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MEASUREMENT OF TURBULENCE IN THREE-DIMENSIONAL MEAN FLOW

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ABSTRACT

A hot-wire anemometer for measuring turbulence in three-dimensional mean flow is presented. Effect of three-dimensionality of mean flow on a yawed wire's sensitivity to longitudinal, vertical and lateral fluctuations is brought out. A four-wire probe is shown to be suitable for measuring all the mean flow and turbulent quantities of interest.

Errors due to the cross flow component on turbulence measurements in two dimensional flows using conventional hot-wire techniques are estimated. Measurements of shear are shown to be very sensitive to even small amounts of cross flow that might be present in many laboratory and field flows of interest.

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MEASUREMENT OF TURBULENCE IN THREE-DIMENSIONAL MEAN FLOW

1. Introduction

Conventional hot-wire techniques assume the mean flow to be one-dimensional, the direction of mean velocity vector at any point being taken to be coincident with that of the general flow. Although, most of the turbulent flows encountered in the laboratory, e.g., those of boundary layers, mixing layers, wakes and jets, are never one-dimensional, the secondary flow components are usually so small that no appreciable errors due to them are introduced. On the other hand, if the stream lines deviate significantly from the general flow direction, e.g., in the regions of abrupt changes of surface conditions, these errors may no longer be negligible.

There are other fluid flows of interest in which mean flow is clearly two- or three-dimensional in nature, e.g., flow around large obstacles, diverging or converging flows, etc. To our knowledge, no turbulence measurements have been reported, in which the three-dimensional nature of the mean flow has been considered. A quantitative evaluation of the errors due to secondary flow components on measurements using conventional hot-wire techniques is also lacking. It is the purpose of this report to determine these errors, as well as, to describe a technique of measuring turbulence in three-dimensional flows.

2. Derivation of Hot-Wire Response Equations

Although, the dynamic response equation for a hot-wire placed in any general manner with respect to the coordinate axes can be derived, for simplicity of algebra we will consider only two most convenient orientations viz., (a) when the wire is in x,z plane and is yawed by an angle θ to the x-axis, and (b) when the wire is in x,y plane and is yawed by an angle ϕ to the x-axis.

Let U, V and W be the mean flow components and u', v', and w' the fluctuations in x, y and z directions, respectively. As in conventional hot-wire technique, we assume that fluctuations are small in comparison to the mean velocity, so that second and higher order terms in u'/U, v'/U and w'/U can be neglected.

For a given hot-wire anemometer and operating conditions, heat transfer and hence, the voltage output across the wire will depend on the total velocity vector U_{tot} and the angle α between the velocity vector and the axis of the wire. That is,

$$E = f(U_{tot}, \alpha) \quad . \tag{1}$$

It is convenient to combine the two variables U_{tot} and α, into what is called the "effective cooling velocity" U_{eff}, so that one can write

$$E = f_1(U_{eff}) \quad , \tag{2}$$

and in the differential form

$$dE = \left(\frac{\partial E}{\partial U_{eff}}\right) d\,U_{eff} \tag{3}$$

There has been much discussion in the past as to what should be the effective cooling velocity. The so-called cosine-law is based on the assumption that only that component of the velocity vector which is normal to the wire affects the heat transfer from the wire. This would have been true in case of a very long wire. It is now generally recognized that for a finite length wire (say, $1/d < 1000$), the component of the velocity parallel to the wire is also, to some extent, significant in heat transfer. Recent works by Webster (1962), Delleur (1966) and Champagne et al., (1967), all point out the merits of using the following expression for U_{eff}, which was suggested by Hinze (1959):

$$U^2_{eff} = U^2_{tot} \; (\sin^2\alpha + a^2 \cos^2 \alpha) = U^2_n + a^2 \, U^2_t \tag{4}$$

in which, 'a' is an empirical constant with a value between 0.1 and 0.3, and U_n and U_t are the normal and parallel components of the velocity vector. Precise measurements of heat transfer from hot-wires by Champagne et al., (1967) show that a is essentially independent of the material of the wire and the yaw angle, but it depends on length to diameter ratio. An experimental plot of a vs $1/d$ has been given by these authors. In what follows, we will assume U_{eff} to be given by Eq. (4) in which a is known.

A Wire in x, z - plane

For wire configuration of Fig. 1 (a), we have

$$U^2_n = \Big[(U+u') \sin\theta + (W+w') \cos\theta\Big]^2 + (V+v')^2 \tag{5}$$

$$U^2_n = \Big[(U+u') \cos\theta - (W+w') \sin\theta\Big]^2 . \tag{6}$$

Substituting in Eq. (4) then, we obtain

$$U^2_{eff} = \Big[(U+u') \sin\theta + (W+w') \cos\theta\Big]^2 + (V+v')^2$$

$$+ \, a^2 \Big[(U+u') \cos\theta - (W+w') \sin\theta\Big]^2 \tag{7}$$

which, after simplification and neglecting second order terms can be written as

$$\frac{U^2_{eff}}{U^2} = (\sin\theta + q\cos\theta)^2 + p^2 + a^2(\cos\theta - q\sin\theta)^2$$
$$+ 2\frac{u'}{U}\left[\sin^2\theta + q\sin\theta\cos\theta + a^2(\cos^2\theta - q\sin\theta\cos\theta)\right]$$
$$+ 2\frac{v'}{U}p + 2\frac{w'}{U}\left[q\cos^2\theta + \sin\theta\cos\theta + a^2(q\sin^2\theta - \sin\theta\cos\theta)\right] \tag{8}$$

in which $p = \frac{V}{U}$, (9)

and $q = \frac{W}{U}$. (10)

For the sake of brevity, we let

$$F = (\sin\theta + q\cos\theta)^2 + p^2 + a^2(\cos\theta - q\sin\theta)^2 \tag{11}$$

$$G = \sin^2\theta + q\sin\theta\cos\theta + a^2(\cos^2\theta - q\sin\theta\cos\theta) \tag{12}$$

and

$$H = q\cos^2\theta + \sin\theta\cos\theta + a^2(q\sin^2\theta - \sin\theta\cos\theta) . \tag{13}$$

Equation (8) can then be written as

$$U_{eff} = U\left[F + 2\frac{u'}{U}G + 2\frac{v'}{U}p + 2\frac{w'}{U}H\right]^{1/2} \tag{14}$$

which, after differentiation gives

$$dU_{eff} = UF^{1/2}\left(1 + 2\frac{u'}{U}\frac{G}{F} + 2\frac{v'}{U}\frac{p}{F} + 2\frac{w'}{U}\frac{H}{F}\right)^{-1/2}$$
$$\left\{\frac{G}{F}\frac{du'}{U} + \frac{p}{F}\frac{dv'}{U} + \frac{H}{F}\frac{dw'}{U}\right\} . \tag{15}$$

We can write Eq. (3) as

$$dE = \left(\frac{\partial E}{\partial U}\right)\left(\frac{\partial U_{eff}}{\partial U}\right)^{-1} d\,U_{eff} \quad . \tag{16}$$

After expanding Eq. (14) in powers of u'/U etc., neglecting second and higher order terms and differentiating with respect to U, one obtains

$$\frac{\partial U_{eff}}{\partial U} = F^{1/2} \quad . \tag{17}$$

Substituting from Eqs. (15) and (17) into Eq. (16), we get

$$dE = \frac{\partial E}{\partial U}\left(1 + 2\frac{u'}{U}\frac{G}{F} + 2\frac{v'}{U}\frac{p}{F} + 2\frac{w'}{U}\frac{H}{F}\right)^{-1/2}\left(\frac{G}{F}du' + \frac{p}{F}dv' + \frac{H}{F}dw'\right) \tag{18}$$

or,

$$dE = \frac{\partial E}{\partial U}\left(1 - \frac{u'}{U}\frac{G}{F} - \frac{v'}{U}\frac{p}{F} - \frac{w'}{U}\frac{H}{F} + \text{higher order terms}\right)\left(\frac{G}{F}du' + \frac{p}{F}dv' + \frac{H}{F}dw'\right) \quad . \tag{19}$$

