifer data from the Denver basin (Arapahoe aquifer) will be used to allow for the natural heterogenity of a confined aquifer.

3. This study will employ a conjugate gradient based nonlinear programming algorithm to solve the intertemporal resource allocation problem. This technique requires considerably less computer storage than other more commonly used gradient search based algorithms. It was therefore possible to implement the models on a personal computer as

opposed to on a super computer as has been the case with previous studies. This could be achieved due to the unique combination of linear response matrix and conjugated gradient based solution algorithm. This is also believed to be an important empirical findings of this study.

4. The study will primarily investigate the economic and hydrologic tradeoffs between the two extreme possible resource extraction schemes. They are the so called 'social optimal' and 'common pool' scenarios. Although in reality, neither of the two situations is likely to exist in its pure form, they act as baseline scenarios for the best and worst possible outcomes. So if the divergence between the two turns out to be small (as sugggested by some researchers above), then the planner really need not worry too much about minimizing the cost of externality because any such institutional measures themselves are also costly.

5. A third application of the model will simulate the economic and hydrologic effects of municipal pumping during a five-year long drought from an aquifer which is primarily used for agriculture. Finally, outline will be provided on how important operational scheduling could also be performed using only the hydrologic and optimization parts of the integrated model.

CHAPTER 3

ECONOMIC SUBMODEL: VALUE OF WATER IN AGRICULTURE

The economic value of water is derived from its intended use or demand, which could be agricultural, municipal, industrial, or even non-consumptive instream use. In this chapter a model for agricultural water use will be developed and relevant parameters will be estimated.

Agricultural water value is a function of a number of underlying factors such as types of crops being irrigated, production functions, prices of crops, variable and fixed costs associated with the production activity, and costs related to the irrigation technology. There are other factors which are not controllable - soil type and different weather parameters determine the yield of a crop to a great extent. All these factors jointly, and in a complex manner, determine the value of water.

As mentioned earlier, the area overlying the Arapahoe aquifer of the Denver groundwater basin will serve as the study area for the economic submodel. Robson (1987) reports that between 1958 to 1978, about 80 to 85 percent of the bedrock pumpage from the Denver basin came from this Aquifer. So, the economic model to be derived below could be integrated

with appropriate hydrologic model for important policy analysis for the Denver groundwater basin.

3.1 Crop type

The main irrigated crops in the Arapahoe basin area are corn, dry beans, sugar beets, barley and alfalfa. Of course, there are other crops which are also irrigated, but they make up a small percentage of the total irrigated acreage. Moreover, data on some of these crops are not reported separately for each county (fruits and vegetables fall into this group). Therefore, only the five main irrigated crops as mentioned above will be considered in deriving the agricultural water demand function.

3.2 Crop production function

The first step towards deriving the marginal value of agricultural water is to establish the functional relationship between the amount of irrigation water applied and the corresponding crop yield. Since this study intends to focus on long run policy analysis, only seasonal production functions will be estimated.

Many different forms of seasonal production function have been reported in the literature which relate inputs of production to the crop yield or output. The form to be used really depends on the purpose of the study. Chang, et al.

(1973) proposed a relationship for sugarcane in Hawaii where the ratio of actual to potential yield (maximum yield under the best possible field condition and input use) is a quadratic function of the ratio of actual to potential evapotranspiration. Hargreaves (1975) uses a similar function where the independent variable is the ratio of available soil moisture to amount of moisture needed for maximum yield. Hexem (1974) presents production functions for many different crops (also specified by site and season) in terms of two control variables - water applied and nitrogen applied. In this study, it is assumed that all other inputs except water are being applied at the optimal level so that the production function could be expressed only in terms of actual water applied at a specific application efficiency.

3.3 Derivation of the quadratic production function

The derivation below follows closely the derivation of a regional production function by English and Dvoskin (1977). It has been shown by the researchers mentioned above that the regional crop production function can be expressed as:

$$\frac{Y_a}{Y_p} = \beta_0 + \beta_1 A + \beta_2 A^2 \qquad (3.1)$$

$$A = \frac{\epsilon W + R_e}{E_p} \qquad (3.2)$$

where:
    Y_a=actual yield
    Y_p=potential yield
    W=amount of water applied
    ϵ=water application efficiency
    R_e=effective rainfall
    E_p=potential evapotranspiration
    A=water adequacy ratio, and
    β_0, β_1, and β_2 are the intercept term, linear and quadratic coefficients respectively.

Now substituting (3.2) in (3.1) and rearranging, we get:

$$Y_a = a + b(\epsilon W) + c(\epsilon W)^2 \tag{3.3}$$

where a, b and c are parameters specific to crop cultivar, soil type and climate conditions and are defined as:

$$a = (\beta_0 + \frac{\beta_1 R_e}{E_p} + \frac{\beta_2 R_e^2}{E_p^2}) Y_p \tag{3.4}$$

$$b = (\frac{\beta_1}{E_p} + \frac{2\beta_2 R_e}{E_p^2}) Y_p \tag{3.5}$$

$$c = (\frac{\beta_2}{E_p^2}) Y_p \tag{3.6}$$

Procedures of estimating long run average R_e and E_p are beyond the scope of this study. For technical details on these climate and crop related factors, see English and Dvoskin (1977) or Doorenbos and Pruitt (1975). The application efficiency, ϵ is defined as the amount of water stored in the root zone of a crop for beneficial plant use divided by the amount of water applied to the field (Hoyt, 1984). Another factor that is not being explicitly considered in the above formulation is the soil moisture content at the time of

planting. The implied assumption is that the soil profile will normally be recharged to the field capacity due to spring snow melt and early seasonal precipitation. In fact the actual amount of soil moisture is not that important as long as it remains fairly constant from year to year. Again, if the derived production functions are to be used for an entirely different area, this assumption may be violated and the functions will make erroneous predictions about crop yield.

For this study, (3.3) will be used as the standard form of the crop production function. This has the advantage of having water applied, as opposed to crop evapotranspiration, as the independent variable which is easier to measure and control.

3.4 Economic properties of the quadratic production function

Let a, b and c be the intercept term, linear and quadratic coefficients of a quadratic function (as in equation 3.3, assume that $\epsilon = 1$ for simplicity). Obviously, the marginal product is linear in the input of production, W (water applied). Typically for a crop production function, b is positive and c is negative. Therefore, when W is close to zero and increasing, marginal product is positive but decreasing. Eventually, W reaches the optimum value W^* where the output is maximum. At this point $dQ/dW = 0$ and $W^* = -b/2c$. Beyond that, the marginal product becomes negative.

Sometimes the production function is expressed in terms of two inputs (say, water, W and nitrogen, N). In that case, a unique maximum output is defined by the optimal values of inputs. Both isoquants and isoclines converge to the point of maximum output. Isoclines are linear but do not radiate from the origin (with a sole exception). This means that the proportion of W and N changes along the expansion path. Also two special isoclines become the ridge lines and define the economically feasible region of production. Beyond the ridge lines, the marginal product of one or the other input becomes negative.

3.5 Production functions used in this study

A number of different sources have been used to extract or estimate the production functions for corn, dry beans, sugar beets, barley and alfalfa. All the functions (except the one for barley) have been derived on basis of experiments conducted at the Agronomy Research Center of the Colorado State University, located at Fort Collins, Colorado.

The experimentation site, at an elevation of 5000 ft, has a semiarid and continental climate. The average 'killing frost'-free season, as reported by Stewart, et al. (1977), spans for 144 days from May 8 to September 29. Average seasonal (March-October) precipitation is about 14-15 inches. The soil at the experiment site has been classed as Nunn clay-loam, which is calcareous and moderately well drained soil

with relatively uniform texture to a depth of 4 t 5 feet. All these climatic and soil factors, along with the specific cultivar planted, jointly determine the site specific parameters of the production function. This is why such production functions have limited applicability and they are only valid for other regions with similar crop, soil and climatic conditions. It is quite likely that the Arapahoe groundwater basin will reasonably meet such preconditions (due to its proximity to the Fort Collins area).

Thus, production functions based on data from Fort Collins will be used for this study without any modification In reality, crop production functions for a specific geographic area are very difficult to come by, and unmodified use of such functions for areas with similar characteristics is quite common in regional studies (Hoyt, 1982, 1984; Ayer et al., 1983).

3.5.1 Corn (Zea maze)

An earlier but well documented production function for corn based on Colorado data was reported by Huszar, et al. (1970). Later an updated regional function was presented by Hoyt (1984) in the form of (3.3) which is usable for this study with little modification. The production function proposed by Hoyt was based on experiments conducted in 1974 and 1975. Since mid eighties, introduction of new high yielding varieties has increased both potential grain corn

yield and water demand significantly, and old functions are no more representative of today's technology.

More recent experiments on corn production function were performed by Vigil (1983) and San (1986). These studies estimate potential yield and water demand for the newer varieties which are consistent with the field observations as reported in recent annual publications of the USDA(1984-1991).

For this study, two sets of data have been used (1982 data from Vigil and 1985 data from San) to estimate the corn production function of the form (3.3). Unfortunately, two other sets were not usable due to experimental/measurement errors which produced highly inconsistent data points compared to field observations or the data sets used in this study. So, the resulting function was estimated from only nine data points and not all the parameters obtained were statistically significant (see Appendix A). However, the function predicts potential yield and water demand which match quite closely with the same reported in a more recent study (Michelsen, 1988). The function, being quadratic in form, also matches the general shape of previously proposed functions. In the absence of any better estimate, this function will be used for the current study. Table 3.1 lists this function, along with other production functions used in this study. Note that all production function coefficients have been modified to reflect 100% application efficiency so that later on, water applied, W could be substituted by irrigation water applied times the application efficiency.

3.5.2 Dry beans (Phaseolus vulgaris L.)

A number of studies exist on the relationship
between irrigation water applied and the yield of dry beans.
These studies were all conducted at the Agronomy Research
Center of the Colorado State University. For the current
study, data from Kisugite (1974), Karim (1986) and
Bandaranayake (1990) have been used to estimate the production
function for dry beans. Since data points represent three
different time periods, dummy variables were initially
introduced to account for any time specific factors. Later,
these dummies were discarded as they came out to be
statistically insignificant (also one data point in the
combined sample was dropped because it clearly appeared to be
a distant outlier). The estimated coefficients are all
statistically significant at one and five percent levels (see
Table 3.1 and Appendix A).

Table 3.1
Regional crop production functions for the
Arapahoe groundwater basin

| Crop Type | Coefficients | | | Y_p | W_{max} ($\epsilon = 1$) |
	a	b	c	per acre	ac-in/acre
Alfalfa	1.070	0.123	0.0	4 ton	23.88
Barley	42.093	13.368	-0.6355	113 bu	10.52
Dry beans	9.048	2.856	-0.1142	27 cwt	12.51
Corn	68.640	8.210	-0.1854	160 bu	22.14
Sugar beets	10.974	2.532	-0.0963	28 ton	13.15

Notes:
1. All Y_p values have been rounded to the nearest integers.
2. Production function has the general form:
   $Y_a = a + b * (\epsilon W) + c * (\epsilon W)^2$

3.5.3 Barley (Hordeum vulgare L.)

The barley production function has been directly adapted from the study done by Jakicic (1983). Jakicic reports three different production functions for three most popular cultivars - Pirouette, Kimberly, and Golden Promise. The production functions are very similar to each other (Kimberly seems to have somewhat higher potential yield and water demand). Since it is not known as to what percent of irrigated land under barley will be allocated for a certain variety, production function coefficients used in this study are the average of the three sets. The production functions used were derived on the basis of experiments conducted in San Luis Valley (south-central Colorado). Although Jakicic reported similar functions for Fort Collins area, due to problems in timely control of irrigation water, the resulting production functions performed very poorly.

The cultivars mentioned above are mainly cropped as malt barley. Another major use of barley is for livestock feeding. The difference between the two lies, not necessarily in the variety being cropped, but in how the crop is attended. Malt barley requires more and timely irrigation water to ensure the right degree of plumpness, color and protein content. It needs less nitrogen than feed barley. On the other hand, feed barley is not so much sensitive to water, but needs more fertilizer. Since no separate estimate of production function for feed barley was available, only one function had to be used to represent both malt and feed barley. However, most malt barley

is irrigated and significant part of feed barley is nonirrigated, so the bias introduced is probably small.

Also, all the cost figures used in this study are for malt barley. It is assumed that higher cost of water for malt barley will be approximately offset by higher cost of fertilizer for feed barley. The price of malt barley is historically about 20 cents (per bushel) higher than that of feed barley and this causes another accounting problem. Part of the problem is offset by higher per acre yield of feed barley due to higher fertilizer use. The price used in the benefit estimation is the weighted average of the prices for malt and feed barley as reported in Colorado agricultural statistics. This might counter part of the upward bias that could have resulted from using only malt barley price. In any case, the actual nature of bias for the study area is indeterminate due to lack of readily available data. It is hoped that the approach taken above will produce cost and benefit estimates which are close to the true ones for a mixed cropping pattern of malt and feed barley.

The production function as reported by Jakicic included nitrogen as an additional variable. This has been taken care of by assuming that optimal level of nitrogen will be applied which is about 225 kg/hectare for a very wide range of water application. Jakicic's water applied term included both irrigation and rainfall, so coefficients were further adjusted to separate out the contribution of effective rainfall.

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3.5.4 Sugar beets (Beta vulgaris L.)

The production function for sugar beets has been adapted from Hoyt (1984). Hoyt's derivation was based on original experiments done by Flack (1981). Some minor transformations of the coefficients as proposed by Hoyt were necessary to conform the final form of the production function to (3.3).

Hoyt's function gave yield in pounds of sucrose per acre, which had to be converted to ton per acre of fresh root material (this latter unit is more widely used in statistical sources). Is has been assumed that 15 percent of the fresh root material could be converted into sucrose when processed (Hexem, 1977). Data from Flack's study has also indicated that 15 percent sucrose content is a genetic property of sugar beets which remains unaffected except for very severe water stress conditions. As before, the coefficients had to be adjusted to reflect 100 percent application efficiency and contribution of effective rainfall.

3.5.5 Alfalfa (Medicago sativa L.)

Alfalfa is a rather unique crop compared to other crops considered in the economic submodel. Generally alfalfa can grow throughout the year in warmer climates, and throughout the killing-frost free season in the semi-arid areas like Colorado. Its growth slows down noticeably when soil moisture deficit drops below 25% of field capacity level, but the plant recuperate quickly if water becomes available within a few weeks. It also grows under a wide variety of water stress and

salinity conditions. Unlike other crops, it may have very deep root system which can penetrate even to a depth of 30 feet. Shallower depths of 10 to 15 feet is more common in areas of shorter growing seasons and low water availability. The crop can survive on deep percolated water from previous irrigation or elevated groundwater table. Moreover, the hay is cut a number of times during the entire growing season, usually at 30 to 40 days interval. Because of all these factors, it is difficult to estimate a general production function for alfalfa.

A thumb-rule for determining irrigation water requirement for alfalfa is to assume that about 6 acre-inch per acre of water is required to produce a ton of field dried hay (Peterson, in Hanson,1972). So for an expected potential yield of 4 tons/acre (used in this study as suggested by Booker (1992)), water applied net of application loss has to be 24 acre-inches per acre. In fact, this thumb-rule based projection is very close to the estimated seasonal net irrigation water requirement of 23.88 acre-inches per acre for the Fort Collins area (Michelsen, 1988). For this study, a simple linear production function similar to Hanks (1974) will be used:

$$\frac{Y_a}{Y_p} = \frac{\epsilon W + R_e}{E_p} \qquad (3.7)$$

Equation (3.7) could be rearranged to separate out the contributions of effective rainfall and irrigation water applied as:

$$Y_a = a + b(\epsilon W) \hspace{4cm} (3.8)$$

where $a = Y_p \cdot R_e / E_p$ and $b = Y_p / E_p$, all the symbols have same meanings as before. Since the coefficients could be calculated directly from available crop and weather data, no curve fitting or parameter estimation was necessary for alfalfa. Table 3.1 shows the coefficients used in this study.

3.6 Crop production cost

To derive the regional demand function for agricultural water, benefit from water use has to be expressed as net of all production related expenses except the irrigation water related costs. These costs could be categorized into two groups - fixed cost and variable cost. Fixed costs are not dependent on the production activity of a particular year and are unavoidable in the short run. For example, opportunity cost (lost income in the form of secure interest) of owning land and buildings, machinery depreciation, tax and insurance, general farm overhead are all fixed costs. So, farmers should consider fixed costs as sunk costs while making cropping and irrigated related decisions in the short run. Variable costs, on the other hand, depend entirely on the extent of production which typically include seed, fertilizer, water, pesticides, labor for land preparation, irrigation and harvesting, and other operation and maintenance related costs.

Estimates of fixed and variable costs for alfalfa,
barley, dry beans and corn have been taken from Michelsen
(1988), which were based on farm enterprise budgets as
compiled by Dalsted et al. (1987). This latter source along
with some estimates of Michelsen (1988) and Booker (1992) were
used to determine fixed and variable costs for sugar beets.

Michelsen made necessary adjustments to alfalfa
production costs to reflect the fact that once planted,
alfalfa could be harvested for the next four years before
switching to another crop. So, cost of land preparation and
planting in the first year was amortized over a period of four
years. All costs (as well as benefits) used in this study are
in 1988 constant dollars. Table 3.2 shows itemized breakdown
of all costs except irrigation water costs.

Table 3.2
Crop production costs
(1988 constant dollars)

Cost type ($/acre)	Alfalfa	Barley	Dry Beans	Corn	Sugar Beets
Fixed costs					
Owner labor	7.50	15.20	18.86	28.62	37.70
Mangmnt. 6%	13.10	10.00	13.80	16.05	34.50
Bldg 7%	2.57	4.63	3.33	6.32	6.35
Machinery depr. 20%	0.51	3.33	2.73	3.73	7.12
Mach. tax & insur.	0.43	2.95	2.46	3.22	5.54
Real estate tax	12.00	12.00	12.00	12.00	12.00
Land return 7%	21.00	21.00	21.00	21.00	21.00
Overhead	10.00	10.00	10.00	10.00	10.00
Total fixed cost	67.11	79.11	84.18	100.94	134.21
Variable costs					
Operating	117.88	68.72	127.60	137.13	412.29
Mach. depr. .8	2.06	13.33	10.94	14.90	28.48
Total variable cost	119.94	82.05	138.54	152.03	440.77

Note: irrigation related fixed / variable costs not included.

3.7 Crop price

Crop prices used in the demand function estimation are very important determinant of the marginal value of water. These estimates should reflect recent prices actually paid to the farmers and should be consistent with other cost figures used in the analysis. Michelsen (1988) presents a lengthy discussion on this issue, particularly on merits of different price projection methods as well as the price indices that could be used to convert all prices to a common base year. It seems to be the case that complex econometric price projection methods are not likely to be any better than simple average of recent prices (or of moving average, when significant yearly fluctuation is observed). This is true for crops with relatively stable historic prices. This is the approach taken in this study. Also implicit GNP deflator has been used to convert all prices to 1988 constant dollars.

Crop prices used were average of real crop prices from 1981 through 1987. Prices for alfalfa, dry beans, corn and sugar beets were extracted from annual Colorado agricultural statistics published by the USDA. Barley prices were proprietary and were not reported in the above source. Michelsen have reported malt barley prices based on northern Colorado contract prices. All the crop prices along with total fixed and variable costs used in this study are shown in Table 3.3.

Table 3.3
Crop production costs and prices
(1988 constant dollars)

Crop type	Variable cost ($/acre)	Fixed cost ($/acre)	Price ($/unit)
Alfalfa	119.94	67.11	73.38 /ton
Barley	82.05	79.11	3.01 /bu
Dry beans	138.54	84.18	17.38 /cwt
Corn	152.03	100.94	2.75 /bu
Sugar beets	440.77	134.21	27.12 /ton

Note: irrigation costs are not included; corn: 1 bu=56 lbs, barley: 1 bu=48 lbs, dry beans: 1 cwt=112 lbs.

3.8 Crop acreage

It is necessary to know the acreage devoted to different crops in the study area so that realistic constraints could be included in the nonlinear demand function estimation model. For example, total acreage under all crops has to be fixed, or alternately, upper and lower limits on the same has to be established. This has been done by estimating sum of maximum and minimum acreage for all the crops based on seven years of data from 1984 to 1991 (longer time series was avoided because older acreage may not reflect today's technology). Also sum the of average acreage for all the crops has been calculated for the same time period. However, using both upper and lower bounds to restrict the total acreage has a different implication than using the sum of average acreage for the same purpose.

When sum of average acreage is used to restrict the total acreage (allowing individual crop acreage to vary within an upper and a lower bound), the implied assumption is that only

the land which received irrigation in the recent past will be allowed to receive irrigation in the future. This allows some interchange of land among the crops keeping the total acreage fixed to the historical average. But this leaves no room for previously nonirrigated land to come under irrigation even when water price is close to zero. For a long run policy simulation, particularly when sufficient nonirrigated arable land is available for agriculture, such restriction seems unrealistic.

On the other hand, using upper and lower limits to restrict the total acreage has some interesting implications. The sum of maximum acreage for all the crops (based on 1984-1991 data) is likely to emulate the maximum total acreage under most favorable conditions. This is because all the maximum acreage did not occur in the same year, in fact they were quite dispersed. This reflects a situation where all the land which could be irrigated have been brought under irrigation. Further addition to irrigated acreage may not be possible due to lack of suitable land, adequate water, or both. In any case, it seems reasonable to have an upper limit on the irrigated acreage for each crop, but it has to be less restrictive than the historical average to allow inclusion of previously nonirrigated land.

Whether or not to use a lower limit on the irrigated acreage raises a more engrossing issue. Michelsen has argued that a lower limit should be included 'to reflect contractual obligations, diversification of crops for risk hedging, crop

specific equipment limitations and other constraints such as livestock demands.' In the short run, such restrictions are no doubt valid. But in the long run, the question remains open as to what extent such factors will prevent substitution or retirement of land as water becomes dearer. Perhaps, the only economic rationale for having a minimum limit in the long run is crop diversification if the farmer insists on practicing irrigated farming.

It is also worth noticing that sudden shocks such as energy price hike of the seventies, change of agricultural policies (price support, special loan rate), catastrophic flood or drought, international crisis - some or all of these are almost bound to occur in a long planning horizon of forty years. But it is virtually impossible to anticipate them a priori. These factors are likely to affect the expected revenue from a crop in a more profound and long lasting way than the usual market and weather related random factors. On the other hand, technological breakthroughs can drastically change the notion of relative riskiness of different crops and can have a significant positive impact on the expected net revenue. The point being that in the long run, simply switching to dryland farming or making a secure investment elsewhere may be as good a strategy as crop diversification.

Of course, from the regional point of view, and for reasons other than economic (self sufficiency, preserving traditional way of living etc.), an argument could be made that certain minimum acreage be allocated to a specific crop

which can not grow without irrigation. Theoretically speaking, this imposes an infinite penalty for violating the nonzero lower bounds in the nonlinear demand estimation model. The outcome is a demand function which may discontinue beyond a specific price level to avoid negative net return.

In the absence of a clear guideline, two different demand functions will be estimated below. The first one is based on zero lower bounds for all crops, the other one based on non-zero lower bounds set equal to the minimum acreages observed during the 1984-91 period.

The absence of a lower limit on irrigated acreage (or equivalently, lower limit of zero) actually simulates an ideal long run scenario where cropping decisions are made solely on the basis of profit maximization. This will serve as the baseline scenario for subsequent analysis and comparison. The other demand function with nonzero lower bounds will then provide an estimate of the premium that the society must pay to continue some minimum level of irrigated farming.

