CONSIDERING CANAL POOL RESONANCE IN CONTROLLER DESIGN

Albert J. Clemmens[1]

ABSTRACT

The Integrator-Delay (**ID**) model (Schuurmans et al 1999) is a simple model of canal response that is used for design of various canal controllers. It describes the change in water depth at the downstream end of a canal pool as a function of flow changes at the upstream and downstream gates. Canal pools are characterized by a **Delay** time and a backwater surface area (**Integrator**). This model works very well for canal pools where water is flowing under normal depth conditions for a portion of the length, or where there are drops. For canal pools where the upstream flow depth is influenced by the downstream flow depth (that is, where canal pool is under backwater) the ID model often does not properly represent the water-level response. Changes in gate flows often cause a step change in water level. Schuurmans (1997) and Miltenburg (2008) propose the use of filters to account for this step change (ID-F), where the filter effectively causes a delay in response. Litrico and Fromion (2004) proposed the IDZ model, where a gate flow change causes a step change in downstream water level, after which the water level response follows the integrator of the ID model. The IDZ model does not fully account for resonance. An IDZ model with Filtering (IDZ-F) is proposed to account for additional resonance. In this paper, we compare the resulting water level response when the ID-F and IDZ-F models are used to design canal controllers for canal pools under backwater. It is shown that controllers designed with the IDZ-F model provide slightly better control than when designed with the ID-F model, although differences are not significant.

INTRODUCTION

Canal controller design requires a mathematical model of canal pool response. Most canal control methods rely on a linear model of canal pool response. Schuurmans (1997) suggest that there are two types of canal pools: 1) pools in which flow is governed by normal depth for some length at the upstream end and under back water at the downstream end and 2) pools which are entirely under backwater. For 1), Schuurans, et al (1999) propose the use of the Integrator-Delay (ID) model to describe canal response. The ID model determines the response of the water level at the downstream end of the pool to changes in gate discharge at either the upstream or downstream end. Resonance is usually not a problem for this type of canal pool. The model has two parameters: a delay time, τ, generally associated with the section under normal depth, and a backwater surface area, A, which functions as the integrator of the flow change. Clemmens and Schuurmans (2004) use state-transition equations to describe the ID model and then use Linear Quadratic Regulator (LQR) design to develop canal controllers. For 2), Litrico and Fromion (2004) propose the Integrator-Delay-Zero (IDZ) model to predict the water level response associated with canal pools under backwater. They use the same integrator and delay as in the ID model. The Zero essentially describes the influence of the celerity

[1] Center Director, U.S. Arid Land Agricultural Research Center, USDA/ARS, 21881 N. Cardon Lane, Maricopa, AZ 85138 bert.clemmens@ars.usda.gov

wave on the water level response. The purpose of this paper is to develop state-transition equations for the IDZ model and to compare canal controllers developed with the ID and IDZ models (with filters).

ID MODEL

The ID model assumes the water level at the downstream end of a pool responds linearly to changes in flow from steady state. It assumes immediate response to downstream flow change and a delayed response to upstream flow change. The ID model in continuous form is:

$$\begin{aligned} y_j(t) &= y_j(0) - \frac{tQ_j}{A_j} && t \le \tau_j \\ y_j(t) &= y_j(0) + \frac{(t-\tau_j)Q_{j-1}}{A_j} - \frac{tQ_j}{A_j} && t > \tau_j \end{aligned} \tag{1}$$

where y is water depth at the downstream end of a pool, Q is discharge, j represents the pool number and the gate at the downstream end, t is time, τ is the delay time for a wave to travel from the upstream to the downstream end of a pool, and A is the backwater surface area of a pool.

A discrete form of the ID model is:

$$\begin{aligned} \Delta y_j(k+1) &= \Delta y_j(k) - \frac{\Delta t}{A_j}\Delta Q_j(k) && t \le \tau_j \\ \Delta y_j(k+1) &= \Delta y_j(k) + \frac{\Delta t}{A_j}[\phi_{0j}\Delta Q_{j-1}(k) + \phi_{1j}\Delta Q_{j-1}(k-1) + \ldots] - \frac{\Delta t}{A_j}\Delta Q_j(k) && t > \tau_j \end{aligned} \tag{2}$$