After neglecting terms like $u'du'$ etc., and other higher order terms, one obtains

$$dE = \frac{\partial E}{\partial U}\left(\frac{G}{F}du' + \frac{p}{F}dv' + \frac{H}{F}dw'\right) \quad . \tag{20}$$

Replacing differentials by fluctuations themselves in Eq. (20) as is done in conventional hot-wire anemometry, we arrive at the following response equation of the hot-wire in the form it can be used for actual measurements.

$$e' = S_{u_\theta} u' + S_{v_\theta} v' + S_{w_\theta} w' \tag{21}$$

in which sensitivities S_{u_θ}, S_{v_θ} and S_{w_θ} are given after substituting from Eqs. (11)-(13).

$$S_{u_\theta} = \frac{\partial E}{\partial U}\left[\frac{\sin\theta(\sin\theta + q\cos\theta) + a^2\cos\theta(\cos\theta - q\sin\theta)}{(\sin\theta + q\cos\theta)^2 + p^2 + a^2(\cos\theta - q\sin\theta)^2}\right] \tag{22}$$

$$S_{v_\theta} = \frac{\partial E}{\partial U}\left[\frac{p}{(\sin\theta + q\cos\theta)^2 + p^2 + a^2(\cos\theta - q\sin\theta)^2}\right] \tag{23}$$

$$S_{w_\theta} = \frac{\partial E}{\partial U}\left[\frac{\cos\theta(\sin\theta + q\cos\theta) - a^2\sin\theta(\cos\theta - q\sin\theta)}{(\sin\theta + q\cos\theta)^2 + p^2 + a^2(\cos\theta - q\sin\theta)^2}\right]. \tag{24}$$

As a check on our procedure, we see that by substituting for $p=q=0$ in the above, our response equation reduces to that of a conventional yawed wire (see Arya, 1968) viz.,

$$e' = \frac{\partial E}{\partial U}\left[u' + cw'\cot\theta\right] \tag{25}$$

in which

$$c = \frac{1-a^2}{1+a^2\cot^2\theta}, \tag{26}$$

appears as a correction factor in the otherwise simple relation obtainable from the "cosine-law" assumption.

From consideration of Eqs. (21) through (24), we see that with non-negligible secondary flow components, even a normal wire $(\theta = 90^\circ)$ is sensitive to all three fluctuating components.

B. Wire in x, y plane

Following the same procedure as in (A) we can obtain the following equations for a hot-wire placed in the x,y plane at an angle of ϕ with x-axis (Fig. 1(b))

$$e' = S_{u_\phi} u' + S_{v_\phi} v' + S_{w_\phi} w' \tag{27}$$

where

$$S_{u_\phi} = \frac{\partial E}{\partial U}\left[\frac{\sin\phi(\sin\phi + p\cos\phi) + a^2\cos\phi(\cos\phi - p\sin\phi)}{(\sin\phi + p\cos\phi)^2 + q^2 + a^2(\cos\phi - p\sin\phi)^2}\right] \tag{28}$$

$$S_{v_\phi} = \frac{\partial E}{\partial U}\left[\frac{\cos\phi(\sin\phi + p\cos\phi) - a^2\sin\phi(\cos\phi - p\sin\phi)}{(\sin\phi + p\cos\phi)^2 + q^2 + a^2(\cos\phi - p\sin\phi)^2}\right] \tag{29}$$

$$S_{w_\phi} = \frac{\partial E}{\partial U}\left[\frac{q}{(\sin\phi + p\cos\phi)^2 + q^2 + a^2(\cos\phi - p\sin\phi)^2}\right] . \tag{30}$$

3. Effect of Cross-Flow Component on Hot-Wire Measurements in Two-Dimensional Flows

Before we outline a method of measuring turbulence in three-dimensional flows, it will be of interest to investigate the effect of cross-flow component of the mean motion on turbulence measurements in two-dimensional flows such as boundary layers, jets, etc. We consider different wire arrangements which are commonly used. Let the flow be two-dimensional in x,y plane, so that $q = 0$.

A. Vertical normal wire

In this case, $\phi = 90^\circ$, and the wire response equation reduces to

$$e' = \frac{\partial E}{\partial U}\left[\frac{1}{1+a^2p^2}u' + \frac{a^2p}{1+a^2p^2}v'\right] . \tag{31}$$

Normally, root mean square of longitudinal fluctuations is evaluated as

$$(\overline{u'^2})_m^{1/2} = \frac{(\overline{e'^2})^{1/2}}{\dfrac{\partial E}{\partial U}} \tag{32}$$

which we have suffixed by m indicating it as measured value as against the actual value given by Eq. (31). A correction factor defined by the ratio of measured to actual value is obtained as

$$C_{u'} = (\overline{u'^2})^{1/2}/(\overline{u'^2})_m^{1/2} = \Big[(1 + a^2p^2)^2 - a^4p^2(\overline{v'^2}/\overline{u'^2_m}) - 2\,a^2p(\overline{u'v'}/\overline{u'^2_m})\Big]^{1/2} \tag{33}$$

In the first approximation of the correction, measured values can be used for $\overline{v'^2}$ and $\overline{u'v'}$ on the right-hand side of Eq. (33). This, then, can be further refined in successive steps.

B. Horizontal normal wire

In this case, we have $\theta = 90^\circ$, and Eq. (27) reduces to

$$e' = \frac{\partial E}{\partial U}\left[\frac{1}{1+p^2}u' + \frac{p}{1+p^2}v'\right] \tag{34}$$

A correction factor can again be obtained in the form

$$C_{u'} = \Big[(1+p^2)^2 - p^2(\overline{v'^2}/\overline{u'^2_m}) - 2p(\overline{u'v'}/\overline{u'^2_m})\Big]^{1/2} . \tag{35}$$

C. X-wires

Let us now consider the effect of vertical mean velocity on measurements of $(\overline{v'^2})^{1/2}$, $\overline{u'v'}$ and $(\overline{u'^2})^{1/2}$ using a pair of matched x-wires ($\phi = \pm 45^\circ$). We have from Eqs. (27)-(30)

$$e_{+45} = \frac{\partial E}{\partial U}\left[\left\{\frac{1+p+a^2(1-p)}{(1+a^2)(1+p^2) + 2p(1-a^2)}\right\}u' + \left\{\frac{1+p-a^2(1-p)}{(1+a^2)(1+p^2) + 2p(1-a^2)}\right\}v'\right] \tag{36}$$

$$e_{-45} = \frac{\partial E}{\partial U}\left[\left\{\frac{1-p+a^2(1+p)}{(1+a^2)(1+p^2) - 2p(1-a^2)}\right\}u' - \left\{\frac{1-p-a^2(1+p)}{(1+a^2)(1+p^2) - 2p(1-a^2)}\right\}v'\right] . \tag{37}$$

Although, algebra is much involved, the correction factors for measurements of $(v'^2)^{1/2}$, $\overline{u'v'}$ and $(\overline{u'^2})^{1/2}$ using x-wires technique can also be obtained in a straight forward manner. These are given as

$$C_{v'}^2 = \overline{v'^2}/\overline{v'^2_m} = \left[1 - p^2 + \frac{16\,p^2a^2}{(1-p^2)(1+a^2)^2}\right]^2 - p^2(\overline{u'^2}/\overline{v'^2_m}) + 2p(\overline{u'v'}/\overline{v'^2_m}) \tag{38}$$

$$\begin{aligned} C_{u'v'} = \overline{u'v'}/\overline{u'v'_m} = {} & \frac{1}{1+p^2}\left[1 - p^2 + \frac{16\,p^2a^2}{(1-p^2)(1+a^2)^2}\right]^2 \\ & + \frac{p}{1+p^2}\left[1 + \frac{8\,p^2a^2}{(1-p^2)(1+a^2)^2}\right](\overline{u'^2}/\overline{u'v'_m}) \\ & - \frac{p}{1+p^2}\left[1 + \frac{2p^2}{1-p^2} - \frac{2}{1-p^2}\left(\frac{1-a^2}{1+a^2}\right)^2\right](\overline{v'^2}/\overline{u'v'_m}) \end{aligned} \tag{39}$$