Table 3.4 summarizes irrigated acreage statistics for the Arapahoe basin. Note that some subjective judgements had to be made while estimating the irrigated land for each county within the Arapahoe basin boundary. The general assumption was that the irrigated area within the basin boundary (for a particular county) was proportional to the ratio of the area within the basin to the total county area. Further adjustments were made to exclude areas such as mountains, forests, parks

Table 3.4
Average, minimum and maximum acreages on the
basis of crop type and county in the Arapahoe basin

Crop type	Adams 0.6	Arapahoe 0.73	Boulder 0.08	Douglus 0.85	Elbert 0.65	El Paso 0.44	Jefferson 0.28	Weld 0.1	Total by crop (acres)
Alfalfa	5265	1405	1092	2061	3356	2244	424	8273	24120
	4560	876	1000	1700	1885	2024	168	7900	20113
	6000	2190	1184	3400	5525	2860	896	9000	31055
Barley	1050	9	244	0	0	0	14	1743	3060
	240	0	168	0	0	0	0	1280	1688
	1560	73	304	0	0	0	28	1950	3915
Dry beans	593	27	142	0	0	0	0	3275	4037
	300	0	80	0	0	0	0	2650	3030
	1020	146	224	0	0	0	0	4470	5860
Corn	3405	110	648	53	0	55	0	15925	20196
	2100	0	480	0	0	0	0	12980	15560
	5100	365	800	170	0	88	0	19450	25973
Sugar beets	427	0	65	0	0	0	0	2032	2524
	258	0	56	0	0	0	0	1725	2039
	678	0	73	0	0	0	0	2148	2899
Total by county (acres)	10740	1551	2191	2114	3356	2299	438	31248	53937
	7458	876	1784	1700	1885	2024	168	26535	42430
	14358	2774	2585	3570	5525	2948	924	37018	69702

Number below the county name represents fraction of the total irrigated land of that county that is within the aquifer boundary.

reservations and large lakes. Maps published by the USGS were used for this purpose.

3.9 Example of derivation of the short run demand curve

All the crop related information presented so far will be used later in Chapter 7 in the combined economic-hydrologic model for long run policy analysis. The integrated model does not require a separate estimation of regional or local water demand function. However the aggregate short run demand curve for water has an informative value of its own. It generates informaton on water demanded and revenue generated by all the major crops in the study area. When a number of irrigation technologies are available, the weighted average of the application efficiencies could be used to generate approximate estimate of water demand and acreage allocation (it is not necessary to use equivalent irrigation technology, the idea is used here to simplify the comparative statics of water allocation presented later in this section). The procedure also serves as an example of using a simple nonlinear model for regional demand function estimation.

The Arapahoe basin is irrigated by four major irrigation methods: flooding, siphon, gated pipe and sprinkler. It is assumed that statewise percent shares of these technologies also prevail in the study area which are approximately - flooding 60%, siphon 20%, gated pipe 5% and sprinkler 15% (Wilson and Ayer, 1982). The efficiencies for these irrigation

methods are assumed to be 0.5, 0.6, 0.75 and 0.85 respectively. So the weighted average of the efficiencies is 0.585 which would be used as a proxy for an equivalent hypothetical irrigation technology. This number and other regional crop and acreage data have been used below to demonstrate the derivation of a short run aggregate demand curve. The general formulation of the problem could be presented as follows.

```
Let:
Z=objective function (dollars)
Y_i=production function for the ith crop
   =a_i + b_i(ε_j·w_{i,j}) + c_i(ε_j·w_{i,j})^2  (unit/acre)
w_{i,j}=water applied for the ith crop (ac-inch/acre)
    using the jth technology
x_{i,j}=acreage under ith crop and jth tech.(acres)
ε_j=efficiency of the jth technology
r_i=price of the ith crop ($/unit)
p=price of water ($/acre-inch)
cv_i=variable cost for the ith crop ($/acre)
xmin_i=minimum acreage for the ith crop (acres)
xmax_i=maximum acreage for the ith crop (acres)
xmax_j=maximum acreage under the jth tech. (acres)
alfamax=max. seasonal net irrigation for alfalfa
n=number of crops
m=number of technologies.
```

As mentioned before, all the cost terms exclude any component which is related to the irrigation water applied. Also fixed costs are excluded and considered as sunk in the short run. Values of all the parameters above except the price of water, p could be found in Tables 3.1, 3.2 and 3.3. Now, the first step towards computing one point on the regional demand curve is to solve the optimization problem below.

For a given value of unit price of water, p and a predetermined level of crop related investment and irrigation technology,

$$Max \quad Z = \sum_{i=1}^{n} [\sum_{j=1}^{m} [(r_i \cdot Y_i(\epsilon_j w_{ij}) - p \cdot w_{ij}) x_{ij}] \quad (3.9)$$
$$- cv_i \cdot \sum_{j=1}^{m} (x_{ij})]$$

subject to the following constraints:

$$Y_i(\epsilon_j w_{ij}) = a_i + b_i(\epsilon_j w_{ij}) + c_i(\epsilon_j w_{ij})^2, \quad \forall \ i \quad (3.10)$$

$$\sum_{j=1}^{m} (x_{ij}) \geq xmin_i, \quad \forall \ i \quad (3.11)$$

$$\sum_{j=1}^{m} (x_{ij}) \leq xmax_i, \quad \forall \ i \quad (3.12)$$

$$\sum_{i=1}^{n} (x_{ij}) \leq xmax_j, \quad \forall j \quad (3.13)$$

$$\epsilon_j w_{ij} \leq alfamax, \quad \forall j, \ i=alfalfa \quad (3.14)$$

$$w_i \geq 0, \quad \forall \ i \quad (3.15)$$

$$x_i \geq 0, \quad \forall \ i \quad (3.16)$$

There is one more condition which has to be imposed for a not so obvious reason: since the model treats both x_i and w_i as decision variables and is not aware of the common sensical association between the two, it has to be checked that when x_i

is zero, w_i must also be set to zero. There is no standard way of implementing such a conditional constraint. A simple trick commonly used in LP based models has been used in this study:

$$w_{ij} \le M.x_{ij}, \qquad \forall i, \forall j \qquad (3.17)$$

where, M is a suitably selected constant.

When M in (3.16) is sufficiently large, w_{ij} will not be constrained by x_{ij}, but when the latter assumes a value of zero, the former will be forced to become zero as well. In theory, the formulation seems simple and sound, but its numerical implementation requires some care. Due to rounding off error and the discrete nature of steps taken by the nonlinear solver, x_{ij} in reality may never be exactly zero. Then if M is large, w_{ij} may still assume a significant nonzero value. On the other hand, too small an M will prematurely constrain w_{ij} from reaching its optimal level. So some trial and error is necessary.

After one run of the model, the quantity of water demanded for price p is:

$$Q(p) = \sum_{i=1}^{n} \left(\sum_{j=1}^{m} (w_{ij}^{*} x_{ij}^{*}) \right) \qquad (3.17)$$

where, optimal values of w_{ij} and x_{ij} have been used in (3.17). The inverse of this relationship for a series of p (say increasing from zero to some value where Z tends to zero) will give the desired demand function.

The optimization problem described above has been solved for j=1 case with the equivalent irrigation application efficiency of 0.585. It is likely to provide a close estimate of marginal value of water for the region as a whole. It also produces optimal acreage figures for different crops. Figures 3.1 and 3.3 show the regional demand functions for the Arapahoe basin, with zero and nonzero (set equal to the historical minimum) lower bounds on crop acreage respectively. Figure 3.2 and 3.4 show the variation of total acreage and benefit with respect to the price of water. Figure 3.5 shows a comparative plot of total benefits for the cases with zero and non-zero minimum bounds on crop acreage. The discussion to follow will only consider the scenario with zero lower bound on acreage.

3.10 Interpretation of the demand function

The demand function in Figure 3.1 has been estimated for a region with five different crops and the equivalent irrigation technology. The algorithm used to numerically solve the problem is quite complex (Chapter 6 will deal with this issue in detail). But considerable insights could be gained about the dynamics of water allocation using simple economic reasoning.

To begin with, assume that there is only one crop and one irrigation technology. So, using the same symbols as before
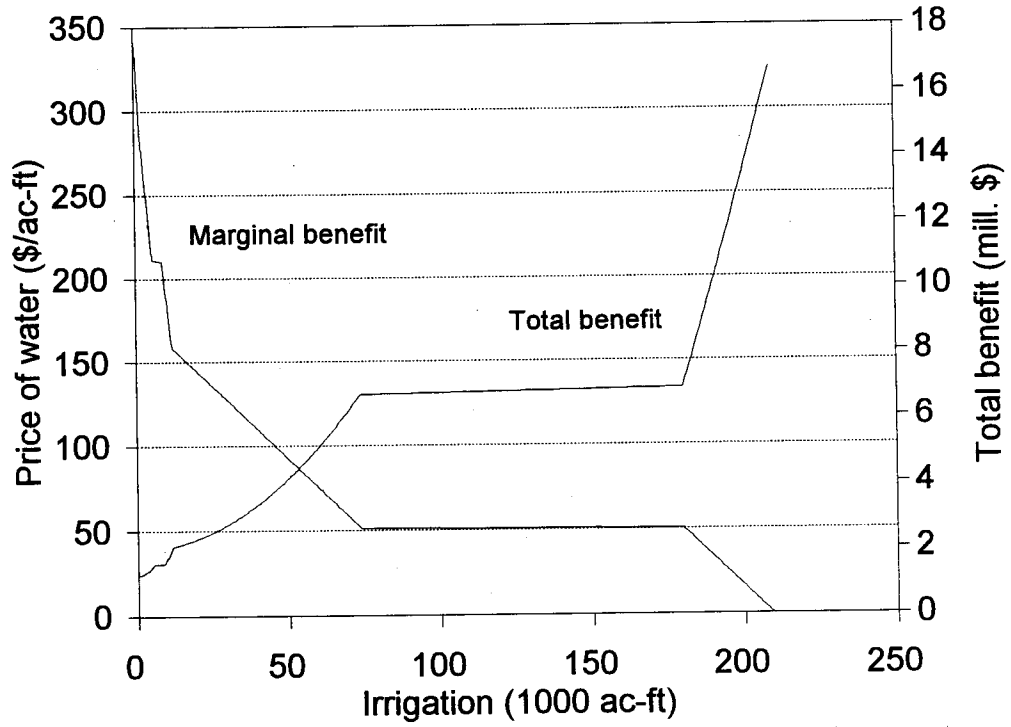
Figure 3.1: Marginal and total value of water
for the Arapahoe basin.
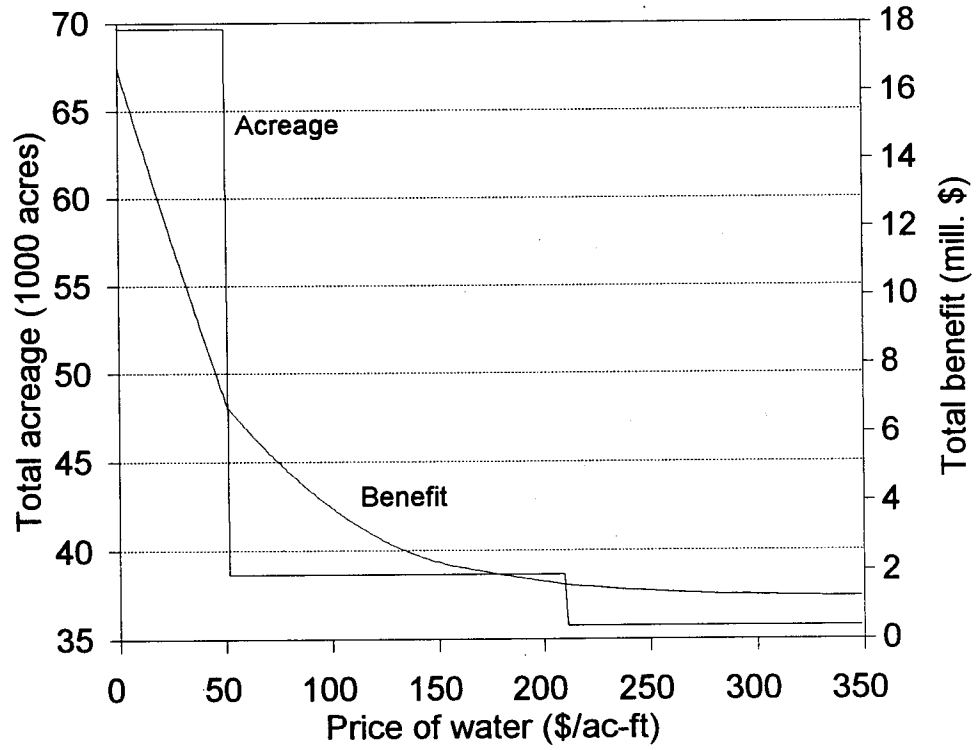
Figure 3.2: Total acreage and benefit against
price of water.
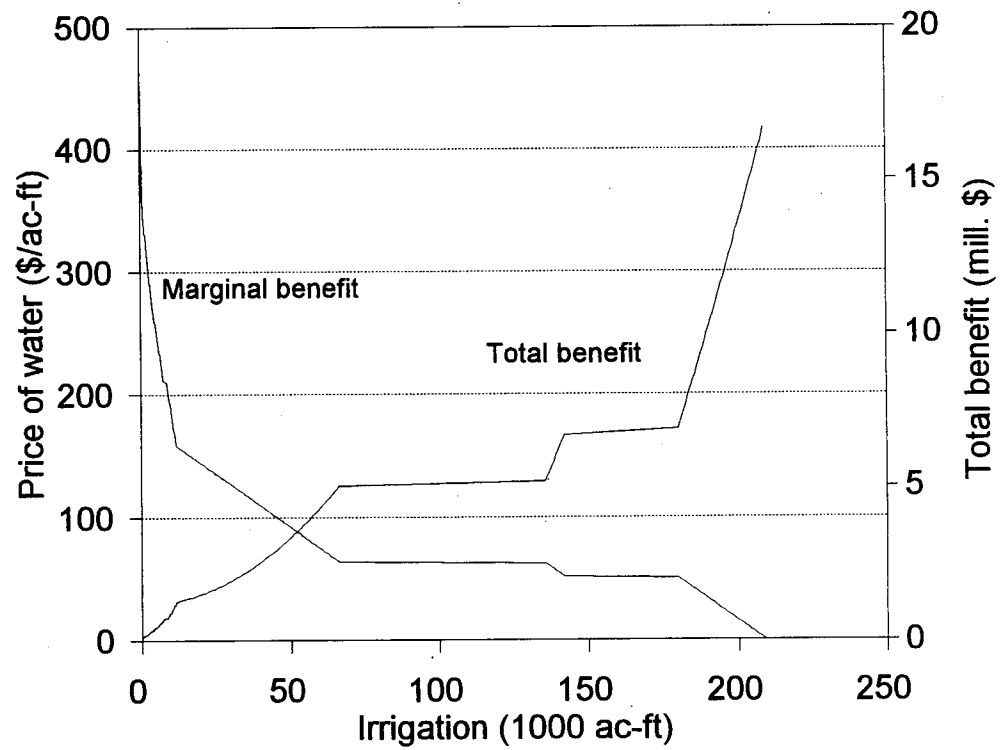
Figure 3.3: Marginal and total value of water
with minimum bounds.
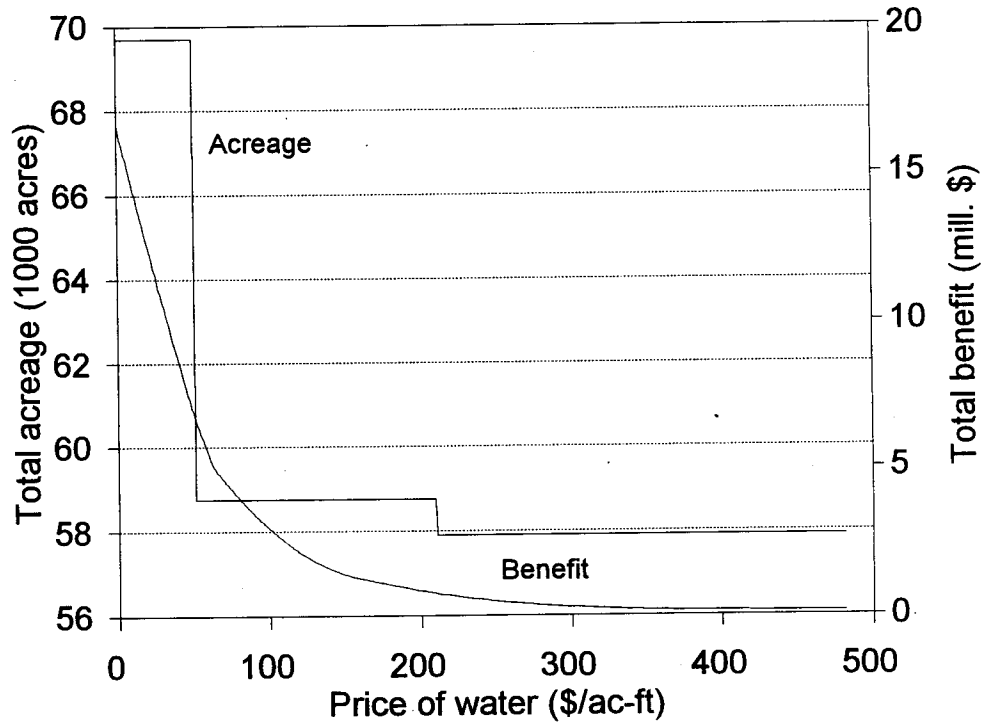
Figure 3.4: Total acreage and benefit against
price of water with min. acreage
bounds.
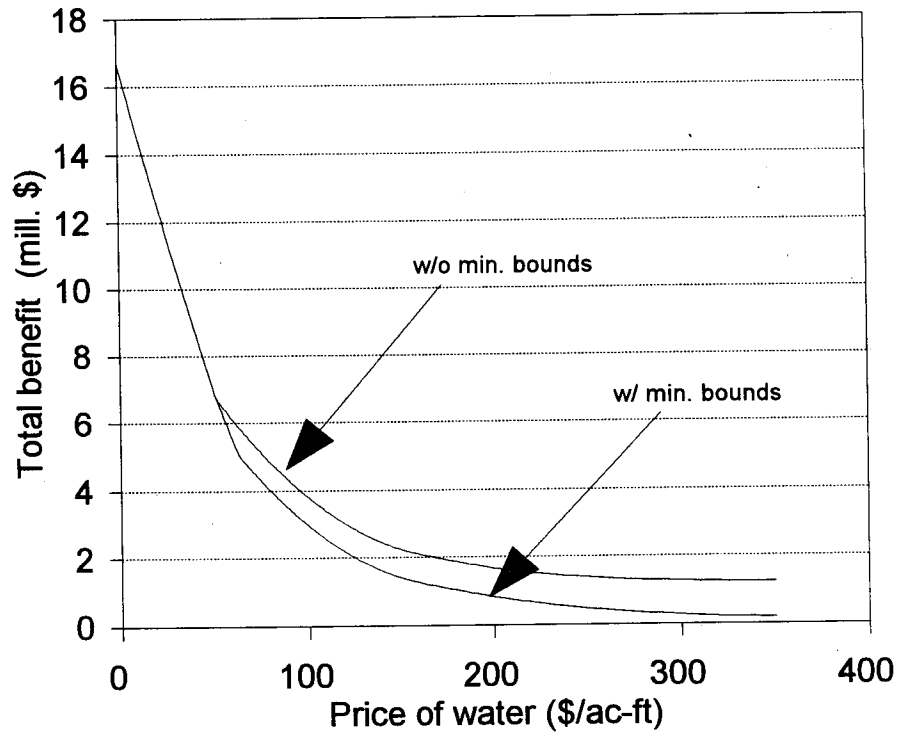
Figure 3.5: Comparative plots of total benefit vs. water price.

(without any index for crop or technology), the objective function Z could be expressed as:

$$Z = (rY(\epsilon W) - p\bar{W}) x - cv.x \qquad (3.18)$$

Let C=cv=total per acre variable cost (excluding water cost), and NR=r.Y(ϵW)-pW= per acre revenue net of water cost.

Then the gradient of Z with respect to x is:

$$\frac{\partial Z}{\partial x} = NR - C \qquad (3.19)$$

From (3.18), Z is clearly linear in x (acreage). This along with (3.19), implies that as long as (NR-C) is positive, it pays to increase x all the way to the maximum allowed acreage, xmax. And this decision rule is invariant to the degree of the polynomial Y(ϵW) (linear for alfalfa and quadratic for others). Table 3.5 confirms this. Until x^* and q^* (optimal acreage and total water applied for a crop) became zero, x^* or the optimal acreage remained constant and equal to xmax for all the crops.

The decision rule for W could also be investigated:

$$\frac{\partial Z}{\partial W} = x[(rb\epsilon - p) - 2r|c|\epsilon^2 W] \qquad (3.20)$$

Now, if the production function is linear, the term associated with 'c' in (3.20) will drop out and (3.20) will simplify to:

$$\frac{\partial Z}{\partial W} = x(rb\epsilon - p) \qquad (3.21)$$

This last condition says that rate of change of Z with respect to W does not depend on W. So, as long as the right

hand side of (3.19) is positive, Z will increase linearly with W until W reaches the maximum allowed limit. But when p increases sufficiently and $\partial Z/\partial W$ becomes negative, optimal policy would be to apply no irrigation at all. This is why input allocation for a crop with linear production function is all or nothing deal.

The point is made clear by examining optimal acreage and irrigation for alfalfa from Table 3.5. At zero price, irrigation applied for alfalfa is 105,640 ac-ft. Right before switching to zero irrigation, water applied for alfalfa is still 105,640 ac-ft for 31,055 acres of land. This is equivalent to 23.88 acre-inch/acre at 58.5% application efficiency - the maximum seasonal irrigation requirement for alfalfa. This is the condition enforced by (3.14) in the demand function estimation model.

Table 3.5
Summary of switch-over price, optimal acreage
and optimal irrigation

Crop	p $/ac-ft	x acres	w ac-inch	q=w*x 1000 ac-ft
Alfalfa	0.0	31055	40.82	105.64
	50.4	31055	40.82	105.64
	52.8	0	0.0	0.00
Corn	0.0	25973	37.85	81.92
	158.4	25973	0.023	0.04
	160.8	25973	0.0	0.00
Sugar beets	0.0	2899	22.478	5.43
	208.8	2899	12.74	3.08
	211.2	0	0.0	0.00
Barley	0.0	3915	18.00	5.87
	280.8	3915	0.13	0.04
	283.2	3915	0.0	0.00
Dry beans	0.0	5860	21.38	10.44
	348.0	5860	0.06	0.03
	350.4	5860	0.0	0.00

For a quadratic production function, the decision rule is slightly more involved. Assume that for the time being, (NR-C) is positive, hence x=xmax until it switches to zero. Then maximizing Z is really the same as maximizing NR with respect to the decision variable W. For this simple problem, the optimal W (for a given p) could be derived from the first order condition $\partial NR/\partial W=0$ (the second order condition is clearly negative ensuring the maximum).

The first order condition, after some rearrangements, gives:

$$W = \frac{b}{2|c|\epsilon} - \frac{1}{2|c|\epsilon^2 r}p \qquad (3.22)$$

Equation (3.22) is really the demand function for irrigation water per acre provided (NR-C) is positive. It shows that for a particular crop, the maximum water demanded at zero price is given by W (max)=b/(2|c|ϵ) which is, not surprisingly, the same as the yield maximizing irrigation per acre for a quadratic production function. It also shows that as the price of water increases, optimal irrigation will drop linearly until it becomes zero at p=rbϵ. However, the actual switch-over may occur at a lower price where NR-C (or for that matter Z), changes sign from positive to negative.

Again the point could be exemplified by using Table 3.5. Consider the case of corn which has a quadratic production function. At zero price, corn gets all the water it needs to maximize production (since this also maximizes the profit). As the price increases, irrigation will continue to drop

linearly. At p=158.4 $/ac-ft, the optimal per acre irrigation could be estimated from (3.22) to be 0.00187 ac-ft. This is equivalent to a total irrigation requirement of 48.57 acre-ft for 25,973 acres of corn, which matches exactly with the model estimate as shown in Table 3.5.

Equation (3.22) also explains the general shape of the aggregate demand curve which looks like a mosaic of multiple linear sections with steps. As price of water increases from zero to higher values, less profitable crops drop out from irrigation at switch-over prices causing horizontal shifts in the aggregate demand curve. Note that generalization to multiple irrigation technology and crop is straight forward in this case because the aggregate objective function is additively separable.