where Δ represents the change in conditions over one time step, Δt, and φ are weighting coefficients that describe the water level slope change at each time step. The φ coefficients for any pool j sum to one. If a delay time falls in the middle of a time step, a portion of the water surface slope (and associated water level increase) occurs. The rest of the water surface slope is added at the next times step so that future water level slopes match the ID model slope ($\Delta Q/A$). The follow expression defines the φ terms for any pool j:

$$\begin{aligned} &\tau = 0 && \phi_0 = 1 \quad \phi_1 = 0 \quad \phi_2 = 0 \quad \ldots \\ &0 < \tau < \Delta t && \phi_0 = \frac{(\Delta t - \tau)}{\Delta t} \quad \phi_1 = \frac{\tau}{\Delta t} \quad \phi_2 = 0 \quad \ldots \\ &\Delta t < \tau < 2\Delta t && \phi_0 = 0 \quad \phi_1 = \frac{(2\Delta t - \tau)}{\Delta t} \quad \phi_2 = \frac{(\tau - \Delta t)}{\Delta t} \quad \phi_3 = 0 \quad \ldots \\ &2\Delta t < \tau < 3\Delta t && \phi_0 = 0 \quad \phi_1 = 0 \quad \phi_2 = \frac{(3\Delta t - \tau)}{\Delta t} \quad \phi_3 = \frac{(\tau - 2\Delta t)}{\Delta t} \quad \phi_4 = 0 \quad \ldots \\ &\ldots \end{aligned} \tag{3}$$

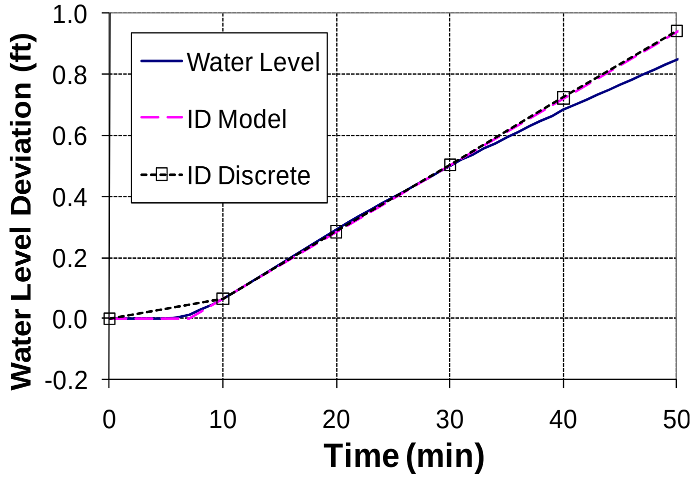

Figure 1. ID model in discrete and continuous form fit for canal pool CD-12. Step change in upstream discharge.

Eqs (2) and (3) were used by Clemmens and Schuurman (2004) to develop state-transition equations as input into controller design as Linear Quadratic Regulators (LQR) [linear model with quadratic penalties]. Figure 1 shows the downstream-water-level response in a canal pool for a step increase in flow at the upstream gate. The water level was computed from simulation with Sobek (2000). This pool does not display resonance. The ID model shown was fit by trial and error. Changes in cross section with depth cause the model to deviate from the simple ID model. The water level shown is for canal pool CD-12 at the Central Arizona Irrigation and Drainage District (CAIDD), Eloy, AZ. The pool has a bottom width of 4 ft, side slopes 1.5:1 (horizontal to vertical), slope 0.0012 m/m, length 3025 ft, a downstream set point depth of 5.05 ft, and a capacity of 110 cfs. The flow change was 5.5 cfs, and the backwater area was 0.346 ac. The ID discrete model in Figure 1 shows the water surface slope which is initially shallower than the ID model because of the discrete data points used (i.e., every 10 minutes). The change in slope is reflected by values of φ. The delay time, τ, for this model is 7 minutes, giving $\varphi_0 = 0.3$ and $\varphi_1 = 0.7$. Thus the initial slope of the response curve is 0.3 times the slope of the ID model.