$$\begin{aligned} C_{u'}^2 = \overline{u'^2}/\overline{u'^2_m} = {} & \left[\frac{1 + p^2 - \dfrac{4p^2}{1+p^2}\left(\dfrac{1-a^2}{1+a^2}\right)^2}{1 - \dfrac{2p^2}{1+p^2}\left(\dfrac{1-a^2}{1+a^2}\right)^2}\right]^2 \\ & - p^2\left[\frac{1 - \dfrac{2}{1+p^2}\left(\dfrac{1-a^2}{1+a^2}\right)^2}{1 - \dfrac{2p^2}{1+p^2}\left(\dfrac{1-a^2}{1+a^2}\right)^2}\right]^2(\overline{v'^2}/\overline{u'^2_m}) \\ & - 2p\left[\frac{1 - \dfrac{2}{1+p^2}\left(\dfrac{1-a^2}{1+a^2}\right)^2}{1 - \dfrac{2p^2}{1+p^2}\left(\dfrac{1-a^2}{1+a^2}\right)^2}\right](\overline{u'v'}/\overline{u'^2_m}) \quad . \end{aligned} \tag{40}$$

In order to have an idea of under what conditions the above derived corrections would become important, we represent them as in

Figs. 2, 3 and 4 as functions of the cross-flow parameter p , for particular values of $(\overline{v'^2}/\overline{u'^2}) = 0.25$, and $\overline{u'v'}/(\overline{u'^2})^{1/2} (\overline{v'^2})^{1/2} =$ $- 0.4$, which are typical for a boundary layer.

We note that the horizontal normal wire, which is otherwise less vulnerable to other common errors such as those due to large gradients of mean velocity and turbulent intensities along the wire, finite wire length, proximity of a solid boundary, etc., is more affected by the vertical component of mean velocity than the vertical wire. It can be seen from Fig. 2 that for value of p up to 0.1, errors in longitudinal turbulent intensity measurements are within 3% and can be neglected. For more significantly diverging or converging two-dimensional flows, conventional techniques can still be used, but proper corrections must be applied.

Figures 3 and 4 indicate that the errors due to cross-flow component are more significant in measurements of traverse velocity fluctuations and turbulent shear stress using x-wire technique. In particular, shear measurements can be very much in error which is about 10% for $V/U = .02$, and increases proportionately with V/U . This fact has not been recognized previously.

By quantitatively expressing the effect of cross flow on turbulence measurements using conventional hot-wire techniques, we have in fact discovered a rather simple method of measuring turbulence in diverging or converging two-dimensional flows.

4. Measurement Technique in Three-Dimensional Mean Flow

In the case of three-dimensional mean flow, one can still extend the method of the preceding section whereby, expressions are obtained for corrections to be applied to the measurements made by assuming one-dimensional mean flow. These are going to be much more complicated, however, and may not be convenient to use. Root-mean-square voltage output of a wire, now, contains information about six Reynolds stresses in varying order of their magnitude. In principle, one can operate the wire in six different positions (yaw angles) and then determine the unknowns from the solution of six simultaneous equations so obtained. In practice, however, it would not be possible to determine, to any reasonable accuracy, more than two or three of these quantities.

Another and perhaps much better method will be to use a three-wire probe, record the fluctuating voltage signals from three wires simultaneously on a magnetic tape and, then, analyze them digitally. This method has been successfully used in our Laboratory for measuring the joint statistics of velocity and temperature fluctuations from the output of three hot wires placed in a thermally stratified boundary layer (results not yet published).

In the design of a probe for measuring turbulence in three-dimensional flows, it will be of interest to plot wire sensitivities S_{u_ϕ} , S_{v_ϕ} and S_{w_ϕ} as functions of ϕ for different values of the cross-flow parameters p and q . This has been done in Figs. 5, 6 and 7 in which S_{u_ϕ} , etc., have been normalized by a reference sensitivity $(\partial E/\partial U)_n$, given by the calibration of the wire placed normal to the flow. We have assumed $a=0$, and

$$\frac{\partial E}{\partial U} = \left(\frac{\partial E}{\partial U}\right)_n |\sin\phi| \quad . \tag{41}$$

In Fig. 8 are represented the ratios S_{v_ϕ}/S_{u_ϕ} and S_{w_ϕ}/S_{u_ϕ} . For convenience in the graphical representation of Figs. 5 through 8,

$$\psi = \pm(90 - |\phi|) \tag{42}$$

has been chosen for the abscissa; ψ represents the angle which the normal to the wire will make with the x-axis. We note that for $\psi = \pm 45^{\circ}$ $(\phi = \pm 45^{\circ})$, the wire is almost equally sensitive to both u' and v' irrespective of the magnitude of cross flow. Similarly, a wire in x,z-plane yawed at $\theta = 45^{\circ}$ will be equally sensitive to u' and w'. After considering several probe combinations, we have chosen a four-wire probe which has two wires arranged in V-form in x,y-plane and the two wires in V-form in x,z-plane as shown in Fig. 9. This choice was also dictated by the fact that it is most suitable for determining cross-flow parameters p and q as shown in the following section.

5. Measurement of Mean Flow Components

In the previous sections we have assumed that the mean-flow parameters p and q are known from other set of measurements for each point in the flow field where turbulence measurements are intended. It will be most desirable, of course, if the same probe can be used for measurement of turbulent, as well as mean-flow quantities. This is what can be accomplished by our four-wire probe as shown in the following.

Let us consider the response of a yawed wire to mean flow. We have, after integrating Eq. (3) and making linearization assumption,

$$E = \left(\frac{\partial E}{\partial U_{eff}}\right) U_{eff} \tag{41}$$

or,

$$E = \left(\frac{\partial E}{\partial U} U_{eff}\right)\left(\frac{\partial U_{eff}}{\partial U}\right)^{-1} . \tag{42}$$

For a yawed wire in x-y plane and considering mean flow only, we have

$$U_{eff} = \left[(U \sin\phi + V \cos\phi)^2 + a^2(U \cos\phi - V \sin\phi)^2 + W^2\right]^{1/2} \tag{43}$$

and

$$U_{eff}\left(\frac{\partial U_{eff}}{\partial U}\right) = U \sin\phi + V \sin\phi \cos\phi + a^2(U \cos\phi - V \sin\phi \cos\phi) . \tag{44}$$

Substituting from Eqs. (43) and (44) in Eq. (42), one obtains

$$E_\phi = U \frac{\partial E}{\partial U}\left[\frac{(\sin\phi + p \cos\phi)^2 + a^2(\cos\phi - p \sin\phi)^2 + q^2}{\sin\phi + p \sin\phi \cos\phi + a^2(\cos\phi - p \sin\phi \cos\phi)}\right] \tag{45}$$

Similarly for a yaw angle of -ϕ, one obtains

$$E_{-\phi} = U \frac{\partial E}{\partial U}\left[\frac{(\sin\phi - p \cos\phi)^2 + a^2(\cos\phi + p \sin\phi)^2 + q^2}{\sin\phi - p \sin\phi \cos\phi + a^2(\cos\phi + p \sin\phi \cos\phi)}\right] . \tag{46}$$

Similar equations can be derived for yawed wires in x,z-plane. We can consider now the effect of mean flow on our four-wire probe.