In short, the results from the nonlinear optimization are not just a set of numbers generated by a blackbox, but they make perfect economic sense. However, a word of caution is warranted here. The fact that simple marginal analysis has gone a long way in explaining the results does not trivialize the optimization process itself. The analysis above was presented to provide some economic insights without formal mathematical rigor. But such analysis becomes increasingly difficult as the number of decision variables and constraints increase. The degree of nonlinearity can make such simple interpretation almost impossible for higher order problems. Moreover, when numerous different scenarios have to be analyzed or the model has to be run recursively (this is how

the demand function was estimated), heuristic calculation quickly becomes an impractical and infeasible option. In such cases, a robust optimization tool is essential for serious inquiry of the problem.

CHAPTER 4

COST OF GROUNDWATER IRRIGATION

Cost of groundwater pumping and delivery to the field must be determined before the net benefit of irrigation could be estimated. In general, groundwater is costlier than surface water due to the fact that water has to be pumped from a considerable depth. Additionally, pumping plant and associated irrigation technology require a sizeable initial capital investment. Since numerous pumping configurations and many irrigation technologies could be used to deliver water to the field, some simplifying assumptions have to be made at this point to limit the number of choices to a representative few.

The total cost related to groundwater irrigation is made up of two components: fixed or investment costs and, variable or operating costs. Together they make up the cost function for a particular pump and irrigation technology combination. Therefore, unlike the benefit function which is unique for the entire region or a county, there will be several cost functions for each subarea based on the irrigation methods available. All cost figures mentioned below are in 1988 constant dollar unless other wise stated.

4.1 Fixed costs

4.1.1 Pump related fixed cost

This category includes well, casing, pump, head, drive, strainer, power unit, and all other components related to the pump and the well including cost of installation and testing. The cost will also depend on the capacity, location and depth of the pumping plant. Since it is practically impossible to incorporate all different kinds of plants observed in the field into the decision model (unless the study area is very small and homogeneous), some sort of representative well has to be selected as this point.

Sharp (1979) gives an average pump related fixed cost estimate of $5276 per well for northern Colorado with the representative capacity of 900 gpm. This corresponds to an approximate well density of 4 wells per square mile of irrigated land, or about 2 to 3 wells per square mile of land area. This well density is likely to satisfy the legal restriction that wells must be located at least one-half mile apart.

In this study, an annual amount of $6000 per well for a representative capacity of 900 gpm will be used as a reasonable average measure of pumping plant related investment cost. This figure is higher than Sharp's estimate to account for some additional costs. The average lift from the hypothetical basin is likely to be greater than the same for the shallower aquifers of northern Colorado. Also it is assumed that annual maintenance and repair cost is included in

this estimate which is about $300-$400 per well for electrically operated pumps (it is assumed that all pumps are electrically powered). Note that unless the well is permanently decommissioned, some annual maintenance cost will be incurred even if no water is pumped in the short term.

4.1.2 Irrigation related fixed cost

It has been assumed that four kinds of irrigation methods are available to the farmers in the study area:

1. Flooding

2. Ditch and siphon

3. Gated pipe

4. Sprinkler

The first one, flooding requires no capital investment. It is assumed that whatever tools may be necessary for breaching and remaking the dikes are generally available to the farmers. Capital investment for siphons is fairly low and assumed to be $2.88 per irrigated acre due to Booker (1992). In both cases, it is assumed that no major land leveling cost is involved.

Capital cost of gated pipe was estimated to be $12.88 per acre by Booker for part of the area overlying the South Platte alluvial aquifer. But this estimate varies considerably from the estimate of Sharp (1979) of $59.14 per irrigated acre based on data from north-eastern Colorado (see Table 4.1 for the estimates provided by sharp expresses in 1988 dollar). The large difference could possibly be attributed to the absence

of land leveling and reservoir and reuse system cost in Booker's estimate. On the other hand, Sharp assumed zero salvage value at the end of 15 years life time of the gated pipe system which might have inflated his estimate to some extent.

In this study, an annualized initial capital cost estimate of $35 per acre will be used for the gated pipe system as a reasonable appraisal for the hypothetical problem. It is also assumed that the estimate above includes reuse system and some minor land leveling, so the gated pipe system will have a higher application efficiency of 75% as opposed to 60% for systems without a reuse system.

Table 4.1
Annual added cost for gated pipe with reuse system
(1988 constant dollars)

Item	Initial Cost	Annual Cost
Land leveling	79,650	8745.14
Pipe (two miles gated pipe+3/4 miles connecting pipe)	64,251	7054.41
Reservoir and reuse system	17,700	1943.36
TOTAL	161,601	17742.91

Notes:
1. Above system irrigates 300 acres of land.
2. Annual costs are based on 15 years of life and 7% interest.
3. Land leveling charge is assumed to be $265.5/acre.
4. Gated and connecting pipe at $4.43/foot
5. Per acre cost: 17742.91/300=59.14 per year.

Investment costs for the center pivot system was given by Sharp to be $68.12 per irrigated acre per year. Table 4.2 shows the breakdown as given by Sharp. This estimate differs

from the same provided by Booker, which is $52.3 per acre. As before, the intermediate value of $60 per acre will be used for this study as a reasonable approximation for the sprinkler irrigation related capital cost.

Table 4.2
Annual added cost due to center pivot system
(1988 constant dollar)

Item	Cost	Life	Annual Cost
Mainline	13,275	15	1457.52
Sprinkler system (47,790 each x 2)	95,580	10	13608.49
Pressure pump (4867.5 each x 2)	9,735	10	1386.05
Transportation, installation and assembly	8,850	10	1260.05
TOTAL	127,440		17712.12

Notes:
1. Annual costs are based on useful life and 7% interest.
2. Mainline is 8 inch PVC pipe (80 psi), 3750 feet long for two pivot points, at $3.54/foot.
3. Sprinkler is electricity driven,1299 feet long, irrigates 260 acres, cost includes buried wire and hookup charge.
4. Per acre cost: 17712.12/260=68.12 per year.

So, the investment cost estimates used in this study are likely to fall within the reasonable range of values although no attempt was made to compare the estimates with the actual data collected from the Arapahoe basin area due to hypothetical nature of the study. As before it is assumed that annual operation and maintenance costs are small compared to large but tentative fixed cost components, so they were not considered separately. In fact except for the sprinkler system, all other irrigation methods will have negligible or

zero annual per acre maintenance cost. Table 4.3 below

summarizes the fixed costs.

Table 4.3
Summary of fixed costs (1988 dollar)

Item	Amount	Unit	Efficiency
Representative well+pump	6000.0	$/well/year	n/a
Flooding	0.00	$/acre/year	0.50
Siphon	2.88	$/acre/year	0.60
Gated pipe w/ reuse	35.00	$/acre/year	0.75
Sprinkler	60.00	$/acre/year	0.85

4.2 Variable costs

4.2.1 Energy cost

The cost of pumping one acre-inch of water using

electrical power is given by (Young et al., 1982):

$$P_c = \frac{1}{12} \frac{1.025 * TDH}{PP_{eff}} E_r \qquad (4.1)$$

where,
    P_c=power cost, dollar per acre-inch,
    TDH=total dynamic head, feet,
    PP_{eff}=pumping plant efficiency (fraction),
    E_r=electric rate, dollar per KWH.

Also, the total dynamic head is defined as (Stringham et al., 1979):

$$TDH = Lift + p.s.i_j*(2.31) \qquad (4.2)$$

where,
    Lift= static or initial level depth + drawdown + pipe
    friction loss + elevation difference from well head
    to the lateral in case of center pivot,
    $p.s.i_j$=operating pressure in pounds per square inch.

The constant 2.31 in (4.2) is a conversion factor to

translate pressure in p.s.i. into feet of head. It is assumed

that elevation difference from well head to center pivot lateral is 10 feet and for all other methods this term is zero. The operating pressure for center pivot and gated pipe systems are assumed to be 75 p.s.i. and 5 p.s.i. respectively. Also it is assumed that for all systems (whenever pump irrigation is used) there will be some frictional loss involved in lifting and transporting water through columns and pipes. On average, this loss is assumed to be 12 feet for all systems due to Young et al.

The pumping plant efficiency, PP_{eff} in (4.1) could further be defined as:

$$PP_{eff} = \frac{GPM * TDH}{Input\ HP * 3960} \qquad (4.3)$$

which is the ratio of electric energy input and water energy output in horsepower (Sharp, 1979). This could be viewed as the product of efficiency of the power unit and efficiency of the pump.

Theoretical analysis done by Miles and Longenbaugh (1968) indicates that a new electric pumping plant is likely to have an efficiency of 64-71% with an average of 66.4%. This estimate is valid for new and well designed plants only without any attached distribution system (such as gated pipe and sprinkler). Actual average efficiency prevailing in the field is somewhat lower due to reasons such as variation in irrigation systems, improper pump selection and installation, poor well and pump maintenance, and temporal increase in headloss due to compaction of the aquifer and clogging of

pipes. For this study it is assumed that flooding and siphon methods have PP_{eff} of 0.65. The same for gated pipe and sprinkler are assumed to be 0.55 and 0.57 respectively due to Young et al. (1982).

4.2.2 Irrigation labor costs

Very little data is available on pre-season, post-season and direct labor requirements for different irrigation methods. Such estimates will also vary based on soil type, climate, topography, and crop type. Table 4.4 shows field survey based estimates of irrigation labor requirements for the eastern high plains of Colorado as reported by Young et al. Table 4.5 provides estimates for the north-eastern Colorado as given by Sharp. As before, Sharp's estimates are considerably higher than the former. Booker on the other hand uses a generic estimate of $12.07 per acre-foot as the measure of labor cost for pump irrigation (for a fifty miles long reach of the South Platte alluvial aquifer) which is close to Sharp's estimates for the gated pipe.

In this study Sharp's estimates will be used due to their proximity to Booker's estimate. No separate estimate could be located for labor requirements for siphon and flooding methods. So it is assumed that they have the same labor requirements as the gated pipe system. This assumption is likely to be valid for ditch and siphon, but flooding may require more or less labor depending on the general topography, and the number and size of the plots being irrigated.

Table 4.4
Irrigation labor requirement
(Given by Young et al., 1982)

Irrigation method	Labor (hour/acre-inch)
Center pivot sprinkler	0.030
Gated pipe	0.075

Table 4.5
Irrigation labor requirement
(Given by Sharp, 1979)

Irrigation method	Labor (hour/acre-inch)
Center pivot sprinkler	0.0796
Gated pipe	0.1731

Finally it is assumed that labor is available at $5 per hour and electric power is purchased at a constant rate of 6.5 cents per KWH. It is recognized at this point that high capacity pumps may actually enjoy a declining block rate and therefore a constant rate assumption will overestimate the cost to a certain degree.

CHAPTER 5

HYDROLOGIC SUBMODEL: GENERATION OF DISCRETE KERNELS

This chapter will elaborate on the theory and method of generating unit response functions or discrete kernels for a hypothetical study area based on hydro-geological data from the Arapahoe aquifer of the Denver basin system. The Arapahoe aquifer is the third aquifer from the top in the Denver basin system and is the principal contributor of groundwater to the overlying area. So the hypothetical aquifer used in this study will have all the nuances of a real and complex groundwater basin.

In fact most of the parameters used in this study for the hypothetical basin came from a previously calibrated model as reported by Robson (1987). Two simplifications have been made however. One, the original model by Robson included all four aquifers of the Denver basin as an interconnected system of aquifers. In this study only the Arapahoe aquifer is being considered (although vertical leakage was compensated by equivalent recharge term). Second, a coarser grid has been used to limit the number of cells but no attempt has been made to recalibrate the model (this is one of the things that make the model hypothetical). Some error has been introduced due to aggregation of parameter values. Such aggregation is not

uncommon in management related studies of large aquifers (such as Young et al., 1982). When coupled with appropriate economic data, this can still provide significant insight about relative merits of various policy options.

5.1 Hydro-geologic description

The Arapahoe aquifer is part of the Denver basin system which underlies a 6,700 square miles area of Colorado neighboring the city of Denver. The aquifer is approximately 97 miles long (north-south) and about 72 miles wide (east-west) but not all the area within the rectangle is part of the aquifer. Figure 5.1.a shows the finite difference grid superimposed on the simplified version of the Arapahoe aquifer which has been used in this study. Figure 5.1.b shows two cross sectional views along sections A-A and B-B of Figure 5.1.a. Together they give some idea about the cup shaped aquifer which is mostly confined except for the outcrop or recharge areas along the boundaries.

Stratigraphic data for the Arapahoe basin indicate that the aquifer is about 400 to 700 feet thick, and consists of interbeded conglomerate, sandstone, siltstone, and shale. Altitude of the base of the aquifer varies widely - from more than 6000 feet near the southern end to about 4000 feet near the northern end. The top of the aquifer mostly runs parallel to the bottom and so, there is a predominant direction of flow (to the north and north-east) caused by the natural gradient.
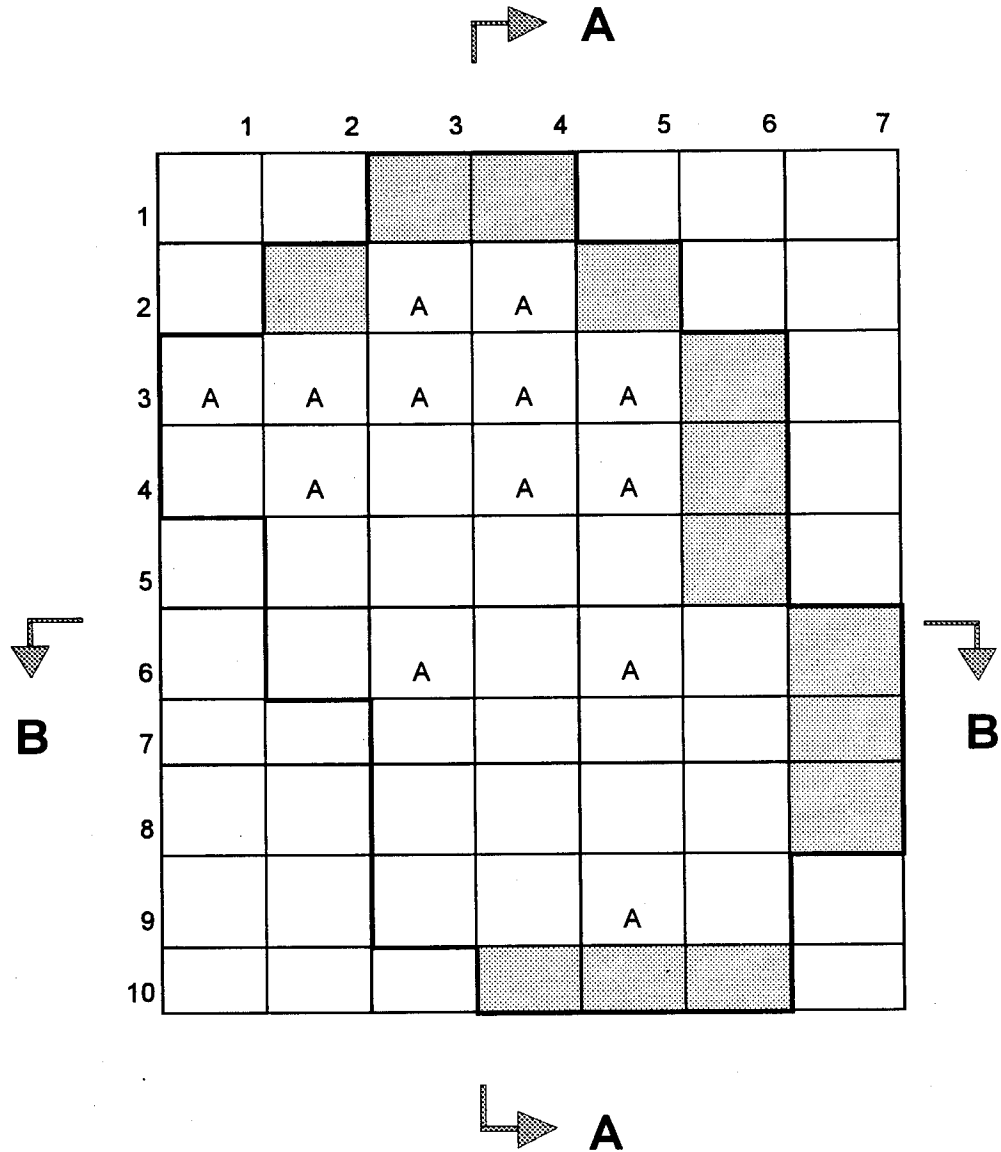
Fifure 5.1.a: Schematic representation of the hypothetical basin, shaded cells are outcrop areas, 'A' stands for active cell.
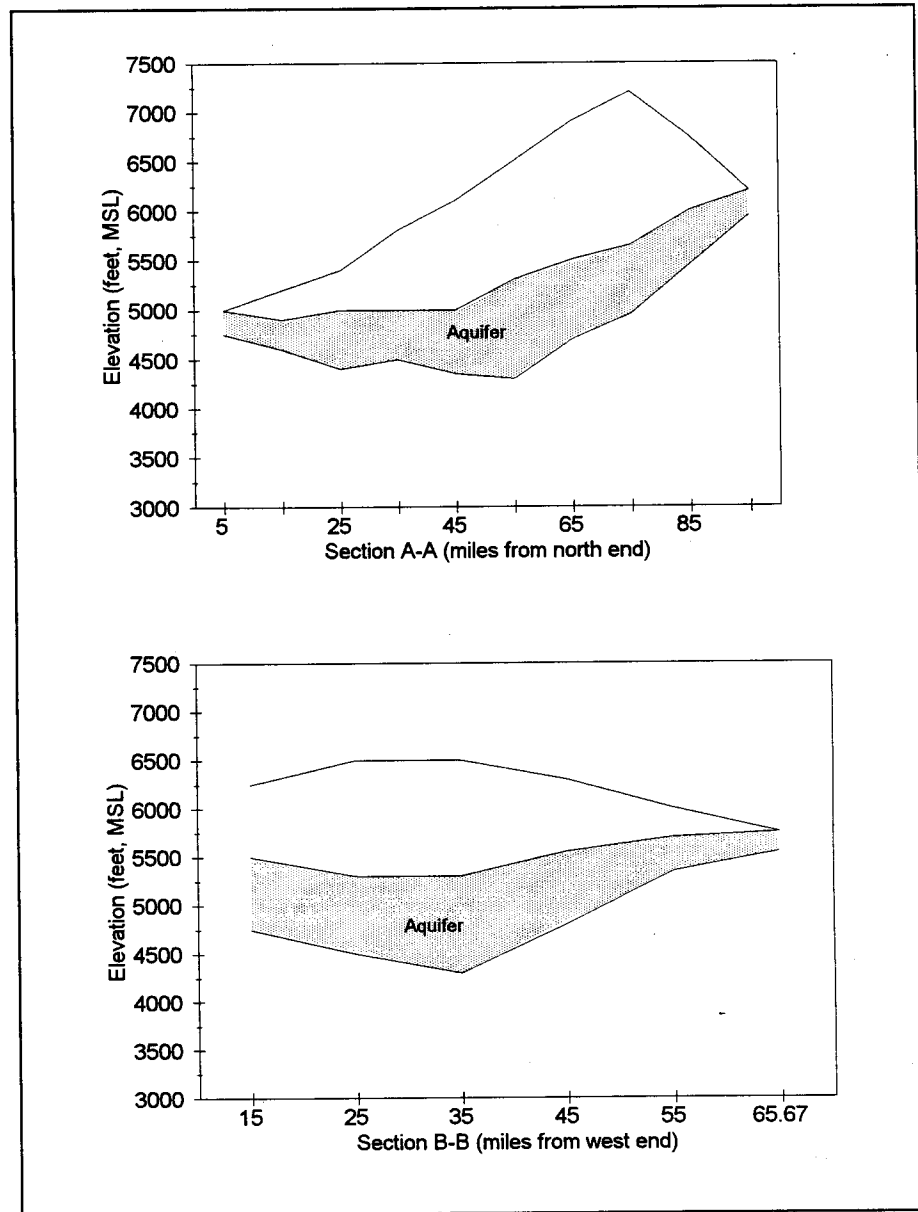
Figure 5.1.b: Cross-sectional view of sections A-A and B-B.

This particular feature along with the heterogeneity of the aquifer causes some interesting aquifer response when external excitation is applied (this will be further elaborated later in this chapter). Also, the Palmer Divide separates the northern and southern flow regimes of the basin (the divide outlines the highest points in the basin). Streams and groundwater south of the divide generally flow in the south and southeastern direction.

The Arapahoe aquifer has a mean porosity of 30 percent and specific yield of 18 percent (Robson, 1983). The confined storage coefficient as reported by Robson ranges from 2×10^{-4} to 8×10^{-4}. The hydraulic conductivity value also varies widely ranging from 7 ft/day at a location south of Littleton to 0.5 ft/day in the central part of the aquifer. Figures 5.2, 5.3 and 5.4 show the contours of storage coefficient, aquifer thickness and hydraulic conductivity values used to build the hypothetical model.