ID-F MODEL

In canal pools under backwater, the dynamic wave created from an upstream flow change arrives suddenly at the downstream end, causing a sudden change in water level there. This can cause difficulty with water-level controllers. Schuurmans (1997) proposed the use of proportional-integral (PI) controllers with filtered water levels (PI-F) to assure control stability. This is currently the primary method for dealing with resonance in the design of canal controllers. A linear filter is used where

$$y_f(k) = F_c y_f(k-1) + (1-F_c)y(k) \tag{4}$$

where $y_f(k)$ is the filtered water level at time step k and F_c is the filter constant. The filter constant is determined from

$$F_c = e^{-T_s/T_f} \tag{5}$$

where T_f is the filter time constant and T_s is the time step for water level sampling. For Schuurman's (1997) PI-F controller based on the ID model, T_f is found from

$$T_f = \sqrt{\frac{AR_p}{\omega_r}} \tag{6}$$

where R_p is the resonance peak height, ω_r is the resonant frequency. The resonant frequency is 2π divided by the time for a wave to travel the length of a pool and back. Any disturbance in a canal pool causes a wave. These waves travel at the speed of celerity

$$c = \sqrt{gD} \tag{7}$$

where g is the acceleration of gravity and D is the hydraulic depth (area divided by top width). The wave travel time can be estimated from

$$\tau_r = \tau_z + \tau_{zu} = \frac{L}{v+c} + \frac{L}{v-c} \tag{8}$$

where L is the pool length, v is the average flow velocity, τ_z is the time for the wave to travel to the downstream end, and τ_{zu} is the time for the wave to travel to the upstream end. Then $\omega_r = 2\pi / \tau_r$.

The filter causes a delay in the water level response, which can be estimated from

$$t_{delay} = \frac{F_c}{1-F_c} T_s \tag{9}$$

For controller design, this delay time is added to the ID model time delay. The filter constant computed above based on Eq. 6 is for Schuurmans' PI-F controller, and may or may not be appropriate for other controllers.

Figure 2 shows the simulated (Sobek 2000) downstream-water-level response in a pool for a step increase in flow at the upstream gate. This canal pool clearly shows resonance. The ID model is fit to the long-term response. The oscillations of the water level around the ID model line indicate resonance. The response in Figure 2 is for pool CM-1 for the Central Main Canal of CAIDD, which has a capacity of 900 cfs, bottom width 12 ft, sides slopes 1.5 to 1 horizontal to vertical, length 17,119 ft, slope 0.00013 ft/ft, downstream water level set point depth 11.0 ft. The step change was 35.3 cfs.

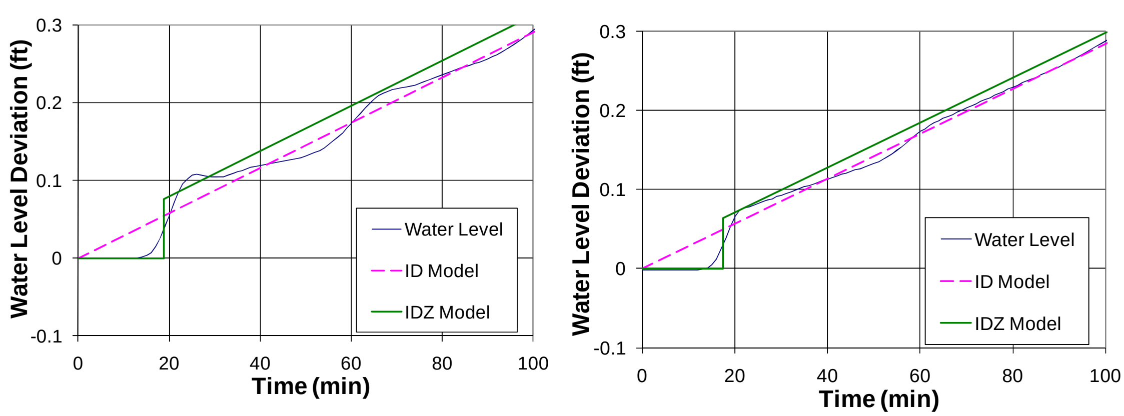

Figure 2. IDZ model in discrete and continuous form for canal pool CM-1 at a) 20% and b) 50% flow. Step change in upstream discharge.