Let E_1 and E_2 be the d.c. voltages across two wires ($\phi = \pm 45^\circ$) in x,y-plane. Then,

$$E_1 = U\left(\frac{\partial E}{\partial U}\right)_1 \left[\frac{(1+p)^2 + a^2(1-p)^2 + q^2}{(\sqrt{2}+p) + a^2 (\sqrt{2}-p)}\right] \tag{47}$$

$$E_2 = U\left(\frac{\partial E}{\partial U}\right)_2 \left[\frac{(1-p)^2 + a^2 (1+p)^2 + q^2}{(\sqrt{2}-p) + a^2 (\sqrt{2}+p)}\right] . \tag{48}$$

It is easy to show from Eqs. (47) and (48) that

$$p = \left(\frac{\sqrt{2}}{2\sqrt{2}-1}\right)\left(\frac{1+a^2}{1-a^2}\right)\left(\frac{U_1-U_2}{U_1+U_2}\right)\left[\frac{1-p^2\left(\sqrt{2}\,\frac{1-a^2}{1+a^2}-1\right) + \frac{q^2}{1+a^2}}{1 - \frac{p^2}{(2\sqrt{2}-1)} - \frac{q^2}{(2\sqrt{2}-1)(1+a^2)}}\right] . \tag{49}$$

in which $U_1 = E_1/\left(\frac{\partial E}{\partial U}\right)_1$ and $U_2 = E_2/\left(\frac{\partial E}{\partial U}\right)_2$. Similarly, we will have for other two wires in x,z plane

$$q = \left(\frac{\sqrt{2}}{2\sqrt{2}-1}\right)\left(\frac{1+a^2}{1-a^2}\right)\left(\frac{U_3-U_4}{U_3+U_4}\right)\left[\frac{1-q^2\left(\sqrt{2}\,\frac{1-a^2}{1+a^2}-1\right)+\frac{p^2}{1+a^2}}{1-q^2\left(\frac{1}{2\sqrt{2}-1}\right)-\frac{p^2}{(2\sqrt{2}-1)(1+a^2)}}\right] \quad . \tag{50}$$

Mean flow parameters p and q can be determined using Eqs. (49) and (50) by writing them in the form

$$p = C_p\left(\frac{\sqrt{2}}{2\sqrt{2}-1}\right)\left(\frac{U_1-U_2}{U_1+U_2}\right) \tag{51}$$

$$q = C_q\left(\frac{\sqrt{2}}{2\sqrt{2}-1}\right)\left(\frac{U_3-U_4}{U_3+U_4}\right) \tag{52}$$

in which C_p and C_q are corrective factors, which may be assumed to be equal to unity in the first approximation, and then can be evaluated for obtaining second or higher order approximation of p and q . Main component U of the mean flow can then be determined from any one of the Eqs. (47), (48), etc.

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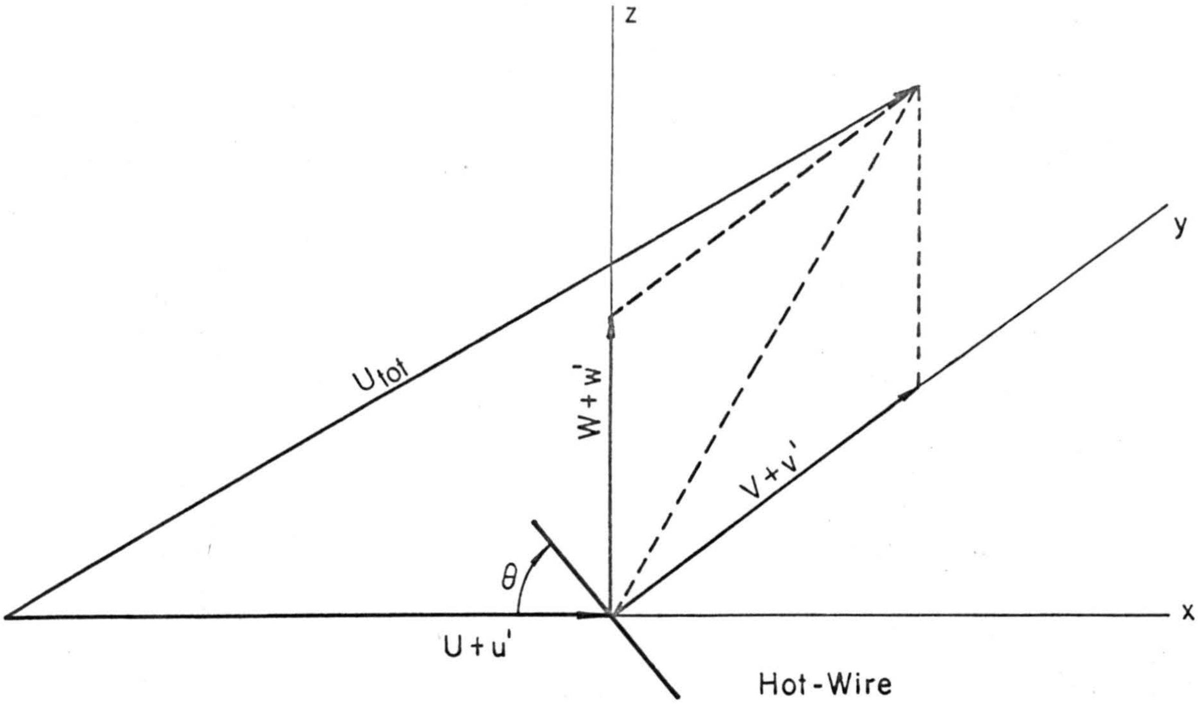