Recharge to the Arapahoe aquifer comes from two sources - precipitation in the outcrop area and vertical leakage from the overlying Denver and Dawson aquifers. Average precipitation in the Denver basin area is about 14 inches per year (with some areal variation) and less than 1 percent of this contributes to recharging the bedrock aquifers (Robson, 1987). For this study it has been assumed that 0.112 inches of recharge occurs in the outcrops area of the Arapahoe aquifer. Moreover, recharge has also been applied to the confined cells at a rate of 0.042 inches per year to account for the vertical
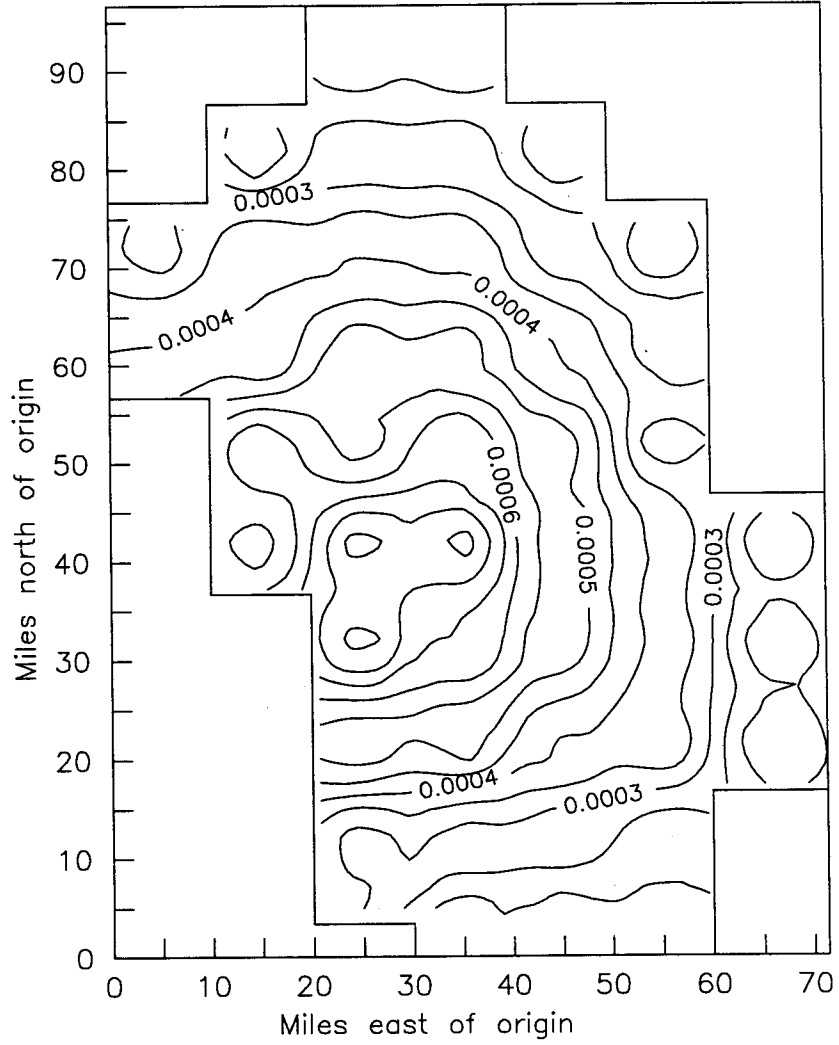
Figure 5.2: Storage coefficients.
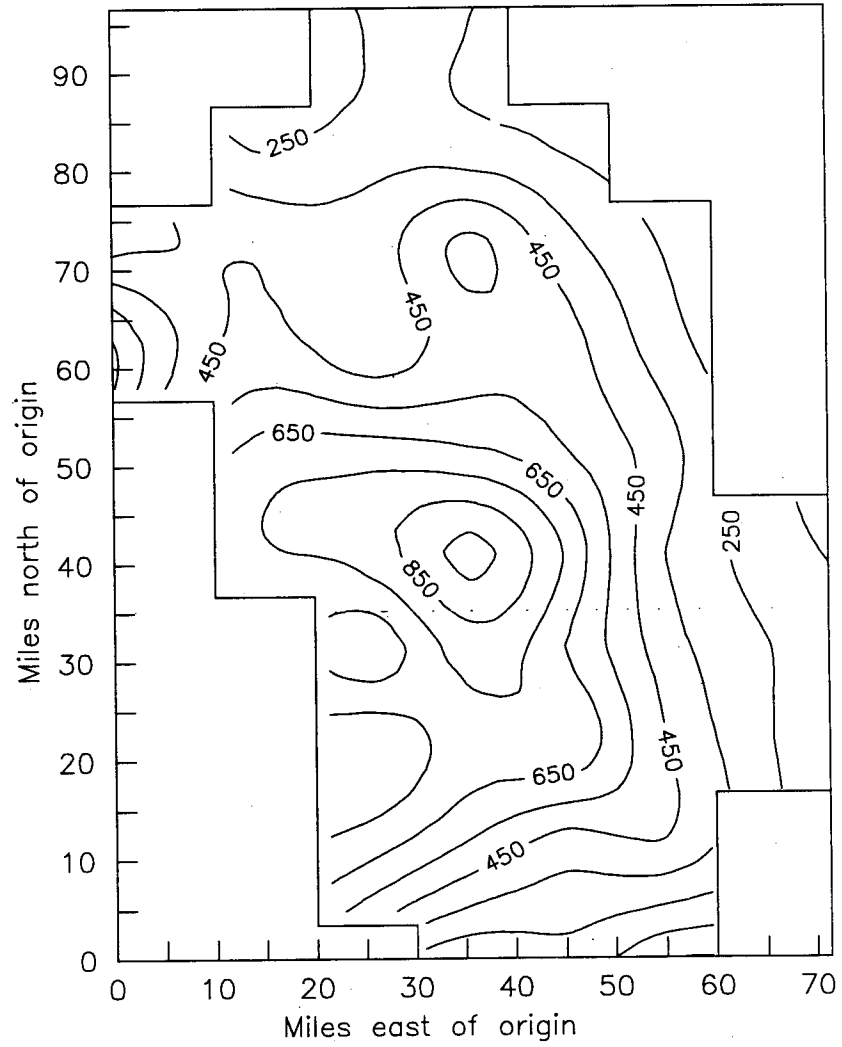
Figure 5.3: Aquifer thickness (feet).
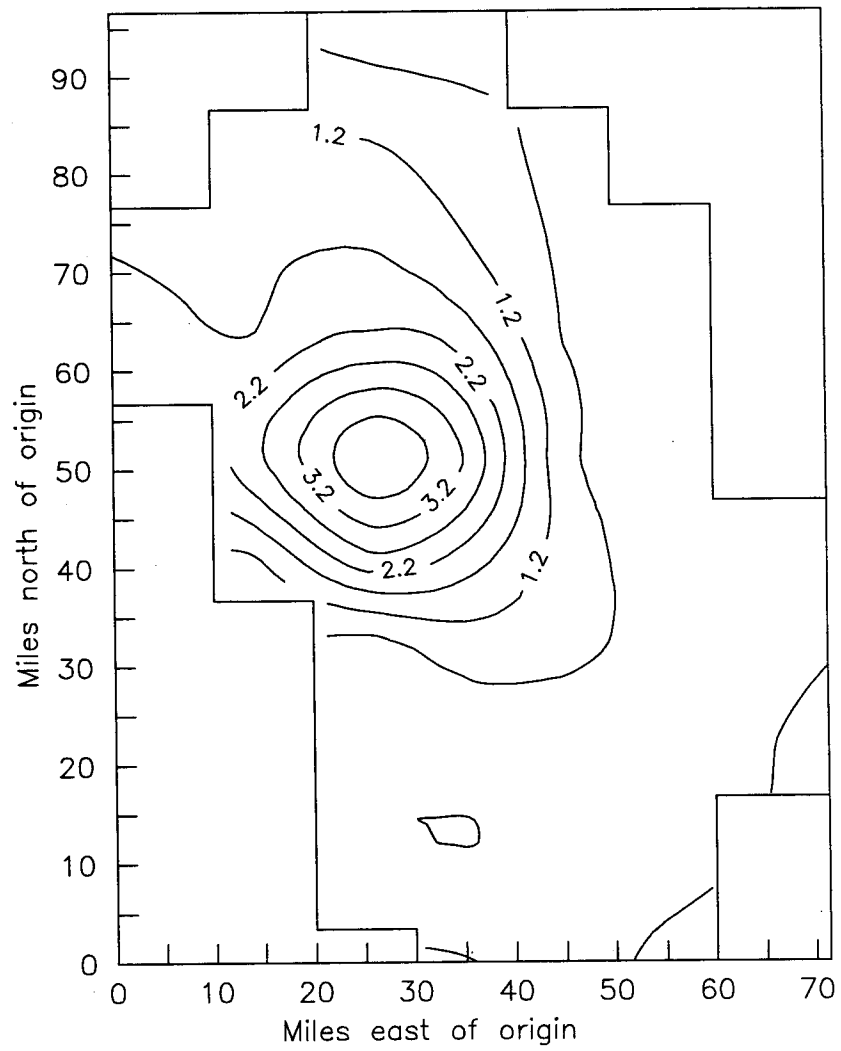
Figure 5.4: Hydraulic conductivity (ft/day).

leakage. These estimates are based on Robson's average annual estimates for recharge from precipitation and leakage for the Arapahoe aquifer.

Natural discharge from the aquifer takes place through the alluvial aquifers and stream valleys. These streams are connected to the aquifer in the outcrop areas primarily along the northern and eastern boundaries. They primarily act as drains and collect the discharge which occurs due to existing natural gradient. Of course, any man made domestic, municipal or irrigation well will also act as a source of discharge form the aquifer.

Due to hypothetical nature of the model, initial heads have been simply estimated by running the model for the steady state. In the original model study by Robson, heads prevailing in 1958 were taken as representative of the pristine state. Later, further adjustments were made based on unsteady state simulation using the historical pumping pattern. In this study, it will be assumed that currently the basin is at the steady state. So the conclusions to be drawn later on merits of different policy options will be contingent on this initial steady state assumption.

5.2 The numerical model and generation of discrete kernels

5.2.1 The numerical model and MODFLOW

The flow of groundwater through three dimensional porous media could be described by the governing partial differential equation as follows (McDonald and Harbaugh, 1987):

$$\frac{\partial}{\partial x}(K_{xx}\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_{yy}\frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(K_{zz}\frac{\partial h}{\partial z}) - W = S_s\frac{\partial h}{\partial t} \quad (5.1)$$

where,

K_{xx}, K_{yy}, K_{zz} are hydraulic conductivities along x, y, and z direction which are assumed to be collinear to the principal directions of flow (LT^{-1}),

h   is the potentiometric head (L),

W   is the volumetric flux per unit volume representing sources and/or sinks of water (T^{-1}),

S_s is the specific storage of the porous media (L^{-1}), and

t   is time (T).

The equation above describes the unsteady groundwater flow in heterogeneous and anisotropic medium. For steady state, the right hand side of (5.1) will be zero. Also, in general, S_s and K's are function of space (x,y,z) and W could be function of both space and time (x,y,z,t).

In the conceptual groundwater model, the aquifer is subdivided into a number of cells (in finite difference based numerical scheme). Equation 5.1 is applied to each of these cells and appropriate initial and boundary conditions are specified. Together they make up the mathematical model of the aquifer. Except for a few simple cases, the resulting system of nonlinear partial differential equations can not be solved

analytically for real world problems. So, numerical techniques are used to linearize the problem (say, by replacing differential equations by difference equations) and eventually a set of approximate linear system of equations is solved via some iterative scheme (such as strongly implicit method). The solution vector consists of a set of potentiometric heads at discrete points and times as $h(x,y,z,t)$. These head values, when substituted back into (5.1), will satisfy the equation and any associated initial and boundary conditions.

Since the process of describing and solving a numerical groundwater model is problem independent, many commercial softwares are available for this purpose. In this study, the widely used computer package MODFLOW has been used to solve the groundwater model and generate the discrete kernels. MODFLOW, which has been developed by the U.S. Geological Survey, is capable of solving three dimensional groundwater flow problems using a finite difference based grid and a number of iterative solvers. See McDonald and Harbaugh (1987) for detailed description of the package.

5.2.2 Generation of the initial steady state heads

The groundwater simulation package MODFLOW requires a number of separate input modules. A complete description of these modules is beyond the scope of this report (see the reference cited above). For the hypothetical model of this study, following modules were prepared for both the steady and unsteady state simulations.

AR.BAS: basic module, contains information on the grid, initial and boundary conditions, type of simulation etc.;

AR.BCF: information on hydro-geologic properties;

AR.RIV: description of streams (constant head);

AR.RAH: description of recharge pattern;

AR.WEL: well location and capacity;

AR.SIP: solver parameters (strongly implicit);

AR.OCT: output control parameters.

The AR.WEL module is not necessary for the steady state simulation (but needed for generating the discrete kernels). Information on the 'state' of the simulation is contained in AR.BAS, which was modified accordingly. AR.BCF also requires slightly different specifications for steady and transient states.

The steady state heads generated by MODFLOW are shown in Figure 5.5 below. These heads are used as the initial heads for generating the discrete kernels through a set of forty years long transient simulations.

5.2.3 Definition of the discrete kernel and the principle of superposition

The discrete kernel, which is a function of location and time, is defined as the *response* of a groundwater aquifer exclusively due to unit excitation or *stress*. The response is usually measured as the change of potentiometric head from a known pre-existing steady state condition. It is theoretically possible to determine the discrete kernel with respect to a transient state if the time-path of the state without the
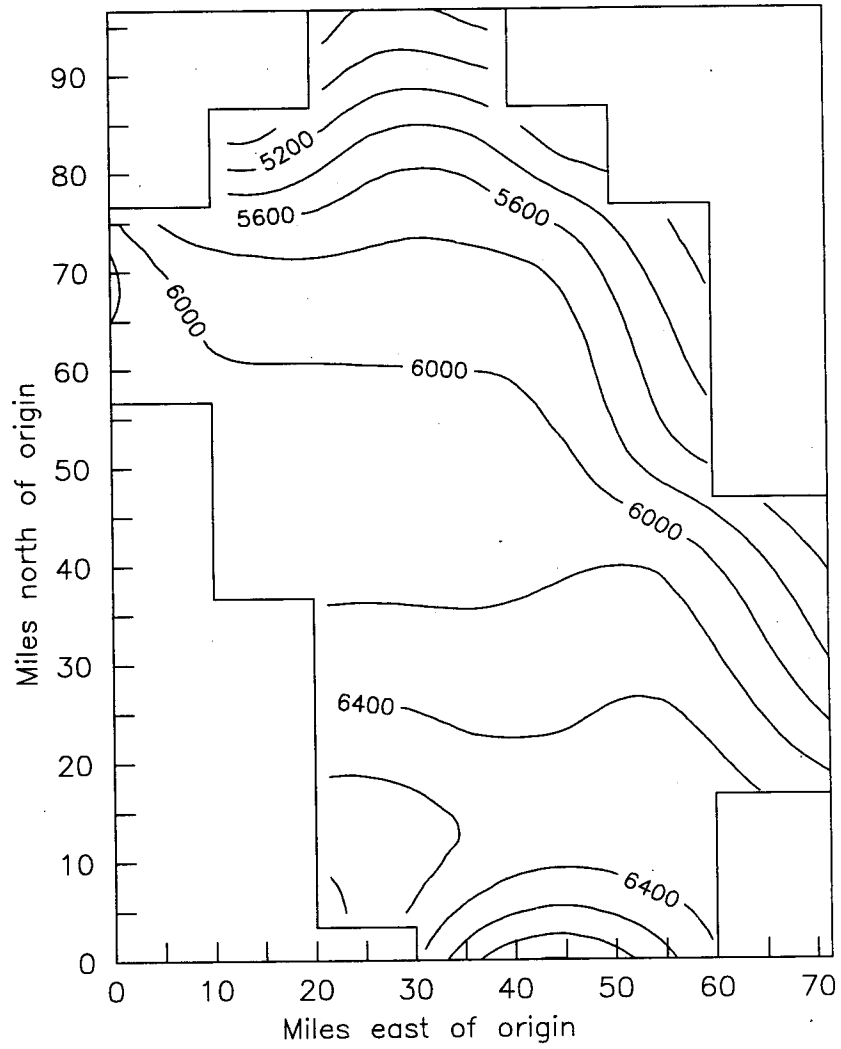
Figure 5.5: Steady state head (feet).

excitation is known. Additionally in both cases, the aquifer must be governed by linear system of equations. A stress is defined as removal (injection) of unit volume of water from (into) the aquifer at a certain location and time.

For the discrete kernels to be usable, the aquifer must be governed by linear flow equations. This allows application of the principle of superposition which can be defined as follows (after Reilly et al., 1988):

The principle of superposition means that for linear systems, the solution to a problem involving multiple inputs (or stresses) is equal to the sum of the solutions to a set of simpler individual problems that form the composite problem.

In the context of groundwater hydraulics, it means that if the flow equation (5.1) is linear, then discrete kernels could be used to estimate the space and time dependent aquifer response without running a groundwater simulation model (assuming of course that the kernels are already available). The idea could be mathematically described as follows.

Let $s_{k,t}$ be the cumulative change of potentiometric head or drawdown at location k at time t from some initial steady state condition. Let $Q_{l,n}$ be the stress at location l, at time n (n<=t) and L be the maximum number of stress locations. Also let $\beta_{l,k,t-n+1}$ be the discrete kernel representing the effect of a stress of unit magnitude. Subscripts l and k imply the location of stress and the location of change of head respectively. The third subscript (t-n+1) is the lag between the time of stress inducement (n) and the time when its effect is being measured (t). A lag of 1 (n=t) then means that $\beta_{l,k,1}$ is the immediate or current period effect of unit stress at

location l on another location k (note that k may or may not be the same as l). Then using the principle of superposition, drawdown $s_{k,t}$ can be estimated as a linear function of $Q_{l,n}$, $\forall$ n: n=1,2,..,t and $\forall$ l: l=1,2,..,L as:

$$s_{k,t} = \sum_{l=1}^{L} \sum_{n=1}^{t} Q_{l,n} \beta_{l,k,t-n+1} \qquad (5.2)$$

It is this equation which will allow the integrated economic-hydrologic model to accurately represent the aquifer response without embedding a groundwater simulation model. It should be mentioned here that discrete kernels are strictly valid only for an aquifer which is confined. For an unconfined aquifer, discrete kernels could still be used as long as the drawdown remains a small fraction of the saturated thickness at that location (as a rule of thumb, 15 percent or less). The hypothetical basin of this study is a large aquifer, which is primarily confined except at the outcrop areas along the boundaries. Since the points where stress will be applied and the points where kernel will be generated are all within the confined part of the aquifer, it has been assumed that the principle of superposition is applicable within the context of this study.

5.2.4 Generation of the discrete kernels

The first step in generating the kernels is to decide which cells (in a finite difference grid) will be stressed and at which locations effects of these stresses will be measured.

The cell under stress will be called active cell in this study.

Based on economic and agricultural data of the counties that overly the aquifer, thirteen cells have been isolated as the potentially active cells. They are shown in Figure 5.1.a as cells with an 'A' at the center. These are the locations where pumping is most likely to occur. Note that due to limited irrigation practices in the counties of Douglas, Elbert and El Paso, it was simply assumed that all the irrigation activities in these counties take place in three specific cells. Economic and hydrologic consequences of this assumption is likely to be small. Of course, in a more realistic case study, additional cells could always be added. Also cell 4 and cell 8 are potential locations for municipal pumping for the Denver Metro area from where no water is being pumped for agricultural purposes. It has been assumed that these thirteen cells are the only points of interest for the hypothetical case study. So discrete kernels will be generated for these cells only.

Discrete kernels are generated by repeated application of the groundwater simulation model. One simulation is required for each of the potentially active cells. The model is initialized with the steady state heads, and a unit excitation is applied at the cell for which kernels are to be generated. The model is then allowed to run under transient mode throughout the planning horizon. Finally the output is processed which contains information on the immediate drawdown

and subsequent recovery of the aquifer as caused by the unit stress. These space and time varying changes of the potentiometric head (with respect to the steady state condition) constitute the discrete kernels for that particular unit stress.

Due to rather large size of the cells, a volumetric withdrawal of 1000 acre-feet has been assumed to be one unit of stress. This is also an appropriate unit based on average irrigation water demands for the active cells. It has been assumed that irrigation water is pumped continuously for 120 days during the irrigation season (alternative irrigation patterns showed little influence on the discrete kernels). The very first kernel is estimated at the end of the irrigation season (120 days after the pumping started). All subsequent kernels have been generated to represent the residual drawdowns at the end of future irrigation seasons for the entire planning period (40 years in this case). So within the basic data module of MODFLOW, the first time interval was 120 days long. All other intervals were 360 days long (approximating a year by 12 months, each 30 days long). As long as all scenarios to be investigated use the same definition of a year, this should not introduce any bias.

Figure 5.6 shows one typical response function or a set of discrete kernels for cell (6,3). This figure shows the own or local effect of the unit stress of 1000 acre-feet at cell (6,3). It is clear that the maximum drawdown occurs at the end of the irrigation season. The aquifer then starts recovering.
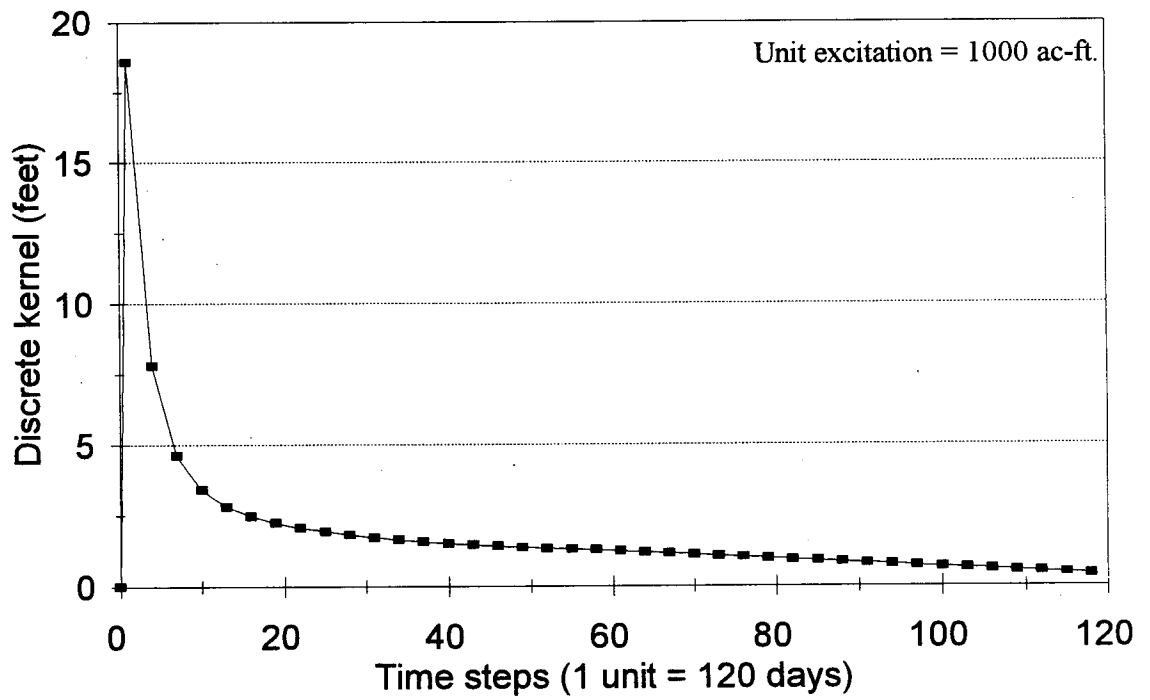
Figure 5.6: A typical profile of discrete kernels for cell (6,3).

Most of the recovery seems to have occurred in first twenty seasons, or in about seven years. There after the recovery is slow but gradual. Simple extrapolation suggests that a full recovery probably occurs approximately after fifty years. Similar response functions have been generated for the effect of unit stress at cell (6,3) on other active locations. And the process has been repeated for all the potentially active cells. Together, these kernels make up what is called the response matrix of the aquifer which is used in the integrated economic-hydrologic model later in this study.

5.3 Lagged aquifer response: an interesting observation

It has been observed that some of the response matrix coefficients have unexpected magnitudes suggesting the possibility of a lag in aquifer response. It is a generally held notion that an aquifer should exhibit the maximum response at the point of excitation, both during drawdown and recovery. Also, it appears to be correct to assume that once the stress is withdrawn, all cells should start recovering, or at least no cell should demonstrate further increase in drawdown. However, numerical results indicate that both these notions could be wrong for a complex heterogenous aquifer.

First, it was necessary to confirm that these unexpectedly valued kernels were not the outcome of accumulation of roundoff errors. Since an analytical solution of a set of partial differential equations with complex

initial and boundary conditions is generally not a valid option, a number of simple numerical test problems were constructed. One such test problem is schematically shown in Figure 5.7. This is a fairly simple one dimensional aquifer, but with highly heterogenous vertical layers. Another key factor is the down sloping (from left to right) part of the aquifer (a complete specification of this test problem is given in Appendix B). This particular combination of heterogeneity and natural gradient was able to reproduce the presumably 'lagged' response of the aquifer quite clearly, beyond the range of roundoff errors.