IDZ MODEL

Litrico and Fromion (2004) proposed the IDZ model to account for the sudden rise in water level that occurs for pools which are under backwater, as shown in Figure 2. They developed a set of frequency-based relationships to describe the IDZ model. A time-based version in continuous form is:

$$
\begin{aligned}
y_j(t) &= y_j(0) - \left(\frac{t}{A_j} + p_{22}\right)Q_j & t \le \tau_{zj} + \tau_j \\
y_j(t) &= y_j(0) + \left(\frac{t-\tau_{zj}-\tau_j}{A_j} + p_{21}\right)Q_{j-1} - \left(\frac{t}{A_j} + p_{22}\right)Q_j & t > \tau_{zj} + \tau_j
\end{aligned}
\tag{10}
$$

where τ_{zj} is the wave travel time through pool j (Trd in Eq.8), and p_{21} and p_{22} are the transfer function for the sudden change in downstream water level for a change in upstream and downstream flow, respectively.

The height of the water level above the ID model line in Figure 2 (i.e., the first cycle) is an indicator of the resonance peak, although a higher wave would result if the input (upstream flow change) were cycled at the resonance frequency. The IDZ model response shown in Figure 2 was computed with equations for p_{21} and p_{22} from the Litrico and Fromion (2004). Note that for the lower flow rate, the peak height was under predicted. (This is not unexpected. Litrico 2010, personal communication). And this model over-predicts long-term changes in water level response. I also found that the estimates for this step height were very sensitive to estimates for the downstream water surface slope. Thus I decided to determine p_{22} and p_{21} based on matching the ID model, and then account for the additional resonance with a water level filter, as in the ID-F model. This approach can be considered a filtered IDZ model, or IDZ-F.

IDZ-F MODEL

For step changes in upstream discharge, we assume that the IDZ-F model steps up to the ID model line when the wave arrives. The step height is thus

$$\Delta y_j(step) = \frac{\tau_{zj}}{A_j}\Delta Q_j \tag{11}$$

Thus $p_{21} = \tau_z / A$. For a change in discharge at the downstream gate, the solution is less obvious. If we assume that the backwater area equals the pool length times the top width ($A_j \approx L_j B_j$) and that celerity is much larger than the average velocity ($\tau_{zj} \approx L_j/c_j$) substituting these relationships into Eq. 11 gives

$$\Delta y_j(step) \approx \frac{\Delta Q_j}{B_j c_j} \tag{12}$$

(This matches the solution for p_{22} by Litrico and Fromion (2004) for a Froude number of zero.) Figure 3 shows simulation results (Sobek 2000) where a step change in downstream discharge of 17.2 cfs (0.5 m^3/s) was made in pool CM-1. Initial flow was 450 cfs (50% of capacity). The value of the step change computed for p_{22} from Litrico and Fromion (2004) was 0.0272 ft. Eq (12) gives 0.0269 ft. The initial drop in level is well predicted (Figure 3). For simplicity, we can use Eq. 11 for changes in either the downstream or upstream gate.

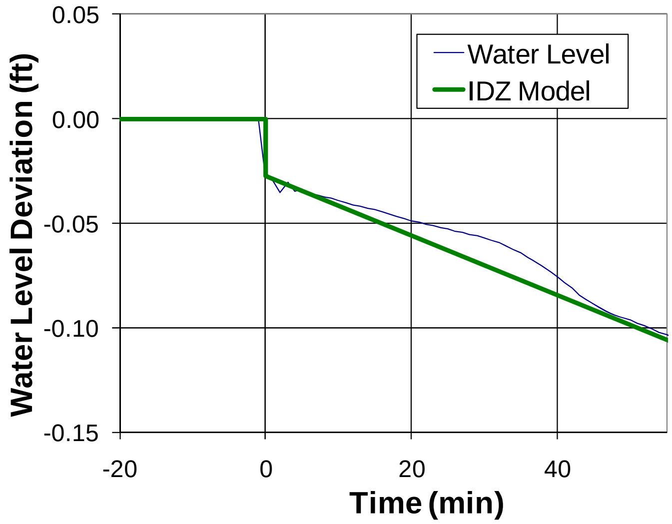

Figure 3. IDZ model response for CM-1 at 50% flow. Step change in downstream discharge.