Fig. 1(a). Yawed wire in x, z - plane

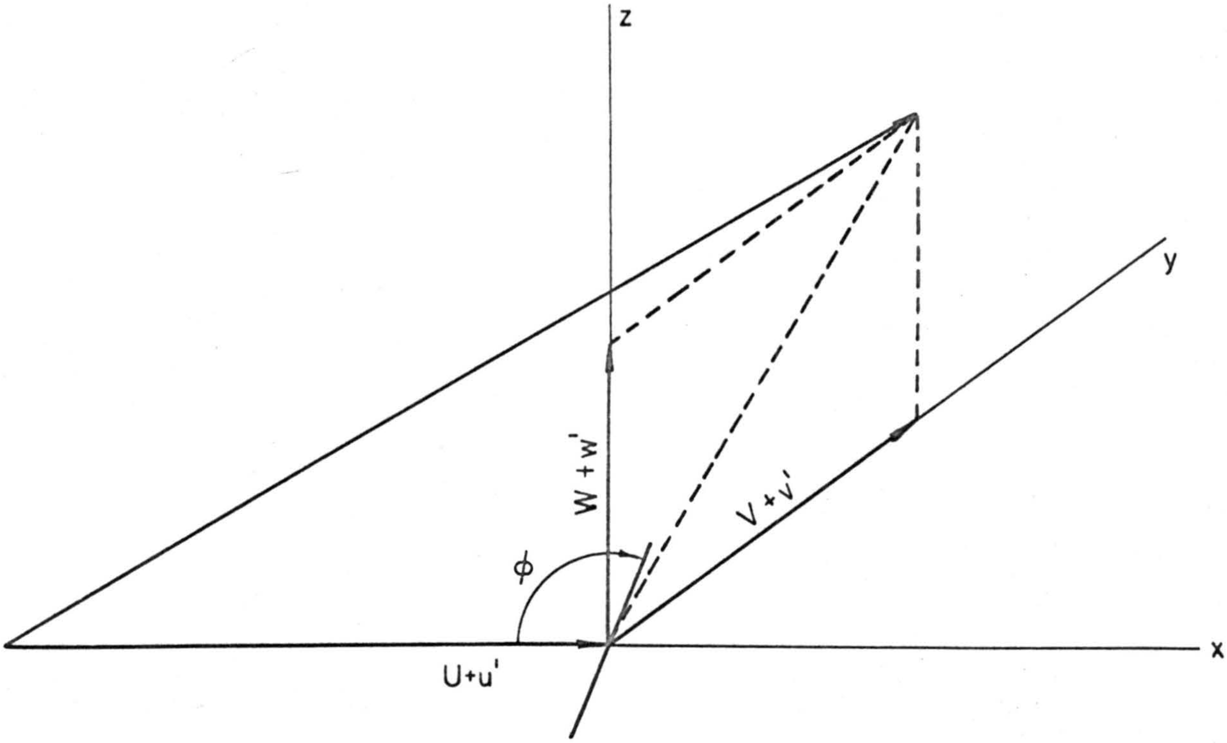

Fig. 1(b). Yawed wire in x, y - plane

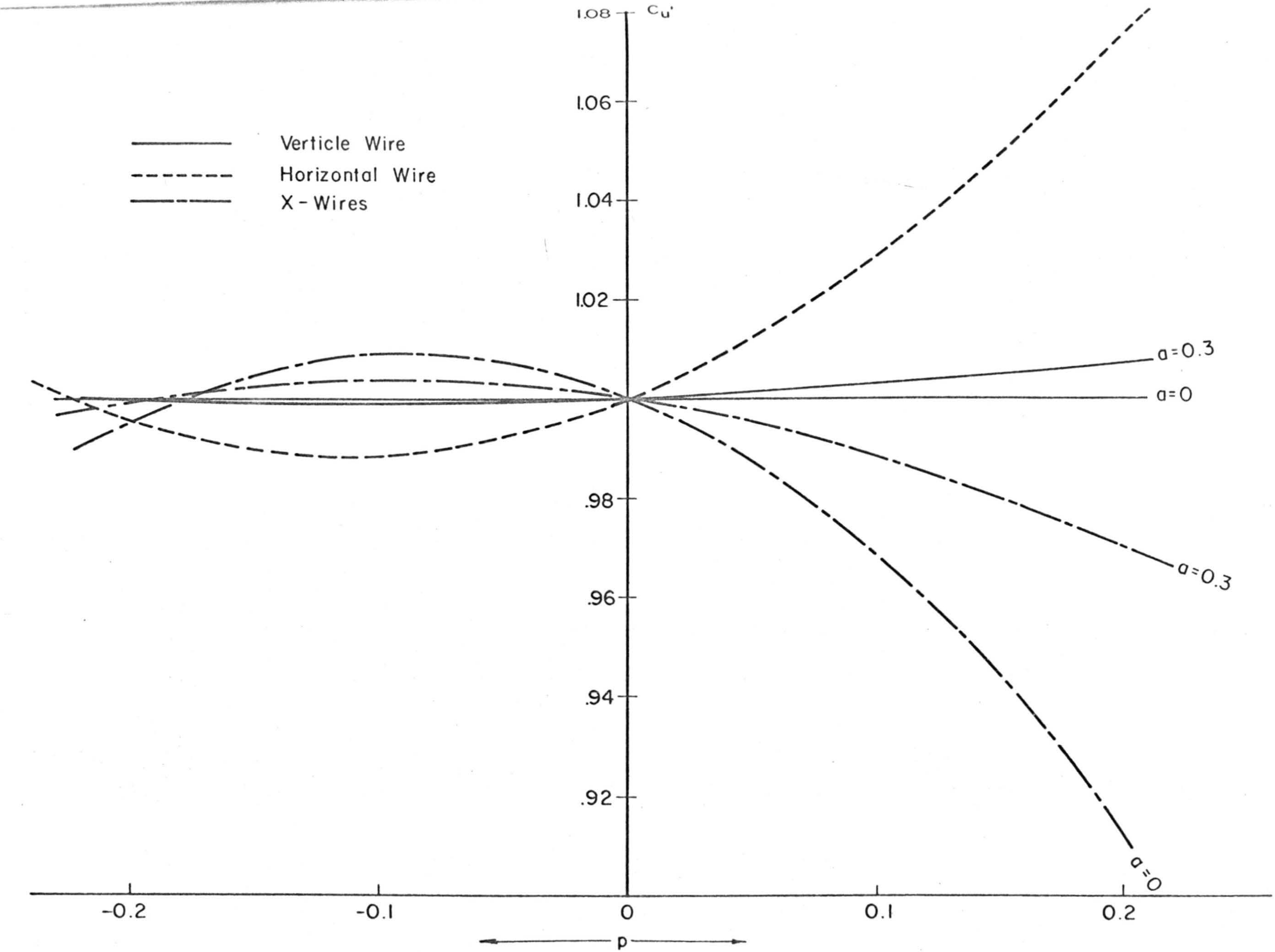

Fig. 2. Effect of cross flow on measurements of $(\overline{u'^2})^{1/2}$ using conventional hot-wire techniques.

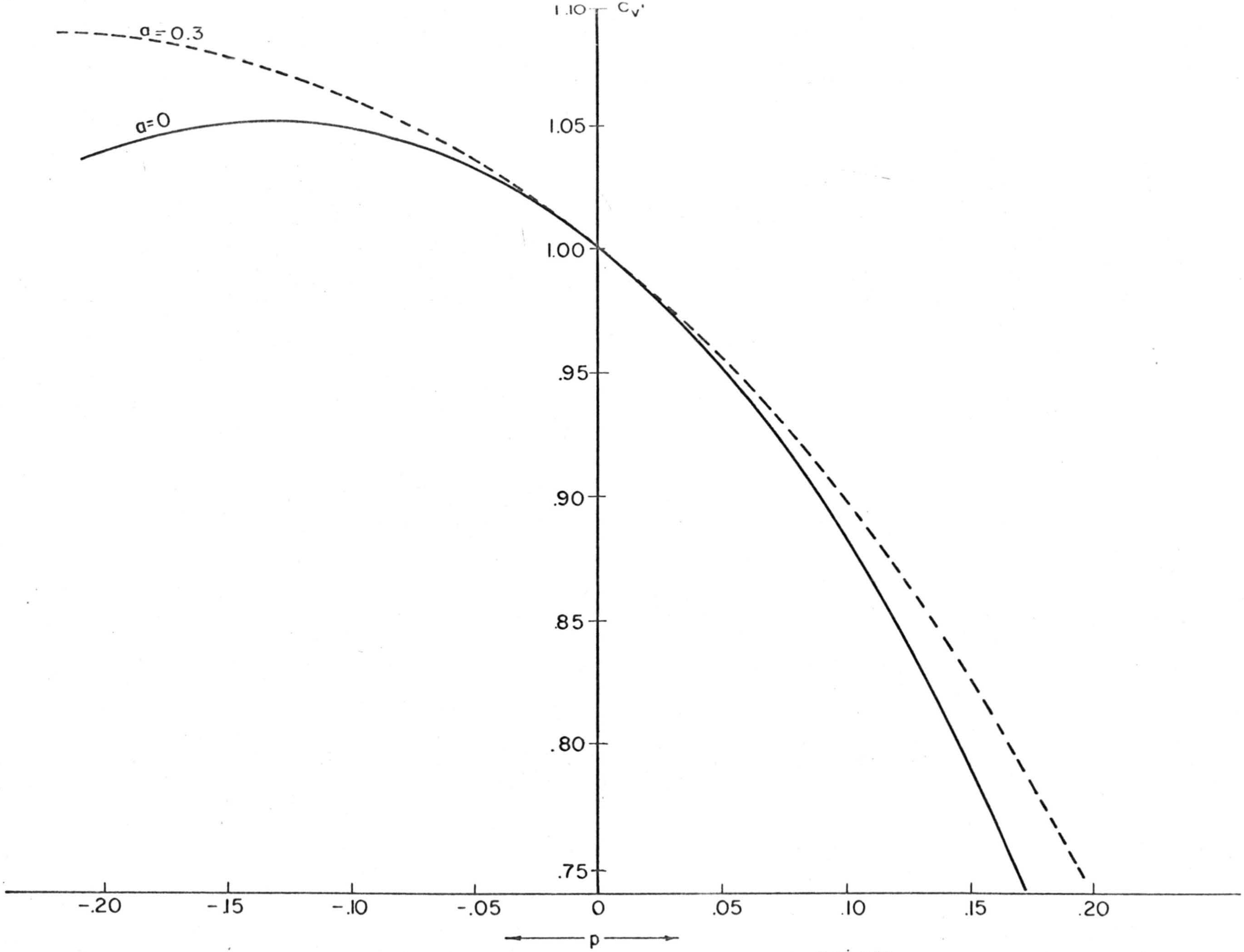

Fig. 3. Effect of cross flow on measurements of $(\overline{v'^2})^{1/2}$ using conventional x-wires technique.