For example, when a single stress is applied at the well location (shown as Q in Figure 5.7), initially the potentiometric surface drops all over the aquifer. Then it starts to recover. Figure 5.8 shows the potentiometric surface profile at the end of days 1, 2, 3 and 4 (note that the aquifer has been divided into 55 equal cells, each 25 feet long, from west to east). Clearly some interesting things happen during the recovery process. First, drawdown in cells 1 through 12 continues to increase till the end of day 2, even though pumping had stopped at the end of day 1. So, while the rest of the aquifer is recovering, these cells are still responding to the stress caused in the previous period. This is a demonstration of temporal lag in aquifer response. Second, note that at the end of day 2, 3 and 4 (and in fact for all subsequent periods), the point of excitation (cell 38) does not have the maximum residual drawdown. All cells to the

Bottle-neck region

Initial steady state head: 250 ft

210 ft

constant head

160 ft

Q

100 ft

1375 ft
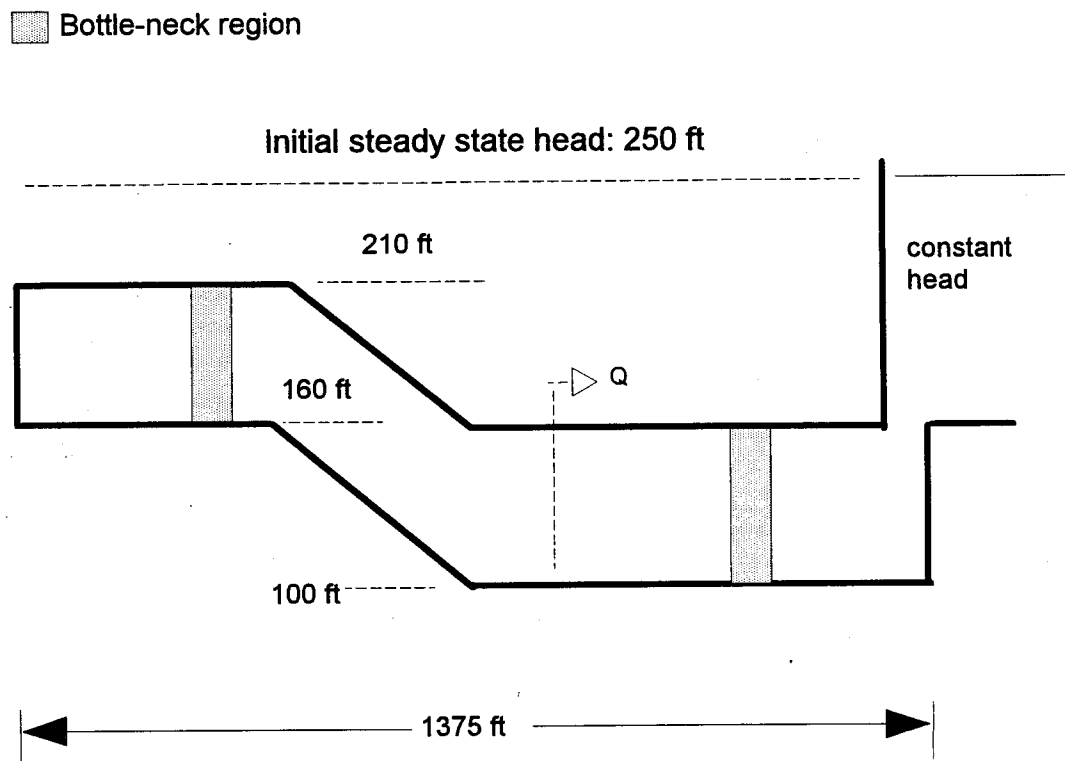
Figure 5.7: Schematic diagram of the test problem.
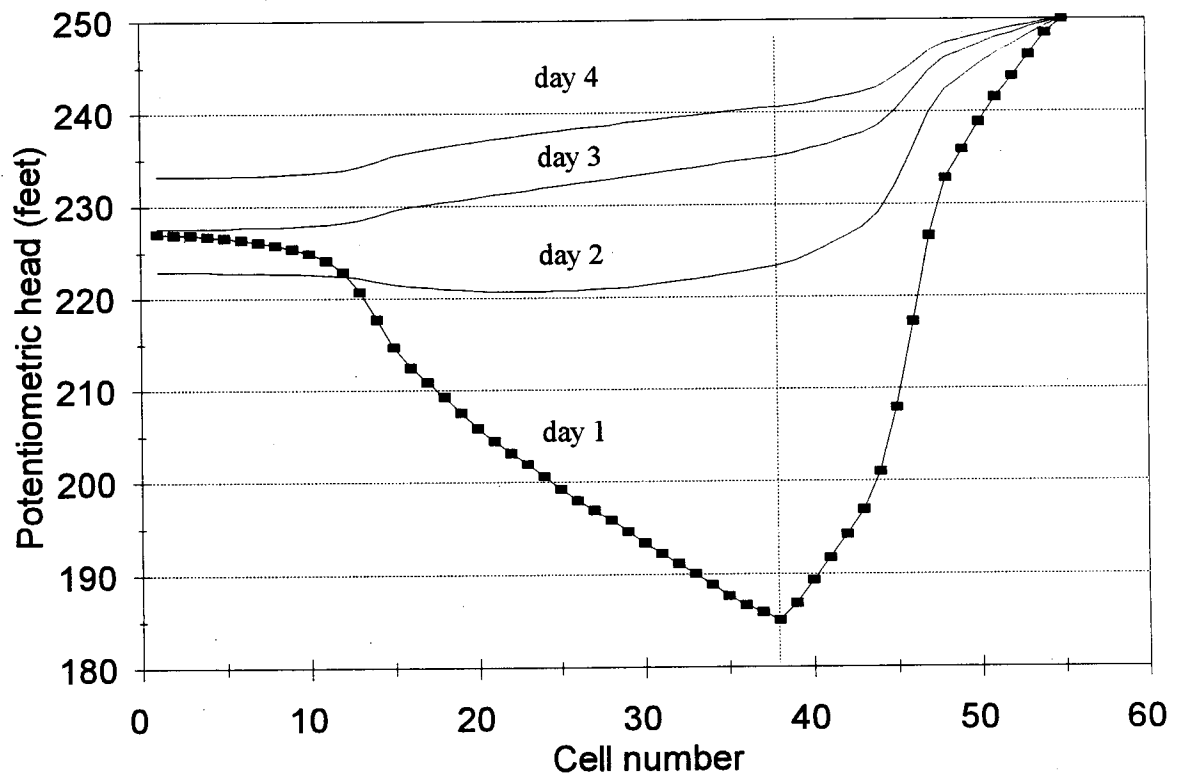
Figure 5.8: Lagged aquifer response in space and time.

east of the well (cells 1 through 37) show greater residual drawdown than cell 38. In other words, these cells are slow to recover. This is particularly true for cells 1 through 12. They indicate that cells which are slow to respond are also likely to be slow to recover. For further confirmation, these results were also reproduced for a two dimensional problem similar to the basin under study.

Due to the numerical nature of these simulations, it was not possible to analytically link these behaviors to any specific cause. But the test problems were constructed based on certain propositions which have successfully reproduced the results. Hence they may provide some intuitive explanation as to why such lag in response might be observed in the real world.

Basically, when an aquifer is subjected to a momentary stress, it acts as a shock to the system (causing sudden local change in the potentiometric surface from its equilibrium state). The shock wave then tries to travel through the system in the form of readjustment of the potentiometric surface. Since the wave travels though a medium (the aquifer), its propagation velocity is likely to be dependent on the properties of the medium. So, heterogeneity in the storage coefficient (S) and the hydraulic conductivity (K) plays an important role. Also, since water flows towards the direction of decreasing head, any presence of downhill slope along that direction will accelerate the flow, and an uphill slope will retard it. Therefore a carefully selected heterogeneity and

slope combination may produce exceedingly complex flow dynamics.

In the test problem, cell 38 (well location) has the largest S and K values (90 and 90 ft/day respectively). The neighboring cells on both sides also have comparable S and K values. But they decrease quickly on both sides eventually to a pair of 'bottle-neck' regions where S and K values drop to 10 and 10 ft/day respectively. Once the bottle-necks are passed, S and K begin to increase again and assume considerably higher values.

So, when a momentary stress is applied at cell 38 on day 1, initially most of the water come from the immediate vicinity of the well. Lowering of the potentiometric surface creates a pulling action to occur on both ends. The pull on cells 1 through 12 is further increased due to the downhill slope right after cell 16. However the cells near the ends can not respond freely due the bottle-neck region. This causes the temporal lag. By the time contributions from the west-end cells reach the stress location, it is already in the process of recovery. So recuperation of the potentiometric head in the well location is accompanied by a drop of the same in cells 1 through 12.

This temporal lag however persists only briefly. It vanishes by the end of the third day letting the recovery process to take over the aquifer completely. Since the location of stress is close to the constant head boundary, it shows quick recovery. But for the cells to the west of it,

particularly for cells 1 through 12 (beyond the bottle-neck), the dynamics is reversed. Water is now needed to be pushed back into these cells against the uphill slope and the resistance of the bottle-neck cells. So, residual drawdown in these cells tend to be greater than the cells near the constant head boundary. More importantly, after day 2 when the temporal lag dissipates, residual drawdown in these cells remain greater than the same at the location of the stress. This is how the spacial lag is established.

To sum up, lag in aquifer response seems to be a viable phenomenon. It is a matter of interest which should be further investigated for a variety of aquifer types and boundary conditions.

CHAPTER 6

NUMERICAL SOLUTION ALGORITHM FOR THE INTEGRATED MODEL

The integrated groundwater management problem to be
formulated in the next chapter could be solved in a number of
ways. In general, problems related to intertemporal resource
allocation could be described as optimal control problems.
However, from a numerical point of view, such problems could
be described as two-point boundary value problems of either
continuous or discrete nature. Since most resource management
decisions are made at discrete times, the integrated model in
this study is described as a discrete time resource allocation
problem.

A discrete time optimal control problem could be solved
in a number of ways. One method is to use the discrete maximum
principle and a penalty function based iterative solution
scheme as suggested by Sage and White (1977). It is also
possible to cast the problem into a nonlinear programming
problem, where both the control and state variables are
treated as variables of the nonlinear optimization problem.
State equations and terminal conditions are then introduced as
equality and/or inequality constraints. State and decision
space constraints could usually be accommodated by simple

upper and lower bounds, although for complex problems additional constraints may have to be specified.

It may also be possible to express the objective function solely in terms of control variables by internalizing the state equations. This is done by expressing the state variables at time (t+1) in terms of the control variables at t=1,2,...t through repeated application of the state equations. This way state equations get embedded into the objective function and separate specification of state variables become unnecessary. Similar substitutions could be used to express complex state space constraints in terms of control variables only.

This latter approach will be used in this study to minimize the number of variables and run-time memory requirement. As will be shown in the next chapter, this formulation also renders first order conditions with interesting economic interpretations. Following is a brief description of the numerical solution algorithm used in this study which could be used to solve a general nonlinear optimization problem with linear/nonlinear equality and inequality constraints.

6.1 Nonlinear programming by multiplier penalty function method

The general nonlinear optimization problem could be mathematically presented as follows:

$$\text{Minimize } f(\mathbf{x}) \qquad\qquad (6.1)$$

subject to:

$$c_i(\mathbf{x}) = 0, \quad c_j(\mathbf{x}) \geq 0,$$

$$\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$$

In the above formulation, $\mathbf{x}$ is the decision vector of size n, $\mathbf{u}$ and $\mathbf{l}$ are vectors of upper and lower bounds on $\mathbf{x}$, and c_i and c_j are vectors of equality and inequality constraints respectively of size m_1 and m_2. Since pre-multiplication of a maximization function by (-1) converts it into a minimization problem, the following discussion will only deal with the minimizaton problem.

The multiplier penalty method is an enhanced version of the penalty function method where the basic idea is to convert the problem into an unconstrained optimization by modifying the objective function (OF). This is done by adding penalty terms to the OF. Penalties are formed from a sum of squares of constraint violations multiplied by the penalty vector so that when a constraint is violated, the OF is penalized. By sequentially increasing the penalty values, it is theoretically possible to force an exact line search algorithm to converge to an optimal solution which satisfies all the constraints[1]. However, this original penalty function approach suffers from a major drawback - as the penalty term becomes bigger, the Hessian matrix associated with the problem becomes

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[1]It is assumed here that the objective function is smooth and convex and the constraints are smooth and concave and constraint qualification is met for all the constraints.

increasingly ill-conditioned and the minimization algorithm fails to converge.

Solutions to this problem have been suggested in a number of papers: Powell (1969) and Hestens (1969) independently proposed a new method for incorporating the equality constraints. Later, Rockafellar (1974) and Fletcher (1975) extended the method for inequality constraints. Together this new approach is called the *multiplier penalty method*. The multiplier penalty function, which is the unconstrained equivalent of (6.1), can be expressed as follows (see references above for details):

$$\phi(x, \lambda, \sigma) = f(x) + \sum_i (-\lambda_i c_i + \frac{1}{2}\sigma_i c_i^2)$$
$$+ \frac{1}{2}\sum_j \sigma_j [\min(c_j - \frac{\lambda_j}{\sigma_j}, 0)^2 - \frac{\lambda_j}{\sigma_j}] \tag{6.2}$$

where,

λ_i, λ_j = multipliers for equality and inequality constraints;

σ_i, σ_j = penalty coefficients for equality and inequality constraints;

and $\phi(x, \lambda, \sigma)$ = the multiplier penalty function.

The upper and lower bounds could be included in (6.2) as either inequality or equality constraints, or could be handled directly within the minimization algorithm. If an equality constraint is used to accommodate upper and lower bounds, then the following formulation could be used for variable x_i:

$$c_i = \min(x_i - l_i, 0) + \min(u_i - x_i, 0) \tag{6.3}$$

where l_i and u_i are lower and upper bounds of x_i respectively.

The basic idea of this approach, as described by Fletcher, is to shift 'the origin of the penalty term.' Fletcher also mentions in his paper that this new penalty function 'is well conditioned, without singularities, and it is not necessary for the control parameters (σ_i and σ_j) to tend to infinity in order to force convergence.' Also one interesting outcome of this method is that the optimal values of the multipliers, λ^*, are in fact the Lagrange multipliers at the optimal solution. This is why (6.2) is often called the augmented Lagrangian function.

Note that the optimal multipliers are not known *a priori* and therefore a sequential minimization algorithm is necessary. Powell (1969) gives the following major steps for problems with only equality constraints (expressed as vector $\mathbf{c}$) :

i. Set initial guess for λ and σ, set $\|\mathbf{c}^{(0)}\|_\infty = \infty$.

ii. Find a local minimizer, $\mathbf{x}(\lambda^k, \sigma^k)$ of $\phi(\mathbf{x}, \lambda, \sigma)$ and denote $\mathbf{c} = \mathbf{c}(\mathbf{x}(\lambda, \sigma))$.

iii. If $\|\mathbf{c}\|_\infty > \frac{1}{4}\|\mathbf{c}^k\|_\infty$ set $\sigma_i = 10\sigma_i$ $\forall i: |c_i| > \frac{1}{4}\|\mathbf{c}^k\|_\infty$ and go to step ii.

iv. set k=k+1, $\lambda^k = \lambda$, $\sigma^k = \sigma$, $\mathbf{c}^k = \mathbf{c}$.

v. Update multiplier vector according to a sequence so that $\{\lambda^k\} \rightarrow \lambda^*$.

In the iteration scheme above, λ and σ are the primary and secondary control parameters respectively. This is because of the fact that, if second order sufficient conditions are met at $(\mathbf{x}^*, \lambda^*)$, then there exists a $\sigma' \geq 0$ such that for any $\sigma > \sigma'$, $\mathbf{x}^*$ is an isolated local minimizer of $\phi(\mathbf{x}, \lambda^*, \sigma)$, that is $\mathbf{x}^* = \mathbf{x}(\lambda^*)$. Simply put, as the penalty coefficients exceed a

certain threshold, optimal solution is obtained by changing λ only, keeping σ fixed.

A very similar set of steps have been proposed for inequality constrained problems. In this study, the steps outlined by Wanakule et al. (1986) have been used without any modification. It should be mentioned here that the updating scheme (step v above) for multiplier vector is slightly different for equality and inequality constraints. Following are the two schemes proposed by Fletcher (1987):

$$\lambda_i^{(k+1)} = \lambda_i^{(k)} - \sigma_i c_i^{(k)} \qquad \forall i \qquad (6.4)$$

$$\lambda_j^{(k+1)} = \lambda_j^{(k)} - \min(\sigma_j c_j^{(k)}, \lambda_j^{(k)}), \qquad \forall j \qquad (6.5)$$

Subscript 'i' is used for equality constraints and 'j' is used for inequality constraints as in the original formulation (6.1).

6.2 The inner loop of unconstrained minimization

The task of step (ii) above is to find a local minimizer, $x(\lambda^k, \sigma^k)$ of $\phi(x, \lambda, \sigma)$ using a suitable unconstrained minimization technique. This of course requires further elaboration. There are many different algorithms available to do the job. Some rely only on function evaluation and others may require first and/or second derivative information. Obviously, methods which require and use derivative information provide more reliable solution and have faster

convergence rate. But it comes at the expense of increased memory requirement and the precondition that such derivatives exist and could be computed at a reasonable cost (in terms of cpu time).

In this study, the *conjugate gradient method* as proposed by Fletcher and Reeves (1964) and slightly modified by Polak (1971) will be used for multidimensional function minimization. The choice of conjugate gradient over variable metric methods is critically important due the size of the integrated economic-hydrologic management model. This method requires first derivatives but does not require or store the Hessian matrix. It is quite economic in terms of memory requirement and is practically the only gradient-based method available for large nonlinear optimization problems. Also, in the neighborhood of the minimum, the method has a quadratic rate of convergence. For pure quadratic functions, the minimum is guaranteed to be located (within the margin of roundoff errors) in at most n exact line searches. For more general functions, as the minimum is approached, the function is more closely approximated by a quadratic function, and so, at least a super linear convergence is achieved.

A detailed discussion of the conjugate gradient method is beyond the scope of this study and interested readers should consult references cited in this section. So, only the working algorithm is outlined below.

Let **h** be the minimum to be located numerically for the n-variable function f(**x**). Also let **g**(**x**) be the gradient vector that could be estimated at any point **x** within the feasible decision space. Most minimization procedures try to locate **h** as the limit of a sequence $\{x^k\} \rightarrow$ **h**. Here k=0 corresponds to the initial guess to the minimum. Also for each iteration index, k >= 0, x^{k+1} is the position of the minimum of f(**x**) along the line x^k in some specified direction p^k. Mathematically,

$$x^{k+1} = x^k + \alpha^k p^k \qquad (6.6)$$

where, α^k is some scalar parameter. It is this p^k which determines the directional search properties of the algorithm. In case of steepest descent, $p^k = (-g(x^k))$. In the conjugate gradient method, directions $\mathbf{p}^0, \mathbf{p}^1, \ldots$ are generated in such a way that $\mathbf{p}^{k+1}$ is a linear combination of $-\mathbf{g}(x^{k+1})$ and all the preceding p-vectors $\mathbf{p}^0, \mathbf{p}^1, \ldots, \mathbf{p}^k$. Additionally, it is ensured that **A**-conjugacy conditions as defined below are satisfied:

$$p_i^T A p_j = 0 \quad \forall i \neq j \qquad (6.7)$$

where, 'A' is a symmetric positive definite matrix of size n (here subscripts are used to denote iteration index). It has been shown by Beckman (1960) that a simple updating rule for $\mathbf{p}^{k+1}$ could indeed be derived which satisfies all the conditions mentioned above. Based on Beckman's study, Fletcher and Reeves (1964) proposed an updating scheme for the search direction. Later on, Polak and Ribiere (Polak (1971)) have made a minor

but important change to the Fletcher-Reeves scheme which could be presented as follows:

$$p^{k+1} = -g^{k+1} + \gamma^k p^k \qquad (6.8)$$

$$\gamma^k = \frac{(g^{k+1} - g^k)\, g^{k+1}}{g^k g^k} \qquad (6.9)$$

where, g^k and g^{k+1} are shortcuts for gradients at x^k and x^{k+1} respectively.

It is worth mentioning at this point that even though the conjugate gradient method does not explicitly use second derivatives, it performs far better than the steepest descent method. This is because of the fact that the information content embodied in current and preceding search directions are used during estimation of a new search direction. More importantly, the new search direction is constructed to be 'conjugate' to the old direction, and also as far as possible, to all directions traversed so far. Mathematically it means that if line minimization is conducted along a conjugate set of directions, then it is unnecessary to travel along a particular direction more than once. This is how the method economizes on the number of line searches needed to arrive at the local minima.

In reality, however, it may be necessary to periodically reset the new direction to the corresponding steepest descent direction. One reason is that the function under consideration may not be a quadratic one for which the theory has been developed. Also, an exact line search is not possible in

reality, due to limited precision of the computer and limited time available to the analyst. So it has been suggested that the direction should be reset to the steepest gradient after every 'n' iterations.

Powell (1975)(in Jacobs, 1977) argues that such resetting may not be required if the Polak-Ribiere version of the updating scheme is used. As evident from (6.8) and (6.9) above, that as the method becomes saturated (γ becomes too small), the updating rule becomes:

$$p^{k+1} \approx -g^{k+1}, \qquad as \quad \gamma^k \to 0 \qquad\qquad (6.10)$$

So, the Polak-Ribiere scheme resets the search direction automatically. Fletcher on the other hand, based on numerical experiments, maintains that 'for some large problems .....it may be appropriate to reset (the search direction) more frequently than on every n iterations.' In this study, (6.8) and (6.9) have been used as the default setting. But the computer code allows the user to reset the direction as frequently as desired.

To sum up, the following major steps have been used in this study to implement the conjugate gradient method.

i. Start with an initial guess, $\mathbf{x}^0$.

ii. Set $\mathbf{g}^0 = \mathbf{g}(\mathbf{x}^0)$, and $\mathbf{p}^0 = -\mathbf{g}^0$.

iii. Find the minimum $\mathbf{x}^{k+1}$ of $f(\mathbf{x})$ on the line through $\mathbf{x}^k$ in the direction $\mathbf{p}^k$.

iv. If converged, stop; else update search direction as $\mathbf{p}^{k+1} = -\mathbf{g}^{k+1} + \gamma^k \mathbf{p}^k$ and go to (iii).

6.3 The inner-most loop: multidimensional line search

The core of any minimization algorithm is a line search module which locates the minimum of the function $f(\mathbf{x})$ once the search direction is specified. Since line search is dominated by function evaluation, and sometimes by derivative evaluation, computational speed and efficiency of the entire optimization process critically depend on the search method.

Since no particular method is ideal for the variety of function types that might be encountered in the real world, three different search methods have been incorporated within the line search option of the computer code.

The first method is known as the *golden section search*. This minimum finding algorithm is analogous to the bisection method for root finding. The method is linearly convergent and is designed to tackle the worst possible situation. As vividly described by Press et al. (1990), the method hunts down and corners an uncooperative minimum 'like a scared rabbit.' This is only recommended for cross-checking purposes in the context of the current study.

The second method is called the *Brent's method* after its designer (see Brent, 1973 for details). This is based on quadratic interpolation, with a switch over mechanism to the golden section in case a near linearity is encountered. Since in the neighborhood of the minimum, the function is likely to be closely approximated by a quadratic form, Brent's method is

considerably faster than the golden section method for well behaved functions.

The third method is an enhanced version of the Brent's method which also uses derivative information. For well behaved functions with easy to calculate derivatives, this is likely to be the fastest method. However, for large and complex problems, derivative evaluation may not be economic and Brent's method may perform better. In this study, Brent's method has been used all along due to fairly involved derivative expressions of the integrated economic-hydrologic model. Figure 6.1 below summarizes all the major steps of the entire optimization process.