A discrete version of the IDZ-F model is:

$$\Delta y_j(k+1) = \Delta y_j(k) - \left(\frac{\Delta t + \tau_{zj}}{A_j}\right)\Delta Q_j(k) + \frac{\tau_{zj}}{A_j}\Delta Q_j(k-1) \qquad t \le \tau_{zj} + \tau_j$$

$$\Delta y_j(k+1) = \Delta y_j(k) + \frac{\Delta t}{A_j}[\phi_{0j}\Delta Q_{j-1}(k) + \phi_{1j}\Delta Q_{j-1}(k-1) + ...] \qquad t > \tau_{zj} + \tau_j \qquad (13)$$

$$- \left(\frac{\Delta t + \tau_{zj}}{A_j}\right)\Delta Q_j(k) + \frac{\tau_{zj}}{A_j}\Delta Q_j(k-1)$$

The last term in the first of Eq. (13) brings the water level slope back to the ID model after the initial shock from moving the downstream gate. The filter delay is included in τ_j.

The following equation defines the φ terms for the IDZ-F model for any pool *j*. This version of the IDZ-F model is essentially a piece-wise linear model of dy/dt over time, where the response over the duration of one time step is linear. The functions described by φ essentially changes dy/dt. The slope dy/dt returns to the simple ID model after the step increase.

$$\tau_f + \tau = 0 \qquad \phi_0 = 1 \quad \phi_1 = 0 \quad \phi_2 = 0 \quad \ldots$$

$$0 < \tau_f + \tau < \Delta t \qquad \phi_0 = \frac{(\Delta t - \tau)}{\Delta t} \qquad \phi_1 = \frac{\tau}{\Delta t} \quad \phi_2 = 0 \quad \ldots$$

$$\Delta t < \tau_f + \tau < 2\Delta t \qquad \phi_0 = 0 \quad \phi_1 = \frac{(2\Delta t - \tau)}{\Delta t} \quad \phi_2 = \frac{(\tau - \Delta t)}{\Delta t} \quad \phi_3 = 0 \quad \ldots \qquad (14)$$

$$2\Delta t < \tau_f + \tau < 3\Delta t \qquad \phi_0 = 0 \quad \phi_1 = 0 \quad \phi_2 = \frac{(3\Delta t - \tau)}{\Delta t} \quad \phi_3 = \frac{(\tau - 2\Delta t)}{\Delta t} \quad \phi_4 = 0 \quad \ldots$$

$$\ldots$$

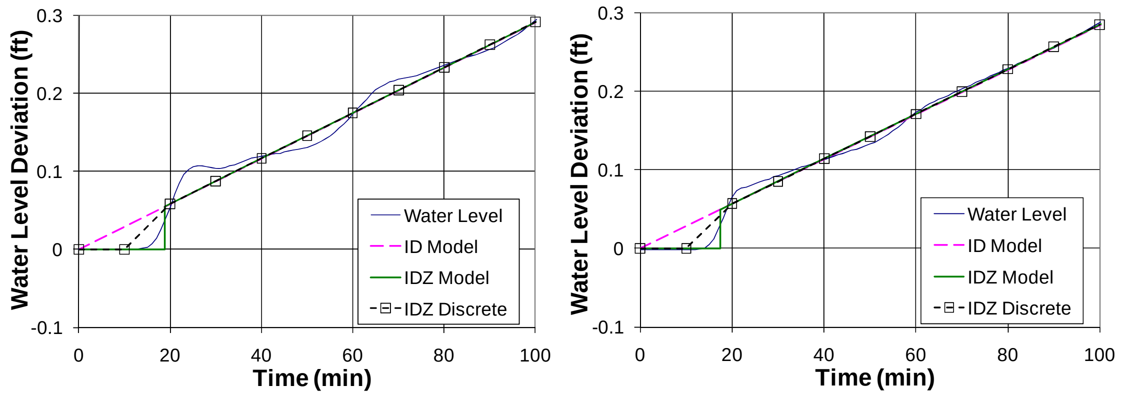

Figure 4. IDZ-F model in discrete and continuous form fit for canal pool CM-1 at 50% flow. (Filter time delay not shown).

Eqs. (3) and (14) are very similar. The only difference is that τ_{zj} influences the time step in which the step increase in water level occurs, while τ has essentially the same influence as in the ID model. A large difference occurs if τ_{zj} is large. Values of the first non-zero φ-term can be larger than unity, but the second non-zero φ-term is then negative to bring the sum back to unity, which brings the water level slope back to the ID model slope. An example of the step change for the discrete IDZ-F model (without filter delay) is shown in Figure 4, for the same scenario as given for Figure 2.