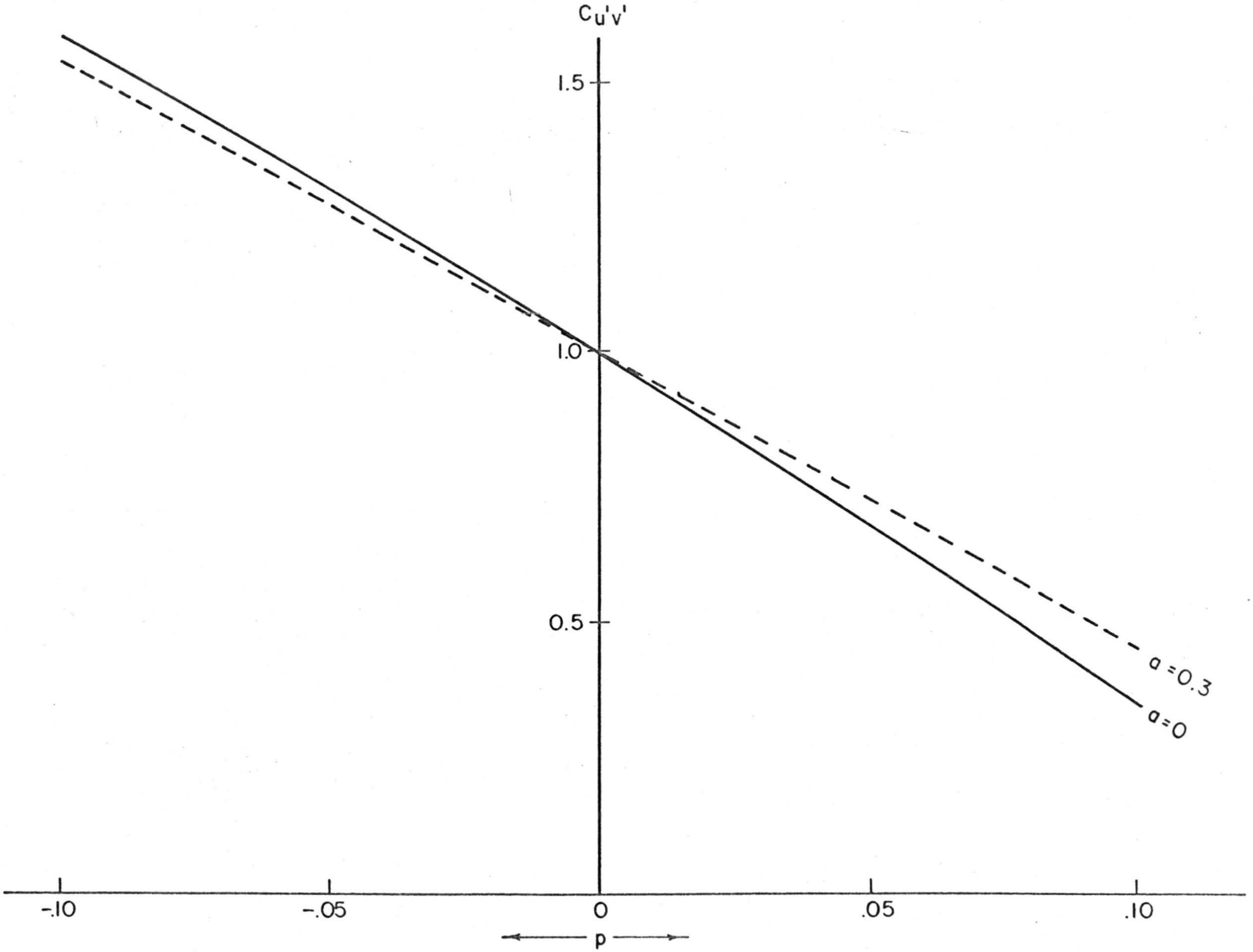

Fig. 4. Effect of cross flow on shear measurements using conventional x-wires technique.

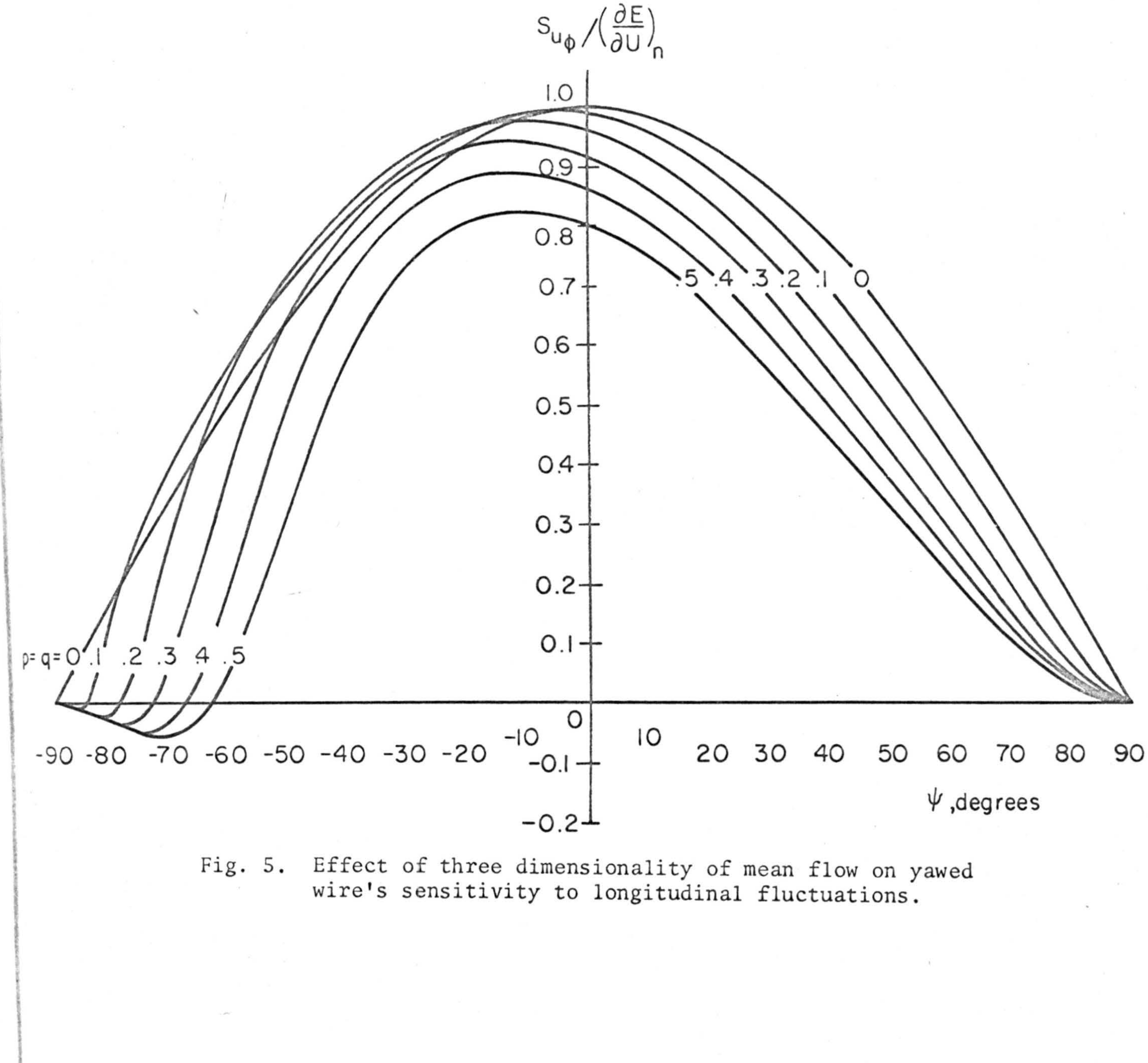

Fig. 5. Effect of three dimensionality of mean flow on yawed wire's sensitivity to longitudinal fluctuations.