6.4 Coding and validation of the nonlinear solver

The nonlinear programming algorithm described above has been implemented by the author into a computer code using a personal computer based 32-bit C-language compiler by Watcom. The 32-bit programming allows full access to all the physical memory of the computer, and therefore breaks the barrier of 640K limitation of the DOS operating system. Also due to the flat memory model, the program can dynamically allocate huge arrays of size greater than 64K, another data segment limitation of the DOS (it is not possible to provide a complete description of the computer program in this chapter; the author intends to document the program in a separate report in future).
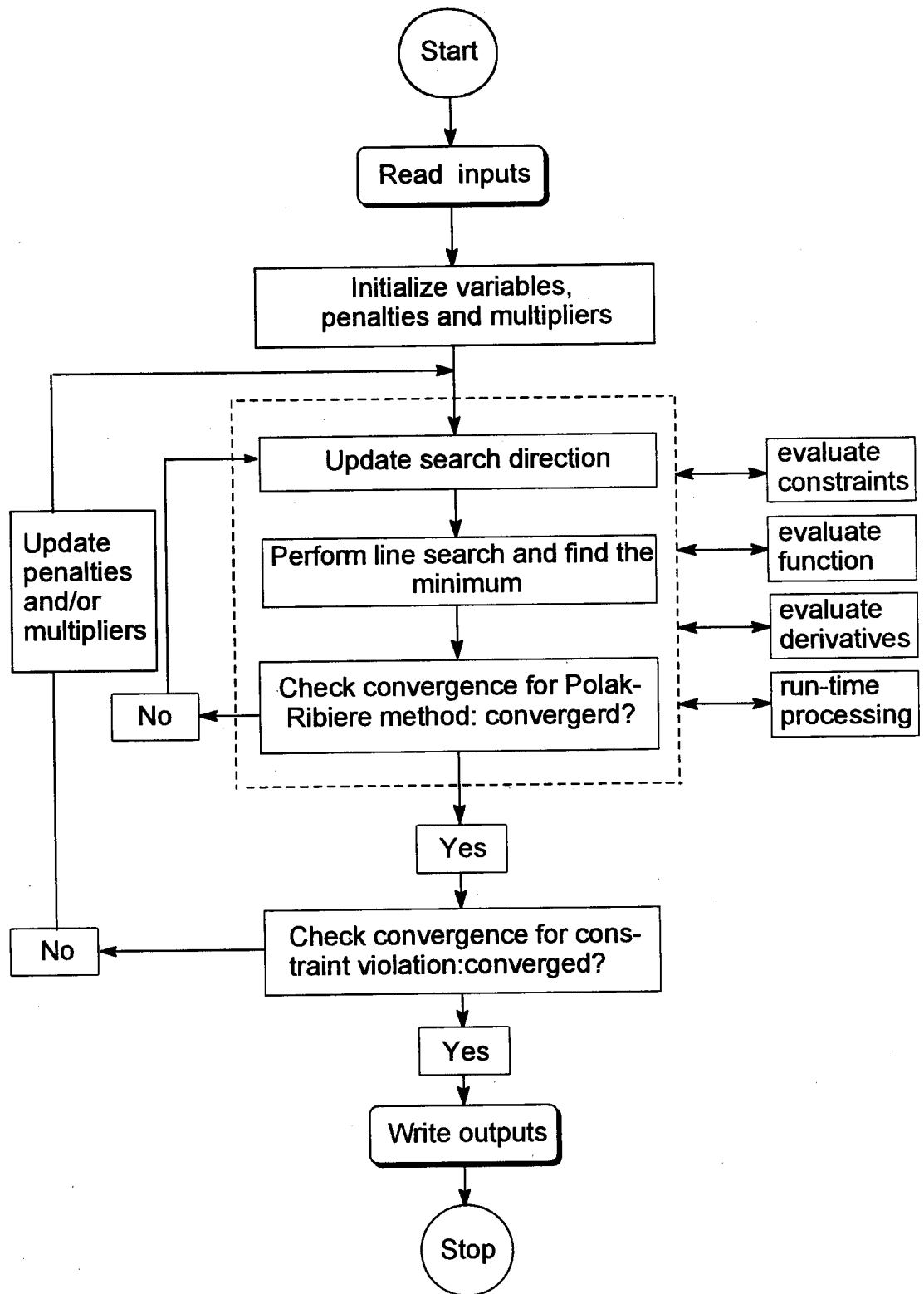
Figure 6.1: Program architecture.

As far as validation of the code is concerned, the program has been tested on many test problems, both unconstrained and constrained. For example, the regional marginal benefit curves for water in Chapter 3 have been derived using this code before solving the integrated problem. The solutions have been verified by generating identical results using GAMS/MINOS - a widely used optimization package developed at the Stanford optimization laboratory (see Brooke et al. (1992)). However, the pc-based MINOS can not be used to solve the integrated model because MINOS uses a variable-metric type minimization algorithm which stores the Hessian matrix. The resulting memory requirement for the integrated model turns out to be many times greater than what is currently available on today's high end pc's and work stations.

It should be mentioned here that like most other nonlinear solvers, the computer program (or the embedded algorithm of conjugate gradient) developed in this study does not guarantee global convergence. In general, it is practically impossible to check global convergence criteria for large problems even after the solution is obtained. Also many practical problems involve nonconvexity or discontinuity in the decision space. So the best that could be done is to compare the solution obtained via optimization with the heuristic management schemes. If the former gives a better solution, then there is no reason why optimization should not be performed (provided it could be performed economically)

even if only local optimality is expected. Again, according to Fletcher (1987), 'the only simple advice in practice (not guaranteed to work) is to solve the problem from a number of different starting points and take the local best solution that is obtained.'

CHAPTER 7

INTEGRATED MODEL AND CASE STUDIES

This chapter will describe the development of the integrated economic-hydrologic model and its application to a number of case studies. The case studies will include development/simulation of long run groundwater extraction profiles under 'social optimal' and 'common pool' scenarios. Trade-off between the two will also be studied. A third scenario will simulate the effect of municipal pumping during a five-year long drought on the long run agricultural return. Finally, suggestions will be provided on how operational decisions could also be made using only the hydrologic and optimization part of the model.

7.1 Mathematical representation of the social optimal case

The term 'social optimal' in the context of this study qualifies any outcome derived by maximizing the sum of discounted net benefits accruing to the society. Net benefit is defined as the sum of consumer surplus and producer surplus. The definition therefore is only concerned with the efficiency of resource use, and not with the equity of distribution. If equity related factors are to be incorporated

into the decision model, this could be done by introducing additional constraints.

The objective function of the social optimal case (SOPT) can be algebraically expressed as:

$$Max \quad Z = \sum_{t=1}^{tmax} \frac{(GB_t - VC_t - FC_t)}{(1+r)^t} \qquad (7.1)$$

where, GB_t, VC_t, and FC_t are gross benefit, variable cost and fixed cost at time t respectively, all discounted at a rate r. For convenience (7.1) will be re-written as:

$$Max \quad Z = \sum_{t=1}^{tmax} \frac{(CB_t - VWC_t - FC_t)}{(1+r)^t} \qquad (7.2)$$

where, CB_t is the benefit from agriculture net of crop variable cost, and VWC_t is the variable water cost at time t. Each of the components of (7.2) can be further expanded as follows.

$$CB_t = \sum_{k=1}^{kmax} \sum_{i=1}^{imax} \sum_{j=1}^{jmax} [r_i (a_i + b_i (\epsilon_j w_{jikt}) + \qquad (7.3)$$
$$c_j (\epsilon_j w_{jikt})^2) - vcc_i] x_{jikt}$$

$$VWC_t = \sum_{k=1}^{kmax} \sum_{i=1}^{imax} \sum_{j=1}^{jmax} (vct_{jkt} + vhl_j ulc) w_{jikt} x_{jikt} \qquad (7.4)$$

$$FC_t = \sum_{k=1}^{kmax} \sum_{i=1}^{imax} \sum_{j=1}^{jmax} (fcc_i + fct_j) x_{jikt} \qquad (7.5)$$

Symbols used above represent the following:

j,i,k,t j is technology, i is crop, k is location, and t is time index respectively;

x_{jikt} acreage allocated for crop i, irrigated by technology j, at location k during year t;

w_{jikt} water allocated for x_{jikt};

a_i, b_i, c_i coefficients of ith crop production function;

r_i farm gate price for the ith crop;

vcc_i, fcc_i variable and fixed costs for the ith crop;

vct_{jkt} variable cost per unit of water pumped using jth technology from cell k at time t;

fct_j technology related fixed cost;

ϵ_j efficiency of the jth technology;

vhl_j variable labor hours per unit of water applied using technology j;

ulc labor cost per hour.

In the numerical representation of (7.2), the objective function (OF) has been expressed in terms of decision variables and parameters only, by substituting (7.3) through (7.5) in (7.2). The decision variables in the integrated model are x_{jikt} and w_{jikt}. All other terms in the above equations are parameters except the term vct_{jkt}. This is the term which embodies the response matrix coefficients generated earlier. The following steps link vct_{jkt} to the decision variables:

$$vct_{jkt} = \frac{1.025}{12} \frac{uec}{ppe_j} tdh_{jkt} \qquad (7.6)$$

$$tdh_{jkt} = initd_k + s_{kt} + 2.31 psih_j + add_j + 12 \qquad (7.7)$$

$$S_{kt} = \sum_{l=1}^{kmax} \sum_{n=1}^{t} Q_{ln} \beta_{l,k,t-n+1} \qquad (7.8)$$

$$Q_{ln} = \sum_{i=1}^{imax} \sum_{j=1}^{jmax} X_{jiln} W_{jiln} \qquad (7.9)$$

where,

thd_{jkt}	is the total dynamic head related to vct_{jkt};
ppe_j	pumping plant efficiency using tech. j;
uec	unit energy cost;
$initd_k$	initial depth to steady state potentiometric surface at location k;
add_j	additional lift above the surface for tech. j;
$psih_j$	pressure head required for tech. j;
l,n	alias to k and t respectively.

All other symbols have been defined earlier.

The OF is maximized subject to the following constraints:

$$\sum_i X_{jikt} \leq tjcmax_{jk}, \qquad \forall j,k,t \qquad (7.10)$$

$$\sum_j X_{jikt} \leq cellmax_{ik}, \qquad \forall i,k,t \qquad (7.11)$$

$$S_{kt} \leq ddmax_k, \qquad \forall k,t \qquad (7.12)$$

$$W_{jikt} \leq wmax_i/\epsilon_j, \qquad \forall j,i,k,t \qquad (7.13)$$

$$W_{jikt} \leq MX_{jikt}, \qquad \forall j,i,k,t \qquad (7.14)$$

$$X_{jikt}, W_{jikt} \geq 0, \qquad \forall j,i,k,t \qquad (7.15)$$

where symbols not defined earlier are:

$tjcmax_{jk}$	upper bound on acreage irrigated by technology j in cell k;

$cellmax_{ik}$ upper bound on acreage allocated to crop i in cell k;

$ddmax_k$ maximum drawdown allowed for cell k;

$wmax_{ji}$ upper limit on per acre irrigation.

Note that each equation above actually represents a set of similar constraints. For example (7.10) describes the technology related acreage constraints, a total of j*k*t of them. Technology constraints are likely to be present due to incompatibility between a certain technology and a topography-soil-crop combination. It has been arbitrarily assumed that all land (meaning potentially irrigable land) could be irrigated by flooding or siphon. But gated pipe can not be used to irrigate more than 50% of land in any cell, and no more that 30% of land in any cell could be irrigated by sprinkler. These restrictions are quite liberal compared to the state-wide technology use statistics for Colorado. Wilson and Ayer (1982) report that in Colorado, the methods of water application as percent of total irrigated land are: flooding 60%, siphon 19%, gated pipe 2%, sprinkler 17% and others 2%.

Crop related acreage constraints are incorporated in (7.11). Implications of these constraints have already been discussed in Chapter 3 in the section on the regional demand curve for water. They essentially act as upper bounds for acreage allocated to different crops.

Drawdown related constraints are summarized in (7.12). This equation makes sure that potentiometric head does not fall below the top confining layer of the aquifer. This is the

most prominent limitation of the response matrix approach of modeling aquifer response. Although the approach could be used to model unconfined aquifer to a limited extent, this does not include dynamic change of local state of the aquifer from confined to unconfined.

Constraint (7.13) sets reasonable upper limits to per acre irrigation applied. This limit is a must for alfalfa due to its linear production function. As before, upper limit for water requirement of alfalfa has been set to $wmax_{j,alfalfa,k,t}=walfamax/\epsilon_j$, where walfamax is the maximum per acre seasonal water demand of alfalfa. Constraint (7.14) is the same conditional constraint used in Chapter 3 which ensures that if x_{jikt} is zero then w_{jikt} must also be zero. And constraint (7.15) is the non-negativity restriction on all the decision variables.

7.2 Mathematical representation of the common pool case

The common pool scenario can be defined as a situation where instead of maximizing the present value of the stream of incomes, farmers simply pump water each year, satisfying the condition that the marginal cost of pumping equals the value of marginal physical product. So the common pool scheme could be described as 'myopic' and 'selfish'. The term 'myopic' implies that farmers only make short run decisions. And 'selfish' means that no attention is given to the externality imposed on others due to mutual interference and hydraulic

linkage. It is said that under pure form of common pool, each farmer (or miner for that matter) is dominated by the rule of capture - if he does not use the water, someone else will and there may not be anything left for the next year. From the society's point of view, the inevitable outcome of this is over-extraction of the resource in a short period of time, accompanied by excessive and wasteful capital investment .

Thus, the optimization problem under the common pool is not an inter-temporal problem. Moreover, within a specific decision period, production decisions are made independently by each farmer without any cooperation with others. Since it is not possible to model every single farmer separately, each cell of the hypothetical basin will be treated as a separate entity under the common pool.

The OF for the kth cell at time period t under common pool situation can be expressed as:

$$Max \quad Z_k = (CB_k - VWC_k) \tag{7.16}$$

where, Z_k is the short-run benefit to cell k; CB_k and VWC_k are crop benefit net of crop variable cost and variable water cost at cell k respectively. As before, these terms could be further expanded as shown below (all symbols used before have the same meaning).

$$CB_k = \sum_{i=1}^{imax} \sum_{j=1}^{jmax} [r_i (a_i + b_i (\epsilon_j w_{jikt}) + \qquad (7.17)$$
$$c_j (\epsilon_j w_{jikt})^2) - vcc_i] x_{jikt}$$

$$VWC_k = \sum_{i=1}^{imax} \sum_{j=1}^{jmax} (vct_{jkt} + vhl_j ulc) w_{jikt} x_{jikt} \qquad (7.18)$$

Note that due to the assumed short run nature of the decision making, no fixed cost is included in the OF. But of course, fixed costs have to be paid for in the long run. It has been assumed that under common pool situation, farmers start up with a given stock of cropping and irrigation technologies. It is difficult to conceive exactly how this initial stock is determined. In this study, it is assumed that farmers make the first period decision based on full consideration of possible variable and fixed costs (fixed cost in terms of amortized annual cost). But once the initial stock is determined, farmers must continue with that level of technology at least for five additional years before any adjustment could me made. This assumption is rather arbitrary but some such assumptions are necessary if any technology adjustment is to be allowed at all.

The common pool scenario has the same constraints as the social optimal case. The total benefit generated in period t Z_t is simply $\Sigma(Z_k)$, and the cumulative total benefit, Z is:

$$Z = \sum_{t=1}^{tmax} \frac{Z_t}{(1+r)^t} \qquad (7.19)$$

7.3 The drought pumping scenario

The third scenario to be investigated is the groundwater extraction scheme in the presence of a five year long drought pumping by the city of Denver at the beginning of the planning period. Both the social optimal and common pool schemes are subjected to the drought pumping. It is assumed that during this drought, the city pumps 20,000 acre-feet (approximately ten percent of its annual demand) to cover the shortage of surface water. Municipal pumping is assumed to occur in cells (3,2) and (4,2) (see Figure 5.1.a) due to their proximity to the city and favorable aquifer characteristics. For convenience (so that the same discrete kernels could be used), it is also assumed that the municipal pumping occurs during the irrigation season (late spring to early fall). Since water demanded in the months of May through August makes up the peak load and about half of the annual demand, this assumption is quite realistic. Besides, a more distributed pattern of pumping is unlikely to affect the long run cumulative benefit in any significant way.

This third case study serves as an illustration of how an exogenous 'shock' could be incorporated into the integrated model. It also demonstrates that when severity of demand on a limited resource increases due to uncontrolled natural or man-made phenomena, a socially optimal scheme is likely to perform better than the common pool or competitive schemes. This is because the former has the ability of dissipating the

aftermath of the shock throughout the planning period, while the latter continues to pursue the 'myopic' strategy (or a strategy with very limited foresight and cooperation). Also, no matter what scheme of extraction is followed, municipal pumping will increase the cost of pumping for agricultural purposes. The consequent loss of benefit to the farmers could be used as a measure of externality that the city imposes on the farmers. Ideally, the city consumers should pay for this externality in addition to the cost of pumping and transporting the water to their households. If income redistribution is an issue of concern, then this loss of agricultural benefit can also serve as an estimate of potential compensation that the city should be paying to the farmers.

Numerical implementation of the drought within the integrated model is very simple. For each of the first five years, exogenous demand of 10,000 acre-feet has been assigned to cells (3,2) and (4,2). Since pumping from these cells is conducted by the city, they are parameters of the integrated model, and not to be confused as additional decision variables. The effect is internalized during the computation of drawdowns when municipal pumping enters into equation (7.8) as fixed exogenous withdrawal.

7.4 Some additional assumptions/preconditions

The issue of discounting is very important in any economic analysis. The discount rate is usually used to reflect the public and private sector opportunity costs of investment. However, when a positive (non-zero) discount rate is used in an inter-temporal resource allocation problem, it also reflects the time preference of consumption. Since one of the goals of the social management of resources is to generate a relatively steady stream of net benefits, a non-zero discount rate may act against that objective. For example, in the social optimal groundwater extraction case, when a positive discount rate is directly used in the estimation of the objective function and the first order conditions, the resulting optimal solution will be biased towards the present. The solution will recommend irrigation intensive cropping during the initial years and very little irrigation near the tail end of the planning period. This is unlikely to be acceptable to the farmers who would like to have a relatively stable stream of incomes throughout the planning horizon. Thus, it may be preferable to first generate the optimal groundwater pumping pattern without using any discount rate, and then convert the stream of optimal net benefits into net present value using appropriate social discount rate. This latter approach will be used in this study while generating the social optimal profiles of groundwater extraction. Later,

a separate analysis will be carried out to show the differences of direct and indirect discounting.

The maximum potential irrigable land in the integrated model has been set equal to one-third the value used during derivation of the irrigation water demand in Chapter 3. This means that the maximum water demand in the integrated model is only one-third of the potential demand. This is due to the fact that in reality, only about one-third of the irrigation water in Colorado come from groundwater aquifers, the rest come from surface water sources (Wilson and Ayers, 1982). Surface water sources are likely to be used first since surface water is cheaper. Additionally, there are two other aquifers above the Arapahoe aquifer (after which the test basin is built) which are also being used. Moreover, there are a number of alluvial aquifers in the study area along the major streams. So if the entire potential demand is specified for the hypothetical basin, it may produce some very unrealistic results.

The assumption of dynamic technology adjustment can not be used in relation to the well and the pumping plant. Installing a well requires considerable preparation, resources and time, and once in place, it can not be de-installed in the next period to recover all the associated fixed costs. It has been assumed that there is a ten-year adjustment period for well and pump related fixed costs for both the social optimal and common pool schemes. So in a planning period of 40 years, a maximum of four such adjustments were allowed.

Another qualification is required in relation to the use of constant prices for inputs and outputs throughout the planning period. The implication is that all input and output prices will be affected by inflation by the same amount, and so their relative magnitudes will not change. Additionally, it could be argued that technological innovations will drive output prices down relative to input prices, and increased production sold at lower unit price approximately equates to constant revenue per acre.

This assumption is by no means required by the integrated model. Constant prices have been used in this study because consistent projections for all the parameters for a planning period of forty years were not readily available.

7.5 The hydrologic-economic link

Incorporation of the discrete kernels within an optimization model establishes the missing link between the economic objective and the physical response of the groundwater basin. Understanding this link is crucial for proper formulation of the case studies and interpretation of the results. So, before the results of the case studies are presented, this section will make a close examination of the hydrologic-economic link for the social optimal case.

All gradient based algorithms, including the conjugate gradient method, require some means of estimating the first partial derivatives of the OF with respective to the decision

variables. Some models may numerically estimate the derivatives, but most models require users to specify them. The integrated model of this study takes the latter approach. And it is through this process of estimating the first partial derivatives that the linkage between the hydrologic and economic components become evident.

Although the actual number of decision variables in the integrated model depends on the scenario being studied, basically the model has only two kinds of variables - acreage allocated, x_{jikt} and irrigation water applied, w_{jikt}. Since they always occur together in the hydrology-related part, it is only necessary to derive the first order conditions for one of them. To allow the minimal derivation of the first order condition, it is assumed at this point that the problem under consideration has an interior minimum. Extension to the more general case is straightforward. Also for the sake of brevity, details of the derivation will be skipped, only the important steps will be outlined.

Let $\partial OF/\partial w_{jikt}$ be the generic first derivative of the OF with respect to the decision variable w_{jikt}. Based on (7.2), this could be broken down as:

$$\frac{\partial Z}{\partial w_{jikt}} = \frac{\partial}{\partial w_{jikt}} \sum_{t=1}^{tmax} \left[\frac{CB_t}{(i+r)^t} - \frac{VWC_t}{(i+r)^t} - \frac{FC_t}{(i+r)^t} \right] \qquad (7.20)$$

where the CB_t and FC_t related terms are evaluated first as:

$$\frac{\partial}{\partial w_{jikt}} \sum_{t=1}^{tmax} \frac{CB_t}{(1+r)^t} = \tag{7.21}$$

$$(r_i b_i \epsilon_j + 2 r_i c_i \epsilon_j^2 w_{jikt}) \frac{X_{jikt}}{(1+r)^t}$$

$$\frac{\partial}{\partial w_{jikt}} \sum_{t=1}^{tmax} \frac{FC_t}{(1+r)^t} = 0 \tag{7.22}$$

The middle term of the RHS of (7.20) is now expanded in a number of steps for clarity:

$$\frac{\partial}{\partial w_{jikt}} \sum_{t=1}^{tmax} \frac{VWC_t}{(1+r)^t} = \tag{7.23}$$

$$\left(\frac{\partial VWC_t}{\partial w_{jikt}} + \frac{\partial}{\partial w_{jikt}} \sum_{\delta t=1}^{tmax-t} \frac{VWC_{t+\delta t}}{(1+r)^{\delta t}} \right) \frac{1}{(1+r)^t}$$

Now there are two different terms on the RHS related to VWC_t and $VWC_{t+\delta t}$ which are further expanded as:

$$\frac{\partial VWC_t}{\partial w_{jikt}} = [(vct_{jkt} + vhl_j ulc) X_{jikt} + \tag{7.24}$$

$$\sum_{m_3=1}^{kmax} \sum_{m_2=1}^{imax} \sum_{m_1=1}^{jmax} X_{m_1 m_2 m_3 t} W_{m_1 m_2 m_3 t} (tdc_{m_1} \frac{\beta_{k,m_3,1} X_{jikt}}{12})]$$

$$\frac{\partial}{\partial w_{jikt}} \sum_{\delta t=1}^{tmax-t} \frac{VWC_{t+\delta t}}{(1+r)^{\delta t}} = \sum_{\delta t=1}^{tmax-t} \frac{1}{(1+r)^{\delta t}} [\tag{7.25}$$

$$\sum_{m_3=1}^{kmax} \sum_{m_2=1}^{imax} \sum_{m_1=1}^{jmax} X_{m_1 m_2 m_3 t+\delta t} W_{m_1 m_2 m_3 t+\delta t} (tdc_{m_1} \frac{\beta_{k,m_3,1+\delta t} X_{jikt}}{12})]$$

where, $tcd_j = (1.025/12) * (uec/ppe_j)$.

Equations (7.21) through (7.25) can now be substituted back into (7.20) to obtain the first partial derivative. The resulting expression will evaluate to zero at the optimal

solution due to the first order necessary condition. So
setting (7.20) to zero will allow derivation of the decision
rule for optimal irrigation for the integrated model. After
necessary rearrangements, and assuming that x_{jikt} is nonzero
(otherwise constraint (7.14) will ensure that w_{jikt} is also
zero), the following decision rule is obtained:

$$w_{jikt} = \frac{b_i}{2|c_i|\epsilon_j} - \frac{1}{2|c_i|r_i\epsilon_j^2} [P_{gw}+S+T] \qquad (7.26)$$

where,

$$P_{gw} = (vct_{jkt} + vhl_julc) \qquad (7.27)$$

$$S = \sum_{m_3=1}^{kmax} \sum_{m_2=1}^{imax} \sum_{m_1=1}^{jmax} x_{m_1m_2m_3t}w_{m_1m_2m_3t} (tdc_{m_1}\frac{\beta_{k,m_3,1}}{12}) \qquad (7.28)$$

$$T = \sum_{\delta t=1}^{tmax-t} \frac{1}{(1+r)^{\delta t}} [\sum_{m_3=1}^{kmax} \sum_{m_2=1}^{imax} \sum_{m_1=1}^{jmax}$$

$$x_{m_1m_2m_3t+\delta t}w_{m_1m_2m_3t+\delta t} (tdc_{m_1}\frac{\beta_{k,m_3,1+\delta t}}{12})] \qquad (7.29)$$

This completes the hydrologic-economic linkage and
demonstrates how decision rules are internally created by the
model. Equation (7.26) is particularly interesting. This has
the same form as (3.22) where the term (P_{gw}+S+T) could be
interpreted as the social cost of extracting unit volume of
groundwater. More importantly, the total social cost is
expressed as the sum of three separate components. The first
component, P_{gw} is the direct cost of pumping unit volume of
water from the aquifer. The second term, S is the spatial

externality caused by w_{jikt}. Finally, T is the corresponding temporal externality, the temporal costs are discounted back to the current period t. So the response matrix based integration of economic and hydrologic components has yielded lucid identification of direct, spacial and temporal costs of groundwater extraction. This is an important finding of this study.