CONTROL EXAMPLE

The details of the Central Main Canal, the delay times and the filter constants are presented in Clemmens and Strand (2010). The parameters for the ID model were determined from steady-state simulation with Sobek (2000). For the ID-F model, filters were designed with the procedures of Schuurmans (1997). The filter delay for the ID-F model response was 14 minutes for all but pool 3, which had a delay of 8.7 min. For the IDZ-F model, we used a much smaller filter, essentially for antialiasing, with a 4.4

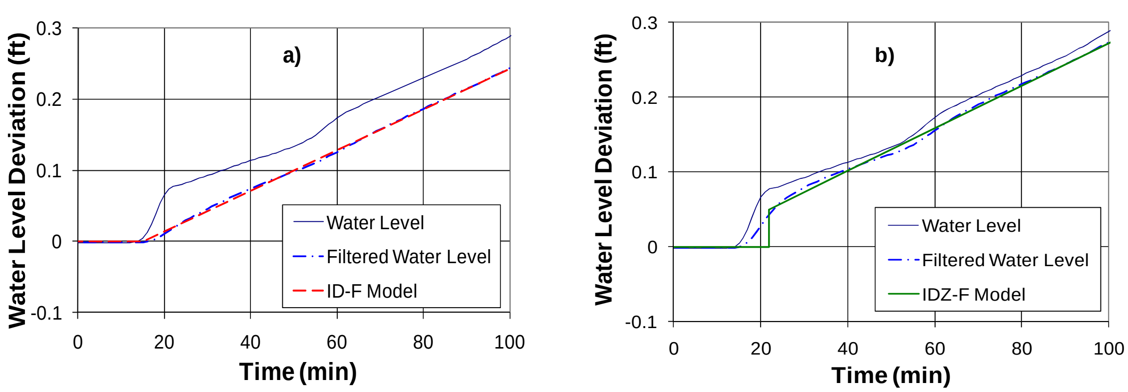

Figure 5. Water level response to upstream flow changes for pool CM-1 at 50% flow.
a) Filtered water level and delayed ID-F model (14 minute delay) and
b) Filtered water level and delayed IDZ-F model (4.4 minute delay).

minute delay. Figure 5 shows the approximate match between the filtered water level for pool CM-1 and the ID-F and IDZ-F models. These models fit the filtered water levels pretty well.

Downstream-water-level controllers were developed with the procedures of Clemmens and Schuurmans (2004) with either Eqs (2) and (3) for the ID-F model or Eqs (13) and (14) for the IDZ-F model. Controller tuning (i.e., tradeoffs between water level deviations and flow rate changes) was the same for both. Steady flow was established, and a turnout in pool CM-5 was suddenly increased by 17.6 cfs (0.5 m^3/s) without knowledge by the controller. Figures 6 and 7 show the water level response for these controllers. Overall, both controllers performed well. The controller designed with the IDZ-F model had a smaller deviation in water level than ID-F. However, the difference was not great. This is expected, since this controller had the most mechanistic response and shorter filter delays. Deviations in neighboring pools differed somewhat in which controller responded better. A more careful analysis of filter constants and tuning would be required to give a more definitive comparison.