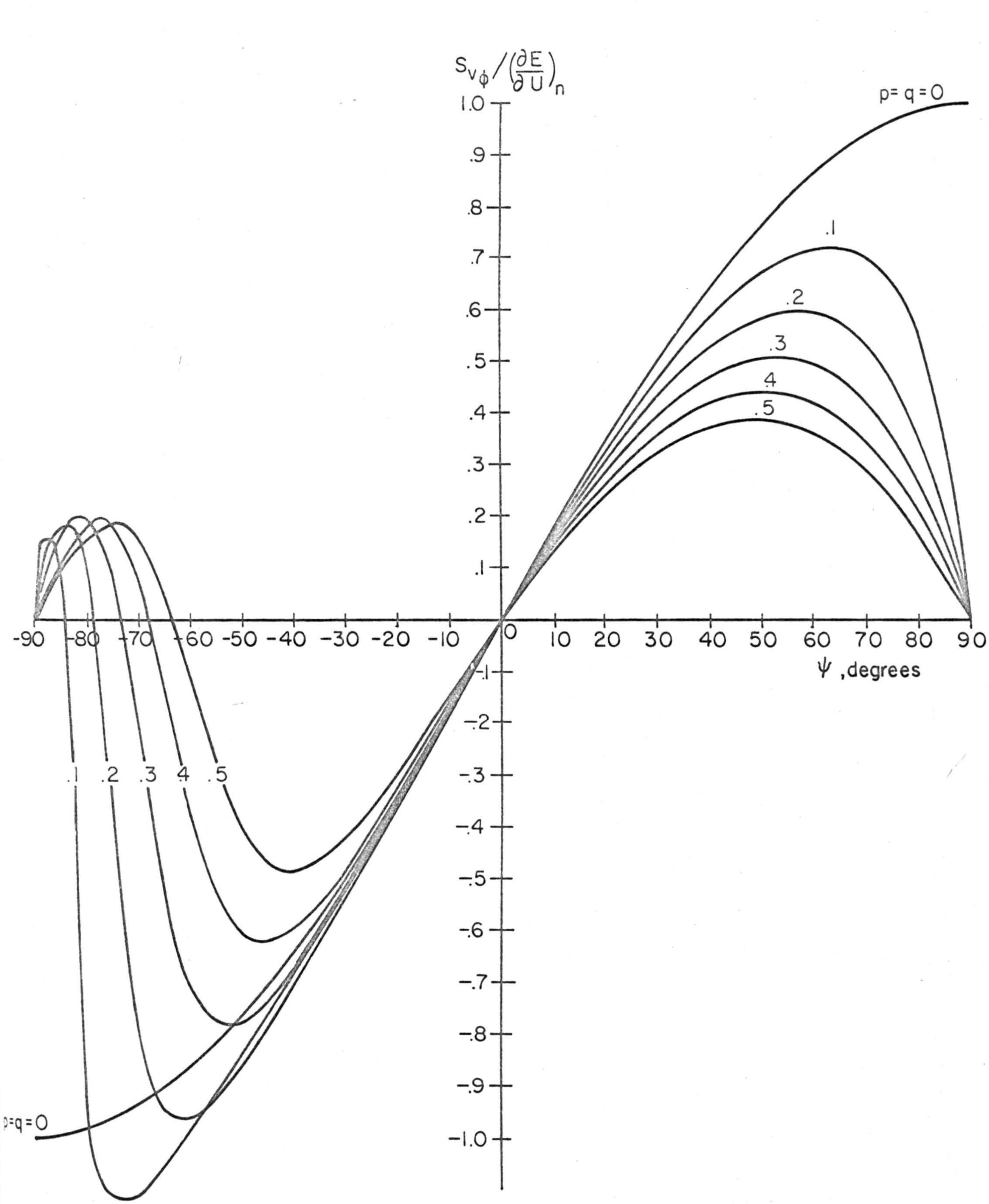

Fig. 6. Effect of three dimensionality of mean flow on yawed wire's sensitivity to vertical fluctuations.

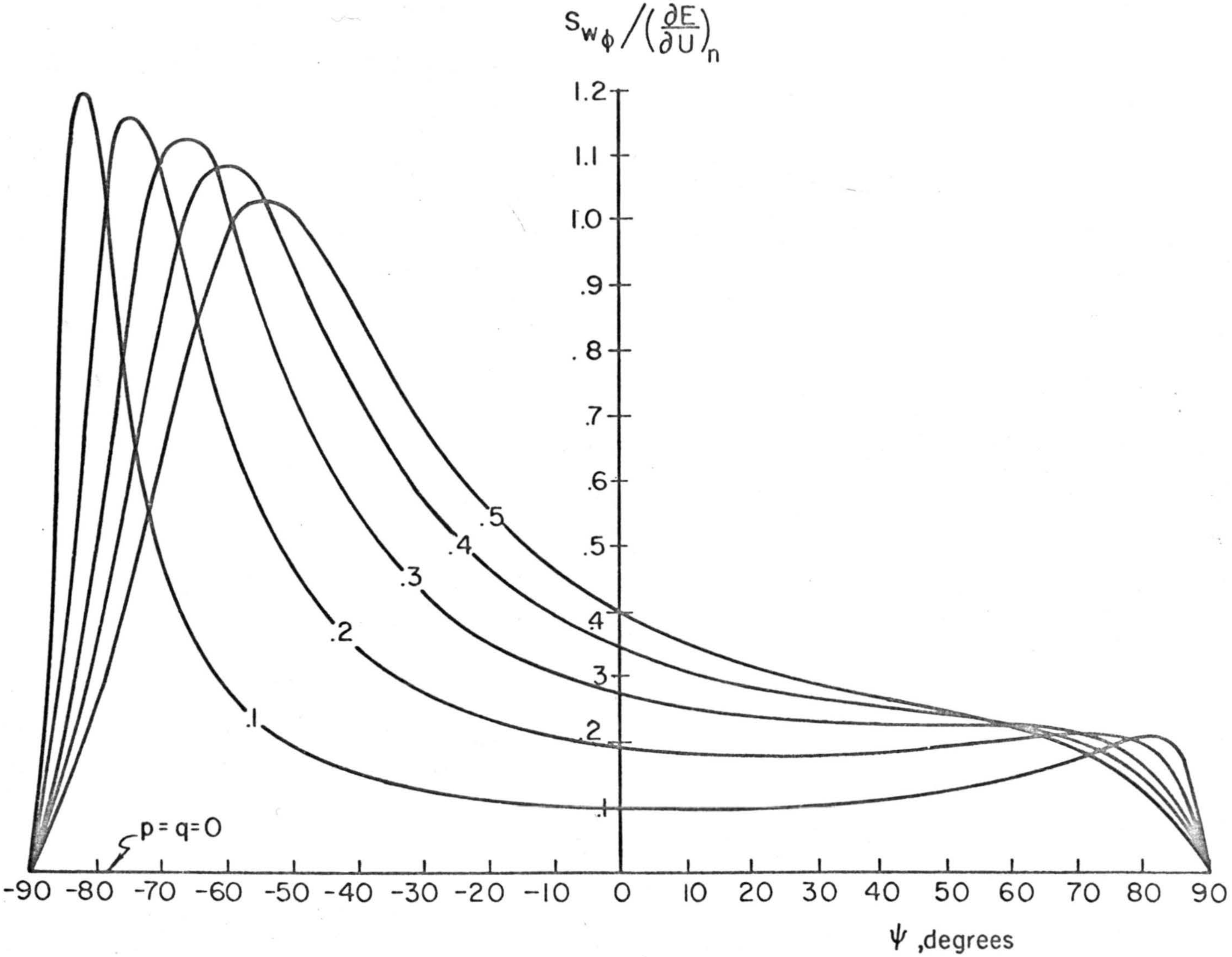

Fig. 7. Effect of three dimensionality of mean flow on yawed wire's sensitivity to lateral fluctuation.