Similar first order conditions could be derived for the variable x_{jikt} which differs in minor ways from the corresponding expression for w_{jikt}. The exercise could be repeated for the common pool case to examine the decision rule for that scenario. Since these derivations closely follow the steps outlined above, they will not be repeated. It should be mentioned here that the actual integrated models solved in this study had more involved derivative expressions due to the presence of binding constraints.

7.6 Results and discussions

7.6.1 Social optimal and common pool cases

Figure 7.1 shows the discounted net benefit profiles of the social optimal (SOPT; as discussed earlier, no intrinsic discounting was used, stream of net benefits were converted to the present value at 7%) and the common pool (CP) cases. The SOPT profile starts off at a lower level than the CP profile, but soon crosses it after the fourth year. After that SOPT stays above CP all along till the end of the planning period. This supports the proposition that although the CP scheme may
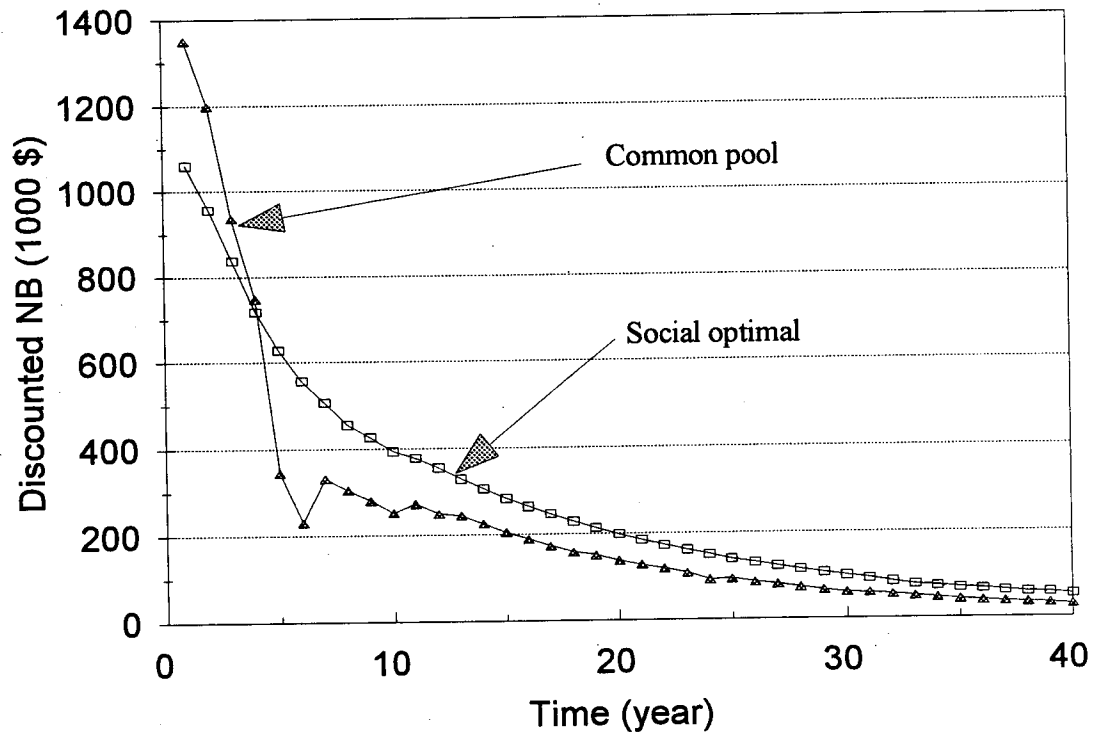
Figure 7.1: Net benefit profiles for the social optimal and the common pool scenarios.

be more profitable in the short run, it is outperformed by the SOPT in a longer planning period.

Also note that the CP profile has a much sharper drop initially than the SOPT profile. This is due to rapid and uncontrolled initial extraction under CP. This myopic policy soon begins to penalize the farmers in the form of high pumping cost due to greater drawdown under CP. Also note the initial uneven nature of the CP profile compared to the smooth profile of the SOPT scheme. This is due to the short run nature of decision making under the CP scheme. Part of the unevenness is caused by the inefficient technology adjustment process where initial heavy capital investment soon results in carry over of nonproductive fixed costs.

The net present value generated by the SOPT and CP schemes are 11.45 and 9.33 million dollars respectively for the forty year long planning period. In other words, SOPT scheme has generated approximately 20% more net benefit compared to the CP scheme for the hypothetical basin.

Figure 7.2 shows the potentiometric head profiles for the active cells at the end of the planning period for both SOPT and CP. Clearly, CP causes faster depletion of the aquifer storage and as a result causes greater drawdown. In most cells, as seen from this figure, CP induced drawdowns are about 100 to 200 feet greater than the SOPT induced drawdowns. Also note that in cells 1 and 2 (here active cells are numbered from west to east, starting at the north end, so cell 1 is the same as cell (2,1) in Figure 5.1.a), CP induced
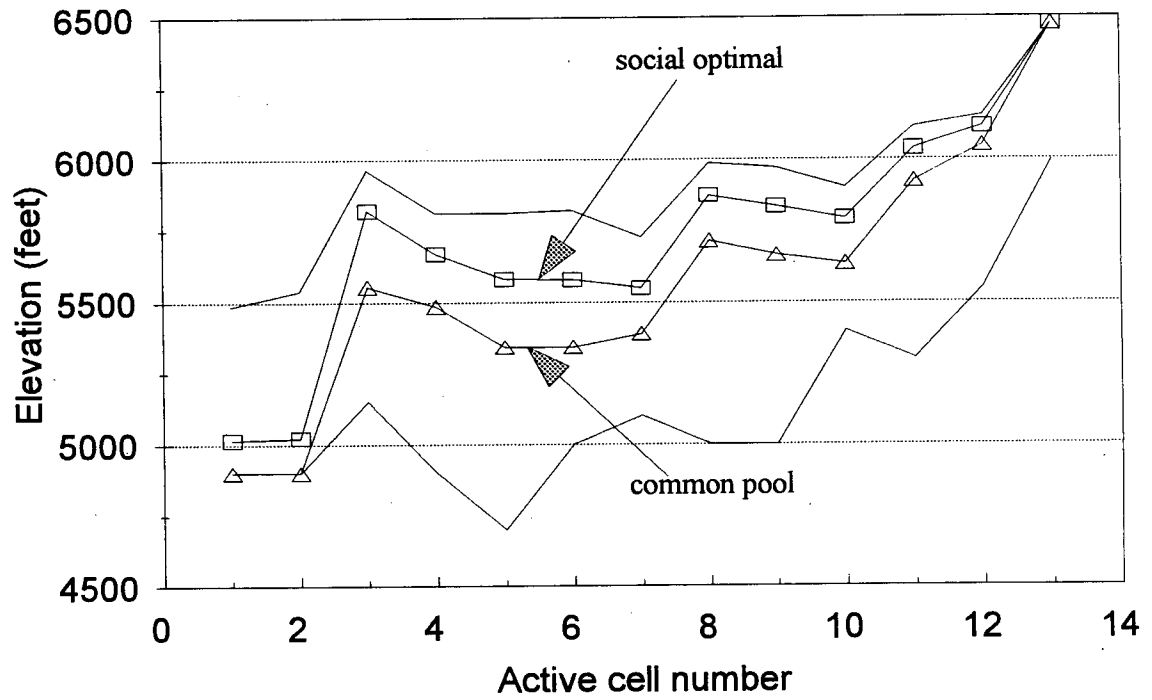
Figure 7.2: Potentiometric head at active cells
at the end of the planning period.

potentiometric head has reached it allowable limit (top of the confined aquifer) below which it is not allowed to fall. This is a major drawback of the discrete kernel approach of modeling the aquifer response. Although totally unconfined aquifers are also sometimes modeled by the response matrix, this does not include dynamic change of state from confined to unconfined. However note that SOPT induced profile does not reach this limit even after forty years of pumping.

Another important point to note is that although no water was pumped from cells 4 and 8, these cells still exhibit significant drop of potentiometric head. This again underscores the assertion that due to the pervasive hydraulic linkage, no location within the groundwater basin should be treated as a separate entity in a decision model.

The total volume of groundwater pumped from all the active cells under SOPT is 512.65×10^3 acre-feet. The same for CP is 902.14×10^3 acre-feet. So the CP pumps out almost twice as much water as demanded by the SOPT scheme. SOPT irrigates a total of 12,512 acres of land which remain unchanged throughout the planning period, whereas total land irrigated under CP varies from 15,590 (period 1) to 10,680 acres (period 40). Average irrigation per acre per year for SOPT is 1.03 acre-feet/acre and the same for CP is 1.91 acre-feet/acre.

Figure 7.3 summarizes the average irrigated land use patterns as generated by SOPT and CP schemes. It should be mentioned here that SOPT land use pattern remains the same throughout the planning period, but both the percentages of
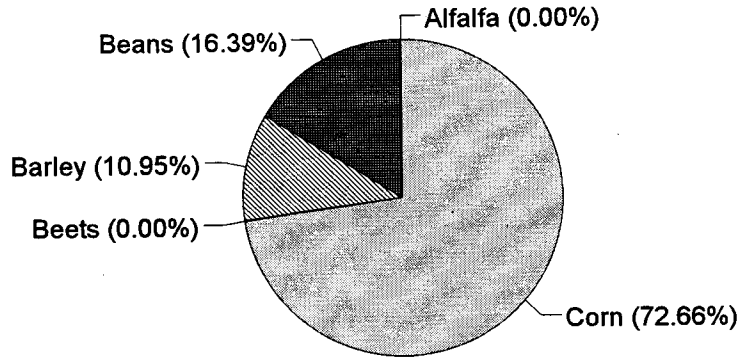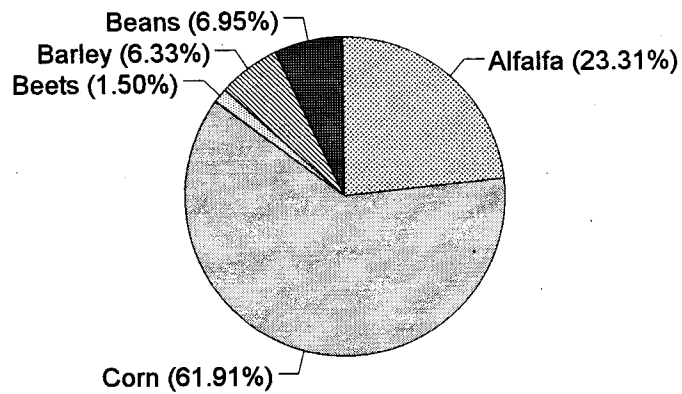
Figure 7.3: Acreage allocation for crops under social optimal and common pool scenarios.

land allocated to different crops and the total amount of land irrigated undergo considerable changes under CP. Annual land use patterns for SOPT and CP cases (along with drought pumping case) are shown later in Figure 7.8. It is clear from Figure 7.8 that in the long run, SOPT is capable of providing uninterrupted irrigation to aproximately 15% more area than what is supported by CP at the end of the planning period. So, if ensuring stable income to the farmers is a concern, policy designed after SOPT is likely to perform much better in the long run.

Finally, average levels of technology use under SOPT and CP are shown in Figure 7.4. Siphon irrigation appears to be the method of choice for both SOPT and CP, followed by flooding. This however, sharply contradicts the statewide technology use pattern: flooding 60% and siphon 20% (approximately). This is probably due to the very low annual fixed cost of 2.88 dollars/acre for siphon as given by Booker (1992). Also it was assumed that siphon has an application efficiency of 60% (10% higher that flooding). Together these two assumptions might have prompted the model to generate this rather unlikely technology use pattern.

Sprinkler irrigation seems to be the least popular among the four choices. In fact SOPT does not use sprinkler at all. This implies that although farmers are prompted to switch to more efficient irrigation methods as the cost of pumping increases, this may not be a desirable course of action if the associated variable and fixed costs are too high.

Sprinkler (0.00%)

Gpipe (1.07%)

Flood (17.96%)

Siphon (80.97%)

Social optimal (average)

Sprinkler (5.89%)

Flood (7.58%)

Gpipe (16.98%)
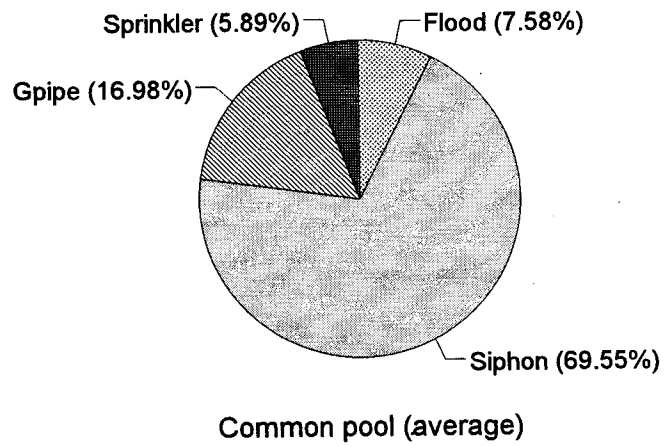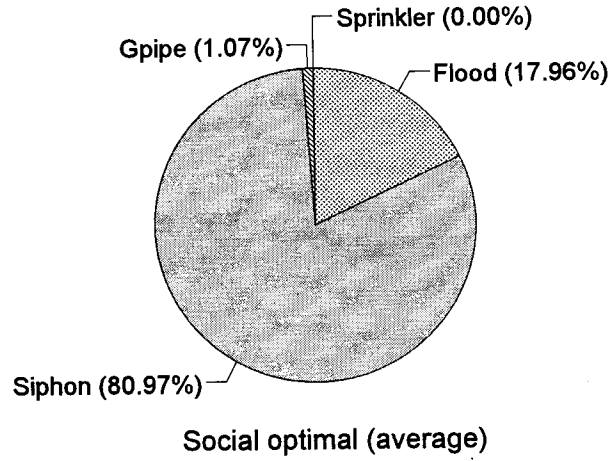
Siphon (69.55%)

Common pool (average)

Figure 7.4: Technology used for crops under social optimal and common pool scenarios.

7.6.2 The drought pumping scenario

Figures 7.5 and 7.6 show the discounted net benefit (DNB) profiles for the SOPT and CP schemes respectively. The presence of drought induced municipal pumping clearly reduces the NPVs of both the schemes. But notice that under CP, net benefit becomes negative momentarily during years 5 and 6. This points to the hidden danger of CP type uncontrolled extraction strategies. If the area gets hit by a second drought before it recovers sufficiently from the impact of the first, it may serve as a major disincentive for irrigated farming to many farmers. As a result, a large number of farmers may switch to dryland farming, causing major shift in cropping practices and considerable loss of agricultural benefit to the society in the long run.

NPVs for SOPT and CP in the presence of the drought are 8.7 and 6.41 million dollars respectively. As before, SOPT has performed better, but the relative performance of SOPT has actually improved in the presence of the drought. The ratio of NPVs for CP and SOPT is 0.81 without the drought, but it is equal to 0.74 in the presence of the drought. So it could be said that SOPT is better able to deal with exogenous shocks due to its intertemporal nature of decision making.

Figure 7.7 shows the potentiometric head profiles at the end of the planning period in the presence of the drought. These profiles exhibit similar characteristics as before. But note that both SOPT and CP induced drawdowns in cells 1 and 2 have reached the allowable limit. Total volume pumped
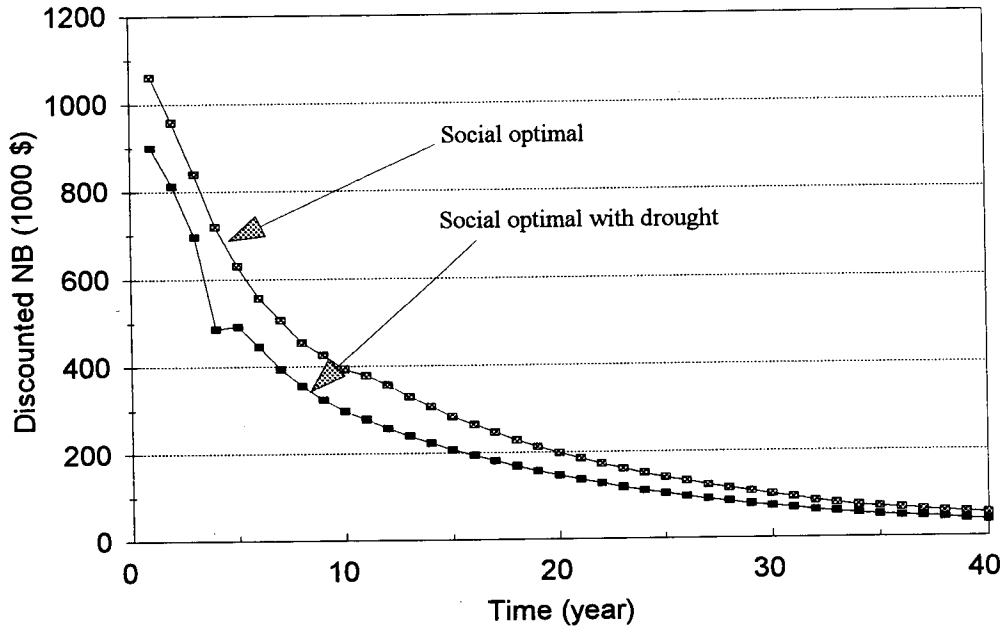
Figure 7.5: Social optimal net benefit profiles, with and
without drought.
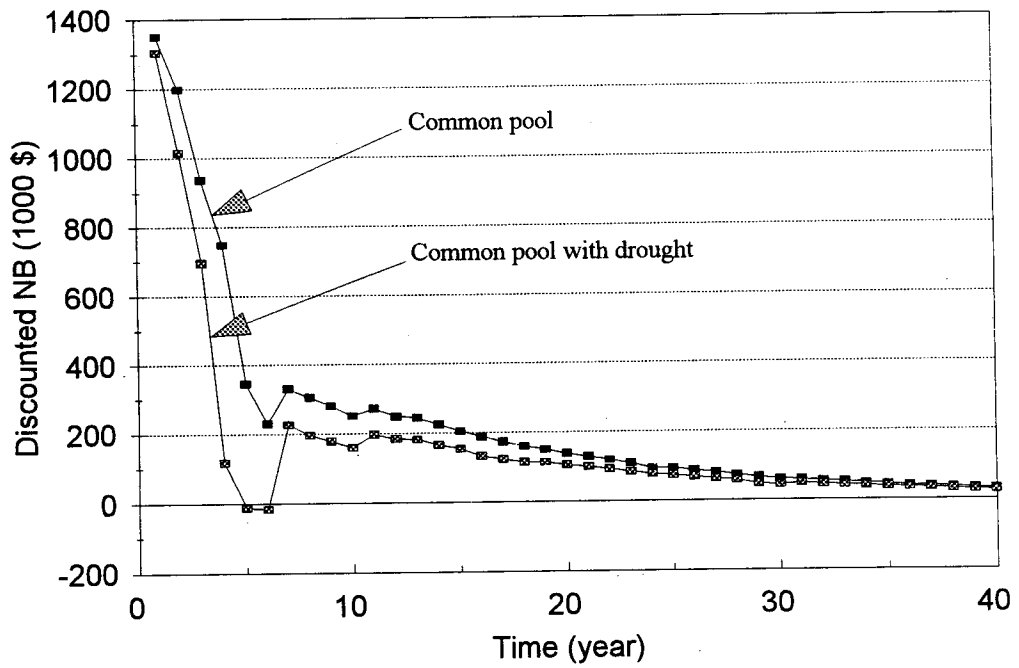
Figure 7.6: Common pool net benefit profiles, with and
without drought (discounted at 7%.).
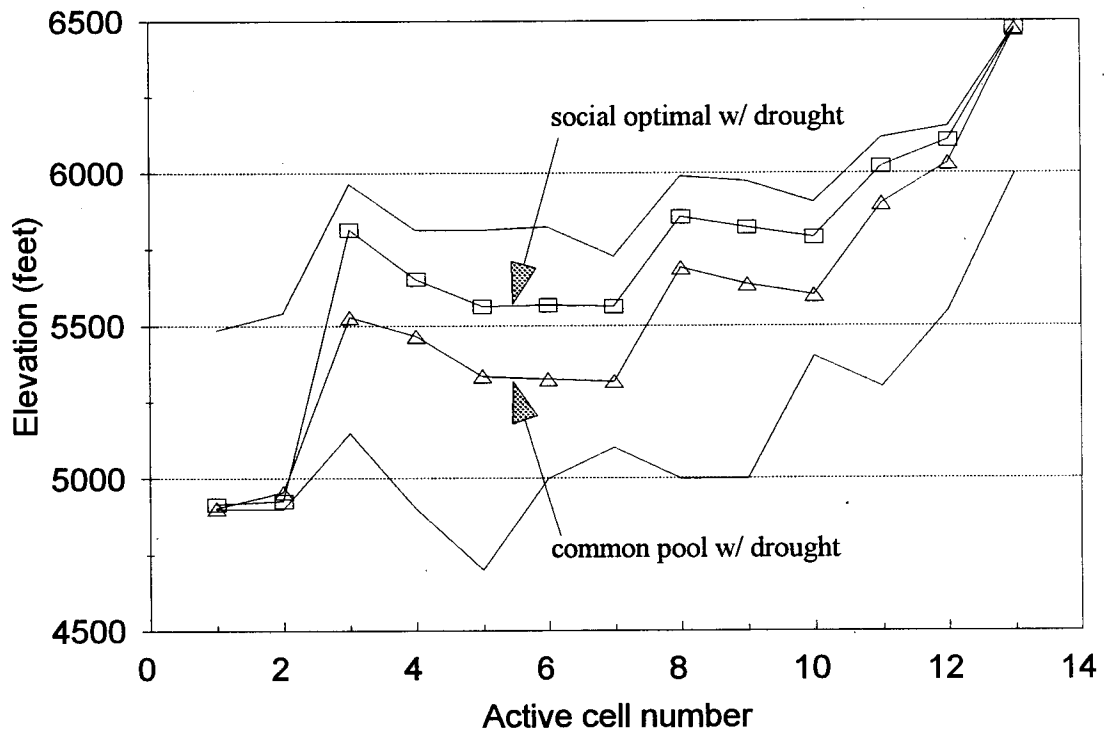
Figure 7.7: Potentiometric head at active cells (with drought)
at the end of the planning period.

(including municipal pumping) under SOPT and CP are 627.92×10^3 and 1004.39×10^3 acre-feet respectively.

As mentioned before, Figure 7.8 shows the annual irrigated land profiles for all the three scenarios. The SOPT land use pattern is not affected by the drought, but irrigated land under the CP scheme drops considerably in the presence of the drought. Again, SOPT promises a more stable income stream and cropping practice for the farmers.

Finally, Figure 7.9 shows the annual groundwater extraction profiles for all the scenarios. The basic difference between the SOPT and CP extraction profiles is that the latter starts with high initial pumping and then undergoes a sudden steep drop. The SOPT exhibits a gradual decline and therefore generates a much smoother profile. The sudden drop of pumping under CP causes excess pumping capacity after the first few years which contributes significantly to the overall economic inefficiency of the CP scheme.

This concludes the discussion of the case studies. Major results for all the scenarios are also summarized in Table 7.1 below.