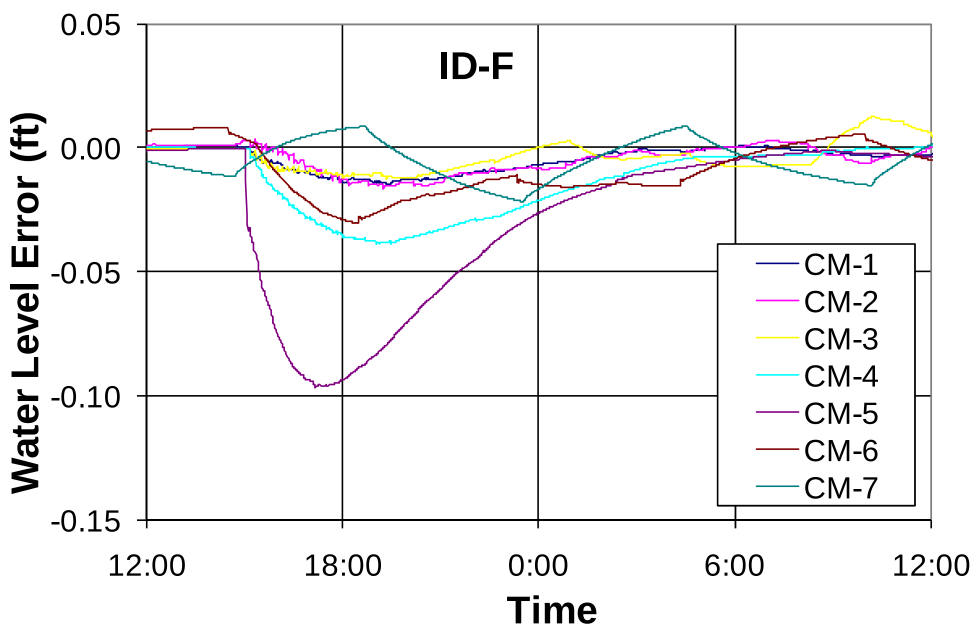

Figure 6. Response of water level to disturbance in pool CM-5: controller designed from ID-F model. (14 minute filter delay).

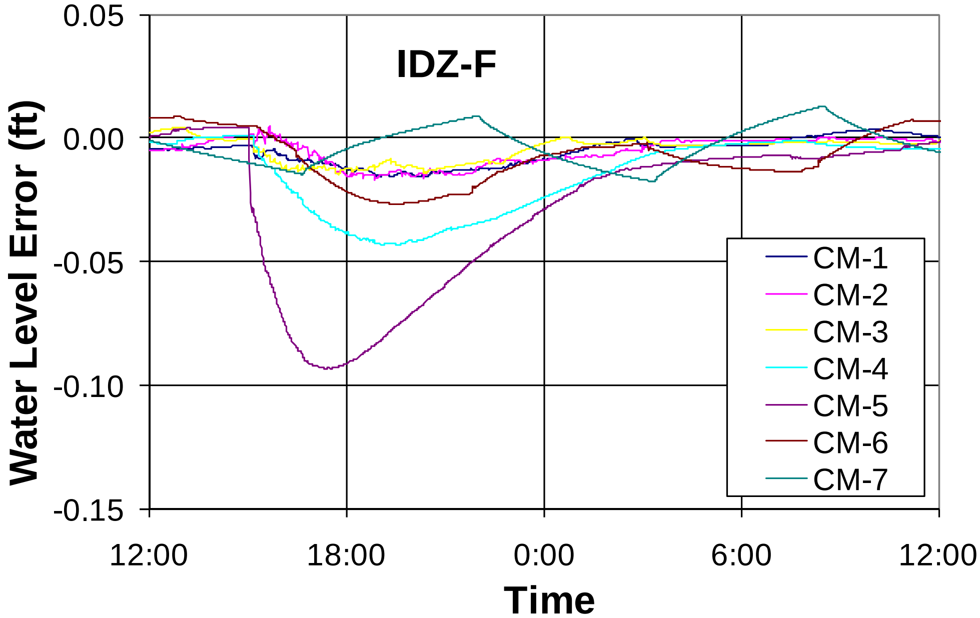

Figure 7. Response of water level to disturbance in pool CM-5: controller designed from IDZ-F model. (4.4 minute delay)

REFERENCES

Clemmens, A. J. and Schuurmans, J. (2004). "Simple optimal downstream feedback canal controllers: Theory." *J. Irrig. and Drain. Eng.,* 130(1), 26-34.

Clemmens, A.J. and Strand, R.J. (2010). Distributing flow mismatches in supply-constrained irrigation canals through feedback control. Proceedings USCID Water Management Conference. March 23-26, 2010. CD-ROM pp 41-52.

Litrico, X., Fromion, V. (2004). "Simplified modeling of irrigation canals for controller design," *J. Irrig. and Drain. Eng*, 130(5), 373-383.

Miltenburg, I.J. (2008), *Determination of canal pool properties by experimental modeling*, Master-thesis Delft University of Technology, The Netherlands.

Schuurmans, J. (1997). *Control of water levels in open channels*. Ph.D.-dissertation Delft University of Technology, The Netherlands.

Schuurmans, J., Clemmens, A. J., Dijkstra, S., Ahmed, R. H., Bosgra, O. H., and Brouwer, R. (1999). "Modeling of irrigation and drainage canals for controller design." *J. Irrig. and Drain. Eng.,* 125(6), 338-344.

Sobek (2000) "Manual and Technical Reference." WL|Delft Hydraulics, Delft, The Netherlands.