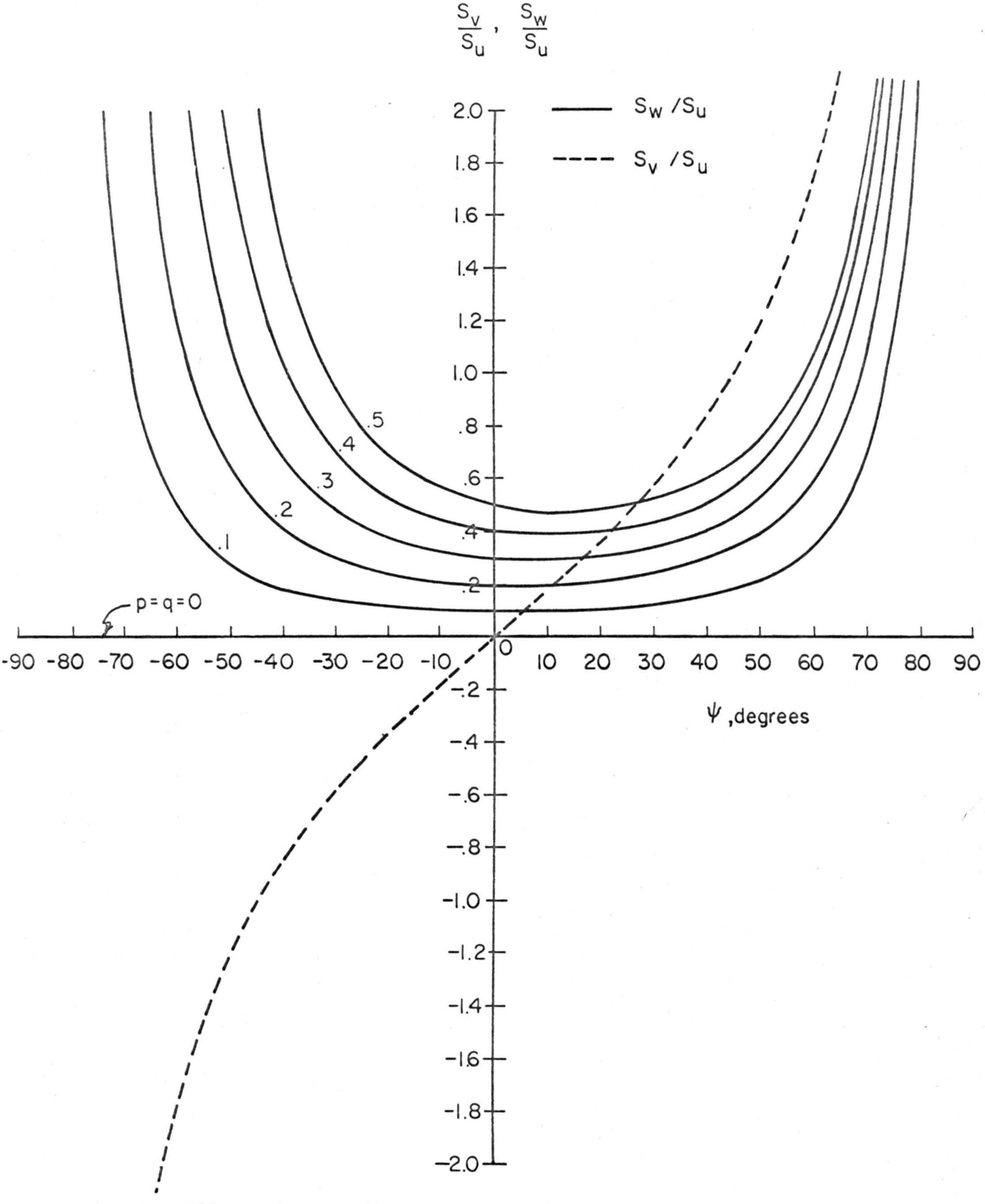

Fig. 8. Effect of three dimensionality of mean flow on the sensitivity ratios S_{v_ϕ}/S_{u_ϕ} and S_{w_ϕ}/S_{u_ϕ}.

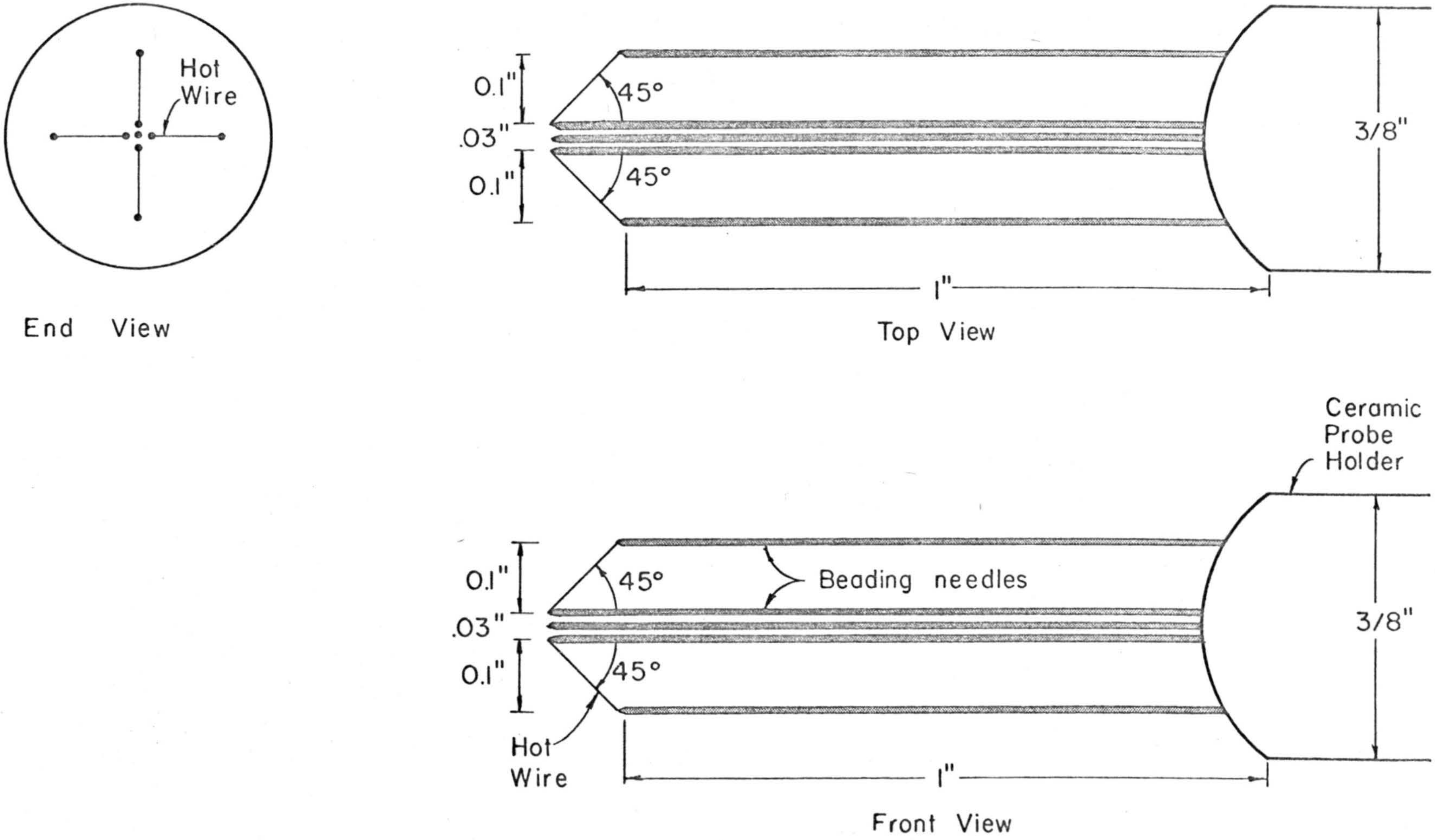

Fig. 9. The probe for measuring turbulence in three dimensional mean flow.

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13. ABSTRACT

A hot-wire anemometer technique for measuring turbulence in three-dimensional mean flow is presented. Effect of three-dimensionality of mean flow on a yawed wire's sensitivity to longitudinal, vertical and lateral fluctuations is brought out. A four-wire probe is shown to be suitable for measuring all the mean flow and turbulent quantities of interest.

Errors due to the cross flow component on turbulence measurements in two dimensional flows using conventional hot-wire techniques are estimated. Measurements of shear are shown to be very sensitive to even small amounts of cross flow that might be present in many laboratory and field flows of interest.

14. KEY WORDS	LINK A		LINK B		LINK C	
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