7.6.3 The tail-end effect

An interesting observation could be made for the social optimal groundwater extraction profiles by close examination of Figure 7.9. Note that near the tail end of the planning period, pumping for the SOPT case reverses its trend and begins to increase. At the same time, no such trend is
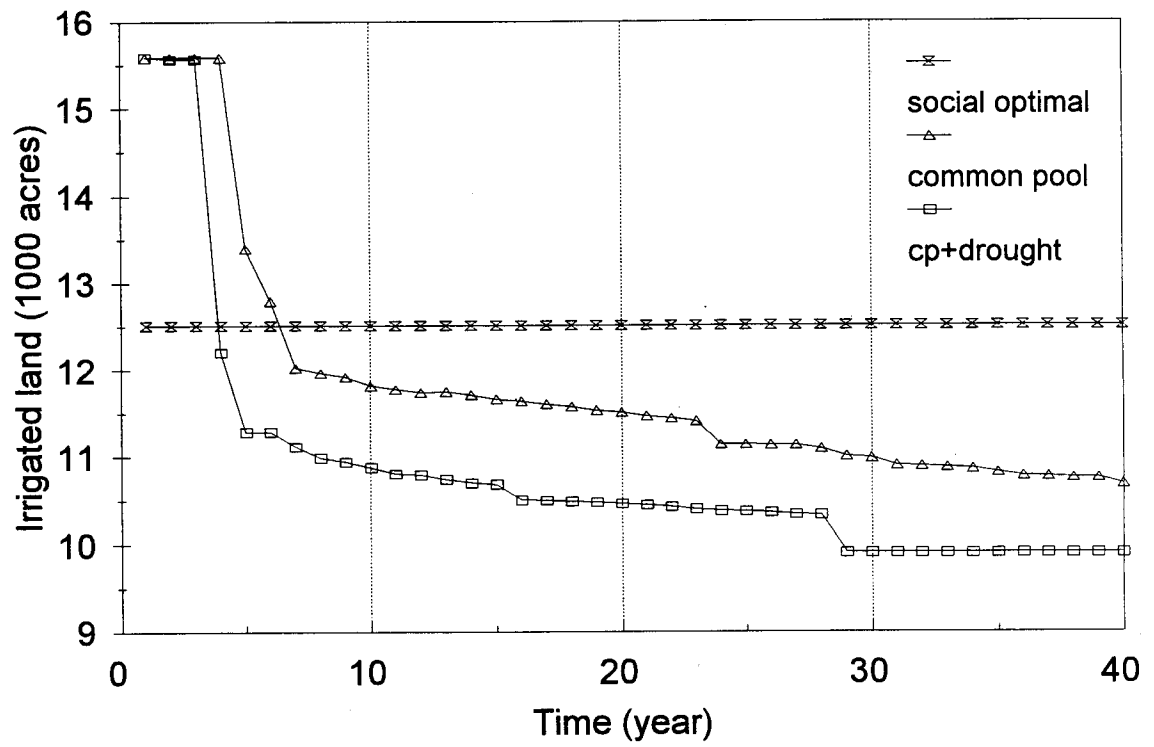
Figure 7.8: Annual irrigated land profile.
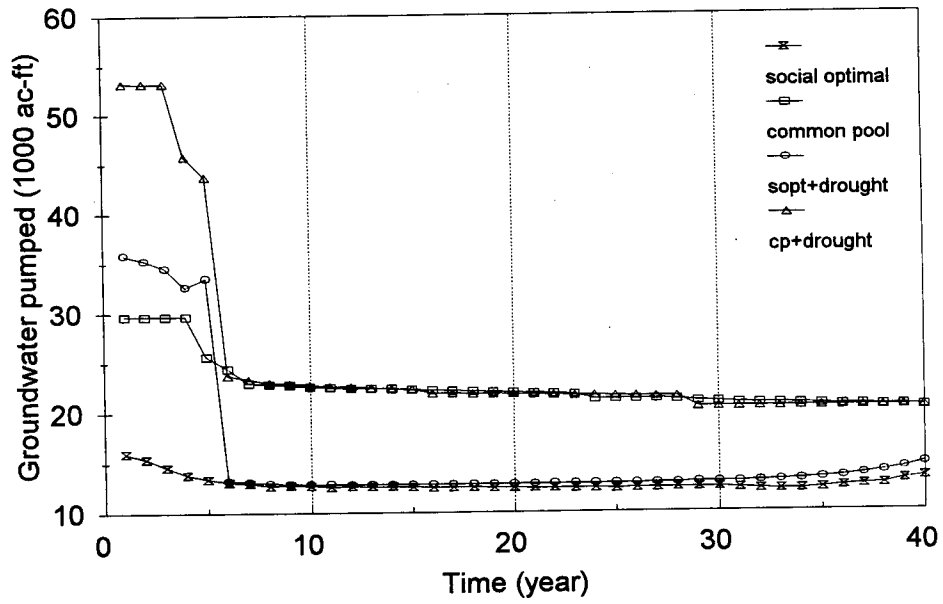
Figure 7.9: Annual groundwater extraction profile.
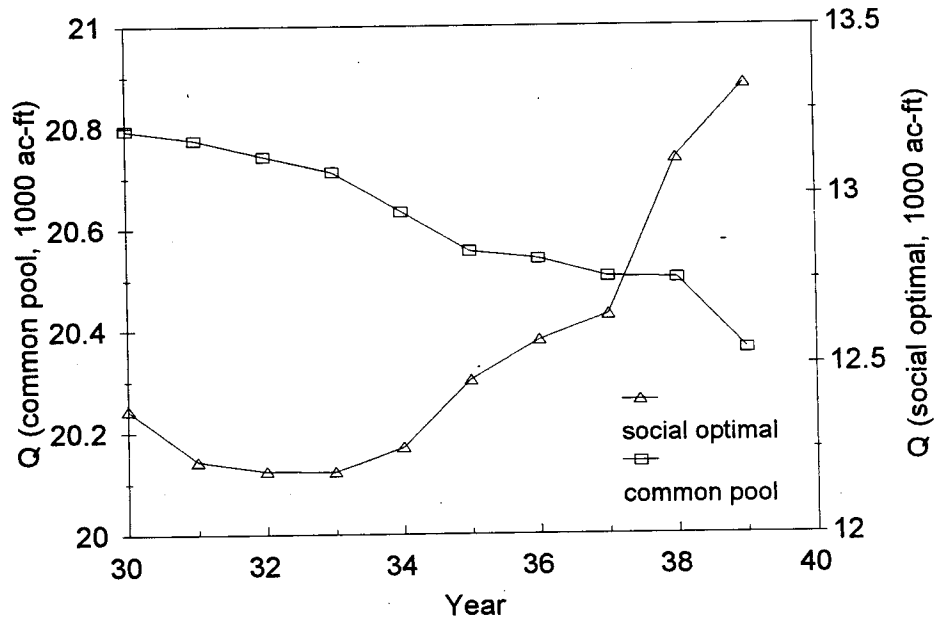
Figure 7.10: Magnified tail-end effect.

SIMULATION SUMMARY

	Q1000	NPV	$/Q
SOPT	512.65	11447.39	22.33
CP	902.14	9329.41	10.34
SOPT+DR	627.92	8700.08	13.86
CP+DR	1004.39	6412.47	6.38

RATIO

CP / SOPT	0.81
CP+DR / SOPT+DR	0.74

Q1000: groundwater pumped in 1000 ac-ft;
NPV: net present value at 7% discount rate
(1000 $);

SOPT: social optimal;
SOPT+DR: social optimal plus drought;
CP: common pool;
CP+DR: common pool plus drought;

Table 7.1: Summary table for the case studies.

observed in the CP related profiles. Figure 7.10 (see page 139 below Figure 7.9) shows this phenomenon more clearly. This 'tail-end' effect of the SOPT profile can be explained by examining the temporal externality component. The temporal effect of the decision w_{jikt}, Tw_{jikt} on the OF, can be expressed as:

$$Tw_{jikt} = - \sum_{\delta t=1}^{tmax-t} [\sum_{m_3=1}^{kmax} \sum_{m_2=1}^{imax} \sum_{m_1=1}^{jmax} X_{m_1 m_2 m_3 t+\delta t} W_{m_1 m_2 m_3 t+\delta t} (tdc_{m_1} \frac{\beta_{k,m_3,1+\delta t} X_{jikt}}{12})] \frac{1}{(1+r)^{\delta t}} \qquad (7.20)$$

Symbols used in (7.20) have the same meanings defined earlier. Now it is obvious that as $t \rightarrow tmax$, the upper limit of the sum over $\delta t \rightarrow 0$ and when $t=tmax$, there is no temporal component at all. So, initially the integrated model pumps water carefully to avoid both the spatial and temporal externality. But when the tail end is approached, temporal externality tends to zero and the model can pump water a little bit more liberally.

7.6.4 Sensitivity to discount rate and unit energy cost

This section is not intended to be a full fledged sensitivity analysis. Here, two additional simulation results will be reported, primarily to illustrate how the social optimal groundwater extraction profile might deviate as the discount rate and unit energy cost change.

The first simulation or scenario uses an intrinsic discount rate (discounting directly affects the OF and

derivatives reflecting time preference of consumption) of 7%. The second scenario uses no intrinsic discounting (as the original model of section 7.6.1) but has a unit energy cost of 0.0975 dollar/KWH (a 50% increase from the original value).

As shown in Figure 7.11, the annual groundwater extraction patterns under these two scenarios differ considerably from the original profile. The intrinsic discounting causes more water to be pumped in the near future as expected. This is because it weights the net benefits generated in the near future quite heavily against those generated at the end of the planning period. This kind of change in yearly irrigation is unlikely to be popular among farmers. Also, the NPV for this case is 10.17 million dollars, which is about 11% less than the NPV with no intrinsic discounting.

The increase in energy cost causes a downward shift of the annual extraction profile as expected. In this case a 50% increase in energy cost decreases the total volume of water pumped to 489.32×10^3 acre-feet from the original value of 512.65×10^3 acre-feet. Also, the NPV drops approximately 22% from 11.45 to 8.92 million dollars.

These two scenarios give some indication about how the optimal annual groundwater extraction might respond under different discount rate and unit energy price. Similar simulations could be conducted to study the effect of crop price variations.
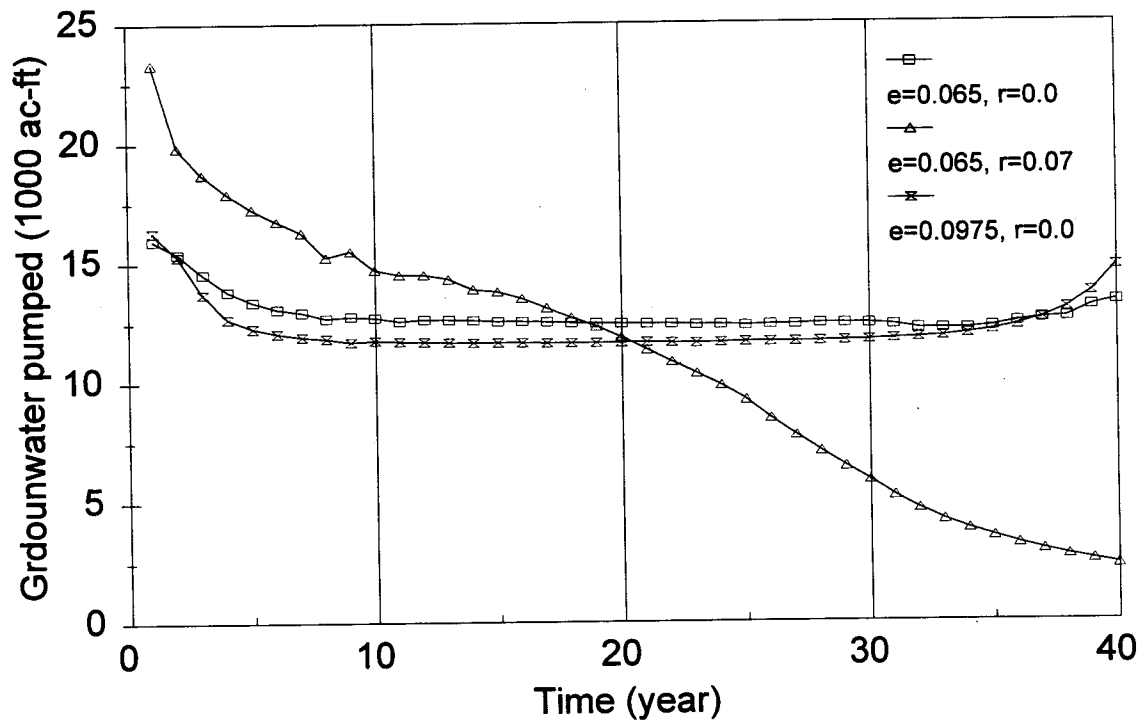
Figure 7.11: Response of annual groundwater extraction profile to discount rate and energy cost.

7.7 Operational decision making

The discrete kernel based methodology can also be used to solve a variety of operational problems with non-economic objectives provided the aquifer response is linear or nearly linear. It has been mentioned before that due to the difficulty of benefit estimation, engineers prefer simpler, and easier to define objective functions. The resulting decision problems are then formulated as linear/non-linear programming problems. Two most commonly used formulations are discussed below.

7.7.1 minimize drawdown

Let d_{kt} is the drawdown at location k at time t, and dt_{kt} is the corresponding target drawdown. Then the general form of the decision problem could be described as:

$$Min \quad Z = \sum_{t=1}^{tmax} \sum_{k=1}^{kmax} (d_{kt} - dt_{kt})^n \qquad (7.21)$$

subject to:

$$d_{kt} = \sum_{l=1}^{kmax} \sum_{n=1}^{t} Q_{ln} \beta_{l,k,t-n+1} \qquad (7.22)$$

$$Q_{kt} \geq QT_{kt} \qquad \forall \, k,t \qquad (7.23)$$

$$d_{kt} \leq dmax_k \qquad \forall \, k,t \qquad (7.24)$$

where symbols not defined earlier are:

n   exponent, usually 2;

QT_{kt} target or minimum withdrawal from cell k at t;

$dmax_k$ maximum allowed drawdown at location k at all times.

This rather simple OF is a surrogate for cost minimization since cost of pumping directly varies with the lift. Of course, the control variable now is Q_{kt}. Often times dt_{kt}'s are set to zero. The OF then becomes minimizing the sum of squared drawdowns. The number of decision variables in this formulation is likely to be considerably smaller than any of the integrated case studies presented above. So a great many cells could be easily included in the optimization model. In fact, if the aquifer is not too big, all major wells (say, about 200-300 of them) can be individually included in the model without requiring expensive computing facilities. Additional constraints can be added to accommodate other institutional and legal requirements.

7.7.2 minimize deviation from target

Formulation (a) above sometimes causes problem when (7.23) and (7.24) become mutually exclusive. An alternate OF could be formulated as:

$$Min \quad Z = \sum_{t=1}^{tmax} \sum_{k=1}^{kmax} (Q_{kt} - QT_{kt})^n \qquad (7.24)$$

This problem is solved subject to the same set of constraints except (7.22). The main difference between (a) and (b) is that the OF of (a) tries to minimize the cost of

pumping, while OF of (b) tries to minimize the variance of irrigation applied (or water supplied in general). So, this is likely to generate a smoother extraction scheme. Also, due to elimination of the hard bound (7.22), there are no constraints in the model which may become mutually exclusive. Again, additional constraints could be added as required.

Two operational problems formulated above further demonstrate the point that a discrete kernel based optimization model (given linear aquifer response) is indeed a simple but very efficient decision making tool for groundwater management. More importantly, it is likely to be a faster and more efficient approach of integrating economic and hydrologic components than the previously used embedding based scheme. Also, the resulting integrated model could be easily implemented on personal computers as opposed to on main frames for small to moderate sized problems.

CHAPTER 8

SUMMARY AND RECOMMENDATIONS

8.1 Summary

This study has presented and investigated an integrated modeling approach for groundwater basin management. It has combined economic objectives with realistic aquifer responses through the use of discrete kernels. The integrated model has been formulated as an intertemporal resource allocation problem which has been solved via a conjugate gradient based nonlinear programming algorithm. The algorithm, though iterative, uses an augmented Lagrangian based penalty function technique which automatically updates penalties and multipliers. Overall, the unique combination of the response matrix and the conjugate gradient method has allowed the integrated optimization model to be defined and solved in an economic and efficient manner (in terms of memory requirement and computational time). This approach has also allowed explicit identification of direct, spatial and temporal cost of pumping groundwater from a confined aquifer. When drawdown is not a significant part of the saturated thickness, the technique can also be applied for optimal management of unconfined aquifers.

The method has been applied to a hypothetical groundwater basin having characterisitcs similar to the Arapahoe aquifer of the Denver basin system. Real economic and agronomic data from the same area have been used in the economic part of the integrated model. As a corollary, regional demand function for water for agricultural use has also been estimated for the area overlying the Arapahoe aquifer. Three case studies and additional discussions on operational management have been presented to demonstrate that a diverse group of problems could be investigated using this decision making tool. The method is capable of handling very large problems when simple operational objectives are used and economic considerations are perhaps relegated to a separate economic submodel.

Optimal long run groundwater extraction policies have been generated and compared for the 'social optimal' and the 'common pool' cases. Later, the effect of a five year long drought induced municipal pumping has also been studied. In general, the social optimal scheme has performed better than the uncontrolled common pool situation. It has generated smoother water extraction and land use profiles compared to the common pool profiles. Also, the social optimal scheme has opted for less efficient but considerably cheaper irrigation methods such as flooding and siphon in the long run. The third case study of drought pumping indicates that as the severity of the demand increases, performance of the social optimal case over the common pool scenario shows further improvement. So, a planned extraction scheme is likely to perform better

when competition for the limited groundwater resource increases.

Two additional scenarios indicate that the optimal annual groundwater extraction profile is quite sensitive to the discount rate and unit energy price. Particularly, more stable extraction scehme (which also generates greater NPV) is obtained by using zero intrinsic discounting. The resulting stream of net benefits could still be converted to NPV externally using appropriate discount rate to reflect the social opportunity cost of investment.

The integrated model has been implemented on a 50Mhz 486 personal computer. The memory requirement varied between 1 to 4 MB depending on the problem being studied. Solution time varied between 30 minutes to 6 hours depending on the problem, length of the planning period, and the convergence criteria. So unlike the embedding based models of the past, integrated model of this study is accessible to practically any interested groundwater manager.

8.2 Recommendations for future research

Following is a list of recommendations for future research:

1. The response matrix approach provides accurate aquifer response for a confined aquifer only. It would be interesting to see as to what extent the method is applicable to unconfined aquifers.

2. The conjugate gradient based nonlinear solver could also be used to solve groundwater quality related problems. Since most previous studies have used variable metric methods, this will provide considerable savings in terms of memory requirement. Computational speed and convergence properties could be compared with other gradient based and heuristic methods.

3. Other more innovative optimization schemes could be tried. When the objective function is nonlinear but the constraints are linear, genetic algorithms may provide better solutions. Trade-off between the quality of the solution and the computational efficiency could be studied.

4. Effectiveness of different economic policies could be investigated. The model is capable of generating cell by cell benefits and costs. Therefore, distributional or equity consequence of different policies could also be investigated.

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APPENDIX A

Alfalfa: The production function for alfalfa is estimated directly from the relationship (3.8) and corresponding long run weather data for the northern Colorado area. It is assumed that seasonal R_e and E_p for alfalfa are 8.69 and 32.57 inches respectively. Also Y_p has been assumed to be 4 tons/acre based on field data. These values give a=1.07 and b=0.123 provided irrigation water applied is in inch/acre.

Corn: The estimated coefficients based on 9 data points are (figures within parentheses are t statistics):

    a=2.6325272   (0.0406)
    b=10.785775   (1.9524)
    c=-0.185372   (-1.6785) R^2=0.574

Independent variable was water applied at the root zone (or irrigation applied at 100% application efficiency) plus effective rainfall. Later effective rainfall (6.95 inches) contributions were separated out to express the production function in the form (3.3).

Dry beans: Estimated coefficients from 24 data points are:

    a=-12.709   (-1.9188)
    b=4.25379   (3.8632)
    c=-0.1142   (-2.6362) R^2=0.7697

Discussion for corn applies for dry beans too; effective rainfall is assumed to be 6.12 inches.

Other production function coefficients were directly adopted from the respective sources as mentioned in Chapter 3 after appropriate transformations. Regression statistics are not available for sugar beets; the same for barley as reported by Jakicic (1983) are not usable since the coefficients used in this study are averages of three different sets.

APPENDIX B

The complete data set and the relevant results for the test problem of section 5.3 are given below.

Description / results for the test problem

Pumping at cell 38: 700 cubic-ft/day

cell lenght: 25 feet, cell width: 20 feet

K: (K/S) col. x 10^{-5} , ft/day

S: (K/S) col. x 10^{-1} .

					Poten. head at the end of:			
cell	init. hd.	top	bot.	K/S	day1	day2	day3	day4
1	250	210	160	80	227	222.9	227.6	233.2
2	250	210	160	80	226.9	222.9	227.6	233.2
3	250	210	160	80	226.9	222.9	227.6	233.2
4	250	210	160	80	226.7	222.9	227.6	233.2
5	250	210	160	80	226.6	222.8	227.6	233.2
6	250	210	160	70	226.4	222.8	227.7	233.3
7	250	210	160	70	226.1	222.8	227.7	233.3
8	250	210	160	60	225.8	222.7	227.7	233.4
9	250	210	160	60	225.4	222.7	227.8	233.5
10	250	210	160	40	224.9	222.6	227.9	233.6
11	250	210	160	30	224.1	222.5	228	233.7
12	250	210	160	20	222.9	222.4	228.2	233.9
13	250	210	160	10	220.7	222.2	228.5	234.3
14	250	210	160	10	217.7	221.8	229	234.8
15	250	210	160	10	214.7	221.5	229.5	235.4
16	250	210	160	20	212.5	221.3	229.9	235.8
17	250	207	157	20	210.9	221.2	230.1	236.1
18	250	204	154	20	209.3	221	230.4	236.4
19	250	201	151	20	207.6	220.9	230.6	236.7
20	250	198	148	20	205.9	220.8	230.9	237
21	250	195	145	30	204.5	220.7	231.2	237.2
22	250	192	142	30	203.2	220.7	231.4	237.4
23	250	189	139	30	202	220.7	231.6	237.7
24	250	186	136	30	200.7	220.7	231.9	237.9
25	250	183	133	30	199.3	220.8	232.1	238.1
26	250	180	130	40	198.1	220.8	232.4	238.4
27	250	177	127	40	197	220.9	232.6	238.5
28	250	174	124	40	195.9	221	232.8	238.7

APPENDIX B (continued)

cell	init. hd.	top	bot.	K/S	day1	day2	day3	day4
					Poten. head at the end of:			
29	250	171	121	40	194.7	221.1	233.1	239
30	250	168	118	40	193.4	221.3	233.3	239.2
31	250	165	115	50	192.3	221.5	233.6	239.4
32	250	162	112	50	191.2	221.7	233.8	239.6
33	250	159	109	50	190.1	221.9	234	239.7
34	250	156	106	50	188.9	222.2	234.3	239.9
35	250	153	103	50	187.7	222.5	234.6	240.1
36	250	150	100	80	186.7	222.8	234.8	240.3
37	250	150	100	80	185.9	223.1	235	240.5
38	250	150	100	90	185.1	223.4	235.2	240.6
39	250	150	100	40	186.9	223.9	235.6	240.8
40	250	150	100	40	189.4	224.7	236.1	241.1
41	250	150	100	40	191.8	225.6	236.5	241.5
42	250	150	100	40	194.3	226.5	237.1	241.8
43	250	150	100	30	197	227.5	237.7	242.2
44	250	150	100	20	201	229.1	238.5	242.7
45	250	150	100	10	208	232	240.1	243.8
46	250	150	100	10	217.3	235.9	242.3	245.1
47	250	150	100	10	226.6	239.8	244.4	246.5
48	250	150	100	30	232.8	242.4	245.9	247.4
49	250	150	100	30	235.9	243.8	246.6	247.9
50	250	150	100	30	238.9	245.1	247.3	248.3
51	250	150	100	40	241.6	246.3	248	248.7
52	250	150	100	40	243.9	247.3	248.5	249.1
53	250	150	100	40	246.2	248.3	249.1	249.4
54	250	150	100	40	248.5	249.3	249.6	249.8
55	250	150	100	40	250	250	250	250